

NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2018 (II)

PAPER I : Mathematics

1. What is the value of $\log_7 \log_7$

$\sqrt{7\sqrt{7\sqrt{7}}}$ equal to ?

- (a) $3 \log_2 7$ (b) $1 - 3 \log_2 7$
(c) $1 - 3 \log_7 2$ (d) $\frac{7}{8}$

- ⊙ (c) We have,

$$\sqrt{7\sqrt{7\sqrt{7}}} = 7^{\frac{1}{2}} \cdot 7^{\frac{1}{4}} \cdot 7^{\frac{1}{8}} = 7^{\frac{7}{8}}$$

$$\text{Now, } \log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$$

$$= \log_7 \log_7 (7)^{\frac{7}{8}}$$

$$= \log_7 \left(\frac{7}{8} \right)$$

$$= \log_7 7 - \log_7 8$$

$$[\because \log \frac{m}{n} = \log m - \log n]$$

$$= \log_7 7 - \log_7 2^3 = 1 - 3 \log_7 2$$

$$[\because \log_b a^n = n \log_b a]$$

2. If an infinite GP has the first term x and the sum 5, then which one of the following is correct?

- (a) $x < -10$
(b) $-10 < x < 0$
(c) $0 < x < 10$
(d) $x > 10$

- ⊙ (c) Given that first term of an infinity GP is x and sum = 5

$$\therefore \frac{x}{1-r} = 5$$

$$[\because \text{sum of infinity GP} = \frac{a}{1-r}]$$

$$\Rightarrow \frac{x}{5} = 1 - r$$

$$\Rightarrow r = 1 - \frac{x}{5}$$

Where, $|r| < 1$

$$\Rightarrow -1 < -\frac{x}{5} < 1$$

$$\Rightarrow -2 < -\frac{x}{5} < 0$$

$$\Rightarrow -10 < -x < 0$$

$$\Rightarrow 10 > x > 0$$

3. Consider the following expressions

$$1. x + x^2 - \frac{1}{x}$$

$$2. \sqrt{ax^2 + bx + x - c + \frac{d}{x} - \frac{e}{x^2}}$$

$$3. 3x^2 - 5x + ab$$

$$4. \frac{2}{x^2 - ax + b^3}$$

$$5. \frac{1}{x} - \frac{2}{x+5}$$

Which of the above are rational expressions?

- (a) 1, 4 and 5 (b) 1, 3, 4 and 5
(c) 2, 4 and 5 (d) 1 and 2

- ⊙ (a) We know that, rational expressions are those expression which can be write in the form of $\frac{p(x)}{q(x)}$, $q(x) \neq 0$

So, 1, 4, 5 are rational expressions

4. A square matrix A is called orthogonal if

- (a) $A = A^2$ (b) $A' = A^{-1}$
(c) $A = A^{-1}$ (d) $A = A'$

where A' is the transpose of A

- ⊙ (b) A square matrix is called an orthogonal matrix if $AA' = I$ multiply by A^{-1}

$$\therefore A^{-1}(AA') = A^{-1}I \Rightarrow IA' = A^{-1}$$

$$\Rightarrow A' = A^{-1}$$

5. If A , B and C are subsets of a universal set, then which one of the following is not correct?

- (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A' \cup (A \cap B) = (B' \cap A')' \cup A$
(c) $A' \cup (B \cap C) = (C' \cap B)' \cap A'$
(d) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

where A' is the complement of A

- ⊙ (c) Let A , B and C are subsets of a universal set.

$$\text{Let } A = \{1\}, B = \{2\}, C = \{3\}$$

$$\cup = \{1, 2, 3\}, A' = \{2, 3\}, B' = \{1, 3\},$$

$$C' = \{1, 2\}$$

by checking options, we get

$$\text{LHS} = A' \cup (B \cap C)$$

$$= \{2, 3\} \cup \{ \}$$

$$= \{2, 3\}$$

$$\text{RHS} = (C' \cap B)' \cap A'$$

$$= (\{1, 2\} \cap \{1, 3\})' \cap \{2, 3\}$$

$$= (\{2\})' \cap \{2, 3\}$$

$$= \{1, 3\} \cap \{2, 3\}$$

$$= \{3\}$$

$$\text{LHS} \neq \text{RHS}$$

So, option (c) is wrong

6. Let x be the number of integers lying between 2999 and 8001 which have at least two digits equal. Then x is equal to

- (a) 2480 (b) 2481
(c) 2482 (d) 2483

- ⊙ (b) We have, x be the number lying between 2999 and 8001

if repetition allowed

$$\text{total numbers} = 5 \times 10 \times 10 \times 10 = 5000$$

if repetition not allowed

$$\therefore \text{total numbers} = 5 \times 9 \times 8 \times 7 = 2520$$

So, x = atleast two digit repeated
 $= 5000 - 2520 + 1$
 $= 2481$
 $\therefore$ add 1 because of number 8000]

7. The sum of the series

$3 - 1 + \frac{1}{3} - \frac{1}{9} + \dots$ is equal to

- (a) $\frac{20}{9}$ (b) $\frac{9}{20}$
 (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

⊙ (c) Given series

$3 - 1 + \frac{1}{3} - \frac{1}{9} + \dots$ are in GP

$$\therefore r = \frac{-1}{3}$$

$$S_n = \frac{3}{1 - \left(-\frac{1}{3}\right)} \left[\because S_n = \frac{a}{1-r} \right]$$

$$= \frac{3}{\frac{4}{3}} = \frac{9}{4}$$

Directions (Q. Nos. 8 and 9)

Consider the information given below and answer the two items that follow.

A survey was conducted among 300 students. It was found that 125 students like to play cricket, 145 students like to play football and 90 students like to play tennis. 32 students like to play exactly two games out of the three games.

8. How many students like to play all the three games?

- (a) 14 (b) 21
 (c) 28 (d) 35

⊙ (a) Let,

A be the set of students like to play cricket
 B be the set of students like to play football.

C be the set of students like to play tennis.

We have,

$$n(A \cup B \cup C) = 300$$

$$n(A) = 125$$

$$n(B) = 145$$

$$n(C) = 90$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$$

$$\Rightarrow 300 = 125 + 145 + 90$$

$$- [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$$

$$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 60 + n(A \cap B \cap C) \quad \dots(i)$$

Again,

$$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C) = 32$$

$$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 32 + 3n(A \cap B \cap C) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$60 + n(A \cap B \cap C) = 32 + 3n(A \cap B \cap C)$$

$$\Rightarrow 2n(A \cap B \cap C) = 28$$

$$\Rightarrow n(A \cap B \cap C) = 14$$

9. How many students like to play exactly one game?

- (a) 196 (b) 228
 (c) 254 (d) 268

⊙ (c) Number of students like to play exactly one

$$\text{game} = n(A) + n(B) + n(C)$$

$$- 2[n(A \cap B) + n(B \cap C) + n(C \cap A)] + 3n(A \cap B \cap C)$$

$$= 125 + 145 + 90 - 2[32 + 3 \times 14] + 3 \times 14$$

$$= 360 - 106$$

$$= 254$$

10. If α and β ($\neq 0$) are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$, where $x \in R$ has

- (a) Least value $-\frac{1}{4}$
 (b) Least value $-\frac{9}{4}$
 (c) Greatest value $\frac{1}{4}$
 (d) Greatest value $\frac{9}{4}$

⊙ (d) α and β are the roots of quadratic equation.

$$x^2 + \alpha x - \beta = 0$$

$$\text{So, } (\alpha\beta = -\beta) \Rightarrow \alpha\beta + \beta = 0$$

$$\Rightarrow \beta(\alpha + 1) = 0$$

$$\alpha = -1 \quad [\because \beta \neq 0]$$

$$\alpha + \beta = -\alpha$$

$$\Rightarrow 2\alpha + \beta = 0$$

$$\Rightarrow \beta = 2$$

$$\therefore -x^2 + \alpha x + \beta \quad [\because \alpha = -1, \beta = 2]$$

$$= -x^2 - x + 2$$

$$\text{Greatest value} = 2 - \left[\frac{(-1)^2}{4(-1)} \right]$$

$$[\because \text{Greatest value} = c - \left(\frac{b^2}{4a} \right)]$$

$$= 2 - \left(\frac{1}{-4} \right)$$

$$= 2 + \frac{1}{4}$$

$$= \frac{9}{4}$$

11. What is the coefficient of the middle term in the binomial expansion of $(2+3x)^4$?

- (a) 6 (b) 12
 (c) 108 (d) 216

⊙ (d) We have, $(2+3x)^4$

Here, $n = 4$, so middle term is

$$\left(\frac{4}{2} + 1 \right)^{\text{th}} = 3^{\text{rd}} \text{ term}$$

$$T_3 = {}^4C_2 \times 2^2 \times (3x)^2$$

$$[T_{r+1} = {}^nC_r a^r b^{n-r}]$$

$$= \frac{4 \times 3}{2 \times 1} \times 4 \times 9x^2$$

$$T_3 = 216x^2$$

Hence, coefficient of middle term is 216.

12. For a square matrix A , which of the following properties hold?

1. $(A^{-1})^{-1} = A$

2. $\det(A^{-1}) = \frac{1}{\det A}$

3. $(\lambda A)^{-1} = \lambda A^{-1}$, where λ is a scalar

Select the correct answer using the code given below.

- (a) 1 and 2 (b) 2 and 3
 (c) 1 and 3 (d) 1, 2 and 3

⊙ (d) For a square matrix A

Statement 1

$$(A^{-1})^{-1} = A$$

Statement 1 is true

Statement 2

$$\det(A^{-1}) = \frac{1}{\det A}$$

Statement 2 is correct

Statement 3

$$(\lambda A)^{-1} = \lambda A^{-1}, \text{ where } \lambda \text{ is a scalar.}$$

So, Statement 3 is correct.

13. Which one of the following factors does the expansion of the determinant

$$\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix} \text{ contain ?}$$

- (a) $x - 3$
 (b) $x - y$
 (c) $y - 3$
 (d) $x - 3y$

⊙ (a) We have,

$$\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix}$$

$$[C_1 \rightarrow C_1 - C_3 \text{ से.}]$$

$$= \begin{vmatrix} x-3 & y & 3 \\ x^2-9 & 5y^3 & 9 \\ x^3-27 & 10y^5 & 27 \end{vmatrix}$$

$$= (x-3) \begin{vmatrix} 1 & y & 3 \\ x+3 & 5y^3 & 9 \\ x^2+9+3x & 10y^5 & 27 \end{vmatrix}$$

14. What is the adjoint of the matrix

$$\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix}?$$

(a) $\begin{pmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

(c) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

⊗ (a) We have,

$$A = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$A = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Now, $C_{11} = \cos \theta$

$C_{12} = -\sin \theta$

$C_{21} = -\sin \theta$

$C_{22} = \cos \theta$

$$\text{adj } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

15. What is the value of

$$\left(\frac{-1+i\sqrt{3}}{2} \right)^{3a} + \left(\frac{-1-i\sqrt{3}}{2} \right)^{3n}$$

where $i = \sqrt{-1}$?

- (a) 3 (b) 2
(c) 1 (d) 0

⊗ (b) We have,

$$\left(\frac{-1+i\sqrt{3}}{2} \right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2} \right)^{3n}$$

$$= (\omega)^{3n} + (\omega^2)^{3n}$$

$$\left[\because \omega = \frac{-1+i\sqrt{3}}{2}, \omega^2 = \frac{-1-i\sqrt{3}}{2} \right]$$

$$= (\omega^3)^n + (\omega^3)^n$$

$$(1)^n + (1)^n$$

$$= 1 + 1 = 2$$

$$[\because \omega^3 = 1]$$

16. There are 17 cricket players, out of which 5 players can bowl. In how many ways can a team of 11 players be selected so as to include 3 bowlers?

- (a) $C(17, 11)$
(b) $C(12, 8)$
(c) $C(17, 5) \times C(5, 3)$
(d) $C(5, 3) \times C(12, 8)$

⊗ (d) There are 17 cricket players, out of which 5 players can bowl.

$$\text{required number of ways} = {}^{12}C_8 \times {}^5C_3$$

$$= C(12, 8) \times C(5, 3)$$

17. What is the value of

$$\log_9 27 + \log_8 32?$$

- (a) $\frac{7}{2}$ (b) $\frac{19}{6}$ (c) 4 (d) 7

⊗ (b) We have,

$$\log_9 27 + \log_8 32$$

$$= \log_{3^2} 3^3 + \log_2 2^5$$

$$= \frac{3}{2} \log_3 3 + \frac{5}{3} \log_2 2$$

$$\left[\because \log_{a^m} b^n = \frac{n}{m} \log_a b \right]$$

$$= \frac{3}{2} + \frac{5}{3} = \frac{19}{6}$$

18. If A and B are two invertible square matrices of same order, then what is $(AB)^{-1}$ equal to?

- (a) $B^{-1}A^{-1}$ (b) $A^{-1}B^{-1}$
(c) $B^{-1}A$ (d) $A^{-1}B$

⊗ (a) If A and B are two invertible square matrices of same order, then

$$(AB)^{-1} = B^{-1}A^{-1}$$

19. If $a + b + c = 0$, then one of the solutions of

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0 \text{ is}$$

(a) $x = a$

(b) $x = \sqrt{\frac{3(a^2 + b^2 + c^2)}{2}}$

(c) $x = \sqrt{\frac{2(a^2 + b^2 + c^2)}{3}}$

(d) $x = 0$

⊗ (d) We have,

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\Rightarrow \begin{vmatrix} a+b+c-x & a+b+c-x & a+b+c-x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -x & -x & -x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow (-x) \begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow x = 0$$

Hence, $x = 0$ is a solution

20. What should be the value of x , so that the matrix $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$ does not

have an inverse?

- (a) 16 (b) -16
(c) 8 (d) -8

⊗ (b) Let, $A = \begin{bmatrix} 2 & 4 \\ -8 & x \end{bmatrix}$

Matrix does not have any solution if

$$|A| = 0$$

$$2x + 32 = 0$$

$$2x = -32$$

$$x = -\frac{32}{2}$$

$$x = -16$$

21. The system of equations

$$2x + y - 3z = 5$$

$$3x - 2y + 2z = 5$$

and $5x - 3y - z = 16$

- (a) is inconsistent
(b) is consistent, with a unique solution
(c) is consistent, with infinitely many solutions
(d) has its solution lying along X-axis in three-dimensional space

⊗ (b) The system of equations

$$2x + y - 3z = 5$$

$$3x - 2y + 2z = 5$$

and $5x - 3y - z = 0$

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & -2 & 2 \\ 5 & -3 & -1 \end{bmatrix}$$

$$|A| = 2[-2(-1) - 2(-3)] - 1[3(-1) - 2(5)]$$

$$+ (-3)[3(-3) - (-2)(5)]$$

$$= 2(8) - 1(-13) - 3(1)$$

$$= 16 + 13 - 3 = 26 \neq 0$$

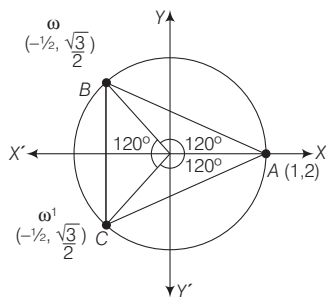
So, System is consistent with unique solution.

22. Which one of the following is correct in respect of the cube roots of unity?

- (a) They are collinear
(b) They lie on a circle of radius $\sqrt{3}$
(c) They form an equilateral triangle
(d) None of the above

⊗ (c) We know that, cube roots of unity is $1, \omega, \omega^2$, where $\omega = \frac{-1+i\sqrt{3}}{2}$ and

$$\omega^2 = \frac{-1-i\sqrt{3}}{2}$$



They form an equilateral triangle.

23. If u , v and w (all positive) are the p^{th} , q^{th} and r^{th} terms of a GP, then the determinant of the matrix
- $$\begin{vmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{vmatrix}$$

- (a) 0
(b) 1
(c) $(p-q)(q-r)(r-p)$
(d) $\ln u \times \ln v \times \ln w$

24. Given that u , v and w are the p^{th} , q^{th} and r^{th} term of GP
 $\therefore u = aR^{p-1}$, $v = aR^{q-1}$ [$\because a_n = aR^{n-1}$]
 and $w = aR^{r-1}$

We have, $\begin{vmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{vmatrix}$

$$= \begin{vmatrix} \ln a R^{p-1} & p & 1 \\ \ln a R^{q-1} & q & 1 \\ \ln a R^{r-1} & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \ln a + p-1 \ln R & p & 1 \\ \ln a + q-1 \ln R & q & 1 \\ \ln a + r-1 \ln R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \ln a & p & 1 \\ \ln a & q & 1 \\ \ln a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1) \ln R & p & 1 \\ (q-1) \ln R & q & 1 \\ (r-1) \ln R & r & 1 \end{vmatrix}$$

$$= \ln a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \ln R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_3$$

$$= 0 + \ln R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$$

$$= 0$$

24. Let the coefficient of the middle term of the binomial expansion of $(1+x)^{2n}$ be α and those of two middle terms of the binomial expansion of $(1+x)^{2n-1}$ be β and γ . Which one of the following relations is correct?

- (a) $\alpha > \beta + \gamma$ (b) $\alpha < \beta + \gamma$
(c) $\alpha = \beta + \gamma$ (d) $\alpha = \beta\gamma$

25. We have, $(1+x)^{2n}$

$$\text{Middle term} = \left(\frac{2n}{2} + 1 \right)^{\text{th}} \text{ term}$$

$$= (n+1)^{\text{th}} \text{ term}$$

$$\text{Coefficient of } (n+1)^{\text{th}} \text{ term} = {}^{2n}C_n$$

$$\alpha = {}^{2n}C_n$$

Again, we have binomial expansion of $(1+x)^{2n-1}$ coefficient of middle terms are,

$$\therefore \beta = {}^{2n-1}C_n$$

$$\text{and } \gamma = {}^{2n-1}C_{n-1}$$

$$\text{Now, } \beta + \gamma = {}^{2n-1}C_n + {}^{2n-1}C_{n-1}$$

$$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

$$= {}^{2n}C_n$$

25. Let $A = [x \in \mathbf{R} : -1 \leq x \leq 1]$,

$B = [y \in \mathbf{R} : -1 \leq y \leq 1]$ and S be the subset of $A \times B$, defined by

$$S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}.$$

Which one of the following is correct?

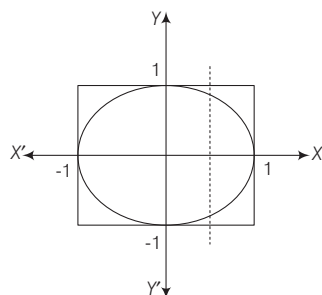
- (a) S is a one-one function from A into B
(b) S is a many-one function from A into B
(c) S is a bijective mapping from A into B
(d) S is not a function

26. Given that,

$$A = \{x \in \mathbf{R} : -1 \leq x \leq 1\},$$

$$B = \{y \in \mathbf{R} : -1 \leq y < 1\}$$

$$\text{and } S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$$



By vertical line test, when we draw a vertical line, then line cuts the circle in two points. Hence, S is not a function.

26. Let T_r be the r^{th} term of an AP for $r = 1, 2, 3, \dots$. If for some distinct positive integers m and n we have $T_m = 1/n$ and $T_n = 1/m$, then what is T_{mn} equal to?

- (a) $(mn)^{-1}$
(b) $m^{-1} + n^{-1}$
(c) 1
(d) 0

27. Let first term of an AP is a and common difference is d

Given that,

$$T_m = \frac{1}{n}$$

$$a + (m-1)d = \frac{1}{n} \quad \dots(i)$$

and

$$T_n = \frac{1}{m}$$

$$a + (n-1)d = \frac{1}{m} \quad \dots(ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(m-1)d - (n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow (m-n)d = \frac{m-n}{mn}$$

$$\Rightarrow d = \frac{1}{mn}$$

Put in Eq. (i),

$$a + (m-1) \frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow a = \frac{1}{mn}$$

$$\text{Now, } T_{mn} = a + (mn-1)d$$

$$= \frac{1}{mn} + (mn-1) \frac{1}{mn}$$

$$= \frac{1}{mn} + 1 - \frac{1}{mn}$$

$$T_{mn} = 1$$

27. Suppose $f(x)$ is such a quadratic expression that it is positive for all real x .

If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x

- (a) $g(x) < 0$ (b) $g(x) > 0$
(c) $g(x) = 0$ (d) $g(x) \geq 0$

28. Given that $f(x)$ is a quadratic expression

$$\text{Let } f(x) = ax^2 + bx + c, a > 0$$

$$\therefore b^2 - 4ac < 0 \quad [\because f(x) > 0]$$

$$\Rightarrow b^2 < 4ac$$

$$\text{Now, } f'(x) = 2ax + b$$

$$\text{and } f''(x) = 2a$$

We have,

$$g(x) = f(x) + f'(x) + f''(x)$$

$$= ax^2 + bx + c + 2ax + b + 2a$$

$$= ax^2 + (b+2a)x + 2a + b + c$$

$$\text{Now, } (b+2a)^2 - 4a(2a+b+c)$$

$$= b^2 + 4ab + 4a^2 - 8a^2 - 4ab - 4ac$$

$$= b^2 - 4ac - 4a^2 < 0$$

$$[\because b^2 - 4ac < 0]$$

$$\Rightarrow g(x) > 0$$

28. Consider the following in respect of matrices A, B and C of same order.

1. $(A + B + C)' = A' + B' + C'$
2. $(AB)' = A' B'$
3. $(ABC)' = C' B' A'$

Where A' is the transpose of the matrix A . Which of the above are correct?

- (a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

- ⊙ (c) Given that A, B and C are matrices of same order

Statement 1

$$(A' + B' + C)' = A' + B' + C'$$

$$[\because (A + B)' = A' + B']$$

So, Statement 1 is correct

Statement 2

We know that,

$$(AB)' = B' A'$$

Hence, Statement 2 is incorrect

Statement 3

$$(ABC)' = C' B' A' [\because (AB)' = B' A']$$

Hence, Statement 3 is correct.

29. The sum of the binary numbers $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is the binary numbers $(101101101)_2$. What are the values of x and y ?

- (a) $x = 1, y = 1$ (b) $x = 1, y = 0$
(c) $x = 0, y = 1$ (d) $x = 0, y = 0$

- ⊙ (b) Sum of the binary number $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is $(101101101)_2$

So, (101101101)

$$\begin{array}{r} 101101101 \\ - 10110110 \\ \hline 10110111 \\ - 11011 \\ \hline 10011100 \end{array}$$

Compare with $(10011x0y)_2$

We get, $x = 1$ and $y = 0$

30. Let matrix B be the adjoint of a square matrix A , I be the identity matrix of same order as A . If $k (\neq 0)$ is the determinant of the matrix A , then what is AB equal to?

