NDA/NA SOLVED PAPER 2024-II

MATHEMATICS

1. Let X be a matrix of order 3 × 3, Y be a matrix of order 2 × 3 and Z be a matrix of order 3 × 2. Which of the following statements are correct?
   A. (ZY)X is defined and is a square matrix of order 3.
   B. Y(XZ) is defined and is a square matrix of order 2.
   C. X(YZ) is not defined.
   Select the answer using the code given below.
   (a) A and B only (b) B and C only
   (c) A and C only (d) A, B and C
2. Consider the following statements :
   A. The set of all irrational numbers between $\sqrt{12}$ and $\sqrt{15}$ is an infinite set.
   B. The set of all odd integers less than 1000 is a finite set.
   Which of the statements given above is/are correct?
   (a) A only (b) B only
   (c) Both A and B (d) Neither A nor B
3. How many 4-digit numbers are there having all digits as odd?
   (a) 625 (b) 400
   (c) 196 (d) 120
4. If $\omega \neq 1$ is a cube root of unity, then what is $(1 + \omega - \omega^2)^{100} + (1 - \omega + \omega^2)^{100}$ equal to?
   (a) $2^{100}\omega^2$ (b) $2^{100}\omega$
   (c) 2^{100} (d) -2^{100}
5. Let A and B be two square matrices of same order. If AB is a null matrix, then which one of the following is correct?
   (a) Both A and B are null matrices
   (b) Either A or B is a null matrix
   (c) B is a null matrix if A is a non-singular matrix
   (d) Both A and B are singular matrices
6. In the expansion of $(1 + x)^p (1 + x)^q$, if the coefficient of x^3 is 35, then what is the value of $(p + q)$?
   (a) 5 (b) 6
   (c) 7 (d) 8
7. If p times the p^{th} term of an AP is equal to q times the q^{th} term $(p \neq q)$, then what is the $(p + q)^{th}$ term equal to?
   (a) 0 (b) $p + q$
   (c) pq (d) $pq(p + q)$
8. Let $p = \text{In}(x)$, $q = \ln(x^3)$ and $r = \ln(x^5)$, where $x > 1$. Which of the following statements is/are correct?
   A. p, q and r are in AP.
   B. p, q and r can never be in GP.
   Select the answer using the code given below:
   (a) A only (b) B only
   (c) Both A and B (d) Neither A nor B
9. If $Z = \frac{1}{3}\begin{vmatrix} i & 2i & 1 \\ 2i & 3i & 2 \\ 3 & 1 & 3 \end{vmatrix} = x + iy; i = \sqrt{-1}$
   then what is modulus of Z equal to?
   (a) 1 (b) $\sqrt{2}$
   (c) 2 (d) $\sqrt{3}$
10. What is the value of the sum $\sum_{n=1}^{20}(i^{n-1} + i^n + i^{n+1})$ where $i = \sqrt{-1}$?
    (a) $-2i$ (b) 0
    (c) 1 (d) $2i$
11. Let $x > 1, y > 1, z > 1$ be in GP. Then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are
    (a) in AP
    (b) in GP
    (c) in HP
    (d) neither in AP nor in GP nor in HP
12. If $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ then what is
    $\begin{vmatrix} 1+\omega & 1+\omega^2 & \omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \frac{1}{\omega} & \frac{1}{\omega^2} & 1 \end{vmatrix}$ equal to?
    (a) 0 (b) ω
    (c) ω^2 (d) $1 - \omega^2$
13. If the sum of the first n terms of a series is $n(2n + 1)$, then what is the nth term?
    (a) $4n - 1$ (b) $4n$
    (c) $4n + 1$ (d) $4n + 3$
14. In how many ways can the letters of the word INDIA be permutated such that in each combination, vowels should occupy odd positions?
    (a) 3 (b) 6
    (c) 9 (d) 12

15. The letters of the word EQUATION are arranged in such a way that all vowels as well as consonants are together. How many such arrangements are there?

(a) 240 (b) 720
(c) 1440 (d) 1620

16. If n is a root of the equation $x^2 + px + m = 0$ and m is a root of the equation $x^2 + px + n = 0$, where $m \neq n$, then what is the value of $p + m + n$?

(a) -1 (b) 0
(c) 1 (d) 2

17. In how many ways can a student choose $(n - 2)$ courses out of n courses if 2 courses are compulsory $(n > 4)$?

(a) $(n-3)(n-4)$ (b) $(n-1)(n-2)$
(c) $\frac{(n-3)(n-4)}{2}$ (d) $\frac{(n-2)(n-3)}{2}$

18. If $D_n = \begin{vmatrix} n & 20 & 30 \\ n^2 & 40 & 50 \\ n^3 & 60 & 70 \end{vmatrix}$ then what is the value of $\sum_{n=1}^{4} D_n$?

(a) -10000 (b) -10
(c) 10 (d) 10000

19. Consider the following in respect of the matrices

$$P = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

A. PQ is null a matrix.
B. QP is an identity matrix of order 3.
C. PQ = QP

which of the above is/are correct?

(a) A only (b) B only
(c) A and C (d) B and C

20. If P is a skew-symmetric matrix of order 3, then what is det(P) equal to?

(a) -1 (b) 0
(c) 1 (d) 3

21. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then what is $\sin^{-1} x + 4 \cos^{-1} x$ equal to?

(a) $\frac{\pi}{2}$ (b) π
(c) $\frac{3\pi}{2}$ (d) 2π

22. What is $\cot^2(\sec^{-1} 2) + \tan^2(\operatorname{cosec}^{-1} 3)$ equal to?

(a) $\frac{11}{12}$ (b) $\frac{11}{24}$
(c) $\frac{7}{24}$ (d) $\frac{1}{24}$

23. In a triangle ABC, $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$

What is the area of the triangle if a = 6 cm?

(a) $9\sqrt{3}$ square cm (b) 12 square cm
(c) $18\sqrt{3}$ square cm (d) 24 square cm

24. The roots of the equation $7x^2 - 6x + 1 = 0$ are $\tan\alpha$ and $\tan\beta$, where 2α and 2β are the angles of a triangle. Which one of the following is correct?

(a) The triangle is equilateral
(b) The triangle is isosceles but not right-angled
(c) The triangle is right-angled
(d) The triangle is right-angled isosceles

25. In a triangle ABC, $\angle A = 75°$ and $\angle B = 45°$. What is $2a - b$ equal to?

(a) c (b) $\sqrt{2}c$
(c) $2c$ (d) $2\sqrt{2}c$

26. What is the number of solutions of the equation $\cot 2x \cdot \cot 3x = 1$ for $0 < x < \pi$?

(a) Only one (b) Only two
(c) Only five (d) More than five

27. What is the general solution of $\cos^{100} x - \sin^{100} x = 1$?

(a) $n\pi$ (b) $(2n + 1)\pi$
(c) $2n\pi$ (d) $(2n+1)\frac{\pi}{2}$

where n is an integer.

28. In a triangle ABC, tanA + tanB + tanC = k

What is the value of cotA cot B cotC ?

(a) 0.5k (b) $\frac{1}{k}$
(c) $\frac{3}{k}$ (d) $\frac{1}{k^3}$

29. What is sin12° sin48° equal to?

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{4}$
(c) $\frac{\sqrt{5}-1}{8}$ (d) $\frac{\sqrt{5}+1}{8}$

30. What is $\frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$ equal to?

(a) tan 34° (b) cot 34°
(c) tan 62° (d) cot 62°

31. Consider the following numbers :

A. tan 22.5°
B. cot 22.5°
C. tan 22.5° – cot 22.5°

How many of the above are irrational numbers?

(a) None (b) Only one
(c) Only two (d) All three

32. If $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\frac{2\pi}{3} - \theta\right)} = \frac{z}{\cos\left(\frac{2\pi}{3} + \theta\right)}$

then what is $x + y + z$ equal to?

(a) -1 (b) 0
(c) 1 (d) 3

33. If $p \tan(\theta - 30^\circ) = q \tan(\theta + 120^\circ)$, then what is $\frac{(p+q)}{(p-q)}$ equal to?

(a) $\sin 2\theta$ (b) $\cos 2\theta$

(c) $2 \sin 2\theta$ (d) $2 \cos 2\theta$

34. Let P and Q be two non-void relations on a set A. Which of the following statements are correct?

A. P and Q are reflexive $\Rightarrow P \cap Q$ is reflexive.

B. P and Q are symmetric $\Rightarrow P \cup Q$ is symmetric.

C. P and Q are transitive $\Rightarrow P \cap Q$ is transitive.

Select the answer using the code given below.

(a) A and B only

(b) B and C only

(c) A and C only

(d) A, B and C

35. If A and B are two non-empty sets having 10 elements in common, then how many elements do $A \times B$ and $B \times A$ have in common?

(a) 10 (b) 20

(c) 40 (d) 100

36. What is the remainder when $7^n - 6n$ is divided by 36 for $n = 100$?

(a) 0 (b) 1

(c) 2 (d) 6

37. What is the maximum number of possible points of intersection of four straight lines and a circle (intersection is between lines as well as circle and lines)?

(a) 6 (b) 10

(c) 14 (d) 16

38. In an AP, the ratio of the sum of the first p terms to the sum of the first q terms is $p^2 : q^2$. Which one of the following is correct?

(a) The first term is equal to the common difference

(b) The first term is equal to twice the common difference

(c) The common difference is equal to twice the first term

(d) The first term is equal to square of the common difference

39. What is the number of real roots of the equation $(x-1)^2 + (x-3)^2 + (x-5)^2 = 0$?

(a) None (b) Only one

(c) Only two (d) Three

40. In a class of 240 students, 180 passed in English, 130 passed in Hindi and 150 passed in Sanskrit. Further, 60 passed in only one subject, 110 passed in only two subjects and 10 passed in none of the subjects. How many passed in all three subjects?

(a) 60 (b) 55

(c) 40 (d) 35

DIRECTIONS (Qs 41-42): *Consider the following for the two (02) items that follow :*

Let Z_1 and Z_2 be any two complex numbers such that

$$Z_1^2 + Z_2^2 + Z_1 Z_2 = 0$$

41. What is the value of $\left|\frac{Z_1}{Z_2}\right|$?

(a) 1 (b) 2

(c) 3 (d) 4

42. What is the value of $\frac{1}{2} + \text{Re}\left(\frac{Z_1}{Z_2}\right)$?

(a) –1 (b) 0

(c) 1 (d) 2

DIRECTIONS (Qs. 43-44): *Consider the following for the two (02) items that follow :*

The product of 5 consecutive terms of an AP is 229635. The first, second and fifth terms are in GP.

43. What is the common difference?

(a) 3 (b) 4

(c) 5 (d) 6

44. What is the sum of all five terms?

(a) 60 (b) 65

(c) 75 (d) 80

DIRECTIONS (Qs. 45-46): *Consider the following for the two (02) items that follow :*

Let $(8+3\sqrt{7})^{20} = U + V$ and $(8-3\sqrt{7})^{20} = W$, where U is an integer and $0 < V < 1$.

45. What is V + W equal to?

(a) 8 (b) 4

(c) 2 (d) 1

46. What is the value of (U + V) W?

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

DIRECTIONS (Qs. 47-48): *Consider the following for the two (02) items that follow :*

The roots of the quadratic equation $a^2(b^2-c^2)x^2 + b^2(c^2-a^2)x + c^2(a^2-b^2) = 0$ are equal $(a^2 \neq b^2 \neq c^2)$.

47. Which one of the following statements is correct?

(a) a^2, b^2, c^2 are in AP.

(b) a^2, b^2, c^2 are in GP.

(c) a^2, b^2, c^2 are in HP.

(d) a^2, b^2, c^2 are neither in AP nor in GP nor in HP.

