NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2018 (II)

PAPER I : Mathematics

1. What is the value of $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$ equal to ?

(a) $3 \log_2 7$ (b) $1 - 3 \log_2 7$
(c) $1 - 3 \log_7 2$ (d) $\frac{7}{8}$

(c) We have,

$$\sqrt{7\sqrt{7\sqrt{7}}} = 7^{\frac{1}{2}} \cdot 7^{\frac{1}{4}} \cdot 7^{\frac{1}{8}} = 7^{\frac{7}{8}}$$

Now, $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$

$= \log_7 \log_7 (7)^{\frac{7}{8}}$

$= \log_7 \left(\frac{7}{8}\right)$

$= \log_7 7 - \log_7 8$

$[\because \log \frac{m}{n} = \log m - \log n]$

$= \log_7 7 - \log_7 2^3 = 1 - 3 \log_7 2$

$[\because \log_b a^n = n \log_b a]$

2. If an infinite GP has the first term x and the sum 5, then which one of the following is correct?

(a) $x < -10$
(b) $-10 < x < 0$
(c) $0 < x < 10$
(d) $x > 10$

(c) Given that first term of an infinity GP is x and sum = 5

$\therefore \quad \frac{x}{1-r} = 5$

[$\because$ sum of infinity GP = $\frac{a}{1-r}$]

$\Rightarrow \quad \frac{x}{5} = 1 - r$

$\Rightarrow \quad r = 1 - \frac{x}{5}$

Where, $|r| < 1$

$\Rightarrow \quad -1 < -\frac{x}{5} < 1$

$\Rightarrow \quad -2 < -\frac{x}{5} < 0$

$\Rightarrow \quad -10 < -x < 0$

$\Rightarrow \quad 10 > x > 0$

3. Consider the following expressions

1. $x + x^2 - \frac{1}{x}$
2. $\sqrt{ax^2 + bx + x - c + \frac{d}{x} - \frac{e}{x^2}}$
3. $3x^2 - 5x + ab$
4. $\frac{2}{x^2 - ax + b^3}$
5. $\frac{1}{x} - \frac{2}{x+5}$

Which of the above are rational expressions?

(a) 1, 4 and 5 (b) 1, 3, 4 and 5
(c) 2, 4 and 5 (d) 1 and 2

(a) We know that, rational expressions are those expression which can be write in the form of $\frac{p(x)}{q(x)}, q(x) \neq 0$

So, 1, 4, 5 are rational expressions

4. A square matrix A is called orthogonal if

(a) $A = A^2$ (b) $A' = A^{-1}$
(c) $A = A^{-1}$ (d) $A = A'$

where A' is the transpose of A

(b) A square matrix is called an orthogonal matrix if $AA' = I$

multiply. by A^{-1}

$\therefore \quad A^{-1}(AA') = A^{-1}I \Rightarrow IA' = A^{-1}$

$\Rightarrow \quad A' = A^{-1}$

5. If A, B and C are subsets of a universal set, then which one of the following is not correct?

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A' \cup (A \cup B) = (B' \cap A)' \cup A$
(c) $A' \cup (B \cup C) = (C' \cap B)' \cap A'$
(d) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

where A' is the complement of A

(c) Let A, B and C are subsets of a universal set.

Let $A = \{1\}$, $B = \{2\}$, $C = \{3\}$
$\cup = \{1, 2, 3\}$, $A' = \{2, 3\}$, $B' = \{1, 3\}$,
$C' = \{1, 2\}$

by checking options, we get

LHS = $A' \cup (B \cup C)$

$= \{2, 3\} \cup \{\}$

$= \{2, 3\}$

RHS = $(C' \cap B)' \cap A'$

$= (\{1, 2\} \cap \{1, 3\}) \cap \{2, 3\}$

$= (\{2\})' \cap \{2, 3\}$

$= \{1, 3\} \cap \{2, 3\}$

$= \{3\}$

LHS $\neq$ RHS

So, option (c) is wrong

6. Let x be the number of integers lying between 2999 and 8001 which have at least two digits equal. Then x is equal to

(a) 2480 (b) 2481
(c) 2482 (d) 2483

(b) We have, x be the number lying between 2999 and 8001

if repetition allowed

total numbers = $5 \times 10 \times 10 \times 10 = 5000$

if repetition not allowed

$\therefore$ total numbers = $5 \times 9 \times 8 \times 7 = 2520$

So, x = atleast two digit repeated
$= 5000 - 2520 + 1$
$= 2481$
[$\because$ add 1 because of number 8000]

7. The sum of the series $3 - 1 + \frac{1}{3} - \frac{1}{9} + ...$ is equal to

(a) $\frac{20}{9}$ (b) $\frac{9}{20}$
(c) $\frac{9}{4}$ (d) $\frac{4}{9}$

(c) Given series

$3 - 1 + \frac{1}{3} - \frac{1}{9} + ...$ are in GP

$\therefore \quad r = \frac{-1}{3}$

$$S_n = \frac{3}{1 - \left(-\frac{1}{3}\right)} \quad \left[\because S_n = \frac{a}{1-r}\right]$$

$$= \frac{3}{\frac{4}{3}} = \frac{9}{4}$$

Directions (Q. Nos. 8 and 9)

Consider the information given below and answer the two items that follow.

A survey was conducted among 300 students. It was found that 125 students like to play cricket, 145 students like to play football and 90 students like to play tennis. 32 students like to play exactly two games out of the three games.

8. How many stdudents like to play all the three games ?

(a) 14 (b) 21
(c) 28 (d) 35

(a) Let,

A be the set of students like to play cricket

B be the set of students like to play football.

C be the set of students like to play tennis.

We have,

$n(A \cup B \cup C) = 300$
$n(A) = 125$
$n(B) = 145$
$n(C) = 90$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$

$\Rightarrow 300 = 125 + 145 + 90 - [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 60 + n(A \cap B \cap C)$ ...(i)

Again,

$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3\,n(A \cap B \cap C) = 32$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 32 + 3n(A \cap B \cap C)$ ...(ii)

From Eqs. (i) and (ii), we get

$60 + n(A \cap B \cap C) = 32 + 3n(A \cap B \cap C)$

$\Rightarrow \quad 2n(A \cap B \cap C) = 28$

$\Rightarrow \quad n(A \cap B \cap C) = 14$

9. How many students like to play exactly only one game?

(a) 196 (b) 228
(c) 254 (d) 268

(c) Number of students like to play exactly one

game $= n(A) + n(B) + n(C) - 2[n(A \cap B) + n(B \cap C) + n(C \cap A)] + 3n(A \cap B \cap C)$

$= 125 + 145 + 90 - 2[32 + 3 \times 14] + 3 \times 14$

$= 360 - 106$

$= 254$

10. If α and β $(\neq 0)$ are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$, where $x \in R$ has

(a) Least value $-\frac{1}{4}$

(b) Least value $-\frac{9}{4}$

(c) Greatest value $\frac{1}{4}$

(d) Greatest value $\frac{9}{4}$

(d) α and β are the roots of quadratic equation.

$x^2 + \alpha x - \beta = 0$

So, $(\alpha\beta = -\beta) \quad \Rightarrow \alpha\beta + \beta = 0$

$\Rightarrow \quad \beta(\alpha + 1) = 0$

$\alpha = -1 \quad [\because \beta \neq 0]$

$\alpha + \beta = -\alpha$

$\Rightarrow \quad 2\alpha + \beta = 0$

$\Rightarrow \quad \beta = 2$

$\therefore \quad -x^2 + \alpha x + \beta \quad [\because \alpha = -1, \beta = 2]$

$= -x^2 - x + 2$

Greatest value $= 2 - \left[\frac{(-1)^2}{4(-1)}\right]$

$\left[\because \text{Greatest value} = c - \left(\frac{b^2}{4a}\right)^2\right]$

$= 2 - \left(\frac{1}{-4}\right)$

$= 2 + \frac{1}{4}$

$= \frac{9}{4}$

11. What is the coefficient of the middle term in the binomial expansion of $(2 + 3x)^4$?

(a) 6 (b) 12
(c) 108 (d) 216

(d) We have, $(2 + 3x)^4$

Here, $n = 4$, so middle term is

$\left(\frac{4}{2} + 1\right)^{\text{th}} = 3^{\text{rd}}$ term

$T_3 = {}^4C_2 \times 2^2 \times (3x)^2$

$[T_{r+1} = {}^nC_r a^r b^{n-r}]1$

$= \frac{4 \times 3}{2 \times 1} \times 4 \times 9x^2$

$T_3 = 216x^2$

Hence, coefficient of middle term is 216.

12. For a square matrix A, which of the following properties hold?

1. $(A^{-1})^{-1} = A$

2. $\det(A^{-1}) = \frac{1}{\det A}$

3. $(\lambda A)^{-1} = \lambda A^{-1}$, where λ is a scalar

Select the correct answer using the code given below.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(d) For a square matrix A

Statement 1

$(A^{-1})^{-1} = A$

Statement 1 is true

Statement 2

$\det(A^{-1}) = \frac{1}{\det A}$

Statement 2 is correct

Statement 3

$(\lambda A)^{-1} = \lambda A^{-1}$, where λ is a scalar.

So, Statement 3 is correct.

13. Which one of the following factors does the expansion of the determinant

$$\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix}$$ contain ?

(a) $x - 3$
(b) $x - y$
(c) $y - 3$
(d) $x - 3y$

(a) We have,

$$\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix}$$

[$C_1 \to C_1 - C_3$ से,]

$$= \begin{vmatrix} x-3 & y & 3 \\ x^2-9 & 5y^3 & 9 \\ x^3-27 & 10y^5 & 27 \end{vmatrix}$$

$$= (x-3)\begin{vmatrix} 1 & y & 3 \\ x+3 & 5y^3 & 9 \\ x^2+9+3x & 10y^5 & 27 \end{vmatrix}$$

14. What is the adjoint of the matrix

$$\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix}?$$

(a) $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(a) We have,

$$A = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now, $C_{11} = \cos\theta$

$C_{12} = -\sin\theta$

$C_{21} = -\sin\theta$

$C_{22} = \cos\theta$

$$\text{adj } A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

15. What is the value of

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^{3a} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$$

where $i = \sqrt{-1}$?

(a) 3 (b) 2

(c) 1 (d) 0

(b) We have,

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$$

$$= (\omega)^{3n} + (\omega^2)^{3n}$$

$$\left[\because \omega = \frac{-1+i\sqrt{3}}{2}, \omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)\right]$$

$$= (\omega^3)^n + (\omega^3)^{2n}$$

$$(1)^n + (1)^{2n} \qquad [\because \omega^3 = 1]$$

$$= 1 + 1 = 2$$

16. There are 17 cricket players, out of which 5 players can bowl. In how many ways can a team of 11 players be selected so as to include 3 bowlers?

(a) $C(17, 11)$

(b) $C(12, 8)$

(c) $C(17, 5) \times C(5, 3)$

(d) $C(5, 3) \times C(12, 8)$

(d) There are 17 cricket players, out of which 5 players can bowl.

required number of ways $= {}^{12}C_8 \times {}^5C_3$

$= C(12, 8) \times C(5, 3)$

17. What is the value of $\log_9 27 + \log_8 32$?

(a) $\frac{7}{2}$ (b) $\frac{19}{6}$ (c) 4 (d) 7

(b) We have,

$\log_9 27 + \log_8 32$

$$= \log_{3^2} 3^3 + \log_{2^3} 2^5$$

$$= \frac{3}{2}\log_3 3 + \frac{5}{3}\log_2 2$$

$$\left[\because \log_{a^m} b^n = \frac{n}{m}\log_a b\right]$$

$$= \frac{3}{2} + \frac{5}{3} = \frac{19}{6}$$

18. If A and B are two invertible square matrices of same order, then what is $(AB)^{-1}$ equal to?

(a) $B^{-1}A^{-1}$ (b) $A^{-1}B^{-1}$

(c) $B^{-1}A$ (d) $A^{-1}B$

(a) If A and B are two invertible square matrices of same order, then

$$(AB)^{-1} = B^{-1}A^{-1}$$

19. If $a + b + c = 0$, then one of the solutions of

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0 \text{ is}$$

(a) $x = a$

(b) $x = \sqrt{\frac{3(a^2+b^2+c^2)}{2}}$

(c) $x = \sqrt{\frac{2(a^2+b^2+c^2)}{3}}$

(d) $x = 0$

(d) We have,

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} a+b+c-x & a+b+c-x & a+b+c-x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -x & -x & -x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow (-x)\begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$\Rightarrow x = 0$

Hence, $x = 0$ is a solution

20. What should be the value of x, so that the matrix $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$ does not have an inverse?

(a) 16 (b) –16

(c) 8 (d) –8

(b) Let, $A = \begin{bmatrix} 2 & 4 \\ -8 & x \end{bmatrix}$

Matrix does not have any solution if

$|A| = 0$

$2x + 32 = 0$

$2x = -32$

$x = -\frac{32}{2}$

$x = -16$

21. The system of equations

$2x + y - 3z = 5$

$3x - 2y + 2z = 5$

and $5x - 3y - z = 16$

(a) is inconsistent

(b) is consistent, with a unique solution

(c) is consistent, with infinitely many solutions

(d) has its solution lying along X-axis in three-dimensional space

(b) The system of equations

$2x + y - 3z = 5$

$3x - 2y + 2z = 5$

and $5x - 3y - z = 0$

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & -2 & 2 \\ 5 & -3 & -1 \end{bmatrix}$$

$|A| = 2[-2(-1) - 2(-3)] - 1[3(-1) - 2(5)] + (-3)[3(-3) - (-2)(5)]$

$= 2(8) - 1(-13) - 3(1)$

$= 16 + 13 - 3 = 26 \neq 0$

So, System is consistent with unique solution.