- (a) I (b) kI (c) $k^2 I$ (d) $(1/k)I$

- ⊙ (b) Given,

$$B = \text{adj}A, I = \text{identity Matrix}$$

$$(A) = K$$

$$\therefore AB = A(\text{adj}A) = (A)I = kI$$

31. If $(0.2)^x = 2$ and $\log_{10} 2 = 0.3010$, then what is the value of x to the nearest tenth?

- (a) -10.0 (b) -0.5
(c) -0.4 (d) -0.2

- ⊙ (c) We have,

$$(0.2)^x = 2$$

taking $\log_{10}$ both side

$$x \log_{10} 0.2 = \log_{10} 2$$

$$\Rightarrow x \log_{10} \left(\frac{2}{10} \right) = \log_{10} 2$$

$$\Rightarrow x [\log_{10} 2 - \log_{10} 10] = \log_{10} 2$$

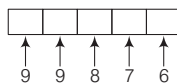
$$\Rightarrow x [0.3010 - 1] = 0.3010 \quad [\because \log_a a = 1]$$

$$\Rightarrow x = -\frac{0.3010}{0.6990} \approx -0.43$$

32. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is

- (a) 45360 (b) 30240
(c) 27216 (d) 15120

- ⊙ (c) 5-digit number that can be composed by distinct digits from 0 to 9 is given as



$$\text{required number} = 9 \times 9 \times 8 \times 7 \times 6 = 27216$$

33. What is the determinant of the matrix

$$\begin{pmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{pmatrix} ?$$

- (a) $(x-y)(y-z)(z-x)$
(b) $(x-y)(y-z)$
(c) $(y-z)(z-x)$
(d) $(z-x)^2(x+y+z)$

- ⊙ (d) We have,

$$\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 2 \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - 2C_1$$

$$= (x+y+z) \begin{vmatrix} 1 & 0 & 0 \\ z & x-z & x-z \\ y & z-y & x-y \end{vmatrix}$$

$$= (x+y+z) [1(x-z)(x-y) - (x-z)(z-y)]$$

$$= (x+y+z) [(x-z)(x-y-z+y)]$$

$$= (x+y+z)(x-z)(x-z)$$

$$= (x+y+z)(z-x)^2$$

34. If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ I + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0,$$

then which one of the following is correct?

- (a) The triangle ABC is isosceles
(b) The triangle ABC is equilateral
(c) The triangle ABC is scalene
(d) No conclusion can be drawn with regard to the nature of the triangle

- ⊙ (a) We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin^2 A - 1 & \sin^2 B - 1 & \sin^2 C - 1 \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 + R_1$$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1 & 1 & 1 \\ -\cos^2 A & -\cos^2 B & -\cos^2 C \end{vmatrix} = 0$$

$$R_3 \rightarrow R_3 + R_2$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ 1 - \cos^2 A & 1 - \cos^2 B & 1 - \cos^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0$$

$$[C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3]$$

$$\Rightarrow \begin{vmatrix} \sin A - \sin B & \sin B - \sin C & \sin C \\ 0 & 0 & 1 \\ \sin^2 A - \sin^2 B & \sin^2 B - \sin^2 C & \sin^2 C \end{vmatrix}$$

$$= 0$$

$$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)$$

$$\begin{vmatrix} 1 & 1 & \sin C \\ 0 & 0 & 0 \\ \sin A + \sin C & \sin B + \sin C & \sin^2 C \end{vmatrix}$$

$$\therefore \sin A - \sin B = 0$$

$$\text{or } \sin B - \sin C = 0$$

$$\Rightarrow \sin A = \sin B$$

$$\text{and } \sin B = \sin C$$

$$\Rightarrow A = B$$

$$\text{and } B = C$$

So, ABC is an isosceles triangle

35. Consider the following in respect of matrices A and B of same order

$$1. A^2 - B^2 = (A + B)(A - B)$$

$$2. (A - I)(I + A) = O \Leftrightarrow A^2 = I$$

where I is the identity matrix and O is the null matrix.

Which of the above is/are correct?

- (a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

➤ (b)

Statement 1

$$\begin{aligned} \text{RHS} &= (A + B)(A - B) \\ &= A^2 - AB + BA - B^2 \quad [\because AB \neq BA] \\ &\neq A^2 - B^2 \end{aligned}$$

Statement 1 is not correct

Statement 2

$$\begin{aligned} (A - I)(I + A) &= 0 \\ \Rightarrow A I + A^2 - I^2 - I A &= 0 \quad [\because A I = I A = A] \\ \Rightarrow A + A^2 - I^2 - A &= 0 \\ \Rightarrow A^2 &= I \quad [\because I^2 = I] \end{aligned}$$

Statement 2 is correct

36. What is $\frac{2 \tan \theta}{1 + \tan^2 \theta}$ equal to ?

- (a) $\cos 2\theta$ (b) $\tan 2\theta$
(c) $\sin 2\theta$ (d) $\csc 2\theta$

➤ (c) We have,

$$\begin{aligned} \frac{2 \tan \theta}{1 + \tan^2 \theta} &= \frac{2 \sin \theta}{\cos \theta} \cdot \frac{1}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= 2 \sin \theta \cos \theta = \sin 2\theta \end{aligned}$$

37. If $\sec(\theta - \alpha)$, $\sec \theta$ and $\sec(\theta + \alpha)$ are in AP, where $\cos \alpha \neq 1$, then what is the value of $\sin^2 \theta + \cos \alpha$?

- (a) 0 (b) 1
(c) -1 (d) $\frac{1}{2}$

➤ (a) If $\sec(\theta - \alpha)$, $\sec \theta$ and $\sec(\theta + \alpha)$ are in AP, then

$$\begin{aligned} 2 \sec \theta &= \sec(\theta - \alpha) + \sec(\theta + \alpha) \\ \Rightarrow \frac{2}{\cos \theta} &= \frac{1}{\cos(\theta - \alpha)} + \frac{1}{\cos(\theta + \alpha)} \\ \Rightarrow \frac{2}{\cos \theta} &= \frac{\cos(\theta + \alpha) + \cos(\theta - \alpha)}{\cos(\theta - \alpha) \cos(\theta + \alpha)} \\ \Rightarrow \frac{2 \cos\left(\frac{\theta + \alpha + \theta - \alpha}{2}\right) \cos\left(\frac{\theta + \alpha - \theta - \alpha}{2}\right)}{\cos^2 \theta - \sin^2 \alpha} \end{aligned}$$

$$\left[\because \cos C + \cos D = 2 \cos\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right) \right]$$

[and $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$]

$$\begin{aligned} \Rightarrow \frac{2}{\cos \theta} &= \frac{2 \cos \theta \cos \alpha}{\cos^2 \theta - \sin^2 \alpha} \\ \Rightarrow \cos^2 \theta \cos \alpha &= \cos^2 \theta - \sin^2 \alpha \\ \Rightarrow \sin^2 \alpha &= \cos^2 \theta - \cos^2 \theta \cos \alpha \\ \Rightarrow \sin^2 \alpha &= \cos^2 \theta (1 - \cos \alpha) \\ \Rightarrow \cos^2 \theta &= \frac{\sin^2 \alpha}{1 - \cos \alpha} \\ &= \frac{1 - \cos^2 \alpha}{1 - \cos \alpha} = \frac{(1 + \cos \alpha)(1 - \cos \alpha)}{1 - \cos \alpha} \\ \Rightarrow \cos^2 \theta &= 1 + \cos \alpha \\ \Rightarrow 1 - \sin^2 \theta &= 1 + \cos \alpha \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\ \Rightarrow 1 - \sin^2 \theta &= 1 + \cos \alpha \\ \Rightarrow \sin^2 \theta + \cos \alpha &= 0 \end{aligned}$$

38. If $A + B + C = 180^\circ$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to?

- (a) $-4 \sin A \sin B \sin C$
(b) $-4 \cos A \sin B \cos C$
(c) $-4 \cos A \cos B \sin C$
(d) $-4 \sin A \cos B \cos C$

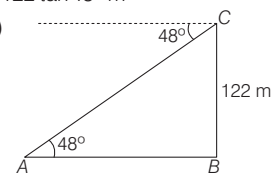
➤ (d) We have,

$$\begin{aligned} \sin 2A - \sin 2B - \sin 2C &= \sin 2A - [\sin 2B + \sin 2C] \\ &= \sin 2A - \left[2 \sin\left(\frac{2B + 2C}{2}\right) \cos\left(\frac{2B - 2C}{2}\right) \right] \\ &= 2 \sin A \cos A - [2 \sin(B + C) \cos(B - C)] \\ &= 2 \sin A \cos A - [2 \sin(180^\circ - A) \cos(B - C)] \\ &= 2 \sin A \cos A - [2 \sin(180^\circ - A) \cos(B - C)] \\ &= 2 \sin A \cos A - 2 \sin A \cos(B - C) \quad [\because \sin(180^\circ - \theta) = \sin \theta] \\ &= 2 \sin A [\cos A - \cos(B - C)] \\ &= 2 \sin A [\cos(180^\circ - (B + C)) - \cos(B - C)] \\ &= 2 \sin A [-\cos(B + C) - \cos(B - C)] \\ &= -2 \sin A [\cos(B + C) + \cos(B - C)] \\ &= -2 \sin A [2 \cos B \cos C] \quad [\because \cos C + \cos D = 2 \cos\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right)] \\ &= -4 \sin A \cos B \cos C \end{aligned}$$

39. A balloon is directly above one end of a bridge. The angle of depression of the other end of the bridge from the balloon is 48° . If the height of the balloon above the bridge is 122 m, then what is the length of the bridge?

- (a) $122 \sin 48^\circ$ m
(b) $122 \tan 42^\circ$ m
(c) $122 \cos 48^\circ$ m
(d) $122 \tan 48^\circ$ m

➤ (b)



$$\begin{aligned} \tan 48^\circ &= \frac{BC}{AB} \\ \tan 48^\circ &= \frac{122}{AB} \\ AB &= \frac{122}{\tan 48^\circ} = 122 \cot 48^\circ \\ &= 122 \cot(90^\circ - 42^\circ) \\ &= 122 \tan 42^\circ \end{aligned}$$

40. A is an angle in the fourth quadrant it satisfies the trigonometric equation $3(3 - \tan^2 A - \cot A)^2 = 1$. Which one of the following is a value of A ?

- (a) 300° (b) 315° (c) 330° (d) 345°

➤ (a) We have,

$$3(3 - \tan^2 A - \cot A)^2 = 1$$

Checking through options

$$A = 300$$

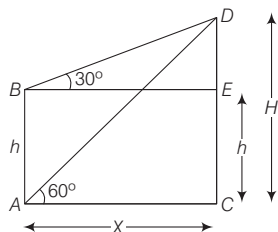
$$\begin{aligned} \text{So, } 3[3 - \tan^2 300^\circ - \cot(300^\circ)]^2 &= 3[3 - \tan^2(360^\circ - 60^\circ) - \cot(360^\circ - 60^\circ)]^2 \\ &= 3[3 - \tan^2 60^\circ + \cot 60^\circ]^2 \\ &= 3\left[3 - 3 + \frac{1}{\sqrt{3}}\right]^2 = 3 \times \frac{1}{3} = 1 \end{aligned}$$

So, option (a) is correct

41. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\frac{\pi}{6}$ and $\frac{\pi}{3}$ respectively. What is the height of the hill?

- (a) $2h$ (b) $\frac{3h}{2}$
(c) h (d) $\frac{h}{2}$

- ② (b) height of building be h and let height of hill is H



In $\triangle ACD$

$$\tan 60^\circ = \frac{H}{x}$$

$$\Rightarrow \sqrt{3} = \frac{H}{x}$$

$$\Rightarrow x = \frac{H}{\sqrt{3}} \quad \dots(i)$$

In $\triangle BDE$

$$\tan 30^\circ = \frac{H-h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H-h}{x}$$

$$\Rightarrow x = \sqrt{3}(H-h)$$

$$\Rightarrow \frac{H}{\sqrt{3}} = \sqrt{3}(H-h)$$

$$\Rightarrow H = 3(H-h)$$

$$\Rightarrow H = 3H - 3h$$

$$\Rightarrow 2H = 3h$$

$$\Rightarrow H = \frac{3}{2}h$$

42. What is/are the solution (s) of the trigonometric equation $\operatorname{cosec} x + \cot x = \sqrt{3}$ where

$$0 < x < 2\pi$$

(a) $\frac{5\pi}{3}$ only (b) $\frac{\pi}{3}$ only

(c) π only (d) $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

- ② (b) We have,

$$\operatorname{cosec} x + \cot x = \sqrt{3} \quad \dots(i)$$

Now,

$$\Rightarrow \frac{(\operatorname{cosec} x + \cot x)(\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x} = \sqrt{3}$$

$$\Rightarrow \frac{\operatorname{cosec}^2 x - \cot^2 x}{\operatorname{cosec} x - \cot x} = \sqrt{3}$$

$$\Rightarrow \frac{1}{\operatorname{cosec} x - \cot x} = \sqrt{3}$$

$$\Rightarrow \operatorname{cosec} x - \cot x = \frac{1}{\sqrt{3}} \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we get

$$2 \operatorname{cosec} x = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$\Rightarrow 2 \operatorname{cosec} x = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \operatorname{cosec} x = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$$

$$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$$

43. If $\theta = \frac{\pi}{8}$, then what is the value of

$$(2 \cos \theta + 1)^{10} (2 \cos 2\theta - 1)^{10} (2 \cos \theta - 1)^{10}$$

$$(2 \cos 4\theta - 1)^{10} ?$$

(a) 0 (b) 1
(c) 2 (d) 4

- ② (b) If $\theta = \frac{\pi}{8}$, then

$$(2 \cos \theta + 1)^{10} (2 \cos 2\theta - 1)^{10}$$

$$(2 \cos \theta - 1)^{10} (2 \cos 4\theta - 1)^{10}$$

$$\Rightarrow (2 \cos \theta + 1)^{10} (2 \cos \theta - 1)^{10}$$

$$(2 \cos 2\theta - 1)^{10} (2 \cos 4\theta - 1)^{10}$$

$$\Rightarrow (4 \cos^2 \theta - 1)^{10} (2 \cos 2\theta - 1)^{10}$$

$$(2 \cos 4\theta - 1)^{10}$$

$$\Rightarrow [2(2 \cos^2 \theta) - 1]^{10} (2 \cos 2\theta - 1)^{10}$$

$$(2 \cos 4\theta - 1)^{10}$$

$$\Rightarrow [2(1 + \cos 2\theta) - 1]^{10} (2 \cos 2\theta - 1)^{10}$$

$$(2 \cos 4\theta - 1)^{10}$$

$$\Rightarrow (2 \cos 2\theta + 1)^{10} (2 \cos 2\theta - 1)^{10}$$

$$(2 \cos 4\theta - 1)^{10}$$

$$\Rightarrow \left[2 \cos \frac{\pi}{4} + 1\right]^{10} \left(2 \cos \frac{\pi}{4} - 1\right)^{10}$$

$$\left(2 \cos \frac{\pi}{2} - 1\right)^{10} \quad \left[\because \theta = \frac{\pi}{8}\right]$$

$$\Rightarrow$$

$$\left[2 \times \frac{1}{\sqrt{2}} + 1\right]^{10} \left(2 \times \frac{1}{\sqrt{2}} - 1\right)^{10} (0 - 1)^{10}$$

$$\Rightarrow (\sqrt{2} + 1)^{10} (\sqrt{2} - 1)^{10} \times 1$$

$$\Rightarrow ((\sqrt{2})^2 - (1)^2)^{10} \times 1$$

$$\Rightarrow (2 - 1)^{10}$$

$$\Rightarrow 1$$

44. If $\cos \alpha$ and $\cos \beta$ ($0 < \alpha < \beta < \pi$) are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec \alpha \times \sec \beta$?

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

- ② (a) Given,

$$4x^2 - 3 = 0$$

$$\Rightarrow 4x^2 = 3$$

$$\Rightarrow x^2 = \frac{3}{4}$$

$$\Rightarrow x = \pm \frac{\sqrt{3}}{2}$$

So, $\cos \alpha = \frac{\sqrt{3}}{2}$

and $\cos \beta = \frac{-\sqrt{3}}{2}$

$$\text{Now, } \sec \alpha \cdot \sec \beta = \frac{2}{\sqrt{3}} \times \left(\frac{-2}{\sqrt{3}}\right) = \frac{-4}{3}$$

45. Consider the following values of x :

1. 8 2. -4

3. $\frac{1}{6}$ 4. $-\frac{1}{4}$

Which of the above values of x is/are the solution (s) of the equation

$$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$$

(a) 3 only (b) 2 and 3
(c) 1 and 4 (d) 4 only

- ② (a) We have,

$$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{2x + 3x}{1 - 2x \cdot 3x}\right) = \frac{\pi}{4}$$

$$[\because \tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right), ab \leq 1]$$

and $\tan^{-1} a + \tan^{-1} b = \pi + \tan^{-1}$

$$\left(\frac{a+b}{1-ab}\right), ab > 1$$

$$\Rightarrow \frac{5x}{1 - 6x^2} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1 - 6x^2} = 1$$

$$\Rightarrow 1 - 6x^2 = 5x$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow 6x^2 + 6x - x - 1 = 0$$

$$\Rightarrow (6x - 1)(x + 1) = 0$$

$$\Rightarrow x = -1, \frac{1}{6}$$

$$x = -1 \text{ is not possible}$$

$$\text{So, } x = \frac{1}{6}$$

46. If the second term of a GP is 2 and the sum of its infinite terms is 8, then the GP is

(a) 8, 2, $\frac{1}{2}$, $\frac{1}{8}$, ...

(b) 10, 2, $\frac{2}{5}$, $\frac{2}{25}$, ...

(c) 4, 2, 1, $\frac{1}{2}$, $\frac{1}{2^2}$, ...

(d) 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, ...

- ② (c) Let first term of a GP is a and common ratio is r

$$\therefore a_2 = 2$$

$$\Rightarrow ar = 2 \quad \dots(i)$$

$$\text{and } S_n = 8$$

$$\Rightarrow \frac{a}{1-r} = 8$$

From Eq. (i)

$$\Rightarrow \frac{2}{r(1-r)} = 8$$

$$\Rightarrow 8r - 8r^2 = 2$$

$$\Rightarrow 4r^2 - 4r + 1 = 0$$

$$\Rightarrow 4r^2 - 2r - 2r + 1 = 0$$

$$\Rightarrow 2r(2r-1) - 1(2r-1) = 0$$

$$\Rightarrow (2r-1)(2r-1) = 0$$

$$\Rightarrow r = \frac{1}{2}$$

$\therefore$ Put in Eq. (i)

$$a\left(\frac{1}{2}\right) = 2$$

$$\Rightarrow a = 4$$

$$\therefore \text{GP is } 4, 2, 1, \frac{1}{2} \dots$$

47. If a, b, c are in AP or GP or HP, then

$\frac{a-b}{b-c}$ is equal to

- (a) $\frac{b}{a}$ or 1 or $\frac{b}{c}$ (b) $\frac{c}{a}$ or $\frac{c}{b}$ or 1
(c) 1 or $\frac{a}{b}$ or $\frac{a}{c}$ (d) 1 or $\frac{a}{b}$ or $\frac{c}{a}$

⊙ (c) a, b, c are in AP, then

$$\therefore a - b = b - c$$

$$\Rightarrow \frac{a-b}{b-c} = 1$$

If a, b, c are in GP, then

$$\therefore \frac{a}{b} = \frac{b}{c} = \frac{a-b}{b-c}$$

If a, b, c are in HP, then

$$\therefore b = \frac{2ac}{a+c}$$

$$\Rightarrow ab + bc = 2ac$$

$$\Rightarrow ab - ac = ac - bc$$

$$\Rightarrow a(b-c) = c(a-b)$$

$$\Rightarrow \frac{a-b}{b-c} = \frac{a}{c}$$

48. What is the sum of all three digit numbers that can be formed using all the digits 3, 4 and 5, when repetition of digits is not allowed?

- (a) 2664 (b) 3382
(c) 4044 (d) 4444

⊙ (a) Three digit number. that can be formed using 3, 4 and 5 when repetition not allowed are 543, 534, 453, 435, 354, 345

Sum

$$= 543 + 534 + 453 + 435 + 354 + 345$$

$$= 2664$$

49. The ratio of roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are equal. If D_1 and D_2 are respective discriminants, then what is $\frac{D_1}{D_2}$ equal

to ?

- (a) $\frac{a^2}{p^2}$ (b) $\frac{b^2}{q^2}$
(c) $\frac{c^2}{r^2}$ (d) None of these

⊙ (b) Let α, β are the roots of quadratic equation.

$$ax^2 + bx + c = 0$$

$$\text{So, } \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

and let γ, δ are the roots of quadratic equation

$$px^2 + qx + r = 0$$

$$\gamma + \delta = -\frac{q}{p}$$

$$\text{and } \gamma\delta = \frac{r}{p}$$

According to the question,

$$\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$$

according to componendo and dividendo

$$\frac{\alpha + \beta}{\alpha - \beta} = \frac{\gamma + \delta}{\gamma - \delta}$$

$$\Rightarrow \frac{\alpha + \beta}{\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}} = \frac{\gamma + \delta}{\sqrt{(\gamma + \delta)^2 - 4\gamma\delta}}$$

$$\Rightarrow \frac{-\frac{b}{a}}{\sqrt{\left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)}} = \frac{-\frac{q}{p}}{\sqrt{\left(-\frac{q}{p}\right)^2 - 4\left(\frac{r}{p}\right)}}$$

$$\Rightarrow \frac{\frac{b}{a}}{\sqrt{\frac{b^2}{a^2} - 4\frac{ac}{a^2}}} = \frac{\frac{q}{p}}{\sqrt{\frac{q^2}{p^2} - 4\frac{pr}{p^2}}}$$

$$\Rightarrow \frac{b}{\sqrt{b^2 - 4ac}} = \frac{q}{\sqrt{q^2 - 4pr}}$$

$$\Rightarrow \frac{b}{\sqrt{D_1}} = \frac{q}{\sqrt{D_2}}$$

$$\Rightarrow \frac{\sqrt{D_1}}{\sqrt{D_2}} = \frac{b}{q}$$

$$\Rightarrow \frac{D_1}{D_2} = \frac{b^2}{q^2}$$

50. If $A = \sin^2 \theta + \cos^4 \theta$, then for all real θ , which one of the following is correct?

- (a) $1 \leq A \leq 2$ (b) $\frac{3}{4} \leq A \leq 1$
(c) $\frac{13}{16} \leq A \leq 1$ (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

⊙ (b) We have

$$A = \sin^2 \theta + \cos^4 \theta$$

$$= \sin^2 \theta + (1 - \sin^2 \theta)^2$$

$$\text{Let } \sin^2 \theta = x$$

$$= x^2 - x + 1, 0 \leq x \leq 1$$

$$= x^2 - x + 1$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\text{Now, } A(0) = A(1) = 1$$

$$\text{So, } \frac{3}{4} \leq A \leq 1$$

51. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is

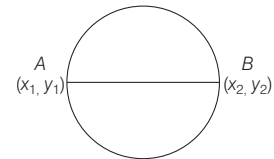
$$(a) (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$$

$$(b) (x - x_1)^2 + (y - y_1)^2 = x_2 y_2$$

$$(c) x^2 + y^2 + 2x_1 x_2 + 2y_1 y_2 = 0$$

$$(d) (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

⊙ (d) Equation of circle if end points of diameter are (x_1, y_1) and (x_2, y_2) are



$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

52. The second degree equation

$$x^2 + 4y^2 - 2x - 4y + 2 = 0$$

represents

- (a) A point
(b) An ellipse of semi-major axis 1
(c) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$
(d) None of the above

⊙ (d) Given that,

$$x^2 + 4y^2 - 2x - 4y + 2 = 0$$

Compare with

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

Then,

$$a = 1, b = 4, h = 0, g = -1, f = -2$$

and $c = 2$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 1 \times 4 \times 2 + 2 \times (-2) \times (-1) \times 0$$

$$- 1 \times (-2)^2 - 4 \times (-1)^2 - 2(0)$$

$$= 8 + 0 - 4 - 4 = 0$$

$$= 8 - 8$$

$$= 0$$

Equation, represent a pair of straight line.

