NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2021 (II)

PAPER I : Mathematics

1. If $x^2 + x + 1 = 0$, then what is the value of $x^{199} + x^{200} + x^{201}$?

(a) – 1 (b) 0
(c) 1 (d) 3

(*b*) Given that,

$$x^2 + x + 1 = 0 \quad \text{...(i)}$$

$$\therefore x^{199} + x^{200} + x^{201} = x^{199}(1 + x + x^2)$$
$$= x^{199} \times 0$$
$$= 0$$

2. If x, y, z are in GP, then which of the following is/are correct?

1. $\ln(3x), \ln(3y), \ln(3z)$ are in AP.
2. $xyz + \ln(x), xyz + \ln(y), xyz + \ln(z)$ are in HP.

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(*a*) Given that x, y, z are in GP.

$$\Rightarrow \quad y^2 = xz \quad \text{...(i)}$$

(1) If $\log(3x), \log(3y), \log(3z)$ are in AP

Then, $2\log(3y) = \log(3x) + \log(3z)$

$$9y^2 = (9xz)$$
$$9y^2 = (9xz)$$
$$y^2 = xz$$

Hence, statement (1) is correct.

Hence, we can say if x, y, z are in GP.

$\therefore \log x, \log y, \log z$ are in AP.

$\Rightarrow xyz + \log x, xyz + \log y, xyz + \log z$ are in AP.

Hence, Statement (2) is wrong.

$\therefore$ Option (a) is correct.

3. If $\log_{10} 2, \log_{10}(2^x - 1), \log_{10}(2^x + 3)$ are in AP, then what is x equal to?

(a) 0 (b) 1
(c) $\log_2 5$ (d) $\log_5 2$

(*c*) Given that, $\log_{10} 2, \log_{10}(2^x - 1)$, $\log_{10}(2^x + 3)$ are in AP.

$\therefore 2\log_{10}(2^x - 1) = \log_{10} 2 + \log_{10}(2^x + 3)$

$\log_{10}(2^x - 1)^2 = \log_{10} 2(2^x + 3)$

$\Rightarrow \quad 2^{2x} + 1 - 2 \cdot 2^x = 2 \cdot 2^x + 6$

$\Rightarrow (2^x)^2 - 4(2^x) - 5 = 0$

Let $\quad 2^x = y$

$\Rightarrow \quad y^2 - 4y - 5 = 0$

$(y - 5)(y + 1) = 0$

$\Rightarrow \quad y = 5$ or $y = -1$

(Ignore because 2^x cannot be negative)

$\Rightarrow \quad y = 5 \Rightarrow 2^x = 5$

$x = \log_2 5$

Hence, option (c) is correct.

4. Let $S = \{2, 3, 4, 5, 6, 7, 9\}$. How many different 3-digit numbers (with all digits different) from S can be made which are less than 500?

(a) 30 (b) 49
(c) 90 (d) 147

(*c*) Let $\quad S = \{2, 3, 4, 5, 6, 7, 9\}$

$\Rightarrow \quad n(S) = 7$

Three digit number less than

500 = [ ][ ][ ]
↓ ↓ ↓
3 6 5

$= 3 \times 6 \times 5 = 90$

$\therefore$ Option (c) is correct.

Note Hundreds digit can be filled with 3 choices that are 2, 3, 4.
Similarly, tens digit can be filled with 6 ways and unit digit can be filled with 5 ways.

5. If $p = (1111 \ldots$ up to n digits), then what is the value of $9p^2 + p$?

(a) $10^n p$ (b) $2p \cdot 10^n$
(c) $10^n p - 1$ (d) $10^n p + 1$

(*a*) Given that,

$p = (1111\ldots$ upto n digits)

$= 1 + 10 + 10^2 + \ldots + 10^{n-1}$

$= \dfrac{1(10^n - 1)}{10 - 1}$

$\left[\because a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(r^n - 1)}{r - 1}\right]$

$\Rightarrow \quad p = \dfrac{10^n - 1}{9}$

$\Rightarrow \quad 9p = 10^n - 1$

$\Rightarrow \quad 9p + 1 = 10^n$

$\Rightarrow \quad 9p^2 + p = 10^n \cdot p$

$\therefore$ Hence, option (a) is correct.

6. The quadratic equation $3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$ has real roots of equal magnitude and opposite sign. Which one of the following is correct?

(a) $0 < k < \dfrac{5}{3}$

(b) $0 < k < \dfrac{3}{5}$ only

(c) $\dfrac{3}{5} < k < \dfrac{5}{3}$

(d) No such value of k exists.

(*d*) Since, we know that if a quadratic equation $ax^2 + bx + c = 0$ has real roots of equal magnitude and opposite sign.

Then, $\quad b = 0 \quad$ ...(i)

and product of roots $< 0 \quad$ ...(ii)

In the given quadratic equation,

$3x^2 - (k^2 - 5k)x + 3k^2 - 5k = 0$

$a = 3, b = -(k^2 + 5k), c = 3k^2 - 5k$

By Eq. (i), $b = 0$

$\Rightarrow -(k^2 + 5k) = 0$

$\Rightarrow k(k + 5) = 0$

$\therefore k = 0, -5$

By Eq. (ii), Product of roots < 0

$$\frac{c}{a} < 0$$

$$\Rightarrow \frac{3k^2 - 5k}{3} < 0$$

$$\Rightarrow k(3k - 5) < 0$$

$$\therefore 0 < k < \frac{5}{3}$$

From (i) and (ii) no such values of k exists.

Hence, option (d) is correct.

7. If $a_n = n(n!)$, then what is $a_1 + a_2 + a_3 + \ldots + a_{10}$ equal to?

(a) 10! – 1 (b) 11! + 1
(c) 10! + 1 (d) 11! – 1

(*d*) Given, $a_n = n(n!)$

$= (n + 1 - 1)(n!)$

$= (n + 1)n! - n!$

$= (n + 1)! - n!$

$\therefore a_1 = 2! - 1!$

$a_2 = 3! - 2!$

... ...

$a_{10} = 11! - 10!$

$\therefore a_1 + a_2 + a_3 + \ldots + a_{10}$

$= 2! + 1! + 3! - 2! + 4! - 3! + \ldots + 11! - 10!$

$= 11! - 1!$

$= 11! - 1$

$\therefore$ Option (d) is correct.

8. If p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then how many possible values can q have?

(a) Nil (b) One
(c) Two (d) Three

(*b*) Given quadratic equation

$$x^2 + px + q = 0$$

and roots are p and q (non zero)

$$\therefore \text{Sum of roots} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$p + q = -p \quad \text{...(i)}$$

$$\therefore \text{Product of roots} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$pq = q \quad \text{...(ii)}$$

$\Rightarrow pq - q = 0$

$q(p - 1) = 0$

$\because q \neq 0 \Rightarrow p - 1 = 0$

$p = 1$

From Eq. (i)

$p + q = -p$

$q = -2p = -2(1)$

$q = -2$

$\therefore$ Option (b) is correct.

9. If $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

then what is $\begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix}$ equal to?

(a) Δ (b) 7Δ
(c) 72Δ (d) -72Δ

(*d*) Given, $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$$= \begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix}$$

$$= 6\begin{vmatrix} 3d + 5g & 4a + 7g & g \\ 3e + 5h & 4b + 7h & h \\ 3f + 5i & 4c + 7i & i \end{vmatrix}$$

By $C_1 \to C_1 - 5C_3, C_2 \to C_2 - 7C_3$

$$= 6\begin{vmatrix} 3d & 4a & g \\ 3e & 4b & h \\ 3f & 4c & i \end{vmatrix}$$

$$= 6 \times 3 \times 4\begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix} \text{ By } C_2 \leftrightarrow C_1$$

$$= -72\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} \quad \text{By } R \leftrightarrow C$$

$$= -72\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -72\Delta$$

Hence, option (d) is correct.

10. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in HP, then which of the following is/are correct?

1. a, b, c are in AP
2. $(b+c)^2, (c+a)^2, (a+b)^2$ are in GP.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*a*) Given that,

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in HP.

$\Rightarrow b + c, c + a, a + b$ are in AP.

$\Rightarrow (a + b + c) - (b + c), (a + b + c)$

$-(c + a), (a + b + c) - (a + b)$ are in AP.

$\Rightarrow a, b, c$ are in AP.

2. From 1; a, b, c are in AP.

$\therefore b = a + d, c = a + 2d$

where, d is common difference.

$\therefore (b + c)^2 = (a + d + a + 2d)^2$

$= (2a + 3d)^2$

$(c + a)^4 = (a + 2d + a)^4 = (2a + 2d)^4$

$(a + b)^2 = (a + a + d)^2 = (2a + d)^2$

Here, $(c + a)^4 \neq (b + c)^2.(a + b)^2$

So, $(b+c)^2, (c+a)^2, (a+b)^2$ are not in G.P.

Hence, option (a) is correct.

11. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$,

where $a \in \mathbb{N}$, then what is $A^{100} - A^{50} - 2A^{25}$ equal to?

(a) $-2I$ (b) $-I$
(c) $2I$ (d) I

where I is the identity matrix.

(*a*) $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} a \in \mathbb{N}$

The sequence for given matrix A is

$$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{100} - A^{50} - 2A^{25} = \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 1 - 2 & 100a - 50a - 50a \\ 0 - 0 - 0 & 1 - 1 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = -2I$$

Hence, option (a) is correct.

12. If $\begin{vmatrix} a & -b & a - b - c \\ -a & b & -a + b - c \\ -a & -b & -a - b + c \end{vmatrix} - kabc = 0$

$(a \neq 0, b \neq 0, c \neq 0)$

then what is the value of k?

(a) – 4 (b) – 2
(c) 2 (d) 4

(*a*) Given that,

$$\begin{vmatrix} a & -b & a - b - c \\ -a & b & -a + b - c \\ -a & -b & -a - b + c \end{vmatrix} - kabc = 0$$

$(a \neq 0, b \neq 0, c \neq 0)$

$R_1 \to R_1 + R_2$

$$\begin{vmatrix} 0 & 0 & -2c \\ -a & b & -a + b - c \\ -a & -b & -a - b + c \end{vmatrix} - kabc = 0$$

$-2c[(-a)(-b) - (-a)b] - kabc = 0$

$-2c(2ab) - kabc = 0$

$-kabc = 4abc$

$\Rightarrow k = -4$

Hence, option (a) is correct.

13. What is $\sum_{n=1}^{8n+7} i^n$ equal to, where $i = \sqrt{-1}$?

(a) -1 (b) 1
(c) i (d) $-i$

(**a**) Let $S = \sum_{n=1}^{8n+7} i^n$

$$S = i + i^2 + i^3 + \ldots + i^{8n+7}$$
$$= i\left[\frac{(i)^{8n+7}-1}{i-1}\right] = i\left[\frac{i^{4(2n+1)+3}-1}{i-1}\right]$$
$$= i\left(\frac{i^3-1}{i-1}\right) \quad [\because i^{4n+r} = i^r]$$
$$= i\left[\frac{-i-1}{i-1}\right] = \frac{-i^2-i}{i-1}$$
$$= \frac{1-i}{i-1} = -1$$

14. If $z = x + iy$, where $i = \sqrt{-1}$, then what does the equation $z\bar{z} + |z|^2 + 4(z+\bar{z}) - 48 = 0$ represent?

(a) Straight line
(b) Parabola
(c) Circle
(d) Pair of straight lines

(**c**) Given, $z = x + iy$

$\therefore \quad \bar{z} = x - iy$

$\therefore \quad z + \bar{z} = 2x$

and $\quad |z|^2 = x^2 + y^2$

$\therefore \quad z\bar{z} + |z|^2 + 4(z+\bar{z}) - 48 = 0$

$$(x+iy)(x-iy) + x^2 + y^2 + 4(2x) - 48 = 0$$
$$x^2 + y^2 + x^2 + y^2 + 8x - 48 = 0$$
$$2x^2 + 2y^2 + 8x - 48 = 0$$
$$x^2 + y^2 + 4x - 24 = 0$$

which represents circle.

Hence, option (c) is correct.