48. Which one of the following is a root of the equation?

(a) $\frac{b^2(c^2-a^2)}{a^2(c^2-b^2)}$ (b) $\frac{b^2(c^2-a^2)}{a^2(b^2-c^2)}$

(c) $\frac{b^2(c^2-a^2)}{2a^2(c^2-b^2)}$ (d) $\frac{b^2(c^2-a^2)}{2a^2(b^2-c^2)}$

DIRECTIONS (Qs. 49-50): *Consider the following for the two (02) items that follow :*

Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

49. What is A(adj A) equal to?

(a) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

50. What is A^{-1} equal to?

(a) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & 0 \\ -1 & \frac{3}{2} & -2 \\ -1 & \frac{3}{2} & \frac{-3}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -2 & 0 \\ -4 & 6 & -8 \\ -4 & 6 & -6 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{5} & \frac{-1}{5} & 0 \\ \frac{-2}{5} & \frac{3}{5} & \frac{-4}{5} \\ \frac{-2}{5} & \frac{3}{5} & \frac{-3}{5} \end{bmatrix}$

51. What is $3\alpha+2\beta$ equal to if

$(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\alpha\hat{j}+\beta\hat{k})$

is a null vector?

(a) 36 (b) 33
(c) 30 (d) 27

52. For what value of the angle between the vectors $\vec{a}$ and $\vec{b}$ is the quantity $|\vec{a}\times\vec{b}|+\sqrt{3}|\vec{a}.\vec{b}|$ maximum?

(a) 0° (b) 30°
(c) 45° (d) 60°

53. Let θ be the angle between two unit vectors $\vec{a}$ and $\vec{b}$. If $\vec{a}+2\vec{b}$ is perpendicular to $5\vec{a}-4\vec{b}$, then what is cosθ + cos2θ equal to ?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{\sqrt{3}+1}{2}$

54. Let ABCDEF be a regular hexagon. If $\overrightarrow{AD} = m\overrightarrow{BC}$ and $\overrightarrow{CF} = n\overrightarrow{AB}$, then what is mn equal to?

(a) – 4 (b) – 2
(c) 2 (d) 4

55. The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are of the same length. If taken pairwise, they form equal angles. If $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$, then what can $\vec{c}$ be equal to?

A. $\hat{i}+\hat{k}$

B. $\frac{-\hat{i}+4\hat{j}-\hat{k}}{3}$

Select the correct answer using the code given below.

(a) A only (b) B only
(c) Both A and B (d) Neither A nor B

56. The diagonals of a quadrilateral ABCD are along the lines $x-2y=1$ and $4x+2y=3$. The quadrilateral ABCD may be a

(a) rectangle (b) cyclic quadrilateral
(c) parallelogram (d) rhombus

57. The foci of the ellipse $4x^2+9y^2=1$ are at Q and R. If $P(x, y)$ is any point on the ellipse, then what is PQ + PR equal to?

(a) 2 (b) 1

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

58. If P(2, 4), Q(8, 12), R(10, 14) and $S(x,y)$ are vertices of a parallelogram, then what is $(x+y)$ equal to?

(a) 8 (b) 10
(c) 12 (d) 14

59. The equation of a circle is

$(x^2-4x+3)+(y^2-6y+8)=0$

Which of the following statements are correct?

A. The end points of a diameter of the circle are at (1, 2) and (3, 4).

B. The end points of a diameter of the circle are at (1, 4) and (3, 2).

C. The end points of a diameter of the circle are at (2, 4) and (4, 2).

Select the answer using the code given below.

(a) A and B only (b) B and C only
(c) A and C only (d) A, B and C

60. Consider the points $P(4k,4k)$ and $Q(4k,-4k)$ lying on the parabola $y^2=4kx$. If the vertex is A, then what is ∠PAQ equal to?

(a) 60° (b) 90°
(c) 120° (d) 135°

DIRECTIONS (Qs. 61-62): *Consider the following for the two (02) items that follow :*

A triangle ABC is inscribed in the circle $x^2+y^2=100$. B and C have coordinates (6, 8) and (– 8, 6) respectively.

61. What is $\angle$ BAC equal to?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (d) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

62. What are the coordinates of A?

(a) (– 6, 8)

(b) (– 6, – 8)

(c) $(5\sqrt{2}, 5\sqrt{2})$

(d) Cannot be determined due to insufficient data

DIRECTIONS (Qs. 63-64): *Consider the following for the two (02) items that follow :*

ABCD is an isosceles trapezium and AB is parallel to DC. Let A(2, 3), B(4, 3), C(5, 1) be the vertices.

63. What are the coordinates of vertex D?

(a) (2, 1) (b) (1, 2)

(c) (1, 1) (a) (3, 1)

64. What is the point of intersection of the diagonals of the trapezium?

(a) $\left(3, \frac{7}{2}\right)$ (b) $\left(3, \frac{7}{3}\right)$

(c) $\left(\frac{7}{2}, 2\right)$ (a) $\left(\frac{5}{2}, 2\right)$

DIRECTIONS (Qs. 65-66): *Consider the following for the two (02) items that follow :*

Let $2x^2 + 2y^2 + 2z^2 + 3x + 3y + 3z - 6 = 0$ be a sphere.

65. What is the diameter of the sphere?

(a) $\frac{5\sqrt{3}}{4}$ (b) $\frac{5\sqrt{3}}{2}$

(c) $\frac{3\sqrt{5}}{4}$ (d) $\frac{3\sqrt{5}}{2}$

66. The centre of the sphere lies on the plane

(a) $2x + 2y + 2z - 3 = 0$

(b) $4x + 4y + 4z - 3 = 0$

(c) $4x + 8y + 8z - 15 = 0$

(d) $4x + 8y + 8z + 15 = 0$

DIRECTIONS (Qs. 67-68): *Consider the following for the two (02) items that follow :*

Let S be the line of intersection of two planes $x + y + z = 1$ and $2x + 3y - 4z = 8$.

67. Which of the following are the direction ratios of S?

(a) <–7, –6, 1> (b) <–7, 6, 1>

(c) <–6, 5, 1> (d) <6, 5, 1>

68. If (l, m, n) are direction cosines of S, then what is the value of $43\,(l^2 - m^2 - n^2)$?

(a) 6 (b) 5

(c) 4 (d) 1

DIRECTIONS (Qs. 69-70): *Consider the following for the two (02) items that follow :*

Let L : $x + y + z + 4 = 0 = 2x - y - z + 8$ be a line and P : $x + 2y + 3z + 1 = 0$ be a plane

69. What are the direction ratios of the line?

(a) <2, 1, – 1> (b) <0, – 1, 2>

(c) <0, 1, – 1> (d) <2, 3, – 3>

70. What is the point of intersection of L and P?

(a) (4, 3, – 3) (b) (4, – 3, 3)

(c) (– 4, – 3, – 3) (d) (– 4, – 3, 3)

71. Let $z = [y]$ and $y = [x] - x$, where [.] is the greatest integer function. If x is not an integer but positive, then what is the value of z?

(a) – 1 (b) 0

(c) 1 (d) 2

72. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $f \, o \, g(x) = g \, o \, f(x)$, then what is the value of k?

(a) 7 (b) 5

(c) 4 (c) 3

73. What is the minimum value of the function $f(x) = \log_{10}(x^2 + 2x + 11)$?

(a) 0 (b) 1

(c) 2 (d) 10

74. Which one of the following is correct regarding $\lim_{x \to 3} \frac{|x-3|}{x-3}$?

(a) Limits exists and is equal to 1

(b) Limits exists and is equal to 0

(c) Limits exists and is equal to –1

(d) Limit does not exist

75. What is the maximum value of the $a \cos x + b \sin x + c$?

(a) $\sqrt{a^2 + b^2 + c}$ (b) $\sqrt{a^2 + b^2} + c$

(c) $\sqrt{a^2 + b^2} - c$ (d) $\sqrt{a^2 + b^2}$

76. If $f(2x) = 4x^2 + 1$, then for how many real values of x will $f(2x)$ be the GM of $f(x)$ and $f(4x)$?

(a) Four (b) Two

(c) One (d) None

77. If $f(x) = [x]^2 - 30[x] + 221 = 0$, where $[x]$ is the greatest integer function, then what is the sum of all integer solutions?

(a) 13 (b) 17

(c) 27 (d) 30

78. If $f(x) = 9x - 8\sqrt{x}$ such that $g(x) = f(x) - 1$, then which one of the following is correct?

(a) $g(x) = 0$ has no real roots

(b) $g(x) = 0$ has only one real root which is an integer

(c) $g(x) = 0$ has two real roots which are integers

(a) $g(x) = 0$ has only one real root which is not an integer

79. What is $\lim_{x \to \frac{\pi}{2}} (\sec\theta - \tan\theta)$ equal to?

(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 1

80. Let $f(x) f(y) = f(xy)$ for all real x, y. If $f(2) = 4$, then what is the value of $f\left(\frac{1}{2}\right)$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 4

DIRECTIONS (Qs. 81-82): *Consider the following for the two (02) items that follow :*

Let $f \, o \, g(x) = \cos^2 \sqrt{x}$ and $g \, o \, f(x) = |\cos x|$.

81. Which one of the following is $f(x)$?

(a) $\cos x$ (b) $\cos x^2$

(c) $\cos^2 x$ (d) $\cos |x|$

82. Which one of the following is $g(x)$?

(a) $\sqrt{x}$ (b) $|x|$

(c) x^2 (d) $x|x|$

DIRECTIONS (Qs. 83-84): *Consider the following for the two (02) items that follow :*

Let $f(x) = [x]^2 - [x^2]$.

83. What is $f(0.999) + f(1.001)$ equal to?

(a) -1 (b) 0

(c) 1 (d) 2

84. Consider the following statements :

A. $f(x)$ is continuous at $x = 0$.

B. $f(x)$ is continuous at $x = 1$

Which of the statements given above is/are correct?

(a) A only

(b) B only

(c) Both A and B

(d) Neither A nor B

DIRECTIONS (Qs. 85-86): *Consider the following for the two (02) items that follow :*

Let $f(x) = \cos 2x + x$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

85. What is the greatest value of $f(x)$?

(a) $\frac{\sqrt{3}}{2} - \frac{\pi}{12}$ (b) $\frac{\sqrt{3}}{2} + \frac{\pi}{12}$

(c) $\frac{\sqrt{3}}{2} + \frac{\pi}{9}$ (d) $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

86. What is the least value of $f(x)$?

(a) $-\left(1 + \frac{\pi}{2}\right)$ (b) $-\left(\frac{1}{2} + \frac{\pi}{2}\right)$

(c) $-\left(1 + \frac{\pi}{4}\right)$ (d) $-2\left(\frac{1}{2} - \frac{\pi}{4}\right)$

DIRECTIONS (Qs. 87-88): *Consider the following for the two (02) items that follow :*

The area bounded by the parabola $y^2 = kx$ and the line $x = k$, where $k > 0$, is $\frac{4}{3}$ square units.

87. What is the value of k?

(a) $\frac{1}{2}$ (b) 1

(c) $\sqrt{2}$ (d) 2

88. What is the area of the parabola bounded by the latus rectum?

(a) $\frac{1}{6}$ square unit (b) $\frac{2}{3}$ square unit

(c) 1 square unit (d) $\frac{4}{3}$ square unit

DIRECTIONS (Qs. 89-90): *Consider the following for the two (02) items that follow :*

Let $y\,dx + (x - y^3)dy = 0$ be a differential equation.

89. What are the order and degree respectively of the differential equation?

(a) 1 and 1 (b) 1 and 2

(c) 2 and 1 (d) 1 and 3

90. What is the solution of the differential equation?

(a) $y^4 + 2x = c$ (b) $y^4 + 3x = c$

(c) $2xy^4 + x = c$ (d) $4xy - y^4 = c$

DIRECTIONS (Qs. 91-92): *Consider the following for the two (02) items that follow :*

Let $f(x) = |x^2 - x - 2|$.

