TS EAMCET 2025 - 2nd May Evening Shift

Mathematics

Question1

If $f : R - \{0\} \rightarrow R$ is defined by $3f(x) + 4f\left(\frac{1}{x}\right) = \frac{2-x}{x}$ then $f(3) =$

Options:

A.

6

B.

12

C.

9

D.

3

Answer: D

Solution:

We have, $3f(x) + 4f\left(\frac{1}{x}\right) = \frac{2}{x} - 1$

$\therefore \quad 3f(3) + 4f\left(\frac{1}{3}\right) = \frac{2}{3} - 1 = \frac{-1}{3} \qquad \ldots(i)$

Also, $3f\left(\frac{1}{3}\right) + 4f(3) = 6 - 1 = 5 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$f(3) = 3$

Question2

The inverse of the function $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} + 1$ is $x =$

Options:

A.

$\log\left(\frac{y}{2-y}\right)$

B.

$\log_{10}\left(\frac{y}{2-y}\right)$

C.

$\frac{1}{10}\log\left(\frac{y}{1-y}\right)$

D.

$\frac{1}{2}\log_{10}\left(\frac{y}{2-y}\right)$

Answer: D

Solution:

$$\begin{aligned}
&\text{We have, } y=\frac{10^x-10^{-x}}{10^x+10^{-x}}+1\\
&\Rightarrow \quad y-1=\frac{10^x-10^{-x}}{10^x+10^{-x}}\\
&\text{Let} \quad a=10^x\\
&\therefore \quad 10^{-x}=\frac{1}{a}\\
&\therefore \quad y-1=\frac{a-\frac{1}{a}}{a+\frac{1}{a}}=\frac{a^2-1}{a^2+1}\\
&\Rightarrow \quad (y-1)\left(a^2+1\right)=a^2-1 \Rightarrow (y-1)a^2+(y-1)=a^2-1\\
&\Rightarrow \quad a^2(y-1-1)=-y \Rightarrow a^2(y-2)=-y\\
&\Rightarrow \quad a^2=\frac{-y}{y-2} \Rightarrow 10^{2x}=\frac{-y}{y-2}\\
&\Rightarrow \quad 2x=\log_{10}\left(\frac{-y}{y-2}\right) \Rightarrow 2x=\log_{10}\left(\frac{y}{2-y}\right)\\
&\Rightarrow \quad x=\frac{1}{2}\log_{10}\left(\frac{y}{2-y}\right)
\end{aligned}$$

Question3

The value of the greatest positive integer k, such that 49^k+1 is a factor of $48\left(49^{125}+49^{124}+\ldots+49^2+49+1\right)$ is

Options:

A.

32

B.

63

C.

65

D.

60

Answer: B

Solution:

We have, 49^k+1 is a factor of

$48\left(49^{125}+49^{124}+\ldots+49^2+49+1\right)$

Let $S=49^{125}+49^{124}+\ldots+49+1$

$\Rightarrow \quad S=\frac{49^{126}-1}{49-1}=\frac{49^{126}-1}{48} \Rightarrow 48S=49^{126}-1$

Since, 49^k+1 divides $49^{126}-1$

$\therefore k$ divides 126 .

$\Rightarrow \quad 2k$ divides 126

$\Rightarrow \quad$ Greatest possible integer $k=63$

Question4

If $\left|\begin{array}{ccc}1 & 2 & 3-\lambda \\ 0 & -1-\lambda & 2 \\ 1-\lambda & 1 & 3\end{array}\right|=A\lambda^3+B\lambda^2+C\lambda+D$**, then** $D+A=$

Options:

A.

1

B.

-4

C.

-5

D.

3

Answer: D

Solution:

We have, $\left|\begin{array}{ccc}1 & 2 & 3-\lambda \\ 0 & -1-\lambda & 2 \\ 1-\lambda & 1 & 3\end{array}\right|=A\lambda^3+B\lambda^2+C\lambda+D$

Now, $\left|\begin{array}{ccc}1 & 2 & 3-\lambda \\ 0 & -1-\lambda & 2 \\ 1-\lambda & 1 & 3\end{array}\right|$

$$\begin{aligned}
& =1(-3-3\lambda-2)+(1-\lambda)(4+(1+\lambda)(3-\lambda)) \\
& =(-3\lambda-5)+(1-\lambda)\left(4+3-\lambda+3\lambda-\lambda^2\right) \\
& =(-3\lambda-5)+(1-\lambda)\left(7+2\lambda-\lambda^2\right) \\
& =(-3\lambda-5)+\left(7+2\lambda-\lambda^2-7\lambda-2\lambda^2+\lambda^3\right) \\
& =\lambda^3-3\lambda^2-8\lambda+2
\end{aligned}$$

$\therefore \lambda^3-3\lambda^2-8\lambda+2=A\lambda^3+B\lambda^2+C\lambda+D$

On comparing both sides, we get

$A=1, B=-3, C=-8, D=2$

$\therefore \quad D+A=2+1=3$

Question5

If $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ **and** $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$**, then** $\operatorname{tr}(A)-\operatorname{tr}(B)=$

Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

We have

$$A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix} \qquad \ldots(i)$$

And $$2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix} \qquad \ldots(ii)$$

Now, $$2A+4B=\begin{bmatrix}2 & 4 & 0\\ 12 & -6 & 6\\ -10 & 6 & 2\end{bmatrix} \qquad \ldots(iii)$$

Substract Eqs. (ii) from (iii), we get

$$5B=\begin{bmatrix}2 & 4 & 0\\ 12 & -6 & 6\\ -10 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}=\begin{bmatrix}0 & 5 & -5\\ 10 & -5 & 0\\ -10 & 5 & 0\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}0 & 1 & -1\\ 2 & -1 & 0\\ -2 & 1 & 0\end{bmatrix}$$

$\therefore$ From Eqs. (ii), we get

$$2A=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 1 & -1\\ 2 & -1 & 0\\ -2 & 1 & 0\end{bmatrix}$$

$$\Rightarrow 2A=\begin{bmatrix}2 & 0 & 4\\ 4 & -2 & 6\\ -2 & 2 & 2\end{bmatrix}\Rightarrow A=\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ -1 & 1 & 1\end{bmatrix}$$

$$\begin{aligned}\therefore & \operatorname{tr}(A)-\operatorname{tr}(B)\\ & =(1-1+1)-(0-1+0)=1+1=2\end{aligned}$$

Question6

A, C are 3×3 matrices B, D are 3×1 matrices. If $AX = B$ has unique solution and $CX = D$ has infinite number of solutions, then

Options:

A.

rank of $[A : D]$ = rank of $[C : B]$

B.

rank of A = rank of C

C.

rank of $[A : B]$ < rank of $[B : D]$

D.

rank of $[A : D] \geq$ rank of $[C : B]$

Answer: D

Solution:

We have,

A, C are 3×3 matrices

$B \times D$ are 3×1 matrices

$AX = B$ has unique solution

$CX = D$ has infinite solution.

Since, $AX = B$ have a unique solution.

$\Rightarrow \text{rank}(A) = \text{number of unknowns} = 3$

$\therefore \quad \text{rank}([A : B]) = \text{rank}(A) = 3 \qquad \ldots(i)$

Since, $CX = D$ have infinite many solutions

$\therefore \quad \text{Rank}(C) = \text{not unique} \Rightarrow \text{rank}(C) < 3$

$\Rightarrow \quad \text{Rank}([C : D]) < 3 \qquad \ldots(ii)$

From Eqs. (i) and (ii), we have

$\text{Rank}([A : D]) \geq \text{rank}[(C : B)]$

Question7

A and B are two non-square matrices. If $P = A + B, Q = A^T B, R = AB^T$, then the matrices whose order is equal to the order of A are

Options:

A.

PQ and QR

B.

RQ and QP

C.

PQ and RP

D.

PQR and RPQ

Answer: C

Solution:

We have, A and B are two non-square matrices such that

$P = A + B, Q = A^T B, R = AB^T$

Since, $A + B$ is defined,

So order of A and B are same.

Let order of A and B be $m \times n$

$\therefore$ Order of A^T is $n \times m$ and order of B^T is $n \times m$

Now, order of $P =$ order of $A + B = m \times n$

Order of $Q =$ order of $A^T B = n \times n$

And order of $R =$ order of $AB^T = m \times m$

Now, order of $PQ = m \times n$

Order of $QR =$ not defined

Order of $RQ =$ not defined

Order of $QP =$ not defined

Order of $RP = m \times n$

$\therefore$ Order of $A =$ order of PQ and RP

Question8

ω is a complex cube root of unity and Z is a complex number satisfying $|Z - 1| \leq 2$. The possible values of r such that $|Z - 1| \leq 2$ and $\left|\omega Z - 1 - \omega^2\right| = r$ have no common solution are

Options:

A.

$0 \leq r \leq 4$

B.

$r = |\omega|$ only

C.

$r > 4$

D.

1

Answer: C

Solution:

We have, $|z-1| \leq 2$

$$\begin{aligned} &And \left|\omega z-1-\omega^2\right|=r \\ &\quad |\omega|\left|z-\omega^2-\omega\right|=r \\ &\quad |z+1|=r \\ &\qquad \left[\because|\omega|=1,1+\omega+\omega^2=0\right] \\ &\quad |z-1| \leq 2 \Rightarrow|z+1-2| \leq 2 \\ &\quad |z+1|-2 \leq 2 \Rightarrow|z+1| \leq 4 \end{aligned}$$

For no solution

$\therefore \quad |z+1|>4 \Rightarrow r>4$

Question9

If $|Z|=2, Z_1=\frac{Z}{2} e^{i \alpha}$ and θ is the $\operatorname{amp}(Z)$, then $\frac{Z_1^n-Z_1^{-n}}{Z_1^n+Z_1^{-n}}=$

Options:

A.

$2^n i \tan (n \theta+n \alpha)$

B.

$i \tan (n \theta-n \alpha)$

C.

$i \tan (n \theta+n \alpha)$

D.

$\tan (n \theta+n \alpha)$

Answer: C

Solution:

Given, $|z|=2, z_1=\frac{z}{2} e^{i \alpha}$

$\therefore \quad z=2 e^{i \theta}$

(θ is the argument of z)

And $z_1=\frac{2 e^{i \theta}}{2} e^{i \alpha}=e^{i(\theta+\alpha)}$

Now, $\quad z_1^n=e^{i n(\theta+\alpha)}$

$\Rightarrow \quad z_1^{-n}=e^{-i n(\theta+\alpha)}$

We know that $\frac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}} = i\tan x$

$$\begin{aligned}\therefore \frac{z_1^n - z_1^{-n}}{z_1^n + z_1^{-n}} &= \frac{e^{in(\theta+\alpha)} - e^{-in(\theta+\alpha)}}{e^{in(\theta+\alpha)} + e^{-in(\theta+\alpha)}} \\ &= i\tan(n(\theta+\alpha)) \\ &= i\tan(n\theta+n\alpha)\end{aligned}$$

Question10

If $n, K \in N$ such that $n \neq 3K$, then $(\sqrt{3}+i)^{2n} + (\sqrt{3}-i)^{2n} =$

Options:

A.

$(-1)^n 2^{2n+1}$

B.

$(-1)^{n+1} 2^{2n+1}$

C.

$(-1)^{n+1} 2^{2n}$

D.

$(-1)^{n+1} 2^n$

Answer: C

Solution:

Given, $n, K \in N$ such that $n \neq 3K$

Let $z = \sqrt{3}+i$, then $\bar{z} = \sqrt{3}-i$

$|z| = |\bar{z}| = \sqrt{3+1} = 2$
$\arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$
$\arg(z) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$
$\therefore \quad z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
$\Rightarrow z^{2n} = 2^{2n}\left(\cos\frac{2n\pi}{6} + i\sin\frac{2n\pi}{6}\right)$
And $\bar{z} = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$
$\Rightarrow \bar{z}^{2n} = 2^{2n}\left(\cos\frac{2n\pi}{6} - i\sin\frac{2n\pi}{6}\right)$
$\therefore z^{2n} + \bar{z}^{2n} = 2\cdot 2^{2n}\cos\left(\frac{n\pi}{3}\right)$

$= 2^{2n+1}\cos\left(\frac{n\pi}{3}\right)$

Since, $n \neq 3K \Rightarrow n$ is not a multiple of 3 $\cos\left(\frac{n\pi}{3}\right) \neq \cos(k\pi) = \pm 1 = (-1)^{n+1}2^{2n}$

Question11

In argand plane, no value of $\sqrt[3]{1 - i\sqrt{3}}$ lie in

Options:

A.

First quadrant

B.

second quadrant

C.

Third quadrant

D.

Fourth quadrant

Answer: A

Solution:

Let $z = 1 - i\sqrt{3}$

$$
\begin{aligned}
\therefore \quad & |z| = \sqrt{1 + (\sqrt{3})^2} = 2 \\
\arg(z) &= \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\tan\frac{\pi}{3} \\
\therefore \quad & z = 2\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) \\
& = 2e^{-i\pi/3} \\
\therefore \quad & \sqrt[3]{z} = 2^{1/3} \cdot e^{4 - i\pi/3 + 2k\pi i} \\
& = 2^{1/3} e^{i\left(-\frac{\pi}{9} + \frac{2k\pi}{3}\right)}
\end{aligned}
$$

For any integer $k \in \{0, 1, 2\}$

$\omega_0 =$ lies in 4th quadrant

$\omega_1 =$ lies in 2nd quadrant

$\omega_2 =$ lies in 3rd quadrant

Question12

If l is the maximum value of $-3x^2 + 4x + 1$ and m is the minimum value of $3x^2 + 4x + 1$, then the equation of the hyperbola having foci at $(l, 0), (7m, 0)$ and eccentricity as 2 is

Options:

A.

$36x^2 - 12y^2 = 49$

B.

$49x^2 - 36y^2 = 12$

C.

$2x^2 - 5y^2 = 1$

D.

$36x^2 - 12y^2 = 1$

Answer: A

Solution:

Given, l is the maximum value of $-3x^2 + 4x + 1$ and m is the minimum value of $3x^2 + 4x + 1$

For $-3x^2 + 4x + 1$, $\frac{-b}{2a} = \frac{-4}{2(-3)} = \frac{2}{3}$

$$\begin{aligned} \therefore \text{ Maximum value } = \quad & -3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) + 1 \\ & = -3 \times \frac{4}{9} + \frac{8}{3} + 1 \\ \Rightarrow \quad & l = \frac{-4}{3} + \frac{8}{3} + 1 = \frac{7}{3} \end{aligned}$$

For $3x^2 + 4x + 1$, $\frac{-b}{2a} = \frac{-4}{2 \times 3} = \frac{-2}{3}$

$\therefore$ Minimum value

$$\begin{aligned} & = 3\left(\frac{-2}{3}\right)^2 + 4\left(\frac{-2}{3}\right) + 1 \\ & = 3 \times \frac{4}{9} - \frac{8}{3} + 1 \\ \Rightarrow \quad m & = \frac{4}{3} - \frac{8}{3} + 1 = \frac{-1}{3} \end{aligned}$$

Now, foci of hyperbola $= (l, 0)$

$= \left(\frac{7}{3}, 0\right)$

And $(7m, 0) = \left(\frac{-7}{3}, 0\right)$

$$\begin{aligned} & \therefore \text{ Distance between foci } = \\ & \Rightarrow 2a \times 2 = 2 \times \frac{7}{3} \Rightarrow a = \frac{7}{6} \\ & \text{Also, } b^2 = a^2\left(e^2 - 1\right) \\ & \Rightarrow \quad b^2 = \frac{49}{36}(4 - 1) \\ & \Rightarrow \quad b^2 = \frac{49}{36} \times 3 = \frac{49}{12} \end{aligned}$$

$$\begin{aligned} & \therefore \text{ Equation of hyperbola, } \frac{x^2}{\frac{49}{36}} - \frac{y^2}{\frac{49}{12}} = 1 \\ & \Rightarrow \quad 36x^2 - 12y^2 = 49 \end{aligned}$$

Question13

If the equation $x^2 - 3ax + a^2 - 2a - k = 0$ has different real roots for every rational number a, then k lies in the interval

Options:

A.

$0 < K < \frac{4}{5}$

B.

$-\infty < K < \frac{4}{5}$

C.

$\frac{4}{5} < K < \infty$

D.

$-\infty < K < \infty$

Answer: C

Solution:

Given equation is

$x^2 - 3ax + a^2 - 2a - k = 0$

For distinct real roots, $D > 0$

$$\begin{aligned} &= (-3a)^2 - 4 \times 1 \times \left(a^2 - 2a - k\right) > 0 \\ &= 9a^2 - 4a^2 + 8a + 4k > 0 \\ &= 5a^2 + 8a + 4k > 0 \end{aligned}$$

Since, leading coefficient of a^2 is $5 > 0$

So, $(8)^2 - 4 \times (5)(4k) < 0 = 64 - 80k < 0$

$$\begin{aligned} \Rightarrow \quad & 64 < 80k \\ & = k > \frac{4}{5} \Rightarrow k \in \left(\frac{4}{5}, \infty\right) \end{aligned}$$

Question14

The number of all common roots of the equation $x^4 - 10x^3 + 37x^2 - 60x + 36 = 0$ and the transformed equation of it obtained by increasing any two distinct roots of it by 1 , keeping the other two roots fixed, is

Options:

A.

1

B.

3

C.

4

D.

2

Answer: B

Solution:

Let

$f(x)=x^4-10x^3+37x^2-60x+36=0$
At $x=1, f(1)=1-10+37-60+36 \neq 0$
At $x=2, f(2)=\left(2^4-10\left(2^3+37\left(2^2\right.\right.\right.$
$-60(2+36=0$
$=16-80+148-120+36$
$=200-200=0$

So, $x=2$ is a factor of $f(x)$

Similarly $x=3$ is a factor of $f(x)$

$\therefore f(x)=(x-2)^2(x-3)^2$

$\therefore$ Original roots are $2,2,3,3$

The distinct roots are 2 and 3

There are three possible pairs of distinct roots

(i) Increase the two $2 s$ by 1

(ii) Increase the two $3 s$ by 1

(iii) Increase one 2 and one 3 by 1

Let us assume we choose one root 2 and one root 3 to increase by 1

Since, these distinct roots.

Increase one ' 2 ' by $1: 2+1=3$

Increase one ' 3 ' by $1: 3+1=4$

Other two roots (2 and 3) are kept fixed

$\therefore$ New sets of roots of he transformed equation is $3,4,2,3$

Let the transformed equation be $g(x)$. its roots are $2,3,3,4$

$$\begin{aligned} \therefore g(x) & =(x-2)(x-3)(x-3)(x-4) \\ & =(x-2)(x-3)^2(x-4) \end{aligned}$$

The common roots are value of x for which $f(x)=0$ and $g(x)=0$

$\therefore$ Common roots are $2,3,3$

$\therefore$ Number of common roots $=3$

Question15

If α, β, γ are the roots of the equation $x^3-P x^2+Q x-R=0$ and $(\alpha-2)^2,(\beta-2)^2,(\gamma-2)^2$ are the roots of the equation $x^3-5 x^2+4 x=0$, then the possible least value of $P+Q+R$ is

Options:

A.

5

B.

-7

C.

-1

D.

1

Answer: A

Solution:

Given, α, β, γ are the roots of the equations $x^3 - Px^2 + Qx - R = 0$

$$\therefore \begin{aligned} \alpha+\beta+\gamma &= P \\ \alpha\beta+\beta\gamma+\gamma\alpha &= Q \end{aligned}$$

And $\alpha\beta\gamma = R$

Also, $(\alpha-2)^2, (\beta-2)^2, (\gamma-2)^2$ are the roots of equations, $x^3 - 5x^2 + 4x = 0$

Now, $x^2 - 5x^2 + 4x = 0$

$$\begin{aligned} \Rightarrow \quad & x\left(x^2-5x+4\right) = 0 \\ \Rightarrow \quad & x(x-1)(x-4) = 0 \\ \Rightarrow \quad & x = 0, 1, 4 \\ \therefore \quad & (\alpha-2)^2 = 0 \Rightarrow \alpha = 2 \\ & (\beta-2)^2 = 1 \Rightarrow \beta - 2 = \pm 1 \Rightarrow \beta = 1, 3 \end{aligned}$$

And $(\gamma-2)^2 = 4 \Rightarrow \gamma - 2 = \pm 2 \Rightarrow \gamma = 0, 4$

Now,

$P+Q+R = (\alpha+\beta+\gamma) + (\alpha\beta+\beta\gamma+\gamma\alpha) + \alpha\beta\gamma$

Case I when $\alpha = 2, \beta = 1, \gamma = 0$

then,

$$\begin{aligned} P+Q+R &= (2+1+0)+(2+0+0) \\ &= 5 \end{aligned}$$

Case II when $\alpha = 2, \beta = 1, \gamma = 4$

then,

$P+Q+R = (2+1+4)+(2+4+8)+(2\times 1\times 4) = 29$

Case III when $\alpha = 2, \beta = 3, \gamma = 0$ then

$$\begin{aligned} P+Q+R &= (2+3+0)+(6+0+0)+0 \\ &= 11 \end{aligned}$$

Case IV when $\alpha = 2, \beta = 3, \gamma = 4$

then

$$\begin{aligned} P+Q+R &= (2+3+4)+(6+12+8)+24 \\ &= 59 \end{aligned}$$

Clearly, least value of $P+Q+R$ is 5 .

Question16

The number of non-negative integral solutions of the equation $x + y + z + t = 10$ when $x \geq 2, z \geq 5$ is

Options:

A.

80

B.

20

C.

50

D.

10

Answer: B

Solution:

We have, $x + y + z + t = 10 \qquad \ldots (i)$

When $x \geq 2, z \geq 5$

let $\quad x' = x - 2$

$\Rightarrow \quad x = x' + 2$

$z' = z - 5$

$\Rightarrow \quad z = z' + 5$

Here, $x', z', y \geq 0$ and $t \geq 0$ are non-negative integers.

$\therefore$ Substitude these value in Eq. (i), we get

$$(x' + 2) + y + (z' + 5) + t = 10$$
$$\Rightarrow \quad x' + y + z' + t = 3 \qquad \ldots (ii)$$

$\therefore$ Number of non-negative integral solution for Eq. (ii)

$$= {}^{3+4-1}C_{4-1} = {}^{6}C_{3}$$
$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

Question17

The number of integers lying between 1000 and 10000 such that the sum of all the digits in each of those numbers becomes 30 is

Options:

A.

84

B.

96

C.

45

D.

75

Answer: A

Solution:

Integers lying between 1000 and 10000 such that the sum of all the digits in each of those numbers becomes 30 .

$\therefore$ Coefficient of x^{30} in the expansion of

$$\begin{aligned}\left(x^1+x^2+\ldots+x^9\right)\left(x^0+x^1+\ldots+x^9\right)^3 \\ =x\left(\frac{1-x^9}{1-x}\right)\left(\frac{1-x^{10}}{1-x}\right)^3 \\ =x\left(1-x^9\right)\left(1-x^{10}\right)^3(1-x)^{-4}\end{aligned}$$

$\Rightarrow$ Coefficient of x^{29} in

$\left(1-x^9\right)\left(1-x^{10}\right)^3(1-x)^{-4}$

$$\begin{aligned}&=\left(1-x^9\right)\left(1-3 x^{10}+3 x^{20}-x^{30}\right)(1-x)^{-4} \\ &=\left(1-x^9-3 x^{10}+3 x^{19}+3 x^{20}-3 x^{29}\right. \\ &\quad\left.-x^{30}+x^{39}\right)(1-x)^{-4}\end{aligned}$$

Here, $(1-x)^{-4}=\sum_{k=0}^{\infty} k+{ }^3 C_3 x^k$

$\therefore$ Coefficient of x^{29} from $1 \cdot(1-x)^{-4}$

$={ }^{29+3} C_3={ }^{32} C_3=4960$

Coefficient of x^{20} from $-x^9 \cdot(1-x)^{-4}$

$=-{ }^{20+3} C_3=-{ }^{23} C_3=-1771$

Coefficient of x^{19} from $-3 x^{10} \cdot(1-x)^{-4}$

$$\begin{aligned}&=-3\left({ }^{19+3} C_3\right) \\ &=-3(1540)=-4620\end{aligned}$$

Coefficient of x^{10} from $3 x^{19}(1-x)^{-4}$

(Means form is $3 x^{19} \Sigma^{k+3} C_3 x^k$ need

$x^{29}, k=10)$

$$=3^{10+^3} C_3=3\left({ }^{13} C_3\right)=858$$

Coefficient of x^9 from $3 x^{20} \cdot(1-x)^{-4}$

$=3\left({ }^{9+3} C_3\right)=3(220)=660$

Coefficient of x^0 from $-3 x^{29} \cdot(1-x)^{-4}$

$=-3\left({ }^{0+3} C_3\right)=-3(1)=-3$

$\therefore$ Total numbers

$$\begin{aligned}&=4960-1771-4620+858+660-3 \\ &=84\end{aligned}$$

Question18

If all the letters of the word MOST are permuted and the words (with or without meaning) thus obtained are arranged in the dictionary order, then the rank of the words STOM when counted from the rank of the word MOST, is

Options:

A.

24

B.

21

C.

12

D.

18

Answer: D

Solution:

Given word in most

Words start with $M = 3! = 6$

Words start with O $= 3! = 6$

Words start with SM $= 2! = 2$

Words start with SO $= 2! = 2$

Words start with STM = 1 ! = 1

Words start with STO is STOM

Rank of STOM is $(6 + 6 + 2 + 2 + 1) + 1 = 18$

Question19

The constant term in the expansion of $\left(1 + \frac{1}{x}\right)^{20} \left(30x(1 + x)^{29} + (1 + x)^{30}\right)$ is

Options:

A.

${}^{50}\mathrm{C}_{20} + 30 \cdot {}^{50}\mathrm{C}_{29}$

B.

${}^{50}\mathrm{C}_{19} + 30 \cdot {}^{49}\mathrm{C}_{19}$

C.

${}^{50}\mathrm{C}_{20} + 30 \cdot {}^{49}\mathrm{C}_{20}$

D.

$^{50}\mathrm{C}_{20} + 30 \cdot {}^{49}\mathrm{C}_{19}$

Answer: D

Solution:

Let the given expression,

$E = \left(1 + \frac{1}{x}\right)^{20} \left[30x(1+x)^{29} + (1+x)^{30}\right]$

Here, $30x(1+x)^{29} + (1+x)^{30}$

$$\begin{aligned} &= (1+x)^{29}[30x + (1+x)] \\ &= (1+x)^{29}(31x+1) \\ \therefore \quad E &= \left(1+\frac{1}{x}\right)^{20}(1+x)^{29}(31x+1) \\ &= \frac{(x+1)^{20}}{x^{20}}(1+x)^{29}(31x+1) \\ &= \frac{(x+1)^{49}}{x^{20}}(31x+1) \\ &= (x+1)^{49}\left(31x^{-19} + x^{-20}\right) \\ &= 31x^{-19}(x+1)^{49} + x^{-20}(x+1)^{49} \end{aligned}$$

For constant term x^0

$\therefore$ Constant term in $31x^{-19}(x+1)^{49}$

i.e., $x^{-19} \cdot x^k = x^0 \Rightarrow k = 19$

$\therefore$ Constant term $= 31^{49}C_{19}$

Constant term in $x^{-20}(x+1)^{49}$

i.e., $x^{-20} \cdot x^k = x^0 \Rightarrow k = 20$

$\therefore$ Constant term $= 1 \times {}^{49}C_{20}$

$\therefore$ Sum of constant term

$$\begin{aligned} &= 31^{49}C_{19} + {}^{49}C_{20} \\ &= 30^{49}C_{19} + {}^{49}C_{19} + {}^{49}C_{20} \\ &= 30 \cdot {}^{49}C_{19} + {}^{50}C_{20} \end{aligned}$$

Question20

When $|x| > 3$, then coefficient of $\frac{1}{x^n}$ in the expansion of $x^{3/2}(3+x)^{1/2}$ is

Options:

A.

$(-1)^n \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n n!} 3^n$

B.

$(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+2}(n+2)!} 3^{n+2}$

C.

$(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n n!} 3^{n+1}$

D.

$(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+3}(n+2)!} 3^{n+1}$

Answer: B

Solution:

We have, $x^{3/2}(3+x)^{1/2}$

$$= x^{3/2} x^{1/2} \left(1+\frac{3}{x}\right)^{1/2}$$
$$= x^2 \left(1+\frac{3}{x}\right)^{1/2}$$

Now, coefficient of general term of $\left(1+\frac{3}{x}\right)^{1/2}$ is

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)\ldots\left(\frac{1}{2}-k+1\right)}{k!}$$
$$\frac{(-1)^{k+1} 1 \cdot 3 \cdot 5 \ldots (2k-3)}{2kk!}$$

On multiply by x^2, then

$x^2\left(1+\frac{3}{x}\right)^{1/2} = \sum_{k=0}^{\infty} {}^{1/2}C_k 3^k x^{2-k}$

For coefficient of $\frac{1}{x^n}, 2-k=-n$

$\Rightarrow \quad k=n+2$

Coefficient of $\frac{1}{x^n}$ is

$$\frac{(-1)^{n+3} 1 \cdot 3 \cdot 5 \ldots (2(n+2)-3) 3^{n+2}}{2^{n+2}(n+2)!}$$
$$= \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+1}(n+2)!} 3^{n+2}$$

Question21

If $\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+1)}$, then $3A+2B-C=$

Options:

A.

$\frac{8}{5}$

B.

$\frac{16}{5}$

C.

$\frac{3}{5}$

D.

$\frac{19}{5}$

Answer: D

Solution:

We are given:

$\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

First, get a common denominator to combine the right side:

$\frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

Set the numerators equal to each other:

$x^2 - 3 = A(x^2+1) + (Bx+C)(x+2)$

Expand the right side:

$A(x^2+1) = Ax^2 + A$
$(Bx+C)(x+2) = Bx^2 + 2Bx + Cx + 2C = Bx^2 + (2B+C)x + 2C$

Combine all terms on the right:

$Ax^2 + Bx^2 = (A+B)x^2$
$(2B+C)x$
$A + 2C$

So we get:

$x^2 - 3 = (A+B)x^2 + (2B+C)x + (A+2C)$

Now, match the coefficients for x^2, x, and the constant term:

For x^2: $A + B = 1$
For x: $2B + C = 0$
For constant: $A + 2C = -3$

Write the equations:

1. $A + B = 1$
2. $2B + C = 0$
3. $A + 2C = -3$

Now solve these equations step by step:

From equation 2: $2B + C = 0 \rightarrow C = -2B$

Substitute $C = -2B$ into equation 3: $A + 2(-2B) = -3$
$A - 4B = -3$

Now use $A + B = 1$. Substitute $A = 1 - B$ into the last equation:
$1 - B - 4B = -3$
$1 - 5B = -3$
$-5B = -4$
$B = \frac{4}{5}$

Now $C = -2B = -2 \times \frac{4}{5} = \frac{-8}{5}$
$A = 1 - B = 1 - \frac{4}{5} = \frac{1}{5}$

Now find $3A + 2B - C$:

$3\left(\frac{1}{5}\right) + 2\left(\frac{4}{5}\right) - \left(\frac{-8}{5}\right)$
$= \frac{3}{5} + \frac{8}{5} + \frac{8}{5}$
$= \frac{3+8+8}{5}$
$= \frac{19}{5}$

Question22

If $5\sin\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ lies between α and β (including α, β also), then $(\alpha - \beta)(\alpha + \beta - 6) =$

Options:

A.

