NDA/NA SOLVED PAPER 2023-II

MATHEMATICS

1. If $z\bar{z} = |z + \bar{z}|$, where $z = x + iy$, $i = \sqrt{-1}$ then the locus of z is a pair of :
   (a) Straight lines (b) rectangular hyperbolas
   (c) parabolas (d) circles

2. If $1! + 3! + 5! + 7! + \ldots + 199!$ is divided by 24, what is the remainder?
   (a) 3 (b) 6
   (c) 7 (d) 9

3. What is the value of $\sqrt{12+5i} + \sqrt{12-5i}$, where $i = \sqrt{-1}$?
   (a) 24 (b) 25
   (c) $5\sqrt{2}$ (d) $5(\sqrt{2}-1)$

4. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then what is value of $\det(I + AA')$, where I is the 3×3 identity matrix?
   (a) 15 (b) 6
   (c) 0 (d) –1

5. If A, B and C are square matrices of order 3 and det(BC) = 2det(A), then what is the value of $\det(2A^{-1}BC)$?
   (a) 16 (b) 8
   (c) 4 (d) 2

6. If the n^{th} term of a sequence is $\frac{2n+5}{7}$, then what is the sum of its first 140 terms?
   (a) 2840 (b) 2780
   (c) 2920 (d) 5700

7. Let A be a skew-symmetric matrix of order 3. What is the value of
   $$\det(4A^4) - \det(3A^3) + \det(2A^2) - \det(A) + \det(-I)$$
   Where I is the identity matrix of order 3?
   (a) –1 (b) 0
   (c) 1 (d) 2

8. If $A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$, then which one of the following statements is correct?
   (a) A^2 is symmetric matrix with $\det(A^2) = 0$
   (b) A^2 is symmetric matrix with $\det(A^2) \neq 0$
   (c) A^2 is skew-symmetric matrix with $\det(A^2) = 0$
   (d) A^2 is skew-symmetric matrix with $\det(A^2) \neq 0$

9. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then which of the following statements are correct?
   1. A^n will always be singular for any positive integer n.
   2. A^n will always be a diagonal matrix for any positive integer n.
   3. A^n will always be a symmetric matrix for any positive integer n.

   Select the correct answer using the code given below:
   (a) 1 and 2 only (b) 2 and 3 only
   (c) 1 and 3 only (d) 1, 2 and 3

10. If (a + b), 2b, (b + c) are in HP, then which one of the following is correct?
   (a) a, b and c are in AP
   (b) a – b, b – c and c – a are in AP
   (c) a, b and c are in GP
   (d) a – b, b – c and c – a are in GP

11. Let $t_1, t_2, t_3, \ldots$ be in GP. What is $(t_1 t_3 \ldots t_{21})^{\frac{1}{11}}$ equal to?
   (a) t_{10} (b) t_{10}^2
   (c) t_{11} (d) t_{11}^2

12. Which one of the following is a square root of $-\sqrt{-1}$?
   (a) $1 + i$ (b) $\frac{1-i}{\sqrt{2}}$
   (c) $\frac{1+i}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}i$

13. What is the maximum number of points of intersection of 10 circles?
   (a) 45 (b) 60
   (c) 90 (d) 120

14. A set S contains (2n +1) elements. There are 4096 subset of S which contain at most n elements. what is n equal to
   (a) 5 (b) 6
   (c) 7 (d) 8

15. If $\begin{vmatrix} x^2-3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, then what is the value of e?
   (a) –1 (b) 0 (c) 1 (d) 2

16. If all elements of a third order determinant are equal to 1 or –1, then the value of the determinant is:
(a) 0 only
(b) an even number but not necessarily 0
(c) an odd number
(d) 0, 1 or –1

17. If $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then what is the value of det[adj(adjA)]?
(a) 5 (b) 25
(c) 125 (d) 625

18. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then what is $23A^3 - 19A^2 - 4A$ equal to?
(a) Null matrix of order 3
(b) Identity matrix of order 3
(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
(d) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

19. The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of C^2?
(a) 3 (b) 9
(c) 81 (d) 729

20. If $A_k = \begin{bmatrix} k-1 & k \\ k-2 & k+1 \end{bmatrix}$, then what is

$\det(A_1) + \det(A_2) + \det(A_3) + \ldots + \det(A_{100})$

equal to?
(a) 100 (b) 1000
(c) 9900 (d) 10000

21. The Cartesian product A × A has 16 elements among which are (0, 2) and (1, 3). Which of the following statements is/are correct?
1. It is possible to determine set A
2. A × A contains the element (3, 2).
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

22. Let A = {1, 2, 3, …20}. Define a relation R from A to A by R = {(x, y) : 4x – 3y = 1}, where x, y ∈ A. Which of the following statements is/are correct?
1. The domain of R is {1, 4, 7, 10, 13, 16}.
2. The range of R is {1, 5, 9, 13, 17}.
3. The range of R is equal to codomain of R.
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) 1 and 2 (d) 2 and 3

23. Consider the following statements:
1. The relation f defined by
$f(x) = \begin{cases} x^3, & 0 \le x \le 2 \\ 4x, & 2 \le x \le 8 \end{cases}$ is a function
2. The relation g defined by
$g(x) = \begin{cases} x^2, & 0 \le x \le 4 \\ 3x, & 4 \le x \le 8 \end{cases}$ is a function.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

24. Consider the following statements:
1. $A = (A \cup B) \cup (A - B)$
2. $A \cup (B - A) = (A \cup B)$
3. $B = (A \cup B) - (A - B)$
Which of the statements given above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

25. A function satisfies $f(x-y) = \dfrac{f(x)}{f(y)}$, where $f(y) \neq 0$. If f(1) = 0.5, then what is f(2) + f(3) + f(4) + f(5) + f(6) equal to?
(a) $\frac{15}{32}$ (b) $\frac{17}{32}$
(c) $\frac{29}{64}$ (d) $\frac{31}{64}$

26. What is $2\cot\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ equal to?
(a) –1 (b) 1
(c) $3+\sqrt{5}$ (d) $3-\sqrt{5}$

27. If $\sec^{-1} p - \operatorname{cosec}^{-1} q = 0$, where p > 0, q > 0; then what is the value of $p^{-2} + q^{-2}$?
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{2}}$

28. What is $1 + \sin^2\left(\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right)$ equal to?
(a) $\frac{25}{17}$ (b) $\frac{8}{17}$
(c) $\frac{9}{17}$ (d) $\frac{47}{17}$

29. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta), 0 < \theta < \frac{\pi}{2}$; then what is the value of $8\sin^2\left(\theta + \frac{\pi}{4}\right)$

(a) 16 (b) 2

(c) 1 (d) $\frac{1}{2}$

30. If $\tan\alpha = \frac{1}{7}$, $\sin\beta = \frac{1}{\sqrt{10}}$; $0 < \alpha,\ \beta < \frac{\pi}{2}$ then what is the value of $\cos(\alpha + 2\beta)$?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Consider the equation $(1 - x)^4 + (5 - x)^4 = 82$.

31. What is the number of real roots of the equation?

(a) 0 (b) 2

(c) 4 (d) 8

32. What is the sum of all the roots of the equation?

(a) 24 (b) 12

(c) 10 (d) 6

DIRECTIONS : *Consider the following for the next tree (03) items that follow:*

Consider equation-I: $z^3 + 2z^2 + 2z + 1 = 0$ and equation -II: $z^{1985} + z^{100} + 1 = 0$

33. What are the roots of equation – I?

(a) $1, \omega, \omega^2$ (b) $-1, \omega, \omega^2$

(c) $1, -\omega, \omega^2$ (d) $-1, -\omega, -\omega^2$

34. Which one of the following is a root of equation-II?

(a) -1 (b) $-\omega$

(c) $-\omega^2$ (d) ω

35. What is the number of common roots of equation-I and equation-II?

(a) 0 (b) 1

(c) 2 (d) 3

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

A quadratic equation is given by

$(a+b)x^2 - (a+b+c)x + k = 0$, where *a, b, c* are real.

