HCF, LCM and Simplification

HCF AND LCM

HIGHEST COMMON FACTOR (HCF) OR GREATEST COMMON DIVISOR (GCD)

The highest (i.e. largest) number that divides two or more given numbers is called the highest common factor (HCF) of those numbers.

Methods to Find the HCF or GCD

There are two methods to find HCF of the given numbers.

(i) Prime Factorization Method

When a number is written as the product of prime numbers, then it is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Here, $2 \times 2 \times 2 \times 3 \times 3$ or $2^3 \times 3^2$ is called prime factorization of 72.

To find the HCF of given numbers by this methods, we perform the prime factorization of all the numbers and then check for the common prime factors in it. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given numbers. The HCF is the product of all such prime factors with their respective least indices.

(ii) Division Method

To find the HCF of two numbers by division method, we divide the larger number by the smaller number. Then we divide the smaller number by the first remainder, then first remainder by the second remainder and so on, till the remainder becomes 0. The last divisor is the required HCF.

LEAST COMMON MULTIPLE (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

Methods to Find the LCM

There are two methods to find the LCM.

(i) Prime Factorization Method

After performing the prime factorization of all the given numbers, we find the highest index of all the prime factors among the given numbers. The LCM is the product of all these prime factors with their respective highest indices because LCM must be divisible by all of the given numbers.

(ii) Division Method

Check whether any prime number that divides at least two of all the given numbers. If there is no such prime number, then the product of all these numbers is the required LCM, otherwise write all the given numbers in a row separating them by putting the comma ',' in between the numbers and find the smallest prime number that divides at least two of the given numbers.

RULE FOR FINDING HCF AND LCM OF FRACTIONS

(I) HCF of two or more fractions

$$= \frac{\text{HCF of numerator of all fractions}}{\text{LCM of denominator of all fractions}}$$

(II) LCM of two or more fractions

$$= \frac{\text{LCM of numerator of all fractions}}{\text{HCF of denominator of all fractions}}$$

Example 1. Find the HCF and LCM of $\frac{4}{5}, \frac{6}{11}$ and $\frac{3}{5}$.

Sol. $\text{HCF} = \frac{\text{HCF of 4, 6, 3}}{\text{LCM of 5, 11, 5}} = \frac{1}{55}$

$\text{LCM} = \frac{\text{LCM of 4, 6, 3}}{\text{HCF of 5, 11, 5}} = \frac{12}{1} = 12$

The product of the H.C.F and L.C.M of any two numbers is always equal to the product of these two numbers. However the same pattern is not applicable to three or more numbers.

Thus, for any two numbers a and b.

$$a \times b = \text{H.C.F.} \times \text{L.C.M.}$$

SIMPLIFICATION

FUNDAMENTAL OPERATIONS

1. Addition

(a) Sum of two positive numbers is a positive number.
For example : $(+5) + (+2) = +7$

(b) Sum of two negative numbers is a negative number.
For example : $(-5) + (-3) = -8$

(c) Sum of a positive and a negative number is the difference between their magnitudes with the sign of the number with greater magnitude.

For example : $(-3) + (+5) = 2$ and $(-7) + (+2) = -5$

2. Subtractions

Subtraction of two numbers is same as the sum of a positive and a negative number.

For Example : $(+9) - (+2) = (+9) + (-2) = 7$

$(-3) - (-5) = (-3) + 5 = +2.$

3. Multiplication

(a) Product of two positive numbers is positive.

(b) Product of two negative numbers is positive.

(c) Product of a positive number and a negative number is negative.

(d) Product of more than two numbers is positive or negative depending upon the presence of negative quantities.

If the number of negative numbers is even then product is positive and if the number of negative numbers is odd then product is negative.

For Example : $(-3) \times (+2) = -6$

$(-5) \times (-7) = +35$

$(-2) \times (-3) \times (-5) = -30$

$(-2) \times (-3) \times (+5) = +30$

4. Division

(a) If both the dividend and the divisor are of same sign, then quotient is always positive.

(b) If the dividend and the divisor are of different sign, then quotient is negative,

For Example : $(-36) \div (+9) = -4$

$(-35) \div (-7) = +5$

5. Brackets

Types of brackets are :

(i) Vinculum or bar : '_'

(ii) Parenthesis or small or common brackets : ( )

(iii) Curly or middle brackets : { }

(iv) Rectangular or big brackets : [ ]

The order for removal of brackets is : ( ), { }, [ ]

Note

If there is a minus (–) sign before the bracket then while removing bracket, sign of each term will change + to – and – to +.

