NDA/NA SOLVED PAPER 2022-II

MATHEMATICS

1. How many four-digit natural numbers are there such that all of the digits are odd?
   (a) 625 (b) 400
   (c) 196 (d) 120

2. What is $\sum_{r=0}^{n} 2^r C(n,r)$ equal to?
   (a) 2^n (b) 3^n
   (c) 2^{2n} (d) 3^{2n}

3. If different permutations of the letters of the word 'MATHEMATICS' are listed as in a dictionary, how many words (with or without meaning) are there in the list before the first word that starts with C ?
   (a) 302400 (b) 403600
   (c) 907200 (d) 1814400

4. Consider the following statements :
   1. If f is the subset of $Z \times Z$ defined by $f = \{(xy, x-y); x, y \in Z\}$, then f is a function from Z to Z.
   2. If f is the subset of $N \times N$ defined by $f = \{(xy, x+y); x, y \in N\}$, then f is a function from N to N.

   Which of the statements given above is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

5. Consider the determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

   If $a_{13} = yz$, $a_{23} = zx$, $a_{33} = xy$ and the minors of a_{13}, a_{23}, a_{33} are respectively $(z-y)$, $(z-x)$, $(y-x)$ then what is the value of Δ ?
   (a) $(z-y)(z-x)(y-x)$
   (b) $(x-y)(y-z)(x-z)$
   (c) $(x-y)(z-x)(y-z)(x+y+z)$
   (d) $(xy+yz+zx)(x+y+z)$

6. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{pmatrix}$, then which of the following are correct ?
   1. $A + adj\,A$ is a null matrix
   2. $A^{-1} + adj\,A$ is a null matrix
   3. $A - A^{-1}$ is a null matrix

   Select the correct answer using the code given below :
   (a) 1 and 2 only (b) 2 and 3 only
   (c) 1 and 3 only (d) 1, 2 and 3

7. If X is a matrix of order 3×3, Y is a matrix of order 2×3 and Z is a matrix of order 3×2, then which of the following are correct ?
   1. $(ZY)\,X$ is a square matrix having 9 entries.
   2. $Y\,(XZ)$ is a square matrix having 4 entries.
   3. $X\,(YZ)$ is not defined.

   Select the correct answer using the code given below :
   (a) 1 and 2 only (b) 2 and 3 only
   (c) 1 and 3 only (d) 1, 2 and 3

8. For how many quadratic equations, the sum of roots is equal to the product of roots?
   (a) 0 (b) 1
   (c) 2 (d) Infinitely many

9. Consider the following statements :
   1. The set of all irrational numbers between $\sqrt{2}$ and $\sqrt{5}$ is an infinite set.
   2. The set of all odd integers less than 100 is a finite set.

   Which of the statements given above is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

10. Consider the following statements :
    1. $2 + 4 + 6 + ... + 2n = n^2 + n$
    2. The expression $n^2 + n + 41$ always gives a prime number for every natural number n

    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

11. Let p, q $(p > q)$ be the roots of the quadratic equation $x^2 + bx + c = 0$ where $c > 0$. If $p^2 + q^2 - 11pq = 0$, then what is $p - q$ equal to?
    (a) $3\sqrt{c}$ (b) $3c$
    (c) $9\sqrt{c}$ (d) $9c$

12. What is the diameter of a circle inscribed in a regular polygon of 12 sides, each of length 1 cm ?
    (a) $1 + \sqrt{2}$ cm (b) $2 + \sqrt{2}$ cm
    (c) $2 + \sqrt{3}$ cm (d) $3 + \sqrt{3}$ cm

13. Let $A = \{7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$ and let $f : A \rightarrow N$ be defined by $f(x)$ = the highest prime factor of x.
    How many elements are there in the range of f ?
    (a) 4 (b) 5
    (c) 6 (d) 7

14. Let R be a relation from N to N defined by $R = \{(x, y) : x, y \in N \text{ and } x^2 = y^3\}$. Which of the following are ***not*** correct?
   1. $(x, x) \in R$ for all $x \in N$
   2. $(x, y) \in R \Rightarrow (y, x) \in R$
   3. $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

   Select the correct answer using the code given below:
   (a) 1 and 2 only (b) 2 and 3 only
   (c) 1 and 3 only (d) 1, 2 and 3
15. Consider the following :
   1. $A \cap B = A \cap C \Rightarrow B = C$
   2. $A \cup B = A \cup C \Rightarrow B = C$

   Which of the above is/are correct ?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

DIRECTIONS: *Consider the following for the next three (03) items that follow:*

Let $z = \dfrac{1+i\sin\theta}{1-i\sin\theta}$ where $i = \sqrt{-1}$

16. What is the modulus of z ?
   (a) 1 (b) $\sqrt{2}$
   (c) $1 + \sin^2\theta$ (d) $\dfrac{1+\sin^2\theta}{1-\sin^2\theta}$
17. What is angle θ such that z is purely real?
   (a) $\dfrac{n\pi}{2}$ (b) $\dfrac{(2n+1)\pi}{2}$
   (c) $n\pi$ (d) $2n\pi$ only
   where n is an integer
18. What is angle θ such that z is purely imaginary ?
   (a) $\dfrac{n\pi}{2}$ (b) $\dfrac{(2n+1)\pi}{2}$
   (c) $n\pi$ (d) $2n\pi$
   where n is an integer

DIRECTIONS: *Consider the following for the next* **three (03)** *items that follow:*

Let P be the sum of first n positive terms of an increasing arithmetic progression A. Let Q be the sum of first n positive terms of another increasing arithmetic progression B.
Let $P : Q = (5n + 4) : (9n + 6)$

19. What is the ratio of the first term of A to that of B ?
   (a) $\dfrac{1}{3}$ (b) $\dfrac{2}{5}$
   (c) $\dfrac{3}{4}$ (d) $\dfrac{3}{5}$
20. What is the ratio of their 10th terms ?
   (a) $\dfrac{11}{29}$ (b) $\dfrac{22}{49}$
   (c) $\dfrac{33}{59}$ (d) $\dfrac{44}{69}$
21. If d is the common difference of A, and D is the common difference of B, then which one of the following is always correct?
   (a) $D > d$ (b) $D < d$
   (c) $7D > 12d$ (d) None of the above

DIRECTIONS: *Consider the following for the next three (03) items that follow :*

Consider the binomial expansion of $(p + qx)^9$:

22. What is the value of q if the coefficients of x^3 and x^6 are equal ?
   (a) p (b) $9p$
   (c) $\dfrac{1}{p}$ (d) p^2
23. What is the ratio of the coefficients of middle terms in the expansion (when expanded in ascending powers of x)?
   (a) pq (b) $\dfrac{p}{q}$
   (c) $\dfrac{4p}{5q}$ (d) $\dfrac{1}{(pq)}$
24. Under what condition the coefficients of x^2 and x^4 are equal ?
   (a) $p : q = 7 : 2$
   (b) $p^2 : q = 7 : 2$
   (c) $p : q = 2 : 7$
   (d) $p^2 : q^2 = 2 : 7$

DIRECTIONS: *Consider the following for the next three (03) items that follow:*

Consider the word 'QUESTION' :

25. How many 4-letter words each of two vowels and two consonants with or without meaning, can be formed ?
   (a) 36 (b) 144
   (c) 576 (d) 864
26. How many 8-letter words with or without meaning, can be formed such that consonants and vowels occupy alternate positions?
   (a) 288 (b) 576
   (c) 1152 (d) 2304
27. How many 8-letter words with or without meaning, can be formed so that all consonants are together ?
   (a) 5760 (b) 2880
   (c) 1440 (d) 720

DIRECTIONS: *Consider the following for the next three (03) items that follow :*

Let Δ be the determinant of a matrix A,

where $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ and C_{11}, C_{12}, C_{13} be the cofactors of a_{11}, a_{12}, a_{13} respectively.

28. What is the value of
   $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$?
   (a) 0 (b) 1
   (c) Δ (d) $-\Delta$
29. What is the value of $a_{21}C_{11} + a_{22}C_{12} + a_{23}C_{13}$?
   (a) 0 (b) 1
   (c) Δ (d) $-\Delta$

30. What is the value of $\begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{23} & a_{33} & a_{13} \\ a_{22} & a_{32} & a_{12} \end{vmatrix}$?

(a) 0 (b) 1

(c) Δ (d) $-\Delta$

DIRECTIONS: *Consider the following for the next three (03) items that follow:*

Let $f(x)$ be a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$ such that $f(1) = 2$:

31. If $\sum_{x=2}^{n} f(x) = 2044,$ then what is the value of n ?

(a) 8 (b) 9

(c) 10 (d) 11

32. What is $\sum_{x=1}^{5} f(2x-1)$ equal to ?

(a) 341 (b) 682

(c) 1023 (d) 1364

33. What is $\sum_{x=1}^{6} 2^x f(x)$ equal to?

(a) 1365 (b) 2730

(c) 4024 (d) 5460

DIRECTIONS: *Consider the following for the next* **three (03)** *items that follow:*

A university awarded medals in basket ball, football and volleyball. Only x students ($x < 6$) got medal in all the three sports and the medals went to a total of $15x$ students. It awarded $5x$ medals in basketball, $(4x + 15)$ medals in football and $(x + 25)$ medals in volleyball.

34. How many received medals in exactly two of the three sports ?

(a) $30 - 4x$ (b) $35 - 7x$

(c) $40 - 7x$ (d) $45 - 5x$

35. How many received medals in at least two of three sports ?

(a) $30 - 6x$ (b) $35 - 6x$

(c) $40 - 5x$ (d) $40 - 6x$

36. How many received medals in exactly one of three sports ?

(a) $21x - 40$ (b) $21x - 35$

(c) $20x - 35$ (d) $20x - 25$

DIRECTIONS: *Consider the following for the next three (03) items that follow :*

Let $A = \begin{pmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{pmatrix}$ and $A = P + Q$ where P is symmetric matrix and Q is skew-symmetric matrix.

