Clock and Calendar

CLOCK

The concept of clock is the same as the concept of circular motion. Just like in circular motion, in case of clocks, the hour hand and the minute hand are continuously racing against each other around a circle.

The hour hand travels 5 minutes or 30° in 1 hour. Similarly, the minute hand travels 60 minutes or 360° in 1 hour. Therefore, in one hour, the minute hand moves 55 minutes or 330° more than the hour hand. This can also be called the relative speed of minute hand with respect to the hour hand.

Minute and Hour hand of a clock			
Actual Time	Movement		Relative speed of minute hand with respect to hour hand
	Minute Hand	Hour Hand	
60 minutes	60 minutes or 360°	5 minutes or 30°	55 minutes or 330°

Other Important Concepts

(i) The two hands of a clock coincide once every hour, but in 12 hours they coincide only 11 times. This is because the coincide position at 12 is counted in both 11 to 12 and 12 to 1.

(ii) The hands are at right angles twice in every hour, but in 12 hours, they are at right angles only 22 times. This is because the right-angle position at 3 p.m. is common to both 2 p.m. to 3 p.m. and 3 p.m. to 4 p.m. Similarly, the right-angled position at 9 p.m. is common to both 8 p.m. to 9 p.m. and 9 p.m. to 10 p.m.

(iii) The hands point in the opposite direction once in every hour, but in 12 hours they are opposite only 11 times. This is because, the opposite position of at 6 p.m. is counted in both 5 p.m. to 6 p.m. and 6 p.m. to 7 p.m.

(iv) The hands are said to be in same straight line whenever they are coincident or opposite to each other.

(v) Two hands of a clock will coincide again when the minute hand takes a lead of 60 minutes from the hour hand. The minute hand takes a lead of 55 minutes from hour hand in 60 minutes.

Concept of Incorrect Clocks

Two hands of every correct clock coincide after every $\left(65+\frac{5}{11}\right)$ minutes.

An incorrect clock can either be a fast clock which gains time or a slow clock which loses time.

Fast clock : In fast clock, two hands of a clock coincide in every x minutes, where x is less than $\left(65+\frac{5}{11}\right)$.

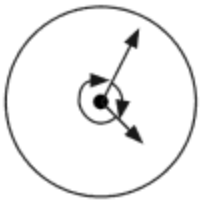
$$\text{Time gained per minute} = \frac{65+\frac{5}{11}-x}{x} \text{ minute}$$

Slow clock : In slow clock, two hands of a clock coincide in every x minutes, where x is greater than $\left(65+\frac{5}{11}\right)$ minutes.

$$\text{Time loss per minute} = \frac{x-\left(65+\frac{5}{11}\right)}{x} \text{ minute}$$

Example 1. Mrs. Veena Gupta goes for marketing between 5 P.M. and 6 P.M. When she comes back, she finds that the hour hand and the minute hand have interchanged their positions. For how much time was she out of her house ?

Sol.: Since two hands are interchange their positions, so sum of the angles subtended at the centre by hour hand and minute hand = 360°



Let us suppose that she was out of house for 't' minutes.

So, the sum of the angles subtended at the centre by the hour hand and minute hand $= (0.5 \times t)° + (6t)°$

$\because \quad 0.5t + 6t = 360$

$\Rightarrow \quad 6.5t = 360 \quad \Rightarrow \quad t = 55.4$ (app.)

Hence required time = 55.4 minutes.

CALENDAR

The topic of calendar includes concepts such as odd days, leap year and finding the day of the day week for a given date.

Leap Year

An end of the century year (the last year of the century, e.g. 1500, 1800, 2000 etc.) is a leap year only if divisible by 400. For all the other years, check the divisibility by 4, and if the year is divisible by 4 it is said to be a *leap year* and will have 366 days. This would mean that the year 1500 and 1800 inspite of being divisible by 4 not a leap year.

Concept of Odd Days : When a given number of days converted into week(s) by dividing 7, then the remaining days which are not covered in week(s) are called *odd days*.

