NDA / NA

National Defence Academy / Naval Academy

MATHEMATICS-I

QUESTION PAPER 2025

Time Allowed : 2 hrs 30 min **Total Marks : 300**

Instructions

1. *This Test Booklet contains **120** items (questions). Each item is printed in **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to the various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i). *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii). *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii). *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If the sum of binomial coefficients in the expansion of $(x + y)^n$ is 256, then the greatest binomial coefficient occurs in which one of the following terms?
(a) Third **(b)** Fourth **(c)** Fifth **(d)** Ninth

2. If $k < (\sqrt{2}+1)^3 < k+2$, where k is a natural number, then what is the value of k?
(a) 11 **(b)** 13 **(c)** 15 **(d)** 17

3. If $[x \quad 1 \quad 1]\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}\begin{bmatrix}1\\ 1\\ x\end{bmatrix} = [45]$

then which one of the following is a value of x?
(a) –2 **(b)** –1 **(c)** 0 **(d)** 1

4. If $A = \begin{bmatrix}y & z & x\\ z & x & y\\ x & y & z\end{bmatrix}$

Where x, y and z are integers, is an orthogonal matrix, then what is the value of $x^2 + y^2 + z^2$?
(a) 0 **(b)** 1 **(c)** 4 **(d)** 14

5. Consider the following in respect of a non-singular matrix M:
I. $|M^2| = |M|^2$
II. $|M| = |M^{-1}|$
I. $|M| = |M^T|$
How many of the above are correct?
(a) None **(b)** One **(c)** Two **(d)** All three

6. If $f(\theta) = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then what is $(f(\pi))^2$ equal to?

(a) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ **(b)** $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$ **(d)** $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

7. If $A = \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ then what is $A^2 - 4A$ equal to?

(a) $-5I_3$ **(b)** $-I_3$ **(c)** I_3 **(d)** $5I_3$

Where I_3 is the identity matrix of order 3.

8. If the number of selections of r as well as $(n + r)$ things from $5n$ different things are equal, then what is the value of r?
(a) n **(b)** $2n$
(c) $3n$ **(d)** $4n$

9. What is the number of selections of at most 3 things from 6 different things?
(a) 20 **(b)** 22 **(c)** 41 **(d)** 42

10. If $A = \begin{bmatrix}x & y & z\\ y & z & x\\ z & x & y\end{bmatrix}$ where x, y and z are integers

is an orthogonal matrix, then what is A^2 equal to?
(a) Null matrix **(b)** Identity matrix
(c) A **(d)** $-A$

Direction for Questions (11-13): Consider the following for the three (03) items that follow:
Let $p = \tan 2\alpha - \tan\alpha$ and $q = \cot\alpha - \cot 2\alpha$.

11. What is (p/q) equal to?
(a) $-\tan\alpha.\tan 2\alpha$ **(b)** $-\cot\alpha.\cot 2\alpha$
(c) $\tan\alpha.\tan 2\alpha$ **(d)** $\cot\alpha.\cot 2\alpha$

12. What is $(p + q)$ equal to?
(a) $\sec 4\alpha$ **(b)** $\operatorname{cosec} 4\alpha$
(c) $2\sec 4\alpha$ **(d)** $2\operatorname{cosec} 4\alpha$

13. What is $\tan^2\alpha$ equal to?
(a) $\frac{(pq)}{(p+q)}$ **(b)** $\frac{(p+2q)}{p}$
(c) $\frac{p}{(p+2q)}$ **(d)** $\frac{p}{(2p+q)}$

Direction for Questions (14-15): Consider the following for the two (02) times that follow:
Let $2\sin\alpha + \cos\alpha = 2$, where $0 < \alpha < 90°$

14. What is $\tan\alpha$ equal to?
(a) $\frac{1}{2}$ **(b)** 1
(c) $\frac{3}{4}$ **(d)** 2

15. What is $2\sin 2\alpha + \cos 2\alpha$ equal to?
(a) $\frac{11}{10}$ **(b)** $\frac{11}{5}$ **(c)** $\frac{12}{5}$ **(d)** $\frac{13}{5}$

Direction for Questions (16-17): Consider the following for the two (02) times that follow:
In a triangle ABC, two sides BC and CA are in the ratio 2 : 1, and their opposite corresponding angles are in the ratio 3 : 1.

16. One of the angles of the triangle is
(a) 15° **(b)** 30°
(c) 45° **(d)** 75°

17. Consider the following statements:
I. The triangle is right-angled.
II. One of the sides of the triangle is 3 times the other.
III. The angles A, C and B of the triangle are in AP.
Which of the statements give above is/are correct?
(a) I only **(b)** II and III only
(c) I and III only **(d)** I, II and III

18. A man at M standing 100 m away from the base (P) of a chimney of height 50 m. He observes the angle of elevation of the highest point (Q) of the smoke to be 45°. The highest point of the chimney is at R. Further P, R and Q are in a straight line and the straight line is perpendicular to PM. What is the angle RMQ equal to?
(a) $\tan^{-1}\left(\frac{1}{2}\right)$ **(b)** $\tan^{-1}\left(\frac{1}{3}\right)$
(c) $\tan^{-1}\left(\frac{2}{3}\right)$ **(d)** $\tan^{-1}\left(\frac{3}{4}\right)$

19. If k is a root of $x^2 - 4x + 1 = 0$, then what is $\tan^{-1} k + \tan^{-1}\frac{1}{k}$ equal to?
(a) $\frac{-\pi}{2}$ **(b)** 0 **(c)** $\frac{\pi}{4}$ **(d)** $\frac{\pi}{2}$

20. If $\tan^{-1} k + \tan^{-1}\frac{1}{2} = \frac{\pi}{4}$
(a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{1}{4}$

21. If a line in 3 dimensions makes angles α, β and γ with the positive directions of the coordinate axes, then what is $\cos(\alpha + \beta)\cos(\alpha - \beta)$ equal to?
(a) $\cos^2\gamma$ **(b)** $-\cos^2\gamma$
(c) $\sin^2\gamma$ **(d)** $-\sin^2\gamma$

22. A(1, 2, –1), B(2, 5, –2) and C(4, 4, –3) are three vertices of a rectangle. What is the area of the rectangle?
(a) 8 square units
(b) 9 square units
(c) $\sqrt{66}$ square units
(d) $\sqrt{68}$ square units

23. ABC is a triangle right-angled at B. If $A(k, 1, -1)$, $B(2k, 0, 2)$ and $C(2 + 2k, k, 1)$ are the vertices of the triangle, then what is the value of k?
(a) –3 **(b)** –1 **(c)** 1 **(d)** 3

24. If a line $\frac{x+1}{p} = \frac{y-1}{q} = \frac{z-2}{r}$
Where $p = 2q = 3r$ makes an angle with the position direction of the y-axis then what is $\cos 2\theta$ equal to?
(a) $\frac{-31}{49}$ **(b)** $\frac{-37}{49}$ **(c)** $\frac{31}{49}$ **(d)** $\frac{37}{49}$

25. What is the equation of the plane passing through the point (1, 1, 1) and perpendicular to the line whose direction ratio is (3, 2, 1)?
(a) $x + 2y + 3z = 6$
(b) $3x + 2y + z = 6$
(c) $x + y + z = 3$
(d) $3x + 2y + z = 0$

26. A line makes angles α, β and γ with the positive directions of the coordinate axes.
If $\vec{a} = (\sin^2\alpha)\hat{i} + (\sin^2\beta)\hat{j} + (\sin^2\gamma)\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ then what is $\vec{a}.\vec{b}$ equal to?
(a) –2 **(b)** –1 **(c)** 1 **(d)** 2

27. Consider the following statements with respect to a vector $\vec{d} = (\vec{a} \times \vec{b}) \times \vec{c}$:
I. $\vec{d}$ is coplanar with $\vec{a}$ and $\vec{d}$
II. $\vec{d}$ is perpendicular to $\vec{c}$

Which of the statements given above is/are correct?

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

28. The position vectors of three points A, B and C are $\vec{a}, \vec{b}$ and $\vec{c}$, respectively, such that $3\vec{a} - 4\vec{b} + \vec{c} = \vec{0}$. What is AB : BC equal to?

(a) 3 : 1 **(b)** 1 : 3 **(c)** 3 : 4 **(d)** 1 : 4

29. The position vectors of three points A, B and C are $\vec{a}.\vec{b}$ and $\vec{c}$ respectively, where, $\vec{c} = (\cos^2\theta)\vec{a} + (\sin^2\theta)\vec{b}$. What is $(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{a})$ equal to?

(a) $\vec{o}$ **(b)** $2\vec{c}$
(c) $3\vec{c}$ **(d)** unit vector

30. Let $\vec{a}$, $\vec{b}$ and $(\vec{a}\times\vec{b})$ be unit vectors. What is $(\vec{a}.\vec{b})$ equal to?

(a) 0 **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** 3

31. The sum of the first k terms of a series is $3k^2 + 5k$. Which one of the following is correct?

(a) The terms of S form an arithmetic progression with common difference 14.
(b) The terms of S form an arithmetic progression with common difference 6.
(c) The terms of S form a geometric progression with a common ratio $\frac{10}{7}$.
(d) The terms of S form a geometric progression with a common ratio $\frac{11}{4}$.

32. The sum of the first 8 terms of a GP is 5 times the sum of its first 4 terms. If $r \neq 1$ is the common ratio, then what is the number of possible real values of k?

(a) One **(b)** Two
(c) Three **(d)** More than three

33. If one root of the equation $x^2 - kx + k = 0$ exceeds the other by $2\sqrt{3}$ then which one of the following is a value of k?

(a) 3 **(b)** 6
(c) 9 **(d)** 12

34. If $x + \frac{5}{y} = 4$ and $y + \frac{5}{x} = -4$, then what is $(x + y)$ equal to?

(a) 0 **(b)** 1 **(c)** 4 **(d)** 5

35. If the 5^{th}, 7^{th} and 13^{th} terms of an AP are in GP, then what is the ratio of its first term to its common difference?

(a) –3 **(b)** –2 **(c)** 2 **(d)** 3

36. If p, 1 and q are in AP and p and 2 q are in GP, then which of the following statements is/are correct?

I. p, 4, q are in HP.
II. $\left(\frac{1}{p}\right), \frac{1}{4}, \left(\frac{1}{q}\right)$ are in AP.

Select the answer using the code given below:

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

37. If $x = (1111)_2$, $y = (1001)_2$ and $z = (110)_2$, then what is $x^3 - y^3 - z^3 - 3xyz$ equal to?

(a) $(1111001)_2$ **(b)** $(1001111)_2$
(c) $(1)_2$ **(d)** $(0)_2$

38. If $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ and A, B, C, D and G are the cofactors of the elements a, b, c, d and g respectively, then what is $bB + cC - dD - gG$ equal to?

