Number System

DIGITS

The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *digits*, which can represent any number.

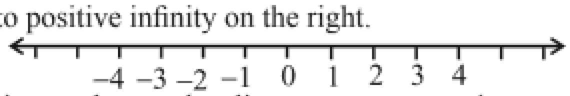
Face Value and Place Value

Face value is absolute value of a digit. Place value is value of a digit in relation to its position in the number. For example, face value and place value of 9 in 14921 are 9 and 900 respectively.

Note that to find the place value of a digit in a number, we put as many zero (0) after the digit as the number of digits after the digit whose place value is to be find in the given number.

THE NUMBER LINE

The number line is a straight line between negative infinity on the left to positive infinity on the right.



Each point on the number line represents a unique number on the number line.

NUMBERS

Natural Numbers

These are the numbers (1, 2, 3, etc.) that are used for counting.

It is denoted by N.

There are infinite natural numbers and the smallest natural number is 1 (one).

Whole Numbers

The natural numbers along with zero (0), form the system of whole numbers.

It is denoted by W.

There is no largest whole number and
the smallest whole number is 0.

Integers

The number system consisting of natural numbers, their negative and zero is called integers.

It is denoted by Z or I.

The smallest and the largest integers cannot be determined.

Even Numbers

An integers which are divisible by 2 are even numbers. It is denoted by E.

$E = -24, -4, -2, 2, 4, 6, 8, \ldots$

Smallest even natural number is 2. There is no largest even number.

Odd Numbers

Integers which are not divisible by 2 are odd numbers.

It is denoted by O.

$O = 1, 3, 5, 7, \ldots$

Smallest odd natural number is 1.

There is no largest odd number.

Prime Numbers

Natural numbers which have exactly two factors, 1 and the number itself are called prime numbers.

The lowest prime number is 2.

2 is also the only even prime number.

All prime number (Except 2 and 3) can be expressed in the form of $6N \pm 1$, where $N = 1, 2, 3, \ldots$..

All prime numbers less than 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Composite Numbers

It is a natural number that has atleast one divisor different from unity and itself.

Every composite number can be factorised into its prime factors.

For Example : $24 = 2 \times 2 \times 2 \times 3$. Here, 24 is a composite number and 2, 3 are prime numbers.

The smallest composite number is 4.

CO-prime Numbers

Those numbers are said to be co-prime if they do not have any common factors other than 1.

For Example : 14 and 15 are co-prime numbers. 14 and 15 have only common factor as 1.

Remember...

- 1 is neither prime nor composite.
- 1 is an odd integer.
- 0 is neither positive nor negative.
- 0 is an even integer.
- 2 is prime & even both.
- All prime numbers (except 2) are odd.

REAL NUMBERS

All numbers that can be represented on the number line by the points on it are called real numbers.

It is denoted by R.

R^+ : Positive real numbers and

R^- : Negative real numbers.

Real numbers = Rational numbers + Irrational numbers.

(i) Rational Numbers

Any number that can be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

It is denoted by Q.

Every integer is a rational number.

Zero (0) is also a rational number. The smallest and largest rational numbers cannot be determined.

Note that every integer is a rational number. Average of any two rational numbers is also a rational number. There are infinite number of rational number between any two rational numbers.

Decimal representation of any rational number is either terminating decimal number or non-terminating recurring (repeating) decimal number. So, all terminating decimal numbers (2.348, 0.07315, etc.) and all non-terminating repeating (recurring) decimal numbers $(54.3\overline{42}, 0.06\overline{3}, 5.\overline{2}$, etc.) are also rational numbers.

Remember...

- If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.
- An infinite number of rational numbers can be determined between any two rational numbers.

(ii) Irrational Numbers

The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called irrational number.

It is denoted by Q' or Q^c.

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

Decimal representation of any irrational number is always non-terminating non-recurring (repeating) decimal number. So, all non-terminating and non-recurring (repeating) decimal numbers (68.3010010001......, 0.233223332223333...... etc.) are irrational numbers.

Note

- $\sqrt{2}+\sqrt{3} \neq \sqrt{5}$
- $\sqrt{5}-\sqrt{3} \neq \sqrt{2}$
- $\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$
- $\sqrt{6} \div \sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$
- Some times, product of two irrational numbers is a rational number.

  For example : $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = 2$

  $(2+\sqrt{3}) \times (2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$

- Both rational and irrational numbers can be represented on number line. Thus real numbers is the set of the union of rational and irrational numbers.

  $R = Q \cup Q'$
- Every real number is either rational or irrational.

Example 1. Convert $2.\overline{46102}$ in the $\frac{p}{q}$ form of rational number.

Sol. Required $\frac{p}{q}$ form $= \frac{246102-2}{99999} = \frac{246100}{99999}$

Example 2. Convert $0.167\overline{3206}$ in the $\frac{p}{q}$ form of rational number.