91. What is $\int_0^2 f(x)\,dx$ equal to?

(a) 0 (b) 1

(c) $\frac{5}{3}$ (d) $\frac{10}{3}$

92. What is $\int_1^3 f(x)\,dx$ equal to?

(a) 2 (b) 3

(c) 4 (d) 5

DIRECTIONS (Qs. 93-94): *Consider the following for the two (02) items that follow :*

Let $f(t) = \ln\left(t + \sqrt{1 + t^2}\right)$ and $g(t) = \tan(f(t))$.

93. Consider the following statements :

A. $f(t)$ is an odd function.

B. $g(t)$ is an odd function.

Which of the statements given above is/are correct?

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

94. What is $\int_{-\pi}^{\pi} g(t)\,dt$ equal to?

(a) -1 (b) 0
(c) $\frac{1}{2}$ (d) 1

DIRECTIONS (Qs. 95-96): *Consider the following for the two (02) items that follow:*

Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$.

95. What is $h'(0)$ equal to?

(a) -2 (b) -1
(c) 0 (d) 2

96. What is $g'(0)$ equal to?

(a) -4 (b) -2
(c) 0 (d) 4

DIRECTIONS (Qs. 97-98): *Consider the following for the two (02) items that follow :*

Let $I = \int_0^{\frac{\pi}{2}} \frac{f(x)}{g(x)} dx,$ where $f(x) = \sin x$ and $g(x) = \sin x + \cos x + 1$.

97. What is $\int_0^{\frac{\pi}{2}} \frac{dx}{g(x)}$ equal to ?

(a) $\frac{\ln 2}{2}$ (b) $\frac{\ln 2}{4}$
(c) $\ln 2$ (d) $2 \ln 2$

98. What is I equal to?

(a) $\frac{\pi}{4} + \ln 2$ (b) $\frac{\pi}{4} - \ln 2$
(c) $\frac{\pi}{4} - \frac{\ln 2}{2}$ (d) $\frac{\pi}{4} + \frac{\ln 2}{2}$

DIRECTIONS (Qs. 99-100): *Consider the following for the two (02) items that follow :*

Let $2\int \frac{x^2 - 1}{\sqrt{x^2 + 1}} dx = U(x)\,V(x) - 3 \ln \{U(x) + V(x)\} + c$

99. What is $|U^2(x) - V^2(x)|$ equal to?

(a) 0 (b) 1
(c) 2 (d) 3

100. What is $U(x)\,V(x)$ equal to?

(a) $\sqrt{x^2 + x^4}$ (b) $\sqrt{x + x^3}$
(c) $\frac{\sqrt{x^2 + x^4}}{2}$ (d) $2\sqrt{x^2 + x^4}$

101. Let $x - 3y + 4 = 0$ and $2x - 7y + 8 = 0$ be two lines of regression computed from some bivariate data. If b_{yx} and b_{xy} are regression coefficients of lines of regression of y on x and x on y respectively, then what is the value of $b_{xy} + 7b_{yx}$?

(a) -2 (b) 1
(c) 2 (d) 5

102. The mean of n observations $1, 4, 9, 16, \ldots, n^2$ is 130. What is the value of n?

(a) 18 (b) 19
(c) 20 (d) 21

103. Three distinct natural numbers are chosen at random from 1 to 10. What is the probability that they are consecutive?

(a) $\frac{1}{12}$ (b) $\frac{3}{40}$
(c) $\frac{1}{15}$ (d) $\frac{7}{120}$

104. A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. If 3P (B) = 4P (A) and 3P(C) = 2P(B), then what is P(A) equal to?

(a) $\frac{7}{29}$ (b) $\frac{8}{29}$
(c) $\frac{9}{29}$ (d) $\frac{10}{29}$

105. A die has two faces with number 4, three faces with number 5 and one face with number 6. If the die is rolled once, then what is the probability of getting 4 or 5?

(a) – (b) $\frac{2}{3}$
(c) $\frac{5}{6}$ (d) $\frac{1}{2}$

106. A box contains 2 black, 4 yellow and 6 white balls. Three balls are drawn in succession with replacement. What is the probability that all three are of the same colour?

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$
(c) $\frac{1}{12}$ (d) $\frac{5}{12}$

107. A can hit a target 5 times in 6 shots, B can hit 4 times in 5 shots and C can hit 3 times in 4 shots. What is the probability that A and C may hit but B may lose?

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

108. The letters of the word ZOOLOGY are arranged in all possible ways. What is the probability that the consonants and vowels occur alternatively?

(a) $\frac{6}{35}$ (b) $\frac{3}{35}$
(c) $\frac{2}{35}$ (d) $\frac{1}{35}$

109. A natural number x is chosen at random from the first 100 natural numbers. What is the probability that $x^2 + x > 50$?

(a) $\frac{93}{100}$ (b) $\frac{47}{50}$

(c) $\frac{24}{25}$ (d) $\frac{23}{25}$

110. What is the mean deviation of the first 10 natural numbers?

(a) 2 (b) 2.5

(c) 3 (d) 3.5

111. Let $\sum_{i=1}^{9} x_i^2 = 855$. If M is the mean and σ is the standard deviation of $x_1, x_2, \ldots\ldots x_9$, then what is the value of $M^2 + \sigma^2$?

(a) 100 (b) 95

(c) 90 (d) 85

112. The mean of the series $x_1, x_2, \ldots, x_n$ is $\bar{x}$. If x_n is replaced by k, then what is the new mean?

(a) $\bar{x} - x_n + k$ (b) $\frac{n\bar{x} - \bar{x} + k}{n}$

(c) $\frac{\bar{x} - x_n - k}{n}$ (d) $\frac{n\bar{x} - x_n + k}{n}$

113. A fair coin is tossed till two heads occur in succession. What is the probability that the number of tosses required is less than 6?

(a) $\frac{5}{64}$ (b) $\frac{15}{32}$

(c) $\frac{31}{64}$ (d) $\frac{19}{32}$

114. Urn A contains 2 white and 2 black balls while urn B contains 3 white and 2 black balls. One ball is transferred from urn A to urn B and then a ball is drawn out of urn B. What is the probability that the ball is white?

(a) $\frac{11}{20}$ (b) $\frac{7}{12}$

(c) – (d) 1

115. For two events A and B, P(A) = P(A|B) = 0.25 and P(B|A) = 0.5. Which of the following are correct?

A. A and B are independent.

B. $P(A^C \cup B^C) = 0.875$

C. $P(A^C \cap B^C) = 0.375$

Select the answer using the code given below.

(a) A and B only (b) B and C only

(c) A and C only (d) A, B and C

116. Two perfect dice are thrown. What is the probability that the sum of the numbers on the faces is neither 9 nor 10?

(a) $\frac{1}{36}$ (b) $\frac{5}{36}$

(c) $\frac{7}{36}$ (d) $\frac{29}{36}$

117. The occurrence of a disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random, 4 or more will suffer from the disease?

(a) $\frac{53}{3125}$ (b) $\frac{63}{3125}$

(c) $\frac{73}{3125}$ (d) $\frac{83}{3125}$

118. Three perfect dice are rolled. Under the condition that no two show the same face, what is the probability that one of the faces shown is an ace (one)?

(a) $\frac{5}{9}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

119. Three perfect dice D_1, D_2 and D_3 are rolled. Let x, y and z represent the numbers on D_1, D_2 and D_3 respectively. What is the number of possible outcomes such that $x < y < z$?

(a) 20 (b) 18

(c) 14 (d) 10

120. In a binomial distribution, if the mean is 6 and the standard deviation is $\sqrt{2}$, then what are the values of the parameters n and p respectively?

(a) 18 and $\frac{1}{3}$ (b) 9 and $\frac{1}{3}$

(c) 18 and $\frac{2}{3}$ (d) 9 and $\frac{2}{3}$

HINTS & SOLUTIONS

MATHEMATICS

1. (d) Since we know that matrix multiplication is only possible when column of first matrix is equal to row of the second matrix.

$A_{m\times n} \cdot B_{n\times p} = (AB)_{m\times p}$

Thus $[Z_{3\times 2} \cdot Y_{2\times 3}]\, X_{3\times 3} = (zyx)_{3\times 3}$

and $Y_{2\times 3} \cdot [X_{3\times 3} \cdot Z_{3\times 2}] = (yxz)_{2\times 2}$

and $x_{3\times 3}\, [Y_{2\times 3} \cdot Z_{3\times 2}] = X_{3\times 3} \cdot (YZ)_{2\times 2}$

$\because$ No. of columns in X = no. of rows in (yz)

Hence x(yz) is not defined.

2. (a) Since between any two irrational numbers, infinite number of irrational numbers are there. Hence, the set of all irrational numbers between $\sqrt{12}$ and $\sqrt{15}$ is having infinite number of elements.

$\because$ Set of all odd integers less than 1000 is {...... – 5, – 3, 1, 3, ... 999} which is an infinite set.

3. (a) Odd digits can be 1, 3, 5, 7, 9

$5 \times 5 \times 5 \times 5$

$\because$ All five odd digits can be filled at 4 places.

Hence total required 4-digits numbers are $5 \times 5 \times 5 \times 5 = 625$.

4. (d) Let $I = (1 + \omega - \omega^2)^{100} + (1 - \omega + \omega^2)^{100}$

$= (-\omega^2 - \omega^2)^{100} + (-\omega - \omega)^{100}$ $\{\because 1 + \omega + \omega^2 = 0\}$

$= 2^{100}(\omega^{200} + \omega^{100})$

$= 2^{100}(\omega^2 + \omega)$ $\{\because \omega^3 = 1\}$

$= -2^{100}$.

5. (c) We have, AB = 0

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

So, $AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Comparing both sides, we have

$ap + br = 0$ ...(i)

$aq + bs = 0$ ...(ii)

$cp + dr = 0$ ...(iii)

$cq + ds = 0$ ...(iv)

Solving equation (i) and (iii), we get

$(ad - bc)\, p = 0$ and $(ad - bc)\, r = 0$

Solving equation (ii) amd (iv), we get

$(ad - bc)q = 0$ and $(ad - bc)s = 0$

Now, If A is non-singular $\Rightarrow ad - bc \neq 0$.

So, we get, $p = q = r = s = 0$

So, B is a null matrix.

6. (c) Here, $(1 + x)^p (1 + x)^q = (1 + x)^{p+q}$

Given, Coeff. of x^3 in the expansion of $(1 + x)^{p+q} = 35$

$\Rightarrow {}^{(p+q)}C_3 = 35 = {}^7C_3$

$\Rightarrow p + q = 7$ (on comparison)

7. (a) Suppose 'a' and 'd' be the first term and common difference of AP respectively.

$\because$ n^{th} term $= T_n = a + (n - 1) \cdot d$

$\therefore$ $(p + q)^{th}$ term $= T_{p+q} = a + (p + q - 1)\, d$ ...(i)

Given, $p \cdot T_p = q \cdot T_q$

$\Rightarrow p \cdot [a + (p - 1)\, d] = q \cdot [a + (q - 1)d]$

$\Rightarrow (p - q)\, [a + (p + q - 1)\, d] = 0$

$\Rightarrow [a + (p + q - 1)\, d] = 0$ $\{\because p \neq q\}$

$\Rightarrow T_{p+q} = 0$ {from (i)}

8. (c) We have given, $p = \ell n\, x$, $q = \ell n\, x^3 = 3\ell n\, x$

$r = \ell n\, x^5 = 5\, \ell n\, x$

Clearly, $q - p = r - q = 2\, \ell nx$

$\Rightarrow$ p, q and r are in A.P.

Also, Since $\frac{q}{p} \neq \frac{r}{q} \Rightarrow$ p, q and r can never be in GP.