- 53.** The angle between the two lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ is given by $\tan^{-1}\theta$. What is θ equal to?

(a) $\left| \frac{lm' - l'm}{ll' - mm'} \right|$ (b) $\left| \frac{lm' + l'm}{ll' + mm'} \right|$
 (c) $\left| \frac{lm' - l'm}{ll' + mm'} \right|$ (d) $\left| \frac{lm' + l'm}{ll' - mm'} \right|$

- ⊙ (c) Given straight lines,

$$lx + my + n = 0 \quad \dots\dots(i)$$

$$m_1 = -\frac{l}{m}$$

and $l'x + m'y + n' = 0$

$$m_2 = -\frac{l'}{m'}$$

angle between both lines is θ

$$\therefore \tan \theta = \left| 1 + \frac{-\frac{l}{m} - \frac{l'}{m'}}{\frac{ll' - mm'}{mm'}} \right|$$

$$\left[\because \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \right]$$

$$= \left| \frac{-\frac{lm' - l'm}{mm'}}{\frac{ll' - mm'}{mm'}} \right| = \left| \frac{lm' - l'm}{ll' - mm'} \right|$$

- 54.** Consider the following statements

- The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$
- The distance between the lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$
- The distance between the lines $x = c_1$ and $x = c_2$ is $|c_1 - c_2|$.

Which of the above statements are correct?

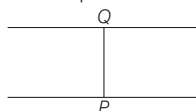
- (a) 1 and 2 (b) 2 and 3
 (c) 1 and 3 (d) 1, 2 and 3

- ⊙ (b) **Statement 1** Given lines,

$$y = mx + c_1$$

and $y = mx + c_2$

both lines are parallel



$$PQ = \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$$

So, Statement 1 is correct

Statement 2 for $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ both lines are parallel so,

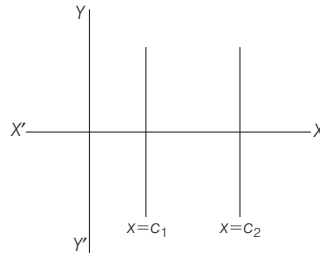
$$\therefore D = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

Statement 3 $x = c_1$

$$x = c_2$$

$$D = |c_1 - c_2|$$

Statement 3 is correct



- 55.** What is the equation of straight line passing through the point of intersection of the lines $\frac{x}{2} + \frac{y}{3} = 1$

and $\frac{x}{3} + \frac{y}{2} = 1$ and parallel to the line

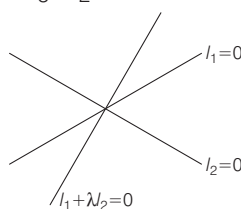
$$4x + 5y - 6 = 0?$$

- (a) $20x + 25y - 54 = 0$
 (b) $25x + 20y - 54 = 0$
 (c) $4x + 5y - 54 = 0$
 (d) $4x + 5y - 45 = 0$

- ⊙ (a) Given lines,

$$\frac{x}{2} + \frac{y}{3} = 1$$

and $\frac{x}{3} + \frac{y}{2} = 1$



Equation of line passing through the point of intersection are $l_1 + \lambda l_2 = 0$

$$(3x + 2y - 6) + \lambda (2x + 3y - 6) = 0 \dots(i)$$

Slope of line

$$\therefore \frac{-(3 + 2\lambda)}{2 + 3\lambda} = -\frac{4}{5}$$

$$\Rightarrow +15 + 10\lambda = 8 + 12\lambda$$

$$\Rightarrow 7 = 2\lambda$$

$$\Rightarrow \lambda = \frac{7}{2}$$

$$(3x + 2y - 6) + \frac{7}{2}(2x + 3y - 6) = 0$$

$$\Rightarrow 2(3x + 2y - 6) + 14x + 21y - 42 = 0$$

$$\Rightarrow 6x + 4y - 12 + 14x + 21y - 42 = 0$$

$$\Rightarrow 20x + 25y - 54 = 0$$

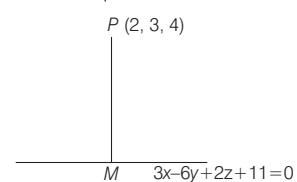
- 56.** What is the distance of the point (2, 3, 4) from the plane

$$3x - 6y + 2z + 11 = 0?$$

- (a) 1 unit (b) 2 units
 (c) 3 units (d) 4 units

- ⊙ (a) Distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is

$$d = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$



$$PM = \left| \frac{3(2) - 6(3) + 2(4) + 11}{\sqrt{(3)^2 + (-6)^2 + (2)^2}} \right|$$

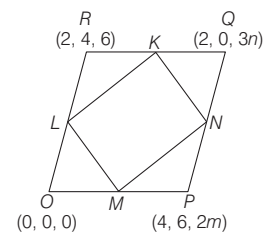
$$= \left| \frac{6 - 18 + 8 + 11}{\sqrt{9 + 36 + 4}} \right| = \left| \frac{7}{\sqrt{49}} \right|$$

$$PM = 1 \text{ unit}$$

- 57.** Coordinates of the points O, P, Q and R respectively: (0, 0, 0), (4, 6, 2m), (2, 0, 2n) and (2, 4, 6) L, M, N and K OR, OP, PQ and QR respectively such that LMNK is a parallelogram whose two adjacent sides LK and LM are each of length $\sqrt{2}$?

- (a) 6, 2
 (b) 1, 3
 (c) 3, 1
 (d) None of the above

- ⊙ (c) LMNK is a parallelogram we know that, if we join mid-point of any quadrilateral we get a parallelogram



So, M, N, K, L are mid-points of OP, PQ, QR and RO respectively

$\therefore$ Coordinate of M

$$= \left(\frac{0 + 4}{2}, \frac{0 + 6}{2}, \frac{0 + 2m}{2} \right)$$

$$= (2, 3, m)$$

[$\because$ Coordinate of mid-point

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)]$$

Coordinate of L = (1, 2, 3)

Coordinate of K = (2, 2, 3 + n)

Now, $LM = \sqrt{2}$

$$\Rightarrow \sqrt{(2-1)^2 + (3-2)^2 + (m-3)^2} = \sqrt{2}$$

$$\Rightarrow \sqrt{1+1+(m-3)^2} = \sqrt{2}$$

Squaring both side,

$$\Rightarrow 2 + (m-3)^2 = 2$$

$$\Rightarrow (m-3)^2 = 0$$

$$\Rightarrow m = 3$$

$$\text{Again, } LK = \sqrt{2}$$

$$\Rightarrow \sqrt{(2-1)^2 + (2-2)^2 + (3+n-3)^2} = \sqrt{2}$$

$$\Rightarrow \sqrt{1+0+n^2} = \sqrt{2}$$

Squaring both side,

$$\Rightarrow 1 + n^2 = 2$$

$$\Rightarrow n^2 = 1$$

$$\Rightarrow n = 1$$

58. The line $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}$ is

given by

$$(a) x + y + z = 6, x + 2y - 3z = -4$$

$$(b) x + 2y - 2x = -1, 4x + 4y - 5z - 3 = 0$$

$$(c) 3x + 2y - 3z = 0, 3x - 6y + 3z = -2$$

$$(d) 3x + 2y - 3z = -2, 3x - 6y + 3z = 0$$

⊗ (d) Given lines,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ (let)}$$

$$\therefore \frac{x-1}{2} = \lambda$$

$$\Rightarrow x = 2\lambda + 1$$

$$\frac{y-2}{3} = \lambda$$

$$\Rightarrow y = 3\lambda + 2$$

$$\text{and } \frac{z-3}{4} = \lambda$$

$$\Rightarrow z = 4\lambda + 3$$

by checking options if $3x + 2y - 3z$, then

$$3(2\lambda + 1) + 2(3\lambda + 2) - 3(4\lambda + 3)$$

$$= 6\lambda + 3 + 6\lambda + 4 - 12\lambda - 9$$

$$= -2$$

$$3(2\lambda + 1) - 6(3\lambda + 2) + 3(4\lambda + 3)$$

$$\Rightarrow 6\lambda + 3 - 18\lambda - 12 + 12\lambda + 9 = 0$$

So, option (d) is correct

59. Consider the following statements

1. The angle between the planes $2x - y + z = 1$ and $x + y + 2z = 3$

$$\text{is } \frac{\pi}{3}$$

2. The distance between the planes $6x - 3y + 6z + 2 = 0$ and

$$2x - y + 2z + 4 = 0 \text{ is } \frac{10}{9}$$

Which of the above statement is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

⊗ (c) Statements 1 Given,

$$2x - y + z = 1 \text{ and } x + y + 2z = 3$$

$$\text{Here, } a_1 = 1, b_1 = -1, c_1 = 1$$

$$\text{and } a_2 = 1, b_2 = 1, c_2 = 2$$

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 \times 1 + (-1 \times 1) + 1 \times 2}{\sqrt{2^2 + (-1)^2 + (1)^2} \sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{4-1}{\sqrt{4+1+1} \sqrt{4+1+1}} = \frac{3}{\sqrt{6} \sqrt{6}} = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

So, Statement 1 is correct.

Statement 2 Distance between two planes

$$ax + by + cz + d_1 = 0$$

$$ax + by + cz + d_2 = 0$$

$$\text{distance } S = \left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$d_1 = \frac{2}{3}, d_2 = 4$$

$$\text{distance} = \left| \frac{4 - \frac{2}{3}}{\sqrt{2^2 + 1^2 + 2^2}} \right| = \left| \frac{\frac{10}{3}}{\sqrt{9}} \right|$$

$$= \frac{10}{3 \times 3} = \frac{10}{9}$$

60. Consider the following statements :

Statement I : If the line segment joining the points $P(m, n)$ and $Q(r, s)$ subtends an angle α at the origin,

$$\text{then } \cos \alpha = \frac{ms - nr}{\sqrt{(m^2 + n^2)(r^2 + s^2)}}$$

Statement II. In any triangle ABC , it is true that $a^2 = b^2 + c^2 - 2bc \cos A$.

Which one of the following is correct in respect of the above two statements?

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of statement I.

(b) Both Statement I and Statement II are true, but Statement II is not the correct explanation of Statement I

(c) Statement I is true, but Statement II is false

(d) Statement I is false, but Statement II is true

⊗ (d) Statement I If the line segment joining the point $P(m, n)$ and $Q(r, s)$ subtends angle α at origin, then

$$\cos \alpha = \frac{mr + ns}{\sqrt{m^2 + n^2} \sqrt{r^2 + s^2}}$$

So, Statement I is not correct

Statement II

In any triangle ABC

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statement II is correct

61. What is the area of the triangle with vertices

$$\left(x_1, \frac{1}{x_1}\right), \left(x_2, \frac{1}{x_2}\right), \left(x_3, \frac{1}{x_3}\right) ?$$

$$(a) |(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)|$$

$$(b) 0$$

$$(c) \left| \frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1 x_2 x_3} \right|$$

$$(d) \left| \frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1 x_2 x_3} \right|$$

⊗ (d) Area of ΔABC

$$A \left(x_1, \frac{1}{x_1}\right), B \left(x_2, \frac{1}{x_2}\right), C \left(x_3, \frac{1}{x_3}\right)$$

$$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} x_1 & \frac{1}{x_1} & 1 \\ x_2 & \frac{1}{x_2} & 1 \\ x_3 & \frac{1}{x_3} & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} x_1 & \frac{1}{x_1} & 1 \\ x_2 & \frac{1}{x_2} & 1 \\ x_3 & \frac{1}{x_3} & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2x_1 x_2 x_3} \begin{vmatrix} x_1^2 & 1 & x_1 \\ x_2^2 & 1 & x_2 \\ x_3^2 & 1 & x_3 \end{vmatrix}$$

$$= \left| \frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1 x_2 x_3} \right|$$

62 If Y-axis touches the circle

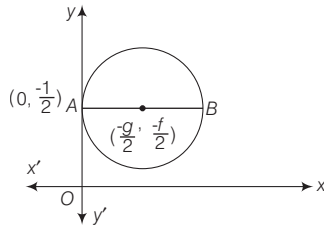
$$x^2 + y^2 + gx + fy + \frac{c}{4} = 0, \text{ then the}$$

normal at this point intersects the circle at the point.

$$(a) \left(-\frac{g}{2}, \frac{f}{2}\right) \quad (b) \left(-g, -\frac{f}{2}\right)$$

$$(c) \left(-\frac{g}{2}, f\right) \quad (d) (-g, -f)$$

⑤ (b)



The equation of circle is

$$x^2 + y^2 + gx + fy + \frac{c}{4} = 0$$

$$\therefore \text{centre} = \left(-\frac{g}{2}, -\frac{f}{2}\right) \text{ and}$$

$$\text{radius} = \sqrt{\frac{g^2}{4} + \frac{f^2}{4} - \frac{c}{4}}$$

Since, circle touches Y-axis, then

$$AC = \text{radius}$$

$$\Rightarrow \left|\frac{g}{2}\right| = \sqrt{\frac{g^2}{4} + \frac{f^2}{4} - \frac{c}{4}}$$

$$\Rightarrow \frac{f^2}{4} = \frac{c}{4}$$

$\therefore$ circle touches Y-axis at

$$y^2 + fy + \frac{f^2}{4} = 0$$

$$\Rightarrow \left(y + \frac{f}{2}\right)^2 = 0$$

$$\Rightarrow y = -\frac{f}{2}$$

$$\therefore A \left(0, -\frac{f}{2}\right)$$

$\therefore$ Normal at A will pass through centre C and intersect circle again at B.

$$\therefore \text{Coordinates of B are } \left(-g, -\frac{f}{2}\right)$$

63. Let $|\vec{a}| \neq 0, |\vec{b}| \neq 0. (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$

$$= |\vec{a}|^2 + |\vec{b}|^2 \text{ holds if and only if}$$

- (a) $\vec{a}$ and $\vec{b}$ are perpendicular
(b) $\vec{a}$ and $\vec{b}$ are parallel
(c) $\vec{a}$ and $\vec{b}$ are inclined at an angle of 45°
(d) $\vec{a}$ and $\vec{b}$ are anti-parallel

⑤ (a) Given,

$$|\vec{a}| \neq 0, |\vec{b}| \neq 0$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \perp \vec{b}$$

64. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then what is $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k})$ equal to ?

- (a) x (b) $x + y$
(c) $-(x + y + z)$ (d) $(x + y + z)$

⑤ (d) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Now, } \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) \quad \hat{i} \cdot \hat{i} = 1$$

$$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$= (x + y + z)$$

65. A unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$ is

(a) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

(b) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$

(c) $\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

(d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

⑤ (a) We know that,

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -4 & -1 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 5\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = 5\sqrt{3}$$

$$\therefore \vec{n} = \frac{5(\hat{i} + \hat{j} - \hat{k})}{5\sqrt{3}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$$

66. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} - \vec{b}| = 5$, then what is the value of $|\vec{a} + \vec{b}|$?

- (a) 8 (b) 6 (c) $5\sqrt{2}$ (d) 5

⑤ (d) Given that,

$$|\vec{a}| = 3$$

$$|\vec{b}| = 4$$

$$\text{and } |\vec{a} - \vec{b}| = 5$$

$$\therefore \vec{a} \cdot \vec{b} = 0$$

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 0 = 25$$

$$|\vec{a} + \vec{b}| = 5$$

67. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three mutually perpendicular vectors each of unit magnitude. If

$$\vec{A} = \vec{a} + \vec{b} + \vec{c}, \vec{B} = \vec{a} - \vec{b} + \vec{c} \quad \text{and}$$

$$\vec{C} = \vec{a} - \vec{b} - \vec{c}, \text{ then which one of the following is correct?}$$

(a) $|\vec{A}| > |\vec{B}| > |\vec{C}|$

(b) $|\vec{A}| = |\vec{B}| \neq |\vec{C}|$

(c) $|\vec{A}| = |\vec{B}| = |\vec{C}|$

(d) $|\vec{A}| \neq |\vec{B}| \neq |\vec{C}|$

⑤ (c) Given that,

$$\vec{A} = \vec{a} + \vec{b} + \vec{c}$$

$$\vec{B} = \vec{a} - \vec{b} + \vec{c}$$

$$\vec{C} = \vec{a} - \vec{b} - \vec{c}$$

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$|\vec{a} \cdot \vec{b}| = |\vec{b} \cdot \vec{c}| = |\vec{c} \cdot \vec{a}| = 0$$

$$\text{Now, } |\vec{A}| = \sqrt{a^2 + b^2 + c^2}$$

$$= \sqrt{1+1+1} = \sqrt{3}$$

$$|\vec{B}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$|\vec{C}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$\Rightarrow |\vec{A}| = |\vec{B}| = |\vec{C}|$$

68. What is $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$ equal to?

(a) $\vec{0}$

(b) $\vec{a} \times \vec{b}$

(c) $2(\vec{a} \times \vec{b})$

(d) $|\vec{a}|^2 - |\vec{b}|^2$

⑤ (c) We have,

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$$

$$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$$

$$= \vec{a} \times \vec{b} - \vec{b} \times \vec{a}$$

$$[\because \vec{a} \times \vec{a} = 0, \vec{b} \times \vec{b} = 0]$$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} [\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$$

$$= 2(\vec{a} \times \vec{b})$$

69. A spacecraft located at $\hat{i} + 2\hat{j} + 3\hat{k}$ is subjected to a force $\lambda\hat{k}$ by firing a rocket. The spacecraft is subjected to a moment of magnitude

(a) λ

(b) $\sqrt{3}\lambda$

(c) $\sqrt{5}\lambda$

(d) None of these

⑤ (c) We have,

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{and } \vec{F} = \lambda\hat{k}$$

$$\text{We know that,}$$

$$\text{Moment} = \vec{r} \times \vec{F}$$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (\lambda\hat{k})$$

$$= -\lambda\hat{j} + 2\lambda\hat{i} \begin{bmatrix} \therefore \hat{i} \times \hat{k} = -\hat{j} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{k} = 0 \end{bmatrix}$$

$$\text{Magnitude of moment} = \sqrt{(-\lambda)^2 + (2\lambda)^2}$$

$$= \sqrt{\lambda^2 + 4\lambda^2} = \sqrt{5\lambda^2} = \sqrt{5}\lambda$$

70. In a triangle ABC, if taken in order, consider the following statements

1. $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$

2. $\vec{AB} + \vec{BC} - \vec{CA} = \vec{0}$

3. $\vec{AB} - \vec{BC} + \vec{CA} = \vec{0}$

4. $\vec{BA} - \vec{BC} + \vec{CA} = \vec{0}$

How many of the above statement are correct?

(a) One

(b) Two

(c) Three

(d) Four

③ (a) We know that,

In a triangle ABC

$$\vec{AB} + \vec{BC} + \vec{CA} = \vec{O} \quad [\because \text{by triangle law}]$$

So, only first statement is correct.

71. Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan \theta$. Then the value of θ in the interval $(0, \pi)$ is

- (a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

③ (b) We have,

$$y = \cos^{-1}(\sin x)$$

differentiation w.r.t. x , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{-1}{\sqrt{1 - \sin^2 x}} \cos x \\ &= \frac{-\cos x}{\cos x} \quad [\because \sin^2 x + \cos^2 x = 1] \\ \frac{dy}{dx} &= -1 \end{aligned}$$

$$\frac{dy}{dx} = -1$$

Slope of the curve = $\tan \theta$

$$\therefore \tan \theta = -1$$

$$\Rightarrow \tan \theta = -\tan \frac{\pi}{4}$$

$$\Rightarrow \tan \theta = \tan \left(\pi - \frac{\pi}{4} \right)$$

$$[\because \theta \in (0, \pi)]$$

$$\Rightarrow \theta = \frac{3\pi}{4}$$

72. If $f(x) = \frac{\sqrt{x-1}}{x-4}$, defines a function

on $\mathbf{R}$, then what is its domain?

- (a) $(-\infty, 4) \cup (4, \infty)$
(b) $[4, \infty)$
(c) $(1, 4) \cup (4, \infty)$
(d) $[1, 4) \cup (4, \infty)$

③ (d) We have,

$$f(x) = \frac{\sqrt{x-1}}{x-4}$$

$$\therefore x-1 \geq 0$$

$$\text{and } x-4 \neq 0$$

$$\Rightarrow x \geq 1$$

$$x \neq 4$$

$$\text{So, } x \in [1, 4) \cup (4, \infty)$$

$$\text{Domain} = [1, 4) \cup (4, \infty)$$

73. Consider the function

$$f(x) = \begin{cases} \frac{\sin 2x}{5x}, & \text{if } x \neq 0 \\ \frac{2}{15}, & \text{if } x = 0 \end{cases}$$

Which one of the following is correct in respect of the function?

- (a) It is not continuous at $x = 0$
(b) It is continuous at every x
(c) It is not continuous at $x = \pi$
(d) It is continuous at $x = 0$

③ (a) We have,

$$f(x) = \begin{cases} \frac{\sin 2x}{5x}, & x \neq 0 \\ \frac{2}{15}, & x = 0 \end{cases}$$

at $x = 0$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\sin 2x \times 2}{5x \times 2} = \frac{2}{5}$$

$$f(0) = \frac{2}{15}$$

$$\therefore \lim_{x \rightarrow 0} f(x) \neq f(0)$$

at $x = 0$ function is discontinuous.

74. For the function $f(x) = |x-3|$, which one of the following is not correct?

- (a) The function is not continuous at $x = -3$
(b) The function is continuous at $x = 3$
(c) The function is differentiable at $x = 0$
(d) The function is differentiable at $x = -3$

③ (a) We have,

$$f(x) = |x-3|$$

We know that, modulus function is continuous in $\mathbf{R}$.

So, option (a) is incorrect.