$28 - 5\sqrt{3}$

B.

0

C.

3

D.

$28 + 5\sqrt{3}$

Answer: B

Solution:

We have the expression: $5\sin\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

Step 1: Expand the cosine term

$3\cos\left(\theta + \frac{\pi}{3}\right)$ can be written using the formula: $\cos(A+B) = \cos A\cos B - \sin A\sin B$.

$\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

So, $3\cos\left(\theta + \frac{\pi}{3}\right) = 3\left(\cos\theta\cdot\frac{1}{2} - \sin\theta\cdot\frac{\sqrt{3}}{2}\right) = \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta$

Step 2: Combine all terms

Now put this back into the main expression:

$5\sin\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Combine the $\sin\theta$ terms:

$5\sin\theta - \frac{3\sqrt{3}}{2}\sin\theta = \left(5 - \frac{3\sqrt{3}}{2}\right)\sin\theta$

So the expression becomes: $\left(5 - \frac{3\sqrt{3}}{2}\right)\sin\theta + \frac{3}{2}\cos\theta + 3$

Step 3: Find the maximum and minimum values

This is now in the form $a\sin\theta + b\cos\theta + c$, where $a = 5 - \frac{3\sqrt{3}}{2}$, $b = \frac{3}{2}$, $c = 3$.

The largest and smallest values of $a\sin\theta + b\cos\theta$ are R and $-R$, where $R = \sqrt{a^2 + b^2}$.

Calculate R: $R = \sqrt{\left(5 - \frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

Expanding: $= \sqrt{25 + \frac{27}{4} - 15\sqrt{3} + \frac{9}{4}} = \sqrt{25 - 15\sqrt{3} + 9} = \sqrt{34 - 15\sqrt{3}}$

Step 4: Substitute the extreme values

The minimum value is $-R + 3 = \alpha$. The maximum value is $R + 3 = \beta$.

So, $\alpha = -R + 3$ and $\beta = R + 3$.

Step 5: Calculate the final result

Find $\alpha + \beta$: $\alpha + \beta = (-R + 3) + (R + 3) = 6$

Find $\alpha - \beta$: $\alpha - \beta = (-R + 3) - (R + 3) = -2R$

Now, $(\alpha-\beta)(\alpha+\beta-6)=(-2R)\times(6-6)=(-2R)\times 0=0$

Question23

$\frac{\sin 1^\circ+\sin 2^\circ+\ldots\sin 89^\circ}{2(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ)+1}=$

Options:

A.

2

B.

$\frac{1}{\sqrt{2}}$

C.

$\frac{1}{2}$

D.

$\sqrt{2}$

Answer: B

Solution:

We have,

$$\begin{aligned}&\frac{\sin 1^\circ+\sin 2^\circ+\ldots+\sin 89^\circ}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1}\\&=\frac{\left(\sin 1^\circ+\sin 89^\circ\right)+\left(\sin 2^\circ+\sin 88^\circ\right)+\ldots+\left(\sin 44^\circ+\sin 46^\circ\right)+\sin 45^\circ}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1}\\&=\frac{2\sin\frac{90^\circ}{2}\cos\frac{88^\circ}{2}+2\sin\frac{90^\circ}{2}\cos\frac{86^\circ}{2}+\ldots+2\sin\frac{90^\circ}{2}\cos\frac{2^\circ}{2}+\frac{1}{\sqrt{2}}}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1}\\&=\frac{\sqrt{2}\left(\cos 44^\circ+\cos 42^\circ+\ldots+\cos 2^\circ+\cos 1^\circ\right)+\frac{1}{\sqrt{2}}}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1}\end{aligned}$$

Let $x=\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ$

$=\frac{\sqrt{2}x+\frac{1}{\sqrt{2}}}{2x+1}=\frac{\frac{(2x+1)}{\sqrt{2}}}{2x+1}=\frac{1}{\sqrt{2}}$

Question24

If $3\sin(\alpha-\beta)=5\cos(\alpha+\beta)$ and $\alpha+\beta\neq\frac{\pi}{2}$, then $\frac{\tan\left(\frac{\pi}{4}-\alpha\right)}{\tan\left(\frac{\pi}{4}-\beta\right)}=$

Options:

A.

0

B.

-4

C.

$-\frac{1}{4}$

D.

$\frac{1}{2}$

Answer: C

Solution:

We have, $3\sin(\alpha-\beta)=5\cos(\alpha+\beta)$

$\Rightarrow 3[\sin\alpha\cos\beta-\cos\alpha\sin\beta]$
$\quad =5[\cos\alpha\cos\beta-\sin\alpha\sin\beta]$

On dividing by $\cos\alpha\cos\beta$, we get

$$\begin{aligned}&3[\tan\alpha-\tan\beta]=5(1-\tan\alpha\tan\beta)\\ \Rightarrow\quad &3\tan\alpha-3\tan\beta=5-5\tan\alpha\tan\beta\\ \Rightarrow\quad &3\tan\alpha+5\tan\alpha\tan\beta=5+3\tan\beta\\ \Rightarrow\quad &\tan\alpha(3+5\tan\beta)=5+3\tan\beta\\ \Rightarrow\quad &\tan\alpha=\tfrac{5+3\tan\beta}{3+5\tan\beta}\end{aligned}$$

Now, $\tan\left(\frac{\pi}{4}-\alpha\right)=\frac{1-\tan\alpha}{1+\tan\alpha}$

And $\tan\left(\frac{\pi}{4}-\beta\right)=\frac{1-\tan\beta}{1+\tan\beta}$

$$\begin{aligned}\therefore\quad &\frac{\tan\left(\frac{\pi}{4}-\alpha\right)}{\tan\left(\frac{\pi}{4}-\beta\right)}=\frac{(1-\tan\alpha)(1+\tan\beta)}{(1+\tan\alpha)(1-\tan\beta)}\\ \therefore\quad &1-\tan\alpha=1-\frac{5+3\tan\beta}{3+5\tan\beta}=\frac{2(\tan\beta-1)}{3+5\tan\beta}\\ \text{And}\quad &1+\tan\beta=1+\frac{5+3\tan\beta}{3+5\tan\beta}=\frac{8(1+\tan\beta)}{3+5\tan\beta}\\ \therefore\quad &\frac{\tan\left(\frac{\pi}{4}-\alpha\right)}{\tan\left(\frac{\pi}{4}-\beta\right)}=\frac{\frac{2(\tan\beta-1)}{3+5\tan\beta}\times(1+\tan\beta)}{8(1+\tan\beta)}\times(1-\tan\beta)\\ &=\frac{-2}{8}=\frac{-1}{4}\end{aligned}$$

Question25

$1+\cos x+\cos^2 x+\cos^3 x+\ldots$ to $\infty=4+2\sqrt{3}$, then $x=$

Options:

A.

$\frac{n\pi}{6}$

B.

$(4n\pm1)\frac{\pi}{3}$

C.

$(12n\pm1)\frac{\pi}{6}$

D.

$(3n \pm 1)\frac{\pi}{3}$

Answer: C

Solution:

Given equation is

$$1+\cos x+\cos^2 x+\ldots+\infty=4+2\sqrt{3}$$
$$\Rightarrow \quad \frac{1}{1-\cos x}=4+2\sqrt{3}$$
$$\Rightarrow \quad 1-\cos x=\frac{1}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$
$$\Rightarrow \quad 1-\cos x=\frac{4-2\sqrt{3}}{4} \Rightarrow 1-\cos x=1-\frac{\sqrt{3}}{2}$$
$$\Rightarrow \quad \cos x=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}$$
$$\Rightarrow \quad x=2n\pi\pm\frac{\pi}{6} \Rightarrow x=\frac{12n\pi\pm\pi}{6}$$
$$\Rightarrow \quad x=(12n\pm1)\frac{\pi}{6}$$

Question26

Consider the following statements

Assertion (A) : When x, y, z are positive numbers, then

$$\tan^{-1}\left(\sqrt{\frac{x(x+y+z)}{yz}}\right)+\tan^{-1}\left(\sqrt{\frac{y(x+y+z)}{xz}}\right)+\tan^{-1}\left(\sqrt{\frac{z(x+y+z)}{xy}}\right)=\pi$$

Reason (R) : $\tan^{-1}a+\tan^{-1}b=\tan^{-1}\left(\frac{a+b}{1-ab}\right)$, if $a>0$ and $b>0$

The correct answer is

Options:

A.

Both (A) and (R) are true, (R) is the correct explanation of (A).

B.

Both (A) and (R) are true, (R) is not the correct explanation of (A).

C.

(A) is true, but (R) is false.

D.

(A) is false, but (R) is true.

Answer: C

Solution:

By LHS, $\tan^{-1}\left(\sqrt{\frac{x(x+y+z)}{yz}}\right)+\tan^{-1}\left(\sqrt{\frac{y(x+y+z)}{xz}}\right)+\tan^{-1}\left(\sqrt{\frac{z(x+y+z)}{xy}}\right)$

Let $a=\sqrt{\frac{x(x+y+z)}{yz}}, b=\sqrt{\frac{y(x+y+z)}{xz}}$ and $z=\sqrt{\frac{z(x+y+z)}{xy}}$

Here, $a+b+c=\frac{(x+y+z)^{1/2}(x+y+z)}{\sqrt{xyz}}=\frac{(x+y+z)^{3/2}}{\sqrt{xyz}}$

And $abc=\frac{(x+y+z)^{3/2}}{\sqrt{xyz}}$

Since, $a+b+c=abc$

$\therefore \quad \text{LHS} =\pi$

$\Rightarrow$ Assertion is true and since, $\tan^{-1} x+\tan^{-1} y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

if $xy<1$

$\Rightarrow$ Reason is false.

Question27

If $e^{\left(\sinh^{-1} 2+\cosh^{-1} \sqrt{6}\right)}=(a+(b+\sqrt{c})\sqrt{a}+b\sqrt{c})$, then $a+b+c=$

Options:

A.

13

B.

15

C.

17

D.

11

Answer: A

Solution:

We know that

$\sinh^{-1} x=\log\left(x+\sqrt{x^2+1}\right)$

$\therefore \sinh^{-1} 2=\log(2+\sqrt{4+1})=\log(2+\sqrt{5})$

$\cosh^{-1} x=\log\left(x+\sqrt{x^2-1}\right)$

$\therefore \cosh^{-1} \sqrt{6}=\log(\sqrt{6}+\sqrt{6-1})=\log(\sqrt{6}+\sqrt{5})$

$\therefore \sinh^{-1} 2+\cosh^{-1} \sqrt{6}$

$=\log(2+\sqrt{5})+\log(\sqrt{6}+\sqrt{5})$

$=\log[(2+\sqrt{5})(\sqrt{6}+\sqrt{5})]$

$e^{\left(\sinh^{-1} \sqrt{2}+\cosh^{-1} \sqrt{6}\right)}=e^{\log((2+\sqrt{5})(\sqrt{6}+\sqrt{5})]}$

$=(2+\sqrt{5})(\sqrt{6}+\sqrt{5})=2\sqrt{6}+2\sqrt{5}+\sqrt{30}+5$

$=5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}=5+2\sqrt{5}+2\sqrt{6}+\sqrt{5}\sqrt{6}$

$=5+(2+\sqrt{6})\sqrt{5}+2\sqrt{6}=a+(b+\sqrt{c})\sqrt{a}+b\sqrt{c}$

$\therefore a=5, b=2, c=6$

$\therefore \quad a+b+c=5+2+6=13$

Question28

In a $\triangle ABC$, if $r_1 = 4, r_2 = 8$ and $r_3 = 24$, then $a : b : c =$

Options:

A.

$4 : 7 : 9$

B.

$2 : 3 : 5$

C.

$1 : 2 : 6$

D.

$6 : 2 : 1$

Answer: A

Solution:

We have, $r_1 = 4, r_2 = 8$ and $r_3 = 24$

$$\therefore \quad 4 = \frac{\Delta}{s-a}, 8 = \frac{\Delta}{s-b} \text{ and } 24 = \frac{\Delta}{s-c}$$

$$\Rightarrow \quad s - a = \frac{\Delta}{4} \quad \ldots (i)$$

$$s - b = \frac{\Delta}{8} \quad \ldots (ii)$$

$$\text{And} \quad s - c = \frac{\Delta}{24} \quad \ldots (iii)$$

On adding these three equations, we get

$$3s - (a + b + c) = \frac{\Delta}{4} + \frac{\Delta}{8} + \frac{\Delta}{24}$$

$$\Rightarrow 3s - 2s = \frac{6\Delta + 3\Delta + \Delta}{24} = \frac{10\Delta}{24} \Rightarrow s = \frac{5}{12}\Delta$$

From Eq. (i), $a = s - \frac{\Delta}{4} = \frac{5\Delta}{12} - \frac{\Delta}{4} = \frac{2\Delta}{12} = \frac{\Delta}{6}$

From Eq. (ii), $b = s - \frac{\Delta}{8} = \frac{5}{12}\Delta - \frac{\Delta}{8} = \frac{7}{24}\Delta$

From Eq. (iii), $c = s - \frac{\Delta}{24} = \frac{5}{12}\Delta - \frac{\Delta}{24} = \frac{9}{24}\Delta$

$\therefore a : b : c = \frac{\Delta}{6} : \frac{7}{24}\Delta : \frac{9}{24}\Delta = 4 : 7 : 9$

Question29

In a $\triangle ABC$, $(r_2 + r_3) \operatorname{cosec}^2\left(\frac{A}{2}\right) =$

Options:

A.

$4R \cot \left(\frac{A}{2}\right)$

B.

$2R \cot^2 \left(\frac{A}{2}\right)$

C.

$\frac{4R}{\tan^2\left(\frac{A}{2}\right)}$

D.

$\frac{2R}{\tan\left(\frac{A}{2}\right)}$

Answer: C

Solution:

We know that

$$r_2 = 4R \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$$
$$r_3 = 4R \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$$
$$\therefore \quad (r_2 + r_3) = 4R \cos \frac{A}{2} \left[\sin \frac{B}{2} \cos \frac{C}{2} + \cos \frac{B}{2} \sin \frac{C}{2}\right]$$
$$\Rightarrow \quad (r_2 + r_3) = 4R \cos \frac{A}{2} \sin \left(\frac{B+C}{2}\right)$$
$$\Rightarrow \quad (r_2 + r_3) = 4R \cos \frac{A}{2} \sin \left(90^\circ - \frac{A}{2}\right) = 4R \cos \frac{A}{2} \cos \frac{A}{2}$$
$$\therefore (r_2 + r_3) \operatorname{cosec}^2 \left(\frac{A}{2}\right) = 4R \cos^2 \frac{A}{2} \operatorname{cosec}^2 \frac{A}{2}$$
$$= \frac{4R}{\tan^2 \frac{A}{2}}$$

Question30

A, B, C and D, are any four points. If E and F are mid-points of AC and BD respectively, then $\mathbf{AB} + \mathbf{CB} + \mathbf{CD} + \mathbf{AD} =$

Options:

A.

EF

B.

$2EF$

C.

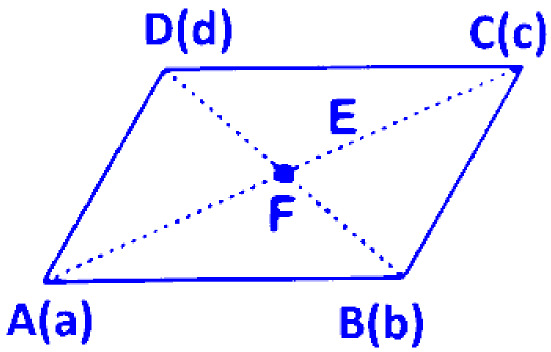
3 EF

D.

$4EF$

Answer: D

Solution:



Let position vector of A, B, C and D be respectively $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$

$$\therefore \quad AB = b - a, \quad CB = b - c$$
$$CD = d - c, \quad AD = d - a$$
$$\therefore AB + CB + CD + AD$$
$$= 2b + 2d - 2a - 2c$$
$$= 2(b + d - a - c) \qquad \ldots (i)$$

And let position vector of E and F are $\mathbf{e}$ and $\mathbf{f}$

$$\therefore \quad \mathbf{e} = \frac{\mathbf{a}+\mathbf{c}}{2}$$
$$\Rightarrow \quad \mathbf{a} + \mathbf{c} = 2\mathbf{e}$$
$$\text{And} \quad \mathbf{f} = \frac{\mathbf{b}+\mathbf{d}}{2} \Rightarrow \mathbf{b} + \mathbf{d} = 2\mathbf{f}$$

$\therefore$ From Eq. (i), we get

$$\mathbf{AB} + \mathbf{CB} + \mathbf{CD} + \mathbf{AD} = 2(2\mathbf{f} - 2\mathbf{c})$$
$$= 4(\mathbf{f} - \mathbf{e}) = 4\mathbf{EF}$$

Question31

The four points whose position vectors are given by $2a + 3b - c, a - 2b + 3c, 3a + 4b - 2c$ and $a - 6b + 6c$ are

Options:

A.

collinear

B.

coplanar

C.

Vertices of a square

D.

Vertices of a rectangle

Answer: B

Solution:

We have, $\mathbf{OA} = 2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$

$$\begin{aligned} OB &= a - 2b + 3c \\ OC &= 3a + 4b - 2c \\ OD &= a - 6b + 6c \\ \therefore AB &= -a - 5b + 4c \\ AC &= a + b - c \\ AD &= -a - 9b + 7c \end{aligned}$$

$\therefore [\mathbf{AB} \quad \mathbf{AC} \quad \mathbf{AD}]$

$$= \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ -1 & -9 & 7 \end{vmatrix}$$
$$= (-1)(7-9) + 5(7-1) + 4(-9+1)$$
$$= 2 + 30 - 32 = 0$$

Given, point are coplanar.

Question32

If $a = |\mathbf{a}|; b = |\mathbf{b}|$, then $\left(\frac{\mathbf{a}}{a^2} - \frac{\mathbf{b}}{b^2}\right)^2$

Options:

A.

$\left(\frac{a-b}{a^2b^2}\right)^2$

B.

$\left(\frac{\mathbf{a}-\mathbf{b}}{\mathbf{ab}}\right)^{\mathbf{2}}$

C.

$\left(\frac{b\mathbf{a}-a\mathbf{b}}{ab}\right)^2$

D.

$\left(\frac{a\mathbf{a}-b\mathbf{b}}{a^2b^2}\right)^2$

Answer: B

Solution:

Given, $a = |\mathbf{a}|, b = |\mathbf{b}|$

$$\left(\frac{\mathbf{a}}{a^2} - \frac{\mathbf{b}}{b^2}\right)^2 = \left(\frac{\mathbf{a}}{a^2} - \frac{\mathbf{b}}{b^2}\right)\left(\frac{\mathbf{a}}{a^2} - \frac{\mathbf{b}}{b^2}\right)$$
$$= \left|\frac{\mathbf{a}}{a^2}\right|^2 + \left|\frac{\mathbf{b}}{b^2}\right|^2 - 2\left(\frac{\mathbf{a}}{a^2}\right)\cdot\left(\frac{\mathbf{b}}{b^2}\right)$$
$$= \frac{|\mathbf{a}|^2}{a^4} + \frac{|\mathbf{b}|^2}{b^4} - \frac{2(\mathbf{a}\cdot\mathbf{b})}{a^2b^2}$$
$$= \frac{1}{a^2} + \frac{1}{b^2} - \frac{2(\mathbf{a}\cdot\mathbf{b})}{a^2b^2} = \frac{a^2+b^2-2(\mathbf{a}\cdot\mathbf{b})}{a^2b^2}$$
$$= \frac{(\mathbf{a}-\mathbf{b})^2}{a^2b^2} = \left(\frac{\mathbf{a}-\mathbf{b}}{ab}\right)^2$$

Question33

$\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three unit vectors such that $x\mathbf{a}+y\mathbf{b}+z\mathbf{c}=p(\mathbf{b}\times\mathbf{c})+q(\mathbf{c}\times\mathbf{a})+r(\mathbf{a}\times\mathbf{b})$. If $(\mathbf{a},\mathbf{b})=(\mathbf{b},\mathbf{c})=(\mathbf{c},\mathbf{a})=\frac{\pi}{3}$, $(\mathbf{a},\mathbf{b}\times\mathbf{c})=\frac{\pi}{6}$ and $\mathbf{a},\mathbf{b},\mathbf{c}$ form a right-handed system, then $\frac{x+y+z}{p+q+r}=$

Options:

A.

$\frac{3}{4}$

B.

$\frac{1}{\sqrt{2}}$

C.

$2\sqrt{2}$

D.

$\frac{3}{8}$

Answer: D

Solution:

We have, $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1$

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b}&=|\mathbf{a}||\mathbf{b}|\cos\frac{\pi}{3}\\&=(1)(1)\left(\frac{1}{2}\right)=\frac{1}{2}\\\mathbf{b}\cdot\mathbf{c}&=|\mathbf{b}||\mathbf{c}|\cos\frac{\pi}{3}\\&=(1)(1)\left(\frac{1}{2}\right)=\frac{1}{2}\end{aligned}$$

And $\mathbf{c}\cdot\mathbf{a}=|\mathbf{c}||\mathbf{a}|\cos\frac{\pi}{3}=(1)(1)\left(\frac{1}{2}\right)=\frac{1}{2}$

$$\begin{aligned}[\mathbf{abc}]&=\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\\&=|\mathbf{a}||\mathbf{b}\times\mathbf{c}|\cos\frac{\pi}{6}\end{aligned}$$

$=(1)\,|\mathbf{b}\times\mathbf{c}|\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{2}|\mathbf{b}\times\mathbf{c}|$

Now, $|\mathbf{b}\times\mathbf{c}|=|\mathbf{b}||\mathbf{c}|\sin\frac{\pi}{3}$

$$=(1)(1)\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{2}$$

$$\therefore\quad[\mathbf{abc}]=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{3}{4}$$

Given, $x\mathbf{a}+y\mathbf{b}+z\mathbf{c}=p(\mathbf{b}\times\mathbf{c})+q(\mathbf{c}\times\mathbf{a})+\mathbf{r}(\mathbf{a}\times\mathbf{b})$

$$\begin{aligned}\therefore\quad&x(\mathbf{a}\cdot\mathbf{a})+y(\mathbf{b}\cdot\mathbf{a})+z(\mathbf{c}\cdot\mathbf{a})\\&=p((\mathbf{b}\times\mathbf{c})\cdot\mathbf{a})\\\Rightarrow\quad&x+y\left(\tfrac{1}{2}\right)+z\left(\tfrac{1}{2}\right)=p[\mathbf{abc}]\\\Rightarrow\quad&2x+y+z=\tfrac{3}{2}p\qquad\ldots(i)\end{aligned}$$

Similarly,

$x(\mathbf{a}\cdot\mathbf{b})+y(\mathbf{b}\cdot\mathbf{b})+z(\mathbf{c}\cdot\mathbf{b})=q((\mathbf{c}\times\mathbf{a})\cdot\mathbf{b})$

$$\Rightarrow \quad x\left(\frac{1}{2}\right)+y+z\left(\frac{1}{2}\right)=q[\mathbf{abc}]$$

$$\Rightarrow \quad x+2y+z=\frac{3}{2}q \qquad \ldots(ii)$$

And $x(\mathbf{a}\cdot\mathbf{c})+y(\mathbf{b}\cdot\mathbf{c})+z(\mathbf{c}\cdot\mathbf{c})=r[\mathbf{abc}]$

$$\Rightarrow \quad x\left(\frac{1}{2}\right)+y\left(\frac{1}{2}\right)+z=r\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad x+y+2z=\frac{3}{2}r \qquad \ldots(ii)$$

On adding Eqs. (i), (ii) and (iii), we get

$$4(x+y+z)=\frac{3}{2}(p+q+r)$$

$$\Rightarrow \quad \frac{x+y+z}{p+q+r}=\frac{3}{8}$$

Question34

Let A be a point having position vector $\hat{\mathbf{i}}-3\hat{\mathbf{j}}$ and $\mathbf{r}=(\hat{\mathbf{i}}-3\hat{\mathbf{j}})+t(\hat{\mathbf{j}}-2\hat{\mathbf{k}})$ be a line. If P is a point on this line and is at a minimum distance from the plane $\mathbf{r}.(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}})=0$, then the equation of the plane through P and perpendicular to AP, is

Options:

A.

$\mathbf{r}\cdot(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=8$

B.

$\mathbf{r}\cdot(\hat{\mathbf{j}}+\hat{\mathbf{k}})=4$

C.

$\mathbf{r}\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=8$

D.

$\mathbf{r}\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}})=12$

Answer: A

Solution:

Given, normal vector to the plane is $\mathbf{n}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

And direction vector to the line is $\mathbf{b}=\mathbf{j}-2\hat{\mathbf{k}}$

Since, $\mathbf{b}\cdot\mathbf{n}=(\hat{\mathbf{j}}-2\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}})$

$=3-10=-7\neq 0$

$\Rightarrow$ line is not perpendicular to the plane Now, $\mathbf{P}(t)=\hat{\mathbf{i}}+(-3+t)\hat{\mathbf{j}}-2t\hat{\mathbf{k}}$

for minimum distance, $\mathbf{p}(t)\cdot\mathbf{n}=0$

$\Rightarrow \quad 2+(-9+3t)-10t=0$
$\Rightarrow \quad -7t-7=0 \Rightarrow t=-1$

$\therefore \quad \mathbf{p}=\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$$\therefore \quad AP=(\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}})-(\hat{\mathbf{i}}-3\hat{\mathbf{j}})$$
$$=-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$

$\therefore$ Equation of plane

$$(\mathbf{r}-(\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}}))\cdot(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=0$$
$$\Rightarrow \quad ((x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})-(\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}}))$$
$$((-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=0$$
$$\Rightarrow \quad ((x-1)\hat{\mathbf{i}}+(y+4)\hat{\mathbf{j}}+(z-2\hat{\mathbf{k}})$$
$$(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=0$$

$\Rightarrow \quad -(y+4)+2(z-2)=0$
$\Rightarrow \quad -y-4+2z-4=0$
$\Rightarrow \quad -y+2z-8=0$
$\Rightarrow \quad -y+2z=8$
$\Rightarrow \quad \mathbf{r}\cdot(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=8$

Question35

If the variance of the numbers $9, 15, 21, \ldots, (6n+3)$ is P, then the variance of the first n even numbers is

Options:

A.

$9P$

B.

$3P$

C.

$\frac{P}{9}$

D.

$\frac{P}{3}$

Answer: C

Solution:

Given, $a=9, d=6$

$\therefore$ Variance,

$$P=\frac{6^2\left(n^2-1\right)}{12}$$
$$=3\left(n^2-1\right)$$
$$\Rightarrow \quad n^2-1=\frac{P}{3} \qquad \ldots(i)$$

Now, variance of $2, 4, 6, \ldots, 2n$

$$a=2, d=2$$

Variance, $\frac{\left(4^2\left(n^2-1\right)\right.}{12}$

$$=\frac{4\left(n^2-1\right)}{12}=\frac{n^2-1}{3}$$

$$=\frac{\frac{P}{3}}{3}=\frac{P}{9}$$

Question36

Let $P=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$ be a matrix. Three elements of this matrix P are selected at random. A is the event of having the three elements whose sum is odd. B is the event of selecting the three elements which are in a row or column. Then, $P(A)+P\left(\frac{A}{B}\right)=$

Options:

A.

$\frac{221}{420}$

B.

$\frac{17}{21}$

C.

$\frac{21}{20}$

D.

$\frac{3}{2}$

Answer: B

Solution:

Given, matrix $P=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$

Odd elements $=\{1,3,5,7,9\}$

Total = 5

Even elements $=\{2,4,6,8\}$

Total =4

Case I 3 odd numbers numbers of ways $={ }^5 C_3=10$

Case II 1 odd and 2 even numbers of ways $={ }^5 C_1 \times{ }^4 C_2$

$=5 \times 6=30$

Total favourable cases for A

$=10+30=40$

And total ways for selecting 3 elements from 9 elements $={ }^9 C_3=84$

$\therefore \quad P(A)=\frac{40}{84}$

Selecting 3 elements in a row or column.

3 rows $\times 1$ way $=3$ ways

3 column $\times 1$ way $=3$ ways

total $=6$ ways

Now, Row 1 : $1+2+3=6$ (even)

Row 2 : $4+5+6=15$ (odd)

Row 3 : $7+8+9=24$ (even)

Column I : $1+4+7=12$ (even)

Column II : $2+5+8=15$ (odd)

Column III : $3+6+9=18$ (even)

$$\therefore P(A \cap B)=\frac{2}{84}$$
$$P(B)=\frac{6}{84}$$

$$\therefore \quad P\left(\frac{A}{B}\right)=\frac{P(A \cap B)}{P(B)}=\frac{\frac{2}{84}}{\frac{6}{84}}=\frac{1}{3}$$

$$\begin{aligned} \therefore P(A)+P\left(\frac{A}{B}\right) &=\frac{40}{84}+\frac{1}{3} \\ &=\frac{40+28}{84} \\ &=\frac{68}{84}=\frac{17}{21} \end{aligned}$$

Question37

A, B_1, B_2, B_3 are the events in a random experiment. If $P\left(B_1\right)=0.25, P\left(B_2\right)=0.30, P\left(B_3\right)=0.45, P\left(\frac{A}{B_1}\right)=0.05$, $P\left(\frac{A}{B_2}\right)=0.04, P\left(\frac{A}{B_3}\right)=0.03$, then $P\left(\frac{B_2}{A}\right)=$

Options:

A.

$\frac{6}{19}$

B.

$\frac{8}{19}$

C.

$\frac{12}{19}$

D.