(i) An ordinary year has 365 days, that is 52 weeks and one odd day. This means out of the 365 days in an ordinary year, 364 days will get converted into 52 weeks and one day will remain. This one day is referred to 1 odd day. This means that when we proceed from ordinary year to the next year, the name of the first day of the next year will be

the name of the day which will come just after the first day of previous (ordinary) year.

For example, if 2 October 2015 is a Friday, then 2 October 2016 is a Sunday, that is a shift of two days because 2016 is a leap year.

(ii) A leap year has 366 days, that is 52 weeks and two odd days. This means that when we proceed from leap year to the next year, the name of the first day of the next year will be the name of the day which will come two days after the first day of previous (leap) year.

For example, if 26th January 2016 is a Saturday, then 26th January 2017 would be a Monday, that is a shift of two days because 2016 is a leap year.

(iii) If name of the day of a particular date is given and we have to find the name of the day of an other particular date, then we need to look at not only the years but also the date from which we are moving to the other given date. If the 29th of February falls between the two dates, there will be a shift of two days, otherwise there will be a shift of one day.

Counting Odd Days

1st January AD from where our calendar started was a Monday. That's why our week starting on a Monday. Saturday and Sunday are called **weekends.**

Therefore, if we are calculating from 1st January AD and after converting into weeks, whenever we have 1 odd day left, it would be a Monday. If there are 2 odd days left, then the first one would be a Monday, then second a Tuesday and so on.

First odd day = Monday
Second odd day = Tuesday
Third odd day = Wednesday
Fourth odd day = Thursday
Fifth odd day = Friday
Sixth odd day = Saturday
Seventh or Zeroth odd day = Sunday

In ordinary year,

1 year = 365 days = 52 weeks + 1 day (i.e. one odd day)

In leap year,

1 year = 366 days = 52 weeks + 2 days (i.e. two odd days)

Concept of Total Odd Days in 100, 200, 300 and 400 Years

If we take 100 consecutive years from 1st AD, there will be 24 leap years (remember the 100th year will not be a leap year) and 76 ordinary years.

24 leap years = 24 × 2 odd days = 48 odd days

Now 48 odd days = 6 weeks + 6 odd days

Hence 24 leap years have 6 ood days

76 ordinary years = 76 × 1 odd days = 76 odd days

Now 76 odd days = 10 weeks and 6 odd days

Hence 76 ordinary years have 6 ood days

Now 6 odd days + 6 odd days = 1 week + 5 odd days

Therefore, 100 consecutive years from 1st January AD have 5 odd days.

Similarly,

- 200 consecutive years contain 10 odd days i.e. 3 odd days.
- 300 consecutive years contain 15 odd days i.e. 1 odd day.
- But 400 consecutive years contain (20 + 1) odd days i.e. 0 odd day (this is because 400th year will be a leap year and contribute 1 extra day.)

Note that the last day of a century be one day out of Friday, Wednesday, Monday or Saturday. First day of a new century will be a Saturday, Thursday, Tuesday or Monday.

Example 2. What day of the week was 15th August 1949?

Sol.: 15th August 1949 means

1948 complete years + first 7 months of the year

1949 + 15 days of August.

1600 years give no odd days.

300 years give 1 odd day.

48 years give {48 + 12} = 60 = 4 odd days.

[∵ For ordinary years → 48 odd days and for leap year 1 more day; (48 ÷ 4) = 12 leap year; 60 = 7 × 8 + 4]

From 1st January to 15th August 1949, Odd days :

January → 3, February → 0, March → 3, April → 2,

May → 3, June → 2, July → 3, August → 1.

17 ⇒ 3 odd days.

∴ 15th August 1949 → 1 + 4 + 3 = 8 = 1 odd day.

This means that 15th Aug. fell on 1st day of the week. Therefore, the required day was Monday.

Example 3. How many times does the 29th day of the month occur in 400 consecutive years?

Sol.: In 400 consecutive years, there are 97 leap years. Hence, in 400 consecutive years, February has the 29th day 97 times and the each remaining eleven months have the 29th day

400 × 11 = 4400 times

∴ The 29th day of the month occurs (4400 + 97) or 4497 times.