75. If the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, then what is the value of $f(0)$?

- (a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

③ (b) We have,

$$f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$$

Function is continuous at each point

$$\therefore f(0) = \lim_{x \rightarrow 0} f(x)$$

$$\text{Now, } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{2 - \frac{1}{\sqrt{1-x^2}}}{2 + \frac{1}{1+x^2}} \\ &= \frac{2-1}{2+1} = \frac{1}{3} \end{aligned}$$

[by L-Hospital]

$$= \frac{2-1}{2+1} = \frac{1}{3}$$

$$\therefore f(0) = \frac{1}{3}$$

76. If $f(x) = \sqrt{25-x^2}$, then what is

$$\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x-1} \text{ equal to?}$$

$$(a) -\frac{1}{\sqrt{24}}$$

$$(b) \frac{1}{\sqrt{24}}$$

$$(c) -\frac{1}{4\sqrt{3}}$$

$$(d) \frac{1}{4\sqrt{3}}$$

③ (a) We have,

$$f(x) = \sqrt{25-x^2}$$

$$\text{Now, } \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{25-x^2} - \sqrt{24}}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{-2x}{2\sqrt{25-x^2}} - 0$$

$$= \lim_{x \rightarrow 1} \frac{-2x}{2\sqrt{25-x^2}} - 0 \quad [\text{by L-Hospital}]$$

$$= -\frac{1}{\sqrt{24}}$$

77. If $y = \tan^{-1} \left(\frac{5-2\tan\sqrt{x}}{2+5\tan\sqrt{x}} \right)$, then

what is $\frac{dy}{dx}$ equal to?

$$(a) -\frac{1}{2\sqrt{x}}$$

$$(b) 1$$

$$(c) -1$$

$$(d) \frac{1}{2\sqrt{x}}$$

③ (a) We have,

$$y = \tan^{-1} \left(\frac{5-2\tan\sqrt{x}}{2+5\tan\sqrt{x}} \right)$$

$$y = \tan^{-1} \left(\frac{5 - \tan\sqrt{x}}{1 + \frac{5}{2}\tan\sqrt{x}} \right)$$

$$\text{Let, } \frac{5}{2} = \tan A$$

$$= \tan^{-1} \left(\frac{\tan A - \tan\sqrt{x}}{1 + \tan A \tan\sqrt{x}} \right)$$

$$= \tan^{-1} [\tan(A - \sqrt{x})] = A - \sqrt{x}$$

$$y = \tan^{-1} \frac{5}{2} - \sqrt{x}$$

differentiation w.r.t x , we get

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

78. Which one of the following is correct in respect of the function

$$f(x) = x \sin x + \cos x + \frac{1}{2} \cos^2 x?$$

(a) It is increasing in the interval $\left(0, \frac{\pi}{2}\right)$

(b) It remains constant in the interval $\left(0, \frac{\pi}{2}\right)$

(c) It is decreasing in the interval $\left(0, \frac{\pi}{2}\right)$

(d) It is decreasing in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

③ (a) We have,

$$f(x) = x \sin x + \cos x + \frac{1}{2} \cos^2 x$$

$$f'(x) = x(\cos x) + \sin x - \sin x + \frac{1}{2} \cdot 2 \cos x (-\sin x)$$

$$f'(x) = x \cos x - \sin x \cos x$$

By checking options, we put

$$x = \frac{\pi}{4}$$

$$\begin{aligned} f'(x) &= \frac{\pi}{4} \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos \frac{\pi}{4} \\ &= \frac{\pi}{4} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\ &= \frac{\pi}{4\sqrt{2}} - \frac{1}{2} > 0 \end{aligned}$$

So, $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$

79. What is $\lim_{\theta \rightarrow 0} \frac{\sqrt{1 - \cos \theta}}{\theta}$ equal to?

- (a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

③ (c) We have,

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{\sqrt{1 - \cos \theta}}{\theta} &= \lim_{\theta \rightarrow 0} \frac{\sqrt{1 - \left(1 - 2 \sin^2 \frac{\theta}{2}\right)}}{\theta} \\ &= \lim_{\theta \rightarrow 0} \frac{\sqrt{2 \sin^2 \frac{\theta}{2}}}{\theta} \\ &= \lim_{\theta \rightarrow 0} \frac{\sqrt{2} \sin \frac{\theta}{2}}{\theta} \\ &= \lim_{\theta \rightarrow 0} \frac{\frac{\theta}{2} \times 2}{\theta} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

80. A function $f: A \rightarrow \mathbf{R}$ is defined by the equation $f(x) = x^2 - 4x + 5$, where $A = (1, 4)$. What is the range of the function?

- (a) (2, 5) (b) (1, 5)
(c) [1, 5] (d) [1, 5]

③ (c) We have,

A function $f: A \rightarrow \mathbf{R}$ is defined by

$$f(x) = x^2 - 4x + 5,$$

Where, $A = (1, 4)$

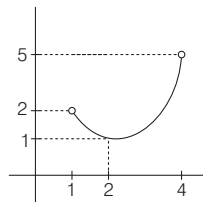
Let, $y = x^2 - 4x + 5$

$$\frac{dy}{dx} = 2x - 4$$

Now, $\frac{dy}{dx} = 0$

$$\Rightarrow 2x - 4 = 0$$

$$\Rightarrow x = 2$$



At $x = 2, y = 1$

At $x = 1, y = (1)^2 - 4(1) + 5 = 2$

At $x = 4, y = (4)^2 - 4(4) + 5 = 5$

So, $y \in [1, 5)$

Range = $[1, 5)$

81. What is $\int_a^b [x] dx + \int_a^b [-x] dx$ equal to, where $[.]$ is the greatest integer function?

- (a) $b - a$ (b) $a - b$
(c) 0 (d) $2(b - a)$

③ (b) We have,

$$\begin{aligned} \int_a^b [x] dx + \int_a^b [-x] dx &= \int_a^b ([x] + [-x]) dx \\ &= \int_a^b (-1) dx \quad [\because [x] + [-x] = -1, \text{ if } x \notin \mathbf{Z}] \\ &= \int_a^b (-1) dx \\ &= -(x)_a^b = -(b - a) = a - b \end{aligned}$$

82. What is $\int_2^8 |x - 5| dx$ equal to?

- (a) 2 (b) 3 (c) 4 (d) 9

③ (d) We have,

$$\begin{aligned} \int_2^8 |x - 5| dx &= \int_2^5 |x - 5| dx + \int_5^8 |x - 5| dx \\ &= \int_2^5 -(x - 5) dx + \int_5^8 (x - 5) dx \\ &= -\int_2^5 (x - 5) dx + \int_5^8 (x - 5) dx \\ &= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8 \\ &= -\left[\left(\frac{25}{2} - 25\right) - \left(2 - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right] \\ &= \frac{25}{2} - 8 - 8 + \frac{25}{2} = 9 \end{aligned}$$

83. What is $\int \sin^3 x \cos x dx$ equal to?

- (a) $\cos^4 x + C$
(b) $\sin^4 x + C$
(c) $\frac{(1 - \sin^2 x)^2}{4} + C$
(d) $\frac{(1 - \cos^2 x)^2}{4} + C$

Where C is the constant of integration.

③ (d) We have,

$$\int \sin^3 x \cos x dx$$

Let, $\sin x = t$

$$\cos x dx = dt$$

$$= \int t^3 dt$$

$$= \frac{t^4}{4} + C$$

$$= \frac{\sin^4 x}{4} + C$$

$$= \frac{(\sin^2 x)^2}{4} + C$$

$$= \frac{(1 - \cos^2 x)^2}{4} + C$$

$$[\because \sin^2 x + \cos^2 x = 1]$$

84. What is $\int e^{\ln(\tan x)} dx$ equal to

- (a) $\ln |\tan x| + C$
(b) $\ln |\sec x| + C$
(c) $\tan x + C$
(d) $e^{\tan x} + C$

Where C is the constant of integration.

③ (b) We have,

$$\int e^{\ln(\tan x)} dx \quad [\because e^{\ln(x)} = x]$$

$$= \int \tan x dx$$

$$= \log |\sec x| + C$$

85. What is $\int_{-1}^1 \left\{ \frac{d}{dx} \left(\tan^{-1} \frac{1}{x} \right) \right\} dx$ equal

to ?

- (a) 0 (b) $-\frac{\pi}{4}$
(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

③ (d) We have,

$$\begin{aligned} \int_{-1}^1 \left[\frac{d}{dx} \left(\tan^{-1} \frac{1}{x} \right) \right] dx &= \left[\tan^{-1} \left(\frac{1}{x} \right) \right]_{-1}^1 \\ &= \tan^{-1}(1) - \tan^{-1}(-1) \\ &= \frac{\pi}{4} + \frac{\pi}{4} \\ &= \frac{\pi}{2} \end{aligned}$$

86. In which one of the following intervals is the function $f(x) = x^2 - 5x + 6$ decreasing?

- (a) $(-\infty, 2]$ (b) $[3, \infty)$
(c) $(-\infty, \infty)$ (d) $(2, 3)$

③ (a) We have,

$$f(x) = x^2 - 5x + 6$$

$$f'(x) = 2x - 5$$

For decreasing

$$\begin{aligned} f'(x) &< 0 \\ 2x - 5 &< 0 \\ x &< \frac{5}{2} \\ x &< 2.5 \\ x &\in (-\infty, 2.5) \end{aligned}$$

- 87.** The differential equation of the family of curves $y = p \cos(ax) + q \sin(ax)$, where p, q are arbitrary constants, is

- (a) $\frac{d^2y}{dx^2} - a^2y = 0$
 (b) $\frac{d^2y}{dx^2} - ay = 0$
 (c) $\frac{d^2y}{dx^2} + ay = 0$
 (d) $\frac{d^2y}{dx^2} + a^2y = 0$

⊙ (d) We have,

$$\begin{aligned} y &= p \cos(ax) + q \sin(ax) \\ \text{differentiation. w. r. t. } x, \text{ we get} \\ \frac{dy}{dx} &= -pa \sin ax + qa \cos ax \end{aligned}$$

Again, differentiation. w. r. t. x , we get

$$\begin{aligned} \frac{d^2y}{dx^2} &= -pa^2 \cos ax - qa^2 \sin ax \\ \Rightarrow \frac{d^2y}{dx^2} &= -a^2(p \cos ax + q \sin ax) \\ \Rightarrow \frac{d^2y}{dx^2} &= -a^2y \\ \Rightarrow \frac{d^2y}{dx^2} + a^2y &= 0 \end{aligned}$$

- 88.** The equation of the curve passing through the point $(-1, -2)$, which satisfies $\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$, is

- (a) $17x^2y - 6x^2 + 3x^5 - 2 = 0$
 (b) $6x^2y + 17x^2 + 2x^5 - 3 = 0$
 (c) $6xy - 2x^2 + 17x^5 + 3 = 0$
 (d) $17x^2y + 6xy - 3x^5 + 5 = 0$

⊙ (b) We have,

$$\begin{aligned} \frac{dy}{dx} &= -x^2 - \frac{1}{x^3} \\ \Rightarrow dy &= \left(-x^2 - \frac{1}{x^3}\right) dx \end{aligned}$$

Integrating both sides

$$\begin{aligned} \int dy &= \int \left(-x^2 - \frac{1}{x^3}\right) dx \\ \Rightarrow y &= -\frac{x^3}{3} - \left(\frac{x^{-2}}{-2}\right) + C \\ \Rightarrow y &= -\frac{x^3}{3} + \frac{1}{2x^2} + C \quad \dots(i) \end{aligned}$$

given curve is passing through $(-1, -2)$

$$\begin{aligned} -2 &= \frac{1}{3} + \frac{1}{2} + C \\ \Rightarrow -2 &= \frac{5}{6} + C \\ \Rightarrow C &= -2 - \frac{5}{6} \\ \Rightarrow C &= -\frac{17}{6} \end{aligned}$$

Put in Eq. (i)

$$\begin{aligned} y &= -\frac{x^3}{3} + \frac{1}{2x^2} - \frac{17}{6} \\ \Rightarrow y &= \frac{-2x^5 + 3 - 17x^2}{6x^2} \\ \Rightarrow 6x^2y &= -2x^5 + 3 - 17x^2 \\ \Rightarrow 6x^2y + 2x^5 + 17x^2 - 3 &= 0 \end{aligned}$$

- 89.** What is the order of the differential equation whose solution is $y = a \cos x + b \sin x + ce^{-x} + d$, where a, b, c and d are arbitrary constants?

- (a) 1 (b) 2 (c) 3 (d) 4

⊙ (d) We have,

$$\begin{aligned} y &= a \cos x + b \sin x + ce^{-x} + d, \\ a, b, c \text{ and } d &\text{ are arbitrary constants.} \\ \text{We know that,} \\ \text{order} &= \text{number of arbitrary constant} \\ \text{So, order} &= 4 \end{aligned}$$

- 90.** What is the solution of the differential equation

$$\ln\left(\frac{dy}{dx}\right) = ax + by?$$

- (a) $ae^{ax} + be^{by} = C$
 (b) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{by} = C$
 (c) $ae^{ax} + be^{-by} = C$
 (d) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{-by} = C$

⊙ (d) We have,

$$\begin{aligned} \ln\left(\frac{dy}{dx}\right) &= ax + by \\ \frac{dy}{dx} &= e^{ax+by} \\ \frac{dy}{dx} &= e^{ax} \cdot e^{by} \\ \frac{dy}{e^{by}} &= e^{ax} dx \end{aligned}$$

Integrating both sides

$$\begin{aligned} \int e^{-by} dy &= \int e^{ax} dx \\ \Rightarrow \frac{e^{-by}}{-b} &= \frac{e^{ax}}{a} + C \\ \Rightarrow \frac{e^{ax}}{a} + \frac{e^{-by}}{b} + C &= 0 \end{aligned}$$

- 91.** If $u = e^{ax} \sin bx$ and $v = e^{ax} \cos bx$, then what is $u \frac{du}{dx} + v \frac{dv}{dx}$ equal to?

- (a) ae^{2ax} (b) $(a^2 + b^2)e^{ax}$
 (c) $ab e^{2ax}$ (d) $(a + b)e^{ax}$

⊙ (a) We have,

$$u = e^{ax} \sin bx$$

differentiation. w. r. t. x , we get

$$\begin{aligned} \frac{du}{dx} &= e^{ax} (b \cos bx) + \sin bx (ae^{ax}) \\ \frac{dv}{dx} &= e^{ax} (b \cos bx + a \sin bx) \quad \dots(i) \end{aligned}$$

Now, $v = e^{ax} \cos bx$

differentiation. w. r. t. x , we get

$$\begin{aligned} \frac{dv}{dx} &= e^{ax} (-b \sin bx) + \cos bx (ae^{ax}) \\ &= e^{ax} (-b \sin bx + a \cos bx) \end{aligned}$$

Now we have,

$$\begin{aligned} u \frac{du}{dx} + v \frac{dv}{dx} &= e^{ax} \sin bx [e^{ax} (b \cos bx + a \sin bx)] \\ &+ e^{ax} \cos bx [e^{ax} (-b \sin bx + a \cos bx)] \\ &= e^{2ax} [b \sin bx \cos bx + a \sin^2 bx] + e^{2ax} \\ &\quad (-b \cos bx \sin bx + a \cos^2 bx) \\ &= e^{2ax} [a \sin^2 bx + a \cos^2 bx] \\ &= ae^{2ax} \end{aligned}$$

- 92.** If $y = \sin(\ln x)$, then which one of the following is correct?

- (a) $\frac{d^2y}{dx^2} + y = 0$
 (b) $\frac{d^2y}{dn^2} = 0$
 (c) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$
 (d) $x \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$

Where C is the constant of integration

⊙ (c) We have,

$$y = \sin(\log_e x)$$

differentiation. w. r. t. x , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos(\log_e x)}{x} \\ x \frac{dy}{dx} &= \cos(\log_e x) \end{aligned}$$

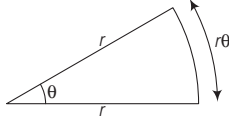
Again, differentiation. w. r. t. x , we get

$$\begin{aligned} x \frac{d^2y}{dx^2} + \frac{dy}{dx} &= \frac{-\sin(\log_e x)}{x} \\ \Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} &= -\sin(\log_e x) \\ \Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} &= -y \\ \Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y &= 0 \end{aligned}$$

93. A flower-bed in the form of a sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible area, then what is the radius of the sector?

(a) 25 m (b) 20 m
(c) 10 m (d) 5 m

- ⊙ (c) Let radius of sector be r and angle subtended at centre be θ .



$$\begin{aligned}\therefore r + r + r\theta &= 40 \\ \Rightarrow 2r + r\theta &= 40 \\ \Rightarrow \theta &= \frac{40 - 2r}{r}\end{aligned}$$

$$\text{Area of the sector} = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} r^2 \left(\frac{40 - 2r}{r} \right)$$

$$A = \frac{1}{2} r (40 - 2r)$$

$$A = \frac{1}{2} (40r - 2r^2)$$

differentiating w. r. t. r ,

$$\frac{dA}{dr} = \frac{1}{2} (40 - 4r)$$

$$\text{Now, } \frac{dA}{dr} = 0$$

$$\Rightarrow 40 - 4r = 0$$

$$\Rightarrow r = 10 \text{ m}$$

94. What is the minimum value of

$$[x(x-1)+1]^{\frac{1}{3}}, \text{ where } 0 \leq x \leq 1?$$

(a) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$ (b) 1
(c) $\frac{1}{2}$ (d) $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

- ⊙ (a) We have,

$$y = (x^2 - x + 1)^{\frac{1}{3}}$$

$$y = \left(x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 \right)^{\frac{1}{3}}$$

$$\text{at } x = \frac{1}{2}, y \text{ is minimum } y = \left(\frac{3}{4} \right)^{\frac{1}{3}}$$

95. If $y = |\sin x|^{x|}$, then what is the

$$\text{value of } \frac{dy}{dx} \text{ at } x = -\frac{\pi}{6}?$$

$$(a) \frac{2^{-\frac{\pi}{6}} (6 \ln 2 - \sqrt{3}\pi)}{6}$$

$$(b) \frac{2^{\frac{\pi}{6}} (6 \ln 2 + \sqrt{3}\pi)}{6}$$

$$(c) \frac{2^{-\frac{\pi}{6}} (6 \ln 2 + \sqrt{3}\pi)}{6}$$

$$(d) \frac{2^{-\frac{\pi}{6}} (6 \ln 2 - \sqrt{3}\pi)}{6}$$

- ⊙ (a) We have,

$$y = |\sin x|^{x|}$$

$$\therefore x = -\frac{\pi}{6},$$

$$\sin x < 0, x < 0$$

$$y = (-\sin x)^{-x}$$

$$\therefore y = f(x)^{g(x)}, \text{ So}$$

$$\frac{dy}{dx} = f(x)^{g(x)} \left[\frac{g(x)}{f(x)} f'(x) + \log |f(x)| \cdot g'(x) \right]$$

$$\frac{dy}{dx} = (-\sin x)^{-x}$$

$$\left[\frac{(-x)}{(-\sin x)} \cdot (-\cos x) + \log |(-\sin x)| \cdot (-1) \right]$$

$$\frac{dy}{dx} \Big|_{x = -\pi/6} = \left[-\sin \left(-\frac{\pi}{6} \right) \right]^{\frac{\pi}{6}}$$

$$\left\{ \frac{\pi/6}{-\sin(-\pi/6)} - \cos \left(-\frac{\pi}{6} \right) \right\}$$

$$- \log |-\sin \left(-\frac{\pi}{6} \right)|$$

$$= \left(\sin \frac{\pi}{6} \right)^{\frac{\pi}{6}} \left[\frac{-\frac{\pi}{6}}{\frac{1}{2}} \left(\frac{\sqrt{3}}{2} \right) - \log \left(\frac{1}{2} \right) \right]$$

$$= \left(\frac{1}{2} \right)^{\frac{\pi}{6}} \left[\frac{-\sqrt{3}\pi}{6} - \log \frac{1}{2} \right]$$

$$= 2^{-\pi/6} \left[\log^2 - \frac{\sqrt{3}\pi}{6} \right]$$

$$= 2^{-\pi/6} \left[\frac{6 \log 2 - \sqrt{3}\pi}{6} \right]$$

96. What is $\frac{d\sqrt{1-\sin 2x}}{dx}$ equal to,

$$\text{where } \frac{\pi}{4} < x < \frac{\pi}{2}?$$

$$(a) \cos x + \sin x$$

$$(b) -(\cos x + \sin x)$$

$$(c) \pm (\cos x + \sin x)$$

$$(d) \text{None of the above}$$

- ⊙ (a) We have,

$$\frac{d}{dx} \sqrt{1-\sin 2x}, \frac{\pi}{4} < x < \frac{\pi}{2}$$

$$= \frac{d}{dx} \sqrt{\cos^2 x + \sin^2 x - 2 \sin x \cos x}$$

$$\left[\because \sin^2 x + \cos^2 x = 1 \right. \\ \left. \text{and } \sin 2x = 2 \sin x \cos x \right]$$

$$= \frac{d}{dx} |\cos x - \sin x|$$

$$= \frac{d}{dx} (\sin x - \cos x)$$

$$\left[\because \frac{\pi}{4} < x < \frac{\pi}{2} \therefore \sin x > \cos x \right]$$

$$= \cos x - (-\sin x)$$

$$= \cos x + \sin x$$

97. What is $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ equal to?

$$(a) C + \frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right)$$

$$(b) C - \frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right)$$

$$(c) C + \frac{1}{ab} \tan^{-1} \left(\frac{b \tan x}{a} \right)$$

$$(d) \text{None of the above}$$

Where C is the constant of integration.

- ⊙ (a) We have,

$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

Divide numerator and denominator by $\cos^2 x$

$$I = \int \frac{\sec^2 x dx}{a^2 \tan^2 x + b^2}$$

$$\text{Let } a \tan x = t$$

$$a \sec^2 x dx = dt$$

$$\sec^2 x dx = \frac{dt}{a}$$

$$I = \frac{1}{a} \int \frac{dt}{t^2 + b^2}$$

$$= \frac{1}{a} \times \frac{1}{b} \tan^{-1} \left(\frac{t}{b} \right) + C$$

$$\left[\because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \right]$$

$$= \frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right) + C$$

98. Let $f(x+y) = f(x)f(y)$ and

$$f(x) = 1 + xg(x)\phi(x), \text{ where}$$

$$\lim_{x \rightarrow 0} g(x) = a \text{ and } \lim_{x \rightarrow 0} \phi(x) = b.$$

Then, what is $f'(x)$ equal to?

$$(a) 1 + abf(x)$$

$$(b) 1 + ab$$

$$(c) ab$$

$$(d) abf(x)$$

- ⊙ (d) Let, $f(x+y) = f(x)f(y)$

$$\text{and } f(x) = 1 + xg(x) \cdot \phi(x)$$

$$\text{Thus, } f(y) = 1 + yg(y)\phi(y)$$

$$\text{Where } \lim_{x \rightarrow 0} g(x) = a \text{ and } \lim_{x \rightarrow 0} \phi(x) = b$$

using first principal

$$f'(x) = \lim_{y \rightarrow 0} \frac{f(x+y) - f(x)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{f(x) \cdot f(y) - f(x)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{f(x)[f(y) - 1]}{y}$$

$$\begin{aligned}\lim_{y \rightarrow 0} f(x) \left[\frac{1 + yg(y)\phi(y) - 1}{y} \right] \\&= f(x) \lim_{y \rightarrow 0} g(y) \lim_{y \rightarrow 0} \phi(y) \\&= f(x) \cdot a \cdot b \\&= ab f(x)\end{aligned}$$

- 99.** What is the solution of the differential equation $\frac{dx}{dy} = \frac{x+y+1}{x+y-1}$?

- (a) $y - x + 4 \ln(x+y) = C$
(b) $y + x + 2 \ln(x+y) = C$
(c) $y - x + \ln(x+y) = C$
(d) $y + x + 2 \ln(x+y) = C$

Where C is an arbitrary constant.

- ⊙ (c) We have, $\frac{dx}{dy} = \frac{x+y+1}{x+y-1}$

$$\begin{aligned}\text{Let } x+y &= u \\ \frac{dx}{dy} + 1 &= \frac{du}{dy} \\ \Rightarrow \frac{dx}{dy} &= \frac{du}{dy} - 1 \\ \Rightarrow \frac{du}{dy} - 1 &= \frac{u+1}{u-1} \\ \Rightarrow \frac{du}{dy} &= \frac{u+1}{u-1} + 1 \\ \Rightarrow \frac{du}{dy} &= \frac{u+1+u-1}{u-1} \\ \Rightarrow \frac{du}{dy} &= \frac{2u}{u-1} \\ \Rightarrow \left(\frac{u-1}{u} \right) du &= 2 dy\end{aligned}$$

Integrating both sides,

$$\begin{aligned}\int \left(1 - \frac{1}{u} \right) du &= \int 2 dy \\ \Rightarrow u - \log u &= 2y + C \\ &[\because u = x+y] \\ \Rightarrow x + y - \log(x+y) &= 2y + C \\ \Rightarrow x - \log(x+y) &= y + C \\ \Rightarrow y - x + \log(x+y) &= C\end{aligned}$$

- 100.** What is $\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$

equal to?