$\frac{5}{19}$

Answer: A

Solution:

Given, $P(B_1) = 0.25, P(B_2) = 0.30,$
$P(B_3) = 0.45, P\left(\frac{A}{B_1}\right) = 0.05$
$P\left(\frac{A}{B_2}\right) = 0.04, P\left(\frac{A}{B_3}\right) = 0.03$

$$\therefore P\left(\frac{B_2}{A}\right) = \frac{P\left(\frac{A}{B_2}\right)P(B_2)}{P\left(\frac{A}{B_2}\right)P(B_2)+P\left(\frac{A}{B_3}\right)P(B_3)}$$
$$+P\left(\frac{A}{B_1}\right)P(B_1)$$
$$= \frac{0.04 \times 0.30}{0.04 \times 0.30 + 0.03 \times 0.45 + 0.05 \times 0.25}$$
$$= \frac{0.012}{0.012 + 0.0135 + 0.0125}$$
$$= \frac{0.012}{0.038} = \frac{6}{19}$$

Question38

A, B are the events in a random experiment.

If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{A^c}{B^c}\right) + P\left(\frac{A}{B}\right) =$

Options:

A.

1

B.

$\frac{4}{5}$

C.

$\frac{11}{8}$

D.

$\frac{7}{3}$

Answer: C

Solution:

Given, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3},$

$$P(A \cap B) = \frac{1}{4}$$
$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$
$$P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')}$$
$$= \frac{P((A \cup B)')}{P(B')}$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$
$= \frac{6+4-3}{12} = \frac{7}{12}$

$\therefore P\left((A \cup B)' = 1 - \frac{7}{12} = \frac{5}{12}\right.$

And $P(B') = \frac{2}{3}$

$\therefore P\left(\frac{A'}{B'}\right) = \frac{\frac{5}{12}}{\frac{2}{3}} = \frac{5}{12} \times \frac{3}{2} = \frac{5}{8}$

$\therefore P\left(\frac{A'}{B'}\right) + P\left(\frac{A}{B}\right) = \frac{5}{8} + \frac{3}{4} = \frac{11}{8}$

Question39

Two persons A and B play a game by throwing two dice. If the sum of the numbers appeared on the two dice is even, A will get $\frac{1}{2}$ point and B will get $\frac{1}{2}$ point.

If the sum is odd, A will get one point and B will get no point. The arithmetic mean of the random variable of the number of points of A is

Options:

A.

$1/2$

B.

$1/4$

C.

1 .

D.

$3/4$

Answer: D

Solution:

Total outcomes when two dice are thrown $= 36$

Even sum : happen when sum

$= 2, 4, 6, 8, 10, 12$

Odd sum : happens when sum

$= 3, 5, 7, 9, 11$

Sum	Combination	Even
2	$(1,1)$	1
4	$(1,3),(2,2),(3,1)$	3
6	$(1,5),(2,4),(3,3),(4,2),(5,1)$	5
8	$(2,6),(3,5),(4,4),(5,3),(6,2)$	5
10	$(4,6),(5,5),(6,4)$	3
12	$(6,6)$	1

$\therefore$ Even sum $= 18$ outcomes

Odd sum $= 36 - 18 = 18$ outcomes

Sum is even, A gets $= \frac{1}{2}$, point with probability

$= \frac{18}{36} = \frac{1}{2}$

Sum is odd, A gets $= \frac{1}{2}$, point with probability

$= \frac{18}{36} = \frac{1}{2}$

Expected value $E(X) = \frac{1}{2} \times \frac{1}{2} + 1 \times \frac{1}{2}$

$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

Question40

A typist claims that he prepares a typed page with typo errors of 1 per 10 pages. In a typing assignment of 40 pages, if the probability that the typo errors are at most 2 is p, then $e^2 p =$

Options:

A.

5

B.

13

C.

$13e^{-2}$

D.

$5e^{-2}$

Answer: C

Solution:

Step 1: Finding the average number of errors

The typist makes 1 mistake for every 10 pages. For 40 pages, expected errors = $\frac{40}{10} = 4$.

Step 2: Setting up the Poisson distribution

We use the Poisson formula to find the chance of getting up to 2 errors when the average (λ) is 4.

$P(X = k) = \frac{e^{-4} 4^k}{k!}$

Step 3: Find probability of at most 2 errors

We need to calculate $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$:

$P(X=0) = \frac{e^{-4} \cdot 4^0}{0!} = e^{-4}$
$P(X=1) = \frac{e^{-4} \cdot 4^1}{1!} = 4e^{-4}$
$P(X=2) = \frac{e^{-4} \cdot 4^2}{2!} = 8e^{-4}$

Add them up:
$p = e^{-4} + 4e^{-4} + 8e^{-4} = 13e^{-4}$

Step 4: Multiply by e^2 as asked in the question

$e^2 p = 13e^{-4} \times e^2 = 13e^{-2}$

Question41

A line segment joining a point A on X-axis to a point B on Y-axis is such that $AB = 15$. If P is a point on AB such that $\frac{AP}{PB} = \frac{2}{3}$, then the locus of P is

Options:

A.

$x = 9\cos\theta, y = 6\sin\theta$

B.

$x = 6\cos\theta, y = 9\sin\theta$

C.

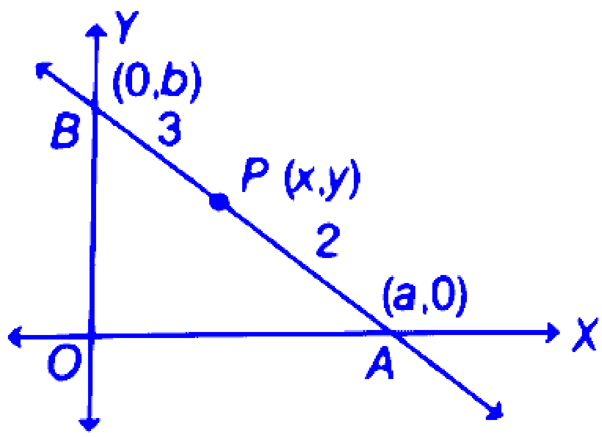
$x = 6\cos\theta, y = 6\sin\theta$

D.

$x = 9\cos\theta, y = 9\sin\theta$

Answer: A

Solution:



Here, $P \equiv \left(\frac{3a}{5}, \frac{2b}{5}\right) = (x, y)$

$\Rightarrow \quad x = \frac{3a}{5}, y = \frac{2b}{5}$

$\Rightarrow \quad a = \frac{5x}{3}, b = \frac{5y}{2}$

$\therefore \quad a^2 + b^2 = (15)^2$

$\Rightarrow \quad \left(\frac{5x}{3}\right)^2 + \left(\frac{5y}{2}\right)^2 = 225$

$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 9$

$\Rightarrow \quad \frac{x^2}{81} + \frac{y^2}{36} = 1$

which represent equation of ellipse

$\therefore \quad x = 9\cos\theta$

And $\quad y = 6\sin\theta$

Question42

The point $P(\alpha, \beta)(\alpha > 0, \beta > 0)$ undergoes the following transformations successively.

(a) Translation to a distance of 3 units in positive direction of X-axis.

(b) Reflection about the line $y = -x$.

(c) Rotation of axes through an angle of $\frac{\pi}{4}$ about the origin in the positive direction.

If the final position of that point P is $(-4\sqrt{2}, -2\sqrt{2})$, then $(\alpha + \beta) =$

Options:

A.

5

B.

7

C.

$6\sqrt{2}$

D.

$2\sqrt{2}$

Answer: A

Solution:

Rotation by angle θ in the positive direction

$x' = x\cos\theta + y\sin\theta$
$y' = -x\sin\theta + y\cos\theta$

Let $(x_2, y_2) = (-4\sqrt{2}, -2\sqrt{2})$

Point before rotation (x_1, y_1),

$$x_1 = x_2 \cos\left(-\frac{\pi}{4}\right) + y_2 \sin\left(-\frac{\pi}{4}\right)$$
$$= x_2\left(\frac{\sqrt{2}}{2}\right) - y_2\left(\frac{\sqrt{2}}{2}\right)$$

And $y_1 = x_2\left(\frac{\sqrt{2}}{2}\right) + y_2\left(\frac{\sqrt{2}}{2}\right)$

$$\Rightarrow \quad x_1 = \frac{\sqrt{2}}{2}(-4\sqrt{2}) - \frac{\sqrt{2}}{2}(-2\sqrt{2})$$
$$= -4 + 2 = -2$$
$$\text{And } y_1 = \frac{\sqrt{2}}{2}(-4\sqrt{2}) + \frac{\sqrt{2}}{2}(-2\sqrt{2})$$
$$= -4 - 2 = -6$$

So, point before rotation is $(-2, -6)$ reflection about $y = -x$

$\Rightarrow$ Reflection of point (x, y) is $(-y, -x)$ i.e., $(6, 2)$

Translation by +3 in X-axis

$$\therefore \quad (\alpha, \beta) = (6, -3, 2)$$
$$= (3, 2)$$
$$\therefore \quad \alpha + \beta = 3 + 2 = 5$$

Question43

If the line passing through the point $(4, -3)$ and having negative slope makes an angle of 45° with the line joining the points $(1, 1)$, $(2, 3)$, then the sum of intercepts of that line is

Options:

A.

$\frac{7}{3}$

B.

1

C.

12

D.

$\frac{26}{3}$

Answer: C

Solution:

Let slope of line passing through the point $(4, -3)$ be m

And slope of the line joining point $(1, 1)$ and $(2, 3)$ is $= \frac{3-1}{2-1} = 2$

$$\therefore \quad \tan 45^\circ = \left|\frac{m-2}{1+2m}\right|$$
$$\Rightarrow \quad 1 = \left|\frac{m-2}{1+2m}\right|$$

taking positive sign, we get

$$1=\frac{m-2}{1+2m}$$
$$\Rightarrow 1+2m=m-2$$
$$\Rightarrow \quad m=-3$$

Taking negative sign, we get

$$-1=\frac{m-2}{1+2m}$$
$$\Rightarrow \quad m-2=-1-2m \Rightarrow 3m=1$$

$\Rightarrow \quad m=\frac{1}{3}$

Since, slope is negative.

$\therefore \quad m=-3$

Equation of line is

$$y+3=(-3)(x-4)$$
$$\Rightarrow \quad y+3=-3x+12$$
$$\Rightarrow \quad 3x+y=9$$

$\therefore$ x-intercept, when $y=0$, is $3x=9$

$\Rightarrow \quad x=3$

and y-intercept, when $x=0$ is $y=9$

$\therefore$ Sum of intercepts $=3+9=12$

Question44

$O(0,0), B(-3,-1)$ and $C(-1,-3)$ are vertices of a $\triangle OBC$. D is a point on OC and E is a point on OB. If the equation of DE is $2x+2y+\sqrt{2}=0$, then the ratio in which the line DE divides the altitude of the $\triangle OBC$ is

Options:

A.

$\sqrt{2}: 4\sqrt{2}+2$

B.

$1: 4\sqrt{2}+1$

C.

$\sqrt{2}: 4\sqrt{2}-2$

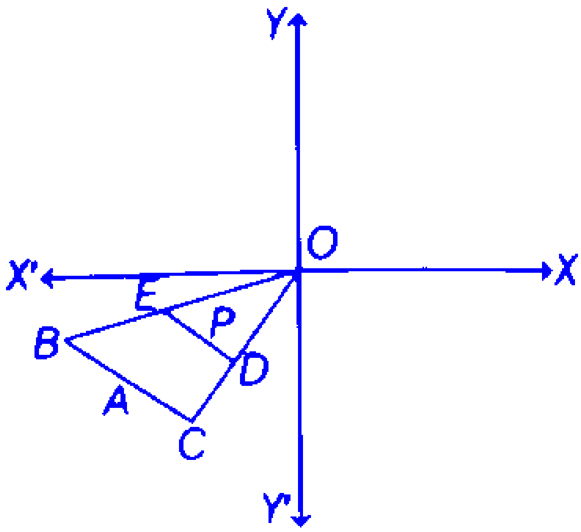
D.

$1: 4\sqrt{2}-1$

Answer: D

Solution:

We have $O(0,0), B(-3,-1), C(-1,-3)$ are vertices of a $\triangle OBC$ and equation of DE is $2x+2y+\sqrt{2}=0$



Slope of BC is $\frac{-3-(-1)}{-1-(-3)} = \frac{-2}{2} = -1$

$\therefore$ Slope of attitude from 0 to BC is 1 ($\perp$ to BC)

$\therefore$ Equation of altitude with slope passes through $(0, 0)$

$$y - 0 = 1(x - 0)$$
$$\Rightarrow \quad y = x$$

Equation of BC

$y + 1 = (-1)(x + 3) \Rightarrow y = -x - 4$

Point of intersection of line BC and altitude

$$x = -x - 4$$
$$\Rightarrow \quad 2x = -4$$
$$\Rightarrow \quad x = -2$$
$$\therefore \quad y = -2$$

$\therefore$ Foot of perpendicular $A = (-2, -2)$

Now, $2x + 2y + \sqrt{2} = 0$

$$\Rightarrow \quad x + y = -\frac{\sqrt{2}}{2}$$
$$\Rightarrow \quad x + x = \frac{-\sqrt{2}}{2} \quad (\text{ altitude } y = x)$$
$$\Rightarrow \quad 2x = \frac{-\sqrt{2}}{2}$$
$$\Rightarrow \quad x = \frac{-\sqrt{2}}{4}$$

Also, $\quad y = \frac{-\sqrt{2}}{4}$

$\therefore$ Intersection of altitude and DE is P. $\left(\frac{-\sqrt{2}}{4}, \frac{-\sqrt{2}}{4}\right)$

$$\therefore AP = \sqrt{\left(-2 + \frac{\sqrt{2}}{4}\right)^2 + \left(-2 + \frac{\sqrt{2}}{4}\right)^2}$$
$$= \sqrt{2\left(-2 + \frac{\sqrt{2}}{4}\right)^2} = \sqrt{2}\left(-2 + \frac{\sqrt{2}}{4}\right)$$

And $PO = \sqrt{2\left(\frac{\sqrt{2}}{4}\right)^2}$

$$= \sqrt{2 \times \frac{2}{16}} = \frac{1}{2}$$
$$\therefore \frac{PO}{AP} = \frac{\frac{1}{2}}{\sqrt{2}\left(-2 + \frac{\sqrt{2}}{4}\right)} = 1 : 4\sqrt{2} - 1$$

Question45

Every point on the curve $3x + 2y - 3xy = 0$ is the centroid of a triangle formed by the coordinate axes and a line (L) intersecting both the coordinates axes. Then, all such lines (L)

Options:

A.

are parallel

B.

are concurrent

C.

intersect each other at different points

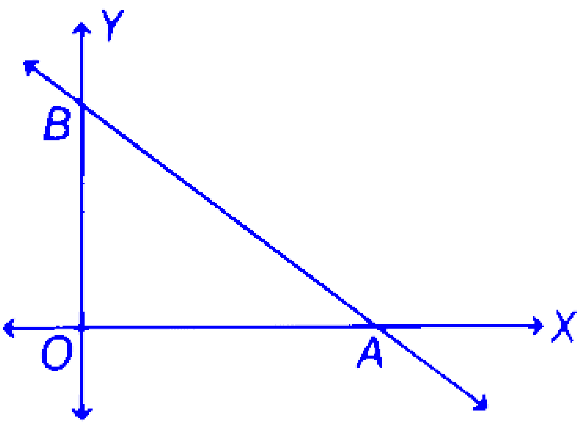
D.

are perpendicular to the tangents to the curve

Answer: B

Solution:

Given equation of curve $3x + 2y - 3xy = 0$ every point on the curve is the centroid of a triangle formed by the coordinate axes and a line (L) Since, L intersect the coordinate axes



$\therefore$ let $A = (a, 0)$
$B = (0, b)$

$\therefore$ Centroid of $\triangle OAB$,

$$(x, y) = \left(\frac{0+0+a}{3}, \frac{0+b+0}{3}\right) = \left(\frac{a}{3}, \frac{b}{3}\right)$$
$$\Rightarrow a = 3x, b = 3y$$

Equation of line

$$\frac{X}{3x} + \frac{Y}{3y} = 1$$
$$\Rightarrow \frac{X}{x} + \frac{Y}{y} = 3 \qquad \ldots (i)$$

Since, centroid (x, y) lies on given curve

$$3x+2y-3xy=0$$
$$\Rightarrow \quad 3x+2y=3xy$$
$$\Rightarrow \quad 3xy=3x+2y \Rightarrow 3=\frac{3x}{xy}+\frac{2y}{xy}$$
$$\Rightarrow \quad \frac{X}{x}+\frac{Y}{y}=\frac{3}{y}+\frac{2}{x} \quad \text{(Using Eq. (i))}$$
$$\Rightarrow \quad \frac{X-2}{x}+\frac{Y-3}{y}=0$$

$\Rightarrow$ Does not represents to a single line

Now, $3x+2y=3xy$

$$\Rightarrow \quad \frac{3x}{3xy}+\frac{2y}{3xy}=1$$
$$\Rightarrow \quad \frac{1}{y}+\frac{2}{3x}=1$$
$$\Rightarrow \quad \frac{1}{\frac{b}{3}}+\frac{2}{3\left(\frac{a}{3}\right)}=1$$
$$\Rightarrow \quad \frac{3}{b}+\frac{2}{a}=1 \Rightarrow 3a+2a=ab$$
$$\Rightarrow \quad ab-3a-2b=0$$
$$\Rightarrow \quad a=\frac{2b}{b-3}$$

Consider the equation of line L

$$bX+aY=ab$$
$$\Rightarrow \quad bX+\frac{2b}{b-3}Y=b\left(\frac{2b}{b-3}\right)$$
$$\Rightarrow \quad X+\frac{2}{b-3}Y=\frac{2b}{b-3}$$
$$\Rightarrow \quad X(b-3)+2Y=2b$$
$$\Rightarrow \quad b(X-2)+(-3X+2Y)=0$$

For all such lines L

$X-2=0 \Rightarrow X=2$

and $\quad -3X+2Y=0$

$\Rightarrow \quad Y=3$

$\Rightarrow \quad L$ is passes through $(2,3)$

$\Rightarrow \quad L$ are concurrent.

Question46

The value of ' a ' for which the equation $\left(a^2-3\right)x^2+16xy-2ay^2+4x-8y-2=0$ represents a pair of perpendicular lines is

Options:

A.

2

B.

-1

C.

3

D.

4

Answer: C

Solution:

Given, equation

$\left(a^2-3\right)x^2+16xy-2ay^2+4x-8y-2=0$

represents a pair of perpendicular line

$$\begin{aligned} \therefore & \left(a^2-3\right)+(-2a)=0 \\ \Rightarrow \quad & a^2-2a-3=0 \\ \Rightarrow \quad & a^2-3a+a-3=0 \\ \Rightarrow \quad & a(a-3)+1(a-3)=0 \\ \Rightarrow \quad & (a-3)(a+1)=0 \\ \Rightarrow \quad & a=3,-1 \end{aligned}$$

$\Rightarrow a=3$, since for $a=-1$ lines does not exists.

Question47

The slope of a common tangent to the circles $x^2+y^2=16$ and $(x-9)^2+y^2=16$ is

Options:

A.

$\frac{8}{\sqrt{13}}$

B.

$\frac{4}{\sqrt{13}}$

C.

$\frac{\sqrt{17}}{8}$

D.

$\frac{8}{\sqrt{17}}$

Answer: D

Solution:

Given, equation of circles

$x^2+y^2=16 \qquad \ldots(i)$

And $(x-9)^2+y^2=16 \qquad \ldots(ii)$

$$C_1=(0,0), r_1=4$$
$$C_2=(9,0), r_2=4$$
$$\therefore \quad C_1C_2=\sqrt{(9-0)^2+(0-0)^2}$$
$$=\sqrt{(9)^2}=9$$

And $r_1+r_2=4+4=8$

Since $C_1C_2>r_1+r_2$

Circles are separate to each other, any tangent to circle (i) with slope (m) is

$$y=mx\pm r\sqrt{1+m^2}$$
$$\Rightarrow \quad y=mx+4\sqrt{1+m^2}$$
$$\Rightarrow \quad mx-y\pm 4\sqrt{1+m^2}=0 \qquad \ldots(ii)$$

Equation (iii) to be tangent to circle (ii)

$$\therefore \quad \left|\frac{m(9)-0\pm 4\sqrt{1+m^2}}{\sqrt{m^2+1}}\right|=4$$
$$\Rightarrow \quad 16\left(m^2+1\right)=\left(9m\pm 4\sqrt{1+m^2}\right)^2$$
$$\Rightarrow \quad 16m^2+16=81m^2+16\left(1+m^2\right)$$
$$\pm 72m\sqrt{1+m^2}$$
$$\Rightarrow \quad 81m^2\pm 72m\sqrt{1+m^2}=0$$
$$\Rightarrow \quad 9m\left(9m\pm 8\sqrt{1+m^2}\right)=0$$

When $9m=0\Rightarrow m=0$

Then Eq. (iii) $y=\pm 4$ (horizontal lines)

If, $9m\pm 8\sqrt{1+m^2}=0$

$$\Rightarrow \quad (9m)^2=\left(\pm 8\sqrt{1+m^2}\right)^2$$
$$\Rightarrow \quad 81m^2=64\left(1+m^2\right)$$
$$\Rightarrow \quad 81m^2-64m^2=64$$
$$\Rightarrow \quad 17m^2=64$$
$$\Rightarrow \quad m=\pm\sqrt{\frac{64}{17}}$$
$$\Rightarrow \quad m=\pm\frac{8}{\sqrt{17}}$$

Question48

The equation of the circle whose radius is 3 and which touches the circle $x^2+y^2-4x-6y-12=0$ internally at $(-1,-1)$ is

Options:

A.

$5x^2+5y^2-8x-14y-32=0$

B.

$x^2+y^2-12x-14y-28=0$

C.

$3x^2+3y^2-8x-14y-31=0$

D.

$x^2+y^2-5x-7y-14=0$

Answer: A

Solution:

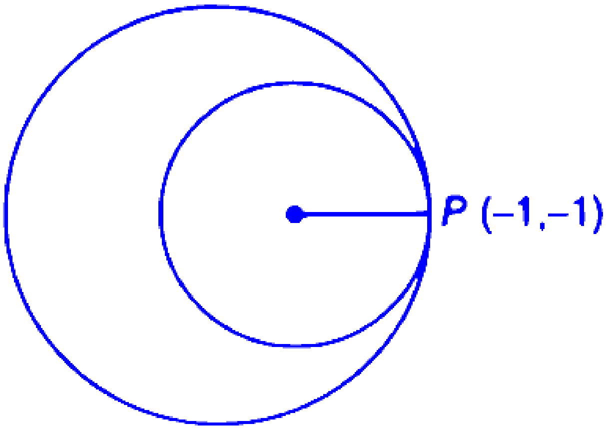
Given, equation of circle

$x^2+y^2-4x-6y-12=0 \qquad \ldots(i)$

centre $(-g,-f)=(2,3)$

And radius $=\sqrt{g^2+f^2-c}$

$=\sqrt{4+9+12}=\sqrt{25}=5$



Equation of new circle whose radius is 3

$(x-h)^2+(y-k)^2=(3)^2$

$\Rightarrow \quad (x-h)^2+(y-k)^2=9 \qquad \ldots(ii)$

Since circle (i) and (ii) touches each other internally at $(-1,-1)$

$\therefore \quad C_1C_2=|r_1-r_2|$

where $c_2=(h,k)$

$\therefore \quad C_1C_2=|5-3|=2$

P divides the line segment C_1C_2 in the ratio $r_1:r_2$

$\therefore C_1P=r_1$ and $C_2P=r_2$

$C_1P=\sqrt{(-1-2)^2+(-1-3)^2}$

$=\sqrt{(-3)^2+(-4)^2}$

$=\sqrt{9+16}=\sqrt{25}=5$

and $C_2P=r_2=3$

$\therefore$ The point of tangency P divides the line segment C_1C_2 externally in the ratio $r_1:-r$

$\therefore \quad P=\frac{r_1C_2-rC_1}{r_1-r}$

$=\frac{5C_2-3C_1}{5-3}=\frac{5C_2-3C_1}{2}$

$\Rightarrow \quad 2P=5C_2-3C_1$

$\Rightarrow \quad 5C_2=2P+3C_1$

$\Rightarrow \quad 5(h,k)=2(-1,-1)+3(2,3)$

$\Rightarrow \quad 5(h,k) = (-2,-2) + (6,9)$

$\Rightarrow \quad 5(h,k) = (4,7) \Rightarrow (h,k) = \left(\frac{4}{5}, \frac{7}{5}\right)$

$\therefore$ Equation of required circle

$$\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = (3)^2$$

$$\Rightarrow \quad \left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 9$$

$$\Rightarrow \quad x^2 + \frac{16}{25} - \frac{8}{5}x + y^2 + \frac{49}{25} - \frac{14}{5}y = 9$$

$$x^2 + y^2 - \frac{8}{5}x - \frac{14}{5}y + \frac{13}{5} = 9$$

$$\Rightarrow \quad 5x^2 + 5y^2 - 8x - 14y - 32 = 0$$

Question49

Suppose C_1 and C_2 are two circles having no common points, then

Options:

A.

There will be 3 common tangents to C_1 to C_2

B.

There will be exactly two common tangents to C_1 and C_2

C.

There will be no common tangent or there will be exactly two common tangents to C_1 and C_2

D.

There will be no common tangents or there will be four common tangents to C_1 and C_2

Answer: D

Solution:

If C_1 and C_2 are two circles having no common points, it means they do not intersect and do not touch each other then there will be no common tangents or there will be four common tangents C_1 and C_2

Question50

The locus of the centre of the circle touching the X-axis and passing through the point $(-1, 1)$ is

Options:

A.

a circle with centre at $\left(-1, \frac{1}{2}\right)$

B.

a pair of lines intersecting at $(-1, 1)$

C.

a parabola with focus at $(-1, 1)$

D.

a hyperbola with centre at $(-1, 1)$

Answer: C

Solution:

Equation of the circle touching the X-axis and passing through the point $(-1, 1)$

$$
\begin{aligned}
& (-1-h)^2+(1-k)^2=k^2 \\
\Rightarrow \quad & 1+h^2+2h+1+k^2-2k=k^2 \\
\Rightarrow \quad & 1+2h+h^2+1-2k+k^2=k^2 \\
\Rightarrow \quad & 1+2h+h^2+1-2k=0 \\
\Rightarrow \quad & h^2+2h-2k+2=0 \\
\Rightarrow \quad & x^2+2x-2y+2=0 \\
\Rightarrow \quad & 2y=x^2+2x+2 \\
\Rightarrow \quad & y=\frac{1}{2}\left(x^2+2x\right)+1 \\
\Rightarrow \quad & y=\frac{1}{2}(x+1)^2-\frac{1}{2}+1 \\
\Rightarrow \quad & y=\frac{1}{2}(x+1)^2+\frac{1}{2} \\
\Rightarrow \quad & (x+1)^2=2\left(y-\frac{1}{2}\right)
\end{aligned}
$$

Which represents parabola with focus $(-1, 1)$

Question51

The centres of all circles passing through the points of intersection of the circles $x^2+y^2+2x-2y+1=0$ and $x^2+y^2-2x+2y-2=0$ and having radius $\sqrt{14}$ lie on the curve

Options:

A.

$x+y=0$

B.

$y^2=4x-2$

C.

$3x^2+5x=y$

D.

$2x^2+3y^2=7$

Answer: A

Solution:

Given, equation of circles

$x^2+y^2+2x-2y+1=0$ and
$x^2+y^2-2x+2y-2=0$

$\therefore$ Equation of circle passing through the point of intersection of two given circle is

$$\left(x^2+y^2+2x-2y+1\right)+\lambda\left(x^2+y^2-2x+2y-2\right)=0$$
$$\Rightarrow \quad (1+\lambda)x^2+(1+\lambda)y^2+(2-2\lambda)x+(-2+2\lambda)y+(1-2\lambda)=0$$
$$\Rightarrow \quad x^2+y^2+\frac{(2-2\lambda)}{1+\lambda}x+\frac{(-2+2\lambda)}{1+\lambda}y+\frac{1-2\lambda}{1+\lambda}=0$$

Coordinate of centre of circle

$=(-g,-f)=\left(-\frac{1-\lambda}{1+\lambda},-\frac{-1+\lambda}{1+\lambda}\right)$

$\therefore$ Radius of

$$\left(-\frac{1-\lambda}{1+\lambda}\right)^2+\left(-\frac{-1+\lambda}{1+\lambda}\right)^2-\left(\frac{1-2\lambda}{1+\lambda}\right)=14$$
$$\Rightarrow \quad \frac{(1-\lambda)^2}{(1+\lambda)^2}+\frac{(\lambda-1)^2}{(1+\lambda)^2}-\frac{1-2\lambda}{1+\lambda}=14$$
$$\Rightarrow \quad (1-\lambda)^2+(\lambda-1)^2-(1+\lambda)(1-2\lambda)$$
$$=14(1+\lambda)^2$$
$$\Rightarrow \quad 2(\lambda-1)^2-\left(1-2\lambda+\lambda-2\lambda^2\right)$$
$$=14\left(1+\lambda^2+2\lambda\right)$$
$$\Rightarrow \quad 2\left(\lambda^2+1-2\lambda-\left(1-\lambda-2\lambda^2\right)\right.$$
$$=14\left(1+\lambda^2+2\lambda\right)$$
$$\Rightarrow \quad 4\lambda^2+1-3\lambda=14+14\lambda^2+28\lambda$$
$$\Rightarrow \quad 10\lambda^2+31\lambda+13=0$$

$\Rightarrow$ This is a quadrate equation is λ the existence of real values for λ conforms that such circle exists.

Coordinates of centre

$$h=\frac{\lambda-1}{1+\lambda}, k=-\frac{\lambda-1}{1+\lambda}$$
$$\Rightarrow \quad h=-k \Rightarrow h+k=0$$
$$\Rightarrow \quad x+y=0$$

Question52

A circle S given by $x^2+y^2-14x+6y+33=0$ cuts the X-axis at A and $B(OB>OA)$. C is mid-point of AB. L is a line through C and having slope (-1). If L is the diameter of a circle S' and also the radical axis of the circles S and S', then the equation of the circle S' is

Options:

A.

$x^2+y^2-17x+3y+54=0$

B.

$x^2+y^2+17x-3y-54=0$

C.