22. Which one of the following is correct in respect of the cube roots of unity?

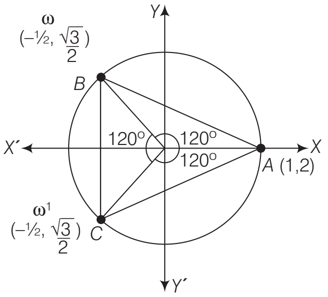
(a) They are collinear

(b) They lie on a circle of radius $\sqrt{3}$

(c) They form an equilateral triangle

(d) None of the above

(c) We know that, cube roots of unity is $1, \omega, \omega^2$, where $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$



They form an equilateral triangle.

23. If u, v and w (all positive) are the p^{th}, q^{th} and r^{th} terms of a GP, then the determinant of the matrix $\begin{pmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{pmatrix}$ is

(a) 0
(b) 1
(c) $(p-q)(q-r)(r-p)$
(d) $\ln u \times \ln v \times \ln w$

(a) Given that u, v and w are the p^{th},q^{th} and r^{th} term of GP

$\therefore u = aR^{p-1}, v = aR^{q-1}$ $[\because a_n = aR^{n-1}]$

and $w = aR^{r-1}$

We have, $\begin{vmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \ln aR^{p-1} & p & 1 \\ \ln aR^{q-1} & q & 1 \\ \ln aR^{r-1} & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \ln a + p-1 \ln R & p & 1 \\ \ln a + q-1 \ln R & q & 1 \\ \ln a + r-1 \ln R & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \ln a & p & 1 \\ \ln a & q & 1 \\ \ln a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\ln R & p & 1 \\ (q-1)\ln R & q & 1 \\ (r-1)\ln R & r & 1 \end{vmatrix}$

$= \ln a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \ln R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$

$C_2 \to C_2 - C_3$

$= 0 + \ln R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$

$= 0$

24. Let the coefficient of the middle term of the binomial expansion of $(1+x)^{2n}$ be α and those of two middle terms of the binomial expansion of $(1+x)^{2n-1}$be β and γ. Which one of the following relations is correct?

(a) $\alpha > \beta + \gamma$ (b) $\alpha < \beta + \gamma$
(c) $\alpha = \beta + \gamma$ (d) $\alpha = \beta\gamma$

(c) We have,$(1+x)^{2n}$

Middle term $= \left(\dfrac{2n}{2}+1\right)^{th}$ term

$= (n+1)^{th}$ term

Coefficient of $(n+1)^{th}$ term $= {}^{2n}C_n$

$\alpha = {}^{2n}C_n$

Again, we have binomial expansion of $(1+x)^{2n-1}$ coefficient of middle terms are,

$\therefore$ $\beta = {}^{2n-1}C_n$

and $\gamma = {}^{2n-1}C_{n-1}$

Now, $\beta + \gamma = {}^{2n-1}C_n + {}^{2n-1}C_{n-1}$

$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$= {}^{2n}C_n$

25. Let $A = [x \in \mathbf{R} : -1 \le x \le 1]$, $B = [y \in \mathbf{R} : -1 \le y \le 1]$ and S be the subset of $A \times B$, defined by

$S = [(x, y) \in A \times B : x^2 + y^2 = 1]$.

Which one of the following is correct ?

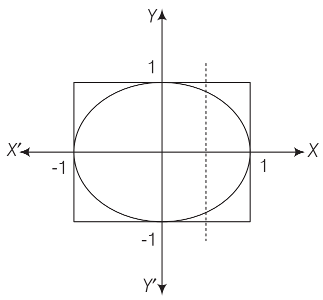
(a) S is a one-one function from A into B
(b) S is a many-one function from A into B
(c) S is a bijective mapping from A into B
(d) S is not a function

(d) Given that,

$A = \{ x \in R : -1 \le x \le 1 \}$,

$B = \{ y \in R : -1 \le y < 1 \}$

and $S = \{ (x, y) \in A \times B : x^2 + y^2 = 1 \}$



By vertical line test. when we draw a vertical line, then line cuts the circle in two points. Hence, S is not a function.

26. Let T_r be the r^{th}term of an AP for $r = 1, 2, 3, \ldots$. If for some distinct positive integers m and n we have $T_m = 1/n$ and $T_n = 1/m$, then what is T_{mn} equal to ?

(a) $(mn)^{-1}$
(b) $m^{-1} + n^{-1}$
(c) 1
(d) 0

(c) Let first term of an AP is a and common difference is d

Given that,

$T_m = \dfrac{1}{n}$

$a + (m-1)d = \dfrac{1}{n}$ ...(i)

and $T_n = \dfrac{1}{m}$

$a + (n-1)d = \dfrac{1}{m}$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$(m-1)d - (n-1)d = \dfrac{1}{n} - \dfrac{1}{m}$

$\Rightarrow$ $(m-n)d = \dfrac{m-n}{mn}$

$\Rightarrow$ $d = \dfrac{1}{mn}$

Put in Eq. (i),

$a + (m-1)\dfrac{1}{mn} = \dfrac{1}{n}$

$\Rightarrow$ $a + \dfrac{1}{n} - \dfrac{1}{mn} = \dfrac{1}{n}$

$\Rightarrow$ $a = \dfrac{1}{mn}$

Now, $T_{mn} = a + (mn-1)d$

$= \dfrac{1}{mn} + (mn-1)\dfrac{1}{mn}$

$= \dfrac{1}{mn} + 1 - \dfrac{1}{mn}$

$T_{mn} = 1$

27. Suppose $f(x)$ is such a quadratic expression that it is positive for all real x.

If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x

(a) $g(x) < 0$ (b) $g(x) > 0$
(c) $g(x) = 0$ (d) $g(x) \ge 0$

(b) Given that $f(x)$ is a quadratic expression

Let $f(x) = ax^2 + bx + c, a > 0$

$\therefore$ $b^2 - 4ac < 0$ $[\therefore f(x) > 0]$

$\Rightarrow$ $b^2 < 4ac$

Now, $f'(x) = 2ax + b$

and $f''(x) = 2a$

We have,

$g(x) = f(x) + f'(x) + f''(x)$

$= ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (b+2a)x + 2a + b + c$

Now, $(b+2a)^2 - 4a(2a + b + c)$

$= b^2 + 4ab + 4a^2 - 8a^2 - 4ab - 4ac$

$= b^2 - 4ac - 8a^2 < 0$

$[\because b^2 - 4ac < 0]$

$\Rightarrow$ $g(x) > 0$

28. Consider the following in respect of matrices A, B and C of same order.

1. $(A+B+C)' = A' + B' + C'$
2. $(AB)' = A'B'$
3. $(ABC)' = C'B'A'$

Where A' is the transpose of the matrix A. Which of the above are correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(c) Given that A, B and C are matrices of same order

Statement 1

$$(A' + B + C)' = A' + B' + C'$$

$$[\because (A + B)' = A' + B']$$

So, Statement 1 is correct

Statement 2

We know that,

$$(AB)' = B'A'$$

Hence, Statement 2 is incorrect

Statement 3

$$(ABC)' = C'B'A' \; [\because (AB)' = B'A']$$

Hence, Statement 3 is correct.

29. The sum of the binary numbers $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is the binary numbers $(101101101)_2$. What are the values of x and y ?

(a) $x = 1, y = 1$ (b) $x = 1, y = 0$
(c) $x = 0, y = 1$ (d) $x = 0, y = 0$

(b) Sum of the binary number $(11011)_2$, $(10110110)_2$ and $(10011xoy)_2$ is $(101101101)_2$

So, (101101101)
 − 10110110
 ―――――――
 10110111
 − 11011
 ―――――――
 10011100

Compare with $(10011x\,o\,y)_2$

We get, $x = 1$ and $y = 0$

30. Let matrix B be the adjoint of a square matrix A, I be the identity matrix of same order as A. If $k(\neq 0)$ is the determinant of the matrix A, then what is AB equal to ?

(a) I (b) kI (c) k^2I (d) $(1/k)I$

(b) Given,

$B = adjA, I =$ identity Matrix

$(A) = K$

$\therefore \; AB = A(\text{adj}A) = (A)I = kI$

31. If $(0.2)^x = 2$ and $\log_{10} 2 = 0.3010$, then what is the value of x to the nearest tenth?

(a) −10.0 (b) −0.5
(c) −0.4 (d) −0.2

(c) We have,

$$(0.2)^x = 2$$

taking $\log_{10}$ both side

$$x \log_{10} 0.2 = \log_{10} 2$$

$$\Rightarrow \quad x \log_{10}\left(\frac{2}{10}\right) = \log_{10} 2$$

$$\Rightarrow \; x\,[\log_{10} 2 - \log_{10} 10] = \log_{10} 2$$

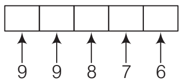
$$\Rightarrow x\,[0.3010 - 1] = 0.3010 \quad [\because \log_a a = 1]$$

$$\Rightarrow \quad x = -\frac{0.3010}{0.6990} \approx -0.43$$

32. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is

(a) 45360 (b) 30240
(c) 27216 (d) 15120

(c) 5-digit number that can be composed by distinct digits from 0 to 9 is given as



required number $= 9 \times 9 \times 8 \times 7 \times 6$
$= 27216$

33. What is the determinant of the matrix

$$\begin{pmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{pmatrix}?$$

(a) $(x-y)(y-z)(z-x)$
(b) $(x-y)(y-z)$
(c) $(y-z)(z-x)$
(d) $(z-x)^2(x+y+z)$

(d) We have,

$$\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - 2C_1$

$$= (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ z & x-z & x-z \\ y & z-y & x-y \end{vmatrix}$$

$$= (x+y+z)\,[1\,(x-z)(x-y) - (x-z)(z-y)]$$

$$= (x+y+z)\,[(x-z)(x-y-z+y)]$$

$$= (x+y+z)(x-z)(x-z)$$

$$= (x+y+z)(z-x)^2$$

34. If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ I+\sin A & 1+\sin B & 1+\sin C \\ \text{Sin}A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$$

then which one of the following is correct?

(a) The triangle ABC is isosceles
(b) The triangle ABC is equilateral
(c) The triangle ABC is scalene
(d) No conclusion can be drawn with regard to the nature of the triangle

(a) We have,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \text{Sin}\, C+\text{Sin}^2 C \end{bmatrix} = 0$$

$R_1 \to R_1 - R_2,\; R_3 \to R_3 - R_2$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin^2 A - 1 & \sin^2 B - 1 & \sin^2 C - 1 \end{vmatrix} = 0$$

$R_2 \to R_2 + R_1$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1 & 1 & 1 \\ -\cos^2 A & -\cos^2 B & -\cos^2 C \end{vmatrix} = 0$$

$R_3 \to R_3 + R_2$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ 1-\cos^2 A & 1-\cos^2 B & 1-\cos^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0$$

$[C_1 \to C_1 - C_2 C_2 \to C_2 - C_3]$

$$\Rightarrow \begin{vmatrix} \sin A - \sin B & \sin B - \sin C & \sin C \\ 0 & 0 & 1 \\ \sin^2 A - \sin^2 B & \sin^2 B - \sin^2 C & \sin^2 C \end{vmatrix}$$

$= 0$

$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)$

$$\begin{bmatrix} 1 & 1 & \sin C \\ 0 & 0 & 0 \\ \sin A+\sin C & \sin B+\sin C & \sin^2 C \end{bmatrix}$$

$\therefore$ Sin $A - \sin B = 0$

or $\sin B - \sin C = 0$

$\Rightarrow \quad \sin A = \sin B$

and $\quad \sin B = \sin C$

$\Rightarrow \quad A = B$

and $\quad B = C$

So, ABC is an isosceles triangle

35. Consider the following in respect of matrices A and B of same order

1. $A^2 - B^2 = (A+B)(A-B)$
2. $(A-I)(I+A) = O \Leftrightarrow A^2 = I$

where I is the identity matrix and O is the null matrix.

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(b)

Statement 1

RHS $= (A+B)(A-B)$

$= A^2 - AB + BA - B^2$ [$\because AB \neq BA$]

$\neq A^2 - B^2$

Statement 1 is not correct

Statement 2

$(A-I)(I+A) = 0$

$\Rightarrow AI + A^2 - I^2 - IA = 0$ [$\because AI = IA = A$]

$\Rightarrow A + A^2 - I^2 - A = 0$

$\Rightarrow A^2 = I$ [$\because I^2 = I$]

Statement 2 is correct

36. What is $\dfrac{2\tan\theta}{1+\tan^2\theta}$ equal to ?

(a) $\cos 2\theta$ (b) $\tan 2\theta$
(c) $\sin 2\theta$ (d) $\operatorname{cosec} 2\theta$

(c) We have,

$$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{\dfrac{2\sin\theta}{\cos\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= 2\sin\theta\cos\theta = \sin 2\theta$

37. If $\sec(\theta-\alpha)$, $\sec\theta$ and $\sec(\theta+\alpha)$ are in AP, where $\cos\alpha \neq 1$, then what is the value of $\sin^2\theta + \cos\alpha$?