36. If $k = \frac{c}{2}, (c \neq 0)$, then the roots of the equation are:

(a) Real and equal

(b) Real and unequal

(c) Real iff $a > c$

(d) Complex but not real

37. If k = c, then the roots of the equation are:

(a) $\frac{a+c}{a+b}$ and $\frac{b}{a+b}$ (b) $\frac{a+c}{a+b}$ and $-\frac{b}{a+b}$

(c) 1 and $\frac{c}{a+b}$ (d) -1 and $-\frac{c}{a+b}$

DIRECTIONS : *Consider the following for the next three (03) items that follow:*

Let $(1+x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + \ldots + T_nx^n$

38. What is $T_1 + 2T_2 + 3T_3 + \ldots + nT_n$ equal to?

(a) 0 (b) 1

(c) 2^n (d) $n2^{n-1}$

39. What is $1 - T_1 + 2T_2 - 3T_3 + \ldots + (-1)^n nT_n$ equal to?

(a) 0 (b) -2^{n-1}

(c) $n2^{n-1}$ (d) 1

40. What is $T_1 + T_2 + T_3 + \ldots + T_n$ equal to ?

(a) 2^n (b) $2^n - 1$

(c) 2^{n-1} (d) $2^n + 1$

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $f(x) = x^2 - 1$ and $gof(x) = x - \sqrt{x} + 1$

41. Which one of the following is a possible expression for g(x)?

(a) $\sqrt{x+1} - \sqrt[4]{x+1}$ (b) $\sqrt{x+1} - \sqrt[4]{x+1} + 1$

(c) $\sqrt{x+1} + \sqrt[4]{x+1}$ (d) $x + 1 - \sqrt{x+1} + 1$

42. What is g(15) equal to?

(a) 1 (b) 2

(c) 3 (d) 4

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let a function f be defined on $\mathbb{R} - \{0\}$ and

$2f(x) + f\left(\frac{1}{x}\right) = x + 3.$

43. What is $f(0.5)$ equal to?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 1 (d) 2

44. If f is differentiable, then what is '(0.5) equal to?

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$

(c) 2 (d) 4

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

A function is defined by

$$f(x)=\begin{vmatrix} x+1 & 2 & 3 \\ 2 & x+4 & 6 \\ 3 & 6 & x+9 \end{vmatrix}$$

45. The function is decreasing on:

(a) $\left[-\frac{28}{3}, 0\right]$ (b) $\left[0, \frac{28}{3}\right]$

(c) $\left[0, \frac{50}{3}\right]$ (d) $\left[0, \frac{56}{3}\right]$

46. The function attains local minimum value at:

(a) $x=-\frac{28}{3}$ (b) $x=-1$

(c) $x=0$ (d) $x=\frac{28}{3}$

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Given that $4x^2+y^2=9$

47. What is the maximum value of y?

(a) $\frac{3}{2}$ (b) 3

(c) 4 (d) 6

48. What is the maximum value of xy?

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$

(c) $\frac{4}{9}$ (d) $\frac{2}{3}$

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

A function is defined by $f(x)=\pi+\sin^2 x$

49. What is the range of the function?

(a) [0, 1] (b) $[\pi, \pi+1]$

(c) $[\pi-1, \pi+1]$ (d) $[\pi-1, \pi-1]$

50. What is the period of the function?

(a) 2π

(b) π

(c) $\frac{\pi}{2}$

(d) The function is non-periodic

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

A parabola pases through (1, 2) and satisfies the differential equation $\frac{dy}{dx}=\frac{2y}{x}, x>0, y>0.$

51. What is the directrix of the parabola?

(a) $y=-\frac{1}{8}$ (b) $y=\frac{1}{8}$

(c) $x=-\frac{1}{8}$ (d) $x=\frac{1}{8}$

52. What is the length of latus rectum of the parabola?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $f(x)$ $\dfrac{a^{x-1} \quad b^{x-1}}{}$ and $g(x)=x-1.$

53. What is $\lim\limits_{x\to 1}\frac{f(x)-1}{g(x)}$ equal to?

(a) $\frac{\ln(ab)}{4}$ (b) $\frac{\ln(ab)}{2}$

(c) ln (ab) (d) 2 ln (ab)

54. What is $\lim\limits_{x\to 1} f(x)^{\frac{1}{g(x)}}$ equal to?

(a) $\sqrt{ab}$ (b) ab

(c) $2ab$ (d) $\frac{\sqrt{ab}}{2}$.

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $f(x)=\sqrt{2-x}+\sqrt{2+x}$.

55. What is the domain of the function?

(a) $(-2, 2)$ (b) $[-2, 2]$

(c) $R-(-2, 2)$ (d) $R-[-2, 2]$

56. What is the greatest value of the function?

(a) $\sqrt{3}$ (b) $\sqrt{6}$

(c) $\sqrt{8}$ (d) 4

DIRECTIONS : *Consider the following for the next two (02) items that follows:*

Let $f(x)=|x|$ and $g(x)=[x]-1,$ where [.] is the greatest integer function.

Let $h(x)=\frac{f(g(x))}{g(f(x))}$.

57. What is $\lim\limits_{x\to 0+} h(x)$ equal to?

(a) -2 (b) -1

(c) 0 (d) 1

58. What is $\lim\limits_{x\to 0-} h(x)$ equal to?

(a) -2 (b) -1

(c) 0 (d) 2

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $f(x)=\begin{cases}\dfrac{x-3}{|x-3|}+a; & x<3\\ a-b; & x=3 \text{ and}\\ \dfrac{x-3}{|x-3|}+b; & x>3\end{cases}$

$f(x)$ be continuous at $x = 3$.

59. What is the value of a ?
    (a) -1 (b) 1
    (c) 2 (d) 3
60. What is the value of b?
    (a) -1 (b) 1
    (c) 2 (d) 3

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $I=\int_{-2\pi}^{2\pi}\frac{\sin^4 x+\cos^4 x}{1+3^x}dx$

61. What is $\int_0^{\pi}(\sin^4 x+\cos^4 x)dx$ equal to?
    (a) $\frac{3\pi}{8}$ (b) $\frac{3\pi}{4}$
    (c) $\frac{3\pi}{2}$ (d) 3π
62. What is I equal to?
    (a) 0 (b) $\frac{3\pi}{4}$
    (c) $\frac{3\pi}{2}$ (d) 3π

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $f(x)=\begin{cases}ax(x+1)+b, & x<1\\ x-1 & 1\le x\le 2\end{cases}$

63. If the function $f(x)$ is differentiable at $x = 1$, then what is the value of $(a + b)$?
    (a) $-\frac{1}{3}$ (b) -1
    (c) 0 (d) 1
64. What is $\lim_{x\to 0} f(x)$ equal to?
    (a) $-\frac{1}{3}$ (b) $-\frac{2}{3}$
    (c) 0 (d) 1
65. If $f(x)=|\ln|x||$ where $0 < x < 1$, then what is $f'(0.5)$ equal to?
    (a) -2 (b) -1
    (c) 0 (d) 2
66. If $f'(x)=\cos(\ln x)$ and $y \quad f\left(\frac{2 \quad 3}{\quad}\right)$, then what is $\frac{dy}{dx}$ equal to?
    (a) $\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$ (b) $-\frac{3}{x^2}\sin\left(\ln\left(\frac{2x-3}{x}\right)\right)$
    (c) $\frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$ (d) $-\frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$
67. What is $\int_0^{8\pi}|\sin x|\,dx$ equal to?
    (a) 2 (b) 4
    (c) 8 (d) 16
68. What is the area between the curve $f(x)=x|x|$ and x-axis for $x\in[-1, 1]$?
    (a) $\frac{2}{3}$ (b) $\frac{1}{2}$
    (c) $\frac{1}{4}$ (d) $\frac{1}{3}$
69. What are the order and the degree respectively of the differential equation $x^2\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{dy}{dx}\right)^4+\sin x=0$?
    (a) 3, 4 (b) 1, 4
    (c) 2, 2 (d) 3, 2
70. What is the differential equation of all parabolas of the type $y^2=4a(x-b)$?
    (a) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ (b) $\frac{d^2y}{dx^2}+x^2\left(\frac{dy}{dx}\right)^2=0$
    (c) $y^2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ (d) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

DIRECTIONS : *Consider the following for the next two (02) items that follow:*

Let $a_1, a_2, a_3 \ldots$ be in AP such that

$a_1+a_5+a_{10}+a_{15}+a_{20}+a_{25}+a_{30}+a_{34}=300.$

71. What is
    $a_1+a_5-a_{10}-a_{15}-a_{20}-a_{25}+a_{30}+a_{34}$
    equal to?
    (a) 0 (b) 25
    (c) 125 (d) 250
72. What is $\sum_{n=1}^{34} a_n$ equal to?
    (a) 900 (b) 1025
    (c) 1200 (d) 1275

DIRECTIONS : *Consider the following for the next two(02) items that follow:*

Let $p = \cos\left(\frac{\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right)$ and $q = \cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)$.

73. What is the value of $p + q$?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) 0 (d) $\frac{1}{2}$

74. What is the value of pq?

(a) $-\frac{1}{16}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

DIRECTIONS: *Consider the following for the next two (02) items that follow:*

Let $p = \frac{1}{3} - \frac{\tan 3x}{\tan x}$ and $q = 1 - 3\tan^2 x,\ 0 < x < \pi,\ x \neq \frac{\pi}{2}$.

75. What is pq equal to

(a) 1 (b) 2

(c) $\frac{8}{3}$ (d) $-\frac{8}{3}$

76. For how many values of x does $\frac{1}{p}$ become zero?

(a) No value (b) Only one value

(c) Only two values (d) Only three values

DIRECTIONS: *Consider the following for the next two (02) items that follow:*

Let $\sin x + \sin y = \sqrt{3}(\cos y - \cos x); x + y = \frac{\pi}{2}$,

$0 < x,\ y < \frac{\pi}{2}$.