Sol. Required $\frac{p}{q}$ form $= \frac{1673206-167}{9999000} = \frac{1673039}{9999000}$

TEST OF A PRIME NUMBER

A prime number is only divisible by 1 and by the number itself. The first prime number is 2. All other prime number are odd. To test whether any given number p is a prime number of not, following steps are to be considered :

Step 1 : If p is odd, then go to step 2. If even, then it is not a prime.

Step 2 : Find an integer $x > \sqrt{p}$.

Step 3 : Test the divisibility of the given number p by every prime number less than x.

Step 4 :

- If the given number is divisible by any of them in Step 3, then the given number is NOT a prime number.
- If the given number is not divisible by any of them in Step 3, then the given number is a **prime number**.

Example 3. Consider a number 437. Test if it is a prime number or not.

Step 1 : It is odd

Step 2 : The approximate square root 437 is 20 plus. Take $x = 21$.

Step 3 : Check the divisibility of 437 by the prime number less than 21 i.e., by 2, 3, 5, 7, 11, 13, 17, 19.

Step 4 : 437 is divisible by 19. Thus 437 is not a prime number.

Law of Indices

If a and b are any two real numbers and m and n are positive integers, then

(i) $a^m \times a^n = a^{m+n}$ (Example: $5^3 \times 5^4 = 5^{3+4} = 5^7$)

(ii) $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$ $\left(\text{Example} : \frac{6^5}{6^2} = 6^{5-2} = 6^3\right)$

$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if $m < n$ $\left(\text{Example} : \frac{4^3}{4^8} = \frac{1}{4^{8-3}} = \frac{1}{4^5}\right)$

and $\frac{a^m}{a^n} = a^0 = 1$, if $m = n$ $\left(\text{Example}: \frac{3^4}{3^4} = 3^{4-4} = 3^0 = 1\right)$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$ (Example: $(6^2)^4 = 6^{2 \times 4} = 6^8 = (6^4)^2$)

(iv) (a) $(ab)^n = a^n \cdot b^n$ (Example: $(6 \times 4)^3 = 6^3 \times 4^3$)

(b) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\text{Example}: \left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}\right)$

(v) $a^{-n} = \frac{1}{a^n}$ $\left(\text{Example}: 5^{-3} = \frac{1}{5^3}\right)$

(vi) For any real number a, $a^0 = 1$ (Example : $8^0 = 1$)

Law of Surds

(i) $(a^{1/n})^n = a$ (Example: $(8^{1/3})^3 = 8$)

(ii) $a^{1/n} \cdot b^{1/n} = (ab)^{1/n}$ (Example: $5^{1/3} \cdot 8^{1/3} = (5 \times 8)^{1/3} = (40)^{1/3}$)

(iii) $(a^{1/n})^{1/m} = a^{\frac{1}{mn}}$ (Example : $(5^{1/3})^{1/5} = 5^{\frac{1}{3\times5}} = 5^{1/15}$)

(iv) $a^{1/n} = \sqrt[n]{a}$ (Example : $5^{1/3} = \sqrt[3]{5}$)

(v) $a^{m/n} = \sqrt[n]{a^m}$ (Example : $5^{3/7} = \sqrt[7]{5^3}$)

ADDITION AND SUBTRACTION OF SURDS

Example:

$$5\sqrt{2} + 20\sqrt{2} - 3\sqrt{2} = (5 + 20 - 3)\sqrt{2} = 22\sqrt{2}$$

Example:

$$\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} = (3 - 6 + 4)\sqrt{5} = \sqrt{5}$$

GENERAL OR EXPANDED FORM OF 2 AND 3 DIGITS NUMBERS

(i) In a two digits number AB, A is the digit of tenth place and B is the digit of unit place, therefore AB is written using place value in expanded form as

$AB = 10A + B$

For example: $35 = 10 \times 3 + 5$

(ii) In a three digits number ABC, A is the digit of hundred place, B is the digit of tenth place and C is the digit of unit place, therefore ABC is written using place value in expanded form as

$ABC = 100A + 10B + C$

For example: $247 = 100 \times 2 + 10 \times 4 + 7$

These expanded forms are used in forming equations related to 2- and 3- digits numbers.

Example 4. In a two digit prime number, if 18 is added, we get another prime number with reversed digits. How many such numbers are possible ?

Sol. Let a two-digit number be $10p + q$.

$\therefore \quad 10p + q + 18 = 10q + p$

$\Rightarrow \quad -9p + 9q = 18 \Rightarrow q - p = 2$

Satisfying this condition and also the condition of being a prime number there are 2 numbers 13 and 79.

PRIME FACTORISATION

It is a process of representing a given number as a product of two or more prime numbers.

Each prime number which is present in the product is called a **prime factor** of the given number.

For example: 12 is expressed in the factorised form in terms of its prime factors as $12 = 2\times2\times3 = 2^2 \times 3$.