(a) 0 **(b)** 1 **(c)** Δ **(d)** $-\Delta$

39. Consider the following statements in respect of the determinant $\Delta = \begin{vmatrix} k(k+2) & 2k+1 & 1 \\ 2k+1 & k+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

I. Δ is positive if $k > 0$.
II. Δ is negative if $k < 0$.
III. Δ is zero if $k = 0$.

How many of the statements given above are correct?

(a) None **(b)** One
(c) Two **(d)** All three

40. If $\begin{vmatrix} 2 & 3+i & 1 \\ 3-i & 0 & i-1 \\ -1 & -1-i & 1 \end{vmatrix} = A + iB$ where $i = \sqrt{-1}$ then what is $A + B$ equal to?

(a) –10 **(b)** –6 **(c)** 0 **(d)** 6

Direction for Questions (41-43): Consider the following for the three (03) items that follow:
Let $p = \sin 35°$, $q = \sin 25°$ and $r = \sin(-95°)$.

41. What is $(p + q + r)$ equal to?

(a) –1 **(b)** 0 **(c)** 2sin 5° **(d)** 2cos 5°

42. What is $(pq + qr + rp)$ equal to?

(a) $\frac{-3}{4}$ **(b)** 0 **(c)** $\frac{1}{4}$ **(d)** $\frac{3}{4}$

43. What is $(p^2 + q^2 + r^2)$ equal to?

(a) $\frac{1}{2}$ **(b)** 1 (c) $\frac{3}{2}$ **(d)** 2

Direction for Questions (44-45): Consider the following for the two (02) items that follow:
Let $p = [\sin\alpha - \sin(\alpha - 90°)]$.

44. What is the minimum value of p?

(a) 0 **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{\sqrt{2}}$ **(d)** 1

45. What is the maximum value of p?

(a) 1 **(b)** $\sqrt{2}$ **(c)** $\sqrt{3}$ **(d)** 2

Direction for Questions (46-48): Consider the following for the three (03) items that follow:
The sides of a triangle ABC are $AB = 3$ cm, $BC = 5$ cm and $CA = 7$ cm.

46. Consider the following statements:

I. The triangle is an obtuse-angled triangle.

II. The sum of acute angles of the triangle is also acute.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

47. What is $\angle B$ equal to?

(a) 60° **(b)** 105° **(c)** 120° **(d)** 150°

48. What is the area of the area of the triangle?

(a) $\frac{15\sqrt{3}}{4}$ square cm **(b)** $\frac{15\sqrt{3}}{2}$ square cm

(c) $15\sqrt{3}$ square cm **(d)** $30\sqrt{3}$ square cm

Direction for Questions (49-50): Consider the following for the two (02) items that follow:
The top (M) of a tower is observed from three points P, Q and R lying in a horizontal straight line which passes directly a long the foot (N) of the tower. The angles of elevations of M from P, Q and R are 30°, 45° and 60°, respectively. Let $PQ = a$ and $QR = b$.

49. What is PN equal to?

(a) $\left(\frac{3-\sqrt{3}}{2}\right)a$ **(b)** $\left(\frac{3+\sqrt{3}}{2}\right)a$

(c) $\left(\frac{3-\sqrt{3}}{4}\right)a$ **(d)** $\left(\frac{3+\sqrt{3}}{4}\right)a$

50. What is MN equal to?

(a) $\left(\frac{3+\sqrt{3}}{2}\right)b$ **(b)** $\left(\frac{3-\sqrt{3}}{2}\right)b$

(c) $\left(\frac{3-\sqrt{3}}{4}\right)b$ **(d)** $\left(\frac{3+\sqrt{3}}{4}\right)b$

Direction for Questions (51-52): Consider the following for the two (02) items that follow:
The probabilities that A, B and C become managers are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$, respectively. The probabilities that the bonus scheme will be introduced if A, B and C become managers are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{1}{3}$, respectively.

51. What is the probability that the bonus scheme will be introduced?

(a) $\frac{17}{45}$ **(b)** $\frac{19}{45}$ **(c)** $\frac{23}{45}$ **(d)** $\frac{26}{45}$

52. If the bonus scheme has been introduced, then what is the probability that the manager appointed was B?

(a) $\frac{5}{23}$ **(b)** $\frac{6}{23}$ **(c)** $\frac{7}{23}$ **(d)** $\frac{8}{23}$

53. The arithmetic mean of 100 observations is 50. If 5 is subtracted from each observation and then divided by 20, then what is the new arithmetic mean?

(a) 2.25 **(b)** 3.5 **(c)** 4.25 **(d)** 5.5

54. The standard deviation of 100 observations is 10. If 5 is added to each observation and then divided by 20, then what will be the new standard deviation?

(a) 0.25 **(b)** 0.5 **(c)** 0.75 **(d)** 1.00

55. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then what is the value of $P(B|A^c)$?

(a) $\frac{1}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{5}{8}$ **(d)** $\frac{7}{8}$

56. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then what is the value of $P(A^c \cap B^c)$?

(a) $\frac{1}{4}$ **(b)** $\frac{5}{12}$ **(c)** $\frac{7}{12}$ **(d)** $\frac{11}{12}$

57. If two fair dice are tossed then what is the probability that the sum of the numbers on the faces of the dice is strictly greater than 7?

(a) $\frac{1}{3}$ **(b)** $\frac{5}{12}$

(c) $\frac{7}{12}$ **(d)** $\frac{3}{4}$

58. The probability of a man hitting a target is $\frac{1}{5}$. If the man fires 7 times, then what is the probability that he hits the target at least twice?

(a) $1-\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)^6$ **(b)** $1-\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)^7$

(c) $1-\left(\frac{11}{5}\right)\left(\frac{4}{5}\right)^6$ **(d)** $1-\left(\frac{11}{5}\right)\left(\frac{4}{5}\right)^7$

59. Let X be a random variable following binomial distribution whose mean and variance are 200 and 160, respectively. What is the value of the number of trials (n)?

(a) 500 **(b)** 1000 **(c)** 1500 **(d)** 2000

60. What is the arithmetic mean of 8^2, 9^2, 10^2, ... 15^2?

(a) 133.5 **(b)** 135.5
(c) 137.5 **(d)** 139.5

Direction for Questions (61-62): Consider the following for the two (02) items

Let $y = \sin^{-1}\left(x - \frac{4x^3}{27}\right)$

61. What is y equal to?

(a) $\sin^{-1} x$ **(b)** $\sin^{-1}\left(\frac{x}{3}\right)$

(c) $3\sin^{-1} x$ **(d)** $3\sin^{-1}\left(\frac{x}{3}\right)$

62. What is $\frac{dy}{dx}$ equal to?

(a) $\frac{1}{\sqrt{9-x^2}}$ **(b)** $\frac{1}{\sqrt{3-x^2}}$

(c) $\frac{3}{\sqrt{9-x^2}}$ **(d)** $\frac{9}{\sqrt{9-x^2}}$

Direction for Questions (63-64): Consider the following for the two (02) items that follow:
Let the function $f(x) = x^2 + 9$.

63. What if $\lim_{x\to 0} \frac{\sqrt{f(x)}-3}{\sqrt{f(x)+7}-4}$ equal to?

(a) $\frac{2}{3}$ **(b)** 2 **(c)** $\frac{4}{3}$ **(d)** 2

64. Consider the following statements:
I. $f(x)$ is an increasing function.
II. $f(x)$ has local maximum at $x = 0$
Which of the statements given above is/are correct?

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

Direction for Questions (65-66): Consider the following for the two (02) items that follow:

The function $f(x)$ satisfies $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all positive real values of x and y, and $f(2) = 3$

65. What is $f(16)$ equal to?

(a) 18 **(b)** 27 **(c)** 54 **(d)** 81

66. What is $f(1)$ s $f(4)$ equal to?

(a) 4 **(b)** 8 **(c)** 9 **(d)** 18

Direction for Questions (67-68): Consider the following for the two (02) items that follow:
A function f is such that $f(xy) = f(x + y)$ for all real values of x and y and $f(5) = 10$

67. What is $f(0)$ equal to?

(a) 0 **(b)** 1 **(c)** 5 **(d)** 10

68. What if $f(20) + f(-20)$ equal to?

(a) 0 **(b)** 10 **(c)** 20 **(d)** 40

Direction for Questions (69-70): Consider the following for the two (02) items that follow:
Let $f(x) = [x^2]$, where [,] is the greater integer function.

69. What is $\int_{\sqrt{2}}^{\sqrt{3}} f(x)$ equal to?

(a) $\sqrt{3}-\sqrt{2}$ **(b)** $2(\sqrt{3}-\sqrt{2})$
(c) $3-\sqrt{2}$ **(d)** 1

70. What is $\int_{\sqrt{2}}^{2} f(x)\,dx$ equal to?

(a) $6-\sqrt{3}-2\sqrt{2}$ **(b)** $6-\sqrt{3}-\sqrt{2}$
(c) $6-\sqrt{3}+2\sqrt{2}$ **(d)** $6+\sqrt{3}-2\sqrt{2}$

71. If $A^2 + B^2 + C^2 = 0$, then what is the value of the following?

$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$$

(a) –1 **(b)** 0 **(c)** 1 **(d)** 2

72. If ω is a non-real cube root of unity, then what is a root of the following equation?

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

(a) $x = 0$ **(b)** $x = 1$
(c) $x = \omega$ **(d)** $x = \omega^2$

73. What is $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^3$ equal to?

(a) –1 **(b)** 0 **(c)** 1 **(d)** 3

74. If $x^2 - x + 1 = 0$, then what is $\left(x-\frac{1}{x}\right)^2 + \left(x-\frac{1}{x}\right)^4 + \left(x-\frac{1}{x}\right)^8$ is equal to?

(a) 81 **(b)** 85 **(c)** 87 **(d)** 90

75. How many 7-letter words (with or without meaning) can be constructed using all the letters of the word CAPITAL so that all consonants come together in each word?

(a) 360 **(b)** 300 **(c)** 288 **(d)** 240

76. If $z \neq 0$ is a complex number, then what is amp (z) + amp $(\bar{z})$ equal to?

(a) 0 **(b)** $\frac{\pi}{2}$ **(c)** π **(d)** 2π

77. How many sides are there in a polygon which has 20 diagonals?

(a) 6 **(b)** 7 **(c)** 8 **(d)** 10

78. In how many ways can the letters of the word DELHI be arranged keeping the positions of vowels and consonants unchanged?

