Average and Ages

AVERAGE

Average is a very simple and effective way of representing an entire group by a single value.

$$\textbf{Average or Mean} = \frac{\text{Sum of given quantities}}{\text{Number of quantities}}$$

To calculate the sum of quantities, they should be in the same unit.

📝 Note

If X is the average of $x_1, x_2, x_3 \ldots x_n$ then

- The average of $x_1 + a, x_2 + a, x_3 + a, \ldots x_n + a$ is $X + a$.
- The average of $x_1 - a, x_2 - a, x_3 - a \ldots x_n - a$ is $X - a$
- The average of $ax_1, ax_2, \ldots ax_n$ is aX, provide $a \neq 0$
- The average of $\dfrac{x_1}{a}, \dfrac{x_2}{a}, \dfrac{x_3}{a}, \ldots \dfrac{x_n}{a}$ is $\dfrac{X}{a}$, provided $a \neq 0$

Average of a Group Consisting Two Different Groups When Their Averages are Known

Let group A contains m quantities and their average is a and group B contains n quantities and their average is b, then **average of group C containing these $(m + n)$ quantities** $= \dfrac{ma + nb}{m + n}$

Weighted Average

If we have two or more groups of members whose individual averages are known, then combined average of all the members of all the groups is known as weighted average. Thus if there are k groups having member of number $n_1, n_2, n_3, \ldots n_k$ with averages $A_1, A_2, A_3 \ldots \ldots A_k$ respectively then weighted average.

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \ldots + n_k A_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

Average Speed if Equal Distances are Travelled by Two Different Speeds

If a car travels at a speed S_1 from A to B and at a speed S_2 from B to A. Then

$$\text{Average speed} = \frac{2 S_1 . S_2}{S_1 + S_2}$$

The above formula can be found out as follows:

If distance between A and B is d, then

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{\dfrac{d}{S_1} + \dfrac{d}{S_2}}$$

$$= \frac{2}{\dfrac{1}{S_1} + \dfrac{1}{S_2}} = \frac{2 S_1 . S_2}{S_1 + S_2}$$

Average Speed if Equal Distances are Travelled by Three Different Speeds

$$\text{Average speed} = \frac{3xyz}{xy + yz + zx}$$

Where x, y and z are three different speeds.

👆 Remember...

- Average of first n natural numbers $= \dfrac{(n+1)}{2}$
- Average of first n consecutive even numbers $= (n + 1)$
- Average of first n consecutive odd numbers $= n$
- Average of any number of consecutive numbers
  $$= \frac{\text{First number} + \text{Last number}}{2}$$
- Average of first n consecutive odd numbers
  $$= \frac{\text{Last odd number} + 1}{2}$$
- Average of first n consecutive even numbers
  $$= \frac{\text{Last even number} + 2}{2}$$
- Average of squares of first n natural numbers
  $$= \frac{(n+1)(2n+1)}{6}$$
- Average of the cubes of first n natural numbers
  $$= \frac{n(n+1)^2}{4}$$
- Average of n consecutive multiples of any number
  $$= \frac{\text{Number} \times (n+1)}{2}$$
- If n is odd, then the average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.

AGES AND AVERAGES

If the average age of a group of persons is x years today then after n years their average age will be $(x + n)$ years because for a group of people, 1 year is added to each person's age every year. Similarly, n years ago their average age would have been $(x - n)$ years, because 1 year is subtracted from each person's age before every year.

