Mixture and Alligation

MIXTURE

Mixture problems involving creating a mixture from two or more things and then determining some quantity (Percent, price etc.) of the resulting mixture.

ALLIGATION

Alligation is nothing but a faster technique of solving problems based on the weighted average situation which applied to the case of two groups being mixed together.

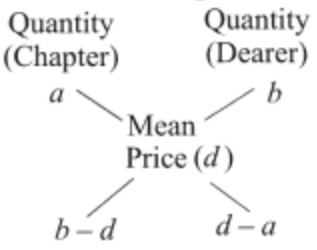
The word 'Alligation' literally means 'linking'.

Alligation Rule

It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, then

$$\frac{\text{Quantity of Cheaper ingredient}}{\text{Quantity of Dearer ingredient}} = \frac{\text{Price of Dearer ingredient} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper ingredient}}$$

Graphical Representation of Alligation Rule



$$\frac{\text{Quantity of } a}{\text{Quantity of } b} = \frac{b-d}{d-a}$$

REMOVAL AND REPLACEMENT

(i) Let a vessel contains Q unit of mixture of ingredients A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only.

If this process is repeated n times, then after n operations,

$$\frac{\text{Quantity of } A \text{ left}}{\text{Quantity of } A \text{ originally present}} = \left(1 - \frac{R}{Q}\right)^n$$

and Quantity of B left = Q – Quantity of A Left

(ii) Let a vessel contains Q unit of ingredient A only. From this R unit of ingredient A is taken out and replaced by an equal amount of ingredient B.

If this process is repeated n times, then after n operations,

$$\text{Quantity of } A \text{ left} = Q\left(1 - \frac{R}{Q}\right)^n$$

Quantity of B left = Q – Quantity of A left

(iii) A container has milk and water in the ratio $a : b$, a second container has milk and water in the ratio $c : d$. If both the mixtures are emptied into a third container, then the ratio of milk to water in third container is given by

$$\left(\frac{a}{a+b} + \frac{c}{c+d}\right) : \left(\frac{b}{a+b} + \frac{d}{c+d}\right)$$

(iv) If in x litres mixture of A and B, the ratio of A and B is $a : b$, the quantity of B to be added in order to make the ratio $c : d$ is $\frac{x(ad-bc)}{c(a+b)}$

(v) If x glasses of equal size are filled with a mixture of milk and water. The ratio of milk and water in each glass are as follows: $a_1 : b_1, a_2 : b_2, a_3 : b_3 \ldots a_x : b_x$

If the content of all the x glasses are emptied into a single large vessel, then proportion of milk and water in it is given by $\left(\frac{a_1}{a_1+b_1} + \frac{a_2}{a_2+b_2} + \ldots + \frac{a_x}{a_x+b_x}\right)$:

$$\left(\frac{b_1}{a_1+b_1} + \frac{b_2}{a_2+b_2} + \ldots + \frac{b_x}{a_x+b_x}\right)$$

(vi) A mixture contains A and B in the ratio $a : b$. If x litres of B is added to the mixture, A and B become in the ratio $a : c$. Then the quantity of A in the mixture is given by $\frac{ax}{c-b}$ and that of B is given by $\frac{bx}{c-b}$.

EXERCISE

1. In a mixture of milk and water the proportion of water by weight was 75%. If in the 60 gms mixture 15 gms. water was added, what would be the percentage of water in the new mixture?
(a) 75% (b) 88%
(c) 90% (d) 100%
(e) None of these

2. A jar has 60 litres of milk. From the jar, 12 litres of milk was taken out and replaced by an equal amount of water. If 12 litres of the newly formed mixture is taken out of the jar, what is the final quantity of milk left in the jar?
(a) 38.4 litres (b) 40 litres
(c) 36 litres (d) 28.6 litres
(e) 36.5 litres

3. Jar A has 60 litres of mixture of milk and water in the respective ratio of 2 : 1. Jar B which had 40 litres of mixture of milk and water was emptied into jar A, as a result in jar A, the respective ratio of milk and water became 13 : 7. What was the quantity of water in jar B?
(a) 8 litre (b) 15 litre (c) 32 litre (d) 7 litre
(e) 1 litre