$x^2+y^2-17x+3y+51=0$

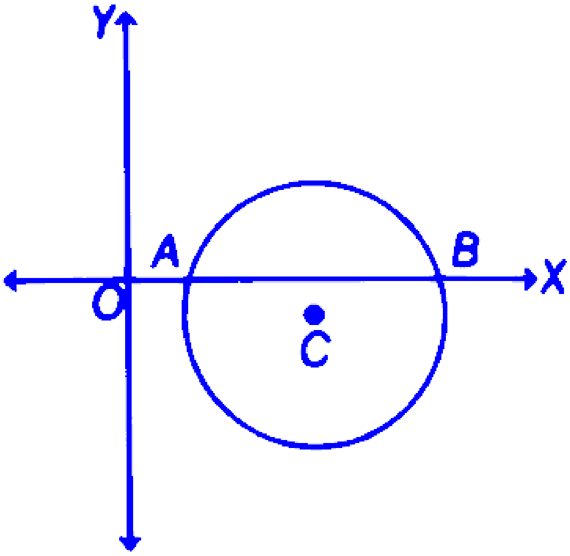
D.

$x^2+y^2-3x+17y-51=0$

Answer: A

Solution:

Given equation of circle
$x^2 + y^2 - 14x + 6y + 33 = 0 \ldots$ (i)



Centre of circle $= (-g, -f)$
$= (7, -3)$
and radius of circle $= \sqrt{g^2 + f^2 - c}$
$= \sqrt{49 + 9 - 33}$
$= \sqrt{25} = 5$

Since, circle cuts the X-axis, $y = 0$

$$
\begin{aligned}
& x^2 - 14x + 33 = 0 \\
\Rightarrow \quad & x^2 - 11x - 3x + 33 = 0 \\
\Rightarrow \quad & x(x - 11) - 3(x - 11) = 0 \\
\Rightarrow \quad & (x - 3)(x - 11) = 0 \\
\Rightarrow \quad & x = 3, 11 \\
\therefore \quad & A = (3, 0) \text{ and } B = (11, 0)
\end{aligned}
$$

Since, C is the mid-point of AB

$$
\begin{aligned}
\therefore \quad C &= \left(\frac{3 + 11}{2}, \frac{0 + 0}{2}\right) \\
&= (7, 0)
\end{aligned}
$$

$\therefore$ Equation of line L, which passes through C and having slope (-1) is

$$
\begin{aligned}
& y - 0 = (-1)(x - 7) \\
\Rightarrow \quad & y = -x + 7 \\
\Rightarrow \quad & x + y - 7 = 0 \qquad \ldots (ii)
\end{aligned}
$$

Since, L is the radical axis of the circles s and S' $(S - S' = 0)$

$$
\begin{aligned}
\therefore \quad & (x^2 + y^2 - 14x + 6y + 33) - (x^2 + y^2 + 2g'x + 2f'y + c') = 0 \\
\Rightarrow & (-14 - 2g')x + (6 - 2f')y + (33 - c') = 0
\end{aligned}
$$

Which is identical with line (ii)

$$
\begin{aligned}
\therefore \quad & \tfrac{-14 - 2g'}{1} = \tfrac{6 - 2f'}{1} = \tfrac{33 - c'}{-7} = k \\
\Rightarrow \quad & -14 - 2g' = k \text{ and } 6 - 2f' = k \\
\therefore \quad & -14 - 2g' = 6 - 2f' \\
\Rightarrow \quad & g' - f' = -10 \qquad \ldots (iii)
\end{aligned}
$$

Since, centre of S' lies on (ii)

$$\therefore \quad -g' - f' = 7$$
$$\Rightarrow \quad g' + f' = -7 \quad \ldots(iv)$$

Solving Eqs. (iii) and (iv), we get

$$g' = \frac{-17}{2}, f' = \frac{3}{2}$$
$$\therefore \quad -14 - 2\left(-\frac{17}{2}\right) = k$$
$$\Rightarrow \quad k = 3$$

Also, $\frac{33-c'}{-7} = k = 3$

$$\Rightarrow \quad 33 - c' = -21$$
$$\Rightarrow \quad c' = 54$$

$\therefore$ Equation of required circle is

$$x^2 + y^2 + 2\left(\frac{-17}{2}\right)x + 2\left(\frac{3}{2}\right)y + 54 = 0$$
$$\Rightarrow x^2 + y^2 - 17x + 3y + 54 = 0$$

Question53

For the parabola $y = x^2 - 3x + 2$, match the items in List I to that of the items in List II. S is a focus, Z is intersection of axis and directrix, P is one end of latus rectum, Q is the point on the parabola at which tangent is parallel to X-axis.

	List I		List II
A.	P	I.	$(2, 0)$
B.	Q	II.	$\left(\frac{3}{2}, -\frac{1}{4}\right)$
C.	S	III.	$\left(\frac{3}{2}, 0\right)$
D.	Z	IV.	$\left(\frac{3}{2}, -\frac{1}{2}\right)$
		V.	$\left(0, \frac{3}{2}\right)$

Options:

A.

A-I, B-II, C-III, D-IV

B.

A-I, B-II, C-V, D-IV

C.

A-II, B-V, C-III, D-IV

D.

A-IV, B-V, C-III, D-I

Answer: A

Solution:

Given, equation of parabola

$$y = x^2 - 3x + 2$$
$$\Rightarrow \quad y = x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2$$
$$\Rightarrow \quad y = \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$$
$$\Rightarrow \quad y + \frac{1}{4} = \left(x - \frac{3}{2}\right)^2$$

which is in the form of

$$(x-h)^2 = 4a(y-k)$$
$$\therefore \text{Vertex}(h, k) = \left(\frac{3}{2}, \frac{-1}{4}\right)$$
$$\therefore \quad Z = \left(\frac{3}{2}, \frac{-1}{2}\right)$$

And $4a = 1 \Rightarrow a = \frac{1}{4}$

$$\therefore \text{ Focus } S = (h, k+a)$$
$$= \left(\frac{3}{2}, \frac{-1}{4} + \frac{1}{4}\right) = \left(\frac{3}{2}, 0\right)$$

Tangent parallel to the X-axis

$\therefore$ Its slope is 0

Now, $y = x^2 - 3x + 2$

$$\Rightarrow \quad \frac{dy}{dx} = 2x - 3 = 0$$
$$\Rightarrow \quad 2x = 3 \Rightarrow x = \frac{3}{2}$$

$$\therefore \quad y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2$$
$$= \frac{9}{4} - \frac{9}{2} + 2$$
$$\Rightarrow \quad y = \frac{9-18+8}{4} = \frac{-1}{4}$$
$$\therefore \quad Q = \left(\frac{3}{2}, -\frac{1}{4}\right).$$

The end points of the latus rectum are
$(h \pm 2a, k+a)$

$$= \left(\frac{3}{2} \pm \frac{1}{2}, 0\right)$$
$$= \left(\frac{3}{2} + \frac{1}{2}, 0\right) \text{ and } \left(\frac{3}{2} - \frac{1}{2}, 0\right)$$
$$= (2, 0) \text{ and } (1, 0)$$
$$\therefore \quad P = (2, 0) \text{ or } (1, 0).$$

Question54

The locus of a point which divides the line segment joining the focus and any point on the parabola $y^2 = 12x$ in the ratio $m : n(m + n \neq 0)$ is a parabola.

Then, the length of the latus rectum of that parabola is

Options:

A.

$\frac{m}{m+n}$

B.

$\frac{12m}{m+n}$

C.

$\frac{m}{12(m+n)}$

D.

$\frac{n}{12(m+n)}$

Answer: B

Solution:

Given, equations of parabola

$$\begin{aligned} & y^2 = 12x \\ \therefore \quad & 4a = 12 \Rightarrow a = 3 \end{aligned}$$

$\therefore$ Parametric equation of parabola

$x = 3t^3, y = 2(3)t = 6t$

Any point on the parabola is $P\left(3t^2, 6t\right)$. and focus of parabola is $F(3,0)$ Let $Q(x,y)$ be the point that divides the line segment FP in the ratio $m:n$

$$\begin{aligned} \therefore \quad & x = \frac{m(3t^2)+n(3)}{m+n} = \frac{3mt^2+3n}{m+n} \\ \text{And} \quad & y = \frac{m(6t)+n(0)}{m+n} = \frac{6mt}{m+n} \\ \Rightarrow \quad & y(m+n) = 6mt \\ \Rightarrow \quad & t = \frac{y(m+n)}{6m} \\ \therefore \quad & x = \frac{3m\left(\frac{y(m+n)}{6m}\right)^2+3n}{m+n} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & x = \frac{\frac{y^2(m+n)^2}{12m}+3n}{m+n} \\ \Rightarrow \quad & x = \frac{36mn+y^2(m+n)^2}{12m(m+n)} \\ \Rightarrow \quad & 12m(m+n)x = 36mn + y^2(m+n)^2 \\ \Rightarrow \quad & y^2(m+n)^2 = 12m(m+n)x - 36mn \\ \Rightarrow \quad & y^2 = \frac{12m(m+n)}{(m+n)}x - \frac{36mn}{(m+n)^2} \\ \Rightarrow \quad & y^2 = \frac{12m}{m+n}x - \frac{36mn}{(m+n)^2} \end{aligned}$$

Which is in the form of $Y^2 = 4AX + B$

$\therefore$ Length of latusrectum $= \dfrac{12m}{m+n}$

Question55

The curve represented by $\frac{x^2}{12-\alpha} + \frac{y^2}{\alpha-10} = 1$ is

Options:

A.

a hyperbola for some values of α in $(10,12)$

B.

an ellipse for all values of α in $(10, 12)$

C.

a circle for some value of α in $(10, 12)$

D.

a hyperbola for all values of α in $(10, 12)$

Answer: C

Solution:

Given, equation of curve

$\frac{x^2}{12-\alpha} + \frac{y^2}{\alpha-10} = 1$

for circle, $12 - \alpha = \alpha - 10$

$\Rightarrow \quad 22 = 2\alpha \Rightarrow \alpha = 11$

$$\begin{aligned} &and\ 12 - \alpha > 0 \Rightarrow \alpha < 12 \\ &\quad \alpha - 10 > 0 \Rightarrow \alpha > 10 \end{aligned}$$

$\Rightarrow$ Curve represents a circle for some value of α in (10,12).

Option (c) is correct.

Similarly, for ellipse $12 - \alpha \neq \alpha - 10$

$\Rightarrow \quad 22 \neq 2\alpha \Rightarrow \alpha \neq 11$

Option (a) is wrong. for hyperbola one denominator must be positive and other negative

$\therefore \quad 12 - \alpha > 0$ and $\alpha - 10 < 0$

$\Rightarrow \quad \alpha < 12$ and $\alpha < 10$

$\Rightarrow \quad$ Means $\alpha < 10$

or $\quad 12 - \alpha < 0$ and $\alpha - 10 > 0$

$\Rightarrow \quad \alpha > 12$ and $\alpha > 10$

$\Rightarrow \quad$ Means $\alpha > 12$

Options (b) and (d) is wrong.

Question56

If any tangent drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ touches one of the circles $x^2 + y^2 = \alpha^2$, then the range of α is

Options:

A.

$9 \leq \alpha \leq 16$

B.

$16 \leq \alpha \leq 25$

C.

$3 \leq \alpha \leq 4$

D.

$4 \leq \alpha \leq 6$

Answer: C

Solution:

Given equation of ellipse is

$\frac{x^2}{16} + \frac{y^2}{9} = 1 \qquad \ldots(i)$

where, $a^2 = 16, b^2 = 9$

$\Rightarrow \quad a = 4, b = 3$

$\therefore$ Equation of tangent to the ellipse (i) is

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$
$$\Rightarrow \quad y = mx \pm \sqrt{16m^2 + 9}$$
$$\Rightarrow \quad y - mx \mp \sqrt{16m^2 + 9} = 0 \qquad \ldots(ii)$$

Given equation of circle $x^2 + y^2 = \alpha^2$

Centre $= (0, 0)$, radius $= \alpha$

Since, Eq. (ii) touches the given circle

$$\therefore \quad \alpha = \left| \frac{0 - 0 \mp \sqrt{16m^2+9}}{\sqrt{1+m^2}} \right|$$
$$\Rightarrow \quad \alpha = \left| \frac{\sqrt{16m^2+9}}{\sqrt{1+m^2}} \right|$$
$$\Rightarrow \quad \alpha^2 = \frac{16m^2+9}{1+m^2}$$
$$\text{Let} \quad f(m) = \frac{16m^2+9}{1+m^2}$$
$$\Rightarrow \quad f(m) = \frac{16m^2+16-16+9}{1+m^2}$$
$$\Rightarrow \quad f(m) = \frac{16(m^2+1)-7}{m^2+1}$$
$$= 16 - \frac{7}{m^2+1}$$

Here, $m^2 \geq 0$

$\Rightarrow \quad m^2 + 1 \geq 1$

$$\Rightarrow \quad 0 \leq \frac{1}{m^2+1} \leq 1$$
$$\Rightarrow \quad 0 \leq \frac{7}{m^2+1} \leq 7$$
$$\Rightarrow \quad 0 \geq -\frac{7}{m^2+1} \geq -7$$
$$\Rightarrow \quad 16 \geq 16 - \frac{7}{m^2+1} \geq 16 - 7$$
$$\Rightarrow \quad 16 \geq 16 - \frac{7}{m^2+1} \geq 9$$
$$\Rightarrow \quad 9 \leq f(m) \leq 16$$
$$\Rightarrow \quad 9 \leq \alpha^2 \leq 16$$
$$\Rightarrow \quad 3 \leq |\alpha| \leq 4$$

Question57

Let x be the eccentricity of a hyperbola whose transverse axis is twice its conjugate axis. Let y be the eccentricity of another hyperbola for which the distance between the focii is

3 times the distance between its directrices. Then $y^2 - x^2 =$

Options:

A.

$\frac{23}{16}$

B.

$\frac{7}{4}$

C.

$\frac{4}{7}$

D.

$\frac{16}{23}$

Answer: B

Solution:

Let equation of hyperbola

$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$

Given, $2a = 2(2b)$

$$\begin{aligned} \Rightarrow \quad & 2a = 4b \\ \Rightarrow \quad & a = 2b \end{aligned}$$

And $x^2 = 1 + \frac{b^2}{a^2}$

$$\begin{aligned} &= 1 + \frac{b^2}{4b^2} \\ &= 1 + \frac{1}{4} = \frac{5}{4} \end{aligned}$$

Also, $2ay = 3\left(\frac{2a}{y}\right)$

(for another hyperbola)

$$\begin{aligned} \Rightarrow \quad & y^2 = 3 \\ \therefore \quad & y^2 - x^2 = 3 - \frac{5}{4} = \frac{12 - 5}{4} = \frac{7}{4} \end{aligned}$$

Question58

$O(0,0,0), A(3,1,4), B(1,3,2)$ and $C(0,4,-2)$ are the vertices of a tetrahedron. If G is the centroid of the tetrahedron and G_1 is the centroid of its face ABC, then the point which divides GG_1 in the ratio $1:2$ is

Options:

A.

$\left(\frac{10}{3}, \frac{20}{3}, \frac{10}{3}\right)$

B.

$\left(\frac{20}{9}, \frac{10}{9}, \frac{10}{9}\right)$

C.

$\left(\frac{10}{9}, \frac{20}{9}, \frac{10}{9}\right)$

D.

$\left(\frac{20}{3}, \frac{10}{3}, \frac{10}{3}\right)$

Answer: C

Solution:

We have, vertices of a tetrahedron $O(0,0,0), A(3,1,4), B(1,3,2)$ and $C(0,4,-2)$

$\therefore$ Centroid of tetrahedron, G

$$= \left(\frac{0+3+1+1+0}{4}, \frac{0+1+3+4}{4}, \frac{0+4+2-2}{4}\right)$$
$$= \left(\frac{4}{4}, \frac{8}{4}, \frac{4}{4}\right) = (1,2,1)$$

and vertices of face ABC

$A(3,1,4), B(1,3,2)$ and $C(0,4,-2)$

$\therefore$ Centroid of face ABC,

$$G_1 = \left(\frac{3+1+0}{3}, \frac{1+3+4}{3}, \frac{4+2-2}{3}\right)$$
$$= \left(\frac{4}{3}, \frac{8}{3}, \frac{4}{3}\right)$$

Let $P(x, y)$ be the point which divides GG_1 in the ratio $1:2$ is

$\left(\frac{1\times\frac{4}{3}+2\times1}{1+2}, \frac{1\times\frac{8}{3}+2\times2}{1+2}, \frac{1\times\frac{4}{3}+2\times1}{1+2}\right)$

$$\therefore P(x, y, z) = \left(\frac{4+6}{3(3)}, \frac{8+12}{3(3)}, \frac{4+6}{3(3)}\right)$$
$$\Rightarrow P(x, y, z) = \left(\frac{10}{9}, \frac{20}{9}, \frac{10}{9}\right)$$

Question59

If L is a line common to the planes $3x+4y+7z=1$, $x-y+z=5$, then the direction ratios of the line L are

Options:

A.

$(16, 0, -1)$

B.

$(11, 4, -7)$

C.

$(2, 5, 1)$

D.

$(4, -7, 11)$

Answer: B

Solution:

Given, equation of planes

$3x + 4y + 7z = 1 \qquad \ldots (i)$

And $\quad x - y + z = 5 \qquad \ldots (ii)$

$\therefore$ Direction ratio of normal to the plane (i) is $3, 4, 7$.

and direction ratio of normal to the plane (ii) is $1, -1, 1$

$\therefore$ Direction ratio of L,

$$\begin{aligned}\mathbf{n} &= \mathbf{n}_1 \times \mathbf{n}_2 \\ &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 4 & 7 \\ 1 & -1 & 1 \end{vmatrix} \\ &= \hat{\mathbf{i}}(4+7) - \hat{\mathbf{j}}(3-7) + \hat{\mathbf{k}}(-3-4) \\ &= 11\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 7\hat{\mathbf{k}}\end{aligned}$$

$\therefore$ Direction ratio of L are $11, 4, -7$

Question60

If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane $3x + 4y - 12z + 13 = 0$, then the values of λ are

Options:

A.

$-1, \frac{7}{3}$

B.

$1, \frac{-7}{3}$

C.

$-1, \frac{-7}{3}$

D.

$1, \frac{7}{3}$

Answer: D

Solution:

Given, points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane

$3x+4y-12z+13=0$

$$\therefore \quad \left|\frac{3(1)+4(1)-12(\lambda)+13}{\sqrt{9+16+144}}\right|$$
$$=\left|\frac{3(-3)+4(0)-12(1)+13}{\sqrt{9+16+144}}\right|$$
$$\Rightarrow \quad \frac{|20-12\lambda|}{\sqrt{169}}=\frac{|-8|}{\sqrt{169}}$$
$$\Rightarrow \quad |20-12\lambda|=8$$

Taking positive sign, we get

$$20-12\lambda=8$$
$$\Rightarrow \quad -12\lambda=-12 \Rightarrow \lambda=1$$

And taking negative sign, we get

$$\Rightarrow \quad 20-12\lambda=-8 \Rightarrow -12\lambda=-28$$
$$\Rightarrow \quad 12\lambda=28 \Rightarrow \lambda=\frac{7}{3}$$
$$\therefore \quad \lambda=1, \frac{7}{3}$$

Question61

If $f(x)=\frac{x(a^x-1)}{1-\cos x}$ and $g(x)=\frac{x(1-a^x)}{a^x\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)}$, then $\lim_{x\to0}(f(x)-g(x))=$

Options:

A.

$3\log a$

B.

e^a

C.

$2\log a$

D.

$\log a$

Answer: D

Solution:

$$\text{Given, } f(x)=\frac{x\left(a^x-1\right)}{1-\cos x}$$
$$\text{And } g(x)=\frac{x\left(1-a^x\right)}{a^x\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)}$$
$$\therefore \lim_{x\to0} f(x)=\lim_{x\to0}\frac{x\left(a^x-1\right)}{1-\cos x}$$
$$=\lim_{x\to0}\frac{x\frac{(a^x-1)}{x}}{\frac{1-\cos x}{x}}$$
$$=\lim_{x\to0}\frac{a^x-1}{x}\times\frac{x^2}{1-\cos x}$$
$$=\lim_{x\to0}\frac{a^x-1}{x}\times\lim_{x\to0}\frac{x^2}{1+\cos x}$$

$= \log a \times 2$
$= 2 \log a$
Also, $\lim_{x \to 0} g(x)$
$$= \lim_{x \to 0} \frac{x(1-a^x)}{a^x\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)}$$
$$= \lim_{x \to 0} \frac{x(1-a^x)\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)}{a^x\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)}$$
$\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)$

$$= \lim_{x \to 0} \frac{x(1-a^x)\left(\sqrt{1-x^2}+\sqrt{1-x^2}\right)}{a^x(1-x^2-1+x^2)}$$
$$= \lim_{x \to 0} \frac{x(1-a^x)\left(\sqrt{1-x^2}+\sqrt{1-x^2}\right)}{a^x(-2x^2)}$$
$$= \lim_{x \to 0} \frac{(a^x-1)\left(\sqrt{1-x^2}+\sqrt{1-x^2}\right)}{a^x(2x)}$$
$$= \lim_{x \to 0} \frac{a^x-1}{x} \times \lim_{x \to 0} \frac{\sqrt{1-x^2}+\sqrt{1-x^2}}{2a^x}$$
$$= \log a \times \frac{1+1}{2(1)}$$
$= \log_a a(1) = \log a$
$\therefore \lim_{x \to 0}(f(x)-g(x))$
$= \lim_{x \to 0} f(x) - \lim_{x \to 0} g(x)$
$= 2\log a - \log a = \log a$

Question62

If $f(x) = \begin{cases} \frac{a \sin x - bx + cx^2 + x^3}{2\log(1+x) - 2x^3 + x^4} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

is continuous at $x = 0$, then

Options:

A.

$a = 2b$

B.

$a = b$

C.

$a = b = c$

D.

$b = c$

Answer: B

Solution:

Given,

$$f(x)=\begin{cases}\frac{a\sin x-bx+cx^2+x^3}{2\log(1+x)-2x^3+x^4}, x\neq 0\\ 0, \quad x=0\end{cases}$$

is continuous at $x=0$

$$\therefore \lim_{x\to 0} f(x)=f(0).$$
$$\Rightarrow \lim_{x\to 0}\frac{a\sin x-bx+cx^2+x^3}{2\log(1+x)-2x^3+x^4}\left(\frac{0}{0}\right)=f(0)$$
$$\Rightarrow \lim_{x\to 0}\frac{a\cos x-b+2cx+3x^2}{\frac{2}{(1+x)}-6x^2+4x^3}=0$$
$$\Rightarrow \quad \frac{a-b}{2}=0\Rightarrow a-b=0$$
$$\Rightarrow \quad a=b$$

Question63

If the function $g(x)=\begin{cases}K\sqrt{x+1} & ,0\leq x\leq 3\\ mx+2 & ,3<x\leq 5\end{cases}$ is differentiable, then $K+m=$

Options:

A.

4

B.

2

C.

6

D.

0

Answer: B

Solution:

Given, $g(x)=\begin{cases}K\sqrt{x+1}, & 0\leq x\leq 3\\ mx+2, & 3<x\leq 5\end{cases}$ is differentiable

Since, $g(x)$ is differentiable therefore it is continuous.

at $x=3$

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$$
$$\Rightarrow \lim_{h\to 0} f(3-h)=\lim_{h\to 0} f(3+h)$$
$$\Rightarrow \lim_{h\to 0}(K\sqrt{(3-h+1)})=\lim_{h\to 0}(m(3+h)+2)$$
$$\Rightarrow K\sqrt{4}=m(3)+2$$

$$\Rightarrow \quad 2K=3m+2 \qquad \ldots(i)$$

for $0\leq x\leq 3$

$$g'(x)=K\frac{1}{2\sqrt{x+1}}$$

And for $3

$g'(x) = m$

for differentiability at $x = 3$

$$\begin{aligned} &Lg'(3) = Rg'(3) \\ &\Rightarrow \frac{K}{2\sqrt{4}} = m \end{aligned}$$

$$\Rightarrow K = 4m \qquad \ldots(ii)$$

On Solving Eqs. (i) and (ii), we get

$$\begin{aligned} & 2(4m) = 3m + 2 \\ \Rightarrow \quad & 8m = 3m + 2 \\ \Rightarrow \quad & 5m = 2 \Rightarrow m = \frac{2}{5} \end{aligned}$$

And $\quad K = \frac{8}{5}$

$\therefore \quad K + m = \frac{8}{5} + \frac{2}{5} = \frac{10}{5} = 2$

Question64

Consider the following statements

Assertion (A) For $x \in R - \{1\}$;

$\frac{d}{dx}\left(\tan^{-1}\left(\frac{1+x}{1-x}\right)\right) = \frac{d}{dx}\left(\tan^{-1} x\right)$

Reason (R) For $x < 1, \tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x$, for

$x > 1, \tan^{-1}\left(\frac{1+x}{1-x}\right) = -\frac{3\pi}{4} + \tan^{-1} x$

The correct answer is

Options:

A.

Both (A) and (R) are true, (R) is the correct explanation of (A).

B.

Both (A) and (R) are true, (R) is not the correct explanation of (A).

C.

(A) is true, but (R) is false.

D.

(A) is false, but (R) is true.

Answer: A

Solution:

Given,

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{1+x}{1-x}\right)\right)=\frac{d}{dx}\left(\tan^{-1}x\right)$$

By LHS, $\frac{d}{dx}\left(\tan^{-1}\left(\frac{1+x}{1-x}\right)\right)$

$$=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$=\frac{(1-x)^2}{(1-x)^2+(1+x)^2}\left[\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right]$$

$$=\frac{(1-x)^2}{2\left(1+x^2\right)}\left[\frac{1-x+1+x}{(1-x)^2}\right]=\frac{1}{(1+x)^2}$$

And by RHS, $\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$

$\therefore$ Assertion is true.

For reason (R)

when $x<1$, let $x=\tan\theta$

$\Rightarrow\quad \theta=\tan^{-1}x$

$\therefore\quad \tan^{-1}\left(\frac{1+x}{1-x}\right)=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)=\frac{\pi}{4}+\theta$

$=\frac{\pi}{4}+\tan^{-1}x$

When $x>1$, then $\tan^{-1}x>1$

$\therefore\quad \frac{1+x}{1-x}<0$

so, $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ lies in $\left(-\frac{\pi}{2},0\right)$

$\therefore \tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{-3\pi}{4}+\tan^{-1}x$

$\therefore$ Reason is also true.

Question65

If $\frac{d}{dx}\left\{\left(\frac{x-1}{x-\sqrt{x}}\right)e^{2x+1}\right\}=\frac{x-1}{x-\sqrt{x}}e^{2x+1}f(x)$, then $f(4)=$

Options:

A.

0

B.

1

C.

$\frac{35}{24}$

D.

$\frac{47}{24}$

Answer: D

Solution:

Given, $\frac{d}{dx}\left\{\left(\frac{x-1}{x-\sqrt{x}}\right)e^{2x+1}\right\}$

$= \frac{x-1}{x-\sqrt{x}}e^{2x+1}f(x)$

Here, $\frac{x-1}{x-\sqrt{x}} = \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}$

$$= \frac{\sqrt{x}+1}{\sqrt{x}} = 1 + \frac{1}{\sqrt{x}}$$

$$\therefore \frac{d}{dx}\left(1+\frac{1}{\sqrt{x}}\right)e^{2x+1} = \left(1+\frac{1}{\sqrt{x}}\right)e^{2x+1}f(x)$$

Now, $\frac{d}{dx}\left(1+\frac{1}{\sqrt{x}}\right)e^{2x+1}$

$$= \frac{-1}{2x\sqrt{x}}e^{2x+1} + \left(1+\frac{1}{\sqrt{x}}\right)e^{2x+1}(2)$$

$$= e^{2x+1}\left[\frac{-1}{2x\sqrt{x}} + 2\left(1+\frac{1}{\sqrt{x}}\right)\right]$$

$$\therefore f(x) = \frac{\frac{-1}{2x\sqrt{x}} + 2\left(1+\frac{1}{\sqrt{x}}\right)}{1+\frac{1}{\sqrt{x}}}$$

$$= \frac{-1+4x\sqrt{x}+4x}{2x\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}+1}$$

$$\Rightarrow f(x) = \frac{-1+4x\sqrt{x}+4x}{2x(\sqrt{x}+1)}$$

$$\therefore f(4) = \frac{-1+4(4)(2+4(4)}{2(4)(2+1)}$$

$$= \frac{-1+32+16}{24} = \frac{47}{24}$$

Question66

If $y = \left(\sin^{-1} x\right)^2$, then $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

Options:

A.

$\frac{1}{2}$

B.

2

C.

$-\frac{1}{2}$

D.

4

Answer: B

Solution:

We have, $y = \left(\sin^{-1} x\right)^2$

$$\Rightarrow \quad \frac{dy}{dx} = 2\left(\sin^{-1} x\right) \frac{1}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad \sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1} x$$
$$\Rightarrow \quad \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = 4\left(\sin^{-1} x\right)^2$$
$$\Rightarrow \quad \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = 4y \text{ (Using Eq.}$$

Again, differentiating w.r.t x, we get

$$(-2x)\left(\frac{dy}{dx}\right)^2 + \left(1-x^2\right)\left(2\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 4\frac{dy}{dx}$$
$$\Rightarrow (-2x)\frac{dy}{dx} + 2\left(1-x^2\right)\frac{d^2y}{dx^2} = 4$$
$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

Question67

The radius of a cone of height 9 units is changed from 2 units to 2.12 units. The exact change and approximate change in the volume of the cone are respectively

Options:

A.

$(1.4437)\pi, (1.44)\pi$

B.

$(1.4832)\pi, (1.479)\pi$

C.

$(1.4842)\pi, (1.48)\pi$

D.

$(1.4832)\pi, (1.44)\pi$

Answer: D

Solution:

Volume of cone $(V) = \frac{1}{3}\pi r^2 h$

$h = 9$ units
$r_1 = 2$
$r_2 = 2.12$

$$\therefore \quad V_1 = \frac{1}{3}\pi\left(2^2 \times 9 = 12\pi \text{ cubic units}\right.$$
$$V_2 = \frac{1}{3}\pi(2.12)^2 \times 9$$

$= 3\pi(4.4944)$
$= 13.4832\pi$ cubic unit

$\therefore$ Exact change, $\Delta V = V_2 - V_1$

$= 13.4832 - 12\pi$
$= 1.4832\pi$ cubic units

Now, $\frac{dV}{dr}=\frac{1}{3}\pi(2r)(h)=\frac{2}{3}\pi rh$

$$\begin{aligned}\therefore\quad dV &= \frac{dV}{dr}\Delta r=\left(\frac{2}{3}\pi rh\right)(0.12)\\ &= \frac{2}{3}\pi(2)(9)(0.12)\\ &= 1.44\pi \text{ cubic units.}\end{aligned}$$

Question68

The local maximum value l and local minimum value m of $f(x)=\frac{x^2+2x+2}{x+1}$ in $R-\{-1\}$ exist at α,β respectively, then $\frac{l+m}{\alpha+\beta}=$

Options:

A.