EXERCISE

1. A cricketer whose bowling average is 24.85 runs per wicket, takes 5 wickets for 52 runs in next inning and thereby decreases his average by 0.85. The number of wickets taken by him till the last match was :
   (a) 75   (b) 85   (c) 80   (d) 96

2. The arithmetic mean of the scores of a group of students in a test was 52. The brightest 20% of them secured a mean score of 80 and the dullest 25% a mean score of 31. The mean score of remaining 55% is.
   (a) 45   (b) 50   (c) 51.4   (d) 54.6

3. The average monthly income (in ₹) of certain agricultural workers is S and that of other workers is T. The number of agricultural workers is 11 times that of other workers. Then the average monthly income (in ₹) of all the workers is :
   (a) $\dfrac{S+11T}{12}$ (b) $\dfrac{S+T}{12}$

   (c) $\dfrac{11S+T}{12}$ (d) $\dfrac{1}{11S}+T$

4. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of the husband is :
   (a) 40 years   (b) 35 years
   (c) None of the options   (d) 50 years

5. The ratio of the number of players on the three cricket teams A, B and C is 2:5:3. If the average number of runs scored per player for each of the three teams, A, B and C, is 30 : 17 : 25 respectively, then what is the average number of runs scored per player across all three teams collectively?
   (a) 20   (b) 21   (c) 22   (d) 23

6. The average of three consecutive odd numbers is 12 more than one third of the first of these numbers. What is the last of the three numbers ?
   (a) 15   (b) 17
   (c) 19   (d) Data inadequate

7. The average of 5 consecutive numbers is n. If the next two numbers are also included, the average of the 7 numbers will
   (a) increase by 2   (b) increase by 1
   (c) remain the same   (d) increase by 1.4

8. A batsman in his 12th innings makes a score of 63 runs and there by increases his average scores by 2. What is his average after the 12th innings?
   (a) 13   (b) 41   (c) 49   (d) 87

9. The average weight of 12 crewmen in a boat is increased by $\dfrac{1}{3}$ kg, when one of the crewmen whose weight is 55 kg is replaced by a new man. What is the weight of that new man ?
   (a) 58 kg   (b) 60 kg
   (c) 57 kg   (d) 59 kg

10. The batting average for 30 innings of a cricket player is 40 runs. His highest score exceeds his lowest score by 100 runs. If these two innings are not included, the average of the remaining 28 innings is 38 runs. The lowest score of the player is:
    (a) 15   (b) 18   (c) 20   (d) 12

11. The average salary of all the workers in a workshop is ₹ 8,000. The average salary of 7 technicians is ₹ 12,000 and the average salary of the rest is ₹ 6,000. The total number of workers in the workshop is
    (a) 20   (b) 21   (c) 22   (d) 23

12. The average of some natural numbers is 15. If 30 is added to first number and 5 is subtracted from the last number the average becomes 17.5 then the number of natural number is:
    (a) 20   (b) 30   (c) 15   (d) 10

13. The average monthly income of A and B is ₹ 15,050, the average monthly income of B and C is ₹ 15,350 and the average income of A and C is ₹ 15,200. The monthly income of A is
    (a) ₹ 15,900   (b) ₹ 15,200
    (c) ₹ 14,900   (d) ₹ 15,500

14. The average weight of 45 students in a class was calculated as 36 kg. It was later found that the weight of two students in the class was wrongly calculated. The actual weight of one of the boys in the class was 32 kg., but it was calculated as 34 kg and the weight of another boy in the class was 45 kg.; whereas it was calculated as 40 kg. What is the actual average weight of the 45 students in the class? (Rounded off to two-digits after decimal).
    (a) 36.07 kg.   (b) 36.16 kg.
    (c) 35.84 kg.   (d) Cannot be determined
    (e) None of these

15. Seema's present age is four times her son's present age and four-seventh of her father's present age. The average of the present age of all three of them is 32 years. What is the difference between the Seema's son's present age and Seema's father's present age?
    (a) 44 years   (b) 48 years
    (c) 46 years   (d) Cannot be determined
    (e) None of these

16. The sum of 8 consecutive odd numbers is 656. Also average of four consecutive even numbers is 87. What is the sum of the smallest odd number and second largest even number?
    (a) 165   (b) 175
    (c) 163   (d) Cannot be determined
    (e) None of these

17. In Arun's opinion his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Arun and he thinks that Arun's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight

cannot be greater than 68 kg. If all of them are correct in their estimation, what is the average of different probable weights of Arun?