37. What is P equal to?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\cos 2\theta \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $\cos 2\theta \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

38. What is Q equal to ?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\cos 2\theta \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $\cos 2\theta \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

39. What is the minimum value of determinant of A ?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) 1

DIRECTIONS: *Consider the following for the next three (03) items that follow:*

ABC is a triangular plot with AB = 16 m, BC = 10 m and CA = 10 m. A lamp post is situated at the middle point of the side AB. The lamp post subtends an angle 45° at the vertex B.

40. What is the height of the lamp post ?

(a) 6 m (b) 7 m

(c) 8 m (d) 9 m

41. What is $\frac{AB}{\sin C}$ equal to?

(a) 17 m (b) $\frac{50}{3}$ m

(c) $\frac{40}{3}$ m (d) 16 m

42. What is $\cos A + \cos B + \cos C$ equal to ?

(a) 1 (b) $\frac{41}{25}$

(c) $\frac{37}{25}$ (d) $\frac{33}{25}$

DIRECTIONS: *Consider the following for the next three (03) items that follow :*

There are two points P and Q due south of a leaning tower, which leans towards north. P is at a distance x and Q is at a distance y from the foot of the tower ($x > y$). The angles of elevation of the top of the tower from P and Q are 15° and 75° respectively.

43. At what height is the top of the tower above the ground level ?

(a) $\frac{x-y}{2\sqrt{3}}$ (b) $\frac{x-y}{4\sqrt{3}}$

(c) $\frac{x-y}{4}$ (d) $\frac{x-y}{2}$

44. If θ is the inclination of the tower to the horizontal, then what is $\cot\theta$ equal to?

(a) $2+\frac{\sqrt{3}(x-y)}{x+y}$ (b) $2-\frac{\sqrt{3}(x-y)}{x+y}$

(c) $2+\frac{\sqrt{3}(x+y)}{x-y}$ (d) $2-\frac{\sqrt{3}(x+y)}{x-y}$

45. What is the length of the tower ?

(a) $\frac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2+\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(b) $\frac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2-\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(c) $\frac{x-y}{4\sqrt{3}}\sqrt{1+\left\{2+\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(d) $\frac{x-y}{4\sqrt{3}}\sqrt{1+\left\{2-\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

46. What is the value of cosec $\left(-\frac{73\pi}{3}\right)$?

(a) $\frac{2}{\sqrt{3}}$ (b) $-\frac{2}{\sqrt{3}}$

(c) 2 (d) -2

47. What is the value of

$\cos\left(\frac{5\pi}{17}\right)+\cos\left(\frac{7\pi}{17}\right)+2\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$?

(a) 0 (b) 1

(c) $4\cos\left(\frac{6\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$ (d) $4\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$

48. What is the value of $\tan\left(\frac{3\pi}{8}\right)$?

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$

(c) $1-\sqrt{2}$ (d) $-(\sqrt{2}+1)$

49. What is $\tan^{-1}\cot(\text{cosec}^{-1}2)$ equal to?

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

50. In a triangle ABC, $a = 4$, $b = 3$, $c = 2$. What is $\cos 3C$ equal to?

(a) $\frac{7}{128}$ (b) $\frac{11}{128}$

(c) $\frac{7}{64}$ (d) $\frac{11}{64}$

51. What is cos 36° – cos 72° equal to?

(a) $\frac{\sqrt{5}}{2}$ (b) $-\frac{\sqrt{5}}{2}$

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

52. If $\sec x = \frac{25}{24}$ and x lies in the fourth quadrant, then what is the value of $\tan x + \sin x$?

(a) $-\frac{625}{168}$ (b) $-\frac{343}{600}$

(c) $\frac{625}{168}$ (d) $\frac{343}{600}$

53. What is the value of $\tan^2 165° + \cot^2 165°$?

(a) 7 (b) 14

(c) $4\sqrt{3}$ (d) $8\sqrt{3}$

54. What is the value of $\sin\left(2n\pi+\frac{5\pi}{6}\right)\sin\left(2n\pi-\frac{5\pi}{6}\right)$, where $n \in Z$?

(a) $\frac{1}{4}$ (b) $-\frac{3}{4}$

(c) $-\frac{1}{4}$ (d) $\frac{3}{4}$

55. If $1 + 2(\sin x + \cos x)(\sin x - \cos x) = 0$ where $0 < x < 360°$, then how many values does x take?

(a) Only one value (b) Only two values

(c) Only three values (d) Four values

56. Consider the following statements in respect of the line passing through origin and inclining at an angle of 75° with the positive direction of x-axis :

1. The line passes through the point $\left(1, \frac{1}{2-\sqrt{3}}\right)$.

2. The line entirely lies in first and third quadrants.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

57. If $P(3, 4)$ is the mid-point of a line segment between the axes, then what is the equation of the line ?
(a) $3x + 4y - 25 = 0$ (b) $4x + 3y - 24 = 0$
(c) $4x - 3y = 0$ (d) $3x - 4y + 7 = 0$

58. The base AB of an equilateral triangle ABC with side 8 cm lies along the y-axis such that the mid-point of AB is at the origin and B lies above the origin. What is the equation of line passing through (8, 0) and parallel to the side AC?
(a) $x - \sqrt{3}\,y - 8 = 0$ (b) $x + \sqrt{3}\,y - 8 = 0$
(c) $\sqrt{3}\,x + y - 8\sqrt{3} = 0$ (d) $\sqrt{3}\,x - y - 8\sqrt{3} = 0$

59. The centre of the circle passing through origin and making positive intercepts 4 and 6 on the coordinate axes, lies on the line
(a) $2x - y + 1 = 0$ (b) $3x - 2y - 1 = 0$
(c) $3x - 4y + 6 = 0$ (d) $2x + 3y - 26 = 0$

60. The centre of an ellipse is at (0, 0), major axis is on the y-axis. If the ellipse passes through (3, 2) and (1, 6), then what is its eccentricity ?
(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$
(c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{5}$

61. An equilateral triangle is inscribed in a parabola $x^2 = \sqrt{3}\,y$ where one vertex of the triangle is at the vertex of the parabola. If p is the length of side of the triangle and q is the length of the latus rectum, then which one of the following is correct ?
(a) $p = q$ (b) $p = \sqrt{3}\,q$
(c) $p = 2\sqrt{3}\,q$ (d) $2\sqrt{3}\,p = q$

62. Consider the points $A(2, 4, 6)$, $B(-2, -4, -2)$, $C(4, 6, 4)$ and $D(8, 14, 12)$. Which of the following statements is/are correct?
1. The points are the vertices of a rectangle $ABCD$.
2. The mid-point of AC is the same as that of BD.
Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

63. Consider the equation of a sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z - 16 = 0$.
Which of the following statements is/are correct ?
1. z-axis is tangent to the sphere.
2. The centre of the sphere lies on the plane $x + y + z - 9 = 0$.
Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 21.

64. A plane cuts intercepts 2, 2, 1 on the coordinate axes. What are the direction cosines of the normal to the plane ?
(a) $< 2/3, 2/3, 1/3 >$ (b) $< 1/3, 2/3, 2/3 >$
(c) $\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \rangle$ (d) $\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$

65. Consider the following statements :
1. The direction ratios of y-axis can be $< 0, 4, 0 >$
2. The direction ratios of a line perpendicular to z-axis can be $< 5, 6, 0 >$
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

66. $PQRS$ is a parallelogram. If $\overrightarrow{PR} = \vec{a}$ and $\overrightarrow{QS} = \vec{b}$, then what is $\overrightarrow{PQ}$ equal to?
(a) $\vec{a} + \vec{b}$ (b) $\vec{a} - \vec{b}$
(c) $\frac{\vec{a} + \vec{b}}{2}$ (d) $\frac{\vec{a} - \vec{b}}{2}$

67. Let $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular. What is the angle between $\vec{a}$ and $\vec{b}$?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

68. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors lying on the same plane. What is $\left\{\left(3\vec{a} + 2\vec{b}\right) \times \left(5\vec{a} - 4\vec{c}\right)\right\} . \left(\vec{b} + 2\vec{c}\right)$ equal to?
(a) -8 (b) -32
(c) 8 (d) 0

69. What are the values of x for which the angle between the vectors $2x^2\hat{i} + 3x\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + x^2\hat{k}$ is obtuse?
(a) $0 < x < 2$ (b) $x < 0$
(c) $x > 2$ (d) $0 \le x \le 2$

70. The position vectors of vertices A, B and C of triangle ABC are respectively $\hat{j} + \hat{k}, 3\hat{i} + \hat{j} + 5\hat{k}$ and $3\hat{j} + 3\hat{k}$. What is angle C equal to ?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

71. Let $z = [y]$ and $y = [x] - x$, where [.] is the greatest integer function. If x is not an integer but positive, then what is the value of z ?
(a) -1 (b) 0
(c) 1 (d) 2

72. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $fog(x) = gof(x)$, then what is the value of k ?
(a) 7 (b) 5
(c) 4 (d) 3

73. What is the minimum value of the function $f(x) = \log_{10}(x^2 + 2x + 11)$?
(a) 0 (b) 1
(c) 2 (d) 10

74. What is $\int (x^x)^2(1 + lnx)dx$ equal to?

(a) $x^{2x} + c$ (b) $\frac{1}{2}x^{2x} + c$

(c) $2x^{2x} + c$ (d) $\frac{1}{2}x^x + c$

75. What is $\int e^x (1 + lnx + xlnx)dx$ equal to?

(a) $xe^x lnx + c$ (b) $x^2e^x lnx + c$

(c) $x + e^x lnx + c$ (d) $xe^x + lnx + c$

76. What is $\int \frac{(\cos x)^{1.5} - (\sin x)^{1.5}}{\sqrt{\sin x.\cos x}}dx$ equal to?

(a) $\sqrt{\sin x} - \sqrt{\cos x} + c$

(b) $\sqrt{\sin x} + \sqrt{\cos x} + c$

(c) $2\sqrt{\sin x} + 2\sqrt{\cos x} + c$

(d) $\frac{1}{2}\sqrt{\sin x} + \frac{1}{2}\sqrt{\cos x} + c$

77. If $y = \frac{x\sqrt{x^2-16}}{2} - 8\ln\left|x + \sqrt{x^2-16}\right|$, then what is $\frac{dy}{dx}$ equal to?

(a) $x\sqrt{x^2-16}$ (b) $x - \sqrt{x^2-16}$

(c) $\sqrt{x^2-16}$ (d) $4\sqrt{x^2-16}$

78. If $y = (x^x)^x$ then which one of the following is correct ?

(a) $\frac{dy}{dx} + xy\,(1 + 2lnx) = 0$

(b) $\frac{dy}{dx} - xy\,(1 + 2lnx) = 0$

(c) $\frac{dy}{dx} - 2xy\,(1 + 2lnx) = 0$

(d) $\frac{dy}{dx} - 2xy\,(1 + 2lnx) = 0$

79. What is the maximum value of $3(\sin x - \cos x) + 4(\cos^3 x - \sin^3 x)$?

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

80. What is the area of the region (in the first quadrant) bounded by $y = \sqrt{1-x^2}$, $y = x$ and $y = 0$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{12}$

81. What is the area of the region bounded by $x - |y| = 0$ and $x - 2 = 0$?

(a) 1 (b) 2

(c) 4 (d) 8

82. If $f(\alpha) = \sqrt{\sec^2 \alpha - 1}$, then what is $\frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)}$ equal to?

