Algebraic Expression,

Polynomial & Rational Expression

CONSTANT

A symbol having a fixed numerical value is called a **constant**.

For example : $5, -2, 3.14, \frac{7}{9}, \sqrt{3}, 0, \pi$ or $\frac{22}{7}$ etc. are constant.

VARIABLES

A symbol which may be assigned different numerical values is known as a **variable**.

For example : In the formula of circumference of a circle, $C = 2\pi r$, 2 and π are constants and C and r are variables. Here, C is the circumference and r is the radius of the circle.

ALGEBRAIC EXPRESSIONS

A combination of variables and constants connected by some or all of the mathematical operations $+, -, \times$ and $\div$ is known as an **algebraic expression**.

For example : $2x + 7$, $x^2 - 3x$, $2y^3 - 7xy + 5$ etc.

Algebraic Expression	Term	Coefficient
$4x + 3y$	$4x$	4
	$3y$	3
$3xy^2 - 4x$	$3xy^2$	3
	$-4x$	-4
$3p^2q + 7pq - 8pq^2$	$3p^2q$	3
	$7pq$	7
	$-8pq^2$	-8

POLYNOMIALS

A polynomial is an algebraic expression in which no variable is in denominator and the variables involved have only non-negative integral powers.

For example : $2x^2 + 3x + 4, \frac{2}{5}y^2 + 6, \sqrt{3}x^2 + y^2$ are polynomials.

Example 1. Find the sum of $5x^3 - 2x^2 + x + 7$, $4x^2 - 3x + 2$ and $x^4 + 3x^2 - x - 3$.

Sol. Required sum $= (5x^3 - 2x^2 + x + 7) + (4x^2 - 3x + 2) + (x^4 + 3x^2 - x - 3).$

$= x^4 + 5x^3 + (-2x^2 + 4x^2 + 3x^2) + (x - 3x - x) + (7 + 2 - 3)$

$= x^4 + 5x^3 + (-2 + 4 + 3)x^2 + (1 - 3 - 1)x + 6$

$= x^4 + 5x^3 + 5x^2 + (-3x) + 6 = x^4 + 5x^3 + 5x^2 - 3x + 6$

Multiplication of Polynomials

In the multiplication of algebraic expressions, we are using the following rules :

(i) The product of two factors with like signs is positive and the product of two factors with unlike signs is negative

i.e. $(+) \times (+) = +$; $(+) \times (-) = -$

$(-) \times (+) = -$; $(-) \times (-) = +$

(ii) If a is any variable and m, n are positive integers, then $a^m \times a^n = a^{(m+n)}$

For example : $x^3 \times x^6 = x^{3+6} = x^9$

(iii) Distributive law of multiplication

i.e. $a(b + c) = ab + ac$

$(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$

Example 2. Multiply :

(i) $5x^2, 17x^8$ **(ii)** $-3y^3, y^2$

(iii) $2p, p^2q^3$ **(iv)** $3x^4, -3x, 2xy^3$

Sol. (i) $5x^2 \times 17x^8 = (5 \times 17) \times (x^2 \times x^8) = 85 \times x^{2+8} = 85x^{10}$

(ii) $-3y^3 \times y^2 = -3 \times y^{3+2} = -3y^5$

(iii) $2p \times p^2q^3 = 2 \times (p \times p^2q^3) = 2 \times (p^{1+2}q^3) = 2p^3q^3$

(iv) $3x^4 \times (-3x) \times 2xy^3 = \{3 \times (-3) \times 2\} \times (x^4 \times x \times xy^3)$

$= -18 \times (x^{4+1+1}y^3) = -18x^6y^3$

REMAINDER THEOREM

Let $f(x)$ be a polynomial of degree greater than or equal to 1. Then if $f(x)$ is divided by $(x - a)$, where a be any real number, then the remainder is $f(a)$.

Note that in $f(a)$, a is the value of x when $x - a = 0$.

