NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2019 (II)

PAPER I : Mathematics

1. If both p and q belong to the set $\{1, 2, 3, 4\}$, then how many equations of the form $px^2 + qx + 1 = 0$ will have real roots?

(a) 12 (b) 10
(c) 7 (d) 6

(**d**) Equation $px^2 + qx + 1 = 0$, has real roots, where p and q belong to the set $\{1, 2, 3, 4\}$.

$\therefore \quad q^2 - 4p \geq 0$

[$\because$ for real roots of a quadratic equation $b^2 - 4ac \geq 0$]

It is possible if value of $(p, q) = (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)$ and $(3, 4)$

Hence, the number of equations are 6.

2. What is the value of $1 - 2 + 3 - 4 + 5 - ..... + 101$?

(a) 51 (b) 55
(c) 110 (d) 111

(**a**) Given series,

$= 1 - 2 + 3 - 4 + 5 - ..... + 101$

$= (1 + 3 + 5 + ... + 101) - (2 + 4 + 6 + ... + 100)$

$= (1 + 3 + 5 + ... 51 \text{ terms}) - (2 + 4 + 6 + ... 50 \text{ terms})$

$= \frac{51}{2}[2 + (51 - 1) \times 2] - \frac{50}{2}[4 + (50 - 1) \times 2]$

[$\because$ both series are AP and $S_n = \frac{n}{2}[2a + (n - 1)d]$]

$= \frac{51}{2} \times 102 - \frac{50}{2} \times 102$

$= 2601 - 2550 = 51$

3. If A, B and C are subsets of a given set, then which one of the following relations is not correct?

(a) $A \cup (A \cap B) = A \cup B$
(b) $A \cap (A \cup B) = A$
(c) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
(d) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

(**a**) Let U be the set and A, B and C are the subset of U.

We know that, $A \cup (A \cap B) = A$,

So option (a) is not correct.

$A \cap (A \cup B) = A$, so option (b) is correct.

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$,

so option (c) is correct.

and $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

so option (d) is correct.

4. If the sum of first n terms of a series is $(n + 12)$, then what is its third term?

(a) 1 (b) 2
(c) 3 (d) 4

(**a**) Sum of first n term of a series $= n + 12$

$\Rightarrow a_1 + a_2 + a_3 + ...+ a_n = n + 12$

Put $n = 1$, $a_1 = 1 + 12 = 13$

Put $n = 2, a_1 + a_2 = 2 + 12 \Rightarrow a_1 + a_2 = 14$

$\Rightarrow 13 + a_2 = 14 \Rightarrow a_2 = 1$

Put $n = 3$

$a_1 + a_2 + a_3 = 3 + 12$

$\Rightarrow \quad 13 + 1 + a_3 = 15$

$\Rightarrow \quad a_3 = 15 - 14 = 1$

5. What is the value of k for which the sum of the squares of the roots of $2x^2 - 2(k - 2)x - (k + 1) = 0$ is minimum?

(a) -1 (b) 1
(c) $\frac{3}{2}$ (d) 2

(**c**) Let α, β be the roots of equation.

$2x^2 - 2(k - 2)x - (k + 1) = 0$

$\because \alpha + \beta = \frac{2(k - 2)}{2} = k - 2,$

$\alpha\beta = \frac{-(k + 1)}{2}$

We know that

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (k - 2)^2 + 2 \times \frac{k + 1}{2}$

$= k^2 + 4 - 4k + k + 1$

$= k^2 - 3k + 5$

$= k^2 - 3k + \frac{9}{4} - \frac{9}{4} + 5$

$= \left(k - \frac{3}{2}\right)^2 + \frac{11}{4}$

$\alpha^2 + \beta^2$ is minimum, if $\left(k - \frac{3}{2}\right) = 0$

$\Rightarrow \quad k = \frac{3}{2}$

6. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then which one of the following is correct?

(a) a, b and c are in AP
(b) a, b and c are in GP
(c) a, b and c are in HP
(d) a, b and c do not follow any regular pattern

(**c**) The roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal.

$\therefore b^2(c - a)^2 - 4a(b - c).c(a - b) = 0$

[$\because ax^2 + bx + c = 0$ of roots are real if $b^2 - 4ac \geq 0$

$\Rightarrow b^2(c^2 + a^2 - 2ca) - 4ac\ (ab - b^2 - ac + bc) = 0$

$\Rightarrow b^2c^2 + a^2b^2 - 2ab^2c - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2 = 0$

$\Rightarrow b^2c^2 + a^2b^2 + 2ab^2c - 4a^2bc - 4abc^2 + 4a^2c^2 = 0$

$\Rightarrow b^2(c^2 + a^2 + 2ac) - 4abc\,(a + c) + 4a^2c^2 = 0$

$\Rightarrow b^2(c + a)^2 - 4abc\,(a + c) + (2ac)^2 = 0$

$\Rightarrow [b(c + a) - 2ac]^2 = 0$

$\Rightarrow b\,(c + a) - 2ac = 0$

$\Rightarrow b(c + a) = 2ac \Rightarrow b = \dfrac{2ac}{c + a}$

So, a, b and c are is HP.

7. If $|x^2 - 3x + 2| > x^2 - 3x + 2$, then which one of the following is correct?

(a) $x \le 1$ or $x \ge 2$ (b) $1 \le x \le 2$
(c) $1 < x < 2$
(d) x is any real value except 3 and 4

(**c**) $|x^2 - 3x + 2| > x^2 - 3x + 2$

$\Rightarrow -(x^2 - 3x + 2) > x^2 - 3x + 2$

[if $x^2 - 3x + 2 < 0$, and $x^2 - 3x + 2 > 0$ not possible]

$\Rightarrow -2(x^2 - 3x + 2) > 0$

$\Rightarrow x^2 - 3x + 2 > 0$

$\Rightarrow x^2 - 2x - x + 2 > 0$

$\Rightarrow (x - 2)(x - 1) > 0$

$\therefore 1 < x < 2$ is correct.

8. A geometric progression (GP) consists of 200 terms. If the sum of odd terms of the GP is m, and the sum of even terms of the GP is n, then what is its common ratio?

(a) m / n (b) n / m
(c) $m + (n / m)$ (d) $n + (m / n)$

(**b**) Let a, ar, ar^2..... 200 terms be a geometric progression.

Where, a is the first terms and r be the common ratio.

GP of odd terms a, ar^2, ar^4, ... 100 terms.

GP of even terms ar, ar^3, ar^5, ... 100 terms.

$\therefore$ Sum of odd terms of the GP = m

$\Rightarrow \dfrac{a\{r^{200} - 1\}}{r - 1} = m$ ...(i)

Sum of even terms of the GP = n

$\Rightarrow \dfrac{ar(r^{200} - 1\}}{r - 1} = n$ ...(ii)

Dividing of Eq. (i) by Eq. (ii),

$\Rightarrow \dfrac{1}{r} = \dfrac{m}{n} \Rightarrow r = \dfrac{n}{m}$

Hence, the common ratio of the GP is $\dfrac{n}{m}$.

9. If a set A contains 3 elements and another set B contains 6 elements, then what is the minimum number of elements that $(A \cup B)$ can have?

(a) 3 (b) 6
(c) 8 (d) 9

(**b**) $n(A) = 3$, $n(B) = 6$

$\therefore$ The minimum number of elements in

$A \cup B = 6$

i.e $n(A \cup B) = 6$

(because max $n(A \cap B) = 3$

10. What is the number of diagonals of an octagon?

(a) 48
(b) 40
(c) 28
(d) 20

(**d**) The number of vertices of an octagon = 8

$\therefore$ The number of points in a plane = 8

$\therefore$ Total number of straight line form by 8 points = ${}^8C_2$

[$\because$ 1 straight line form by 2 points]

$= \dfrac{8!}{2!6!} = \dfrac{8 \times 7}{2} = 28$

$\therefore$ The number of diagonals of an octagon = Total number of straight line form by 8 points – number of sides of octagon

$= 28 - 8 = 20$

11. What is the value of the determinant $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$?

(a) 0 (b) 12
(c) 24 (d) 36

(**c**) Given determinant

$= \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 6 & 12 & 48 \end{vmatrix}$

[by $C_2 \to C_2 - 2C_1, C_3 \to C_3 - 3C_2$]

$= 1(96 - 72) - 0 + 0$

[expression w.r.t. first row]

$= 24$

12. What are the values of x that satisfy the equation $\begin{vmatrix} x & 0 & 2 \\ 2x & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 0 & 2 \\ x^2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$?

(a) $-2 \pm \sqrt{3}$
(b) $-1 \pm \sqrt{3}$
(c) $-1 \pm \sqrt{6}$
(d) $-2 \pm \sqrt{6}$

(**d**) Given equation,

$\begin{vmatrix} x & 0 & 2 \\ 2x & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 0 & 2 \\ x^2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow x(2 - 1) - 0 + 2(2x - 2) + 3x(2 - 1) - 0 + 2(x^2 - 0) = 0$

[expression w.r.t. first row]

$\Rightarrow x + 4x - 4 + 3x + 2x^2 = 0$

$\Rightarrow 2x^2 + 8x - 4 = 0$

$\Rightarrow x^2 + 4x - 2 = 0$

$\Rightarrow x = \dfrac{-4 \pm \sqrt{16 - 4\,(1)\,(-2)}}{2}$

$= \dfrac{-4 \pm \sqrt{24}}{2} = \dfrac{-4 \pm 2\sqrt{6}}{2}$

$= -2 \pm \sqrt{6}$

13. If $x + a + b + c = 0$, then what is the value of $\begin{vmatrix} x + a & b & c \\ a & x + b & c \\ a & b & x + c \end{vmatrix}$?

(a) 0 (b) $(a + b + c)^2$
(c) $a^2 + b^2 + c^2$ (d) $a + b + c - 2$

(**a**) Given, $x + a + b + c = 0$

$\begin{vmatrix} x + a & b & c \\ a & x + b & c \\ a & b & x + c \end{vmatrix}$

$= \begin{vmatrix} x + a + b + c & b & c \\ x + a + b + c & x + b & c \\ x + a + b + c & b & x + c \end{vmatrix}$

[by $C_1 \to C_1 + C_2 + C_3$]

$= (x + a + b + c) \begin{vmatrix} 1 & b & c \\ 1 & x + b & c \\ 1 & b & x + c \end{vmatrix}$

[$x + a + b + c$ common from C_1] $= 0$

[$\because x + a + b + c = 0$]

14. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then the expression $A^3 - 2A^2$ is

(a) a null matrix (b) an identity matrix
(c) equal to A (d) equal to $-A$

(**a**) $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + 1 & -1 - 1 \\ -1 - 1 & 1 + 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

and $A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 + 2 & -2 - 2 \\ -2 - 2 & 2 + 2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$

Now,

$$A^3 - 2A^2 = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & -4+4 \\ -4+4 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= a null matrix

15. Let m and $n(m < n)$ be the roots of the equation $x^2 - 16x + 39 = 0$. If four terms p, q, r and s are inserted between m and n to form an AP, then what is the value of $p + q + r + s$?

(a) 29 (b) 30
(c) 32 (d) 35

(**c**) m and n be the roots of the equation $x^2 - 16x + 39 = 0$ $(m < n)$.

$\therefore$ $m + n = 16$ …(i)

and $mn = 39$ …(ii)

We know that, $n - m = \sqrt{(m+n)^2 - 4mn}$

$(\because m < n)$

$= \sqrt{256 - 156} = \sqrt{100}$

$n - m = 10$ …(iii)

Solving the Eqs. (ii) and (iii), $n = 13, m = 3$

Four terms p, q, r and s are inserted between m and n to form an AP.

$\therefore$ AP is 3, p, q, r, s, 13

Here, $a = 3, l = 13, n = 6$

$\therefore$ $l = a + (n-1)d$

$13 = 3 + (6-1)d$

$\Rightarrow$ $d = 2$

$\therefore$ $p = a + d = 3 + 2 = 5,$

$q = a + 2d = 3 + 4 = 7$

$r = a + 3d = 3 + 6 = 9,$

$d = a + 4d = 3 + 8 = 11$

Now, $p + q + r + s = 5 + 7 + 9 + 11$

$= 32$

16. Under which one of the following conditions will the quadratic equation $x^2 + mx + 2 = 0$ always have real roots?

(a) $2\sqrt{3} \le m^2 < 8$ (b) $\sqrt{3} \le m^2 < 4$
(c) $m^2 \ge 8$ (d) $m^2 \le \sqrt{3}$

(**c**) The quadratic equation

$x^2 + mx + 2 = 0,$

have real roots.

$\therefore$ $m^2 - 4(1)(2) \ge 0$

[quadratic equation $ax^2 + bx + c = 0$ have real roots if $b^2 - 4ac \ge 0$]

$\Rightarrow$ $m^2 - 8 \ge 0$

$\Rightarrow$ $m^2 \ge 8$

17. What is the value of

$$\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019} ?$$

(a) 1
(b) – 1
(c) $2i$
(d) $-2i$

(**c**) $\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019}$

$$= \left[\frac{\sqrt{3}}{2} + \frac{1}{2}i\right]^{2019} - \left[\frac{\sqrt{3}}{2} - \frac{1}{2}i\right]^{2019}$$

$$= \left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]^{2019} - \left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]^{2019}$$

$$= \cos\frac{2019\pi}{6} + i\sin\frac{2019\pi}{6} - \cos\frac{2019\pi}{6} + i\sin\frac{2019\pi}{6}$$

[De-moivre's theorem

$(\cos\theta \pm i\sin\theta)^n = \cos n\theta \pm i\sin n\theta$]

$$= 2i\sin\frac{2019\pi}{6}$$

$$= 2i\sin\left(168 \times 2\pi + \frac{3\pi}{6}\right)$$

$$= 2i\sin\frac{3\pi}{6}$$

[$\because \sin(2n\pi + \theta) = \sin\theta$, n is an integer]

$$= 2i\sin\frac{\pi}{2} = 2i$$

18. If α and β are the roots of $x^2 + x + 1 = 0$, then what is $\sum_{j=0}^{3}(\alpha^j + \beta^j)$ equal to?

(a) 8 (b) 6
(c) 4 (d) 2

(**d**) α and β are the roots of the equation

$x^2 + x + 1 = 0$

$\therefore$ $\alpha + \beta = -1$

and $\alpha\beta = 1$

Now, $\sum_{j=0}^{3}(\alpha^j + \beta^j) = (\alpha^0 + \beta^0)$

$+ (\alpha^1 + \beta^1) + (\alpha^2 + \beta^2) + (\alpha^3 + \beta^3)$

$= (1+1) + (-1) + \{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta\}$

$+ (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= 2 - 1 + \{(\alpha+\beta)^2 - 2\alpha\beta\} + (-1)$

$\{\alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha\beta\}$

$= 1 + \{(-1)^2 - 2(1)\} - \{(\alpha+\beta)^2 - 3(1)\}$

$= 1 - 1 - \{(-1)^2 - 3\}$

$= -(1-3) = 2$

19. In a school, 50% students play cricket and 40% play football. If 10% of students play both the games, then what per cent of students play neither cricket nor football?

(a) 10% (b) 15% (c) 20% (d) 25%

(**c**) Students, who play cricket = 50%

Students, who play football = 40%

Students who play both games = 10%

Students who play only cricket

$= 50 - 10 = 40\%$

Students who play only football

$= 40 - 10 = 30\%$

$\therefore$ Total students who play any game

$= 40 + 30 + 10 = 80\%$

$\therefore$ Students who play neither cricket nor football = 100 – 80 = 20%

20. If $A = \{x : 0 \le x \le 2\}$ and $B = \{y : y$ is a prime number$\}$, then what is $A \cap B$ equal to?

(a) ϕ (b) $\{1\}$ (c) $\{2\}$ (d) $\{1, 2\}$

(**c**) $A = \{x : 0 \le x \le 2\} = \{0, 1, 2\}$

and $B = \{y : y$ is a prime number$\}$

$= \{2, 3, 5, 7, 11, \ldots\ldots\}$

$\therefore A \cap B = \{0, 1, 2\} \cap \{2, 3, 5, 7, 11, \ldots\ldots\}$

$= \{2\}$

21. If $x = 1 + i$, then what is the value of $x^6 + x^4 + x^2 + 1$?

(a) $6i - 3$ (b) $-6i + 3$
(c) $-6i - 3$ (d) $6i + 3$

(**c**) Given, $x = 1 + i$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Now, $x^6 + x^4 + x^2 + 1$

$= x^4(x^2+1) + 1(x^2+1)$

$= (x^2+1)(x^4+1)$

$$= \left[(\sqrt{2})^2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^2 + 1\right]\left[(\sqrt{2})^4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^4 + 1\right]$$

$$= \left[2\left(\cos\frac{2\pi}{4} + i\sin\frac{2\pi}{4}\right) + 1\right]\left[4\left(\cos\frac{4\pi}{4} + i\sin\frac{4\pi}{4}\right) + 1\right]$$

[$\because (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$]

$$= \left[2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) + 1\right][4(\cos\pi + i\sin\pi) + 1]$$

$= [2(0+i) + 1][4(-1+0) + 1]$

$= (2i+1)(-4+1) = -6i - 3$

22. What is the value of $2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\infty}}}$?

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) 3 (d) 4

(*b*) Let, $x=2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\infty}}}$

$\Rightarrow \quad x=2+\frac{1}{x} \Rightarrow x^2=2x+1$

$\Rightarrow \quad x^2-2x-1=0$

$\Rightarrow \quad x=\frac{2\pm\sqrt{(-2)^2-4(1)(-1)}}{2}$

$=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$

$=\sqrt{2}+1 \quad (\because x>2)$

23. If $P(n,r)=2520$ and $C(n,r)=21$, then what is the value of $C(n+1,r+1)$?

(a) 7 (b) 14
(c) 28 (d) 56

(*c*) If $P(n, r) = 2520$ and $C(n, r) = 21$,

$\therefore \quad {}^{n}p_r = 2520$

$\Rightarrow \quad \frac{n!}{(n-r)!}=2520 \quad \ldots(i)$

and $\quad {}^{n}C_r = 21$

$\Rightarrow \quad \frac{n!}{r!\,(n-r)!}=21 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$\frac{2520}{r!}=21$

$\Rightarrow \quad r!=\frac{2520}{21}=120$

$\Rightarrow \quad r!=5!$

$\therefore \quad r=5$

Putting the value of r in Eq. (i),

$\frac{n!}{(n-5)!}=2520$

$\Rightarrow \quad n(n-1)(n-2)(n-3)(n-4)$

$=7\times6\times5\times4\times3$

$\therefore \quad n=7$

Now, $C(n+1, r+1) = {}^{n+1}C_{r+1}$

$={}^{7+1}C_{5+1}={}^{8}C_6$

$=\frac{8!}{6!2!}=\frac{8\times7}{2}$

$=28$

24. How many terms are there in the expansion of $(1+2x+x^2)^5+(1+4y+4y^2)^5$?

(a) 12 (b) 20
(c) 21 (d) 22

(*d*) Given expansion,

$(1+2x+x^2)^5+(1+4y+4y^2)^5$

$=[(1+x)^2]^5+[(1+2y)^2]^5$

$=(1+x)^{10}+(1+2y)^{10}$

$\therefore$ Total number of terms in given expansion.

$=(10+1)+(10+1)=22$

[$\because$ total number of terms in expansion of $(1+x)^n=n+1$]

25. If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$ is $184756x^{10}$, then what is the value of n?

(a) 10 (b) 8
(c) 5 (d) 4

(*a*) The middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$

$=\left(\frac{2n}{2}+1\right)$th term [$\because 2n$ is even]

$=(n+1)$th term.

According to the question,

Value of middle term $=184756x^{10}$

$\Rightarrow \quad {}^{2n}C_n\,(x^2)^{2n-n}\left(\frac{1}{x}\right)^n=184756x^{10}$

[$\because T_{r+1}={}^{n}C_r\,x^{n-r}a^r$ in expansion of $(x+a)^n$]

$\Rightarrow {}^{2n}C_n(x)^{4n-2n-n}=184756x^{10}$

$\Rightarrow \quad {}^{2n}C_n(x)^n=184756x^{10}$

Comparing the power of x both sides

$n=10$

26. If $A=\begin{pmatrix}1&2\\2&3\\3&4\end{pmatrix}$ and $B=\begin{pmatrix}1&2\\2&1\end{pmatrix}$, then which one of the following is correct?

(a) Both AB and BA exist
(b) Neither AB nor BA exists
(c) AB exists but BA does not exist
(d) AB does not exist but BA exists

(*c*) We have, $A=\begin{pmatrix}1&2\\2&3\\3&4\end{pmatrix}$ and $B=\begin{pmatrix}1&2\\2&1\end{pmatrix}$

order of $A=3\times2$ and order of $B=2\times2$

$\because$ Number of column of A = Number of row of B

$\therefore AB$ exists.

and number of column of $B \neq$ Number of raw of A

$\therefore BA$ does not exist.

Hence, AB exists but BA does not exist.

27. If $n!$ has 17 zeros, then what is the value of n?

(a) 95 (b) 85
(c) 80
(d) No such value of n exists

(*b*) We know that each interval of 5! is one zero.

i.e. 5! has one zero.

10! has two zeros.

$\therefore$ 85! has 17 zeros.

Hence, the value of n is 85.