77. What is a value of $\sin 3x + \sin 3y$?

(a) -1 (b) 0

(c) 1 (d) 3

78. What is a value of $\cos^3 x + \cos^3 y$?

(a) $\frac{3\sqrt{3}}{8}$ (b) $\frac{3\sqrt{6}}{8}$

(c) $\frac{3\sqrt{6}}{4}$ (d) 1

DIRECTIONS: *Consider the following for the next two (02) items that follow:*

The angles A, B and C of a triangle ABC are in the ratio 3 : 5 : 4.

79. What is the value of $a + b + \sqrt{2}c$ equal to?

(a) 3a (b) 2b

(c) 3b (d) 2c

80. What is the ratio of $a^2 : b^2 : c^2$?

(a) $2 : 2 + \sqrt{3} : 3$ (b) $2 : 2 - \sqrt{3} : 2$

(c) $2 : 2 + \sqrt{3} : 2$ (d) $2 : 2 - \sqrt{3} : 3$

81. What is the equation of directrix of parabola $y^2 = 4bx$, where $b < 0$ and $b^2 + b - 2 = 0$?

(a) $x + 1 = 0$ (b) $x - 2 = 0$

(c) $x - 1 = 0$ (d) $x + 2 = 0$

82. The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are:

(a) lying on the same circle

(b) vertices of a square

(c) vertices of a parallelogram that is not a square

(d) collinear

83. Given that $16p^2 + 49q^2 - 4r^2 - 56pq = 0$, which one of the following is a point on a pair of straight lines $(px + qy + r)(px + qy - r) = 0$?

(a) $\left(2, \frac{7}{2}\right)$ (b) $\left(2, -\frac{7}{2}\right)$.

(c) $(4, -7)$ (d) $(4, 7)$

84. If $3x + y - 5 = 0$ is the equation of a chord of the circle $x^2 + y^2 - 25 = 0$, then what are the coordinates of the mid-point of the chord?

(a) $\left(\frac{3}{4}, \frac{1}{4}\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{3}{4}, -\frac{1}{4}\right)$ (d) $\left(\frac{3}{2}, -\frac{1}{2}\right)$

85. Consider the following in respect of the equation $\frac{x^2}{24-k} + \frac{y^2}{k-16} = 2$.

1. The equation represents an ellipse if k = 19.
2. The equation represents a hyperbola if k = 12
3. The equation represents a circle if k =20

How many of the statements given above are correct?

(a) Only one (b) Only two

(c) All three (d) None

86. Consider the following statements in respect of hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$:

1. The two foci are independent of θ.
2. The eccentricity is sec θ.
3. The distance between the two foci is 2 units.

How many of the statements given above are correct?

(a) Only one (b) Only two

(c) All three (d) None

87. Consider the following in respect of the circle $4x^2+4y^2-4ax-4ay+a^2=0$:

1. The circle touches both the axes.
2. The diameter of the circle is 2a.
3. The centre of the circle lies on the line x + y = a.

How many of the statements given above are correct

(a) Only one (b) Only two
(c) All three (d) None

88. For what value of k is the line $(k-3)x-(5-k^2)y+k^2-7k+6=0$ parallel to the line $x+y=1$?

(a) – 1, 1 (b) – 1, 2
(c) 1, – 2 (d) 2, – 2

89. The line $x+y=4$ cuts the line joining $P(-1, 1)$ and Q (5, 7) at R. What is PR : RQ equal to ?

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 3

90. What is the sum of the intercepts of the line whose perpendicular distance from origin is 4 units and the angle which the normal makes with positive DIRECTIONS of x-axis is 15°?

(a) 8 (b) $4\sqrt{6}$
(c) $8\sqrt{6}$ (d) 16

91. What is the length of projection of the vector $\hat{i}+2\hat{j}+3\hat{k}$ on the vector $2\hat{i}+3\hat{j}-2\hat{k}$?

(a) $\frac{1}{\sqrt{17}}$ (b) $\frac{2}{\sqrt{17}}$
(c) $\frac{3}{\sqrt{17}}$ (d) $\frac{2}{\sqrt{14}}$

92. If $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=144$ and $|\vec{b}|=4$, then what is the value of $|\vec{a}|$?

(a) 3 (b) 4
(c) 5 (d) 6

93. If θ is the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}\cdot\vec{b}\geq 0$, then which one of the following is correct?

(a) $0\leq\theta\leq\pi$ (b) $\frac{\pi}{2}\leq\theta\leq\pi$
(c) $0\leq\theta\leq\frac{\pi}{2}$ (d) $0<0<\frac{\pi}{2}$

94. The vectors $60\hat{i}+3\hat{j}$, $40\hat{i}-8\hat{j}$ & $\beta\hat{i}-52\hat{j}$ are collinear if:

(a) $\beta=20$ (b) $\beta=40$
(c) $\beta=-40$ (d) $\beta=26$

95. Consider the following in respect of the vectors $\vec{a}=(0,1,1)$ and $\vec{b}=(1,0,1)$:

1. The number of unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ is only one.
2. The angle between the vectors is $\frac{\pi}{3}$.

Which of the statement given above is/are correct?

(a) 1 only (b) 2 only
(c) both 1 and 2 (d) Neither 1 nor 2

96. If L is the line with DIRECTIONS ratios < 3, – 2, 6 > and passing through (1, – 1, 1), then what are the coordinates of the points on L whose distance from (1, –1, 1) is 2 units?

(a) $\left(-\frac{11}{7},\frac{13}{7},\frac{19}{7}\right)$ and $\left(\frac{1}{7},\frac{3}{7},\frac{5}{7}\right)$

(b) $\left(\frac{19}{7},-\frac{11}{7},\frac{13}{7}\right)$ and $\left(-\frac{1}{7},\frac{3}{7},-\frac{5}{7}\right)$

(c) $\left(\frac{13}{7},\frac{11}{7},\frac{19}{7}\right)$ and $\left(-\frac{1}{7},-\frac{3}{7},\frac{5}{7}\right)$

(d) $\left(\frac{13}{7},-\frac{11}{7},\frac{19}{7}\right)$ and $\left(\frac{1}{7},-\frac{3}{7},-\frac{5}{7}\right)$

97. Which one of the planes is parallel to the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$?

(a) $2x+2y+z-1=0$ (b) $2x-y-2z+5=0$
(c) $2x+2y-2z+1=0$ (d) $x-2y+z-1=0$

98. What is the angle between the lines $2x=3y=-z$ and $6x=-y=-4z$?

(a) 0° (b) 30°
(c) 60° (d) 90°

99. What is the equation of the sphere concentric with the sphere $x^2+y^2+z^2-2x-6y-8z-5=0$ and which passes through the origin?

(a) $x^2+y^2+z^2-2x-8z=0$
(b) $x^2+y^2+z^2-2x-6y=0$
(c) $x^2+y^2+z^2-6y-8z=0$
(d) $x^2+y^2+z^2-2x-6y-8z=0$

100. A point P lies on the line joining A (1, 2, 3) and B (2, 10, 1). If z-coordinate of P is 7, what is the sum of other two coordinates?

(a) – 15 (b) – 13
(c) – 11 (d) – 9

101. The sum of deviations of n numbers from 10 and 20 are p and q respectively. If $(p-q)^2=10000$, then what is the value of n?

(a) 10 (b) 20
(c) 50 (d) 100

102. If $\overline{X} = 20$ is the mean of 10 observations $x_1, x_2, \ldots, x_{10}$, then what is the value of $\sum_{i=1}^{10}\left(\frac{3x_i - 4}{5}\right)$?

(a) 0 (b) 12

(c) 112 (d) 1012

103. If the mean and the sum of squares of 10 observations are 40 and 16160 respectively, then what is the standard deviation?

(a) 16 (b) 6

(c) 5 (d) 4

104. Three dice are thrown. What is the probability of getting a sum which is a perfect square?

(a) $\frac{17}{108}$ (b) $\frac{5}{108}$

(c) $\frac{19}{108}$ (d) $\frac{23}{108}$

105. A, B. C and D are mutually exclusive and exhaustive events.
If 2P(A) = 3P(B) = 4P(C) = 5P(D), then what is 77P(A) equal to?

(a) 12 (b) 15

(c) 20 (d) 30

106. Two distinct natural numbers from 1 to 9 are picked at random. What is the probability that their product has 1 in its unit place?

(a) $\frac{1}{81}$ (b) $\frac{1}{72}$

(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

107. Two dice are thrown. What is the probability that difference of number on them is 2 or 3?

(a) $\frac{7}{36}$ (b) $\frac{7}{18}$

(c) $\frac{5}{18}$ (d) $\frac{11}{36}$

108. What is the mean of the numbers 1, 2, 3, …, 10 with frequencies ${}^9c_0, {}^9c_1, {}^9c_2, \ldots, {}^9c_9$, respectively?

(a) 1.1×2^8 (b) 1.2×7^4

(c) 5.5 (d) 0.55

109. The probability that a person recovers from a disease is 0.8. What is the probability that exactly 2 persons out of 5 will recover from the disease?