Example 3. Find the remainder when

(i) $x^3 + 2x^2 - 5x + 3$ **is divided by** $(x - 2)$

(ii) $-x^4 - x^2 + 1$ **is divided by** $(x + 1)$

(iii) $2x^3 - x^2 + 5x$ **is divided by** $(2x - 1)$

Sol. (i) Let $p(x) = x^3 + 2x^2 - 5x + 3$

Put $x - 2 = 0 \Rightarrow x = 2$

Remainder $= p(2) = 2^3 + 2 \times 2^2 - 5 \times 2 + 3$

$= 8 + 8 - 10 + 3 = 9.$

(ii) Let $p(x) = -x^4 - x^2 + 1$

Put $x + 1 = 0 \Rightarrow x = -1$

Hence remainder $= p(-1) = -(-1)^4 - (-1)^2 + 1 = -1$

(iii) Let $p(x) = 2x^3 - x^2 + 5x$

Put $2x - 1 = 0$, then $x = \frac{1}{2}$

So, remainder $= p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right)$

$= 2 \times \frac{1}{8} - \frac{1}{4} + \frac{5}{2} = \frac{1}{4} - \frac{1}{4} + \frac{5}{2} = \frac{5}{2}$

FACTOR THEOREM

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be any real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

Let us understand this through the following example.

Example 4. Is $(x - 2)$ is a factor of $x^3 + 3x^2 - 12x + 4$?

Sol. Let $p(x) = x^3 + 3x^2 - 12x + 4$

Now $p(2) = (2)^3 + 3(2)^2 - 12 \times 2 + 4 = 8 + 12 - 24 + 4 = 0$.

Hence $(x - 2)$ is a factor of $x^3 + 3x^2 - 12x + 4$.

ZERO OF A POLYNOMIAL

A number α is a zero of a polynomial $f(x)$, if $f(\alpha) = 0$.

Number of zero(s) of a polynomial = Degree of the polynomial.

Hence number of zero(s) of a linear polynomial, quadratic polynomial, cubic polynomial are 1, 2, 3 respectively.

Relation Between Zero(s) and Coefficient of a Polynomial

(i) Linear Polynomial ($ax + b, a \neq 0$) :

If α be the zero of linear polynomial $ax + b$, then $\alpha = -\frac{b}{a}$.

(ii) Quadratic Polynomial ($ax^2 + bx + c, a \neq 0$):

If α and β are zeros of quadratic polynomial $ax^2 + bx + c$, then

$\alpha + \beta = -\frac{b}{a}$ and $\alpha \cdot \beta = \frac{c}{a}$

(iii) Cubic Polynomial ($ax^3 + bx^2 + cx + d, a \neq 0$) :

If α, β, γ are the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$; then

$\alpha + \beta + \gamma = -\frac{b}{a}$; $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ and $\alpha\beta\gamma = -\frac{d}{a}$

Factorisation of Polynomial using Algebraic Identities

You can also factorise the polynomial using the algebraic identities:

(i) $a^2 - b^2 = (a+b)(a-b)$

(ii) $a^2 + b^2 + 2ab = (a+b)^2$

(iii) $a^2 + b^2 - 2ab = (a-b)^2$

(iv) $a^3 + b^3 + 3a^2b + 3ab^2 = a^3 + b^3 + 3ab(a+b) = (a+b)^3$

(v) $a^3 - b^3 - 3a^2b + 3ab^2 = a^3 - b^3 - 3ab(a-b) = (a-b)^3$

(vi) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(vii) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(viii) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$

(ix) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(x) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

HCF (OR GCD) OF TWO POLYNOMIALS

HCF of two polynomials $p(x)$ and $q(x)$ is that divisor which has the highest degree among all common divisors.

Note that the coefficient of the highest degree divisor is positive.

