NDA/NA SOLVED PAPER 2024-I

MATHEMATICS

1. Let A and B be matrices of order 3×3. If $|A| = \dfrac{1}{2\sqrt{2}}$ and $|B| = \dfrac{1}{729}$, then what is the value of $|2B(adj(3A)|$?
   (a) 27
   (b) $\dfrac{27}{2\sqrt{2}}$
   (c) $\dfrac{27}{2}$
   (d) 1

2. If z is any complex number and $iz^3 + z^2 - z + i = 0$, where $i = \sqrt{-1}$, then what is the value of $(|z| + 1)^2$?
   (a) 1
   (b) 4
   (c) 81
   (d) 121

3. What is the sum of all four digit numbers formed by using all digits 0, 1, 4, 5 without repetition of digits?
   (a) 44440
   (b) 46460
   (c) 46440
   (d) 64440

4. If x, y and z are the cube roots of unity, then what is the value of $xy + yz + zx$?
   (a) 0
   (b) 1
   (c) 2
   (d) 3

5. A man has 7 relatives (4 women and 3 men). His wife also has 7 relatives (3 women and 4 men). In how many ways can they invite 3 women and 3 men so that 3 of them are man's relatives and 3 of them are his wife's relatives?
   (a) 340
   (b) 484
   (c) 485
   (d) 469

6. A triangle PQR is such that 3 points lie on the side PQ, 4 points on QR and 5 points on RP respectively. Triangles are constructed using these points as vertices. What is the number of triangles so formed ?
   (a) 205
   (b) 206
   (c) 215
   (d) 220

7. If $\log_b a = p, \log_d c = 2p$ and $\log_f e = 3p$, then what is $(ace)^{\frac{1}{p}}$ equal to?
   (a) bd^2f^3
   (b) bdf
   (c) b^3d^2f
   (d) $b^2d^2f^2$

8. If $-\sqrt{2}$ and $\sqrt{3}$ are roots of the equation $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$ where a_0, a_1, a_2, a_3 are integers, then which one of the following is correct?
   (a) $a_2 = a_3 = 0$
   (b) $a_2 = 0$ and $a_3 = -5$
   (c) $a_0 = 6, a_3 = 0$
   (d) $a_1 = 0$ and $a_2 = 5$

9. Let z_1 and z_2 be two complex numbers such that $\left|\dfrac{z_1 + z_2}{z_1 - z_2}\right| = 1$, then what is $\mathrm{Re}\left(\dfrac{z_1}{z_2}\right) + 1$ equal to?
   (a) -1
   (b) 0
   (c) 1
   (d) 5

10. If $26! = n8^k$, where k and n are positive integers, then what is the maximum value of k?
    (a) 6
    (b) 7
    (c) 8
    (d) 9

11. Consider the following statements in respect of two non-singular matrices A and B of the same order n:
    A. $adj(AB) = (adjA)(adjB)$
    B. $adj(AB) = adj(BA)$
    C. $(AB)adj(AB) - |AB|I_n$ is a null matrix of order n
    How many of the above statements are correct?
    (a) None
    (b) Only one statement
    (c) Only two statements
    (d) All three statements

12. Consider the following statements in respect of a non-singular matrix A of order n:
    A. $A(adjA^T) = A(adjA)^T$
    B. If $A^2 = A$, then A is identity matrix of order n
    C. If $A^3 = A$, then A is identity matrix of order n
    Which of the statements given above are correct?
    (a) A and B only
    (b) B and C only
    (c) A and C only
    (d) A, B and C

13. How many four-digit natural numbers are there such that all of the digits are even?
    (a) 625
    (b) 500
    (c) 400
    (d) 256

14. If $\omega \neq 1$ is a cube root of unity, then what are the solutions of $(z - 100)^3 + 1000 = 0$?
    (a) $10(1 - \omega)$, $10(10 - \omega^2)$, 100
    (b) $10(10 - \omega)$, $10(10 - \omega^2)$, 90
    (c) $10(1 - \omega)$, $10(10 - \omega^2)$, 1000
    (d) $(1 + \omega)$, $(10 + \omega^2)$, -1

15. What is $(1 + i)^4 + (1 - i)^4$ equal to, where $i = \sqrt{-1}$?
    (a) 4
    (b) 0
    (c) -4
    (d) -8

16. Consider the following statements in respect of a skew-symmetric matrix A of order 3 :
    A. All diagonal elements are zero.
    B. The sum of all the diagonal elements of the matrix is zero.
    C. A is orthogonal matrix.
    Which of the statements given above are correct ?
    (a) A and B only
    (b) B and C only
    (c) A and C only
    (d) A, B and C

17. Four digit numbers are formed by using the digits 1, 2, 3, 5 without repetition of digits. How many of them are divisible by 4 ?
    (a) 120   (b) 24
    (c) 12   (d) 6

18. What is the remainder when 2^{120} is divided by 7 ?
    (a) 1   (b) 3
    (c) 5   (d) 6

19. For what value of n is the determinant
$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix} = 0$$
    for every $m > n$?
    (a) 4   (b) 5
    (c) 6   (d) 7

20. If ABC is a triangle, then what is the value of the determinant
$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix} ?$$
    (a) −1   (b) 0
    (c) 1   (d) 3

21. What is the number of different matrices, each having 4 entries that can be formed using 1, 2, 3, 4 (repetition is allowed)?
    (a) 72   (b) 216
    (c) 254   (d) 768

22. Let $A = \{x \in R : -1 < x < 1\}$. Which of the following is/are bijective functions from A to itself?
    (A) $f(x) = x\,|x|$
    (B) $g(x) = \cos(\pi x)$
    Select the correct answer using the code given below:
    (a) A only   (b) B only
    (c) Both A and B   (d) Neither A nor B

23. Let R be a relation on the open interval $(-1, 1)$ and is given by
    $R = \{(x, y) : |x + y| < 2\}$. Then which of the following is correct?
    (a) R is reflexive but neither symmetric nor transitive
    (b) R is reflexive and symmetric but not transitive
    (c) R is reflexive and transitive but not symmetric
    (d) R is an equivalence relation

24. For any three non-empty sets A, B, C, what is
    $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$ is equal to?
    (a) Null set   (b) A
    (c) B   (d) $(A \cup B) - (A \cap B)$

25. If a, b, c are the sides of triangle ABC, then what is
$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix} \text{ equal to?}$$
    (a) Zero   (b) Area of triangle
    (c) Perimeter of triangle   (d) $a^2 + b^2 + c^2$

26. If a, b, c are in AP; b, c, d are in GP; c, d, e are in HP, then which of the following is/ are correct?
    (A) a, c and e are in GP
    (B) $\dfrac{1}{a}, \dfrac{1}{c}, \dfrac{1}{e}$ are in GP
    Select the correct answer using the code given below:
    (a) A only   (b) B only
    (c) Both A and B   (d) Neither A nor B

27. What is the number of solutions of
$$\log_4(x-1) = \log_2(x-3) ?$$
    (a) Zero   (b) One
    (c) Two   (d) Three

28. For $x \geq y > 1$,
    let $\log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right) = k$,
    then the value of k can never be equal to
    (a) −1   (b) $-\dfrac{1}{2}$
    (c) 0   (d) 1

29. If $A = \begin{vmatrix} \sin 2\theta & 2\sin^2\theta - 1 & 0 \\ \cos 2\theta & 2\sin\theta\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$, then
    which of the following statements is/ are correct?
    (A) $A^{-1} = adjA$
    (B) A is skew-symmetric matrix
    (C) $A^{-1} = A^T$
    Select the correct answer using the code given below:
    (a) A only   (b) A and B only
    (c) A and C (c) B and C

30. What is the coefficient of x^{10} in the expansion of
$$\left(1 - x^2\right)^{20}\left(2 - x^2 - \dfrac{1}{x^2}\right)^{-5} ?$$
    (a) −1   (b) 1
    (c) 10
    (d) Coefficient of x^{10} does not exist

31. If the 4th term in the expansion of $\left(mx + \dfrac{1}{x}\right)^n$ is $\dfrac{5}{2}$, then what is the value of mn?
    (a) −3   (b) 3
    (c) 6   (d) 12

32. If a, b and $c(a > 0, c > 0$)are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$:
    (A) The equation has imaginary roots.
    (B) The ratio of the roots of the equation is $1 : \omega$ where ω is a cube root of unity.
    (C) The product of roots of the equation is $\left(\dfrac{b^2}{a^2}\right)$.

Which of the statements given above are correct?
(a) (A) and (B) only   (b) (B) and (C) only
(c) (A) and (C) only   (d) (A), (B) and (C)

33. If $x^2 + mx + n$ is an integer for all integral values of x, then which of the following is/ are correct?
(A) m must be an integer
(B) n must be an integer
Select the correct answer using the code given below:
(a) A only   (b) B only
(c) Both (A) and (B)   (d) Neither (A) nor B

34. In a binomial expansion of $(x+y)^{2n+1}(x-y)^{2n+1}$, the sum of middle terms is zero. What is the value of $\left(\dfrac{x^2}{y^2}\right)$?

(a) 1   (b) 2
(c) 4   (d) 8

35. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7\}$. What is the number of onto functions from A to B?
(a) 10   (b) 20
(c) 30   (d) 32

36. What is $\dfrac{\sqrt{3}\cos 10° - \sin 10°}{\sin 25° \cos 25°}$ equal to ?

(a) 1   (b) $\sqrt{3}$
(c) 2   (d) 4

37. What is $(\sin 9° - \cos 9°)$ is equal to?

(a) $-\dfrac{\sqrt{5-\sqrt{5}}}{2}$ (b) $-\dfrac{\sqrt{5-\sqrt{3}}}{2}$

(c) $\dfrac{\sqrt{5-\sqrt{5}}}{2}$ (d) $\dfrac{\sqrt{5-\sqrt{5}}}{4}$

38. If in a triangle ABC, $\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \sin B \sin C$, then what is the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$;

where a, b, c are sides of the triangle?
(a) $a + b + c$ (b) $ab + bc + ca$
(c) $(a + b)(b + c)(c + a)$ (d) 0

39. If $\cos^{-1}x = \sin^{-1}x$, then which one of the following is correct?

(a) $x = 1$ (b) $x = \dfrac{1}{2}$

(c) $x = \dfrac{1}{\sqrt{2}}$ (d) $x = \dfrac{1}{\sqrt{3}}$

40. What is the number of solutions of $(\sin\theta - \cos\theta)^2 = 2$ where $-\pi < \theta < \pi$?
(a) Only one   (b) Only two
(c) Four   (d) No solution

41. ABC is a triangle such that angle C = 60°, then what is $\dfrac{\cos A + \cos B}{\cos\left(\dfrac{A-B}{2}\right)}$ equal to?

(a) 2   (b) $\sqrt{2}$

(c) 1   (d) $\dfrac{1}{\sqrt{2}}$

42. What is $\sqrt{15 + \cot^2\left(\dfrac{\pi}{4} - 2\cot^{-1}3\right)}$ equal to?

(a) 1   (b) 7
(c) 8   (d) 16

43. What is the value of $\sin 10°\cdot\sin 50° + \sin 50°\cdot\sin 250° + \sin 250°\cdot\sin 10°$ equal to?

(a) $-\dfrac{1}{4}$ (b) $-\dfrac{3}{4}$

(c) $\dfrac{3\sin 10°}{4}$ (d) $-\dfrac{3\cos 10°}{4}$

44. What is $\tan^{-1}\left(\dfrac{a}{b}\right) - \tan^{-1}\left(\dfrac{a-b}{a+b}\right)$ eqaul to

(a) $-\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{4}$

(c) $\tan^{-1}\left(\dfrac{a^2 - b^2}{a^2 + b^2}\right)$ (d) $\tan^{-1}\left(\dfrac{2ab}{a^2 + b^2}\right)$

45. Under which one of the following conditions does the equation $(\cos\beta - 1)x^2 + (\cos\beta)x + \sin\beta = 0$ in x have a real root for $\beta \in [0, \pi]$?
(a) $1 - \cos\beta < 0$ (b) $1 - \cos\beta \leq 0$
(c) $1 - \cos\beta > 0$ (d) $1 - \cos\beta \geq 0$

46. In a triangle ABC, $AB = 16$ cm, $BC = 63$ cm and $AC = 65$ cm. What is the value of $\cos 2A + \cos 2B + \cos 2C$?
(a) -1 (b) 0

(c) 1   (d) $\dfrac{76}{65}$

47. If $f(\theta) = \dfrac{1}{1 + \tan\theta}$ and $\alpha + \beta = \dfrac{5\pi}{4}$, then what is the value of $f(\alpha) f(\beta)$?

(a) $-\dfrac{1}{2}$ (b) $\dfrac{1}{2}$

(c) 1   (d) 2

48. If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 - 6x + 8 = 0$, then what is the value of $\cos(2\alpha + 2\beta)$?

(a) $\dfrac{13}{75}$ (b) $\dfrac{13}{85}$

(c) $\dfrac{17}{85}$ (d) $\dfrac{19}{85}$

49. What is the value of
$\tan 65° + 2\tan 45° - 2\tan 40° - \tan 25°$?
(a) 0   (b) 1
(c) 2   (d) 4

50. Consider the following statements :
   A. In a triangle ABC, if $\cot A \cdot \cot B \cdot \cot C > 0$, then the triangle is an acute angled triangle.
   B. In a triangle ABC, if $\tan A \cdot \tan B \cdot \tan C > 0$, then the triangle is an obtuse angled triangle.
   Which of the statements given above is/are correct ?
   (a) A Only
   (b) B Only
   (c) Both A and B
   (d) Neither A nor B

51. If (a, b) is the centre and c is the radius of the circle $x^2 + y^2 + 2x + 6y + 1 = 0$, then what is the value of $a^2 + b^2 + c^2$?
   (a) 19
   (b) 18
   (c) 17
   (d) 11

52. If $(1, -1, 2)$ and $(2, 1, -1)$ are the end points of a diameter of a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$, then what is $u + v + w$ equal to ?
   (a) −2
   (b) −1
   (c) 1
   (d) 2

53. The number of points represented by the equation $x = 5$ on the xy-plane is
   (a) Zero
   (b) One
   (c) Two
   (d) Infinitely many

54. If $< l, m, n >$ are the directions cosines of a normal to the plane $2x - 3y + 6z + 4 = 0$, then what is the value of $49(7l^2 + m^2 - n^2)$?
   (a) 0
   (b) 1
   (c) 3
   (d) 71

55. A line through $(1, -1, 2)$ with directions ratios $< 3, 2, 2 >$ meets the plane $x + 2y + 3z = 18$. What is the point of intersection of line and plane ?
   (a) (4, 4, 1)
   (b) (2, 4, 1)
   (c) (4, 1, 4)
   (d) (3, 4, 7)

56. If p is the perpendicular distance from origin to the plane passing through $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, then what is $3p^2$ equal to ?
   (a) 4
   (b) 3
   (c) 2
   (d) 1

57. If the directions cosines $< l, m, n >$ of a line are connected by relation $l + 2m + n = 0$, $2l - 2m + 3n = 0$, then what is the value of $l^2 + m^2 - n^2$?
   (a) $\dfrac{1}{101}$
   (b) $\dfrac{29}{101}$
   (c) $\dfrac{41}{101}$
   (d) $\dfrac{92}{101}$