(a) 0 (b) 1
(c) -1 (d) $\dfrac{1}{2}$

(a) If $\sec(\theta-\alpha)$, $\sec\theta$ and $\sec(\theta+\alpha)$ are in AP, then

$2\sec\theta = \sec(\theta-\alpha) + \sec(\theta+\alpha)$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{1}{\cos(\theta-\alpha)} + \frac{1}{\cos(\theta+\alpha)}$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{\cos(\theta+\alpha)+\cos(\theta-\alpha)}{\cos(\theta-\alpha)(\cos(\theta+\alpha)}$$

$\Rightarrow$

$$\frac{2}{\cos\theta} = \frac{2\cos\left(\dfrac{\theta+\alpha+\theta-\alpha}{2}\right)\cos\left(\dfrac{\theta+\alpha-\theta+\alpha}{2}\right)}{\cos^2\theta - \sin^2\alpha}$$

$$\left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

[and $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$]

$$\Rightarrow \frac{2}{\cos\theta} = \frac{2\cos\theta\cos\alpha}{\cos^2\theta - \sin^2\alpha}$$

$\Rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$

$\Rightarrow \sin^2\alpha = \cos^2\theta - \cos^2\theta\cos\alpha$

$\Rightarrow \sin^2\alpha = \cos^2\theta\,(1-\cos\alpha)$

$$\Rightarrow \cos^2\theta = \frac{\sin^2\alpha}{1-\cos\alpha}$$

$$= \frac{1-\cos^2\alpha}{1-\cos\alpha} = \frac{(1+\cos\alpha)(1-\cos\alpha)}{1-\cos\alpha}$$

$\Rightarrow \cos^2\theta = 1+\cos\alpha$

$\Rightarrow 1-\sin^2\theta = 1+\cos\alpha$

[$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow 1-\sin^2\theta = 1+\cos\alpha$

$\Rightarrow \sin^2\theta + \cos\alpha = 0$

38. If $A+B+C = 180°$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to?

(a) $-4\sin A\sin B\sin C$
(b) $-4\cos A\sin B\cos C$
(c) $-4\cos A\cos B\sin C$
(d) $-4\sin A\cos B\cos C$

(d) We have,

$\sin 2A - \sin 2B - \sin 2C$

$= \sin 2A - [\sin 2B + \sin 2C]$

$$= \sin 2A - \left[2\sin\left(\frac{2B+2C}{2}\right)\cos\left(\frac{2B-2C}{2}\right)\right]$$

$$\left[\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

$= 2\sin A\cos A - [2\sin(B+C)\cos(B-C)]$

[$\because \sin 2A = 2\sin A\cos A$]

$= 2\sin A\cos A - [2\sin(180° - A)\cos(B-C)]$

[$\because A+B+C = 180$]

$= 2\sin A\cos A - 2\sin A\cos(B-C)$

[$\because \sin(180° - \theta) = \sin\theta$]

$= 2\sin A[\cos A - \cos(B-C)]$

$= 2\sin A[\cos(180-(B+C)) - \cos(B-C)]$

$= 2\sin A[-\cos(B+C) - \cos(B-C)]$

$= -2\sin A[\cos(B+C) + \cos(B-C)]$

$= -2\sin A[2\cos B\cos C]$

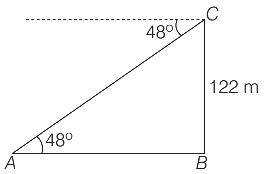
$$\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$= -4\sin A\cos B\cos C$

39. A balloon is directly above one end of a bridge. The angle of depression of the other end of the bridge from the ballon is 48°. If the height of the balloon above the bridge is 122 m, then what is the length of the bridge?

(a) $122\sin 48°$ m
(b) $122\tan 42°$ m
(c) $122\cos 48°$ m
(d) $122\tan 48°$ m

(b)



$$\tan 48° = \frac{BC}{AB}$$

$$\tan 48° = \frac{122}{AB}$$

$$AB = \frac{122}{\tan 48°} = 122\cot 48°$$

$= 122\cot(90° - 42°)$

[$\because \cot(90° - \theta) = \tan\theta$]

$= 122\tan 42°$

40. A is an angle in the fourth quadrant it satisfies the trigonometric equation $3(3-\tan^2 A - \cot A)^2 = 1$. Which one of the following is a value of A ?

(a) 300° (b) 315° (c) 330° (d) 345°

(a) We have,

$3(3-\tan^2 A - \cot A)^2 = 1$

Checking through options

$A = 300$

So, $3[3-\tan^2 300° - \cot(300)°]^2$

$= 3[3-\tan^2(360° - 60°) - \cot(360° - 60)°]^2$

$= 3[3-\tan^2 60° + \cot 60°]^2$

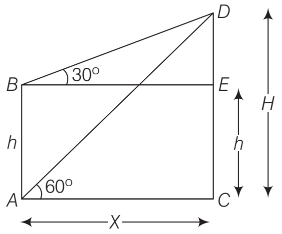
$$= 3\left[3-3+\frac{1}{\sqrt{3}}\right]^2 = 3\times\frac{1}{3} = 1$$

So, option (a) is correct

41. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\dfrac{\pi}{6}$ and $\dfrac{\pi}{3}$ respectively. What is the height of the hill?

(a) $2h$ (b) $\dfrac{3h}{2}$
(c) h (d) $\dfrac{h}{2}$

(b) height of building be h and let height of hill is H



In ΔACD

$$\tan 60° = \frac{H}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{H}{x}$$

$$\Rightarrow \quad x = \frac{H}{\sqrt{3}} \quad \text{...(i)}$$

In ΔBDE

$$\tan 30° = \frac{H-h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{H-h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,(H-h)$$

$$\Rightarrow \quad \frac{H}{\sqrt{3}} = \sqrt{3}\,(H-h)$$

$$\Rightarrow \quad H = 3\,(H-h)$$

$$\Rightarrow \quad H = 3H - 3h$$

$$\Rightarrow \quad 2H = 3h$$

$$\Rightarrow \quad H = \frac{3}{2}h$$

42. What is/are the solution (s) of the trigonometric equation $\operatorname{cosec} x + \cot x = \sqrt{3}$ where $0 < x < 2\pi$?

(a) $\frac{5\pi}{3}$ only (b) $\frac{\pi}{3}$ only

(c) π only (d) $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

(b) We have,

$$\operatorname{cosec}x + \cot x = \sqrt{3} \quad \text{...(i)}$$

Now,

$$\Rightarrow \frac{(\operatorname{cosec}x + \cot x)(\operatorname{cosec}x - \cot x)}{\operatorname{cosec}x - \cot x} = \sqrt{3}$$

$$\Rightarrow \quad \frac{\operatorname{cosec}^2 x - \cot^2 x}{\operatorname{cosec}x - \cot x} = \sqrt{3}$$

$$\Rightarrow \quad \frac{1}{\operatorname{cosec}x - \cot x} = \sqrt{3}$$

$$\Rightarrow \quad \operatorname{cosec}x - \cot x = \frac{1}{\sqrt{3}} \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2\operatorname{cosec}x = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 2\operatorname{cosec}x = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \quad \operatorname{cosec}x = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \sin x = \frac{\sqrt{3}}{2}$$

$$\therefore \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$$

43. If $\theta = \frac{\pi}{8}$, then what is the value of

$$(2\cos\theta + 1)^{10}(2\cos 2\theta - 1)^{10}(2\cos\theta - 1)^{10}$$
$$(2\cos 4\theta - 1)^{10}?$$

(a) 0 (b) 1

(c) 2 (d) 4

(b) If $\theta = \frac{\pi}{8}$, then

$(2\cos\theta + 1)^{10}(2\cos 2\theta - 1)^{10}$

$(2\cos\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$

$\Rightarrow (2\cos\theta + 1)^{10}(2\cos\theta - 1)^{10}$

$(2\cos 2\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$

$\Rightarrow (4\cos^2\theta - 1)^{10}(2\cos 2\theta - 1)^{10}$

$(2\cos 4\theta - 1)^{10}$

$\Rightarrow [2(2\cos^2\theta) - 1]^{10}(2\cos 2\theta - 1)^{10}$

$(2\cos 4\theta - 1)^{10}$

$\Rightarrow [2(1 + \cos 2\theta) - 1]^{10}(2\cos 2\theta - 1)^{10}$

$(2\cos 4\theta - 1)^{10}$

$\Rightarrow (2\cos 2\theta + 1)^{10}(2\cos 2\theta - 1)^{10}$

$(2\cos 4\theta - 1)^{10}$

$$\Rightarrow \left[2\cos\frac{\pi}{4} + 1\right]^{10}\left(2\cos\frac{\pi}{4} - 1\right)^{10}$$

$$\left(2\cos\frac{\pi}{2} - 1\right)^{10} \quad \left[\because \theta = \frac{\pi}{8}\right]$$

$\Rightarrow$

$$\left[2 \times \frac{1}{\sqrt{2}} + 1\right]^{10}\left(2 \cdot \frac{1}{\sqrt{2}} - 1\right)^{10}(0-1)^{10}$$

$$\Rightarrow \quad (\sqrt{2}+1)^{10}(\sqrt{2}-1)^{10} \times 1$$

$$\Rightarrow \quad ((\sqrt{2})^2 - (1)^2)^{10} \times 1$$

$$\Rightarrow \quad (2-1)^{10}$$

$$\Rightarrow \quad 1$$

44. If $\cos\alpha$ and $\cos\beta$ $(0 < \alpha < \beta < \pi)$ are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec\alpha \times \sec\beta$?

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

(a) Given,

$$4x^2 - 3 = 0$$

$$\Rightarrow \quad 4x^2 = 3$$

$$\Rightarrow \quad x^2 = \frac{3}{4}$$

$$\Rightarrow \quad x = \pm\frac{\sqrt{3}}{2}$$

So, $\quad \cos\alpha = \frac{\sqrt{3}}{2}$

and $\quad \cos\beta = \frac{-\sqrt{3}}{2}$

$$\text{Now, } \sec\alpha \cdot \sec\beta = \frac{2}{\sqrt{3}} \times \left(\frac{-2}{\sqrt{3}}\right) = \frac{-4}{3}$$

45. Consider the following values of x :

1. 8 2. −4

3. $\frac{1}{6}$ 4. $-\frac{1}{4}$

Which of the above values of x is/are the solution (s) of the equation

$$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\ ?$$

(a) 3 only (b) 2 and 3

(c) 1 and 4 (d) 4 only

(a) We have,

$$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{2x + 3x}{1 - 2x \cdot 3x}\right) = \frac{\pi}{4}$$

$$\left[\because \tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right),\ ab \le 1\right.$$

and $\tan^{-1}a + \tan^{-1}b = \pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right), ab > 1$

$$\Rightarrow \quad \frac{5x}{1-6x^2} = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{5x}{1-6x^2} = 1$$

$$\Rightarrow \quad 1 - 6x^2 = 5x$$

$$\Rightarrow \quad 6x^2 + 5x - 1 = 0$$

$$\Rightarrow \quad 6x^2 + 6x - x - 1 = 0$$

$$\Rightarrow \quad (6x-1)(x+1) = 0$$

$$\Rightarrow \quad x = -1, \frac{1}{6}$$

$x = -1$ is not possible

So, $\quad x = \frac{1}{6}$

46. If the second term of a GP is 2 and the sum of its infinite terms is 8, then the GP is

(a) $8, 2, \frac{1}{2}, \frac{1}{8}, \ldots$

(b) $10, 2, \frac{2}{5}, \frac{2}{25}, \ldots$

(c) $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}\ldots$

(d) $6, 3, \frac{3}{2}, \frac{3}{4}, \ldots$

(c) Let first term of a GP is a and common ratio is r

$$\therefore \quad a_2 = 2$$

$\Rightarrow \quad ar = 2 \quad \ldots$(i)

and $\quad S_n = 8$

$\Rightarrow \quad \frac{a}{1-r} = 8$

From Eq. (i)

$\Rightarrow \quad \frac{2}{r(1-r)} = 8$

$\Rightarrow \quad 8r - 8r^2 = 2$

$\Rightarrow \quad 4r^2 - 4r + 1 = 0$

$\Rightarrow \quad 4r^2 - 2r - 2r + 1 = 0$

$\Rightarrow \quad 2r(2r-1) - 1(2r-1) = 0$

$\Rightarrow \quad (2r-1)(2r-1) = 0$

$\Rightarrow \quad r = \frac{1}{2}$

$\therefore$ Put in Eq. (i)

$a\left(\frac{1}{2}\right) = 2$

$\Rightarrow \quad a = 4$

$\therefore$ GP is $4, 2, 1, \frac{1}{2} \ldots$

47. If a, b, c are in AP or GP or HP, then $\frac{a-b}{b-c}$ is equal to

(a) $\frac{b}{a}$ or 1 or $\frac{b}{c}$ (b) $\frac{c}{a}$ or $\frac{c}{b}$ or 1

(c) 1 or $\frac{a}{b}$ or $\frac{a}{c}$ (d) 1 or $\frac{a}{b}$ or $\frac{c}{a}$

(c) a, b, c are in AP, then

$\therefore \quad a - b = b - c$

$\Rightarrow \quad \frac{a-b}{b-c} = 1$

If a, b, c are in GP, then

$\therefore \quad \frac{a}{b} = \frac{b}{c} = \frac{a-b}{b-c}$

If a, b, c are in HP, then

$\therefore \quad b = \frac{2ac}{a+c}$

$\Rightarrow \quad ab + bc = 2ac$

$\Rightarrow \quad ab - ac = ac - bc$

$\Rightarrow \quad a(b-c) = c(a-b)$

$\Rightarrow \quad \frac{a-b}{b-c} = \frac{a}{c}$

48. What is the sum of all three digit numbers that can be formed using all the digits 3, 4 and 5, when repetition of digits is not allowed?