Example 5. Find the HCF of following pair of polynomials:

$p(x) = (x^2 - 9)(x - 3)$; $q(x) = x^2 + 6x + 9$

Sol. $p(x) = (x^2 - 9)(x - 3)$

$= (x + 3)(x - 3)(x - 3) = (x + 3)(x - 3)^2$

$q(x) = x^2 + 6x + 9$

$= x^2 + 3x + 3x + 9$

$= (x + 3)(x + 3) = (x + 3)^2$

HCF of $p(x)$ and $q(x) = x + 3$

LCM OF TWO POLYNOMIALS

LCM of two polynomials $p(x)$ and $q(x)$ is a polynomial of lowest degree of which $p(x)$ and $q(x)$ both are multiples.

Example 6. Find the LCM of $p(x) = (x + 3)(x - 2)^2$ and $q(x) = (x - 2)(x - 6)$.

Sol. $p(x) = (x + 3)(x - 2)^2$

$q(x) = (x - 2)(x - 6)$

HCF of $p(x)$ and $q(x) = (x - 2)$

LCM of $p(x)$ and $q(x) = (x + 3)(x - 2)^2(x - 6)$

EXERCISE

1. If $\frac{4x-3}{x}+\frac{4y-3}{y}+\frac{4z-3}{z}=0$ then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is

(a) 9 (b) 3 (c) 4 (d) 6

2. If $2+x\sqrt{3}=\frac{1}{2+\sqrt{3}}$, then the simplest value of x is :

(a) 1 (b) −2 (c) 2 (d) −1

3. If $x-\sqrt{3}-\sqrt{2}=0$ and $y-\sqrt{3}+\sqrt{2}=0$, then value of $(x^3-20\sqrt{2})-(y^3+2\sqrt{2})$

(a) 2 (b) 3 (c) 1 (d) 0

4. If $\frac{2p}{P^2-2P+1}=\frac{1}{4}$, then the value of $\left(p+\frac{1}{p}\right)$ is

(a) 7 (b) 1 (c) $\frac{2}{5}$ (d) 10

5. If $x^2+\frac{1}{x^2}=1$ then the value of $x^{102}+x^{96}+x^{90}+x^{84}+x^{78}+x^{72}+5$ is

(a) 0 (b) 5 (c) 3 (d) 1

6. If $a^2+b^2+c^2=2a-2b-2$, then the value of $3a-2b+c$ is

(a) 0 (b) 3 (c) 5 (d) 2

7. If $x=2+\sqrt{3}$, then $x^2+\frac{1}{x^2}$ is equal to

(a) 10 (b) 12 (c) −12 (d) 14

8. If $x+y+z=0$, then the value of $\frac{x^2+y^2+z^2}{x^2-yz}$ is

(a) −1 (b) 0 (c) 1 (d) 2

9. If $p=\frac{5}{18}$ then $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ is equal to

(a) $\frac{4}{27}$ (b) $\frac{5}{27}$ (c) $\frac{8}{27}$ (d) $\frac{10}{27}$

10. If $a^x=b$, $b^y=c$ and $c^z=a$; then the value of $xyz=$

(a) 0 (b) 1 (c) −1 (d) 2

11. If $x+\frac{1}{x}=5$, then the value of $x^3+\frac{1}{x^3}$ is :

(a) 125 (b) 110 (c) 45 (d) 75

12. If $x+\frac{1}{y}=1$ and $y+\frac{1}{z}=1$, what is the value of xyz?

(a) 1 (b) −1 (c) 0 (d) $\frac{1}{2}$

13. If $x+\frac{4}{x}=4$, find the value of $x^3+\frac{4}{x^3}$.

(a) 8 (b) $8\frac{1}{2}$ (c) 16 (d) $16\frac{1}{2}$