4. A vessel contains 180 litres of mixture of milk and water in the respective ratio of 13 : 5. Fifty-four litres of this mixture was taken out and replaced with 6 litres of water, what is the approximate percentage of water in the resultant mixture?
(a) 41% (b) 31% (c) 24% (d) 9%
(e) 17%

5. When one litre of water is added to a mixture of acid and water, the new mixture contains 20% acid. When one litre of acid is added to the new mixture, then the resulting mixture contains $33\frac{1}{3}$% acid. Then percentage of acid in the original mixture was
(a) 20% (b) 22% (c) 24% (d) 25%

6. A jar contained a mixture of two liquids *A* and *B* in the ratio 4 : 1. When 10 litres of the mixture was taken out and 10 litres of liquid *B* was poured into the jar. This ratio became 2:3. The quantity of liquid *A* contained in the jar initially was
(a) 4 litres (b) 8 litres
(c) 16 litres (d) 40 litres

7. A shopkeeper bought 15 kg of rice at the rate of ₹ 29 per kg and 25 kg of rice at the rate of ₹ 20 per kg. He sold the mixture of both types of rice at the rate of ₹ 27 per kg. His profit in this transaction is
(a) ₹ 125 (b) ₹ 150
(c) ₹ 140 (d) ₹ 145

8. Nikita bought 30 kg of wheat at the rate of ₹ 9.50 per kg and 40 kg of wheat at the rate of ₹ 8.50 per kg and mixed them. She sold the mixture at the rate of ₹ 8.90 per kg. Her total profit or loss in the transaction was
(a) ₹ 2 loss (b) ₹ 2 profit
(c) ₹ 7 loss (d) ₹ 7 profit

9. Lala has lent some money to Arun at 5% p.a. and Bhatia at 8% p.a. At the end of the year, he has gained an overall interest of 6%. In what ratio has he lent the money to Arun and Bhatia ?
(a) 2 : 1 (b) 1 : 2
(c) 3 : 2 (d) 3 : 1

10. Two vessels *A* and *B* contain spirit and water mixed in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture in vessel *C* containing spirit and water in the ratio 8 : 5 ?
(a) 4 : 3 (b) 3 : 4
(c) 5 : 6 (d) 7 : 9
(e) None of these

11. A container has 30 litres of water. If 3 litres of water is replaced by 3 litres of spirit and this operation is repeated twice, what will be the quantity of water in the new mixture?
(a) 24 litres (b) 23 litres
(c) 24.3 litres (d) 23.3 litres
(e) None of these

12. A vessel contains 100 litres mixture of milk and water in the respective ratio of 22 : 3. 40 litres of the mixture is taken out from the vessel and 4.8 litres of pure milk and pure water each is added to the mixture. By what percent is the quantity of water in the final mixture less than the quantity of milk?
(a) $78\frac{1}{2}$ (b) $79\frac{1}{6}$ (c) $72\frac{5}{6}$ (d) 76
(e) $77\frac{1}{2}$

13. 18 litres of pure water was added to a vessel containing 80 litres of pure milk. 49 litres of the resultant mixture was then sold and some more quantity of pure milk and pure water was added to the vessel in the respective ratio of 2 : 1. If the resultant respective ratio of milk and water in the vessel was 4 : 1. If the resultant respective ratio of milk and water in the vessel was 4 : 1, what was the quantity of pure milk added in the vessel? (in litres)
(a) 4 (b) 8
(c) 10 (d) 12
(e) 2

14. A vessel contains a mixture of Grape, Pineapple and Banana juices in the respective ratio of 4 : 6 : 5. 15 litres of this mixture is taken out and 8 litres of grape juice and 2 litres of pineapple juice is added to the vessel. If the resultant quantity of grape juice is 10 litres less than the resultant quantity of pineapple juice. What was the initial quantity of mixture in the vessel? (in litres)