0

B.

-4

C.

-2

D.

2

Answer: A

Solution:

Given, $f(x)=\frac{x^2+2x+2}{x+1}$

$$\Rightarrow f'(x)=\frac{(x+1)(2x+2)-\left(x^2+2x+2\right)(1)}{(x+1)^2}$$
$$\begin{aligned}\Rightarrow f'(x) &= \frac{\left(2x^2+2x+2x+2\right)-\left(x^2+2x+2\right)}{(x+1)^2}\\ &= \frac{\left(x^2+2x\right)}{(x+1)^2}\end{aligned}$$

For critical points, put $f'(x)=0$

$$\begin{aligned}&\Rightarrow\quad x^2+2x=0\Rightarrow x(x+2)=0\\ &\Rightarrow\quad x=0,-2\\ &\therefore\quad \alpha=-2,\beta=0\end{aligned}$$

Now,

$$\begin{aligned}f''(x) &= \frac{(x+1)^2\left(2x+2-2(x+1)\left(x^2+2x\right)\right.}{(x+1)^4}\\ &= \frac{2}{(x+1)^3}\end{aligned}$$

At $x=0, f''(x)>0$

$\Rightarrow\beta=0$ is the point of minima at $x=-2$

$f''(x) < 0$

$\Rightarrow \alpha = -2$ is the point of maxima

$\therefore \quad l = \frac{(-2)^2+2(-2)+2}{(-2)+1} = \frac{4-4+2}{-1} = -2$

And $m = \frac{0+0+2}{0+1} = 2$

$\therefore \frac{l+m}{\alpha+\beta} = \frac{-2+2}{-2+0} = 0$

Question69

$P(5,2)$ is a point on the curve $y = f(x)$ and $\frac{7}{2}$ is the slope of the tangent to the curve at P. The area of the triangle (in sq. units) formed by the tangent and the normal to the curve at P with X-axis is

Options:

A.

35

B.

$\frac{35}{2}$

C.

$\frac{53}{7}$

D.

$\frac{53}{14}$

Answer: C

Solution:

Given, the slope of the tangent at $P(5,2)$ is $\frac{7}{2}$

$\therefore$ Equation of tangent at P is

$$\begin{aligned} & y-2 = \frac{7}{2}(x-5) \\ \Rightarrow \quad & 2y-4 = 7x-35 \\ \Rightarrow \quad & 7x-2y = 31 \end{aligned}$$

$\therefore$ The tangent intersect the X-axis,

$$\begin{aligned} & y = 0 \\ \therefore \quad & 7x = 31 \Rightarrow x = \frac{31}{7} \end{aligned}$$

$\therefore$ Tangent intersect the X-axis at $A\left(\frac{31}{7}, 0\right)$

Now, slope of normal at P is $-\frac{2}{7}$

Equation of normal at P is

$$\begin{aligned} & y-2 = -\frac{2}{7}(x-5) \\ \Rightarrow \quad & 7y-14 = -2x+10 \Rightarrow 2x+7y = 24 \end{aligned}$$

$\therefore$ x-intercept of normal, $y = 0$

$\Rightarrow \quad x = 12$

$\Rightarrow$ The normal intersects the X-axis at $B(15, 0)$

Base of the triangle $= \left|\left(12 - \frac{31}{7}\right)\right| = \frac{53}{7}$

$$\begin{aligned} \therefore \text{ Area of triangle } & = \frac{1}{2} \times \frac{53}{7} \times 2 \\ & = \frac{53}{7} \text{ sq. units} \end{aligned}$$

Question70

If a particle is moving in a straight line so that after t seconds its distance S (in cms) from a fixed point on the line is given by $S = f(t) = t^3 - 5t^2 + 8t$, then the acceleration of the particle at $t = 5\text{sec}$ is (in cm/sec^2)

Options:

A.

10

B.

30

C.

20

D.

40

Answer: C

Solution:

Given, $S = f(t) = t^3 - 5t^2 + 8t$

$$\begin{aligned} \Rightarrow \quad \text{Velocity } & = \frac{dS}{dt} = t^2 - 10t + 8 \\ \therefore \text{ Acceleration } & = \frac{d^2S}{dt^2} = 6t - 10 \\ \text{at} \quad t & = 5 \end{aligned}$$

Acceleration of the particle

$$\begin{aligned} & = 6 \times 5 - 10 = 30 - 10 \\ & = 20 \text{ cm/sec}^2 \end{aligned}$$

Question71

If $f : [a, b] \to [c, d]$ is a continuous and strictly increasing function, then $\frac{d-c}{b-a}$ is

Options:

A.

value of the function at a point $t \in (a, b)$

B.

value of the function at $t \in (a, b)$ such that $f'(t) = 0$

C.

Slope of the tangent drawn to the curve $y = f(t)$ at a point $t \in (c, d)$

D.

Slope of the tangent drawn to the curve $y = f(t)$ at a point $t \in (a, b)$

Answer: D

Solution:

Given, $f : [a, b] \to [c, d]$ is a continuous and strictly increasing function.

$\therefore \quad f(a) = c, f(b) = d$

Now, $\frac{d-c}{b-a} = \frac{f(b)-f(a)}{b-a}$

$\Rightarrow$ Slope of tangent at a point $t \in (a, b)$

Question72

$$\int \left(\frac{1}{x^2} + \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \right) dx =$$

Options:

A.

$\frac{(\sin x - \cos x)x - \sin x \cos x}{x \sin x \cos x} + C$

B.

$-\frac{1}{x} + \frac{\sin x + \cos x}{\cos x - \sin x} + c$

C.

$-\frac{1}{x} + \frac{\sin x - \cos x}{\sin^2 x \cos^2 x} + C$

D.

$\frac{(\sin x - \cos x)x - \sin x - \cos x}{x(\sin x + \cos x)} + C$

Answer: A

Solution:

Let $I=\int\left(\frac{1}{x^2}+\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}\right)dx$

again let $I_1=\int\frac{1}{x^2}dx$

And $I_2=\int\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx$

$\therefore I_1=\int x^{-2}dx=\frac{x^{-2+11}}{-2+1}+c_1=-\frac{1}{x}+c_1$

And $I_2=\int\left(\frac{\sin^3 x}{\sin^2 x\cos^2 x}+\frac{\cos^3 x}{\sin^2 x\cos^2 x}\right)dx$

$=\int(\tan x\sec x+\cot x\operatorname{cosec} x)dx$

$=\int\tan x\sec x dx+\int\cot x\operatorname{cosec} x dx$

$=\sec x+(-\operatorname{cosec} x)+c_2$

$\therefore I=I_1+I_2=-\frac{1}{x}+\sec x-\operatorname{cosec} x+C$

$\Rightarrow I=-\frac{1}{x}+\frac{1}{\cos x}-\frac{1}{\sin x}+C$

$=\frac{-(\cos x\sin x)+x\sin x-x\cos x}{x\cos x\sin x}+C$

$=\frac{x(\sin x-\cos x)-\cos x\sin x}{x\cos x\sin x}+C$

Question73

If $I_n=\int\frac{1}{(x^2+1)^n}dx$, then $2nI_{n+1}-(2n-1)I_n=$

Options:

A.

$\frac{(x^2+1)^n}{x}+C$

B.

$\frac{x}{(x^2+1)^n}+C$

C.

$x(x^2+1)^{n-1}+C$

D.

$\frac{x}{(x^2+1)^{n-1}}+C$

Answer: B

Solution:

Given, $I_n = \int \frac{1}{(x^2+1)^n} dx$

$= \frac{x}{(x^2+1)^n} - \int \left(\frac{d}{dx} \left(\frac{1}{(x^2+1)^n} \right) \int dx \right) dx$

$= \frac{x}{(x^2+1)^n} + 2n \int \frac{x^2}{(x^2+1)^{n+1}} dx$

$\Rightarrow I_n = \frac{x}{(x^2+1)^n} + 2n \int \left(\frac{1}{(x^2+1)^n} - \frac{1}{(x^2+1)^{n+1}} \right) dx$

$\Rightarrow I_n = \frac{x}{(x^2+1)^n} + 2n \left(I_n - I_{n+1} \right) + C$

$\Rightarrow 2n I_{n+1} = \frac{x}{(x^2+1)^n} + (2n-1) I_n + C$

$\Rightarrow 2n I_{n+1} - (2n-1) I_n = \frac{x}{(x^2+1)^n} + C$

Question74

$\int \frac{x^3}{x^4+3x^2+2} dx =$

Options:

A.

$\log \left(\frac{x^2+2}{\sqrt{x^2+1}} \right) + C$

B.

$\log \left(x^2+2 \right) - 2 \log \left(x^2+1 \right) + C$

C.

$\log \left(\frac{(x^2+2)x}{\sqrt{x^2+1}} \right) + C$

D.

$\log \left(\frac{x^2+1}{\sqrt{x^2+2}} \right) + C$

Answer: A

Solution:

Let $I = \int \frac{x^3 dx}{x^4+3x^2+2}$

put $x^2 = t \Rightarrow 2x dx = dt \Rightarrow x dx = \frac{dt}{2}$

$\therefore I = \int \frac{t \cdot \frac{dt}{2}}{t^2+3t+2}$

$I = \frac{1}{2} \int \frac{t dt}{t^2+3t+2} = \frac{1}{2} \int \frac{t dt}{(t+1)(t+2)}$

Now, $\frac{t}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2}$

$\Rightarrow \quad t = A(t+2) + B(t+1)$

When, $t+2=0 \Rightarrow t=-2$

$\therefore \quad -2=B(-1) \Rightarrow B=2$

when, $t+1=0 \Rightarrow t=-1$

$\therefore \quad -1=A(1)+B(0) \Rightarrow A=-1$

$\therefore \frac{t}{(t+1)t+2)}=\frac{-1}{t+1}+\frac{2}{t+2}$

$\therefore I=\frac{1}{2} \int\left(\frac{-1}{t+1}+\frac{2}{t+2}\right) d t$

$I=\frac{1}{2}[-\log (t+1)+2 \log (t+2)]+C$

$=-\frac{1}{2} \log \left(x^2+1\right)+\log \left(x^2+2\right)]+C$

$=\log \left(x^2+2\right)+\log \left(x^2+1\right)^{-1 / 2}+C$

$=\log \left(\frac{x^2+2}{\sqrt{x^2+1}}\right)+C$

Question75

If $\int \frac{dx}{(x^2+9)\sqrt{x^2+16}}=\frac{1}{3\sqrt{7}}\tan^{-1}\left(K\frac{x}{\sqrt{16+x^2}}\right)+c$, then $K=$

Options:

A.

$\frac{\sqrt{7}}{3}$

B.

$3\sqrt{7}$

C.

$\frac{3}{\sqrt{7}}$

D.

$\frac{3}{7}$

Answer: A

Solution:

Let $I=\int \frac{d x}{\left(x^2+9\right) \sqrt{x^2+16}}$

put $x=4 \tan \theta \Rightarrow d x=4 \sec ^2 \theta d \theta$

$\therefore I=\int \frac{4 \sec ^2 \theta d \theta}{\left(16 \tan ^2 \theta+9\right) \sqrt{16 \tan ^2 \theta+16}}$

$=\int \frac{4 \sec ^2 \theta d \theta}{\left(16 \tan ^2 \theta+9\right) 4 \sec \theta}$

$=\int \frac{\sec \theta d \theta}{16 \tan ^2 \theta+9}$

$$= \int \frac{\cos\theta d\theta}{16\sin^2\theta + 9\cos^2\theta} = \int \frac{\cos\theta d\theta}{7\sin^2\theta + 9}$$

Again put $\sin\theta = t$

$\Rightarrow \cos\theta d\theta = dt$

$$\therefore I = \int \frac{dt}{7t^2 + 9}$$

$$I = \int \frac{dt}{7\left(t^2 + \frac{9}{7}\right)} = \frac{1}{7}\int \frac{dt}{t^2 + \left(\frac{3}{\sqrt{7}}\right)^2}$$

$$= \frac{1}{7} \cdot \frac{1}{\frac{3}{\sqrt{7}}} \tan^{-1}\left(\frac{t}{\frac{3}{\sqrt{7}}}\right) + C$$

$$= \frac{1}{3\sqrt{7}} \tan^{-1}\left(\frac{\sqrt{7}t}{3}\right) + C$$

$$= \frac{1}{3\sqrt{7}} \tan^{-1}\left(\frac{\sqrt{7}\sin\theta}{3}\right) + C$$

$$= \frac{1}{3\sqrt{7}} \tan^{-1}\left(\frac{\sqrt{7}x}{3\sqrt{x^2+16}}\right) + C$$

$$\left(\because x = 4\tan\theta \Rightarrow \sin\theta = \frac{x}{\sqrt{x^2+16}}\right)$$

$$\therefore \quad K = \frac{\sqrt{7}}{3}$$

Question76

$$\lim_{n\to\infty} \frac{1}{n^2}\left[e^{1/n} + 2e^{2/n} + 3e^{3/n} + \ldots + 2ne^2\right] =$$

Options:

A.

$e^2 - 1$

B.

$e^2 + 1$

C.

$2e^2 - 2$

D.

$2e^2 + 1$

Answer: B

Solution:

We have,

$$\lim_{n\to\infty} \frac{1}{n^2}\left[e^{1/n} + 2e^{2/n} + 3e^{3/n} + \ldots + 2ne^2\right]$$

$$= \lim_{n \to \infty} \frac{1}{n^2} \left[\sum_{k=1}^{2n} k e^{k/n} \right]$$
$$= \lim_{n \to \infty} \frac{1}{n} \left[\sum_{k=1}^{2n} \left(\frac{k}{n} \right) e^{k/n} \right]$$
$$= \int_0^2 x e^x dx = (x e^x - e^x)_0^2$$
$$= (2e^2 - e^2) - (0 - 1) = e^2 + 1$$

Question77

Let m, n, p, q be four positive integers. If

$$\int_0^{2\pi} \sin^m x \cos^n x dx = 4 \int_0^{\pi/2} \sin^m x \cos^n x dx \int_0^{2\pi} \sin^p x \cos^n x dx = 0$$
$$\int_0^{\pi} \sin^p x \cos^q x dx = 0, a = m + n + p \text{ and } b = m + n + q, \text{ then}$$

Options:

A.

a is even number and b is odd number

B.

a is odd number and b is even number

C.

Both a and b are even numbers

D.

Both a and b are odd numbers

Answer: D

Solution:

Given, m, n, p, q be four positive inegers

$$\int_0^{2\pi} \sin^m x \cos^n x dx = 4 \int_0^{\pi/2} \sin^m x \cos^n x dx \quad \ldots (i)$$
$$\int_0^{2\pi} \sin^p x \cos^n x dx = 0 \quad \ldots (ii)$$
$$\text{And } \int_0^{2\pi} \sin^p x \cos^q x dx = 0 \quad \ldots (iii)$$
$$a = m + n + p, b = m + n + q$$

From Eq. (i), we know that

$\int_0^{2\pi} f(x) dx = 4 \int_0^{\pi/2} f(x)$

If f is periodic with π and symmetric in all four quadrants

$\Rightarrow \quad m$ and n are even

$\int_0^{2\pi} \sin^p x \cos^n x dx = 0$

$\Rightarrow$ Integrand is odd over symmetric interval $[0, 2\pi]$

Since, n is even so, p is odd also,

$$\int_0^{\pi} \sin^p x \cos^q x dx = 0$$

$\Rightarrow \quad \sin^p x \cos^q x$ is odd about $x = \dfrac{\pi}{2}$

$\Rightarrow \quad q$ is odd

$\therefore a = m + n + p$

$= \text{ even } + \text{ even } + \text{ odd } = \text{ odd}$

And $b = m + n + p$

$= \text{ even } + \text{ even } + \text{ odd } = \text{ odd}$

Question78

The area of the region bounded by the curves $y = x^3, y = x^2$ and the lines $x = 0$ and $x = 2$ is

Options:

A.

$\frac{4}{3}$

B.

$\frac{3}{2}$

C.

$\frac{2}{3}$

D.

$\frac{5}{3}$

Answer: B

Solution:

Given, curves, $y = x^3, y = x^2$

for point of intersection

$x^3 = x^2$

$\Rightarrow \quad x^2(x-1) = 0 \Rightarrow x = 0, 1$

$\therefore \quad y = 0, 1$

point of intersection are $(0, 0)$ and $(1, 1)$

$\therefore$ Required area

$$= \int_0^1 (x^2 - x^3) dx + \int_1^2 (x^3 - x^2) dx$$
$$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)_0^1 + \left(\frac{x^4}{4} - \frac{x^3}{3}\right)_1^2$$
$$= \left(\frac{1}{3} - \frac{1}{4}\right) + \left[\left(\frac{16}{4} - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right]$$
$$= \frac{1}{12} + \frac{15}{4} - \frac{7}{3}$$
$$= \frac{1 + 45 - 28}{12} = \frac{18}{12} = \frac{3}{2} \text{ sq. units}$$

Question79

The substitution required to reduce the differential equation $t^2 dx + (x^2 - tx + t^2) dt = 0$ to a differential equation which can be solved by variables separable method is

Options:

A.

$t = V_x$

B.

$ax + bt = Z$

C.

$V = tx^2$

D.

$x = tV^2$

Answer: A

Solution:

Given, $t^2 dx + (x^2 - tx + t^2) dt = 0$

$\Rightarrow \quad \frac{dt}{dx} = -\frac{t^2}{x^2 - tx + t^2}$

On putting $t = Vx$

$$\Rightarrow \quad \frac{dt}{dx} = V + x\frac{dV}{dx}$$
$$\therefore \quad \left(V + x\frac{dV}{dx}\right) = -\frac{V^2 x^2}{x^2 - Vx^2 + V^2 x^2}$$
$$\Rightarrow \quad V + x\frac{dV}{dx} = -\frac{V^2}{1 - V + V^2}$$
$$\Rightarrow \quad x\frac{dV}{dx} = -\frac{V^2}{1 - V + V^2} - V$$
$$\Rightarrow \quad x\frac{dV}{dx} = \frac{-V^2 - V + V^2 - V^3}{1 - V + V^2}$$
$$\Rightarrow \quad x\frac{dV}{dx} = -\frac{V + V^3}{1 - V + V^2}$$
$$\Rightarrow \quad \frac{1 - V + V^2}{V + V^3} dV = -\frac{dx}{x}$$

Which can be solve by variables separable method.

Question80

The equation which represents the system of parabolas whose axis is parallel to Y-axis satisfies the differential equation.

Options:

A.

$\frac{d^3y}{dx^3} = 0$

B.

$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = x + y$

C.

$\frac{d^2y}{dx^2} + xy = 4ax$

D.

$\frac{dy}{dx} + xy = x^2$

Answer: A

Solution:

Equation of parabola with its axis parallel to the Y-axis.

$$
\begin{aligned}
& y = ax^2 + bx + c, a \neq 0 \\
\Rightarrow & \frac{dy}{dx} = 2ax + b \Rightarrow \frac{d^2y}{dx^2} = 2a \\
\Rightarrow & \frac{d^3y}{dx^3} = 0
\end{aligned}
$$

Chemistry

Question1

The electron in hydrogen atom undergoes transition from higher orbits to an orbit of radius 476.1 pm . This transition corresponds to which of the following series?

Options:

A.

Lyman

B.

Paschen

C.

Balmer

D.

Pfund

Answer: B

Solution:

For hydrogen atom, radius is given by

$r_n = n^2 \cdot a_0 \qquad \ldots (i)$
$a_0 = 52.9\text{pm}$ (Bohr's radius)
$r_n = 476.1\text{pm}$

Substituting the given values in Eq. (i),

$n^2 \approx 9, n = 3$

Transition is to $n = 3$, so it is a Paschen series.

Question2

Identify the incorrect statement from the following?

Options:

A.

m, designates the orientation of the orbital.

B.

The probability density of electron is expressed by $|\psi|^3$.

C.

The total information about electron in atom is stored in its ψ.

D.

Total number of orbitals in a sub level is equal to $(2l + 1)$.

Answer: B

Solution:

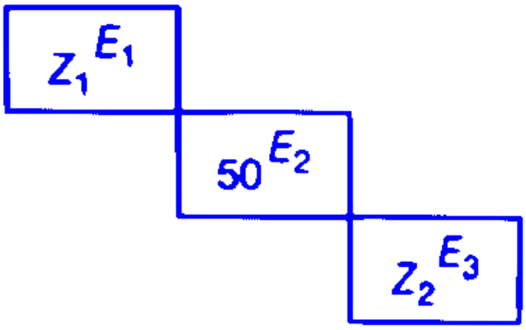
Among the given statements, statement given in option

(b) is incorrect. It's correct form is,

The probability density of electron is expressed by $|\psi|^2$.

Question3

Atomic numbers of three elements E_1, E_2 and E_3 of periodic table are $Z_1, 50$ and Z_2 respectively. From the position of the elements shown in figure, the values of $(Z_2 - Z_1)$ is



Options:

A.

52

B.

46

C.

64

D.

34

Answer: A

Solution:

From periodic table,

E_1 will be Gallium with $Z_1 = 31$

E_2 will be Bismuth with $Z_2 = 83$

$Z_2 - Z_1 = 83 - 31 = 52$

Question4

Electron gain enthalpy values $(\Delta_{cg} H)$ (in $\mathrm{kJmol^{-1}}$) of elements X, Y and Z are -349, -200 and -295 respectively. X, Y and Z are respectively

Options:

A.

Cl, I, S

B.

Cl, S, I

C.

S, Se, Te

D.

$\mathrm{Na}, \mathrm{K}, \mathrm{Rb}$

Answer: B

Solution:

Element $X=\mathrm{Cl}, Y=\mathrm{S}, Z=\mathrm{I}$

More the value of electron gain enthalpy, greater is the tendency to gain electron.

Question5

Observe the following list of molecules. Number of polar and non-polar molecules are respectively

$\mathrm{NH}_3, \mathrm{BF}_3, \mathrm{NF}_3, \mathrm{H}_2 \mathrm{~S}_2, \mathrm{CO}_2, \mathrm{CH}_4, \mathrm{CHCl}_3, \mathrm{H}_2 \mathrm{O}$

Options:

A.

4,4

B.

3,5

C.

5,3

D.

2,6

Answer: C

Solution:

Among the given molecules, total five molecules are polar in nature

$\mathrm{NH}_3, \mathrm{NF}_3, \mathrm{H}_2 \mathrm{~S}, \mathrm{CHCl}_3, \mathrm{H}_2 \mathrm{O}$

Total three, are non-polar molecules

$\mathrm{BF}_3, \mathrm{CO}_2, \mathrm{CH}_4$

Question6

The molecule ' X ' has see-saw shape with central atom in sp^3d hybridisation. What is ' X '?

Options:

A.

ClF_3

B.

XeF_4

C.

SF_4

D.

BrF_5

Answer: C

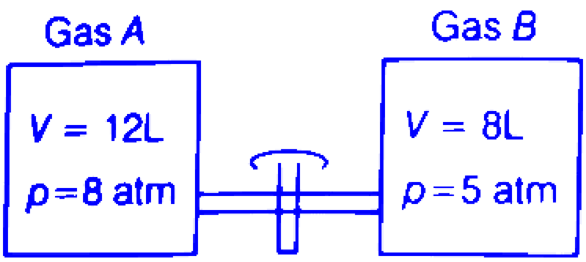
Solution:

$$\begin{aligned} ClF_3 &= T\text{-shape} \\ &= sp^3d \text{ hybridisation} \\ XeF_4 &= \text{ square planar } = sp^3d^2 \text{ hybridisation} \\ SF_4 &= see\text{-saw } = sp^3d \text{ hybridisation} \\ BrF_5 &= \text{ square pyramidal } = sp^3d^2 \end{aligned}$$

Question7

Two vessels are filled with ideal gases A and B and are connected through a pipe of zero volume as shown in figure. The stop cock is opened and the gases are allowed to mix homogeneously and the temperature is

kept constant. The partial pressures of A and B respectively (in atm) are



Options:

A.

8.0, 5

B.

9.6, 4

C.

6.4, 4

D.

4.8, 2

Answer: D

Solution:

Using formula for partial pressure

$$p_i = \frac{n_i}{n_{\text{total}}} \times p_{\text{total}}$$

$$p_{\text{total}} = \frac{n_{\text{total}} \cdot R \cdot T}{V} = \frac{136}{20} = 6.8 \text{ atm}$$

$$p_A = \frac{96}{136} \times 6.8 = 4.8 \text{ atm}$$

$$p_B = \frac{40}{136} \times 6.8 = 2 \text{ atm}$$

Question8

If the number of moles of Fe^{2+} ions oxidised by one mole of acidified MnO_4^- is x, the number of moles of Fe^{2+} ions oxidised by one mole of acidified $\mathrm{Cr}_2\mathrm{O}_7^{2-}$ is

Options:

A.

$\frac{5x}{8}$

B.

$\frac{6x}{5}$

C.

$\frac{8x}{5}$

D.

$\frac{5x}{6}$

Answer: B

Solution:

The reactions that happen are:

$$\mathrm{MnO}_4^- + 5e^- \longrightarrow \mathrm{Mn}^{2+} \quad (1 \text{ mole of } \mathrm{MnO}_4^- \text{ takes 5 electrons})$$
$$\mathrm{Cr}_2\mathrm{O}_7^{2-} + 14\mathrm{H}^+ + 6e^- \longrightarrow 2\mathrm{Cr}^{3+} + 7\mathrm{H}_2\mathrm{O} \quad (1 \text{ mole of } \mathrm{Cr}_2\mathrm{O}_7^{2-} \text{ takes 6 electrons})$$
$$\mathrm{Fe}^{2+} \longrightarrow \mathrm{Fe}^{3+} \quad (\text{Each } \mathrm{Fe}^{2+} \text{ loses 1 electron.})$$

Step 1: How many Fe^{2+} ions does 1 mole of MnO_4^- oxidize?

1 mole of MnO_4^- can take in 5 electrons. Each Fe^{2+} gives up 1 electron to get oxidized. So, 1 mole of MnO_4^- can oxidize 5 moles of Fe^{2+}. This number is called x.

Step 2: How many Fe^{2+} ions does 1 mole of $Cr_2O_7^{2-}$ oxidize?

1 mole of $Cr_2O_7^{2-}$ can take in 6 electrons. That means it can oxidize 6 moles of Fe^{2+}.

Step 3: What is the answer in terms of x?

Since $x = 5$, and $Cr_2O_7^{2-}$ can oxidize 6 moles, the answer is $\frac{6x}{5}$ moles of Fe^{2+} ions.

Question9

One mole of an ideal gas at 300 K and 20 atm expands to 2 atm under isothermal and reversible conditions. The work done by the gas is $-x\ \mathrm{kJ\ mol^{-1}}$. The value of x is $\left(R = 8.3\ \mathrm{J\ K^{-1}\ mol^{-1}}\right)$

Options:

A.

5.73

B.

7.37

C.

3.75

D.

4.57

Answer: A

Solution:

Work done in isothermal reversible expansion,

$W = -nRT \ln\left(\frac{p_2}{p_1}\right)$

$W = -(1)(8.3)(300)\ln\left(\frac{2}{20}\right) = 5730\ \mathrm{J/mole}$ or $5.73\ \mathrm{kJ/mol}$

$x = 5.73$

Question10

At 1000 K , the equilibrium constant for the reaction, $\mathrm{CO_2(g) + H_2(g) \rightleftharpoons CO(g) + H_2O(g)}$ is 0.53 . In a one litre vessel, at equilibrium the mixture contains 0.25 mole of CO, 0.5 mole of $\mathrm{CO_2}$, 0.6 mole of $\mathrm{H_2}$ and x moles of $\mathrm{H_2O}$. The value of x is

Options:

A.

0.563

B.

0.363

C.

0.636

D.

0.736

Answer: C

Solution:

Given reaction,

$CO_2(g) + H_2(g) \rightleftharpoons CO(g) + H_2O(g); K = 0.53$

At equilibrium, $[CO] = 0.25 \text{ mol/L}$,

$[CO_2] = 0.5 \text{ mol/L}$
$[H_2] = 0.6 \text{ mol/L}, [H_2O] = x$

Rate reaction; $K = \frac{[CO][H_2O]}{[CO_2][H_2]} = \frac{0.25 \times x}{0.5 \times 0.6}$

$x = 0.636$

Question11

Match the following.

List I (Reactions)	List II (Methods)
(A) $Mg(HCO_3)_2 \longrightarrow Mg(OH)_2 \downarrow + 2CO_2 \uparrow$	(I) Clark's method
(B) $M^{2+} + Na_4P_6O_{18}^{2-} \longrightarrow [Na_2MP_6O_{18}]^{2-} + 2Na^+$	(II) Ion exchange method
(C) $Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3 + 2H_2O$	(III) Boiling
(D) $2NaZ + Ca^{2+}(aq) \longrightarrow 2Na^+ + CaZ(Z = \text{ Zeolite })$	(IV) Calgon's method

The correct answer is

Options:

A.

A-III, B-IV, C-I, D-II

B.

A-IV, B-II, C-I, D-III

C.

A-III, B-IV, C-II, D-I

D.

A-II, B-IV, C-I, D-III

Answer: A

Solution:

- **(A) $Mg(HCO_3)_2 \rightarrow Mg(OH)_2\downarrow + 2CO_2\uparrow$**

  This reaction shows the decomposition of magnesium bicarbonate upon heating (boiling) to form insoluble magnesium hydroxide and carbon dioxide. This process removes temporary hardness from water and is achieved by **Boiling**.

  So, (A) matches with **(III) Boiling**.

- **(B) $M^{2+} + Na_4P_6O_{18}^{2-} \rightarrow [Na_2MP_6O_{18}]^{2-} + 2Na^+$**

  In this reaction, hard water ions (M^{2+}, typically Ca^{2+} or Mg^{2+}) react with sodium hexametaphosphate ($Na_4P_6O_{18}^{2-}$), commonly known as Calgon. The Calgon forms a soluble complex with the metal ions, thereby sequestering them and preventing them from causing hardness. This is the principle behind **Calgon's method**.

  So, (B) matches with **(IV) Calgon's method**.

- **(C) $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3\downarrow + 2H_2O$**

  This reaction shows calcium bicarbonate reacting with a calculated amount of slaked lime ($Ca(OH)_2$) to precipitate insoluble calcium carbonate. This method is specifically known as **Clark's method** for removing temporary hardness.

  So, (C) matches with **(I) Clark's method**.

- **(D) $2NaZ + Ca^{2+}(aq) \rightarrow 2Na^+ + CaZ$ (Z = Zeolite)**

  This reaction illustrates the exchange of calcium ions (Ca^{2+}) from hard water with sodium ions (Na^+) from sodium zeolite (NaZ). Zeolites are ion-exchange resins that replace hardness-causing ions with non-hardness-causing ions. This is the basis of the **Ion exchange method** (also known as the Zeolite process or Permutit process).