28. Let $A\cup B=\{x\,|(x-a)(x-b)>0,$ where $a<b\}$. What are A and B equal to?

(a) $A=\{x|\,x>a\}$ and $B=\{x|\,x>b\}$
(b) $A=\{x|\,x<a\}$ and $B=\{x|\,x>b\}$
(c) $A=\{x|\,x<a\}$ and $B=\{x|\,x<b\}$
(d) $A=\{x|\,x>a\}$ and $B=\{x|\,x<b\}$

(*c*) Let $A\cup B=\{x:(x-a)(x-b)>0,$ where $a<b\}$.

It is possible if $x-a<0$ and $x-b<0$

or $x<a$ and $x<b$

$\therefore A=\{x:x<a\}$ and $B=\{x:x<b\}$

29. If the constant term in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then what can be the values of k?

(a) ±2 (b) ±3
(c) ±5 (d) ±9

(*b*) Let $(r+1)$th term in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is constant.

$\therefore \quad T_{r+1}={}^{10}C_r\,(\sqrt{x})^{10-r}\left(\frac{-k}{x^2}\right)^r$

[$\because T_{r+1}={}^{n}C_r x^{n-r}a^r$ in expansion of $(x+a)^n$]

$\Rightarrow 405={}^{10}C_r\,(x)^{\frac{10-r}{2}-2r}\cdot(-k)^r$

$\Rightarrow 405={}^{10}C_r\,(x)^{\frac{10-5r}{2}}\cdot(-k)^r \quad \ldots(i)$

For constant term

$\frac{10-5r}{2}=0\Rightarrow10-5r=0$

$\therefore \quad r=2$

Putting the value of r, in Eq. (i),

$405={}^{10}C_2\cdot(-k)^2$

$\Rightarrow \quad 405=\frac{10!}{2!8!}\times k^2$

$\Rightarrow \quad 405=\frac{10\times9}{2}.k^2$

$\Rightarrow \quad k^2=\frac{405}{45}$

$\Rightarrow \quad k^2=9$

$\Rightarrow \quad k=\pm3$

30. What is $C(47, 4) + C(51, 3) + C(50, 3) + C(49, 3) + C(48, 3) + C(47, 3)$ equal to?

(a) $C(47, 4)$ (b) $C(52, 5)$
(c) $C(52, 4)$ (d) $C(47, 5)$

(**c**) $C(47, 4) + C(51, 3) + C(50, 3) + C(49, 3) + C(48, 3) + C(47, 3)$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$[\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r]$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4$

$= {}^{52}C_4 = C(52, 4)$

31. Let a, b, c be in AP and $k \neq 0$ be a real number. Which of the following are correct?

1. ka, kb, kc are in AP
2. $k - a, k - b, k - c$ are in AP
3. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(**d**) a, b, c are in AP.

We know that equal number addition, subtraction and multiply, divide, by equal number of each term of an AP, the resultent, series be an AP.

$\therefore ka, kb, kc$ are in AP (multiplying by k).

$k - a, k - b, k - c$ are in AP (subtraction from k) and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP (divide by k)

Hence, option (d) is correct answer.

32. How many two-digit numbers are divisible by 4?

(a) 21 (b) 22
(c) 24 (d) 25

(**b**) Series of two-digit number that divisible by 4 is

12, 16, 20, ..., 96

This series is an AP

Here, $A = 12, d = 4, l = 96$

Let total number of terms be n.

$\therefore \quad l = a + (n-1)d$

$\Rightarrow \quad 96 = 12 + (n-1)4$

$\Rightarrow \quad 84 = (n-1)4$

$\Rightarrow \quad n - 1 = 21$

$\Rightarrow \quad n = 21 + 1 = 22$

33. Let S_n be the sum of the first n terms of an AP. If $S_{2n} = 3n + 14n^2$, then what is the common difference?

(a) 5 (b) 6
(c) 7 (d) 9

(**c**) $S_{2n} = 3n + 14n^2$ (S_n be the sum of first n terms of an AP)

$\Rightarrow \quad S_{2n} = \frac{3}{2}.(2n) + \frac{7}{2}(2n)^2$

Put $2n = n$

we get, $S_n = \frac{3n}{2} + \frac{7n^2}{2}$

$\therefore \quad T_n = S_n - S_{n-1}$

$= \frac{3}{2}n + \frac{7n^2}{2} - \frac{3}{2}(n-1) - \frac{7}{2}(n-1)^2$

$= \frac{3}{2}n + \frac{7}{2}n^2 - \frac{3}{2}n + \frac{3}{2} - \frac{7}{2}n^2 - \frac{7}{2} + \frac{7}{2}2n$

$T_n = 7n - 2$

Put $n = 1, 2, ...$

$T_1 = 7(1) - 2 = 5$

$T_2 = 7(2) - 2 = 12$

$\therefore \quad d = T_2 - T_1 = 12 - 5 = 7$

34. If 3rd, 8th and 13th terms of a GP are p, q and r respectively, then which one of the following is correct?

(a) $q^2 = pr$ (b) $r^2 = pq$
(c) $pqr = 1$ (d) $2q = p + r$

(**a**) Let first term and common ratio of a GP be a and R.

$\therefore \quad T_3 = aR^2 = p \quad ...(i)$

$T_8 = aR^7 = q \quad ...(ii)$

$T_{13} = aR^{12} = r \quad ...(iii)$

Multiplying of Eqs. (i) and (iii)

$(aR^2)(aR^{12}) = pr$

$\Rightarrow \quad a^2R^{14} = pr$

$\Rightarrow \quad (aR^7)^2 = pr$

$\Rightarrow \quad q^2 = pr$ [from Eq. (ii)]

35. What is the solution of $x \leq 4, y \geq 0$ and $x \leq -4, y \leq 0$?

(a) $x \geq -4, y \leq 0$ (b) $x \leq 4, y \geq 0$
(c) $x \leq -4, y = 0$ (d) $x \geq -4, y = 0$

(**c**) Given inequalities

$x \leq 4, y \geq 0 \quad ...(i)$

and $x \leq -4, y \leq 0 \quad ...(ii)$

Possible value of x and y.

$x = \{4, 3, 2, 1, 0, -1, -2, -3, -4, -5, ...\}$

$y = \{0, 1, 2, 3, 4 .....\} \quad ...(i)$

and $x = \{-4, -5, -6, -7, ...\}$,

$y = \{0, -1, -2, -3, -4 ...\} \quad ...(ii)$

Take combine (i) and (ii),

$x = \{-4, -5, -6, -7 ... \}, y = 0$

or $x \leq -4, y = 0$.

36. If $x^{\log_7 x} > 7$ where $x > 0$, then which one of the following is correct?

(a) $x \in (0, \infty)$ (b) $x \in \left(\frac{1}{7}, 7\right)$
(c) $x \in \left(0, \frac{1}{7}\right) \cup (7, \infty)$
(d) $x \in \left(\frac{1}{7}, \infty\right)$

(**b**) $x^{\log_7 x} > 7$ where $x > 0$.

Taking log on base 7 both sides

$\log_7 x . \log_7 x > \log_7 7$

$[\because \log_a m^n = n \log_a m]$

$\Rightarrow \quad (\log_7 x)^2 > 1 \quad [\because \log_a a = 1]$

$\Rightarrow \quad \log_7 x > (\pm 1)$

$\therefore \quad x > 7^1 \Rightarrow x > 7$

and $x < 7^{-1} \Rightarrow x < \frac{1}{7}$

Hence, $x \in \left(\frac{1}{7}, 7\right)$

37. How many real roots does the equation $x^2 + 3|x| + 2 = 0$ have?

(a) Zero (b) One
(c) Two (d) Four

(**a**) Given equation, $x^2 + 3|x| + 2 = 0$

Case I $x^2 + 3x + 2 = 0$ (when $x > 0$)

$\Rightarrow \quad x^2 + x + 2x + 2 = 0$

$\Rightarrow \quad x(x+1) + 2(x+1) = 0$

$\Rightarrow \quad (x+1)(x+2) = 0$

$\therefore \quad x = -1, -2$

Hence, no real roots because $x > 0$.

Case II $x^2 - 3x + 2 = 0$ (when $x < 0$)

$\Rightarrow \quad x^2 - 2x - x + 2 = 0$

$\Rightarrow \quad x(x-2) - 1(x-2) = 0$

$\Rightarrow \quad (x-2)(x-1) = 0$

$\therefore \quad x = 1, 2$

Hence, no real roots because $x < 0$.

$\therefore$ The number of real roots of given equation is zero.

38. Consider the following statements in respect of the quadratic equation $4(x-p)(x-q) - r^2 = 0$, where p, q and r are real numbers.

1. The roots are real.
2. The roots are equal, if $p = q$ and $r = 0$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) Given quadratic equation,

$4(x-p)(x-q) - r^2 = 0$

$\Rightarrow \quad 4x^2 - (4q + 4p)x + 4pq - r^2 = 0$

Comparing it Eq. by $ax^2 + bx + c = 0$

$a = 4, b = -4(p+q), c = 4pq - r^2$

$b^2 - 4ac = 16(p+q)^2 - 4 \times 4(4pq - r^2)$

$= 16p^2 + 16q^2 + 32pq - 64pq + 16r^2$

$= 16p^2 + 16q^2 - 32pq + 16r^2$

$= 16(p-q)^2 + 16r^2$

$\because b^2 - 4ac$ will be positive.

So, the roots are real.

If $p = q$ and $r = 0$, then $b^2 - 4ac = 0$

So, the roots are equal.

Hence, the statements both 1 and 2 are correct.

39. Let $S = \{2, 4, 6, 8, ...,20\}$.
What are the maximum number of subsets of S?
(a) 10 (b) 20
(c) 512 (d) 1024

(***d***) $S = \{2, 4, 6, 8, ..., 20\}$
Here, number of elements of set
$S = 10\ (n)$
$\therefore$ Maximum number of subsets of set
$S = 2^n = 2^{10} = 1024$

40. A binary number is represented by $(cdccddcccddd)_2$, where $c > d$. What is its decimal equivalent?
(a) 1848 (b) 2048
(c) 2842 (d) 2872

(***d***) Binary number $= (cdccddcccddd)_2$ where, $c > d$. We know that only two bit (digits) 0 and 1 be any binary number.
$\therefore$ Given binary number
$= (101100111000)_2$
$= (1\times2^{11} + 0\times2^{10} + 1\times2^9 + 1\times2^8 +$
$+ 0\times2^7 + 0\times2^6 + 1\times2^5 + 1\times2^4$
$+ 1\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0)_{10}$
$= (2048 + 512 + 256 + 32 + 16 + 8)_{10}$
$= (2872)_{10}$

41. If $\text{cosec}\,\theta = \frac{29}{21}$, where $0 < \theta < 90°$, then what is the value of $4\sec\theta + 4\tan\theta$?
(a) 5 (b) 10 (c) 15 (d) 20

(***b***) Given, $\text{cosec}\,\theta = \frac{29}{21}$
where, $0 < \theta < 90°$
$\because \text{cosec}\,\theta = \frac{H}{P} = \frac{29}{21} = k$ (let)
$\therefore H = 29k, P = 21k$
$\therefore B = \sqrt{(H)^2 - (P)^2} = \sqrt{(29k)^2 - (21k)^2}$
$= \sqrt{841k^2 - 441k^2}$
$= \sqrt{400k^2} = 20k$
$\therefore \sec\theta = \frac{H}{B} = \frac{29k}{20k} = \frac{29}{20}$
and $\tan\theta = \frac{P}{B} = \frac{21k}{20k} = \frac{21}{20}$
Now, $4\sec\theta + 4\tan\theta$
$= 4 \times \frac{29}{20} + 4 \times \frac{21}{20}$
$= 4 \times \frac{50}{20} = 10$

42. Consider the following statements
1. $\cos\theta + \sec\theta$ can never be equal to 1.5.
2. $\tan\theta + \cot\theta$ can never be less than 2.

Which of the above statements is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***b***) We know that, $-1 \le \cos\theta \le 1$ and $-1 \le \sec\theta \le \infty$ but $\cos\theta = \sec\theta$ if $\theta = 0$ and $\theta = 180°$
$\therefore \quad -2 \le \cos\theta + \sec\theta \le \infty$
So, $\cos\theta + \sec\theta = 1.5$ is possible.
and again $0 \le \tan\theta \le \infty$ and
$0 \le \cot\theta \le \infty$, but $\tan\theta = \cot\theta$
if $\theta = 45°$
$\therefore \quad 2 \le \tan\theta + \cot\theta \le \infty$
So, $\tan\theta + \cot\theta$ can never be less than 2.
Hence, only the Statement 2 is correct.

43. A ladder 9 m long reaches a point 9 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the flagstaff is 60°. What is the height of the flagstaff?
(a) 9 m (b) 10.5 m
(c) 13.5 m (d) 15 m

(*) Let AP be a ladder and QR be a vertical flagstaff. P is a point 9 m below the top on flagstaff. A is the foot of ladder and h is the height of point P from the ground.
$\therefore AP = 9$ m, $PR = 9$ m, $PQ = h$m
In $\Delta\ AQP$, $\sin\theta = \frac{PQ}{AP}$
$\Rightarrow \sin 60° = \frac{h}{9} \Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{9}$
$\Rightarrow \quad 9\sqrt{3} = 2h \Rightarrow h = \frac{9\sqrt{3}}{2}$
$= \frac{9 \times 1.73}{2} = \frac{15.57}{2} = 7.7$ m
$\therefore$ Height of flagstaff
$= h + 9 = 7.7 + 9$
$= 16.7$ m

44. What is the length of the chord of a unit circle which subtends an angle θ at the centre?
(a) $\sin\left(\frac{\theta}{2}\right)$ (b) $\cos\left(\frac{\theta}{2}\right)$
(c) $2\sin\left(\frac{\theta}{2}\right)$ (d) $2\cos\left(\frac{\theta}{2}\right)$

(***c***) Given, radius of circle = 1 unit
Angle subtends at the centre of circle by chord = θ
We know that, length of chord
$= 2r\sin\frac{\theta}{2} = 2 \times 1\sin\frac{\theta}{2} = 2\sin\frac{\theta}{2}$

45. What is $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$ equal to?
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$
(d) $\frac{1}{9}$

(***b***) $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$
$= \tan\left\{\tan^{-1}\frac{2 \times \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2}\right\}$
$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$
$= \tan\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right) = \frac{2 \times 9}{3 \times 8} = \frac{3}{4}$

46. What is the scalar projection of $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ on $\mathbf{b} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$?
(a) $\frac{\sqrt{6}}{9}$ (b) $\frac{19}{9}$ (c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

(***b***) $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$
Projection of $\mathbf{a}$ on $\mathbf{b}$
$= \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|}$
$= \frac{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}).(4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}})}{\sqrt{16 + 16 + 49}}$
$= \frac{4 + 8 + 7}{\sqrt{81}} = \frac{19}{9}$

47. If the magnitude of the sum of two non-zero vectors is equal to the magnitude of their difference, then which one of the following is correct?
(a) The vectors are parallel
(b) The vectors are perpendicular
(c) The vectors are anti-parallel
(d) The vectors must be unit vectors

(***b***) Let $\mathbf{a}$ and $\mathbf{b}$ are the two non-zero vectors.
According to the question,
$|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$
$\Rightarrow \mathbf{a}^2 + \mathbf{b}^2 + 2\mathbf{a}\cdot\mathbf{b} = \mathbf{a}^2 + \mathbf{b}^2 - 2\mathbf{a}\cdot\mathbf{b}$
$\Rightarrow \quad 4\mathbf{a}\cdot\mathbf{b} = 0 \Rightarrow \mathbf{a}\cdot\mathbf{b} = 0$
So, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

48. Consider the following equations for two vectors $\mathbf{a}$ and $\mathbf{b}$.

1. $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2$
2. $(|\mathbf{a}+\mathbf{b}|)(|\mathbf{a}-\mathbf{b}|) = |\mathbf{a}|^2 - |\mathbf{b}|^2$
3. $|\mathbf{a}\cdot\mathbf{b}| + |\mathbf{a}\times\mathbf{b}| = |\mathbf{a}^2|\,|\mathbf{b}|^2$

Which of the above statements are correct?

(a) 1, 2 and 3 (b) Only 1 and 2
(c) Only 1 and 3 (d) Only 2 and 3

(*c*) I : $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$

$= \mathbf{a}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{a} - \mathbf{b}\cdot\mathbf{b}$

$|\mathbf{a}|^2 - \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{b} - |\mathbf{b}|^2$

$[\because \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}]$

$= |\mathbf{a}|^2 - |\mathbf{b}|^2$

So, Statement 1 is correct.

2. $(|\mathbf{a}+\mathbf{b}|)(|\mathbf{a}-\mathbf{b}|) = |\mathbf{a}+\mathbf{b}||\mathbf{a}-\mathbf{b}|$

$\neq |\mathbf{a}|^2 - |\mathbf{b}|^2$

So, Statement 2 is not correct.

3. $|\mathbf{a}\cdot\mathbf{b}|^2 + |\mathbf{a}\times\mathbf{b}|^2 = |\,|\mathbf{a}|\,|\mathbf{b}|\cos\theta|^2$

$+ ||\mathbf{a}|\,|\mathbf{b}|\sin\theta|^2$

$= |\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta + |\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta$

$= |\mathbf{a}|^2|\mathbf{b}|^2(\cos^2\theta + \sin^2\theta)$

$= |\mathbf{a}|^2|\mathbf{b}|^2 \quad [\because \cos^2\theta + \sin^2\theta = 1]$

So, statement 3 is correct.

Hence, only Statements 1 and 3 are correct.

49. Consider the following statements.

1. The magnitude of $\mathbf{a}\times\mathbf{b}$ is same as the area of a triangle with sides $\mathbf{a}$ and $\mathbf{b}$
2. If $\mathbf{a}\times\mathbf{b} = 0$, where $\mathbf{a}\neq 0, \mathbf{b}\neq 0$, then $\mathbf{a} = \lambda\,\mathbf{b}$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*b*) 1. We know that,

Area of triangle with sides $\mathbf{a}$ and $\mathbf{b}$

$= \frac{1}{2}|\mathbf{a}\times\mathbf{b}|$

So, statement 1 is not correct.

2. $\mathbf{a}\times\mathbf{b} = 0$, where $\mathbf{a}\neq 0, \mathbf{b} = 0$,

So, $\mathbf{a}$ and $\mathbf{b}$ are parallel.

$\Rightarrow \quad \mathbf{a} = \lambda\,\mathbf{b}$

So, Statement 2 is correct.

Hence, only statement 2 is correct.

50. If $\mathbf{a}$ and $\mathbf{b}$ are unit vectors and θ is the angle between them, then what is $\sin^2\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\mathbf{a}+\mathbf{b}|^2}{4}$ (b) $\frac{|\mathbf{a}-\mathbf{b}|^2}{4}$

(c) $\frac{|\mathbf{a}+\mathbf{b}|^2}{2}$ (d) $\frac{|\mathbf{a}-\mathbf{b}|^2}{2}$

(*b*) Given, $|\mathbf{a}| = 1, |\mathbf{b}| = 1$

We know that,

$|\mathbf{a}-\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a}\cdot\mathbf{b}$

$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = 1+1-2|\mathbf{a}||\mathbf{b}|\cos\theta$

$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = 2 - 2\cos\theta$

$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = 2(1-\cos\theta)$

$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = 2\left(1 - 1 + 2\sin^2\frac{\theta}{2}\right)$

$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = 2\cdot\left(2\sin^2\frac{\theta}{2}\right)$

$\Rightarrow \quad \sin^2\frac{\theta}{2} = \frac{|\mathbf{a}-\mathbf{b}|^2}{4}$

51. The equation $ax + by + c = 0$ represents a straight line

(a) for all real numbers, a, b and c
(b) only when $a \neq 0$
(c) only when $b \neq 0$
(d) only when at least one of a and b is non-zero.

(*d*) The equation $ax + by + c = 0$ represents a straight line only when at least one of a and b is non zero.