15. Which one of the following is a square root of $2a + 2\sqrt{a^2+b^2}$, where $a, b \in \mathbb{R}$?

(a) $\sqrt{a+ib} + \sqrt{a-ib}$
(b) $\sqrt{a+ib} - \sqrt{a-ib}$
(c) $2a + ib$
(d) $2a - ib$, where $i = \sqrt{-1}$

(**a**) $2a + 2\sqrt{a^2+b^2}$

$$= 2a + ib - ib + 2\sqrt{a^2 - i^2b^2}$$
$$= (a+ib) + (a-ib) + 2\sqrt{(a+ib)(a-ib)}$$
$$= (\sqrt{a+ib} + \sqrt{a-ib})^2$$

Hence, square root of

$$2a + 2\sqrt{a^2+b^2} = \sqrt{a+ib} + \sqrt{a-ib}$$

16. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a^2 + b^2 - 2ac = 0$
(b) $-a^2 + b^2 + 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$
(d) $a^2 + b^2 + 2ac = 0$

(**c**) Given, equation $ax^2 + bx + c = 0$ ...(i)

$\because$ Roots are $\sin\theta$ and $\cos\theta$

$\therefore \quad \sin\theta + \cos\theta = -\frac{b}{a}$

and $\quad \sin\theta \cdot \cos\theta = \frac{c}{d}$

On squaring both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$$
$$1 + 2\frac{c}{a} = \frac{b^2}{a^2}$$
$$a(a+2c) = b^2$$
$$\Rightarrow \quad a^2 - b^2 + 2ac = 0$$

$\therefore$ Option (c) is correct.

17. If $C(n, 4)$, $C(n, 5)$ and $C(n, 6)$ are in AP, then what is the value of n?

(a) 7 (b) 8 (c) 9 (d) 10

(**a**) ${}^nC_4 \cdot {}^nC_5$ and ${}^nC_6$ are in AP.

$$\Rightarrow \quad 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$
$$\frac{2n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$
$$\frac{2(n!)}{5(4!)(n-5)(n-6)!}$$
$$= \frac{n!}{4!(n-6)!}\left[\frac{1}{(n-4)(n-5)} + \frac{1}{6\times 5}\right]$$
$$\frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$
$$\frac{2n-8-5}{5(n^2-9n+20)} = \frac{1}{30}$$
$$30(2n-13) = 5n^2 - 45n + 100$$
$$5n^2 - 105n + 490 = 0$$
$$n^2 - 21n + 98 = 0$$
$$(n-14)(n-7) = 0$$
$$n = 14 \text{ or } n = 7$$

$\therefore$ Option (a) is correct.

18. How many 4-letter words (with or without meaning) containing two vowels can be constructed using only the letters (without repetition) of the word 'LUCKNOW'?

(a) 240 (b) 200
(c) 150 (d) 120

(**a**) In LUCKNOW, there are 2 vowels and 5 consonants.

$\therefore$ 4 letter words $= {}^5C_2 \cdot {}^2C_2 \cdot 4!$

$$= 10 \times 1 \times 24 = 240$$

$\therefore$ Option (a) is correct.

19. Suppose 20 distinct points are placed randomly on a circle. Which of the following statements is/are correct?

1. The number of straight lines that can be drawn by joining any two of these points is 380.
2. The number of triangles that can be drawn by joining any three of these points is 1140.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(**b**) Given, that there are 20 distinct points on a circle and we have to draw a straight line by joining any two of these points.

Hence, number of straight lines

$$= {}^{20}C_2 = \frac{20\times 19}{2} = 190$$

$\therefore$ Statement (1) is wrong.

and Number of triangle

$$= {}^{20}C_3 = \frac{20\times 19\times 18}{1\times 2\times 3} = 1140$$

$\therefore$ Statement (2) is correct.

Hence, option (b) is correct.

20. How many terms are there in the expansion of $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21}$ where $a \neq 0, b \neq 0$?

(a) 21 (b) 22
(c) 42 (d) 43

(**d**) $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21}$

$$\Rightarrow \quad \left[\left(\frac{a}{b} + \frac{b}{a}\right)^2\right]^{21} = \left(\frac{a}{b} + \frac{b}{a}\right)^{42}$$

Since, we know that number of terms in the expansion of $(a+b)^n = n + 1$

Hence, total number of terms

$$= 42 + 1 = 43$$

$\therefore$ Option (d) is correct.

21. For what values of k is the system of equations $2k^2x + 3y - 1 = 0$, $7x - 2y + 3 = 0$, $6kx + y + 1 = 0$ consistent?

(a) $\frac{3\pm\sqrt{11}}{10}$ (b) $\frac{21\pm\sqrt{161}}{10}$
(c) $\frac{3\pm\sqrt{7}}{10}$ (d) $\frac{4\pm\sqrt{11}}{10}$

(**b**) Given equations,

$$2k^2x + 3y - 1 = 0$$
$$7x - 2y + 3 = 0$$
$$6kx + y + 1 = 0$$

For consistency, determinant formed by the equations

$$\begin{vmatrix} 2k^2 & 3 & -1 \\ 7 & -2 & 3 \\ 6k & 1 & 1 \end{vmatrix} = 0$$

$$2k^2(-2-3) - 3(7-18k)$$
$$-1(7+12k) = 0$$
$$-10k^2 - 21 + 54k - 7 - 12k = 0$$
$$-10k^2 - 42k - 28 = 0$$
$$5k^2 - 21k + 14 = 0$$
$$k = \frac{21 \pm \sqrt{441 - 280}}{10}$$
$$k = \frac{21 \pm \sqrt{161}}{10}$$

Hence, option (b) is correct.

22. The inverse of a matrix A is given by $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

What is A equal to?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

(**a**) $A = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

$$\therefore \quad |A| = (-2)\left(-\frac{1}{2}\right) - \frac{3}{2} = -\frac{1}{2} \neq 0$$

$$A_{11} = -\frac{1}{2}, A_{12} = -\frac{3}{2}$$
$$A_{21} = -1, A_{22} = -2$$

$$\therefore \text{adj } A = \begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-\frac{1}{2}} \begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}$$

$$= 2\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{3}{2} & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Hence, option (a) is correct.

23. What is the period of the function $f(x) = \ln(2 + \sin^2 x)$?

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) 3π

(**b**) $f(x) = \ln(2 + \sin^2 x)$

$\because$ Period of $\sin^2 x$ is π.

and $f(\pi + x) = \log\{2 + \sin^2(\pi + x)\}$
$= \log\{2 + \sin^2 x\}$
$= f(x)$

Hence, period of $\ln(2 + \sin^2 x) = \pi$

$\therefore$ Option (b) is correct.

24. If $\sin(A + B) = 1$ and $2\sin(A - B) = 1$, where $0 < A, B < \frac{\pi}{2}$, then what is $\tan A : \tan B$ equal to?

(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 3 : 1

(**d**) Given, $\sin(A + B) = 1$

and $2\sin(A - B) = 1$

$$0 < A, B < \frac{\pi}{2}$$

$$\because \quad \sin(A + B) = 1 = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad A + B = \frac{\pi}{2} \quad \text{...(i)}$$

$$2\sin(A - B) = 1$$

$$\Rightarrow \quad \sin(A - B) = \frac{1}{2} = \sin\frac{\pi}{6}$$

$$\Rightarrow \quad A - B = \frac{\pi}{6} \quad \text{...(ii)}$$

Now, adding Eq. (i) and Eq. (ii), we get

$$2A = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6}$$

$$A = \frac{\pi}{3}, B = \frac{\pi}{6}$$

$$\therefore \quad \tan A : \tan B = \tan\frac{\pi}{3} : \tan\frac{\pi}{6}$$

$$= \sqrt{3} : \frac{1}{\sqrt{3}} = 3 : 1$$

$\therefore$ Option (d) is correct.

25. Consider a regular polygon with 10 sides. What is the number of triangles that can be formed by joining the vertices which have no common side with any of the sides of the polygon?

(a) 25 (b) 50
(c) 75 (d) 100

(**b**) Given number of sides $(n) = 10$

Number of triangles which have no common side with any of the sides of the polygon $= \frac{n(n-4)(n-5)}{3!}$

$\therefore$ Number of triangles

$$= \frac{10(10-4)(10-5)}{6}$$
$$= \frac{10 \times 6 \times 5}{6}$$
$$= 50$$

Hence, option (b) is correct.

26. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$.

What is the sum of the absolute values of the roots?

(a) 4 (b) 6
(c) 8 (d) 10

(**c**) Given equation,

$$x^4 - 10x^2 + 9 = 0$$

Let $y = x^2$

$$\therefore \quad y^2 - 10y + 9 = 0$$
$$(y - 9)(y - 1) = 0$$
$$\Rightarrow \quad y = 9 \text{ or } y = 1$$
$$x^2 = 9 \text{ or } y = 1$$
$$x^2 = 9 \text{ or } x^2 = 1$$
$$x = \pm 3, x = \pm 1$$
$$\therefore \quad \text{Sum} = |3| + |-3| + |1| + |-1|$$
$$= 8$$

Hence, option (c) is correct.

27. Consider the expansion of $(1 + x)^n$. Let p, q, r and s be the coefficients of first, second, nth and $(n + 1)$th terms respectively. What is $(ps + qr)$ equal to?

(a) $1 + 2n$ (b) $1 + 2n^2$
(c) $1 + n^2$ (d) $1 + 4n$

(**c**) Given, $(1 + x)^n$

In the above expansion, $(r + 1)$th term

$$T_{r+1} = {}^nC_r x^r$$
$$\therefore \quad T_1 = {}^nC_0 x^0, T_2 = {}^nC_1 x'$$
$$p = 1 \quad \text{...(i)}$$
$$\therefore \quad {}^nC_1 = q$$
$$n = q \quad \text{...(ii)}$$
$$T_n = {}^nC_{n-1} x^{n-1},$$
$$T_{n+1} = {}^nC_n x^n$$
$$\therefore \quad r = n \quad \text{...(iii)}$$
$$\therefore \quad s = 1 \quad \text{...(iv)}$$
$$\therefore \quad (ps + qr) = 1 \cdot 1 + n \cdot n = 1 + n^2$$
$$= (1 + n^2)$$

Hence, option (c) is correct.

28. Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ for $0 \le x, y, z \le 1$. What is the value of $x^{1000} + y^{1001} + z^{1002}$?

(a) 0 (b) 1
(c) 3 (d) 6

(**c**) Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

Which is only possible when,

$$\sin^{-1} x = \frac{\pi}{2}, \sin^{-1} y = \frac{\pi}{2},$$
$$\sin^{-1} z = \frac{\pi}{2}$$
$$\Rightarrow \quad x = 1, y = 1, z = 1$$
$$\therefore \quad x^{1000} + y^{1001} + z^{1002}$$
$$= 1 + 1 + 1 = 3$$

Hence, option (c) is correct.

29. Let $\sin x + \sin y = \cos x + \cos y$ for all $x, y \in \mathbb{R}$. What is $\tan\left(\frac{x}{2}+\frac{y}{2}\right)$ equal to?

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

(**a**) Given that,

$\sin x + \sin y = \cos x + \cos y \; \forall \; x \in \mathbb{R}$

$$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)} = 1$$

$$\tan\left(\frac{x+y}{2}\right) = 1 \text{ or } \tan\left(\frac{x}{2}+\frac{y}{2}\right) = 1$$

Hence, option (a) is correct.

30. Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

and $(mI + nA)^2 = A$, where m, n are positive real numbers and I is the identity matrix. What is $(m+n)$ equal to?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{3}{2}$

(**d**) Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $(mI + nA)^2 = A$

where, I is Identify matrix

$$\because mI + nA = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} + \begin{bmatrix} 0 & 2n \\ -2n & 0 \end{bmatrix}$$

$$= \begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}$$

$\therefore (mI + nA)^2 = A$

$$\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} m^2 - 4n^2 & 4mn \\ -4mn & m^2 - 4n^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$\Rightarrow \quad 4mn = 2$ and $m^2 - 4n^2 = 0$

$mn = \frac{1}{2}$ and $m = \pm 2n$

When, $m = 2n$

$$(2n)(n) = \frac{1}{2}$$

$$n = \pm\frac{1}{2} \Rightarrow m = \pm 1$$

$$\therefore \quad m + n = 1 + \frac{1}{2} = \frac{3}{2}$$

Hence, option (d) is correct.

31. What is the value of the following?

$$\cot\left[\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$$

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$

(c) $\frac{16}{7}$ (d) $\frac{17}{6}$

(**a**) $\cot\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$

$$\Rightarrow \cot\left[\cot^{-1}\left(\frac{\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right) + \cot^{-1}\frac{3}{2}\right]$$

$$\left[\because \sin^{-1}x = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right.$$

$$\left.\text{and } \cot x + \cot^{-1}y = \cot^{-1}\left(\frac{xy-1}{x+y}\right)\right]$$

$$\Rightarrow \cot\left[\cot^{-1}\left(\frac{4}{3}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$$

$$\Rightarrow \cot\left[\cot^{-1}\left(\frac{\frac{4}{3}\times\frac{3}{2}-1}{\frac{4}{3}+\frac{3}{2}}\right)\right]$$

$$\Rightarrow \frac{1}{\frac{17}{6}} = \frac{6}{17}$$

Hence, option (a) is correct.