14. If $x=3+2\sqrt{2}$, then the value of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$ is

(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

15. If $x+\frac{1}{x}=2$, then $x^{2013}+\frac{1}{x^{2014}}=?$

(a) 0 (b) 1 (c) −1 (d) 2

16. If $x+\frac{1}{x}=2$, then the value of $\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$ is

(a) 20 (b) 4 (c) 8 (d) 16

17. If $x=\frac{1}{2+\sqrt{3}}$, $y=\frac{1}{2-\sqrt{3}}$, then the value of $8xy(x^2+y^2)$ is

(a) 112 (b) 194 (c) 290 (d) 196

18. If $x+\frac{1}{x}=3$, where $x\neq 0$, then the value of

$$\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$$

(a) 3 (b) 5 (c) 7 (d) 2

19. If $x^2+x+\frac{1}{x^2}+\frac{1}{x}<0$, then which of the following is true?

(a) $x+\frac{1}{x}>-2$ (b) $x+\frac{1}{x}<-2$

(c) $x+\frac{1}{x}<1$ (d) Both (a) and (c)

20. If $x=\frac{4ab}{a+b}$, then the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ is equal to:

(a) 0 (b) 1
(c) 2 (d) None of these

21. If $x+\frac{1}{x}=\sqrt{3}$, then the value of $x^{18}+x^{12}+x^6+1$ is

(a) 0 (b) 1 (c) 2 (d) 3

22. If $5a+\frac{1}{3a}=5$, then the value of $9a^2+\frac{1}{25a^2}$ is

(a) $\frac{51}{5}$ (b) $\frac{29}{5}$ (c) $\frac{52}{5}$ (d) $\frac{39}{5}$

Hints & Solutions

1. (c) $\frac{4x-3}{x}+\frac{4y-3}{y}+\frac{4z-3}{z}=0$

$\Rightarrow \frac{4x}{x}-\frac{3}{x}+\frac{4y}{y}-\frac{3}{y}+\frac{4z}{z}-\frac{3}{z}=0$

$\Rightarrow 4-\frac{3}{x}+4-\frac{3}{y}+4-\frac{3}{z}=0$

$\Rightarrow 12-3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=0$

$-3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=-12$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=4$

2. (d) From question, $2+x\sqrt{3}=\frac{1}{2+\sqrt{3}}$

$\Rightarrow 2+\sqrt{3}x=\frac{1}{2+\sqrt{3}}$

$\Rightarrow 2+\sqrt{3}\times x=\frac{2-\sqrt{3}}{1}$

$\Rightarrow 2+x\sqrt{3}=2-\sqrt{3} \Rightarrow x=-1$

3. (d) According to the question

$x=\sqrt{3}+\sqrt{2}$

$y=\sqrt{3}-\sqrt{2}$

$\therefore (x^3-20\sqrt{2})-(y^3+2\sqrt{2})$

$=[(\sqrt{3}+\sqrt{2})^3-20\sqrt{2}-(\sqrt{3}-\sqrt{2})^3-2\sqrt{2}]$

$=3\sqrt{3}+2\sqrt{2}+9\sqrt{2}+6\sqrt{3}-20\sqrt{2}-3\sqrt{3}+2\sqrt{2}+9\sqrt{2}-6\sqrt{3}-2\sqrt{2}$

$=9\sqrt{3}-9\sqrt{2}-9\sqrt{3}+9\sqrt{2}=0$

4. (d) $\frac{2p}{p^2-2p+1}=\frac{1}{4}$

(Divide p both in nu. & de.)