(a) $f(\alpha - \beta)$ (b) $f(\alpha + \beta)$

(c) $f(\alpha)\ f(\beta)$ (d) $f(\alpha\beta)$

83. If $f(x) = \ln(x + \sqrt{1+x^2})$, then which one of the following is correct ?

(a) $f(x) + f(-x) = 0$ (b) $f(x) - f(-x) = 0$

(c) $2f(x) = 2f(-x)$ (d) $f(x) = 2f(-x)$

84. What is $\lim_{x\to 0} \frac{x}{\sqrt{1-\cos^4 x}}$ equal to?

(a) $\frac{1}{2\sqrt{2}}$ (b) $-\frac{1}{2\sqrt{2}}$

(c) $\sqrt{2}$ (d) Limit does not exist

85. What is $\lim_{x\to \frac{x}{2}} \frac{4x - 2\pi}{\cos x}$ equal to?

(a) -4 (b) -2

(c) 2 (d) 4

86. If $f(x) = \frac{x2 + x + |x|}{x}$ then what is $\lim_{x\to 0} f(x)$ equal to?

(a) 0 (b) 1

(c) 2 (d) $\lim_{x\to 0} f(x)$ does not exist

87. What is $\lim_{h\to 0} \frac{\sin^2(x+h) - \sin^2 x}{h}$ equal to?

(a) $\sin^2 x$ (b) $\cos^2 x$

(c) $\sin 2x$ (d) $\cos 2x$

88. Let $f(x)$ be a function such that $f'(x) = g(x)$ and $f''(x) = -f(x)$. Let $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. Then consider the following statements :

1. $h'(3) = 0$
2. $h(1) = h(2)$

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

89. In $y = \ln^2\left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)$, then what is $\frac{dy}{dx}$ at $x = 0$ equal to?

(a) -2 (b) 0

(c) 1 (d) 2

90. If $\frac{d}{dx}\left(\frac{1+x^4+x^8}{1-x^2+x^4}\right) = ax + bx^3$, then which one of the following is correct ?

(a) $a = b$ (b) $a = 2b$

(c) $a + b = 0$ (d) $2a = b$

91. Under which one of the following conditions does the function $f(x) = (p \sec x)^2 + (q \operatorname{cosec} x)^2$ attain minimum value ?

(a) $\tan^2 x = \frac{q}{p}$ (b) $\cot^2 x = \frac{q}{p}$

(c) $\tan^2 x = pq$ (d) $\cot^2 x = pq$

92. Where does the function $f(x) = \sum_{j=1}^{7}(x-j)^2$ attain its minimum value ?

(a) $x = 3.5$ (b) $x = 4$

(c) $x = 4.5$ (d) $x = 4$

93. Consider the following statements in respect of the function $f(x) = \begin{cases} |x|+1, & 0<|x|\le 3 \\ 1, & x=0 \end{cases}$

1. The function attains maximum value only at $x = 3$
2. The function attains local minimum only at $x = 0$

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

94. What is $\int_0^1 \ln\left(\frac{1}{x}-1\right)dx$ equal to?

(a) – 1 (b) 0

(c) 1 (d) $ln2$

95. If $\int_0^{\pi/2}(\sin^4 x+\cos^4 x)dx = k$, then what is the value of $\int_0^{20\pi}(\sin^4+\cos^4 x)dx$?

(a) k (b) $10\,k$

(c) $20\,k$ (d) $40\,k$

96. What is $\int_{-\pi/2}^{\pi/2}(e^{\cos x}\sin x + e^{\sin x}\cos x)dx$ equal to?

(a) $\frac{e^2-1}{e}$ (b) $\frac{e^2+1}{e}$

(c) $\frac{1-e^2}{e}$ (d) 0

97. What is the area of the region enclosed in the first quadrant by $x^2 + y^2 = \pi^2$, $y = \sin x$ and $x = 0$?

(a) $\frac{\pi^3}{4}-1$ (b) $\frac{\pi^3}{4}-2$

(c) $\frac{\pi^3}{2}-1$ (d) $\frac{\pi^2}{4}-2$

98. Consider the following statements :

1. The degree of the differential $\frac{dy}{dx}+\cos\left(\frac{dy}{dx}\right)=0$ is 1.
2. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\cos\left(\frac{dy}{dx}\right)=0$ is 2.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

99. What is the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis ?

(a) $x\frac{dy}{dx}+2y=0$ (b) $x\frac{dy}{dx}-2y=0$

(c) $y\frac{dx}{dy}+2x=0$ (d) $y\frac{dx}{dy}-2x=0$

100. What is the solution of the differential equation $(dy - dx) + \cos x(dy + dx) = 0$?

(a) $y=\tan\left(\frac{x}{2}\right)-x+c$ (b) $y=\frac{1}{2}\tan\left(\frac{x}{2}\right)-x+c$

(c) $y=2\tan\left(\frac{x}{2}\right)-x+c$ (d) $y=\tan\left(\frac{x}{2}\right)-2x+c$

101. Let x be the mean of squares of first n natural numbers and y be the square of mean of first n natural numbers.

If $\frac{x}{y}=\frac{55}{42}$, then what is the value of n ?

(a) 24 (b) 25

(c) 27 (d) 30

102. What is the probability of getting a composite number in the list of natural numbers from 1 to 50 ?

(a) $\frac{7}{10}$ (b) $\frac{17}{25}$

(c) $\frac{18}{25}$ (d) $\frac{33}{50}$

103. If $n > 7$, then what is the probability that $C(n, 7)$ is a multiple of 7 ?

(a) 0 (b) $\frac{1}{7}$

(c) – (d) 1

104. Two numbers x and y are chosen at random from a set of first 10 natural numbers. What is the probability that $(x + y)$ is divisible by 4 ?

(a) $\frac{1}{5}$ (b) $\frac{2}{9}$

(c) $\frac{8}{45}$ (d) $\frac{7}{45}$

105. A number x is chosen at random from first n natural numbers. What is the probability that the number chosen satisfies $x+\frac{1}{x}>2$?

(a) $\frac{1}{n}$ (b) $\frac{1}{(2n)}$

(c) $\frac{(n-1)}{n}$ (d) 1

106. Three fair dice are tossed once. What is the probability that they show different numbers that are in *AP* ?

(a) $\frac{1}{12}$ (b) $\frac{1}{18}$

(c) $\frac{1}{36}$ (d) $\frac{1}{72}$

107. If $P(A) = 0{\cdot}5$, $P(B) = 0{\cdot}7$ and $P(A \cap B) = 0{\cdot}3$, then what is the value of $P(A' \cap B') + P(A' \cap B) + P(A \cap B')$?

(a) 0.6 (b) 0.7

(c) 0.8 (d) 0.9

108. Five coins are tossed once. What is the probability of getting at most four tails?

(a) $\frac{31}{32}$ (b) $\frac{15}{16}$

(c) $\frac{29}{32}$ (d) $\frac{7}{8}$

109. Three fair dice are thrown. What is the probability of getting a total greater than or equal to 15 ?

(a) $\frac{19}{216}$ (b) $\frac{1}{12}$

(c) $\frac{17}{216}$ (d) $\frac{5}{54}$

110. The probability that a person hits a target is $0{\cdot}5$. What is the probability of at least one hit in 4 shots ?

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$

(c) $\frac{15}{16}$ (d) $\frac{7}{8}$

111. A box contains 2 white balls, 3 black balls and 4 red balls. What is the number of ways of drawing 3 balls from the box with at least one black ball ?

(a) 84 (b) 72

(c) 64 (d) 48

112. During war one ship out of 5 was sunk on an average in making a certain voyage. What is the probability that exactly 3 out of 5 ships would arrive safely?

(a) $\frac{16}{625}$ (b) $\frac{32}{625}$

(c) $\frac{64}{625}$ (d) $\frac{128}{625}$

113. A card is drawn from a pack of 52 cards. A gambler bets that it is either a spade or an ace. The odds against his winning are

(a) 9 : 4 (b) 35 : 17

(c) 17 : 35 (d) 4 : 9

114. The coefficient of correlation between ages of husband and wife at the time of marriage for a given set of 100 couples was noted to be $0{\cdot}7$. Assume that all these couples survive to celebrate the silver jubilee of their marriage. The coefficient of correlation at that point of time will be

(a) 1 (b) 0.9

(c) 0.7 (d) 0.3

115. The completion of a construction job may be delayed due to strike. The probability of strike is $0{\cdot}6$. The probability that the construction job gets completed on time if there is no strike is $0{\cdot}85$ and the probability that the construction job gets completed on time if there is a strike is $0{\cdot}35$. What is the probability that the construction job will not be completed on time ?