9. (b) We have given, $Z = \frac{1}{3}\begin{vmatrix} i & 2i & 1 \\ 2i & 3i & 2 \\ 3 & 1 & 3 \end{vmatrix}$

$\Rightarrow Z = \frac{1}{3}\, [i(9i - 2) - 2i\,(6i - 6) + 1\,(2i - 9i)]$

$\Rightarrow Z = \frac{1}{3}\,(3 + 3i) = 1 + i$

$\Rightarrow$ Modulus of $z = \sqrt{1^2 + 1^2} = \sqrt{2}$

10. (b) Suppose $A = \sum_{n=1}^{20}\left(i^{n-1} + i^n + i^{n+1}\right)$

$\Rightarrow A = \sum_{n=1}^{20} i^n \cdot \left(\frac{1}{i} + 1 + i\right) = \sum_{n=1}^{20} i^n\,(-i + 1 + i)$

$= \sum_{n=1}^{20} i^n = i^1 + i^2 + i^3 + i^4 \ldots\ldots + i^{20}$

$= 5 \cdot (i + i^2 + i^3 + i^4)$

$= 5 \cdot (0) = 0$ $\{\because i + i^2 + i^3 + i^4 = 0\}$

11. (c) Since, x, y, z are in G.P.

$\Rightarrow y^2 = xz \Rightarrow 2 \ln y = \ln x + \ln z$

$\Rightarrow \ln y - \ln x = \ln z - \ln y$

$\Rightarrow \ln x, \ln y, \ln z$ are in AP.

$\Rightarrow (1 + \ln x), (1 + \ln y), (1 + \ln z)$ are in AP

$\Rightarrow \frac{1}{(1+\ln x)}, \frac{1}{(1+\ln y)}, \frac{1}{(1+\ln z)}$ are in AP.

12. (a) Suppose $A = \begin{vmatrix} 1+\omega & 1+\omega^2 & \omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \frac{1}{\omega} & \frac{1}{\omega^2} & 1 \end{vmatrix}$

Now $C_1 \to C_1 + C_2 + C_3$, we get

$$\Rightarrow A = \begin{vmatrix} 2(1+\omega+\omega^2) & 1+\omega^2 & \omega+\omega^2 \\ (1+\omega+\omega^2) & \omega & \omega^2 \\ \frac{(1+\omega+\omega^2)}{\omega^2} & \frac{1}{\omega^2} & 1 \end{vmatrix}$$

$$\Rightarrow A = \begin{vmatrix} 2(0) & 1+\omega^2 & \omega+\omega^2 \\ 0 & \omega & \omega^2 \\ 0 & \frac{1}{\omega^2} & 1 \end{vmatrix} = 0$$

{$\because$ all element of C_1 is zero}.

13. (a) Here, $S_n = n(2n+1) \Rightarrow s_{n-1} = (n-1)(2n-1)$
n^{th} term $= T_n = S_n - S_{n-1}$
$T_n = n(2n+1) - (n-1)(2n-1) = 4n - 1$

14. (b) Here word 'INDIA' contains 3 vowels (I, I, A) and two consonants (N, D)
Now, 3 vowels (I, I, A) can be arranged at odd position in $\frac{3!}{2!} = 3$ ways and 2 consonants (N, D) can be arranged in $2!$ ways = 2 ways

odd	even	odd	even	odd

Hence, required number of ways = (3) (2) = 6

15. (c) Here, word, 'EQUATION' contains 5 vowels (A, E, I, O, U) and 3 consonants (Q, T, N).
Two cases are possible here
Case (I):

N	Q	T	A	E	I	O	U

Consonants — Vowels

Number of ways for this arrangement = (3!) . (5!)
Case(II):

A	E	I	O	U	N	Q	T

Vowels — Consonants

Number of ways for this arrangement = (3!) · (5!)
Total required number of ways = (3!) (5!) + (3!) (5!) = 1440

16. (c) Given, n is a root of $x^2 + px + m = 0$
$\Rightarrow n^2 + pn + m = 0$ ....(i)
Given, m is a root of the equation $x^2 + px + n = 0$
$\Rightarrow m^2 + pm + n = 0$ ...(ii)
Now subtracting Eq (ii) from Eq (i), we get
$(n^2 - m^2) + p(n - m) - (n - m) = 0$
$\Rightarrow (n - m)[(n + m) + p - 1] = 0$
$\Rightarrow n + m + p - 1 = 0$ {given, $n \neq m$}
$\Rightarrow n + m + p = 1$

17. (d) We have given that 2 courses are compulsory.
Total remaining courses available for selection = n – 2 and total remaining courses that can be chosen from (n – 2) courses is (n – 2) – 2 = 0 (n – 4).
Thus, required number of ways $= {}^{(n-2)}C_{(n-4)}$
$= \frac{(n-2)(n-3)}{2}$

18. (a) Given,

$$D_n = \begin{vmatrix} n & 20 & 30 \\ n^2 & 40 & 50 \\ n^3 & 60 & 70 \end{vmatrix} = (20)(10)\begin{vmatrix} n & 1 & 3 \\ n^2 & 2 & 5 \\ n^3 & 3 & 7 \end{vmatrix}$$

$\Rightarrow D_n = 200[n(14 - 15) - 1(7n^2 - 5n^3) + 3(3n^2 - 2n^3)]$
$\Rightarrow D_n = 200(-n + 2n^2 - n^3)$

Now, $\sum_{n=1}^{4} D_n = 200\sum_{n=1}^{4}\left(-n + 2n^2 - n^3\right)$

$\because \sum n = \frac{n(n+1)}{2}, \sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Hence,

$$\sum_{n=1}^{4} D_n = 200\left[-\frac{(4)(5)}{2} + 2.\frac{(4)(5)(9)}{6} - \left(\frac{(4)(5)}{2}\right)^2\right]$$

$= 200(-50) = -10000$

19. (c) We have given,

$$P = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}, Q = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$\Rightarrow PQ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{and } QP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow PQ = QP \Rightarrow$ option (c) is correct.

20. (b) We know that, determinant of any skew symmetric matrix with odd order is always zero.
Hence, determinant of skew symmetric matrix (3 × 3) will be zero.

21. (c) We have given, $4\sin^{-1}x + \cos^{-1}x = \pi$.

$$\Rightarrow 4\left(\frac{\pi}{2} - \cos^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}x\right) = \pi$$

$$\left\{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right\}$$

$$\Rightarrow\ 2\pi - 4\cos^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$$

$$\Rightarrow\ 4\cos^{-1}x + \sin^{-1}x = \frac{3\pi}{2}$$

22. (b) Suppose $I = \cot^2(\sec^{-1}2) + \tan^2(\text{cosec}^{-1}3)$

$$\Rightarrow\ I = \cot^2(60°) + \tan^2 + \left(\cot^{-1}\sqrt{9-1}\right)$$

$$\left\{\because\ \text{cosec}^{-1}t = \cot^{-1}\sqrt{t^2-1}\right\}$$

$$\Rightarrow\ I = \frac{1}{3} + \tan^2\left(\tan^{-1}\frac{1}{\sqrt{8}}\right) = \frac{1}{3} + \frac{1}{8} = \frac{11}{24}$$

23. (a) We have given,

$$\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\text{Cos}C}$$

$$\Rightarrow\ \frac{a}{\left(\frac{b^2+c^2-a^2}{2bc}\right)} = \frac{b}{\left(\frac{a^2+c^2-b^2}{2ac}\right)} = \frac{c}{\left(\frac{a^2+b^2-c^2}{2ab}\right)}$$

$\Rightarrow\ b^2 + c^2 - a^2 = c^2 + a^2 - b^2 = a^2 + b^2 - c^2$

$\Rightarrow\ a^2 = b^2 = c^2 \Rightarrow a = b = c$

Thus, ΔABC is an equilateral triangle

So, Area of $\Delta ABC = \frac{\sqrt{3}}{4}.a^2$ (Given, a = 6 cm)

$$= \frac{\sqrt{3}}{4}.(6)^2 = 9\sqrt{3}\text{cm}^2$$

24. (c) Since $\tan\alpha$, $\tan\beta$ are roots of $7x^2 - 6x + 1 = 0$

$\therefore$ Sum of roots $= \tan\alpha + \tan\beta = \frac{6}{7}$ ......(i)

Product of roots $= \tan\alpha\,.\,\tan\beta = \frac{1}{7}$ ......(ii)

$$\because \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \frac{\left(\frac{6}{7}\right)}{1-\frac{1}{7}} = 1$$

$\Rightarrow\ \alpha + \beta = 45° \Rightarrow 2\alpha + 2\beta = 90°$

Which means 3rd angle of triangle is 90°.

Hence, triangle is right-angled triangle.

Now consider,

$$(\tan\alpha - \tan\beta) = \sqrt{(\tan\alpha + \tan\beta)^2 - 4\tan\alpha.\tan\beta}$$

$$= \sqrt{\left(\frac{6}{7}\right)^2 - 4\left(\frac{1}{7}\right)} = \frac{2\sqrt{2}}{7}$$

Thus, $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha.\tan\beta} = \frac{1}{2\sqrt{2}}$

and $(\alpha - \beta) = \frac{2\tan(\alpha-\beta)}{1 - \tan^2(\alpha-\beta)} = \left(\frac{4\sqrt{2}}{7}\right)$

$\Rightarrow\ 2\alpha \neq 2\beta \Rightarrow$ triangle is not an isosceles triangles.

25. (b) Here, $\angle A = 75°$, $\angle B = 45° \Rightarrow \angle C = 60°$

We know that,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 75°} = \frac{b}{\sin 45°} = \frac{c}{\sin 60°}$$

Hence,

$$a = \frac{\sin 75°}{\sin 60°}c = \frac{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)c}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{(\sqrt{3}+1)c}{\sqrt{6}}$$

and $b = \frac{\sin 45°}{\sin 60°}.\ c = \frac{\left(\frac{1}{\sqrt{2}}c\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2c}{\sqrt{6}}$

Now,

$$2a - b = \frac{2\sqrt{3}}{\sqrt{6}}c = \sqrt{2}\,c$$

26. (c) We have given, $\cot 2x\,.\,\cot 3x = 1$ (given)

$\Rightarrow\ \cos 2x\,.\,\cot 3x - \sin 2x\,.\,\sin 3x = 0$

$\Rightarrow\ \cos(2x + 3x) = \cos 5x = 0$

Hence $5x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \ldots$

But $0 < x < \pi$

$$\Rightarrow\ x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$$

Thus only five values of x are possible.