METHOD TO FIND THE NUMBER OF DIFFERENT DIVISORS OR FACTORS (INCLUDING 1 AND ITSELF) OF ANY COMPOSITE NUMBER N :

Step I : Express N as a product of prime numbers as

$N = x^a \times y^b \times z^c$ ....

Step II : Number of different divisors (including 1 and itself)

$= (a + 1)(b + 1)(c + 1)$ ....

NUMBER OF WAYS OF EXPRESSING A COMPOSITE NUMBER AS A PRODUCT OF TWO FACTORS

(i) Number of ways of expressing a composite number N which is not a perfect square as a product of two factors

$= \frac{1}{2} \times$ (Number of prime factors of the N)

(ii) Number of ways of expressing a perfect square number M as a product of two factors

$= \frac{1}{2}$ [(Number of prime factors of M) + 1]

Example 5. Find the number of ways of expressing 180 as a product of two factors.

Sol. $180 = 2^2 \times 3^2 \times 5^1$

Number of factors = $(2 + 1)(2 + 1)(1 + 1) = 18$

Since 180 is not a perfect square, hence there are total $\frac{18}{2} = 9$ ways in which 180 can be expressed as a product of two factors.

COUNTING NUMBER OF ZEROS

Sometimes we come across problems in which we have to count number of zeros at the end of factorial of any number. For example:

Number of zeros at the end of 10!

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Here basically we have to count number of fives, because multiplication of five by any even number will result in 0 at the end of final product. In 10! we have 2 fives (as $10 = 2 \times 5$). There are more than two even number also in 10! Thus total number of zeros are 2.

Note

'10!' is read as 'Ten factorial'.

Here '!' indicates 'factorial'.

We find the factorial of only natural numbers.

If 'n' is a natural number, then $n! = n(n-1)(n-2)$ 4.3.2.1

Remember...

Number of zeros at the end of n! is the integral part of the value of

$$\frac{n}{5}+\frac{n}{5^2}+\frac{n}{5^3}+\frac{n}{5^4}+\cdots\cdots\cdots$$

Example 6. Number of zeros at the end of 100!

Sol. Number of zeros at the end of 100!

$$= \text{Integral part of } \frac{100}{5}+\frac{100}{5^2}+\frac{100}{5^3}+\cdots\cdots\cdots$$

$$= 20 + 4 = 24 \text{ zeros.}$$

HIGHEST POWER OF A PRIME NUMBER CONTAINED IN A FACTORIAL

Highest power of a prime number P in $N!$

$$=\left[\frac{N}{P}\right]+\left[\frac{N}{P^2}\right]+\left[\frac{N}{P^3}\right]+---+\left[\frac{N}{P^r}\right],$$

where $[x]$ denotes the greatest integers less than or equal to x and is a natural number. Also $P^r < N$.

Example 7. Find highest power of 7 in 50!

Sol. The highest power 7 in 50!

$$=\left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right]=7+1=8$$

TO FIND THE LAST DIGIT OR DIGIT AT THE UNIT'S PLACE OF a^n.

(i) If the last digit or digit at the unit's place of a is 1, 5 or 6, then whatever be the value of n, it will have the same digit at unit's place, i.e.,

$(.....1)^n = (.....1)$

$(.....5)^n = (.....5)$

$(.....6)^n = (.....6)$

(ii) If the last digit or digit at the units place of a is 2, 3, 7 or 8, then the last digit of a^n depends upon the value of n and follows a repeating pattern in terms of 4 as given below :

n	last digit of $(...2)^n$	last digit of $(...3)^n$	last digit of $(...7)^n$	last digit of $(...8)^n$
1	2	3	7	8
2	4	9	9	4
3	8	7	3	2
4	6	1	1	6
5	2	3	7	8

(iii) If the last digit or digit at the unit's place of a is either 4 or 9, then the last digit of a^n depends upon the value of n and follows repeating pattern in terms of 2 as given below.

n	last digit of $(....4)^n$	last digit of $(....9)^n$
1	4	9
2	6	1
3	4	9

Example 8. Find unit digit of $963^{63} \times 73^{73}$.

Sol. $(963)^{63} = (963)^{4\times15} \times (963)^3$

$\therefore$ Unit digit of $963^{63} = 7$

$(73)^{73} = (73)^{4\times18} \times (73)^1$

Unit digit of $73^{73} = 3$

So unit digit of $963^{63} \times 73^{73} \Rightarrow 7 \times 3 \Rightarrow 21$. i.e., 1.

REMAINDER THEOREM

Remainder of expression $\frac{a\times b\times c}{n}$ [i.e. $a \times b \times c$ when divided by n] is equal to the remainder of expression $\frac{a_r\times b_r\times c_r}{n}$ [i.e. $a_r \times b_r \times c_r$ when divided by n] , where

a_r is remainder when a is divided by n.

b_r is remainder when b is divided by n. and

c_r is remainder when c is divided by n.

Example 9. Find the remainder of 15 × 17 × 19 when divided by 7.

Sol. On dividing 15 by 7, we get 1 as remainder.