(a) 2664 (b) 3382

(c) 4044 (d) 4444

(a) Three digit number. that can be formed using 3, 4 and 5 when repetition not allowed are 543, 534, 453, 435, 354, 345

Sum

$= 543 + 534 + 453 + 435 + 354 + 345$

$= 2664$

49. The ratio of roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are equal. If D_1 and D_2 are respective discriminants, then what is $\frac{D_1}{D_2}$ equal to ?

(a) $\frac{a^2}{p^2}$ (b) $\frac{b^2}{q^2}$

(c) $\frac{c^2}{r^2}$ (d) None of these

(b) Let α, β are the roots of quadratic equation.

$ax^2 + bx + c = 0$

So, $\quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

and let γ , δ are the roots of quadratic equation

$px^2 + qx + r = 0$

$\gamma + \delta = -\frac{q}{p}$

and $\quad \gamma\delta = \frac{r}{p}$

Acoording to the question,

$\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$

according to componendo and dividendo

$\frac{\alpha + \beta}{\alpha - \beta} = \frac{\gamma + \delta}{\gamma - \delta}$

$\Rightarrow \frac{\alpha + \beta}{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}} = \frac{\gamma + \delta}{\sqrt{(\gamma+\delta)^2 - 4\gamma\delta}}$

$\Rightarrow \frac{-\frac{b}{a}}{\sqrt{\left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)}} = \frac{-\frac{q}{p}}{\sqrt{\left(-\frac{q}{p}\right)^2 - 4\left(\frac{r}{p}\right)}}$

$\Rightarrow \quad \frac{\frac{b}{a}}{\frac{\sqrt{b^2 - 4ac}}{a}} = \frac{\frac{q}{p}}{\frac{\sqrt{q^2 - 4pr}}{p}}$

$\Rightarrow \quad \frac{b}{\sqrt{b^2 - 4ac}} = \frac{q}{\sqrt{q^2 - 4pr}}$

$\Rightarrow \quad \frac{b}{\sqrt{D_1}} = \frac{q}{\sqrt{D_2}}$

$\Rightarrow \quad \frac{\sqrt{D_1}}{\sqrt{D_2}} = \frac{b}{q}$

$\Rightarrow \quad \frac{D_1}{D_2} = \frac{b^2}{q^2}$

50. If $A = \sin^2\theta + \cos^4\theta$, then for all real θ, which one of the following is correct?

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$

(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

(b) We have

$A = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + (1 - \sin^2\theta)^2$

Let $\sin^2\theta = x$

$= x^2 - x + 1, 0 \le x \le 1$

$= x^2 - x + 1$

$\Rightarrow \quad \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Now, $\quad A(0) = A(1) = 1$

So, $\quad \frac{3}{4} \le A \le 1$

51. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is

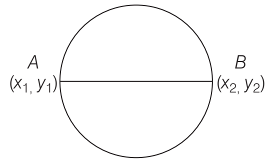
(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$

(b) $(x - x_1)^2 + (y - y_1)^2 = x_2 y_2$

(c) $x^2 + y^2 + 2x_1x_2 + 2y_1y_2 = 0$

(d) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(d) Equation of circle if end points of diameter are (x_1, y_1) and (x_2, y_2) are



$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

52. The second degree equation

$x^2 + 4y^2 - 2x - 4y + 2 = 0$

represents

(a) A point

(b) An ellipse of semi-major axis 1

(c) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$

(d) None of the above

(d) Given that,

$x^2 + 4y^2 - 2x - 4y + 2 = 0$

Compare with

$ax^2 + by^2 + 2hxy + 2gx + 2yx + 2fy + c = 0$

Then,

$a = 1, b = 4, h = 0, g = -1, f = -2$

and $\quad c = 2$

$\Delta = abc + 2fhg - af^2 - bg^2 - ch^2$

$= 1 \times 4 \times 2 + 2 \times (-2) \times (-1) \times 0$
$-1 \times (-2)^2 - 4 \times (-1)^2 - 2(0)$

$= 8 + 0 - 4 - 4 = 0$

$= 8 - 8$

$= 0$

Equation, represent a pair of straight line.

53. The angle between the two lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ is given by $\tan^{-1}\theta$. What is θ equal to?

(a) $\left|\frac{lm' - l'm}{ll' - mm'}\right|$ (b) $\left|\frac{lm' + l'm}{ll' + mm'}\right|$

(c) $\left|\frac{lm' - l'm}{ll' + mm'}\right|$ (d) $\left|\frac{lm' + l'm}{ll' - mm'}\right|$

(c) Given straight lines,

$$lx + my + n = 0 \quad \ldots\ldots(i)$$

$$m_1 = -\frac{l}{m}$$

and $l'x + m'y + n' = 0$

$$m_2 = -\frac{l'}{m'},$$

angle between both lines is θ

$$\therefore \tan\theta = \left|\frac{-\frac{l}{m} - \frac{l'}{m'}}{1 + \frac{ll'}{mm'}}\right|,$$

$$\left[\because \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|\right]$$

$$= \frac{\left|\frac{-lm' - l'm}{mm'}\right|}{\frac{mm' + ll'}{mm'}}, = \left|\frac{lm' - l'm}{ll' + mm'}\right|$$

54. Consider the following statements

1. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$
2. The distance between the lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.
3. The distance between the lines $x = c_1$ and $x = c_2$ is $|c_1 - c_2|$.

Which of the above statements are correct?

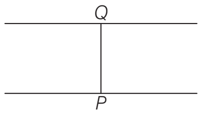
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(b) Statement 1 Given lines,

$$y = mx + c_1$$

and $y = mx + c_2$

both lines are parallel



$$PQ = \left|\frac{c_1 - c_2}{\sqrt{1 + m^2}}\right|$$

So, Statement 1 is correct

Statement 2 for $ax + by + c_1 = 0$
and $ax + by + c_2 = 0$
both lines are parallel so,

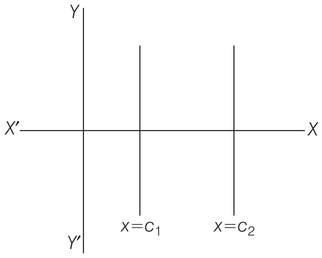
$$\therefore \quad D = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

Statement 3 $x = c_1$

$x = c_2$

$D = |c_1 - c_2|$

Statement 3 is correct



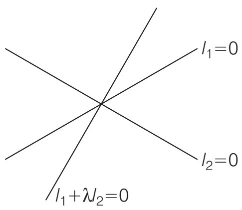
55. What is the equation of straight line passing through the point of intersection of the lines $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$ and parallel to the line $4x + 5y - 6 = 0$?

(a) $20x + 25y - 54 = 0$
(b) $25x + 20y - 54 = 0$
(c) $4x + 5y - 54 = 0$
(d) $4x + 5y - 45 = 0$

(a) Given lines,

$$\frac{x}{2} + \frac{y}{3} = 1$$

and $\frac{x}{3} + \frac{y}{2} = 1$



Equation of line passing through the point of intersection are $l_1 + \lambda l_2 = 0$

$(3x + 2y - 6) + \lambda\ (2x + 3y - 6) = 0$ ...(i)

Slope of line

$$\therefore \quad \frac{-(3 + 2\lambda)}{2 + 3\lambda} = -\frac{4}{5}$$

$\Rightarrow \quad + 15 + 10\lambda = 8 + 12\lambda$

$\Rightarrow \quad 7 = 2\lambda$

$\Rightarrow \quad \lambda = \frac{7}{2}$

$(3x + 2y - 6) + \frac{7}{2}(2x + 3y - 6) = 0$

$\Rightarrow 2\ (3x + 2y - 6) + 14x + 21y - 42 = 0$

$\Rightarrow \quad 6x + 4y - 12 + 14x + 21y - 42 = 0$

$\Rightarrow \quad 20x + 25y - 54 = 0$

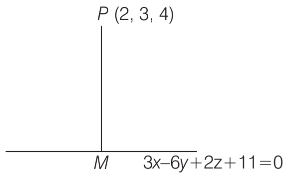
56. What is the distance of the point (2, 3, 4) from the plane

$$3x - 6y + 2z + 11 = 0?$$

(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

(a) Distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is

$$d = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$



$$PM = \left|\frac{3(2) - 6\ (3) + 2\ (4) + 1}{\sqrt{(3)^2 + (-6)^2 + (2)^2}}\right|$$

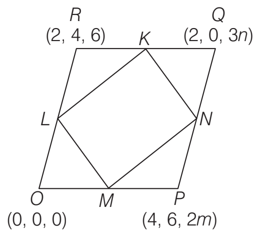
$$= \left|\frac{6 - 18 + 8 + 11}{\sqrt{9 + 36 + 4}}\right| = \left|\frac{7}{\sqrt{49}}\right|$$

$PM = 1$ unit

57. Coordinates of the points O, P, Q and R respectively: (0, 0, 0), (4, 6, 2m), (2, 0, 2n) and (2, 4, 6) L, M, N and K OR, OP, PQ and QR respectively such that $LMNK$ is a parallelogram whose two adjacent sides LK and LM are each of length $\sqrt{2}$?

(a) 6, 2
(b) 1, 3
(c) 3, 1
(d) None of the above

(c) $LMNK$ is a parallelogram we know that, if we join mid- point of any quadrilateral we get a parallelogram



So. M, N, K, L are mid- points of OP, PQ, QR and RO respectively

$\therefore$ Coordinate of M

$$= \left(\frac{0 + 4}{2}, \frac{0 + 6}{2}, \frac{0 + 2m}{2}\right)$$

$= (2, 3, m)$

$$\left[\because \text{Coordinate of mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

Coordinate of L = (1, 2, 3)
Coordinate of $K = (2, 2, 3 + n)$
Now, $LM = \sqrt{2}$

$$\Rightarrow \sqrt{(2-1)^2 + (3-2)^2 + (m-3)^2} = \sqrt{2}$$
$$\Rightarrow \sqrt{1 + 1 + (m-3)^2} = \sqrt{2}$$

Squaring both side,

$$\Rightarrow 2 + (m-3)^2 = 2$$
$$\Rightarrow (m-3)^2 = 0$$
$$\Rightarrow m = 3$$

Again, $LK = \sqrt{2}$

$$\Rightarrow \sqrt{(2-1)^2 + (2-2)^2 + (3+n-3)^2} = \sqrt{2}$$
$$\Rightarrow \sqrt{1 + 0 + n^2} = \sqrt{2}$$

Squaring both side,

$$\Rightarrow 1 + n^2 = 2$$
$$\Rightarrow n^2 = 1$$
$$\Rightarrow n = 1$$

58. The line $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}$ is given by

(a) $x + y + z = 6, x + 2y - 3z = -4$
(b) $x + 2y - 2x = -1, 4x + 4y - 5z - 3 = 0$
(c) $3x + 2y - 3z = 0, 3x - 6y + 3z = -2$
(d) $3x + 2y - 3z = -2, 3x - 6y + 3z = 0$

(d) Given lines,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ (let)}$$
$$\therefore \quad \frac{x-1}{2} = \lambda$$
$$\Rightarrow x = 2\lambda + 1$$
$$\frac{y-2}{3} = \lambda$$
$$\Rightarrow y = 3\lambda + 2$$
$$\text{and} \quad \frac{z-3}{4} = \lambda$$
$$\Rightarrow z = 4\lambda + 3$$

by checking options if $3x + 2y - 3z$, then

$3(2\lambda + 1) + 2(3\lambda + 2) - 3(4\lambda + 3)$
$= 6\lambda + 3 + 6\lambda + 4 - 12\lambda - 9$
$= -2$

$3(2\lambda + 1) - 6(3\lambda + 2) + 3(4\lambda + 3)$
$\Rightarrow 6\lambda + 3 - 18\lambda - 12 + 12\lambda + 9 = 0$

So, option (d) is correct

59. Consider the following statements

1. The angle between the planes $2x - y + z = 1$ and $x + y + 2z = 3$ is $\frac{\pi}{3}$
2. The distance between the planes $6x - 3y + 6z + 2 = 0$ and $2x - y + 2z + 4 = 0$ is $\frac{10}{9}$

Which of the above statement is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Statements 1 Given,

$2x - y + z = 1$ and $x + y + 2z = 3$
Here, $a_1 = 1, b_1 = -1, c_1 = 1$
and $a_2 = 1, b_2 = 1, c_2 = 2$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$= \frac{2 \times 1 + (-1 \times 1) + 1 \times 2}{\sqrt{2^2 + (-1)^2 + (1)^2}\sqrt{1^2 + 1^2 + 2^2}}$$
$$= \frac{4-1}{\sqrt{4+1+1}\sqrt{4+1+1}} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6}$$
$$= \frac{1}{2}$$
$$\therefore \quad \theta = \frac{\pi}{3}$$

So, Statement 1 is correct.