(a) 0.35 (b) 0.45

(c) 0.55 (d) 0.65

DIRECTIONS : *Consider the following for the next* **two (02)** *items that follow :*

The mean and standard deviation (SD) of marks obtained by 50 students of a class in 4 subjects are given below:

Subject	Mathematics	Physics	Chemistry	Biology
Mean Marks	40	28	38	36
SD	15	12	14	16

116. Which one of the following subjects shows highest variability of marks ?

(a) Mathematics (b) Physics

(c) Chemistry (d) Biology

117. What is the coefficient of variation marks in Mathematics?

(a) 37.5% (b) 38.0%

(c) 38.5% (d) 39.0%

DIRECTIONS: *Consider the following for the next* **three (03)** *items that follow :*

Consider the following grouped frequency distribution :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	1	2	4	6	4	3

118. What is the median of the distribution?

(a) 34 (b) 34.5

(c) 35 (d) 35.5

119. What is mean deviation about the median?

(a) 11.4 (b) 11.1

(c) 10.8 (d) 10.5

120. What is the mean deviation about the mean?

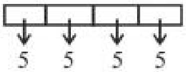
(a) 10.15 (b) 10.65

(c) 11.15 (d) 11.65

HINTS & SOLUTIONS

MATHEMATICS

1. (a) Odd digits = 1, 3, 5, 7, 9



Number of four digits number with all odd digits
$= 5 \times 5 \times 5 \times 5 = 625$

2. (b) $\sum_{r=0}^{n} 2^r \ {}^nCr = {}^nC_0 2^0 + {}^nC_1 2^1 + {}^nC_2 2^2 + \ldots + {}^nC_2 2^n$

$= (1 + 2)^n = 3^n$

3. (c) Given word 'MATHEMATICS'

No. of words start with $A = \frac{10!}{2!2!}$

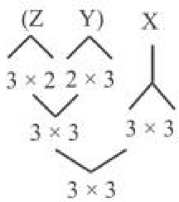
$\therefore$ No. of words before the first word that start with

$c = \frac{10!}{2!2!} = 907200$

4. (d) For statement 1:

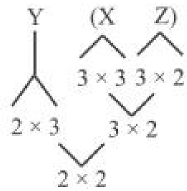
Let $xy = 6 = 1.6 \quad \Rightarrow x - y = 1 - 6 = -5$
and $xy = 6 = 2.3 \quad \Rightarrow x - y = 2 - 3 = -1$
$\therefore$ 6 related to both -5 and -1
So f is not a function from Z to Z.
For statement 2:
Let $xy = 6 = 1.6 \quad \Rightarrow x + y = 1 + 6 = 7$
and $xy = 6 = 2.3 \quad \Rightarrow x + y = 2 + 3 = 5$
$\therefore$ 6 related to both 7 and 5
So f is not a function from N to N.

5. (a)
6. (d)
7. (d) 1.



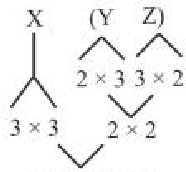
Total entries = 9

2.



Total entries = 4

3.



Not defined

8. (d) Sum of roots = product of roots

$\frac{-b}{a} = \frac{c}{a} \Rightarrow b = -c \qquad \therefore$ Quadratic equation is

$ax^2 - cx + c = 0$
So, roots of quadratic equation depend on $a, c \in R$.
Hence, infinitely many quadratic equations.

9. (a) There are infinitely many irrational numbers lies between $\sqrt{2}$ and $\sqrt{5}$.

Odd integers less than 100
$= \ldots -5, -4, -3, -2, -1, 1, 2, 3, \ldots. 99$
Which is infinite.

10. (a) Let $S = 2 + 4 + 6 + \ldots + 2n$

$= 2\,(1 + 2 + 3 + \ldots + n) = 2.\frac{n(n+1)}{2} = n^2 + n$

$\therefore$ Statement 1 is true.
Putting $n = 41$ in $n^2 + n + 41$
$= 41^2 + 41 + 41 = 41.(41 + 2) = 41.43$
Which is not prime.
So, expression not always prime for $n \in N$

11. (a) Given that roots of $x^2 + bx + c = 0$ are p and q.

$\therefore \quad p + q = -b$ and $pq = c$

Given that

$p^2 + q^2 - 11pq = 0$

$\Rightarrow \quad (p + q)^2 - 13pq = 0$

$\Rightarrow \quad b^2 - 13c = 0$

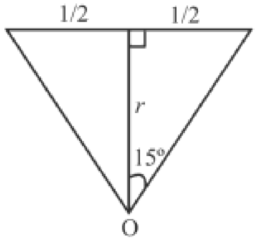
$\Rightarrow \quad b^2 - 13c$ ...(i)

Now $(p - q)^2 = (p + q)^2 - 4pq$

$= b^2 - 4c = 13c - 4c = 9c$ [from (i)]

$p - q = 3\sqrt{c}$

12. (c) Central angle subtend by side $= \dfrac{360}{12} = 30°$



$\tan 15° = \dfrac{1/2}{r}$

$\Rightarrow \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{1}{2r}$

$\Rightarrow 2r = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \dfrac{4+2\sqrt{3}}{2}$

$\Rightarrow \quad d = 2 + \sqrt{3}$ cm

13. (c) $\therefore \quad f(x)$ = the highest prime factor of x,

$\therefore \quad f(7) = 7, f(8) = 2, f(9) = 3, f(10) = 5, f(11) = 11,$
$f(12) = 3, f(14) = 7, f(15) = 5, f(16) = 2, f(13) = 13$

$\therefore$ Range = {2, 3, 5, 7, 11, 13}

14. (d)

15. (d) Let $A = \{1, 2\}, B = \{2, 3\}, C = \{2, 4\}$

$\therefore \quad A \cap B = A \cap C = \{2\}$

But $B \neq C$

So, statement 1 is wrong

Let $A = \{1, 2\}, B = \{2, 3\}, C = \{1, 2, 3\}$

$\therefore \quad A \cup B = A \cup C = \{1, 2, 3\}$

But $B \neq C$.

So, statement 2 is wrong.

16. (a) $\because z = \dfrac{1+i\sin\theta}{1-i\sin\theta}$

$\therefore |z| = \dfrac{|1+i\sin\theta|}{|1-i\sin\theta|} = \dfrac{\sqrt{1+\sin^2\theta}}{\sqrt{1+\sin^2\theta}} = 1$

17. (c) $\because \quad z = \dfrac{1+i\sin\theta}{1-i\sin\theta} \times \dfrac{1+i\sin\theta}{1+i\sin\theta}$

$= \dfrac{1-\sin^2\theta + 2i\sin\theta}{1+\sin^2\theta}$

$= \dfrac{\cos^2\theta}{1+\sin^2\theta} + \dfrac{2\sin\theta}{1+\sin^2\theta}\hat{i}$

If z is purely real

$\therefore \dfrac{2\sin\theta}{1+\sin^2\theta} = 0 \quad \Rightarrow \quad \sin\theta = 0$

$\Rightarrow \quad \theta = n\pi$, where n is an integer.

18. (b) If z is purely imaginary.

$\therefore \dfrac{\cos^2\theta}{1+\sin^2\theta} = 0 \quad \Rightarrow \quad \cos^2\theta = 0$

$\Rightarrow \quad \cos\theta = 0 \quad \Rightarrow \quad \theta = (2n+1)\dfrac{\pi}{2}$

Where n is an integer.

19. (d) Given that $\dfrac{S_n}{S'_n} = \dfrac{5n+4}{9n+6}$

Put $n = 1$

$\dfrac{S_1}{S'_1} = \dfrac{5+4}{9+6} = \dfrac{a_1}{a'_1}$

$\Rightarrow \dfrac{a_1}{a'_1} = \dfrac{9}{15} = \dfrac{3}{5}$

20. (c) $\dfrac{S_n}{S'_n} = \dfrac{5n+4}{9n+6}$

$\Rightarrow \dfrac{2a+(n-1)d}{2a'+(n-1)d'} = \dfrac{5n+4}{9n+6}$

Put $n = 19$

$\Rightarrow \dfrac{2a+18d}{2a'+18d'} = \dfrac{95+4}{171+6}$

$\Rightarrow \dfrac{a+9d}{a'+9d'} = \dfrac{99}{177} \Rightarrow \dfrac{a_{10}}{a_{10}{}^1} = \dfrac{33}{59}$

21. (c)

22. (a) Given expansion is $(p + qx)^9$

$T^{r+1} = {}^9Cr\, p^{9-r} (qx)^r = {}^9Cr\, p^{q-r} q^r x^r$

$\because$ coefficient of x^3 = coefficient x^6

${}^9C_3 p^6 q^3 = {}^9C_6 p^3 q^6$

$\Rightarrow \quad p^3 = q^3$

$\Rightarrow \quad p = q$

23. (b) Middle terms

$T_5 = T_{4+1} = {}^9C_4 p^5 q^4 x^4$

$T_6 = T_{5+1} = {}^9C_5 p^4 q^5 x^5$

Ratio of coefficients $= \dfrac{{}^9C_5 p^5 q^4}{{}^9C_4 p^4 q^5} = \dfrac{p}{q}$

24. (b) coefficient of x^2 = coefficient of x^4

${}^9C_2 p^7 q^2 = {}^9C_4 p^5 q^4$

$\dfrac{9.8}{2.1} p^2 = \dfrac{9.8.7.6}{4.3.2.1} q^2$

$\dfrac{p^2}{q^2} = \dfrac{7}{2}$

25. (d) Number of words contains two vowels and two consonants = ${}^4C_2.{}^4C_2 \times 4!$
$= 6.6.24 = 864$

26. (c) C V C V C V/ CV or VC VC VC VC
Number of 8 letter words such that vowels and consonants occupy alternate positions.
$= 2 \times 4!\,4! = 2 \times 24 \times 24 = 1152$

27. (b) Number 8 letter words such that all consonants are together
$= 4!\,5! = 2880$

28. (c) We know that
$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = \Delta$