(a) 0.00512 (b) 0.02048

(c) 0.2048 (d) 0.0512

110. Suppose that there is a chance for a newly constructed building to collapse, whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the building collapses is 95% if the design is faulty, otherwise it is 45%. If it is seen that the building has collapsed, then what is the probability that it is due to faulty design?

(a) 0.10 (b) 0.19

(c) 0.45 (d) 0.95

111. If r is the coefficient of correlation between x and y, then what is the correlation coefficient between $(3x + 4)$ and $(-3y + 3)$?

(a) $-r$ (b) r

(c) $\sqrt{3}r$ (d) $-\sqrt{3}r$

112. A fair coin is tossed 6 times. What is the probability of getting a result in the 6th toss which is different from those obtained in the first five tosses?

(a) $\frac{7}{16}$ (b) $\frac{1}{16}$

(c) $\frac{1}{32}$ (d) $\frac{1}{64}$

113. If H is the Harmonic Mean of three numbers ${}^{10}C_4$, ${}^{10}C_5$, and ${}^{10}C_6$, then what is the value of $\frac{270}{H}$?

(a) 1 (b) $\frac{14}{17}$

(c) $\frac{17}{14}$ (d) $\frac{1}{31}$

114. In a class, there are n students including the students P and Q. What is the probability that P and Q sit together if seats are assigned randomly?

(a) $\frac{1}{n}$ (b) $\frac{2}{n}$

(c) $\frac{4}{n}$ (d) $\frac{1}{2n}$

115. In a Binomial distribution B(n, p), n = 6 and 9P(X = 4) = P(X = 2). What is p equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

DIRECTIONS: *Consider the following for the next five(05) items that follow:*

Three boys, P, Q, R an three girls S, T, U are to be arranged in a row for a group photograph.

116. What is the probability that all three boys sit together?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{12}$

117. What is the probability that boys and girls sit alternatively?

(a) $\frac{4}{5}$ (b) $\frac{1}{10}$

(c) $\frac{5}{6}$ (d) $\frac{1}{7}$

118. What is the probability that no two girls sit together?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{18}$ (d) $\frac{1}{5}$

119. What is the probability that P and Q take the two end positions?

(a) $\frac{1}{15}$ (b) $\frac{7}{15}$

(c) $\frac{14}{15}$ (d) $\frac{11}{45}$

120. What is the probability that Q and U sit together?

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$

(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

HINTS & SOLUTIONS

MATHEMATICS

1. (d) Since, $z = x + iy$

Now, $(x + iy)(x - iy) = |x + iy + x - iy|$

$\Rightarrow x^2 - (iy)^2 = |2x|$

$\Rightarrow x^2 + y^2 = 2x \Rightarrow x^2 - 2x + 1 + y^2 = 1$

$\Rightarrow (x-1)^2 + y^2 = 1$

So locus of z is circle

2. (c) Since, $\frac{1!+3!+5!+7!+.............+199!}{24}$

$= \frac{1+6+5\times24+7\times6\times5\times24+...........+199!}{24}$

$= \frac{1+6}{24}+\frac{24k}{24}=\frac{7}{24}+k$ (Where $k\in Z$) $= 7$

3. (c) Let $\sqrt{12+5i} = x + iy \Rightarrow 12 + 5i = (x + iy)^2$

$\Rightarrow x^2 - y^2 = 12$ .....(i)

and $2xy = 5$ ....(ii)

On solving (i) and (ii), we get

$x = \frac{5}{\sqrt{2}}, \; y = \frac{1}{\sqrt{2}}$

So, $\sqrt{12+5i} = \frac{5}{\sqrt{2}}+\frac{i}{\sqrt{2}}$

Similarly, $\sqrt{12-5i} = \frac{5}{\sqrt{2}}-\frac{i}{\sqrt{2}}$

Now, $\sqrt{12+5i}+\sqrt{12-5i} = \frac{5}{\sqrt{2}}+\frac{i}{\sqrt{2}}+\frac{5}{\sqrt{2}}-\frac{i}{\sqrt{2}}$

$= 5\sqrt{2}$

4. (a) Since, $I + AA'$

$= \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1&2&3\end{bmatrix}$

$= \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}1&2&3\\2&4&6\\3&6&9\end{bmatrix}=\begin{bmatrix}2&2&3\\2&5&6\\3&6&10\end{bmatrix}$

Now, $|I + AA'| = 2(50-36) - 2(20-18) + 3(12-15)$
$= 15$

5. (a) Given $|BC| = 2|A|$

Now, $|2A^{-1}BC| = 2^3|A^{-1}||BC|$

$= 8 \times \frac{1}{|A|} \times 2|A| = 16$

6. (c) Given, $t_n = \frac{2n+5}{7}$

Since, $t_{140} = \frac{2\times140+5}{7} = \frac{285}{7}$

$t_1 = \frac{2\times1+5}{7} = \frac{7}{7}$

Now, $S_{140} = \frac{140}{2}(t_1 + t_{140}) = 70\left(\frac{7}{7}+\frac{285}{7}\right) = 2920$

7. (a) Given A is skew symmetric matrix of order 3 (odd)

So $|A| = 0$

Now, $|4A^4| - |3A^3| + |2A^2| - |A| + |-I|$

$= 4^3|A|^4 - 3^3|A|^3 + 2^2|A|^2 - |A| + (-1)^3|I|$

$= 0 - 0 + 0 - 0 - 1 = -1$

8. (a) Since the matrix A is skew symmetric of order 3 (odd)
So, $|A| = 0$ and A^2 is symmetric matrix
Now $|A^2| = |A|^2 = 0$
Similarly, $|A^3| = 0$, $|A^4| = 0$
Hence $|4A^4| - |3A^3| + |2A^2| - |A| + (-1)^3 |I_3| = -1$

9. (b) Given $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a non-zero diagonal matrix

So A^n is also non-zero diagonal matrix for any $n \in N$ and diagonal elements of A are positive so A^n is also symmetric thus the statements 2 and 3 are correct.

10. (c) since, a + b, 2b, b + c are in H.P

$\Rightarrow \frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P

$\Rightarrow \frac{2}{a+b}, \frac{1}{b}, \frac{2}{b+c}$ are in A.P

$\Rightarrow \frac{1}{b} - \frac{2}{a+b} = \frac{2}{b+c} - \frac{1}{b} \Rightarrow \frac{1}{b} = \frac{a+2b+c}{(a+b)(b+c)}$

$\Rightarrow ab + ac + b^2 + bc = ab + 2b^2 + bc$
$\Rightarrow b^2 = ac$
$\Rightarrow$ a, b, c are in G.P.

11. (c) since, t_1, t_2, t_3 ..... in G.P
Let $t_n = ar^{n-1}$

Now, $(t_1.t_3 .....t_{21})^{\frac{1}{11}}$

$= (a.ar^2.ar^4 .....ar^{20})^{\frac{1}{11}} = (a^{11}r^{2+4+6 \,.....\, +20})^{\frac{1}{11}}$

$= (a^{11}r^{110})^{\frac{1}{11}} = ar^{10} = t_{11}$

12. (b) since, $(-\sqrt{-1})^{\frac{1}{2}} = \sqrt{-i} = x + iy$ (let)

$-i = x^2 - y^2 + 2xyi \Rightarrow 2xy = -1, x^2 - y^2 = 0$
after solving these equation, we get

$x = \frac{-1}{\sqrt{2}}, y = \frac{\pm 1}{\sqrt{2}}$

$\Rightarrow \sqrt{-i} = \pm\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) \pm \frac{1-i}{\sqrt{2}} = \frac{1-i}{\sqrt{2}}$ or $\frac{-1+i}{\sqrt{2}}$

13. (c) since, maximum number of points of intersection of 10 circle

$= {}^{10}C_2 \times 2 = \frac{10 \times 9}{2} \times 2 = 90$

14. (b) Since, n(s) = 2n + 1
Now, ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow 2({}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n) = 2^{2n+1} = 4096$

$\Rightarrow 2^{2n} = 2^{12} \Rightarrow 2n = 12 \Rightarrow n = 6$

15. (b) If we put x = 0

$\begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = 0 + 0 + 0 + 0 + e$

it is skew-symmetric of order 3 × 3
$\Rightarrow$ So, e = 0

16. (b) Since, all the elements of 3 × 3 matrix is –1 or 1 then determinent of the matrix is always an even number.