Statement 2 Distance between two planes

$$ax + by + cz + d_1 = 0$$
$$ax + by + cz + d_2 = 0$$
$$\text{distance } S = \left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$$
$$d_1 = \frac{2}{3}, d_2 = 4$$
$$\text{distance} = \left|\frac{4 - \frac{2}{3}}{\sqrt{2^2 + 1^2 + 2^2}}\right| = \left|\frac{\frac{10}{3}}{\sqrt{9}}\right|$$
$$= \frac{10}{3 \times 3} = \frac{10}{9}$$

60. Consider the following statements :

Statement I : If the line segment joining the points $P(m, n)$ and $Q(r, s)$ subtends an angle α at the origin, then $\cos\alpha = \frac{ms - nr}{\sqrt{(m^2 + n^2)(r^2 + s^2)}}$.

Statement II. In any triangle ABC, it is true that $a^2 = b^2 + c^2 - 2bc\cos A$.

Which one of the following is correct in respect of the above two statements?

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of statement I.
(b) Both Statement I and Statement II are true, but Statement II is not the correct explanation II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

(d) Statement I If the line segment joining the point $P(m, n)$ and $Q(r, s)$ subtends angle α at origin, then

$$\cos\alpha = \frac{mr + ns}{\sqrt{m^2 + n^2}\sqrt{r^2 + s^2}}$$

So, Statement I is not correct

Statement II

In any triangle ABC

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statement II is correct

61. What is the area of the triangle with vertices $\left(x_1, \frac{1}{x_1}\right), \left(x_2, \frac{1}{x_2}\right), \left(x_3, \frac{1}{x_3}\right)$?

(a) $|(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)|$
(b) 0
(c) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1x_2x_3}\right|$
(d) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1x_2x_3}\right|$

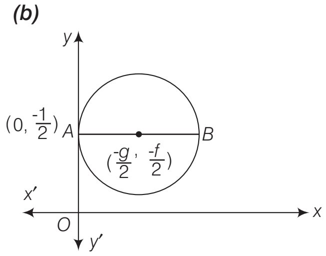
(d) Area of ΔABC

$$A\left(x_1, \frac{1}{x_1}\right), B\left(x_2, \frac{1}{x_2}\right), C\left(x_3, \frac{1}{x_3}\right)$$
$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
$$= \frac{1}{2}\begin{vmatrix} x_1 & \frac{1}{x_1} & 1 \\ x_2 & \frac{1}{x_2} & 1 \\ x_3 & \frac{1}{x_3} & 1 \end{vmatrix}$$
$$= \frac{1}{2}\begin{vmatrix} x_1 & \frac{1}{x_1} & 1 \\ x_2 & \frac{1}{x_2} & 1 \\ x_3 & \frac{1}{x_3} & 1 \end{vmatrix}$$
$$\Rightarrow \frac{1}{2x_1x_2x_3}\begin{vmatrix} x_1^2 & 1 & x_1 \\ x_2^2 & 1 & x_2 \\ x_3^2 & 1 & x_3 \end{vmatrix}$$
$$= \left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1x_2x_3}\right|$$

62 If Y-axis touches the circle $x^2 + y^2 + gx + fy + \frac{c}{4} = 0$, then the normal at this point intersects the circle at the point.

(a) $\left(-\frac{g}{2}, \frac{f}{2}\right)$ (b) $\left(-g, -\frac{f}{2}\right)$
(c) $\left(-\frac{g}{2}, f\right)$ (d) $(-g, -f)$

(b)



The equation of circle is

$$x^2 + y^2 + gx + fy + \frac{c}{4} = 0$$

$\because$ centre $= \left(-\frac{y}{2}, \frac{-f}{2}\right)$ and

radius $= \sqrt{\frac{g^2}{4} + \frac{f^2}{4} - \frac{c}{4}}$

Since, circle touches Y-axis, then

AC = radious

$$\Rightarrow \quad \left|\frac{g}{2}\right| = \sqrt{\frac{g^2}{4} + \frac{f^2}{4} \frac{-c}{4}}$$

$$\Rightarrow \quad \frac{f^2}{4} = \frac{c}{4}$$

$\therefore$ circle touches Y-axis at

$$y^2 + fy + \frac{f^2}{4} = 0$$

$$\Rightarrow \quad \left(y + \frac{f}{2}\right)^2 = 0$$

$$\Rightarrow \quad y = \frac{-f}{2}$$

$$\therefore \; A\left(0, \frac{-f}{2}\right)$$

$\therefore$ Normal at A will pass through centre C and intersect circle again at B.

$\therefore$ Coordinates of B are $\left(-g, \frac{-f}{2}\right)$

63. Let $|\vec{\mathbf{a}}| \neq 0, |\vec{\mathbf{b}}| \neq 0$. $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2$ holds if and only if

(a) $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are perpendicular
(b) $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are parallel
(c) $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are inclined at an angle of 45°
(d) $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are anti-parallel

(a) Given,

$$|\vec{\mathbf{a}}| \neq 0, |\vec{\mathbf{b}}| \neq 0$$

$$(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$$

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$$

$$\Rightarrow \quad \vec{\mathbf{a}} \perp \vec{\mathbf{b}}$$

64. If $\vec{\mathbf{r}} = x\hat{i} + y\hat{j} + z\hat{k}$, then what is $\vec{\mathbf{r}} \cdot (\hat{i} + \hat{j} + \hat{k})$ equal to ?

(a) x (b) $x + y$
(c) $-(x + y + z)$ (d) $(x + y + z)$

(d) $\vec{\mathbf{r}} = \hat{x}i + y\hat{j} + z\hat{k}$

Now, $\vec{\mathbf{r}} \cdot (\hat{i} + \hat{j} + \hat{k}) \qquad \hat{i} \cdot \hat{i} = 1$

$$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$= (x + y + z)$$

65. A unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$ is

(a) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$
(b) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$
(c) $\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$
(d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

(a) We know that,

$$\vec{\mathbf{n}} = \frac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|}$$

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -4 & -1 \end{vmatrix}$$

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = 5\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 5\sqrt{3}$$

$$\therefore \quad \vec{\mathbf{n}} = \frac{5(\hat{i} + \hat{j} - \hat{k})}{5\sqrt{3}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$$

66. If $|\vec{\mathbf{a}}| = 3, |\vec{\mathbf{b}}| = 4$ and $|\vec{\mathbf{a}} - \vec{\mathbf{b}}| = 5$, then what is the value of $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|$?

(a) 8 (b) 6 (c) $5\sqrt{2}$ (d) 5

(d) Given that,

$$|\vec{\mathbf{a}}| = 3$$

$$|\vec{\mathbf{b}}| = 4$$

and $\quad |\vec{\mathbf{a}} - \vec{\mathbf{b}}| = 5$

$$\therefore \quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$$

$$|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 | + |\vec{\mathbf{b}}|^2 + 0 = 25$$

$$|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 5$$

67. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be three mutually perpendicular vectors each of unit magnitude. If $\vec{\mathbf{A}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}, \vec{\mathbf{B}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} + \vec{\mathbf{c}}$ and $\vec{\mathbf{C}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} - \vec{\mathbf{c}}$, then which one of the following is correct?

(a) $|\vec{\mathbf{A}}| > |\vec{\mathbf{B}}| > |\vec{\mathbf{C}}|$
(b) $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}| \neq |\vec{\mathbf{C}}|$
(c) $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}| = |\vec{\mathbf{C}}|$
(d) $|\vec{\mathbf{A}}| \neq |\vec{\mathbf{B}}| \neq |\vec{\mathbf{C}}|$

(c) Given that,

$$\vec{\mathbf{A}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} + c$$

$$\vec{\mathbf{C}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} - \vec{\mathbf{c}}$$

$$|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1$$

$$|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}| = |\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}| = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$$

Now, $|\vec{\mathbf{A}}| = \sqrt{a^2 + b^2 + c^2}$

$$= \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$|\vec{\mathbf{B}}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$|\vec{\mathbf{C}}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$\Rightarrow \quad |\vec{\mathbf{A}}| = |\vec{\mathbf{B}}| = |\vec{\mathbf{C}}|$$

68. What is $(\vec{\mathbf{a}} - \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}})$ equal to?

(a) $\vec{\mathbf{0}}$ (b) $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$
(c) $2(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$ (d) $|\vec{\mathbf{a}}|^2 - |\vec{\mathbf{b}}|^2$

(c) We have,

$(\vec{\mathbf{a}} - \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}})$

$$= \vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}} - \vec{\mathbf{b}} \times \vec{\mathbf{b}}$$

$$= \vec{\mathbf{a}} \times \vec{\mathbf{b}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}}$$

$$[\because \vec{\mathbf{a}} \times \vec{\mathbf{a}} = 0, \vec{\mathbf{b}} \times \vec{\mathbf{b}} = 0]$$

$$= \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} \quad [\because \vec{\mathbf{a}} \times \vec{\mathbf{b}} = -\vec{\mathbf{b}} \times \vec{\mathbf{a}}]$$

$$= 2(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$$

69. A spacecraft located at $\hat{i} + 2\hat{j} + 3\hat{k}$ is subjected to a force $\lambda\hat{k}$ by firing a rocket. The spacecraft is subjected to a moment of magnitude

(a) λ (b) $\sqrt{3}\lambda$
(c) $\sqrt{5}\lambda$ (d) None of these

(c) We have,

$$\vec{\mathbf{r}} = \hat{i} + 2\hat{j} + 3\hat{k}$$

and $\quad \vec{\mathbf{F}} = \lambda\hat{k}$

We know that,

Moment $= \vec{\mathbf{r}} \times \vec{\mathbf{F}}$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (\lambda\hat{k})$$

$$= -\lambda\hat{j} + 2\lambda\hat{i} \quad \left[\begin{array}{l} \therefore \hat{i} \times \hat{k} = -\hat{j} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{k} = 0 \end{array}\right]$$

Magnitude of moment $= \sqrt{(-\lambda)^2 + (2\lambda)^2}$

$$= \sqrt{\lambda^2 + 4\lambda^2} = \sqrt{5\lambda^2} = \sqrt{5}\,\lambda$$

70. In a triangle ABC, if taken in order, consider the following statements

1. $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{0}}$
2. $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{0}}$
3. $\overrightarrow{\mathbf{AB}} - \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{0}}$
4. $\overrightarrow{\mathbf{BA}} - \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{0}}$

How many of the above statement are correct?

(a) One (b) Two
(c) Three (d) Four

(a) We know that,

In a triangle ABC

$\vec{AB} + \vec{BC} + \vec{CA} = \vec{O}$ [$\because$ by triangle law]

So, only first statement is correct.

71. Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$. Then the value of θ in the interval $(0, \pi)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(b) We have,

$y = \cos^{-1}(\sin x)$

differentiation w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\sin^2 x}}\cos x$$

$$= \frac{-\cos x}{\cos x} \quad [\because \sin^2 x + \cos^2 x = 1]$$

$$\frac{dy}{dx} = -1$$

Slope of the curve = $\tan\theta$

$\therefore$ $\tan\theta = -1$

$\Rightarrow$ $\tan\theta = -\tan\frac{\pi}{4}$

$\Rightarrow$ $\tan\theta = \tan\left(\pi - \frac{\pi}{4}\right)$

[$\because \theta \in (0, \pi)$]

$\Rightarrow$ $\theta = \frac{3\pi}{4}$

72. If $f(x) = \frac{\sqrt{x-1}}{x-4}$, defines a function on $\boldsymbol{R}$, then what is its domain ?

(a) $(-\infty, 4) \cup (4, \infty)$
(b) $[4, \infty)$
(c) $(1, 4) \cup (4, \infty)$
(d) $[1, 4) \cup (4, \infty)$

(d) We have,

$$f(x) = \frac{\sqrt{x-1}}{x-4}$$

$\therefore$ $x - 1 \geq 0$

and $x - 4 \neq 0$

$\Rightarrow$ $x \geq 1$

$x \neq 4$

So, $x \in [1, 4) \cup (4, \infty)$

Domain = $[1, 4) \cup (4, \infty)$

73. Consider the function

$$f(x) = \begin{cases} \frac{\sin 2x}{5x}, & \text{if } x \neq 0 \\ \frac{2}{15}, & \text{if } x = 0 \end{cases}$$

Which one of the following is correct in respect of the function?

(a) It is not continuous at $x = 0$
(b) It is continuous at every x
(c) It is not continuous at $x = \pi$
(d) It is continuous at $x = 0$

(a) We have,

$$f(x) = \begin{cases} \frac{\sin 2x}{5x} & , x \neq 0 \\ \frac{2}{15} & , x = 0 \end{cases}$$

at $x = 0$

$$\lim_{x \to 0} f(x) = \lim_{x\to 0}\frac{\sin 2x \times 2}{5x \times 2} = \frac{2}{5}$$

$$f(0) = \frac{2}{5}$$

$\therefore$ $\lim_{x\to 0} f(x) \neq f(0)$

at $x = 0$ function is discontinuous.

74. For the function $f(x) = |x - 3|$, which one of the following is not correct?

(a) The function is not continuous at $x = -3$
(b) The function is continuous at $x = 3$
(c) The function is differentiable at $x = 0$
(d) The function is differentiable at $x = -3$

(a) We have,

$f(x) = |x - 3|$

We know that, modulus function is continuous in R.