32. Let $4\sin^2 x = 3$, where $0 \le x \le \pi$. What is $\tan 3x$ equal to?

(a) -2 (b) -1

(c) 0 (d) 1

(**c**) Given that $4\sin^2 x = 3, 0 \le x \le \pi$

$$\therefore \quad \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \quad \sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \text{ or } \sin\frac{2\pi}{3}$$

$$\therefore \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\therefore \quad \tan 3x = \tan\frac{3\pi}{3} = \tan\pi = 0$$

$$\text{Also} \quad \tan 3x = \tan 3\left(\frac{2\pi}{3}\right) = \tan 2\pi = 0$$

$\therefore$ Option (c) is correct.

33. Let p, q and 3 be respectively the first, third and fifth terms of an AP. Let d be the common difference. If the product (pq) is minimum, then what is the value of d?

(a) 1 (b) $\frac{3}{8}$

(c) $\frac{9}{8}$ (d) $\frac{9}{4}$

(**c**) Given that first term of AP $= p$

$\Rightarrow \quad a = p$ ...(i)

Where, a denotes first term.

and $a_3 = q, a_5 = 3$

$\Rightarrow \quad a + 2d = q$ ...(ii)

$a + 4d = 3$ ...(iii)

$$\therefore \quad pq = a(a+2d) = (3-4d)(3-4d+2d) = (3-4d)(3-2d) = 9 - 18d + 8d^2$$

Let $f = 9 - 18d + 8d^2$

$f' = 0 - 18 + 16d = -18 + 16d$

For maxima and minima

$f' = 0$

$\Rightarrow \; -18 + 16d = 0$

$\Rightarrow \quad d = \frac{18}{16} = \frac{9}{8}.$

Now, $f'' = 16$ (Positive)

So, f will be maximum at $d = \frac{9}{8}$.

Hence, option (c) is correct.

34. Consider the following statements in respect of the roots of the equation $x^3 - 8 = 0$

1. The roots are non-collinear.
2. The roots lie on a circle of unit radius.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

(**a**) $x^3 - 8 = 0$

$$\Rightarrow (x-2)(x^2 + 2x + 4) = 0$$

$$x = 2, 2\omega, 2\omega^2$$

Where, $\omega = \frac{-1+\sqrt{3}i}{2}$

Hence, roots are non-collinear and will lie on a circle of 2 unit radius.

Hence, option (a) is correct.

35. Let the equation $\sec x \cdot \operatorname{cosec} x = p$ have a solution, where p is a positive real number. What should be the smallest value of p?

(a) $\frac{1}{2}$ (b) 1

(c) 2

(d) Minimum does not exist

(**c**) $\sec x \cdot \operatorname{cosec} x = p$

$$\Rightarrow \quad \frac{1}{\sin x \cdot \cos x} = p$$

$$\Rightarrow \quad \frac{2}{2\sin x\cos x} = p$$

$$\frac{2}{\sin 2x} = p$$

Where, $\sin 2x \in [-1, 1]$

If $\sin 2x = 1$

Then $p = 2$ will be the smallest value.

Hence, option (c) is correct.

36. For what value of θ, where $0 < \theta < \frac{\pi}{2}$, does $\sin\theta + \sin\theta\cos\theta$ attain maximum value?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(***b***) Let $P = \sin\theta + \sin\theta \cdot \cos\theta$

$\therefore \quad \frac{dP}{d\theta} = \cos\theta + \cos^2\theta - \sin^2\theta$

For maxima-minima $= \frac{dP}{d\theta} = 0$

$\cos\theta + \cos^2\theta - \sin^2\theta = 0$

$\cos + \cos^2\theta - 1 + \cos^2\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(\cos\theta + 1)(2\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = -1 \quad \text{or} \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \pi \quad \text{or} \quad \theta = \frac{\pi}{3}$

$\theta = \pi$ can be neglected as $\theta \in \left(0, \frac{\pi}{2}\right)$.

$\therefore \quad \theta = \frac{\pi}{3}$

Hence, option (b) is correct.

37. Consider the following statements in respect of sets.

1. The union over intersection of sets is distributive.
2. The complement of union of two sets is equal to intersection of their complements.
3. If the difference of two sets is equal to empty set, then the two sets must be equal.

Which of the above statements are correct ?

(a) 1 and 2 (b) 2 and 3

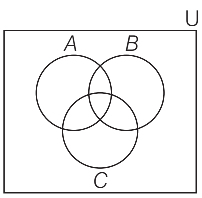
(c) 1 and 3 (d) 1, 2 and 3

(***a***) Since, we know that distributive property for sets A, B and C.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

and $(A \cup B)' = A' \cap B'$

(By De Morgan's Law)



Also, if $A - B = \phi$

$\Rightarrow$ We cannot say $A = B$

e.g., if $A = \phi$ and $B = \{1, 2\}$

$\Rightarrow A - B = \phi$ and $A \neq B$

$\therefore$ Option (a) is correct.

38. Consider three sets X, Y and Z having 6, 5 and 4 elements respectively. All these 15 elements are distinct. Let $S = (X - Y) \cup Z$. How many proper subsets does S have?

(a) 255 (b) 256

(c) 1023 (d) 1024

(***c***) Given, $n(X) = 6, n(Y) = 5, n(Z) = 4$

$S = (X - Y) \cup Z$

Since, all 15 elements are different.

Hence, $n(X - Y) = 6$

and $n(S) = 6 + 4 = 10$

$\Rightarrow$ Number of proper subsets of S

$= 2^{10} - 1$

$= 1024 - 1$

$= 1023$

$\therefore$ Option (c) is correct.

39. Consider the following statements in respect of relations and functions.

1. All relations are functions but all functions are not relations.
2. A relation from A to B is a subset of Cartesian product $A \times B$.
3. A relation in A is a subset of Cartesian product $A \times A$.

Which of the above statements are correct?

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

(***b***) Since, we know that relations can be function iff every element has unique image.

Hence, first statement is wrong.

If $R : A \to A$ then $R \subseteq A \times A$

and if $R : A \to B$ then $R \subseteq A \times B$

Hence, 2nd and 3rd statements are correct.

$\therefore$ Option (b) is correct.

40. If $\log_{10} 2 \log_2 10 + \log_{10}(10^x) = 2$, then what is the value of x?

(a) 0 (b) 1

(c) $\log_2 10$ (d) $\log_5 2$

(***b***) Given that,

$\log_{10} 2 \cdot \log_2 10 + \log_{10}(10)^x = 2$

$\log_{10} 2 \times \frac{1}{\log_{10} 2} + x \log_{10} 10 = 2$

$1 + x = 2$

$\Rightarrow \quad x = 1$

$\therefore$ Option (b) is correct.

41. Let ABC be a triangle.

If $\cos 2A + \cos 2B + \cos 2C = -1$, then which one of the following is correct?

(a) $\sin A \sin B \sin C = 0$

(b) $\sin A \sin B \cos C = 0$

(c) $\cos A \sin B \sin C = 0$

(d) $\cos A \cos B \cos C = 0$

(***d***) Given that, ABC is a triangle and

$\cos 2A + \cos 2B + \cos 2C = -1$

$\Rightarrow 1 + \cos 2A + \cos 2B + \cos 2C = 0$

$\Rightarrow 2\cos^2 A + 2\cos\left(\frac{2B + 2C}{2}\right) \cdot \cos\left(\frac{2B - 2C}{2}\right) = 0$

$\Rightarrow 2\cos^2 A + 2\cos(B + C) \cdot \cos(B - C) = 0$

$\{\because A + B + C = 180\}$

$\Rightarrow 2\cos^2 A + 2\cos(180° - A) \cdot \cos(B - C) = 0$

$\Rightarrow 2\cos^2 A - 2\cos A \cdot \cos(B - C) = 0$

$\Rightarrow 2\cos A[\cos A - \cos(B - C)] = 0$

$\Rightarrow 2\cos A[\cos(180° - (B + C)) - \cos(B - C)] = 0$

$\Rightarrow -2\cos A[\cos(B + C) + \cos(B - C)] = 0$

$\Rightarrow -2\cos A\left(2\cos\frac{B + C + B - C}{2} \cdot \cos\frac{B + C - B + C}{2}\right) = 0$

$-4\cos A \cdot \cos B \cdot \cos C = 0$

$\Rightarrow \quad \cos A \cdot \cos B \cdot \cos C = 0$

$\therefore$ Option (d) is correct.

42. What is the value of the following determinant?

$$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$

(a) -1

(b) 0

(c) $2\tan A \sin B \sin C$

(d) $-2\tan A \sin B \sin C$

(***b***) Let $\Delta = \begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$

$\Delta = \cos C\,[0 + \sin B \tan A] - \tan A\,[\sin B \cos C - 0]$

$= \tan A \sin B \cos C - \tan A \sin B \cos C$

$\therefore \Delta = 0$

Hence, option (b) is correct.

43. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements does $A \cup B$ have?

(a) 324 (b) 364
(c) 384 (d) 400

(**c**) Given that, A consists of first 250 natural numbers that are multiple of 3.

$\therefore \quad A = \{3, 6, 9, 12, ..., 750\}$,

$n(A) = 250$

Set B consists of first 200 even natural numbers.

$\therefore \quad B = \{2, 4, 6, 8, ..., 400\}$

$\therefore \quad A \cap B = \{6, 12, ..., 750\}$

$\because \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 250 + 200 - 66$

$n(A \cup B) = 384$

Hence, option (c) is correct.

44. Let S_k denote the sum of first k terms of an AP. What is $\dfrac{S_{30}}{S_{20} - S_{10}}$ equal to?

(a) 1 (b) 2
(c) 3 (d) 4

(**c**) Let's take first K terms are first K natural numbers.

$$\therefore \quad S_K = \frac{K(K+1)}{2}$$

$$\text{Consider } \frac{S_{30}}{S_{20} - S_{10}} = \frac{\frac{30(31)}{2}}{\frac{20(21)}{2} - \frac{10(11)}{2}}$$

$$= \frac{930}{310} = 3$$

$\therefore$ Option (c) is correct.

45. If the roots of the equation $4x^2 - (5k+1)x + 5k = 0$ differ by unity, then which one of the following is a possible value of k?

(a) -3 (b) -1
(c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

(**c**) Given equation,

$4x^2 - (5K + 1)x + 5K = 0$ ...(i)

Let the roots are α and β.

$$\alpha + \beta = \frac{-(-(5K+1))}{4} = \frac{5K+1}{4}$$

$$\alpha \cdot \beta = \frac{5K}{4}$$

Given that, $\alpha - \beta = 1$

$$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1$$

$$\left(\frac{5K+1}{4}\right)^2 - 4\left(\frac{5K}{4}\right) = 1$$

$$\frac{25K^2 + 1 + 10K}{16} = 1 + 5K$$

$$25K^2 + 10K + 1 = 80K + 16$$

$$\Rightarrow \quad 25K^2 - 70K - 15 = 0$$

$$5K^2 - 14K - 3 = 0$$

$$5K^2 - 15K + K - 3 = 0$$

$$5K(K-3) + 1(K-3) = 0$$

$$(K-3)(5K+1) = 0$$

$$\Rightarrow \quad K = 3 \text{ or } -\frac{1}{5}$$

Hence, option (c) is correct.

46. Consider the digits 3, 5, 7, 9. What is the number of 5-digit numbers formed by these digits in which each of these four digits appears?

(a) 240 (b) 180
(c) 120 (d) 60

(**a**) Given digits are 3, 5, 7, 9.

Since, the number of ways to find 5-digit numbers = 5!

but using 3, 5, 7, 9 every time one-digit will be repeated.

Hence number of 5-digit numbers with digit 3 repeated $= \frac{5!}{2!}$

Number of 5-digit numbers with digit 5 repeated $= \frac{5!}{2!}$

Number of 5-digit numbers with digit 7 repeated $= \frac{5!}{2!}$

Number of 5-digit numbers with digit 9 repeated $= \frac{5!}{2!}$

$\therefore$ Total 5-digit numbers

$$= \frac{5!}{2!} + \frac{5!}{2!} + \frac{5!}{2!} + \frac{5!}{2!}$$

$$= 4 \times \left(\frac{5 \times 4 \times 3 \times 2!}{2!}\right)$$

$$= 4 \times \left(\frac{5 \times 4 \times 3 \times 2!}{2!}\right)$$

$$= 240$$

Hence, option (a) is correct.

47. How many distinct matrices exist with all four entries taken from {1, 2}?

(a) 16 (b) 24
(c) 32 (d) 48

(**a**) Given digits are 1, 2.

Let matrix $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\because$ Each entries can filled with 2 ways.

Therefore, number of distinct matrices

$= 2 \times 2 \times 2 \times 2$

$= 16$

Hence, option (a) is correct.