$\frac{2}{p-2+\frac{1}{p}}=\frac{1}{4}$

$p+\frac{1}{p}-2=8$

$p+\frac{1}{p}=10$

5. (b) If $x^2+\frac{1}{x^2}=1$

Then, $x+\frac{1}{x}=\sqrt{3}$

$\Rightarrow x^3+\frac{1}{x^3}=(\sqrt{3})^3-3\sqrt{3}=0$

$\Rightarrow x^6=-1$, or $x^6+1=0$

then $x^{102}+x^{96}+x^{90}+x^{84}+x^{78}+x^{72}+5$

$x^{96}(x^6+1)+x^{84}(x^6+1)+x^{72}(x^6+1)+5=5$

6. (c) $a^2+b^2+c^2=2a-2b-2$

$(a^2-2a+1)+(b^2+2b+1)+c^2=0$

$(a-1)^2+(b+1)^2+c^2=0$

This equation is possible if

$a-1=0, b+1=0$ and $c=0$

$a=1, b=-1, c=0$

$3a-2b+c=3\times 1-2\times(-1)+0=3+2=5$

7. (d) $x=2+\sqrt{3}$

$\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=2-\sqrt{3}$

$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

$=\left(2+\sqrt{3}+2-\sqrt{3}\right)^2-2$

$=16-2=14$

8. (d) $x+y+z=0$

$y+z=-x$

$y^2+z^2+2yz=x^2$

$\Rightarrow y^2+z^2=x^2-2yz$ ...(1)

$\frac{x^2+y^2+z^2}{x^2-yz}=\frac{x^2-2yz+x^2}{x^2-yz}=\frac{2(x^2-yz)}{x^2-yz}=2$

9. (c) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$=(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2.\frac{1}{6}+3(3p)\left(\frac{1}{6}\right)^2$

$=\left(3P-\frac{1}{6}\right)^3=\left(3\times\frac{5}{18}-\frac{1}{6}\right)^3=\frac{8}{27}$

10. (b) $a^x=b, b^y=c$ and $c^z=a$

$\Rightarrow a^xb^yc^z=abc$

On comparing the powers of a, b, c we get $x=1, y=1$ and $z=1 \Rightarrow xyz=1$

11. (b) Using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \quad (5)^3 = \left(x^3 + \frac{1}{x^3}\right) + 15$$

or $\quad x^3 + \frac{1}{x^3} = 125 - 15 = 110$

12. (b) Given that, $x + \frac{1}{y} = 1$

$\Rightarrow \quad xy + 1 = y$ ...(i)

and $y + \frac{1}{z} = 1$

$\Rightarrow \quad 1 - \frac{1}{z} = y \Rightarrow \quad \frac{z-1}{z} = y$ ...(ii)

From eq. (ii),

$y = \frac{z-1}{z}$

Comparing eqn. (i) with (ii)

$xy + 1 = \frac{z-1}{z}$

$\Rightarrow \quad xyz + z = z - 1 \Rightarrow \quad xyz = -1$

13. (b) $x + \frac{4}{x} = 4$

$x^2 + 4 = 4x \quad \Rightarrow \quad x^2 - 4x + 4 = 0 \quad \Rightarrow \quad (x-2)^2 = 0$

$x = 2$

$$x^3 + \frac{4}{x^3} = (2)^3 + \frac{4}{(2)^3} \Rightarrow 8 + \frac{4}{8} \Rightarrow 8 + \frac{1}{2} \Rightarrow 8\frac{1}{2}$$

14. (b) $x = 3 + 2\sqrt{2}$

$x = 2 + 1 + 2\sqrt{2}$

$x = (\sqrt{2})^2 + (1)^2 + 2.1.\sqrt{2}$

$x = (\sqrt{2}+1)^2$

$\sqrt{x} = (\sqrt{2}+1)$ ...(1)

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

Now,

$$\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{2}+1-\left(\sqrt{2}-1\right) = \sqrt{2}+1-\sqrt{2}+1$$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 2$$

15. (d) $x + \frac{1}{x} = 2 \quad \Rightarrow \quad x + \frac{1}{x} - 2 = 0$

$\Rightarrow \quad x^2 - 2x + 1 = 0$; $(x-1)^2 = 0$; $x = 1$

Now, $x^{2013} + \frac{1}{x^{2014}} = 1 + 1 = 2$

16. (b) $x + \frac{1}{x} = 2$ ...(I)

Squaring both sides

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4 \Rightarrow x^2 + \frac{1}{x^2} = +2$

Cubing equation (I)