So, option (a) is incorrect.

75. If the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, then what is the value of $f(0)$?

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

(b) We have,

$$f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$$

Function is continuous at each point

$\therefore$ $f(0) = \lim_{x\to 0} f(x)$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$

$$= \lim_{x\to 0}\frac{2 - \frac{1}{\sqrt{1-x^2}}}{2 + \frac{1}{1+x^2}}$$

[by L-Hospital]

$$= \frac{2-1}{2+1} = \frac{1}{3}$$

$\therefore$ $f(0) = \frac{1}{3}$

76. If $f(x) = \sqrt{25 - x^2}$, then what is $\lim_{x\to 1}\frac{f(x) - f(1)}{x - 1}$ equal to?

(a) $-\frac{1}{\sqrt{24}}$ (b) $\frac{1}{\sqrt{24}}$
(c) $-\frac{1}{4\sqrt{3}}$ (d) $\frac{1}{\sqrt{4\sqrt{3}}}$

(a) We have,

$$f(x) = \sqrt{25 - x^2}$$

Now, $\lim_{x\to 1}\frac{f(x) - f(1)}{x - 1}$

$$= \lim_{x \to 1}\frac{\sqrt{25 - x^2} - \sqrt{24}}{x - 1}$$

$$= \lim_{x\to 1}\frac{\frac{-2x}{2\sqrt{25 - x^2}} - 0}{1 - 0}$$

[by L-Hosptial]

$$= -\frac{1}{\sqrt{24}}$$

77. If $y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$, then what is $\frac{dy}{dx}$ equal to?

(a) $-\frac{1}{2\sqrt{x}}$ (b) 1
(c) -1 (d) $\frac{1}{2\sqrt{x}}$

(a) We have,

$$y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$$

$$y = \tan^{-1}\left(\frac{\frac{5}{2} - \tan\sqrt{x}}{1 + \frac{5}{2}\tan\sqrt{x}}\right)$$

Let, $\frac{5}{2} = \tan A$

$$= \tan^{-1}\left(\frac{\tan A - \tan\sqrt{x}}{1 + \tan A \tan\sqrt{x}}\right)$$

$$= \tan^{-1}[\tan(A - \sqrt{x})] = A - \sqrt{x}$$

$$y = \tan^{-1}\frac{5}{2} - \sqrt{x}$$

differentiation w.r.t x, we get

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

78. Which one of the following is correct in respect of the function $f(x) = x\sin x + \cos x + \frac{1}{2}\cos^2 x$?

(a) It is increasing in the interval $\left(0, \frac{\pi}{2}\right)$
(b) It remains constant in the interval $\left(0, \frac{\pi}{2}\right)$
(c) It is decreasing in the interval $\left(0, \frac{\pi}{2}\right)$
(d) It is decreasing in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(a) We have,

$$f(x) = x\sin x + \cos x + \frac{1}{2}\cos^2 x$$

$$f'(x) = x(\cos x) + \sin x - \sin x + \frac{1}{2} \cdot 2\cos x(-\sin x)$$

$$f'(x) = x\cos x - \sin x \cos x$$

By checking options, we put

$$x = \frac{\pi}{4}$$

$$f'(x) = \frac{\pi}{4}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{4}$$

$$= \frac{\pi}{4}\cdot\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{4\sqrt{2}} - \frac{1}{2} > 0$$

So, $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$

79. What is $\lim\limits_{\theta\to 0}\dfrac{\sqrt{1-\cos\theta}}{\theta}$ equal to?

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

(c) We have,

$$\lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta}$$

$$= \lim_{\theta\to 0}\frac{\sqrt{1-\left(1-2\sin^2\frac{\theta}{2}\right)}}{\theta}$$

$$= \lim_{\theta\to 0}\frac{\sqrt{2\sin^2\frac{\theta}{2}}}{\theta}$$

$$= \lim_{\theta\to 0}\frac{\sqrt{2}\sin\frac{\theta}{2}}{\frac{\theta}{2}\times 2}$$

$$= \frac{1}{\sqrt{2}}$$

80. A function $f : A \to \mathbf{R}$ is defined by the equation $f(x) = x^2 - 4x + 5$, where $A = (1, 4)$. What is the range of the function?

(a) (2, 5) (b) (1, 5)
(c) [1, 5) (d) [1, 5]

(c) We have,

A function $f : A \to R$ is defined by

$$f(x) = x^2 - 4x + 5,$$

Where, $A = (1, 4)$

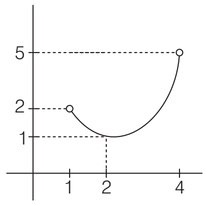
Let, $y = x^2 - 4x + 5$

$$\frac{dy}{dx} = 2x - 4$$

Now, $\frac{dy}{dx} = 0$

$\Rightarrow$ $2x - 4 = 0$

$\Rightarrow$ $x = 2$



At $x = 2$, $y = 1$

At $x = 1$, $y = (1)^2 - 4(1) + 5 = 2$

At $x = 4$ $y = (4)^2 - 4(4) + 5 = 5$

So, $y \in [1, 5)$

Range $= [1, 5)$

81. What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where [,] is the greatest integer function?

(a) $b - a$ (b) $a - b$
(c) 0 (d) $2(b - a)$

(b) We have,

$$\int_a^b [x]\,dx + \int_a^b [-x]\,dx$$

$$= \int_a^b ([x] + [-x])\,dx$$

$$[\because [x] + [-x] = -1, \text{ if } x \notin z]$$

$$= \int_a^b (-1)\,dx$$

$$= -(x)_a^b = -(b - a) = a - b$$

82. What is $\int_2^8 |x - 5|\,dx$ equal to?

(a) 2 (b) 3 (c) 4 (d) 9

(d) We have,

$$\int_2^8 |x - 5|\,dx$$

$$= \int_2^5 |x - 5|\,dx + \int_5^8 |x - 5|\,dx$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x) = dx\right]$$

$$= -\int_2^5 (x - 5)\,dx + \int_5^8 (x - 5)\,dx$$

$$= -\left(\frac{x^2}{2} - 5x\right)_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\left(\frac{25}{2} - 25\right) - (2 - 10)\right] + \left[(32 - 40) - \left(\frac{25}{2} - 25\right)\right]$$

$$= \frac{25}{2} - 8 - 8 + \frac{25}{2} = 9$$

83. What is $\int \sin^3 x \cos x\,dx$ equal to ?

(a) $\cos^4 x + C$
(b) $\sin^4 x + C$
(c) $\frac{(1 - \sin^2 x)^2}{4} + C$
(d) $\frac{(1 - \cos^2 x)^2}{4} + C$

Where C is the constant of integration.

(d) We have,

$$\int \sin^3 x \cos x\,dx$$

Let, $\sin x = t$

$$\cos x\,dx = dt$$

$$= \int t^3\,dt$$

$$= \frac{t^4}{4} + C$$

$$= \frac{\sin^4 x}{4} + C$$

$$= \frac{(\sin^2 x)^2}{4} + C$$

$$= \frac{(1 - \cos^2 x)^2}{4} + C$$

$$[\because \sin^2 x + \cos^2 x = 1]$$

84. What is $\int e^{\ln(\tan x)}\,dx$ equal to

(a) $\ln|\tan x| + C$
(b) $\ln|\sec x| + C$
(c) $\tan x + C$
(d) $e^{\tan x} + C$

Where C is the constant of integration.

(b) We have,

$$\int e^{\ln(\tan x)}\,dx \qquad [\because e^{\ln(x)} = x]$$

$$= \int \tan x\,dx$$

$$= \log|\sec x| + C$$

85. What is $\int_{-1}^{1}\left\{\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right\}dx$ equal to ?

(a) 0 (b) $-\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

(d) We have,

$$\int_{-1}^{1}\left[\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right]dx$$

$$= \left[\tan^{-1}\left(\frac{1}{x}\right)\right]_{-1}^{1}$$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{\pi}{2}$$

86. In which one of the following intervals is the function $f(x) = x^2 - 5x + 6$ decreasing?

(a) $(-\infty, 2]$ (b) $[3, \infty)$
(c) $(-\infty, \infty)$ (d) (2, 3)

(a) We have,

$$f(x) = x^2 - 5x + 6$$

$$f'(x) = 2x - 5$$

For decreasing

$$f'(x) < 0$$
$$2x - 5 < 0$$
$$x < \frac{5}{2}$$
$$x < 2.5$$
$$x \in (-\infty, 2.5)$$

87. The differential equation of the family of curves $y = p\cos(ax) + q\sin(ax)$, where p, q are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} - a^2y = 0$

(b) $\frac{d^2y}{dx^2} - ay = 0$

(c) $\frac{d^2y}{dx^2} + ay = 0$

(d) $\frac{d^2y}{dx^2} + a^2y = 0$

(d) We have,

$$y = p\cos(ax) + q\sin(ax)$$

differentiation. w. r. t. x, we get

$$\frac{dy}{dx} = -pa\sin ax + qa\cos ax$$

Again, differentiation. w. r. t. x, we get

$$\frac{d^2y}{dx^2} = -pa^2\cos ax - qa^2\sin ax$$

$$\Rightarrow \frac{d^2y}{dx^2} = -a^2(p\cos ax + q\sin ax)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -a^2y$$

$$\Rightarrow \frac{d^2y}{dx^2} + a^2y = 0$$

88. The equation of the curve passing through the point $(-1, -2)$, which satisfies $\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$, is

(a) $17x^2y - 6x^2 + 3x^5 - 2 = 0$
(b) $6x^2y + 17x^2 + 2x^5 - 3 = 0$
(c) $6xy - 2x^2 + 17x^5 + 3 = 0$
(d) $17x^2y + 6xy - 3x^5 + 5 = 0$

(b) We have,

$$\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$$

$$\Rightarrow dy = \left(-x^2 - \frac{1}{x^3}\right)dx$$

Integrating both sides

$$\int dy = \int\left(-x^2 - \frac{1}{x^3}\right)dx$$

$$\Rightarrow y = -\frac{x^3}{3} - \left(\frac{x^{-2}}{-2}\right) + c$$

$$\Rightarrow y = \frac{-x^3}{3} + \frac{1}{2x^2} + C \quad \text{...(i)}$$

given curve is passing through $(-1, -2)$

$$-2 = \frac{1}{3} + \frac{1}{2} + C$$

$$\Rightarrow -2 = \frac{5}{6} + C$$

$$\Rightarrow C = -2\,\frac{-5}{6}$$

$$\Rightarrow C = \frac{-17}{6}$$

Put in Eq. (i)

$$y = \frac{-x^3}{3} + \frac{1}{2x^2} - \frac{17}{6}$$

$$\Rightarrow y = \frac{-2x^5 + 3 - 17x^2}{6x^2}$$

$$\Rightarrow 6x^2y = -2x^5 + 3 - 17x^2$$

$$\Rightarrow 6x^2y + 2x^5 + 17x^2 - 3 = 0$$

89. What is the order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x} + d$, where a, b, c and d are arbitrary constants?

(a) 1 (b) 2 (c) 3 (d) 4

(d) We have,

$$y = a\cos x + b\sin x + ce^{-x} + d,$$

a, b, c and d are arbitrary constants.

We know that,

order = number of arbitrary constant

So, order = 4

90. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) = ax + by$?

(a) $ae^{ax} + be^{by} = C$
(b) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{by} = C$
(c) $ae^{ax} + be^{-by} = C$
(d) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{-by} = C$

(d) We have,

$$\ln\left(\frac{dy}{dx}\right) = ax + by$$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{dx} = e^{ax}\cdot e^{by}$$

$$\frac{dy}{e^{by}} = e^{ax}\,dx$$

Integrating both sides

$$\int e^{-by}dy = \int e^{ax}dx$$

$$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + C$$

$$\Rightarrow \frac{e^{ax}}{a} + \frac{e^{-by}}{b} + C = 0$$

91. If $u = e^{ax}\sin bx$ and $v = e^{ax}\cos bx$, then what is $u\frac{du}{dx} + v\frac{dv}{dx}$ equal to?

(a) ae^{2ax} (b) $(a^2 + b^2)e^{ax}$
(c) $ab\,e^{2ax}$ (d) $(a + b)e^{ax}$

(a) We have,

$$u = e^{ax}\sin bx$$

differentiation. w. r. t. x, we get

$$\frac{du}{dx} = e^{ax}(b\cos bx) + \sin bx\,(ae^{ax})$$

$$\frac{du}{dx} = e^{ax}(b\cos bx + a\sin bx) \quad \text{...(i)}$$

Now, $v = e^{ax}\cos bx$

differentiation. w. r. t. x, we get

$$\frac{dv}{dx} = e^{ax}(-b\sin bx) + \cos bx\,(ae^{ax})$$

$$= e^{ax}(-b\sin bx + a\cos bx)$$

Now we have,

$$u\frac{du}{dx} + v\frac{dv}{dx}$$

$$= e^{ax}\sin bx\,[e^{ax}(b\cos bx + a\sin bx)]$$

$$+ e^{ax}\cos bx\,[e^{ax}(-b\sin bx + a\cos bx)]$$

$$= e^{2ax}[b\sin bx\cos bx + a\sin^2 bx] + e^{2ax}(-b\cos bx\sin bx + a\cos^2 bx)$$

$$= e^{2ax}[a\sin^2 bx + a\cos^2 bx]$$

$$= ae^{2ax}$$

92. If $y = \sin(ln\,x)$, then which one of the following is correct?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dn^2} = 0$

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(d) $x\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Where C is the constant of integration

(c) We have,

$$y = \sin(\log_e x)$$

differentiation. w. r. t. x, we get

$$\frac{dy}{dx} = \frac{\cos(\log_e x)}{x}$$

$$x\frac{dy}{dx} = \cos(\log_e x)$$

Again, differentiation. w. r. t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-\sin(\log_e x)}{x}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\sin(\log_e x)$$

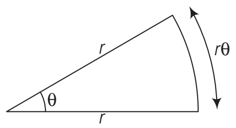
$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

93. A flower-bed in the form of a sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible area, then what is the radius of the sector?