(a) 6 **(b)** 9 **(c)** 12 **(d)** 24

79. What is the number of positive integer solutions of $x + y + z = 5$?

(a) 3 **(b)** 5 **(c)** 6 **(d)** 9

80. What is the number of rational terms in the expansion of $(3^{1/2} + 5^{1/4})^{12}$?

(a) 2 **(b)** 3 **(c)** 4 **(d)** 6

81. Under what condition will the lines $m^2x + ny - 1 = 0$ and $n^2x - my + 2 = 0$ be perpendicular?

(a) $mn - 1 = 0$ **(b)** $mn + 1 = 0$

(c) $m + n = 0$ **(d)** $m - n = 0$

82. If p and q are real numbers between 0 and 1 such that the points $(p, 1)$, $(1, q)$ and $(0, 0)$ form an equilateral triangle, then what is $(p + q)$ equal to?

(a) $\sqrt{2}$ **(b)** $\sqrt{2} - 1$

(c) $2 - \sqrt{3}$ **(d)** $4 - 2\sqrt{3}$

83. The vertices of a triangle are $A(1, 1)$, $B(0, 0)$ and $C(2, 0)$. The angular bisectors of the triangle meet at P. What are the coordinates of P?

(a) $(1, \sqrt{2} - 1)$ **(b)** $(1, \sqrt{3} - 1)$

(c) $\left(1, \frac{1}{2}\right)$ **(d)** $\left(\frac{1}{2}, \sqrt{2} - 1\right)$

84. Let $A(3, -1)j$ and $B(1, 1)$ be the end points of line segment AB. Let P be the middle point of the line segment AB. Let Q be the point situated at a distance of $\sqrt{2}$ units from P on the perpendicular bisector line of AB. What are the possible coordinates of Q?

(a) (2, 1) **(b)** (3, 1) **(c)** (2, 2) **(d)** (1, 3)

85. ABC is an equilateral triangle and AD is the altitude on BC. If the coordinates of A are (1, 2) and that of D are (–2, 6) then what is the equation of BC?

(a) $3x + 4y - 18 = 0$ **(b)** $4x + 3y - 1 = 0$

(c) $4x - 3y = 26$ **(d)** $3x - 4y + 30 = 0$

86. What is the equation of the circle whose diameter is 10 cm and the equation of two of its diameters are $x + y = 0$ and $x - y = 0$?

(a) $x^2 + y^2 = 0$

(b) $x^2 + y^2 = 25$

(c) $x^2 + y^2 = 100$

(d) $x^2 + y^2 - 2x - 2y - 23 = 0$

87. A square is inscribed in a circle $x^2 + y^2 + 2x + 2y + 1 = 0$ and its sides are parallel to coordinate axes. Which one of the following is a vertex of the square?

(a) (–2, 2)

(b) (–2, –2)

(c) $\left(-1 + \frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}\right)$

(d) None of the above

88. A tangent to the parabola $y^2 = 4x$ is inclined at an angle of 45° with the positive direction of x-axis. What is point of contact of the tangent and the parabola?

(a) (1, 1) **(b)** $(2, \sqrt{2})$

(c) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ **(d)** (1, 2)

89. What is the distance between the two foci of the hyperbola $25x^2 - 75y^2 = 225$?

(a) $2\sqrt{3}$ units **(b)** $4\sqrt{3}$ units

(c) $\sqrt{6}$ units **(d)** $2\sqrt{6}$ units

90. If any point on an ellipse is $(3\sin\alpha, 5\cos\alpha)$, then what is the eccentricity of the ellipse?

(a) $\frac{4}{3}$ **(b)** $\frac{4}{5}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{1}{2}$

Direction for Questions (91-94): Consider the following for the four (04) times that follow: The frequency distribution of height of students of a class is given below:

Height (in cm)	Number of students
160–162	12
162–164	15
164–166	24
166–168	13

91. What is the total number of students whose height is less than or equal to 165 cm?

(a) 15 **(b)** 39

(c) 51 **(d)** None of the above

92. What is the median height of the class?

(a) 162.41 cm **(b)** 163.41 cm

(c) 164.41 cm **(d)** 165.41 cm

93. The height which occurs most frequently in the class is:

(a) 163.5 cm **(b)** 163.9 cm
(c) 164.5 cm **(d)** 164.9 cm

94. The most appropriate graphical representation of the given frequency distribution is:

(a) bar chart
(b) percentage bar chart
(c) histogram
(d) pie chart

Direction for Questions (95-96): Consider the following for the two (02) items that follow:
The sum and the sum of squares of the observation corresponding to length X (in cm) and weight Y (in gm) of 50 tropical tubers are given as $\sum X = 200$, $\sum Y = 250$, $\sum X^2 = 900$ and $\sum Y^2 = 1400$.

95. Which of the following is correct?

(a) Variance (X) > Variance (Y)
(b) Variance (X) < Variance (Y)
(c) Variance (X) = Variance (Y)
(d) Cannot be determined from the given data

96. Which one of the following statements is correct?

(a) Coefficient of variation of X is strictly more than coefficient of variation of Y.
(b) Coefficient of variation of X is strictly less than coefficient of variation of Y.
(c) Coefficient of variation of X is the same as coefficient of variation of Y.
(d) Coeficient of variation cannot be determined from the given data.

Direction for Questions (97-98): Consider the following for the two (02) items that follow:
Let X be a random variable following binomial distribution with parameters $n = 6$ and $p = k$. Further, $9P(X = 4) = P(X = 2)$.

97. What is the value of k?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{5}$

98. What is the value of $P(X = 3)$?

(a) $\frac{135}{1024}$ **(b)** $\frac{5}{128}$
(c) $\frac{45}{1024}$ **(d)** $\frac{70}{1024}$

Direction for Questions (99-100): Consider the following for the two (02) items that follow:
A committee of 6 members is formed from a group of 7 gentlemen and 4 ladies.

99. What is the probability that the committee includes exactly 3 gentlemen?

(a) $\frac{10}{33}$ **(b)** $\frac{30}{77}$ **(c)** $\frac{100}{231}$ **(d)** $\frac{5}{11}$

100. What is the probability that the committee includes at least 2 ladies?

(a) $\frac{41}{66}$ **(b)** $\frac{47}{66}$ **(c)** $\frac{49}{66}$ **(d)** $\frac{53}{66}$

Direction for Questions (101-102): Consider the following for the two (02) items that follow:
Let the function $y = (1 - \cos x)^{-1}$, where $x \neq 2n\pi$ and n is an integer.

101. What is the range of the function?

(a) $[0, \infty)$ **(b)** $(0.5, \infty)$
(c) $[1, \infty)$ **(d)** $(-\infty, 0.5]$

102. What is $\int y dx$ equal to?

(a) $-\tan\left(\frac{x}{2}\right) + c$ **(b)** $-\cot\left(\frac{x}{2}\right) + c$
(c) $\tan\left(\frac{x}{2}\right) + c$ **(d)** $\cot\left(\frac{x}{2}\right) + c$

where c is the constant of integration.

Direction for Questions (103-104): Consider the following for the two (02) items that follow:
Let the function $f(x) = \sin[x]$, where [.] is the greatest integer function and $g(x) = [x]$.

103. What is $\lim_{x \to 0}\{f(x)g(x)\}$ equal to?

(a) –1 **(b)** 0
(c) 1 **(d)** Limit does not exist

104. What is $\lim_{x \to 0}\frac{f(x)}{g(x)}$ equal to?

(a) –sin 1 **(b)** sin 1
(c) 0 **(d)** Limit does not exist

Direction for Questions (105-106): Consider the following for the two (02) items that follow:
Let the curve $f(x) = |x - 3|$.

105. What is the domain of the function $f(x)$?

(a) $(0, \infty)$ **(b)** $(3, \infty)$
(c) $(-\infty, \infty)$ **(d)** $(-\infty, \infty) - \{3\}$

106. What is the area bounded by the curved $f(x)$ and $y = 3$?

(a) 3 square units **(b)** 4.5 square units
(c) 7.5 square units **(d)** 9 square units

Direction for Questions (107-108): Consider the following for the two (02) items that follow:
Let $f = \{(1, 1), (2, 4), (3, 7), (4, 10)\}$

107. If $f(x) = px + q$, then what is the value of $(p + q)$?

(a) –1 **(b)** 0 **(c)** 1 **(d)** 5

108. Consider the following statements:

I. f is an one-one function.

II. f is onto function if the codomain is the set of natural numbers.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

Direction for Questions (109-110): Consider the following for the two (02) items that follow: Let the function $f(x) = x^2 - 1$.

109. What is $\lim_{x \to 1}\{f.f((x))\}$ equal to?

(a) –1 **(b)** 0 **(c)** 1 **(d)** 2

110. What is the area bounded by the function $f(x)$ and the x-axis?

(a) $\frac{1}{3}$ square units **(b)** $\frac{2}{3}$ square unit

(c) $\frac{4}{3}$ square units **(d)** 2 square units

Direction for Questions (111-112): Consider the following for the two (02) items that follow: Let $x = \sec\theta - \cos\theta$ and $y = \sec^4\theta - \cos^4\theta$,

111. What is $\left(\frac{dy}{dx}\right)^2$ equal to?

(a) $\frac{4(y^2+4)}{(x^2+4)}$ **(b)** $\frac{4(y^2-4)}{(x^2-4)}$

(c) $\frac{16(y^2+4)}{(x^2+4)}$ **(d)** $\frac{16(y^2-4)}{(x^2-4)}$

112. What is $\left(\frac{x^2+4}{y^2+4}\right)\frac{dy}{dx}\left[(x^2+4)\frac{d^2y}{dx^2} - 16y\right]$ equal to?

(a) $16x$ **(b)** $16y$ **(c)** $-16x$ **(d)** $-16y$

Direction for Questions (113-114): Consider the following for the two (02) items that follow: Let ABC be a triangle right-angled at B and $AB + AC = 3$ units.

113. What is $\angle A$ equal to if the area of the triangle is maximum?

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{3}$ **(d)** $\frac{5\pi}{12}$

114. What is the maximum area of the triangle?

(a) $\frac{\sqrt{3}}{2}$ square unit **(b)** $\sqrt{3}$ square unit

(c) $\frac{\sqrt{6}}{2}$ square unit **(d)** $\sqrt{6}$ square unit

Direction for Questions (115-116): Consider the following for the two (02) items that follow: Let $(x + y)^{p+q} = x^p y^p$, where p, q are positive integers.