58. If a variable line passes through the point of intersection of the lines $x + 2y - 1 = 0$ and $2x - y - 1 = 0$ and meets the coordinate axes in A and B, then what is the locus of the mid-point of AB?
   (a) $3x + y = 10\,xy$
   (b) $x + 3y = 10\,xy$
   (c) $3x + y = 10$
   (d) $x + 3y = 10$

59. What is the equation to the straight line passing through the point $(-\sin\theta, \cos\theta)$ and perpendicular to the line $x\cos\theta + y\sin\theta = 9$?
   (a) $x\sin\theta - y\cos\theta - 1 = 0$
   (b) $x\sin\theta - y\cos\theta + 1 = 0$
   (c) $x\sin\theta - y\cos\theta = 0$
   (d) $x\cos\theta - y\sin\theta + 1 = 0$

60. Two points P and Q lie on line $y = 2x + 3$. These two points P and Q are at a distance 2 units from another point $R(1, 5)$. What are the coordinates of the points P and Q?
   (a) $\left(1 + \dfrac{2}{\sqrt5}, 5 + \dfrac{4}{\sqrt5}\right)\left(1 - \dfrac{2}{\sqrt5}, 5 - \dfrac{4}{\sqrt5}\right)$
   (b) $\left(3 + \dfrac{2}{\sqrt5}, 5 + \dfrac{4}{\sqrt5}\right), \left(-1 - \dfrac{2}{\sqrt5}, 5 - \dfrac{4}{\sqrt5}\right)$
   (c) $\left(1 - \dfrac{2}{\sqrt5}, 5 + \dfrac{4}{\sqrt5}\right), \left(1 + \dfrac{2}{\sqrt5}, 5 - \dfrac{4}{\sqrt5}\right)$
   (d) $\left(3 - \dfrac{2}{\sqrt5}, 5 + \dfrac{4}{\sqrt5}\right), \left(-1 + \dfrac{2}{\sqrt5}, 5 - \dfrac{4}{\sqrt5}\right)$

61. If two sides of a square lie on the lines $2x + y - 3 = 0$ and $4x + 2y + 5 = 0$, then what is the area of the square in square units?
   (a) 6.05
   (b) 6.15
   (c) 6.25
   (d) 6.35

62. ABC is a triangle with $A(3, 5)$. The mid-points of sides AB, AC are at $(-1, 2)$, $(6, 4)$ respectively. What are the coordinates of centroid of the triangle ABC ?
   (a) $\left(\dfrac{8}{3}, \dfrac{11}{3}\right)$
   (b) $\left(\dfrac{7}{3}, \dfrac{7}{3}\right)$
   (c) $\left(2, \dfrac{8}{3}\right)$
   (d) $\left(\dfrac{8}{3}, 2\right)$

63. ABC is an acute angled isosceles triangle. Two equal sides AB and AC lie on the lines $7x - y - 3 = 0$ and $x + y - 5 = 0$. If θ is one of the equal angles, then what is $\cot\theta$ equal to ?
   (a) $\dfrac{1}{3}$
   (b) $\dfrac{1}{2}$
   (c) $\dfrac{2}{3}$
   (d) 2

64. In the parabola $y^2 = 8x$, the focal distance of a point P lying on it is 8 units. Which of the following statements is/are correct?
   (A) The coordinates of P can be $\left(6, 4\sqrt3\right)$
   (B) The perpendicular distance of P from the directrix of parabola is 8 units.
   Select the correct answer using the code given below :
   (a) A only
   (b) B only
   (c) Both A and B
   (d) Neither A nor B

65. What is the eccentricity of the ellipse if the angle between the straight lines joining the foci to an extremity of the minor axis is 90° ?
   (a) $\dfrac{1}{3}$
   (b) $\dfrac{1}{2}$
   (c) $\dfrac{1}{\sqrt3}$
   (d) $\dfrac{1}{\sqrt2}$

66. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$.

If $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$, then what is the value of $\alpha + \beta + \gamma$?

(a) 8   (b) 7

(c) 6   (d) 1

67. If a vector of magnitude 2 units makes an angle $\frac{\pi}{3}$ with $2\hat{i}$, $\frac{\pi}{4}$ with $3\hat{j}$ and an acute angle θ with $4\hat{k}$ then what are the components of the vector?

(a) $(1, \sqrt{2}, 1)$ (b) $(1, -\sqrt{2}, 1)$

(c) $(1, -\sqrt{2}, -1)$ (d) $(1, \sqrt{2}, -1)$

68. Consider the following in respect of moment of a force :

(A) The moment of force about a point is independent of point of application of force.

(B) The moment of a force about a line is a vector quantity.

Which of the statements given above is/are correct ?

(a) A only   (b) B only

(c) Both A and B   (d) Neither A nor B

69. For any vector $\vec{r}$, what is

$(\vec{r} \cdot \hat{i})(\vec{r} \times \hat{i}) + (\vec{r} \cdot \hat{j})(\vec{r} \times \hat{j}) + (\vec{r} \cdot \hat{k})(\vec{r} \times \hat{k})$ equal to

(a) $\vec{0}$ (b) $\vec{r}$

(c) $2\vec{r}$ (d) $3\vec{r}$

70. Let $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 4 inclined at an angle $\frac{\pi}{3}$, then what is the angle between $\vec{a}$ and $\vec{a} - \vec{b}$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

71. Let $y_1(x)$ and $y_2(x)$ be two solutions of the differential equation $\frac{dy}{dx} = x$. If $y_1(0) = 0$ and $y_2(0) = 4$, then what is the number of points of intersection of the curves $y_1(x)$ and $y_2(x)$?

(a) No point   (b) One point

(c) Two points   (d) More than two points

72. The differential equation, representing the curve $y = e^x(a \cos x + b \sin x)$ where a and b are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} + 2y = 0$ (b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

(c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (d) $\frac{d^2y}{dx^2} + y = 0$

73. If $f(x) = ax - b$ and $g(x) = cx + d$ are such that $f(g(x)) = g(f(x))$, then which one of the following holds?

(a) $f(d) = g(b)$

(b) $f(b) + g(d) = 0$

(c) $f(a) + g(c) = 2a$

(d) $f(d) + g(b) = 2d$

74. What is $\int_{-1}^{1} (3\sin x - \sin 3x)\cos^2 x\,dx$ equal to?

(a) $-\frac{1}{4}$ (b) 0

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

75. What are the order and degree respectively of the differential equation

$$\left\{2 - \left(\frac{dy}{dx}\right)^2\right\}^{0.6} = \frac{d^2y}{dx^2}?$$

(a) 2, 2   (b) 2, 3

(c) 5, 2   (d) 2, 5

76. If $\frac{dy}{dx} = 2e^x y^3$, $y(0) = \frac{1}{2}$ then what is $4y^2(2 - e^x)$ equal to?

(a) 1   (b) 2

(c) 3   (d) 4

77. Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|dx$. If $f(x) = e^{-x}$, then which one of the following is correct?

(a) $p = 2q$ (b) $p = -q$

(c) $4p = q$ (d) $p = q$

78. What is $\int_0^{\frac{\pi}{2}} \frac{a + \sin x}{2a + \sin x + \cos x}dx$ equal to?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) 1   (d) 0

79. The non-negative values of b for which the function $\frac{16x^3}{3} - 4bx^2 + x$ has neither maximum nor minimum in the range $x > 0$ is

(a) $0 < b < 1$ (b) $1 < b < 2$

(c) $b > 2$ (d) $0 \le b < 1$

80. Which one of the following is correct in respect of $f(x) = \frac{1}{\sqrt{|x| - x}}$ and $g(x) = \frac{1}{\sqrt{x - |x|}}$?

(a) $f(x)$ has some domain and $g(x)$ has no domain

(b) $f(x)$ has no domain and $g(x)$ has some domain

(c) $f(x)$ and $g(x)$ have the same domain

(d) $f(x)$ and $g(x)$ do not have any domain

Given that $\int \dfrac{3\cos x + 4\sin x}{2\cos x + 5\sin x}dx =$

$$\dfrac{\alpha x}{29} + \dfrac{\beta}{29}\ln|2\cos x + 5\sin x| + c$$

81. What is the value of α?
    (a) 7   (b) 13
    (c) 17   (d) 26

82. What is the value of β?
    (a) 7   (b) 13
    (c) 17   (d) 26

Consider the following for the next two (02) items that follow:

Let $f(x) = \dfrac{x}{\ln x}; (x > 1)$

83. Consider the following statements:
    (A) $f(x)$ is increasing in the interval (e, ∞)
    (B) $f(x)$ is decreasing in the interval $(1, e)$
    (C) $9ln7 > 7ln9$
    Which of the statements given above are correct?
    (a) (A) and (B) only   (b) (B) and (C) only
    (c) (A) and (C) only   (d) (A), (B) and (C)

84. Consider the following statements:
    (A) $f''(e) = \dfrac{1}{e}$
    (B) $f(x)$ attain local minimum value at $x = e$
    (C) A local minimum value of $f(x)$ is e
    Which of the statements given above are correct?
    (a) (A) and (B) only
    (b) (B) and (C) only
    (c) (A) and (C) only
    (d) (A), (B), and (C)

Consider the following for the next two (02) items that follow:
Let f(x) and g(x) be two functions such that

$$g(x) = x - \dfrac{1}{x} \text{ and } fog(x) = x^3 - \dfrac{1}{x^3}.$$

85. What is $g[f(x) - 3x]$ equal to ?
    (a) $x^3 - \dfrac{1}{x^3}$ (b) $x^3 + \dfrac{1}{x^3}$
    (c) $x^2 - \dfrac{1}{x^2}$ (d) $x^2 + \dfrac{1}{x^2}$

86. What is $f''(x)$ equal to?
    (a) $-\dfrac{2}{x^3}$ (b) $2x + \dfrac{2}{x^3}$
    (c) $6x + 3$ (d) $6x$

Consider the following for the next two (02) items that follow:

Let $f(x) = |x| + 1$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

Let $h(x) = \dfrac{f(x)}{g(x)}$

87. Consider the following statements:
    (A) $f(x)$ is differentiable for all $x < 0$
    (B) $g(x)$ is continuous at $x = 0.0001$
    (C) The derivative of $g(x)$ at $x = 2.5$ is 1
    Which of the statements given above are correct?
    (a) A and B only   (b) B and C only
    (c) A and C only   (d) A, B and C

88. What is $\lim\limits_{x \to 0^-} h(x) + \lim\limits_{x \to 0^+} h(x)$ equal to?
    (a) $-\dfrac{3}{2}$ (b) $-\dfrac{1}{2}$
    (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{2}$

Consider the following for the next two (02) items that follow:

Let $\varphi(a) = \int_a^{a+100\pi} |\sin x| dx$

89. What is $\varphi(a)$ equal to?
    (a) 0   (b) a
    (c) $100\,a$ (d) 200

90. What is $\varphi'(a)$ equal to?
    (a) 0   (b) π
    (c) 100   (d) 200

A differentiable function $f(x)$ has a local maximum at $x = 0$. Let $y = 2f(x) + ax - b$.

91. Which of the following is/are correct?
    (A) $f'(0) = 0$
    (B) $f''(0) < 0$
    Select the correct answer using the code given below:
    (a) A only   (b) B only
    (c) Both A and B   (d) Neither A nor B

92. The function y has a relative maxima at $x = 0$ for
    (a) $a > 0, b = 0$ (b) for all b and $a = 0$
    (c) for all $b > 0$ only   (d) for all a and $b = 0$

Let $f(x) = |x - 1|$, $g(x) = [x]$ and
$h(x) = f(x)g(x)$ where [.] is greatest integer function.

93. What is $\int_{-1}^0 h(x)dx$ equal to?
    (a) $-\dfrac{3}{2}$ (b) -1
    (c) 0   (d) $\dfrac{1}{2}$

94. What is $\int_0^2 h(x)dx$ equal to?

(a) $-\dfrac{3}{2}$

(b) -1

(c) 0

(d) $\dfrac{1}{2}$

DIRECTIONS (Qs. 95-96): *Consider the following for the next two (02) items that follow:*

Let $\int \dfrac{dx}{\sqrt{x+1}-\sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + c$

95. What is the value of α ?

(a) $\dfrac{1}{3}$

(b) $\dfrac{2}{3}$

(c) 1

(d) $\dfrac{4}{3}$

96. What is the value of β?

(a) $-\dfrac{2}{3}$

(b) $-\dfrac{1}{3}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{2}{3}$

DIRECTIONS (Qs. 97-98): *Consider the following for the next two (02) items that follow:*

The circle $x^2 + y^2 - 2x = 0$ is partitioned by line $y = x$ in two segments. Let A_1, A_2 be the areas of major and minor segments respectively.

97. What is the value of A_1?

(a) $\dfrac{\pi-2}{4}$

(b) $\dfrac{\pi+2}{4}$

(c) $\dfrac{3\pi-2}{4}$

(d) $\dfrac{3\pi+2}{4}$

98. What is the value of $\dfrac{2(A_1 + A_2)}{A_1 - 3A_2}$?

(a) π

(b) 1

(c) -1

(d) $-\pi$

DIRECTIONS (Qs. 99-100): *Consider the following for the next two (02) items that follow:*

Let $3f(x) + f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 1$

99. What is $f(x)$ equal to?

(a) $\dfrac{1}{8x} - \dfrac{x}{8} + \dfrac{1}{4}$

(b) $\dfrac{3}{8x} + \dfrac{x}{8} + \dfrac{3}{4}$

(c) $\dfrac{3}{8x} + \dfrac{x}{8} + \dfrac{1}{4}$

(d) $\dfrac{3}{8x} - \dfrac{x}{8} + \dfrac{1}{4}$

---

100. What is $8\int_1^2 f(x)dx$ equal to?

(a) $\ln(8\sqrt{e})$

(b) $\ln(4\sqrt{e})$

(c) $\ln 2$

(d) $\ln 2 - 1$

101. A bag contains 5 black and 4 white balls. A man selects two balls at random. What is the probability that both of these are of the same colour?

(a) $\dfrac{1}{6}$

(b) $\dfrac{5}{108}$

(c) $\dfrac{4}{9}$

(d) $\dfrac{5}{18}$

102. If a random variable (x) follows binomial distribution with mean 5 and variance 4, $5^{23}P(X=3) = \lambda 4^\lambda$, then what is the value of λ?

(a) 3

(b) 5

(c) 23

(d) 25

103. From data $(-4, 1), (-1, 2), (2, 7)$ and $(3, 1)$, the regression line of y on x is obtained as $y = a + bx$, then what is the value of $2a + 15b$?

(a) 6

(b) 11

(c) 17

(d) 21

104. Let $x + 2y + 1 = 0$ and $2x + 3y + 4 = 0$ are two lines of regression computed from some bivariate data. If θ is the acute angle between them, then what is the value of $488 \tan 3\theta$?

(a) 191

(b) 161

(c) 131

(d) 121

105. If two random variables X and Y are connected by relation $\dfrac{2X-3Y}{5X+4Y} = 4$ and X follows Binomial distribution with parameters $n = 10$ and $p = \dfrac{1}{2}$, then what is the variance of Y?