17. (d) Since, $|A| = 1(6 - 1) = 5$
Now, $|adj\ (adj\ A)| = |A|^{(n-1)^2} = 5^{(3-1)^2} = 5^4 = 625$

18. (a) Since, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow A^n = I$ (identity matrix)
Now, $23A^3 - 19A^2 - 4A$
$23I - 19I - 4I = O$

19. (c) Given $|A| = 3$, ($\therefore$ where $A_{3\times3}$)
Since, $C = (adj(A))'$
Now, $|C| = |adj\ A| = |A|^{n-1} = 3^2 = 9$
So, $|C^2| = 9^2 = 81$

20. (d) Since, $A_K = \begin{bmatrix} K-1 & K \\ K-2 & K+1 \end{bmatrix}$

$\Rightarrow |A_K| = (K - 1)(K + 1) - K(K - 2)$
$= K^2 - 1 - K^2 + 2K = 2K - 1$
Now, $|A_1| + |A_2| + |A_3| + ... + |A_{100}|$
$= (2 \times 1 - 1) + (2 \times 2 - 1) + (2 \times 3 - 1) + ...$
$+ (2 \times 100 - 1)$

$= 1 + 3 + 5 + ... + 199$

$= \frac{100}{2}(1 + 199) = 100 \times 100 = 10000$

21. (c) Since, n(A × A) = 16
and A × A contain (0, 2) and (1, 3)
So A = {0, 1, 2, 3}
Thus statements 1 and 2 are correct.

22. (b) Since, R = {(x, y) : 4x – 3y = 1}
Now, 4x – 3y = 1

$\Rightarrow x = \frac{1+3y}{4}$

So, R = {(1, 1), (4, 5), (7, 9), (10, 13) (13, 17)}
$\because$ Domain of R = {1, 4, 7, 10, 13}
Range of R = {1, 5, 9, 13, 17}

23. (a)

25. (d) Since, $f(2 - 1) = f(1) = \frac{f(2)}{f(1)}$

$\Rightarrow f(2) = (f(1))^2 = \frac{5}{10} \times \frac{5}{10} = \frac{1}{4}$

$f(3 - 2) = f(1) = \frac{f(3)}{f(2)} \Rightarrow \frac{1}{2} = \frac{f(3)}{\frac{1}{4}} \Rightarrow f(3) = \frac{1}{8}$

Similarly we get $f(4) = \frac{1}{16}$, $f(5) = \frac{1}{32}$, $f(6) = \frac{1}{64}$

Now, $f(2) + f(3) + f(4) + f(5) + f(6)$
$= \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = \frac{31}{64}$

26. (c) Let, $\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right) = \theta \Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3}$

Now, $2\cot\theta = 2\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$

$$2\sqrt{\frac{1+\frac{\sqrt{5}}{3}}{1-\frac{\sqrt{5}}{3}}} = 2\sqrt{\frac{3+\sqrt{5}}{3-\sqrt{5}}} = 3+\sqrt{5}$$

27. (a) Given, $\sec^{-1} p - \text{cosec}^{-1} q = 0$

$\Rightarrow \cos^{-1}\left(\frac{1}{p}\right) = \sin^{-1}\left(\frac{1}{q}\right)$

$\Rightarrow \left(\frac{1}{p}\right)^2 + \left(\frac{1}{q}\right)^2 = 1$

$\Rightarrow p^{-2} + q^{-2} = 1$

28. (a) Given, $1 + \sin^2\left(\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right)$

$= 1+1-\left(\cos\left(\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right)\right)^2$

$= 2-\left(\frac{3}{\sqrt{17}}\right)^2 = 2-\frac{9}{17} = \frac{25}{17}$

29. (c) Given, $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$

Let $\tan(\pi \cos\theta) = \cot(\pi \sin\theta) = k$

$\Rightarrow \pi \cos\theta = \tan^{-1} k,\ \pi \sin\theta = \cot^{-1} k$

Since, $\pi \cos\theta + \pi \sin\theta = \frac{\pi}{2}$

$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

$\Rightarrow 8\sin^2\left(\theta + \frac{\pi}{4}\right) = 1$

30. (c) Since, $\cos(\alpha + 2\beta)$

$= \cos\alpha \cos 2\beta - \sin\alpha \sin 2\beta$

$= \cos\alpha\,(1 - 2\sin^2\beta) - \sin\alpha(2\sin\beta\cos\beta)$

$= \frac{7}{\sqrt{50}}\left(1-2\times\frac{1}{10}\right) - \left(2\times\frac{1}{\sqrt{10}}\times\frac{3}{\sqrt{10}}\right)\frac{1}{\sqrt{50}}$

$= \frac{7}{\sqrt{50}}\times\frac{8}{10} - \frac{6}{10\sqrt{50}} = \frac{50}{10\sqrt{50}} = \frac{1}{\sqrt{2}}$

31. (b) Given, $(1 - x)^4 + (5 - x)^4 = 82$

Let, $3 - x = y$

$\Rightarrow ((y - 2)^2)^2 + ((y + 2)^2)^2 = 82$

$\Rightarrow (y^2 - 4y + 4)^2 + (y^2 + 4y + 4)^2 = 82$

$\Rightarrow 2[(y^2 + 4)^2 + (4y)^2] = 82$

$\Rightarrow y^4 + 8y^2 + 16 + 16y^2 = 41$

$\Rightarrow y^4 + 24y^2 - 25 = 0$

$\Rightarrow (y^2 + 25)(y^2 - 1) = 0$

$\Rightarrow y = \pm1, \pm5i$

If $y = 1 \Rightarrow x = 2$

If $y = -1 \Rightarrow x = 4$

If $y = 5i \Rightarrow x = 3 - 5i$

If $y = -5i \Rightarrow x = 3 + 5i$

So, number of real roots = 2

32. (b) Since, sum of all roots

$= 4 + 2 + 3 - 5i + 3 + 5i = 12$

33. (b) Given the equation-I is

$z^3 + 2z^2 + 2z + 1 = 0 \Rightarrow z^3 + z^2 + z^2 + z + z + 1 = 0$

$\Rightarrow z^2(z + 1) + z(z + 1) + (z + 1) = 0$

$\Rightarrow (z + 1)(z^2 + z + 1) = 0$

$\Rightarrow z = -1, \omega, \omega^2$

34. (d) Since, the given equation-II is

$z^{1985} + z^{100} + 1 = 0$

If $z = -1$, $-1 + 1 + 1 \neq 0$

If $z = -\omega$, $\omega^{1985} + \omega^{100} + 1 \neq 0$

If $z = -\omega^2$, $(-\omega^2)^{1985} + (-\omega^2)^{100} + 1 \neq 0$

If $z = \omega$, $\omega^{1985} + \omega^{100} + 1 = 0$

So, option (d) is correct.

35. (c) Since, the common roots of equation-I and equation-II are ω, ω^2

$\Rightarrow$ Number of common roots = 2

36. (b) If $k = \frac{c}{2}, (c \neq 0)$

then, the given equation is

$(a + b)x^2 - (a + b + c)x + \frac{c}{2} = 0$

Now, $D = \{-(a + b + c)\}^2 - 4(a + b).\ \frac{c}{2}$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 2ac - 2bc$

$(a + b)^2 + c^2 > 0$ (since, $c \neq 0$)

So, roots are real and unequal.

37. (c)

38. (d) Given, $(1 + x)^n = 1 + T_1 x + T_2x^2 + ... + T_nx^n$

differentiating both side with respect to x, we get

$\Rightarrow n(1 + x)^{n-1} = T_1 + 2T_2x^1 + 2T_3x^2 + ... + nT_nx^{n-1}$ ....(i)

$\Rightarrow n(1 + 1)^{n-1} = T_1 + 2T_2 + 3T_3 + ... + nT_n$ {Putting x = 1}

$\Rightarrow T_1 + 2T_2 + 3T_3 + ... + nT_n = n2^{n-1}$

39. (d) Putting, x = –1 in equation (i) in privious problem, we get

$n(1 - 1)^{n-1} = T_1 - 2T + 3T_3 - 4T_4 + ... (-1)^{n-1} T_n$

$\Rightarrow 1 - T_1 + 2T_2 - 3T_3 + 4T_4 + ... + (-1)^nT_n = 0 + 1 = 1$

40. (b) Putting x = 1 in given statement $(1 + 1)^n = 1 + T_1 + T_2 + T_3 + ... + T_n$

$\Rightarrow T_1 + T_2 + T_3 + ... + T_n = 2^n - 1$

41. (b) Given, $f(x) = x^2 - 1$, $g \circ f(x) = x - \sqrt{x} + 1$

$\Rightarrow g\{f(x)\} = x - \sqrt{x} + 1$

$\Rightarrow g(x^2 - 1) = x - \sqrt{x} + 1$

Let, $x^2 - 1 = t \Rightarrow x^2 = 1 + t \Rightarrow x = (1 + t)^{1/2}$

$\Rightarrow g(t) = (1 + t)^{1/2} - (1 + t)^{1/4} + 1$

$\Rightarrow g(x) = \sqrt{1+x} - \sqrt[4]{1+x} + 1$

42. (c) Since, $g(x) = \sqrt{1+x} - \sqrt[4]{1+x} + 1$

$\Rightarrow g(15) = \sqrt{16} - \sqrt[4]{16} + 1 = 4 - 2 + 1 = 3$

43. (b)