(a) 69 kg (b) 67 kg
(c) 68 kg (d) Data inadequate
(e) None of these

18. The average age of 80 girls was 20 years, the average age of 20 of them was 22 years and that of another 20 was 24 years. Find the average age of the remaining girls.

(a) 17 years (b) 19 years
(c) 21 years (d) 15 years
(e) None of these

19. A batsman played three matches in a tournament. The respective ratio between the scores of 1st and 2nd matches was 5 : 4 and that between the scores of 2nd and 3rd matches was 2 : 1. The difference between the 1st and 3rd matches was 48 runs. What was the batsman's average score in all the three matches?

(a) 45 (b) $58\frac{2}{3}$

(c) 70 (d) $40\frac{2}{3}$

(e) $50\frac{1}{4}$

20. After replacing an old member by a new member, it was found that the average age of five members of a club is the same as it was 3 years ago. The difference between the ages of the replaced and the new members is

(a) 2 years (b) 4 years
(c) 8 years (d) 15 years

21. The average age of a husband and wife, who were married 4 years ago, was 25 years at the time of their marriage. The average age of the family consisting of husband, wife and a child, born during the interval is 20 years today. The age of the child is :

(a) 1 years (b) 2 years
(c) 2.5 years (d) 3 years

22. A man purchases milk for three consecutive years. In the first year, he purchases milk at the rate of ₹ 7.50 per litre, in the second year, at the rate of ₹ 8.00 per litre and in the third year, at ₹ 8.50 per litre. If he purchases milk worth ₹ 4080 each year, the average price of milk per litre for the three year is :

(a) ₹ 7.68 (b) ₹ 7.98
(c) ₹ 7.54 (d) ₹ 7.83

23. The average weight of three men A, B and C is 84 kg. D joins them and the average weight of the four becomes 80 kg. If E whose weight is 3 kg more than that of D, replaces A, the average weight of B, C, D and E becomes 79 kg. The weight of A is :

(a) 65 kg (b) 70 kg
(c) 75 kg (d) 80 kg

24. An elevator can carry maximum of 16 passengers with an average weight of 80 kg. However, four boys more than the maximum carrying capacity of the elevator entered it making the average weight as 86 kg and overloading the elevator. What is the average weight of those four boys ?

(a) 112 kg (b) 108 kg
(c) 110 kg (d) 98 kg

25. Out of three given numbers, the first number is twice the second and thrice the third. If the average of the three numbers is 154, what is the difference between the first and the third number?

(a) 126
(b) 42
(c) 166
(d) 52
(e) None of these

Hints & Solutions

1. (b) Let the no. of wickets $= x$
According to question
$24.85x + 52 = 24(x + 5)$
$24.85x + 52 = 24x + 120$
$0.85x = 68$
$$x = \frac{68 \times 100}{85}$$
$x = 80$
No. of wickets till the last match is
$x + 5 = 80 + 5 = 85$

2. (c) Let the total no. of students $= 100$
According to question
$20 \times 80 + 25 \times 31 + 55 \times x = 52 \times 100$
$1600 + 775 + 55x = 5200$
$55x = 5200 - 1600 - 775$
$55x = 2825$
$x = 51.4$

3. (c) According to question
$$\frac{\text{Agricultural}}{\text{other workers}} = \frac{11}{1}$$
Average of monthly Income of all workers
$$= \frac{11 \times S + 1 \times T}{12} = \frac{11S + T}{12}$$

4. (a) Total age of husband, wife & their child at present
$= 3 \times 27 + 3 \times 3 = 90$ years.
Total age of wife & child at present
$= 20 \times 2 + 2 \times 5 = 50$ years
$\therefore$ Present age of husband $= 90 - 50 = 40$ years

5. (c) $A : B : C = 2 : 5 : 3$
$$\text{Avg} = \frac{30 \times 2 + 17 \times 5 + 25 \times 3}{10} = \frac{60 + 85 + 75}{10} = 22$$