On dividing 17 by 7, we get 3 as remainder.

On dividing 19 by 7, we get 5 as remainder.

$$\text{Remainder of } \frac{15\times17\times19}{7} = \text{Remainder of } \frac{1\times3\times5}{7}$$

$$= \text{Remainder of } \frac{15}{7} = \text{Remainder of } 2\frac{1}{7} \text{ i.e. } 1$$

POLYNOMIAL THEOREM

When $(x + a)^n$ is divided by x, then remainder is equal to the remainder of the expression $\frac{a^n}{x}$.

Example 10. Find the remainder of $\frac{9^{99}}{8}$.

Sol. $\frac{9^{99}}{8} = \frac{(8+1)^{99}}{8}$

According to polynomial theorem, remainder of $\frac{9^{99}}{8}$ will be equal to remainder of the expression $\frac{1^{99}}{8} \Rightarrow \frac{1}{8}$ i.e., Remainder =1

SHORTCUTS

Shortcut Approach – 1

When two numbers are divided by a third number, leave the same remainder, then the difference of these two numbers is always perfectly divisible by third number.

Example 11. 24345 and 33334 are divided by certain number of three digits and the remainder is the same in both the cases. Find the divisor and the remainder.

(a) 103, 6 **(b)** 809, 3 **(c)** 101, 4 **(d)** 109, 5

Sol. (c) Difference = 33334 – 24345 = 8989

Since, 8989 = 101 × 89

$\therefore$ 101 is the required 3 digit divisor
On dividing any of the given numbers by 101, we get 4 as remainder.

Shortcut Approach – 2

Sum of the digits of a given two digit number is S. When its digits are interchange their places, the number decreased by D. Then,

Given number $= 5\left(S + \frac{D}{9}\right) + \frac{1}{2}\left(S - \frac{D}{9}\right)$

Example 12. Sum of the digits of a given 2–digit number is 12. When its digits interchange their places, the number decreases by 54. Find the number.

(a) 93 **(b)** 84 **(c)** 75 **(d)** 66

Sol. (a) $\therefore$ Given number $= 5\left[S + \frac{D}{9}\right] + \frac{1}{2}\left[S - \frac{D}{9}\right]$

$$= 5\left[12 + \frac{54}{9}\right] + \frac{1}{2}\left[12 - \frac{54}{9}\right] = 5 \times 18 + \frac{1}{2} \times 6 = 93$$

Shortcut Approach – 3

(i) $(a^n + b^n)$ is divisible by $(a + b)$ when n is odd

(ii) $(a^n - b^n)$ is divisible by both $(a + b)$ and $(a - b)$ when n is even

(iii) $(a^n - b^n)$ is divisible by only $(a - b)$ when n is odd

Example 13. If $(67^{67} + 67)$ is dividing by 68, the remainder is :

(a) 61 **(b)** 67 **(c)** 63 **(d)** 66

Sol. (d) $\because$ $(x^n + y^n)$ is divisible by $(x + y)$ when n is odd
So, $((67^{67} + 1^{67}) + 66)$ $(67^{67} + 1^{67})$ is divisible by 68,
Hence remainder is 66.

Shortcut Approach – 4

When $(x^n + k)$ is divided by $(x - 1)$,

(a) Remainder $= 1 + k$; if $k < (x - 1)$

(b) Remainder $= 1 +$ (Remainder obtained when k is divided by $x - 1$); if $k > x - 1$

Example 14. Find the remainder on dividing $(9^{16} + 6)$ by 8.

(a) 5 **(b)** 7 **(c)** 2 **(d)** 3

Sol. (b) Here $k = 6$ and $x - 1 = 8$
$\therefore$ $k < (x - 1)$
So, Remainder $= 1 + k = 1 + 6 = 7$

Shortcut Approach – 5

To find the value of $\sqrt{x + \sqrt{x + \sqrt{x + ...}}}$, find the factors of x, such that the difference between the factors is 1, then the larger factor will be the result.

Example 15. $\sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}}$ is equal to

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

Sol. (a) $\sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}}$
The factors of 6 with difference one are 2 and 3
Here 3 is the larger factors.
Hence $\sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}} = 3$

Example 16. $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12.....}}}} = ?$

(a) 3 **(b)** 4 **(c)** 6 **(d)** 12

Sol. (b) $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + ...}}}}$
The two factors of 12 with difference one are 4 and 3.
Here, 4 is the bigger factor.
Hence, $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + ...}}}} = 4$

Shortcut Approach – 6

To find the value of $\sqrt{x - \sqrt{x - \sqrt{x - .....}}}$ find the factors of x, such that the difference between the factors is 1, then the smaller factor will be the result.