(a) 25 m (b) 20 m
(c) 10 m (d) 5 m

(c) Let radius of sector be r and angle subtended at centre be θ.



$$\therefore \quad r + r + r\theta = 40$$
$$\Rightarrow \quad 2r + r\theta = 40$$
$$\Rightarrow \quad \theta = \frac{40 - 2r}{r}$$

Area of the sector $= \frac{1}{2}r^2\theta$

$$A = \frac{1}{2}r^2\left(\frac{40-2r}{r}\right)$$
$$A = \frac{1}{2}r(40 - 2r)$$
$$A = \frac{1}{2}(40r - 2r^2)$$

differentiating. w. r. t. r,

$$\frac{dA}{dr} = \frac{1}{2}(40 - 4r)$$

Now, $\frac{dA}{dr} = 0$

$$\Rightarrow \quad 40 - 4r = 0$$
$$\Rightarrow \quad r = 10\text{m}$$

94. What is the minimum value of $[x(x-1)+1]^{\frac{1}{3}}$, where $a \le x \le 1$?

(a) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$ (b) 1

(c) $\frac{1}{2}$ (d) $\left(\frac{3}{8}\right)^{1/3}$

(a) We have,

$$y = (x^2 - x + 1)^{\frac{1}{3}}$$
$$y = \left(x^2 - x + \frac{1}{4} - \frac{1}{4} + 1\right)^{\frac{1}{3}}$$

at $x = \frac{1}{2}$, y is minimum $y = \left(\frac{3}{4}\right)^{\frac{1}{3}}$

95. If $y = |\sin x|^{|x|}$, then what is the value of $\frac{dy}{dx}$ at $x = -\frac{\pi}{6}$?

(a) $\frac{2^{-\frac{\pi}{6}}(6\ln 2 - \sqrt{3}\pi)}{6}$

(b) $\frac{2^{\frac{\pi}{6}}(6\ln 2 + \sqrt{3}\pi)}{6}$

(c) $\frac{2^{-\frac{\pi}{6}}(6\ln 2 + \sqrt{3}\pi)}{6}$

(d) $\frac{2^{-\frac{\pi}{6}}(6\ln 2 - \sqrt{3}\pi)}{6}$

(a) We have,

$$y = |\sin x|^{|x|}$$
$$\therefore \quad x = -\frac{\pi}{6},$$
$$\sin x < 0,\ x < 0$$
$$y = (-\sin x)^{-x}$$
$$\because \quad y = f(x)^{g(x)}, \text{ So}$$
$$\frac{dy}{dx} = f(x)^{g(x)}\left[\frac{g(x)}{f(x)}f'(x) + \log|f(x)|\cdot g'(x)\right]$$
$$\frac{dy}{dx} = (-\sin x)^{-x}$$
$$\left[\frac{(-x)}{(-\sin x)}\cdot(-\cos x) + \log|(-\sin x)|(-1)\right]$$
$$\left.\frac{dy}{dx}\right|_{x=-\pi/6} = \left[-\sin\left(\frac{-\pi}{6}\right)\right]^{\frac{\pi}{6}}$$
$$\left\{\frac{\pi/6}{-\sin(-\pi/6)} - \cos\left(-\frac{\pi}{6}\right)\right\}$$
$$\left.-\log\left|-\sin\left(-\frac{\pi}{6}\right)\right|\right]$$
$$= \left(\sin\frac{\pi}{6}\right)^{\frac{\pi}{6}}\left[\frac{-\frac{\pi}{6}}{\frac{1}{2}}\left(\frac{\sqrt{3}}{2}\right) - \log\left(\frac{1}{2}\right)\right]$$
$$= \left(\frac{1}{2}\right)^{\frac{\pi}{6}}\left[\frac{-\sqrt{3}\pi}{6} - \log\frac{1}{2}\right]$$
$$= 2^{-\pi/6}\left[\log^2 - \frac{\sqrt{3}\pi}{6}\right]$$
$$= 2^{-\frac{\pi}{6}}\left[\frac{6\log 2 - \sqrt{3}\pi}{6}\right]$$

96. What is $\frac{d\sqrt{1-\sin 2x}}{dx}$ equal to, where $\frac{\pi}{4} < x < \frac{\pi}{2}$?

(a) $\cos x + \sin x$
(b) $-(\cos x + \sin x)$
(c) $\pm(\cos x + \sin x)$
(d) None of the above

(a) We have,

$$\frac{d}{dx}\sqrt{1-\sin 2x},\ \frac{\pi}{4} < x < \frac{\pi}{2}$$
$$= \frac{d}{dx}\sqrt{\cos^2 x + \sin^2 x - 2\sin x\cos x}$$
$$\left[\begin{array}{l}\because \sin^2 x + \cos^2 x = 1 \\ \text{and } \sin 2x = 2\sin x\cos x\end{array}\right]$$
$$= \frac{d}{dx}|\cos x - \sin x|$$
$$= \frac{d}{dx}(\sin x - \cos x)$$
$$\left[\because \frac{\pi}{4} < x < \frac{\pi}{2}\ \therefore \sin x > \cos x\right]$$
$$= \cos x - (-\sin x)$$
$$= \cos x + \sin x$$

97. What is $\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$ equal to ?

(a) $C + \frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$

(b) $C - \frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$

(c) $C + \frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{b}\right)$

(d) None of the above

Where C is the constant of integration.

(a) We have,

$$\int\frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$$

Divide numerator and denominator by $\cos^2 x$

$$I = \int\frac{\sec^2 x\,dx}{a^2\tan^2 x + b^2}$$

Let $a\tan x = t$

$$a\sec^2 x\,dx = dt$$
$$\sec^2 x\,dx = \frac{dt}{a}$$
$$I = \frac{1}{a}\int\frac{dt}{t^2 + b^2}$$
$$= \frac{1}{a}\times\frac{1}{b}\tan^{-1}\left(\frac{t}{b}\right) + C$$
$$\left[\because \int\frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$$
$$= \frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + C$$

98. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + xg(x)\phi(x)$, where $\lim_{x\to 0} g(x) = a$ and $\lim_{x\to 0}\phi(x) = b$. Then, what is $f'(x)$ equal to?

(a) $1 + abf(x)$ (b) $1 + ab$
(c) ab (d) $abf(x)$

(d) Let, $f(x+y) = f(x)f(y)$

and $f(x) = 1 + x\,g(x)\cdot\phi(x)$

Thus, $f(y) = 1 + yg(y)\phi(y)$

Where $\lim_{x\to 0} g(x) = a$ and $\lim_{x\to 0}\phi(x) = b$

using first principal

$$f'(x) = \lim_{y\to 0}\frac{f(x+y) - f(x)}{y}$$
$$= \lim_{y\to 0}\frac{f(x)\cdot f(y) - f(x)}{y}$$
$$= \lim_{y\to 0}\frac{f(x)[f(y) - 1]}{y}$$

$$\lim_{y\to 0} f(x)\left[\frac{1+yg(y)\phi(y)-1}{y}\right]$$
$$= f(x)\lim_{y\to 0} g(y)\lim_{y\to 0}\phi(y)$$
$$= f(x)\cdot a\cdot b$$
$$= ab\, f(x)$$

99. What is the solution of the differential equation $\frac{dx}{dy}=\frac{x+y+1}{x+y-1}$?

(a) $y-x+4\ln(x+y)=C$
(b) $y+x+2\ln(x+y)=C$
(c) $y-x+\ln(x+y)=C$
(d) $y+x+2\ln(x+y)=C$

Where C is an arbitrary constant.

(c) We have, $\frac{dx}{dy}=\frac{x+y+1}{x+y-1}$

Let $x+y=u$

$$\frac{dx}{dy}+1=\frac{du}{dy}$$
$$\Rightarrow \frac{dx}{dy}=\frac{du}{dy}-1$$
$$\Rightarrow \frac{du}{dy}-1=\frac{u+1}{u-1}$$
$$\Rightarrow \frac{du}{dy}=\frac{u+1}{u-1}+1$$
$$\Rightarrow \frac{du}{dy}=\frac{u+1+u-1}{u-1}$$
$$\Rightarrow \frac{dy}{dy}=\frac{2u}{u-1}$$
$$\Rightarrow \left(\frac{u-1}{u}\right)du=2\,dy$$

Integrating both sides,

$$\int\left(1-\frac{1}{u}\right)du=\int 2\,dy$$
$$\Rightarrow u-\log u=2y+C \quad [\because u=x+y]$$
$$\Rightarrow x+y-\log(x+y)=2y+C$$
$$\Rightarrow x-\log(x+y)=y+C$$
$$\Rightarrow y-x+\log(x+y)=C$$

100. What is $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$ equal to?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$
(c) -2 (d) -3

(d) We have,

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$$
$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+2\sin x-\sin x-1}{2\sin^2 x-2\sin x-\sin x+1}$$
$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x(\sin x+1)-(\sin x+1)}{2\sin x(\sin x-1)-1(\sin x-1)}$$
$$=\lim_{x\to\frac{\pi}{6}}\frac{(\sin x+1)(2\sin x-1)}{(\sin x-1)(2\sin x-1)}$$
$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin x+1}{\sin x-1}$$
$$=\frac{\frac{1}{2}+1}{\frac{1}{2}-1}=\frac{\frac{3}{2}}{-\frac{1}{2}}$$
$$=-3$$

101. If two dice are thrown and atleast one of the dice shows 5, then the probability that the sum is 10 or more is

(a) $\frac{1}{6}$ (b) $\frac{4}{11}$
(c) $\frac{3}{11}$ (d) $\frac{2}{11}$

(c) Let A be event of dice shows 5 and B be the event that the sum is 10 or more

Here, $n(S)=36$

$n(A)=\{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,6)\}$

$n(B)=\{(5,5),(6,4),(4,6),(6,5),(5,6),(6,6)\}$

$n(A\cap B)=\{(5,5),(6,5),(5,6)\}$

$$P\left(\frac{B}{A}\right)=\frac{\frac{3}{36}}{\frac{11}{36}} \quad \left[\because P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}\right]$$
$$=\frac{3}{11}.$$

102. The correlation coefficient computed from a set of 30 observations is 0.8. Then the percentage of variation not explained by linear regression is

(a) 80% (b) 20%
(c) 64% (d) 36%

(b) Given that,

correlation coefficient = 0.8 = 80%, if the relation is 80% explained, then 20% of variation will not explained by near regression.

103. The average age of a combined group of men and women is 25 yr. If the average age of the group of men is 26 yr and that of the group of women is 21 yr, then the percentage of men and women in the group is respectively

(a) 20, 80 (b) 40, 60
(c) 60, 40 (d) 80, 20

(d) In group of men, let number of men $=a$

$\overline{X}_1=26$ yr and $n_1=a$ (let)

and in group of women, number of women $=b$ (let)

$\overline{X}_2=21$ yr

and combined mean $\overline{X}=25$

Now, $\overline{X}=\frac{n_1\overline{X}_1+n_2\overline{X}_2}{n_1+n_2}$

$$\Rightarrow 25=\frac{26a+21b}{a+b}$$
$$\Rightarrow 25a+25b=26a+21b$$
$$\Rightarrow 4b=a$$
$$\Rightarrow \frac{a}{b}=\frac{4}{1}$$

By checking options (d) is correct.

104. If $\sin\beta$ is the harmonic mean of $\sin\alpha$ and $\cos\alpha$ and $\sin\theta$ is the arithmetic mean of $\sin\alpha$ and $\cos\alpha$, then which of the following is/are correct?

1. $\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)\sin\beta=\sin 2\alpha$
2. $\sqrt{2}\sin\theta=\cos\left(\alpha-\frac{\pi}{4}\right)$

Select the correct answer using the code given below.

(a) 1only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Given that,

$\sin\beta$ is HM of $\sin\alpha$ and $\cos\alpha$

So, $\sin\beta=\frac{2\sin\alpha\cos\alpha}{\sin\alpha+\cos\alpha}$

$$\Rightarrow \sin\beta(\sin\alpha+\cos\alpha)=\sin 2\alpha$$
$$\Rightarrow \sin\beta\times\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\alpha+\frac{1}{\sqrt{2}}\cos\alpha\right)=\sin 2\alpha$$
$$\Rightarrow \sin\beta\times\sqrt{2}\left(\cos\frac{\pi}{4}\sin\alpha+\sin\frac{\pi}{4}\cos\alpha\right)=\sin 2\alpha$$
$$\Rightarrow \sqrt{2}\sin\beta\left[\sin\left(\alpha+\frac{\pi}{4}\right)\right]=\sin 2\alpha$$
$$\Rightarrow \sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)\sin\beta=\sin 2\alpha$$

Statement I is true.