48. If $i = \sqrt{-1}$, then how many values does i^{-2n} have for different $n \in \mathbb{Z}$?

(a) One (b) Two
(c) Four (d) Infinite

(**b**) Given that, $i = \sqrt{-1}$

To find $(i)^{-2n}$

Let $i = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad r = 1, \theta = \frac{\pi}{2}$

$$\therefore \quad i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$\therefore (i)^{-2n} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{-2n}$$

$$= \cos\left(\frac{-2n\pi}{2}\right) + i\sin\left(\frac{-2n\pi}{2}\right)$$

$$= \cos(n\pi) - i\sin(n\pi)$$

$$= (-1)^n = \begin{cases} -1; & \text{if } n \text{ is odd} \\ 1; & \text{if } n \text{ is even} \end{cases}$$

$\therefore$ Option (b) is correct.

49. If $x = \dfrac{a}{b-c}, y = \dfrac{b}{c-a}, z = \dfrac{c}{a-b}$, then what is the value of the following?

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

(a) 0 (b) 1
(c) abc (d) $ab + bc + ca$

(**a**) Given, $x = \dfrac{a}{b-c}, y = \dfrac{b}{c-a}, z = \dfrac{c}{a-b}$

$$\therefore \begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & -x & x \\ 0 & 1+x & -y-x \\ 0 & z+x & 1-x \end{vmatrix}$$

$$= (1+x)(1-x) - (-y-x)(z+x)$$

$$= 1 - x^2 + x^2 + (y+z)x + yz$$

$$= 1 + \left(\frac{b}{c-a} + \frac{c}{a-b}\right)\left(\frac{a}{b-c}\right) + \left(\frac{b}{c-a} \times \frac{c}{a-b}\right)$$

$$= 1 + \left(\frac{ab - b^2 + c^2 - ac}{(a-b)(c-a)}\right)\left(\frac{a}{b-c}\right) + \left(\frac{bc}{(c-a)(a-b)}\right)$$

$$= 1 + \frac{(b-c)(a-b-c)a}{(a-b)(c-a)(b-c)} + \frac{bc}{(c-a)(a-b)}$$

$$= \frac{(a-b)(c-a) + a^2 - ab - ac + bc}{(a-b)(c-a)}$$

$= \dfrac{ac - a^2 - bc + ab + a^2 - ab - ac + bc}{(a - b)(c - a)}$

$= 0$

Hence, option (a) is correct.

50. Consider the following in respect of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. Inverse of A does not exist
2. $A^3 = A$
3. $3A = A^2$

Which of the above are correct ?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(*c*) Given matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\because |A| = 1(1 - 1) - 1(1 - 1) + 1(1 - 1) = 0$

$\therefore A^{-1}$ doesn't exist.

Now, $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$

and $A^3 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \neq A$

Hence, option (c) is correct.

Directions (Q.Nos. 51 and 52)

Consider the following for the next two questions that follow.

A circle is passing through the points $(5, -8)$, $(-2, 9)$ and $(2, 1)$.

51. What are the coordinate of the centre of the circle?

(a) $(-2, -50)$ (b) $(-50, -20)$
(c) $(-24, -58)$ (d) $(-58, -24)$

(*d*) Given that, circle is passing through the points $(5, -8)$, $(-2, 9)$ and $(2, 1)$.

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

If Eq. (i) passes through $(5, -8)$

$\therefore$ $25 + 64 + 10g - 16f + c = 0$

$\Rightarrow$ $10g - 16f + c + 89 = 0$ ...(ii)

If Eq. (i) passes through $(-2, 9)$

$4 + 81 - 4g + 18f + c = 0$

$-4g + 18f + c + 85 = 0$ ...(iii)

If Eq. (i) passes through (2, 1)

$\Rightarrow$ $4 + 1 + 4g + 2f + c = 0$

$4g + 2f + c + 5 = 0$ ...(iv)

On solving Eqs. (ii), (iii) and (iv)

Eqs. (ii) – Eq. (iii)

$\Rightarrow$ $14g - 34f + 4 = 0$

$\Rightarrow$ $7g - 17f + 2 = 0$ ...(v)

Eq. (iv) – Eq. (iii)

$\Rightarrow$ $8g - 16f - 80 = 0$

$g - 2f - 10 = 0$ ...(vi)

Eq. (v) – 7 × Eq. (vi)

$-3f + 72 = 0$

$f = 24$

From Eq. (vi)

$g = 2f + 10$

$g = 58$

From Eq. (iv) $c = -4g - 2f - 5$

$= -232 - 48 - 5$

$c = -285$

$\therefore$ Centre $= (-g, -f)$

$= (-58, -24)$

$\therefore$ Option (d) is correct.

52. If r is the radius of the circle, then which one of the following is correct?

(a) $r < 10$ (b) $10 < r < 30$
(c) $30 < r < 60$ (d) $r > 60$

(*d*) Since, the centre of the above circle

$= (-58, -24)$

$g = 58, f = 24$ and $c = -285$

$\therefore$ Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(58)^2 + (24)^2 - (-285)}$

$= \sqrt{3364 + 576 + 285}$

$= \sqrt{4225}$

$r = 65$ unit.

$\therefore$ Option (d) is correct.

Directions (Q.Nos. 53 and 54)

Consider the following for the next two questions that follow.

The two vertices of an equilateral triangle are (0, 0) and (2, 2).

53. Consider the following statements.

1. The third vertex has atleast one irrational coordinate.
2. The area is irrational.

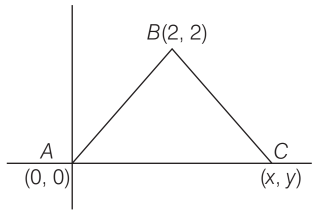
Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(*c*) Let vertex of $A = (0, 0)$

$B = (2, 2)$

$C = (x, y)$



$\because \Delta ABC$ is equilateral triangle.

$\therefore$ $AB = BC$

$\Rightarrow \sqrt{(2 - 0)^2 + (2 - 0)^2}$

$= \sqrt{(2 - x)^2 + (2 - y)^2}$

$\Rightarrow$ $8 = 4 + x^2 - 4x + 4 + y^2 - 4y$

$\Rightarrow$ $x^2 + y^2 - 4x - 4y = 0$ ...(i)

and $AB = AC$

$\Rightarrow \sqrt{(2 - 0)^2 + (2 - 0)^2}$

$= \sqrt{(x - 0)^2 + (y - 0)^2}$

$\Rightarrow$ $8 = x^2 + y^2$ ...(ii)

and $AC = BC$

$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{(x - 2)^2 + (y - 2)^2}$

$\Rightarrow$ $x^2 + y^2 = x^2 + y^2 - 4x - 4y + 8$

$\Rightarrow$ $x + y = 2$ ...(iii)

From Eqs. (ii) and (iii)

$8 = x^2 + (2 - x)^2$

$x^2 + 4 + x^2 - 4x = 8$

$x^2 - 2x - 2 = 0$

$x = \dfrac{2 \pm \sqrt{4 + 8}}{2}$

$= \dfrac{1 \pm \sqrt{3}}{2}$

Hence, third vertex atleast one irrational coordinate.

$\Rightarrow$ Area will also be irrational.

Hence, option (c) is correct.

54. The difference of coordinates of the third vertex is

(a) 0 (b) $\sqrt{3}$
(c) $2\sqrt{2}$ (d) $2\sqrt{3}$

(*d*) Since, $x = \dfrac{1 \pm \sqrt{3}}{2}$

and $y = 2 - x$

$y = 2 - \dfrac{1 \pm \sqrt{3}}{2}$

$y = \dfrac{3 \pm \sqrt{3}}{2}$

If $x = \dfrac{1 + \sqrt{3}}{2}$, $y = \dfrac{3 - \sqrt{3}}{2}$

$\therefore$ $|x - y| = 2\sqrt{3}$

Hence, option (d) is correct.

Directions (Q. Nos. 55 and 56)

Consider the following for the questions that follow.

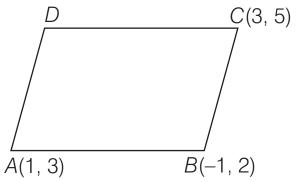
The coordinates of three consecutive vertices of a parallelogram ABCD are A(1, 3), B(– 1, 2) and C(3, 5).

55. What is the equation of the diagonal *BD*?

(a) $2x - 3y + 2 = 0$
(b) $3x - 2y + 5 = 0$
(c) $2x - 3y + 8 = 0$
(d) $3x - 2y - 5 = 0$

(***c***) Given, vertices of parallelogram are

$A = (1, 3), B = (-1, 2), C = (3, 5)$



ABCD is a parallelogram, then

Mid-point of *AC* = Mid-point of *BD*

$$\Rightarrow \left(\frac{1+3}{2}, \frac{3+5}{2}\right) = \left(\frac{-1+x}{2}, \frac{2+y}{2}\right)$$

$$\Rightarrow \quad \frac{-1+x}{2} = \frac{4}{2} \Rightarrow x = 5$$

and $\quad \frac{2+y}{2} = \frac{8}{2} \Rightarrow y = 6$

$\therefore$ Point = (5, 6)

$\therefore$ Equation of *BD*,

where $B = (-1, 2)$ and $D = (5, 6)$

$$y - 2 = \frac{6-2}{5-(-1)}(x+1)$$

$$y - 2 = \frac{4}{6}(x+1)$$

$$6y - 12 = 4x + 4$$

$$\Rightarrow \quad 2x - 3y + 8 = 0$$

$\therefore$ Option (c) is correct.

56. What is the area of the parallelogram?

(a) 1 sq. unit (b) $\frac{3}{2}$ sq. units
(c) 2 sq. units (d) $\frac{5}{2}$ sq. units

(***c***) The vertices of parallelogram are $A(1, 3), B(-1, 2), C(3, 5)$ and $D(5, 6)$.

$\therefore$ Area = | Area of ΔABC + Area of ΔACD |

$$= \frac{1}{2}|1(2-5) - 1(5-3) + 3(3-2)|$$

$$+ \frac{1}{2}|1(5-6) + 3(6-3) + 5(3-5)|$$

$$= \frac{1}{2}|-3 - 2 + 3 + (-1) + 9 - 10|$$

= 2 sq. units

$\therefore$ Option (c) is correct.

Directions (Q. Nos. 57 and 58)

Consider the following for the next two questions that follow.

The equations of the sides AB, BC and CA of a triangle ABC are x – 2 = 0 , y + 1 = 0 and x + 2y – 4 = 0 respectively.

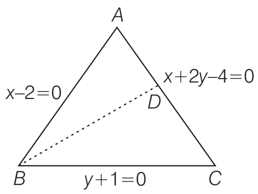
57. What is the equation of the altitude through *B* on *AC*?

(a) $x - 3y + 1 = 0$ (b) $x - 3y + 4 = 0$
(c) $2x - y + 4 = 0$ (d) $2x - y - 5 = 0$

(***d***) Equation of $AB \Rightarrow x - 2 = 0$ ...(i)

Equation of $BC \Rightarrow y + 1 = 0$ ...(ii)

Equation of $AC \Rightarrow x + 2y - 4 = 0$ ...(iii)



On solving Eq. (i) and Eq. (ii), we get

$$x = 2, y = -1$$

$\therefore \quad B = (2, -1)$

$$\text{Slope of } AC = \frac{-\text{ coefficient of } x}{\text{coefficient of } y}$$

$$m_1 = -\frac{1}{2}$$

$$\therefore \text{Slope of altitude } BD = \frac{-1}{m_1} = \frac{-1}{-\frac{1}{2}} = 2$$

$\therefore$ Equation of altitude *BD* drawn from *B* on *AC* having slope 2.

$$y + 1 = 2(x - 2)$$

$$y + 1 = 2x - 4$$

$$\Rightarrow \quad 2x - y - 5 = 0$$

Hence, option (d) is correct.

58. What are the coordinates of circumcentre of the triangle?

(a) (4, 0) (b) (2, 1)
(c) (0, 4) (d) (2, – 1)

(***a***) Slope of line *AB*

$$\Rightarrow \frac{-\text{ coefficient of } x}{\text{coefficient of } y} = \frac{-1}{0} = \infty$$

$$\text{Slope of line } BC = -\frac{0}{1} = 0$$

$\therefore$ Angle between *AB* and *BC*

$$= \left|\frac{\infty - (0)}{1 + \infty \cdot (0)}\right|$$

$$\Rightarrow \quad \tan\theta = \infty \qquad \left|\theta = \frac{\pi}{2}\right|$$

$\because \Delta ABC$ is right angled triangle.

$\therefore$ Circumcentre will lie on Hypotenuse *AC* i.e. $x + 2y - 4 = 0$ at mid point.

Equation of $AB : x - 2 = 0$ ...(i)

Equation of $AC : x + 2y - 4 = 0$ ...(ii)

Equation of $BC : y + 1 = 0$ . ..(iii)

On solving Eqs. (i) and (ii)

$$x = 2, y = 1$$

$\therefore \quad A = (2, 1)$

On solving Eqs. (ii) and (iii)

$$y = -1, x = 6$$

$\therefore \quad c = (6, -1)$

$\therefore$ Circumcentre will be mid-point of

$$AC = \left(\frac{2+6}{2}, \frac{1-1}{2}\right) = (4, 0)$$

$\therefore$ Option (a) is correct.