115. The derivative of y with respect to x:

(a) depends on p only

(b) depends on q only

(c) depends on both p and q

(d) is independent of both p and q

116. If $p + q = 10$, then what is $\frac{dy}{dx}$ equal to?

(a) $\frac{y}{x}$ **(b)** xy

(c) $x^{10}y^{10}$ **(d)** $\left(\frac{y}{x}\right)^{10}$

Direction for Questions (117-118): Consider the following for the two (02) items that follow: The slope of the tangent of the curve $y = f(x)$ at $(x, f(x))$ is 4 for every real number x and the curve passes through the origin.

117. What is the nature of the curve?

(a) A straight line passing through (1, 4)

(b) A straight line passing through (–1, 4)

(c) A parabola with vertex at origin and focus at (2, 0)

(d) A parabola with vertex at origin and focus at (1, 0)

118. What is the area bounded by the curve the x-axis and the line $x = 4$?

(a) 8 square units **(b)** 16 square units

(c) 32 square units **(d)** 64 square units

Direction for Questions (119-120): Consider the following for the two (02) items that follow:

Let $f(x) = \begin{cases} x^3, & x^2 < 1 \\ x^2, & x^2 \geq 1 \end{cases}$

119. What is $\lim_{x \to 0} f'(x)$ equal to?

(a) 2 **(b)** 1

(c) 0 **(d)** Limit does not exist

120. Consider the following statements:

I. The function is continuous at $x = -1$

II. The function is differentiable at $x = 1$

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

Answer Key

Q. No	Answer Key	Q. No	Answer Key	Q. No	Answer Key	Q. No	Answer Key
1	c	31	b	61	d	91	c
2	b	32	b	62	c	92	c
3	d	33	a	63	c	93	d
4	b	34	a	64	d	94	c
5	c	35	b	65	d	95	b
6	d	36	c	66	c	96	b
7	d	37	d	67	d	97	c
8	b	38	a	68	c	98	a
9	c	39	a	69	b	99	a
10	b	40	b	70	a	100	d
11	c	41	b	71	b	101	b
12	d	42	a	72	a	102	b
13	c	43	c	73	a	103	b
14	b	44	a	74	c	104	d
15	b	45	b	75	c	105	c
16	b	46	c	76	a	106	d
17	c	47	c	77	c	107	c
18	a	48	a	78	c	108	a
19	d	49	b	79	c	109	a
20	c	50	a	80	c	110	c
21	b	51	c	81	a	111	c
22	c	52	b	82	d	112	c
23	d	53	a	83	a	113	c
24	a	54	b	84	b	114	a
25	b	55	b	85	d	115	d
26	d	56	b	86	b	116	a
27	c	57	b	87	c	117	a
28	b	58	c	88	d	118	c
29	a	59	b	89	b	119	c
30	a	60	c	90	a	120	d

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MATHEMATICS-I

QUESTION PAPER 2025

ANSWERS WITH EXPLANATIONS

1. Option (c) is correct.

Explanation: Put $x = 1$ and $y = 1$, we get

Sum of coefficients $= 2^n = 256$

Total terms $= 8 + 1 = 9$

$$\text{Greatest term} = \text{middle term} = \frac{9+1}{2}$$

$$= 5^{\text{th}} \text{ term}$$

2. Option (b) is correct.

Explanation: $k < (\sqrt{2}+1)^3 < k+2$

$\Rightarrow k < 2\sqrt{2}+1+6+3\sqrt{2} < k+2$

$\Rightarrow k < 5\sqrt{2} + 7 < k+2$

$\Rightarrow k < 14.07 < k+2$

$\Rightarrow k < 14.07$ and $k > 12.07$

$\therefore \quad k = 13, 14$

3. Option (d) is correct.

Explanation:

$$\begin{bmatrix} x & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [45]$$

$$\Rightarrow [x + 11 + 2x + 13 + 3x + 15]\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [45]$$

$\Rightarrow x + 11 + 2x + 13 + 3x^2 + 15x = 45$

$\Rightarrow \quad 3x^2 + 18x + 24 = 45$

$\Rightarrow \quad x^2 + 6x - 7 = 0$

$\Rightarrow \quad x = -7$ and 1

4. Option (b) is correct.

Explanation: For orthogonal matrix.

$A^T = A^{-1} \Rightarrow AA^T = I$

$A^2 = I$ ($\therefore$ A is symmetric)

$$\begin{bmatrix} y & z & x \\ z & x & y \\ x & y & z \end{bmatrix}\begin{bmatrix} y & z & x \\ z & x & y \\ x & y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} y^2+z^2+x^2 & yz+zx+xy & xy+yz+xz \\ yz+zx+xy & x^2+y^2+z^2 & xy+yz+zx \\ xy+yz+zx & xy+yz+zx & x^2+y^2+z^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x^2 + y^2 + z^2 = 1$$

5. Option (c) is correct.

Explanation: For non-singular matrix we know that

$$|M^n| = |M|^n, |M^{-1}| = \frac{1}{|M|}$$

and $|M^T| = |M|$

6. Option (d) is correct.

Explanation:

$$f(\pi) = \begin{bmatrix} \cos\pi & \sin\pi \\ -\sin\pi & \cos\pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[f(\pi)]^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

7. Option (d) is correct.

Explanation:

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= 5I_3$$

8. Option (b) is correct.

Explanation: ${}^{5n}C_r = {}^{5n}C_{n+r}$

$n + r + r = 5n \Rightarrow 2r = 4n \Rightarrow r = 2n$

9. Option (d) is correct.

Explanation: Number of selections (at most 3 things)

$$= {}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3$$

$$= 1 + 6 + \frac{6 \cdot 5}{2 \cdot 1} + \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$$

$$= 1 + 6 + 15 + 20 = 42$$

10. Option (b) is correct.

Explanation: We know that for orthogonal matrix

$$A^T = A^{-1} \Rightarrow AA^T = AA^{-1} = I$$

$$\Rightarrow \quad A^2 = I \ (\because A = A^T)$$

11. Option (c) is correct.

Explanation:

$$\frac{p}{q} = \frac{\tan 2\alpha - \tan\alpha}{\cot\alpha - \cot 2\alpha} = \frac{\tan 2\alpha - \tan\alpha}{\frac{1}{\tan\alpha} - \frac{1}{\tan 2\alpha}}$$

$$= \frac{(\tan 2\alpha - \tan\alpha)\tan\alpha\cdot\tan 2\alpha}{(\tan 2\alpha - \tan\alpha)}$$

$$= \tan\alpha \,.\, \tan 2\alpha$$

12. Option (d) is correct.

Explanation: $p + q = \tan 2\alpha - \tan\alpha + \cot\alpha - \cot 2\alpha$

$$= \left(\frac{\sin^2 2\alpha - \cos^2 2\alpha}{\sin 2\alpha\cdot\cos 2\alpha}\right) + \left(\frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cdot\cos\alpha}\right)$$

$$= \frac{\sin^2 2\alpha - \cos^2 2\alpha}{\sin 2\alpha\cdot\cos 2\alpha} + \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cdot\cos\alpha}$$

$$= \frac{-2\cos 4\alpha}{\sin 4\alpha} + \frac{2\cos 2\alpha}{\sin 2\alpha}$$

$$= \frac{2(\sin 4\alpha\cdot\cos 2\alpha - \cos 4\alpha\cdot\sin 2\alpha)}{\sin 4\alpha\cdot\sin 2\alpha}$$

$$= \frac{2\sin(4\alpha - 2\alpha)}{\sin 4\alpha\cdot\sin 2\alpha} = \frac{2.\sin 2\alpha}{\sin 4\alpha\cdot\sin 2\alpha}$$

$$= 2\text{cosec } 4\alpha.$$

13. Option (c) is correct.

Explanation: $\frac{p}{p+2q} = \frac{1}{1+\frac{2q}{p}} = \frac{1}{1+\frac{2}{\tan\alpha\cdot\tan 2\alpha}}$

$$= \frac{\sin\alpha\cdot\sin 2\alpha}{\sin\alpha\cdot\sin 2\alpha + \cos\alpha\cdot\cos 2\alpha + \cos\alpha\cdot\cos 2\alpha}$$

$$= \frac{\sin\alpha\cdot\sin 2\alpha}{\cos\alpha + \cos\alpha\cdot\cos 2\alpha} = \frac{\sin\alpha\cdot 2\sin\alpha\cdot\cos\alpha}{\cos\alpha\cdot 2\cdot\cos^2\alpha}$$

$$= \tan^2\alpha.$$

14. Option (c) is correct.

Explanation: $\cos\alpha = 2(1 - \sin\alpha)$

$$\Rightarrow \quad \frac{\cos^2\alpha}{(1-\sin\alpha)^2} = 4$$

$$\frac{1+\sin\alpha}{1-\sin\alpha} = \frac{4}{1} \Rightarrow \sin\alpha = \frac{3}{5}$$

$$\tan\alpha = \frac{3}{\sqrt{25-9}} = \frac{3}{4}$$

15. Option (b) is correct.

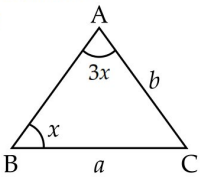
Explanation: $\therefore \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

$$\therefore 2\sin 2\alpha + \cos 2\alpha = 4\sin\alpha.\cos\alpha + 1 - 2\sin^2\alpha$$

$$= \frac{48}{25} + 1 - \frac{18}{25} = \frac{48+25-18}{25} = \frac{55}{25} = \frac{11}{5}$$

16. Option (b) is correct.

Explanation:



$$\frac{a}{\sin 3x} = \frac{b}{\sin x}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{\sin 3x}{\sin x}$$

$$\Rightarrow \quad 2 = \frac{\sin 3x}{\sin x} \Rightarrow 2\sin x = 3\sin x - 4\sin^3 x$$

$$\Rightarrow \sin x\,(1 - 4\sin^2 x) = 0 \Rightarrow \sin x = \frac{1}{2}$$

$$\Rightarrow \quad x = 30°$$

17. Option (c) is correct.

Explanation: $\angle B = x = 30°$, $\angle A = 3x = 90°$, $\angle C = 180° - 120° = 60°$

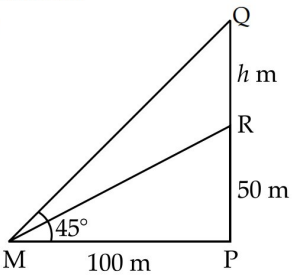
$\therefore$ Triangle is right-angled.

$\therefore$ 30°, 60° and 90° are in A.P.

So options (I) and (III) are only correct.