6. (c) Let 3 consecutive odd no. be $x, x + 2$ and $x + 4$
ATQ
$$\frac{x + x + 2 + x + 4}{3} = 12 + \frac{1}{3}x$$
$$\frac{3x + 6}{3} - \frac{x}{3} = 12 = 2x + 6 = 36$$
$$\Rightarrow \quad x = \frac{36 - 6}{2} = 15$$
last No $= 15 + 4 = 19$

7. (b) Let the numbers be $n - 2, n - 1, n, n + 1$ and $n + 2$.
Their average $= n$.
Next two consecutive numbers are $n + 3$ and $n + 4$.
Therefore the average of 7 consecutive numbers
$$= \frac{(n-2) + (n-1) + n + (n+1) + (n+2) + (n+3) + (n+4)}{7}$$
$$= \frac{5n + 2n + 7}{7} = n + 1$$

8. (b) Let the average of batsman after 11th innings $= A$
$$\frac{\text{Total score made by batsman at the end of } 11^{\text{th}} \text{ innings}}{11} = A$$
$\therefore$ Total score after 11th innings $= 11A$
Now,
$$\frac{\text{Total score after } 11^{\text{th}} \text{ innings} + \text{score made in } 12^{\text{th}} \text{ innings}}{12} = A + 2$$
$\Rightarrow \quad 11A + 63 = (A + 2) \times 12$
$\Rightarrow \quad 11A - 12A = 24 - 63$
$\Rightarrow \quad A = 39$
12th innings average $= 39 + 2 = 41$

9. (d) Weight of new crewmen
$= $ Replace man weight $+$ [No. of crewmen $\times$ increased average]
$$= 55 + 12 \times \frac{1}{3} = 59 \text{ kg}$$

10. (b) Lowest score $= x$
Highest score $= x + 100$
$\therefore \quad 28 \times 38 + x + x + 100 = 30 \times 40$
$\Rightarrow \quad 1064 + 2x + 100 = 1200$
$\Rightarrow \quad 2x = 1200 - 1164 = 36$
$\Rightarrow \quad x = 18$

11. (b) Let total number of workers be n
total salary of all workers $= 8000 \, n$
total salary of 7 technicians $= 7 \times 12000 = 84,000$
total salary of remaining workers $= (n - 7) \times 6000$
$84000 + (n - 7) \times 6000 = 8000 \, n$
$84 + 6n - 42 = 8n$
$42 = 2n$
$n = 21$

12. (d) Let number of natural numbers be x
$\therefore \quad 15x + 30 - 5 = 17.5 \, x$
$\qquad 25 = 2.5x$
$$x = \frac{25}{2.5} = 10$$
So 10 natural numbers are there.

13. (c) Sum of monthly income of $A + B = 30100$ ...(i)
Sum of monthly income of $B + C = 30700$ ...(ii)
Sum of monthly income of $A + C = 30400$ ...(iii)
Subtracting eqn. (i) from (ii)
$B + C - A - B = 30700 - 30100$
$C - A = 600$ ...(iv)
$C + A = 30400$ ...(v)
Subtracting eqn. (iv) from (v)
$C - A - C - A = 600 - 30400$
$-2A = -29800$
$A = ₹\,14900$

14. (a) Actual weight of all the students.
$$= (36 \times 45) - (34 - 32) - (40 - 45)$$
$$= 1620 + 3 = 1623 \text{ kg}$$
∴ Actual average weight
$$= \frac{1623}{45} = 36.07 \text{ kg}$$

15. (b) Let Seema's son's present age $= x$ years
∴ Seema's present age $= 4x$ years
∴ Seema's father's present age
$$= 4x \times \frac{7}{4} = 7x \text{ years}$$
Now $x + 4x + 7x = 32 \times 3$
$\Rightarrow \quad 12x = 96$
$$\Rightarrow \quad x = \frac{96}{12} = 8$$
∴ Required difference
$$= 7x - x = 6x = 6 \times 8 = 48 \text{ years}$$