Directions (Q. Nos. 59 and 60)

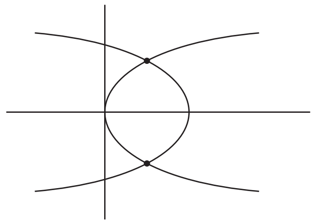
Consider the following for the next two questions that follow.

The two ends of the latus rectum of a parabola are (– 2, 4) and (– 2, – 4).

59. What is the maximum number of parabolas that can be drawn through these two points as end points of latusrectum?

(a) Only one (b) Two
(c) Four (d) Infinite

(***b***) The maximum number of parabolas that can be drawn through



These two points as end points of latusrectum = two

$\therefore$ Option (b) is correct.

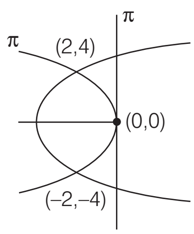
60. Consider the following statements in respect of such parabolas

1. One of the parabolas passes through the origin (0, 0).
2. The focus of one of the parabolas lies at (– 2, 0).

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) Let parabola-1 passes through origin.



It's equation will be

$$y^2 = -4ax$$

Whose leading points of latusrectum will be $(-a, 2a)$ and $(-a, -2a)$

$\therefore \quad a = 2$

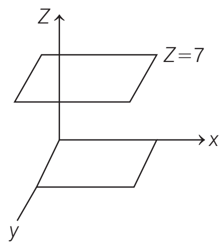
$\therefore \quad$ Focus $= (-2, 0)$

Hence, option (a) is correct.

61. The locus of a point $P(x, y, z)$ which moves in such a way that $z = 7$ is a

(a) line parallel to X-axis
(b) line parallel to Y-axis
(c) line parallel to Z-axis
(d) plane parallel to xy-plane

(***d***) Since, point moves in a plane $z = 7$ which will be parallel to xy-plane.



Hence, option (d) is correct.

62. Consider the following statements

1. A line in space can have infinitely many direction ratios.
2. It is possible for certain line that the sum of the squares of direction cosines can be equal to sum of its direction cosines.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***c***) Since, we know that A line in space can have infinitely many direction ratio and also it is possible for certain line that the sum of the squares of direction cosine can be equal to sum of its direction cosines.

For example, (1, 0, 0) is the direction cosines for X-axis.

$\therefore l = 1, m = 0, n = 0$, then

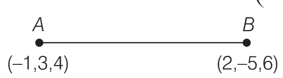
$l^2 + m^2 + n^2 = 1^2 + 0^2 + 0^2$

Hence, option (c) is correct.

63. The xy-plane divides the line segment joining the points $(-1, 3, 4)$ and $(2, -5, 6)$.

(a) internally in the ratio 2 : 3
(b) internally in the ratio 3 : 2
(c) externally in the ratio 2 : 3
(d) externally in the ratio 2 : 1

(***c***) Since, we know that xy-plane divides the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\left(-\frac{z_1}{z_2}\right)$.



Hence, $\quad -\frac{4}{6} = -\frac{2}{3}$

where (–) indicates externally division.

Hence, option (c) is correct.

64. The number of spheres of radius r touching the coordinate axes is

(a) 4 (b) 6
(c) 8 (d) infinite

(***c***) Since, we know that the number of spheres of radius r touching the coordinate axes is 8.

Hence, option (c) is correct.

65. $ABCDEFGH$ is a cuboid with base $ABCD$. Let $A(0, 0, 0)$, $B(12, 0, 0)$, $C(12, 6, 0)$ and $G(12, 6, 4)$ be the vertices. If α is the angle between AB and AG. β is the angle between AC and AG, then what is the value of $\cos 2\alpha + \cos 2\beta$?

(a) $\frac{40}{49}$ (b) $\frac{64}{49}$
(c) $\frac{120}{49}$ (d) $\frac{160}{49}$

(***b***) Given, $ABCDEFGH$ is a cuboid.

$\because$ Angle between AB and $AG = \alpha$

d.r's of $AB = (12 - 0, 0 - 0, 0 - 0)$

$= (12, 0, 0)$

d.r's of $AG = (12 - 0, 6 - 0, 4 - 0)$

$= (12, 6, 4)$

$$\therefore \quad \cos\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{144 + 0 + 0}{\sqrt{12^2}\sqrt{12^2 + 6^2 + 4^2}}$$

$$\cos\alpha = \frac{144}{12 \times 14} = \frac{6}{7}$$

Now, d.r's of $AC = (12, 6, 0)$

d.r's of $\quad AG = (12, 6, 4)$

$$\therefore \quad \cos\beta = \frac{144 + 36}{\sqrt{180} \times 14}$$

$$= \frac{180}{\sqrt{180} \times 14} = \frac{\sqrt{180}}{14}$$

$\therefore \cos 2\alpha + \cos 2\beta$

$$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1$$

$$= 2\left(\left(\frac{6}{7}\right)^2 + \left(\frac{\sqrt{180}}{14}\right)^2\right) - 2$$

$$= 2\left(\frac{36}{49} + \frac{180}{196}\right) - 2$$

$$= \frac{72}{49} + \frac{90}{49} - 2$$

$$= \frac{162 - 98}{49} = \frac{64}{49}$$

Hence, option (b) is correct.

66. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$. If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, then which of the following is/are correct?

1. $\mathbf{a} \times \mathbf{b} = \sin\theta\, \mathbf{c}$
2. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$

Select the correct answer using the code given below.

(a) only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***c***) Given, that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$.

angle between $\mathbf{a}$ and $\mathbf{b} = \theta$

$\therefore \quad \mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta \cdot \mathbf{c}$

$\{\because \mathbf{a} \times \mathbf{b}$ is the vector perpendicular to $\mathbf{a}$ and $\mathbf{b}\}$

$= 1 \cdot 1 \cdot \sin\theta \cdot \mathbf{c}$

$= \sin\theta\, \mathbf{c}$

Since, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are lying on the same plane.

$\therefore \quad \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$

Hence, option (c) is correct.

67. If $\mathbf{a} + 3\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $2\mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, then what is the angle between $\mathbf{a}$ and $\mathbf{b}$?

(a) 0 (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(***d***) Given, $\mathbf{a} + 3\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} \quad$ ...(i)

and $\quad 2\mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} \quad$ ...(ii)

Eq. (i) × 2 – Eq. (ii)

$(2\mathbf{a} + 6\mathbf{b}) - (2\mathbf{a} + \mathbf{b}) = 2(3\hat{\mathbf{i}} - \hat{\mathbf{j}}) - (\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$

$5\mathbf{b} = 5\hat{\mathbf{i}}$

$\therefore \quad \mathbf{b} = \hat{\mathbf{i}}$

From Eq. (i)

$\mathbf{a} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}}) - 3\mathbf{b}$

$\mathbf{a} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{i}}$

$\mathbf{a} = -\hat{\mathbf{j}}$

$\therefore \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

$(-\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}}) = 1 \cdot 1 \cdot \cos\theta$

$0 = \cos\theta$

$\therefore \quad \theta = \frac{\pi}{2}$

Hence, option (d) is correct.

68. If $(\mathbf{a} + \mathbf{b})$ is perpendicular to $\mathbf{a}$ and magnitude of $\mathbf{b}$ is twice that of $\mathbf{a}$, then what is the value of $(4\mathbf{a} + \mathbf{b}) \cdot \mathbf{b}$ equal to?

(a) 0 (b) 1
(c) $8|\mathbf{a}|^2$ (d) $8|\mathbf{b}|^2$

(a) Given, $\mathbf{a}+\mathbf{b}$ is perpendicular to $\mathbf{a}$.

$\therefore \quad (\mathbf{a}+\mathbf{b})\cdot\mathbf{a}=0$

$\Rightarrow \quad |\mathbf{a}|^2+\mathbf{b}\cdot\mathbf{a}=0$

$\mathbf{a}\cdot\mathbf{b}=-|\mathbf{a}|^2$

and $\quad |\mathbf{b}|=2|\mathbf{a}|$

$\therefore \quad (4\mathbf{a}+\mathbf{b})\cdot\mathbf{b}=4\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{b}$

$=4(-|\mathbf{a}|^2)+|\mathbf{b}|^2$

$=-4|\mathbf{a}|^2+(2|\mathbf{a}|)^2$

$=-4|\mathbf{a}|^2+4|\mathbf{a}|^2$

$=0$

Hence, option (a) is correct.

69. Let **a**, **b** and **c** be three vectors such **a**, **b** and **c** are coplanar. Which of the following is/are correct?

1. $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is coplanar with **a** and **b**
2. $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to $\mathbf{a}\times\mathbf{b}$

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Given that **a**, **b**, **c** are coplanar.

$\Rightarrow \quad \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=0=\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

$=\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$

$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]=[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]=0$

$\because \quad (\mathbf{a}\times\mathbf{b})\times\mathbf{c}=-\mathbf{c}\times(\mathbf{a}\times\mathbf{b})$

$=-[(\mathbf{c}\cdot\mathbf{b})\mathbf{a}-(\mathbf{c}\cdot\mathbf{a})\mathbf{b}]$

$=-[[\mathbf{c}\ \mathbf{b}\ \mathbf{a}]-[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]]$

$=2[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]=0$

Hence, $\{(\mathbf{a}\times\mathbf{b})\times\mathbf{c}\}\cdot(\mathbf{a}\times\mathbf{b})=0$

$\Rightarrow (\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to $\mathbf{a}\times\mathbf{b}$.

and coplanar with **a** and **b**.

Hence, option (c) is correct.

70. If the position vectors of A and B are $(\sqrt{2}-1)\,\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\hat{\mathbf{i}}+(\sqrt{2}+1)\hat{\mathbf{j}}$ respectively, then what is the magnitude of **AB**?

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$
(c) $2\sqrt{3}$ (d) $3\sqrt{3}$

(c) Given that, $\mathbf{OA}=(\sqrt{2}-1)\hat{\mathbf{i}}-\hat{\mathbf{j}}$

and $\quad \mathbf{OB}=\hat{\mathbf{i}}+(\sqrt{2}+1)\hat{\mathbf{j}}$

$\therefore \quad \mathbf{AB}=\mathbf{OB}-\mathbf{OA}$

$=(1-\sqrt{2}+1)\hat{\mathbf{i}}+(\sqrt{2}+1+1)\hat{\mathbf{j}}$

$\mathbf{AB}=(2-\sqrt{2})\hat{\mathbf{i}}+(\sqrt{2}+2)\hat{\mathbf{j}}$

$\therefore |\mathbf{AB}|=\sqrt{(2-\sqrt{2})^2+(2+\sqrt{2})^2}$

$=\sqrt{4+2-4\sqrt{2}+4+2+4\sqrt{2}}$

$=\sqrt{12}=2\sqrt{3}$

Hence, option (c) is correct.

71. If $y=(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$, then what is $\dfrac{dy}{dx}$ at $x=0$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

(b) Given, $y=(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$

$\therefore \dfrac{dy}{dx}=(1+x)(1+x^2)(1+x^4)(1+x^8)\cdot(16x^{15})$
$+(1+x)(1+x^2)(1+x^4)(8x^7)(1+x^{16})$
$+(1+x)(1+x^2)(4x^3)(1+x^8)(1+x^{16})$
$+(1+x)(2x)(1+x^4)(1+x^8)(1+x^{16})$
$+(1)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$

$\therefore \left.\dfrac{dy}{dx}\right|_{x=0}=0+0+0+0+1=1$

Hence, option (b) is correct.

72. If $y=\cos x\cdot\cos 4x\cdot\cos 8x$, then what is $\dfrac{1}{y}\dfrac{dy}{dx}$ at $x=\dfrac{\pi}{4}$ equal to?

(a) –1 (b) 0 (c) 1 (d) 3

(a) Given, $y=\cos x\cdot\cos 4x\cdot\cos 8x$

$\therefore \log y=\log\cos x+\log\cos 4x+\log\cos 8x$

On differentiating w.r.t 'x'.

$\dfrac{1}{y}\cdot\dfrac{dy}{dx}=\dfrac{1}{\cos x}(-\sin x)+\dfrac{1}{\cos 4x}(-4\sin 4x)+\dfrac{1}{\cos 8x}(-8\sin 8x)$

$=-\tan x-4\tan 4x-8\tan 8x$

$\therefore \left(\dfrac{1}{y}\dfrac{dy}{dx}\right)_{\text{at } x=\frac{\pi}{4}}=-\tan\dfrac{\pi}{4}-4\tan\pi-8\tan 2\pi$

$=-1-0-0=-1$

Hence, option (a) is correct.