(a) 120 (b) 150
(c) 105 (d) 135
(e) 90

15. A container contains a mixture of milk and water in the respective ratio of 2 : 3. 20 litres of the mixture is replaced with same quantity of milk. The respective ratio of milk and water now becomes 4 : 1. Find the initial quantity of the mixture in the container. (In litres)

(a) 30 (b) 32
(c) 35 (d) 36
(e) 40

16. A jar had 120 litres mixture of milk and water in the respective ratio of 5 : 1. 30 litres of this mixture is taken out and '*X*' litres of each milk and water is added to the jar. The respective ratio between milk and water in the jar was 4 : 1 respectively. What was the total quantity of both milk and water added to the jar? (in litres)

(a) 12 (b) 10
(c) 16 (d) 20
(e) 18

17. A jar contains a mixture of milk and water in the respective ratio of 3 : 1. When 4 litres of the mixture is taken out and thereafter 3 litres of milk is added to the remaining mixture, the respective ratio of milk and water in the resultant mixture thus formed is 4 : 1. What was the initial quantity of water in the mixture?

(a) 1 litres (b) 6 litres
(c) 4 litres (d) 2 litres
(e) 3 litres

18. A vessel contains a mixture of milk and water in the respective ratio of 10 : 3. Twenty-six litres of this mixture was taken out and replaced with 10 litres of water. If the respective ratio of milk and water in the resultant mixture was 5 : 2, what was the initial quantity of mixture in the vessel? (in litres)

(a) 143 (b) 182
(c) 169 (d) 156
(e) 130

19. A jar contains mixture of milk and water in the respective ratio of 3 : 1. $\frac{1}{25}$ th of the mixture is taken out and 24 litre water was added to it. If the resultant ratio between milk and water in the jar was 2 : 1, what was the initial quantity of mixture in the jar? (in litre)

(a) 160 (b) 180
(c) 200 (d) 250
(e) None of these

20. Vessel *A* contains a mixture of milk and water in the respective quantities of 20 litres and 4 litres. Vessel *B* contains a mixture of milk and water in the respective ratio of 3 : 1. Mixtures from vessel *A* and *B* are both mixed together in vessel *C*. If the resultant percentage of water in vessel *C* was 20%, what was the initial quantity of mixture in vessel *B*? (in litres)

(a) 16 (b) 20
(c) 12 (d) 24
(e) 28

21. In an alloy, zinc and copper are in the ratio 1 : 2. In the second alloy, the same elements are in the ratio 2 : 3. If these two alloys be mixed to form a new alloy in which two elements are the ratio 5 : 8, the ratio of these two alloys in the new alloys is

(a) 3 : 10 (b) 3 : 7
(c) 10 : 3 (d) 7 : 3

22. Three glasses of equal volume contain acid mixed with water. The ratio of acid and water are 2 : 3, 3 : 4 and 4 : 5 respectively. Contents of these glasses are poured in a large vessel. The ratio of acid and water in the large vessel is

(a) 417 : 564 (b) 401 : 544
(c) 407 : 560 (d) 411 : 540

23. A vessel contains 60 litre of milk. 12 litres of milk taken out from it and replaced by water. Then again from mixture, 12 litres are taken out and replaced by water. The ratio of milk and water in the resultant mixture is :

(a) 16 : 9 (b) 15 : 10
(c) 16 : 10 (d) 9 : 5

24. A can is full of a mixture of two liquids *A* and *B* in the ratio of 7 : 5. When 9 litres of mixture are drawn off from the can and replaced by the same quantity of liquid *B*, the ratio of *A* and *B* in the can becomes 7 : 9. The capacity of the can is

(a) 21 litres (b) 20 litres
(c) 10 litres (d) 36 litres

25. Three containers whose volumes are in the ratio of 2 : 3 : 4 are full of mixture of spirit and water. In the I^{st} container the ratio of spirit and water is 4 : 1 in 2^{nd} container the ratio is 11 : 4 and in the 3^{rd} container ratio is 7 : 3. All the three mixtures are mixed in a big container. The ratio of spirit and water in the resultant mixture is :

(a) 4 : 9 (b) 9 : 5
(c) 11 : 4 (d) 5 : 10

Hints & Solutions

1. (e) In 60 gm. of mixture,

Quantity of water = $60 \times \frac{75}{100}$ = 45 gm

Quantity of milk = 15 gm

After mixing 15 gm of more water.