29. (a) We know that
$a_{21}C_{11} + a_{22}C_{12} + a_{23}C_{13} = 0$

30. (d) $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$

Apply $C_1 \longleftrightarrow C_3$ Apply $C_1 \longleftrightarrow C_2$

$= -\begin{vmatrix} a_{31} & a_{21} & a_{11} \\ a_{32} & a_{22} & a_{12} \\ a_{33} & a_{23} & a_{13} \end{vmatrix} = +\begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{22} & a_{32} & a_{12} \\ a_{23} & a_{33} & a_{13} \end{vmatrix}$

Apply $R_2 \longleftrightarrow R_3$

$= -\begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{23} & a_{33} & a_{13} \\ a_{22} & a_{32} & a_{12} \end{vmatrix}$

31. (c) Given that $f(x+y) = f(x)\,.\,f(y)$ and $f(1) = 2$
$\therefore\ f(2) = f(1+1) = f(1)f(1) = 2.2 = 4$
$f(3) = f(1+2) = f(1)\,.\,f(2) = 2.4 = 8$
$f(4) = f(1+3) = f(1)\,.\,f(3) = 2.8 = 16$, So on

Now, $\sum_{x=2}^{n} f(x) = 2044$
$f(2) + f(3) + f(4) + \ldots + f(n) = 2044$
$4 + 8 + 16 \ldots (n-1)$ terms $= 2044$
$\frac{4(2^{n-1}-1)}{2-1} = 2044 \Rightarrow 2^{n-1}{}_{-1} = 511$
$\Rightarrow\ 2^{n-1} = 512 = 2^9$
$\Rightarrow\ n = 10$

32. (b) $\sum_{x=1}^{5} f(2x-1) = f(1) + f(3) + f(5) + f(7) + f(9)$
$= 2 + 8 + 32 + \ldots$ 5 terms
$= \frac{2(4^5-1)}{4-1} = \frac{2 \times 1023}{3} = 682$

33. (d) $\sum_{x=1}^{6} 2^x f(x) = 2f(1) + 2^2 f(2) + 2^3 f(3) \ldots + 2^6 f(6)$
$= 2.(2) + 4.(4) + 8.(8) + \ldots 64.64$
$= 4 + 16 + 64 \ldots 6$ terms
$= \frac{4(4^6-1)}{4-1} = \frac{4 \times 4095}{3} = 5460$

34. (c)

35. (d) At least two of three sports
$= b + e + f + d = 40 - 7x + x = 40 - 6x$

36. (a) Exactly one of three sports
$= a + c + g = 15x - (40 - 6x)$
$= 21x - 40$

37. (a) $\therefore A = \begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & \cos^2\theta & \sin^2\theta \\ \sin^2\theta & 0 & \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 0 \end{bmatrix}$

$P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

38. (d) $Q = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{bmatrix} 0 & -\cos 2\theta & \cos 2\theta \\ \cos 2\theta & 0 & -\cos 2\theta \\ -\cos 2\theta & \cos 2\theta & 0 \end{bmatrix}$

$= \cos 2\theta \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$

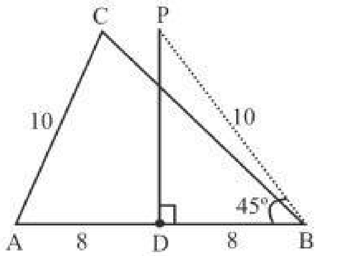
39. (a) $|A| = \begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix}$

$= 0 - \sin^2\theta\,(0 - \sin^4\theta) + \cos^2\theta\,(\cos^4\theta)$
$= \sin^6\theta + \cos^6\theta$
$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta.\cos^2\theta)$
$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$
$= 1 - \frac{3}{4}\sin^2 2\theta = \frac{1}{4}\left[4 - 3\frac{(1-\cos 4\theta)}{2}\right]$
$= \frac{1}{8}\,[5 + 3\cos 4\theta]$

Minimum value of $\cos 4\theta$ is -1.

$\therefore\ \ |A|_{\min} = \frac{1}{8}[5-3] = \frac{2}{8} = \frac{1}{4}$

40. (c)



In ΔPDB,

$\tan 45° = \frac{PD}{BD} \Rightarrow PD = BD = 8$

$\therefore$ Height of the lamp post = 8 m

41. (b) $\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{100+100-256}{200} = \frac{-56}{200} = \frac{-7}{25}$

$\sin C = \sqrt{1-\cos^2 C} = \sqrt{1-\frac{49}{625}} = \frac{24}{25}$

$\therefore \frac{AB}{\sin C} = \frac{16\times 25}{24} = \frac{50}{3}$ m

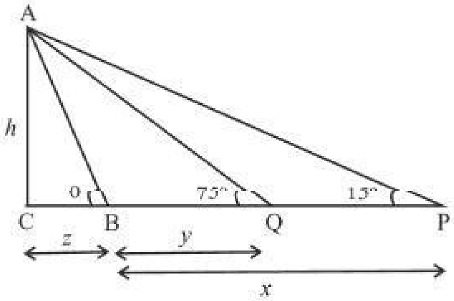
42. (d) $\cos A + \cos B + \cos C$

$\frac{b^2+c^2-a^2}{2bc} + \frac{a^2+c^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab}$

$= \frac{100+256-100}{2\times 160} + \frac{100+256-100}{2\times 160} + \frac{100+100-256}{200}$

$= \frac{256}{160} - \frac{56}{200} = \frac{1280-224}{800} = \frac{1056}{800} = \frac{33}{25}$

43. (a)



In ΔACQ $\tan 75° = \frac{h}{z+y}$

$\Rightarrow z = \frac{h}{\tan 75°} - y$ ...(i)

In ΔACP $\tan 15° = \frac{h}{z+x}$

$\Rightarrow z = \frac{h}{\tan 15°} - x$ ...(ii)

From (i) and (ii)

$\frac{h}{\tan 15°} - \frac{h}{\tan 75°} = x - y$

$\Rightarrow h\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = x - y$

$\left[\because \tan 75° = \frac{\sqrt{3}-1}{\sqrt{3}+1} \quad \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1}\right]$

$\Rightarrow h\left(\frac{3+1+2\sqrt{3}-3-1+2\sqrt{3}}{3-1}\right) = x - y$

$\Rightarrow h(2\sqrt{3}) = x - y \Rightarrow h = \frac{x-y}{2\sqrt{3}}$

44. (d) $\therefore z = \frac{h}{\tan 75°} - y = \frac{x-y}{2\sqrt{3}} \cdot \frac{\sqrt{3}-1}{\sqrt{3}+1} - y$

$= \frac{(x-y)}{2}\left[\frac{\sqrt{3}-1}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}\right] - y$

$= \frac{x-y}{2}\left[\frac{3\sqrt{3}-3-3+\sqrt{3}}{9-3}\right] - y$

$= (x-y)\frac{[2\sqrt{3}-3]}{6} - y = \frac{2\sqrt{3}(x-y)-3(x+y)}{6}$

$\cot\theta = \frac{z}{h} = \frac{2\sqrt{3}(x-y)-3(x+y)}{6} \times \frac{2\sqrt{3}}{x-y}$

$= 2 - \frac{\sqrt{3}(x+y)}{x-y}$

45. (b) Length of the tower, $AB = \sqrt{h^2+z^2}$

$= \sqrt{\frac{(x-y)^2}{12} + \left(\frac{2\sqrt{3}(x-y)-3(x+y)}{6}\right)^2}$

$= \frac{(x-y)}{2\sqrt{3}}\sqrt{1+\left(2-\frac{\sqrt{3}(x+y)}{x-y}\right)^2}$

46. (b) $\text{cosec}\left(-\frac{73\pi}{3}\right) = -\text{cosec}\left(\frac{73\pi}{3}\right)$

$= -\text{cosec}\left(24\pi + \frac{\pi}{3}\right) = -\text{cosec}\frac{\pi}{3} = -\frac{2}{\sqrt{3}}$

47. (a)

48. (b) $\because \tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right) = -1$

Let $\frac{3\pi}{4} = \theta \Rightarrow \tan\theta = -1$

Now, $\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2}$

Let $\tan\theta/2 = x$

$\therefore\quad -1 = \dfrac{2x}{1-x^2}$

$\Rightarrow\ x^2 - 2x - 1 = 0 \quad \Rightarrow\ x = 1 \pm \sqrt{2}$

$\therefore\quad \tan\dfrac{3\pi}{8} = 1+\sqrt{2} \qquad (\because 0 < \dfrac{3\pi}{8} < \dfrac{\pi}{2})$

49. (d) $\tan^{-1}\cot(\text{cosec}^{-1} 2)$

$= \tan^{-1}\left(\cot\dfrac{\pi}{6}\right) = \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

50. (a) $\because\ a = 4, b = 3, c = 2$

$\therefore \cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{16+9-4}{24} = \dfrac{21}{24}$

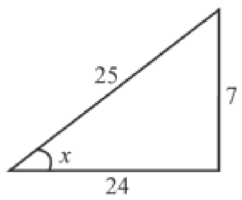
$\therefore\quad \cos 3C = 4\cos^3 C - 3\cos C$

$= 4\left(\dfrac{21}{24}\right)^3 - 3\left(\dfrac{21}{24}\right) = \dfrac{21}{24}\left(\dfrac{4\times 441}{576} - 3\right) = \dfrac{7}{128}$

51. (c) $\cos 36° - \cos 72° = -2\sin 54° \,.\, \sin(-18°)$

$= 2\sin 54°.\sin 18° = 2\left(\dfrac{\sqrt{5}+1}{4}\right)\left(\dfrac{\sqrt{5}-1}{4}\right) = \dfrac{1}{2}$

52. (b)



$\sec x = \dfrac{25}{24} = \dfrac{h}{b}$

$P = \sqrt{(25)^2 - (24)^2} = 7$

$\therefore \tan x + \sin x = \left(-\dfrac{7}{24}\right) + \left(-\dfrac{7}{25}\right)$

$= -7\left[\dfrac{1}{24} + \dfrac{1}{25}\right] = \dfrac{-343}{600}$

53. (b)

54. (a) $\sin\left(2n\pi + \dfrac{5\pi}{6}\right)\sin\left(2n\pi - \dfrac{5\pi}{6}\right)$

$= -\sin\dfrac{5\pi}{6}.\sin\left(\dfrac{5\pi}{6}\right) = -\sin^2\left(\pi - \dfrac{\pi}{6}\right)$

$= -\sin^2\dfrac{\pi}{6} = -\dfrac{1}{4}$

55. (d) $1 + 2(\sin x + \cos x)(\sin x - \cos x) = 0$

$\Rightarrow\ 1 + 2(\sin^2 x - \cos^2 x) = 0$

$\Rightarrow\ 1 - 2\cos^2 2x = 0$

$\Rightarrow\ \cos^2 2x = \dfrac{1}{2} \qquad \Rightarrow\ \cos 2x = \dfrac{1}{\sqrt{2}}$

$\Rightarrow\ x = n\pi \pm \dfrac{\pi}{8} \qquad \therefore\ x = 0, \pi - \dfrac{\pi}{8},$

$\pi + \dfrac{\pi}{8}, 2\pi - \dfrac{\pi}{8}$. Four values.