Example 17. $\sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2.....}}}} = ?$

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

Sol. (b) As given expression is
$\sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2}}}}$
The factors of 2 with difference one are 1 and 2.
Here 1 is the smaller factor
Hence, $\sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2}}}} = 1$

Shortcut Approach – 7

$$\sqrt{x\sqrt{x\sqrt{x..........\ n \text{ times}}}} = (x)^{\frac{2^n - 1}{2^n}}$$

Example 18. The value of $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ will be

(a) 2 **(b)** $2^{15/32}$ **(c)** $2^{31/32}$ **(d)** 4

Sol. (c) $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = (2)^{\left[\frac{2^5 - 1}{2^5}\right]} = 2^{(31/32)}$

Shortcut Approach – 8

$$\sqrt{x\sqrt{x\sqrt{x\sqrt{x.......\infty}}}} = x$$

Example 19. $\sqrt{3\sqrt{3\sqrt{3...}}}$ is equal to

(a) $\sqrt{3}$ **(b)** 3 **(c)** $2\sqrt{3}$ **(d)** $3\sqrt{3}$

Sol. (b) $\sqrt{3\sqrt{3\sqrt{3.....}}} = 3$

Alternate Method :

Let $\sqrt{3\sqrt{3\sqrt{3.....}}} = x$

or, $\sqrt{3x} = x$

Squaring both side

$3x = x^2$ $\quad 0 = x^2 - 3x$ $\quad 0 = x(x - 3)$

$x = 3$

EXERCISE

1. The difference between a two-digit number and the number obtained by interchanging the two digits of the number is 18. The sum of the two digits of the number is 12. What is the product of the two digits of the two digits number ?
   (a) 35 (b) 27
   (c) 32 (d) Cannot be determined
   (e) None of these

2. The prime number 1999 can be written as $a^2 - b^2$, where a and b are natural numbers. Then the value of $a^2 + b^2$ is
   (a) 1998000 (b) 1998001
   (c) 1999000 (d) 1999001

3. If a two digits number is k times the sum of its digits, then the number formed by interchanging the digits is the sum of the digits multiplied by:
   (a) $9 + k$ (b) $10 + k$ (c) $11 - k$ (d) $k - 1$

4. Which is the smallest number by which 4320 be divided to make it a perfect cube?
   (a) 15 (b) 20 (c) 24 (d) 25

5. In every 30 minutes the time of a watch increases by 3 minutes. After setting the correct time at 5 am., what time will the watch show after 6 hours?
   (a) 10 : 54 a.m. (b) 11 : 30 a.m.
   (c) 11 : 36 a.m. (d) 11 : 42 a.m.
   (e) 11 : 38 p.m.

6. If two numbers are each divided by the same divisor, the remainders are respectively 3 and 4. If the sum of the two numbers be divided by the same divisor, the remainder is 2. The divisor is :
   (a) 9 (b) 7 (c) 5 (d) 3

7. If the digits in the unit and the ten's places of a Two digit number are interchanged, a new number is formed, which is greater than the original number by 63. Suppose the digit in the unit place of the original number the x. Then, all the possible values of x are
   (a) 7, 8, 9 (b) 2, 7, 9 (c) 0, 1, 2 (d) 1, 2, 8

8. The numerator of a fraction is 4 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 1, the denominator becomes eight times the numerator. Find the fraction.
   (a) $\frac{3}{8}$ (b) $\frac{3}{7}$ (c) $\frac{4}{8}$ (d) $\frac{2}{7}$

9. Given that, three numbers are such that the second number is twice the first and thrice the third. Also the average of the three numbers is 44. Then the difference of the first and the third is :
   (a) 10 (b) 11 (c) 12 (d) 13

10. Let x be an odd natural number. If x is divided by 6, it leaves a remainder y. If y^2 is divided by 4, it leaves remainder of z. Which of the following must be true for z?
    (a) $z = 3$ (b) $z = 5$
    (c) $z = 1$ (d) z is even

11. When 9 is subtracted from a two digit number, the number so formed is reverse of the original number. Also, the average of the digits of the original number is 7.5. What is definitely the original number?
    (a) 87 (b) 92 (c) 90 (d) 69
    (e) 96

12. The sum of a series of 5 consecutive odd numbers is 195. The second lowest number of this series is 9 less than the second highest number of another series of 5 consecutive even numbers. What is 40% of the second lowest number of the series of consecutive even numbers?
    (a) 16.8 (b) 18.8 (c) 19.4 (d) 17.6
    (e) 16.4

13. The sum of a series of 5 consecutive odd numbers is 225. The second number of this series is 15 less than the second lowest number of another series of 5 consecutive even numbers. What is 60% of the highest number of this series of consecutive even numbers?
    (a) 36.0 (b) 34.6 (c) 38.4 (d) 40.8
    (e) 39.2

14. $(x^{2a})^b = \sqrt{x^{\frac{4b}{c}}}$ and $\frac{x^{4b}}{x^{3a}} = x^{3(a-b)}.x^b$. a, b and c being natural numbers.
    (a) $a \neq b \neq c$ (b) $a = b < c$
    (c) $a < b = c$ (d) $a = b = c$
    (e) None of these