EXERCISE

1. Time appears in the mirror 11 : 09 Then what time will be appear in clock?
(a) 1 : 51 (b) 12 : 09 (c) 12 : 51 (d) 1 : 09

2. If reflecting time is 3 :43 then the real time of clock is?
(a) 3 : 17 (b) 7 : 17 (c) 8 : 17 (d) 8 : 43

3. What angle is made by minute and hour hand at 4 : 12?
(a) 66° (b) 44° (c) 54° (d) 60.5°

4. At what time between 6 to 7 O' clock minute and hour hand will coincide?
(a) $6 : 38\frac{2}{11}$ (b) $6 : 43\frac{7}{11}$
(c) $6 : 32\frac{8}{11}$ (d) $6 : 5\frac{5}{11}$

5. At what time between 3 to 4 O' clock minute and hour hand are opposite to each other?

(a) $3:43\frac{7}{11}$ (b) $3:38\frac{2}{11}$ (c) $3:49\frac{1}{11}$ (d) $3:54\frac{6}{11}$

6. When did the minute and hour hand makes 180° angle between 6 to 7 O' clock?

(a) $6:54\frac{6}{11}$ (b) 6 : 60 (c) 6 : 00 (d) $6:5\frac{5}{11}$

7. At what time between 8 to 9 O' clock the minute and hour will apart 7 minutes to each other?

(a) $8:42, 8:51\frac{3}{11}$ (b) $8:36, 8:51\frac{3}{11}$

(c) $8:09, 8:47\frac{4}{11}$ (d) $8:17, 8:28\frac{9}{11}$

8. The minute hand of a clock overtakes the hour hand at in tervals of 64 minutes of correct time. How much a day does the clock gain or lose?

(a) $43\frac{9}{11}$ minute loss (b) $32\frac{8}{11}$ minute gain

(c) $33\frac{9}{11}$ minute gain (d) $32\frac{8}{11}$ minute loss

9. The minute hand of a clock overtakes the hour hand at intervals of 66 minute of correct time. How much a day does the clock gain or lose?

(a) $11\frac{109}{121}$ minute gain (b) $11\frac{109}{121}$ minute loss

(c) $11\frac{117}{121}$ minute gain (d) $11\frac{117}{121}$ minute loss

10. A watch which gains uniformaly is 4 minute slow at 9 A.M on Sunday and is 4 minute 15 sec. fast at 9 P.M on upcoming Friday. When was it correct?

(a) 2 A.M Thursday (b) 6 P.M Wednesday
(c) 1 A.M Wednesday (d) 6 P.M Thursday

11. A watch which loses uniformaly is 3 minute fast at 6 A.M. on Thursday and is 3 minute 12 sec. slow at 5 P.M. on upcoming Wednesday. When was it correct?

(a) 9 P.M on Sunday (b) 9 A.M on Monday
(c) 9 A.M on Sunday (d) 8 A.M on Sunday

12. By looking in a mirror it ap-pears that it is 6 : 30 in the clock. What is the real time?

(a) 6 : 30 (b) 5 : 30 (c) 6 : 00 (d) 5 : 00

13. After 9'O clock at what time between 9 p.m. and 10 p.m will the hour and minute hands of a clock point in opposite direction?

(a) 15 minutes past 9 (b) 16 minutes past 9

(c) $16\frac{4}{11}$ minutes past 9 (d) $17\frac{1}{11}$ minutes past 9

14. At what time are the hand of clocks together between 6 and 7?

(a) $32\frac{8}{11}$ minutes past 6 (b) $34\frac{8}{11}$ minutes past 6

(c) $30\frac{8}{11}$ minutes past 6 (d) $32\frac{5}{7}$ minutes past 6

15. A clock only with dots marking 3, 6, 9, and 12 O' clock positions has been kept upside down in front of a mirror. A person reads the time in the reflection of the clock as 10:20. What is the actual time?

(a) 07: 10 (b) 02 : 40 (c) 04 : 50 (d) 10 : 20

16. A clock goes slow from midnight by 5 min. at the end of the first hour, by 10 min. at the end of the second hour, by 15 min. and the end of the 3rd hour and so on. What will be the time by this clock after 6 hours?