Quantity of water in the new mixture = 45 + 15 = 60 gm

$\because$ Quantity of water in 75 gm of mixture = 60 gm

$\therefore$ 100 gm of mixture will contain

$= \frac{60}{75} \times 100 = 80\%$ of water

2. (a) Remaining quantity of milk

$$= \text{Initial quantity} \times \left(1 - \frac{\text{quantity taken out}}{\text{Initial quantity}}\right)^n$$

$$= 60\left(1 - \frac{12}{60}\right)^2 = 60\left(1 - \frac{1}{5}\right)^2 = 60\left(\frac{4}{5}\right)^2$$

$$= \frac{60 \times 4 \times 4}{5 \times 5} = 38.4 \text{ litres}$$

3. (b) In jar A

Quantity of milk = $\frac{2}{3} \times 60$ = 40 litres

Quantity of water = 20 litres

Let the quantity of water in jar B be x litres.

$\therefore$ Quantity of milk = $(40 - x)$ litres

According to the question,

$$\frac{40 + 40 - x}{20 + x} = \frac{13}{7} \quad \Rightarrow \quad \frac{80 - x}{20 + x} = \frac{13}{7}$$

$\Rightarrow$ $260 + 13x = 560 - 7x$ $\Rightarrow$ $13x + 7x = 560 - 260$

$\Rightarrow$ $20x = 300$ $\Rightarrow$ $x = \frac{300}{20} = 15$ litres

4. (b) Remaining mixture = 180 – 54 = 126 litres

Quantity of water = $\left(\frac{5}{18} \times 126\right)$ litres = 35 litre

On adding 6 litres of water,

Total quantity of water = 41 litres

Quantity of mixture = 126 + 6 = 132 litres

$\therefore$ Percentage of water = $\frac{41}{132} \times 100 \approx 31$

5. (d) If there be 1 litre of acid in 4 litres of mixture, then in

Case I. Percentage of acid = $\frac{1}{4+1} \times 100 = 20\%$

Case II. Percentage of acid = $\frac{2}{6} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$

$\therefore$ Percentage of acid in original mixture

$= \frac{1}{4} \times 100 = 25\%$

6. (c) Let the initial quantity liquid A and $B = 4x$ and x

According to question,

$$\frac{4x - 8}{x - 2 + 10} = \frac{2}{3}$$

$12x - 24 = 2x + 16$

$10x = 40$

$x = 4$

$\Rightarrow$ Initial quantity = $4x = 4 \times 4 = 16$ litres

7. (d) According to the question,

CP of the mixture = $15 \times 29 + 25 \times 20$ = ₹ 935

SP of the mixture = 27×40 = ₹ 1080

Profit = SP – CP

Profit = 1080 – 935 = ₹ 145

8. (a) According to the question,

CP of the mixture = $30 \times 9.5 + 40 \times 8.5$

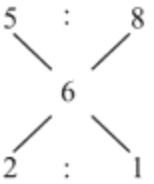
= 285 + 340 = ₹ 625

SP of the mixture = 8.90×70 = ₹ 623

loss = CP – SP

loss = 625 – 623 = ₹ 2

9. (a)



Required ratio = 2 : 1

10. (d) Let the C.P. of spirit be ₹ 1 per litre.

Spirit in 1 litre mix. of $A = \frac{5}{7}$ litre; C.P. of a litre mix. in $A = ₹\frac{5}{7}$.

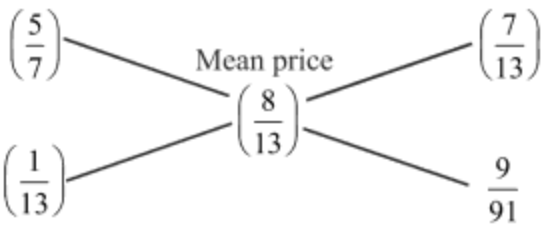
Spirit in 1 litre mix. of $B = \frac{7}{13}$ litre;

C.P. of 1 litre mix. in $B = ₹\frac{5}{13}$.