- (a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$
(c) -2 (d) -3

- ⊙ (d) We have,

$$\begin{aligned}\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} \\&= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + 2 \sin x - \sin x - 1}{2 \sin^2 x - 2 \sin x - \sin x + 1} \\&= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x (\sin x + 1) - (\sin x + 1)}{2 \sin x (\sin x - 1) - 1 (\sin x - 1)}\end{aligned}$$

$$\begin{aligned}&= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\sin x + 1)(2 \sin x - 1)}{(\sin x - 1)(2 \sin x - 1)} \\&= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin x + 1}{\sin x - 1} \\&= \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} = \frac{\frac{3}{2}}{-\frac{1}{2}} \\&= -3\end{aligned}$$

- 101.** If two dice are thrown and atleast one of the dice shows 5, then the probability that the sum is 10 or more is

- (a) $\frac{1}{6}$ (b) $\frac{4}{11}$
(c) $\frac{3}{11}$ (d) $\frac{2}{11}$

- ⊙ (c) Let A be event of dice shows 5 and B be the event that the sum is 10 or more

$$\text{Here, } n(S) = 36$$

$$n(A) = \{ (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) \}$$

$$n(B) = \{ (5, 5), (6, 4), (4, 6), (6, 5), (5, 6), (6, 6) \}$$

$$n(A \cap B) = \{ (5, 5), (6, 5), (5, 6) \}$$

$$\begin{aligned}P\left(\frac{B}{A}\right) &= \frac{\frac{3}{36}}{\frac{11}{36}} \\&= \frac{3}{11}\end{aligned}$$

- 102.** The correlation coefficient computed from a set of 30 observations is 0.8. Then the percentage of variation not explained by linear regression is

- (a) 80% (b) 20%
(c) 64% (d) 36%

- ⊙ (b) Given that,

correlation coefficient = 0.8 = 80%, if the relation is 80% explained, then 20% of variation will not explained by linear regression.

- 103.** The average age of a combined group of men and women is 25 yr. If the average age of the group of men is 26 yr and that of the group of women is 21 yr, then the percentage of men and women in the group is respectively

- (a) 20, 80 (b) 40, 60
(c) 60, 40 (d) 80, 20

- ⊙ (d) In group of men, let number of men = a

$$\bar{X}_1 = 26 \text{ yr and } n_1 = a \text{ (let)}$$

and in group of women, number of women = b (let)

$$\bar{X}_2 = 21 \text{ yr}$$

and combined mean $\bar{X} = 25$

$$\text{Now, } \bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$$

$$\Rightarrow 25 = \frac{26a + 21b}{a + b}$$

$$\Rightarrow 25a + 25b = 26a + 21b$$

$$\Rightarrow 4b = a$$

$$\Rightarrow \frac{a}{b} = \frac{4}{1}$$

By checking options (d) is correct.

- 104.** If $\sin \beta$ is the harmonic mean of $\sin \alpha$ and $\cos \alpha$ and $\sin \theta$ is the arithmetic mean of $\sin \alpha$ and $\cos \alpha$, then which of the following is/are correct?

$$1. \sqrt{2} \sin \left(\alpha + \frac{\pi}{4} \right) \sin \beta = \sin 2\alpha$$

$$2. \sqrt{2} \sin \theta = \cos \left(\alpha - \frac{\pi}{4} \right)$$

Select the correct answer using the code given below.

- (a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

- ⊙ (c) Given that,

$\sin \beta$ is HM of $\sin \alpha$ and $\cos \alpha$

$$\text{So, } \sin \beta = \frac{2 \sin \alpha \cos \alpha}{\sin \alpha + \cos \alpha}$$

$$\Rightarrow \sin \beta (\sin \alpha + \cos \alpha) = \sin 2\alpha$$

$$\Rightarrow \sin \beta \times \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin \alpha + \frac{1}{\sqrt{2}} \cos \alpha \right) = \sin 2\alpha$$

$$\Rightarrow \sin \beta \times \sqrt{2} \left(\cos \frac{\pi}{4} \sin \alpha + \sin \frac{\pi}{4} \cos \alpha \right)$$

$$= \sin 2\alpha$$

$$\Rightarrow \sqrt{2} \sin \beta \left[\sin \left(\alpha + \frac{\pi}{4} \right) \right] = \sin 2\alpha$$

$$\Rightarrow \sqrt{2} \sin \left(\alpha + \frac{\pi}{4} \right) \sin \beta = \sin 2\alpha$$

Statement I is true.

Now, $\sin \theta$ is AM of $\sin \alpha$ and $\cos \alpha$

$$\sin \theta = \frac{\sin \alpha + \cos \alpha}{2}$$

$$\Rightarrow 2 \sin \theta = \sin \alpha + \cos \alpha$$

$$\Rightarrow 2 \sin \theta = \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin \alpha + \frac{1}{\sqrt{2}} \cos \alpha \right)$$

$$\Rightarrow$$

$$2 \sin \theta = \sqrt{2} \left(\sin \frac{\pi}{4} \sin \alpha + \cos \frac{\pi}{4} \cos \alpha \right)$$

$$\Rightarrow \sqrt{2} \sin \theta = \cos \left(\alpha - \frac{\pi}{4} \right)$$

Hence, Statement II is also correct.

- 105.** Let A, B and C be three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = 1.5 P(A)$ and $P(C) = 0.5 P(B)$, then $P(A)$ is equal to

- (a) $\frac{3}{4}$ (b) $\frac{4}{13}$
(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

⊙ (b) We have,

$$P(B) = 1.5 P(A) = \frac{3}{2} P(A)$$

$$\text{and } P(C) = 0.5 P(B) = \frac{1}{2} P(B)$$

$$= \frac{1}{2} \times \frac{3}{2} P(A)$$

$$= \frac{3}{4} P(A)$$

Now, A, B and C are mutually exclusive and exhaustive events

So,

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow P(A) + \frac{3}{2} P(A) + \frac{3}{4} P(A) = 1$$

$$\Rightarrow P(A) \left[1 + \frac{3}{2} + \frac{3}{4} \right] = 1$$

$$\Rightarrow P(A) \left(\frac{13}{4} \right) = 1$$

$$\Rightarrow P(A) = \frac{4}{13}$$

- 106.** In a bolt factory, machines X, Y, Z manufacture bolts that are respectively 25%, 35% and 40% of the factory's total output. The machines X, Y, Z respectively produce 2%, 4% and 5% defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine X ?

- (a) $\frac{5}{39}$ (b) $\frac{14}{39}$
(c) $\frac{20}{39}$ (d) $\frac{34}{39}$

⊙ (a) Let

A : bolt manufactured from machine

B : bolt manufactured from machine

C : bolt manufactured from machine

and E : bolt is defective

$$\therefore P(A) = 25\% = \frac{25}{100}$$

$$P(B) = 35\% = \frac{35}{100}$$

$$P(C) = 40\% = \frac{40}{100}$$

$$\text{and } P\left(\frac{E}{A}\right) = 2\% = \frac{2}{100}$$

$$P\left(\frac{E}{B}\right) = 4\% = \frac{4}{100}$$

$$P\left(\frac{E}{C}\right) = 5\% = \frac{5}{100}$$

Probability of defective bulb that was manufactured by machine X ,

$$P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right) + P(C) \cdot P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{2}{100}}{\frac{25}{100} \times \frac{2}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{5}{100}}$$

$$= \frac{25 \times 2}{25 \times 2 + 35 \times 4 + 40 \times 5}$$

$$= \frac{50}{50 + 140 + 200}$$

$$= \frac{50}{390} = \frac{5}{39}$$

- 107.** 8 coins are tossed simultaneously. The probability of getting atleast 6 heads is

- (a) $\frac{7}{64}$ (b) $\frac{57}{64}$
(c) $\frac{37}{256}$ (d) $\frac{229}{256}$

⊙ (c) We have,

8 coins are tossed simultaneously i.e $n = 8$ probability of getting head $p = \frac{1}{2}$,

so $q = 1 - p = \frac{1}{2}$. Probability of getting atleast 6 heads.

$$= {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 ({}^8C_6 + {}^8C_7 + {}^8C_8)$$

$$= \left(\frac{1}{2}\right)^8 (28 + 8 + 1) = \frac{37}{256}$$

- 108.** Three groups of children contain 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consist of 1 girl and 2 boys is

- (a) $\frac{13}{32}$ (b) $\frac{9}{32}$
(c) $\frac{3}{32}$ (d) $\frac{1}{32}$

⊙ (a) In first group, 3 girls and 1 boy

$$P(G) = \frac{3}{4} \text{ and } P(B) = \frac{1}{4}$$

In second group, 2 girls and 2 boys

$$P(G) = \frac{2}{4} = \frac{1}{2}$$

$$\text{and } P(B) = \frac{2}{4} = \frac{1}{2}$$

One child is selected at random from each group and consists of 1 girl and 2 boys are GBB or BGB or BBG.

So, required probability

$$= \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4}$$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$$

- 109.** Consider the following statements :

1. If 10 is added to each entry on a list, then the average increases by 10.
2. If 10 is added to each entry on a list, then the standard deviation increases by 10.
3. If each entry on a list is doubled then the average doubles.

Which of the above statements are correct?

- (a) 1, 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 2 and 3

⊙ (c) **Statement 1** The average is affected by the change of the origin.

So, if 10 is added to each entry on list then average increase by 10. Statement 1 is correct.

Statement 2 Standard deviation is independent on change in origin. Statement 2 is incorrect.

Statement 3 The average is affected by change in scale in same ratio as each entry is changes.

So, Statement 3 is correct.

- 110.** The variance of 25 observations is 4.

If 2 is added to each observation, then the new variance of the resulting observations is

- (a) 2 (b) 4
(c) 6 (d) 8

⊙ (b) We know that,

Variance is independent on change in origin.

So, if 2 is added to each observation, then variance is remain same

Hence, variance = 4

- 111.** If $x_i > 0, y_i > 0 (i = 1, 2, 3, \dots, n)$ are the values of two variables X and Y with geometric means P and Q respectively, then the geometric mean of $\frac{X}{Y}$ is

- (a) $\frac{P}{Q}$
 (b) $\text{antilog}\left(\frac{P}{Q}\right)$
 (c) $n(\log P - \log Q)$
 (d) $n(\log P + \log Q)$

- ⊙ (a) We have, $x_i > 0, y_i > 0$,
 ($i = 1, 2, 3, \dots, n$)

$$P = (x_1 \times x_2 \times x_3 \times \dots \times x_n)^{\frac{1}{n}}$$

$$\text{and } Q = (y_1 \times y_2 \times y_3 \times \dots \times y_n)^{\frac{1}{n}}$$

Now, geometric mean of $\frac{X}{Y}$

$$= \left(\frac{x_1}{y_1} \times \frac{x_2}{y_2} \times \dots \times \frac{x_n}{y_n} \right)^{\frac{1}{n}}$$

$$= \frac{P}{Q}$$

- 112.** If probability of simultaneous occurrence of two events A and B is p and the probability that exactly one of A, B occurs is q , then which of the following is/are correct?

1. $P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$
 2. $P(\bar{A} \cap \bar{B}) = 1 - p - q$

Select the correct answer using the code given below.

- (a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

- ⊙ (c) Given that,

$$P(A \cap B) = p$$

$$\text{and } P(A) + P(B) - 2P(A \cap B) = q$$

Now, **Statement 1**

Given,

$$P(A) + P(B) - 2P(A \cap B) = q$$

$$\Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) - 2p = q$$

$$\Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$$

Statement 1 is correct

For **Statement 2**

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [P(A) + P(B) - 2P(A \cap B) + P(A \cap B)]$$

$$= 1 - [q + p]$$

$$= 1 - q - p$$

Statement 2 is also correct.

- 113.** If the regression coefficient of Y on X is -6 and the correlation coefficient between X and Y is $-\frac{1}{2}$, then the regression coefficient of X on Y would be

- (a) $\frac{1}{24}$ (b) $-\frac{1}{24}$
 (c) $-\frac{1}{6}$ (d) $\frac{1}{6}$

- ⊙ (b) Given that,

$$b_{yx} = -6$$

$$\text{and } r_{xy} = -\frac{1}{2}$$

We know that,

$$r_{xy} = \sqrt{b_{yx} \times b_{xy}}$$

$$\Rightarrow -\frac{1}{2} = \sqrt{-6 \times b_{xy}}$$

Squaring both sides,

$$\Rightarrow \frac{1}{4} = -6 \times b_{xy}$$

$$\Rightarrow b_{xy} = -\frac{1}{24}$$

- 114.** The set of bivariate observations $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ are such that all the values are distinct and all the observations fall on a straight line with non-zero slope. Then the possible values of the correlation coefficient between x and y are

- (a) 0 and 1 (b) 0 and -1
 (c) 0, 1 and -1 (d) -1 and 1

- ⊙ (d) Given that,

All the observations fall on a straight line with non-zero, slope then if slope is positive then $r = 1$

and if slope is negative then $r = -1$

So, values of the correlation coefficient between x and y are -1 and 1 .

- 115.** Two integers x and y are chosen with replacement from the set $[0, 1, 2, \dots, 10]$. The probability that $|x - y| > 5$ is

- (a) $\frac{6}{11}$ (b) $\frac{35}{121}$
 (c) $\frac{30}{121}$ (d) $\frac{25}{121}$

- ⊙ (c) Given that,

$$S = \{0, 1, 2, \dots, 10\}$$

$$n(S) = 11 \times 11 = 121$$

Now, $x - y > 5$

Now, E is the set of element such that $|x - y| > 5$

$$E = \{(6, 0), (0, 6), (7, 1), (1, 7), (8, 2), (2, 8), (9, 3), (3, 9), (10, 4), (4, 10), (7, 0), (0, 7), (8, 1), (1, 8), (9, 2), (2, 9), (10, 3), (3, 10), (8, 0), (0, 8), (9, 1),$$

$$(1, 9), (10, 2), (2, 10), (9, 0), (0, 9), (10, 1), (1, 10), (10, 0), (0, 10)\}$$

$$n(E) = 30$$

$$\text{So, required probability} = \frac{30}{121}$$

- 116.** An analysis of monthly wages paid to the workers in two firms A and B belonging to the same industry gives the following result

	Firm A	Firm B
Number of workers	500	600
Average monthly wage	₹ 1860	₹ 1750
Variance of distribution of wages	81	100

The average of monthly wage and variance of distribution of wages of all the workers in the firms A and B taken together are

- (a) ₹ 1860, 100
 (b) ₹ 1750, 100
 (c) ₹ 1800, 81
 (d) None of the above

- ⊙ (d) For firm A

$$n_1 = 500, \bar{X}_1 = 1860$$

$$\text{and variance} = \sigma_1^2 = 81$$

$$\text{So, S.D.} = \sigma_1 = 9$$

For firm B

$$n_2 = 600, \bar{X}_2 = 1750$$

$$\text{and variance } \sigma_2^2 = 100$$

$$\text{So, S.D.} = \sigma = 10$$

$$\text{Now, combined mean } \bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$$

$$= \frac{500 \times 1860 + 600 \times 1750}{500 + 600}$$

$$\bar{X} = 1800$$

Combined variance

$$= \frac{n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{500 [81 + (-60)^2] + 600 [100 + (50)^2]}{500 + 600}$$

$$\therefore d_1 = \bar{X} - X_1 = 1800 - 1860 = -60$$

$$\text{and } d_2 = 1800 - 1750 = 50$$

$$= \frac{500 (81 + 3600) + 600 (100 + 2500)}{1100}$$

$$= \frac{500 (3681) + 600 (2600)}{1100}$$

$$= 3091.36$$

- 117.** Three dice having digits 1, 2, 3, 4, 5 and 6 on their faces are marked I, II, and III and rolled. Let x , y and z represent the number on die-I, die-II and die-III, respectively. What is the number of possible outcomes such that $x > y > z$?

(a) 14 (b) 16
(c) 18 (d) 20

- ⊗ (d) Three dice having digit 1, 2, 3, 4, 5 and 6 and given that $x > y > z$.

So, possibilities are

Case I If $x = 6$

$$x > y > z$$

Possible ways

$$= \{ (6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 4, 1), (6, 4, 2), (6, 4, 3), (6, 3, 2), (6, 3, 1), (6, 2, 1) \}$$

So, possible ways = 10

Case II If $x = 5$

Then, possible ways

$$= \{ (5, 4, 3), (5, 4, 2), (5, 4, 1), (5, 3, 2), (5, 3, 1), (5, 2, 1) \}$$

So, possible ways = 6

Case III If $x = 4$

Then, possible ways

$$= \{ (4, 3, 2), (4, 3, 1), (4, 2, 1) \}$$

So, possible ways = 3

Case IV If $x = 3$

Then, possible ways = 1 (3, 2, 1)

So, required possible outcomes

$$= 10 + 6 + 3 + 1 = 20$$

- 118.** Which one of the following can be obtained from an ogive?

(a) Mean
(b) Median
(c) Geometric Mean
(d) Mode

- ⊗ (b) We know that, median is always calculated from less than or more than type ogive curve where both curve cuts each other at that point median.

- 119.** In any discrete series (when all values are not same), if x represents mean deviation about mean and y represents standard deviation, then

which one of the following is correct?

(a) $y \geq x$ (b) $y \leq x$
(c) $x = y$ (d) $x < y$

- ⊗ (d) We know that,

$$MD = \frac{4}{5} S.D.$$

$$\Rightarrow 5MD = 4S.D.$$

$$\Rightarrow 5x = 4y$$

$$[\because MD = x \text{ and } SD = y]$$

$$\therefore x < y$$

- 120.** In which one of the following cases would you expect to get a negative correlation?

(a) The ages of husbands and wives
(b) Shoe size and intelligence
(c) Insurance companies profits and the number of claims they have to pay
(d) Amount of rainfall and yield of crop

- ⊗ (c) In negative correlation, if x is increases then y is decreases by checking options Insurance companies profits and the number of claims they have to pay are negatively correlated.

PAPER II English Language and General Studies

Part A (English Language)

Directions (Q.Nos. 1-5) *Each item in this section has a sentence with three underlined parts labelled (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined parts. If you find no error, your response should be indicated as (d).*

- 1.** He appears to be a honest man.

(a) (b) (c)

No error

(c)

- ⊗ (c) Before honest article 'an' should be used as 'h' is mute here and the word gives us vowel sound.

- 2.** One of the members expressed doubt if

(a) (b)

the Minister was an atheist.

(c)

No error

(d)

- ⊗ (b) In place of 'if' we should use 'that' as it is a noun clause.

- 3.** This view has been taken

(a)

by one of the ablest persons

(b)

who has written on this subject.

(c)

No error

(c)

- ⊗ (c) Use of verb 'has' should be replaced by plural 'have' as to agree with the subject 'persons'.

- 4.** Slow and steady owns the race,

(a) (b)

as the wise would say. No error

(c) (d)

- ⊗ (b) Part (b) has error as owns will be replaced by wins. If is a popular proesh.

- 5.** We often hear people say

(a)

that most human beings have not evovled

(b)

or grow up enough to stick to truth or non-violence. No error

(c) (d)

- ⊗ (c) Part (c) has error of correct use of verb. As noun clause is in present perfect tense, use of 'grow up' is incorrect. Will be used Past participle of 'grow' ie 'grown up'.

Directions (Q.Nos. 6-10) *Each item in this section consists of a sentence with an underlined word followed by four words or groups of words. Select the word or group of words that is nearest in meaning to the underlined word.*

- 6.** The Industrial Revolution saw a massive rise in the population of Europe.

(a) enormous (b) erroneous
(c) hazardous (d) perilous

- ⊗ (a) 'enormous' is nearest in meaning to the word 'massive' which means large and heavy.

- 7.** I had some deepest convictions reflected in my work.

(a) ideas and opinions
(b) firm beliefs
(c) prejudices
(d) biases

- ⊗ (b) 'firm beliefs' is nearest meaning to the word 'conviction' as both words mean a firmly held belief or opinion. Other options are not suitable here.

8. This boy is very timid.

- (a) courageous (b) shy
(c) clever (d) dull

- ⊗ (b) 'shy' is nearest in meaning to the word 'timid' which means showing lack of courage or confidence.

9. My friend is as stubborn as a mule.

- (a) observant (b) obnoxious
(c) obstinate (d) corpulent

- ⊗ (c) 'obstinate' is nearest in meaning to the word 'stubborn' which means a person who is determined not to change one's attitude or who is headstrong or uncompromising.

10. His behaviour was deliberately provocative.

- (a) exciting desire (b) infuriating
(c) pitiable (d) creating frustration

- ⊗ (b) 'infuriating' is nearest in meaning to the word 'provocative' which means causing anger.

Directions (Q.Nos 11-15) Each item in this section consists of a sentence with an underlined word followed by four words or groups of words. Select the word or group of words that is opposite in meaning to the underlined word.

11. Too many cooks spoil the broth.

- (a) tarnish (b) wreck
(c) embellish (d) upset

- ⊗ (c) The word spoil means to diminish or destroy. 'Embellish' means to improve or enhance. So, it is correct antonym of 'spoil'.

12. He is biased against the students from cities.

- (a) open (b) prejudiced
(c) liked (d) impartial

- ⊗ (d) The word 'biased' means inclined to favour one over another. The word 'impartial' means treating everyone equally.

13. It is easy to be an orthodox.

- (a) idolatrous (b) intelligent
(c) malignant (d) heterodox

- ⊗ (d) 'Orthodox' means following or conforming to the traditions whereas its antonym is 'heterodox' which means a person who does not conform to accepted standards or traditions.

14. Permit me to present you with a book.

- (a) allow (b) enclose
(c) prohibit (d) persuade

- ⊗ (c) 'permit' means to allow whereas prohibit means to disallow or deny. So, it is the antonym of permit.

15. None but the brave deserves the fair.

- (a) ugly (b) coward
(c) jealous (d) weak

- ⊗ (b) 'coward' is correct antonym of the word 'brave'. 'Brave' means courageous or ready to face danger whereas 'coward' means a person who lacks courage and turns away from danger.

Directions (Q.Nos. 16-25) In this section, you have two short passages. After each passage, you will find some items based on the passage. First, read a passage and answer the items based on it. You are required to select your answers based on the contents of the passage and opinion of the author only.

Passage-1

"Now, ladies and gentlemen," said the conjuror, 'having shown you that the cloth is absolutely empty. I will proceed to take from it a bowl of goldfish. Presto !'"

All around the hall people were saying, "Oh, how wonderful ! How does he do it?"

But the Quick Man on the front seat said in a big whisper to the people near him, "He-had-it-up-his-sleeve."

Then the people nodded brightly at the Quick Man and said, 'Oh, of course'; and everybody whispered round the hall, "He-had-it-up-his-sleeve."

"My next trick," said the conjuror, "is the famous Hindostanee rings. You will notice that the rings are apparently separate; at a blow they all join (clang, clang, clang)—Presto !"

There was a general buzz of stupefaction till the Quick Man was heard to whisper, "He-must-have-had-another-lot-up-his-sleeve".

Again everybody nodded and whispered, "The-rings-were-up-his-sleeve."

The brow of the conjuror was clouded with a gathering frown.

"I will now," he continued, "show you a most amusing trick by which I am enabled to take any number of eggs from a hat. Will some gentleman kindly lend me his hat?" Ah, thank you—Presto !"

He extracted seventeen eggs and for thirty-five seconds the audience began to think that he was wonderful. Then the Quick Man whispered along the front bench,

"He-has-a-hen- up-his-sleeve," and all the people whispered it on. "He-has-a-lot-of-hens-up-his- sleeve".