18. Option (a) is correct.

Explanation:



$$\tan 45° = \frac{50+h}{100}$$

$$100 = 50 + h \Rightarrow h = 50 \text{ m}$$

$$\tan\angle RMQ = \frac{50}{100} = \frac{1}{2}$$

$$\angle RMQ = \tan^{-1}\frac{1}{2}$$

19. Option (d) is correct.

Explanation: $\because x^2 - 4x + 1 = 0$

$\Rightarrow x = 2 \pm \sqrt{3} > 0$

$$\therefore \tan^{-1} k + \tan^{-1}\frac{1}{k} = \tan^{-1}\left(\frac{k+\frac{1}{k}}{1-k\times\frac{1}{k}}\right)$$

$$\tan^{-1}\infty = \frac{\pi}{2}$$

20. Option (c) is correct.

Explanation: $\tan^{-1}k + \tan^{-1}\frac{1}{2} = \frac{\pi}{4} = \tan^{-1}1$

$$\tan^{-1}k = \tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right) = \tan^{-1}\left(\frac{1}{2}\times\frac{2}{3}\right)$$

$$k = \frac{1}{3}$$

21. Option (b) is correct.

Explanation:

We know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

and $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$

$= \cos^2\alpha + \cos^2\beta - 1 = 1 - \cos^2\gamma - 1 = -\cos^2\gamma.$

22. Option (c) is correct.

Explanation:

$$\text{Area} = AB \times BC$$
$$= \sqrt{1^2+3^2+1^2}\times\sqrt{2^2+1^2+1^2}$$
$$= \sqrt{11}\times\sqrt{6}$$
$$= \sqrt{66} \text{ sq. units}$$

23. Option (d) is correct.

Explanation: D.rs of AB = $k, -1, 3$

D.rs of BC = $2, k, -1$

$\because$ AB $\perp$ BC

$\therefore \quad 2k - k - 3 = 0 \Rightarrow k = 3$

24. Option (a) is correct.

Explanation: D.rs of line are p, q, r

where $p = 2q = 3r \Rightarrow \frac{p}{6} = \frac{q}{3} = \frac{r}{2}$

$\therefore$ D.rs of line are 6, 3, 2

$$\therefore \quad \cos\theta = \frac{3}{\sqrt{6^2+3^2+2^2}} = \frac{3}{7}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = \frac{18}{49} - 1 = \frac{-31}{49}$$

25. Option (b) is correct.

Explanation: Point (1, 1, 1) and d.rs of normal are <3, 2, 1>

$\therefore$ Equation of the plane is

$3(x-1) + 2(y-1) + 1(z-1) = 0$

$\Rightarrow \quad 3x + 2y + z = 6$

26. Option (d) is correct.

Explanation: $\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Now, $\vec{a}.\vec{b}. = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

27. Option (c) is correct.

Explanation: $\vec{d} = (\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$ ...(i)

Since $\vec{d}$ is a linear combination of $\vec{a}$ and $\vec{b}$.

Therefore $\vec{d}$ lies in the plane formed by $\vec{a}$ and $\vec{b}$.

So, $\vec{d}$ is coplanar with $\vec{a}$ and $\vec{b}$.

$\vec{d}.\vec{c} = \left[(\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}\right].\vec{c}$

$= (\vec{a}.\vec{c})(\vec{b}.\vec{c}) - (\vec{b}.\vec{c})(\vec{a}.\vec{c}) = 0$

So, $\vec{d}$ is perpendicular to $\vec{c}$

Hence both statements are true.

28. Option (b) is correct.

Explanation: $\because \quad 3\vec{a} - 4\vec{b} + \vec{c} = 0$

$\Rightarrow \quad 3\vec{a} - 3\vec{b} - \vec{b} + \vec{c} = 0$

$\Rightarrow \quad 3(\vec{a}-\vec{b}) = \vec{b}-\vec{c} \Rightarrow 3(\vec{b}-\vec{a}) = \vec{c}-\vec{b}$

$$\Rightarrow \quad \frac{\vec{b}-\vec{a}}{\vec{c}-\vec{b}} = \frac{1}{3} \Rightarrow \frac{\overline{AB}}{\overline{BC}} = \frac{1}{3}$$

29. Option (a) is correct.

Explanation: $(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{a})$

$= (\vec{a}\times\vec{b}) + (\vec{b}\times\vec{a})\times\vec{c}$

$= (\vec{a}\times\vec{b}) + (\vec{b}\times\vec{a})\times(\cos^2\theta\vec{a} + \sin^2\theta\vec{b})$

$= (\vec{a}\times\vec{b}) + \cos^2\theta(\vec{b}\times\vec{a}) - \sin^2\theta(\vec{a}\times\vec{b})$

$= (\vec{a}\times\vec{b}) - \cos^2\theta(\vec{a}\times\vec{b}) - \sin^2\theta(\vec{a}\times\vec{b})$

$= (\vec{a}\times\vec{b})\left[1 - (\cos^2\theta + \sin^2\theta)\right] = 0$

30. Option (a) is correct.

Explanation: $\therefore \quad |\vec{a}| = |\vec{b}| = |\vec{a}\times\vec{b}| = 1$

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = 1.$$

$$\therefore \quad \theta = \frac{\pi}{2}$$

Now, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

31. Option (b) is correct.

Explanation:

$S_k = 3k^2 + 5k$

$S_1 = 3 + 5 = 8$

$S_2 = 12 + 10 = 22$

$S_3 = 27 + 15 = 42$

$a_1 = S_1 = 8$

$a_2 = S_2 - S_1 = 22 - 8 = 14$

$$a_3 = S_3 - S_2 = 42 - 22 = 20$$

$\therefore$ Series = 8 + 14 + 20 ..... are in A.P. with common difference = 14 – 8 = 6.

32. Option (b) is correct.

Explanation: $S_8 = 5S_4$

$$\frac{a(r^8-1)}{r-1} = \frac{5.a(r^4-1)}{r-1}$$

$$\Rightarrow \quad \frac{r^8-1}{r^4-1} = 5 \Rightarrow \frac{(r^4-1)(r^4+1)}{r^4-1} = 5$$

$$\Rightarrow \quad r^4 = 4 \Rightarrow r = \pm\sqrt{2}, \pm\sqrt{2}i$$

33. Option (b) is correct.

Explanation: Here, $\alpha + \beta = k$, $\alpha\beta = k$

$$\text{and } \alpha - \beta = (2\sqrt{3})$$

$$\therefore \quad (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

$$\Rightarrow \quad (2\sqrt{3})^2 = k^2 - 4k$$

$$\Rightarrow \quad k^2 - 4k - 12 = 0 \Rightarrow k^2 - 6k + 2k - 12 = 0$$

$$\Rightarrow \quad (k-6)(k+2) = 0 \Rightarrow k = -2, 6$$

34. Option (a) is correct.

Explanation:

$$xy + 5 = 4y$$

$$xy + 5 = -4x$$

$$- \quad - \quad +$$

$$0 = 4(x + y)$$

$$\Rightarrow \quad x + y = 0$$

35. Option (a) is correct.

Explanation:

$a_5 = a + 4d$, $a_7 = a + 6d$ and $a_{13} = a + 12d$

$\therefore$ a_5, a_7 and a_{13} are in G.P

$$(a_7)^2 = a_5 . a_{13}$$

$$(a + 6d)^2 = (a + 4d)(a + 12d)$$

$$a^2 + 12ad + 36d^2 = a^2 + 16ad + 48d^2$$

$$12d^2 + 4ad = 0 \Rightarrow 3d + a = 0$$

$$a = -3d \Rightarrow \frac{a}{d} = -3$$

36. Option (c) is correct.

Explanation: $\therefore$ p, 1 and q are A.P.

$$\Rightarrow \quad p + q = 2 \qquad \text{.... (i)}$$

p, 2 and q are G.P $\Rightarrow pq = 4$ ...(ii)

Now, $$\frac{2pq}{p+q} = \frac{8}{2} = 4$$

$$\therefore \quad \frac{p+q}{2pq} = \frac{1}{4}$$

$\therefore$ $\frac{1}{p}, \frac{1}{4}$ and $\frac{1}{q}$ are in A.P.

$\therefore$ p, 4 and q are in H.P.

Both statements are correct.

37. Option (d) is correct.

Explanation:

$$x = (1111)_2 = 2^3 + 2^2 + 2^1 + 1 = 15$$

$$y = (1001)_2 = 2^3 + 0 + 0 + 1 = 9$$

$$z = (110)_2 = 2^2 + 2^1 + 0 = 6$$

Now, $x^3 - y^3 - z^3 - 3xyz$

$$= (15)^3 - (9)^3 - (6)^3 - 3 \times 15 \times 9 \times 6$$

$$= 3375 - 729 - 216 - 2430$$

$$= 0$$

38. Option (a) is correct.

Explanation:

Cofactors: A = $ei - hf$, B = $gf - di$, C = $dh - ge$

D = $ch - bi$ and G = $bf - ec$

Now, $bB + cC - dD - gG$

$= bgf - bdi + cdh - cge - cdh + bdi - gbf + gec = 0$

39. Option (a) is correct.

Explanation:

$$\Delta = \begin{vmatrix} k^2+2k & 2k+1 & 1 \\ 2k+1 & k+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$= (k^2 + 2k)(k + 2 - 3) - (2k + 1)(2k + 1 - 3) + (6k + 3 - 3k - 6)$