73. Let $f(x)$ be a polynomial function such that $fof(x)=x^4$. What is $f'(1)$ equal to?

(a) 0 (b) 1 (c) 2 (d) 4

(c) Given, $f(x)$ be a polynomial such that

$fof(x)=x^4$

To find $\quad f'(1)=?$

$\because \quad fof(x)=x^4 \Rightarrow f(x)=x^2$

$\therefore \quad f'(x)=2x$

$\Rightarrow \quad f'(1)=2\times 1=2$

Hence, option (c) is correct.

74. What is $\lim\limits_{n\to\infty}\dfrac{a^n+b^n}{a^n-b^n}$ where $a>b>1$, equal to?

(a) –1 (b) 0
(c) 1
(d) Limit does not exist

(c) Given, $\lim\limits_{n\to\infty}\dfrac{a^n+b^n}{a^n-b^n}$ where $a>b>1$

$\Rightarrow \quad \lim\limits_{n\to\infty}\dfrac{a^n\left[1+\left(\dfrac{b}{a}\right)^n\right]}{a^n\left[1-\left(\dfrac{b}{a}\right)^n\right]} \qquad \left[\because \dfrac{b}{a}<1\right]$

$\therefore \quad =\dfrac{1+0}{1-0}=1$

$\therefore \quad \left(\dfrac{b}{a}\right)^\infty=0$

Hence, option (c) is correct.

75. Let $f(x)=\begin{cases}1+\dfrac{x}{2k}, & 0<x<2\\ kx, & 2\le x<4\end{cases}$

If $\lim\limits_{x\to 2}f(x)$ exists, then what is the value of k?

(a) –2 (b) –1
(c) 0 (d) 1

(d) Let $f(x)\begin{cases}1+\dfrac{x}{2k}; & 0<x<2\\ kx; & 2\le x<4\end{cases}$

$\because \lim\limits_{x\to 2}f(x)$ exists

$\Rightarrow \quad \lim\limits_{x\to 2^-}f(x)=\lim\limits_{x\to 2^+}(kx)$

$\lim\limits_{n\to 2^-}\left(1+\dfrac{x}{2k}\right)=\lim\limits_{x\to 2^+}(kx)$

$1+\dfrac{2}{2k}=2k$

$\dfrac{2}{2k}=2k-1$

$2=4k^2-2k$

$4k^2-2k-2=0$

$2k^2-k-1=0$

$(2k+1)(k-1)=0$

$\Rightarrow \quad k=1$ or $k=-\dfrac{1}{2}$

Hence, option (d) is correct.

76. Consider the following statements in respect of $f(x)=|x|-1$:

1. $f(x)$ is continuous at $x=1$.
2. $f(x)$ is differentiable at $x=0$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Given, $f(x)=|x|-1$

Since, modulus function is continuous.

$\Rightarrow f(x)$ is continuous at $x=1$

and $|x|$ is not differentiable if $x=0$

$\therefore f(x)=|x|-1$ is not differentiable at $x=0$

Hence, statement (1) is correct and (2) is false.

Hence, option (a) is correct.

77. If $f(x)=\dfrac{[x]}{[x]}$, $x\neq 0$,

where [ ] denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x=1$?

(a) – 1
(b) 0
(c) 1
(d) Right-hand limit of $f(x)$ at $x=1$ does not exist

(**c**) Given that, $f(x)=\dfrac{[x]}{|x|}$, $x\neq 0$

$$=\lim_{x\to 1^+}\frac{[x]}{|x|}$$

$x=1+h$, where $h\to 0$

$$\therefore \quad \lim_{h\to 0}=\frac{[1+h]}{[1+h]}=\frac{1}{|1+0|}=1$$

Hence, option (c) is correct.

78. Consider the following statements in respect of the function.

$f(x)=\sin\left(\dfrac{1}{x^2}\right)$, $x\neq 0$.

1. It is continuous at $x=0$, if $f(0)=0$.
2. It is continuous at $x=\dfrac{2}{\sqrt{x}}$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(**b**) Given that, $f(x)=\sin\left(\dfrac{1}{x^2}\right)$, $x\neq 0$

At $x=0$,

LHL $\lim\limits_{x\to 0^-}\sin\left(\dfrac{1}{x^2}\right)$

= value in between – 1 and + 1

RHL $\lim\limits_{x\to 0^+}\sin\left(\dfrac{1}{x^2}\right)$

= value in between – 1 and + 1

$\therefore$ Limit doesn't exist $\Rightarrow f(x)$ is not continuous at $x=0$.

At $x=\dfrac{2}{\sqrt{\pi}}$,

$$\lim_{x\to\frac{2}{\sqrt{\pi}}}\sin\left(\frac{1}{x^2}\right)=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

Also $\quad f\left(\dfrac{2}{\sqrt{\pi}}\right)=\sin\left(\dfrac{1}{\left(\dfrac{2}{\sqrt{\pi}}\right)^2}\right)=\dfrac{1}{\sqrt{2}}$

$\therefore f(x)$ is continuous at $x=\dfrac{2}{\sqrt{\pi}}$

Hence, option (b) is correct.

79. What is the range of the function $f(x)=1-\sin x$ defined on entire real line?

(a) (0, 2) (b) [0, 2]
(c) (– 1, 1) (d) [– 1, 1]

(**b**) Given that, $f(x)=1-\sin x$

Since, the range of $\sin x$ is $[-1, 1]$.

$$-1\le \sin x\le 1$$
$$-1\le -\sin x\le 1$$
$$1-1\le 1-\sin x\le 1+1$$
$$0\le 1-\sin x\le 2$$

$\therefore$ Range = [0, 2]

Hence, option (b) is correct.

80. What is the slope of the tangent of $y=\cos^{-1}(\cos x)$ at $x=-\dfrac{\pi}{4}$?

(a) – 1 (b) 0
(c) 1 (d) 2

(**a**) Given that, $y=\cos^{-1}(\cos x)$

Since range of $\cos^{-1}x$ is $[0,\pi]$.

$\therefore \quad y=\cos^{-1}(\cos x)=-x$,

if $\quad x\in(-\pi, 0)$

$\because \quad x=-\dfrac{\pi}{4}$

$\therefore \quad y=-x$

$\Rightarrow \quad \dfrac{dy}{dx}=-1$

$\therefore$ Slope of tangent = – 1

Hence, option (a) is correct.

81. What is the integral of $f(x)=1+x^2+x^4$ with respect to x^2?

(a) $x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+C$

(b) $\dfrac{x^3}{3}+\dfrac{x^5}{5}+C$

(c) $x^2+\dfrac{x^4}{4}+\dfrac{x^6}{6}+C$

(d) $x^2+\dfrac{x^4}{2}+\dfrac{x^6}{3}+C$

(**d**) Given function, $f(x)=1+x^2+x^4$

$\therefore$ Integral of $f(x)$ w.r.t x^2.

$$=\int(1+x^2+x^4)\cdot 2x\,dx$$
$$=\int(2x+2x^3+2x^5)dx$$
$$=x^2+\frac{x^4}{2}+\frac{x^6}{3}+C$$

Hence, option (d) is correct.

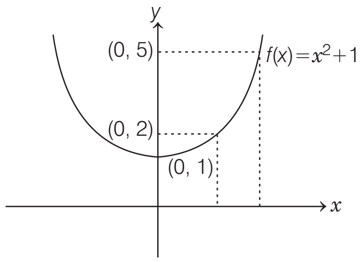
82. Consider the following statements in respect of the function $f(x)=x^2+1$ in the interval (1, 2).

1. The maximum value of the function is 5.
2. The minimum value of the function is 2.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) Given function,



$f(x)=x^2+1 \quad$ in (1, 2)

$\Rightarrow \quad y=x^2+1$

$x^2=(y-1)$

Which is the equation of parabola with vertex (0, 1).

At $\quad x=1$

$f(1)=1^2+1=2$

$f(2)=2^2+1=5$

Hence, maximum value of the function in (1, 2) is 5 and minimum value is 2.

Hence, option (c) is correct.

83. If $f(x)$ satisfies $f(1)=f(4)$, then what is $\int_1^4 f'(x)\,dx$ equal to?

(a) – 1 (b) 0
(c) 1 (d) 2

(**b**) $\quad f(1)=f(4)$

$$\therefore \int_1^4 f'(x)dx=[f(x)]_1^4=f(4)-f(1)$$
$$=f(1)-f(1)=0$$

84. What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}dx$ equal to?

(a) – 1 (b) 0
(c) 1 (d) 2

(**c**) Let $I=\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\,dx=\int_0^{\frac{\pi}{2}}(\cos x)dx$

$$=[\sin x]_0^{\frac{\pi}{2}}=\sin\frac{\pi}{2}-\sin 0=1$$

Hence, option (c) is correct.

85. If $\int\sqrt{1-\sin 2x}\,dx=A\sin x+B\cos x+C$, where $0<x<\dfrac{\pi}{4}$, then which one of the following is correct?

(a) $A+B=0$ (b) $A+B-2=0$
(c) $A+B+2=0$ (d) $A+B-1=0$

(**b**) Given that,

$\int\sqrt{1-\sin 2x}\,dx=A\sin x+B\cos x+C$,

where $0\le x\le\dfrac{\pi}{4}$.

Let

$I=\int\sqrt{\cos^2x+\sin^2x-2\sin x\cdot\cos x}\;dx$

$I=\int\sqrt{(\cos x-\sin x)^2}\;dx$

$\left\{\because \cos x>\sin x \text{ when } 0<x<\dfrac{\pi}{4}\right\}$

$$I = \int (\cos x - \sin x)\, dx$$

$$I = \sin x + \cos x + C$$

$$= A \sin x + B \cos x + C$$

$\therefore \quad A = 1, B = 1$

$\therefore A + B - 2 = 1 + 1 - 2 = 0$

Hence, option (b) is correct.

86. What is the order of the differential equation of all ellipses whose axes are along the coordinate axes?

(a) 1 (b) 2
(c) 3 (d) 4

(***b***) Since, the equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\therefore$ There are 2 variable a and b.

$\therefore$ Order of the differential equation = 2

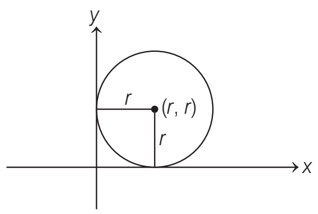
Hence option (d) is correct.

87. What is the degree of the differential equation of all circles touching both the coordinate axes in the first quadrant?

(a) 1 (b) 2
(c) 3 (d) 4

(***b***) If r be the radius of circle.

Since, the circle touching both the coordinate axes in the first quadrant.



$\therefore$ Centre = (r, r) and radius = r

$\therefore$ Equation of circle

$$(x - r)^2 + (y - r)^2 = r^2$$

$$x^2 + y^2 - 2xr - 2yr + r^2 = 0 \quad \ldots\text{(i)}$$

$$2x + 2yy' - 2r - 2ry' = 0$$

$$r(1 + y') = x + yy'$$

$$r = \frac{x + yy'}{1 + y'}$$

Putting the value of r in Eq. (i)

$$x^2 + y^2 - 2x\frac{(x + yy')}{1 + y'} - 2y\frac{(x + yy')}{1 + y'} + \left(\frac{x + yy'}{1 + y'}\right)^2 = 0$$

$$(1 + y')^2 x^2 + (1 + y')^2 y^2 - 2x(x + yy')(1 + y') - 2y(x + yy')(1 + y') + (x + yy')^2 = 0$$

$$(1 + y')^2 (x^2 + y^2) - 2(x - y)(x + yy')(1 + y') + (x + yy')^2 = 0$$

Hence, the degree of the differential equation is 2.

88. What is the differential equation of $y = A - \frac{B}{x}$?

(a) $xy_2 + y_1 = 0$ (b) $xy_2 + 2y_1 = 0$
(c) $xy_2 - 2y_1 = 0$ (d) $2xy_2 + y_1 = 0$

(***b***) Given, $y = A - \frac{B}{x}$

On differentiating w.r.t 'x'

$$\frac{dy}{dx} = 0 - B\left(-\frac{1}{x^2}\right) = \frac{B}{x^2}$$

$$x^2\frac{dy}{dx} = B$$

On differentiating again w.r.t 'x'.

$$x^2 \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot(2x) = 0$$

$$\Rightarrow \quad x(xy_2 + 2y_1) = 0$$

$$\Rightarrow \quad xy_2 + 2y_1 = 0$$

Hence, option (b) is correct.