  So, (D) matches with **(II) Ion exchange method**.

Combining the matches:

A - III

B - IV

C - I

D - II

Comparing this with the given options:

Option A: A-III, B-IV, C-I, D-II (Matches our findings)

Option B: A-IV, B-II, C-I, D-III

Option C: A-III, B-IV, C-II, D-I

Option D: A-II, B-IV, C-I, D-III

The correct option is A.

The final answer is $\boxed{\text{A-III, B-IV, C-I, D-II}}$.

Question12

Observe the following statements.

Statement I Both LiF and CsI have low solubility in water.

Statement II Low solubility of LiF in water is due to smaller hydration enthalpy of ions and that of CsI is due to its high lattice enthalpy.

The correct answer is

Options:

A.

Both Statement I and II are correct.

B.

Statement I is correct, but Statement II is not correct.

C.

Statement I is not correct, but Statement II is correct.

D.

Both Statements I and II are not correct.

Answer: B

Solution:

Statement I is correct but Statement II is not correct. It's correct form is,

Low solubility of LiF in water is due to its high lattice enthalpy and low solubility of CsI in water is due to its smaller hydration enthalpy.

Question13

In which of the following the s-block elements are arranged in the correct order of their melting points?

Options:

A.

$\mathrm{Mg} > \mathrm{Be} > \mathrm{Na} > \mathrm{Li}$

B.

$\mathrm{Li} > \mathrm{Be} > \mathrm{Mg} > \mathrm{Na}$

C.

$\mathrm{Be} > \mathrm{Mg} > \mathrm{Li} > \mathrm{Na}$

D.

$\mathrm{Li} > \mathrm{Mg} > \mathrm{Na} > \mathrm{Be}$

Answer: C

Solution:

The correct order of melting point of s-block element is,

$\mathrm{Be} > \mathrm{Mg} > \mathrm{Li} > \mathrm{Na}$

Melting point decreases down the group. Group 2 has higher melting point than group 1 due to high charge density and strong metallic bonding.

Question14

The correct statements about the compounds of boron are

I. In borax bead test, the colour of cobalt metaborate is blue.

II. Diborane is prepared by the oxidation of sodium borohydride with iodine.

III. In diborane oxidation state of hydrogen is +1 .

IV. Boric acid is a tribasic acid.

Options:

A.

I and II

B.

III and IV

C.

I and III

D.

II and IV

Answer: A

Solution:

Statement given in I and II are correct regarding boron, while statement III and IV are incorrect.

Their correct forms are,

III. in diborane oxidation state of hydrogen is -1 .

IV. boric acid is not a tribasic acid, it is monobasic acid.

Question15

Dehydration of an organic acid X with concentrated $\mathrm{H_2SO_4}$ at 373 K gives $\mathrm{H_2O}$ and gas Y. The hybridisation of the carbon in Y and nature of Y are respectively.

Options:

A.

sp^2, Neutral

B.

sp, Neutral

C.

sp^2, Acidic

D.

sp, Acidic

Answer: B

Solution:

The reaction involved is,

$$\underset{\substack{\text{(Organic acid)} \\ \text{X}}}{\text{HCOOH}} \xrightarrow[373\text{ K}]{H_2SO_4} \underset{\text{(Gas Y)}}{\text{CO}\uparrow} + H_2O$$

Thus, the hybridisation of Y is sp and its nature is neutral.

Question16

Identify the correct statements from the following

I. Photochemical smog has high concentration of oxidising agents.

II. NO_2 is present in classical smog.

III. Higher concentration of SO_2 in air can cause stiffness of flower buds.

IV. pH of rain water is approximately 7.5 .

Options:

A.

I and III

B.

I and II

C.

III and IV

D.

II and III

Answer: A

Solution:

The correct statements are given in I and III. While II and IV are incorrect. Their correct forms are,

II. NO_2 is not present in classical smog. It consist of SO_2, fog, smoke.

IV. pH of rain water is around 5.6 .

Question17

Consider the given sequence of reactions,

$$C_2H_6 + \frac{3}{2}O_2 \xrightarrow[\Delta]{(CH,COO)_2Mn} X \xrightarrow{Na} Y$$

Electrolysis of aqueous solution of Y gives gases P and Q at anode. P and Q are respectively

Options:

A.

C_2H_6, CO_2

B.

CH_4, CO_2

C.

C_2H_6, H_2

D.

CH_4, CO

Answer: A

Solution:

The complete reaction is as follows,

$$C_2H_6 + \frac{3}{2}O_2 \xrightarrow{(CH_3COO)_2Mn, \Delta} \underset{(\text{Product } X)}{2CH_3COOH} \xrightarrow{\text{Na}} \underset{(\text{Product } Y)}{CH_3COO^-}$$

Now Kolbe's electrolysis,

$$2CH_3COO^- \xrightarrow{\text{Electrolysis}} \underset{\substack{(P) \\ \text{Ethane}}}{C_2H_6} + \underset{(\text{Q gas})}{2CO_2} + 2e^-$$

Question18

When sodium fusion extract of an organic compound is boiled with iron (II)sulphate solution followed by addition of concentrated H_2SO_4, gives prussian blue colour. This confirms the presence of the element

Options:

A.

sulphur

B.

chlorine

C.

phosphorus

D.

nitrogen

Answer: D

Solution:

First, when the sodium fusion extract reacts with iron(II) sulfate, a chemical reaction happens that forms a compound called sodium ferrocyanide.

$$6NaCN + FeSO_4 \longrightarrow Na_4[Fe(CN)_6] + Na_2SO_4$$

Next, when you add concentrated H_2SO_4, the iron(II) changes to iron(III). This iron(III) reacts with sodium ferrocyanide to make a blue colored compound called Prussian blue.

$$3Na_4[Fe(CN)_6] + 4Fe^{3+} \xrightarrow{xH_2O} Fe_4[Fe(CN)_6]_3 \cdot xH_2O$$

The blue color (Prussian blue) shows that nitrogen is present in the organic compound.

Question19

Which of the following is an example of electrophilic substitution reaction?

Options:

A.

$$CH_3CHO + HCN \longrightarrow CH_3CH(OH)CN$$

B.

$$(CH_3)_3CX + H_2O \longrightarrow (CH_3)_3C - OH + HX$$

C.

$$C_6H_6 + CH_3COCl \xrightarrow{AlCl_3} C_6H_5(COCH_3) + HCl$$

D.

$$BrCH_2CH_2Br + Zn \xrightarrow[\Delta]{Alcohol} CH_2 = CH_2 + ZnBr_2$$

Answer: C

Solution:

Among the given options (c) is an example of electrophilic substitution reaction.

$$C_6H_6 + CH_3COCl \xrightarrow{AlCl_3} C_6H_5(COCH_3) + HCl$$

The given reaction is Friedal-Crafts acylation reaction, here the reaction of benzene with an arylhalide or acid anhydride is the presence of Lewis acid ($AlCl_3$) yields acyl benzene. Here, the attacking reagent is as electrophile.

Question20

An alkene X on ozonolysis gives a mixture of simplest ketone (Y) and 3-pentanone. The IUPAC name of the alkene X is

Options:

A.

2, 3-dimethylbut-2-ene

B.

3-ethyl-4-methylpent-3-ene

C.

3-ethyl-2-methylpent-2-ene

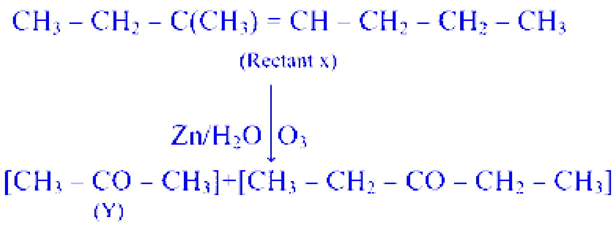
D.

2-methyl-3-ethylpent-2-ene

Answer: C

Solution:

The complete reaction is



The IUPAC name of $X =$ 3-ethyl-2-methylpent-2-ene

Question21

A solid contains elements A and B. Anions of B form ccp lattice. Cations of A occupy 50\% of octahedral voids and 50\% of tetrahedral voids. What is the molecular formula of the solid?

Options:

A.

AB_3

B.

A_3B_2

C.

A_2B_3

D.

AB

Answer: B

Solution:

Let the number of B atom in $ccp = N$

Number of octahedral voids $= N$

Number of tetrahedral voids $= 2N$

Number of A atoms from octahedrals voids $= \frac{N}{2}$

Number of A atoms from tetrahedral voids $= \frac{1}{2} \times 2N = N$

Total number of A atoms $= \frac{N}{2} + N = \frac{3N}{2}$

Ratio of $A : B = \frac{3N}{2} : N$ or $3 : 2$

Thus molecular formula A_3B_2.

Question22

The osmotic pressure (in atm) of an aqueous solution containing 0.01 mol of NaCl (degree of dissociation 0.94) and 0.03 mol of glucose in 500 mL at $27^{\circ}\mathrm{C}$ is $\left(R = 0.082 \mathrm{~L~atm~K^{-1}~mol^{-1}}\right)$

Options:

A.

2.43

B.

4.23

C.

3.24

D.

3.42

Answer: A

Solution:

$\mathrm{NaCl} \rightleftharpoons \mathrm{Na^{+}} + \mathrm{Cl^{-}}$

$$i_{\mathrm{NaCl}} = 1 + \alpha(n-1)$$
$$n = 2; i_{\mathrm{NaCl}} = 1 + 0.94(2-1) = 1.94$$
$$i_{\text{glucose}} = 1$$

Concentration of NaCl $= \frac{0.01}{0.5} = 0.02\mathrm{M}$

Effective concentration of NaCl $= iC_{\mathrm{NaCl}}$

$$\begin{aligned} &= 1.94 \times 0.02\mathrm{M} = 0.0388\mathrm{M} \\ C_{\text{glucose}} &= \frac{0.03}{0.5} = 0.06\mathrm{M} \end{aligned}$$

Effective concentration $= 0.06\mathrm{M}$

$$\begin{aligned} iC_{\text{total}} &= 0.0388 + 0.06 = 0.0988\mathrm{M} \\ \pi &= iC_{\text{total}} \quad RT \\ &= 0.098 \times 0.082 \times 300.15 = 2.43 \mathrm{~atm} \end{aligned}$$

Question23

Electrolysis of aqueous copper (II) sulphate between Pt electrodes gives ' X' at anode and ' Y' at cathode. X and Y are respectively.

Options:

A.

$\mathrm{Cu}, \mathrm{O}_2$

B.

$\mathrm{O}_2, \mathrm{Cu}$

C.

$\mathrm{SO}_2, \mathrm{H}_2$

D.

$\mathrm{O}_2, \mathrm{H}_2$

Answer: B

Solution:

Reaction at anode is,

$2\mathrm{H}_2\mathrm{O}(l) \longrightarrow \mathrm{O}_2(g) + 4\mathrm{H}^+(aq) + 4e^-$

This produce gas $= \mathrm{O}_2(X)$

Reaction at cathode

$\mathrm{Cu}^{2+} + 2e^- \longrightarrow \mathrm{Cu}(s)$

Metal deposited = $\mathrm{Cu}(Y)$

Question24

Consider a general first order reaction,

$$A(g) \longrightarrow B(g) + C(g)$$

If the initial pressure is 200 mm and after 20 minutes it is 250 mm , then the half-life period of the reaction (in minutes) is ($\log 2 = 0.30, \log 3 = 0.48, \log 4 = 0.60$)

Options:

A.

40.2

B.

50.2

C.

20.5

D.

60.5

Answer: B

Solution:

Given reaction is, $A(g) \longrightarrow B(g)+C(g)$

Let x be the decrease in pressure of A.

The pressure of A at time t is $p_A=p_0-x$

The total pressure at time t is

$$p_t=p_0-x+x+x=p_0+x$$
$$250=200+x ; x=50 \text{ mm}$$
$$p_A=200-50=150 \text{ mm}$$

Use the integrated rate law,

$k=\frac{2.303}{t} \log \left(\frac{p_0}{p_A}\right)$

Substitute the value,

$$k=\frac{2.303}{20} \log \left(\frac{200}{150}\right) \Rightarrow k=0.0138 \text{ min}^{-1}$$
$$t_{1/2}=\frac{0.693}{k}$$
$$t_{1/2}=\frac{0.693}{0.0138}=50.2 \text{ minutes}$$

Question25

The most effective coagulating agent for antimony sulphide sol is

Options:

A.

Na_2SO_4

B.

$CaCl_2$

C.

NH_4Cl

D.

$Al_2(SO_4)_3$

Answer: D

Solution:

Most effective coagulating agent for $Sb_2 S_3$ sol is $Al_2(SO_4)_3$. This is because $Sb_2 S_3$ forms a negative charge sol, and according to the Hardy-Schulze rule, the coagulating power increases with oppositely charged ions.

Question26

Metal X obtained from sphalarite ore can be purified by which of the following method?

Options:

A.

Distillation

B.

Poling

C.

Zone refining

D.

Vapour phase refining

Answer: A

Solution:

Zinc is obtained from sphalerite ore by distillation method.

Question27

An oxoacid of phosphorus ' X ' reduces silver nitrate solution to metallic silver and gets oxidised to another compound Y. X and Y respectively are

Options:

A.

HPO_3, H_3PO_4

B.

H_3PO_2, H_3PO_4

C.

H_3PO_3, H_3PO_2

D.

H_3PO_2, HNO_3

Answer: B

Solution:

The complete reaction is as follows,

$$\underset{\text{Hypophosphorus acid}}{\underset{(X)}{H_3PO_2(aq)}} + 2AgNO_3(aq) \xrightarrow{H_2O} \underset{(Y)}{H_3PO_4} + 2Ag + 2HNO_3$$

Question28

Zinc on reaction with concentrated nitric acid gives an oxide of nitrogen (A). Zinc with dilute nitric acid gives another oxide of nitrogen (B). Oxidation numbers of nitrogen in (A) and (B) are respectively

Options:

A.

$+4, +1$

B.

$+4, +2$

C.

$+2, +4$

D.

$+1, +4$

Answer: A

Solution:

The reaction is,

$\mathrm{Zn} + \underset{\text{(conc.)}}{4\mathrm{HNO}_3} \longrightarrow \mathrm{Zn}(\mathrm{NO}_3)_2 + \underset{(A)}{2\mathrm{NO}_2} + 2\mathrm{H}_2\mathrm{O}$

$4\mathrm{Zn} + \underset{\text{(dil.)}}{10\mathrm{HNO}_3} \longrightarrow 4\mathrm{Zn}(\mathrm{NO}_3)_2 + \underset{(B)}{\mathrm{N}_2\mathrm{O}} + 5\mathrm{H}_2\mathrm{O}$

Oxidation state of nitrogen,

(A) $\mathrm{NO}_2 = +4$

(B) $\mathrm{N}_2\mathrm{O} = +1$

Question29

Identify the reaction related to Deacon's process

Options:

A.

$2\mathrm{H}_2\mathrm{O} + 2\mathrm{Cl}_2 \xrightarrow{\text{sunight}} 4\mathrm{HCl} + \mathrm{O}_2$

B.

$4\mathrm{HCl} + \mathrm{O}_2 \xrightarrow[\text{723 K}]{\mathrm{CuaC}_2} 2\mathrm{Cl}_2 + 2\mathrm{H}_2\mathrm{O}$

C.

$2\mathrm{NaCl} + \mathrm{H}_2\mathrm{SO}_4 \xrightarrow{\text{823 K}} \mathrm{Na}_2\mathrm{SO}_4 + 2\mathrm{HCl}$

D.

$Na_2\ S_2O_3 + Cl_2 + H_2O \longrightarrow Na_2SO_4 + 2HCl + S$

Answer: B

Solution:

Deacon's process is a chemical method use to produce chlorine gas from hydrochloric acid.

The reaction is, $4HCl + O_2 \xrightarrow[723\ K]{CuCl_2} 2Cl_2 + 2H_2O$

Question30

Identify the correct statements about lanthanoids

I. Ce^{4+} and Tb^{4+} act as oxidising agents.

II. Eu^{2+} and Yb^{2+} act as oxidising agents.

III. Misch metal is an alloy of 95% iron and 5% lanthanoid metal.

IV. La^{3+} and Ce^{4+} are diamagnetic in nature.

Options:

A.

I and II only

B.

I and IV only

C.

II, III and IV only

D.

I, II and IV only

Answer: B

Solution:

Statements I and IV are correct, while II and III are incorrect. Their correct form is,

II Eu^{2+} and Yb^{2+} acts as reducing agent.

IV Misch metal is primarily composed of lanthanoids metal ($\sim 95\%$) and iron ($\sim 5\%$) and traces of S, C, Ca and Al.

Question31

When 100 mL of 0.2 M solution of $\mathrm{CoCl_3 \cdot xNH_3}$ is treated with excess of $\mathrm{AgNO_3}$ solution, 3.6×10^{22} ions are precipitated. The value of x is $\left(N = 6 \times 10^{23}\ \mathrm{mol^{-1}}\right)$

Options:

A.

5

B.

6

C.

4

D.

3

Answer: B

Solution:

Moles of ions precipitated.

$$\begin{aligned} n_{\text{ions}} &= \frac{\text{Number of ions}}{\text{Avogadro's number}} = \frac{3.6 \times 10^{22}}{6 \times 10^{23}} \\ n_{\text{ions}} &= 0.06\ \text{mol} \\ n_{\text{complex}} &= \text{Molarity} \times \text{Volume} = 0.2 \times 0.1 = 0.02\ \text{mol} \end{aligned}$$

Number of $\mathrm{Cl^-}$ ions per complex units,

$Y = \frac{\text{Moles of precipitated ions}}{\text{Moles of complex}} = 3$

The complex is $[\mathrm{Co(NH_3)_x Cl_{3-y}}]\mathrm{Cl_y}$.

Coordination number of $\mathrm{Co} = 6$.

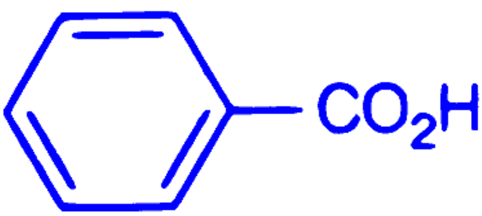
$x = 6$

Question32

Ethylene on reaction with cold, dilute alkaline $\mathrm{KMnO_4}$ at 273 K gives a compound ' P '. This on polymerisation with which of the following gives dacron?

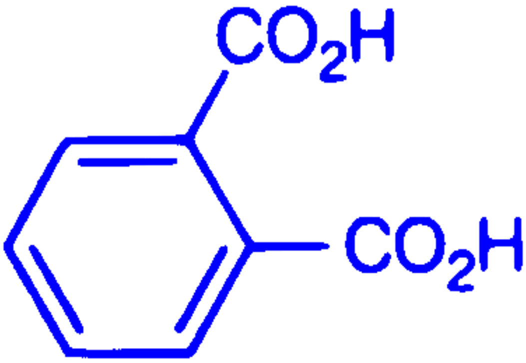
Options:

A.

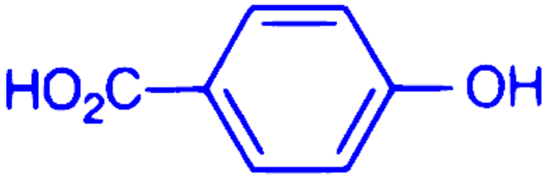


B.

C.

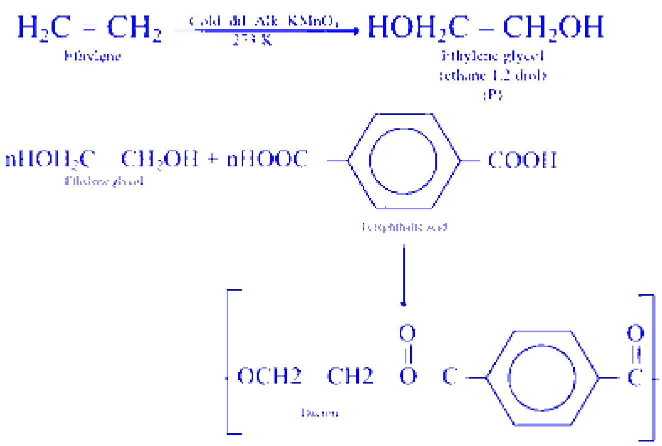


D.



Answer: B

Solution:



Question33

A carbohydrate (A), when treated with dilute HCl in alcoholic solution gives two isomers (B) and (C). B on reaction with bromine water gives a monocarboxylic acid ' Z ' and ' C ' is a ketohexose. What is A ?

Options:

A.

Starch

B.

Maltose

C.

Sucrose

D.

Lactose

Answer: C

Solution:

The reaction involved is as follows,

$$\underset{\substack{\text{Sucrose} \\ (A)}}{C_{12}H_{22}O_{11}} + \underset{\text{Alcoholic solution}}{\text{dil HCl in}} \longrightarrow \underset{\substack{\text{Glucose} \\ (B)}}{C_6H_{12}O_6} + \underset{\substack{\text{Fructose} \\ (C)}}{C_6H_{12}O_6}$$

$$\underset{(B)}{C_6H_{12}O_6} + \underset{\text{Water}}{Br_2} \longrightarrow \underset{\substack{\text{Gluconic acid} \\ (Z)}}{C_6H_{12}O_7} + 2HBr$$

Question34

The incorrect statement about chloramphenicol is

Options:

A.

it is a broad spectrum antibiotic.

B.

it is a bacteriostatic antibiotic.

C.

it is a bactericidal antibiotic.

D.

it is used to cure pneumonia.

Answer: C

Solution:

The incorrect statement about chloramphenicol is given in option (c). It's correct form is, It is primarily a bacteriostatic antibiotic.

Question35

The number of chlorine (Cl) atoms in the structure of DDT molecule is

Options:

A.

4

B.

3

C.

2

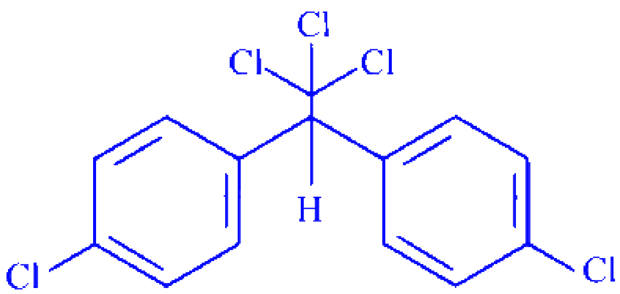
D.

5

Answer: D

Solution:

The structure of DDT is,

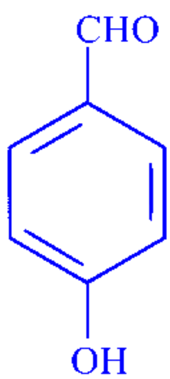


Question36

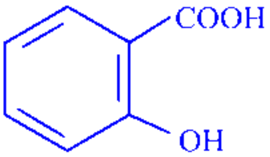
The major product in Reimer-Tiemann reaction is X. The reactants are Y and Z, X, Y and Z are respectively

Options:

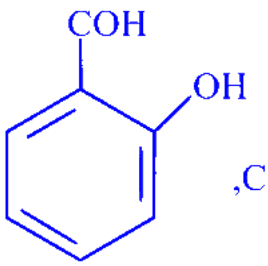
A.



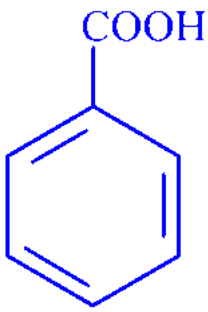
$,CHCl_3$, aq. NaOH

B. 

,CCl_4, aq. $Ba(OH)_2$

C. 

,$CHCl_3$, aq. NaOH

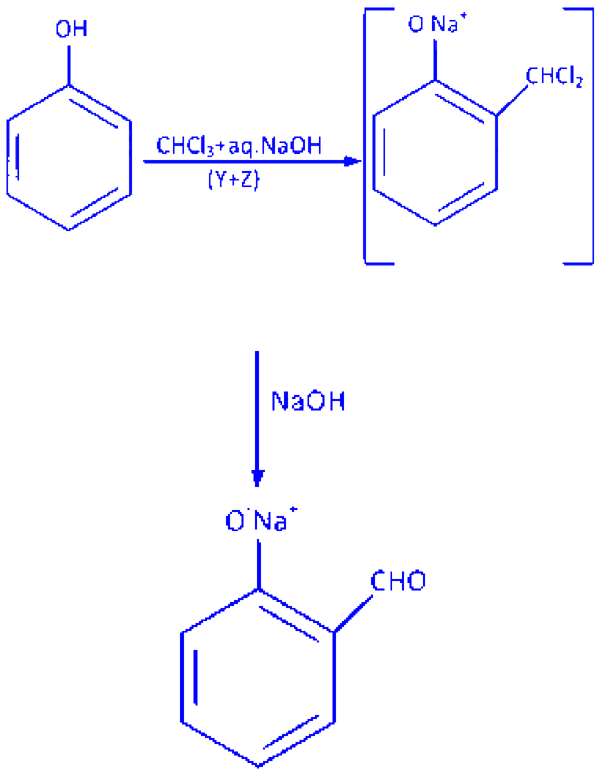
D. 

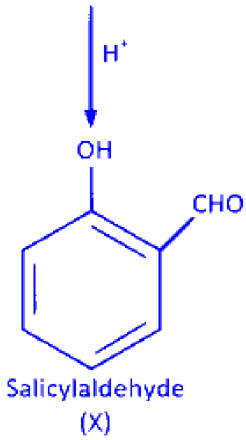
,CCl_4, aq. KOH

Answer: C

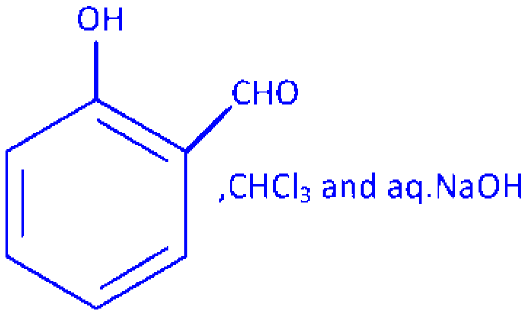
Solution:

The complete Reimer-Tiemann reaction is as follows,





Thus, X, Y and Z are



Question37

$$\text{Toluene} \xrightarrow[\text{(ii)} \quad H_3O^+]{\text{(i)} \quad CrO_2Cl_2/CS_2} X \xrightarrow{\text{Conc.} \quad \text{NaOH}} Y + Z$$

The correct statements about Y and Z are

A. Y is a secondary alcohol.

B. Y is the reduction product of X.

C. Z on heating with sodalime gives benzene.

D. Y does not give H_2 gas with Na metal.

Options:

A.

B and C only

B.

A and B only

C.

A and D only

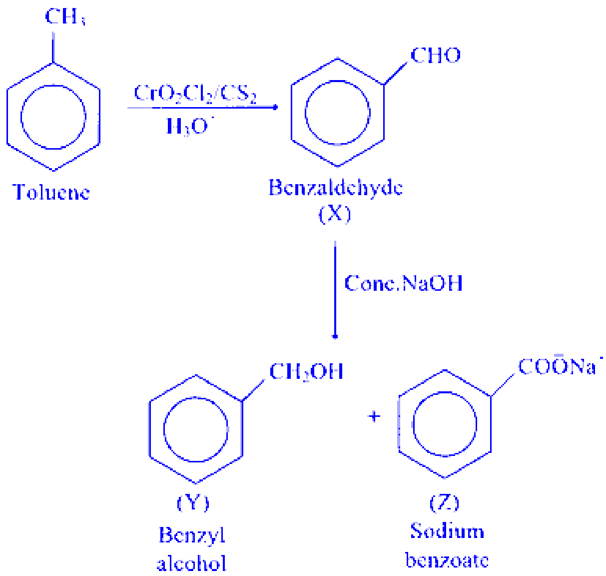
D.

B and D only

Answer: A

Solution:

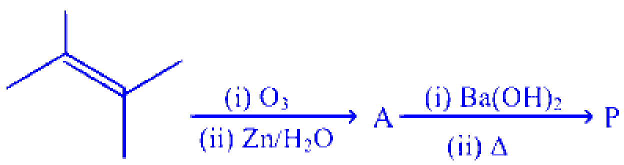
The complete reaction sequence is as follows.



Thus, statement given in B and C are correct. i.e., Y is the reduction product of X and Z on heating with sodalime gives benzene.

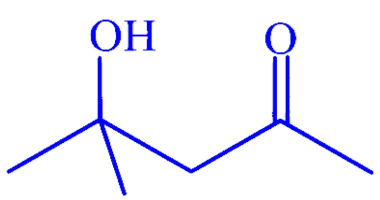
Question38

Identify the product ' P ' in the given reaction sequence.

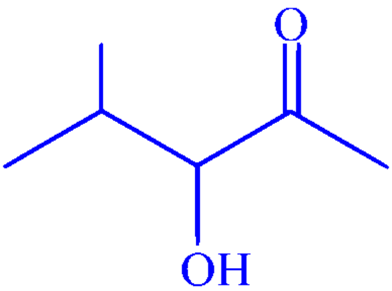


Options:

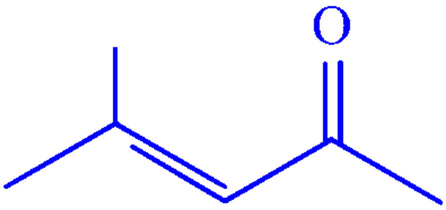
A.



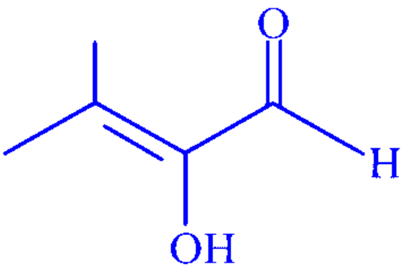
B.



C.



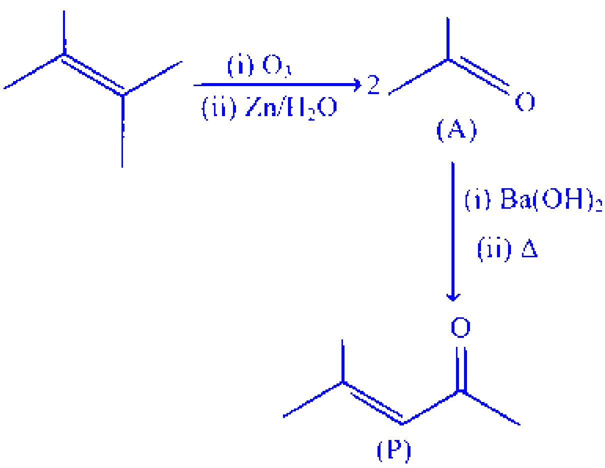
D.