44. (c) Since, $2f(x) + f\left(\frac{1}{x}\right) = x + 3$

$\Rightarrow 2f'(x) + f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = 1$

Put $x = 2, \frac{1}{2}$

$\Rightarrow 2f'(2) - \frac{1}{4}f'\left(\frac{1}{2}\right) = 1$ ...(i)

and, $2f'\left(\frac{1}{2}\right) - 4f'(2) = 1$ ...(ii)

after solving (i) & (ii), we get

$f'\left(\frac{1}{2}\right) = 2, f'(2) = \frac{3}{4}$

So, $f'(0.5) = 2$

45. (a)

46. (c) For local minima

$f'(x) = 0 \Rightarrow 3x^2 + 28x = 0 \Rightarrow x = \frac{-28}{3}, 0$

Now, $f''(x) = 6x + 28$

Now, $f''(0) = 28 > 0$ & $f''\left(\frac{-28}{3}\right) > 0$

So, f(x) has local minimum value at x = 0

47. (b) Since, $4x^2 + y^2 = 9$

$\Rightarrow y = \pm\sqrt{9-4x^2}$

So, max value of y = 3

48. (a) Since, $y = \pm\sqrt{9-4x^2}$

let $f(x) = xy = x\sqrt{9-4x^2}$

So, maximum value of f(x) is $\frac{9}{4}$

49. (b) Given a function $f(x) = \pi + \sin^2 x$

Since, $0 \le \sin^2 x \le I \Rightarrow \pi \le \pi + 1 \sin^2 x \le 1 + \pi$

So Range = $[\pi, \pi + 1]$

50. (b) Since, period of $\sin^2 x$ is π

$\Rightarrow$ period of $f(x) = \pi + \sin^2 x$ is π

(Sol). 51-52:

Since, $\frac{dy}{dx} = \frac{2y}{x}$

$\int \frac{dy}{y} = \int \frac{2dx}{x} \Rightarrow \ln y = 2\ln x + \ln c \Rightarrow y = x^2 c$

it satisfy (1,2)

$\Rightarrow 2 = 1\ C \Rightarrow C = 2 \Rightarrow y = 2x^2$

51. (a) Since, $y = 2x^2 \Rightarrow x^2 = \frac{1}{2}\ y \Rightarrow x^2 = 4.\frac{1}{8}\ y$

So, directrix of Parabola is $y = -\frac{1}{8}$

52. (b) Since, length of lotus Rectum of Parabola = $4 \times \frac{1}{8} = \frac{1}{2}$

53. (b) Since, $\lim_{x\to 1} \frac{f(x)-1}{g(x)}$

$= \lim_{x\to 1} \frac{\frac{a^{x-1}+b^{x-1}}{2} - 1}{x-1} = \lim_{x\to 1} \frac{a^{x-1}+b^{x-1}-2}{2(x-1)}$

$= \lim_{x\to 1}\left(\frac{a^{x-1}-1}{2(x-1)} + \frac{b^{x-1}-1}{2(x-1)}\right) = \frac{1}{2}\ln a + \frac{1}{2}\ln(b)$

$= \frac{1}{2}\ln(ab)$

54. (a) Since, $\lim_{x\to 1} f(x)^{\frac{1}{g(x)}}$

$= \lim_{x\to 1}\left(\frac{a^{x-1}+b^{x-1}}{2}\right)^{\frac{1}{x-1}}$ (it is form 1^∞)

$= e^{\lim_{x\to 1}\left(\frac{a^{x-1}+b^{x-1}}{2} - 1\right)\frac{1}{x-1}} = e^{\lim_{x\to 1}\left(\frac{a^{x-1}-1}{2(x-1)} + \frac{b^{x-1}-1}{2(x-1)}\right)}$

$= e^{\left(\frac{1}{2}\ln a + \frac{1}{2}\ln b\right)} = e^{\frac{1}{2}\ln(ab)} = \sqrt{ab}$

55. (b) Given, $f(x) = \sqrt{2-x} + \sqrt{2+x}$

for Domain,

$2 - x \ge 0$ and $2 - x \ge 0$

$\Rightarrow 2 \ge x$ and $x \ge -2 \Rightarrow -2 \le x \le 2$

$\Rightarrow x \in [-2, 2]$

56. (c) If we put x = 0

then $f(x) = 2\sqrt{2} = \sqrt{8}$ is greatest value of f (x)

(57-58) Since, $h(x) = \frac{f(g(x))}{g(f(x))}$

$= \frac{f([x]-1)}{g(|x|)} = \frac{|[x]-1|}{[|x|]-1}$

57. (b) Since, $= \lim_{x\to 0^+} \frac{|[x]-1|}{[x]-1} = \frac{|-1|}{-1} = -1$

58. (a) Since, $= \lim_{x\to 0-} h(x)$

$= \lim_{x\to 0^-} \frac{|[x]-1|}{[x]-1} = \lim_{x\to 0^-} = \frac{|-1-1|}{-1} = -2$

(Sol. 59-60): Since, the function

$$f(x) = \begin{cases} -1+a & ; \ x < 3 \\ a-b & ; \ x = 3 \\ 1+b & ; \ x > 3 \end{cases}$$

Since, f(x) is continuous at x = 3

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$-1 + a = a - b = 1 + b$

59. (d) $-1 + a = 1 + 1 \Rightarrow a = 3$

60. (b) $\Rightarrow -1 + a = a - b \Rightarrow b = 1$

61. (b) Since, $\int_0^{\pi}\left(\sin^4 x + \cos^4 x\right)dx$

$$= \int_0^{\pi}\left(\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x\cos^2 x\right)dx$$

$$= \int_0^{\pi}\left(1 - \frac{\sin^2 2x}{2}\right)dx = \int_0^{\pi}\left(1 - \frac{1-\cos 4x}{4}\right)dx$$

$$= \frac{1}{4}\int_0^{\pi}(3+\cos 4x)\,dx = \frac{1}{4}\left(3x + \frac{\sin 4x}{4}\right)_0^{\pi} = \frac{3\pi}{4}$$

62. (c) Since, $I = \int_{-2\pi}^{2\pi}\frac{\sin^4 x + \cos^4 x}{1+3^x}dx$

$$I = \int_{-2\pi}^{2\pi}\frac{3^x\left(\sin^4 x + \cos^4 x\right)}{1+3^x}dx$$

Since, $2I = \int_{-2\pi}^{2\pi}\left(\sin^4 x + \cos^4 x\right)dx$

$$\Rightarrow 2I = \frac{1}{4}\left(3x + \frac{\sin 4x}{4}\right)_{-2\pi}^{2\pi}$$

$$\Rightarrow 2I = \frac{1}{4}\left(3x(4\pi)\right) \Rightarrow I = \frac{3\pi}{2}$$

63. (a) Since,

$$f(x) = \begin{cases} ax(x+1)+b; & x<1 \\ x-1 \quad ; & 1 \le x \le 2 \end{cases}$$

Since, f(x) is differential at x = 1

$$f'(x) = \begin{cases} a(2x+1); & x<1 \\ 1 \quad ; & 1 \le x \le 2 \end{cases}$$

$\Rightarrow a(3) = 1 \Rightarrow a = \frac{1}{3}$

since f(x) is continuous at x = 1

$\Rightarrow 2a + b = 0 \Rightarrow b = \frac{-2}{3}$

so, $a + b = \frac{-1}{x}$

64. (b) Since, $\lim_{x\to 0} f(x)$

$$= \lim_{x\to 0} ax(x+1) + b = b = \frac{-2}{3}$$

65. (a) $f(x) = \left|\ln|x|\right|$; $0 < x < 1 \Rightarrow f(x) = -\ln x$

Now, $f'(x) = \frac{-1}{x} \Rightarrow f'(0.5) = -2$

66. (c) Since, $y = f\left(\frac{2x-3}{x}\right)$

$$\Rightarrow \frac{dy}{dx} = f'\left(\frac{2x-3}{x}\right).\frac{d}{dx}\left(\frac{2x-3}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = f'\left(\frac{2x-3}{x}\right).\left(\frac{3}{x^2}\right) \quad \text{(i)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$$

67. (d) Since, $|\sin x|$ is periodic function with period π

So, $\int_0^{8\pi}|\sin x|\,dx = 8\int_0^{\pi}|\sin x|\,dx$

$$= 8\int_0^{\pi}|\sin x|\,dx = 0$$

$$= 8(-\cos x)_0^{\pi} = 8(+1+1) = 16$$

68. (d)

69. (d) Order = 3
degree = 2

70. (d) Since, $y^2 = 4ax - 4ab$

$$\Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

71. (a) Since,

$a_1 + a_5 - a_{10} - a_{15} - a_{20} - a_{25} + a_{30} + a_{34}$
$= a_1 + a_1 + 4d - a_1 - 9d - a_1 - 14d - a_1 - 19d$
$- a_1 - 24d + a_1 + 29d + a_1 + 33d$
$= 0$