52. What is the angle between the lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\beta - y\cos\beta = a$?

(a) $\beta - \alpha$ (b) $\pi + \beta - \alpha$
(c) $\frac{(\pi + 2\beta + 2\alpha)}{2}$ (d) $\frac{(\pi - 2\beta + 2\alpha)}{2}$

(*d*) The equations of given lines

$x\cos\alpha + y\sin\alpha = a$ … (i)

and $x\sin\beta - y\cos\beta = a$ … (ii)

Slope of Eq. (i), $m_1 = \frac{-\cos\alpha}{\sin\alpha} = -\cot\alpha$

$= \tan\left(\frac{\pi}{2} + \alpha\right)$

Slope of Eq. (ii), $m_2 = \frac{-\sin\beta}{-\cos\beta} = \tan\beta$

Let θ be the angle between the lines, then

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \frac{\tan\left(\frac{\pi}{2} + \alpha\right) - \tan\beta}{1 + \tan(\pi - \alpha)\tan\beta}$

$\tan\theta = \tan\left(\frac{\pi}{2} + \alpha - \beta\right)$

$\theta = \frac{\pi}{2} + \alpha - \beta = \frac{\pi + 2\alpha - 2\beta}{2}$

53. What is the distance between the points $P(m\cos 2\alpha, m\sin 2\alpha)$ and $Q(m\cos 2\beta, m\sin 2\beta)$?

(a) $|2m\sin(\alpha-\beta)|$ (b) $|2m\cos(\alpha-\beta)|$
(c) $|m\sin(2\alpha - 2\beta)|$
(d) $|m\sin(2\alpha - 2\beta)|$

(*a*) Given points, $p(m\cos 2\alpha, m\sin 2\alpha)$ and $Q(m\cos 2\beta, m\sin 2\beta)$

$\therefore PQ = |\sqrt{(m\cos 2\beta - m\cos 2\alpha)^2 + (m\sin 2\beta - m\sin 2\alpha)^2}|$

[$\because$ Distance between two points (x_1, y_1) and (x_2, y_2)

$= |\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}|$

$= |\sqrt{\begin{matrix} m^2\cos^2 2\beta + m^2\cos^2 2\alpha \\ - 2m^2\cos 2\beta\cos 2\alpha + m^2\sin^2 2\beta \\ + m^2\sin^2 2\alpha - 2m^2\sin 2\beta\sin 2\alpha \end{matrix}}|$

$= |m\sqrt{\begin{matrix} (\cos^2 2\beta + \sin^2 2\beta) \\ + (\cos^2 2\alpha + \sin^2 2\alpha) \\ - 2(\cos 2\beta\cos 2\alpha \\ + \sin 2\beta\sin 2\alpha) \end{matrix}}|$

$= |m\sqrt{1 + 1 - 2\cos(2\alpha - 2\beta)}|$

$= |m\sqrt{2[1 - \cos 2(\alpha - \beta)]}|$

$= |m\sqrt{2[1 - 1 + 2\sin^2(\alpha - \beta)]}|$

$= |m\sqrt{2\times 2\sin^2(\alpha - \beta)}|$

$= |2m\sin(\alpha - \beta)|$

54. An equilateral triangle has one vertex at $(-1, -1)$ and another vertex at $(-\sqrt{3}, \sqrt{3})$. The third vertex may lie on

(a) $(-\sqrt{2}, \sqrt{2})$ (b) $(\sqrt{2}, -\sqrt{2})$
(c) (1, 1) (d) (1, – 1)

(*c*) Consider two vertices of an equilateral triangle are $A(-1, -1)$ and $B(-\sqrt{3}, \sqrt{3})$. Let third vertex x be $C(x, y)$.

$\because \Delta ABC$ is equilateral

$\therefore \quad AC = AB$

$\sqrt{(x+1)^2 + (y+1)^2} = \sqrt{(-\sqrt{3}+1)^2 + (\sqrt{3}+1)^2}$

$\Rightarrow \quad x^2 + 1 + 2x + y^2 + 1 + 2y$

$= 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}$

$\Rightarrow \quad x^2 + y^2 + 2x + 2y + 2 = 8$

$\Rightarrow \quad x^2 + y^2 + 2x + 2y = 6$

From option only point (1, 1) is satisfying of it equation. Hence, the third vertex may lie on (1, 1).

55. If the angle between the lines joining the end points of minor axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with one of the its foci is $\frac{\pi}{2}$, then what is the eccentricity of the ellipse?

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

(*b*) Equation of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

End points of minor axis are $(0, b)$, $(0, -b)$ and one foci is $(ae, 0)$

Slope of line $BS = \frac{0 - b}{ae - 0} = -\frac{b}{ae}\,(m_1)$

Slope of line $B'S = \frac{0 + b}{ae - 0} = \frac{b}{ae}\,(m_2)$

According to the question, angle between BS and $B'S$ is $\frac{\pi}{2}$.

i.e. BS and $B'S$ are perpendicular,

$\therefore \quad m_1 m_2 = -1$

$$\frac{-b}{ae} \times \frac{b}{ae} = -1 \Rightarrow b^2 = a^2e^2 \ \ldots \text{(i)}$$

We know that, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow \quad a^2e^2 = a^2 - b^2$

$\Rightarrow \quad a^2e^2 = a^2 - a^2e^2$ [from Eq. (i)]

$\Rightarrow \quad 2a^2e^2 = a^2 \Rightarrow e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$

56. A point on a line has coordinates $(p+1, p-3, \sqrt{2}p)$ where p is any real number. What are the direction cosines of the line?

(a) $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}, \frac{1}{2}, -\frac{1}{2}$

(d) Cannot be determined due to insufficient data

(**d**) Coordinate of a point on a line is $(p+1, p-3, \sqrt{2}p)$, p is any real number.

Equation of a line, whose direction ratios are a, b and c and passing through the point (x_1, y_1, z_1)

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r$$

$\therefore (ar + x_1, br + y_1, cr + z_1)$ any point on the line.

According to the questions,

$(ar + x_1, br + y_1, cr + z_1)$

$= (p+1, p-3, \sqrt{2}p)$

$\therefore \quad ar = p + 1 - x_1 \quad \ldots \text{(i)}$

$br = p - 3 - y_1 \quad \ldots \text{(ii)}$

$cr = \sqrt{2}p - z_1 \quad \ldots \text{(iii)}$

Squaring and adding of (i), (ii) and (iii)

$(a^2 + b^2 + c^2)r^2 = (p + 1 - x_1)^2$

$+ (p - 3 - y_1)^2 + (\sqrt{2}p - z_1)^2$

We can not find the values of a, b and c.

Hence, the direction cosines of the line can not be determined due to insufficient data.

57. A point on the line

$$\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+2}{7}$$

has coordinates

(a) (3, 5, 4) (b) (2, 5, 5)

(c) (– 1, – 1, 5) (d) (2, – 1, 0)

(**b**) Equation of the line

$$\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+2}{7}$$

From option, point (2, 5, 5) is satisfying the given equation of line.

$$\left[\because \frac{2-1}{1} = \frac{5-3}{2} = \frac{5+2}{7} \Rightarrow 1 = 1 = 1\right]$$

Hence, the coordinates of required point (2, 5, 5).

58. If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$, then what is the value of k?

(a) 2 (b) 3

(c) 5 (d) 7

(**d**) Equation of line

$$= \frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2} = r$$

$\therefore (r + 4, r + 2, 2r + k)$ point lies on the line.

This line lies on the plane

$$2x - 4y + z = 7$$

Then, the point $(r + 4, r + 2, 2r + k)$ lies on the plane, we get

$$2(r + 4) - 4(r + 2) + (2r + k) = 7$$

$\Rightarrow \quad 2r + 8 - 4r - 8 + 2r + k = 7$

$\Rightarrow \quad k = 7$

Hence, the value of k is 7.

59. A straight line passes through the point (1, 1, 1) makes an angle 60° with the positive direction of Z-axis, and the cosine of the angles made by it with the positive directions of the Y-axis and the X-axis are in the ratio $\sqrt{3} : 1$. What is the acute angle between the two possible positions of the line?

(a) 90° (b) 60°

(c) 45° (d) 30°

(**b**) Let the straight line makes the angle with X-axis, Y-axis and Z-axis be α, β and γ.

$\therefore \gamma = 60°$ and $\frac{\cos\beta}{\cos\alpha} = \frac{\sqrt{3}}{1}$

If l, m and n are the direction cosines of the lines, then

$$n = \cos\gamma = \cos 60° = \frac{1}{2}$$

and $\frac{m}{l} = \frac{\cos\beta}{\cos\alpha} = \frac{\sqrt{3}}{1}$

$\Rightarrow \quad \frac{m}{l} = \frac{\sqrt{3}}{1} = k$ (Let)

$\therefore \quad m = \sqrt{3}k, l = k$

We know that, $l^2 + m^2 + n^2 = 1$

$k^2 + 3k^2 + \frac{1}{4} = 1$

$\Rightarrow \quad 4k^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \quad k^2 = \frac{3}{16} \Rightarrow k = \pm\frac{\sqrt{3}}{4}$

$\therefore \quad l_1 = \frac{\sqrt{3}}{4}, m_1 = \frac{3}{4}, n_1 = \frac{1}{2}$

and $l_2 = -\frac{\sqrt{3}}{4}, m_2 = -\frac{3}{4}, n_2 = \frac{1}{2}$

We know that,

$\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$

$\Rightarrow \cos\theta = \left|-\frac{3}{16} - \frac{9}{16} + \frac{1}{4}\right|$

$= \left|\frac{-3 - 9 + 4}{16}\right| = \left|\frac{-8}{16}\right|$

$\cos\theta = \frac{1}{2} = \cos 60°$

$\therefore \quad \theta = 60°$

60. If the points $(x, y, -3)$, $(2, 0, -1)$ and $C(4, 2, 3)$ lie on a straight line, then what are the values of x and y respectively?

(a) 1, – 1 (b) –1, 1

(c) 0, 2 (d) 3, 4

(**a**) Points, $A(x, y, -3)$, $B(2, 0, -1)$ and $(4, 2, 3)$. These points lie on a straight line, then direction ratios of $AB = \lambda$ (direction ratios of BC)

$\therefore (2 - x, 0 - y, -1 + 3)$

$= (4 - 2, 2 - 0, 3 + 1)$

$\Rightarrow (2 - x, -y, 2) = (2, 2, 4)$

$\Rightarrow (2 - x, -y, 2) = 2(1, 1, 2)$

Comparing both sides,

$2 - x = 1 \Rightarrow x = 1$

and $-y = 1 \Rightarrow y = -1$

61. What is the minimum value of $\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$ where $a > 0$ and $b > 0$?

(a) $(a + b)^2$ (b) $(a - b)^2$

(c) $a^2 + b^2$ (d) $|a^2 + b^2|$

(*) Let $p = \frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$

$= a^2\sec^2 x + b^2\operatorname{cosec}^2 x$

$- 2ab\sec x \operatorname{cosec} x$

$+ 2ab \sec x \operatorname{cosec} x$

$= (a \sec x - b\operatorname{cosec} x)^2$

$+ 2ab \sec x \operatorname{cosec} x$

For minimum value of p,

$a \sec x - b \operatorname{cosec} x = 0$

$\Rightarrow \quad a \sec x = b \operatorname{cosec} x$

$\Rightarrow \quad \frac{\sec x}{\operatorname{cosec} x} = \frac{b}{a}$

$\Rightarrow \quad \tan x = \frac{b}{a}$

$\therefore \sin x = \frac{b}{\sqrt{a^2 + b^2}}, \cos x = \frac{a}{\sqrt{a^2 + b^2}}$

$\therefore$ Minimum value of p

$= \frac{a^2(a^2 + b^2)}{a^2} + \frac{b^2(a^2 + b^2)}{b^2}$

$= 2(a^2 + b^2)$

62. If the angles of a triangle ABC are in AP and $b : c = \sqrt{3} : \sqrt{2}$, then what is the measure of angle A?

(a) 30° (b) 45°
(c) 60° (d) 75°

(d) Angles of a triangle ABC are in AP, then $2B = A + C$

We know that, $A + B + C = 180°$

$\Rightarrow \quad 3B = 180° \Rightarrow B = 60°$

By sine rule, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin A}{a} = \frac{\sin 60°}{b} = \frac{\sin C}{c}$

Take II and III, $\frac{\sin 60°}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin 60°}{\sin C} = \frac{b}{c} \Rightarrow \frac{\sqrt{3}/2}{\sin C} = \frac{\sqrt{3}}{\sqrt{2}}$

[$\because$ Given, $b : c = \sqrt{3} : \sqrt{2}$]

$\Rightarrow \quad \sin C = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin C = \sin 45° \Rightarrow C = 45°$

$\therefore \quad A = 180° - (B + C)$

$= 180° - (60° + 45°) = 75°$

63. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then what is the value of $\cot (A - B)$?

(a) $\frac{1}{x} + \frac{1}{y}$ (b) $\frac{1}{y} - \frac{1}{x}$
(c) $\frac{xy}{x + y}$ (d) $1 + \frac{1}{xy}$

(a) Given, $\tan A - \tan B = x$ ... (i)

and $\cot B - \cot A = y$ ... (ii)

From Eq. (i), $\tan A - \tan B = x$

$\Rightarrow \quad \frac{1}{\cot A} - \frac{1}{\cot B} = x$

$\Rightarrow \quad \frac{\cot B - \cot A}{\cot A \cot B} = x$

$\Rightarrow \quad \cot A \cot B = \frac{y}{x}$ [from Eq. (ii)]

Now, $\cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

$= \frac{\frac{y}{x} + 1}{y} = \frac{y + x}{xy} = \frac{1}{x} + \frac{1}{y}$

64. What is $\sin (\alpha + \beta) - 2 \sin \alpha \cos \beta + \sin (\alpha - \beta)$ equal to?

(a) 0 (b) $2 \sin \alpha$
(c) $2 \sin \beta$ (d) $\sin \alpha + \sin \beta$

(a) $\sin (\alpha + \beta) - 2 \sin \alpha \cos \beta + \sin (\alpha - \beta)$

$= \sin \alpha \cos \beta + \cos \alpha \sin \beta - 2 \sin \alpha \cos \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta = 0$

65. If $2 \tan A = 3 \tan B = 1$, then what is $\tan (A - B)$ equal to?

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$
(c) $\frac{1}{7}$ (d) $\frac{1}{9}$

(c) Given, $2 \tan A = 3 \tan B = 1$

$\therefore \tan A = \frac{1}{2}, \tan B = \frac{1}{3}$

Now, $\tan (A - B)$

$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\frac{1}{2} - \frac{1}{3}}{1 + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} = \frac{\frac{3-2}{6}}{\frac{6+1}{6}} = \frac{1}{7}$

66. What is $\cos 80° + \cos 40° - \cos 20°$ equal to?

(a) 2 (b) 1
(c) 0 (d) –19

(c) $\cos 80° + \cos 40° - \cos 20°$

$= 2 \cos \frac{80° + 40°}{2} . \cos \frac{80° - 40°}{2} - \cos 20°$

$= 2 \cos 60° \cos 20° - \cos 20°$

$= 2 \times \frac{1}{2} \cos 20° - \cos 20°$

$= \cos 20° - \cos 20° = 0$

67. If angle C of a triangle ABC is a right angle, then what is $\tan A + \tan B$ equal to ?

(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{a^2}{bc}$
(c) $\frac{b^2}{ca}$ (d) $\frac{c^2}{ab}$

(d) In ΔABC, $\angle C = 90°$

$\therefore \quad c^2 = a^2 + b^2$ [by Pythagoras theorem] ... (i)

$\tan A = \frac{a}{b}, \tan B = \frac{b}{a}$

Now, $\tan A + \tan B = \frac{a}{b} + \frac{b}{a}$

$= \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$ [from Eq. (i)]

68. What is $\cot \left(\frac{A}{2}\right) - \tan \left(\frac{A}{2}\right)$ equal to?

(a) $\tan A$ (b) $\cot A$
(c) $2 \tan A$ (d) $2 \cot A$

(d) $\cot \frac{A}{2} - \tan \frac{A}{2}$

$= \frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} - \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}}$

$= \frac{\cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}}{\sin \frac{A}{2} \cos \frac{A}{2}} = \frac{2 \cos A}{2 \sin \frac{A}{2} \cos \frac{A}{2}}$

$= \frac{2 \cos A}{\sin A} = 2 \cot A$

69. What is $\cot A + \operatorname{cosec} A$ equal to?

(a) $\tan \left(\frac{A}{2}\right)$ (b) $\cot \left(\frac{A}{2}\right)$
(c) $2 \tan \left(\frac{A}{2}\right)$ (d) $2 \cot \left(\frac{A}{2}\right)$

(b) $\cot A + \operatorname{cosec} A$

$= \frac{\cos A}{\sin A} + \frac{1}{\sin A} = \frac{\cos A + 1}{\sin A}$

$= \frac{2 \cos^2 \frac{A}{2} - 1 + 1}{2 \sin \frac{A}{2} \cos \frac{A}{2}}$

$= \frac{2 \cos^2 \frac{A}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}} = \frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} = \cot \left(\frac{A}{2}\right)$

70. What is $\tan 25° \tan 15° + \tan 15° \tan 50° + \tan 25° \tan 50°$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

(b) $\because \tan 50° = \tan (90° - 40°)$

$\Rightarrow \tan 50° = \cot 40°$

$\Rightarrow \tan 50° = \frac{1}{\tan 40°}$

$\Rightarrow \tan 50° = \frac{1}{\tan (25° + 15°)}$

$\Rightarrow \tan 50° = \frac{1 - \tan 25° \tan 15°}{\tan 25° + \tan 15°}$

$\Rightarrow \tan 25° \tan 50° + \tan 15° \tan 50° = 1 - \tan 25° \tan 15°$

$\Rightarrow \tan 25° \tan 15° + \tan 15° \tan 50° + \tan 25° \tan 50° = 1$

71. What is the area of the region bounded by $|x| < 5$, $y = 0$ and $y = 8$?

(a) 40 sq units (b) 80 sq units
(c) 120 sq units (d) 160 sq units

(b) Given curve $y = 0$ and $y = 8$ and $|x| < 5$

Case I When $x < 0$, then

area of the region bounded

$= \int_{-5}^{0} 0 \, dx - \int_{-5}^{0} 8 \, dx = 0 - 8 [x]_{-5}^{0}$

$= -8 [0 + 5] = -40$

$= 40$ sq units

[$\because$ area will not be negative]

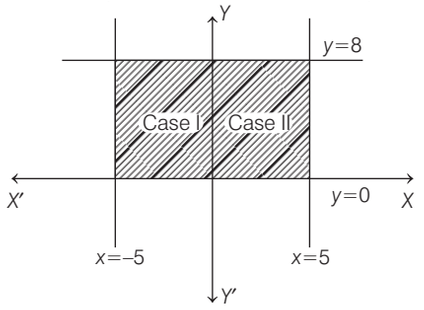
Case II when $x > 0$, then

Area of the region bounded

$= \int_{0}^{5} 0 \, dx - \int_{0}^{5} 8 \, dx = 0 - 8 [x]_{0}^{5}$

$= -8[5 - 0] = -40 = 40$ sq units

∴ Required area = 40 + 40 = 80 sq units



72. Consider the following statements in respect of the function $f(x)=\sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0)=0$:

1. $\lim_{x\to 0} f(x)$ exists
2. $f(x)$ is continuous at $x = 0$

Which of the above statement is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***d***) Given, $f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\text{LHL} = \lim_{x\to 0^-} \sin\left(\frac{1}{x}\right) = \lim_{h\to 0} \sin\left(\frac{1}{0-h}\right)$$

$$= \lim_{h\to 0} -\sin\left(\frac{1}{h}\right) = -\sin\infty$$

= – (a rational number)

[∵ sin θ lies between –1 to 1]

$$\text{RHL} = \lim_{x\to 0^+} \sin\left(\frac{1}{x}\right) = \lim_{h\to 0} \sin\left(\frac{1}{0+h}\right)$$

$$= \lim_{h\to 0} \sin\left(\frac{1}{h}\right) = \sin\infty$$

= a rational number

[∵ sin θ lies between – 1 to 1]

∵ LHL ≠ RHL

So, $f(x)$ does not exists.

∵ $f(x) = 0$ at $x = 0$

∴ LHL ≠ RHL ≠ $f(0)$

So, $f(x)$ is not continuous.

Hence, the statements neither 1 nor 2 correct.

73. What is the value of $\lim_{x\to 0} \frac{\sin x^\circ}{\tan 3x^\circ}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(***b***) $$\lim_{x\to 0}\frac{\sin x^\circ}{\tan 3x^\circ} = \lim_{x\to 0}\left(\frac{x \times \frac{\sin x}{x}}{3x \times \frac{\tan 3x}{3x}}\right)$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)}{\left(\frac{\tan 3x}{3x}\right)} = \frac{1}{3}$$

$$\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \text{ and } \lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1\right]$$

74. What is the degree of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 - x^2\left(\frac{d^4y}{dx^4}\right) = 0?$$

(a) 1 (b) 2
(c) 3 (d) 4

(***a***) Given differential equation,

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 - x^2\left(\frac{d^4y}{dx^4}\right) = 0$$

$$\Rightarrow \frac{d^4y}{dx^4} - \frac{1}{x^2}\left(\frac{d^3y}{dx^3}\right) - \frac{1}{x^2}\left(\frac{dy}{dx}\right)^2 = 0$$

We know that power of the heighest order of differentiation is the degree of differential equation.

So, the degree of it equation is 1.

75. Which one of the following is the second degree polynomial function $f(x)$ where, $f(0) = 5$, $f(-1) = 10$ and $f(1) = 6$?

(a) $5x^2 - 2x + 5$ (b) $3x^2 - 2x - 5$
(c) $3x^2 - 2x + 5$ (d) $3x^2 - 10x + 5$

(***c***) From the option (c),

$f(x) = 3x^2 - 2x + 5$

$f(0) = 3(0)^2 - 2(0) + 5$

$= 5$

$f(-1) = 3(-1)^2 - 2(-1) + 5$

$= 3 + 2 + 5 = 10$

and $f(1) = 3(1)^2 - 2(1) + 5$

$= 3 - 2 + 5 = 6$

Hence, the required polynomial

$f(x) = 3x^2 - 2x + 5$.

Directions (Q. Nos. 76-78) *Read the following information and answer the three items that follow .*

A curve $y = me^{mx}$ where $m > 0$ intersects Y-axis at a point P.