(a) $\dfrac{810}{361}$

(b) $\dfrac{9}{19}$

(c) $\dfrac{21}{361}$

(d) $\dfrac{121}{361}$

106. If a, b, c are in HP, then what is $\dfrac{1}{b-a} + \dfrac{1}{b-c}$ equal to?

(A) $\dfrac{2}{b}$

(B) $\dfrac{1}{a} + \dfrac{1}{c}$

(C) $\dfrac{1}{2}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$

Select the correct answer using the code given below:

(a) A only

(b) B only

(c) C only

(d) A, B and C

107. An edible oil is sold at the rates 150, 200, 250, 300 rupees per litre in four consecutive years. Assuming that an equal amount of money is spent on oil by a family in every year during these years, what is the average price of oil in rupees (approximately) per litre?
    (a) 210   (b) 220
    (c) 230   (d) 240

108. If the letters of the word "TIRUPATI" are written down at random, then what is the probability that both Ts are always consecutive?
    (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$

    (c) $\dfrac{1}{7}$ (d) $\dfrac{1}{14}$

109. Let $m = 77^n$. The index n is given a positive integral value at random. What is the probability that the value of m will have 1 in the units place?
    (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$

    (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{n}$

110. Three different numbers are selected at random from the first 15 natural numbers. What is the probability that the product of two of the numbers is equal to third number?
    (a) $\dfrac{1}{91}$ (b) $\dfrac{2}{455}$

    (c) $\dfrac{1}{65}$ (d) $\dfrac{6}{455}$

DIRECTIONS (Qs. 111-112): *Consider the following for the next two (02) items that follow:*

Let A and B be two events such that $P(A \cup B) \geq 0.75$ and $0.125 \leq P(A \cap B) \leq 0.375$.

111. What is the minimum value of $P(A) + P(B)$?
    (a) 0.625   (b) 0.750
    (c) 0.825   (d) 0.875
112. What is the maximum value of $P(A) + P(B)$?
    (a) 0.75   (b) 1.125
    (c) 1.375   (d) 1.625

DIRECTIONS (Qs. 113-114): *Consider the following for the next two (02) items that follow:*

A, B and C are three events such that P(A) = 0.6, P(B) = 0.4, P(C) = 0.5, $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$

and $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$.

113. What is the minimum value of $P(B \cap C)$?
    (a) 0.1   (b) 0.2
    (c) 0.35   (d) 0.45
114. What is the maximum value of $P(B \cap C)$?
    (a) 0.1   (b) 0.2
    (c) 0.35   (d) 0.45

DIRECTIONS (Qs. 115-116): *Consider the following for the next two (02) items that follow:*

An unbiased coin in tossed n times. The probability of getting at least one tail is p and the probability of at least two tails is q and $p - q = \dfrac{5}{32}$.

115. What is the value of n?
    (a) 4   (b) 5
    (c) 6   (d) 7
116. What is the value of $p + q$?
    (a) $\dfrac{57}{32}$ (b) $\dfrac{53}{32}$

    (c) $\dfrac{51}{32}$ (d) 1

DIRECTIONS (Qs. 117-118): *Consider the following for the next two (02) items that follow:*

x_i	1	2	3	...	n
f_i	1	2^{-1}	2^{-2}	...	$2^{-(n-1)}$

117. What is $\sum_i^n x_i f_i$ equal to?

    (a) $\dfrac{2^{n+1} - n + 2}{2^{n-1}}$ (b) $\dfrac{2^{n+1} - n - 2}{2^{n-1}}$

    (c) $\dfrac{2^{n+1} + n + 2}{2^{n-1}}$ (d) $\dfrac{2^{n+1} - n - 2}{2^n}$

118. What is the mean of the distribution?

    (a) $\dfrac{2^{n+1} - n + 2}{2^n - 1}$

    (b) $\dfrac{2^{n+1} - n - 2}{2^{n-1}}$

    (c) $\dfrac{2^{n+1} - n - 2}{2^n - 1}$

    (d) $\dfrac{2^{n+1} - n + 2}{2^n}$

DIRECTIONS (Qs. 119-120): *Consider the following for th next two (02) items that follow:*

The marks obtained by 10 students in a Statistics test are 24, 47, 18, 32, 19, 15, 21, 35, 50 and 41.

119. What is the mean deviation of the largest five observations?
    (a) 4.8   (b) 5.5
    (c) 6   (d) 7.5
120. What is the variance of the largest five observations?
    (a) 14.6
    (b) 21.8
    (c) 25.2
    (d) 46.8

PART - A: ENGLISH

ANTONYMS

DIRECTIONS: *Each item in this section consists of a sentence with an underlined word followed by four words or groups of words. Select the option that is* **opposite in meaning** *to the underlined word and mark your response on the Answer Sheet accordingly.*

1. Mohan was living in a <u>deplorable</u> condition.
   - (a) reprehensible
   - (b) opprobrious
   - (c) despicable
   - (d) commendable

2. These exercises will <u>counteract</u> the effects of stress and tension.
   - (a) exacerbate
   - (b) foil
   - (c) thwart
   - (d) contravene

3. You have to <u>persevere</u> with difficult students.
   - (a) give up
   - (b) persist
   - (c) carry on
   - (d) stand firm

4. Their new colour scheme is <u>hideous</u>.
   - (a) appalling
   - (b) beautiful
   - (c) grotesque
   - (d) macabre

5. Everywhere they go, they <u>squander</u> their earnings.
   - (a) fritter away
   - (b) splurge
   - (c) squabble
   - (d) manage

CLOZE TEST

DIRECTIONS: *In this section, there is a passage having some blank spaces, each blank space is followed by four individual words or group of words. Select a word or group of words you consider the most appropriate for filling the blank space, and indicate your response on the Answer Sheet accordingly.*

6. The Earth's climate is _____ changing and evolving. Some of
   - (a) passionately
   - (b) continually
   - (c) deadly
   - (d) thoughtfully

7. _____ changes have been due to natural causes but others can be
   - (a) these
   - (b) those
   - (c) this
   - (d) that

8. attributed to _____ activities such as deforestation,
   - (a) practical
   - (b) geological
   - (c) human
   - (d) animal

9. emissions from industry and transport, etc., which have led to the _____
   - (a) assimilation
   - (b) absolution
   - (c) collection
   - (d) accumulation

10. of gases and aerosols in _____ atmosphere. These gases are known
    - (a) an
    - (b) a
    - (c) the
    - (d) no article

11. as greenhouse gases because they trap heat and _____ air temperatures
    - (a) raise
    - (b) hike
    - (c) rise
    - (d) balance

12. near the ground, acting like a greenhouse on the surface of the _____.
    - (a) crust
    - (b) core
    - (c) mantle
    - (d) planet

13. In order to create and strengthen the scientific and analytical capacity _____ assessment
    - (a) for
    - (b) of
    - (c) in
    - (d) to

14. of climate change in the country, different studies under Climate Change Action Programme _____. Many important bilateral and multilateral
    - (a) are developed
    - (b) are initiated
    - (c) have developed
    - (d) have initiated

15. meetings and negotiations are held _____ regular intervals
    - (a) with
    - (b) at
    - (c) in
    - (d) on

SENTENCE COMPLETION

DIRECTIONS : *Each of the following items features one part of a sentence, followed by four alternatives. Complete the sentence by choosing the correct alternative and mark your response on the Answer Sheet accordingly.*

16. He paused
    - (a) with a few moments.
    - (b) for a few moments.
    - (c) by a few moments.
    - (d) into a few moments.

17. Late at night, our car
    - (a) broke down on the highway.
    - (b) broke up on the highway.
    - (c) broke into the highway.
    - (d) broke out on the highway.

18. All the countries signed the treaty
    - (a) beneath Germany and France.
    - (b) except Germany and France.
    - (c) among Germany and France.
    - (d) during Germany and France.

19. I reminded her
    - (a) to turn off the lights at night.
    - (b) to call off the lights at night.
    - (c) to drop off the lights at night.
    - (d) to break off the lights at night.

20. They told us about the woman
    - (a) whom lived next door.
    - (b) which lived next door.
    - (c) who lived next door.
    - (d) whose lived next door.

21. I'll see you
    (a) at the end of February.
    (b) on the end of February.
    (c) along the end of February.
    (d) for the end of February.
22. I dislike films
    (a) who have sad endings.
    (b) that have sad endings.
    (c) whose have sad endings.
    (d) whom have sad endings.
23. The thieves denied
    (a) having stealing the money.
    (b) stealing the money.
    (c) had stolen the money.
    (d) having to steal the money.
24. Did you
    (a) drop out to see Sunita on your way home ?
    (b) drop in to see Sunita on your way home ?
    (c) drop behind to see Sunita on your way home ?
    (d) drop away to see Sunita on your way home ?
25. Would you like to
    (a) meeting her ?   (b) meet her ?
    (c) met her ?   (d) meet with her ?

IDIOMS AND PHRASES

DIRECTIONS : *Given below are some idioms/phrases followed by four alternative meanings to each. Select the most appropriate response from the options (a), (b), (c) or (d), and mark your response on the Answer Sheet accordingly.*

26. Be bad news
    (a) feelings of hate
    (b) someone who behaves in a dishonest way
    (c) someone who is considered undesirable
    (d) to be ill
27. Back to the drawing board
    (a) to draw comparisons
    (b) to be a member of planning group
    (c) to start planning again because the previous plan has failed
    (d) to be fast at understanding a situation
28. Be in the eye of the storm
    (a) to look at something quickly
    (b) to fail miserably at a sport
    (c) to see something that you do not want to see
    (d) to be in the midst of a controversy
29. Life in the fast lane
    (a) to lead a selfless life
    (b) to have a happy life without complaints
    (c) life without money or comforts
    (d) an exciting and eventful lifestyle
30. To pass the buck
    (a) to collect money from a group of people
    (b) to shift the responsibility of something to someone
    (c) to have a short conversation with someone
    (d) to have the credentials for a particular job

ORDERING OF WORDS IN A SENTENCE

DIRECTIONS: *Each of the following items in this section consists of a sentence, the parts of which have been jumbled as P, Q, R and S. Rearrange the jumbled parts of the sentences into sequence and select the correct sequence as (a), (b), (c), and (d) given below and mark your response on the Answer Sheet accordingly.*

31. include basic freedoms   the fundamental rights are
            P   Q
     guaranteed to the individual   basic rights and
            R   S
    (a) QSPR   (b) QRSP
    (c) SQRP   (d) RQSP
32. for many millennia   been in the making
            P   Q
     of a historical process that has
            R
     the Indian nation is the product
            S
    (a) SRQP   (b) SQPR
    (c) PRSQ   (d) RSQP
33. an epoch that was imbued   for its people the start of
            P   Q
     with a new vision
            R
     India's Independence represented
            S
    (a) SQPR   (b) QRPS
    (c) SPQR   (d) RQSP
34. science and mathematics exhibition
            P
     for a discussion on the forthcoming
            Q
     to be present before the Principal
            R
     all students have been instructed
            S
    (a) SQPR   (b) RPQS
    (c) SRQP   (d) QPRS
35. two bowlers and two batsmen respectively
            P
     to the new team   with the selection of another player
            Q   R
     the team now has
            S
    (a) QPRS   (b) SQRP
    (c) RQSP   (d) RPSQ

WORD CLASSES

DIRECTIONS: *Each of the following sentences has an underlined word. Read the sentence carefully and find which word class the underlined word belongs to. Indicate your response on the Answer Sheet accordingly.*

36. You are most welcome to visit my humble <u>abode</u>.
    - (a) Noun
    - (b) Adjective
    - (c) Adverb
    - (d) Participle

37. The situation is <u>beyond</u> our control.
    - (a) Adjective
    - (b) Adverb
    - (c) Conjunction
    - (d) Preposition

38. I didn't tell him anything <u>except</u> that I needed the money.
    - (a) Adjective
    - (b) Adverb
    - (c) Preposition
    - (d) Conjunction

39. He dry-cleaned my coat <u>nicely</u>.
    - (a) Adjective
    - (b) Adverb
    - (c) Relative pronoun
    - (d) Conjunction

40. Would you like to buy <u>this</u> new item ?
    - (a) Determiner
    - (b) Conjunction
    - (c) Interjection
    - (d) Adjective

Passage

DIRECTIONS: *Read the following passage carefully and answer the items based on it. You are required to select your answers based solely on the contents of the passage and the opinion of the author.*

The grouping or assemblage of plants, animals and microbes we observe when we study a natural forest, a grassland, a pond, a coral reef or some other undisturbed area, is referred to as the area's biota or biotic community. The plant portion of the biotic community includes all vegetation, from large trees down through to microscopic algae. Likewise, the animal portion includes everything from large mammals, birds, reptiles and amphibians through to earthworms, tiny insects and mites. Microbes encompass a large array of microscopic bacteria, fungi and protozoans. Thus, the biotic community comprises a plant community, an animal community and a microbial community.

The particular kind of biotic community found in a given area is, in large part, determined by abiotic factors such as the amount of water or moisture present, the temperature, the salinity, or the type of soil in the area. These abiotic factors both support and limit the particular community. For example, a relative lack of available moisture prevents the growth of most species of plants, but supports certain species, such as cacti; these kinds of areas are deserts. Land with plenty of available moisture and a suitable temperature supports forests. The presence of water is the major factor that sustains aquatic communities.

The first step in investigating a biotic community may be simply to catalogue all the species present. Species are the different kinds of plants, animals and microbes in the community. A given species includes all those individuals which have a strong similarity in appearance to one another and which are distinct in appearance from other such groups. Each species in a biotic community is represented by a certain population — that is, by a certain number of individuals that make up the interbreeding, reproducing group.