27. (a) Since, $\cos^2 x \in [0, 1] \Rightarrow \cos^{100}x \in [0, 1]$

and $\sin^2 x \in [0, 1] \Rightarrow \sin^{100}x \in [0, 1]$

Hence, $\cos^{100}x - \sin^{100}x = 1$ is only possible when $\cos^{100}x = 1$

$\Rightarrow\ \cos^2 x = 1 \Rightarrow x = n\pi \pm 0 \Rightarrow x = n\pi$

28. (b) In ΔABC, $A + B + C = \pi$

Taking tan both side, we get

$\Rightarrow\ \tan(A + B + C) = \tan\pi = 0$

$$\Rightarrow\ \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A.\tan B - \tan B.\tan C - \tan C.\tan A} = 0$$

$\Rightarrow\ \tan A\,.\,\tan B\,.\,\tan C = \tan A + \tan B + \tan C$

$$\Rightarrow\ \text{Cot}A\,.\,\text{Cot}B\,.\,\text{Cot}C = \frac{1}{\tan A + \tan B + \tan C}$$

$$= \frac{1}{K}$$

29. (c) Let $I = \sin 12°\,.\,\sin 48° \Rightarrow I = \frac{2\sin 12°\sin 48°}{2}$

$$\Rightarrow\ I = \frac{1}{2}[\cos(36°) - \cos(60°)]$$

$$\Rightarrow\ I = \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right] = \frac{\sqrt{5}-1}{8}$$

30. (d) Let $I = \frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°} = \frac{\cos 17°\left(1 - \frac{\sin 17°}{\cos 17°}\right)}{\cos 17°\left(1 + \frac{\sin 17°}{\cos 17°}\right)}$

$\Rightarrow I = \frac{\tan 45° - \tan 17°}{1 + \tan 45°.\tan 17°}$

$= \tan(45° - 17°) = \tan 28°$

$= \tan(90° - 62°) = \cot 62°$

31. (b) Let $t = \tan 22.5° = \tan\frac{45°}{2}$

Now, $\tan 45° = \tan\left(2 \times \frac{45}{2}\right)^{\circ} = \frac{2\tan\left(\frac{45}{2}\right)^{\circ}}{1 - \tan^2\left(\frac{45}{2}\right)^{\circ}}$

$\Rightarrow 1 = \frac{2t}{1 - t^2} \Rightarrow t^2 + 2t - 1 = 0$

$\Rightarrow t = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

$\Rightarrow \tan\left(\frac{45}{2}\right)^{\circ} = -1 \pm \sqrt{2} \Rightarrow \tan\left(\frac{45}{2}\right)^{\circ} = -1 + \left(\sqrt{2}\right)$

{$\therefore$ In I^{st} quadrant tan is positive}

(II) Now, cot $22.5° = \frac{1}{\tan 22.5°} = \frac{1}{-1+\sqrt{2}} = 1 + \sqrt{2}$

which is also an irrational number.

(III) Since, $\tan 22.5° - \cot 22.5° = -1 + \sqrt{2} - \left(1 + \sqrt{2}\right) = -2$ which is not an irrational number.

32. (b)

33. (d) We have,

$p\tan(\theta - 30°) = q\tan(\theta + 120°)$

$\Rightarrow \frac{p}{q} = \frac{\tan(\theta + 120°)}{\tan(\theta - 30°)}$

$= \frac{\sin(\theta + 120°)\ \cos(\theta - 30°)}{\cos(\theta + 120°)\ \sin(\theta - 30°)}$

Using componendo and dividendo rule,

$\Rightarrow \frac{p+q}{p-q} = \frac{\sin(\theta+120°)\cos(\theta-30°) + \cos(\theta+120°)\sin(\theta-30°)}{\sin(\theta+120°)\cos(\theta-30°) - \cos(\theta+120°)\sin(\theta-30°)}$

$\Rightarrow \frac{p+q}{p-q} = \frac{\sin[(\theta+120°) + (\theta-30°)]}{\sin[(\theta+120°) - (\theta-30°)]} = \frac{\sin(90° + 2\theta)}{\sin(150°)}$

$\Rightarrow \frac{p+q}{p-q} = \frac{\sin(90° + 2\theta)}{\left(\frac{1}{2}\right)} = 2\cos 2\theta$

34. (d)

35. (d) We have,

$A \cap B = 10$

So, $(A \times B) \cap (B \times A) = 10 \times 10 = 100$.

36. (b) Suppose $p(n) = 7^n - 6n = (6+1)^n - 6n$

Using Binomial expansion, we get

$= 1 + 6n + {}^nc_2\, 6^2 + {}^nc_3\, 6^3 + ... + 6^n - 6n$

$= 1 + 6^2\,({}^nc_2 + {}^nc_2 \times 6 + ... + 6^{n-2})$

$= 1 + 36\,({}^{100}c_2 + {}^{100}c_2 \times 6 + ... + 6^{98})$ {$\because$ n = 100}

$= 1 + 36\,m$ where $m = {}^{100}c_2 + {}^{100}c_2 \times 6 + ... + 6^{98}$

So, when $7^n - 6n$ is divided by 36, the remainder will be 1.

37. (c) There are 4 lines, then the maximum number of intersection points are ${}^4c_2 = 6$

And maximum number of intersection of circle with 4 lines are $4 \times 2 = 8$.

Thus, the required intersection point = 6 + 8 = 14.

38. (c) We have,

$\frac{S_p}{S_q} = \frac{p^2}{q^2}$

$\Rightarrow \frac{\frac{p}{2}[2a + (p-1)d]}{\frac{q}{2}[2a + (q-1)d]} = \frac{p^2}{q^2}$ $\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\Rightarrow \frac{2a + (p-1)d}{2a + (q-1)d} = \frac{p}{q}$

$\Rightarrow$ $2aq + q(p-1)d = 2ap + p(q-1)d$

$\Rightarrow$ $2a(q-p) = d(q-p)$

$\Rightarrow$ $2a = d$ or $d = 2a$

This, the common difference is equal to twice of the first term.

39. (a) Given that, $(x-1)^2 + (x-3)^2 + (x-5)^2 = 0$

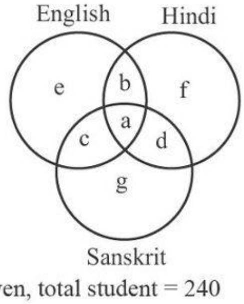
$\Rightarrow$ $x^2 - 2x + 1 + x^2 - 6x + 9 + x^2 - 10x + 25 = 0$

$\Rightarrow$ $3x^2 - 18x + 35 = 0$

Discriminant $D = (-18)^2 - 4 \times 3 \times 35 = -96 < 0$

$\because$ Discriminant is negative. So, the given equation does not have any real roots.

40. (a)



Given, total student = 240

Since, 10 student failed in every subject.

Hence, Remaining total = 230

Now, According to question,

$a + b + c + d + e + f + g = 230$ ...(i)

and $b + c + d = 110$ ...(ii)

and $e + f + g = 60$ ...(iii)

Using equation (ii) and (iii) in equation (i), we get

$\Rightarrow a = 230 - (110 + 60) = 60$

Sol. 41 to 42

Given that,

$Z_1^2 + Z_2^2 + Z_1 Z_2 = 0 \Rightarrow Z_1 = w$ and $Z_2 = w^2$

41. (a) $\left|\frac{Z_1}{Z_2}\right| = \left|\frac{\omega}{\omega^2}\right| = \left|\frac{1}{\omega}\right| = 1$

42. (b) Now,

$$\frac{1}{2} + \text{Re}\left(\frac{Z_1}{Z_2}\right) = \frac{1}{2} + \text{Re}\left(\frac{\omega}{\omega^2}\right) = \frac{1}{2} + \text{Re}\left(\omega^2\right)$$

$$= \frac{1}{2} + \text{Re}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{1}{2} + \left(-\frac{1}{2}\right) = 0$$

$= 0$

43. (d) Let the required terms of the given AP are-

$a - 2d, a - d, a, a + d, a + 2d.$

According to the question,

$(a - 2d)(a - d)a(a + d)(a + 2d) = 229635$

$\Rightarrow a(a^2 - d^2)(a^2 - 4d^2) = 229635$ ...(i)

$\because$ $a - 2d, a - d, a + 2d$ are in G.P.

$\therefore$ $(a - d)^2 = (a - 2d)(a + 2d)$

$\Rightarrow (a - d)^2 = a^2 - 4d^2$

$\Rightarrow 5d^2 - 2ad = 0 \Rightarrow d(5d - 2a) = 0 \Rightarrow 5d = 2a$ ...(ii)

Solving equation (i) and (ii), we have,

$d = 6.$

44. (c) Using the value of d in equation (ii), we have,

$5 \times 6 = 2a \Rightarrow a = 15$

$\therefore$ The required terms of A.P. are 3, 9, 15, 21, 27.

So, required sum = 3 + 9 + 15 + 21 + 27 = 75.

For. 45 and 46:

We have, $(8 + 3\sqrt{7})^{20} = U + V$ ...(i)

and $(8 - 3\sqrt{7})^{20} = W$ ...(ii)

Here, $o < W < 1$

Now, adding equation (i) & (ii), we have,

$$U + V + W = (8 + 3\sqrt{7})^{20} + (8 - 3\sqrt{7})^{20}$$

$$= 2\left[{}^{20}C_0 \cdot 8^{20} + {}^{20}C_2\, 8^{18}.\left(3\sqrt{7}\right)^2 + \ldots.. + \left(3\sqrt{7}\right)^{20}\right]$$

R.H.S. is an even number.

Also, $o < V < 1$, $o < W < 1$ and U is an integer

$\Rightarrow$ V + W is an integer $\Rightarrow V + W = 1$

45. (d) So, $V + W = 1$

48. (b) $(U + V)W = \left(8 + 3\sqrt{7}\right)^{20}\left(8 - 3\sqrt{7}\right)^{20}$

$$= \left[\left(8 + 3\sqrt{7}\right)\left(8 - 3\sqrt{7}\right)\right]^{20}$$

$= (64 - 63)^{20} = 1^{20} = 1$

Sol. 47 and 48 :

The given equation is as follows,

$a^2(b^2 - c^2)x^2 + b^2(c^2 - a^2)x + c^2(a^2 - b^2) = 0$ ...(i)

Since, equation has equal roots $\Rightarrow D = 0$

$\Rightarrow [b^2(c^2 - a^2)]^2 - 4a^2c^2(b^2 - c^2)(a^2 + b^2) = 0$

$\Rightarrow b^4(c^2 - a^2)^2 - 4a^2c^2(b^2 - c^2)(a^2 - b^2) = 0$

Solving above equation, we have,

$b^4(c^2 + a^2)^2 = 4a^4c^4 \Rightarrow b^2(a^2 + c^2)^2 = 2a^2c^2$

$\Rightarrow \frac{2}{b^2} = \frac{1}{a^2} + \frac{1}{c^2}$

$\therefore$ a^2, b^2, c^2 and in H.P.

47. (c) Option (c) is the correct option.

48. (c) Let α be the required root of the given equation.

Now, Sum of roots $= -\frac{B}{A} = -\frac{b^2(c^2 - a^2)}{a^2(b^2 - c^2)}$

$$\Rightarrow \alpha + \alpha = \frac{-b^2(c^2 - a^2)}{a^2(b^2 - c^2)} \Rightarrow \alpha = \frac{b^2(c^2 - a^2)}{2a^2(c^2 - b^2)a}$$

Sol. 49 and 50 :

We have : $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

So, Determinant

$|A| = 3(-3 + 4) - 2(-3 + 4) + 0 = 3 - 2 = 1$

$\because$ $A(\text{adj } A) = |A|\, I = 1.I = I.$

49. (d) Option (d) is correct option.

50. (a) Now,

$$\text{adj}(A) = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|}\left(\text{Adj}(A)\right) = \frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

51. (a) Given that, $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \alpha\hat{j} + \beta\hat{k}) = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \hat{i}(6\beta - 27\alpha) - \hat{j}(2\beta - 27) + \hat{k}(2\alpha - 6) = 0$$

Comparing both sides of above equation, we get:-

$6\beta - 27\alpha = 0 \Rightarrow 2\beta = 9\alpha$

$2\beta - 27 = 0 \Rightarrow \beta = \frac{27}{2}$ and $2\alpha - 6 = 0 \Rightarrow \alpha = 3$

So, $3\alpha + 2\beta = 3 \times 3 + 2 \times \frac{27}{2} = 9 + 27 = 36$

52. (b) Let $m = |\vec{a} \times \vec{b}| + \sqrt{3}|\vec{a}.\vec{b}|$

$= |\vec{a}|.|\vec{b}||\sin\theta| + \sqrt{3}|\vec{a}||\vec{b}||\cos\theta|$

$= |\vec{a}||\vec{b}|\left[|\sin\theta| + \sqrt{3}|\cos\theta|\right]$

m will be max if ($\sin\theta + \sqrt{3}\ \cos\theta$) is maximum

Now, for its maximum, $\frac{d}{d\theta}\left(\sin\theta + \sqrt{3}\cos\theta\right) = 0$

$\Rightarrow \cos\theta - \sqrt{3}\sin\theta = 0$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

53. (a) Since, $\left(\vec{a} + 2\vec{b}\right) \& \left(5\vec{a} - 4\vec{b}\right)$ are perpendicular vectors.