6. 'BODMAS' Rule

Now a days BODMAS becomes 'VBODMAS' where,

'V' stands for "Vinculum" or Bar

'B' stands for "Bracket" order of operation of bracket is ( ), { }, [ ].

'O' stands for "Of" (Calculation is done the same as multiplication)

'D' stands for "Division"

'M' stands for "Multiplication"

'A' stands for "Addition"

'S' stands for "Subtraction"

A given series of calculations or operations is done in a specific order as each letter of VBODMAS in order represent.

So, first of all we solve vinculum then the inner most brackets moving outwards. Then we perform 'of' which means multiplication, then division, addition and subtraction.

- Addition and subtraction can be done together or separately as required.
- Between any two brackets if there is not any sign of addition, subtraction and division it means we have to do multiplication

$(20 \div 5)(7 + 3 \times 2) + 8 = 4(7 + 6) + 8$

$= 4 \times 13 + 8 = 52 + 8 = 60$

Example 2. Simplify : 7 – 2 + 13 – 5 – 2 + 1

Sol. $7 - 2 + 13 - 5 - 2 + 1$

$= 7 + 13 + 1 - 2 - 5 - 2 = 21 - 9 = 12$

$[7 + 13 + 1 = 21 \text{ and } -2 - 5 - 2 = -9]$

Example 3. Find the approximate value of

$234 \div 17 + 15.3 \times 18 - 13 \times 3.7$

(a) 250 (b) 220

(c) 245 (d) 235

Sol. (d) $234 \div 17 + 15.3 \times 18 - 13 \times 3.7$

$= 225 \div 15 + 15 \times 18 - 13 \times 4$

$= 15 + 270 - 52 = 15 + 270 - 50 = 235.$

Example 4. What approximate value should come in place of the question mark?

$11^3 + 0.8^3 + 12^3 + 1.1^3 + 1.2^3 = ?$

(a) 3063 (b) 3060

(c) 3066 (d) 3068

Sol. (a) $11^3 + 0.8^3 + 12^3 + 1.1^3 + 1.2^3$

$= 1331 + 1 + 1728 + 1.331 + 1.728$

$= 1332 + 1728 + 1 + 2 = 3063.$

Example 5. What approximate value should come in place of the question mark (?) in the following question?

$$\frac{256}{\sqrt{17}} + \frac{190}{16} = ?$$

(a) 68 (b) 76 (c) 78 (d) $\frac{446}{16}$

Sol. (b) $\frac{256}{\sqrt{17}} + \frac{190}{16} = \frac{256}{4} + 12 = 64 + 12 = 76.$

ALGEBRAIC IDENTITIES

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\Rightarrow (a + b)^3 = a^3 + 3ab(a + b) + b^3$

(ii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$\Rightarrow (a - b)^3 = a^3 - 3ab(a - b) - b^3$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(v) $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

EXERCISE

1. The smallest five digit number which is divisible by 12, 18 and 21 is:
(a) 10080 (b) 30256 (c) 10224 (d) 50321

2. The H.C.F. and L.C.M. of two numbers are 8 and 48 respectively. If one of the numbers is 24, then the other number is
(a) 48 (b) 36 (c) 24 (d) 16

3. The greatest number, which when subtracted from 5834, gives a number exactly divisible by each of 20, 28, 32 and 35, is
(a) 1120 (b) 4714 (c) 5200 (d) 5600

4. The H.C.F. and L.C.M. of two numbers are 12 and 336 respectively. If one of the numbers is 84, the other is
(a) 36 (b) 48 (c) 72 (d) 96

5. The traffic lights at three different road crossings change after 24 seconds, 36 seconds and 54 seconds respectively. If they all change simultaneously at 10 : 15 :00 AM, then at what time will they again change simultaneously?
(a) 10 : 16 : 54 AM (b) 10 : 18 : 36 AM
(c) 10 : 17 : 02 AM (d) 10 : 22 : 12 AM

6. Four runners started running simultaneously from a point on a circular track. They took 200 seconds, 300 seconds, 360 seconds and 450 seconds to complete one round. After how much time they meet at the starting point for the first time?
(a) 1800 seconds (b) 3600 seconds
(c) 2400 seconds (d) 4800 seconds

7. L.C.M. of $\frac{2}{3}, \frac{4}{9}, \frac{5}{6}$ is
(a) $\frac{8}{27}$ (b) $\frac{20}{3}$ (c) $\frac{10}{3}$ (d) $\frac{20}{27}$

8. Amit, Sucheta and Neeti start running around a circular track and complete one round in 18, 24 and 32 seconds respectively. In how many seconds will the three meet again at the starting point if they all have started running at the same time?
(a) 196 sec. (b) 288 sec.
(c) 324 sec. (d) Cannot be determined
(e) None of these