76. What is the slope of the curve at the point of intersection P?

(a) m (b) m^2
(c) $2m$ (d) $2m^2$

77. How much angle does the tangent at P make with y-axis?

(a) $\tan^{-1} m^2$

(b) $\cot^{-1}(1 + m^2)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+m^4}}\right)$

(d) $\sec^{-1}\sqrt{1+m^4}$

78. What is the equation of tangent to the curve at P?

(a) $y = mx + m$ (b) $y = -mx + 2m$
(c) $y = m^2x + 2m$ (d) $y = m^2x + m$

Solutions (Q. Nos. 76-78)

Given curve $y = me^{mx}$ where $m > 0$

∵ Curve intersects Y-axis at a point P, then $x = 0$

∴ $y = me^0 \Rightarrow y = m$

∴ Point P $(0, m)$

Now, differentiation w.r.t x of given curve,

$$\frac{dy}{dx} = m.e^{mx}.m$$

$$\frac{dy}{dx} = m^2e^{mx}$$

at point $P(0, m)$, $\frac{dy}{dx} = m^2e^0 = m^2$

76. (***b***) Slope of the curve at the point $P(0, m)$

$= \left(\frac{dy}{dx}\right)$ at point P $(0, m) = m^2$

77. (***c***) Let the tangent makes the angle with X-axis be θ, then

$\tan\theta = \left(\frac{dy}{dx}\right)$ at P $(0, m)$

$\Rightarrow$ $\tan\theta = m^2 \Rightarrow \theta = \tan^{-1} m^2$

Now, the tangent will make the angle with Y-axis

$$= \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1} m^2$$

$$= \cot^{-1} m^2 \left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{1+m^4}}\right)$$

$$\left[\because \cot^{-1} x = \sin\left(\frac{1}{\sqrt{1+m^2}}\right)\right]$$

78. (***d***) Equation of tangent to curve at P is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1)$$

$\Rightarrow y - m = m^2(x - 0)$

$\Rightarrow$ $y = m^2x + m$

Directions (Q. Nos. 79 and 80) *Read the following information and answer the two items that follow.*

Let $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$.

79. For $x = \frac{\sqrt{\pi}}{2}$, what is the value of $[ho(gof)](x)$?

(a) 0 (b) 1 (b) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

80. What is $[fo(fof)](2)$ equal to ?

(a) 2 (b) 8
(c) 16 (d) 256

Solutions (Q. Nos. 79 and 80) Given, $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$

79. (*a*) $(gof)(x) = g\{f(x)\} = \tan x^2$

Now, $[ho(gof)](x) = h\{(gof)(x)\}$
$= \log(\tan x^2)$

for $x = \frac{\sqrt{\pi}}{2}$

$[ho(gof)]\left(\frac{\sqrt{\pi}}{2}\right) = \log\tan\left(\frac{\pi}{4}\right)$
$= \log 1 = 0$

80. (*d*) $(fof)(x) = f\{f(x)\}$
$= (x^2)^2 = x^4$

Now, $[fo(fof)](x) = f\{(fof)(x)\}$
$= (x^4)^2 = x^8$

$\therefore [fo(fof)](2) = 2^8 = 256$

81. What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$
(b) $2\tan^{-1}(2x-1) + c$
(c) $\frac{\tan^{-1}(2x+1)}{2} + c$
(d) $\tan^{-1}(2x-1) + c$

(*d*) Let $I = \int \frac{dx}{2x^2 - 2x + 1}$

$= \frac{1}{2}\int \frac{dx}{x^2 - x + \frac{1}{2}}$

$= \frac{1}{2}\int \frac{dx}{x^2 - x + \frac{1}{4} - \frac{1}{4} + \frac{1}{2}}$

$= \frac{1}{2}\int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}}$

$= \frac{1}{2}\int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \frac{1}{2}.2\tan^{-1}\left[\frac{\left(x - \frac{1}{2}\right)}{\frac{1}{2}}\right] + c$

$\left[\because \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$

$= \tan^{-1}(2x - 1) + c$

82. What is $\int \frac{dx}{x(1 + \ln x)^n}$ equal to $(n \neq 1)$?

(a) $\frac{1}{(n-1)(1 + \ln x)^{n-1}} + c$
(b) $\frac{1-n}{(1 + \ln x)^{1-n}} + c$
(c) $\frac{n+1}{(1 + \ln x)^{n+1}} + c$
(d) $-\frac{1}{(n-1)(1 + \ln x)^{n-1}} + c$

(*d*) Suppose,

$I = \int \frac{dx}{x(1 + \ln x)^n}$ (where $n \neq 1$)

Let $1 + \ln x = t$

Diff. w.r.t. x, we get

$0 + \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{dx}{x} = dt$

$\therefore \quad I = \int \frac{dt}{t^n} = \frac{t^{-n+1}}{-n+1} + c$

$= -\frac{1}{(n-1)t^{n-1}} + c$

$= -\frac{1}{(n-1)(1 + \log x)^{n-1}} + c$

83. Which one of the following is the differential equation that represents the family of curves $y = \frac{1}{2x^2 - C}$, where C is an arbitrary constant?

(a) $\frac{dy}{dx} = 4xy^2$ (b) $\frac{dy}{dx} = \frac{1}{y}$
(c) $\frac{dy}{dx} = x^2 y$ (d) $\frac{dy}{dx} = -4xy^2$

(*d*) The differential equation of family of curves $y = \frac{1}{2x^2 - C}$ … (i)

(where, C is any arbitrary constant)

Differentiation w.r.t. x of Eq. (i)

$\frac{dy}{dx} = -\frac{1}{(2x^2 - C)^2}\frac{d}{dx}(2x^2 - C)$

$\Rightarrow \frac{dy}{dx} = -y^2.(4x - 0)$

$\Rightarrow \frac{dy}{dx} = -4xy^2$, it is required differential equation.

Directions (Q. Nos. 84 and 85) *Read the following information and answer the two items that follow.*

Consider the equation $x^y = e^{x-y}$

84. What is $\frac{dy}{dx}$ at $x = 1$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

85. What is $\frac{d^2y}{dx^2}$ at $x = 1$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

Solutions (Q. Nos 84 and 85)

Given equation, $x^y = e^{x-y}$

On taking log both sides, we get

$y\log x = (x - y)\log e$

$\Rightarrow \quad y\log x = x - y[\because \log_e e = 1]$

$\Rightarrow (1 + \log x)y = x \Rightarrow y = \frac{x}{(1 + \log x)}$

Differentiation w.r.t. x, we get

$\frac{dy}{dx} = \frac{(1 + \log x).1 - x\left(0 + \frac{1}{x}\right)}{(1 + \log x)^2}$

$\frac{dy}{dx} = \frac{1 + \log x - 1}{(1 + \log x)^2}$

$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

84. (a) $\because \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

At $x = 1$, $\frac{dy}{dx} = \frac{\log 1}{(1 + \log 1)^2}$

$= \frac{0}{1} = 0$ $[\because \log 1 = 0]$

85. (b) $\because \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

Differentiation. w.r.t. x, we get

$(1 + \log x)^2.\frac{1}{x} - (\log x).$

$\frac{d^2y}{dx^2} = \frac{2(1 + \log x)\left(0 + \frac{1}{x}\right)}{(1 + \log x)^4}$

$= \frac{\frac{1}{x}(1 + \log x)(1 + \log x - 2\log x)}{(1 + \log x)^4}$

$= \frac{\frac{1}{x}(1 + \log x)(1 - \log x)}{(1 + \log x)^4}$

At $x = 1$ $\frac{d^2y}{dx^2} = \frac{1(1+0)(1-0)}{(1+0)^4} = 1$

Directions (Q.Nos. 86-88) *Read the following information and answer the three items that follow.*

Consider the function

$f(x) = g(x) + h(x)$

where, $g(x) = \sin\left(\frac{x}{4}\right)$

and $h(x) = \cos\left(\frac{4x}{5}\right)$

86. What is the period of the function $g(x)$?

(a) π (b) 2π
(c) 4π (d) 8π

87. What is the period of the function $h(x)$?

(a) π (b) $\frac{4\pi}{5}$
(c) $\frac{5\pi}{2}$ (d) $\frac{3\pi}{2}$

88. What is the period of the function $f(x)$?

(a) 10π (b) 20π
(c) 40π (d) 80π

Solutions (Q. Nos. 86-88)
Given, $f(x) = g(x) + h(x)$,
where, $g(x) = \sin\left(\frac{x}{4}\right)$ and

$$h(x) = \cos\left(\frac{4x}{5}\right)$$

86. (*d*) $g(x) = \sin\left(\frac{x}{4}\right)$

$$g(x + 8\pi) = \sin\left(\frac{x + 8\pi}{4}\right)$$

$$= \sin\left(2\pi + \frac{x}{4}\right)$$

$$= \sin\left(\frac{x}{4}\right) = g(x)$$

$\therefore$ Period of the function $g(x) = 8\pi$

87. (*c*) $h(x) = \cos\left(\frac{4x}{5}\right)$

$$h\left(x + \frac{5\pi}{2}\right) = \cos\frac{4}{5}\left(x + \frac{5\pi}{2}\right)$$

$$= \cos\left(2\pi + \frac{4x}{5}\right)$$

$$= \cos\left(\frac{4x}{5}\right) = h(x)$$

$\therefore$ Pperiod of the function $h(x) = \frac{5\pi}{2}$

88. (*c*) $f(x) = g(x) + h(x)$

$$= \sin\left(\frac{x}{4}\right) + \cos\left(\frac{4x}{5}\right)$$

$$f(x + 40\pi) = \sin\left(\frac{x + 40\pi}{4}\right) + \cos\frac{4}{5}(x + 40\pi)$$

$$= \sin\left(10\pi + \frac{x}{4}\right) + \cos\left(32\pi + \frac{4x}{5}\right)$$

$$= \sin\left(5 \times 2\pi + \frac{x}{4}\right) + \cos\left(16 \times 2\pi + \frac{4x}{5}\right)$$

$$= \sin\left(\frac{x}{4}\right) + \cos\left(\frac{4x}{5}\right) = f(x)$$

$\therefore$ Period of the function $f(x) = 40\pi$

Directions (Q. Nos. 89 and 90) *Read the following information and answer the two items that follow.*

consider the function

$$f(x) = 3x^4 - 20x^3 - 12x^2 + 288x + 1$$

89. In which one of the following intervals is the function increasing?

(a) (–2, 3) (b) (3, 4)
(c) (–3, – 2) (d) (–4, – 3)

90. In which one of the following intervals is the function decreasing?

(a) (–2, 3) (b) (3, 4)
(c) (4, 6) (d) (6, 9)

Solutions (Q. Nos. 89 and 90)
Given function,
$f(x) = 3x^4 - 20x^3 - 12x^2 + 288x + 1$
Differentiation w.r.t. x, we get

$$f'(x) = 12x^3 - 60x^2 - 24x + 288$$

89. (*b*) $f(x)$ is increasing, if $f'(x) \geq 0$

$$12x^3 - 60x^2 - 24x + 288 \geq 0$$

$$\Rightarrow \quad x^3 - 5x^2 - 2x + 24 \geq 0$$

$$\Rightarrow \quad (x + 2)(x^2 - 7x + 12) \geq 0$$

$$\Rightarrow \quad (x + 2)(x - 3)(x - 4) \geq 0$$

$$\therefore \quad x \leq -2,\ x \geq 3,\ x \geq 4$$

Hence, $f(x)$ is increasing the interval (3, 4).

90. (*a*) $f(x)$ is decreasing, if $f'(x) \leq 0$

$$12x^3 - 60x^2 - 24x + 288 \leq 0$$

$$\Rightarrow \quad x^3 - 5x^2 - 2x + 24 \leq 0$$

$$\Rightarrow \quad (x + 2)(x^2 - 7x + 12) \leq 0$$

$$\Rightarrow \quad (x + 2)(x - 3)(x - 4) \leq 0$$

$$\therefore \quad x \geq -2,\ x \leq 3,\ x \leq 4$$

Hence $f(x)$ is decreasing the interval (–2, 3).

Directions (Q.Nos. 91-93) *Read the following information and answer the two items that follow .*

Let $f(x) = x^2 + 2x - 5$
and $g(x) = 5x + 30$

91. What are the roots of the equation $g[(f(x)] = 0$?

(a) 1, –1 (b) –1, – 1
(c) 1, 1 (d) 0, 1

92. Consider the following statements.

1. $f[g(x)]$ is a polynomial of degree 3.
2. $g[g(x)]$ is a polynomial of degree 2.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

93. If $h(x) = 5f(x) - xg(x)$, then what is the derivative of $h(x)$?

(a) –40 (b) –20
(c) –10 (d) 0

Solutions (Q. Nos. 91-93) Given,
$f(x) = x^2 + 2x - 5$, $g(x) = 5x + 30$

$$\therefore g[f(x)] = 5(x^2 + 2x - 5) + 30$$

$$= 5x^2 + 10x + 5$$

$$f[g(x)] = (5x + 30)^2 + 2(5x + 30) - 5$$

$$= 25x^2 + 900 + 300x + 10x + 60 - 5$$

$$= 25x^2 + 310x + 955$$

and $g[g(x)] = 5(5x + 30) + 30$

$$= 25x + 180$$

91. (*b*) The equation, $g[f(x)] = 0$

$$5x^2 + 10x + 5 = 0$$

$$\Rightarrow \quad x^2 + 2x + 1 = 0$$

$$\Rightarrow \quad (x + 1)^2 = 0$$

$$\therefore \quad x = -1, -1$$

Hence, the roots of this equation are –1, – 1.

92. (*d*) 1. $f[g(x)] = 25x^2 + 310x + 955$
$f[g(x)]$ is a polynomial of degree 2.
So, Statement 1 is not correct.
2. $g[g(x)] = 25x + 180$
$g[g(x)]$ is a polynomial of degree 1.
So, Statement 2 is not correct.
Hence, the Statement neither 1 nor 2 correct.

93. (*b*) Given, $h(x) = 5f(x) - xg(x)$

$$= 5(x^2 + 2x - 5) - x(5x + 30)$$

$$= 5x^2 + 10x - 25 - 5x^2 - 30x$$

$$= -20x - 25$$

Differentiation w.r.t. x, we get

$$h'(x) = -20$$

Hence, derivative of $h(x)$ is –20.

Directions (Q.Nos. 94 and 95) *Read the following information and answer the questions given below.*

Consider the integrals

$$I_1 = \int_0^{\pi} \frac{x\,dx}{1 + \sin x} \text{ and}$$

$$I_2 = \int_0^{\pi} \frac{(\pi - x)dx}{1 - \sin(\pi + x)}$$

94. What is the value of I_1?

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

95. What is the value of $I_1 + I_2$?

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0

Solutions (Q.Nos. 94 and 95)
Given, $I_1 = \int_0^{\pi} \frac{x\,dx}{1 + \sin x}$,

$$I_2 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1 - \sin(\pi + x)}$$

$$I_1 = \int_0^{\pi} \frac{x\,dx}{1 + \sin x} \quad \ldots(i)$$

$$= \int_0^{\pi} \frac{(\pi - x)\,dx}{1 + \sin(\pi - x)}$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx\right]$$

$$I_1 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1 + \sin x} \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$2I_1 = \int_0^{\pi} \frac{(x + \pi - x)\,dx}{1 + \sin x} = \int_0^{\pi} \frac{\pi\,dx}{1 + \sin x}$$

$$= \pi \int_0^{\pi} \frac{(1 - \sin x)\,dx}{(1 - \sin^2 x)}$$

$$= \pi \int_0^{\pi} \frac{(1 - \sin x)\,dx}{\cos^2 x}$$

$$= \pi \int_0^{\pi} (\sec^2 x - \sec x \tan x)\,dx$$

$$= \pi\,[\tan x - \sec x]_0^{\pi}$$

$$= \pi\,[(\tan \pi - \sec \pi) - (\tan 0 - \sec 0)$$

$= \pi\ [(0+1)-(0-1)]$

$2I_1 = \pi\ (1+1) = 2\pi$

$\Rightarrow \quad I_1 = \pi$

Now, $I_2 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1 - \sin(\pi + x)}$

$= \int_0^{\pi} \frac{(\pi - \pi + x)\,dx}{1 - \sin(\pi - \pi - x)}$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$= \int_0^{\pi} \frac{x\,dx}{1 - \sin(-x)}$

$= \int_0^{\pi} \frac{x\,dx}{1 + \sin x} [\because \sin(-\theta) = \sin\theta]$

$= I_1 = \pi$

94. (**c**) $I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x} = \pi$

95. (**a**) $I_1 + I_2 = \pi + \pi = 2\pi$

96. The differential equation which represents the family of curves given by $\tan y = C(1 - e^x)$ is

(a) $e^x \tan y\,dx + (1 - e^x)\,dy = 0$

(b) $e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$

(c) $e^x (1 - e^x)\,dx + \tan y\,dy = 0$

(d) $e^x \tan y\,dy + (1 - e^x)\,dx = 0$

(**b**) The equation of the family of curves.

$$\tan y = C(1 - e^x) \qquad \ldots (i)$$

Differentiation w.r.t. x, we get

$$\sec^2 y \cdot \frac{dy}{dx} = C(0 - e^x)$$

$$\Rightarrow \sec^2 y \cdot \frac{dy}{dx} = -Ce^x \qquad \ldots (ii)$$

Eleminating C from Eqs. (i) and (ii), we get

$$\sec^2 y \cdot \frac{dy}{dx} = -e^x \frac{\tan y}{(1 - e^x)}$$

$\Rightarrow (1 - e^x)\sec^2 y\,dy = -e^x \tan y.\,dx$

$\Rightarrow e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$

97. What is the derivative of $2^{(\sin x)^2}$ with respect to $\sin x$?

(a) $\sin x\ 2^{(\sin x)^2} \ln 4$

(b) $2\sin x\ 2^{(\sin x)^2} \ln 4$

(c) $\ln(\sin x)\,2^{(\sin x)^2}$

(d) $2\sin x \cos x\ 2^{(\sin x)^2}$

(**a**) Derivative of $2^{(\sin x)^2}$ with respect to $\sin x$

$$= \frac{\frac{d}{dx}2^{(\sin x)^2}}{\frac{d}{dx}(\sin x)} = \frac{2^{(\sin x)^2}\log 2 \frac{d}{dx}(\sin x)^2}{\cos x}$$

$$= \frac{2^{(\sin x)^2}\log 2.\,2\sin x.\cos x}{\cos x}$$

$= 2\log 2.(\sin x).\,2^{(\sin x)^2}$

$= \sin x.\,2^{(\sin x)^2}.\log 4$

98. For what value of k is the function

$$f(x) = \begin{cases} 2x + \frac{1}{4}, & x < 0 \\ k, & x = 0 \\ \left(x + \frac{1}{2}\right)^2, & x > 0 \end{cases}$$ continuous?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

(**a**) Given,

$$f(x) = \begin{cases} 2x + \frac{1}{4}, & x < 0 \\ k, & x = 0 \\ \left(x + \frac{1}{2}\right)^2, & x > 0 \end{cases}$$ continuous.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0^-} \left(2x + \frac{1}{4}\right)$$

$$= \lim_{h \to 0}\left[2(0-h) + \frac{1}{4}\right]$$

$$= \lim_{h \to 0}\left(-2h + \frac{1}{4}\right) = \frac{1}{4}$$

and $f(0) = k$

$\because$ The function $f(x)$ is continuous at $x = 0$

$\therefore \quad \text{LHL} = f(0) = \text{RHL}$

$\Rightarrow \quad \text{LHL} = f(0) \Rightarrow \frac{1}{4} = k$

Hence, $k = \frac{1}{4}$

99. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{2}{3}$ sq unit (b) $\frac{4}{3}$ sq unit

(c) $\frac{1}{3}$ sq unit (d) 1 sq unit

(**a**) Equations of curves,

$$y^2 = 2x \qquad \ldots (i)$$

and $y = x \qquad \ldots (ii)$

From Eqs. (i) and (ii),

$$y^2 = 2y \Rightarrow y(y-2) = 0$$

$\therefore \quad y = 0, 2$

$$\therefore \text{Required area} = \int_0^2 \left(\frac{y^2}{2} - y\right) dy$$

$$= \left[\frac{1}{2}.\frac{y^3}{3} - \frac{y^2}{2}\right]_0^2$$

$$= \left[\frac{1}{6}(2)^3 - \frac{(2)^2}{2}\right] - 0$$

$$= \frac{8}{6} - \frac{4}{2} = \frac{8-12}{6}$$

$$= -\frac{4}{6} = \frac{-2}{3} = \frac{2}{3} \text{ sq unit}$$

($\because$ area will not be negative)

100. If $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$ increases in the interval T and decreases in the interval S, then which one of the following is correct?

(a) $T = (-\infty, 2), \cup (3, \infty)$ and $S = (2, 3)$

(b) $T = \phi$ and $S = (-\infty, \infty)$

(c) $T = (-\infty, \infty)$ and $S = \phi$

(d) $T = (2, 3)$ and $S = (-\infty, 2) \cup (3, \infty)$

(**a**) Given, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$

Differentiating w.r.t x, we get

$$f'(x) = \frac{3x^2}{3} - \frac{5}{2}.2x + 6 = x^2 - 5x + 6$$

$\because f(x)$ is increases in interval T,

$\therefore \quad f'(x) \geq 0$

$\Rightarrow \quad x^2 - 5x + 6 \geq 0$

$\Rightarrow \quad x^2 - 3x - 2x + 6 \geq 0$

$\Rightarrow \quad x(x-3) - 2(x-3) \geq 0$

$\Rightarrow (x-3)(x-2) \geq 0 \Rightarrow x \leq 2,\ x \geq 3$

$\therefore \quad T = (-\infty, 2) \cup (3, \infty)$

Again, $f(x)$ is decreases in interval S.

$\therefore \quad f'(x) \leq 0$

$\Rightarrow \quad x^2 - 5x + 6 \leq 0$

$\Rightarrow x^2 - 3x - 2x + 6 \leq 0$

$\Rightarrow x(x-3) - 2(x-3) \leq 0$

$\Rightarrow \quad (x-3)(x-2) \leq 0$

$\Rightarrow \quad 2 < x < 3 \Rightarrow x \in (2, 3)$

$\therefore \quad S = (2, 3)$

101. A coin is biased so that heads comes up thrice as likely as tails. For three independent tosses of a coin, what is the probability of getting at most two tails?