41. Which one of the following does not belong to the biotic community ?

    - (a) Water
    - (b) Human
    - (c) Protozoa
    - (d) Bush

42. The nature of the biotic community largely depends on :
    1. Biotic components
    2. Abiotic components
    Select the correct answer using the code given below :
    - (a) 1 only
    - (b) 2 only
    - (c) Both 1 and 2
    - (d) Neither 1 nor 2

43. Which of the following is not an abiotic factor ?
    - (a) Temperature
    - (b) Humidity
    - (c) Algae
    - (d) Moisture

44. In the passage, the word 'aquatic community' refers to :
    - (a) Biotic community inhabiting the land
    - (b) Biotic community found in water
    - (c) Biotic community found in the air
    - (d) Assemblage of rain and moisture

45. Species are defined on the basis of:
    - (a) Similarities only
    - (b) Differences only
    - (c) Both similarities and differences
    - (d) None of the above

SYNONYMS

DIRECTIONS : *Each item in this section consists of a sentence with an underlined word followed by four words /group of words. Select the option that is nearest in meaning to the underlined word and mark your response on the Answer Sheet accordingly.*

46. The king's son was <u>banished</u> due to the political rivalry.
    - (a) deported
    - (b) called back
    - (c) accepted
    - (d) notified

47. He gave a <u>dazzling</u> performance at the opera.
    - (a) brilliant
    - (b) dismal
    - (c) pathetic
    - (d) puerile

48. She <u>rebuffed</u> all suggestions from her friends.
    - (a) took
    - (b) rejected
    - (c) criticized
    - (d) granted

49. He <u>disowned</u> his royal legacy and became a commoner.
    - (a) accepted
    - (b) abandoned
    - (c) allowed
    - (d) questioned

50. The public was <u>sceptical</u> about the new government policy.
    - (a) hopeful
    - (b) anxious
    - (c) doubtful
    - (d) expectant

PART- B: GENERAL KNOWLEDGE

51. An infinite combination of resistors, each having resistance $R = 4\,\Omega$, is given below. What is the net resistance between the points A and B? (Each resistance is of equal value, $R = 4$)
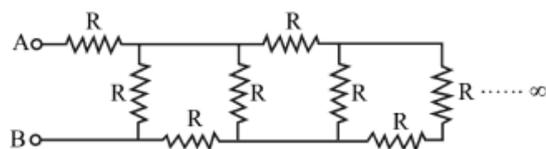
    - (a) $0\,\Omega$
    - (b) $2 + 2\sqrt{5}\,\Omega$
    - (c) $2 + \sqrt{5}\,\Omega$
    - (d) $\infty\,\Omega$

52. An electric circuit is given below. $V_1 = 1$ V and Resistance $R = 1000 \Omega$.
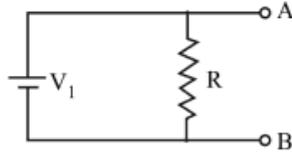
The current through the resistance R is very close to l mA and the voltage across point A and B, $V_{AB} = 1$ V. Now the circuit is changed to:
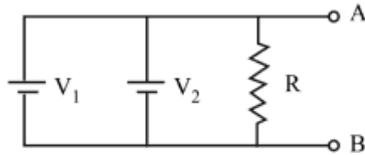
where value of $V_2 = 5$ V. The internal resistances of both the batteries are 0.1Ω. The current through the resistance R is about:

(a) 1.0 mA   (b) 1.2 mA
(c) 3.0 mA   (d) 5.0 mA

53. If the current through an electrical machine running on direct current is 15 A and the machine runs for 10 minutes, the charge that passes through the machine during this time is:

(a) 1.50 C   (b) 150 C
(c) 900 C   (d) 9000 C

54. Which one of the following is the best shape of a solid metal rod to form the top end of a lightning conductor ?
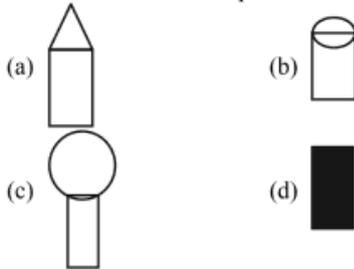
55. Which of the following forces is/are fundamental in nature?
1. Gravitational force
2. Electromagnetic forces
3. Strong and weak nuclear forces
Select the correct answer using the code given below:

(a) 1 only   (b) 1 and 2 only
(c) 2 and 3 only   (d) 1, 2 and 3

56. A spherical shell of outer radius R and inner radius R/2 contains a solid sphere of radius R/2 (see figure). The density of the material of the solid sphere is ρ and that of the shell is $\rho/2$. What is the average mass density of the larger sphere thus formed ?
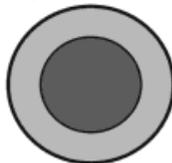
(a) $3\rho/4$ (b) $9\rho/16$
(c) $7\rho/8$ (d) $5\rho/8$

57. The human eye is like a camera that has a lens with:
(a) fixed focal length and fixed aperture size.
(b) variable focal length and fixed aperture size.
(c) fixed focal length and variable aperture size.
(d) variable focal length and variable aperture size.

58. Under Article 371A of the Constitution of India, with respect to which of the following item(s) the Acts of the Parliament of India shall **not** apply to the State of Nagaland, unless the Legislative Assembly of the State of Nagaland by a resolution so decides ?
1. Religious or social practices of Nagas
2. Naga customary law and procedure
3. Ownership and transfer of land and its resources
4. Boundaries of Nagaland with other states
Select the answer using the code given below:
(a) 1 only   (b) 2 and 3 only
(c) 1, 2 and 3 only   (d) 1, 2, 3 and 4

59. Under Article 191 of the Constitution of India, a person shall be disqualified for being chosen as, and for being, a member of the Legislative Assembly or Legislative Council of a state if the person holds any office of profit under:
1. the Government of India.
2. any State Government.
Select the correct answer using the code given below:
(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

60. Consider the following UN Declarations/Covenants:
1. Universal Declaration of Human Rights
2. Convention on the Elimination of All Forms of Discrimination Against Women
3. International Covenant of Economic, Social and Cultural Rights
4. Convention on Refugees
Which one of the following is the correct chronological order (starting with the earliest) of the above ?
(a) 1, 4, 3, 2   (b) 1, 2, 3, 4
(c) 1, 3, 2, 4   (d) 4, 1, 2, 3

61. Which one of the following statements regarding the objectives of the Second Five-Year Plan is **not** correct?
(a) Development of the basic and heavy industry sector
(b) Increase in national income to raise living standards
(c) Expansion of the consumer goods sector
(d) Expansion of employment opportunities

62. Consider the following statements about the emergence of urban centres in India from circa 6th Century BCE:
1. All of them developed away from the capitals of *mahajanpadas*.
2. Major towns were located along routes of communication.
3. Many were bustling centres of commercial, cultural and political activity.
How many of the above statements is/are correct ?
(a) 1   (b) 2
(c) 3   (d) None

63. Consider the following statements about ancient Indian inscriptions:
    1. The earliest inscriptions are in Sanskrit.
    2. Kharosthi script, used in inscriptions in the north-west, was deciphered with the help of coins of Indo-Greek kings who ruled over the area.
    3. Most of the inscriptions mention grand, unique events, and routine agricultural practices do not find mention.

    How many of the above statements is/are correct ?
    (a) 1   (b) 2
    (c) 3   (d) None

64. The relative atomic mass of boron (which exists in two isotopic forms ^{10}B and ^{11}B) is 10.81. What will be the abundance of ^{10}B and ^{11}B, respectively (consider a sample of 100 atoms) ?
    (a) 19% and 81%   (b) 81% and 19%
    (c) 38% and 62%   (d) 62% and 38%

65. Which among the following are essential constituents of Portland cement ?
    (a) Sand, lime, clay
    (b) Silica, alumina, lime
    (c) Silica, lime, graphite powder
    (d) Sand, graphene, clay

66. Litmus, a well-known acid-base indicator, is derived from:
    (a) Fungi   (b) Lichens
    (c) Bacteria   (d) Termite

67. What is the oxidation state of Vanadium in V_2O_5?
    (a) $+2$ (b) $+4$
    (c) $+3$ (d) $+5$

68. Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I	List-II
(Allotrope of Carbon)	(Property)
A. Graphite	1. Thinnest and strongest material
B. Diamond	2. Hardest natural substance
C. Fullerene	3. Very light and strong material
D. Graphene	4. Soft and slippery material

    Code:
       A B C D   A B C D
    (a) 4 2 3 1   (b) 4 3 2 1
    (c) 1 3 2 4   (d) 1 2 3 4

69. Consider the following statements regarding the International Date Line:
    1. It is roughly 180° meridian, which deviates slightly East and West to avoid land areas surrounded by the Atlantic Ocean.
    2. The date to the East of this line is (24 hours) earlier than to the West.

    Which of the statements given above is/are correct ?
    (a) 1 only   (b) 2 only
    (c) Both 1 and 2   (d) Neither 1 nor 2

70. Which of the following statements regarding 'aurora' is/are correct ?
    1. The solar wind upon reaching the Earth's atmosphere is directed towards two magnetic poles and a colourful display of lights is seen in the night sky.
    2. The particles interact with the different gases of the atmosphere and each gas glows with a particular colour.
    3. In April 2023, aurora was captured in the night sky in India by the Indian Astronomical Observatory at Hanle, Ladakh.

    Select the answer using the code given below:
    (a) 1 only   (b) 2 and 3 only
    (c) 1 and 3 only   (d) 1, 2 and 3

71. Which of the following statements regarding Barchan is/are correct?
    1. It is a crescent shaped mound of sand, which is deposited by the wind blowing constantly from one DIRECTIONS in a desert.
    2. The windward side has a convex steep slope with maximum height at the centre.
    3. Two ends of Barchan are called horns and point opposite to the DIRECTIONS the wind blows.

    Select the answer using the code given below:
    (a) 1 only   (b) 1 and 2 only
    (c) 2 and 3 only   (d) 1, 2 and 3

72. Which of the following statements regarding Regur soil is/are correct?
    1. It is a light coloured, clayey and fertile soil.
    2. It is developed on Deccan basaltic lava under hot and humid conditions.
    3. Cotton is extensively cultivated in this soil.

    Select the answer using the code given below:
    (a) 1 and 2   (b) 2 and 3
    (c) 2 only   (d) 3 only

73. Which of the following statements regarding clouds is/are correct?
    1. Clouds are classified on the basis of altitude and their form.
    2. According to altitude they are classified as High, Middle and Low clouds.
    3. Stratus, Nimbostratus and Stratocumulus are high clouds.

    Select the answer using the code given below:
    (a) 1 only   (b) 1 and 2 only
    (c) 2 and 3 only   (d) 1, 2 and 3

74. Consider the following statements regarding 'fronts':
    1. The movement of a front causes a slow change in weather in the area over which it moves.
    2. Cold fronts are associated with thunderstorms.
    3. Warm front is the boundary between an advancing mass of warm air where it is overriding and rising above a mass of colder air.

    Which of the statements given above is/are correct?
    (a) 1 only   (b) 1 and 2 only
    (c) 2 and 3 only   (d) 1, 2 and 3

75. Consider the following statements regarding soils:
   1. Soils having a very high content of sodium and calcium and pH of more than 7 – 0 are alkaline soils.
   2. Black cotton soil had developed on the Deccan basaltic lava under hot and humid conditions.
   3. Laterite soils are characterised by a deep weathered layer from which silica has been leached.

   Which of the statements given above is/are correct?
   (a) 1 only   (b) 1 and 2 only
   (c) 2 and 3 only   (d) 1, 2 and 3

76. Consider the following characteristics of a grassland:
   1. An extensive area of mid latitude grasslands are devoid of trees and shrubs.
   2. The climate is characterised by hot summers, cold winters and relatively low rainfall occurring mainly in spring and summer.
   3. In humid parts, grasses grow to a meter or more in height.

   On the basis of the above characteristics, select the correct grassland from the options given below:
   (a) Steppes   (b) Prairies
   (c) Savanna   (d) Pampas

77. Which of the following statements regarding animal cell membrane is correct ?
   (a) They are composed of phospholipids only.
   (b) They are composed of proteins only.
   (c) They are composed of phospholipids and proteins only.
   (d) They are composed of phospholipids, proteins and cholesterol (lipid).

78. Which of the following is *not* a part of compound microscope?
   (a) Mirror   (b) Stage
   (c) Clip   (d) Retina

79. Which one of the following is the correct sequence in increasing complexity?
   (a) Protein – Organism – Tissue – Organ
   (b) Protein – Organ – Tissue – Organism
   (c) Protein – Organism – Organ – Tissue
   (d) Protein – Tissue – Organ – Organism

80. Consider the following cell types:
   1. Monocyte   2. Chondrocyte
   3. Basophil   4. Lymphocyte

   How many of the above belong to animal cell types?
   (a) 1   (b) 2
   (c) 3   (d) 4

81. In the human digestive system, which one among the following is the role of the pancreas?
   (a) Secretion of surfactants to break up lipid droplets
   (b) Storage and regulated release of bile
   (c) Secretion of lipase, amylase and protease
   (d) Neutralizing stomach acids

82. Consider the following statements :
   1. DNA replication takes place when chromatin is opened up.
   2. Chromatin organises itself into rod-shaped chromosomes before cell division.

3. Both prokaryotes and eukaryotes have the same process for cell division.

Which of the statements given above is/are correct ?
(a) 1 only
(b) 1 and 2 only
(c) 1, 2 and 3
(d) 3 only

83. A uniform meter scale of mass 0.24 kg is made of steel. It is kept on two wedges, W_1 and W_2, in a horizontal position. W_1 is at a distance of 0.2 m from one of its ends, while W_2 is at distance of 0.4 m from the other end. If the force on the scale is N_1 due to W_1 and N_2 due to W_2, then : (take g = 10.0 m s^{-2})
(a) N_1 = 1.6 N and N_2 = 0.8 N
(b) N_1 = 0.8 N and N_2 = 1.6 N
(c) N_1 = 0.6 N and N_2 = 1.8N
(d) N_1 = 1.8 N and N_2 = 0.6 N

84. Escape speed from the Earth is close to 11.2 km s^{-1}. On another planet whose radius is half of the Earth's radius and whose mass density is four times that of the Earth, the escape speed in km s^{-1} will be close to :
(a) 11.2   (b) 15.8
(c) 5.6   (d) 7.9

85. Which one of the following does *not* apply to sound waves in fluids ?
(a) They transport energy
(b) They need a medium to travel
(c) They are transverse
(d) They travel faster in liquids than in gases

86. A ball of 0.1 kg mass is dropped on a hard floor from a height of 0.45 m and rises to a height of 0.20 m. If it was in touch with the floor for 0.1 s, the net force it applied on the floor while bouncing is : (take the gravitational acceleration g = 10 m s^{-2})
(a) 1.0 N   (b) 6.0 N
(c) 3.0 N   (d) 5.0 N

87. Which one of the following statements regarding simple pendulum is correct?

Simple pendulum has a time period independent of amplitude:
(a) only for small amplitudes because then the net force on its bob is independent of its displacement.
(b) for any amplitude because the net force on the bob is always proportional to its displacement.
(c) for any amplitude because the net force on the bob is independent of its displacement.
(d) only for small amplitudes because then the net force on its bob is proportional to its displacement.

88. Which one of the following about different frictional forces is correct ?
(a) Kinetic friction > Static friction > Rolling friction
(b) Static friction > Rolling friction > Kinetic friction
(c) Static friction > Kinetic friction > Rolling friction
(d) Static friction > Kinetic friction = Rolling friction

89. A metallic bob X of mass m is released from position A. It collides elastically with another identical bob Y placed at rest at position B on a horizontal frictionless table. The angle AOB is 30°.
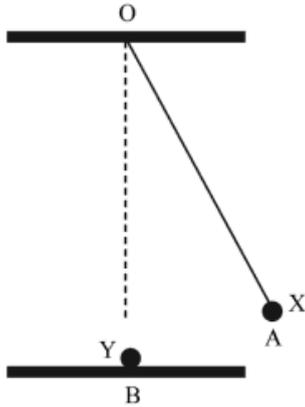
How high does the bob X rise immediately after the collision ?

(a) To the same height as that of position A on the other side in the same trajectory

(b) To half the height as that of position A on the other side along the same trajectory

(c) The same height at position A

(d) It stops at position B

90. Consider the two statements given below:

Statement-1 : Infrared waves are also called heat waves.

Statement-2 : Water molecules readily absorb infrared waves.

Select the correct answer using the code given below:

(a) Both the statements are individually true and Statement-2 is the correct explanation of Statement-1.

(b) Both the statements are individually true, but Statement-2 is *not* the correct explanation of Statement-1.

(c) Statement-1 is true, but Statement-2 is false.

(d) Statement-2 is true, but Statement-1 is false.

91. Which of the following statements are correct?

1. British 'trade surplus' with India in the nineteenth century meant that the value of British exports to India was much higher than the value of British imports from India.

2. India played a crucial role in the late-nineteenth-century world economy by helping Britain balance its deficits.

3. Britain grew opium in India and exported it to China and, therefore, for a while after the 1820s, opium became India's single largest export.