15. A classroom has equal number of boys and girls. Eight girls left to play Kho-Kho, leaving twice as many boys as girls in the classroom. What was the total number of girls and boys present initially?
    (a) Cannot be determined (b) 16
    (c) 24 (d) 32
    (e) None of these

16. If in a three-digit number the last two digits places are interchanged, a new number is formed which is greater than the original number by 45. What is the difference between the last two digits of that number?
    (a) 9 (b) 8 (c) 6 (d) 5

17. In a two digit number the digit in the unit's place is twice the digit in the ten's place and the number obtained by interchanging the digits is more than the original number by 27. What is 50% of the original number?
    (a) 36 (b) 63
    (c) 48 (d) 18
    (e) None of these

18. What is the greater of two numbers whose product is 1092 and the sum of the two numbers exceeds their difference by 42?
    (a) 48 (b) 44
    (c) 52 (d) 54
    (e) None of these

19. Two numbers are such that the sum of twice the first number and thrice the second number is 36 and the sum of thrice the first number and twice the second number is 39. Which is the smaller number?

(a) 9 (b) 5
(c) 7 (d) 3
(e) None of these

20. In a three digit number the digit in the unit's place is twice the digit in the ten's place and 1.5 times the digit in the hundred's place. If the sum of all the three digits of the number is 13, what is the number?

(a) 364 (b) 436
(c) 238 (d) 634
(e) None of these

21. The denominators of two fractions are 5 and 7 respectively. The sum of these fractions is $\frac{41}{35}$. On interchanging the numerators, their sum becomes $\frac{43}{35}$. The fractions are

(a) $\frac{2}{5}$ and $\frac{4}{7}$ (b) $\frac{3}{5}$ and $\frac{4}{7}$
(c) $\frac{4}{5}$ and $\frac{2}{7}$ (d) $\frac{3}{5}$ and $\frac{5}{7}$
(e) None of these

22. The number $25^{64} \times 64^{25}$ is the square of a natural number *n*. The sum of digits of *n* is

(a) 7 (b) 14
(c) 21 (d) 28

23. Let *x* be the product of two numbers 3, 659, 893, 456, 789, 325, 678 and 342, 973, 489, 379, 256. The number of digits in *x* is

(a) 32 (b) 34
(c) 35 (d) 36

24. The number $(2^{48} - 1)$ is exactly divisible by two numbers between 60 and 70. The numbers are:

(a) 63 and 65 (b) 63 and 67
(c) 61 and 65 (d) 65 and 67

25. If $(2^{36} - 1) = 68$ a 19476735, where a is any digit, then the value of a is

(a) 1 (b) 3 (c) 5 (d) 7

26. Given the numbers: 2^{5555}, 3^{3333}, 6^{2222}. These can be written in ascending order as

(a) $2^{5555}, 3^{3333}, 6^{2222}$ (b) $3^{3333}, 2^{5555}, 6^{2222}$
(c) $2^{5555}, 6^{2222}, 3^{3333}$ (d) $6^{2222}, 2^{5555}, 3^{3333}$
(e) None of these

27. Rachita enters a shop to buy ice-creams, cookies and pastries. She has to buy atleast 9 units of each. She buys more cookies than ice-creams and more pastries than cookies. She picks up a total of 32 items. How many cookies does she buy?

(a) Either 12 or 13 (b) Either 11 or 12
(c) Either 10 or 11 (d) Either 9 or 11
(e) Either 9 or 10

28. The fare of a bus is ₹ *x* for the first five kilometres and ₹13 per kilometres thereafter. If a passenger pays ₹ 2,402 for a journey of 187 kilometres, what is the value of *X*?

(a) ₹ 29 (b) ₹ 39
(c) ₹ 36 (d) ₹ 31
(e) None of these

29. In an examination, a student scores 4 marks for every correct answer and losses 1 mark for every wrong answer. A student attempted all the 200 questions and scored in all 200 marks. The number of questions, he answered correctly was :

(a) 82 (b) 80
(c) 68 (d) 60

Hints & Solutions

1. **(a)** Let the two-digit number be = $10x + y$, where $x > y$

According to the question,

$10x + y - 10y - x = 18$ or, $9x - 9y = 18$

or, $x - y = \frac{18}{9} = 2$ ...(i)

and, $x + y = 12$ ...(ii)

From equations (i) and (ii)

$2x = 14 \Rightarrow x = 7$

From equation (i)

$y = 7 - 2 = 5$

$\therefore$ Required product $= xy = 7 \times 5 = 35$

2. **(b)** $a^2 - b^2 = 1999$

$\Rightarrow (a + b)(a - b) = 1999$

$\Rightarrow (1000 + 999)(1000 - 999) = 1999$

$\therefore a^2 + b^2 = (1000)^2 + (999)^2$

$= 1000000 + 998001 = 1998001$

3. **(c)** $10x + y = k(x + y)$

$\therefore 10y + x = 11y + 11x - 10x - y$

$= 11(x + y) - k(x + y) = (11 - k)(x + y)$

4. **(b)**

2	4320
2	2160
2	1080
2	540
2	270
5	135
3	27
3	9
3	3
	4

$\therefore 4320 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 3 \times 3 \times 3 = 2^5 \times 3^3 \times 5$

$\therefore$ Required number $= 2^2 \times 5 = 20$

5. (c) Time gained in 6 hours = 12 × 3 = 36 minutes
$\therefore$ Required time = 11 : 36 a.m.