(a) 0.16 (b) 0.48

(c) 0.58 (d) 0.98

(**d**) Let X be a random variable, it represents of the number tail comes of three tosses of a coin.

$\therefore$ Possible value of X are 0, 1, 2, 3.

According to the question, the coin is biased in which the probability to comes head is thrice as likely as tails.

$\therefore P(\text{H}) = \frac{3}{4}$ and $P(\text{T}) = \frac{1}{4}$

$$P(\text{X} = 0) = P(\{\text{HHH}\}) = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$P(X = 1) = \text{P}$ (2 heads and 1 tail)

$= P(\{\text{HHT}\}) + P(\{\text{HTH}\}) + P(\{\text{THH}\})$

$$= \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$$

$$= \frac{27}{64}$$

$P(X = 2) = P$ (1 head and 2 tails)

$= P(\{\text{HTT}\}) + P(\{\text{THT}\}) + P(\{\text{TTH}\})$

$$= \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}$$

$$= \frac{9}{64}$$

∴ Required probability

$$= P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \frac{27}{64} + \frac{9}{64} + \frac{27}{64}$$

$$= \frac{63}{64} = 0.98$$

102. A bag contains 20 books out of which 5 are defective. If 3 of the books are selected at random and removed from the bag in succession without replacement, then what is the probability that all three books are defective?

(a) 0.009 (b) 0.016
(c) 0.026 (d) 0.047

(**a**) Total books in bag = 20

Defective books = 5

∴ Undefective books = 20 – 5 = 15

∴ Probability to selected three books are defective without replacement

$$= \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18}$$

$$= \frac{6}{684}$$

$$= 0.0087 = 0.009$$

103. The median of the observations 22, 24, 33, 37, $x + 1$, $x + 3$, 46, 47, 57, 58 in ascending order is 42. What are the values of 5th and 6th observations respectively?

(a) 42, 45 (b) 41, 43
(c) 43, 46 (d) 40, 40

(**b**) The observations in ascending order are

22, 24, 33, 37, x + 1, x + 3, 46, 47, 57, 58

Here, $n = 10$

∴ Median

$$= \frac{\text{Value of } \frac{N}{2}\text{th observations} + \text{Values of } \left(\frac{N}{2} + 1\right)\text{th observations}}{2}$$

$$\Rightarrow 42 = \frac{\text{Value of 5th observations} + \text{Value of 6th observation}}{2}$$

$$\Rightarrow 84 = x + 1 + x + 3$$

$$\Rightarrow 2x = 84 - 4$$

$$\Rightarrow x = \frac{80}{2} = 40$$

∴ 5th observation = x + 1 = 40 + 1 = 41

and 6th observation

$$= x + 3 = 40 + 3 = 43$$

104. Arithmetic mean of 10 observations is 60 and sum of squares of deviations from 50 is 5000. What is the standard deviation of the observations?

(a) 20 (b) 21
(c) 22.36 (d) 24.70

(**a**) Arithmetic mean of 10 observations = 60

$$\therefore \Sigma x_i = 60 \times 10 = 600 \qquad \left[\because \bar{x} = \frac{\Sigma x_i}{n}\right]$$

If, $A = 50$, then $\Sigma d_i^2 = 5000$

$\because$ $d_i = x_i - A$

$$\therefore \Sigma d_i = \Sigma(x_i - A) = \Sigma x_i - A\Sigma 1$$

$$= 600 - 50 \times n \qquad [\because \Sigma 1 = n]$$

$$= 600 - 50 \times 10 = 100$$

$$\text{Now, SD} = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{5000}{10} - \left(\frac{100}{10}\right)^2}$$

$$= \sqrt{500 - 100} = \sqrt{400} = 20$$

105. If p and q are the roots of the equation $x^2 - 30x + 221 = 0$, what is the value of $p^3 + q^3$?

(a) 7010 (b) 7110
(c) 7210 (d) 7240

(**b**) Since, p and q are the roots of the equation

$$x^2 - 30x + 221 = 0$$

$$\therefore p + q = 30 \text{ and } pq = 221$$

$$\text{Now, } p^3 + q^3 = (p + q)(p^2 + q^2 - pq)$$

$$= 30[p^2 + q^2 + 2pq - 3pq]$$

$$= 30[(p + q)^2 - 3pq]$$

$$= 30[(30)^2 - 663]$$

$$= 30[900 - 663]$$

$$= 30 \times 237 = 7110$$

106. For the variables x and y, the two regression lines are $6x + y = 30$ and $3x + 2y = 25$. What are the values of $\bar{x}$, $\bar{y}$ and r respectively?

(a) $\frac{20}{3}, \frac{35}{9}, -0.5$ (b) $\frac{20}{3}, \frac{35}{9}, 0.5$
(c) $\frac{35}{9}, \frac{20}{3}, -0.5$ (d) $\frac{35}{9}, \frac{20}{3}, 0.5$

(**c**) Given lines, $6x + y = 30$ …(i)

and $3x + 2y = 25$ …(ii)

where, x and y are two variables.

Solving these equations,

$$x = \frac{35}{9}, \text{ and } y = \frac{20}{3}$$

These lines are regression,

Then, $\bar{x} = \frac{35}{9}$, $\bar{y} = \frac{20}{3}$

and $r = -\frac{3}{6}$ or $-\frac{1}{2}$

$$= -0.5$$

107. The class marks in a frequency table are given to be 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. The class limits of the first five classes are

(a) 3-7, 7-13, 13-17, 17-23, 23-27
(b) 2.5-7.5, 7.5-12.5, 12.5-17.5, 17.5-22.5, 22.5-27.5
(c) 1.5-8.5, 8.5-11.5, 11.5,-18.5, 18.5-21.5, 21.5-28.5
(d) 2-8, 8-12, 12-18, 18-22, 22-28

(**b**) Given, class marks in a frequency table are

5, 10, 15, 20, 25, 30, 35, 40, 45, 50.

Let L_1 and L_2 be the lower limit and upper limit of first interval.

$\because$ Class mark $= \frac{L_1 + L_2}{2}$ $5 = \frac{L_1 + L_2}{2}$

$$\Rightarrow L_1 + L_2 = 10 \qquad \ldots (i)$$

and $L_2 - L_1$ = Class interval

or $L_2 - L_1 = 5$ … (ii)

Solving Eq. (i) and (ii),

$$L_2 = 7.5 \text{ and } L_1 = 2.5$$

∴ Class limit of first classes is 2.5 – 7.5

Similarly find class limit of other classes.

Hence, class limits of the first five classes are

2.5 – 7.5, 7.5 – 12.5, 12.5 – 17.5, 17.5 – 22.5, 22.5 –27.5.

108. The mean of 5 observations is 4.4 and variance is 8.24. If three of the five observations are 1, 2 and 6, then what are the other two observations?

(a) 9, 16 (b) 9, 4
(c) 81, 16 (d) 81, 4

(**b**) Let x_1, x_2, x_3, x_4 and x_5 are five observations.

$$\therefore \quad x_1 = 1,\ x_2 = 2 \text{ , and } x_3 = 6$$

$$\therefore \quad \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$\Rightarrow 4.4 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 22$$

$$\Rightarrow \quad 1 + 2 + 6 + x_4 + x_5 = 22$$

$$\Rightarrow \quad x_4 + x_5 = 22 - 9$$

$$\Rightarrow \quad x_4 + x_5 = 13 \qquad \ldots (i)$$

and variance,

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2 + (x_5 - \bar{x})^2}{5}$$

$$\Rightarrow 8.24 = \frac{(1 - 4.4)^2 + (2 - 4.4)^2 + (6 - 4.4)^2 + (x_4 - 4.4)^2 + (x_5 - 4.4)^2}{5}$$

$\Rightarrow 8.24 \times 5 = 11.56 + 5.76 + 2.56 + (x_4 - 4.4)^2 + (13 - x_4 - 4.4)^2$

[from Eq. (i)]

$\Rightarrow 41.20 = 19.88 + (x_4 - 4.4)^2 + (8.6 - x_4)^2$

$\Rightarrow 41.20 - 19.88 = x_4^2 + 19.36 - 8.8\,x_4 + 73.96 + x_4^2 - 17.2x$

$\Rightarrow 21.32 = 2x_4^2 - 26x_4 + 93.32$

$\Rightarrow 2x_4^2 - 26x_4 + 72 = 0$

$\Rightarrow x_4^2 - 13x_4 + 36 = 0$

$\Rightarrow x_4^2 - 9x_4 - 4x_4 + 36 = 0$

$\Rightarrow x_4(x_4 - 9) - 4(x_4 - 9) = 0$

$\Rightarrow (x_4 - 9)(x_4 - 4) = 0$

$\therefore x_4 = 4, 9$

From Eq. (i), $x_5 = 9, 4$

Hence, other two observations are 9 and 4.

109. If a coin is tossed till the first head appears, then what will be the sample space?

(a) {H} (b) {TH}
(c) {T, HT, HHT, HHHT, ...}
(d) {H, TH, TTH, TTTH, ...}

(***a***) A coin is tossed till the first head appears, then the sample space will be = {H}

110. Consider the following discrete frequency distribution.

x	1	2	3	4	5	6	7	8
f	3	15	45	57	50	36	25	9

What is the value of median of the distribution?

(a) 4 (b) 5 (c) 6 (d) 7

(***b***)

x	*f*	*C*
1	3	3
2	15	18
3	45	63
4	57	120
5	50	170
6	36	236
7	25	261
8	9	270
	N **= 270**	

Here, $N = 270$

$\therefore$ Median

$$= \frac{\text{Value of } \frac{N}{2}\text{th term} + \text{value of } \left(\frac{N}{2} + 1\right)\text{th term}}{2}$$

$$= \frac{\text{Value of 135th term} + \text{Value of 136th term}}{2}$$

$$= \frac{5 + 5}{2} = 5$$

111. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on them is a prime number?

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$
(c) $\frac{7}{12}$ (d) $\frac{2}{3}$

(***a***) Total number of sample space of two dice are thrown, $n(s) = 6 \times 6 = 36$

Total number of favourable outcomes the sum of numbers appearing on them is a prime number.

(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)

$\therefore n(E) = 15$

$\therefore$ Required probability $= \frac{n(E)}{n(S)}$

$$= \frac{15}{36} = \frac{5}{12}$$

112. If 5 of a Company's 10 delivery trucks do not meet emission standards and 3 of them are chosen for inspection, then what is the probability that none of the trucks chosen will meet emission standards?

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$
(c) $\frac{1}{12}$ (d) $\frac{1}{4}$

(***c***) Total trucks of a company's = 10

Number of trucks that do not meet emission standards = 5

Number of trucks that are chosen for inspection = 3

$\therefore$ Required probability $= \frac{{}^5C_3}{{}^{10}C_3}$

$$= \frac{\frac{5!}{3!2!}}{\frac{10!}{3!7!}} = \frac{5!7!}{10!2!}$$

$$= \frac{5 \cdot 4 \cdot 3}{10 \cdot 9 \cdot 8} = \frac{1}{12}$$

113. There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and third is biased coin that comes up heads 75% of time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

(a) $\frac{2}{9}$ (b) $\frac{1}{3}$
(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

(***c***) Let E_1, E_2 and E_3 represent the events of two-headed coin, a fair coin and biased coin respectively.

$\therefore P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P(E_3) = \frac{1}{4}$

$P\left(\frac{E}{E_1}\right) = \frac{1}{2}, P\left(\frac{E}{E_2}\right) = \frac{1}{2}, P\left(\frac{E}{E_3}\right) = \frac{1}{4}$

Apply Baye's theorem,

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{4} + \frac{1}{16}}$$

$$= \frac{\frac{1}{4}}{\frac{4 + 4 + 1}{16}} = \frac{4}{9}$$

114. Consider the following statements:

1. If A and B are mutually exclusive events, then it is possible that $P(A) = P(B) = 0.6$.
2. If A and B are any two events such that $P(A/B) = 1$, then $P(\overline{B}/\overline{A}) = 1$.

Which of the above statement is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***b***) **Statement 1** : A and B are mutually exclusive events, then $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

$= 0.6 + 0.6$

$= 1.2$, it is not possible

So, Statement 1 is not correct.

Statement 2 : A and B are any two events such that

$P\left(\frac{A}{B}\right) = 1$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = 1 \Rightarrow P(A \cap B) = P(B)$ ... (i)

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(B)$

[from Eq. (i)]

$\Rightarrow P(A \cup B) = P(A)$ ... (ii)

Now, $P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{P(\overline{B} \cap \overline{A})}{P(\overline{A})} = \frac{P(\overline{A \cup B})}{P(\overline{A})}$

$$= \frac{1 - P(A \cup B)}{1 - P(A)} = \frac{1 - P(A)}{1 - P(A)} = 1$$

So, Statement 2 is correct.

Hence, only the Statement 2 is correct.

115. If a fair die is rolled 4 times, then what is the probability that there are exactly 2 sixes?

(a) $\frac{5}{216}$ (b) $\frac{25}{216}$

(c) $\frac{125}{216}$ (d) $\frac{175}{216}$

(**b**) Let X be a random variable that represents to appearing 6 of rolled a die.

Probability of to get 6 to rolled a die,

$p = \frac{1}{6}$

∴ Probability of not get 6 to rolled a die,

$$q = 1 - \frac{1}{p} = 1 - \frac{1}{6} = \frac{5}{6}$$

Here, $n = 4$, $r = 2$

∴ Required probability = ${}^nC_r p^r q^{n-r}$

[By Bernoulli distribution]

$$= {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= \frac{4!}{2!2!} \times \frac{1}{36} \times \frac{25}{36}$$

$$= \frac{4 \cdot 3}{2 \cdot 1} \times \frac{1}{36} \times \frac{25}{36} = \frac{25}{216}$$

116. Mean of 100 observations is 50 and standard deviation is 10. If 5 is added to each observation, then what will be the new mean and new standard deviation respectively?

(a) 50, 10 (b) 50, 15

(c) 55, 10 (d) 55, 15

(**c**) Mean of 100 observations = 50

and standard deviation = 10

We know that, if k is added to each observation, then new mean will be more than k and standard deviation no change.

∴ After 5 added to each observation.

mean = 50 + 5 = 55

and standard deviation = 10

117. If the range of a set of observations on a variable X is known to be 25 and if $Y = 40 + 3X$, then what is the range of the set of corresponding observations on Y?

(a) 25 (b) 40

(c) 75 (d) 115

(**c**) Range of set of observations on a variable, $X = 25$

We know that

Range, $R_X = X_{max} = X_{min}$

$\Rightarrow$ $25 = X_{max} - 0$ [$\because X_{min} = 0$]

$\Rightarrow$ $X_{max} = 25$

$\because$ $Y = 40 + 3X$

$\therefore$ $Y_{min} = 40 + 3X_{min}$

$= 40 + 3(0)$ [$\because X_{min} = 0$]

$= 40$

and $Y_{max} = 40 + 3X_{max}$

$= 40 + 3(25)$ [$\because X_{max} = 25$]

$= 40 + 75 = 115$

Now, $R_Y = Y_{max} - Y_{min}$

$= 115 - 40 = 75$

118. If V is the variance and M is the mean of first 15 natural numbers, then what is $V + M^2$ equal to?

(a) $\frac{124}{3}$ (b) $\frac{148}{3}$

(c) $\frac{248}{3}$ (d) $\frac{124}{9}$

(**c**) Mean of first 15 natural numbers, M

$$= \frac{1+2+3+4+5+6+7+8+9+10+11+12+13+14+15}{15}$$

$$= \frac{15(15+1)}{2 \times 15}$$

$$\left[\because 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}\right]$$

$= 8$

Variance of first 15 natural numbers, V

$$= \frac{1}{15}[(1-8)^2 + (2-8)^2 + (3-8)^2 + (4-8)^2 + (5-8)^2 + (6-8)^2 + (7-8)^2 + (8-8)^2 + (9-8)^2 + (10-8)^2 + (11-8)^2 + (12-8)^2 + (13-8)^2 + (14-8)^2 + (15-8)^2]$$

$$= \frac{1}{15}[(-7)^2 + (-6)^2 + (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2 + 0 + (1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2 + (6)^2 + (7)^2]$$

$$= \frac{2}{15}[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2]$$

$$= \frac{2}{15} \times \frac{7(7+1)(14+1)}{6}$$

$$\left[\because 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{2}{15} \times \frac{7 \times 8 \times 15}{6} = \frac{56}{3}$$

Now, $V + M^2 = \frac{56}{3} + 64$

$$= \frac{56 + 192}{3}$$

$$= \frac{248}{3}$$

119. A car travels first 60 km at a speed of 3 v km/h and travels next 60 km at 2 v km/h. What is the average speed of the car?

(a) 2.5 v km/h

(b) 2.4 v km/h

(c) 2.2 v km/h

(d) 2.1 v km/h

(**b**) Time taken for first 60 km with speed 3 v km/h

$$= \frac{60}{3v} = \frac{20}{v} \text{ h} \quad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

Time taken for next 60 km with speed 2v km/h

$$= \frac{60}{2v} = \frac{30}{v} \text{h}$$

$$\therefore \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{60 + 60}{\frac{20}{v} + \frac{30}{v}}$$

$$= \frac{120\,v}{50}$$

$= 2.4\,v$ km/h

120. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls is 55 kg. What are the number of boys and girls respectively in the class?

(a) 75 and 75

(b) 50 and 100

(c) 70 and 80

(d) 100 and 50

(**b**) Let number of boys and girls be x and y respectively.

$\therefore$ $x + y = 150$ … (i)

Mean weight of 150 students = 60 kg

∴ Total weight of 150 students

$= 60 \times 150$

$= 9000$kg.

Mean weight of boys = 70 kg

∴ Total weight of boys = 70x kg

and mean weight of girls = 55 kg

∴ Total weight of girls = 55 y kg

∴ Total weight of 150 students = 9000 kg

$\Rightarrow$ $70x + 55y = 9000$

$\Rightarrow$ $14x + 11y = 1800$ … (ii)

Solving Eqs. (i) and (ii), we get

$x = 50$

$y = 100$

Hence, the number of boys and girls are 50 and 100 respectively.

PAPER II English Language and General Studies

Part A (English Language)

1. He is always anxious.
(a) worried (b) dispassionate
(c) sluggish (d) torpid

(a) 'Worried' is the correct synonym of 'anxious'. Both words mean 'feeling uneasy about something with an uncertain outcome.'
Some other synonyms are - uneasy, agitated, etc.

2. The poems of Kabir are ecstatic in nature.
(a) efficacious (b) eerie
(c) rapturous (d) reverential

(*c*) 'Rapturous' is the correct synonym of 'ecstatic'. Both words mean 'feeling or expression overwhelming happiness'. Some other synonyms are - joyful, overjoyed, etc.

3. Ravi loves seclusion. Therefore, he lives in the mountain.
(a) nature (b) scripture
(c) seafaring (d) solitariness

(d) 'Solitariness' is the correct synonym of 'seclusion'. Both words mean 'being separate and apart from others'.
Some other synonyms are - solitude, isolation, etc.

4. Hitler was a despot.
(a) conservative (b) dictator
(c) passionate (d) monstrous

(*b*) 'Dictator' is the correct synonym of 'despot'. Both words refer to 'a ruler or other person who holds absolute power.'
Some other synonyms are - autocrat, authoritarian, etc.

5. The imagery used in the poem is vivid.
(a) lively (b) inert
(c) ebullient (d) caustic

(*a*) 'Lively' is the correct synonym of 'vivid'. Both words refer to 'something which is very clear and true to life.'
Some other synonyms are - realistic, lifelike, etc.

6. A human being is always vulnerable to other human beings.
(a) resilient (b) elastic
(c) defenceless (d) crude

(c) 'Defenceless' is the correct synonym of 'vulnerable'. Both words mean 'open to attack or damage.'
Some other synonyms are - unprotected, unguarded, etc.

7. His language is political and vitriolic.
(a) imaginative (b) sprightly
(c) vivacious (d) abusive

(d) 'Abusive' is the correct synonym of the word 'vitriolic'. Both words mean 'violent hate and anger expressed through severe criticism'. Some other synonyms are - acrimonious, bitter, etc.

8. The Managing Director of the company declared that he is broke and there is a need to seek support from the government.
(a) bankrupt (b) rich
(c) making profit (d) having liabilities

(a) 'Bankrupt' and 'broke' are synonyms. Both words mean 'having completely run out of money.' Some other synonyms are - insolvent, penniless, etc.

9. He loves doing nasty things.
(a) nice (b) fastidious
(c) foul (d) finicky

(*c*) 'Foul' is the correct synonym of 'nasty'. Both words mean 'something unpleasant or disagreeable'.
Some other synonyms are - awful, disgusting, etc.

10. He could not muster courage to speak the truth before his friend.
(a) injure (b) spoil
(c) gather (d) maim

(*c*) 'Gather' is the correct synonym of 'muster'. Both words mean 'assemble or bring together'.
Some other synonyms are - gather, assemble, etc.

11. His conversations are always absurd.
(a) farcical (b) foolish
(c) preposterous (d) rational

(d) The word 'absurd' means 'unreasonable or inappropriate'. Its antonym is 'Rational' which means 'in accordance with reason or logic.'