(a) 6 : 00 am (b) 5 : 30 am
(c) 6 : 30 am (d) 5 : 15 am

17. A clock goes fast by one minute during the first hour, by two minutes at the end of the second hour, by 4 minutes at the end of 3rd hour, by eight minutes by the end of 4th hour, and so on. At the end of which hour, will it be fast by just over sixty minutes?

(a) Fifth (b) Sixth (c) Seventh (d) Eighth

18. A clock with only dots marking 3,6,9 and 12 positions has been kept upside down in front of a mirror. A person reads the time in the reflection as 9.50. What is the actual time?

(a) 2: 15 (b) 8 : 40 (c) 8 : 50 (d) 4 : 15

19. A clock with only dots marking 3, 6, 9, and 12 positions has been upside down in front of a mirror. A person reads the time in the reflection as 6 : 10 The real time is:

(a) 06 : 50 (b) 12 : 40 (c) 11 : 20 (d) 6 : 10

20. A clock with only dot markings 3, 6, 9 and 12 positions has been kept upside down in front of a mirror. A person reads the time in the reflections of the clock as 12 : 30 the actual that will be

(a) 12 O'clock (b) 12 : 30
(c) 6 O' clock (d) 03 : 45

21. If 50 minutes ago, it was 45 minutes past four O' clock, how many minutes is it until six O' clock?

(a) 45 (b) 15 (c) 25 (d) 35

22. What was the day of week on 2nd October 1869?

(a) Friday (b) Saturday (c) Sunday (d) Monday

23. On what dates of May 1993 did Sunday fall?

(a) 1, 8, 15, 22, 29 (b) 2, 9, 16, 23, 30
(c) 3, 10, 17, 24, 31 (d) 4, 11, 18, 25

24. If the 3rd day of a month is Tuesday, which of the following will be the 6th day from 23rd of that month?

(a) Sunday (b) Saturday
(c) Thursday (d) Friday

25. If the 27th day of a month is Friday, which of the following will be the 4th day of that month?

(a) Sunday (b) Saturday
(c) Wednesday (d) Friday

Hints & Solutions

1. **(c)** Because the time 11:09 lies between 11:00 to 1:00, Hence we subtract that time from 23:60
23:60 –11:09 = 12:51

2 **(c)** Because the time 3:43 lies between 1:00 to 11:00. Hence, we subtract that time from 11:60
11:60 – 3:43 = 8:17

3. **(c)** Formula for Angle = H × 30 = F°

$M \times \frac{11}{2} = \gamma°$

(Subtract smaller from larger)

Hence H = 4, M = 12; $4 \times 30 = 120°$

$12 \times \frac{11}{2} = 66°$; $120° - 66° = 54°$

4. (c) Consider means O° Angle By unique Formula

$= H : \left(H \times 5 \pm \frac{\text{Angle}}{6}\right) \times \frac{12}{11}$

Angle = 0°, h = 6 then $= 6 : \left(6 \times 5 \pm \frac{0}{6}\right) \times \frac{12}{11}$

$= 6 : (30 \pm 0) \times \frac{12}{11} = 6 : \frac{360}{11} = 6 : 32\frac{8}{11}$

5. (c) By Unique Formula

H = 3, Angle = 180º

Note : Hands are opposite means 180°

$3 : \left(3 \times 5 \pm \frac{180}{6}\right) \times \frac{12}{11}$

$3 : (15 \pm 30) \times \frac{12}{11}$

$3 : (15 + 30) \times \frac{12}{11}, 3 : (15 - 30) \times \frac{12}{11}$

$3 : (45) \times \frac{12}{11}, 3 : (-15) \times \frac{12}{11}$

$3 : \frac{540}{11}, 3 : \left(\frac{-180°}{11}\right)$

Angle = 180° = $3 : 49\frac{1}{11}$

6. (c) By unique formula

H = 6 Angle = 180°

$6 : \left(6 \times 5 \pm \frac{180}{6}\right) \times \frac{12}{11}$

$6 : (30 \pm 30) \times \frac{12}{11}$

$6 : (30 + 30) \times \frac{12}{11}, 6 : (30 - 30) \times \frac{12}{11}$

$6 : (60) \times \frac{12}{11}, 6 : (0) \times \frac{12}{11}$

$6 : \frac{720}{11}, 6 : 00$

Not possible

Note : Minute and hour hand does not makes 180°. Angle between 5 to 6 and 6 to 7 O' clock. It makes it correct at 6 O' clock.