6. (c) Divisor = Remainder 1 + Remainder 2 – Remainder 3
= 3 + 4 – 2 = 7 – 2 = 5

7. (c) Let the two digit no. be $10x + y$ where $y > x$
$10y + x - 10x - y = 63$
$9y - 9x = 63$
$y - x = 7$
$y = 7, 8, 9$ and $x = 0, 1, 2$

8. (b) Let 'd' be denominator so numerator be $(d - 4)$
According to question
$\Rightarrow \frac{(d-4)-2}{d+1} = \frac{1}{8} \quad \Rightarrow \frac{d-6}{d+1} = \frac{1}{8}$
$\Rightarrow 8d - 48 = d + 1$
$7d = 49$
$d = 7$
numerator $\Rightarrow d - 4 = 7 - 4 = 3$
denominator (d) $\Rightarrow 7$
$\therefore$ fraction = $\frac{3}{7}$

9. (c) Let three numbers are $3x, 6x, 2x$
Avg. = $\frac{11x}{3} = 44$
$x = 12$
Difference of first and third number
$= 3x - 2x = x = 12.$

10. (c) $x = 6Q + y$
$y^2 = 4Q^1 + z$
The value of z may be 1, 2 or 3.
The value of y may be 1, 3, or 5 as if 2 or 4 be the value, y^2 will be exactly divisible by 4.
$\therefore z = 1$

11. (a) Let the two-digit number be $10x + y$
According to the questions,
$10x + y - 9 = 10y + x$
$\Rightarrow 10x + y - 10y - x = 9$
$\Rightarrow 9x - 9y = 9$
$\Rightarrow x - y = 1$ ...(i)
and, $\frac{x+y}{2} = 7.5$
$\Rightarrow x + y = 15$ ...(ii)
On adding equations (i) and (ii),
$2x = 16$
$\Rightarrow x = 8$
from equation (i),
$8 - y = 1$
$\Rightarrow y = 8 - 7 = 7$
$\therefore$ Required number = 10 × 8 + 7 = 87

12. (a) Five consecutive odd numbers
$\Rightarrow x, x + 2, x + 4, x + 6$ and $x + 8$
$\therefore x + x + 2 + x + 4 + x + 6 + x + 8 = 195$
$\Rightarrow 5x + 20 = 195$
$\Rightarrow 5x = 195 - 20 = 175$
$\Rightarrow x = \frac{175}{5} = 35$
$\therefore$ Second lowest number = 35 + 2 = 47
$\therefore$ Second highest even number = 37 + 9 = 46
$\therefore$ Second lowest even number = 42
$\therefore$ 40% of 42 = $\frac{42 \times 40}{100} = 16.8$

13. (c) Let the odd number be: $x, x + 2, x + 4, x + 6$ and $x + 8$
$\therefore x + x + 2 + x + 4 + x + 6 + x + 8 = 225$
$\Rightarrow 5x + 20 = 225$
$\Rightarrow 5x = 225 - 20 = 205$
$\Rightarrow x = \frac{205}{5} = 41$
$\therefore$ Second number = 43
Second lowest even number = 43 + 15 = 58
$\therefore$ Largest even number = 58 + 6 = 64
$\therefore$ 60% of 64 = $\frac{64 \times 60}{100} = 38.4$

14. (d) $(x^{2a})^b = (x)^{\frac{4b}{2c}}$
$\Rightarrow 2\,ab = \frac{2b}{c} \quad \Rightarrow ac = 1$
$a = c = 1$ [$\because$ Both a and c are natural numbers] ...(i)
$\frac{x^{4b}}{x^{3a}} = x^{3(a-b)}.\, x^b \quad \Rightarrow x^{4b - 3a} = x^{3a - 2b}$
$\Rightarrow 4b - 3a = 3a - 2b \quad \Rightarrow 6b = 6a$
$\Rightarrow a = b$ ....(ii)
From (i) and (ii) we have,
$a = b = c$

15. (d) Let the no. of boys and girls each be x.
$2(x - 8) = x$
$2x - 16 = x$
$x = 16$
$\therefore$ Total no. of boys and girls present initially in the classroom = $2x = 32$

16. (b) Three digit number = $100x + 10y + z$
To make number after changing last two digits
$= 100x + 10z + y$
Now, $100x + 10y + z = 100x + 10z + y - 45$
$9z - 9y = 45$
$z - y = 5$