$= k^3 - k^2 + 2k^2 - 2k - 4k^2 + 4k - 2k + 2 + 3k - 3$

$= k^3 - 3k^2 + 3k - 1$

$= (k - 1)^3$

(i) $(k-1)^3 > 0 \Rightarrow k > 1$

(ii) $(k-1)^3 < 0 \Rightarrow k < 1$

(iii) $(k-1)^3 = 0 \Rightarrow k = 1$

40. Option (b) is correct.

Explanation:

$$\begin{vmatrix} 2 & 3+i & 1 \\ 3-i & 0 & i-1 \\ -1 & -1-i & 1 \end{vmatrix}$$

$= 2(i^2 - 1) - (3 + i)(3 - i + i - 1) + 1(3 - i)(i + 1)$

$= 2(-1 - 1) - (3 + i)\, 2 + 3i - i^2 + 3 - i$

$= -4 - 6 - 2i + 3i + 1 + 3 - i$

$= -6 = -6 + 0i = A + iB$

$\therefore$ A + B = $-6 + 0 = -6$

41. Option (b) is correct.

Explanation: $p + q + r = \sin 35° + \sin 25° + \sin(-95°)$

$= \sin 35° + \sin 25° - \sin 95°$

$= \sin 35° + 2 \cos 60°.\sin(-35°)$

$= \sin 35° - 2 \times \frac{1}{2} \sin 35° = 0$

42. Option (a) is correct.

Explanation: $pq + qr + rp = q(p + r) + rp$

$= -q^2 + rp = -\sin^2 25° - \sin 95°. \sin 35°$

$(\therefore p + q + r = 0)$

$$= \frac{1}{2}\ [-2\sin^2 25° - 2\sin 95°.\ \sin 35°]$$

$$= \frac{1}{2}\ [-1 + \cos 50° + \cos 130° - \cos 60°]$$

$$= \frac{1}{2}\ [-1 + \cos 50° - \cos 50° - \frac{1}{2}]$$

$$= \frac{-3}{4}$$

43. Option (c) is correct.

Explanation:

$$(p + q + r)^2 = p^2 + q^2 + r^2 + 2(pq + qr + rp)$$

$$\Rightarrow \quad 0 = p^2 + q^2 + r^2 + 2\left(\frac{-3}{4}\right)$$

$$\Rightarrow\ p^2 + q^2 + r^2 = \frac{3}{2}$$

44. Option (a) is correct.

Explanation:

$$p = |\sin\alpha - \sin(\alpha - 90°)|$$
$$= |\sin\alpha + \cos\alpha|$$
$$= \sqrt{2}\ |\sin 45° \sin\alpha + \cos 45° \cos\alpha|$$
$$p = \sqrt{2}\ |\sin(45°+\alpha)|$$
$$0 \le |\sin(45°+\alpha)| \le 1$$
$$0 \le \sqrt{2}\ |\sin(45°+\alpha)| \le \sqrt{2}$$

$\therefore$ Minimum value of p is 0

45. Option (b) is correct.

Explanation: $p = \sqrt{2}\ |\sin 45° + \alpha|$

$$\Rightarrow \quad 0 \le p \le \sqrt{2}$$

$\therefore$ Maximum value $= \sqrt{2}$

46. Option (c) is correct.

Explanation: $a = 5$ cm, $b = 7$ cm, $c = 3$ cm

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{25 + 9 - 49}{2 \times 5 \times 3} = \frac{-15}{2 \times 15} = \frac{-1}{2}$$

$$B = 120°$$

$\therefore$ $A + C = 180° - 120° = 60°$ (acute)

So both statements are correct.

47. Option (c) is correct.

Explanation:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{25 + 9 - 49}{2 \times 5 \times 3} = -\frac{1}{2}$$

$$B = 120°$$

48. Option (a) is correct.

Explanation:

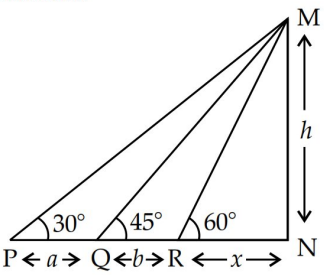
$$\text{Area of triangle} = \frac{1}{2} \times ac \sin B$$

$$= \frac{1}{2} \times 5 \times 3 \times \sin 120° = \frac{1}{2} \times 15 \times \sin 60°$$

$$= \frac{1}{2} \times 15 \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}\text{ cm}^2$$

49. Option (b) is correct.

Explanation:



$$\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}x$$

$$\tan 45° = \frac{h}{QN} \Rightarrow QN = h$$

$$\tan 30° = \frac{h}{PN}$$

$$PN = h\sqrt{3}$$
$$h + a = h\sqrt{3}$$
$$h = \frac{a}{\sqrt{3} - 1} = \frac{a(\sqrt{3}+1)}{2}$$

$$PN = h\sqrt{3} = \frac{a(3+\sqrt{3})}{2}$$

50. Option (a) is correct.

Explanation:

$$\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}x$$

$$\tan 45° = \frac{h}{b + x}$$

$$\Rightarrow \quad h = b + x \Rightarrow \sqrt{3}x = b + x$$
$$x = \frac{b(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad h = MN = \sqrt{3}x = \left(\frac{3+\sqrt{3}}{2}\right)b$$

51. Option (c) is correct.

Explanation: P(Bonus introduced)

$$= \frac{3}{10} \times \frac{4}{9} + \frac{1}{2} \times \frac{2}{9} + \frac{4}{5} \times \frac{1}{3}$$

$$= \frac{2}{15}+\frac{1}{9}+\frac{4}{15} = \frac{6+5+12}{45} = \frac{23}{45}$$

52. Option (b) is correct.

Explanation: F: Bonus introduced

$$\therefore \quad P(A\backslash F) = \frac{\frac{2}{15}}{\frac{23}{45}} = \frac{2}{15}\times\frac{45}{23} = \frac{6}{23}$$

53. Option (a) is correct.

Explanation:

$$\text{New Mean} = \frac{50-5}{20} = \frac{45}{20} = 2.25$$

54. Option (b) is correct.

Explanation: When 5 is added to each observation then the standard deviation is not changed

$$\therefore \text{ New standard deviation} = \frac{10}{20} = 0.5$$

55. Option (b) is correct.

Explanation:

$$P(B/A') = \frac{P(BAA')}{P(A')} = \frac{P(B)-P(B\cap A)}{1-P(A)}$$

$$= \frac{\frac{1}{2}-\frac{1}{4}}{1-\frac{1}{3}} = \frac{1}{4}\times\frac{3}{2} = \frac{3}{8}$$

56. Option (b) is correct.

Explanation:

$$P(A'\cap B') = 1 - P(A\cup B)$$
$$= 1 - [P(A) + P(B) - P(A\cap B)]$$
$$= 1-\left[\frac{1}{3}+\frac{1}{2}-\frac{1}{4}\right] = 1-\left[\frac{4+6-3}{12}\right]$$
$$= 1 - \frac{7}{12} = \frac{5}{12}$$

57. Option (b) is correct.

Explanation:

$$n(s) = 6 \times 6 = 36$$

Sum strictly greater than 7

$= \{(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5,3), (5, 4), (5, 5), (5, 6), (6, 2),(6, 3), (6, 4), (6, 5),(6, 6)\}$

$$n(E) = 15$$

$$\therefore \quad P(E) = \frac{15}{36} = \frac{5}{12}$$

58. Option (c) is correct.

Explanation: $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

$$= 1 - {}^7C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^7 - {}^7C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{7}\right)^6$$

$$= 1-\left(\frac{4}{5}\right)^6\left[\frac{4}{5}+\frac{7}{5}\right] = 1-\left(\frac{11}{5}\right)\left(\frac{4}{5}\right)^6$$

59. Option (b) is correct.

Explanation:

$$\bar{x} = xp = 200 \qquad ...(i)$$
$$\sigma^2 = xpq = 160 ...$$
$$\Rightarrow \quad q = \frac{160}{200} = \frac{4}{5}$$
$$\Rightarrow \quad p = 1 - \frac{4}{5} = \frac{1}{5}$$
$$\Rightarrow \quad n = 1000$$

60. Option (c) is correct.

Explanation:

$$\text{Sum} = S_{15} - S_7$$
$$= \frac{15\times16\times31}{6} - \frac{7\times8\times15}{6}$$
$$= 1240 - 140 = 1100$$

$$\text{So,} \quad \text{mean} = \frac{1100}{8} = 137.5$$

61. Option (d) is correct.

Explanation:

$$y = \sin^{-1}\left(x - \frac{4x^3}{27}\right)$$
$$= \sin^{-1}\left[3.\frac{x}{3} - 4\left(\frac{x}{3}\right)^3\right]$$
$$= 3\sin^{-1}\frac{x}{3}$$

62. Option (c) is correct.

Explanation:

$$y = 3\sin^{-1}\frac{x}{3}$$

$$\frac{dy}{dx} = 3\frac{1}{\sqrt{1-\frac{x^2}{9}}}\times\frac{1}{3} = \frac{3}{\sqrt{9-x^2}}$$

63. Option (c) is correct.

Explanation: $\lim_{x\to0}\frac{\sqrt{x^2+9}-3}{\sqrt{x^2+16}-4}$

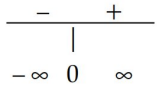
$$= \lim_{x\to0}\frac{x^2+9-9}{x^2+16-16}\times\frac{\sqrt{x^2+16}+4}{\sqrt{x^2+9}+3}$$

$$= \frac{\sqrt{16}+4}{\sqrt{9}+3} = \frac{8}{6} = \frac{4}{3}$$

64. Option (d) is correct.

Explanation: $f(x) = x^2 + 9$

$f'(x) = 2x = 0 \Rightarrow x = 0$



$\therefore f(x)$ is increasing on $[0, \infty]$

and $f(x)$ is decreasing on $(-\infty, 0)$

So, $f(x)$ is minimum at $x = 0$

Hence both statements are wrong.

65. Option (d) is correct.

Explanation:

$$\frac{f(4)}{f(2)} = f\left(\frac{4}{2}\right) = f(2) \Rightarrow f(4) = 3 \times 3 = 9$$

$$\frac{f(16)}{f(4)} = f(4) \Rightarrow f(16) = 9 \times 9 = 81$$

66. Option (c) is correct.

Explanation:

$$\frac{f(4)}{f(2)} = f(2) \Rightarrow f(4) = 3 \times 3 = 9$$

$$\frac{f(2)}{f(2)} = f(1) \Rightarrow f(1) = 1$$

Now, $f(1)\, f(4) = 1 \times 9 = 9$

67. Option (d) is correct.

Explanation:

$f(0) = f(0 \times 5) = f(0 + 5) = f(5) = 10$

68. Option (c) is correct.

Explanation:

$f(0 \cdot 5) = f(0 + 5) = f(0) = f(5) = 10$

$f(0 \cdot 20) = f(0 + 20) \Rightarrow f(0) = f(20) = 10$

$f(0 \times -20) = f(0 - 20) \Rightarrow f(0) = f(-20) = 10$

$\therefore f(20) + f(-20) = 10 + 10 = 20$

69. Option (b) is correct.

Explanation:

$$\int_{\sqrt{2}}^{\sqrt{3}} [x^2]\, dx = \int_{\sqrt{2}}^{\sqrt{3}} 2dx = 2[x]_{\sqrt{2}}^{\sqrt{3}}$$

$$= 2\left(\sqrt{3} - \sqrt{2}\right)$$

70. Option (a) is correct.

Explanation:

$$\int_{\sqrt{2}}^{2} f(x)dx = \int_{\sqrt{2}}^{2} [x^2]dx = \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

$$= 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3[x]_{\sqrt{3}}^{2}$$

$$= 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$= 6 - 2\sqrt{2} - \sqrt{3}$$