89. What is $\int_0^\pi \ln\left(\tan\frac{x}{2}\right)dx$ equal to?

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(***a***) Let $I = \int_0^\pi \ln\left(\tan\frac{x}{2}\right)dx \quad \ldots\text{(i)}$

$$I = \int_0^\pi \ln\left(\cot\left(\frac{\pi - x}{2}\right)\right)dx$$

$$I = \int_0^\pi \ln\left(\cot\left(\frac{x}{2}\right)\right)dx \quad \ldots\text{(ii)}$$

Adding Eq. (i) and Eq. (ii)

$$2I = \int_0^\pi \left\{\ln\left(\tan\frac{x}{2}\right) + \ln\left(\cot\frac{x}{2}\right)\right\}dx$$

$$= \int_0^\pi \ln\left(\tan\frac{x}{2}\cdot\cot\frac{x}{2}\right)dx = \int_0^\pi \ln(1)\, dx$$

$$2I = 0$$

$$\therefore \quad I = 0$$

Hence, option (a) is correct.

90. Where does the tangent to the curve $y = e^x$ at the point (0, 1) meet X-axis?

(a) (1, 0) (b) (– 1, 0)
(c) (2, 0) (d) $\left(-\frac{1}{2}, 0\right)$

(***b***) Given curve, $y = e^x$

$$\therefore \quad \frac{dy}{dx} = e^x$$

$$\left(\frac{dy}{dx}\right)_{\text{at}(0,\,1)} = e^0 = 1$$

$\therefore$ Equation of tangent at (0, 1).

$$y - 1 = \left(\frac{dy}{dx}\right)_{\text{at}(0,\,1)}(x - 0)$$

$$y - 1 = x$$

Since, (– 1, 0) satisfies above equation.

Hence, option (b) is correct.

91. Consider the following statements in respect of the function $f(x) = x + \frac{1}{x}$.

1. The local maximum value of $f(x)$ is less than its local minimum value.
2. The local maximum value of $f(x)$ occurs at $x = 1$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) Given, $f(x) = x + \frac{1}{x}$

$$f'(x) = 1 - \frac{1}{x^2} \text{ and } f''(x) = \frac{2}{x^3}$$

For critical points $f'(x) = 0$

$$1 - \frac{1}{x^2} = 0$$

$$x = \pm 1$$

At $x = 1, \quad f''(x) = 2 > 0$

$\Rightarrow f(x)$ is minimum at $x = 1$

$\Rightarrow f(1) = 2$

At $\quad x = -1\, f''(x) = -2 < 0$

$\Rightarrow f(x)$ is maximum at $x = -1$

$\Rightarrow f(-1) = -2$

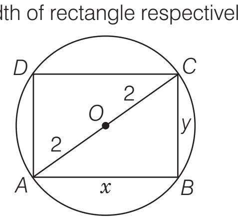
Hence, statement (1) is correct and (2) is false.

$\therefore$ Option (a) is correct.

92. What is the maximum area of a rectangle that can be inscribed in a circle of radius 2 units?

(a) 4 sq. units (b) 6 sq. units
(c) 8 sq. units (d) 16 sq. units

(***c***) Let x and y be the length and breadth of rectangle respectively.



In ΔABC,

$$x^2 + y^2 = 16$$

$$\Rightarrow \quad y = \sqrt{16 - x^2}$$

$\therefore$ Area of rectangle, $A = xy$

$$A = x\sqrt{16 - x^2}$$

$$\frac{dA}{dx} = \sqrt{16 - x^2} + \frac{x}{2\sqrt{16 - x^2}}(-2x)$$

$$= \frac{16 - x^2 - x^2}{\sqrt{16 - x^2}} = \frac{16 - 2x^2}{\sqrt{16 - x^2}}$$

For maximum A,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \frac{16-2x^2}{\sqrt{16-x^2}} = 0 \quad \Rightarrow \quad 16-2x^2 = 0$$

$$\Rightarrow \qquad x^2 = 8 \quad \Rightarrow \quad x = \pm 2\sqrt{2}$$

$$\text{Now, } \frac{d^2A}{dx^2} = \frac{-4x.\sqrt{16-x^2} - (16-2x^2)\dfrac{1}{2\sqrt{16-x^2}}(-2x)}{16-x^2}$$

$$= \frac{-4x(16-x^2) + x(16-2x^2)}{(16-x^2)^{3/2}}$$

$$= \frac{-3x(16-x^2)}{(16-x^2)^{3/2}}$$

$$\left(\frac{d^2A}{dx^2}\right)_{\text{at } x=2\sqrt{2}} = \frac{-3(2\sqrt{2})(16-8)}{(16-8)^{3/2}} \text{ (Negative)}$$

$$\therefore \qquad y = \sqrt{16-(2\sqrt{2})^2} = 2\sqrt{2}$$

Area $= 2\sqrt{2} \times 2\sqrt{2} = 8$ sq. units

Hence, A is maximum at $x = 2\sqrt{2}$.

93. What is $\int \frac{dx}{x(x^2+1)}$ equal to?

(a) $\frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right) + C$

(b) $\ln\left(\frac{x^2}{x^2+1}\right) + C$

(c) $\frac{3}{2}\ln\left(\frac{x^2}{x^2+1}\right) + C$

(d) $\frac{1}{2}\ln\left(\frac{x^2+1}{x^2}\right) + C$

(***a***) Let $I = \int \frac{dx}{x(x^2+1)}$

$$\because \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(by using partial fraction)

$$\frac{1}{x(x^2+1)} = \frac{Ax^2 + A + Bx^2 + Cx}{x(x^2+1)}$$

$$1 = (A+B)x^2 + Cx + A$$

$\Rightarrow A + B = 0, C = 0, A = 1$

$\therefore \qquad B = -A = -1$

$$\therefore I = \int \frac{dx}{x(x^2+1)} = \int \left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx$$

$$= \ln x - \frac{1}{2}\ln(x^2+1) + C$$

$$= \frac{1}{2}(2\ln x) - \frac{1}{2}\ln(x^2+1) + C$$

$$= \frac{1}{2}(\ln x^2 - \ln(x^2+1)) + C$$

$$= \frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right) + C$$

Hence, option (a) is correct.

94. What is the derivative of e^{e^x} with respect to e^x?

(a) e^{e^x} (b) e^x
(c) $e^{e^x}e^x$ (d) ee^x

(***a***) Let $y_1 = e^{e^x}$ and $y_2 = e^x$

$$\therefore \qquad \frac{dy_1}{dx} = e^{e^x}\cdot e^x, \frac{dy_2}{dx} = e^x$$

$$\Rightarrow \qquad \frac{dy_1}{dy_2} = \frac{e^{e^x}\cdot e^x}{e^x} = e^{e^x}$$

$\therefore$ Option (a) is correct.

95. What is the condition that $f(x) = x^3 + x^2 + kx$ has no local extremum?

(a) $4k < 1$ (b) $3k > 1$
(c) $3k < 1$ (d) $3k \le 1$

(***b***) Given that, $f(x) = x^3 + x^2 + kx$

$\because f(x)$ has no local extremum.

$\Rightarrow \qquad f'(x) \ne 0$

$\Rightarrow \quad 3x^2 + 2x + k \ne 0$

for no extremum, $D < 0$

$\Rightarrow \quad (2)^2 - 4(3)(k) < 0$

$\Rightarrow \qquad 4 - 12k < 0$

$\qquad 3k > 1$

$\therefore$ Option (b) is correct.

96. If $f(x) = 2^x$, then what is $\int_2^{10} \frac{f'(x)}{f(x)} dx$ equal to?

(a) 4ln2 (b) ln4
(c) ln5 (d) 8ln2

(***d***) Given, $f(x) = 2^x$

$$\therefore \int_2^{10} \frac{f'(x)}{f(x)} dx = [\ln f(x)]_2^{10} = [\ln 2^x]_2^{10}$$

$$= [x \ln 2]_2^{10}$$

$$= 10\ln 2 - 2\ln 2$$

$$= 8\ln 2$$

$\therefore$ Option (d) is correct.

97. If $\int_{-2}^{0} f(x)\, dx = k$, then $\int_{-2}^{0} |f(x)|\, dx$ is

(a) less than k
(b) greater than k
(c) less than or equal to k
(d) greater than or equal to k

(***d***) Given, $\int_{-2}^{0} f(x) dx = k$

To find $\int_{-2}^{0} |f(x)| dx$

Let $f(x) = x$

$$\therefore \qquad \int_{-2}^{0} x\, dx = \left[\frac{x^2}{2}\right]_{-2}^{0} = -2 = k$$

$$\therefore \qquad \int_{-2}^{0} |x|\, dx = -\int_{-2}^{0} x\, dx$$

$$= -(-2) = 2 \ge k$$

$\therefore$ Option (d) is correct.

98. If the function $f(x) = x^2 - kx$ is monotonically increasing in the interval $(1, \infty)$, then which one of the following is correct?

(a) $k < 2$ (b) $2 < k < 3$
(c) $3 < k < 4$ (d) $k > 4$

(***a***) Let the function $f(x) = x^2 - kx$ is monotonically increasing in $(1, \infty)$.

$\Rightarrow \qquad f'(x) \ge 0$

$\Rightarrow \qquad 2x - k \ge 0$

$\Rightarrow k \le 2x$ in $(1, \infty)$ at lower value

at $\qquad x = 1$

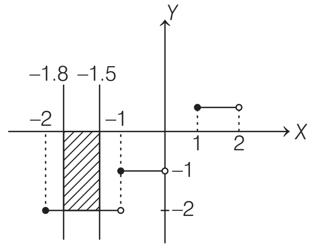
$\qquad k < 2$

Hence, option (a) is correct.

99. What is the area bounded by $y = [x]$, where $[\cdot]$ is the greatest integer function, the X-axis and the lines $x = -1.5$ and $x = -1.8$?

(a) 0.3 sq. unit (b) 0.4 sq. unit
(c) 0.6 sq. unit (d) 0.8 sq. unit

(***c***) Given, $y = [x]$



$$\therefore \qquad \text{Area} = \int_{-1.8}^{-1.5} [x]\, dx = \int_{-1.8}^{-1.5} (-2)\, dx$$

$$= -2(x)_{-1.8}^{-1.5}$$

$$= -2(-1.5 + 1.8) = -0.6$$

$\therefore \qquad$ Area $= 0.6$ sq. unit

Hence, option (c) is correct.

100. The tangent to the curve $x^2 = y$ at $(1, 1)$ makes an angle θ the with the positive direction of X-axis. Which one of the following is correct?

(a) $\theta < \frac{\pi}{6}$ (b) $\frac{\pi}{6} < \theta < \frac{\pi}{4}$
(c) $\frac{\pi}{4} < \theta < \frac{\pi}{3}$ (d) $\frac{\pi}{3} < \theta < \frac{\pi}{2}$

(***d***) Given, curve $y = x^2$

$$\frac{dy}{dx} = 2x$$

$$\left.\frac{dy}{dx}\right|_{\text{at}(1,\,1)} = 2 \times 1 = 2$$

$\Rightarrow \qquad \tan\theta = 2$

$\because \tan\frac{\pi}{3} = \sqrt{3} = 1.732$ and $\tan\frac{\pi}{2} = \infty$

$$\therefore \qquad \frac{\pi}{3} < \theta < \frac{\pi}{2}$$

Hence, option (d) is correct.

101. Consider the following relations for two events E and F.

1. $P(E \cap F) \geq P(E) + P(F) - 1$
2. $P(E \cup F) = P(E) + P(F) + P(E \cap F)$
3. $P(E \cup F) \leq P(E) + P(F)$

Which of the above relations is/are correct?

(a) 1 only (b) 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(*c*) Let E and F be two events.

Then, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ ...(i)

or $P(E \cup F) \leq P(E) + P(F)$

$P(E \cup F) \leq 1$

$-P(E \cap F) \geq -1$

$\Rightarrow P(E) + P(F) - P(E \cup F) \geq P(E) + P(F) - 1$

$\Rightarrow P(E \cap F) \geq P(E) + P(F) - 1$

Hence, option (c) is correct.

102. If $P(A/B) < P(A)$, then which one of the following is correct?

(a) $P(B|A) < P(B)$ (b) $P(B|A) > P(B)$
(c) $P(B|A) = P(B)$ (d) $P(B|A) > P(A)$

(*a*) If $P\left(\frac{A}{B}\right) < P(A)$

$\because \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} < P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(A)} < P(B)$

$\Rightarrow P\left(\frac{B}{A}\right) < P(B)$

Hence, option (a) is correct.

103. When the measure of central tendency is available in the form of mean, which one of the following is the most reliable and accurate measure of variability?

(a) Range
(b) Mean deviation
(c) Standard deviation
(d) Quartile deviation

(*c*) When the measure of central tendency is available in the form of mean then, we know that Standard Deviation is the most reliable and accurate measure of variability.

Hence, option (c) is correct.

104. A problem is given to three students A, B and C, whose probabilities of solving the problem independently are $\frac{1}{2}, \frac{3}{4}$ and p, respectively. If the probability that the problem can be solved is $\frac{29}{32}$, then what is the value of p ?