9. The H.C.F. of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$ is
(a) $\frac{1}{54}$ (b) $\frac{1}{27}$ (c) $\frac{20}{3}$ (d) $\frac{27}{3}$

10. HCF of two numbers each of 4 digits is 103 and their LCM is 19261. Sum of the numbers is
(a) 2884 (b) 2488 (c) 4288 (d) 4882

11. Let x be the least number, which when divided by 5, 6, 7 and 8 leaves a remainder 3 in each case but when divided by 9 leaves remainder 0. The sum of digits of x is
(a) 24 (b) 21 (c) 22 (d) 18

12. The LCM of four consecutive numbers is 60. The sum of the first two numbers is equal to the fourth number. What is the sum of four numbers?
(a) 17 (b) 14 (c) 21 (d) 24

13. The least number which when divided by 48, 64, 90, 120 will leave the remainders 38, 54, 80, 110 respectively, is
(a) 2870 (b) 2860
(c) 2890 (d) 2880

14. Three numbers which are co-prime to one another are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is:
(a) 75 (b) 81
(c) 85 (d) 89

15. If A and B are the HCF and LCM respectively of two algebraic expressions x and y, and $A + B = x + y$, then the value of $A^3 + B^3$ is
(a) $x^3 - y^3$ (b) x^3
(c) y^3 (d) $x^3 + y^3$

16. A milk vendor has 21 litres of cow milk, 42 litres of toned milk and 63 litres of double toned milk. If he wants to pack them in cans so that each can contains same litres of milk and does not want to mix any two kinds of milk in a can, then the least number of cans required is:
(a) 3 (b) 6
(c) 9 (d) 12

17. The LCM of two positive integers is twice the larger number. The difference of the smaller number and the GCD of the two numbers is 4. The smaller number is:
(a) 12 (b) 6
(c) 8 (d) 10

18. The number of prime factors in $6^{333} \times 7^{222} \times 8^{111}$?
(a) 1221 (b) 1222
(c) 1111 (d) 1211

19. $\dfrac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}}{\sqrt[3]{8}} = ?$
(a) 8 (b) 4
(c) 1/2 (d) 2

20. (3x – 2y) : (2x + 3y) = 5 : 6, then one of the value of $\left(\dfrac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$ is
(a) $\frac{1}{25}$ (b) 5
(c) $\frac{1}{5}$ (d) 25

Hints & Solutions

1. **(a)** Lowest 5 digit number = 10,000
The number which is divisible by 12, 18 and 21 is LCM of 12, 18, 21 which is 252.
$\frac{10000}{252}$ gives 172 as remainder
So, 252 – 172 = 80
10,000 + 80 = 10080
If 10080 when divided by 12, 18 and 21 gives 0 as remainder
So, 10080 is the least 5–digit number.

2. **(d)** $p \times q = \text{HCF} \times \text{LCM}$
$\therefore$ Second number $= \frac{8 \times 48}{24} = 16$

3. **(b)** LCM of 20, 28, 32, 35 is 1120
$\therefore$ Required number = 5834 – 1120 = 4714

4. **(b)** First number × second number = HCF × LCM
$q = \frac{12 \times 336}{84} = 48$

5. **(b)** LCM of 24, 36 and 54 seconds = 216 seconds
= 3 minutes 36 seconds
$\therefore$ Required time
= 10 : 15 : 00 + 3 minutes 36 seconds
= 10 : 18 : 36 A.M.

6. **(a)** Required time = LCM of 200, 300, 360 and 450 seconds = 1800 seconds.

7. **(b)** LCM of $\frac{2}{3}, \frac{4}{9}, \frac{5}{6}$
$\frac{\text{LCM of } (2,4,5)}{\text{HCF of } (3,9,6)} = \frac{20}{3}$

8. **(b)** Required time = LCM of 18, 24 and 32 seconds.
= 288 seconds

9. **(b)** $\text{HCF} = \frac{\text{HCF of } 1, 5, 2, 4}{\text{LCM of } 3, 6, 9, 27} = \frac{1}{27}$

10. **(a)** Let the numbers be $103x$ and $103y$ where x and y are prime to each other.
$\therefore$ LCM = $103xy$ $\Rightarrow$ $103xy = 19261$
$\Rightarrow xy = \frac{19261}{103} = 187 \Rightarrow x = 11$ or 17
$y = 17$ or 11
$\therefore$ Numbers = 103 × 11 = 1133
and 103 × 17 = 1751 and Sum = 1751 + 1133 = 2884