4. The nineteenth century saw export of Indian raw materials decline, and that of manufactured goods increase.

Select the answer using the code given below:

(a) 1 and 2 only   (b) 3 and 4 only

(c) 1, 2, 3 and 4   (d) 1, 2 and 3 only

92. Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I (Unit in use in early India)	List-II (Content)
A. *Muhurta*	1. Measure of weight
B. *Raktika*	2. Measure of time
C. *Angula*	3. Metre of poetry
D. *Pada*	4. Measure of length

Code:

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	4	1	2
(d)	3	1	4	2

93. First coins in Indian history bearing the names and images of rulers were issued by the:

(a) Mauryas   (b) Pushyabhutis

(c) Guptas   (d) Indo-Greeks

94. Social theorists in the nineteenth and twentieth centuries emphasised the emergence of industrialisation, urbanisation, secularisation and bureaucratisation as hallmarks of:

(a) Modernity   (b) Feudalism

(c) Kingship   (d) Medievalism

95. Consider the following events:

1. Launch of Non-Cooperation Movement

2. All-India Khilafat Conference in Delhi

3. Passing of the Rowlatt Act

4. Jallianwala Bagh incident

Which one of the following is the correct chronological order of the given events (earliest to latest) ?

(a) 1, 2, 3, 4   (b) 3, 4, 2, 1

(c) 3, 2, 4, 1   (d) 2, 3, 4, 1

96. Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I (System/ Category)	List-II (Description)
A. *Upari*	1. Peasants directly responsible for the payment of revenue
B. *Pattadar*	2. Category of tenancy tenure held under the Marathas
C. *Mirasidar*	3. Gifts of land or land revenue
D. *Inam lands*	4. Co-parcenary title holder and designated revenue payer in southern India

Code:

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	4	1	2
(d)	3	1	4	2

97. Which of the following statements about Vijayanagara king Krishnadevaraya's expeditions are correct ?
    1. In the early years of his reign he marched against the ruler of Orissa.
    2. In 1520 CE, he captured the fort of Raichur from Ismail Adil Khan.
    3. Krishnadevaraya restored Sultan Mahmud Shah to power and assumed the title of *Yavana-rajya-sthapan-acharya*.
    Select the answer using the code given below:
    (a) 1 and 3 only   (b) 2 and 3 only
    (c) 1, 2 and 3   (d) 1 and 2 only

98. Water gas is a mixture of:
    (a) CO_2 and H_2 (b) CO and H_2
    (c) CO_2 and H_2O (d) CO and H_2O

99. Human body works in the pH range of:
    (a) 6.8 – 7.2   (b) 7.0 – 7.8
    (c) 6.5 – 7.5   (d) 7.5 – 8.0

100. Which one among the following statements with reference to the properties of water is **not** correct ?
    (a) The specific heat of water is abnormally high.
    (b) Latent heat of fusion of water is very low.
    (c) Density of water is higher than ice.
    (d) Pure water is a non-conductor of electricity.

101. Which one among the following compounds has a sweet and fruity smell ?
    (a) Ethyl alcohol   (b) Acetic acid
    (c) Ethyl acetate   (d) Acetophenone

102. What is the coordination number of Na^+ and Cl^- ions in NaCl lattice ?
    (a) 6, 1   (b) 1, 6
    (c) 6, 6   (d) 5,5

103. Which one among the following is present in the nettle leaf hairs that causes burning pain ?
    (a) Methanoic acid   (b) Ethanoic acid
    (c) Benzoic acid   (d) Acetic acid

104. Which one among the following is known as Milk of Magnesia ?
    (a) Magnesium bicarbonate
    (b) Magnesium carbonate
    (c) Magnesium sulphate
    (d) Magnesium hydroxide

105. Soap with water forms:
    (a) Metallotropic liquid crystal
    (b) Thermotropic liquid crystal
    (c) Homogeneous solution
    (d) Lyotropic liquid crystal

106. Which among the following is the correct composition of Borax ?
    (a) Sodium, Boron, Magnesium and Hydrogen
    (b) Sodium, Boron, Oxygen and Hydrogen
    (c) Potassium, Boron, Oxygen and Hydrogen
    (d) Sodium, Boron, Nitrogen and Hydrogen

107. Which one among the following oxides has the highest melting point ?
    (a) Na_2O (b) MgO
    (c) Fe_2O_3 (d) CuO

108. Which of the following statements regarding neap tides is/are correct?
    1. It occurs every 14 – 15 days, which coincides with the first and third quarter of the Moon.
    2. This tide has a small tidal range because the gravitational forces of the Moon and the Sun are in quadrature.
    Select the answer using the code given below:
    (a) 1 only   (b) 2 only
    (c) Both 1 and 2   (d) Neither 1 nor 2

109. Which of the following statements regarding earthquakes is/are correct?
    1. The point on the Earth's surface directly above the focus is called the epicentre of the earthquake.
    2. Earthquakes generate Primary and Secondary waves that radiate outward from the earthquake focus.
    3. Deep-focus earthquakes are likely to cause more damage than shallow-focus earthquakes.
    Select the answer using the code given below:
    (a) 1 only   (b) 1 and 2 only
    (c) 2 and 3 only   (d) 1, 2 and 3

110. Which of the following statements regarding air temperature is/are correct?
    1. Air temperature is measured at a standard height of 1.2 m (4.0 feet) above the ground surface.
    2. The average rate of temperature decrease with height is termed as the environmental temperature lapse rate.
    Select the answer using the code given below:
    (a) 1 only   (b) 2 only
    (c) Both 1 and 2   (d) Neither 1 nor 2

111. Which of the following statements regarding humidity is/are correct?
    1. It is a ratio between the actual amount of water vapour present in the atmosphere and the maximum amount it can hold at a given temperature.
    2. When the relative humidity is high, more water evaporates from the skin.
    3. Higher the air temperature lower the relative humidity of the air.
    Select the answer using the code given below:
    (a) 1 only   (b) 2 and 3 only
    (c) 1 and 3 only   (d) 1, 2 and 3

112. Which of the following statements regarding cyclones and anti-cyclones is/are correct?
    1. In the Northern Hemisphere, cyclones rotate counter clockwise and anticyclones rotate clockwise.
    2. Cyclones are often associated with cloudy or rainy weather, whereas anticyclones are often associated with fair weather.
    3. In the Southern Hemisphere, the cyclonic spiral will be clockwise because the Coriolis force acts to the left.
    Select the answer using the code given below:
    (a) 1 only   (b) 1 and 2 only
    (c) 2 and 3 only   (d) 1, 2 and 3

113. Identify the sea-port in India on the basis of the given characteristics:
 1. It is more than a century old port on the west coast of India.
 2. It is a natural harbour protected by a breakwater and also by a mole.
 3. A deep draft channel of about 14 metres depth permits large vessels to enter the harbour.
 Select the correct sea-port from the options given below:
 (a) Mormugao Port   (b) Deendayal Port
 (c) Cochin Port   (d) Jawaharlal Nehru Port

114. Identify the Iron and Steel Plant on the basis of the given characteristics:
 1. It receives coal from Jharia and iron-ore from Sundargarh and Kendujhar.
 2. Power for the electric furnaces is procured from Hirakund.
 3. Water is obtained from Koel and Sankh rivers.
 Select the correct Iron and Steel Plant from the options given below:
 (a) Bhilai Steel Plant   (b) Durgapur Steel Plant
 (c) Rourkela Steel Plant   (d) Bokaro Steel Plant

115. Which of the following statements regarding Earth's internal structure is/are correct ?
 1. The oceanic crust is heavier than the continental crust.
 2. Most of the Earth's internal heat is contained within the mantle.
 3. Large convective cells in the crust circulate heat and drive plate-tectonic processes.
 Select the answer using the code given below:
 (a) 3 only   (b) 1 and 2 only
 (c) 2 and 3 only   (d) 1, 2 and 3

116. Which of the following statements regarding the given rivers is/are correct ?
 1. Tsangpo crosses over into India under the name of Dihang.
 2. Godavari has the second largest river basin covering about 10 percent of the area of India.
 3. Rivers Chambal and Betwa are the important tributaries of river Ganga which join river Ganga directly in Uttar Pradesh.
 Select the answer using the code given below:
 (a) 1 only   (b) 1 and 2 only
 (c) 2 and 3 only   (d) 1, 2 and 3

117. Which of the following statements regarding the World Climate Types and their relation with Inter Tropical Convergence Zone (ITCZ) is/are correct ?
 1. Tropical Wet Climate is one of the world's rainiest due to the dominance of the ITCZ over it.
 2. Tropical Savanna Climate experiences a distinctive alternation of wet and dry seasons, caused chiefly by the seasonal shift in latitude of the subtropical highs and ITCZ.
 3. The Tropical Monsoonal Climate experiences heavy rainfall as a consequence of the nearness of the ITCZ for much of the year.

Select the answer using the code given below:
 (a) 1 only   (b) 1 and 2 only
 (c) 2 and 3 only   (d) 1, 2 and 3

118. Which of the following statements regarding tropical cyclones is/are correct?
 1. These storms develop during the summer and autumn in every tropical ocean except the South Atlantic and eastern South Pacific Oceans.
 2. Tropical cyclones that occur in the North Atlantic and eastern North Pacific Oceans are known as typhoons.
 3. Tropical cyclones that occur in the Indian and South Pacific Oceans are called cyclones.
 Select the answer using the code given below:
 (a) 3 only   (b) 1 and 2 only
 (c) 1 and 3 only   (d) 1, 2 and 3

119. Identify the land biome on the basis of the given characteristics:
 1. Their climates are characterised by high rainfall and temperatures that vary from cold to mild.
 2. These forests contain primarily deciduous trees – including maple, oak, hickory and beechwood.
 3. Raccoons, opossums, bats and squirrels are found in the trees.
 Select the correct land biome from the options given below:
 (a) Tropical forest   (b) Temperate forest
 (c) Boreal forest   (d) Mediterranean forest

120. Leaves of most plants appear green because the chlorophyll present in it:
 (a) absorbs red and blue light while reflecting green light.
 (b) absorbs green light only.
 (c) absorbs green light while reflecting red and blue light.
 (d) reflects red light and absorbs blue and green light.

121. In most prokaryotes, the chromosome number is:
 (a) 4   (b) 3
 (c) 2   (d) 1

122. Bacterial DNA is referred to as naked because it is **not** associated with:
 (a) any scaffold   (b) proteins
 (c) ribozymes   (d) plasmid

123. Which one among the following cells produces antibodies against a foreign antigen ?
 (a) Lymphocytes   (b) Erythrocytes
 (c) Eosinophils   (d) Platelets

124. If the block P as shown in the figure below were to be at rest, what should the magnitude of force F be ?
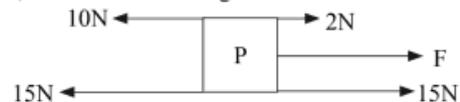
 (a) 5N   (b) 6N
 (c) 8N   (d) 10 N

125. Of the following, which does **not** belong to a nuclear reactor?
 (a) A turbine
 (b) A heat exchanger
 (c) A mechanism to reduce CO_2 emission
 (d) A reaction chamber

126. Consider the following electric circuit:
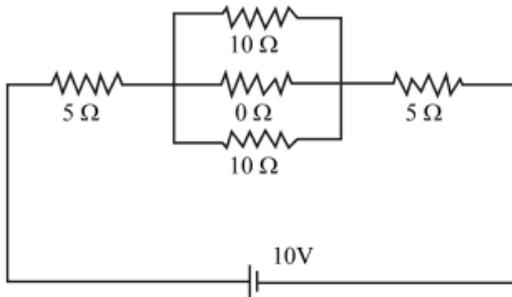
The current in the above electric circuit is
(a) 1A
(b) (10/15) A
(c) 2 A
(d) 1.5 A

127. A microscope may be a combination of:
(a) two convex lenses.
(b) a convex and a concave lens.
(c) two concave lenses.
(d) a convex lens and a convex mirror.

128. For a human eye, where u is the distance of an object from the eye, f is the focal length of the lens and v is the distance of image from the eye, which is the correct schematic graph ?
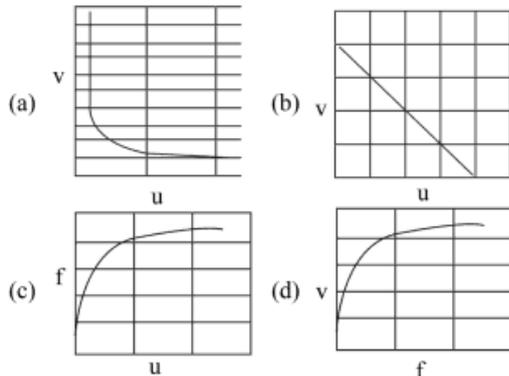
129. Which of the following statements with regard to the phenomenon of the primary rainbow formation by water droplets is/are correct?
1. It involves refraction and one internal reflection of sunlight.
2. It involves refraction of sunlight only.
3. It is formed as the inner bow.
4. It may involve more than one internal reflection as well as refraction of sunlight.
Select the answer using the code given below:
(a) 1 only
(b) 1 and 3
(c) 3 and 4
(d) 2 and 3

130. Which one of the following scientists mentioned that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy?
(a) Ernest Rutherford
(b) J.J. Thomson
(c) Niels Bohr
(d) Albert Einstein

131. LIGO experiment confirmed one of the predictions of:
(a) String theory
(b) Special theory of relativity
(c) Quantum mechanics
(d) General theory of relativity

132. A block of mass 2 kg, moving with the initial speed of 3 m/s comes to rest on a rough horizontal surface after travelling a distance of 3 m. The magnitude of the frictional force is:
(a) 9N
(b) 3N
(c) 18N
(d) 1N

133. Which one of the following is primarily responsible for conduction of current in a metal ?
(a) Bound electrons
(b) Free electrons
(c) Both bound and free electrons
(d) Ions

134. Which of the following were features of the postal system of India as described by Ibn Battuta ?
1. The foot post carries a ringing bell in one hand.
2. The horse post is called *uluq*, stationed at a distance of every four miles.
3. The foot post or *dawa* has one station at every four *kos* of distance.
4. The foot post is quicker than horse post.
Select the correct answer using the code given below:
(a) 1, 2 and 4
(b) 1 and 2 only
(c) 2 and 3 only
(d) 1, 3 and 4

135. Which one of the following texts is a commentary written by Chakrapanidatta (11 Century CE from Bengal) on the *Sushrut Samhita*?
(a) Sabdachandrika
(b) Bhanumati
(c) Nitiratnakara
(d) Lohasarvasava

136. Consider the following statements about the arrival of tobacco in India:
1. Tobacco plant arrived first in the Deccan.
2. Tobacco spread to northern India in the early years of the seventeenth century.
Which of the statements given above is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

137. Consider the following Women's Associations and their important associates in early twentieth century India:
1. Women's Indian Association of 1917 : Margaret Cousins
2. National Council of Women in India of 1925 : Sarala Devi Chaudhurani
3. Bharat Stree Mahamandal of 1910 : Lady Meherbai Tata
Which of the above pairs is/are correctly matched ?
(a) 1, 2 and 3
(b) 1 and 3 only
(c) 1 and 2 only
(d) 1 only

138. Consider the following statements about the issue of separate electorate during the early decades of the twentieth century in India as a means to secure adequate representation for the deprived social categories in the absence of universal adult franchise:
1. The All India Depressed Classes Association of M.C. Rajah was staunchly in favour of joint electorate.
2. The All India Depressed Classes Leaders' Conference demanded separate electorate.
3. The Communal Award in September 1932 recognised the right to separate electorate for the 'untouchables'.
Which of the statements given above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

139. Which of the following statements with regard to the famous 'silk routes' are correct ?
1. Silk routes refer to East-bound European silk cargoes along these routes.
2. There were several silk routes, over land and by sea, joining together vast regions of Asia with Europe and northern Africa.
3. They are known to have existed since before the Christian Era and thrived almost till the fifteenth century.
4. Besides silk, Chinese pottery, textiles and spices from India and South-east Asia travelled the same route.
Select the answer using the code given below:
(a) 1, 2 and 3 (b) 1, 3 and 4
(c) 2, 3 and 4 (d) 1, 2 and 4 .