56. (c) Given point (0, 0)

$m = \tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ ($\therefore$ Equation of straight line)

$y = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}x$

Put $x = 1$ and $y = \dfrac{1}{2-\sqrt{3}}$

$\therefore \dfrac{1}{2-\sqrt{3}} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{3-1}{4-2\sqrt{3}} = \dfrac{1}{2-\sqrt{3}}$

Satisfy equation of straight line. So, statement 1 is correct.

Since $\dfrac{\sqrt{3}+1}{\sqrt{3}-1} > 0$

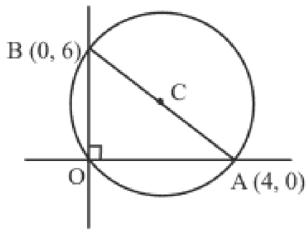
$\therefore$ Straight line passes through first and third quadrants.

So, statement 2 is also correct.

57. (b)

58. (a)

59. (c) Centre $C(x, y)$



$= \left(\dfrac{4+0}{2}, \dfrac{0+6}{2}\right) = (2, 3)$

Which satisfy the equation $3x - 4y + 6 = 0$

60. (a) Let equation of ellipse is $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

$\therefore$ It passes through (3, 2) and (1, 6)

$\therefore \dfrac{9}{b^2} + \dfrac{4}{a^2} = 1$ ...(i)

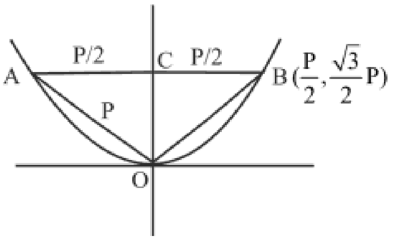
$\dfrac{1}{b^2} + \dfrac{36}{a^2} = 1$ ...(ii)

Solving equations (i) and (ii), we get

$a^2 = 40$ and $b^2 = 10$

Eccentricity of ellipse $= \sqrt{1 - \dfrac{b^2}{a^2}} = \sqrt{1 - \dfrac{10}{40}} = \dfrac{\sqrt{3}}{2}$

61. (c)



$OC = \sqrt{P^2 - \frac{P^2}{4}} = \frac{\sqrt{3}}{2}P$

$\therefore BC\left(\frac{P}{2}, \frac{\sqrt{3}}{2}P\right)$

Which lies on parabola

$x^2 = \sqrt{3}\, y$ i.e., $x^2 = qy$

[$\because$ Length of latus rectum = q]

$\therefore \frac{P^2}{4} = q\frac{\sqrt{3}}{2}P \Rightarrow P = 2\sqrt{3}\,q$

62. (b) $AC = \sqrt{(4-2)^2 + (6-4)^2 + (4-6)^2}$

$= \sqrt{4+4+4} = 2\sqrt{3}$

$BD = \sqrt{(8+2)^2 + (14+4)^2 + (12+2)^2}$

$= 2\sqrt{187}$

$\because AC \neq BD$ (i.e., Diagonals are not equal.)

So, statement is wrong.

Mid point of AC = (3, 5, 5) is equal to mid point of BD = (3, 5, 5)

So, statement 2 is correct

63. (b) $x^2 + y^2 + z^2 - 4x - 6y - 8z - 16 = 0$

$\Rightarrow (x^2 - 4x + 4) + (y^2 - 6y + 9) + (z^2 - 8z + 16)$

$= 16 + 4 + 9 + 16$

$\Rightarrow (x-2)^2 + (y-3)^2 + (z-4)^2 = (3\sqrt{5})^2$

$\therefore$ Centre of sphere = (2, 3, 4)

Radius = $3\sqrt{5}$

Since, radius $3\sqrt{5}$ is not equal to $\sqrt{2^2 + 3^2} = \sqrt{13}$

$\therefore$ Z-axis is not tangent to the sphere, so, statement 1 is wrong.

$\because 2 + 3 + 4 - 9 = 0$

$\therefore$ Centre of the sphere lies on the plane $x + y + z - 9 = 0$

So, statement 2 is correct.

64. (c) Equation plane is $\frac{x}{2} + \frac{y}{2} + \frac{z}{1} = 1$

$\Rightarrow x + y + 2z - 2 = 0$

$\therefore$ direction ratio's of normal are < 1, 1, 2 >

So, Direction cosines of normal are $< \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} >$

65. (c) The direction cosines of y-axis are < 0, 1, 0 >

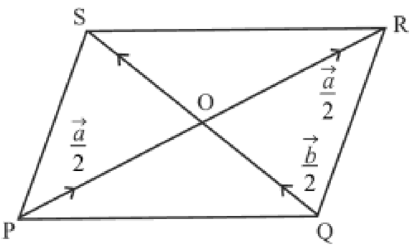
$\therefore$ The direction ratio of y-axis are in the form $< 0, a, 0 >, \forall a \in R$

So, statement 1 is correct.

We have direction ratio of a line perpendicular to Z-axis are in the form $< a, b, 0 >$

So, statement 2 is correct.

66. (d)



$\overrightarrow{PQ} = \overrightarrow{PO} - \overrightarrow{QO}$

$= \frac{\vec{a}}{2} - \frac{\vec{b}}{2} = \frac{1}{2}(\vec{a} - \vec{b})$

67. (c) According to question, $|\vec{a}| = |\vec{c}| = 1$

and $(\vec{a} + 2\vec{c}).(5\vec{a} - 4\vec{c}) = 0$

$\Rightarrow 5(\vec{a})^2 - 4\vec{a}.\vec{c} + 10\vec{c}.\vec{a} - 8|\vec{c}|^2 = 0$

$\Rightarrow 5 + 6\vec{a}.\vec{c} - 8 = 0 \quad \Rightarrow \vec{a}.\vec{c} = \frac{1}{2}$

Now, $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$

68. (d) $\{(3\vec{a} + 2\vec{b}) \times (5\vec{a} - 4\vec{c})\}.(\vec{b} + 2\vec{c})$

$= \{0 - 12\vec{a} \times \vec{b} + 10\vec{b} \times \vec{a} - 8\vec{b} \times \vec{c}\}.(\vec{b} + 2\vec{c})$

$= -12[\vec{a}\ \vec{c}\ \vec{b}] + 10[\vec{b}\ \ \vec{b}] - 8[\vec{b}\ \vec{c}\ \vec{b}]$

$-24[\vec{a}\ \vec{c}\ \vec{c}] + 20[\vec{b}\ \vec{a}\ \vec{c}] - 16[\vec{b}\ \vec{c}\ \vec{c}]$

$= 0$ [$\because \vec{a}, \vec{b}, \vec{c}$ are coplaner so, $[\vec{a}\ \vec{b}\ \vec{c} = 0]$ and $[\vec{b}\ \vec{a}\ \vec{c}]$, $[\vec{b}\ \vec{c}\ \vec{b}]$ all are zero]

69. (a) Given that angle between vectors is obtuse

$\therefore \vec{a}.\vec{c} < 0$

$\Rightarrow 2x^2 - 6x + x^2 < 0 \Rightarrow 3x^2 - 6x < 0$

$\Rightarrow 3x(x-2) < 0 \quad \Rightarrow 0 < x < 2$

70. (d) $\overrightarrow{CA} = (3\hat{j} + 3\hat{k}) - (\hat{j} + \hat{k}) = 2\hat{j} + 2\hat{k}$

$\overrightarrow{CB} = (3\hat{j} + 3\hat{k}) - (3\hat{i} + \hat{j} + 5\hat{k})$

$= -3\hat{i} + 2\hat{j} - 2\hat{k}$

$\overrightarrow{CA}.\overrightarrow{CB} = 0 + 4 - 4 = 0$

$\cos C = \frac{\overrightarrow{CA}.\overrightarrow{CB}}{|\overrightarrow{CA}||\overrightarrow{CB}|} = 0 \quad \Rightarrow C = \frac{\pi}{2}$

71. (a) We have $x = [x] + \{x\}$

$\therefore \quad y = [x] - x = -\{x\}$

Now, $z = [y] = [-\{x\}] \ [\because 0 < \{x\} < 1] = -1$

72. (a) Given that $f(g(x)) = g(f(x))$

$\Rightarrow \ (f(kx + 2) = g(4x + 1)$

$\Rightarrow \ 4(kx + 2) + 1 = k(4x + 1) + 2$

$\Rightarrow \ 4kx + 9 = 4kx + k + 2 \Rightarrow k = 7$

73. (b) Since, $f(x) = \log_{10}(x^2 + 2x + 11)$

$\therefore f'(x) = \frac{2x+2}{x^2+2x+11}\log_{10}^{e} = 0$

$\Rightarrow \ x = -1$ (critical point)

$f''(x) = \frac{2(x^2+2x+11)-(2x+2)^2}{(x^2+2x+11)^2}.\log_{10}^{e}$

$\because \ f''(-1) > 0$

$\therefore$ Minimum value of $f(x)$

$f(-1) = \log_{10}10 = 1$

74. (b) $I = \int (x^x)^2(1+\int lnx)dx$

Let $x^x = t$

$\Rightarrow \ ln\, t = x\, ln\, x \quad \Rightarrow \ dt = x^x(1 + ln\, x)\, dx$

$I = \int t\ dt = \frac{t^2}{2} + C = \frac{1}{2}(x^x)^2 + C$

$\frac{1}{2}x^{2x} + C$

75. (a) $\because \frac{d(xlnx)}{dx} = 1 + lnx$

$\therefore I = \int e^x\{1 + lnx + xlnx\}\,dx$

$= x\, ln\, x\, e^x + c\left[\because \int e^x(f(x) + f'(x)dx = e^x f(x) + c)\right]$

76. (c) $I = \int \frac{\cos^{3/2}x - \sin^{3/2}x}{\sin^{1/2}x.\cos^{1/2}x}dx$

$\int\left(\sin^{-1/2}x.\cos x - \cos^{-1/2}x.\sin x\right)dx$

$= 2.\sin^{1/2}x + 2.\cos^{1/2}x + C \quad = 2\sqrt{\sin x} + 2\sqrt{\cos x} + C$

77. (c) We have

$\int\sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}ln\left|x+\sqrt{x^2-a^2}\right|$

$\therefore \frac{dy}{dx} = \sqrt{x^2-16}.$

78. (b) $y = \left(x^x\right)^x = x^{x^2}$

Taking ln both sides, we get

$ln\, y = x^2 \ln x$

Differentiate w.r.t to x.