Answer: C

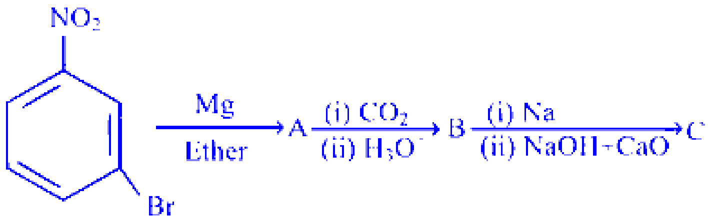
Solution:

The complete sequence is as follows.



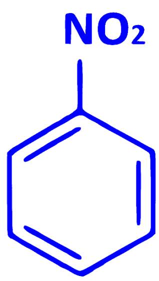
Question39

The product ' C ' in the given reaction sequence is

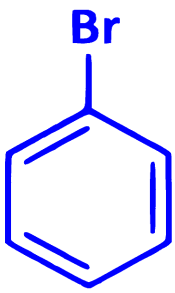


Options:

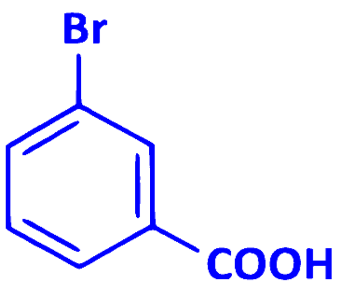
A.



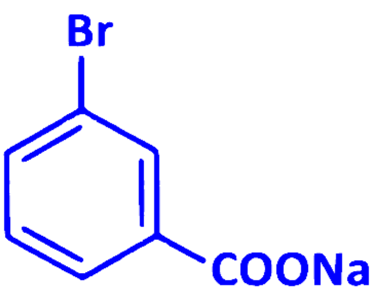
B.



C.



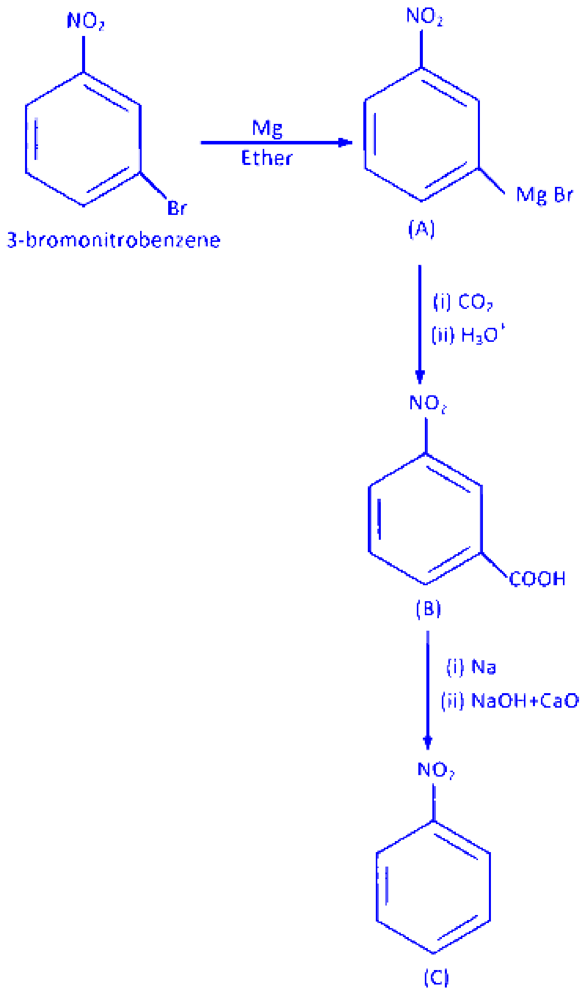
D.



Answer: A

Solution:

The complete reaction sequence involved is as follows

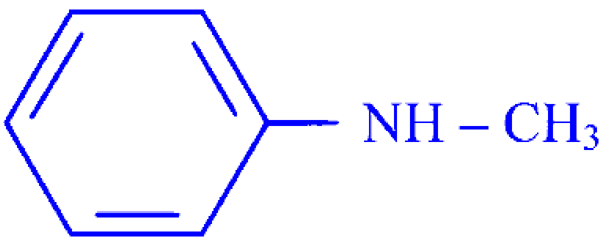


Question40

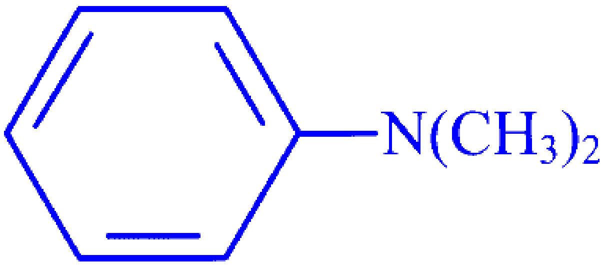
The amine/salt of amine which gives positive test with a mixture of chloroform and alcoholic KOH solution is

Options:

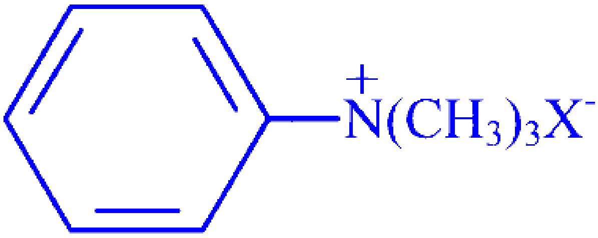
A.



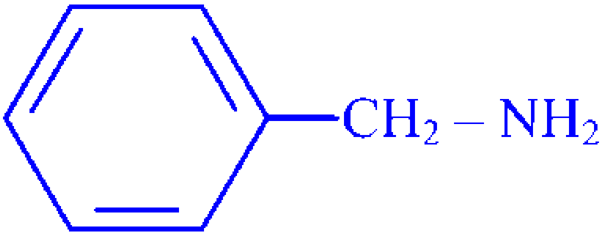
B.



C.



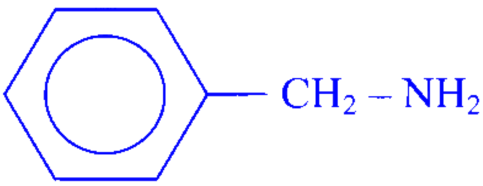
D.



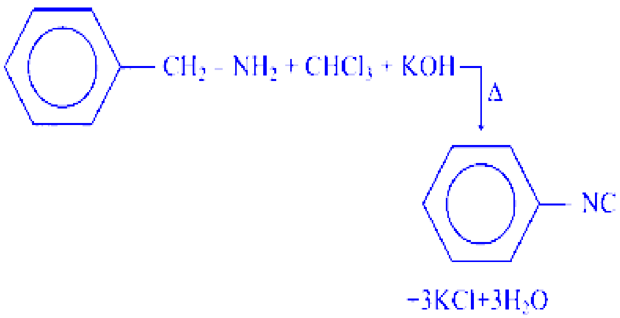
Answer: D

Solution:

Aliphatic and aromatic primary amines give positive carbylamine reaction. Thus, among the given options,



will give positive test with a mixture of chloroform and alc. KOH solution.



Physics

Question1

Bose-Einstein statistics is applicable to particles with

Options:

A.

even integral spin particles only

B.

integral spin particles

C.

half odd integral spin particles

D.

odd integral spin particles only

Answer: B

Solution:

Bose-Einstein statics describes the behaviour of a system of identical, indistinguishable particles with integer spin, known as bosons.

Question2

If L and C are inductance and capacitance respectively, then the dimensional formula of $(\mathrm{LC})^{-\frac{1}{2}}$ is

Options:

A.

$\left[\mathrm{M}^{9}\ \mathrm{L}^{0}\ \mathrm{T}^{-1}\right]$

B.

$\left[\mathrm{M}'\mathrm{L}'\mathrm{T}^{-1}\right]$

C.

$\left[M^{9}L^{1}T^{1}\right]$

D.

$\left[\mathrm{M}^{9} \mathrm{~L}^{0} \mathrm{~T}^{-2}\right]$

Answer: A

Solution:

We know that the formula for resonance angular frequency is:

$\omega=\frac{1}{\sqrt{L C}}$

So, $\omega=(L C)^{-1 / 2}$

This means the dimensional formula for $(L C)^{-1 / 2}$ is the same as the dimensional formula for ω.

The dimensional formula for angular frequency ω is time inverse: $\left[\mathrm{T}^{-1}\right]$.

In full form, this is $\left[\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{-1}\right]$.

Question3

The ratio of times taken by a freely falling body to travel first 5 m , second 5 m , third 5 m distances is

Options:

A.

$1: \sqrt{2}: \sqrt{3}$

B.

$1: \sqrt{2-1}: \sqrt{3-2}$

C.

$1: \sqrt{3}: \sqrt{5}$

D.

$1: \sqrt{2}-1: \sqrt{3}-\sqrt{2}$

Answer: D

Solution:

Time taken $\left(t_{1}\right)$ by freely falling body to travel first 5 m is given as,

$$\begin{aligned}
& s=u t_{1}+\frac{1}{2} g t_{1}^{2} \\
& 5=0 \times t_{1}+\frac{1}{2} \times 10 \times t_{1}^{2} \\
\Rightarrow \quad & t_{1}=1 \mathrm{~s}
\end{aligned}$$

Time taken ($t^{\prime}$) to travel 10 m .

$$10 = 0 \times t_1' + \frac{1}{2} \times 10 \times t'^2$$
$$\Rightarrow \quad t' = \sqrt{2}\,\text{s}$$

$\therefore$ Time taken (t_2) to travel next 5 m ,

$t_2 = t' - t_1 = (\sqrt{2} - 1)\text{s}$

Time taken (t'') to travel 15 m ,

$$15 = 0 \times t'' + \frac{1}{2} \times 16 \times (t'')^2$$
$$15 = 5(t'')^2$$
$$\therefore \quad t'' = \sqrt{3}\,\text{s}$$

Thus, time taken (t_3) to travel lost 5 m

$$t_3 = t'' - t' = \sqrt{3} - \sqrt{2}$$
$$\therefore t_1 : t_2 : t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$$

Question4

Two bodies are projected from the same point with the same initial velocity ' u ' making angles ' θ' and $(90^\circ - \theta)$ with the horizontal in opposite directions. The horizontal distance between their positions when the bodies are at their maximum heights is

Options:

A.

$\frac{u^2}{2g}(\sin^2\theta - \cos^2\theta)$

B.

$\frac{u^2 \sin 2\theta}{2g}$

C.

$\frac{u^2}{g}$

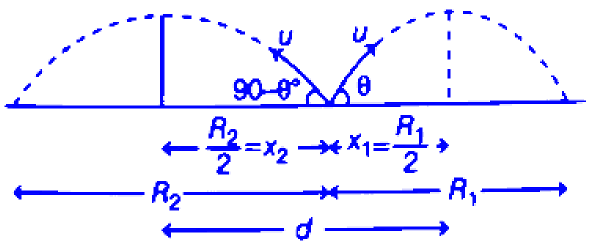
D.

$\frac{u^2 \sin 2(90^\circ - \theta)}{g}$

Answer: D

Solution:

The given situation is shown below in figure.



$x_1 = \frac{R_1}{2} = \frac{u^2 \sin 2\theta}{2g}$

and $x_2 = \frac{R_2}{2} = \frac{u^2 \sin 2(90^\circ - \theta)}{2g}$

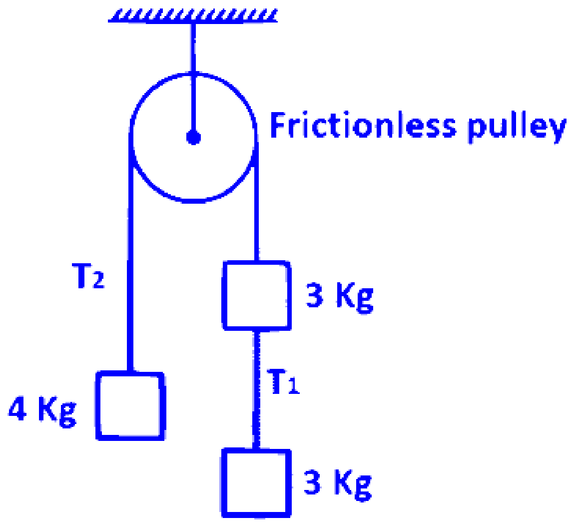
$\therefore \quad = \frac{u^2 \sin(180^\circ - 2\theta)}{2g} = \frac{u^2 \sin 2\theta}{2g}$

$\therefore d = x_1 + x_2$

$$= \frac{u^2 \sin 2\theta}{2g} + \frac{u^2 \sin 2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$
$$= \frac{u^2 \sin(180^\circ - 2\theta)}{g} = \frac{u^2 \sin 2(90^\circ - \theta)}{g}$$

Question5

If the system of blocks shown in the figure is released from rest, the ratio of the tensions T_1 and T_2 is (Neglect the mass of the string shown in the figure)



Options:

A.

1 : 1

B.

1 : 2

C.

1 : 3

D.

3 : 4

Answer: B

Solution:

Tension in the string connected to the 4 kg block $= T_2$

Tension in each side of the movable pulley (connected to 3 kg blocks) $= T_1$

Since, the pulley is ideal and frictionless, the net upward force on the pulley is

$T_2 = 2T_1$

$4g - T_2 = 4a(\text{ downward acceleration } a) \quad \ldots(i)$

$T_2 = 4g - 4a$

$T_1 = 3g - 3a(\text{ acceleration upwards } a) \quad \ldots(ii)$

$T_2 = 2T_1 \quad \ldots(iii)$

From Eqs. (i) and (ii)

$$4g - 4a = 2(3g - 3a)$$
$$4g - 4a = 6g - 6a$$
$$2a = 2g$$
$$\Rightarrow \quad a = g$$

Substitute $a = g$ in Eq. (ii)

$T_1 = 3g - 3g = 0(\text{ invalid })$

Assuming equilibrium $a = 0$

Then, $T_1 = 3g$

(From Eq. (ii))

$T_2 = 2T_1$

$\frac{T_1}{T_2} = \frac{1}{2}$

(From Eq. (iii))

Question6

If the component of the vector $\mathbf{A}$ along the vector $\mathbf{B}$ is twice the component of $\mathbf{B}$ along $\mathbf{A}$, then the ratio of magnitudes of vectors $\mathbf{A}$ and $\mathbf{B}$ is

Options:

A.

1 : 2

B.

3 : 2

C.

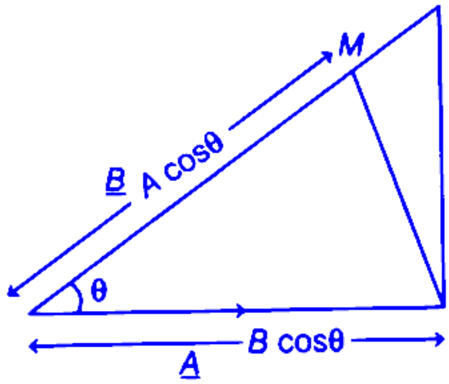
2 : 1

D.

3 : 1

Answer: C

Solution:



According to question,

$$A\cos\theta = 2B\cos\theta$$
$$A = 2B$$
$$\Rightarrow \quad \frac{A}{B} = \frac{2}{1} \Rightarrow A : B = 2 : 1$$

Question7

A body projected vertically up with an initial speed of $10\ \mathrm{ms}^{-1}$ reaches the point of projection after sometime with a speed of $8\ \mathrm{ms}^{-1}$. The maximum height reached by the body is (Acceleration due to gravity $= 10\ \mathrm{ms}^{-2}$)

Options:

A.

5 m

B.

3.2 m

C.

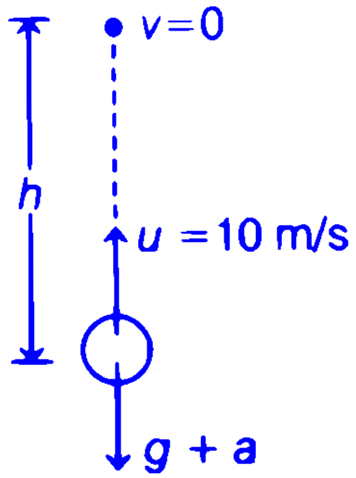
4.1 m

D.

4.5 m

Answer: C

Solution:

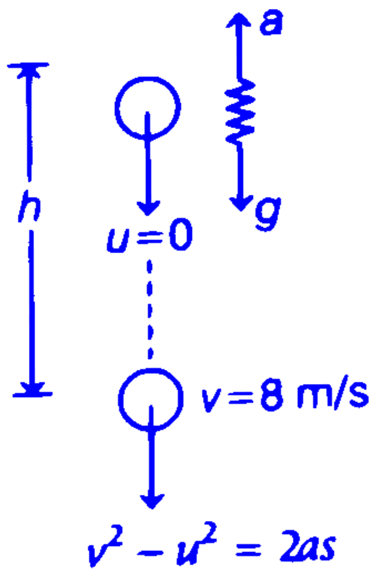


As object return with smaller speed that means there is a loss of energy. This loss of energy occurs due to viscous drag of atmosphere. While object is moving up let viscous drag produces an deceleration ' a '.

Then,

$$v^2 - u^2 = 2as$$
$$\Rightarrow \quad 0-(10)^2 = 2(-(g+a))h$$
$$\text{or} \quad 100 = 2(g+a)h \qquad \ldots(i)$$

While coming down,



$$\Rightarrow \quad 64-0 = 2(-(g-a))(-h)$$
$$\Rightarrow \quad 64 = 2(g-a)h \qquad \ldots(ii)$$

Dividing (i) and (ii) we get

$$\frac{100}{64} = \frac{g+a}{g-a}$$
$$\Rightarrow \quad 100g - 100a = 64g + 64a$$
$$36g = 164a \text{ or } a = \frac{36}{164}g$$

Substituting ' a ' in either (i) or (ii) we get ' h.

From (ii)

$$64=2\left(g-\frac{36}{164} g\right) h$$
$$\Rightarrow \quad 32=\left(\frac{164-36}{164}\right) g h \Rightarrow$$
$$\Rightarrow \quad h=4.1 \mathrm{~m}$$

Question8

Due to global warming, if the ice in the polar region melts and some of this water flows to the equatorial region, then

Options:

A.

angular momentum of the Earth increases and duration of day increases.

B.

angular momentum of the Earth decreases and duration of day decreases.

C.

angular momentum of the Earth is constant and duration of day decreases.

D.

angular momentum of the Earth is constant and duration of day increases.

Answer: D

Solution:

When ice from the polar regions melts and flows towards the equator, the mass distribution of the Earth changes. This shift in mass causes in increases in the Earth's moment of inertia. According to the principle of conservation of angular momentum, if the angular momentum ($L=I \omega$) remains constant and the moment of inertia (I) increases, the angular velocity (ω) must decrease. A decrease in angular velocity means the Earth rotates more slowly, leading to an increase in the duration of a day.

Question9

If the moment of inertia of a thin circular ring about an axis passing through its edge and perpendicular to its plane is I, then the moment of inertia of the ring about its diameter is

Options:

A.

$I / 4$

B.

$4 I$

C.

$I/2$

D.

$2I$

Answer: A

Solution:

Moment of inertia of ring about its center:

$I_{\text{CM}} = MR^2$

Using the parallel axis theorem:

The parallel axis theorem says: If you know the moment of inertia of an object about its center (I_{CM}), then the moment of inertia about a parallel axis a distance R away is:

$I = I_{\text{CM}} + MR^2$

For a thin circular ring, this axis through the edge is a distance R from the center, so:

$I = MR^2 + MR^2 = 2MR^2$

So, $MR^2 = \frac{I}{2} \qquad \cdots (i)$

Finding the moment of inertia about the diameter:

The moment of inertia of a ring about its diameter is:

$I_d = \frac{MR^2}{2}$

Substituting from equation (i):

Replace MR^2 with $\frac{I}{2}$: $I_d = \frac{\frac{I}{2}}{2} = \frac{I}{4}$

Question10

A particle is executing simple harmonic motion. If the force acting on the particle at a position is 86.6% of the maximum force on it, then the ratio of its velocity at that point and its maximum velocity is

Options:

A.

$1 : \sqrt{3}$

B.

$1 : 2$

C.

$\sqrt{3} : 2$

D.

1 : 3

Answer: B

Solution:

The force in SHM is $F = -kx$

The maximum force is $F_{\max} = -kA$

Given, $F = 0.866 F_{\max}$

$$So, \; -kx = 0.866(-kA)$$
$$x = 0.866 \text{ A}$$

The velocity in SHM is $v = \omega\sqrt{(A^2 - x^2)}$

$$v = \omega\sqrt{A^2 - (0.866A)^2}$$
$$v = \omega\sqrt{A^2 - 0.75A^2} = \omega\sqrt{0.25A^2}$$
$$v = \omega(0.5A)$$

The maximum velocity in SHM occurs at $x = 0$, So $v_{\max} = \omega A$

The ratio is $\frac{v}{v_{\max}} = \frac{0.5\omega A}{\omega A}$

$\frac{v}{v_{\max}} = 0.5 = \frac{1}{2}$

The ratio of its velocity at that point and its maximum velocity is $\frac{1}{2}$.

Question11

The ratio of the time periods of a simple pendulum at heights $2R_E$ and $3R_E$ from the surface of the Earth is (R_E is radius of the Earth)

Options:

A.

1 : 2

B.

1 : 3

C.

3 : 4

D.

2 : 3

Answer: C

Solution:

Time Period of a Simple Pendulum:

The time period (how long it takes for one swing) of a simple pendulum is given by: $T = 2\pi\sqrt{\frac{l}{g}}$

This means T depends on $1/\sqrt{g}$, where g is gravity at the pendulum's location.

Gravity at a Height:

Gravity changes as you go away from the Earth's surface. At height h above Earth, gravity is: $g' = \frac{g}{\left(1+\frac{h}{R_E}\right)^2}$ where R_E is Earth's radius.

Time Period Ratio at Two Heights:

Let T_1 be the time period at height $2R_E$ and T_2 at $3R_E$.

So, $h_1 = 2R_E$ and $h_2 = 3R_E$.

The ratio is: $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

Calculate Gravity at Each Height:

At $h_1 = 2R_E$, gravity is $g_1 = \frac{g}{(1+2)^2} = \frac{g}{9}$

At $h_2 = 3R_E$, gravity is $g_2 = \frac{g}{(1+3)^2} = \frac{g}{16}$

Find the Ratio:

So, $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{g/16}{g/9}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Therefore, the ratio of the time periods is 3 : 4.

Question12

Two wires A and B made of same material and areas of cross-section in the ratio $1:2$ are stretched by same force. If the masses of the wires A and B are in the ratio $2:3$, then the ratio of the elongations of the wires A and B is

Options:

A.

1 : 2

B.

8 : 3

C.

1 : 3

D.

4 : 3

Answer: B

Solution:

The mass of a wire is $m = \rho AL$

Since, the material is the same, the density ρ is constant for both wires.

$$\frac{m_A}{m_B} = \frac{\rho A_A L_A}{\rho A_B L_B} \Rightarrow \frac{m_A}{m_B} = \frac{A_A L_A}{A_B L_B}$$
$$\frac{2}{3} = \frac{1}{2} \times \frac{L_A}{L_B}$$
$$\frac{L_A}{L_B} = \frac{2}{3} \times 2 = \frac{4}{3}$$

The ratio of elongations is

$$\frac{\Delta L_A}{\Delta L_B} = \frac{\frac{F L_A}{A_A Y}}{\frac{F L_B}{A_B Y}} = \frac{L_A}{A_A} \times \frac{A_B}{L_B}$$

Substitute $\frac{L_A}{L_B} = \frac{4}{3}$ and $\frac{A_B}{A_A} = \frac{2}{1}$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{4}{3} \times \frac{2}{1} = \frac{8}{3}$$

The ratio of the elongations of the wires A and B is $\frac{8}{3}$.

Question13

Water is filled in a tank up to a height of 20 cm from the bottom of the tank. Water flows through a hole of area 1 mm^2 at its bottom. The mass of the water coming out from the hole in a time of 0.6 s is

Options:

A.

1.8 g

B.

1.2 g

C.

0.6 g

D.

2.4 g

Answer: B

Solution:

Height of water $h = 20 \text{ cm}$

$= 20 \times \frac{1}{100} \text{ m} = 0.2 \text{ m}$

Velocity of efflux

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \text{ m/s}^2 \times 0.2 \text{ m}}$$
$$v = \sqrt{4 \text{ m}^2/\text{s}^2} = 2 \text{ ms}^{-1}$$

Volume flow rate

$$Q = A \times V$$
$$Q = 1 \text{ m}^2 \times 2 \text{ m/s} = 2 \text{ m}^3/\text{s}$$

Volume of water $V = Q \times t$

$V = 2\ \mathrm{m^3/s} \times 0.6\ \mathrm{s} = 1.2\ \mathrm{m^3}$

Mass of the water

$$m = \rho \times V$$
$$m = 1000\ \mathrm{kg/m^3} \times 1.2\ \mathrm{m^3}$$
$$= 1200\ \mathrm{kg} = 1.2\ \mathrm{g}$$

Question14

For which of the following Reynold's number, a flow is streamlined?

Options:

A.

900

B.

2100

C.

2900

D.

4000

Answer: A

Solution:

For stream line flow

$R_e < 1000$

Thus, for $R_e = 900$, flow of liquid is streamlined.

Question15

A body cools from a temperature of $60^\circ\mathrm{C}$ to $50^\circ\mathrm{C}$ in 10 minutes and $50^\circ\mathrm{C}$ to $40^\circ\mathrm{C}$ in 15 minutes. The time taken in minutes for the body to cool from $40^\circ\mathrm{C}$ to $30^\circ\mathrm{C}$ is

Options:

A.

30

B.

20

C.

25

D.

40

Answer: A

Solution:

According to Newton's law of cooling.

$\frac{T_1-T_2}{t}=K\left(\frac{T_1+T_2}{2}-T_0\right)$

For the first case

$$\frac{60-50}{10}=K\left(\frac{60+50}{2}-T_0\right)$$
$$1=K\left(55-T_0\right) \qquad \ldots(i)$$

For the second case

$$\frac{50-40}{15}=K\left(\frac{50+40}{2}-T_0\right)$$
$$\Rightarrow \quad \frac{10}{15}=K\left(45-T_0\right)$$
$$\frac{2}{3}=K\left(45-T_0\right) \qquad \ldots(ii)$$

Dividing Eqs. (i) by Eq. (ii), we get

$$\frac{1}{2/3}=\frac{55-T_0}{45-T_0}$$
$$\frac{3}{2}=\frac{55-T_0}{45-T_0}$$
$$\Rightarrow \quad T_0=25^{\circ}\mathrm{C}$$

$\therefore$ From Eq. (i), $1=K(55-25)$,

$\Rightarrow K=\frac{1}{30}$

If t be the time taken to cool from $40^{\circ}\mathrm{C}$ to $30^{\circ}\mathrm{C}$, then

$$\frac{40-30}{t}=K\left(\frac{40+30}{2}-T_0\right)$$
$$\frac{10}{t}=\left(\frac{1}{30}\right)(35-25)$$
$$\Rightarrow t=30 \text{ minutes}$$

Question16

When the temperature of a gas in a closed vessel is increased by $2.4^{\circ}\mathrm{C}$, its pressure increases by 0.5%. The initial temperature of the gas is

Options:

A.

$120^{\circ}\mathrm{C}$

B.

$240^{\circ}\mathrm{C}$

C.

$480^{\circ}\mathrm{C}$

D.

$207^{\circ}\mathrm{C}$

Answer: D

Solution:

According to Gay-Lussac's law,

$$\begin{aligned} & \frac{P_1}{T_1}=\frac{P_2}{T_2} \\ \Rightarrow \quad & \frac{P_1}{T_1}=\frac{P_1+0.5\% \text{ of } P_1}{T_1+2.4} \\ \Rightarrow \quad & \frac{P_1}{T_1}=\frac{1.005 P_1}{T_1+2.4} \Rightarrow T_1+2.4=1.005 T_1 \\ \Rightarrow \quad & 0.005 T_1=2.4 \\ \Rightarrow \quad & T_1=\frac{2.4}{0.005}=\frac{2400}{5} \\ & \ =480 \mathrm{~K}=480-273^{\circ} \mathrm{C} \\ & \ =207^{\circ} \mathrm{C} \end{aligned}$$

Question17

A gas is suddenly compressed such that its absolute temperature is doubled. If the ratio of the specific heat capacities of the gas is 1.5 , then the percentage decrease in the volume of the gas is

Options:

A.

30

B.

50

C.

25

D.

75

Answer: D

Solution:

For adiabatic process,

$$\begin{aligned} & T_1 V_1^{\gamma-1}=T_2 V_2^{\gamma-1} \\ & T_1 V_1^{1.5-1}=\left(2 T_1\right) V_2^{(1.5-1)} \\ & V_1^{0.5}=2 V_2^{0.5} \\ & \Rightarrow \quad V_1=4 V_2 \Rightarrow V_2=\frac{V_1}{4} \end{aligned}$$

$\therefore$ Percentage decrease in the volume

$= \frac{V_1 - V_2}{V_1} \times 100$

$= \frac{V_1 - \frac{V_1}{4}}{V_1} \times 100 = 75\%$

Question18

If the heat required to increase the rms speed of 4 moles of a diatomic gas from v to $\sqrt{3}v$ is 83.1 kJ , then the initial temperature of the gas is

(universal gas constant $= 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$)

Options:

A.

$377^{\circ}\mathrm{C}$

B.

$327^{\circ}\mathrm{C}$

C.

$227^{\circ}\mathrm{C}$

D.

$277^{\circ}\mathrm{C}$

Answer: C

Solution:

Step 1: How rms speed relates to temperature

The root mean square (rms) speed of a gas, V_{rms}, increases as the square root of its temperature (T). So, $V_{\text{rms}} \propto \sqrt{T}$.

Step 2: Setting up the ratio

Since $V_{\text{rms}}^2 \propto T$, we can write:

$\left[\frac{(V_{\text{rms}})_1}{(V_{\text{rms}})_2}\right]^2 = \frac{T_1}{T_2}$

Step 3: Substituting given values

The rms speed increases from v to $\sqrt{3}v$. So:

$\left(\frac{v}{\sqrt{3}v}\right)^2 = \frac{T_1}{T_2}$

This simplifies to $\frac{1}{3} = \frac{T_1}{T_2}$, so $T_2 = 3T_1$.