140. Consider the following statements about Ajanta paintings:
1. The surface of the rock was coated with clay mixed with rice husk and gum and a coat of lime was applied over this.
2. The outline drawing was done first on the lime coating and colours were added subsequently.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

141. Which of the following statements with regard to the Square Kilometer Array (SKA) project is/are correct ?
1. SKA is a state of the art, mega science international facility to build the world's biggest and most sensitive radio telescope for addressing a wide variety of cutting-edge science goals.
2. The Government of India has accorded its approval for India's participation in the international mega science project.
Select the answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

142. Exercise Desert Cyclone is a joint military exercise between India and:
(a) Kuwait (b) Saudi Arabia
(c) United Arab Emirates (d) Oman

143. Which of the following statements with regard to the constitution of Finance Commission is/are correct ?
1. The Government of India has recently constituted the Sixteenth Finance Commission under the Chairmanship of Dr. Arvind Panagariya.
2. Finance Commission is constituted in pursuance to Article 263 of the Constitution of India.
Select the answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

144. Recently the Ministry of Environment, Forest and Climate Change has submitted three nominations from India for Wetland City Accreditation (WCA) under the Ramsar Convention on Wetlands. Which one among the following cities is **not** one among them ?
(a) Indore (b) Bhopal
(c) Udaipur (d) Jodhpur

145. Which of the following statements is/are correct ?
1. In India, 12th March is celebrated as the National Youth Day.
2. National Youth Day is celebrated to commemorate the birth anniversary of Swami Vivekananda.
Select the answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

146. Consider the following statements about *INS Sagardhwani:*
1. It is an oceanographic research vessel of Naval Physical and Oceanographic Laboratory (NPOL) of DRDO.
2. Recently it embarked on the Sagar Maitri Mission-4 to establish long-term scientific partnerships with Indian Ocean Rim countries in Ocean Research and Development.
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

147. Who among the following is the Chairman of the Committee constituted by the Government of India to examine the issue of holding simultaneous elections in the country and to make recommendations thereon?
(a) Union Home Minister Shri Amit Shah
(b) Chief Justice of India Shri Dhananjaya Yeshwant Chandrachud
(c) Former President of India Shri Ram Nath Kovind
(d) Chief Election Commissioner of India Shri Rajiv Kumar

148. In January 2024, India's first all-girls' Sainik School was inaugurated at:
(a) Bhopal (b) Kohima
(c) Vrindavan (d) Namsai

149. In February 2024, Government of India has decided to scrap the Free Movement Regime (FMR) between India and:
(a) Myanmar (b) Bhutan
(c) Bangladesh (d) Nepal

150. Who among the following is the author of the book Why Bharat Matters ?
(a) L.K. Advani (b) S. Jaishankar
(c) Mohan Bhagwat (d) Pranab Mukherjee

HINTS & SOLUTIONS

MATHEMATICS

1. (d) Given $|A| = \dfrac{1}{2\sqrt{2}}, |B| = \dfrac{1}{729}$

   $|2B \,(\text{adj}\,(3A))| = |2B|\,|\text{adj}\,(3A)|$
   We know that $|\text{adj}\,A| = |A|^{n-1}$ and $|kA| = k^n\,|A|$
   $= 2^3\,|B|\,|3A|^{3-1} = 8\,|B|\,(3^3|A|)^2$

   $= 8|B|\,3^6\,|A|^2 = 8 \cdot \dfrac{1}{729} \times 3^6 \left(\dfrac{1}{2\sqrt{2}}\right)^2 = 8 \times \dfrac{1}{8} = 1$

2. (b) Since that $iz^3 + z^2 - z + i = 0$ put $z = i$
   $i^4 + i^2 - i + i = 0 \Rightarrow 1 - 1 - i + i = 0$
   Satisfied then, $(|z| + 1)^2 = [|i| + 1]^2 = 4$

3. (d) Number of four digit numbers begins with 1.

   $= 3 \times 2 \times 1 = 6$ and repeatation of digit $= \dfrac{6}{3} = 2$

   Digit total for each place $= (0 + 4 + 5) \times 2 = 18$
   Total of all four-digit numbers beginning with 1
   $= 1 \times 6 \times 1000 + 18 \times 100 + 18 \times 10 + 18 = 7998$
   Number of four-digit numbers begins with 4.

   $= 3 \times 2 \times 1 = 6$ and repeatation of digit $= \dfrac{6}{3} = 2$

   Digit total for each place $= (0 + 1 + 5) \times 2 = 12$
   Total of all four-digit numbers beginning with 4.
   $= 4 \times 6 \times 1000 + 12 \times 100 + 12 \times 10 + 12 = 25332.$
   Number of four-digit numbers begins with 5

   $= 3 \times 2 \times 1 = 6$ and repetition of digit $= \dfrac{6}{3} = 2$

   Digit total for each place $= (0 + 1 + 4) \times 2 = 10$
   Total of all four-digit numbers beginning with 5
   $= 6 \times 5 \times 1000 + 10 \times 100 + 10 \times 10 + 10 = 31110$
   Required sum $= 7998 + 25332 + 31110 = 64440.$

4. (a) Considering that - x, y and z are the cube roots of unity
   Assume $x = 1, y = \omega$ and $z = \omega^2$
   So, $xy + yz + zx = \omega + \omega^3 + \omega^2$
   $= \omega + 1 + \omega^2 = 0\ (\because \omega^3 = 1)$

5. (c) Given,

   Man's   Wife

   $\boxed{\begin{matrix} M \\ 3 \end{matrix}}\ \boxed{\begin{matrix} W \\ 4 \end{matrix}}\quad \boxed{\begin{matrix} M \\ 4 \end{matrix}}\ \boxed{\begin{matrix} W \\ 3 \end{matrix}}$

   According to question,
   Case-1 0 3   3 0 $\rightarrow {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0 = 16$
   Case-2 3 0   0 3 $\rightarrow {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3 = 1$

   Case-3 1 2   2 1 $\rightarrow {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 = 324$
   Case-4 2 1   1 2 $\rightarrow {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 = \underline{114}$
                                                  485

6. (a)
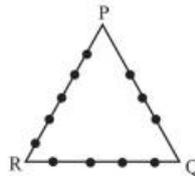
   Required number of triangles
   $= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 220 - (1 + 4 + 10) = 205$

7. (a) Given, $\log_b a = p \Rightarrow a = b^p$
   also, $\log_b c = 2p \Rightarrow c = d^{2p}$
   also, $\log_f e = 3p \Rightarrow e = f^{3p}$
   Now, $(ace)^{1/p} = (b^p\, d^{2p}\, f^{3p})^{1/p} = bd^2 f^3$

8. (c) As irrational roots occurs in pair and $-\sqrt{2}$ and $\sqrt{3}$ are
   roots of the given equation. So, $\sqrt{2}$ and $-\sqrt{3}$ are also
   roots of the given equation.
   Thus, $x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$

   $= \left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$

   $= (x^2 - 2)\,(x^2 - 3) = x^4 - 5x^2 + 6$
   On comparing $a_3 = 0, a_2 = -5, a_1 = 0$ and $a_0 = 6$

9. (c) Consider $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

   Now, $\left|\dfrac{z_1 + z_2}{z_1 - z_2}\right| = 1 \Rightarrow |z_1 + z_2| = |z_1 - z_2|$

   $\Rightarrow |(x_1 + x_2) + i(y_1 + y_2)| = |(x_1 - x_2) + i(y_1 - y_2)|$
   $\Rightarrow (x_1 + x_2)^2 + (y_1 + y_2)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
   $\Rightarrow x_1^2 + x_2^2 + 2x_1 x_2 + y_1^2 + y_2^2 + 2y_1 y_2 = x_1^2 + x_2^2 -$
   $\qquad 2x_1 x_2 + y_1^2 + y_2^2 - 2y_1 y_2$
   $\Rightarrow x_1 x_2 + y_1 y_2 = 0 \qquad \text{...(i)}$

   $\dfrac{z_1}{z_2} = \dfrac{x_1 + iy_1}{x_2 + iy_2} \times \dfrac{x_2 - iy_2}{x_2 - iy_2} = \dfrac{(x_1 x_2 + y_1 y_2) + i(y_1 x_2 - x_1 y_2)}{x_2^2 + y_2^2}$

   $= 0 + \dfrac{i(y_1 x_2 - x_1 y_2)}{x_2^2 + y_2^2} \qquad \therefore\ \text{Re}\left(\dfrac{z_1}{z_2}\right) = 0$

   Required answer $\text{Re}\left(\dfrac{z_1}{z_2}\right) + 1 = 1$

10. (b) Given, $26! = n\, 8^k = n\, 2^{3k}$
    maximum power of 2

    $= \left[\dfrac{26}{2}\right] + \left[\dfrac{26}{4}\right] + \left[\dfrac{26}{8}\right] + \left[\dfrac{26}{16}\right] = 13 + 6 + 3 + 1 = 23$

    As power of 2 is multiple of 3
    So, maximum value of $3k = 21$
    $k = 7.$

11. (b) (1) We know that adj $(AB) = (\text{adj } B) \cdot (\text{adj } A)$
Hence, statement 1 is not correct.
(2) We know that $AB \neq BA$
$\Rightarrow$ adj $(AB) \neq$ adj (BA)
Hence, statement 2 is not correct.
(3) We know that $A \cdot \text{adj } A = |A| I$
$(AB) \text{ adj } (AB) - |AB| I_n$
$= |AB| I_n - |AB| I_n = $ Null matrix
Hence, statement 3 is correct.

12. (d) As, adj $A^T = (\text{Adj } A)^T \Rightarrow A(\text{adj } A^T) = A(\text{Adj } A)^T$
Statement 1 is correct
We know that, If $A^n = A$ then A is identify matrix.
$\therefore$ Statements 2 and 3 are correct
Hence, all statement are correct.

13. (b) All even digit are 0, 2, 4, 6, 8
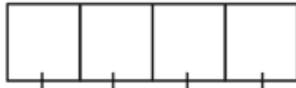
Choice$\rightarrow$ 4   5   5   5
$\therefore$ Total required numbers $= 4 \times 5 \times 5 \times 5 = 500$

14. (b) Given, $(z - 100)^3 + 1000 = 0 \Rightarrow (z - 1)^3 = (-10)^3$
$\therefore$ $z - 100 = -10$ $(\omega, \omega^2, 1)$ $\therefore z - 100 = -10 \Rightarrow z = 90$
$z - 100 = -10w \Rightarrow z = 100 - 10\omega$
$z - 100 = -10w^2 \Rightarrow z = 100 - 10\omega^2$

15. (d) $(1 + i)^4 + (1 - i)^4 = [(1 + i)^2]^2 + [(1 - i)^2]^2$
$= (1 - 1 + 2i)^2 + (1 - 1 - 2i)^2 = (2i)^2 + (-2i)^2$
$= 4i^2 + 4i^2 = -4 - 4 = -8$

16. (a) All the diagonal elements of skew-symmetric matrix are zero.
Hence, statements 1 and 2 are correct we know that if $A A^T = I$ then A is called orthogonal matrix.
But $A A^T = A(-A) = -A^2$ $[A^T = -A]$
Hence, Statement 3 is not correct.

17. (d) We know that a number is divisible by 4 if its last 2 digits is divisible by 4.
Total number divisible by 4
$= 0$ 0   0   2 $1 \times 2 \times 3 \times 1 = 1 \times 2 \times 3 \times 1 = 6$

18. (a) $2^{120} = (2^3)^{40} = 8^{40} = (1 + 7)^{40}$
$= 1 + {}^{40}C_1 7 + {}^{40}C_2 7^2 + \ldots\ldots {}^{40}C_{40} 7^{40}$
$= 1 + 7 [{}^{40}C_1 + {}^{40}C_2 7 + \ldots\ldots + {}^{40}C_{40} 7^{39}]$
$\Rightarrow$ Remainder $= 1$ [using division algorithm]

19. (c) Given,
$$\begin{vmatrix} C(9,4) & C(9,3) & C(10, n-2) \\ C(11,6) & C(11,5) & C(12, n) \\ C(m,7) & C(m,6) & C(m+1, n+1) \end{vmatrix} = 0$$
Applying $C_3 \rightarrow C_1 + C_2 - C_3$
$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,4) - C(10, n-2) \\ C(11,6) & C(11,5) & C(12,6) - (12, n) \\ C(m,7) & C(m,6) & C(m+1, 7) - C(m+1, n+1) \end{vmatrix} = 0$$
Since determinant value is zero. $\therefore C_3 = 0$
So, $n - 2 = 4 \Rightarrow n = 6$

20. (b) Consider
$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix}$$
Expanding along C_1
$= \cos C (-\sin B \cdot \tan (B + C)) - \tan A (\sin B \cos C)$
$= -\sin B . \cos C \left[\dfrac{\sin(B+C)}{\cos(B+C)} + \dfrac{\sin A}{\cos A} \right]$
$= -\sin B . \cos C \left[\dfrac{\sin(B+C).\cos A + \sin A \cos(B+C)}{\cos A . \cos(B+C)} \right]$
$= -\sin B \cos C \dfrac{\sin(A+B+C)}{\cos A . \cos(B+C)}$
$= \dfrac{-\sin B \cos C \sin(\pi)}{\cos A . \cos(B+C)} = 0$

21. (d) Possible order of matrices with 4 entries
$1 \times 4, 2 \times 2, 4 \times 1$
$\therefore$ Total number of matrices $= 4 \times 4 \times 4 \times 4 \times 3 = 768$

22. (a)
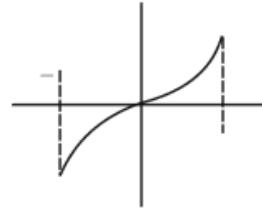
$f(x) = x|x|$
is one-one and onto
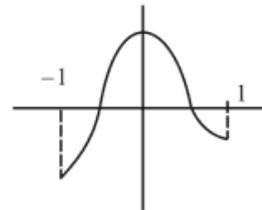
$g(x) = \cos(\pi x)$ is not one-one as any horizontal line passes $g(x)$ twice.

23. (d) Given $xRy \Rightarrow |x + y| < 2$ in $(-1, 1)$
for reflexive
$|x + x| < 2 \Rightarrow |x| < 1$ true $\Rightarrow R$ is reflexive
for symmetric
Let $xRy \Rightarrow |x + y| < 2$
$\Rightarrow$ $|y + x| < 2 \Rightarrow yRx$ $\Rightarrow R$ is symmetric
for transitive $\because -1 < x < 1$
Let $|x + y| < 2$ and $|y + 2| < 2$
then $|x + z| < 2$
$\Rightarrow$ R is transitive.