$\Rightarrow \left(\vec{a} + 2\vec{b}\right).\left(5\vec{a} - 4\vec{b}\right) = 0$

$\Rightarrow 5|\vec{a}|^2 - 4\vec{a}.\vec{b} + 10\vec{a}.\vec{b} - 8|\vec{b}|^2 = 0$

$\Rightarrow 5 \times 1 + 6\vec{a}.\vec{b} - 8 = 0$ Since, $|\vec{a}| = |\vec{b}| = 1$

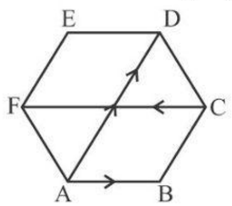
$\Rightarrow 6|\vec{a}||\vec{b}|\cos\theta = 3$

$\Rightarrow 1.1\cos\theta = \frac{1}{2} \Rightarrow \cos\theta = \frac{1}{2}$

So, $\cos 2\theta = 2\cos^2\theta - 1 = 2 \times \frac{1}{4} - 1 = -\frac{1}{2}$

Now, $\cos\theta + \cos 2\theta = \frac{1}{2} - \frac{1}{2} = 0$

54. (a)



In a regular hexagon,

$\overrightarrow{AB} = 2\overrightarrow{FC}$

$\Rightarrow \overrightarrow{AB} = -2\overrightarrow{CF}$

$\therefore\ n = -2$

Also, $\overrightarrow{AD} = 2\overrightarrow{BC}$

$\therefore\ m = 2$

Now, $mn = 2\,(-2) = -4$

55. (c) Given that,

$\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}$

Since, $\vec{a}, \vec{b}, \vec{c}$ has same length.

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{1+1} = \sqrt{2}$

Let θ be the angle between the vectors.

$\therefore \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{0+1+0}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2}$

(I) Take $\vec{c} = \hat{i} + \hat{k} \Rightarrow |\vec{c}| = \sqrt{1+1} = \sqrt{2}$

and $\cos\theta = \frac{\vec{a}.\vec{c}}{|\vec{a}||\vec{c}|} = \frac{1+0+0}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

All $\vec{c}$ the conditions are satisfied. So, it can be vector.

(II) Take $\vec{c} = \frac{\hat{i} + 4\hat{j} - \hat{k}}{3} \Rightarrow |\vec{c}| = \frac{1}{3}\sqrt{1+16+1} = \frac{3\sqrt{2}}{3} = \sqrt{2}$

and $\cos\theta = \frac{\vec{a}.\vec{c}}{|\vec{a}||\vec{c}|} = \frac{\frac{-1}{3} + \frac{4}{3}}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\because$ All the conditions are satisfied, so it can be vector $\vec{c}$

56. (d) The Slope of the diagonal along the line $x - 2y = 1$ is $m_1 = \frac{1}{2}$

and the slope of the diagonal along the line $4x + 2y = 3$ is $m_2 = -2$.

Now, $m_1\ m_2 = \frac{1}{2}(-2) = -1 \Rightarrow m_1\ m_2 = -1$

So, Diagonals are perpendicular to each others.

$\Rightarrow$ The quadrilateral ABCD is a rhombus.

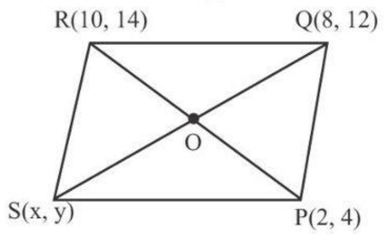
57. (b) Given equation of ellipse, $4x^2 + 9y^2 = 1$

Here, $a = \frac{1}{2}$ and $b = \frac{1}{3}$

Since, P(x, y) is any point on the ellipse.

So, $PQ + PR = 2a = 2 \times \frac{1}{2} = 1$.

58. (b)



Let O is the intersection point of S(x, y)

So, O is mid point of P and R.

$\Rightarrow O \equiv \left(\frac{10+2}{2}, \frac{14+4}{2}\right) = (6, 9)$

Also, O is mid point of S and Q.

$\therefore O \equiv \left(\frac{x+8}{2}, \frac{y+12}{2}\right)$

$\Rightarrow (6, 9) = \left(\frac{x+8}{2}, \frac{y+12}{2}\right)$

Comparing both sides, we have,

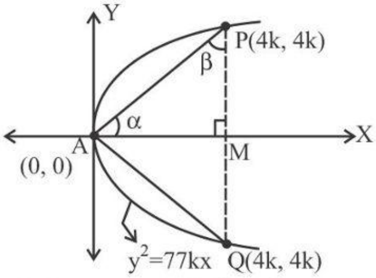
$\frac{x+8}{2} = 6 \Rightarrow x = 4$

and $\frac{y+12}{2} = 9 \Rightarrow y = 6$

Now, $x + y = 4 + 6 = 10$

59. (a) Given, $(x^2 - 4x + 3) + (y^2 - 6x + 8) = 0$
$\Rightarrow$ $(x - 3)(x - 1) + (y - 4)(y - 2) = 0$
So, all the possible ends of the diameters of given circle are (3, 4), (3, 2), (1, 4) and (1, 4).
Also, Radius = $\sqrt{2}$ and center = (2, 3)
So, the required pair of ends of diameters are
(I) (1, 2) & (3, 4) and (II) (1, 4) and (3, 2).

60. (b)



Let $\angle PAM = \alpha$ and $\angle APM = \beta$
From figure, we have,
$\angle PAQ = \angle PAM + \angle QAM$
$= \alpha + \alpha = 2\alpha$
In right angled ΔAMP,
$AM = PM = 4k \Rightarrow \alpha = \beta$
$\therefore$ $\alpha + \beta + 90 = 180 \Rightarrow 2\alpha = 90° \Rightarrow \alpha = 45°$
Hence, $\angle PAQ = 2\alpha = 90°$

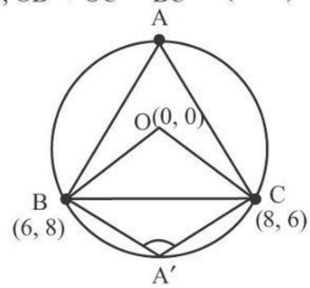
Sol. 61 and 62

61. (c) Let O is centre of circle $x^2 + y^2 = 100$, radius = OB = OC = 10 units.

$BC = \sqrt{(6+8)^2 + (8-6)^2} = \sqrt{200} = 10\sqrt{2}$

$OB^2 + OC^2 = \sqrt{100 + 100} = \sqrt{200} = 10\sqrt{2}$

In ΔBOC, $OB^2 + OC^2 = BC^2 = (10\sqrt{2})^2 = 200$



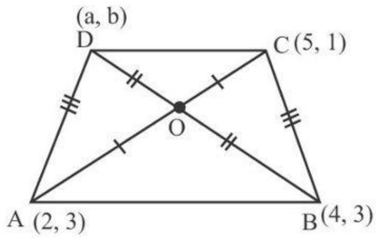
So $\angle BOC = 90°$ (By converse of Pythagoras theorem)

$\therefore$ $\angle BAC = 45°$ or $180° - 45° = \frac{3\pi}{4}$

(Angle subtended by a chord at centre is double than the angle at any point on circle)
Option (c) is correct.

62. (d) Since, the point A can be taken at any point on the circumference of circle which subtends $\frac{\pi}{4}$ to the chord so coordinates of A cannot be determined due to insufficient data.

63. (d)



We know,
In isosceles trapezium diagonal bisect each other. O is mid point of AC and BD.

$O \equiv \left(\frac{7}{2}, \frac{4}{2}\right) = \left(\frac{4+a}{2}, \frac{b+3}{2}\right)$

a = 3, b = 1 (equating x and y coordinates of O)
D (a, b) ≡ (3, 1)

64. (c) Since Diagonal of Isosceles trapezium bisect each other then let intersecting point of diagonal is O.

Coordinates of O is $\left(\frac{2+5}{2}, \frac{3+1}{2}\right)$ i.e $\left(\frac{7}{2}, 2\right)$

65. (b) Given sphere : $2x^2 + 2y^2 + 2z^2 + 3x + 3y + 3z - 6 = 0$ ...(i)
Divide equation (1) by (2)

$\Rightarrow x^2 + y^2 + z^2 + \frac{3}{2}x + \frac{3}{2}y + \frac{3}{2}z - 3 = 0$

radius of sphere = $\sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + 3} = \sqrt{\frac{27}{16} + 3} = \frac{\sqrt{75}}{4}$

radius = $\frac{5\sqrt{3}}{4}$

Diameter of sphere = $\frac{5\sqrt{3}}{2}$

66. (d) Centre of sphere is $C\left(\frac{-3}{4}, \frac{-3}{4}, \frac{-3}{4}\right)$

4x + 8y + 8z + 15 = 0 is the only plane which satisfy the centre of sphere $- 3 + - 6 - 6 + 15 = 0 \Rightarrow 0 = 0$
Option (d) is correct.

Sol. 67 and 68

2 planes are $x + y + z = 1$ and $2x + 3y - 4z = 8$.

67. (b) Let a, b, c are dr's is of lines of formed by intersection of planes then the lines is perpendicular to normal of both planes.
$a + b + c = 0$ ...(i)
and $2a + 3b - 4c = 0$ ...(ii)
$\frac{a}{-4-3} = \frac{b}{2+a} = \frac{c}{3-2} = \lambda$ (Solving (i) and (ii)
$a = -7\lambda$, $b = 6\lambda$, $c = \lambda$
Dr's or < – 7, 6, 1 >.

68. (a) DC's of line : S are l, m, n

$l=\frac{-7}{\sqrt{86}}, m=\frac{6}{\sqrt{86}}, n=\frac{1}{\sqrt{86}}$

$43(l^2-m^2-n^2)=43\left(\frac{49}{86}-\frac{36}{86}-\frac{1}{86}\right)=43\times\frac{12}{86}=6$

Sol. for 69 to 70

We have

$L: x+y+z+4=0=2x-y-z-8$ and

$P: x+2y+3z+1=0$ is a plane.

69. (c) Let Dr's of line < a, b, c >

Since, line L obtained by intersection of planes $x+y+z+4=0$ and $2x-y-z-8=0$

So, $a+b+c=0$ ...(i)

$2a-b-c=0$ ...(ii)

(line is perpendicular to normal of both plane)

$\frac{a}{-1+1}=\frac{b}{2+1}=\frac{c}{-1-2}=\lambda$ (solving (i) and (ii)

$\Rightarrow a=0, b=3\lambda, c=-3\lambda$

Dr's of line is < 0, 3, –3 > or < 0, 1, – 1 >

70. (d) To find point of intersection of L and P.

The point must satisfy both line L and P.

From option (d) (–4, –3, 3), will satisfy both line L and plane P.

So, option (d) is correct answer.

71. (a) We know,

$\{x\}+[x]=x \Rightarrow x-[x]=\{x\}$ (where $\{x\}$ is fractional port of x.)

$0\le x-[x]<1$ $(\because 0\le\{x\}<L)$

$-1<[x]-x\le 0$

$-1<y<0$

$\therefore [y]=-1$

72. (a) Given, $f(x)=4x+1, g(x)=kx+2$

$fog(x)=4(kx+2)+1=4kx+9$

$go f(x)=k(4x+1)+2=4kx+k+2$

$fog(x)=g of (x)\Rightarrow g=k+2\Rightarrow k=7$

73. (b) Given, $f(x)=\log_{10}(x^2+2x+11)$

For minimum and maximum

$f'(x)=0$

$\frac{2x+2}{x^2+2x+11}=0$ $(\sin, x^2+2x+11>0)$

At, $x=-1$, we get minimum value of f(x)

$f(-1)=\log_{10}10=I$

74. (d) Given,

$\lim_{x\to 3}\frac{|x-3|}{x-3}$

Finding LRL and RHL at x = 3

$LHL=\lim_{x\to 3^-}\frac{|x-3|}{x-3}=\frac{-(x-3)}{x-3}=-1$

$LHL=\lim_{x\to 3^+}\frac{|x-3|}{x-3}=\frac{(x-3)}{x-3}=+1$

$LHL\ne RHL$, limit does not exist.