The egg trick was ruined.

It went on like that all through. It transpired from the whispers of the Quick Man that the conjuror must have concealed up his sleeve, in addition to the rings, hens and fish, several packs of cards, a loaf of bread, a doll's Cradle, a live guinea pig, a fifty-cent piece and a rocking chair.

16. "The brow of the conjuror was clouded with a gathering frown."

The sentence means that the conjuror

- (a) was very pleased (b) was very sad
(c) was rather angry (d) was very afraid

- ⊗ (c) 'was rather angry' is correct option. All tricks of conjuror were ruined due to the Quick man's remarks.

17. "The egg trick was ruined". This means that

- (a) eggs were all broken
(b) people were unconvinced
(c) conjuror was disappointed
(d) the trick could not be performed

- ⊗ (b) The egg trick was ruined means that people were unconvinced.

18. According to the Quick Man, the conjuror

- (a) had everything bought for production
(b) produced things with the magic he knew
(c) had things in the large sleeves of his coat
(d) created an illusion of things with his magic

- ⊗ (c) According to the Quick man, the conjuror had things in large sleeves of his coat.

19. The author believes that the Quick Man was really

- (a) foolish (b) clever
(c) wrong (d) right

- ⊗ (a) 'foolish' as last sentence of the author shows that the Quick Man was not intelligent to presume that the conjuror must be having a live hen up his sleeve who on demand will give 17 eggs.

20. The conjuror extracted seventeen eggs from the hat of

- (a) the Quick Man
(b) his own
(c) one gentleman from the audience
(d) None of the above

- ⊗ (c) The conjuror extracted seventeen eggs from the hat of one gentleman from the audience.

Passage-2

Magda Good morning, Smiles. Mrs. It's wet, isn't it?

Mrs. Smiles Yes, it is, isn't it? How are you today? All right? You haven't been studying too hard, have you? You look a bit pale.

Magda I don't, do I? Well, I haven't been out much lately. I don't like this weather a bit. Why, we haven't had a dry day for weeks, have we?

Mrs. Smiles No, I don't think we have. Let's see, you'll be going back home next month, won't you?

Magda Oh no, I'm not going back yet. My sister's coming over first.

Mrs. Smiles Oh, is she? You're looking forward to that, I expect. How long is it since you saw her?

Magda Nearly a year now. Yes, I am looking forward to it very much.

Mrs. Smiles She's coming over here to study?

Magda Yes, that's right, I shan't go back home until she's settled down. I don't think I ought to, ought I?

Mrs. Smiles Well, no, she's younger than you are, is she?

Magda Yes, she's only eighteen.

21. Magda had not been out much lately because

- (a) she had not been keeping well
- (b) she was busy with her studies
- (c) the weather had been unpleasant
- (d) her sister had come over to stay

➤ (c) As stated in the passage Magda had not been out much lately because it has been raining continuously. It also implies that 'the weather has been unpleasant'.

22. Magda was not going back home yet because

- (a) it had been raining heavily
- (b) her sister was coming over
- (c) her studies were not completed
- (d) she was not feeling well

➤ (b) 'her sister was coming over' so Magda was not planning to go back home.

23. Mrs. Smiles says, "You're looking forward to that". This means, Magda was her sister's arrival.

- (a) expecting with pleasure
- (b) preparing hard for
- (c) thinking greatly about
- (d) watching with joy

➤ (a) Magda was expecting with pleasure her sister's arrival

24. Magda's sister was coming to
(a) visit her sister (b) help Mrs. Smiles
(c) settle down in England

(d) pursue her studies

➤ (d) Magda's sister was coming to pursue her studies.

25. Mrs. Smiles and Magda are

- (a) classmates (b) school friends
- (c) neighbours (d) sisters-in-law

➤ (c) From the conversation shown in the passage between Mrs. Smiles and Magda, it appears that both are neighbours.

Directions (Q.Nos. 26-35) Each of the following sentences in this section has a blank space and four words or groups of words given after the sentence. Select whichever word or group of words you consider most appropriate for the blank space.

26. We fail to understand your reasons for the college without completing the degree.

- (a) attending (b) joining
- (c) leaving (d) refusing

➤ (c)

27. She her energy and started shouting only when she heard the noise of bulldozers and cranes.

- (a) checked (b) conserved
- (c) maintained (d) controlled

➤ (b)

28. The British all over Africa and Asia collapsed in the first half of the twentieth century.

- (a) hegemony (b) domicile
- (c) residence (d) inheritance

➤ (a)

29. The football match has to be because of the weather.

- (a) called off (b) continued
- (c) put off (d) turned off

➤ (a)

30. If I were rich, a lot.

- (a) I'll travel
- (b) I can travel
- (c) I would travel
- (d) I travelled

➤ (c)

31. They apologised me for what happened.

- (a) to (b) at
- (c) for (d) with

➤ (a)

32. If you are tired of swimming, just for a while.

- (a) struggle (b) stroke
- (c) float (d) streak

➤ (c) 'float' is appropriate word here as floating in the water will not tire anyone.

33. He had spoken for two minutes when there was a commotion in the crowd.

- (a) even (b) hardly
- (c) often (d) little

➤ (b) 'hardly' is correct here. The word means 'not' much, or 'scarcely'.

34. I would not commit myself that course of action.

- (a) of (b) to
- (c) by (d) with

➤ (b)

35. The soldiers waiting to go into battle for the first time were full of

- (a) apprehension (b) consideration
- (c) anticipation (d) frivolity

➤ (a)

Directions (Q.Nos. 36-40) Given below are some idioms / phrases followed by four alternative meanings to each. Choose the response (a), (b), (c) or (d) which is the most appropriate meaning.

36. A red-letter day

- (a) a trivial day
- (b) a very important or significant day
- (c) a day of bloodshed and violence
- (d) a mourning day

➤ (b)

37. The gift of the gab

- (a) ability to speak easily and confidently
- (b) ability to spoil something
- (c) ability to sell things
- (d) gift from a sacred institution

➤ (a)

38. Walk a tightrope

- (a) to be forced to leave your job
- (b) to be ready to fail
- (c) to act very carefully
- (d) to invite danger

➤ (c)

39. To be in a fix

- (a) to receive strong criticism
- (b) to support oneself
- (c) to fix problems
- (d) to be in a difficult situation

➤ (d)

40. To fish in troubled waters

- (a) to borrow money
- (b) to steal belongings of
- (c) to get benefit in bad situation
- (d) to extend a helping hand

➤ (c)

Directions (Q.Nos. 41-45) *Each of the following items in this section consists of a sentence the parts of which have been jumbled. These parts have been labelled P, Q, R and S. Given below each sentence are four sequences namely (a), (b), (c) and (d). You are required to rearrange the jumbled parts of the sentence.*

- 41.** Domestic fires in Indian villages
P. as the heat is dispersed
Q. in unhealthy smoke accumulates in unventilated houses
R. are wasteful
S. lit in Chulhas

The correct sequence should be

- (a) PQSR (b) PQRS
(c) SPQR (d) SRPQ

➤ (d)

- 42.** Bold rocks near at hand
P. makes a fine forest for the imagination
Q. than distant Alps
R. are more inspiring
S. and the thick fern upon a health

The correct sequence should be

- (a) QRSP (b) RSQP
(c) RQSP (d) SRQP

➤ (c)

- 43.** Newton discovered that
P. due to the gravitational pull
Q. of the earth

- R. the apple falls
S. on the ground

The correct sequence should be

- (a) QPRS (b) RSPQ
(c) QSPR (d) SQRP

➤ (b)

- 44.** The knocking
P. were still in the house
Q. for some time
R. although the echoes of it
S. ceased

The correct sequence should be

- (a) SRPQ (b) SQRP
(c) PQSR (d) RSPQ

➤ (b)

- 45.** The history of mankind is
P. and steady progress
Q. continuous change
R. the history of
S. from barbarism to refinement

The correct sequence should be

- (a) PQRS (b) RQPS
(c) QSPR (d) SQRP

➤ (b)

Directions (Q.Nos. 46-50) *In the following passage there are some blank spaces with four words or groups of words given. Select whichever word or group of words you consider most appropriate for the blank space.*

The honeybee is a very unusual kind of insect. **46.** other insects which live

- (a) Unlike
(b) Similar to
(c) With
(d) Like

➤ (a)

alone, the honeybee lives as a/an **47.** of a community. These bees live

- (a) group (b) individual
(c) member (d) troop

➤ (c)

together in what is known as a bee **48.** The head of the bees is called the

- (a) house. (b) home.
(c) army, (d) colony

➤ (d)

queen bee. She is the **49.** than the rest of the bees. Her main task

- (a) heaviest (b) heavier
(c) largest (d) larger

➤ (d)

is to **50.** eggs.

- (a) lay (b) hatch
(c) make (d) bear

➤ (a)

Part B (General Studies)

- 51. Statement I** The pitch of sound wave depends upon its frequency.
Statement II The loudness of the sound wave depends upon its amplitude.

➤ (b) Pitch of sound wave depends upon its frequency because pitch of a sound is actually its property due, to which we can differentiate between a shrill and grave sound. The amplitude of a sound wave determines its loudness or volume.

- 52. Statement I** Sound wave cannot propagate in vacuum.
Statement II Sound waves are elastic waves and require a medium to propagate.

➤ (a) Sound waves are elastic waves because their propagation depends on the material media, which possess elasticity and inertia. So it cannot propagate in vacuum.

- 53. Statement I** The Government of India Act, 1935 introduced Dyarchy at the centre.

Statement II The provincial autonomy was granted to the Provinces.

➤ (b) Government of India Act of 1935 marked a milestone towards a completely responsible government in India. It was a lengthy and detailed document having 321 Sections and 10 Schedules.

The Act provided for the adoption of dyarchy at the Centre. Consequently, the federal subjects were divided into reserved subjects and transferred subjects. And abolished dyarchy in the provinces and introduced 'provincial autonomy' in its place.

- 54. Statement I** Mughal Painting reached its climax during the reign of Jahangir.

Statement II Aurangzeb's Court was adorned by some of the best known artists of the Mughal School of Paintings.

➤ (c) During the reign of Jahangir (1605–1627) the Mughal painting reached its zenith. He favoured paintings of events from his own life rather than illustrated fiction. He encouraged portraiture and scientific studies of birds, flowers and animals, which were collected in albums. Mansur and Manohar were among his famous painters.

But during the reign of Aurangzeb (1659–1707) the Mughal painting started to decline, although his ornate Pearl Mosque (1662) at Delhi is worthy of mention. Many artists then joined Rajput courts, where their influence on Hindu painting is clearly evident.

- 55. Statement I** Phytoplanktons produce most of the organic carbon in the ocean.

Statement II Algae are produced in the cold water biome.

➤ (d) Algae are a diverse group of aquatic organisms that have the ability to conduct photosynthesis, for example, seaweeds

(such as kelp or phytoplankton), pond scum or the algal blooms in lakes. They are produced in the cold water biome.

Phytoplanktons are similar to terrestrial plants. They contain chlorophyll and require sunlight in order to live and grow. They produce oxygen as a byproduct of photosynthesis.

- 56. Statement I** Geostrophic wind blows above a height of 600 m, parallel to the isobars.

Statement II Geostrophic wind is the horizontal wind velocity, in which the Coriolis force balances the horizontal pressure force.

- ⊙ (c) An air parcel initially at rest, but it moves from high pressure to low pressure because of the Pressure Gradient Force (PGF).

However, as that air parcel begins to move, it is deflected by the Coriolis force to the right in the Northern hemisphere (to the left on the Southern hemisphere).

As the wind gains speed, the deflection increases until the Coriolis force equals the pressure gradient force. At this point, the wind will be blowing parallel to the isobars.

When this happens, the wind is referred to as geostrophic wind. It blows above the height of 600 m at the upper atmosphere.

- 57. Which one of the following groups of cellular organelles contains DNA?**

- (a) Mitochondria, nucleus, chloroplast
(b) Mitochondria, Golgi bodies, nucleus
(c) Mitochondria, plasma membrane, nucleus
(d) Chloroplast, nucleus, ribosomes

- ⊙ (a) The Mitochondria, nucleus and chloroplast contain DNA. Mitochondria and chloroplasts are known as semiautonomous cell organelles because they can replicate themselves.

- 58. One of the additional function of Smooth Endoplasmic Reticulum (SER) is**

- (a) protein synthesis
(b) lipid synthesis
(c) storage of biomolecules
(d) detoxification of toxic substances

- ⊙ (d) Detoxification of toxic substances is the additional function of Smooth Endoplasmic Reticulum (SER).

- 59. Damage to the apical meristem of a growing young plant will affect the**

- (a) length of the plant
(b) colour of the flower
(c) colour of the leaves
(d) taste of the fruits

- ⊙ (a) Damage to the apical meristem of a growing young plant will affect the length of the plant. It is responsible for apical growth.

- 60. Which of the following kingdom(s) has/have only unicellular organisms?**

- (a) Monera (b) Protista
(c) Monera and Protista
(d) Protista and Fungi

- ⊙ (c)

- 61. Which one of the following is a waterborne disease?**

- (a) Jaundice (b) Tuberculosis
(c) Rabies (d) Arthritis

- ⊙ (a) Jaundice is caused by microbial infection present in water and food items.

- 62. The atomic number of an element is 8. How many electrons will it gain to form a compound with sodium?**

- (a) One (b) Two (c) Three (d) Four

- ⊙ (b) The atomic number of an element = 8

Electronic configuration = $2 \text{K} \text{L} \text{M}$

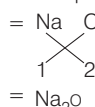
∴ Valency = 8 – 6 = 2

Sodium (Atomic number 11) has electronic configuration = $2 \text{K} \text{L} \text{M}$

∴ Valency = 1

Thus, the given element will gain 2 electrons from two sodium atoms to complete its octet.

Formula of the compound



- 63. A sample of oxygen contains two isotopes of oxygen with masses 16 μ and 18 μ respectively. The proportion of these isotopes in the sample is 3 : 1. What will be the average atomic mass of oxygen in this sample?**

- (a) 17.5 μ (b) 17 μ
(c) 16 μ (d) 16.5 μ

- ⊙ (d) Average atomic mass of an element

$$= \left[\frac{\text{Atomic mass of isotope I} \times \% \text{age of isotope I}}{100} \right] + \left[\frac{\text{Atomic mass of isotope II} \times \% \text{age of isotope II}}{100} \right]$$

As it is given that the proportion of two isotopes of oxygen with masses 16 μ and 18 μ is 3 : 1 respectively. So, they have percentage composition 75% and 25% respectively.

∴ Average atomic mass of oxygen in the sample

$$= \left[16 \times \frac{75}{100} \right] + \left[18 \times \frac{25}{100} \right] = 16.5 \mu$$

- 64. Which one of the following is a heterogeneous mixture?**

- (a) Hydrochloric acid
(b) Vinegar
(c) Milk
(d) Soda water

- ⊙ (c) A mixture that does not have uniform composition is called heterogeneous mixture.

Out of the given compounds, milk is a heterogeneous mixture of fats, carbohydrates, proteins etc. dispersed in water.

- 65. What is the formula mass of anhydrous sodium carbonate? (Given that the atomic masses of sodium, carbon and oxygen are 23 μ, 12 μ and 16 μ respectively)**

- (a) 286 μ (b) 106 μ
(c) 83 μ (d) 53 μ

- ⊙ (b) Chemical formula of (anhydrous) sodium carbonate = Na_2CO_3

∴ Formula unit mass of Na_2CO_3

$$= (2 \times 23 + 12 + 16 \times 3) \mu$$

$$= 106 \mu$$

- 66. Which one of the following is called 'syngas'?**

- (a) C (s) + H_2O (g) (b) CO (g) + H_2O (g)
(c) CO (g) + H_2 (g) (d) NO_2 (g) + H_2 (g)

- ⊙ (c) Syngas or synthesis gas is mixture of Carbon monoxide (CO) gas and Hydrogen (H_2) gas. It may also consist of Carbon Dioxide (CO_2) gas. It is used as a fuel gas in various industries.

- 67. The frequency of ultrasound waves is**

- (a) less than 20 Hz
(b) between 20 Hz and 2 kHz
(c) between 2 kHz and 20 kHz
(d) greater than 20 kHz

- ⊙ (d) Ultrasonic waves range above 20 kHz which is above the audible range of human ears.

- 68. The magnetic field strength of a current carrying wire at a particular distance from the axis of the wire**

- (a) depends upon the current in the wire
(b) depends upon the radius of the wire
(c) depends upon the temperature of the surroundings
(d) None of the above

- ⊙ (a) The magnetic field strength of a current carrying wire at a particular distance from the axis of the wire is given by the Biot-Savart's law.

$$\text{Magnetic field, } dB \propto \frac{id \sin \theta}{r^2}$$

Where i = current in wire.

r = distance of the point from the wire.

$$dB \propto \frac{1}{r^2}$$

Hence option (a) is correct.

69. A stainless steel chamber contains Ar gas at a temperature T and pressure P . The total number of Ar atoms in the chamber is n . Now Ar gas in the chamber is replaced by CO_2 gas and the total number of CO_2 molecules in the chamber is $n/2$ at the same temperature T . The pressure in the chamber now is P' . Which one of the following relations holds true? (Both the gases behave as ideal gases)

- (a) $P' = P$ (b) $P' = 2P$
(c) $P' = P/2$ (d) $P' = P/4$

⊗ (c) For Ar gas $PV = nRT$
$$P = \frac{nRT}{V} \quad \dots(i)$$

For CO_2 gas, $n' = \frac{n}{2}$, $P = P'$

$$P'V = n'RT$$

$$P' = \frac{n'RT}{V}$$

From Eqn. (i), we get $P' = \frac{P}{2}$

70. Which one of the following is the correct relation between Å and nm?

- (a) $1 \text{ nm} = 10^{-1} \text{ Å}$ (b) $1 \text{ nm} = 10 \text{ Å}$
(c) $1 \text{ nm} = 1 \text{ Å}$ (d) $1 \text{ nm} = 10^{-2} \text{ Å}$

⊗ (b) $1 \text{ Å} = 0.1 \text{ nm}$
 $\Rightarrow 10 \text{ Å} = 1 \text{ nm}$

71. The full form of LED is

- (a) Light Emitting Diode
(b) Light Emitting Device
(c) Light Enhancing Device
(d) Light Enhancing Diode

- ⊗ (a) The full form of LED is Light Emitting Diode. It is a heavily doped $p-n$ junction diode which converts electrical energy into light energy.

72. If a free electron moves through a potential difference of 1 kV, then the energy gained by the electron is given by

- (a) $1.6 \times 10^{-19} \text{ J}$ (b) $1.6 \times 10^{-16} \text{ J}$
(c) $1 \times 10^{-19} \text{ J}$ (d) $1 \times 10^{-16} \text{ J}$

⊗ (b) Given, Potential difference,
 $V = 1 \text{ kV} = 1 \times 10^3 \text{ V}$

We know that,

$$\text{Energy gained by electron} = e \cdot V$$

$$= 1.6 \times 10^{-19} \times 10^3 = 1.6 \times 10^{-16} \text{ J}$$

73. Consider the following places of India :

1. Itanagar 2. Imphal
3. Agartala 4. Aizawl

Which one of the following is the correct chronological order of the above places in terms of sunrise time?

- (a) 3 - 2 - 1 - 4 (b) 2 - 1 - 4 - 3
(c) 1 - 4 - 3 - 2 (d) 4 - 3 - 2 - 1

- ⊗ (b) The correct sequence in order of Sunrise timing in the following places are as:

At Imphal, the capital city of Manipur, the sunrise time is 4:59 AM.

At Itanagar, the capital city of Arunachal Pradesh, the sunrise time is 5:00 AM.

At Aizawl, the capital city of Mizoram, the sunrise time is 5:05 AM.

At Agartala, the capital city of Tripura, the sunrise time is 5:10 AM.

74. Which one of the following is known as uplands of delta region?

- (a) Baf (b) Bils
(c) Peh (d) Chars

- ⊗ (d) The Deltaic plain is the extension of the Khadar land. It covers large area in the lower reaches of the Ganga River. In the delta region, the upland is called Chars, while the marshy area is known as Bils.

75. Consider the following Wildlife Sanctuaries of India :

1. Shikari Devi 2. Bhadra
3. Simlipal 4. Pachmarhi

Which one of the following is the correct order of the above Wildlife Sanctuaries in terms of their location from South to North?

- (a) 1-2-3-4 (b) 2-4-3-1
(c) 2-3-4-1 (d) 3-1-2-4

- ⊗ (c) The correct order of the given wildlife sanctuaries in terms of their location from South to North is

Bhadra Wildlife Sanctuary is located at Chikkamagaluru town in Karnataka. It was established in the year 1951. It is a protected area and a tiger reserve as part of Project Tiger.

Simlipal Wildlife Sanctuary, also known as Mayurbhanj Elephant Reserve, is a national park and a tiger reserve. It was established in the year 1980. It is located in the Mayurbhanj district of Odisha.

Pachmarhi Wildlife Sanctuary is also a biosphere reserve in the Satpura Range of Madhya Pradesh. It was established in the year 1999.

Shikari Devi Wildlife Sanctuary is located near the Shikari Devi Temple in Mandi district of Himachal Pradesh. It was established in the year 1974.

76. Which one of the following statements about temperature is correct?

- (a) Temperature decreases with height in the stratosphere.
(b) Temperature is constant at different heights in the stratosphere.
(c) Temperature increases with height in the troposphere at an average rate of 6.5°C per km.
(d) Temperature decreases with height in the troposphere at an average rate of 6.4°C per km.

- ⊗ (d) The temperature of the troposphere is highest near the surface of the Earth and decreases with height or altitude. On average, the temperature gradient of the troposphere declines by 6.5°C per kilometre on going above. However, in the stratosphere, temperature increases with altitude. The reason is that the direct heat source for the stratosphere is the Sun.

77. Which one of the following is known as a zone of sharp salinity change in the vertical section of ocean?

- (a) Thermocline (b) Halocline
(c) Photocline (d) Pycnocline

- ⊗ (b) A halocline is a strong, vertical salinity gradient within a body of water. It is known as the zone of sharp salinity change in the vertical section of ocean. As salinity (in concert with temperature) affects the density of seawater, it can play an important role in its vertical stratification.

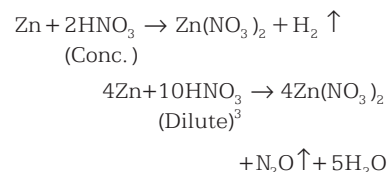
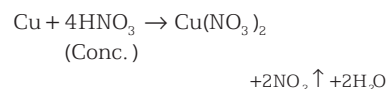
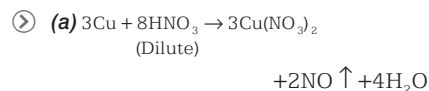
78. Permanent hardness of water cannot be removed by which one of the following methods?

- (a) Treatment with washing soda
(b) Calgon's method
(c) Boiling
(d) Ion exchange method

- ⊗ (c) Permanent hardness of water is due to the presence of chlorides and sulphates of calcium and magnesium ions. These compounds do not decompose on heating. These are removed by ion exchange resin treatment or by Calgon's method or by using a water softener like washing soda, calcium hydroxide, etc.