72. (d) Given,

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{25} + a_{30} + a_{34} = 300$
$= a_1 + a_1 + 4d + a_1 + 9d + a_1 + 14d + a_1 + 19d$
$+ a_1 + 24d + a_1 + 29d + a_1 + 33d = 300$
$= 8a_1 + 132d = 300$

$$=(2a_1 + 33d) = \frac{300}{4} \quad \text{(i)}$$

Since, $\sum_{n=1}^{34} a_n = a_1 + a_2 + \ldots\ldots\ldots + a_{34}$

$$= \frac{34}{2}(2a_1 + 33d)$$

$$= 17 \times \frac{300}{} = 1275$$

73. (c) Since, p + q

$$= \cos\left(\frac{\pi}{5}\right).\cos\left(\frac{2\pi}{5}\right) + \cos\left(\frac{4\pi}{5}\right).\cos\left(\frac{8\pi}{5}\right)$$

$$= \cos\left(\frac{\pi}{5}\right).\cos\left(\frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right).\cos\left(\frac{2\pi}{5}\right) = 0$$

74. (a) Since, pq

$$= -\cos^2\left(\frac{\pi}{5}\right).\cos^2\left(\frac{2\pi}{5}\right)$$

$$= -\left(\frac{\sqrt{5}+1}{4}\right)^2.\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= -\frac{(5-1)^2}{16\times 16} = -\frac{1}{16}$$

75. (d) Since, $P = \frac{1}{3} - \frac{\tan 3x}{\tan x}$

$= \frac{1}{3} - \frac{3\tan x - \tan^3 x}{(1 - 3\tan^2 x)\tan x}$

$= \frac{1 - 3\tan^2 x - 9 + 3\tan^2 x}{3(1 - 3\tan^2 x)} = \frac{-8}{3(1 - 3\tan^2 x)}$

$\Rightarrow pq = \frac{-8(1 - 3\tan^2 x)}{3(1 - 3\tan^2 x)} = \frac{-8}{3}$

76. (c) Since, $\frac{1}{P} = 0$

$\Rightarrow \frac{3(1 - 3\tan^2 x)}{-8} = 0 \Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$

So, there are two values of x.

Sol. (77-78):

77. (b) Since, sin3x + sin3y

$= \sin\left(3.\frac{5\pi}{12}\right) + \sin\left(3.\frac{\pi}{12}\right)$

$= -\sin\left(\pi/4\right) + \sin\left(\pi/4\right) = 0$

78. (b) Since,

$\cos^3\left(\frac{5\pi}{12}\right) + \cos^3\left(\frac{\pi}{12}\right)$

$= \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)^3 + \left(\frac{\sqrt{3} + 1}{2\sqrt{2}}\right)^3 = \frac{3\sqrt{6}}{8}$

Sol. (79-80): Since, $3x + 5x + 4x = \pi \Rightarrow 12x = \pi \Rightarrow x = \frac{\pi}{12}$

So, $A = \frac{\pi}{4}, B = \frac{5\pi}{12}, C = \frac{\pi}{3}$

Let $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = K$

$\Rightarrow a = \frac{\sin A}{K} = \frac{1}{\sqrt{2}K}$

$\Rightarrow b = \frac{\sin B}{K} = \frac{\sqrt{3} + 1}{2\sqrt{2}K}$

$\Rightarrow c = \frac{\sin C}{K} = \frac{\sqrt{3}}{2K}$

79. (c) Since, $a + b + \sqrt{2}\,c$

$= \frac{1}{\sqrt{2}K} + \frac{\sqrt{3} + 1}{2\sqrt{2}K} + \frac{\sqrt{3}}{\sqrt{2}K}$

$= \frac{2 + \sqrt{3} + 1 + 2\sqrt{3}}{2\sqrt{2}K} = \frac{3 + 3\sqrt{3}}{2\sqrt{2}K} = 3b$

80. (a) Since, $a^2 : b^2 : c^2$

$\Rightarrow \left(\frac{1}{\sqrt{2}K}\right)^2 : \left(\frac{\sqrt{3} + 1}{2\sqrt{2}K}\right)^2 : \left(\frac{\sqrt{3}}{\sqrt{2}K}\right)^2$

$\Rightarrow 2 : 2 + \sqrt{3} : 3$

81. (b) Given equation of Parabola $y^2 = 4bx$

Since, $b^2 + b - 2 = 0 \Rightarrow (b + 2)(b - 1) = 0$

$\Rightarrow b = -2, 1$

So, equation of directrix is $x = -b$

$\Rightarrow x = -(-2) \Rightarrow x = +2$ (Since, $b < 0$)

$\Rightarrow x - 2 = 0$

82. (d) Let A (–a, –b), B (0,0), C(a,b), D(a^2,ab)

Since, Slope of AB = $\frac{b}{a}$

Slope of BC = $\frac{b}{a}$

Slope of CD = $\frac{b}{a}$

Slope of DA = $\frac{b}{a}$

So, given points are collinear.

83. (b) Given $16P^2 + 49q^2 - 4r^2 - 56Pq = 0$

$\Rightarrow (4P - 7q)^2 = (2r)^2 \Rightarrow 4P - 7q = \pm 2r$

$\Rightarrow (4P - 7q + 2r)(4P - 7q - 2r) = 0$

$\Rightarrow (2P - \frac{7}{2}q + r)(2P - \frac{7}{2}q - r) = 0$

So $(x, y) = (2, \frac{-7}{2})$

84. (b)

85. (c) Given the equation

$\frac{x^2}{24 - k} + \frac{y^2}{k - 16} = 2$

If k = 19 ⇒ Given equation is ellipse.

If k = 12 ⇒ Given equation is hyperbola

If k = 20 ⇒ Given equation is circle.

86. (c) Given the equation

$\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$

Since $C^2 = \cos^2\theta + \sin^2\theta = 1$

So focus = $(0 \pm 1, 0) = (\pm 1, 0)$

$e = \frac{c}{a} = \frac{1}{\cos\theta} = \sec\theta$

Distance between the two foci = 1 + 1 = 2

87. (b) Given, $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$

$\Rightarrow \left(x - \frac{a}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2$

So, this circle touches both the axis,

Diameter of circle = $\frac{a}{2} + \frac{a}{2} = a$

and the centre lies on the line x + y = a

88. (b) Since, for parallel lines

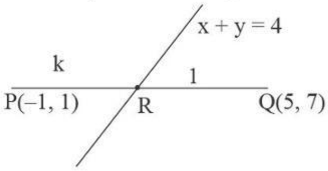
$$\frac{k-3}{1} = \frac{-(5-k^2)}{1}$$

$\Rightarrow k^2 - 5 = k - 3 \Rightarrow k^2 - k - 2 = 0$

$\Rightarrow (k-2)(k+1) = 0 \Rightarrow k = -1, 2$

89. (b) Let the ratio is k : 1

So, $R = \left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$



Now,

$$\frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4$$

$\Rightarrow 12k = 4k + 4 \Rightarrow k = \frac{1}{2}$

So, PR : RQ = 1 : 2

90. (c) Since, equation of normal is

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow x\cos 15° + y\sin 15° = 4$ ...(i)

If $y = 0 \Rightarrow x = \frac{4}{\cos 15°} = a$ = intercept on X–axis

If $x = 0 \Rightarrow y = \frac{4}{\sin 15°} = b$ = intercept on Y-axis

Now, $a + b = \frac{4}{\cos 15°} + \frac{4}{\sin 15°} = 8\sqrt{6}$

91. (b) Length of projection of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$ on the vector

$2\hat{i} + 3\hat{j} - 2\hat{k}$ is

$$\frac{(\hat{i} + 2\hat{j} + 3\hat{k})\cdot(2\hat{i} + 3\hat{j} - 2\hat{k})}{|2\hat{i} + 3\hat{j} - 2\hat{k}|} = \frac{2+6-6}{\sqrt{4+9+4}} = \frac{2}{\sqrt{17}}$$

92. (a) Given, $(\vec{a} \times \vec{b})^2 + (\vec{a}\,.\,\vec{b})^2 = 144$

$\Rightarrow |\vec{a}|^2 |\vec{b}|^2 \to \sin^2\theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta = 144$

$\Rightarrow |\vec{a}|^2 |\vec{b}|^2 = 144$

$|\vec{a}|^2 \times 16 = 144 \Rightarrow |\vec{a}| = 3$

93. (b) Since $\vec{a}\cdot\vec{b} \ge 0 \Rightarrow |\vec{a}||\vec{b}|\cos\theta \ge 0 \Rightarrow \vec{0} \le \theta \le \frac{\pi}{2}$

94. (c) For collinear of given vectors

$$\frac{-8-3}{40-60} = \frac{-52+8}{\beta - 40}$$

$$\Rightarrow \frac{-11}{-20} = \frac{-44}{\beta - 40}$$

$\Rightarrow \beta - 40 = -80$

$\Rightarrow \beta = -40$

95. (b) Number of units vectors perpendiculars to $\vec{a}$ and $\vec{b}$ is two ie., $\frac{\pm\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ and angle between $\vec{a}$ and $\vec{b}$ is

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|}$$

$$\Rightarrow \cos\theta = \frac{(0,1,1)\cdot(1,0,1)}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