75. (b) We know

Maximum value of $a\cos x+b\sin x$ is $\sqrt{a^2+b^2}$

So Maximum value of $a\cos x+b\sin x+c$ is $\sqrt{a^2+b^2}+c$

76. (c) Given

$f(2x)=4x^2+1\Rightarrow f(x)=x^2+1$

$f(4x)=16x^2+1$

$(f(2x))^2=f(x)\times f(4x)$

$(4x^2+1)^2=(x^2+1)\times(16x^2+1)$

(f(2x) is GM of f(x) & f(4x))

$16x^4+1+8x^2=16x^2+16x^2+1+x^2$

$\Rightarrow 9x^2=0$

$x=0$

$\therefore$ One real value of x is possible.

77. (d) We are given.

$f(x)=[x]^2-30[x]+221=0$

$[x]=\frac{30\pm\sqrt{900-884}}{2}=\frac{30\pm 4}{2}$

$[x]=17$ or 13.

Integral value of x and 17 and 13.

$\therefore$ Sum of integral solution of $f(x)=30$.

78. (b) Given, $f(x)=9x-8\sqrt{x}$, $g(x)=9x-8\sqrt{x}-1$

Lets take $x=t^2$,

$g(t^2)=9t^2-8t-1$

$t=\frac{8\pm 10}{18}\Rightarrow t=\frac{18}{18}$ or $\frac{-1}{9}$

$t=1$ or $\frac{-1}{9}$

$\Rightarrow x=1, \frac{1}{81}$

$\therefore$ $g(x)=0$ has only one real root which is an integer.

79. (b) $\lim_{\theta\to\frac{\pi}{2}}(\sec\theta-\tan\theta)$

$\lim_{\theta\to\frac{\pi}{2}}(\sec\theta-\tan\theta)=\lim_{\theta\to\frac{\pi}{2}}\frac{1-\sin\theta}{\cos\theta}\left(\frac{0}{0}\text{form}\right)$

$\lim_{\theta\to\frac{\pi}{2}}\left(\frac{-\cos\theta}{-\sin\theta}\right)=0$ (Using L Hospital Rule)

80. (a) Given $f(x)=f(y)$

Let $f(x)=x^n$ and $f(y)=y^n$.

$f(xy)=(xy)^n$

$f(2)=4\Rightarrow 4=(2)^n\Rightarrow n=2$

$f(xy)=(xy)^2$

$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2=\frac{1}{4}$

Sol. for 81 and 82

81. (c) Given fog(x) = $\cos^2 \sqrt{x}$, go f(x) = |cos x|
If we take option (c)
f(x) = $\cos^2 x$, then g(x) = $\sqrt{x}$
$\text{fog}(x) = \cos^2 \sqrt{x}$ and $\text{g of}(x) = \sqrt{\cos^2 x} = |\cos x|$
$\therefore$ f(x) = $\cos^2 x$
so option (c) is correct.

82. (a) If f(x) = $\cos^2 x$ then g(x) = $\sqrt{x}$,
so that fog (x) = $\cos^2 \sqrt{x}$

Sol. for Q.83 and Q.84

Given f(x) = $[x]^2 - [x^2]$

83. (b) According to the question
$f(0.999) = [0.999]^2 - [(0.999)^2]$
= 0 – [value lies b/w 0 and 1]
f(0.999) = 0 – 0 = 0 ...(i)
$f(1.001) = [1.001]^2 - [(1.o001)^2] = 1 - 1 =$ ...(ii)
Adding (i) and (ii)
f(0.999) + f(1.001) = 0 – 0 = 0

84. (b) Given,
f(x) = $[x]^2 - [x^2]$
Let us check continuity of f(x) at x = 0
LHL = $\lim\limits_{x\to 0^-} = [0-h]^2 - [(0-h)^2] = (-1)^2 - 0 = 1$
RHL = $\lim\limits_{x\to 1^+} = [0+h]^2 - [(0+h)^2] = 0 - 0 = 0$
LHL ≠ RHL, so f(x) is not continous at x = 0
Let us check continuity of f(x) at x = 1
LHL = $\lim\limits_{x\to 1^-} = [1-h]^2 - [(1-h)^2] = 0 - 0 = 0$
RHL = $\lim\limits_{x\to 1^+} = [1+h]^2 - [(1+h)^2] = 1 - 1 = 0$
f(x) is continuous at x = 1 or LHL = RHL

Sol. for 85 to 86

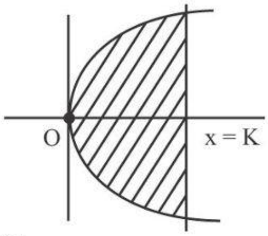
$f(x) = \cos 2x + x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

85. (b) $f'(x) = -2\sin 2x + 1$
for maxima & minima f′(x) = 0
$$\sin 2x = \frac{1}{2}$$
$$x = \frac{\pi}{12} \text{ or } \frac{5\pi}{12} \left(\text{Not in int erval}\right) \text{ of } \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$
So, maximum value of f(x) occurs at $x = \frac{\pi}{12}$ ie
$$f\left(\frac{\pi}{12}\right) = \cos\frac{\pi}{6} + \frac{\pi}{12} = \frac{\sqrt{3}}{2} + \frac{\pi}{12}$$

86. (a) Find the value of f(x) at boundary of interval i.e at $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$
$$f\left(\frac{-\pi}{2}\right) = -1 + \frac{-\pi}{2}$$
$$f\left(\frac{\pi}{2}\right) = -1 + \frac{\pi}{2}$$
So, minimum value of $f(x)^2 = -\left(1 + \frac{\pi}{2}\right)$

Sol. for 87 and 88

87. (b)



Given,
Area bounded = $\frac{4}{3}$squar
$$0 = 2\int_0^k y\, dx$$
$$\frac{4}{3} = 2\int_0^k \sqrt{k}\sqrt{x}dx \Rightarrow \frac{4}{3} = 2\sqrt{k}\,\frac{2}{3}(k)^{\frac{3}{2}} \Rightarrow 1 = k^2$$
$\Rightarrow$ k = ± 1 (since k > 0)
$\Rightarrow$ k = 1

88. (a) So equation of parabola is $y^2 = x$ (k = 1)
$4a = 1,\ a = \frac{1}{4}$
Area of parabola $y^2 = x$ and latus rectum at $x = \frac{1}{4}$ is
$$= 2\int_0^{\frac{1}{4}} \sqrt{x}dx = 2 \times \frac{2}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}} = \frac{4}{3} \times \frac{1}{8} = \frac{1}{6} \text{ sq. units}$$

Sol. for 89 and 90

89. (a) Given differential equation is
$$ydx + \left(x - y^3\right)dy = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{y^3 - x}$$
Order = 1, degree = 1

90. (d) Given differential equation is
$$y\frac{dx}{dy} + x = y^3 \Rightarrow \frac{dx}{dy} + x\frac{1}{y} = y^2$$
(Linear differential equationof form $\frac{dx}{dy} + px = \theta$)
If = $e^{\int \frac{1}{y}dy} = y$
x (I.F) = ∫(If) y^2 dy ⇒ xy = ∫ y^3dy ⇒ $4xy - y^4 + c$

91. (d) Given f(x) = $|x^2 - x - 2|$
$$\begin{cases} x^2 - x - 2; x \in (-\infty, -1] \cup [2, \infty) \\ -\left(x^2 - x - 2\right); x \in [-1, 2] \end{cases}$$

Let $I=\int_0^2 f(x)=-\int_0^2\left(x^2-x-2\right)dx=-\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_0^2$

$=-\left(\frac{8}{3}-\frac{4}{2}-4\right)+0$

$=-\left(\frac{8-18}{3}\right)=\frac{10}{3}$

92. (b) $f(x)=|x^2-x-2|$

$=\begin{cases}x^2-x-2; x\in(-\infty,-1]\cup[2,\infty)\\ -\left(x^2-x-2\right); x\in[-1,2]\end{cases}$

Let $I=\int_1^3 f(x)\,dn=-\int_1^2\left(x^2-x-2\right)dx+\int_2^3\left(x^2-x-2\right)dx$

$=-\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_1^2+\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_2^3$

$=-\left[\frac{8}{3}-\frac{4}{2}-4\right]+\left[\frac{1}{3}-\frac{1}{2}-2\right]+\left[\frac{27}{3}-\frac{9}{2}-6\right]-\left[\frac{8}{3}-\frac{4}{2}-4\right]$

$=\frac{20}{6}-\frac{13}{6}-\frac{9}{6}+\frac{20}{6}=\frac{18}{6}=3$

Sol. For 93 and 94

let $f(t)=\ln\left(t+\sqrt{1+t^2}\right)$

Put t = – t in f(t)

$f(-t)=\ln\left(\sqrt{1+t^2}-t\right)=\ln\left(\frac{1}{t+\sqrt{1+t^2}}\right)$

$=-\ln\left(t+\sqrt{1+t^2}\right)=-f(t)$

$\Rightarrow$ f(–t) = – f(t)

$\therefore$ f(t) is odd function and g(l) = tan f(t)

Put t = – t in g(t)

$\therefore$ g(–t) = tan(f(–t)) = tan (– f(t))

[$\because$ f(t) is odd function]

= – tan (f(t)) = – g(t)

So, g(t) is odd function.

93. (c) Both statements I and II are correct.

94. (b) we know that g(t) is odd function then $\int_{-\pi}^{\pi} g(t)\,dt=0$

Sol. for 95 and 96

Let f : (– 1, 1) $\rightarrow$ R be a differentiable function with f(0) = – 1 and f′ (0) = 1

95. (d) h(x) = f(2f(n) + 2)

Differentiate both side w.r.t x

h′ (x) = f′(2f(x) + 2) 2f′(x)

h′(0) = f′(2f(0) + 2) · 2 f′(0) = f′(– 2 + 2) . 2 (1)

= f′(0) . 2 = (1).2 = 2 [$\because$ f(0) = – 1 and f′(0) = 1]

96. (a) $g(n)=(h(x))^2$

Differentiate both side w.r.t x

g′(x) = 2 (h (x)) . h′(x)

g′(0) = 2(h(0)) . h′(0)

[$\because$h′(0) = 2]

[$\because$f′(0) = – 1]

= 2f(2 + 10) + 2) . 2

= 2f (2 (–1) + 2) = 2 f(0) . 2 = (–2) . 2 = – 4.