24. (a) $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B\}$
From the venn diagram
$(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$
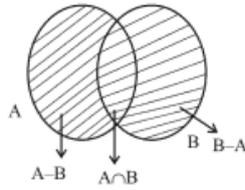
$\Rightarrow \quad (A \cup B) - (A \cup B) = \phi$ (Null set)

25. (a)
$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$

Expanding along R_1
$= a^2(1 - \cos^2 A) - b\sin A(b\sin A - c\cos A\sin A) + c\sin A$
$(b\sin A \cdot \cos A - c\sin A)$
$= a^2\sin^2 A - b^2\sin^2 A + bc\sin^2 A \cdot \cos A + bc\sin^2 A \cdot \cos A$
$- c^2\sin^2 A$
$= a^2\sin^2 A - b^2\sin^2 A - c^2\sin^2 A + 2bc\sin^2 A \cdot \cos A$
$= \sin^2 A(a^2 - b^2 - c^2 + 2bc\cos A) = \sin^2 A \times 0 = 0$

26. (a) Given, a, b, c are in AP $\Rightarrow 2b = a + c$ ...(i)
also, b, c, d are in G.P $\Rightarrow c^2 = bd$ ...(ii)

also, c, d, e are in H.P $\Rightarrow \dfrac{2}{d} = \dfrac{1}{c} + \dfrac{1}{e}$ ... (iii)
from (i) and (ii)

$2c^2 = (a + c)d \Rightarrow \dfrac{2}{d} = \dfrac{a + c}{c^2} \Rightarrow \dfrac{1}{c} + \dfrac{1}{e} = \dfrac{a}{c^2} + \dfrac{1}{c}$ from (iii)

$\dfrac{1}{e} = \dfrac{a}{c^2} \Rightarrow c^2 = ae$

So, a, c, e are in G.P.

27. (b) Given, $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \quad \dfrac{1}{2}\log_2(x - 1) = \log_2(x - 3)$

$\Rightarrow \quad \log_2(x - 1) = \log_2(x - 3)^2$

$\Rightarrow \quad x - 1 = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow \quad x^2 - 5x - 2x + 10 = 0 \Rightarrow (x - 5)(x - 2) = 0$

$x = 2, 5$, but $x - 3 > 0 \Rightarrow x > 3$

$\Rightarrow \quad x = 5$ only solution

28. (d) Given, $\log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right) = k$

$\Rightarrow \quad \log_x x - \log_x y + \log_y y - \log_y x = k$

$= 1 - \log_x y + 1 - \dfrac{1}{\log_x y} = k \ [\because \log_a a = 1]$

Let $\log_x y = t$

$\therefore \quad 2 - t - \dfrac{1}{t} = k \Rightarrow 2t - t^2 - 1 = kt \Rightarrow t^2 + (k - 2)t + 1 = 0$

For solution $D = b^2 - 4ac \geq 0$

$(k - 2)^2 - 4 \geq 0 \Rightarrow (k - 2)^2 \geq 4 \Rightarrow k - 2 \geq 2$ or $k - 2 \leq -2$

$k \geq 4$ or $k \leq 0 \Rightarrow k \neq 1$

29. (c) Given

$$A = \begin{vmatrix} \sin 2\theta & -\cos 2\theta & 0 \\ \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

We know that if $|A| = \pm 1$ then A is orthogonal matrix
So $A^{-1} = A^T$ and $A^{-1} = $ adj A

30. (a) $\left(1 - x^2\right)^{20}\left(-\left(\dfrac{1}{x^2} + x^2 - 2\right)\right)^{-5}$

$= -\left(1 - x^2\right)^{20}\left(x - \dfrac{1}{x}\right)^{-10}$

$= -(1 - x^2)^{20}(x^2 - 1)^{-10} \cdot x^{10} = -(1 - x^2)^{10} \cdot x^{10}$

General term $T_{r+1} = -{}^{10}c_r(1)^{10-r}(-x^2)^{10-r} \cdot x^{10}$

$= (-1)^{11-r} \cdot {}^{10}c_r x^{30-2r}$

$\because \quad 30 - 2r = 10 \Rightarrow r = 10$

So, coefficient $x^{10} = (-1){}^{10}C_{10} = -1$.

31. (b) Given,

$T_4 = T_{3+1} = {}^nC_3(mx)^{n-3}\left(\dfrac{1}{x}\right)^3 = \dfrac{5}{2}$

${}^nC_3(m)^{n-3}x^{n-6} = \dfrac{5}{2}x^0$

On comparing $n - 6 = 0 \Rightarrow n = 6$

and ${}^6C_3 m^3 = \dfrac{5}{2}$

$\Rightarrow \quad \dfrac{6.5.4}{3.2}m^3 = \dfrac{5}{2} \Rightarrow m^3 = \dfrac{1}{8} \Rightarrow m = \dfrac{1}{2}$

$\therefore \quad mn = 6 \times \dfrac{1}{2} = 3$

32. (d) Since a, b, c are in G.P $\Rightarrow b^2 = ac$

Now, for equal $ax^2 + bx + c = 0$

$D = b^2 - 4ac = ac - 4ac = -3ac < 0$

$\therefore$ Roots are imaginary

Let $a = 2, b = 4, c = 8$

$\therefore \quad x^2 + 2x + 4 = 0 \Rightarrow x = \dfrac{-2 \pm \sqrt{4 - 16}}{2} = -1 \pm \sqrt{3}$

Ratio of roots $= \dfrac{-1 + \sqrt{3}}{-1 - \sqrt{3}} = \dfrac{\omega}{\omega^2} = \dfrac{1}{\omega}$

Product of roots $= \dfrac{\left(-1 + \sqrt{3}\right)}{2}\dfrac{\left(-1 - \sqrt{3}\right)}{2} \times 4 = \omega.\omega^2.4$

$= 4 = \dfrac{b^2}{a^2}$

33. (c) $\therefore f(x) = x^2 + mx + n \in I, \forall x \in I$

$f(0) = 0 + 0 + n \in I \Rightarrow x \in I$

$f(1) = 1 + m + n \in I \Rightarrow m + n \in I$

$\Rightarrow m \in I \quad (\because n \in I)$

So, both m and n are integer.

34. (a) Given,

$(x + y)^{2n+1} \cdot (x - y)^{2n+1} = (x^2 - y^2)^{2x+1}$

Middle term $\dfrac{2n + 2}{2}, \dfrac{2n + 4}{2} = (x + 1)^{th}$ term, $(n + 2)^{th}$ term

$T_{n+1} = {}^{2n+1}C_n \left(x^2\right)^{n+1} \left(y^2\right)^n$

$T_{n+2} = {}^{2n+1}C_{n+1} \left(x^2\right)^n \left(y^2\right)^{x+1}$

${}^{2n+1}C_n \left(x^2\right)^{n+1} \left(y^2\right)^n = {}^{2n+1}C_{n+1} \left(x^2\right)^n \left(y^2\right)^{n+1}$

$\dfrac{{}^{2n+1}C_n}{{}^{2n+1}C_{n+1}} = \dfrac{x^{2n} y^{2n+2}}{x^{2n+2} y^{2n}} \Rightarrow \dfrac{(2n+1)!}{(n+1)!n!} \cdot \dfrac{n!(n+1)!}{(2n+1)!} = \dfrac{y^2}{x^2}$

$\Rightarrow \dfrac{y^2}{x^2} = 1$

35. (c) Given, n (A) = 5 and n (B) = 2
Number of onto function from A to B $= 2^5 - {}^2C_1 (2-1)^5$
$= 32 - 2 = 30.$

36. (d)

$\dfrac{\sqrt{3}\cos 10° - \sin 10°}{\sin 25° . \cos 25°} = \dfrac{2\left(\dfrac{\sqrt{3}}{2}.\cos 10° - \dfrac{1}{2}\sin 10°\right)}{\dfrac{1}{2}\left(2\sin 25°.\cos 25°\right)}$

$= \dfrac{4.\sin\left(60° - 10°\right)}{\sin 50°} = \dfrac{4\sin 50°}{\sin 50°} = 4$

37. (c) $\sin 9° - \cos 9° = \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\sin 9° - \dfrac{1}{\sqrt{2}}\cos 9°\right)$

$= \sqrt{2}\sin\left(45° - 9°\right) = \sqrt{2}\sin 36°$

$= \sqrt{2}\sqrt{1 - \cos^2 36°} \qquad \left[\because \cos 36° = \dfrac{\sqrt{5}+1}{4}\right]$

$= \sqrt{2}\sqrt{1 - \left(\dfrac{\sqrt{5}+1}{4}\right)2}$

$= \sqrt{2}.\dfrac{\sqrt{10 - 2\sqrt{5}}}{4} = \dfrac{\sqrt{5 - \sqrt{5}}}{2}$

38. (d) Given
$\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A . \sin B . \sin C$
$\Rightarrow \sin A + \sin B + \sin C = 0$
$\Rightarrow a k + k b + k c = 0 \Rightarrow a + b + c = 0$

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ b+c+a & c & a \\ c+a+b & a & b \end{vmatrix} \left(\text{Applying } C_1 \to C_1 + C_2 + C_3\right)$

$= \begin{vmatrix} 0 & b & c \\ 0 & c & a \\ 0 & a & b \end{vmatrix} = 0$

39. (c) Given $\cos^{-1}x = \sin^{-1}x \Rightarrow \dfrac{\pi}{2} - \sin^{-1}x = \sin^{-1}x$

$\Rightarrow 2\sin^{-1}x = \dfrac{\pi}{2} \Rightarrow \sin^{-1}x = \dfrac{\pi}{4}$

$\Rightarrow x = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

40. (b) Given, $(\sin\theta - \cos\theta)^2 = 2$
$\Rightarrow \sin^2 + \cos^2\theta - 2\sin\theta.\cos\theta = 2$
$\Rightarrow \sin 2\theta = -1$
$\Rightarrow$ Two solution possible in $(-\pi, \pi)$

41. (c) $\dfrac{\cos A + \cos B}{\cos\left(\dfrac{A-B}{2}\right)} = \dfrac{2\cos\dfrac{A+B}{2}.\cos\left(\dfrac{A-B}{2}\right)}{\cos\left(\dfrac{A-B}{2}\right)}$

$= 2\cos\left(\dfrac{\pi - C}{2}\right) = 2\cos\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right) = 2\sin\dfrac{C}{2}$

$= 2\sin 30° = 2 \times \dfrac{1}{2} = 1$

42. (c) $\sqrt{15 + \cot^2\left(\dfrac{\pi}{4} - 2\cot^{-1}3\right)}$

Consider, $2\cot^{-1}3 = 2\tan^{-1}\dfrac{1}{3} = \tan\dfrac{\dfrac{2}{3}}{1 - \dfrac{1}{9}}$

$= \tan^{-1}\left(\dfrac{2}{3} \times \dfrac{9}{8}\right) = \tan^{-1}\dfrac{3}{4}$

So, $\sqrt{15 + \cot^2\left(\tan^{-1}1 - \tan^{-1}\dfrac{3}{4}\right)}$

$= \sqrt{15 + \cot^2\left(\tan^{-1}\left(\dfrac{1 - \dfrac{3}{4}}{1 + \dfrac{3}{4}}\right)\right)}$

$= \sqrt{15 + \cot^2\left(\tan^{-1}\dfrac{1}{7}\right)} = \sqrt{15 + \cot^2\left(\cot^{-1}7\right)}$

$= \sqrt{15 + 49} = 8$

43. (b) $\sin 10° . \sin 50° + \sin 50° . \sin 250° + \sin 250 . \sin 10°$

$= \dfrac{1}{2}\left(\cos 40° - \cos 60° + \cos 200° - \cos 300° + \cos 240°\right.$

$\left. - \cos 260°\right)$

$= \dfrac{1}{2}\left[\cos 40° - \dfrac{1}{2} + \cos(180 + 20)° - \cos(360 - 60)° + \right.$

$\left. \cos(180 + 60)° - \cos(180 + 80)°\right]$

$= \dfrac{1}{2}\left[\cos 40° - \dfrac{1}{2} - \cos 20° - \cos 60° - \cos 60° + \cos 80°\right]$

$$= \frac{1}{2}\left[\cos 40° - \cos 20° + \cos 80° - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[-2\sin 30°.\sin 10° + \cos(90° - 10°) - \frac{3}{2}\right]$$

$$= \frac{1}{2}\left[-\sin 10° + \sin 10° - \frac{3}{2}\right] = -\frac{3}{4}$$

44. **(b)** $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$

$$= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left|\frac{\frac{a}{b}-1}{1+\frac{a}{b}.1}\right|$$

$$= \tan^{-1}\frac{a}{b} - \tan^{-1}\frac{a}{b} + \tan^{-1}1 = \tan^{-1}(1) = \frac{\pi}{4}$$

45. **(d)** For real roots, discriminant ≥ 0
$\Rightarrow$ $(\cos B)^2 - 4\sin B(\cos B - 1) \geq 0$
$(\cos B)^2 + 4\sin B(1 - \cos B) \geq 0$
As $-1 \leq \cos B \leq 1$ and $\sin B \geq 0$ for B $[-0, \pi]$
So, it is only possible when $1 - \cos B \geq 0$

46. **(c)** $\cos 2A + \cos 2B + \cos 2C$ $[\because \cos 2x = 1 - 2\sin^2 x]$
$= 1 - 2\sin^2 A + 1 - 2\sin^2 B + 1 - 2\sin^2 C$
$= 3 - 2(\sin^2 A + \sin^2 B + \sin^2 C)$

$$= 3 - 2\left[\left(\frac{16}{65}\right)^2 + \left(\frac{63}{65}\right)^2 + 0\right]$$ [since, $16^2 + 63^2 = 65^2$

$\Rightarrow [\angle B = 90°] = 3 - 2 = 1.$

47. **(b)** Given $\alpha + \beta = \frac{5\pi}{4}$ $\tan(\alpha + \beta) = \tan\frac{5\pi}{4}$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = 1$$

$\tan\alpha + \tan\beta + \tan\alpha.\tan\alpha = 1$

Also, $f(\theta) = \frac{1}{1+\tan\theta}$

so, $f(\alpha).f(\beta) = \frac{1}{1+\tan\alpha} \times \frac{1}{1+\tan\beta}$

$$= \frac{1}{1+\tan\alpha + \tan\beta + \tan\alpha.\tan\beta} = \frac{1}{1+1} = \frac{1}{2}$$

48. **(b)** $\tan\alpha + \tan\beta = 6$
$\tan\alpha, \tan\beta = 8$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \frac{6}{1-8} = \frac{6}{-7}$$

$\cos(2\alpha + 2\beta) = \cos 2(\alpha + \beta)$

$$= \frac{1-\tan^2(\alpha+\beta)}{1+\tan^2(\alpha+\beta)} = \frac{1-\frac{36}{49}}{1+\frac{36}{49}} = \frac{13}{85}$$

49. **(c)** $\tan(90 - 65)° + 2 - 2\tan 40° - \tan 25°$
$= \cot 25° - \tan 25° - 2\tan 40° + 2$

$$= \frac{\cos^2 25 - \sin^2 25}{\sin 25.\cos 25} - 2\tan 40° + 2$$

$$= \frac{2\cos 50°}{\sin 50°} - 2\tan 40° + 2$$

$= 2\cot 50° - 2\tan 40° + 2 = 2\tan 40° - 2\tan 40° + 2 = 2$

50. **(a)** 1. $\cot A \cot B \cot C > 0$
$\therefore$ $\cot A > 0, \cot B > 0, \cot C > 0$

$$\therefore \quad 0 < A < \frac{\pi}{2}, 0 < B < \frac{\pi}{2}, 0 < C < \frac{\pi}{2}$$

$\therefore$ $\triangle ABC$ is an acute angled triangle.
Hence, statement 1 is correct.
2. $\tan A \tan B \tan C > 0$
If $\triangle ABC$ is obtuse angled triangle then two of tan A, tan B, tan C < 0
$\Rightarrow$ Two of angles are obtuse but it in not possible
Hence, statement 2 is not correct.