12. The politician was a dissident.
(a) Agitator (b) Rebel
(c) Conformist (d) Revolutionary

(c) The word 'dissident' means 'a person who opposes official policy'.
Its antonym from the given option is 'conformist', which means 'someone who conforms to the accepted behaviour or established practices'.

13. He wrote an incisive article on corruption in politics.
(a) vague (b) penetrating
(c) trenchant (d) precise

(a) The word 'incisive' means 'something that is sharp or direct'. Its antonym is 'vague' which means 'uncertain or indistinct'.

14. Rakesh is an eccentric prodigal.
(a) extravagant (b) profligate
(c) wasteful (d) thrifty

(d) The word 'prodigal' means 'spending money or using resources freely and recklessly'. Its antonym is 'thrifty', which means 'using money and other resources carefully'.

15. Friendship is always reciprocal.
(a) mutual (b) one-sided
(c) shared (d) corresponding

(b) The word 'reciprocal' means 'mutual or complementary'. Its antonym is 'one-sided' meaning 'having or occurring on side of something only'.

16. The debate was highly stimulating.
(a) arousing (b) invigorating
(c) boring (d) stirring

(c) The word 'stimulating' means 'exciting or arousing'. Its antonym is 'boring' which means 'not interesting'.

17. Malaria is a widespread disease.
(a) endemic (b) pervasive
(c) common (d) rare

(a) The word 'widespread' means 'extensive or distributed over a large area'.Its antonym is 'endemic' which means 'native and restricted to a certain place'.

18. The bill received a vehement resistance from the opposition party in the Parliament.
(a) animated (b) apathetic
(c) fervent (d) vigorous

(b) The word 'vehement' means 'showing strong feelings or shown with great energy or force'. Its antonym is 'apathetic' which means 'showing no enthusiasm or concern'.

19. He has been obstinate since his childhood.
(a) stubborn (b) dogged
(c) amenable (d) rigid

(c) The word 'obstinate' means 'stubborn or determined'. Its antonym is 'amenable' which means 'responsive to suggestion'.

20. Mahatma Gandhi was one of the most prominent leaders of his times.

(a) renowned (b) distinguishable
(c) eminent (d) unknown

(*d*) The word 'prominent' means 'readily noticeable.' Its antonym is 'unknown' which means 'not known or familiar'.

21. He had arrived at Cairo *(a)* a few months before *(b)* protests shook the Arab world. *(c)* No error *(d)*

(a) Here part (a) has an error. As sentence is in past tense, so we should use Simple Past tense 'arrived' without the use of had.

22. Most of us who are older competitive runners *(a)* are not able to race anywhere at the same speed *(b)* as we do when we were 30. *(c)* No error *(d)*

(b) Here part (b) has an error. 'At the same speed' is incorrect. It should be 'with the same speed', to make the given sentence grammatically correct. Because 'at' is used to show only a definite speed; e.g. at 80mph etc.

23. Work hard *(a)* lest you do not *(b)* fail. *(c)* No error *(d)*

(b) Here part (b) has an error. With 'lest' we should always use 'should'. Replace 'do not' with 'should' to make the given sentence grammatically correct.

24. The Eastern Ghats are home of 2600 plant species *(a)* and this habitat fragmentation *(b)* can pose a serious threat to endemic plants. *(c)* No error (d)

(a) Here part (a) has an error. 'Home of' is incorrect expression. It should be replaced with 'home to' as it refers to the place where you live or from where you belonged.

25. Turbidity current is a fast-moving current *(a)* that sweeps down submarine canyons, *(b)* carrying sand and mud into the deep sea. *(c)* No error *(d)*

(c) Here part (c) has an error. Replace 'carrying sand' with 'that carries sand' as the reference is to a particular thing (Turbidity current).

26. Every one *(a)* of the boys *(b)* love to ride. *(c)* No error *(d)*

(c) Here part (c) has an error. With 'everyone' singular verb should be used. So, use 'loves' in place of 'love' to make the given sentence grammatically correct.

27. Neither praise nor blame *(a)* seem *(b)* to affect him. *(c)* No error *(d)*

(b) Here part (b) has an error of use of verb. 'Seem' should be replaced with 'seems' as the sentence is in Simple Present tense.

28. Many a man *(a)* has succumbed *(b)* to this temptation. *(c)* No error *(d)*

(d) The given sentence has no error.

29. A time slot of fifteen minutes *(a)* are allowed *(b)* to each speaker. *(c)* No error *(d)*

(b) Here part (b) has an error. As 'time slot' given in part (a) is singular noun, so the verb used with it, should also be singular. Hence, 'is allowed' should be used in place of 'are allowed' to make the given sentence grammatically correct.

30. He asked *(a)* whether either of the brothers *(b)* were at home. *(c)* No error *(d)*

(c) Here part (c) has an error. 'Were at home' is incorrect. It should be replaced with 'was at home', as either is always followed by the singular verb.

31. Be over the hill

(a) To be on a mountain top
(b) To travel in the jungle
(c) To be too old to do things
(d) To do something in the most complete way

(*c*) Idiom 'Be over the hill' means 'to be too old to do things'. Hence, option (c) is a suitable choice.

32. Bite your tongue

(a) To stop yourself from saying something because it would be better not to
(b) To bite off your tongue while eating
(c) To feel sorry
(d) To ask someone something that you want

(*a*) Idiom 'Bite your tongue' means 'to stop yourself from saying something because it would be better not to'. So option (a) is a suitable choice.

33. Turn a blind eye

(a) To run away
(b) To begin to be have in a more positive manner
(c) To change a situation
(d) To choose to ignore behaviour that you know is wrong

(*d*) The given idiom 'Turn a blind eye' means 'to ignore something that you know is wrong'. So, option (d) is the correct choice.

34. Yellow journalism

(a) Paid storytelling
(b) Government reports published on yellow coloured papers
(c) Writings in newspapers that try to influence people's opinion by using strong language and false information
(d) Journalism which agrees completely with the policies of the government and capitalists

(*c*) Idiom 'Yellow Journalism' means 'writing in newspapers that try to influence people's opinion by using strong language and false information'. So, option (c) is a suitable choice.

35. To be in seventh heaven

(a) To fly in the ail
(b) To be extremely happy
(c) To be completely alone
(d) To visit a place which is in the lit of the seven wonders of the world.

(***b***) Idiom 'To be in seventh heaven' means 'to be extremely happy'. So, option (b) is the correct choice.

36. closely monitor the situation (P) by the disturbing allegations (Q) we are deeply concerned (R) and will continue to (S)

(a) R Q S P (b) S P Q R
(c) S P R Q (d) P R S Q

(a) RQSP is the correct sequence.

37. relieve nausea, pain and stress (P) aromatherapy may also help (Q) using lavender oil on their skin (R) but patients are cautioned against (S)

(a) R Q P S (b) S Q P R
(c) Q P S R (d) R P S Q

(c) QPSR is the correct sequence.

38. without any mandate of law (P) the plea filed through the advocate (Q) online is illegal and submitted (R) that the sale of drugs and prescription medicines (S)

(a) R Q S P (b) Q S R P
(c) Q R S P (d) P R S Q

(b) Q S R P is the correct sequence.

39. humans and machines have co-existed (P) dating back to the invention (Q) of the potter's wheel in ancient Mesopotamia (R) at the physical level for millennia. (S)

(a) R S P Q (b) Q S P R
(c) Q R P S (d) P S Q R

(d) P S Q R is the correct sequence.

40. a sell-off as US Treasury yields (P) surged to multi-year highs (Q) on robust economic data (R) global markets also witnessed (S)

(a) R S Q P (b) P Q S R
(c) Q R P S (d) S P Q R

(d) SPQR is the correct sequence.

Passage 1

Studies serve for delight, for ornament and for ability. Their chief use for delight, is in privateness and retiring; for ornament, is in discourse; and for ability, is in the judgement and disposition of business. For expert men can execute and perhaps judge of particulars, one by one; but the general counsels and the plots and marshalling of affairs, come best from those that are learned.

To spend too much time in studies is sloth; to use them too much for ornament, is affectation; to make judgement wholly by their rules, is the humor of a scholar. They perfect nature and are perfected by experience: for natural abilities are like natural plants, that need pruning, by study; and studies themselves do give forth directions too much at large, except they be bounded in by experience.

Crafty men condemn studies, simple men admire them and wise men use them; for they teach not their own use; but that is a wisdom without them and above them, won by observation. Read not to contradict and confute; nor to believe and take for granted; nor to find talk and discourse; but to weigh and consider.

41. According to the author, why should one study?

(a) To gather information about the world
(b) To pass the time in a creative way
(c) For pleasure, enhance capability and holistic growth
(d) To become only worldly wise

(***c***) As stated in the passage, studies should be done for pleasure, enhancing capability and for holistic growth of a mind.

42. Why does the author not recommend too much of studies?

(a) Because it reflects idleness
(b) Because it requires too much money
(c) Because it is dangerous for the health of the mind
(d) Because it makes men crafty

(***a***) The author does not recommend too much of studies as it can make a person lazy or idle.

43. According to the author, how can the studies be perfected?

(a) By reflection (b) By application
(c) By conversation (d) By experience

(***d***) According to the author, studies can be perfected via experience.

44. What kind of human beings denounce studies?

(a) Innocent (b) Cunning
(c) Stupid (d) Wise

(***b***) Crafty or Cunning human beings denounce studies.

45. According to the author, one should read to

(a) only argue (b) only believe
(c) evaluate and understand (d) preach and believe

(***c***) As stated in the passage, a person should read to 'evaluate and understand'.

46. The Election Commission on Saturday ..... that the Assembly elections in the five States will be held from November 12th to December 7th.

(a) pronounced (b) announced
(c) promulgated (d) issued

(***b***) 'Announced' is the appropriate word to fill the given blank. Announce means 'make a formal public statement about a fact, occurrence or intention'.

47. The victims were fruit vendors and they were going in an auto when they ... an accident on the way.

(a) met with (b) ran into (c) experienced (d) heard

(***a***) According to the given sentence, 'Met with' is the correct alternative to fill the blank as it means 'to experience something, such as an accident'.

48. Scores of villagers are ..... a sit-in protest against the consruction of a new underpass.

(a) performing (b) sitting
(c) staging (d) standing

(c) (*c*) 'Staging' is the appropriate word for the given blank. Staging means 'the method of presenting a play or other dramatic performance.'

49. It is common for patients to stop ..... medicine as soon as they start feeling better.

(a) earing (b) gulping
(c) swallowing (d) taking

(c) *(d)* According to the given sentence, option (d) 'taking' is the correct alternative to fill the blank as 'taking medicine' refers to swallow medicine. Other words are irrelevant.

50. A four-year-old girl got a new lease of life after doctors at a hospital ..... a cancerous tumour from one of her kidneys.

(a) rejuvenated (b) removed
(c) displaced (d) replaced

(c) *(b)* Here, 'removed' is the correct alternative to fill the given blank as 'removed' means 'take something off or away'. Other words do not fit in the blank.

Part B (General Studies)

51. A thin disc and a thin ring, both have mass M and radius R. Both rotate about axes through their centre of mass and are perpendicular to their surfaces at the same angular velocity. Which one of the following statements is correct?

(a) The ring has higher kinetic energy.
(b) The disc has higher kinetic energy.
(c) The ring and the disc both have the same kinetic energy.
(d) Kinetic energies of both the bodies are zero, since they are not in linear motion.

(c) *(a)* Given, a thin disc and a thin ring have equal mass and radius.

Angular velocity of thin disc
= Angular velocity of thin ring

$\omega_{ring} = \omega_{disc}$

$\therefore$ Kinetic energy of ring
= Kinetic energy of disc

$$\frac{1}{2} I_{ring}\omega_{ring}^2 = \frac{1}{2} I_{disc}\omega_{disc}^2$$

$$\frac{1}{2}(MR^2)\,\omega_{ring}^2 = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega_{disc}^2$$

$$(\because I_{ring} = MR^2 \text{ or } I_{disc} = \frac{1}{2} MR^2)$$

$$\omega_{ring}^2 = \frac{\omega_{disc}^2}{2},\ 2\omega_{ring}^2 = \omega_{disc}^2$$

Hence, $KE_{ring} > KE_{disc}$

So, the ring has higher kinetic energy.

52. Let there be an object having some chemicals in it. It starts moving with a uniform velocity v and a chemical reaction starts happening. In this case, which of the following statement(s) is/are correct?

1. Chemical reactions happening in the system cannot change the velocity (v) of the centre of mass of the object.
2. Chemical reactions happening in the system cannot change kinetic energy of the particles inside with respect to the centre of mass of object.

Select the correct answer using the code given below:

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) (*a*) A chemical reaction is an internal process, so there is no change in momentum and mass (since, mass is conserved in a chemical reaction).

But in a chemical reaction, the kinetic energy of the particle is changes with respect to centre of mass of whole body.

Hence, statement 1 is correct are stakines 2 is incorrect.

53. In which of the following phenomena do heat waves travel along a straight line with the speed of light?

(a) Thermal conduction
(b) Thermal convention
(c) Thermal radiation
(d) Both (a) and (c)

(c) *(c)* Thermal radiations are heat waves (infrared waves) which are electromagnetic waves. Electromagnetic waves travel in straight line with the speed of light. It depends on the temperature of the body and nature of radiating surface of the body.

Hence, option (c) is correct.

54. Which one of the following statements regarding Ohm's law is not correct?

(a) Ohm's law is an assumption that current through a conductor is always directly proportional to the potential difference applied to it.
(b) A conducting device obeys Ohm's law when the resistance of a device is independent of magnitude and polarity of applied potential difference.
(c) A conducting material obeys Ohm's law when the resistance of material is independent of the magnitude and direction of applied electric field.
(d) All homogeneous materials obey Ohm's law irrespective of whether the field is within range or strong.

(c) *(d)* Statement (d) is not correct, because in strong electric field, heating effect will dominant and here is deviations of curve, i.e. it does not obey Ohm's law.

55. When a ball bounces off the ground, which of the following changes suddenly? (Assume no loss of energy to the floor)

(a) Its speed
(b) Its momentum
(c) Its kinetic energy
(d) Its potential energy

(c) *(b)* When a ball bounces off the ground, there will be sudden change in the momentum of the ball.

- It includes the mass and the velocity of the object.
- It shows the sudden change of the velocity of the object according to its mass.
- If the mass is large, the change will be greater and if it is small, then the velocity will be lesser.

56. Which one of the following compounds does not exhibit a different oxidation number of the same element?

(a) Pb_3O_4 (b) Fe_3O_4
(c) Fe_2O_3 (d) Mn_3O_4

(c) (*c*) Among the given species/ compounds, only Fe_2O_3 does not exhibit different oxidation number of Fe, in this species can be calculated as :

Let x be the O.N. of Fe.

$\therefore \quad 2x + 3(-2) = 0,\ x = +3$

Hence, Fe is in only + 3 oxidation state in Fe_2O_3, whereas in rest of the species, the same metal exhibits different oxidation states.

57. Which one of the following statements about glass is not correct?

(a) Glass is often said to be a supercooled liquid.
(b) Glass has no definite melting point.
(c) Soda glass is harder than pyrex glass.
(d) Boron is present in pyrex glass.

(*c*) Pyrex glass is harder than soda lime glass. This is because, pyrex glass, (a particular blend of borosilicate glass) has a higher proportion of silicone dioxide than soda lime glass. Due to which, pyrex glass does not expand on heating, while soda lime glass is not likely to be directly and strongly heated.

Thus, the high heating tolerance of pyrex glass makes it harder than soda lime glass.

Hence, option (c) is incorrect.

Rest of the given statements are correct.

58. Which one of the following statements is correct?

(a) Both boiling and evaporation are surface phenomena.

(b) Boiling is a surface phenomenon, but evaporation is a bulk phenomenon.

(c) Both boiling and evaporation are bulk phenomena.

(d) Boiling is a bulk phenomenon, but evaporation is a surface phenomenon.

(*d*) Boiling is a bulk phenomenon, but evaporation is a surface phenomenon is the correct statement. This is because, during evaporation, the high energy molecules from the liquid go into the gaseous phase only through the surface.

While, in boiling, the vaporisation occurs throughout the bulk of the liquid and the vapours expand freely into the surrounding.

59. Which one of the following statements about diamond and graphite is not correct?

(a) Diamond has a tetrahedral structure, whereas graphite has a hexagonal planar structure.

(b) Both physical and chemical properties of diamond and graphite are different.

(c) Graphite is soft, but diamond is hard.

(d) Graphite is a good conductor of electricity, while diamond is not.

(*b*) Statement given in option (b) is incorrect. The correct statement is, the physical properties of both diamond and graphite are same. But, as the both diamond and graphite are allotropes of same element (carbon).

Thus, they both exhibits similar chemical properties.

So, option (b) is the correct answer.

60. The raw materials used for the manufacture of Portland cement are

(a) lime, silica and sulphur dioxide

(b) lime, silica and carbon dioxide

(c) lime, silica and alumina

(d) lime, silica and boric acid

(*c*) The raw materials used for the manufacture of Portland cement are lime, clay (provides alumina and silica) and gypsum.

Thus, the correct option is (c).

61. Which one of the following statements about water is not true?

(a) Hydrogen bonds are present in liquid water.

(b) Water has a high boiling point.

(c) Water has a high heat of fusion.

(d) Water is a non-polar molecule.

(*d*) The option (d) contains incorrect statement. Water is a polar molecule in which central oxygen atom is sp^3-hybridised.

Out of four sp^3-hybrid orbit, two for σ bond with H-atom and rest two are occupied by lone pair of electrons.

Thus, due to V-shaped structure of H_2O is a polar molecule (μ = 1840).

62. A sample of 'soft soap' contains

(a) caesium (b) potassium

(c) calcium (d) magnesium

(*b*) Soaps are the sodium or potassium salts of long chain fatty acid (RCOONa). e.g. stearic acid ($C_{17}H_{35}COOH$), oleic acid ($C_{17}H_{33}COOH$) and palmitic acid ($C_{15}H_{31}COOH$). Therefore, a sample of soft soap contains sodium or potassium.

Thus, the option (b) is correct.

63. If an object moves at a non-zero constant acceleration for a certain interval of time, then the distance it covers in that time

(a) depends on its initial velocity

(b) is independent of its initial velocity

(c) increases linearly with time

(d) depends on its initial displacement

(*a*) If object moving with non-zero constant acceleration, then the displacement covered by it, is given by equation

$$s = ut + \frac{1}{2}at^2$$

where, s = displacement, u = initial velocity, a = acceleration and t = time.

Hence, from above equation, displacement is dependent to initial velocity. Distance covered is a non-linear curve of time.

64. A solid disc and a solid sphere have the same mass and same radius. Which one has the higher moment of inertia about its centre of mass?

(a) The disc

(b) The sphere

(c) Both have the same moment of inertia

(d) The information provided is not sufficient to answer the question

(*a*) As we know, moment of the inertia

$$I_{disc} = \frac{1}{2}MR^2 \text{ and } I_{sphere} = \frac{2}{5}MR^2$$

$\because$ Given, $M_{disc} = M_{sphere}$ and $R_{disc} = R_{sphere}$

So, $$\frac{I_{disc}}{I_{sphere}} = \frac{\frac{1}{2}MR^2}{\frac{2}{5}MR^2} = \frac{5}{4} \quad \ldots (i)$$

Hence, from Eq. (i), we get

$$I_{disc} > I_{sphere}$$

So, the disc has higher moment of inertia.

65. Two substances of densities ρ_1 and ρ_2 are mixed in equal volume and their relative density is 4. When they are mixed in equal masses, relative density is 3. The values of ρ_1 and ρ_2 respectively, are

(a) 6, 2 (b) 3, 5 (c) 12, 4 (d) 9, 3

(*a*) When substances are mixed in equal volume.

Then, relative density,

$$4 = \frac{V\rho_1 + V\rho_2}{V + V}$$

[$\because$ volume(V) x density (ρ) = mass (M)]

$$\Rightarrow \quad \rho_1 + \rho_2 = 8 \quad \ldots (i)$$

Similarly, when substances are mixed in equal mass.

Relative density,

$$3 = \frac{M + M}{\frac{M}{\rho_1} + \frac{M}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \quad \ldots (ii)$$

Hence, from Eqs. (i) and (ii), we get

$$\rho_1 = 6 \text{ and } \rho_2 = 2$$

66. A car starts from Bengaluru, goes 50 km in a straight line towards South, immediately turns around and returns to Bengaluru. The time taken for this round trip is 2h. The magnitude of the average velocity of the car for this round trip

(a) is zero (b) is 50 km/h

(c) is 25 km/h

(d) cannot be calculated without knowing acceleration

(*a*) According to question, a round trip is mentioned by a car. But as we know, for round trip the displacement is zero. So,

$$v_{av} = \frac{\text{displacement}}{\text{time}} = \frac{0}{2} = 0 \text{ ms}^{-1}$$

Hence, option (a) is correct.

67. "The sum of emf's and potential differences around a closed loop equals zero" is a consequence of

(a) Ohm's law

(b) conservation of charge

(c) conservation of momentum

(d) conservation of energy

(*d*) It is Kirchhoff's second law also known as loop's law.

i.e. $\Sigma E = iR$

It is based on conservation of energy, since work done in a closed path is zero.

68. A student measures certain lengths using a meter scale having least count equal to 1 mm.

Which one of the following measurement is more precise?

(a) 0.50 mm
(b) 29.07 mm
(c) 0.925 mm
(d) 910 mm

(***c***) Using instrument of higher precision improving experimental techniques, etc., we can reduce the least count error.

Here, option (c) 0.925 mm is more precise than other.