7. (b) $\frac{\text{Angle}}{6} = \text{Minutes}$

By unique formula

$8 : (8 \times 5 \pm 7) \times \frac{12}{11}$

$8 : (40 \pm 7) \times \frac{12}{11}$

$8 : (40 + 7) \times \frac{12}{11}, 8(40 - 7) \times \frac{12}{11}$

$8 : 47 \times \frac{12}{11}, 8 : 33 \times \frac{12}{11}$

$8 : \frac{564}{11}, 8 : \frac{396}{11}$

$8 : 51\frac{3}{11}, 8 : 36$

8. (b) Normal watch overtakes in = $65\frac{5}{11}$ Minute

This watch overtakes in = 64 minute

It means in 64 minutes the clock gains

$65\frac{5}{11} - 64 = \frac{16}{11}\text{min}.$

"In one day = 24 × 60 minutes"

Then in 1 minute clock gains = $\frac{16}{11 \times 64}$

In 24 × 60 minute clock gains

$= \frac{16 \times 24 \times 60}{11 \times 64} = \frac{360}{11}$ Minutes $32\frac{8}{11}$ Minutes

9. (b) Total loses in 66 minutes is = $66 - 65\ \frac{5}{11} = 66 - \frac{720}{11}$

$= \frac{726 - 720}{11} = \frac{6}{11}$

In 1 minutes = $\frac{6}{11 \times 66}$

In 24 × 60 Minute = $\frac{6 \times 24 \times 60}{11 \times 66} = \frac{1440}{121}$

$= 11\frac{109}{121}$ Minutes loss

10. (c) Sunday 9 A.M

Upcoming Friday 9 P.M

− 4 minute 4 Minute 15 sec.

Total gains = 8 min. 15 sec

$8 + \frac{15}{60} = 8 + \frac{1}{4} = \frac{33}{4}$ min.

Total hours = 5 days + 12 hr. = 120 + 12 = 132 hours

$\frac{33}{4}$ min. gains in = 132 hour

1 minute gains in = $\frac{132}{33} \times 4$

4 minute gains in = $\frac{132}{33} \times 4 \times 4 = 64$ hour

9 A.M Sunday + 64 hours

9 A.M Sunday + 2 days 16 hours = 1 A.M Wednesday

11. (c) 6 A.M Thursday 5 P.M upcoming Wednesday +3 minutes − (3 minutes + 12 sec.)

Total loses = 6 min. + 12 sec

$= 6 + \frac{12}{60} = 6 + \frac{1}{5} = \frac{31}{5}$ min.

Total hour = 6 days + 11

= 24 × 6 + 11 = 144 + 11 = 155 hour

$\frac{31}{5}$ minutes loss in = 155 hr.

1 minutes loss in = $\frac{155}{31} \times 5$

3 minutes loss in = $\frac{155}{31} \times 5 \times 3 = 75$ hour

75 hour = 3 day + 3 hour

Thursday 6 A.M + (3 day + 3 hour) = Sunday 9 A.M

12. (b) Because the time 6 : 30 lies between 1 : 00 to 11 : 00, hence we substruct that time from 11 : 60