Step 4: Calculating heat required

The heat (Q) needed to raise the temperature is:

$Q = nC_V \Delta T$

For 4 moles of a diatomic gas, $C_V = \frac{5}{2}R$, so:

$Q = 4 \times \frac{5}{2}R(T_2 - T_1)$

We know $T_2 = 3T_1$, so $\Delta T = 3T_1 - T_1 = 2T_1$.

Step 5: Plugging in the numbers

We are given $Q = 83.1$ kJ $= 83,100$ J and $R = 8.31$ J mol^{-1}K^{-1}.

$83,100 = 4 \times \frac{5}{2} \times 8.31 \times 2T_1$

$4 \times \frac{5}{2} \times 2 = 20$, so:

$83,100 = 20 \times 8.31 \times T_1$

$20 \times 8.31 = 166.2$, so: $83,100 = 166.2 \times T_1$

$T_1 = \frac{83,100}{166.2} = 500$ K

Step 6: Converting to Celsius

T_1 in Celsius = $500 - 273 = 227°$C

Question19

If the lengths of the open and closed pipes are in the ratio of $2:3$, then the ratio of the frequencies of the third harmonic of the open pipe and the fifth harmonic of the closed pipe is

Options:

A.

3 : 5

B.

9 : 5

C.

2 : 3

D.

4 : 9

Answer: B

Solution:

Step 1: Write the formula for frequency in an open pipe

The frequency of the nth harmonic in an open organ pipe is:

$f_0 = \frac{nv}{2l_0}$

Step 2: Find the frequency for the 3rd harmonic in the open pipe

For the third harmonic, $n = 3$:

$f_0' = \frac{3v}{2l_0}$

Step 3: Write the formula for frequency in a closed pipe

The frequency of the nth harmonic in a closed pipe is:

$f_c = \frac{nv}{4l_c}$

Step 4: Find the frequency for the 5th harmonic in the closed pipe

For the fifth harmonic, $n = 5$:

$f_c' = \frac{5v}{4l_c}$

Step 5: Write the ratio of the two frequencies

We want to find the ratio $\frac{f_0'}{f_c'}$.

$\frac{f_0'}{f_c'} = \frac{\frac{3v}{2l_0}}{\frac{5v}{4l_c}}$

Step 6: Simplify the ratio

First, simplify by dividing:

$\frac{3v}{2l_0} \times \frac{4l_c}{5v} = \frac{12l_c}{10l_0}$

Step 7: Use the ratio of lengths

It is given that the ratio of lengths of the open and closed pipes is $2:3$, so $\frac{l_0}{l_c} = \frac{2}{3}$.

Step 8: Substitute the value for $\frac{l_0}{l_c}$

$\frac{12}{10} \times \frac{1}{\frac{l_0}{l_c}} = \frac{12}{10} \times \frac{1}{\frac{2}{3}} = \frac{12}{10} \times \frac{3}{2} = \frac{36}{20} = \frac{9}{5}$

Step 9: State the final ratio

The ratio of the frequencies is 9 : 5.

Question20

The equation of a transverse wave propagating on a stretched string is given by $y = 3\sin(4x + 200t)$, where x and y are in metre and the time ' t ' is in second. If the tension applied to the string is 500 N , the linear density of the string is

Options:

A.

0.25 kg m^{-1}

B.

0.4 kg m^{-1}

C.

0.2 kg m^{-1}

D.

0.1 kg m^{-1}

Answer: C

Solution:

The standard form of a travelling wave is

$y = A\sin(kx \pm \omega t)$

Comparing

$y = 3\sin(4x + 200t)$
$k = 4\text{rad/m}$
$\omega = \text{rad/sec}$

wave speed is given by

$v = \frac{\omega}{k} = \frac{200}{4} = 50 \text{ m/s}$

Wave speed for string

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow \mu = \frac{T}{v^2}$$
$$\mu = \frac{500}{(50)^2} = \frac{500}{2500} = 0.2 \text{ kg/m}$$

Question21

A compound microscope has an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm separated by a distance of 7.5 cm . The total magnification produced by the microscope when the final image forms at infinity is

Options:

A.

6.25

B.

30

C.

120

D.

72.5

Answer: A

Solution:

Given, the focal length of the objective lens is $f_0 = 1.25 \text{ cm}$

The focal length of the eyepiece is

$f_e = 5 \text{ cm}$

The separation between the objective and eyepiece is $L = 7.5 \text{ cm}$

The distance of this image from the objective lens is

$v_0 = L - f_e$
$v_0 = 7.5 \text{ cm} - 5 \text{ cm} = 2.5 \text{ cm}$

The lens formula for the objective lens

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$
$$\frac{1}{u_0} = \frac{1}{2.5 \text{ cm}} - \frac{1}{1.25 \text{ cm}}$$
$$u_0 = -2.5 \text{ cm}$$

The magnification of the objective lens is

$M_0 = -\frac{v_0}{u_0} = \frac{-2.5 \text{ cm}}{-2.5 \text{ cm}} = 1$

The magnification of the eyepiece when the final image is at infinity is $M_e = \frac{D}{f_e}$

$M_e = \frac{25 \text{ cm}}{5 \text{ cm}} = 5$

The total magnification is $M = M_0 \times M_e$

$M = 1 \times 5 = 5$

Question22

The property of light that explains the formation of coloured images due to thick lenses is

Options:

A.

refraction

B.

dispersion

C.

reflection

D.

total internal reflection

Answer: B

Solution:

Dispersion is the property of light that causes the separation of light into its component colours when passing through a medium, such as thick lens. This happens because different colours(wavelengths) of light are refracted by different amount.

Question23

For an aperture of 5×10^{-3} m and a monochromatic light of wavelength λ, the distance for which ray optics becomes a good approximation is 50 m , then $\lambda =$

Options:

A.

$5000\overset{o}{A}$

B.

$6000\overset{o}{A}$

C.

$5400\overset{o}{A}$

D.

$6500\overset{o}{A}$

Answer: A

Solution:

According to Fresnel distance

$$Z_F=\frac{a^2}{\lambda}$$

$$\begin{aligned}\lambda=\frac{a^2}{Z_F}&=\frac{\left(5\times10^{-3}\right)^2}{50}\\&=0.5\times10^{-6}\text{ m}\\&=5000\times10^{-10}\text{ m}\\&=5000\overset{o}{A}\end{aligned}$$

Question24

An electron and a positron enter a uniform electric field E perpendicular to it with equal speeds at the same time. The distance of separation between them in the direction of the field after a time ' t ' is

($\frac{e}{m}$ is specific charge of electron)

Options:

A.

$\frac{2Eet^2}{m}$

B.

$\frac{Eet^2}{m}$

C.

$\frac{Eet^2}{2m}$

D.

zero

Answer: B

Solution:

Acceleration of electron in electric field is

$a_e=\frac{F_e}{m}=-\frac{eE}{m}$

Acceleration of positron in electric field.

$a_p=\frac{F_p}{m}=\frac{eE}{m}$

Displacement of electron in electric field

$y_e = 0 \times t + \frac{1}{2} a_e t^2 = -\frac{1}{2} \frac{eE}{m} t^2$

Displacement of positron in electric field

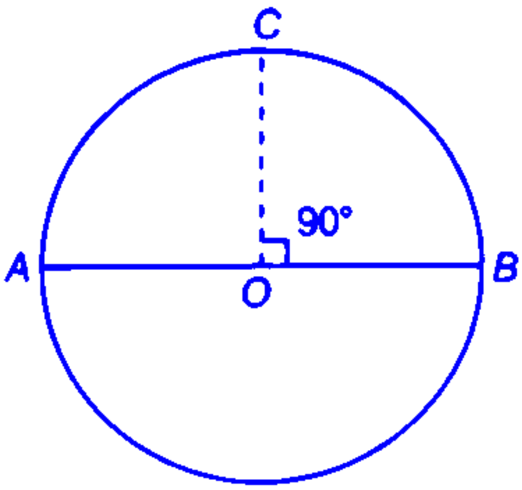
$y_p = \frac{1}{2} a_p t^2 = \frac{1}{2} \frac{eE}{m} t^2$

The distance of separation between electron and positron

$$\begin{aligned} &= |y_p - y_e| \\ &= \left| \frac{\frac{1}{2} eEt^2}{m} - \left(-\frac{1}{2} \frac{eEt^2}{m} \right) \right| \\ &= \left| \frac{eEt^2}{m} \right| = \frac{eEt^2}{m} \end{aligned}$$

Question25

A charge q is placed at the centre ' O ' of a circle of radius R and two other charges q and q are placed at the ends of the diameter AB of the circle. The work done to move the charge at point B along the circumference of the circle to a point C as shown in the figure is



Options:

A.

$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{R}(\sqrt{2})$

B.

zero

C.

$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{R}\left(\frac{\sqrt{2}-1}{2}\right)$

D.

$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{R}\left(\frac{1}{\sqrt{2}}\right)$

Answer: C

Solution:

The potential at points B and C due to the charges at O and A

Potential at B

$V_B = \frac{q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 2R}$

Potential at C

$V_c = \frac{q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 \sqrt{2}R}$

Work done

$$\begin{aligned} W &= q\left(V_C - V_B\right) \\ &= q\left[\begin{array}{l}\left(\frac{q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0\sqrt{2}R}\right) \\ -\left(\frac{q}{4\pi\varepsilon_0 R} - \frac{q}{4\pi\varepsilon_0 2R}\right)\end{array}\right] \\ &= \frac{q^2}{4\pi\varepsilon_0 R}\left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right] \\ &= \frac{q^2}{4\pi\varepsilon_0 R}\left[\frac{\sqrt{2}-1}{2}\right] \end{aligned}$$

Question26

In a potentiometer experiment, a wire of length 10 m and resistance 5Ω is connected to a cell of emf 2.2 V . If the potential difference between two points separated by a distance of 660 cm on potentiometer wire is 1.1 V , then the internal resistance of the cell is

Options:

A.

1.6Ω

B.

1.4Ω

C.

1.2Ω

D.

1Ω

Answer: A

Solution:

The length of the potentiometer wire is $L = 10$ m.

The resistance of the potentiometer wire is

$R_\omega = 5\Omega$

The emf of the cell is $E = 2.2$ V

The potential difference across a length of $l = 660$ cm of the wire is $V_l = 1.1$ V.

$l = 660 \text{ cm} = 6.6 \text{ m}$

The potential gradient, $K = \frac{V_l}{l}$

$K = \frac{1.1 \text{ V}}{6.6 \text{ m}} = \frac{1}{6} \text{ V/m}$

The total potential drop across the wire V_ω is

$$V_\omega = K \times L$$
$$V_\omega = \frac{1}{6} \text{ V/m} \times 10 \text{ m} = \frac{10}{6} \text{ V} = \frac{5}{3} \text{ V}$$

Current, $I = \frac{V_\omega}{R\omega} = \frac{\frac{5}{3} \text{ V}}{5\Omega} = \frac{1}{3} \text{ A}$

The current in the main circuit is also given by

$$I = \frac{E}{R_\omega + r}$$
$$r = \frac{E}{I} - R_\omega = \frac{2.2 \text{ V}}{\frac{1}{3} \text{ A}} - 5\Omega$$
$$r = (2.2 \times 3)\Omega - 5\Omega$$
$$r = 6.6\Omega - 5\Omega = 1.6\Omega$$

Question27

When the right gap of a metre bridge consists of two equal resistors in series, the balancing point is at 50 cm . When one of the resisters in the right gap is removed and is connected in parallel to the resistor in the left gap, the balancing point is at

Options:

A.

60 cm

B.

33.3 cm

C.

25 cm

D.

40 cm

Answer: D

Solution:

First setup:

Let the resistance in the left gap be R', and in the right gap there are two resistors with resistance R, connected in series. So, the right gap has a total resistance of $R + R = 2R$.

The balancing point is at 50 cm. In a metre bridge, the ratio of resistances equals the ratio of the lengths on both sides: $\frac{R'}{2R} = \frac{50}{50}$ So, $\frac{R'}{2R} = 1$ which means $R' = 2R$

Second setup:

Now, one resistor from the right gap is taken out and connected in parallel with the resistor in the left gap.

Now, the left gap contains R' and R in parallel. The resistance of two resistors A and B in parallel is: $R'' = \frac{A \times B}{A+B}$ The parallel resistance is: $R'' = \frac{R' \times R}{R'+R}$

From earlier, we know $R' = 2R$. Substitute this value: $R'' = \frac{2R \times R}{2R+R} = \frac{2R^2}{3R} = \frac{2R}{3}$

Now, the right gap just has R.

Let the new balancing point be l. The bridge principle gives us: $\frac{R''}{R} = \frac{l}{100-l}$ Substitute $R'' = \frac{2R}{3}$: $\frac{\frac{2R}{3}}{R} = \frac{l}{100-l}$ $\frac{2}{3} = \frac{l}{100-l}$

Solve for l: $3l = 2(100-l)$ $3l = 200 - 2l$ $3l + 2l = 200$ $5l = 200$ $l = 40$ cm

----

Question28

Two identical wires, carrying equal currents are bent into circular coils A and B with 2 and 3 turns respectively. The ratio of the magnetic fields at the centres of the coils A and B is

Options:

A.

4 : 9

B.

2 : 3

C.

9 : 4

D.

3 : 2

Answer: A

Solution:

The length of the wire L is constant for both coils

For coil $A, L = N_A \times 2\pi R_A$

For coil $B, L = N_B \times 2\pi R_B$

$N_A \times 2\pi R_A = N_B \times 2\pi R_B$
$N_A R_A = N_B R_B$

So, $\frac{R_A}{R_B} = \frac{N_B}{N_A}$

Magnetic field at the centre of coil A

$B_A = \frac{\mu_0 N_A I}{2R_A}$

Magnetic field at the centre of coil B

$B_B = \frac{\mu_0 N_B I}{2R_B}$

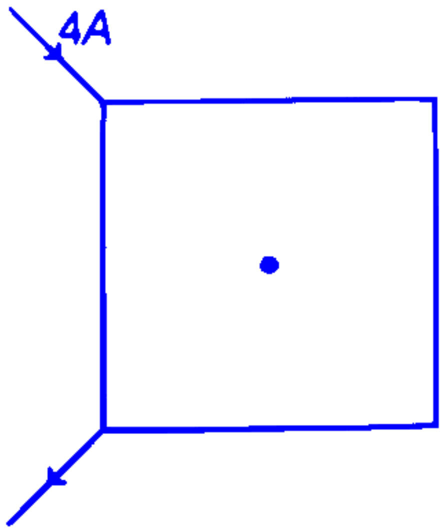
The ratio is $\frac{B_A}{B_B} = \frac{\frac{\mu_0 N_A I}{2R_A}}{\frac{\mu_0 N_B I}{2R_B}}$

$$\frac{B_A}{B_B} = \frac{N_A}{N_B} \times \frac{R_B}{R_A} = \left(\frac{N_A}{N_B}\right) \cdot \left(\frac{N_A}{N_B}\right) = \left(\frac{N_A}{N_B}\right)^2$$
$$\frac{B_A}{B_B} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

The ratio of the magnetic fields at the centres of coils A and B is 4 : 9.

Question29

A current of 4 A is passed through a square loop of side 5 cm made of a uniform manganin wire as shown in the figure. The magnetic field at the centre of the loop is



Options:

A.

$\frac{24\sqrt{2}}{5} \times 10^{-5}$ T

B.

$\frac{3\sqrt{2}}{5} \times 10^{-5}$ T

C.

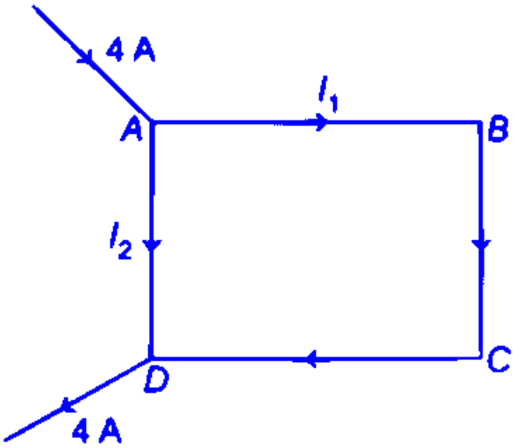
$\frac{6\sqrt{2}}{5} \times 10^{-5}$ T

D.

zero

Answer: D

Solution:

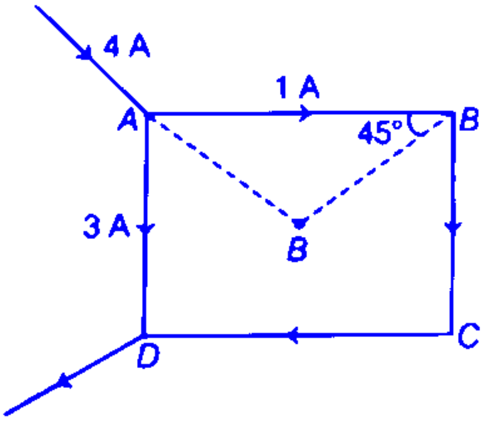


$$V_{AD} = I_1 \cdot R_{ABCD} = I_2 R_{AD}$$
$$\Rightarrow \quad I_1 \times 3R = I_2 \times R \Rightarrow I_2 = 3I_1$$

Also $I_1 + I_2 = 4$ A (given)

$\Rightarrow \quad 4I_1 = 4$ A or $I_1 = 1$ A

So, we have following situation,



$\therefore$ Magnetic field B at centre

$= B_{AB} + B_{BC} + B_{CD} - B_{AD}$

Here, $B_{AB} = B_{BC} = B_{CD}$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{r} \times 2\sin\theta \quad [\theta = 45^\circ]$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{1}{2.5 \times 10^{-2}} \times \frac{2}{\sqrt{2}}$$
$$= \frac{10^{-7} \times \sqrt{2}}{25 \times 10^{-2}} = 4\sqrt{2} \times 10^{-6} \text{ T}$$

And $B_{AD} = 3 \times 4\sqrt{2} \times 10^{-6}$
$$= 12\sqrt{2} \times 10^{-6} \text{ T}$$
$$\therefore \quad B_{\text{centre}} = 3 \times \left(4\sqrt{2} \times 10^{-6}\right) - \left(12\sqrt{2} \times 10^{-6}\right)$$
$$= 0$$

Question30

If B_V and B_H are respectively the vertical and horizontal components of the Earth's magnetic field at a place where the angle of dip is 60°, then the total magnetic field at that place is

Options:

A.

$\sqrt{3} B_H$

B.

$\sqrt{3} B_v$

C.

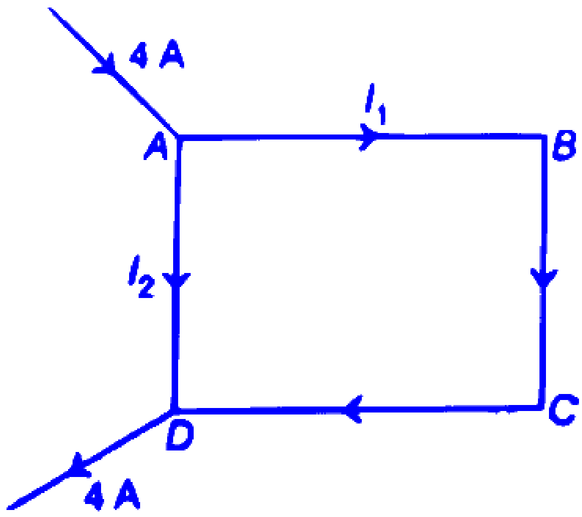
$\frac{2}{\sqrt{3}} B_v$

D.

$\frac{\sqrt{3}}{2} B_H$

Answer: C

Solution:



$$B_H = B \cos 60^{\circ}$$
$$B_H = \frac{B}{2}$$
$$B_V = B \sin 60^{\circ}$$
$$B_V = \frac{\sqrt{3}}{2} B \Rightarrow B = \frac{2}{\sqrt{3}} B_V$$

Question31

A coil of resistance 8Ω, number of turns 250 and area $120 \mathrm{~cm}^2$ is placed in a uniform magnetic field of 2 T such that the plane of the coil makes and angle of $\frac{\pi}{6}$ with the direction of the magnetic field. In a time of 100 ms , the coil is rotated until its plane becomes parallel to the direction of the magnetic field. The current induced in the coil is

Options:

A.

5.25 A

B.

3.75 A

C.

2.75 A

D.

1.25 A

Answer: B

Solution:

Induced emf

$$
\begin{aligned}
E &= -\frac{N \Delta \phi}{\Delta t}=\frac{-N\left(\phi_2-\phi_1\right)}{\Delta t} \\
&= \frac{-N\left(B A \cos \theta_2-B A \cos \theta_1\right)}{\Delta t} \\
&= -250 \frac{\left(B A \cos 90^{\circ}-B A \cos 60^{\circ}\right)}{100 \times 10^{-3}} \\
&= \frac{-250\left(-\frac{B A}{2}\right)}{100 \times 10^{-3}} \\
&= 2500 \times \frac{B A}{2}=1250 \times B A \\
&= 1250 \times 2 \times 120 \times 10^{-4}=30 \text{volt}
\end{aligned}
$$

$\therefore$ Induced current

$I=\frac{E}{R}=\frac{30}{8}=3.75$ A

Question32

An inductor and a resistor are connected in series to an AC supply. If the potential difference across the inductor and the resistor are 180 V and 240 V respectively, then the voltage of the AC supply is

Options:

A.

300 V

B.

420 V

C.

60 V

D.

210 V

Answer: A

Solution:

- In an **L-R series circuit**,
- Voltage across resistor: $V_R = 240\,\text{V}$
- Voltage across inductor: $V_L = 180\,\text{V}$

We are asked to find the **supply voltage** V.

Step 1: Relation between voltages in series L-R circuit

In a series L-R circuit, the resistor voltage V_R and the inductor voltage V_L are **90° out of phase** (since V_R is in phase with current, and V_L leads the current by 90°).

Hence, the total voltage is obtained by the **vector sum**:

$$V = \sqrt{V_R^2 + V_L^2}$$

Step 2: Substitute given values

$$V = \sqrt{(240)^2 + (180)^2}$$

$$V = \sqrt{57600 + 32400} = \sqrt{90000} = 300\,\text{V}$$

✅ **Final Answer:**

300 V

Option A

Question33

If electromagnetic waves of power 600 W incident on a non-reflecting surface, then the total force acting on the surface is

Options:

A.

$12 \times 10^{-6}\,\text{N}$

B.

$9 \times 10^{-9}\,\text{N}$

C.

$6 \times 10^{-6}\,\text{N}$

D.

$2 \times 10^{-6}\,\text{N}$

Answer: D

Solution:

When light waves with a power of 600 W hit a surface that absorbs all the light (does not reflect any), the force on the surface can be found using a formula.

The formula is: $F = \frac{P}{C}$

Here, F is the force, P is the power (600 W), and C is the speed of light (3×10^8 m/s).

Now substitute the values: $F = \frac{600}{3 \times 10^8}$

Solve this to get: $F = 2 \times 10^{-6}$ Newton

Question34

When a photosensitive material is illuminated by photons of energy 3.1 eV , the stopping potential of the photoelectrons is 1.7 V . When the same photosensitive material is illuminated by photons of energy 2.5 eV , the stopping potential of the photoelectrons is

Options:

A.

1.8 V

B.

1.4 V

C.

1.1 V

D.

1.3 V

Answer: C

Solution:

When light hits a photosensitive material, electrons are released. The energy of the incoming light (photon) is used to release these electrons and provide them with some extra energy.

Einstein's photoelectric equation helps us understand this process:

$E_{\max} = h\nu - \phi_0$

This means: the maximum energy of the emitted electrons equals the energy of the incoming photon minus the work function (ϕ_0), which is the energy needed to release the electron.

The stopping potential (V_0) is linked to this energy by:

$eV_0 = h\nu - \phi_0$

or

$V_0 = \frac{h\nu}{e} - \frac{\phi_0}{e}$

First, use the given values for the first case:

For photon energy 3.1 eV and stopping potential 1.7 V:

$V_0 = 3.1 - \frac{\phi_0}{e}$

$1.7 = 3.1 - \frac{\phi_0}{e}$

To find the work function divided by e:

$\frac{\phi_0}{e} = 3.1 - 1.7 = 1.4$

So, $\frac{\phi_0}{e} = 1.4$ volt

Now, use this value for the second case:

For photon energy 2.5 eV:

$V_0 = 2.5 - \frac{\phi_0}{e}$

$V_0 = 2.5 - 1.4 = 1.1$ V

Question35

The ratio of the kinetic energies of the electrons in the third and fourth excited states of hydrogen atom is

Options:

A.

4 : 3

B.

16 : 9

C.

25 : 16

D.

5 : 4

Answer: C

Solution:

The kinetic energy (KE) of an electron in a hydrogen atom depends on the energy level (n) it is in.

Here is the formula for kinetic energy at level n:

$\mathrm{KE}_n = \frac{13.6}{n^2}$ eV

Step 1: Identify the energy levels

The "third excited state" means $n = 4$, and the "fourth excited state" means $n = 5$.

Step 2: Calculate the kinetic energy for $n = 4$

$\mathrm{KE}_3 = \frac{13.6}{4^2} = \frac{13.6}{16}$ eV

Step 3: Calculate the kinetic energy for $n = 5$

$\mathrm{KE}_4 = \frac{13.6}{5^2} = \frac{13.6}{25}$ eV

Step 4: Find the ratio of kinetic energies

$\frac{\mathrm{KE}_3}{\mathrm{KE}_4} = \frac{\frac{13.6}{16}}{\frac{13.6}{25}} = \frac{25}{16}$

So, the ratio of kinetic energies is 25 : 16.

Question36

In β^-decay, a neutron transforms into a proton within the nucleus according to the equation :

neutron $\rightarrow$ proton $+\beta^- + x$

In this equation the particle represented by ' x ' is

Options:

A.

Neutrino

B.

Anti neutrino

C.

Positron

D.

Meson

Answer: B

Solution:

In β^--decay, a neutron transforms into a proton, an electron (which is the β^-particle) and an antinutrino ($\overline{\mathrm{v}}$)

Question37

Two radioactive substances A and B have same number of initial nuclei. If the half lives of A and B are 1.5 days and 4.5 days respectively, then the ratio of the number of nuclei remaining in A and B after 9 days is

Options:

A.

1 : 16

B.

1 : 1

C.

1 : 4

D.

1 : 8

Answer: A

Solution:

Step 1: Formula for Remaining Nuclei

We use the formula $N = N_0\left(\frac{1}{2}\right)^n$, where:

- N = number of nuclei left
- N_0 = starting number of nuclei
- n = number of half-lives passed

Step 2: Setting Up the Ratio

The question asks for the ratio of nuclei left in A and B, so: $\frac{N_A}{N_B} = \frac{\left(\frac{1}{2}\right)^{n_A}}{\left(\frac{1}{2}\right)^{n_B}}$

Step 3: Simplifying the Ratio

This simplifies to: $\frac{N_A}{N_B} = \left(\frac{1}{2}\right)^{n_A - n_B}$

Step 4: Finding Number of Half-lives for Each Substance

The number of half-lives n is found by dividing total time by half-life:

- For A: $n_A = \frac{9}{1.5} = 6$
- For B: $n_B = \frac{9}{4.5} = 2$

Step 5: Plugging In the Values

$\frac{N_A}{N_B} = \left(\frac{1}{2}\right)^{6-2} = \left(\frac{1}{2}\right)^4$

Step 6: Final Answer

$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ So, the ratio of the remaining nuclei in A and B after 9 days is $\frac{1}{16}$.

Question38

10^{10} electrons enter the emitter of a junction transistor in a time of 0.4μ s. If 5% of the electrons are lost in the base, then the collector current is

Options:

A.

3.0 mA

B.

3.2 mA

C.

3.6 mA

D.

3.8 mA

Answer: D

Solution:

Given:

- Number of electrons entering emitter per second (or per given time)

  $N_E = 10^{10}$ electrons in $t = 0.4\,\mu s$

- 5% of electrons are **lost in the base**.

So, only 95% of electrons reach the collector.

Step 1: Find the emitter current

Total charge entering emitter in $0.4\,\mu s$:

$Q_E = N_E \times e = 10^{10} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-9}\,C$

Emitter current:

$I_E = \frac{Q_E}{t} = \frac{1.6\times10^{-9}}{0.4\times10^{-6}} = 4.0\,mA$

Step 2: Find the collector current

Since 5% are lost, 95% reach collector:

$I_C = 0.95\,I_E = 0.95 \times 4.0\,mA = 3.8\,mA$

✅ **Final Answer:**

$\boxed{I_C = 3.8\,mA}$

Correct option: D (3.8 mA)

Question39

An electron in n-region of a $p-n$ junction moves towards the junction with a speed of $5 \times 10^5\ \mathrm{ms}^{-1}$. If the barrier potential of the junction is 0.45 V , then the speed with which the electron enters the p-region after penetration through the barrier is

(Charge of the electron $= 1.6 \times 10^{-19}\mathrm{C}$ and mass of the electron $= 9 \times 10^{-31}$ kg)

Options:

A.

$3 \times 10^5\ \mathrm{ms}^{-1}$

B.

$5 \times 10^5\ \mathrm{ms}^{-1}$

C.

$4 \times 10^5\ \mathrm{ms}^{-1}$

D.

$6 \times 10^5\ \mathrm{ms}^{-1}$

Answer: A

Solution:

According to conservation of energy

$$\frac{1}{2}m_e v_i^2 = \frac{1}{2}m_e v_f^2 + eV_b$$

$$\Rightarrow \quad \frac{1}{2}m_e v_f^2 = \frac{1}{2}m_e v_i^2 - eV_b$$

$$\Rightarrow \quad v_f^2 = v_i^2 = \frac{2eV_b}{m_e}$$

$$= \left(5 \times 10^5\right)^2 - \frac{2 \times 1.6 \times 10^{-19} \times 0.4}{9 \times 10^{-31}}$$

$$= 9 \times 10^{10}$$

$$\Rightarrow v_f = \sqrt{9 \times 10^{10}} = 3 \times 10^5 \text{ m/s}$$

Question40

Coaxial cable, a widely used wire medium offers and approximate frequency bandwidth of

Options:

A.

750 GHz

B.

750 Hz

C.

750 MHz

D.

750 kHz

Answer: C

Solution:

The correct answer is:

✅ **Option C: 750 MHz**

Explanation:

Coaxial cable is a high-frequency transmission medium commonly used for cable television, internet, and long-distance communication.

- It typically offers a bandwidth range from a few kilohertz up to several hundred megahertz.
- The **approximate frequency bandwidth** of a standard coaxial cable used in communications is **up to 750 MHz**.
- Actual usable bandwidth depends on cable type, length, and quality (e.g., RG-6, RG-59, etc.).

Hence, **Option C (750 MHz)** is correct.