17. (d) Let the ten's digit be x, then, unit's digit = $2x$
$\therefore$ Original number = $10x + 2x = 12x$
On interchanging the digits, the new number
$= 10 \times 2x + x = 21x$
According to question,

$21x - 12x = 27$

$\Rightarrow \quad 9x = 27 \quad \Rightarrow \quad x = \frac{27}{9} = 3$

$\therefore$ Original number $= 12x = 12 \times 3 = 36$

Hence, 50% of original number $= 36 \times \frac{50}{100} = 18$

18. (c) Let the numbers be x and y.
According to the question,
$x + y - (x - y) = 42 \quad \Rightarrow \quad 2y = 42$

$\Rightarrow \quad y = \frac{42}{2} = 21 \quad \therefore \quad x = \frac{1092}{21} = 52$

19. (e) Let the numbers be x and y.
According to the question,
$2x + 3y = 36$ ...(i)
$3x + 2y = 39$ ...(ii)
By equation (i) $\times$ 3 – (ii) $\times$ 2,
$6x + 9y - 6x - 4y = 108 - 78$

$\Rightarrow \quad 5y = 30 \quad \Rightarrow \quad y = \frac{30}{5} = 6$

From equation (i), we have,
$2x + 3 \times 6 = 36$
$\Rightarrow \quad 2x = 36 - 18 = 18$

$\Rightarrow \quad x = \frac{18}{2} = 9$ and, $y = 6$

$\therefore$ The smaller number = 6

20. (b) Let the ten's digit be x.
then, Unit's digit $= 2x$

and hundred's digits $= \frac{2x}{1.5}$

According to the question,

$x + 2x + \frac{2x}{1.5} = 13 \quad \Rightarrow \quad \frac{1.5x + 3x + 2x}{1.5} = 13$

$\Rightarrow \quad 6.5x = 13 \times 1.5$

$\Rightarrow \quad x = \frac{13 \times 1.5}{6.5} = 3$

$\therefore$ Unit's digit = 6, ten's digit = 3 and hundred's digit

$= \frac{\quad}{1.5} = 4$

$\therefore$ Number = 436
Note: This question can be solved by oral calculation, taking the alternatives into consideration.

21. (b) From given alternatives,

$\frac{3}{5} + \frac{4}{7} = \frac{21 + 20}{35} = \frac{41}{35}$ and

$\frac{4}{5} + \frac{3}{7} = \frac{28 + 15}{35} = \frac{43}{35}$

22. (b) $n^2 = 25^{64} \times 64^{25} = (5^2)^{64} \times (2^6)^{25}$
$= 5^{128} \times 2^{128} \times 2^{22}$
$\therefore \quad n = 5^{64} \times 2^{64} \times 2^{11}$
$= (5 \times 2)^{64} \times 2048 = 10^{64} \times 2048$
$\therefore$ Sum of digits $= 2 + 0 + 4 + 8 = 14$

23. (b) $365989345689325678 = 37 \times 10^{17}$
and $342973489379256 = 3 \times 10^{14}$
$\therefore \quad 37 \times 10^{17} \times 3 \times 10^{14} = 111 \times 10^{31}$
$\therefore$ Number of digits $= 31 + 3 = 34$

24. (a) $2^{48} - 1 = (2^{24} + 1)(2^{24} - 1)$
$= (2^{24} + 1)(2^{12} + 1)(2^6 + 1)(2^6 - 1)$
$\therefore$ Required numbers $= 2^6 + 1$ and $2^6 - 1 = 65$ and 63.

25. (a) $2^2 - 1 = 4 - 1 = 3$
$2^4 - 1 = 16 - 1 = 15$
$2^6 - 1 = 64 - 1 = 63$
$2^8 - 1 = 256 - 1 = 255$
Hence, if n = even number, then $(2^n - 1)$, is divisible by 3.
We know a number is divisible by 3. If the sum of its digits is divisible by 3.
$\therefore \quad a = 1$

26. (b) $2^{5555} = (2^5)^{1111} = (32)^{1111}$
$3^{3333} = (3^3)^{1111} = (27)^{1111}$
$6^{2222} = (6^2)^{1111} = (36)^{1111}$
$3^{3333} < 2^{5555} < 6^{2222}$

27. (c) According to the question,
Ice-creams + Cookies + Pastries = 32
$\Rightarrow \quad 9 + 11 + 12 = 32$
or, $\quad 9 + 10 + 13 = 32$
$10 + 11 + 12 \neq 32$
Hence, either she but 10 or 11 Cookies.

28. (c) According to the question,
$x + 182 \times 13 = 2402$
$\Rightarrow \quad x + 2366 = 2402$
$\Rightarrow \quad x = 2402 - 2366 = ₹\, 36$

29. (b) Let the correct answer are 'n'
According to question

Correct marks		Incorrect marks		Total marks
$4n$	$-$	$(200 - n) \times 1$		$= 200$

$4n - 200 + n = 200$
$5n = 400$
$n = 80$