Now, $\sin\theta$ is AM of $\sin\alpha$ and $\cos\alpha$

$$\sin\theta=\frac{\sin\alpha+\cos\alpha}{2}$$
$$\Rightarrow 2\sin\theta=\sin\alpha+\cos\alpha$$
$$\Rightarrow 2\sin\theta=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\alpha+\frac{1}{\sqrt{2}}\cos\alpha\right)$$
$$\Rightarrow 2\sin\theta=\sqrt{2}\left(\sin\frac{\pi}{4}\sin\alpha+\cos\frac{\pi}{4}\cos\alpha\right)$$
$$\Rightarrow \sqrt{2}\sin\theta=\cos\left(\alpha-\frac{\pi}{4}\right)$$

Hence, Statement II is also correct.

105. Let A, B and C be three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = 1.5\ P(A)$ and $P(C) = 0.5\ P(B)$, then $P(A)$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{4}{13}$
(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(b) We have,

$$P(B) = 1.5\ P(A) = \frac{3}{2} P(A)$$

and $P(C) = 0.5\ P(B) = \frac{1}{2} P(B)$

$$= \frac{1}{2} \times \frac{3}{2} P(A)$$
$$= \frac{3}{4} P(A)$$

Now, A, B and C are mutually exclusive and exhaustive events

So,

$$P(A) + P(B) + P(C) = 1$$
$$\Rightarrow P(A) + \frac{3}{2} P(A) + \frac{3}{4} P(A) = 1$$
$$\Rightarrow P(A)\left[1 + \frac{3}{2} + \frac{3}{4}\right] = 1$$
$$\Rightarrow P(A)\left(\frac{13}{4}\right) = 1$$
$$\Rightarrow P(A) = \frac{4}{13}$$

106. In a bolt factory, machines X, Y, Z manufacture bolts that are respectively 25%, 35% and 40% of the factory's total output. The machines X, Y, Z respectively produce 2%, 4% and 5% defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine X?

(a) $\frac{5}{39}$ (b) $\frac{14}{39}$
(c) $\frac{20}{39}$ (d) $\frac{34}{39}$

(a) Let

A : bolt manufactured from machine
B : bolt manufactured from machine
C : bolt maufactured from machine
and E : bolt is defective

$$\therefore \quad P(A) = 25\% = \frac{25}{100}$$
$$P(B) = 35\% = \frac{35}{100}$$
$$P(C) = 40\% = \frac{40}{100}$$

and $P\left(\frac{E}{A}\right) = 2\% = \frac{2}{100}$

$$P\left(\frac{E}{B}\right) = 4\% = \frac{4}{100}$$
$$P\left(\frac{E}{C}\right) = 5\% = \frac{5}{100}$$

Probability of defective bulb that was manufactured by machine X,

$$P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right) + P(C) \cdot P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{2}{100}}{\frac{25}{100} \times \frac{2}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{5}{100}}$$

$$= \frac{25 \times 2}{25 \times 2 + 35 \times 4 + 40 \times 5}$$

$$= \frac{50}{50 + 140 + 200}$$

$$= \frac{50}{390} = \frac{5}{39}$$

107. 8 coins are tossed simultaneously. The probability of getting atleast 6 heads is

(a) $\frac{7}{64}$ (b) $\frac{57}{64}$
(c) $\frac{37}{256}$ (d) $\frac{229}{256}$

(c) We have,

8 coins are tossed simultaneously i.e $n = 8$ probability of getting head $p = \frac{1}{2}$, so $q = 1 - p = \frac{1}{2}$. Probability of getting atleast 6 heads.

$$= {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 ({}^8C_6 + {}^8C_7 + {}^8C_8)$$

$$= \left(\frac{1}{2}\right)^8 (28 + 8 + 1) = \frac{37}{256}$$

108. Three groups of children contain 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consist of 1 girl and 2 boys is

(a) $\frac{13}{32}$ (b) $\frac{9}{32}$
(c) $\frac{3}{32}$ (d) $\frac{1}{32}$

(a) In first group, 3 girls and 1 boy

$$P(G) = \frac{3}{4} \text{ and } P(B) = \frac{1}{4}$$

In second group, 2 girls and 2 boys

$$P(G) = \frac{2}{4} = \frac{1}{2}$$

and $P(B) = \frac{2}{4} = \frac{1}{2}$

One child is selected at random from each group and consists of 1 girl and 2 boys are GBB or BGB or BBG.

So, required probability

$$= \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4}$$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$$

109. Consider the following statements :

1. If 10 is added to each entry on a list, then the average increases by 10.
2. If 10 is added to each entry on a list, then the standard deviation increases by 10.
3. If each entry on a list is doubled then the average doubles.

Which of the above statements are correct?

(a) 1, 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 2 and 3

(c) **Statement 1** The average is affected by the change of the origin.

So, if 10 is added to each entry on list then average increase by 10. Statement 1 is correct.

Statement 2 Standard deviation is independent on change in origin. Statement 2 is incorrect.

Statement 3 The average is affected by change in scale in same ratio as each entry is changes.

So, Statement 3 is correct.

110. The variance of 25 observations is 4. If 2 is added to each observation, then the new variance of the resulting observations is

(a) 2 (b) 4
(c) 6 (d) 8

(b) We know that,

Variance is independent on change in origin.

So, if 2 is added to each observation, then variance is remain same

Hence, variance $= 4$

111. If $x_i > 0, y_i > 0 (i = 1, 2, 3, ..., n)$ are the values of two variables X and Y with geometric means P and Q respectively, then the geometric mean of $\frac{X}{Y}$ is

(a) $\frac{P}{Q}$
(b) antilog $\left(\frac{P}{Q}\right)$
(c) $n(\log P - \log Q)$
(d) $n(\log P + \log Q)$

(a) We have, $x_i > 0$, $y_i > 0$,
$(i = 1, 2, 3, ..., n)$

$$P = (x_1 \times x_2 \times x_3 \times ..... x_n)^{\frac{1}{n}}$$

and $Q = (y_1 \times y_2 \times y_3 \times ......\times y_n)^{\frac{1}{n}}$

Now, geometric mean of $\frac{x}{y}$

$$= \left(\frac{x_1}{y_1} \times \frac{x_2}{y_2} \times ....\times \frac{x_n}{y_n}\right)^{\frac{1}{n}}$$

$$= \frac{P}{Q}$$

112. If probability of simultaneous occurrence of two events A and B is p and the probability that exactly one of A, B occurs is q, then which of the following is/are correct?

1. $P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$
2. $P(\bar{A} \cap \bar{B}) = 1 - p - q$

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Given that,

$$P(A \cap B) = P$$

and $P(A) + P(B) - 2P(A \cap B) = q$

Now, **Statement 1**

Given,

$$P(A) + P(B) - 2P(A \cap B) = q$$

$\Rightarrow$ $1 - P(\bar{A}) + 1 - P(\bar{B}) - 2p = q$

$\Rightarrow$ $P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$

Statement 1 is correct

For **Statement 2**

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - [P(A) + P(B) - 2P(A \cap B) + P(A \cap B)]$$
$$= 1 - [q + p]$$
$$= 1 - q - p$$

Statement 2 is also correct.

113. If the regression coefficient of Y on X is -6 and the correlation coefficient between X and Y is $-\frac{1}{2}$, then the regression coefficient of X on Y would be

(a) $\frac{1}{24}$ (b) $-\frac{1}{24}$
(c) $-\frac{1}{6}$ (d) $\frac{1}{6}$

(b) Given that,

$$b_{yx} = -6$$

and $r_{xy} = -\frac{1}{2}$

We know that,

$$r_{xy} = \sqrt{b_{yx} \times b_{xy}}$$

$\Rightarrow$ $-\frac{1}{2} = \sqrt{-6 \times b_{xy}}$

Squaring both sides,

$\Rightarrow$ $\frac{1}{4} = -6 \times b_{xy}$

$\Rightarrow$ $b_{xy} = -\frac{1}{24}$

114. The set of bivariate observations $(x_1, y_1), (x_2, y_2)....(x_n, y_n)$ are such that all the values are distinct and all the observations fall on a straight line with non-zero slope. Then the possible values of the correlation coefficient between x and y are

(a) 0 and 1 (b) 0 and -1
(c) 0, 1 and -1 (d) -1 and 1

(d) Given that,

All the observations fall on a straight line with non-zero, slope then if slope is positive then $r = 1$

and if slope is negative then $r = -1$

So, values of the correlation coefficient between x and y are -1 and 1.

115. Two integers x and y are chosen with replacement from the set $[0, 1, 2, ..., 10]$. The probability that $|x - y| > 5$ is

(a) $\frac{6}{11}$ (b) $\frac{35}{121}$
(c) $\frac{30}{121}$ (d) $\frac{25}{121}$

(c) Given that,

$$S = \{0, 1, 2, ..., 10\}$$
$$n(S) = 11 \times 11 = 121$$

Now, $x - y > 5$

Now, E is the set of element such that $|x - y| > 5$

$E = \{(6, 0), (0, 6), (7, 1), (1, 7), (8, 2), (2, 8), (9, 3), (3, 9), (10, 4), (4, 10), (7, 0), (0, 7), (8,1), (1,8), (9, 2), (2, 9), (10, 3), (3, 10), (8, 0), (0, 8), (9, 1), (1, 9), (10, 2), (2, 10), (9, 0), (0, 9), (10, 1), (1, 10), (10, 0), (0, 10)\}$

$$n(E) = 30$$

So, required probability $= \frac{30}{121}$

116. An analysis of monthly wages paid to the workers in two firms A and B belonging to the same industry gives the following result

	Firm A	Firm B
Number of workers	500	600
Average monthly wage	₹ 1860	₹ 1750
Variance of distribution of wages	81	100

The average of monthly wage and variance of distribution of wages of all the workers in the firms A and B taken together are

(a) ₹ 1860, 100
(b) ₹ 1750, 100
(c) ₹ 1800, 81
(d) None of the above

(d) For firm A

$$n_1 = 500, \bar{X}_1 = 1860$$

and variance $= \sigma_1^2 = 81$

So, S.D. $= \sigma_1 = 9$

For firm B

$$n_2 = 600, \bar{X}_2 = 1750$$

and variance $\sigma_2^2 = 100$

So, S.D. $= \sigma = 10$

Now, combined mean $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$

$$= \frac{500 \times 1860 + 600 \times 1750}{500 + 600}$$

$$\bar{X} = 1800$$

Combined variance

$$= \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{500[81 + (-60)^2] + 600[100 + (50)^2]}{500 + 600}$$

$\because$ $d_1 = \bar{X} - X_1 = 1800 - 1860 = -60$

and $d_2 = 1800 - 1750 = 50$

$$= \frac{500(81 + 3600) + 600(100 + 2500)}{1100}$$

$$= \frac{500(3681) + 600(2600)}{1100}$$

$= 3091.36$

117. Three dice having digits 1, 2, 3, 4, 5 and 6 on their faces are marked I, II, and III and rolled. Let x, y and z represent the number on die-I, die-II and die-III, respectively. What is the number of possible outcomes such that $x > y > z$?

(a) 14 (b) 16
(c) 18 (d) 20

(d) Three dice having digit 1, 2, 3, 4, 5 and 6 and given that $x > y > z$.

So, possiblities are

Case I If $x = 6$

$$x > y > z$$

Possible ways

$= \{ (6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4)$
$(6, 4, 1), (6, 4, 2), (6, 4, 3)$
$(6, 3, 2), (6, 3, 1)(6, 2, 1)$

So, possible ways = 10

Case II If $x = 5$

Then, possible ways

$= \{ (5, 4, 3), (5, 4, 2), (5, 4, 1)$
$(5, 3, 2), (5, 3, 1), (5, 2, 1)\}$

So, possible ways = 6

Case III If $x = 4$

Then, possible ways

$= \{ (4, 3, 2), (4, 3, 1), (4, 2, 1)$

So, possible ways = 3

Case IV If $x = 3$

Then, possible ways = 1 (3, 2, 1)

So, required possible outcomes

$= 10 + 6 + 3 + 1$
$= 20$

118. Which one of the following can be obtained from an ogive?

(a) Mean
(b) Median
(c) Geometric Mean
(d) Mode

(b) We know that, median is always calculated from less than or more than type ogive curve where both curve cuts each other that point median.

119. In any discrete series (when all values are not same), if x represents mean deviation about mean and y represents standard deviation, then which one of the following is correct?

(a) $y \geq x$ (b) $y \leq x$
(c) $x = y$ (d) $x < y$

(d) We know that,

$$\text{MD} = \frac{4}{5}\text{S.D.}$$

$\Rightarrow$ 5MD = 4.S.D.

$\Rightarrow$ $5x = 4y$

[$\because$ MD = x and SD = y]

$\therefore$ $x < y$

120. In which one of the following cases would you expect to get a negative correlation?

(a) The ages of husbands and wifes
(b) Shoe size and intelligence
(c) Insurance companies profits and the number of claims they have to pay
(d) Amount of rainfall and yield of crop

(c) In negative correlation, if x is increases then y is decreases by checking options Insurance companies profits and the number of claims they have to pay are negatively correlated.