$\frac{1}{y}\frac{dy}{dx} = 2xlnx + x^2.\frac{1}{x}$

$\Rightarrow \ \frac{dy}{dx} = (x + 2x\, ln\, x) \qquad \Rightarrow \ \frac{dy}{dx} = xy\,(1 + 2\, ln\, x)$

$\Rightarrow \ \frac{dy}{dx} - xy\,(1 + 2\, ln\, x) = 0$

79. (b) Let $f(x) = 3(\sin x - \cos x) + 4(\cos^3 x - \sin^3 x)$

$f'(x) = 3(\cos x + \sin x) - 12\sin x\cos x(\cos x + \sin x)$

$= 3(\cos x + \sin x)(1 - 2\sin 2x) = 0$

$\Rightarrow x = \frac{3\pi}{4}, \frac{\pi}{6}$

$f''(x) = 3(-\sin x + \cos x)(1 - 2\sin 2x)$

$\qquad - 3(\cos x + \sin x)(-4\cos 2x)$

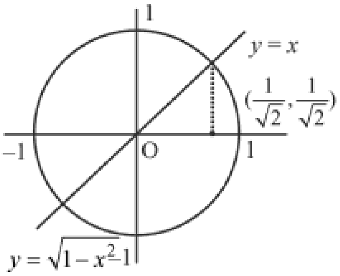
$f''\left(\frac{3\pi}{4}\right) < 0$ and $f''\left(\frac{\pi}{6}\right) > 0$

$\therefore \ f(x)$ is maximum at $x = \frac{3\pi}{4}$

$f\left(\frac{3\pi}{4}\right) = 3\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) + 4\left(\frac{-1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$

$= 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

80. (c)



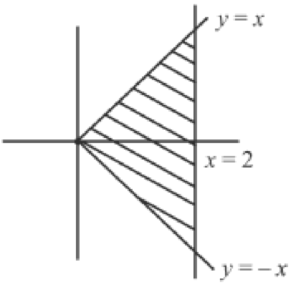
$\Rightarrow \ x^2 + y^2 = 1$

Required area

$\int_0^{1/\sqrt{2}} x\ dx + \int_{1/\sqrt{2}}^{1}\sqrt{1-x^2}\,dx$

$= \left[\frac{x^2}{2}\right]_0^{1/\sqrt{2}} + \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1} \quad = \frac{\pi}{8}$

81. (c)



$x = |y|$

Required area $= 2\int_0^2 x\ dx = 2\left[\frac{x^2}{2}\right]_0^2 = 2 \times 2 = 4$

82. (b) $f(\alpha) = \sqrt{\sec^2\alpha - 1} = \tan\alpha$

$\therefore \dfrac{f(\alpha) + f(\beta)}{1 - f(\alpha).f(\beta)} = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$= \tan(\alpha + \beta) = f(\alpha + \beta)$

83. (a) $\because \quad f(x) = ln(x + \sqrt{1 + x^2})$ ...(i)

$f(-x) = ln(-x + \sqrt{1 + x^2})$ ...(ii)

Adding equations (i) and (ii)

$f(x) + f(-x) = ln(\sqrt{1+x^2} + x) + ln(\sqrt{1+x^2} - x)$

$= ln 1 = 0$

84. (d) $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{1 - \cos 4x}} = \lim\limits_{x\to 0} \dfrac{2x}{\sqrt{2}\,|\sin 2x|} \times \dfrac{1}{2}$

L.H.L $= -\dfrac{1}{2\sqrt{2}}$ and R.H.L. $= \dfrac{1}{2\sqrt{2}}$

$\because$ L.H.L $\neq$ R.H.L.

So, $\lim\limits_{x\to 0} f(x)$ is not exist.

85. (a) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{4x - 2\pi}{\cos x}$

Apply L' Hospital rule.

$\lim\limits_{x\to\frac{\pi}{2}} \dfrac{4}{-\sin x} = -4$

86. (d) L.H.L. $= \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} = \dfrac{x^2 + x - x}{x} = \lim\limits_{x\to 0^-} x = 0$

R.H.L. $= \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \dfrac{x^2 + 2x}{x} = \lim\limits_{x\to 0^+} x + 2 = 2$

$\because$ L.H.L $\neq$ R.H.L.

$\therefore$ $\lim\limits_{x\to 0} f(x)$ does not exist.

87. (c) $\lim\limits_{h\to 0} \dfrac{\sin^2(x+h) - \sin^2 x}{h}$

Apply L' Hospital rule

$\lim\limits_{h\to 0} 2\sin(x+h).\cos(x+h)$

$= 2\sin x.\cos x$ [Take x as constant]

$= \sin 2x$

88. (c) $h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$= \{f(x)\}^2 + \{f'(x)\}^2$

$h'(x) = 2f(x).f'(x) + 2f'(x)f''(x)$

$= 2f(x)f'(x) - 2f(x)f'(x) = 0$

$\because$ h' $(x) = 0$

$\therefore$ $h'(3) = 0$

Statement 1 is correct.

$h'(x) = 0 \Rightarrow h(x) =$ constant function

$\therefore$ $h(1) = f(2)$

Statement 2 is correct.

89. (b) $y = \left[ln\left(\dfrac{x^2 - x + 1}{x^2 + x + 1}\right)\right]^2$

$\dfrac{dy}{dx} = 2ln\left(\dfrac{x^2 - x + 1}{x^2 + x + 1}\right) \times \dfrac{x^2 + x + 1}{x^2 - x + 1}$

$\times \dfrac{(2x-1)(x^2 + x + 1) - (2x+1)(x^2 - x + 1)}{(x^2 + x - 1)^2}$

Put $x = 0$

$\dfrac{dy}{dx} = 0$

90. (d) Let $f(x) = \dfrac{x^8 + x^4 + 1}{x^4 - x^2 + 1}$

$= \dfrac{x^8 + 2x^4 + 1 - x^4}{x^4 - x^2 + 1} = \dfrac{(x^4 + 1)^2 - (x^2)^2}{(x^4 - x^2 + 1)}$

$= \dfrac{(x^4 - x^2 + 1)(x^4 + x^2 + 1)}{x^4 - x^2 + 1}$

$f(x) = x^4 + x^2 + 1$

$f'(x) = 4x^3 + 2x = ax + bx^3$

$\Rightarrow \quad a = 2, b = 4 \quad \Rightarrow \quad b = 2a$

91. (a) $f(x) = (P\sec x)^2 + (q\,\mathrm{cosec}\,x)^2$

$f'(x) = 2P^2\sec^2 x\tan x - 2q^2\,\mathrm{cosec}^2 x.\cot x = 0$

$\Rightarrow \dfrac{p^2}{q^2} = \dfrac{\mathrm{cosec}^2 x.\cot x}{\sec^2 x.\tan x} = \cot^4 x$

$\Rightarrow \quad \cot^2 x = \dfrac{p}{q} \Rightarrow \tan^2 x = \dfrac{q}{p}$

$f'(x) = 2p^2(\tan x + \tan^3 x) - 2q^2(\cot x + \cot^3 x)$

$f''(x) = 2p^2\sec^2 x(1 + 3\tan^2 x) + 2q^2\,\mathrm{cosec}^2 x(1 + 3\cot^2 x)$

For $\tan^2 x = \dfrac{q}{p}$

$f''(x) > 0$

$\therefore$ $f(x)$ is minimum when $\tan^2 x = \dfrac{q}{p}$

92. (b) $f(x) = \sum\limits_{j=1}^{7}(x - j)^2 = \sum\limits_{j=1}^{7}(x^2 - 2xj + j^2)$

$x^2\sum\limits_{j=1}^{7} 1 - 2x\sum\limits_{j=1}^{7} j + \sum\limits_{j=1}^{7} j^2$

$= 7x^2 - 2x\dfrac{7(7+1)}{2} + \dfrac{7(7+1)(14+1)}{6}$

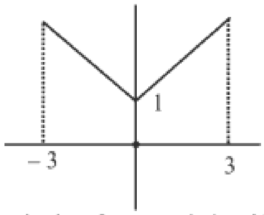
$= 7x^2 - 56x + 140$

$f'(x) = 14x - 56 = 0 \Rightarrow x = 4$

$f''(x) = 14 > 0$

$\therefore$ $f(x)$ is minimum at $x = 4$.

93. (b)



It is clear from graph that $f(x)$ is maximum at 3 and – 3 and minimum at $x = 1$.