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8$$

$x^3 + \frac{1}{x^3} + 6 = 8$

$x^3 + \frac{1}{x^3} = 2$

$$\therefore \quad \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 2 \times 2 = 4$$

17. (a) According to the question,

$$\Rightarrow \quad x = \frac{1}{2+\sqrt{3}}, y = \frac{1}{2-\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$$

$$y = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$$

$\Rightarrow \quad x = 2-\sqrt{3}, y = 2+\sqrt{3}$

$\therefore \quad 8xy(x^2 + y^2)$

$$= 8(2-\sqrt{3})(2+\sqrt{3})\left[(2-\sqrt{3})^2 + (2+\sqrt{3})^2\right]$$

$$\Rightarrow \quad 8 \times 1[7 - 4\sqrt{3} + 7 + 4\sqrt{3}] = 112$$

18. (a) $x + \frac{1}{x} = 3$

$x^2 + 1 = 3x$ ...(i)

$(x^2+1)^2 = 9x^2$

$x^4 + 1 + 2x^2 = 9x^2$

$x^4 + 1 = 7x^2$ ...(ii)

$$\therefore \frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

$$\frac{12x^2 + 3x^3 + 3x}{7x^2}$$

From equation (i)

$$\Rightarrow \quad \frac{12x + 3(x^2+1)}{7x}$$

$$\Rightarrow \quad \frac{12x + 3 \times 3x}{7x} \quad \Rightarrow \quad \frac{21x}{7x} \quad \Rightarrow \quad 3$$

19. (d) Given that $\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) < 0$

$$\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 2 < 0$$

Substituting $x + \frac{1}{x} = y$, we get

$y^2 + y - 2 < 0$

$\Rightarrow \quad (y-1)(y+2) < 0$

$\therefore \quad$ either $y - 1 < 0; y + 2 > 0$

or $\quad y + 2 < 0; y - 1 > 0.$

i.e., $y < 1, y > -2$ or $y < -2$; $\quad y > 1$ $\Downarrow$ (not possible)

Therefore, $-2 < y < 1$

i.e. $-2 < \left(x + \frac{1}{x}\right) < 1.$

20. (c) Given, $x = \frac{4ab}{a+b} \quad \Rightarrow \quad \frac{x}{2a} = \frac{2b}{a+b}$

Applying componendo and dividendo, we get

$$\frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-a-b} = \frac{a+3b}{b-a} \quad \ldots\text{(i)}$$

Also, $\frac{x}{2b} = \frac{2a}{a+b}$

Applying componendo and dividendo, we get

$$\frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \quad \ldots\text{(ii)}$$

Add (i) & (ii),

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{a+3b}{b-a} + \frac{3a+b}{a-b}$$

$$= \frac{1}{b-a}[a+3b-3a-b] = \frac{2(b-a)}{(b-a)} = 2$$

21. (a) $x + \frac{1}{x} = \sqrt{3}$

Cubing both sides,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \left(\sqrt{3}\right)^3$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3} \Rightarrow \quad x^3 + \frac{1}{x^3} = 0$$

Now,

$$x^{18} + x^{12} + y^6 + 1 = x^{12}(x^6 + 1) + 1(x^6 + 1)$$

$$= (x^{12} + 1)(x^6 + 1)$$

$$= (x^{12} + 1).\, x^3 \left(x^3 + \frac{1}{x^3}\right) = 0$$

22. (d) $5a + \frac{1}{3a} = 5$

Multiply by $\frac{3}{5}$ on both sides

$$\frac{3}{5}\left(5a + \frac{1}{3a}\right) = 5 \times \frac{3}{5}$$

$$3a + \frac{1}{5a} = 3$$

Squaring on both sides

$$9a^2 + \frac{1}{25a^2} + 2 \times 3a \times \frac{1}{5a} = 9$$

$$\Rightarrow \quad 9a^2 + \frac{1}{25a^2} = 9 - \frac{6}{5} = \frac{39}{5}$$