16. (c) Average of 8 consecutive odd numbers $= \dfrac{656}{8} = 82$
∴ Fourth number $= 82 - 1 = 81$
∴ First number $= 75$
Average of 4 even numbers $= 87$
∴ Second even number $= 87 - 1 = 86$
Second largest even number $= 88$
∴ Required sum $= 75 + 88 = 163$

17. (b) Arun's opinion: $65 < W < 72$
Brother's opinion: $60 < W < 70$
Mother's opinion: $W \le 68$
Probable weights $= 66, 67, 68$
$$\text{Average} = \frac{66 + 67 + 68}{3} = 67 \text{ kg}$$

18. (a) Total age of remaining 40 girls
$$= (80 + 20 - 20 \times 22 - 20 \times 24) \text{ years}$$
$$= (1600 - 440 - 480) \text{ years} = 680 \text{ years}$$
∴ Required average age $= \dfrac{680}{40} = 17$ years

19. (b) Match I : Match II $= 5 : 4$
Match II : Match III $= 2 : 1 = 4 : 2$
∴ Match I : Match II : Match III $= 5 : 4 : 2$
According to the question.
$5x - 2x = 48$
$$\Rightarrow \quad 3x = 48 \quad \Rightarrow \quad x = \frac{48}{3} = 16$$
Total runs scored in three matches
$$= 5x + 4x + 2x = 11x = 11 \times 16 = 176$$
∴ Required average $= \dfrac{176}{3} = 58\dfrac{2}{3}$

20. (d) Let present average age is $= x$ years
Total age $= 5x$ year
According to question,
$5x - y + z = 5x - 15$
where $y =$ Replaced member

$z =$ New member
$-y + z = -15$
$y - z = 15$
This is required difference.

21. (b) According to question
$$\frac{H + W}{2} = 25 \text{ years}$$
$H + W = 50$ years
$$\frac{H + W + C}{3} = 20$$
$H + W + C = 60$ years and
Sum of present age of $H + W$
$$= 50 + 4 \times 2 = 58 \text{ years}$$
∴ Child age $= 2$ years

22. (b) According to the question
Quantity of Milk in Ist year $= \dfrac{4080}{7.5} = 544$ ltr
Quantity of Milk in 2nd Year $= \dfrac{4080}{8} = 510$ ltr
Quantity of Milk in 3rd Year $= \dfrac{4080}{8.5} = 480$ ltr
Total Milk $= 544 + 510 + 480 = 1534$ ltr
Average price in three year $= \dfrac{4080 \times 3}{1534} = ₹ 7.98$

23. (c) Sum of weight of A, B & $C = 84 \times 3 = 252$ kg.
Sum of weight of A, B, C & $D = 80 \times 4 = 320$ kg.
D's weight $= 320 - 252 = 68$ kg.
E's weight $= 68 + 3 = 71$ kg.
Sum of weight of B, C, D & $E = 4 \times 79 = 316$
$B + C + 68 + 71 = 316$
$B + C = 177$
$A + B + C = 252$
∴ $A = 252 - 177$
∴ $A = 75$ kg.

24. (c)

Passengers	weight	Total weight
$16 \times$	80	$= 1280$
$20 \times$	86	$= 1720$

∴ Weight of 4 Boys $= (1720 - 1280) = 440$
∴ Average weight of 4 Boys $= \dfrac{440}{4} = 110$ kg.

25. (e) Let the first number be $= 6x$
∴ Second number $= 3x$
and the third number $= 2x$
According to the question,
$$6x + 3x + 2x = 154 \times 3$$
or, $11x = 154 \times 3$
∴ $x = \dfrac{154 \times 3}{11} = 42$
∴ Required difference
$$= 6x - 2x = 4x = 4 \times 42 = 168$$