97. (c) Given that g(x) = sinx + cosx + 1

$\int_0^{\frac{\pi}{2}}\frac{dx}{g(x)}=\int_0^{\frac{\pi}{2}}\frac{1}{\sin x+\cos x+1}dx$

$=\int_0^{\frac{\pi}{2}}\frac{1}{\frac{2\tan\frac{\pi}{2}}{1+\tan^2\frac{\pi}{2}}+\frac{1-\tan^2\frac{\pi}{2}}{1+\tan^2\frac{\pi}{2}}+1}dx$

$=\int_0^{\frac{\pi}{2}}\frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}+1+\tan^2\frac{x}{2}}dx$

$=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}}{2\left(1+\tan\frac{x}{2}\right)}dx$

let $1+\tan\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dt$

when x = π/2 then t = 2, and when x = 0 then t = 1

$=\int_1^2\frac{1}{t}dt=[\ln t]_1^2=\ln 2-\ln 1=\ln 2$

98. (c) Given that f(x) = sinx and g(x) sinx + cosx + 1

Let $I=\int_0^{\frac{\pi}{2}}\frac{f(x)}{g(x)}dx=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x+1}dx$ ......(i)

$I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)+1}dx$

$I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x+1}dx$ ....(ii)

Adding equation (i) and (ii), we get

$2I=\int_0^{\frac{\pi}{2}}\frac{\sin x+\cos x}{\sin x+\cos x+1}dx$

$=\int_0^{\frac{\pi}{2}}\left(1-\frac{1}{\sin x+\cos x+1}\right)dx$

$=\int_0^{\frac{\pi}{2}}1\,dn-\int_0^{\frac{\pi}{2}}\frac{1}{\sin x+\cos x+1}dx$

from solution of previous question

$2I=\int_0^{\frac{\pi}{2}}1\,dx-\ln 2=\frac{\pi}{2}-\ln 2$

$\therefore\ I=\frac{\pi}{4}-\frac{\ln 2}{2}$

Sol. for 99 and 100

$$I = 2\int \frac{x^2-1}{\sqrt{x^2+1}}\,dx = 2\int\left(\frac{x^2+1}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+1}}\right)dx$$

$$= 2\int\left(\sqrt{x^2+1} - \frac{2}{\sqrt{x^2+1}}\right)dx$$

$$= 2\left[\frac{x}{2}\sqrt{x^2+1} + \frac{1}{2}\ln\left(x+\sqrt{x^2+1}\right) - 2\ln\left(x+\sqrt{x^2+1}\right) + c\right.$$

$$= x\sqrt{x^2+1} + \ln\left(x+\sqrt{x^2+1}\right) - 4\ln\left(x+\sqrt{x^2+1}\right)$$

$$= x\sqrt{x^2+1} - 3\ln\left(x+\sqrt{x^2+1}\right) + c$$

$= U(x).\,V(x) - 3\ln\{U(x) + V(x)\} + c$

$\therefore\ U(x) = x$ and $V(x) = \sqrt{x^2+1}$

99. (b) Now, $|U^2(x) - V^2(x)|$
$= |x^2 - (x^2+1)| = |x^2 - x^2 - 1| = |-1| = 1$

100. (d) $\therefore U(x).V(x) = x\sqrt{x^2+1} = \sqrt{x^4+x^2} = \sqrt{x^2+x^4}$

101. (d) Line of regression of x on y
$x - 3y + 4 = 0$
$\Rightarrow\ x = 3y - 4$
$\Rightarrow\ b_{xy} = 3$
Line of regression of y on x

$2x - 7y + 8 = 0 \Rightarrow\ y = \frac{2}{7}x + \frac{8}{7}$

$\Rightarrow\ b_{yx} = \frac{2}{7}$

$\therefore\ b_{xy} + 7b_{yx} = 3 = 7\ .\ \frac{2}{7} = 3 + 2 = 5$

102. (b) Mean $= \frac{1+4+9+......+n^2}{n}$

$130 = \frac{n(n+1)(2n+1)}{6n}$

$\Rightarrow\ 780n = n\,(n+1)\,(2n+1)$
$\Rightarrow\ 780 = 2n^2 + 3n + 1$
$\Rightarrow\ 2n^2 + 3n - 779 = 0$
$\Rightarrow\ (2n+41)\,(n-19) = 0$
$\Rightarrow\ x = -\frac{41}{2}, 19$

$\Rightarrow\ n = 19\quad (\because\ n = -\frac{41}{2}$ not possible)

103. (c) Three distinct natural numbers are chosen from 1 to 10.
$n(s) = 10c_3$
Favourable outcomes = (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9), (8, 9, 10)
n(E) = 8

$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{8}{10_3} = \frac{8\times 7!\times 3!}{10!} = \frac{8\times 3\times 2\times 1}{10\times 9\times 8} = \frac{1}{15}$

104. (c) Given that A, B, C are mutually exclusive and exhaustive events

$\therefore\ P(A) + P(B) + P(C) = 1$ ...(i)

Given $3\,P(B) = 4P(A) \Rightarrow P(A) = \frac{3}{4}\,P(B)$ ...(ii)

and $3\,P(C) = 2\,P(B) \Rightarrow P(C) = \frac{2}{3}\,P(B)$ ...(iii)

Put (ii) and (iii) in (i), we get

$\frac{3}{4}P(B) + P(B) + \frac{2}{3}P(B) = 1$

$\Rightarrow\ \frac{9P(B) + 12P(B) + 8P(B)}{12} = 1$

$\Rightarrow\ \frac{29P(B)}{12} = 1 \Rightarrow P(B) = \frac{12}{29}$

Put the value of P(B) in (ii), we get

$P(A) = \frac{3}{4}P(B) = \frac{3}{4}\times\frac{12}{29} = \frac{9}{29}$

105. (c) Number on faces on die are 4, 4, 5, 5, 5, 6
$\therefore\ n(s) = 6$

P(getting 4 or 5) $= \frac{5}{6}$

106. (a)

2B, 4y 6w

To get all 3 balls of same colour, ball drawn must be either yellow or white with replacement.
Required probability

$= \frac{2}{12}\times\frac{2}{12}\times\frac{2}{12} + \frac{4}{12}\times\frac{4}{12}\times\frac{4}{12} + \frac{6}{12}\times\frac{6}{12}\times\frac{6}{12}$

$= \frac{8}{1728} + \frac{64}{1728} + \frac{216}{1728} = \frac{288}{1728} = \frac{1}{6}$

107. (a) Here A, B and C are independent events

$P(A) = \frac{5}{6}, P(B) = \frac{4}{5}, P(C) = \frac{3}{4}$

and $P(\overline{B}) = 1 - P(B) = 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore P(A\cap\overline{B}\cap C) = P(A).P(\overline{B}).P(C) = \frac{5}{6}\times\frac{1}{5}\times\frac{3}{4} = \frac{1}{8}$

108. (a) Vowels = 0, 0, 0 and consonant = Z L G Y
Number of words form from letters of word ZOOLOGY
$= \frac{7!}{3!}$

$\therefore$ Number of words in which consonants and vowels occur alternatively n(E)
$= Z * L * G * Y = 4! \times 3!$

$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{4!\times 3!\times 3!}{7!} = \frac{3\times 2\times 3\times 2}{7\times 6\times 5} = \frac{6}{35}$

109. (b) First 100 natural numbers

$n(s) = 100$

Number from 7 to 100 satisfy

$x^2 + x > 50$

$\therefore$ $n(E) = 100 - 6 = 94$

$$P(E) = \frac{n(E)}{n(S)} = \frac{94}{100} = \frac{47}{50}$$

110. (b) Mean of 10 natural numbers

$$\overline{x} = \frac{1+2+3+........+10}{10} = \frac{10\times11}{2\times10} = 5.5$$

Mean deviation $\frac{|1-5.5|+|2-5.5|+|3-5.5|+....+|10-5.5|}{10}$

$$= \frac{4.5+3.5+2.5+1.5+0.5+0.5+1.5+2.5+3.5+4.5}{10}$$

$$\frac{25}{10} = 2.5$$

111. (b) Given that $\sum_{i=1}^{9} x_1^2 = 855$

We know that variance $(6^2) = \dfrac{\sum_{i=1}^{9} x_1^2}{N} - (\text{Mean})^2$

$$\Rightarrow\ 6^2 = \frac{855}{9} - M^2$$

$$\Rightarrow\ 6^2 + M^2 = \frac{855}{9} = 95$$

112. (d) Given that, $\overline{x} = \dfrac{x_1 + x_2 + x_3 + ...+ x_n}{n}$

$\Rightarrow\ x_1 + x_2 + x_3 + ... + x_n = n\overline{x}$

Mean when x_n is replaced by k.

$$\text{New mean} = \frac{x_1 + x_2 + x_3........+ x_{n-1} + k}{n}$$

$$= \frac{x_1 + x_2 + x_3 + ...x_{n-1} + x_n - x_n + k}{n}$$

$$= \frac{n\overline{x} - x_n + k}{n}$$

113. (b) $\because$ Fair coin is tossed till two heads occur in succession.

$\therefore$ P(the number of tosses required in less than 6)

= P(HH) + P(THH) + P (TTHH) + P(TTTHH)

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5$$

$$= \frac{\left(\frac{1}{2}\right)^2\left[1-\left(\frac{1}{2}\right)^4\right]}{1-\frac{1}{2}} = \frac{\frac{1}{4}\left(1-\frac{1}{16}\right)}{\frac{1}{2}} = \frac{1}{2}\left(\frac{15}{16}\right) = \frac{15}{32}$$

114. (b) From the given information

A	B
White = 2	White = 3
Black = 2	Black = 2

E_1 = One white ball transferred from A to B then urn A contains White = 1, Black = 2 and urn B contains White = 4, Black = 2

E_2 = One Black ball transferred from A to B then urn A contains White = 2, Black = 1 and urn B contains White = 3, Black = 3

F = One white ball is drawn from urn B.

$\therefore$ $P(F) = P(E_1) . P(F/E_1) + P(E_2) . P(F/E_2)$

$$= \frac{2}{4}\times\frac{4}{6}+\frac{2}{4}\times\frac{3}{6} = \frac{8}{24}+\frac{6}{24} = \frac{14}{24} = \frac{7}{12}$$

115. (d)

116. (d) $n(s) = 6 \times 6 = 36$

Sum of the numbers on the faces 9 = (3, 6), (4, 5), (5, 4), (6, 3)

Sum of the numbers on the faces 10 : (4, 6), (5, 5), (6, 4)

$$P\text{ (sum 9 or 10)} = \frac{7}{36}$$

$\therefore$ P (sum is neither 9 nor 10) $= 1 - \dfrac{7}{36} = \dfrac{29}{36}$

117. (a) Here, n = 6

$$p = P(\text{Suffering from disease}) = \frac{20}{100} = \frac{1}{9}$$

$$q = P(\text{Not suffering from disease}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6)$

$$= {}^6c_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + {}^6c_5\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + {}^6c_6\left(\frac{1}{5}\right)^6$$

$$= \frac{6!}{4!2!}\times\frac{16}{5^6} + \frac{6!}{5!}\times\frac{4}{5^6} + \frac{1}{5^6}$$

$$= \frac{1}{(5)^6}[15\times16 + 6\times4 + 1]$$

$$= \frac{240+24+1}{15625} = \frac{26.5}{15625} = \frac{53}{3125}$$

118. (d) When three dice rolled and no two show same faces

$n(s) = 6 \times 5 \times 4$

$n(E)$ = n(one of the faces shown is an one) $= 3\ (1 \times 5 \times 4)$ $= 3 \times 5 \times 4$

$$P(E) = \frac{3\times5\times4}{6\times5\times4} = \frac{1}{2}$$

119. (a) Let x, y, z are represent the numbers on three dice possible outcomes such that $n < y < z$

= (1, 2, 3) (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 3, 4), (1, 3, 5) (1, 3, 6), (1, 4, 5), (1, 4, 6), (1, 5, 6), (2, 3, 4), (2, 3, 5), (2, 3, 6), (2, 4, 5), (2, 4, 6), (2, 5, 6), (3, 4, 5), (3, 4, 6), (3, 5, 6), (4, 5, 6)

$\therefore$ Total possible outcomes = 20

120. (d) Given that, mean $\overline{x} = np = 6$ ....(i)

and S.D = $\sqrt{npq} = \sqrt{2} \Rightarrow npq = 2$ ...(ii)

Divide (ii) by (i), we get

$$\frac{npq}{np} = \frac{2}{6} \Rightarrow q = \frac{1}{3}$$

We know that $p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore \quad P = \frac{2}{3}$

Put the value of p in (i), we get

$$n \times \frac{2}{3} = 6$$

$n = 9$