94. (b) $I = \int_0^1 ln\left(\frac{1}{x}-1\right)dx = \int_0^1 [ln(1-x) - lnx]dx$

$= \int_0^1 ln(1-x)dx - \int_0^1 lnx\ dx$

$= \int_0^1 ln(1-x)dx - \int_0^1 ln(1-x)\ dx = 0$

95. (d)

96. (a) $I = \int_{-\pi/2}^{\pi/2} (e^{\cos x}\sin x + e^{\sin x}\cos x)dx$

$= \int_{-\pi/2}^{\pi/2} e^{\cos x}.\sin x\ dx \pm \int_{-\pi/2}^{\pi/2} e^{\sin x}.\cos x\ dx$

$\because$ $e^{\cos x}$. $\sin x$ is odd function.

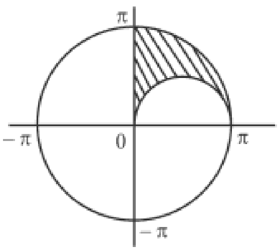
$\therefore \int_{-\pi/2}^{\pi/2} e^{\cos x}.\sin x\ dx = 0$

Let $\sin x = t$, $\cos x\ dx = dt$

when $x = \frac{-\pi}{2}$, $t = -1$, when $x = \frac{\pi}{2}$, $t = 1$

$\therefore$ $I = \int_{-1}^{1} e^t\ dt = e - e^{-1} = \frac{e^2-1}{e}$

97. (b)



Required area

$= \frac{1}{4}\pi(\pi)^2 - \int_0^\pi \sin x\ dx = \frac{\pi^3}{4} - [-\cos x]_0^\pi = \frac{\pi^3}{4} - 2$

98. (b) $1.\frac{dy}{dx} + \cos\left(\frac{dy}{dx}\right) = 0$

Degree is not define, so, statement 1 is wrong.

2. Highest derivative is two therefore order is 2. So, statement 2 is correct.

99. (b) Let equation of parabola is

$x^2 = 4ay$ ...(i)

$\therefore$ $2x = 4a\frac{dy}{dx}$

$\Rightarrow 2x = \frac{x^2}{y}\frac{dy}{dx} \Rightarrow x\frac{dy}{dx} - 2y = 0$

100. (c) $(dy - dx) + \cos x\,(dy + dx) = 0$

$\Rightarrow$ $(1 + \cos x)dy = (1 - \cos x)dx$

$\Rightarrow \frac{dy}{dx} = \frac{1-\cos x}{1+\cos x} = \tan^2\frac{x}{2} = \sec^2\frac{x}{2} - 1$

$\int dy = \int\left(\sec^2\frac{x}{2} - 1\right)dx$

$\Rightarrow$ $y = 2\tan\frac{x}{2} - x + C$

101. (c) Mean of squares of first n natural number

$x = \frac{n(n+1)(2n+1)}{6n} = \frac{(n+1)(2n+1)}{6}$

Square of mean of first n natural numbers

$y = \left(\frac{n(n+1)}{2n}\right)^2 = \frac{(n+1)^2}{4}$

$\therefore \frac{x}{y} = \frac{2(2n+1)}{3(n+1)} = \frac{55}{42}$

$\Rightarrow$ $28\,(2n + 1) = 55(n + 1) \Rightarrow n = 27$

102. (b) Prime number between 1 to 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

1 is not prime and composite number.

$\therefore$ Total number of composite number between

= 50 – 16 = 34

$\therefore$ Probability $= \frac{34}{50} = \frac{17}{25}$

103. (a) $\because$ ${}^nC_7 = \frac{n!}{(n-7)!7!}$

$= \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{7!}$

is not divisible by 7 for $n > 7$

$\therefore$ $n(E) = 0$ $\Rightarrow$ $P(E) = 0$

104. (b) $n(s) = 10 \times 9 = 90$

Sum of $x + y$ is divisible by 4 are

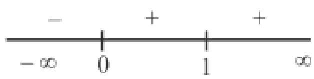
(1, 3), (1, 7), (2, 6), (2, 10), (3, 1), (3, 5), (3, 9), (4, 8), (5, 3), (5, 7), (6, 2), (6, 10), (7, 1), (7, 5), (7, 9), (8, 4), (9, 3), (9, 7), (10, 2), (10, 6)

$n(E) = 20$

$\therefore$ $P(E) = \frac{20}{90} = \frac{2}{9}$

105. (c) $x + \frac{1}{x} > 2 \Rightarrow \frac{x^2+1}{x} - 2 > 0$

$\Rightarrow \frac{x^2-2x+1}{x} > 0 \Rightarrow \frac{(x-1)^2}{x} > 0$



$x \in (0, \infty) - \{1\}$ and $x \in N$

$\therefore$ $n = \{2, 3, 4, ...n)$

$\therefore \quad n(E) = n - 1$

$P(E) = \frac{n-1}{n}$

106. (b) $n(s) = 6 \times 6 \times 6$

Different numbers that are in AP.

(1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (1, 3, 5), (2, 4, 6), (3, 2, 1), (4, 3, 2), (5, 4, 3), (6, 5, 4), (5, 3, 1), (6, 4, 2)

$n(E) = 12$

$P(E) = \frac{12}{6 \times 6 \times 6} = \frac{1}{18}$

107. (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.7 - 0.3 = 0.9$

$\therefore \quad P(A' \cap B') + P(A' \cap B) + P(A \cap B')$

$= 1 - P(A \cup B) + P(B) - P(A \cap B) + P(A) - P(A \cap B)$

$= 1 - P(A \cap B) = 1 - 0.3 = 0.7$

108. (a) $n = 5$

$P = P(T) = \frac{1}{2}, q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(x \leq 4) = 1 - P(x = 5)$

$= 1 - \left(\frac{1}{2}\right)^5 = 1 - \frac{1}{32} = \frac{31}{32}$

109. (d) $n(s) = 6 \times 6 \times 6 = 216$

Sum greater than or equal to 15 i.e. 15, 16, 17, 18 are

(4, 5, 6) → 6 ways;

(5, 5, 6) → 3ways; (5, 5, 5) → 1 way; (6, 6, 3) → 3 ways (6, 6, 4) → 3 ways; (6, 6, 5) → 3 ways; (6, 6, 6) → 1way

$\therefore \quad n(E) = 20$

$P(E) = \frac{20}{216} = \frac{5}{54}$

110. (c) $n = 4, P = 0.5, q = 1 - 0.5 = 0.5$

$P(x \geq 1) = 1 - P(x = 0) = 1 - (0.5)^4$

$= 1 - \frac{1}{16} = \frac{15}{16}$

111. (c) W = 2, B = 3, R = 4

Total balls = 2 + 3 + 4 = 9

∴ Number of ways of drawing 3 balls with at least one black ball

$= {}^3C_1 . {}^6C_2 + {}^3C_2 . {}^6C_1 + {}^3C_3$

$= 45 + 18 + 1 = 64$

112. (d) $n = 5, q = P(\text{sunk}) = \frac{1}{5}, P = 1 - \frac{1}{5} = \frac{4}{5}$

$P(x = 3) = {}^5C_3 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^2 = \frac{128}{625}$

113. (a) Number of favorable outcomes (either spade or ace)

= 13 + 3 = 16

Number of unfavorable outcomes

= 52 – 16 = 36

$\therefore$ Odds against his winnings $= \frac{\text{No. of unfavorable}}{\text{No. of favorable}}$

$= \frac{36}{16} = 9 : 4$

114. (c) $\therefore r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \; \Sigma(y_i - \bar{y})^2}} = 0.7$

At time of silver jubilee new $\bar{x}$ and $\bar{y}$ increase 25 also each xi and yi increase with 25

Therefore, $(xi - \bar{x})$, $(yi - \bar{y})$ not change

So, coefficient of correlation remains same

Hence, new coefficient of correlation = 0.7

115. (b) Let E_1 = strike; E_2 = no strike

F = Job completed on time.

$\therefore \quad P(F) = P(F/E_1).P(E_1) + P(F/E_2).P(E_2)$

$= 0.35 \times 0.6 + 0.4 \times 0.85 = 0.55$

$P(\bar{F}) = P$ (Job not completed on time)

$= 1 - 0.55 = 0.45$

116. (d) C.V. of Mathematics $= \frac{15}{40} \times 100 = 37.5\%$

C.V. of Physics $= \frac{12}{28} \times 100 = 42.86\%$

C.V. of Chemistry $= \frac{14}{38} \times 100 = 36.84\%$

C.V. of Biology $= \frac{16}{36} \times 100 = 44.44\%$

Since C.V. of Biology is greatest, therefore, Biology have more variability.

117. (a) C.V. of mathematics = 37.5%

118. (c)

Class	0-10	10-20	20-30	30-40	40-50	50-60
fi	1	2	4	6	4	3
C.F.	1	3	7	13	17	20

$\frac{N}{2} = \frac{20}{2} = 10$

$\therefore$ Median class = 30 – 40

$\therefore$ Median $= l + \frac{\frac{N}{2} - C}{f} \times h.$

Me $= 30 + \frac{10 - 7}{6} \times 10$

$= 30 + 5 = 35$

119. (d)

Class	0-10	10-20	20-30	30-40	40-50	50-60	Total
xi	5	15	25	35	45	55	
|*xi-Me*|	30	20	10	0	10	20	
fi	1	2	4	6	4	3	20
fi |*xi-Me*|	30	40	40	0	40	60	210

Meadian = $\frac{\sum |x_i - M_e|}{\sum f_i} = \frac{210}{20} = 10.5$

120. (b)

Class	0-10	10-20	20-30	30-40	40-50	50-60	Total
fi	1	2	4	6	4	3	20
xi	5	15	25	35	45	55	
xi fi	5	30	100	210	180	165	690
$\|xi-\bar{x}\|$	29.5	19.5	9.5	0.5	10.5	20.5	
$fi\|xi-\bar{x}\|$	29.5	39	38	3	42	61.5	213

Mean = $\frac{\sum x_i f_i}{\sum f_i} = \frac{690}{20} = 34.5$

Mean deviation about mean

$= \frac{\sum |x_i - \bar{x}| f_i}{\sum} = \frac{213}{20} = 10.65$