11. **(d)** LCM of 5, 6, 7 & 8 = 840
$\frac{840n+3}{9} \Rightarrow \frac{3n+3}{9}$
$\Rightarrow$ Take $n = 2$ $\Rightarrow$ 3(2) + 3
$\Rightarrow \frac{9}{9} =$ Remainder = 0
$\therefore$ Number is = $840n + 3$
$\Rightarrow$ 840(2) + 3 ($n = 2$) $\Rightarrow$ 1683
Sum of digits = 18

12. **(b)** Numbers = $x, x+1, x+2, x+3$
Ist + IInd = IVth
$x + x + 1 = x + 3$
$\therefore$ $x = 2$
$\therefore$ Numbers are 2, 3, 4, 5
$\therefore$ Sum of four numbers = 2 + 3 + 4 + 5 = 14.

13. **(a)** Here, (48 – 38) = 10, (64 – 54) = 10, (90 – 80) = 10 and (120 – 110) = 10.
$\therefore$ Required number
= (L.C.M of 48, 64, 90 and 120) – 10 = 2870

14. **(c)** Let numbers are a, b and c.
= a, b, c are co-prime numbers
HCF of co-prime numbers = 1
$\therefore$ HCF $(a, b, c) = 1$
$\therefore$ $a \times b = 551$, $b \times c = 1073$
$\Rightarrow \frac{a \times b}{b \times c} = \frac{551}{1073} = \frac{19 \times 29}{37 \times 29} \Rightarrow \frac{a}{c} = \frac{19}{37}$
$\therefore$ $a = 19$, $b = 29$ $c = 37$
$\therefore$ Sum of numbers = $a + b + c$ = 19 + 29 + 37 = 85

15. **(d)** HCF = A (given)
LCM = B
Given numbers are x & y respectively.
$\Rightarrow$ (Product of numbers is = Product of LCM × HCF)
$\Rightarrow xy = AB$
Now $\Rightarrow A + B = x + y$ (given)
Take cube on both sides
$(A+B)^3 = (x+y)^3$
$\Rightarrow A^3 + B^3 + 3AB(A+B) = x^3 + y^3 + 3xy(x+y)$
$\Rightarrow A^3 + B^3 + 3xy(x+y) = x^3 + y^3 + 3xy(x+y)$
$\therefore A^3 + B^3 = x^3 + y^3$

16. **(b)** For least or minimum number of canes we should have maximum capacity canes for required quantity
$\Rightarrow$ For this we take HCF of given quantities.
HCF (21, 42, 63) = 21
$\therefore$ Maximum capacity of a cane = 21 litres
$\therefore$ Number of canes of cow milk $= \frac{21}{21} = 1$
$\therefore$ Number of canes of toned milk $= \frac{42}{21} = 2$
$\therefore$ Number of canes of double toned milk $= \frac{63}{21} = 3$
$\therefore$ Total number of canes = 1 + 2 + 3 = 6

17. (c) Let G.C.D. = a

$\therefore$ Let number are ax and ay $(ax > ay)$

LCM = axy

$\Rightarrow$ LCM = 2 × larger number

$\therefore$ $axy = 2 \times ax$ $\quad\therefore\quad y = 2$

Also given that

$\Rightarrow$ Smaller number – G.C.D = 4 $\Rightarrow$ $ay - a = 4$

$2a - a = 4$

$a = 4$

G.C.D. = $a = 4$

$y = 2$

$\therefore$ Smaller number = ay $\Rightarrow 2 \times 4 = 8$

18. (a) $6^{333} \times 7^{222} \times 8^{111} \Rightarrow 2^{333} \times 3^{333} \times 7^{222} \times (2^3)^{111}$

$\Rightarrow 2^{666} \times 3^{333} \times 7^{222}$

$\Rightarrow$ Total factors = 666 + 333 + 222 = 1221

19. (d) $$\frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}}{\sqrt[3]{8}}$$

$$\Rightarrow \frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}}{2}$$

$$\Rightarrow \frac{\sqrt{10+\sqrt{25+\sqrt{121}}}}{2} \Rightarrow \frac{\sqrt{10+\sqrt{36}}}{2}$$

$$\Rightarrow \frac{\sqrt{16}}{2} = \frac{4}{2} = 2$$

20. (d) $\dfrac{3x-2y}{2x+3y} = \dfrac{5}{6}$

$18x - 12y = 10x + 15y$

$8x = 27y$

$\dfrac{x}{y} = \dfrac{27}{8}$

$$\left[\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right]^2 = \left(\frac{\sqrt[3]{27}+\sqrt[3]{8}}{\sqrt[3]{27}-\sqrt[3]{8}}\right)^2$$

$$= \left(\frac{3+2}{3-2}\right)^2 = (5)^2 = 25$$