79. Which one of the following reactions will give NO (Nitric Oxide) gas as one of the products?

- (a) $3\text{Cu} + 8\text{HNO}_3$ (dilute) $\rightarrow$
(b) $\text{Cu} + 4\text{HNO}_3$ (conc) $\rightarrow$
(c) $4\text{Zn} + 10\text{HNO}_3$ (dilute) $\rightarrow$
(d) $\text{Zn} + 4\text{HNO}_3$ (conc) $\rightarrow$



Thus, in option (a) contains the reaction having NO gas as one of the products.

80. Which one of the following is a tribasic acid?

- (a) Hydrochloric acid
- (b) Nitric acid
- (c) Sulphuric acid
- (d) Phosphoric acid

➤ (d) H_3PO_4 , Phosphoric acid can be written as $\text{O}=\text{P}(\text{OH})_3$. It has three acidic hydrogens (attached to oxygen) and is therefore tribasic acid. On the other hand hydrochloric acid, nitric acid are monobasic while sulphuric acid is dibasic.

81. Which one of the following statements is not correct?

- (a) All carbons in diamond are linked by carbon-carbon single bond.
- (b) Graphite is layered structure in which layers are held together by weak van der Waal's forces.
- (c) Graphite layers are formed by hexagonal rings of carbon atoms.
- (d) Graphite layers are held together by carbon-carbon single bond.

➤ (d) Graphite layers are held together by van der Waals forces and not by carbon-carbon single bond.

82. Which one of the following is called Dry Ice?

- (a) Solid carbon dioxide
- (b) Liquid carbon dioxide
- (c) Liquid nitrogen
- (d) Liquid ammonia

➤ (a) Solid carbon dioxide is called Dry Ice or Cardice. It is used as a refrigerant for ice-cream and frozen food.

83. The acidic, semidigested food coming out of the stomach is neutralised by

- (a) pancreatic juice
- (b) duodenal secretion
- (c) large intestine secretion
- (d) bile juice

➤ (d) The acidic, semidigested food coming out of the stomach is neutralised by sodium bicarbonate present in the bile juice secreted by liver. Here it makes the medium of food alkaline.

84. The oxygenated blood from the lungs is received by the

- (a) left auricle
- (b) left ventricle
- (c) right auricle
- (d) right ventricle

➤ (a) The oxygenated blood from the lungs is received by left auricle through pulmonary vein from here it goes to left ventricle and then it is pumped into other body parts.

85. The oxygen evolved during photosynthesis comes from splitting of

- (a) water
- (b) carbon dioxide
- (c) oxygen
- (d) light

➤ (a) Photosynthesis is the process that plants undergo in order to transform solar energy into chemical energy which is stored in the bonds of sugar molecules. During the process, released oxygen comes from the splitting of water in the presence of light-dependent reaction. The products of the light-dependent reactions are ATP and NADPH oxygen production is simply a byproduct of the splitting of water by sunlight).

The overall process of photosynthesis is typically represented by the following equation



86. Which one of the following depicts the correct circuit of a reflex arc?

- (a) Effector → sensory neuron → spinal cord → motor neuron → receptor
- (b) Receptor → sensory neuron → spinal cord → neuron → effector
- (c) Receptor → sensory neuron → brain → motor neuron → effector
- (d) Sensory neuron → receptor → brain → effector → motor neuron

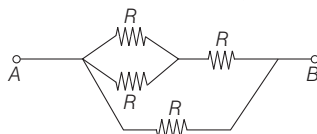
➤ (b) The correct reflex arch is depicted by option (b). It is receptor → sensory neuron → spinal cord → neuron → effector.

87. If one set of chromosomes for a given plant is represented as N ; in case of double fertilisation, the zygote and the endosperm nucleus of a diploid plant would have how many sets of chromosomes respectively?

- (a) N and $2N$
- (b) $2N$ and $2N$
- (c) N and $3N$
- (d) $2N$ and $3N$

➤ (d) The zygote will have $2N$ chromosomes while endosperm nucleus will have $3N$ set of chromosomes after double fertilisation in plant.

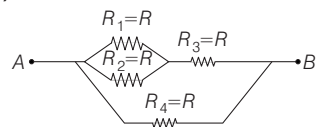
88. Consider the following circuit :



Which one of the following is the value of the resistance between points A and B in the circuit given above?

- (a) $\frac{2}{5}R$
- (b) $\frac{3}{5}R$
- (c) $\frac{3}{2}R$
- (d) $4R$

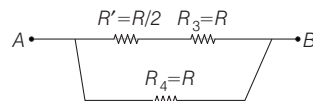
➤ (b)



In the above circuit R_1 and R_2 are connected in the parallel combination. Hence, equivalent resistance,

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} \Rightarrow R' = \frac{R}{2}$$

Now, circuit becomes



In the circuit R' and R are connected in series combination, Hence

$$R'' = R/2 + R = \frac{R+2R}{2}$$

$$R'' = \frac{3R}{2}$$

Now R'' and R_4 are connected in parallel combination. Hence, equivalent resistance between A and B.

$$\frac{1}{R_{AB}} = \frac{1}{\frac{3R}{2}} + \frac{1}{R}$$

$$\frac{1}{R_{AB}} = \frac{2}{3R} + \frac{1}{R} \Rightarrow \frac{1}{R_{AB}} = \frac{2+3}{3R}$$

$$\frac{1}{R_{AB}} = \frac{5}{3R} \Rightarrow R_{AB} = \frac{3R}{5}$$

89. The absolute zero temperature is 0 Kelvin. In $^{\circ}\text{C}$ unit, which one of the following is the absolute zero temperature?

- (a) 0°C
- (b) -100°C
- (c) -273.15°C
- (d) -173.15°C

➤ (c) The absolute zero temperature is 0°K . So, the relation between Celsius and Kelvin scale is $\frac{C}{100} = \frac{K - 273.15}{100}$ where, $K = 0$, $C = -273.15$

90. Consider the following statements about visible light, UV light and X-rays:

1. The wavelength of visible light is more than that of X-rays.
2. The energy of X-ray photons is higher than that of UV light photons.
3. The energy of UV light photons is less than that of visible light photons.

Which of the statement(s) given above is/are correct?

- (a) 1, 2 and 3
- (b) 1 and 2
- (c) 2 and 3
- (d) Only 1

➤ (b) (1) The frequency of X-ray is greater than visible light. So, the wavelength of visible light is more than that of X-rays.

(2) We know that, $E = h\nu$

Energy (E) $\propto \nu$ (frequency)

X-ray have higher frequency so it has higher energy than that of UV light photons.

91. The time period of oscillation of a simple pendulum having length L and mass of the bob m is given as T . If the length of the pendulum is increased to $4L$ and the mass of the bob is increased to $2m$, then which one of the following is the new time period of oscillation?

(a) T (b) $2T$ (c) $4T$ (d) $\frac{T}{2}$

- ⊗ (b) Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \dots(i)$$

According to the question,

$$L' = 4L$$

New time period of simple pendulum,

$$T' = 2\pi\sqrt{\frac{4L}{g}} = 2 \times \left[2\pi\sqrt{\frac{L}{g}} \right]$$

From equation (i), we get
 $T' = 2T$

92. The connecting cable of electrical appliances like electric iron, water heater or room heater contains three insulated copper wires of three different colours—red, green and black. Which one of the following is the correct colour code?

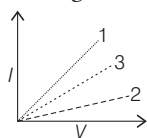
- (a) Red—live wire, Green—neutral wire, Black—ground wire
(b) Red—neutral wire, Green—ground wire, Black—live wire
(c) Red—live wire, Green—ground wire, Black—neutral wire
(d) Red—ground wire, Green—live wire, Black—neutral wire

- ⊗ (c) Red-Live wire (brings the current)

Green — Ground wire

Black — Neutral wire

93. The graphs between current (I) and voltage (V) for three linear resistors 1, 2 and 3 are given below :



If R_1 , R_2 and R_3 are the resistances of these resistors, then which one of the following is correct?

- (a) $R_1 > R_2 > R_3$ (b) $R_1 < R_3 < R_2$
(c) $R_3 < R_1 < R_2$ (d) $R_3 > R_2 > R_1$

- ⊗ (b) We know that,

$$\text{Slope of graph} \propto \frac{1}{\text{Resistance}}$$

In the graph,

$$\text{slope of 1} > \text{slope of 3} > \text{slope of 2}$$

$$R_1 < R_3 < R_2$$

94. Consider the following statements about a microscope and a telescope :

- Both the eyepiece and the objective of a microscope are convex lenses.
- The focal length of the objective of a telescope is larger than the focal length of its eyepiece.
- The magnification of a telescope increases with the increase in focal length of its objective.
- The magnification of a microscope increases with the increase in focal length of its objective.

Which of the statements given above are correct?

- (a) 1 and 3 (b) 1 and 4
(c) 2, 3 and 4 (d) 1, 2 and 3

- ⊗ (d)

- Microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.
- The focal length of the objective of a telescope is larger than the focal length of eyepiece because objective has much larger aperture than the eyepiece.
- Magnification of telescope is directly proportional to the focal length of objective (f_o). Hence, the focal length of telescope increases with the increase in focal length of its objective. Hence, statements (1), (2) and (3) are the correct.

95. A planet has a mass M_1 and radius R_1 . The value of acceleration due to gravity on its surface is g_1 . There is another planet 2, whose mass and radius both are two times that of the first planet. Which one of the following is the acceleration due to gravity on the surface of planet 2?

- (a) g_1 (b) $2g_1$
(c) $g_1/2$ (d) $g_1/4$

- ⊗ (c) Acceleration due to gravity at planet is given by

$$g_1 = \frac{GM_1}{R_1^2} \quad \dots(i)$$

For the second planet (2)

$$M_2 = 2M_1, R_2 = 2R_1$$

Now, acceleration due to gravity,

$$g_2 = \frac{G(2M_1)}{(2R_1)^2}$$

$$g_2 = \frac{1}{2} \times \frac{GM_1}{R_1^2}$$

From Eqn. (i), we get $g_2 = \frac{g_1}{2}$

96. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (River Basin)	List II (Town)
A. Bhagirathi	1. Lansdowne
B. Alaknanda	2. Narendra Nagar
C. Nayar	3. Uttarkashi
D. Ganga	4. Pauri

Code

- A B C D
(a) 3 1 4 2
(b) 3 4 1 2
(c) 2 4 1 3
(d) 2 1 4 3

- ⊗ (b) Uttarkashi lies in Bhagirathi basin. Pauri lies in Alaknanda basin. Lansdowne lies in Nayar basin. Narendra Nagar lies in Ganga basin.

97. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (Local Wind)	List II (Place)
A. Yamo	1. Sudan
B. Black Roller	2. France
C. Bise	3. Japan
D. Haboob	4. North America

Code

- A B C D A B C D
(a) 1 4 2 3 (b) 1 2 4 3
(c) 3 4 2 1 (d) 3 2 4 1

- ⊗ (c) Yamo is warm and dry wind that blows down the steep valleys in Japan.

Black roller is a very strong and dust-laden wind blowing from the South-West or North-West in the Great Plains of North America.

Bise is a cold, dry wind in Switzerland and France.

A haboob is a type of intense dust storm that blows in Sudan.

98. 'Majuli', the river island, is located in which one of the following rivers?

- (a) Jamuna
(b) Padma
(c) Ganga
(d) Brahmaputra

- ⊗ (d) Majuli is a river island, located in Asom. It is formed by river Brahmaputra. In 2016, Majuli became the first island to be made a district in India. It is often claimed to be the world's largest river island.

99. Which one of the following Indian States has recorded negative growth of population as per Census 2011?

- (a) Manipur (b) Mizoram
(c) Tripura (d) Nagaland

➤ (d) As per Census 2011, Nagaland is the only state of the country which has negative growth rate of -0.58%. Growth rate of Indian population was 17.69% between 2001 and 2011.

100. Which one of the following types of cloud is characterised by continuous precipitation?

- (a) Cirrocumulus (b) Cumulus
(c) Nimbostratus (d) Cumulonimbus

➤ (c) Clouds are visible accumulations of tiny water droplets or ice crystals in the Earth's atmosphere. Nimbostratus clouds bring continuous precipitation that can last for many hours. These low-level clouds are full of moisture.

101. Match List I with List II and select the correct answer using the code given below the Lists :

List I (Major Port)	List II (Location)
A. Kolkata	1. Land-locked area
B. Mormugao	2. In the delta region
C. Visakhapatnam	3. On the riverside
D. Paradip	4. On the entrance of the estuary

Code

- | | | | | | | | | | |
|-----|---|---|---|---|-----|---|---|---|---|
| A | B | C | D | A | B | C | D | | |
| (a) | 3 | 1 | 4 | 2 | (b) | 3 | 4 | 1 | 2 |
| (c) | 2 | 4 | 1 | 3 | (d) | 2 | 1 | 4 | 3 |

➤ (b) The Port of Kolkata is a riverine port in the city of Kolkata. It is the oldest operating port and was constructed by the British East India Company.

Mormugao is main port of Goa. It is a natural port and on the entrance of the estuary. Visakhapatnam Port is the deepest land-locked and protected port of India. It is located in Andhra Pradesh. It is India's second largest port by volume of cargo handled.

Paradip Port is a natural, deep-water, and deltaic port of India in Odisha. It is situated at confluence of the Mahanadi River and the Bay of Bengal.

102. Which one of the following is not an agent of metamorphism?

- (a) Heat (b) Compression
(c) Decomposition (d) Solution

➤ (c) The literary meaning of metamorphism is to change form. The agents of metamorphism include heat, pressure

(stress, compression), and chemically active like fluids, solution, etc.

During metamorphism, rocks are often subjected to all three metamorphic agents simultaneously. However, the degree of metamorphism and the contribution of each agent vary greatly from one environment to another.

103. The solution of which one of the following will have pH less than 7?

- (a) NaOH (b) KCl
(c) FeCl₃ (d) NaCl

➤ (c) FeCl₃ is a salt of strong acid (HCl) and weak base Fe(OH)₃. As a result FeCl₃ is an acidic salt and hence in solution form its pH lies below 7.

104. Which one of the following is an oxidation-reduction reaction?

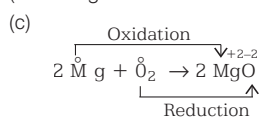
- (a) NaOH + HCl → NaCl + H₂O
(b) CaO + H₂O → Ca(OH)₂
(c) 2Mg + O₂ → 2MgO
(d) Na₂SO₄ + BaCl₂ → BaSO₄ + 2NaCl

➤ (c) (a) $\overset{+1}{\text{Na}}\overset{-1}{\text{OH}} + \overset{+1}{\text{H}}\overset{-1}{\text{Cl}} \rightarrow \overset{+1}{\text{Na}}\overset{-1}{\text{Cl}} + \overset{+1}{\text{H}}\overset{-2}{\text{O}}$

(No change in oxidation states)

(b) $\overset{+2}{\text{Ca}}\overset{-2}{\text{O}} + \overset{+1}{\text{H}}\overset{-2}{\text{O}} \rightarrow \overset{+2}{\text{Ca}}\overset{-2}{\text{O}}\overset{+1}{\text{H}}\overset{-2}{\text{O}}$

(No change in oxidation states)



(d) $\overset{+1}{\text{Na}}\overset{-2}{\text{SO}_4} + \overset{+1}{\text{Ba}}\overset{-2}{\text{Cl}_2} \rightarrow \overset{+2}{\text{Ba}}\overset{-2}{\text{SO}_4} + 2\overset{+1}{\text{Na}}\overset{-1}{\text{Cl}}$

(No change in oxidation states)

∴ It is clear from the above equations, that in option (c), the reaction involves both oxidation and reduction. Thus, it is a redox reaction.

105. Which one of the following is not used as fertilizer?

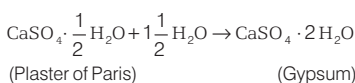
- (a) Ammonium nitrate
(b) Ammonium sulphide
(c) Ammonium phosphate
(d) Ammonium sulphate

➤ (b) Ammonium sulphide is not used as fertilizer. Rest of the given compounds are used as fertilizers to compensate the deficiency of nitrogen, phosphorus and sulphur in the soil.

106. Which one of the following is the chemical formula of gypsum?

- (a) CaSO₄·2H₂O (b) Ca₂SiO₄
(c) 2CaSO₄·H₂O (d) CaSO₄

➤ (a) Gypsum has the chemical formula CaSO₄·2H₂O. It can be prepared by mixing plaster of paris with water.



107. Which one of the following statements about the law of conservation of mass is correct?

- (a) A given compound always contains exactly same proportion of elements.
(b) When gases combine in a reaction, they do so in a simple ratio by volume, provided all gases are at room temperature.
(c) Matter can neither be created nor destroyed.
(d) Equal volumes of all gases at same temperature and pressure contain equal number of molecules.

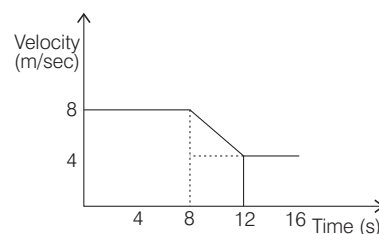
➤ (c) Law of conservation of mass states that matter can neither be created nor be destroyed.

108. The wavelength of X-rays is of the order of

- (a) 1 Å (b) 1 μm (c) 1 mm d. 1 cm

➤ (a) X-Ray is an electromagnetic wave which is discovered by German professor Rontgen. The wavelength of X-rays is of the order of 1 Å. It is used in surgery to detect the fracture, diseased organs, stones in the body.

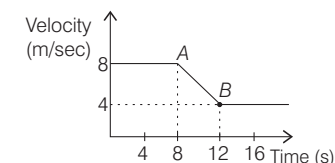
109. Consider the following velocity and time graph



Which one of the following is the value of average acceleration from 8sec to 12sec?

- (a) 8 m/sec² (b) 12 m/sec²
(c) 2 m/sec² (d) -1 m/sec²

➤ (d)



Average acceleration from 8 sec to 12 sec, is given by

$$a_{\text{avg}} = \frac{\text{Velocity at B} - \text{Velocity at A}}{\text{Time Interval}}$$

$$= \frac{4 - 8}{(12 - 8)} = \frac{-4}{4} = -1 \text{ m/sec}^2$$

110. If the focal length of a convex lens is 50 cm, which one of the following is its power?

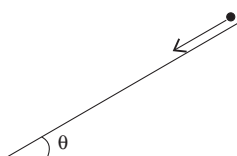
- (a) +2 dioptre (b) +0.02 dioptre
(c) -0.5 dioptre (d) +0.5 dioptre

- ③ (a) Given, Focal length of lens, $f = 50$ cm, $= 0.5$ m.

We know that,

$$\text{Power of lens, } P = \frac{1}{\text{Focal length (m)}} \\ = \frac{1}{0.5} = 2 \text{ dioptre}$$

- 111.** A ball is released from rest and rolls down an inclined plane, as shown in the following figure, requiring 4 sec to cover a distance of 100 cm along the plane :



Which one of the following is the correct value of angle θ that the plane makes with the horizontal?

($g = 1000 \text{ cm/sec}^2$)

- (a) $\theta = \sin^{-1} (1/9.8)$ (b) $\theta = \sin^{-1} (1/20)$
(c) $\theta = \sin^{-1} (1/80)$ (d) $\theta = \sin^{-1} (1/100)$

- ③ (c) Given,

Initial velocity of ball, $u = 0$

distance, $s = 100$ cm, time $t = 4$ sec

We know that

$$S = ut + \frac{1}{2} at^2$$

$$100 = 0 + \frac{1}{2} \times a \times (4)^2$$

$$a = \frac{100 \times 2}{4 \times 4} = \frac{25}{2} \text{ cm/sec}^2$$

Now, $a = g \sin \theta$

$$\sin \theta = \frac{a}{g} = \frac{25/2}{1000}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{1}{80} \right)$$

- 112.** The coefficient of areal expansion of a material is $1.6 \times 10^{-5} \text{ K}^{-1}$. Which one of the following gives the value of coefficient of volume expansion of this material?

- (a) $0.8 \times 10^{-5} \text{ K}^{-1}$ (b) $2.4 \times 10^{-5} \text{ K}^{-1}$
(c) $3.2 \times 10^{-5} \text{ K}^{-1}$ (d) $4.8 \times 10^{-5} \text{ K}^{-1}$

- ③ (b) We know that,

$$\frac{\text{Area expansion coefficient } (\beta)}{\text{Volumetric expansion coefficient } (\alpha)} \\ = \frac{2}{3}$$

$$\alpha = \frac{3}{2} \beta \quad \text{Where } \beta = 1.6 \times 10^{-5} \text{ K}^{-1}$$

$$\alpha = \frac{3}{2} \times 1.6 \times 10^{-5} \text{ K}^{-1}$$

$$\alpha = 2.4 \times 10^{-5} \text{ K}^{-1}$$

- 113.** The refractive indices of two media are denoted by n_1 and n_2 and the velocities of light in these two media are respectively v_1 and v_2 . If n_2/n_1 is

1.5, which one of the following statements is correct?

- (a) v_1 is 1.5 times v_2 (b) v_2 is 1.5 times v_1
(c) v_1 is equal to v_2 (d) v_1 is 3 times v_2

- ③ (a) We know that,

Refractive index =

$$\frac{\text{Velocity in vacuum (C)}}{\text{Velocity in medium (V)}}$$

$$\text{In first medium } n_1 = \frac{C}{v_1} \quad \dots\dots(i)$$

$$\text{Similarly } n_2 = \frac{C}{v_2} \quad \dots\dots(ii)$$

From equation (i) and (ii)

$$\frac{n_2}{n_1} = \frac{v_1}{v_2}$$

$$1.5 = \frac{v_1}{v_2}$$

$$\Rightarrow v_1 = 1.5v_2$$

So, option (a) is correct.

- 114.** Which one of the following greenhouse gases is in largest concentration in the atmosphere?

- (a) Chlorofluorocarbon
(b) Nitrous oxide
(c) Carbon dioxide
(d) Methane

- ③ (c) A greenhouse gas is a gas that absorbs and emits radiant energy within the thermal infrared range. In order, the most abundant greenhouse gases atmosphere are water vapour, carbon dioxide, methane, nitrous oxide, ozone, chlorofluorocarbons and hydro fluorocarbons.

- 115.** Match List-I with List-II and select the correct answer using the codes given below the Lists :

List I (Koppen's Climatic Type)	List I (Letter Code)
A. Tropical wet	1. Af
B. Mid-latitude desert	2. Cs
C. Mediterranean	3. Df
D. Humid Continental	4. BWk

Code

- A B C D A B C D
(a) 1 4 2 3 (b) 1 2 4 3
(c) 3 2 4 1 (d) 3 4 2 1

- ③ (a)

Type	Type	Characteristics
Tropical Wet	Af	No dry season
Mid-latitude arid or dry	BWk	Mid-latitude Desert
Mediterranean	Cs	Dry hot summer
Humid-continental	Df	No dry season, severe winter

- 116.** Match List I with List II and select the correct answer using the codes given below the Lists :

List I (Railway Zone)	List II (Headquarters)
A. West Central	1. Jabalpur
B. South-East Central	2. Gorakhpur
C. East-Central	3. Bilaspur
D. North-Eastern	4. Hajipur

Code

- A B C D A B C D
(a) 2 3 4 1 (b) 1 4 3 2
(c) 2 4 3 1 (d) 1 3 4 2

- ③ (d) Indian Railways is divided into 17 zones, which are further sub-divided into divisions. Each zonal railway is made up of a certain number of divisions, each having a divisional headquarter. There are a total of sixty-eight divisions.