69. If the work done on the system or by the system is zero, then which one of the following statement for a gas kept at a certain temperature is correct?

(a) Change in internal energy of the system is equal to flow of the heat in or out of the system.
(b) Change in internal energy of the system is less than heat transferred.
(c) Change in internal energy of the system is more than the heat flow.
(d) Cannot be determined.

(***a***) For first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

where, ΔQ = heat,

ΔU = change in internal energy

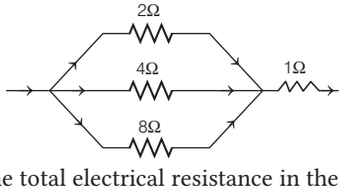
and ΔW = work done.

Since, $\Delta W = 0$

So, $\Delta Q = \Delta U$

Hence, there is change in internal energy of the system is equal to flow of heat in or out of the system, option (a) is correct.

70. Consider the following part of an electric circuit.



The total electrical resistance in the given part of the electric circuit is

(a) $\frac{15}{8}\ \Omega$ (b) $\frac{15}{7}\ \Omega$
(c) $15\ \Omega$ (d) $\frac{17}{3}\ \Omega$

(***b***) There are three resistances (2Ω, 4Ω and 8Ω) are in parallel.

So, $\frac{1}{R_1} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \Rightarrow R_1 = \frac{8}{7}\Omega$

Now, R_1 is in series with $1\ \Omega$ resistance.

So, $R_{eq} = 1\ \Omega + R_1 = 1 + \frac{8}{7} = \frac{15}{7}\ \Omega$

Hence, option (b) is correct.

71. Which one of the following statements regarding lenses is not correct?

(a) A convex lens produces both real and virtual images.
(b) A concave lens produces both real and virtual images.
(c) A convex lens can produce images equal, greater and smaller than the size of the object.
(d) A concave lens always produces images smaller than the size of the object.

(***b***) A concave lens always formes virtual image, whereas convex lens forms both real and virtual images.

Hence, option (b) is incorrect.

72. 10 g of ice at $-10°C$ is mixed with 10 g of water at 0°C. The amount of heat required to raise the temperature of mixture to 10°C is

(a) 400 cal (b) 550 cal
(c) 1050 cal (d) 1200 cal

(***c***) Heat require by 10 g of water at $-10°C$ to 0°/C is

$$H_1 = ms\Delta\theta$$

$$= 10 \times (0.50)\,(0 - (-10) = 50 \text{ cal}$$

Now, latent heat of melting of 10 g of ice

$$H_2 = mL = 10 \times 80 = 800 \text{ cal}$$

Now, there is 20 g of water at 0°C, so heat required to reach the temperature of $+10°C$.

$H_3 = ms\Delta\theta = 20 \times 1 \times (10 - 0) = 200$ cal

Total heat required,

$$H = H_1 + H_2 + H_3$$

$$= 50 + 800 + 200 = 1050 \text{ cal}$$

73. Which one of the following does not represent the salt, calcium carbonate?

(a) Lime water (b) Limestone
(c) Chalk (d) Marble

(***a***) Calcium carbonate is also known as limestone and has a molecular formula is $CaCO_3$. It is found in nature in the form of chalk, marble etc. Thus, limestone. chalk and marble represents calcium carbonate. Whereas, lime water is the aqueous solution of $Ca(OH)_2$.

Thus, the correct option is (a).

74. Soaps clean surfaces on the principle of based on

(a) viscosity (b) floatation
(c) elasticity (d) surface tension

(***d***) Soaps clean surfaces on the principle of based on surface tension. Soap is a surface active agent or surfactant. It helps to reduce the surface tension of water and thereby speed up the cleaning process of soaps.

75. Which one of the following substances is not a mixture?

(a) Tin (b) Sea water
(c) Soil (d) Air

(***a***) A mixture is a type of matter which has variable composition throughout.

Among the given options, only Sn(tin) is not a mixture, while rest are the examples of mixture. Sn is an element which contains only one kind of particles or atoms.

76. Which one of the following is termed as 'dry ice'?

(a) Ice present in ice-cream
(b) Solid water at Antarctica
(c) Solid state of carbon dioxide
(d) Solid water of ionosphere

(***c***) When liquefied CO_2 is allowed to expand rapidly, it gets converted in the form of solid CO_2 which is also called dry ice. Thus, dry ice is solid state of the carbon dioxide.

77. Net movement of water from a dilute to a concentrated solution through a selectively permeable membrane is called

(a) Diffusion (b) Dispersion
(c) Osmosis (d) Absorption

(***c***) The net movement of water from a dilute to concentrated solution through a selectively permeable membrane is called osmosis. This term is used specifically to refers to the diffusions of water across semi or selectively permeable membrane.

Water moves from a region of higher concentration to a region of lower concentration until an equilibrium is reached.

78. Blood is a type of

(a) epithelial tissue
(b) muscular tissue
(c) nervous tissue
(d) connective tissue

(***d***) Blood is a type of connective tissue, which forms 30-32% of total extracellular fluid. The connective tissues have a special function of linking and supporting other tissues/organs of the body.

Blood is a fluid, which helps in the transportation of various substances in the body. It is red because it contains a red pigment called haemoglobin in its red blood cells.

79. Mitochondria are able to produce their own

(a) nucleus
(b) proteins
(c) chloroplasts
(d) digestive enzymes

(b) Mitochondria are able to produce their own protein. It acts as the cell's power producer and also called as powerhouse of the cell.

It possesses single circular DNA molecules, a few RNA molecules, ribosomes and the components required for the synthesis of proteins. It provides energy for energy requiring processes of the cell like muscle contraction, nerve impulse, conduction, etc. It may store and release calcium when required.

80. Which one of the following statements is correct?

(a) Xylem consists of tracheids, vessels, xylem parenchyma and xylem fibres
(b) Flexibility in plants is due to sclerenchyma
(c) Parenchyma have no intercellular spaces
(d) Xylem consists of sieve plate, sieve tube and companion cells

(a) The statement given in option (a) is correct. Rest of the statements are incorrect and can be corrected as

(b) Flexibility in plants is due to the permanent tissue collenchyma. Sclerenchyma provides mechanical support to organs.
(c) Parenchyma cells are thin walled with small intercellular spaces.
(d) Phloem consists of sieve plates, sieve tube and companion cells.

81. Who among the following popularised the use of embryological characters in taxonomy?

(a) Carl Linnaeus
(b) Panchanan Maheshwari
(c) Birbal Sahni
(d) Bentham and Hooker

(b) Panchanan Maheshwari popularised the use of embryological characters in taxonomy. He is also known as 'The Father of Indian Plant Embryology'.

82. Water is heated with a coil of resistance R connected to domestic supply. The rise of temperature of water will depend on

1. supply voltage
2. current passing through the coil
3. time for which voltage is supplied

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 1 and 2
(c) Only 1 (d) 2 and 3

(a) Heat generated by a resistance coil is given by

$$H = I^2Rt$$

From Ohm's law, $I = \frac{V}{R}$

So, $H = I\left(\frac{V}{R}\right)Rt = IVt$

Hence, option (a) is correct.

83. A pendulum clock is lifted to a height where the gravitational acceleration has a certain value of g. Another pendulum clock of same length, but of double the mass of the bob is lifted to another height where the gravitational acceleration is $\frac{g}{2}$.

The time period of the second pendulum would be
(in terms of period T of the first pendulum)

(a) $\sqrt{2}\,T$ (b) $\frac{1}{\sqrt{2}}T$
(c) $2\sqrt{2}\,T$ (d) T

(a) Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{...(i)}$$

When acceleration due to gravity changed to $\frac{g}{2}$, then time period,

$$T' = 2\pi\sqrt{\frac{l}{g/2}} = \sqrt{2}\left(2\pi\sqrt{\frac{l}{g}}\right)$$

[from Eq. (i)]

$$\Rightarrow \quad T' = \sqrt{2}\,T$$

Hence, option (a) is correct.

84. The loudness of sound depends upon the

(a) velocity of sound waves in the medium
(b) amplitude of the sound waves
(c) frequency of the sound waves
(d) frequency and velocity of the sound waves

(b) Loudness of sound wave is depend on its amplitude, while sharpness of sound wave is depend on its frequency.
Hence, option (b) is correct.

85. Two bodies of mass M each are placed R distance apart. In another system, two bodies of mass $2M$ each are placed $\frac{R}{2}$ distance apart. If F be the gravitational force between the bodies in the first system, then the gravitational force between the bodies in the second system will be

(a) 16 F (b) 1 F
(c) 4F (d) None of these

(a) **Situation I,**
Force between the masses

$$F = \frac{GMM}{R^2} = \frac{GM^2}{R^2} \quad \text{... (i)}$$

Situation II,
Force between the masses

$$F' = \frac{G(2M)(2M)}{\left(\frac{R}{2}\right)^2} = \frac{16\,GM^2}{R^2} \text{ ... (ii)}$$

From Eqs. (i) and (ii), we get

$$F' = 16F$$

Hence, option (a) is correct.

86. Light rays move in straight lines. But through an optical fibre, they can move in any type of *zig-zag* path because

(a) the holes through the fibres are extremely fine
(b) light rays are absorbed at the entry end and relieved at the exit end of the fibre
(c) scattering of light occurs inside the fibre
(d) successive total internal reflections occur as a ray moves through the fibre

(d) The working of optical fibre is based on the total internal reflection. Its inner part is core of higher refractive index surrounded by another layer of glass of lower refractive index. It is surrounded by plastic jacket.

When light enters from the one end of the core and moves towards cladding, then total internal reflection takes place again and again and light propagate through it. Optical fibres are used in decorative table lamps.

87. Which one of the following statements regarding cathode rays is not correct?

(a) Cathode ray particles are electrons.
(b) Cathode ray particles start from anode and move towards cathode.
(c) In the absence of electrical and magnetic fields, cathode rays travel in straight lines.
(d) Television picture tubes are cathode ray tubes.

(b) Cathode rays start from cathode and move towards anode, because they are made of negatively charged electrons.
Hence, option (b) is incorrect statement.

88. A very large volume of hydrogen can be accommodated by making

(a) non-metallic hydrides
(b) hydrogen peroxide
(c) non-stoichiometric hydrides
(d) alkali metal hydrides

(c) A very large volume of hydrogen can be accommodated by making non-stoichiometric hydrides.
Such hydrides are formed by some of the metals (e.g. Pd, Pt). This property of hydrogen has high potential for hydrogen storage .

89. Which one of the following is not a monoatomic element?

(a) Copper (b) Helium
(c) Iodine (d) Barium

(***c***) Among the given species, only iodine is not a monoatomic element. It exists in the form of I_2 (diatomic), while rest occur as monoatomic element.

90. Which one of the following represents the correct order of electron releasing tendency of metals?

(a) Zn > Cu > Ag (b) Ag > Cu > Zn
(c) Cu > Zn > Ag (d) Cu > Ag > Zn

(***a***) The correct order of electron releasing tendency of given metals is

Zn> Cu > Ag.

This can be explained on the basis of standard reduction potentials of given elements. More negative be the E°_{red} value, greater is the tendency of element to release electron.

Zn has highly negative E°_{red} value, while Cu has least negative E°_{red} value.

Thus, the correct order is

Zn > Cu > Ag

91. Which one of the following is the major constituent of biogas?

(a) Carbon dioxide (b) Nitrous oxide
(c) Methane (d) Oxygen

(***c***) Biogas usually contains 45-70% methane and 30-45% carbon dioxide. It also contains small amounts of nitrogen, hydrogen sulphide, halogenated compound and organic silicon compounds. Thus, among the given options, methane is the major constituent of biogas.

92. Thyroid gland produces a hormone called 'thyroxine' which

(a) controls blood glucose levels
(b) controls ovulation
(c) controls metabolic rate
(d) maintains pregnancy

(***c***) Thyroid gland produces a hormone called thyroxine, which controls the metabolic rate of the body. It acts to increase the basal metabolic rate and affects protein, fat and carbohydrate metabolism affecting how human cells use energetic compounds.

93. Which one of the following is a bacterium that causes disease in the human body?

(a) *Varicella zoster*
(b) *Trypanosoma gambiense*
(c) *Salmonella typhi*
(d) *Plasmodium falciparum*

(***c***) *Salmonella typhi* is a bacterium which causes typhoid fever in the human body. This fever is contracted by the ingestion of contaminated food and water.*Varicella zoster* is a virus causing chickenpox. Trypanosoma gambiense and Plasmodium falciparum are protozoans which cause sleeping- sickness and malaria, respectively in the human body.

94. Which one of the following statements regarding viruses is not true?

(a) Viruses need living cells to reproduce
(b) All viruses are parasites
(c) Viruses can synthesise their food through photosynthesis
(d) Viruses are similar to chemical substances outside the host

(***c***) The statement given in option (c) is not true and can be corrected as Viruses do not perform photosynthesis to synthesise their food. They are inert, crystalline structure outside the living cell.

Once they infect a cell, they takeover the machinery of the host cell to replicate themselves, killing the host.

Rest of the statements are correct.

95. Which one of the following diseases may be caused by the deficiency of vitamin-C?

(a) Rickets (b) Rabies
(c) Hepatitis (d) Scurvy

(***d***) The deficiency of vitamin-C (ascorbic acid) causes scurvy. This condition is characterised by general weakness, anaemia, gingivitis and skin haemorrhage.

96. Compared to audible sound waves, ultrasound waves have

(a) higher speed (b) higher frequency
(c) Both (a) and (b (d) long wavelength

(***b***) Ultrasound waves have higher frequency than audible sound waves.

The audible range of sound for human beings is form 20 to 20,000 Hz. Children under the age of 5 and some animals such as dogs can hear upto 25,000 Hz. As people grow older, their ears become less sensitive to higher frequency.

Ultrasonic sound of frequency higher than 20,000 Hz, which cannot be heard by human beings.

97. A rigid body of mass 2 kg is dropped from a stationary balloon kept at a height of 50 m from the ground. The speed of the body when it just touches the ground and the total energy when it is dropped from the balloon are respectively.

(Take, acceleration due to gravity, $g = 9.8\ m/s^2$)

(a) 980 ms^{-1} and 980 J
(b) $\sqrt{980}$ ms^{-1} and $\sqrt{980}$ J
(c) 980 ms^{-1} and $\sqrt{980}$ J
(d) $\sqrt{980}$ ms^{-1} and 980 J

(***d***) Given, $M = 2$ kg and height = 50 m

Speed of body at the ground,

$$v^2 - u^2 = 2gH$$

$$v = \sqrt{2gH}\ [\because u = 0]$$

$$= \sqrt{2 \times 9.8 \times 50}$$

$$\Rightarrow \quad v = \sqrt{980}\ ms^{-1}$$

Hence, the total energy = kinetic energy

$$\Rightarrow \quad TE = \frac{1}{2}Mv^2$$

$$= \frac{1}{2} \times 2 \times (\sqrt{980})^2 = 980\ J$$

Hence, option (d) is correct.

98. The temperature of a place on one sunny day is 113 in Fahrenheit scale. The Kelvin scale reading of this temperature will be

(a) 318 K (b) 45 K
(c) 62.8 K (d) 335.8 K

(***a***) Given, temperature,

$$T_f = 113\ F$$

As, we know $T_k = 273 + \frac{10}{18}(T_f - 32)$

$$\Rightarrow \quad T_k = 273 + \frac{10}{18}(113 - 32)$$

$$\Rightarrow \quad T_k = 318\ K$$

99. Radiations coming from the sun are mostly in the form of

(a) light only
(b) light and long wavelength infrared
(c) light and short wavelength infrared
(d) Both short and long wavelength infrared

(***b***) The mode of transfer of heat from one body to another without any actual movement the particles involved and without heating the intervening medium, is called radiation. For radiation, there is no need of any medium because it is of electromagnetic nature.

So, radiation coming from the sun are mostly in form of light and long wavelength infrared.

100. Which of the following are the characteristics of electromagnetic waves?

1. They are elastic waves.
2. They can also move in vacuum.
3. They have electric and magnetic components which are mutually perpendicular.

4. They move with a speed equal to 3 lakh meters per second.

Select the correct answer using the codes given below.

(a) 1, 2, 3 and 4 (b) 1, 2 and 4
(c) 2 and 3 (d) 3 and 4

(c) Electromagnetic waves or EM waves are waves that are created as a result of vibrations between an electric field and a magnetic field. They are formed when an electric field comes in contact with a magnetic field. The electric field and magnetic field of an electromagnetic wave are perpendicular (at right angles) to each other. They are also perpendicular to the direction of the EM wave.

These waves travel with a constant velocity of 3.0×10^8m / s in vacuum. They are deflected neither by the electric field nor by the magnetic field.

Hence-statement 2 and 3 are correct.

101. In the Manusmriti which form of marriage results from the 'Voluntary union of a maiden and her lover'?

(a) Eighth form (b) Fifth form
(c) Seventh form (d) Sixth form

(b) In the Manusmriti, fifth (Gandharva form of marriage) results from the 'Voluntary union of a maiden and her love'. In Gandharva Marriage, a groom and his bride could getmarried without their parents knowledge or sanction. Hence, option (d) is the correct answer.

102. Which one of the following is a Rabi crop in the Northern States of India?

(a) Rice (b) Bajra
(c) Barley (d) Ragi

(c) Rabi crops are winter crops which are sown in October-November and harvested in March. The Rabi crops in Northern states of India includes Barley, wheat, oats, gram, linseed and mustard.

103. The rivers of North-West Europe are good examples of

(a) parallel pattern of drainage
(b) radial pattern of drainage
(c) barbed pattern of drainage
(d) trellis pattern of drainage.

(c) The rivers of North-West Europe are good examples of barbed drainage pattern. **Barbed drainage pattern** is a rare kind of drainage pattern which is formed when the tributaries flow in opposite direction to their master streams.

A parallel pattern of drainage system is a pattern of river caused by steep slopes with some relief. Because of the steep slopes, the streams are swift and straight, with very few tributaries and all flow in the same direction.

Radial drainage patterns form when streams and rivers flow off a central peak or dome like a volcano.

In a **trellis pattern**, the river forms a net like system and the tributaries flow roughly parallel to each other.

104. Which one of the following ocean currents is a cold current?

(a) South Atlantic Drift
(b) Mozambique Current
(c) East Australian Current
(d) Caribbean Current

(a) **South Atlantic Drift/Current** is an eastward flowing cold ocean current, fed by the Brazil Current. It is continuous with the northern edge of the Antarctic Circumpolar Current.

The **Mozambique current** is a warm ocean current flowing along the east coast of Africa in Indian ocean. It flows between Mozambique and Madagascar islands.

The **East Australian Current** (EAC) is a warm ocean current that flows along the eastern cost of Australia.

The **Caribbean Current** is a warm ocean current that flows northwestward through the Caribbean sea along the coast of South America and into the Gulf of Mexico.

105. Paradip Port is located on the delta of river

(a) Rihand (b) Ganga
(c) Mahanadi (d) Bitarani

(c) It is a natural, deep-water port on the East coast of India in Jagatsinghpur district of Odisha. Paradip Port is situated at confluence of the Mahanadi river and the Bay of Bengal.

106. Which one of the following hypothesis/theory explains the origin of the Universe?

(a) Nebular Hypothesis
(b) Binary Theory
(c) Big Bang Theory
(d) Planetesimal Hypothesis

(a) Nebular Hypothesis was given by Immanuel Kant explain about the origin of Universe. According to this theory, plants were formed out of a cloud (nebula) of hydrogen, helium & dust surrounding the sun. Binary theory explain about the existence of companion star of the sun.

According to **Big Bang theory**, all matter or substance forming this universe existed at one place as a tiny ball. This tiny ball had infinite density and temperature. At the Big Bang Theory the tiny ball exploded and started a process of expansion of the universe which continues to a day.

According to Planetesimal Hypothesis, when a planetary system was forming, there was a proto planetary disc with materials form the nebula from which the system came.

107. Which among the following struggles, based on Gandhi's philosophy of Satyagraha, involved the industrial working class?

(a) Champaran (b) Kheda
(c) Ahmedabad (d) Bardoli

(c) Ahmedabad Mill Strike, 1918 was based on Gandhi's philosophy of Satyagraha, that involved the industrial working class. The industrial dispute for wage hike was between the owners and workers of a cotton mill in Ahmedabad. The result of peaceful strike led by Gandhiji was successful and workers got a 35% wage increase.

108. Consider the following statements related to the Virashaiva tradition of Karnataka.

1. The Virashaiva Movement was led by a Brahmana named Basavanna, a minister in the court of the Kalachuri ruler in the twelfth century.
2. The Virashaivas encouraged practices, approved by the Dharmashastras, like post-puberty marriage and remarriage of widows.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) The twelfth century witnessed the emergence of a new movement called Virashaiva Movement in Karnataka. The movement was led by a Brahmana named Basavanna (1106-68) who was initially a Jaina and a minister in the court of a Kalachuri ruler (Chalukya, king of Gujarat). His followers were known as Virashaivas (heroes of Shiva) or Lingayats (wearers of the linga). The Chalukyas were contested for the suzerainty by the kalachuris of Southern India in that period. Hence, statement (1) is correct.

Also, the Dharmashashtras did not permit widow remarriages and post-puberty marriages. But the Virashaivas undertook them as a sign of refusal of the sacred Brahmanical scriptures. Hence, statement (2) is in correct.