71. Option (b) is correct.

Explanation:

$\therefore A^2 + B^2 + C^2 = 0$

$\therefore A = B = C = 0$

$$\text{Now, } \begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

72. Option (a) is correct.

Explanation: $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$

(Applying $C_1 \to C_1 + C_2 + C_3$)

$$\begin{vmatrix} x+1+\omega+\omega^2 & \omega & \omega^2 \\ x+1+\omega+\omega^2 & x+\omega^2 & 1 \\ x+1+\omega+\omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

$$\begin{vmatrix} x & \omega & \omega^2 \\ x & x+\omega^2 & 1 \\ x & 1 & x+\omega \end{vmatrix} = 0$$

$$x\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x+\omega^2 & 1 \\ 1 & 1 & x+\omega \end{vmatrix} = 0$$

$\therefore x = 0$

73. Option (a) is correct.

Explanation:

$$\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^3 = \left(\frac{(\sqrt{3}+i)^2}{3+1}\right)^3$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^6$$

$$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^6$$

$$= \cos\pi + i\sin\pi = -1$$

74. Option (c) is correct.

Explanation: $x^2 - x + 1 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{3}i}{2}$

if $x = \dfrac{1+\sqrt{3}i}{2}$ then $\dfrac{1}{x} = \dfrac{1-\sqrt{3}i}{2}$

Now $x - \dfrac{1}{x} = \sqrt{3}i$

$$\left(x - \frac{1}{x}\right)^2 + \left(x - \frac{1}{x}\right)^4 + \left(x - \frac{1}{x}\right)^8$$

$= -3 + 9 + 81 = 87$

75. Option (c) is correct.

Explanation:

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1 3

Total words $= \frac{4!}{2!} \times 4!$

$= \frac{24 \times 24}{2} = 288$

76. Option (a) is correct.

Explanation: Let $\text{amp}(z) = \theta$

$\therefore \quad \text{amp}(\bar{z}) = -\theta$

Now amp (z) + amp $(\bar{z}) = 0$

77. Option (c) is correct.

Explanation: Number of diagonals

$= {}^{n}C_2 - n = 20$

$= \frac{n(n-1)}{2} - n$

$= 20$

$\Rightarrow \quad n^2 - 3n - 40 = 0$

$\Rightarrow \quad n = 8$

78. Option (c) is correct.

Explanation: Number of arrangements

$= 3!2! = 12$

79. Option (c) is correct.

Explanation:

$\therefore \quad x + y + z = 5$

$(1, 1, 3) \rightarrow \frac{3!}{2!} = 3$ ways

$(1, 2, 2) \rightarrow \frac{3!}{2!} = 3$ ways

Total ways $= 3 + 3 = 6$

80. Option (c) is correct.

Explanation:

$$T_{r+1} = {}^{n}C_r \left(3^{\frac{1}{2}}\right)^{12-r} \left(5^{\frac{1}{4}}\right)^{r}$$

$$= {}^{12}C_r 3^{\frac{12-r}{2}} 5^{\frac{r}{4}}$$

for rational term $\frac{12-r}{2}$ and $\frac{r}{4}$ is integer.

$\therefore$ Possible values of $r = 0, 4, 8, 12$

81. Option (a) is correct.

Explanation:

$m_1 = \frac{-m^2}{n}$ and $m_2 = \frac{-n^2}{-m} = \frac{n^2}{m}$

for perpendicular

$-\frac{m^2}{n} \times \frac{n^2}{m} = -1 \Rightarrow mn - 1 = 0$

82. Option (d) is correct.

Explanation:

$\sqrt{p^2+1} = \sqrt{q^2+1} = \sqrt{(p-1)^2 + (q-1)^2}$

$\sqrt{p^2+1} = \sqrt{q^2+1} \Rightarrow p = q \; [\because p, q \in (0, 1)]$

Now, $\sqrt{q^2+1} = \sqrt{(p-1)^2 + (q-1)^2}$

$\Rightarrow p^2 + 1 = 2(p-1)^2$

$\Rightarrow p^2 - 4p + 1 = 0 \Rightarrow p = 2 \pm \sqrt{3}$

Since, $0 < p < 1 \Rightarrow p = 2 - \sqrt{3} = q$

Now, $p + q = 2(2 - \sqrt{3}) = 4 - 2\sqrt{3}$

83. Option (a) is correct.

Explanation: $a = \text{BC} = \sqrt{(2-0)^2 + (0-0)^2} = 2$

$b = \text{AC} = \sqrt{(2-1)^2 + (0-1)^2} = \sqrt{2}$

$c = \text{AB} = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$

$\therefore$ Coordinate of incentive

$$= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$= \frac{2(1) + \sqrt{2}(0) + \sqrt{2}(2)}{2 + \sqrt{2} + \sqrt{2}}, \frac{2(1) + \sqrt{2}(0) + \sqrt{2}(0)}{2 + \sqrt{2} + \sqrt{2}}$$

$$= \left(\frac{2 + 2\sqrt{2}}{2 + 2\sqrt{2}}, \frac{2}{2 + 2\sqrt{2}}\right) = \left(1, \sqrt{2} - 1\right)$$

84. Option (b) is correct.

Explanation: Coordinate of P $= \left(\frac{3+1}{2}, \frac{-1+1}{2}\right)$

$= \text{P}(2, 0)$

Slope of AB $= \frac{1+1}{1-3} = -1$

Equation of AB is

$y - 1 = -1 (x - 1) \Rightarrow x + y - 2 = 0$

Distance from Q(x, y)

$\frac{x + y - 2}{\sqrt{2}} = \sqrt{2}$

$x + y - 2 = 2 \Rightarrow x + y = 4$ ...(i)

Slope of PQ $= \frac{y}{x-2}$

$\Rightarrow \text{PQ} \perp \text{AB}$

$\therefore (-1)\left(\frac{y}{x-2}\right) = -1 \Rightarrow y = x - 2$

From (i)

$x + x - 2 = 4 \Rightarrow x = 3$ and $y = 1$

Q(3, 1)

85. Option (d) is correct.

Explanation:

$$\text{Slope of AD} = \frac{6-2}{-2-1} = \frac{4}{-3}$$

$$\therefore \quad \text{Slope of BC} = \frac{3}{4}$$

Equation of BC is

$$y - 6 = \frac{3}{4}(x + 2) \Rightarrow 3x - 4y + 30 = 0$$

86. Option (b) is correct.

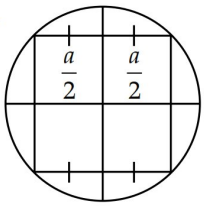
Explanation:

$$\text{Radius} = \frac{10}{2} = 5 \text{ cm}$$

On solving $x + y = 0$ and $x - y = 0$
we get centre (0, 0)
Now, the equation of circle
$x^2 + y^2 = 25$

87. Option (c) is correct.

Explanation:



Equation of circle is
$x^2 + y^2 + 2x + 2y + 1 = 0$

$$\therefore \quad \text{centre} = (-1, -1)$$

$$\text{Radius} = \sqrt{1+1-1} = 1$$

$$\text{Diagonal} = \text{diameter}$$

$$\sqrt{2}a = 2 \Rightarrow a = \sqrt{2}$$

Since the sides of the square are parallel to the co-ordinate axes, the vertices can be found by moving $\frac{a}{2}$ units from the centre along the axes.

$\therefore$ One of the vertices is

$$\left(-1+\frac{1}{\sqrt{2}}, -1-\frac{1}{\sqrt{2}}\right)$$

88. Option (d) is correct.

Explanation: Scope of tangent = $\tan 45° = 1$

$$y^2 = 4x \Rightarrow 2y\frac{dy}{dx} = 4$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{2y} = 1 \Rightarrow y = 2$$

$$\text{putting in } y^2 = 4x \Rightarrow x = 1$$

$\therefore$ Point of contact = (1, 2)

89. Option (b) is correct.

Explanation:

$$25x^2 - 75y^2 = 225$$

$$\frac{x^2}{9} - \frac{y^2}{3} = 1$$

$$\therefore \quad a^2 = 9 \text{ and } b^2 = 3$$

$$c^2 = a^2 + b^2 = 9 + 3 = 12$$

$$c = 2\sqrt{3}$$

Distance between the two foci = $2c = 4\sqrt{3}$

90. Option (a) is correct.

Explanation: We know that the parametric point of ellipse is $(a \sin \alpha)\ b \cos \alpha)$

$\therefore a = 3$ and $b = 5$

$$b^2 = a^2(1 - e^2) \Rightarrow \frac{25}{9} = 1 - e^2$$

$$e^2 = 1 - \frac{25}{9} = \frac{16}{9} \Rightarrow e = \frac{4}{3}$$

91. Option (c) is correct.

Explanation: Total number of students whose height is less than or equal to 165 cm = 12 + 15 + 24 = 51

92. Option (c) is correct.

Explanation:

C.I.	fi	C.F	
160–162	12	12	
162–164	15	27	
164–166	24(M)	51	Median Class
166–168	13	64	

$$\frac{N}{2} = \frac{64}{2} = 32$$

$$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - \text{C.F}}{f} \times h$$

$$= 164 + \frac{32-27}{24} \times 2$$

$$= 164 + \frac{5}{12} = 164.41 \text{ cm}$$

93. Option (d) is correct.

Explanation: Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$= 164 + \frac{24-15}{48-15-13} \times 2$$

$$= 164 + \frac{9}{20} \times 2$$

$$= 164.9 \text{ cm}$$

94. Option (c) is correct.

Explanation: Histogram is made from group frequency data.