Railway Zone	Headquarters
West Central	Jabalpur
South-East Central	Bilaspur
East-Central	Hajipur
North-Eastern	Gokakhpur

- 117.** Which one of the following is the correct descending order of Indian States in terms of sex ratio as per Census 2011?

- (a) Mizoram — Manipur — Tripura — Meghalaya
(b) Tripura — Manipur — Meghalaya — Mizoram
(c) Meghalaya — Manipur — Mizoram — Tripura
(d) Manipur — Meghalaya — Tripura — Mizoram

- ③ (c) Sex ratio is used to describe the number of females per 1000 of males. In India, it is especially significant because the ratio is heavily skewed towards men. In the Population Census of 2011, it was revealed that the population ratio of India 2011 is 943 females per 1000 of males.

As per final data (given by Union Minister of Health and Family Welfare) the correct data is

State	Sex Ratio (2011)
Meghalaya	989
Manipur	987
Mizoram	976
Tripura	960

But according to the provisional data of Census 2011, the correct descending order is Meghalaya (989)-Manipur (987)-Mizoram (976)-Tripura (960).

Hence, option (c) is correct.

118. Steppe (temperate continental) climate is not experienced in which one of the following places?

- (a) Pretoria (b) Saskatchewan
(c) Perth (d) Buenos Aires

➤ (a) Pretoria is a city in the Northern part of Gauteng province in South Africa. Pretoria has a humid sub-tropical climate with long hot rainy summers and short cool to cold, dry winters.

The mid latitude steppe climate spread over temperate grasslands is located in the interiors of the continents, which come in the westerly wind belt, but because of their more interior locations they do not get sufficient rainfall and hence the grasslands are practically treeless.

119. Which one of the following is not a process of chemical weathering?

- (a) Solution (b) Carbonation
(c) Oxidation (d) Exfoliation

➤ (d) Chemical weathering is caused by rainwater reacting with the mineral grains in rock to form new minerals and soluble salts. These reactions occur particularly when the water is slightly acidic. Different types of chemical weathering are solution, oxidation and carbonation. These chemical processes occur more rapidly at higher temperature.

Weathering is the breakdown of rocks at the Earth's surface, by the action of rainwater, extremes of temperature and biological activity. It does not involve the removal of rock material.

120. Which one of the following statements is correct for a plane mirror?

- (a) Its focal length is zero.
(b) The size of the image of an object placed in front of the mirror is slightly less than that of the object.
(c) The image is virtual, erect and laterally inverted.
(d) Its focal length is 200 cm.

➤ (c) In a plane mirror, image formed is virtual, erect at the same distance behind the mirror as the object is in front of mirror the focal length of plane mirror is infinite and power is zero.

121. An object is placed in front of a convex mirror. Which one of the following statements is correct?

- (a) It will never form an inverted image.
(b) The image moves towards the focus when the object moves towards the mirror.
(c) Depending on the position of the object with respect to the mirror, the image can be inverted and real.

(d) The size of the image becomes larger than that of the object when the object is placed at a distance equal to half the focal length.

➤ (a) The convex mirror will never form an inverted image.

122. A circular coil of radius R having N number of turns carries a steady current I . The magnetic induction at the centre of the coil is 0.1 tesla. If the number of turns is doubled and the radius is halved, which one of the following will be the correct value for the magnetic induction at the centre of the coil?

- (a) 0.05 tesla (b) 0.2 tesla
(c) 0.4 tesla (d) 0.8 tesla

➤ (c) Magnetic field at the centers of coil.

$$B = \frac{\mu_0 NI}{2r}$$

$$\Rightarrow 0.1 = \frac{\mu_0 NI}{2r} \quad \dots (i)$$

According to the question,

$$B' = \frac{\mu_0 (2N)(I)}{2 \times \frac{r}{2}}$$

$$\Rightarrow B' = 4 \left(\frac{\mu_0 NI}{2r} \right)$$

From Eqs. (i), we get

$$B' = 4 \times 0.1 \Rightarrow B' = 0.4 \text{ tesla}$$

123. Which one among the following is not a Fundamental Right under the Constitution of India?

- (a) Right to equality
(b) Right to freedom
(c) Right to citizenship
(d) Right against exploitation

➤ (c) Fundamental Rights are the basic rights of the common people and inalienable rights of the people who enjoy it under the charter of rights contained in Part III (Article-12 to 35) of Constitution of India. Right to citizenship is not a fundamental right.

All the Fundamental Rights have been classified under the following six categories

Right to be Equality (Article-14-18)

Right to Freedom (Article 19-22)

Right against Exploitation (23-24)

Right to Freedom of Religion

(Article 25-28)

Cultural and Educational Rights

(Article 29-30)

Right to Constitutional Remedies (Art.32)

The Right to Property is no longer a Fundamental Right.

124. Which one of the following crops was introduced by the Portuguese in India?

- (a) Opium (b) Coffee
(c) Betel leaf (d) Chili

➤ (d) Chili is reported to be an inhabitant of South America and is widely scattered in all tropical and sub-tropical countries including India. It was first introduced in India by the Portuguese towards the end of the 15th Century.

125. Consider the following statements about merchant guilds of South India :

1. Ayyavole merchant guild was originally established in Aihole.
2. Manigraman merchant guild was sub-ordinated to the Anjuvannam merchant guild in the 13th century.

Which of the statement(s) given above is/are correct?

- (a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

➤ (a) Southern Indian trade guilds were formed by merchants in order to organise and expand their trading activities. The two most important merchant guilds of South India were known as the Ayyavole and the Manigraman. Ayyavole were a merchant guild from Aihole that provided trade links between trading communities in Tamil Nadu, Karnataka and Andhra Pradesh. Manigraman guild flourished in Tamil Nadu in the Pallava and Chola periods and was active in South-East Asia. The Manigramam merchants enjoyed a special position as compared to the other merchants.

Anjuvannam was another body of merchants in South India, which almost certainly represented an association of foreign merchants who traded across the Arabian sea. It interacted both with local merchants as well as the Ayyavole and Manigramam organisations.

126. Which one of the following pairs is correctly matched?

Bhakti Saint	Philosophy
(a) Shankara	: Avadhuta
(b) Ramananda	: Kevaladvaita
(c) Ramanuja	: Vishishtadvaita
(d) Chaintanya	: Advaita

➤ (c) Kevaladvaita is another name for the philosophy of advaita which was expounded by Adi Shankara. The meaning of Advaita is non-dualism or belief in one reality.

Philosophy of Vishishtadvaita was pioneered by Ramanujacharya during the 11th century. It literally means the Unique Advaita, i.e., Advaita with some amendments.

Avadhuta is a Sanskrit term used to refer to a person who has reached a stage in their spiritual development in which they are beyond worldly concerns.

127. When did the Stamp Act Congress consisting of delegates from 9 of the 13 colonies of America meet in New York City?

(a) 1763 (b) 1764 (c) 1765 (d) 1766

- (c) The Stamp Act Congress, or First Congress of the American Colonies, was a meeting held between October 7 and 25, 1765, in New York City. It consists of representatives from some of the British colonies in North America. At this meet, the delegates discussed and united against the stamp act, which required the use of specially stamped paper for legal documents, playing cards, calendars, newspapers and dice for virtually all business in the colonies.

128. Who among the following travellers was from Italy and visited Vijayanagar Kingdom in the fifteenth century?

(a) Nikitin (b) Fa-Hien
(c) Bernier (d) Nicolo Conti

- (d) *Important Foreign Travellers Who Visited India*

Nicolo Conti (AD 1420-1421), an Italian traveller, visited Vijayanagar empire during the reign of Devaraya-I (theTuluva dynasty ruler).

Nikitin (AD 1470-1474) was a Russian merchant, who describes the condition of the Bahmani kingdom under Muhammad III (1463-82).

Fa-Hien was a Chinese pilgrim who visited India during the reign of Chandra Gupta II. His primary aim was to visit the Buddhist religious places and to take with him the copies of the Buddhist religious texts.

François Bernier (September 25, 1620-September 22, 1688) was a French physician and traveller. He was briefly personal physician to Mughal prince Dara Shikoh (son of the Mughal emperor Shah Jahan), and was attached to the court of the emperor Aurangzeb.

129. Where did the French East India Company first establish its factory in India?

(a) Calicut (b) Surat
(c) Pondicherry (d) Masulipatnam

- (b) Among the Dutch, Danish, Portuguese and French, the French East India Company was the last to be formed. It was founded by Jean-Baptiste Colbert in 1664. Under Francis Caron, the company established first factory at Surat in 1667 and second factory was established at Masulipattanam a year later. In 1741, Joseph François Dupleix began to cherish the ambition of a French Empire in India. But the problem began to emerge when the conflict of the British and French started. The hopes of the French company were finally ruined with the arrival of Robert Clive(a British General) in India in 1744.

130. The Central Vigilance Commission was established on the recommendation of which one of the following Committees?

(a) Santhanam Committee
(b) Dinesh Goswami Committee
(c) Tarkunde Committee
(d) Narasimham Committee

- (a) The Central Vigilance Commission was set up by the government in February, 1964 on the recommendations of the Committee on Prevention of Corruption, headed by K. Santhanam, to advise and guide Central government agencies in the field of vigilance. CVC is conceived to be the apex vigilance institution, free of control from any executive authority, monitoring all vigilance activity under the Central Government and advising various authorities in Central Government organizations in planning, executing, reviewing and reforming their vigilance work. It consists of A Central Vigilance Commissioner as a chairperson; and not more than two Vigilance Commissioners as its members.

131. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (Author)	List II (Book)
A. Bal Gangadhar Tilak	1. <i>The Arctic Home in the Vedas</i>
B. Dadabhai Naoroji	2. <i>Hind Swaraj</i>
C. Mahatma Gandhi	3. <i>The Discovery of India</i>
D. Jawaharlal Nehru	4. <i>Poverty and Un-British Rule in India</i>

Code

A B C D A B C D
(a) 3 4 2 1 (b) 3 2 4 1
(c) 1 4 2 3 (d) 1 2 4 3

- (c) *The Arctic Home in the Vedas* is a history book on the origin of Aryan People by Bal Gangadhar Tilak. It propounded the theory that the North Pole was the original home of Aryans during the pre-glacial period which they had to leave due to the ice deluge around 8000 BC.

Hind Swaraj or Indian Home Rule is a book written by Mohandas K. Gandhi in 1909. In it he expresses his views on Swaraj, modern civilisation, mechanisation etc.

The Discovery of India was written by India's first Prime Minister Pt. Jawaharlal Nehru during his imprisonment in 1942-46 at Ahmednagar fort in Maharashtra, India. It is an honour paid to the rich cultural heritage of India, its history and its philosophy.

Poverty and Un-British Rule in India was authored by Dadabhai Naoroji. It brought attention to the draining of India's wealth into Britain.

132. Which one of the following is the Official Mascot of the FIFA World Cup 2018?

(a) Fuleco (b) Zakumi
(c) Pille (d) Zabivaka

- (d) The official mascot for the FIFA World Cup 2018 tournament was Zabivaka—an anthropomorphic wolf dressed in the colours of the Russian national team.

The 2018 FIFA World Cup was the 21st FIFA World Cup, an international football tournament contested by the men's national teams of the member associations of FIFA once every four years.

133. The Headquarters of the proposed National Sports University (as per the National Sports University Ordinance, 2018) will be set-up in

(a) Chhattisgarh (b) Manipur
(c) Kerala (d) West Bengal

- (b) National Sports University is a sports university proposed to be set-up in Manipur in India and three outlying campuses, one each at Lucknow, Mohali and Gwalior. It was proposed to be set-up by the National Sports University Ordinance, 2018 (the Ordinance) was designed on the lines of the National Sports University Bill, 2017.

134. Sentosa island, is located in

(a) Singapore (b) China
(c) Australia (d) Sri Lanka

- (a) Sentosa, previously called Pulau Blakang Mati, is a resort island in Singapore. It was once a British military base and a Japanese prisoner of war camp. The island was renamed Sentosa and turned into a tourist destination in 1972.

The island hosted the 2018 North Korea-United States summit between the United States President Donald Trump and Kim Jong-un of North Korea on June 12, 2018, at the Capella Hotel. Sentosa island was gazetted as a 'special event area' by the Singapore government.

135. India, in June, 2018, asserted that any mega connectivity project must respect sovereignty and territorial integrity of the countries. The project referred to above is

(a) North-South Corridor Project
(b) Belt and Road Initiative
(c) Chabahar Port
(d) Panama Canal Expansion

- (b) The One Belt One Road (OBOR) initiative is a Chinese effort which focusses on improving connectivity and cooperation among Asian countries, Africa, China and

Europe. The emphasis is on enhancing land as well as maritime routes through 'Silk Road Economic Belt' and 'Maritime Silk Road' initiatives respectively. With the exception of Bhutan, which has no diplomatic ties with China, almost every other South Asian country has signed into OBOR. But India has not signed it. The main reason behind India's opposition towards the policy is the China-Pakistan Economic Corridor (CPEC), which is a part of OBOR. Keeping this in mind, India in 2018 asserted that to be a part of any mega project, it is necessary that the project must respect sovereignty and territorial integrity of the countries.

136. Who among the following is not a member of G-7?

- (a) France (b) Germany
- (c) Russia (d) Japan

- (c) The Group of Seven (G-7) is a group of seven largest advanced economies in the world, consisting of Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. The 44th edition of G-7 summit 2018 was hosted by Canadian Prime Minister Justin Trudeau in Quebec (Canada) on June 8–9. The summit was dubbed the G-6+1 by the French Government and political commentators because of the United States withdrawal from the Iran Deal along with the Paris Agreement, rampant American protectionism and controversies with French President Emmanuel Macron and Canadian Prime Minister Justin Trudeau. The 45th G-7 summit 2019 will be held in France.

137. Who among the following leaders started the Indian Home Rule League?

- (a) Gopal Krishna Gokhale
- (b) Mahatma Gandhi
- (c) Bal Gangadhar Tilak
- (d) JB Kripalani

- (c) Between the years 1916 and 1918, the Indian independence movement witnessed the growth and spread of the home rule movement started by leaders like Bal Gangadhar Tilak and Annie Besant. The aim of the home rule movement was the attainment of home rule or a dominion status for India under the British Empire. This movement was carried out through the two home rule leagues – Indian Home Rule League and Home Rule League. While Tilak launched the Indian Home Rule League in April 1916 at Belgaum, Annie Besant launched the Home Rule League in September, 1916 at Madras. The two leagues worked closely with one another. However, they did not merge to avoid friction between both the leaders

138. Every Judge of the Supreme Court of India is appointed by

- (a) the Supreme Court Collegium
- (b) the Cabinet
- (c) the President of India
- (d) the Lok Sabha

- (c) Article-124 provides for the establishment and Constitution of Supreme Court of India, which is the Apex Court of India. The sanctioned strength of the judges is 31, consisting of the Chief Justice and 30 other judges. Every Judge of the Supreme Court shall be appointed by the President by warrant under his hand and seal after consultation with such of the Judges of the Supreme Court and of the High Courts in the States as the President may deem necessary for the purpose and shall hold office until he attains the age of 65 years.

139. The Nehru-Mahalanobis Strategy of Development was implemented for the first time by which one of the following Five-Year Plans?

- (a) First Five-Year Plan
- (b) Second Five-Year Plan
- (c) Third Five-Year Plan
- (d) Seventh Five-Year Plan

- (b) At the time of the formulation of the Second Five Year Plan, Prof. PC Mahalanobis (adviser to Late Prime Minister Jawaharlal Nehru), therefore it is called as Nehru-Mahalanobis strategy. It prepared a growth model with which he showed that to achieve a rapid long-term rate of growth it would be essential to devote a major part of the investment outlay to building of basic heavy industries.

140. Which one of the following is not a part of the Directive Principles of State Policy as enshrined in the Constitution of India?

- (a) Equal justice and free legal aid
- (b) Protection of monuments and places and objects of national importance
- (c) Protection of personal law
- (d) Separation of Judiciary from Executive

- (c) Part-IV of Indian Constitution deals with Directive Principles of our State Policy (DPSP). Basic aim of DPSPs is to set-up social and economic goals before the law makers. *Some of the (other than given option DPSP's) important DPSPs are*
- Uniform Civil Code for the citizens.
 - Provide free and compulsory education for children below 14 years.
 - To promote international peace and amity.

- Protection and improvement of environment and safeguarding of forests and wildlife.
- Organisation of Village Panchayats and to promote cottage industry.
- To bring about the prohibition of intoxicating drinks and drugs that are injurious to health.
- Equal pay for equal work.
- Participation of workers in the management of the industries.

141. The word 'socialist' was inserted into the Preamble to the Constitution of India through which one of the following Amendment Acts?

- (a) 41st Amendment Act
- (b) 42nd Amendment Act
- (c) 43rd Amendment Act
- (d) 44th Amendment Act

- (b) The preamble to the constitution of India is a brief introductory statement that sets out the guiding purpose, principles and philosophy of the Constitution. Preamble gives idea about the source of the Constitution, nature of Indian State (democratic, sovereign, socialist, secular, republic) a statement of its objectives and the date of its adoption. The 42nd Amendment Act amended the Preamble and changed the description of India from 'sovereign democratic republic' to a 'sovereign, socialist secular democratic republic', and also changed the words 'unity of the nation' to 'unity and integrity of the nation'.

142. The palace of English East India Company settlement in Madras was known as

- (a) Fort William
- (b) Fort St. George
- (c) Elphinstone Circle
- (d) Marble Palace

- (b) The English East India Company was originally known as 'The Company of Merchants of London trading into the East Indies.' It was founded by a Royal Charter, signed by Queen Elizabeth-I, on 31st December, 1600. Sir Thomas Smith was the Company's first Governor.

Soon, English open their trading centres at several parts of India. In 1613, Mughal emperor Jahangir issued a farman (permission letter) to English to establish their trade centre in India, hence British established their 1st trading factory at Surat in 1613. In 1639 AD, the East India Company obtained the lease of the city of Madras from the local king where it built Fort St. George to protect its factory.

143. Which one of the following Indian States (other than Himalayan or North-Eastern States) ranked first in the Composite Water Management Index as per the report issued by the NITI Aayog in June, 2018?

- (a) Madhya Pradesh (b) Karnataka
(c) Gujarat (d) Maharashtra

- (c) The NITI (National Institution for Transforming India) Aayog launched Composite Water Management Index to assess and improve performance in efficient management of water resources. It aims to inspire States/UTs towards efficient and optimal utilisation of water and recycling thereof with sense of urgency.

The index comprises of nine parameters and 28 indicators. The states were divided into two categories namely North-Eastern and Himalayan states and other States, to take into account different hydrological conditions across these groups. The ranking of States/UTs is given below :

Gujarat topped the list followed by Madhya Pradesh, Andhra Pradesh, Karnataka and Maharashtra in category of General States or Other States. Tripura has topped the list followed by Himachal Pradesh, Sikkim and Assam in category of North-Eastern and Himalayan states.

144. Who among the following is the ex-officio Chairman of the North-Eastern Council?

- (a) The President of India
(b) The Prime Minister of India
(c) The Union Home Minister
(d) The Union Minister of State (Independent Charge), Ministry of Development of North-Eastern Region

- (c) The North-Eastern Council is the nodal agency for the economic and social development of the North-Eastern Region which consists of the eight states of Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim and Tripura. It was constituted in 1971 by an Act of Parliament. The Union Home Minister acts as ex-officio Chairman of North-Eastern Council (NEC). The Secretariat (Headquarters) of the Council is located at Shillong, Meghalaya. The President of India nominates the Chairman of the Council.

145. Which one of the following is the theme of the World Blood Donor Day 2018?

- (a) Blood connects us all
(b) Be there for someone else. Give blood. Share life
(c) Give blood. Give now. Give often
(d) Thank you for saving my life

- (a) The host country for World Blood Donor Day 2018 events was Greece with the theme 'Blood Connects us All'. The global event was held in Athens on 14th June, 2018.

Every year, on 14 June, countries around the world celebrate World Blood Donor Day. The event serves to thank voluntary, unpaid blood donors for their life-saving gifts of blood and to raise awareness of the need for regular blood donations to ensure the quality, safety and availability of blood and blood products for patients in need.

146. 'Seva Bhoj Yojana', a scheme of the government of India that seeks to reimburse Central share of CGST and IGST on Food/Prasad/Langar/Bhandara offered by religious institutions, is introduced recently by which one of the following Ministries?

- (a) The Ministry of Culture
(b) The Ministry of Home Affairs
(c) The Ministry of Consumer Affairs, Food and Public Distribution
(d) The Ministry of Social Justice and Empowerment

- (a) The Ministry of Culture, government of India has introduced a new scheme namely 'Seva Bhoj Yojana' to provide financial assistance on purchase of specific food items by Charitable Religious Institutions (CRIs) for free distribution among people. Under this scheme, Centre's share of Central Goods and Services Tax (CGST) charged on the raw food materials purchased by the religious institutions will be refunded.

147. Dr. Bindeshwar Pathak, who was awarded Nikkei Asia Prize 2018 for Culture and Community, is the founder of

- (a) Bachpan Bachao Andolan
(b) PRS Legislative Research
(c) Sulabh Sanitation and Social Reform Movement
(d) Smile Foundation

- (c) Bindeshwar Pathak, a noted social reformer and founder of Sulabh International, was honoured with Japan's prestigious 'Nikkei Asia Prize for Culture and Community' for his significant work in tackling poor hygiene and discrimination.

Launched in 1996, the award honours people in Asia who have made significant contributions in one of the three areas—regional growth; science, technology and innovation; and culture and community. Former Prime Minister Manmohan Singh and Infosys Chairman Narayan Murthy are among the few Indians who have won the prize in the past.

148. Article-352 of the Constitution of India contains provisions related to

- (a) financial emergency
(b) failure of constitutional machinery in States
(c) suspension of the enforcement of rights conferred in Part III of the Constitution
(d) general emergency

- (d) The part 18 of Indian Constitution deals with the emergency provisions. It contains three types of emergency namely:

National Emergency : Article 352

This Article says that if the president is satisfied that a grave emergency exists whereby the security of India or any part of the territory of India is threatened by war or external aggression or armed rebellion, he may proclaim an emergency.

149. The Theosophical Society was led by

- (a) AO Hume (b) Arthur Griffith
(c) Annie Besant (d) Lord Dufferin

- (c) The Theosophical Society was founded by Madame Blavatsky and Col. Olcott in 1875 in New York. It was only in 1879, that this ideology gained its roots in India. It was crystallised in the Madras Presidency with its headquarters in Adyar. The movement was popularised by Annie Besant in India. It was based on the principles of Universal brotherhood and the study of comparative religion and philosophy. The chief aim of the Theosophical Society in India was to establish the lost glory and honour of Hindu religion. Mrs. Annie Besant became its President and worked for the upliftment of Indian mass.

150. Which one of the following statements about Bipin Chandra Pal is correct?

- (a) He was a member of the moderate group of Congress.
(b) He was a member of the extremist group of Congress.
(c) He was the Minister of Defence in the first government of independent India.
(d) He was the Chief Minister of West Bengal.

- (b) In the beginning of the 20th century, a new class of national leaders emerged in India which was different from the moderate group. They were known as extremists, who took a more aggressive stance against the British Empire and did not believe in the soft, persuasive approach of the moderate leaders. Important extremist leaders were Lala Lajpat Rai, Bal Gangadhar Tilak, Bipin Chandra Pal (Lal-Bal-Pal) leading the extremist cause in Punjab, Bombay and Bengal respectively. Their goal was 'swaraj' and did not stick to constitutional methods of protest and demand rather they resorted to boycotts, strikes, etc.