109. Consider the following statements related to Wellesley's administration.

1. In the six years of Wellesley's administration, the army accounted for 42.5% of the Company's total expenditure.
2. Wellesley's administrative measures were restricted to the affairs to the Company and had nothing to do with the commercial and military affairs of the Indian ruling families.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) During Lord Wellesley's tenure, army was an expensive machinery of the British East India Company. He had waged war on many Princely States and native rulers like Mysore, Marathas, Nizams etc. Hence, to sustain such a larger army, he spent huge amounts (42.5% of the company's total expenditure) on army.He also devised the infamous Subsidiary Alliance System which required to maintain British army in native ruler's courts. Hence, statement (1) is correct.

Wellesley was an excellent administrator. He used his Subsidiary Alliance System to interfere with the commercial and military affairs to Indian rulers by making them mandatorily consult him on important decisions.

Hence, statements (2) is also correct.

110. Consider the following statements concerning the Russian Revolution.

1. The Bolsheviks believed that in a repressive society like Tsarist Russia, the party should be disciplined and should control the number of its members.
2. The Mensheviks believed that the party should be opened to all (as in Germany).

Selects the correct answer using the codes given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) The Bolsheviks and Mensheviks were the two factions of Russian Social Democratic Labour Party formed under leaderships of Vladimir Lenin and Julius Martov respectively.

Bolsheviks believed in limiting the membership of party to a core professional member body as it was easier to manage them which was necessary in a repressive society like Tsarist Russia.

Hence, statement (1) is correct.

On the other hand Mensheviks believed in power of masses and advocated liberal thoughts like entry of common masses in the party. Hence, statement (2) is also correct.

111. Which of the following was/were among the decisions taken by the First Continental Congress held in Philadelphia in September, 1774?

1. It rejected a plan for a colonial union under the British authority.
2. It drew up a statement of grievances.
3. It agreed upon a scheme to stop trade between the Colonies and England.

Which of the statement(s) given above is/are correct?

(a) 1 and 3 (b) Only 2
(c) 1 and 2 (d) 1, 2 and 3

(d) The First Continental Congress held in Philadelphia in September, 1774 is a major landmark in the history of USA. It was attended by representatives of 12 out of 13 British Colonies in USA, at the Carpenter's Hall. It was held in response to British impasses over the historic Boston Tea Party of December, 1773. *The major outcomes of this Congress were*

- Rejection of Union of colonies under British Crown.
- Scheme for boycott of trade between the colonies and the Britishers.
- Mechanism of grievance discussion.

Hence, all the statements are correct.

112. Which one of the following land territories of Indonesia is not touched by the Equator?

(a) Sumatra (b) Sulawesi
(c) Java (d) Kalimantan

(c) Java island is located to the South of equator at around 5°S latitude in a West to East orientation, so Java is not touched by the equator. However, equator passes through the Sumatra, Sulawesi and Kalimantan portion of Borneo island in Indonesia.

113. Which one of the following sedimentary rocks is organically formed?

(a) Shale (b) Chert
(c) Halite (d) Chalk

(d) Organically formed sedimentary rocks are formed from the remains of living organisms such as corals & shell fish (calcareous rock); or from vegetative remains like forests & swamps (carbonaceous rocks). Some examples of organically formed sedimentary rocks are chalk, limestone, peat, lignite & coal.

Shale is a mechanically formed sedimentary rock. Halite & chert are chemically formed sedimentary rocks.

114. According to the Koppen climatic classification, the letter code Cfa denotes.

(a) Tropical wet climate
(b) Humid subtropical climate
(c) Tundra Climate
(d) Tropical wet and dry climate

(b) Koeppen's climate classification is based on annual & monthy average temperatures and precipitation. According to this classification **Cfa** denotes **Humid subtropical** climate.

This type of climate is characterised by warm summer and absence of dry season. Tropical wet climate is denoted by Af; Tundra climate is denoted by ET and Tropical wet & dry climate is denoted by Aw.

115. In which one of the following folds in the axial plane found to be virtually horizontal?

(a) Isoclinal (b) Anticlinal
(c) Recumbent (d) Monoclinal

(c) In **recumbent fold**, aecial plane found to be horizontal. The axial plane of a fold is the plane or surface that divides the fold as symmetrically as possible.

In **Isoclinal fold**, the axial surface and limbs slope in the same direction and at approximately the same angle.

An **Anticlinal fold** is an arch like fold which is convex upward.

Monoclinal fold is a step-like fold consisting of a zone of steeper dip within or otherwise horizontal topography.

116. How many Ramsar convention sites have been enlisted from India so far?

(a) 12 (b) 16 (c) 27 (d) 62

(c) Has 27 Ramsar sites like Dal lake, Pong Dam, Harike Wetland, Chilika lake etc.

Ramsar Convention on Wetlands is an international treaty on conservation and sustainable use of Wetlands. It was signed in Ramsar, a city in Iran in 1971.

India Chilika lake was first to be designated as a Ramsar site in India.

117. The Munda language group belongs to which family?

(a) Austric (b) Dravidian
(c) Sino-Tibetan (d) Indo-European

(a) Munda language belongs to the **Austroasiatic or Austric** group of language. It is a group of language with more than 9 million user from central and eastern India.

Dravidian languages are Tamil, Telugu, Kannada and Malyalam. **Sion-Tibetan languages** includes chinese and the Tibeto-Busman languages. **Indo-European languages** are dialects in Asia and Europe.

118. Which one of the following statements about the Suez Canal is not correct?

(a) The Suez Canal was constructed in 1869.
(b) It links the Mediterranean Sea and the Read Sea.
(c) It has a six lock system and ships cross the different levels through these locks.
(d) It has given Europe a new gateway to the Indian Ocean.

(c) Option, (c) is not the correct statement about the Suez Canal because. It has no lock system as the water level in Mediterranean Sea and Red Sea through Gulf of Suez is nearly same. The Suez Canal in Egypt connects the Red Sea with Mediterranean Sea. It was constructed between 1859 to 1869 and officially opened in 1869.

It gives Europe a new gateway to the Indian Ocean allowing huge trade and cargo movement between the Europe and Asian Countries.

119. Which among the following statements is not correct about the Reign of Terror?

(a) The Jacobins dashed out the Girondists with the help of Sans-Culottes.
(b) The Jacobins were in league with the French clergy.
(c) The arrest for anti-revolutionary activities was by law restricted to the political class.
(d) The Reign of Terror came to an end with the execution of Robespierre by guillotine in July, 1794.

(c) Reign of Terror was the period in French revolution in 1793 to 1794, marked by widespread violence and executions by the Jacobins, in association with the Sans-Culottes. The Jacobins wre in league with the French clergy.

Jacobins dashed out Girondists with help of Sans–Culottes and were actively conspiring against Aristocracy. The laws for arresting were not limited to political class instead it was vaguely defined and anyone could be arrested. Hence, option (c) is not correct about the Reign of Terror. The reign came to an end on 27th July, 1794 with the execution of Robespierre by guillotine.

Hence, option (c) is incorrect.

120. Who among the following was the first Indian to qualify for the Indian Civil Service?

(a) Satyendranath Tagore
(b) Surendranath Banerjee
(c) RC Dutt
(d) Subhas Chandra Bose

(a) Satyendranath Tagore was the first Indian to join the Indian Civil Service. He was selected in 1863 and posted in Bombay Presidency as a Civil Servant.

121. Which among the following was/were among the factors for England to experience the Industrial Revolution first?

1. The scientific inventions had paved the way for Industrial Revolution.
2. It had favourable social and political structures in place.
3. Navigable rivers had made inland transport easier.
4. It had seen the ascendancy of capitalist practices.

Select the correct answer using the codes given below.

(a) 1 and 3 (b) 1, 3 and 4
(c) Only 4 (d) All of these

(d) Industrial Revolution was the technological implementation in mass production during the second half of 18th century in England. It was a period of transition with new machines technologies and steam power driving the production capacity. The causes/ factors for England to experience the Industrial Revolution first were

1. Scientific innovations.
2. Favourable Social and Political Structure
3. Navigable rivers
4. Higher Capital in hands for setting up enterprise.

Hence, all the statements are correct.

122. Article-46 of the Constitution of India refers to the promotion of educational and economic interests of

(a) religious minorities
(b) Scheduled Castes, Scheduled Tribes and other weaker sections
(c) displaced persons from large irrigation projects
(d) the economically deprived

(b) Article 46 in part IV of the Constitution of India is related to the promotion of educational and economic interests of the Scheduled Castes, Scheduled Tribes and other weaker sections. Article 46 also provides that the state shall protect the weaker sections of the society from social in justice and all forms of exploitation.

123. Who was the Education Minister in the first Cabinet of Independent India?

(a) Sarvepalli Radhakrishnan
(b) Sardar Vallabhbhai Patel
(c) Maulana Abul kalam Azad
(d) Acharya Narendra Dev

(c) Maulana Abul Kalam Azad was the Education Minister in the first Cabinet of Independent India.

National Education Day is celebrated on 11th November every year to commemorate the birth anniversary of Maulana Abul Kalam Azad.

124. Who among the following died in exile?

(a) Rani Laxmibai
(b) Bahadur Shah Zafar
(c) Tantia Tope
(d) Nana Saheb

(b) Bahadur Shah Zafar → Died in Rangoon while in exile in 1862.

Rani Laxmibai → Died on the battlefield in Gwalior on 18th June, 1858.

Tantia Tope → Executed by Britishers in 1859 at Shivpuri.

Nana Saheb → Escaped to Nepal and is thoughts to have died in 1859.

125. The normal lapse rate of temperature of Earth's atmosphere drops to 0°C at the

(a) upper part of ionosphere
(b) upper boundary of the tropopause
(c) lower part of mesosphere
(d) upper boundary of stratopause

(b) The normal lapse rate of temperature of Earth's atmosphere drops to 0°C at the upper boundary of the tropopause. The lapse rate is the rate at which temperature in Earth's atmosphere changes with altitude.

126. The headquarters of National Research Laboratory for Conservation of Cultural Property (NRLC) is located at

(a) New Delhi (b) Lucknow
(c) Kolkata (d) Bhopal

(b) The National Research Laboratory for Conservation of Cultural Property (NRLC) is a scientific institution of the Ministry of Culture, Government of India. It is responsible for the Conservation of Cultural Heritage. The NRLC was established in 1976 with its headquarters located at Lucknow.

127. Loktak lake is situated in the State of
(a) Sikkim (b) Manipur
(c) Odisha (d) Mizoram

(***b***) Loktak Lake is located at Moirang in Manipur, India. It is the largest fresh water lake of North-East India & is famous for phumdis (floating vegetative mass). Keibul Lamjao National Park, one of the phumdis of the Lake, is the only floating national park in the world. It is also the last natural habitat of endagered Sangai deer.

128. Stalactite, stalagmite and pillars are the depositional landforms of
(a) running water (b) wind
(c) glacier
(d) underground water

(***d***) Stalactite, stalagmite and pillars are the depositional landforms of underground water. Stalactite and stalagmite are formed when the calcium carbonate dissolved in groundwater.

129. A deep valley characterised by steep step like slope is known as
(a) U-shaped valley (b) Blind valley
(c) Gorge (d) Canyon

(***d***) Canyon is a deep valley characterised by steep step like slopes. A **U-shaped** valley is a glaciated landform having a characteristic U-shape with steep straight sides & a flat or rounded bottom. A **blind valley** is a deep, narrow & flat bottomed valley with an abrupt ending. **A Gorge** is a deep, narrow valley with steep sides.

130. Which one of the following is a major plate?
(a) Pacific Plate (b) Cocos Plate
(c) Arabian Plate (d) Philippine Plate

(***a***) Among the given options, Pacific plate is a major plate. The outer shell of Earth, Lithosphere is broken into tectonic plates.

There are seven major tectonic plates which include Pacific Plate, North American Plate, Eurasian Plate, African Plate, Antarctic Plate, Indo-Australian Plate and South American Plate. There are about 20 minor plates including Cocos Plate, Nazca plate Arabian Plate Philippine plate and Caroline Plate etc.

131. Lithosphere consists of
(a) upper and lower mantle
(b) crust and core
(c) crust and uppermost solid mantle
(d) mantle and core

(***c***) Lithosphere is the solid outer part of the Earth. It consists of the crust and solid outermost layer of the upper mantle. Hence option (c) is correct answer.

132. The principal component(s) of Nehru-Mahalanobis strategy of economic development was/were
1. Restructuring economic dependency on metropolitan capitalism into independent economic development.
2. Transition from semi-feudal agricultural to capitalist farming.

Select the correct answer using the codes given below.
(a) Only 1 (b) Only 2
(d) Both 1 and 2 (d) Neither 1 nor 2

(***c***) The Nehru-Mahalanobis strategy was developed by PC Mahalanobis, under the guidance of Jawaharlal Nehru in 1953. *The Principal components of this development strategy were*
(i) Restructuring economic dependency on metropolitan capitalism into independent economic development.
(ii) State capitalist and capitalist developments in the urban sector and
(iii) Transition from semi-feudal agricultural to capitalist farming.

Hence, Statements 1 and 2 both are correct.

133. The provisions of the Fifth Schedule of the Constitution of India shall not apply to the administration and control of the Scheduled Areas and Scheduled Tribes in
1. Chhattisgarh, Jharkhand and Odisha
2. Assam and Tripura
3. Meghalaya and Mizoram

Select the correct answer using the codes given below.
(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

(***c***) The provisions of the Fifth Schedule of the Constitution of India apply to the administration and control of the Scheduled Areas and Scheduled Tribes in 10 states namely: Andhra Pradesh, Chhattisgarh, Gujarat, Himachal Pradesh, Jharkhand, Madhya Pradesh, Maharashtra, Odisha, Rajasthan and Telangana.

134. Consider the following statements related to Dayanand's idea of social reform
1. Dayanand's reform envisaged a social order, where different castes performed functions suitable to their status determined by merit.
2. Dayanand's 'robust Vedic counterpart' challenged the masculine West that had enslaved the Aryavarta.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***c***) Both the statements are correct as Dayanand was critical of the ancient caste system. He criticised it to its length and breadth. He envisioned a homogenous society where the profession/function performed by the person was determinant of their merit. Also, Dayanand's 'robust Vedic counterpart' challenged the masculine West that had enslaved the Aryavarta.

135. Consider the following statements concerning the American War of Independence.
1. The Americans raised the slogan, 'No taxation without representation'.
2. The American merchants resorted to the boycott of the British products.

Select the correct answer using the codes given below.
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***c***) During the American War of Independence, in 1774, the first Continental Congress was held in Philadelphia which actively advocated three agendas—
1. Rejection of Union of Colonies under British authority.
2. Scheme of boycott of trade with England and Britain.
3. No taxation without representation and redressal of grievances.

So, both statements are correct. Hence, option (c) is correct.

136. The first Utkrisht Double Decker Air-Conditioned Yatri (UDAY) Express train runs between
(a) Delhi and Kalka
(b) Bengaluru and Coimbatore
(c) Chennai and Visakhapatnam
(d) Chennai and Bengaluru

(***b***) The first Utkrisht Double Decker Air-Conditioned Yatri (UDAY) Express was a specially designed trains which started service between Coimbatore and Bengaluru in June, 2018. Second, such service was launched in September, 2019 that runs between Visakhapatnam and Vijayawada.

137. Who among the following were selected for the Rajiv Gandhi Khel Ratna Award, 2019?
(a) Vijay Kumar and Yogeshwar Dutt
(b) Sakshi Malik and Jitu Rai
(c) Virat Kohli and Mirabai Chanu
(d) Bajrang Punia and Deepa Malik

(***d***) Rajiv Gandhi Khel Ratna Award is the highest sporting honour of India. It is annually awarded to players by the Ministry of Youth Affairs and Sports. Bajrang Punia (Wrestling) and Deepa Malik (Parathlete) were awarded with this award in 2019.

138. Betel leaf of which one of the following states has been accorded a GI tag recently?

(a) Andhra Pradesh (b) Madhya Pradesh
(c) Kerala (d) Mizoram

(*c*) The Tirur Vettila (Betel leaf) from Kerala has got its GI tag. The leaf given its unique component i.e., Eugenol in leaves along with high protein and chlorophyll content has several medicinal properties. Apart from this Arnamula Kannadi, Payyannur Pavithra Ring, Navara Rice, Alleppey Coir also gets GI tag in Kerala.

139. SAMARTH is a flagship skill development scheme of which one of the following Ministries?

(a) Ministry of Textiles
(b) Ministry of Agriculture and Farmers Welfare
(c) Ministry of Skill Development and Entrepreneurship
(d) Ministry of Human Resources Development

(*a*) SAMARTH is a flagship skill development scheme of Ministry of Textiles, Government of India. The objective of the scheme is to provide skill development to the youth for gainful and sustainable employment in the textile sector.

140. CORAS is a separate Commando Unit of

(a) National Security Guard
(b) Central Industrial Security Force
(c) Railway Protection Force
(d) Central Reserve Police Force

(*c*) The Ministry of Railways had launched the Commando for Railway Security (CORAS) of Indian railways in August, 2019. CORAS is a separate Commando Unit of Railway Protection Force and are raised keeping in mind the threat from disruptive forces. The first deployment of the unit will be in the naxalism affected Chhattisgarh state.

141. Which one of the following statements is not true about Ijarah system?

(a) It was a system of revenue farming.
(b) It encouraged the role of middlemen.
(c) Under this system, land revenue was fixed for the peasant.
(d) The Ijaradar paid a fixed amount to the state treasury.

(c) The Ijarah system was developed during sultanate period and gained prominence in Mughals after Akbar. It was a system of revenue farming. In this system, the revenue was collected by a middlemen called Ijaradars who had to pay a certain sum to ruler and keep rest with him. This led to ruthless extration of revenue from poor peasants. The revenue was not fixed and varied upon the production. Hence, statement (c) is incorrect.

142. During the Swadeshi movement, a National College was started in Calcutta under the principalship of

(a) Rabindranath Tagore
(b) Aurobindo Ghosh
(c) Rajani Kant Sen
(d) Syed Abu Mohammad

(b) Aurobindo Ghosh founded the Bengal National College at Calcutta on 15th August, 1906 and became its first Principal. But following his trial in the cases like Bande Mataram (Sedition case), 1907 and Alipore Bomb Conspiracy Case, 1908, he resigned from the post.

143. The ruins of Vijayanagar (Hampi) were brought to public light in the year 1800 by the following British surveyor and engineer

(a) James Prinsep (b) Colin Mackenzie
(c) James Rennell (d) Charles Metcalfe

(b) Colin Mackenzie an engineer and antiquarian in East Indian Company found the ruins of Vijayanagar (Hampi) in 1800. He prepared the first survey map of Hampi. James Prinsep, deciphered Kharoshti and Brahmi scripts.

James Rennell, produce first accurate maps of Bengal and the outlines of India.

Charles Metcalfe, appointed as acting Governor-General of India in 1835.

144. Which country among the following was not a part of the Third Coalition against Napoleon?

(a) Russia (b) Prussia
(c) Sweden (d) France

(d) The third coalition was an alliance of Russia, Britain, Prussia, Austria and Sweden against the Napoleon I of France. Hence, France was a party to war, but not a coalition member.

145. Which one of the following pairs of travellers and their nationality is not correctly matched?

(a) Ibn Battuta-Moroccan
(b) Duarte Barbosa-Portuguese
(c) Jean Baptiste Tavernier-Dutch
(d) Niccolao Manucci-Venetian

(c) Option (c) is not correctly matched because Jean Baptiste Tavernier was a French gem merchant and traveller. He made 6 voyages to Persia and India in his lifetime from 1630 to 1668 and has left valuable accounts of Mughal empire, Nizams of Hyderabad, etc.

146. World Humanitarian Day is celebrated on

(a) 26th June (b) 17th July
(c) 19th August (d) 27th September

(*c*) World Humanitarian Day is an international day celebrated on 19th August every year to recognise the humanitarian personnel and those who laid down their lives for the causes of humanity. It was designated by the United Nation General Assembly for the first time in 2009 on 19th August.

147. Which one of the following is the name given to the lander of Chandrayaan-2?

(a) Vigyan (b) Anusandhan
(c) Pragyan (d) Vikram

(*d*) Chandrayaan-2 was the second lunar exploration mission of Indian Space Research Organisation (ISRO). It consisted of a lunar orbiter, lander named 'Vikram' and lunar rover called 'Pragyan'. The Mission's Lander was named after Vikram Sarabhai, who is widely regarded as the founder of Indian Space Programme.

148. Along with Pranab Mukherjee, how many more persons were conferred the Bharat Ratna Award in the year 2019?

(a) One (b) Two
(c) Three (d) Four

(*b*) In the year 2019, along with Pranab Mukherjee, two personalities were awarded Bharat Ratna, the highest civilian award of India. The awardees were:

1. Pranab Mukherjee → Former President of India.
2. Nanaji Deshmukh → Social Activist from Maharashtra.
3. Bhupen Hazarika → Assamese Composer, Musician and Lyricist.

149. Aishwarya Pissay excels in which one of the following sports?

(a) Badminton (b) Boxing
(c) Motorsports (d) Chess

(*c*) Aishwarya Pissay is an Indian circuit and off road motor cycle racer. She became the first ever Indian athlete to won a world title in motorsports.

150. Which one of the following films was adjudged as the Best Feature Film in the 66th National Film Awards?

(a) Bulbul can Sing (b) Hellaro
(c) Badhaai Ho (d) Uri

(*b*) The movie 'Hellaro' won the National Film ward for the Best Feature Film in the 66th National Film Awards. 'Hellaro' is a Gujarati language period drama, co-written and directed by Abhishek Shah.