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(*d*) Given, $P(A) = \frac{1}{2}, P(B) = \frac{3}{4}, P(C) = p$

$\because$ Probability that the problem can not be solved $= P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C})$

$= \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)(1 - p)$

$= \frac{1}{2} \times \frac{1}{4}(1 - p)$

$= \frac{1 - p}{8}$

$\therefore$ Probability that the problem can be solved

$= 1 -$ Probability that the problem cannot be solved

$\frac{29}{32} = 1 - \frac{(1 - p)}{8}$

$\frac{1 - p}{8} = 1 - \frac{29}{32}$

$\frac{1 - p}{8} = \frac{3}{32}$

$1 - p = \frac{3}{4}$

$\therefore \quad p = \frac{1}{4}$

Hence, option (d) is correct.

105. In a cricket match a batsman hits a six 8 times out of 60 balls he plays. What is the probability that on a ball played he does not hit a six?

(a) $\frac{2}{3}$ (b) $\frac{1}{15}$ (c) $\frac{2}{15}$ (d) $\frac{13}{15}$

(*d*) Since, the batsman hits a six 8 times out of 60 balls.

The batsman could not hit sixes in (60-8) balls.

$\therefore$ Probability that on a ball played he does not hit six $= \frac{52}{60}$

$p = \frac{13}{15}$

Hence, option (d) is correct.

Directions (Q. Nos. 106 and 107)

Consider the following for the questions that follow.

Two regression lines are given as
$3x - 4y + 8 = 0$ and $4x - 3y - 1 = 0$

106. Consider the following statements.

1. The regression line of y on x is $y = \frac{3}{4}x + 2$
2. The regression line of x on y is $x = \frac{3}{4}y + \frac{1}{4}$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) Two regression lines are

$3x - 4y + 8 = 0, 4x - 3y - 1 = 0$

for finding the regression line of y on x

$3x - 4y + 8 = 0$

$4y = 3x + 8$

$y = \left(\frac{3}{4}\right)x + 2$ ...(i)

and the regression line of x on y:

$4x - 3y - 1 = 0$

$4x = 3y + 1$

$x = \frac{3}{4}y + \frac{1}{4}$ ...(ii)

Hence, option (c) is correct.

107. Consider the following statements.

1. The coefficient of correlations r is $\frac{3}{4}$
2. The means of x and y are 3 and 4 respectively.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*a*) Since, regression line of y on x

$\Rightarrow \quad y = \frac{3}{4}x + 2$

$\therefore \quad b_{xy} = \frac{3}{4}$

and regression line of x on y

$\Rightarrow \quad x = \frac{3}{4}y + \frac{1}{4}$

$\therefore \quad b_{yx} = \frac{3}{4}$

$\therefore$ Coefficient of correlations

$r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{\frac{3}{4} \times \frac{3}{4}}$

$r = \frac{3}{4}$

Means of x and y are nothing but the solution of regression lines

$3x - 4y + 8 = 0$

and $4x - 3y - 1 = 0$

$3x - 4y = -8$ ...(i)

$4x - 3y = 1$ ...(ii)

Eq. (i) × 4 – Eq. (ii) × 3

$12x - 16y = -32$

$12x - 9y = 3$

$7y = 35 \Rightarrow y = 5$

$\therefore \quad 4x = 1 + 3 \times 5$

$x = 4$

$\therefore$ Statement (2) is wrong.

Hence, option (a) is correct.

Directions (Q. Nos. 108 and 109) *Consider the following for the questions that follow.*

The marks obtained by 60 students in a certain subject out of 75 are given below.

Marks	Number of students
15-20	4
20-25	5
25-30	11
30-35	6
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

108. What is the median?

(a) 35 (b) 38
(c) 39 (d) 40

(*c*)

Marks	Frequency	Cumulative frequency
15-20	4	4
20-25	5	9
25-30	11	20
30-35	6	26 = Cf
35-40	5	31
40-45	8	39
45-50	9	48
50-55	6	54
55-60	4	58
60-65	2	60
	$N = 60$	

$\therefore \frac{N}{2} = 30 \Rightarrow$ model class will be 35 – 40.

$\therefore$ lower limit $(l) = 35$

$h = 40 - 35 = 5$

$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - C.f}{f} \times h$

$= 35 + \frac{30 - 26}{5} \times 5 = 39$

Hence, option (c) is correct.

109. What is the mode?

(a) 27.27 (b) 27.73
(c) 27.93 (d) 28.27

Marks	Frequency
15-20	4
20-25	$5 \to f_0$
25-30	$11 \to f_1$
30-35	$6 \to f_2$
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

(*b*) Highest frequency is given for class 25-30.

$\therefore$ Model class will be 25-30.

$\therefore \quad l = 25, h = 5$

$\therefore \quad f_1 = 11, f_0 = 5, f_2 = 6$

$\therefore \text{ Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 25 + \frac{11 - 5}{22 - 5 - 6} \times 5$

$= 25 + \frac{6}{11} \times 5 = \frac{275 + 30}{11}$

$= 27.73$

Hence, option (b) is correct.

110. What is the mean of natural numbers contained in the interval [15, 64]?

(a) 36.8 (b) 38.3
(c) 39.5 (d) 40.3

(*c*) Mean of natural numbers contained in [15, 64].

$= \frac{15 + 16 + 17 + \dots + 64}{50}$

$= \frac{\sum_{n=1}^{64} n - \sum_{r=1}^{14} r}{50}$

$= \frac{\frac{64 \times 65}{2} - \frac{14 \times 15}{2}}{50}$

$= \frac{2080 - 105}{50} = 39.5$

Hence, option (c) is correct.

111. For the set of numbers $x, x, x+2, x+3, x+10$ where x is a natural number, which of the following is/are correct?

1. Mean > Mode
2. Median> Mean

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(*a*) Given data $x, x, x+2, x+3, x+10$ where $x \in N$.

$\therefore \text{Mean} = \frac{x + x + x + 2 + x + 3 + x + 10}{5}$

$= \frac{5x + 15}{5} = x + 3$

$\therefore$ Mode $= x$

$\text{Median} = \left(\frac{5+1}{2}\right)^{\text{th}}$ term $= 3^{\text{rd}}$ term

$= x + 2$

$\therefore$ Mean > Mode and Median < Mean

Hence, correct option is (a).

112. The mean of 10 observations is 5.5. If each observation is multiplied by 4 and subtracted from 44, then what is the new mean?

(a) 20 (b) 22
(c) 34 (d) 44

(*b*) Given that, the mean of 10 observation is 5.5.

$\because \quad \text{Mean} = \frac{x_1 + x_2 + \dots + x_{10}}{10}$

$5.5 = \frac{x_1 + x_2 + \dots + x_{10}}{10}$

$\therefore \quad \sum_{i=1}^{10} x_i = 55$

Also, given that new observations are obtained by multiplying by 4 and subtracting from 44.

Hence, new mean $= 44 - 4 \times 5.5$

$= 44 - 22 = 22$

Hence, correct option is (b).

113. If g is the geometric mean of 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, then which one of the following is correct?

(a) $8 < g < 16$ (b) $16 < g < 32$
(c) $32 < g < 64$ (d) $g > 64$

(*c*) Given that, geometric mean of 2, 4, 8, 16, 32, 64, 126, 256, 512, 1024 is g.

$\therefore \quad g = \sqrt[10]{2 \times 4 \times 8 \times 16 \times 32 \times 64 \times 128 \times 256 \times 512 \times 1024}$

$= (2^{1+2+3+\dots+10})^{\frac{1}{10}}$

$g = (2^{55})^{\frac{1}{10}} \quad \Rightarrow \quad g = (2)^{\frac{11}{2}}$

$\therefore \quad 2^5 < g < 2^6 \quad \Rightarrow \quad 32 < g < 64$

Hence, option (c) is correct.

114. If the harmonic mean of 60 and x is 48, then what is the value of x?

(a) 32 (b) 36
(c) 40 (d) 44

(*c*) Given, harmonic mean of 60 and x is 48.

$\because \quad H = \frac{2ab}{a + b}$

$48 = \frac{2 \times 60 \times x}{60 + x}$

$2880 + 48x = 120x$

$72x = 2880$

$x = 40$

Hence, option (c) is correct.

115. What is the mean deviation of first 10 even natural numbers?

(a) 5 (b) 5.5
(c) 10 (d) 10.5

(**a**) Mean deviation of first 10 even natural numbers

Since, mean $(\bar{x}) = \dfrac{2+4+6+\ldots 20}{10}$

$= \dfrac{2(10 \times 11)}{20} = 11$

$\therefore$ Mean deviation

$= \dfrac{|2-11| + |4-11| + |6-11| + \ldots + |20-11|}{10}$

$= \dfrac{9+7+5+3+1+1+3+5+7+9}{10}$

$= 5$

Hence, option (a) is correct.

116. If $\sum_{i=1}^{10} x_i = 110$ and $\sum_{i=1}^{10} x_i^2 = 1540$, then what is the variance?

(a) 22 (b) 33
(c) 44 (d) 55

(**b**) Given, $\sum_{i=1}^{10} x_i = 110$

and $\sum_{i=1}^{10} x_i^2 = 1540$

$\because$ Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2$

$= \dfrac{1540}{10} - \left(\dfrac{110}{10}\right)^2$

$= 154 - 121$

$= 33$

Hence, option (b) is correct.

117. 3-digit numbers are formed using the digits 1, 3, 7 without repetition of digits. A number is randomly selected. What is the probability that the number is divisible by 3?

(a) 0 (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

(**a**) Let 3-digit numbers using the digits without repetition 1, 3, 7 are 3!.

Since, the sum of the digits

$= 1 + 3 + 7 = 11$

which is not divisible by 3.

$\therefore P$ (number of divisible by 3) $= \dfrac{0}{3!}$

$= 0$

Hence, option (a) is correct.

118. What is the probability that the roots of the equation $x^2 + x + n = 0$ are real, where $n \in N$ and $n < 4$?

(a) 0 (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(**a**) Given, equation $x^2 + x + n = 0$,

where $n \in N, n < 4$

$\therefore \quad n \in \{1, 2, 3\}$

Since, above equation is quadratic.

So, for each value of n, we have two roots.

$\therefore$ Total number of roots = 6

When $n = 1$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{3}i}{2}$

when $n = 2$,

$x^2 + x + 2 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{1-8}}{2} = \dfrac{-1 \pm \sqrt{7}i}{2}$

and $n = 3$, $x^2 + x + 3 = 0$

$\Rightarrow \quad x = \dfrac{- \pm \sqrt{11}i}{2}$

There are no real roots.

$\therefore P$ (roots are real) $= \dfrac{0}{6} = 0$

Hence, option (a) is correct.

119. If A and B are two events such that $P(\text{not } A) = \dfrac{7}{10}$, $P(\text{not } B) = \dfrac{3}{10}$ and $P(A \mid B) = \dfrac{3}{14}$, then what is $P(B \mid A)$ equal to?

(a) $\frac{11}{14}$ (b) $\frac{9}{14}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

(**d**) Given, $P(\text{not } A) = \dfrac{7}{10}, P(\text{not } B) = \dfrac{3}{10}$

$P\left(\dfrac{A}{B}\right) = \dfrac{3}{14}$

$\therefore \quad P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)} \quad \ldots\text{(i)}$

$\because \quad P(\text{not } A) = \dfrac{7}{10}$

$\therefore \quad P(A) = 1 - \dfrac{7}{10} = \dfrac{3}{10}$

$P(\text{not } B) = \dfrac{3}{10}$

$\therefore \quad P(B) = 1 - \dfrac{3}{10} = \dfrac{7}{10}$

$P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$

$\dfrac{3}{14} = \dfrac{P(A \cap B)}{\frac{7}{10}}$

$\therefore \quad P(A \cap B) = \dfrac{3}{20}$

Eq. (i) $\Rightarrow P\left(\dfrac{B}{A}\right) = \dfrac{\frac{3}{20}}{\frac{3}{10}} = \dfrac{1}{2}$

Hence, option (d) is correct.

120. Seven white balls and three black balls are randomly placed in a row. What is the probability that no two black balls are placed adjacently?

(a) $\frac{7}{15}$ (b) $\frac{8}{15}$
(c) $\frac{11}{15}$ (d) $\frac{13}{15}$

(**a**) There are 10 balls among which 7 are white and 3 are black.

$\therefore$ Number of ways to arrange 10 balls

$= 10!$

If we put the balls in such a way that no two black balls are placed adjacently.

$\therefore$ Number of arrangements $= 7! \times {}^8P_3$

$\therefore \quad P = \dfrac{7! \times 8!}{5! \times 10!}$

$= \dfrac{6 \times 7}{9 \times 10} = \dfrac{7}{15}$

Hence, option (a) is correct.