Spirit in 1 litre mix. of $C = \frac{8}{13}$ litre; Mean price = ₹. $\frac{8}{13}$.

By the rule of alligation, we have :

C.P. of 1 litre mixture in A C.P. of 1 litre mixture in B



$\therefore$ Required ratio = $\frac{1}{13} : \frac{9}{91} = 7 : 9$.

11. (c) Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid

$= x\left(1 - \frac{y}{x}\right)^n$ units

$\therefore$ Remaining water $= 30\left(1-\frac{3}{30}\right)^2 = \frac{30\times9\times9}{100}$

$= 24.3$ litres

12. (b) Remaining mixture in the vessel = 60 litres

Milk $= \frac{22}{25} \times 60 = 52.8$ litres

Water = 7.2 litres

On adding additional milk and water,

Milk = 52.8 + 4.8 = 57.6 litres

Water = 7.2 + 4.8 = 12 litres

$\therefore$ Required percent $= \frac{57.6-12}{57.6} \times 100$

$= \frac{45.6}{57.6} \times 100 = \frac{475}{6} = 79\frac{1}{6}\%$

13. (a) In initial mixture of the vessel,

Milk : Water = 80 : 18 = 40 : 9

In 49 litres of mixtures,

Milk = 40 litres

Water = 9 litres

Let $2x$ litres of milk and x litres of water be added.

According to the question,

$$\frac{40+2x}{9+x} = \frac{4}{1}$$

$\Rightarrow$ $36 + 4x = 40 + 2x$

$\Rightarrow$ $4x - 2x = 40 - 36$ $\Rightarrow$ $2x = 4$

$\Rightarrow$ $x = 2$ litres $\therefore$ Milk added = 4 litres

14. (d) Total initial quantity of juice in the vessel

$= 4x + 6x + 5x = 15x$ litres

In 15 litres of juice,

Grapes's juice = 4 litres

Pineapple's juice = 6 litres

Banana's juice = 5 litres

According to the question,

$(6x - 6 + 2) - (4x - 4 + 8) = 10$

$\Rightarrow$ $6x - 4 - 4x - 4 = 10$ $\Rightarrow$ $2x - 8 = 10$

$\Rightarrow$ $2x = 10 + 8 = 18$ $\Rightarrow$ $x = 9$

$\therefore$ Initial quantity of mixture $= 15x$

$= 15 \times 9 = 135$ litres

15. (a) Initial quantity of milk $= 2x$ litres

Water $= 3x$ litres

In 20 litres of mixture,

Milk $= \frac{2}{5} \times 20 = 8$ litre

Water $= \frac{3}{5} \times 20 = 12$ litre

On adding 20 litres of milk,

$\frac{2x-8+20}{3x-12} = \frac{4}{1}$ $\Rightarrow$ $2x + 12 = 12x - 48$

$\Rightarrow$ $12x - 2x = 12 + 48$ $\Rightarrow$ $10x = 60$

$\Rightarrow$ $x = 6$

Initial quantity of mixture $= 5x = 5 \times 6 = 30$ litres

16. (b) In 90 litres of mixture in the jar.

Milk $\Rightarrow \frac{5}{6} \times 90 = 75$ litres

Water $\Rightarrow$ 15 litres

On adding x litres of milk and water each:

$$\frac{75+x}{15+x} = \frac{4}{1}$$

$\Rightarrow$ $4x + 60 = 75 + x$ $\Rightarrow$ $4x - x = 75 - 60$

$\Rightarrow$ $3x = 15$ $\Rightarrow$ $x = \frac{15}{3} = 5$

$\therefore$ Required quantity that was added $= 2 \times 5 = 10$ litres

17. (c) Initially

Quantity of milk $= 3x$ litres

Quantity of water $= x$ litres

In four litres of mixture,

Milk = 3 litres

Water = 1 litre

According to the question,

$$\frac{3x-3+3}{x-1} = \frac{4}{1}$$

$\Rightarrow$ $4x - 4 = 3x$ $\Rightarrow$ $x = 4$ litres

18. (d) In 26 litres of mixture,

Milk $= \frac{10}{13} \times 26 = 20$ litres

Water = 6 litres

In original mixture,

Milk $= 10x$ litres

Water $= 3x$ litres

According to the question,

$\frac{10x-20}{3x-6+10} = \frac{5}{2}$ $\Rightarrow$ $\frac{2x-4}{3x+4} = \frac{1}{2}$