95. Option (b) is correct.

Explanation:

$$\text{Variance of X} = \frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2$$

$$= \frac{900}{50} - \left(\frac{200}{50}\right)^2$$

$$= 18 - 16 = 2$$

$$\text{Variance of Y} = \frac{\Sigma Y^2}{N} - \left(\frac{\Sigma Y}{N}\right)^2$$

$$= \frac{1400}{50} - \left(\frac{250}{50}\right)^2$$

$$= 28 - 25 = 3$$

$\therefore$ Variance (X) < Variance (Y)

96. Option (b) is correct.

Explanation:

$$\text{C.V(X)} = \frac{\sigma}{M} \times 100$$

$$= \frac{\sqrt{2}}{50} \times 100 = 2\sqrt{2}$$

$$\text{C.V(Y)} = \frac{\sigma}{M} \times 100$$

$$= \frac{\sqrt{3}}{50} \times 100 = 2\sqrt{3}$$

$\therefore$ C.V(X) < C.V(Y)

97. Option (c) is correct.

Explanation: $n = 6, p = k \Rightarrow q = 1 - k$

Now, $9.P(X = 4) = P(X = 2)$

$9.{}^6C_4 k^4 (1-k)^2 = {}^6C_2 k^2 (1-k)^4$

$9k^2 = (1-k)^2 \; (\because {}^6C_4 = {}^6C_2)$

$3k = 1 - k \Rightarrow k = \frac{1}{4}$

98. Option (a) is correct.

Explanation:

$$P(x = 3) = {}^6C_3 (k)^3 (1-k)^3$$

$$= \frac{6.5.4}{3.2.1}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^3$$

$$= 20\left(\frac{3}{16}\right)^3 = 20 \times \frac{27}{16 \times 16 \times 16}$$

$$= \frac{135}{1024}$$

99. Option (a) is correct.

Explanation:

$$n(S) = {}^{11}C_6$$

$$n(E) = {}^7C_3 \times {}^4C_3$$

$$P(E) = \frac{{}^7C_3 \times {}^4C_3}{{}^{11}C_6} = \frac{10}{33}$$

100. Option (d) is correct.

Explanation: P(E)

$$= \frac{{}^4C_2 \times {}^7C_4 + {}^4C_3 \times {}^7C_3 + {}^4C_4 \times {}^7C_2}{{}^{11}C_6}$$

$$= \frac{53}{66}$$

101. Option (b) is correct.

Explanation: $y = (1 - \cos x)^{-1} = \frac{1}{1 - \cos x}$

$$= \frac{1}{2\sin^2\frac{x}{2}} = \frac{1}{2}\text{cosec}^2\frac{x}{2}$$

$\therefore \; \text{cosec}\frac{x}{2} \leftarrow (-\infty, -1) \cup (1, \infty)$

$\text{cosec}^2\frac{x}{2} \leftarrow (1, \infty)$

$\Rightarrow \frac{1}{2}\text{cosec}^2\frac{x}{2} \leftarrow (0.5, \infty)$

Range = $(0.5, \infty)$

102. Option (b) is correct.

Explanation:

$$I = \int y dx = \frac{1}{2}\int \text{cosec}^2\frac{x}{2} dx$$

$$= \frac{-\frac{1}{2}\cot\frac{x}{2}}{\frac{1}{2}} + C$$

$$= -\cot\frac{x}{2} + C$$

103. Option (b) is correct.

Explanation:

$$\lim_{x \to 0}\{f(x).g(x)\} = \lim_{x \to 0}\{f(x).g(x)\}$$

$$\textbf{L.H.L} = \lim_{h \to 0} \sin[0-h]|0-h|$$

$$= \lim_{h \to 0} \sin[0-h].0 = \mathbf{0}$$

$$\textbf{R.H.L} = \lim_{h \to 0} \sin[0+h]|0+h|$$

$$= \lim_{h \to 0} \sin 0.h = \mathbf{0}$$

$\therefore \quad \lim_{x \to 0}\{f(x)g(x)\} = 0$

104. Option (d) is correct.

Explanation: $\lim_{x\to 0}\frac{\sin[x]}{[x]}$

$$\text{L.H.L} = \lim_{h\to 0}\frac{\sin[0-h]}{(0-h)} = \text{infinity}$$

$$\text{R.H.L} = \lim_{h\to 0}\frac{\sin[0+h]}{(0+h)}$$

$$= \lim_{h\to 0}\frac{\sin 0}{(h)} = 0$$

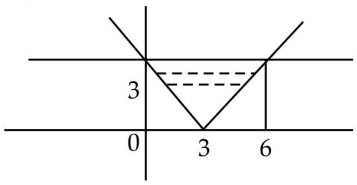
L.H.L ≠ R.H.L, limit does not exist.

105. Option (c) is correct.

Explanation: Domain of $f(x) = |x-3|$ is $(-\infty, \infty)$

106. Option (d) is correct.

Explanation:



Required area

$= 3\times 6 - \frac{1}{2}\times 3\times 3 - \frac{1}{2}\times 3\times 3$

$= 18 - 9 = 9$ square units

107. Option (c) is correct.

Explanation:

$$f(x) = px + q$$

$$\text{put } x = 1 \text{ and } y = 1$$

$$p + q = 1$$

108. Option (a) is correct.

Explanation:

$\therefore f(x) = px + q$ is linear polynomial so, $f(x)$ is one-one

$$\text{Range} = \{1, 4, 7, 10\}$$

$$\text{codomain} = \text{N}$$

So it is not onto.

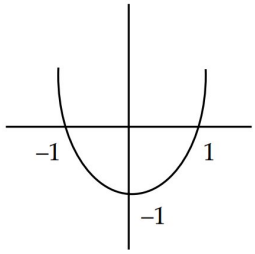
109. Option (a) is correct.

Explanation: $\lim_{x\to 1} f(f(x)) = \lim_{x\to 1}(x^2-1)^2 - 1$

$= (1-1)^2 - 1 = 0 - 1 = -1$

110. Option (c) is correct.

Explanation: Let $y = x^2 - 1 \Rightarrow x^2 = y + 1$



Required area $= \left|\int_{-1}^{1}(x^2-1)dx\right|$

$$= \left|\left[\frac{x^3}{3} - x\right]_{-1}^{1}\right| = \left|\left(\frac{1}{3}-1\right) - \left(-\frac{1}{3}+1\right)\right|$$

$$= \left|-\frac{2}{3} - \frac{2}{3}\right| = \left|-\frac{4}{3}\right| = \frac{4}{3} \text{ sq. units}$$

111. Option (c) is correct.

Explanation: We know that

If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$

then $(x^2+4)\left(\frac{dy}{dx}\right)^2 - n^2(y^2+4) = 0$

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 - (4)^2(y^2+4) = 0$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{16(y^2+4)}{x^2+4}$$

112. Option (c) is correct.

Explanation: $\therefore \left(\frac{dy}{dx}\right)^2 = \frac{16(y^2+4)}{x^2+4}$

$$2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$$

$$= \frac{16(2y)(x^2+4)\frac{dy}{dx} - 16(y^2+4)(2x)}{(x^2+4)^2}$$

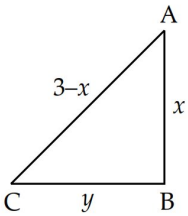
$$\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)(x^2+4)^2 = 16y(x^2+4)\frac{dy}{dx} - 16x(y^2+4)$$

$$\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)(x^2+4)^2 - 16y(x^2+4)\frac{dy}{dx} = -16x(y^2+4)$$

$$\frac{(x^2+4)}{y^2+4}\frac{dy}{dx}\left((x^2+4)\frac{d^2y}{dx^2} - 16y\right) = -16x$$

113. Option (c) is correct.

Explanation: $y = \sqrt{(3-x)^2 - x^2} = \sqrt{9-6x}$



$$\text{Area (A)} = \frac{1}{2}xy = \frac{1}{2}x\sqrt{9-6x}$$

$$A_1 = A^2 = \frac{1}{4}x^2(9-6x)$$

$$A_1 = \frac{1}{4}(9x^2 - 6x^3)$$

$$\frac{dA_1}{dx} = \frac{1}{4}(18x - 18x^2) = 0$$

$$18x(1-x) = 0$$

$$x = 0 \text{ (Not Possible)}$$

$\therefore \quad x = 1$

$\therefore$ Area is maximum

$\therefore \quad y = \sqrt{9-6} = \sqrt{3}$

$$\tan A = \frac{y}{x} = \frac{\sqrt{3}}{1}$$

$$\angle A = \frac{\pi}{3}$$

114. Option (a) is correct.

Explanation: Maximum area

$$= \frac{1}{2} \times y \times x = \frac{\sqrt{3}}{2} \text{ sq. units}$$

115. Option (d) is correct.

Explanation:

$$(x+y)^{p+q} = x^p y^q$$

$$\therefore (p+q)\log(x+y) = p\log x + q\log y$$

$$\frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{p}{x} + \frac{q}{y}\frac{dy}{dx}$$

$$\left(\frac{p+q}{x+y} - \frac{q}{y}\right)\frac{dy}{dx} = \frac{p}{x} - \frac{p+q}{x+y}$$

$$\frac{py+qy-qx-qy}{y(x+y)}\frac{dy}{dx} = \frac{px+py-px-qx}{x(x+y)}$$

$$\frac{py-qx}{y}\frac{dy}{dx} = \frac{py-qx}{x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{y}{x}$$

which is independent of both p and q.

116. Option (a) is correct.

Explanation: Since $\frac{dy}{dx}$ is independent of both p and q. So for $p + q = 10$

$$\frac{dy}{dx} = \frac{y}{x}$$

117. Option (a) is correct.

Explanation:

$$\therefore \quad \frac{dy}{dx} = 4 \Rightarrow \int dy = \int 4dx$$

$$y = 4x + c$$

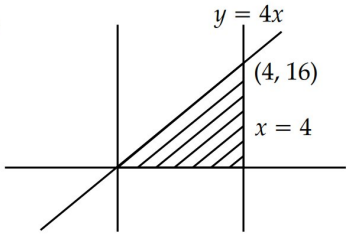
$\therefore$ it passes through (0, 0) then $c = 0$

$$\therefore \quad y = 4x$$

Hence it is an equation of a straight line passing through (1, 4)

118. Option (c) is correct.

Explanation:



$$\text{Required area} = \frac{1}{2} \times 4 \times 16$$

$$= 32 \text{ sq. units}$$

119. Option (c) is correct.

Explanation:

$$f(x) = \begin{cases} x^3 & -1 < x < 1 \\ x^2 & x \in (-\infty, -1) \cup (1, \infty) \end{cases}$$

$$f(x) = \begin{cases} 3x^2 & -1 < x < 1 \\ 2x & x \in (-\infty, -1) \cup (1, \infty) \end{cases}$$

$$\therefore \quad \lim_{x \to 0} f'(x) = 3(0) = 0$$

120. Option (d) is correct.

Explanation:

$$\text{L.H.S} = \lim_{x \to 1^-} x^2 = (-1)^2 = 1$$

$$\text{R.H.S} = \lim_{x \to -1^+} x^3 = (-1)^3 = -1$$

$$\therefore \quad \text{L.H.S} \neq \text{R.H.S}$$

So $f(x)$ is discontinuous at $x = -1$

$$\text{L.H.D} = 3(1) = 3$$

$$\text{R.H.D} = 2(1) = 2$$

$$\therefore \quad \text{L.H.D} \neq \text{R.H.D}$$

So, $f(x)$ is not differentiable at $x = 1$

Hence neither I nor II is true.