96. (d) Since equation of line is

$$\frac{x-1}{3} = \frac{y+1}{-2} = \frac{z-1}{6} = \lambda \text{ (let)}$$

$\Rightarrow (x, y, z) = (3\lambda + 1, -2\lambda - 1, 6\lambda + 1)$

Now,

$$\sqrt{(3\lambda + 1 - 1)^2 + (-2\lambda - 1 + 1)^2 + (6\lambda + 1 - 1)^2} = 2$$

$$= \sqrt{9\lambda^2 + 4\lambda^2 + 36\lambda^2} = 2$$

$$\Rightarrow \sqrt{49\lambda^2} = 2 \Rightarrow \pm 7\lambda = 2 \Rightarrow \lambda = \pm\frac{2}{7}$$

So, $(x, y, z) = \left(\frac{13}{7}, \frac{11}{7}, \frac{19}{7}\right)$ or $\left(\frac{1}{7}, \frac{-3}{7}, \frac{-5}{7}\right)$

97. (d) If a line parallel to a plane

then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

so option (a), $3.2 + 2.4 + 1.5 \ne 0$

(b) $2.3 - 1.4 - 2.5 \ne 0$

(c) $2.3 + 2.4 - 2.5 \ne 0$

(d) $1.3 - 2.4 + 1.5 = 0$

98. (d) Given the lines

$2x = 3y = -z$ and $6x = -y = -4z$

$\Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{2}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{2}{-3}$

So, angle between lines is given by.

$\cos\theta = |3.2 + 2(-12) + (-6).(-3)|$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} = 90°$

99. (d) Since, equation of sphere concentric with the sphere

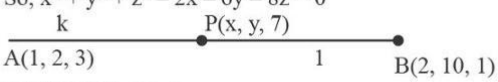
$x^2 + y^2 + z^2 - 2x - 6y - 8z - 5 = 0$ is

$x^2 + y^2 + z^2 - 2x - 6y - 8z + k = 0$

Since, (0, 0, 0) satisfy $\Rightarrow k = 0$

So, $x^2 + y^2 + z^2 - 2x - 6y - 8z = 0$

100.(a)



Let AP : BP = k : 1

$7 = \frac{k+3}{k+1} \Rightarrow k = \frac{-2}{3}$

So ratio is 2 : 3 externally.

Now, $x = \frac{2\times 2 - 3\times 1}{2-3} = -1$

$y = \frac{2\times 10 - 3\times 2}{2-3} = -14$

$\Rightarrow x + y = -15$

101.(a) Since, $\sum(x_i - 10) = P$

and $\sum(x_i - 20) = q$

$\Rightarrow \sum x_i - 10n = P$

$\sum x_i - 20n = q$

Now, $(-10n + 20n)^2 = 10000$

$\Rightarrow 100n^2 = 10000 \Rightarrow n = 10$

102.(c) Since, $\sum_{i=1}^{10} x_i = 10 \times 20 = 200$

Now, $\frac{1}{5}\sum_{i=1}^{10}(3x_i - 4)$

$= \frac{3}{5}\sum_{i=1}^{10} x_i - \frac{4}{5} \times 10 = \frac{3}{5} \times 200 - 4 \times 2 = 112$

103.(d) Since, $\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

$\Rightarrow \sigma = \sqrt{\frac{16160}{10} - (40)^2}$

$= \sqrt{1616 - 1600} = \sqrt{16} = 4$

104.(a) $n(s) = 6 \times 6 \times 6 = 216$

Since, sum is 4 $= (1, 1, 2) \to 3$ ways

sum is 9 $= (1, 2, 6) \to 6$ ways

$= (2, 2, 5) \to 3$ ways

$= (3, 1, 5) \to 6$ ways

$= (1, 4, 4) \to 3$ ways

$= (3, 2, 4) \to 6$ ways

$= (3, 3, 3) \to 1$ way

Sum is 16 $= (4, 6, 6) \to 3$ ways

$= (5, 5, 6) \to 3$ ways

So, required probability $= \frac{34}{216} = \frac{17}{108}$

105.(d) Let $2P(A) = 3P(B) = 4P(C) = 5P(D) = k$

Now $P(A) + P(B) + P(C) + P(D) = 1$

$\Rightarrow \frac{k}{2} + \frac{k}{3} + \frac{k}{4} + \frac{k}{5} = 1$

$\Rightarrow \frac{30k + 20k + 15k + 12k}{60} = 1$

$\Rightarrow 77k = 60 \Rightarrow k = \frac{60}{77}$

$P(A) = \frac{k}{2} = \frac{30}{77}$

$\Rightarrow 77P(A) = 30$

106.(d) There are only two combination are possible (3, 7), (7, 3) such that their product has 1 in its unit place. So, required probability $= \frac{2}{9 \times 8} = \frac{1}{36}$

107.(b) When difference is 2 then

{(1, 3), (2, 4), (3, 5), (4, 6), (3, 1), (4, 2), (5, 3), (6, 4)}

When difference is 3 then

{(1, 4), (2, 5), (3, 6), (4, 1), (5, 2), (6, 3)}

So, required probability $= \frac{14}{36} = \frac{7}{18}$

108.(c) Mean $= \frac{\sum f_i.x_i}{\sum f_i} = \frac{1 \times {}^9C_0 + 2\,{}^9C_1 + ... + 10 \times {}^9C_9}{{}^9C_0 + {}^9C_1 + {}^9C_2 + ... + {}^9C_9}$

$= \frac{11 \times 2^8}{2^9} = 5.5$

109.(d) P (recover from disease) $= 0.8 = \frac{4}{5}$

So, required probability $= {}^5C_2\left(\frac{4}{5}\right)^2\left(1 - \frac{4}{5}\right)^3$

$= \frac{5 \times 4}{2} \times \frac{16}{25} \times \frac{1}{125} = 0.0512$

110.(b) Let E_1 = design is faulty

E_2 = design is not faulty

A = building is collapses

Since, $P(E_1) = 10\% = \frac{1}{10}$

$P(E_2) = 90\% = \frac{9}{10}$

$P(A/E_1) = 95\% = \frac{19}{20}$

$P(A/E_2) = 45\% = \frac{9}{20}$

Now $P\left(\frac{E}{A}\right) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\,P(A/E_2)}$

$= \frac{\frac{1}{10} \times \frac{19}{20}}{\frac{1}{10} \times \frac{19}{20} + \frac{9}{10} \times \frac{9}{20}} = \frac{19}{100} = 0.19$

111.(a) $\because r = \frac{cor(x, y)}{\sqrt{var(x)\cdot var(y)}}$

$cor(3x \times 4, -3y + 3) = \sum(3x - 3x')(-3y + 3y')$

$= -9cor(x, y)$

$\therefore r(3x + 4, -3y + 3) = \frac{-9cor(x, y)}{\sqrt{9cor(x) - \sqrt{9\,var(y)}}}$

$= \frac{-9cor(x, y)}{9\sqrt{var(x)}\cdot\sqrt{var(y)}} = -r$

112.(c) Let E = {HHHHHT, TTTTTH}

Now, $P(E) = \frac{2}{2^6} = \frac{2}{64} = \frac{1}{32}$

113.(c) Since, Harmonic Mean (H)

$H = \frac{3}{\frac{1}{{}^{10}C_4} + \frac{1}{{}^{10}C_5} + \frac{1}{{}^{10}C_6}}$

$$H = \frac{3}{\frac{1}{210} + \frac{1}{252} + \frac{1}{210}} = \frac{3 \times 105 \times 252}{357}$$

Hence, $\frac{270}{H} = \frac{17}{14}$

114. (b) The number of ways in which p and q seat together $= (n-1)! \times 2$

So, required Probability $= \frac{2(n-1)!}{n!} = \frac{2}{n}$

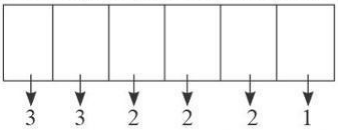
115. (a) Since, given $n = 6$
& $9P(x = 4) = P(x = 2)$
$\Rightarrow 9\ {}^6C_4 P^4 q^2 = {}^6C_2 P^2 q^4$
$\Rightarrow \frac{p^2}{q^2} = \frac{1}{9} \Rightarrow \frac{p}{q} = \frac{1}{3} \Rightarrow q = 3p$

Since $p + q = 1 \Rightarrow p = \frac{1}{4}$

116. (a) The number of ways in which all boys sit together $= 4! \times 3!$

Required probability $= \frac{4! \times 3!}{6!} = \frac{1}{5}$

117. (b) Since boys & girls seats alternatively



Required probability $= \frac{2 \times 3 \times 3 \times 2 \times 2}{6!} = \frac{1}{10}$

118. (d) Required probability $= \frac{3! \times {}^4P_3}{6!} = \frac{1}{5}$

119. (a) Since 

Required probability $= \frac{4! \times 2}{6!} = \frac{1}{15}$

120. (d) Required probability $= \frac{5! \times 2}{6!} = \frac{1}{3}$