$\Rightarrow$ $4x - 8 = 3x + 4$ $\Rightarrow$ $x = 8 + 4 = 12$

$\therefore$ Initial quantity of mixture $= 13x$

$= 13 \times 12 = 156$ litres

19. (c) Let initial quantity of mixture = 25 litres

In 24 litre

Milk = 18 litres

Water = 6 litres

When, 3 litres water is added,

Milk : Water = 2 : 1

$\therefore$ If 3 litres of water is added total mixture = 25 litre

$\therefore$ If 24 litres of water is added total quantity

$= \frac{25\times24}{3} = 200$ litres

20. (a) In vessel B,

Quantity of milk $= 3x$ litres

Quantity of water $= x$ litres

In vessel C,

Quantity of water = $(4 + x)$ litres

Total quantity = $(24 + 4x)$ litres

According to the question,

$$\frac{4+x}{24+4x} = \frac{20}{100} = \frac{1}{5} \quad \Rightarrow \quad 20 + 5x = 24 + 4x$$

$$\Rightarrow \quad 5x - 4x = 24 - 20 \quad \Rightarrow \quad x = 4$$

$\therefore$ Initial quantity of mixture in vessel B $= 4 \times 4 = 16$ litres

21. (a) According to the question

	Zinc	:	Copper
In first alloy	$1_{\times 65}$	:	$2 \times 65 = 3$
Second alloy	$2_{\times 39}$	:	$3 \times 39 = 5$
New alloy	$5_{\times 15}$	:	$8 \times 15 = 13$

65, 39, 15 → 195

First alloy	65	:	130
Second alloy	78	:	117
New alloy	75	:	120

65 : 78 → 75 → 3 : 10

$\therefore$ Required ratio = 3 : 10

22. (b)

	Acid	:	Water	Mixture
Glass 1	[2	:	3 =	$5]_{\times 63}$
Glass 2	[3	:	4 =	$7]_{\times 45}$
Glass 3	[4	:	5 =	$9]_{\times 35}$
Glass 1	126	:	189	= 315
Glass 2	135	:	180	= 315
Glass 3	140	:	175	= 315
	+		+	
	401	:	544	

Required ratio = 401 : 544

23. (a) Total milk = 60 litres

Drawn off = 12 litres

$$\frac{\text{Final Quantity}}{\text{Initial Quantity}} = \left(1 - \frac{x}{c}\right)^t$$

x = Replaced Quantity

C = Capacity

T = number of process

$$\frac{\text{Final Quantity}}{\text{Initial Quantity}} = \left(1 - \frac{12}{60}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

Ratio of milk and water in the resultant mixture = 16 : 9

24. (d) According to the question

A		B	
7	+	5	= 12
			4 units
Final → 7	+	9	= 16

Solution

4 units → 9

1 unit → $\frac{9}{4}$

16 units → $\frac{9}{4} \times 16 = 36$

$\therefore$ The capacity of the can = 36 litres

25. (c)

	Spirit		Milk		Total	Capacity Ratio
I	4	+	1	=	5	2
II	11	+	4	=	15	3
III	7	+	3	=	10	4

	Spirit		Milk	Total
I	$4_{\times 12}$	= 48	$1_{\times 12}$ = 12	$5_{\times 6 \times 2}$
II	$11_{\times 6}$	= 66	$4_{\times 6}$ = 24	$15_{\times 2 \times 3}$
III	$7_{\times 12}$	= 84	$3_{\times 12}$ = 36	$10_{\times 3 \times 4}$

Total Ratio 198 : 72 = 11 : 4