13. (c) oppsite direction means in 180°. Angle by unique Formula H = 9, Angle = 180°

$$9 : \left(9 \times 5 \pm \frac{180}{6}\right) \times \frac{12}{11}$$

$$9 : (45 \pm 30) \times \frac{12}{11}$$

$$9 : (45 + 30) \times \frac{12}{11}, 9 : (45 - 30) \times \frac{12}{11}$$

$$9 : (75) \times \frac{12}{11}, 9 : (15) \times \frac{12}{11}$$

Not possible $9 : \frac{180}{11}$; $9 : 16\frac{4}{11}$

14. (a) Together = 0° Angle

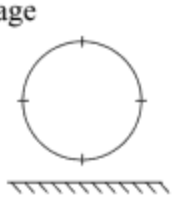
By unique formula H = 6

$$6 : \left(6 \times 5 \pm \frac{0}{6}\right) \times \frac{12}{11}$$

$$6 : (30 \pm 0) \times \frac{12}{11}; \quad 6 : \frac{360}{11} = 6 : 32\frac{8}{11}$$

15. (a) Here the reflection works as a water image

In this time 10 : 20, the hour hand is between 6 : 00 to 12 : 00 and the minute hand is between 12 : 00 to 6 : 00 clockwise. Hence we subtract that time from 17 : 30



17 : 30 − 10 : 20 = 7 : 10

16. (b) It goes a low In 1 hour = 5 mts

then in 6 hour = 5 × 6 = 30 minutes

Then after 6 hour the time will be = 5 : 30 a.m.

17. (c) Every hour it is double fast of given minutes

In 8 minute fast = In 4th hour

× 2

16 minute fast = In 5th hour

× 2

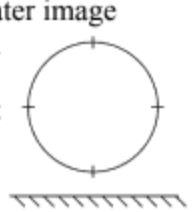
32 minute fast = In 6th hour

× 2

64 minute fast = In 7th hour

18. (b) In this question reflection works as water image

In this time 9 : 50 the both hand of clock are lies between 6 to 12 clockwise. Hence we subtract that time from 17 : 90



17 : 90 − 9 : 50 = 8 : 40

19. (c) In this time 6 : 10 the hour hand lies between 6 : 00 to 12 : 00 clockwise and hour hand lies between 12 : 00 to 600. Hence we subtract that time from 17 : 30

17 : 30 − 6 : 10 = 11 : 20

20. (c) In this time 12 : 30 we subtract that time from

17 : 90 – 12 : 30 = 5 : 60 = 6 : 00

21. (c) before 50 minutes its 4 : 45 means now times = 4:45 + 50 minutes = 5 : 35

22. (b) Total number of odd days till 20 Oct. 1869 ⇒

Year	Year	Year	Month	days
1600 +	200 +	68 +	Jan = 3 +	2
↓	↓	↓	Feb = 0	↓
0	3	1	March = 3	2
			April = 2	
			May = 3	
			June = 2	
			July = 3	
			August = 3	
			September = 2	

Total E.D. = 0 + 3 + 1 + 3 + 0 + 3 + 2 + 3 + 2 + 3 + 3 + 2 + 2 = 27

But = (27) can never be odd days

$\frac{27}{7}$ = 6 (Remainder)

odd days 6 for ⇒ **Saturday**

23. (b) For Sunday in May, 1993 we should find the day on 1st May, 1993

Total add days till 1st May 1993

Year	Year	Year	Month	days
1600 +	30 +	92 +	Jan = 3	1
↓	↓	↓	Feb = 0	↓
0	1	3	March = 3	1
			April = 2	

Total odd days = 0 + 1 + 3 + 3 + 0 + 3 + 2 + 1 = 13

13 can never be odd days, Hence again divide by 7 and find remainder as odd days.

$\frac{13}{7}$ = 6 (odd days)

Odd days on 1st May = 1993 = 6

6 for = Saturday

Then 1 May = Saturday

2 May = Sunday

3 May = Monday

First Sunday of this month is on 2nd day. Then on 2nd, 9th, 16th, 23th, 30th all are Sunday.

24. (b) 3rd day = Tuesday

When was start from 23rd, then the 6th day of the month will be 28th days of the same month

3rd day = Tuesday, Then on 10th, 17th, 24th = Tuesday

24th = Tuesday

25th = Wednesday

26th = Thursday

27th = Friday

28th = **Saturday**

25. (c) 27th day of Month = Friday Similarly 20th, 13th, 6th also are Friday

6th day = Friday

5th day = Thursday

4th day = **Wednesday**