51. **(a)** $x^2 + y^2 + 2x + 6y + 1 = 0$
To obtain the center (a,b) and the radius c of a circle equation, rewrite it as $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the circle's center and r is its radius.
Consider, $x^2 + y^2 + 2x + 6y + 1 = 0$
To obtain the equation in the desired form use completing the square
$(x^2 + 2x + 1) - 1 + (y^2 + 6y + 9) - 9 + 1 = 0$
$\Rightarrow$ $(x+1)^2 + (y+3)^2 = 9$
$\Rightarrow$ $(x-(-1))^2 + (y-(-3))^2 = (3)^2$
Center of the circle is $(-1, -3)$ and radius is 3.
$\therefore$ $a = -1, b = -3$ and $c = 3$
$\therefore$ $a^2 + b^2 + c^2 = (-1)^2 + (-3)^2 + (3)^2 = 1 + 9 + 9 = 19.$

52. **(a)** Given: Equation of sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$ with AB as its diameter and $A \equiv (1, -1, 2)$ and $B \equiv (2, 1, -1)$ So, C will be midpoint of AB.

A (1, −1, 2) ——— C ——— B (2, 1, −1)
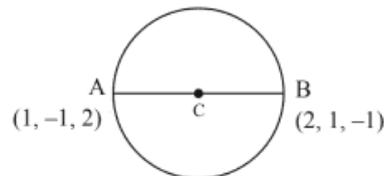
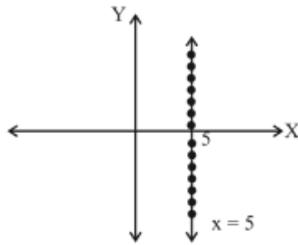$$C = \left(\frac{1+2}{2}, \frac{1-1}{2}, \frac{-1+2}{2}\right) = \left(\frac{3}{2}, 0, \frac{1}{2}\right)$$

Since centre of the sphere is $(-u, -v, -w)$
On comparing we get

$$u = \frac{-3}{2}, v = 0, w = \frac{-1}{2}$$

Hence, $u + v + w = \frac{-3}{2} + 0 - \frac{1}{2} = \frac{-4}{2} = -2$

53. (d)

x = 5 represents a line parallel to y-axis so there are infinite points on x y – plane

54. (b) Equation of plane is $ax + by + cz = d$

Given the plane equation is $2x - 3y + 6z + 4 = 0$

On comparing we get $a = 2, b = -3, c = 6, d = -4$

Here we need to find the direction cosine of a normal to the plane i.e $< l, m, n >$

Direction cosines of normal to plane

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{49}} = \frac{2}{7}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{\sqrt{49}} = \frac{-3}{7}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{\sqrt{49}} = \frac{6}{7}$$

$$\therefore \quad <l, m, n> = <\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}>$$

$$\therefore \quad 49(7l^2 + m^2 - n^2) = 49\left(7\left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2 - \left(\frac{6}{7}\right)^2\right)$$

$$= 49\left(\frac{28}{49} + \frac{9}{49} - \frac{36}{49}\right) = 1$$

55. (c)
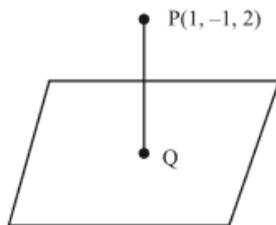
Assume the line from (1, –1, 2) meet the plane at Q

Direction ratios of the line from the point (1, –1, 2) to the given plane is <3, 2, 2>

So the equation of the line passing through P and with direction ratios will be:

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{2} = \lambda$$

$x = 3\lambda + 1, y = 2\lambda - 1, z = 2\lambda + 2$

Now, since Q lies on the plane so it must satisfy the equation of the plane.

i.e $x + 2y + 3z = 18$ $\therefore 3\lambda + 1 + 4\lambda - 2 + 6\lambda + 6 = 18$

$13\lambda + 5 = 18 \Rightarrow \lambda = 1$

Therefore, coordinates of Q are

$(3 + 1, 2 - 1, 2 + 2) = (4, 1, 4)$

56. (d) Let Plane $ax + by + cz + d = 0$ is passing through (1, 0, 0) (0, 1, 0) & (0, 0, 1)

We will find the equation of plane

Hence $a + b = 0, b + d = 0, c + d = 0$

Solving above three equations, we get $a = b = c = -d$

Now Let $a = b = c = -d = \lambda$

So, equation of plane will be $x + y + z - 1 = 0$

Perpendicular distance from origin to the plane

$$(p) = \frac{|-1|}{\sqrt{(1)^2 + (1)^2 + (1)^2}} = \frac{1}{\sqrt{3}} \text{ unit}$$

Hence, $3p^2 = 3 \times \frac{1}{3} = 1$

57. (b) Given $l + 2m + n = 0$ ...(i)

and $2l - 2m + 3n = 0$ ...(ii)

From (i)

$l = -2m - n$ ...(iii)

Substitute (iii) in (ii)

$2(-2m - n) - 2m + 3n = 0$

$-4m - 2n - 2m + 3n = 0 \Rightarrow -6m + n = 0 \Rightarrow n = 6m$

From (iii) we get, $l = -2m - n = -2m - 6m = -8m$

$$\frac{l}{-8m} = \frac{m}{m} = \frac{n}{6m}$$

$$\therefore \quad \frac{l}{-8} = \frac{m}{1} = \frac{n}{6} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{(-8)^2 + (1)^2 + (6)^2}} = \frac{1}{\sqrt{101}}$$

$$\therefore \quad l = -\frac{8}{\sqrt{101}}, n = \frac{1}{\sqrt{101}}, n = \frac{6}{\sqrt{101}}$$

Hence, $l^2 + m^2 - n^2 = \frac{64}{101} + \frac{1}{101} - \frac{36}{101} = \frac{29}{101}$

58. (b)

59. (b) Rewrite the equation as follows:

$y \sin\theta = 9 - x \cos\theta$

$$\Rightarrow \quad y = \frac{9}{\sin\theta} - x\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad y = -x\frac{\cos\theta}{\sin\theta} + \frac{9}{\sin\theta} \quad\quad ...(i)$$

∵ The general equation of line is.

$y = mx + c$ ...(ii)

On comparing (i) and (ii), we get

$$m = -\frac{\cos\theta}{\sin\theta}$$

Since, the slope of perpendicular line are negative inverse of each other.

So, the slope m_1 of the required line can be

$$m_1 = -\left(\frac{1}{-\frac{\cos\theta}{\sin\theta}}\right) \Rightarrow m_1 = \frac{\sin\theta}{\cos\theta}$$

Also, the line passes through the point $(-\sin\theta, \cos\theta)$
So, the equation of the required line is

$$y - \cos\theta = \frac{\sin\theta}{\cos\theta}(x + \sin\theta)$$

$\Rightarrow \quad \cos\theta\, y - \cos^2\theta = x\sin\theta + \sin^2\theta)$
$\Rightarrow \quad x\sin\theta - y\cos\theta + \sin^2\theta + \cos^2\theta = 0$
$\Rightarrow \quad x\sin\theta - y\cos\theta + 1 = 0$

60. (a) Given that P and Q lie on line $y = 2x + 3$
For P put $x = a$ then $y = 2a + 3$
For Q put $x = b$ then $y = 2b + 3$
Coordinates $P = (a, 2a + 3)$ and $Q = (b, 2b + 3)$
From $R = (1, 5)$ using distance formula, we have

$$PR = \sqrt{(a-1)^2 + (2a+3-5)^2} = 2 \quad \text{...(i)}$$

$$QR = \sqrt{(b-1)^2 + (2b+3-5)^2} = 2 \quad \text{...(ii)}$$

From (i) we get
$(a-1)^2 + (2a-2)^2 = 4$
$a^2 - 2a + 1 + 4a^2 + 4 - 8a = 4$
$5a^2 - 10a + 1 = 0$
From (ii), we get
$5b^2 - 10b + 1 = 0$
By using quadratic formula

$$(a, b) = \frac{10 \pm \sqrt{100 - 80}}{10} \Rightarrow a = 1 \pm \frac{2}{\sqrt{5}}$$

If $a = 1 + \dfrac{2}{\sqrt{5}}$ then

Coordinate of $P = (a, 2a + 3) = \left(1 + \dfrac{2}{\sqrt{5}}, 2 + \dfrac{4}{\sqrt{5}} + 3\right)$

$$= \left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

If $a = 1 - \dfrac{2}{\sqrt{5}}$ then

Coordinate of $P = (a, 2a + 3) = \left(1 - \dfrac{2}{\sqrt{5}}, 2 - \dfrac{4}{\sqrt{5}} + 3\right)$

$$= \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

Coordinate of $Q = \left(1 - \dfrac{2}{\sqrt{5}}, 5 - \dfrac{4}{\sqrt{5}}\right)$ or

$$\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

Required coordinates of the point P and Q are

$$\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

61. (a) Given that two side of a square lie on the lines
$2x + y - 3 = 0$ ...(i)
$4x + 2y + 5 = 0$ ...(ii)

Divide (ii) by 2 we get, $2x + y + \dfrac{5}{2} = 0$ ...(iii)

Here $2x + y - 3 = 0$ and $2x + y + \dfrac{5}{2} = 0$ are parallel lines

So, length of the side of the square = Distance between

parallel side $= \dfrac{|C_1 - C_2|}{\sqrt{a_1^2 + b_1^2}}$

where $C_1 = -3$ and $C_2 = \dfrac{5}{2}$

$(a_1, b_1) = (2, 1)$

$$\Rightarrow \frac{\left|-3 - \dfrac{5}{2}\right|}{\sqrt{(2)^2 + (1)^2}} = \frac{\left|\dfrac{11}{2}\right|}{\sqrt{5}} = \frac{11}{2\sqrt{5}}$$

So, length of the side of square $= \dfrac{11}{2\sqrt{5}}$ units

Required area of the square $= (\text{side})^2 = \left(\dfrac{11}{2\sqrt{5}}\right)^2$

$= 6.05$ square units.

62. (b) Given, $A \equiv (3, 5)$ and mid-points of sides AB and AC are $(-1, 2)$ and $(6, 4)$ respectively
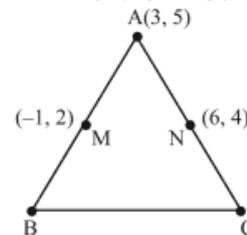
Consider $B = (x_1, y_1)$ and $C = (x_2, y_2)$
By using mid point formula

$$M = (-1, 2) = \left(\frac{x_1 + 3}{2}, \frac{y_1 + 5}{2}\right)$$

$$N = (6, 4) = \left(\frac{x_2 + 3}{2}, \frac{y_2 + 5}{2}\right)$$

By comparing we get

$\dfrac{x_1 + 3}{2} = -1, \dfrac{y_1 + 5}{2} = 2 \Rightarrow x_1 = -5, y_1 = -1$

and $\dfrac{x_2 + 3}{2} = 6, \dfrac{y_2 + 5}{2} = 4$

$\Rightarrow x_2 = 9, y_2 = 3$
So, $B = (-5, -1)$ and $C (9, 3)$

Centroid of $\triangle ABC = \left(\dfrac{3 - 5 + 9}{3}, \dfrac{5 - 1 + 3}{3}\right)$

$$= \left(\frac{7}{3}, \frac{7}{3}\right)$$

63. (b)

64. (c) Equation of parabola is $y^2 = 8x$
$\therefore \quad a = 2$

Since focal distance of point $P_{(x_1, y_1)}$ is $x_1 + a$

So, focal distance from point $\left(6, 4\sqrt{3}\right)$ is $6 + 2 = 8$

Hence, Statement 1 is correct.

Now, distance of point $P\left(6, 4\sqrt{3}\right)$ from direction

$$PF = \sqrt{(6-2)^2 + \left(4\sqrt{3} - 0\right)^2} = \sqrt{16 + 48} = 8$$

Hence, statement 2 is also correct.

65. (d)

66. (Bonus) Given,

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Using vector triple product

$\vec{a} \times (\vec{b} \times \vec{a}) = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$= \left(\left(\hat{i} - \hat{j} + \hat{k}\right).\left(\hat{i} - \hat{j} + \hat{k}\right)\right)\vec{b} - \left(\left(\hat{i} - \hat{j} + \hat{k}\right).\left(\hat{i} + 2\hat{j} - \hat{k}\right)\right)\vec{a}$

$= (1 + 1 + 1)\vec{b} - (1 - 2 - 1)\vec{a}$

$= 3\left(\hat{i} + 2\hat{j} - \hat{k}\right) + 2\left(\hat{i} - \hat{j} + \hat{k}\right)$

$= 5\hat{i} + 4\hat{j} - \hat{k}$ ...(i)

Given $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$

Compare with (i), we get

$\alpha = 5, \beta = -4, \gamma = -1$

$\Rightarrow \alpha + \beta + \gamma = 5 - 4 - 1 = 0$

67. (a)

68. (b) Statement 1 is not correct

We know that $t = \vec{r} \times \vec{F}$

The moment of force about a point is dependent of application of force.

Statement 2 is correct.

The moment of force about a line is a vector quantity. Because gross product of two vector is again a vector

$\vec{t} = \vec{r} \times \vec{F}$

$\downarrow \quad \downarrow \quad \downarrow$

Vec Vec Vec

69. (a) Let $\vec{I} = (\vec{r}.\hat{i})(\vec{r} \times \hat{i}) + (\vec{r}.\hat{j}) + (\vec{r} \times \hat{j}) + (\vec{r}.\hat{k})(\vec{r} \times \hat{k})$ ...(i)

Let $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

$(\vec{r}.\hat{i}) = (a\hat{i} + b\hat{j} + c\hat{k}).\hat{i} = a$

$(\vec{r}.\hat{j}) = b \Rightarrow (\vec{r}.\hat{k}) = c$

and $\vec{r} \times \hat{I} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 1 & 0 & 0 \end{vmatrix} = -\hat{j}(-c) - b\hat{k} = c\hat{j} - b\hat{k}$

Similarly, $\vec{r} \times \hat{j} = -c\hat{i} + a\hat{k}$ and $\vec{r} \times \hat{k} = b\hat{i} - a\hat{j}$

Now substitute in equation (i), we get

$\vec{I} = a\left(c\hat{j} - b\hat{k}\right) + b\left(-c\hat{i} + a\hat{k}\right) + c\left(b\hat{i} - a\hat{j}\right)$

$= ac\hat{j} - ab\hat{k} - cb\hat{i} + ab\hat{k} + bc\hat{i} - ac\hat{j} = \vec{O}$

70. (b)

71. (a)

72. (c) We have,

$y = e^x (a \cos x + b \sin x)$ ...(i)

$\Rightarrow \dfrac{dy}{dx} = e^x(-a \sin x + b \cos x) + (a \cos x + b \sin x)e^x$

$\Rightarrow \dfrac{dy}{dx} = e^x(-a \sin x + b \cos x) + y$ (From (i)) ....(ii)

Again differentiate w.r.t x.

$\dfrac{d^2y}{dx^2} = -e^x(a \cos x + b \sin x) + (-a \sin x + b \cos x)e^x + \dfrac{dy}{dx}$

Eliminating arbitrary constants

$\Rightarrow \dfrac{d^2y}{dx^2} = -y + \dfrac{dy}{dx} - y + \dfrac{dy}{dx}$ [From (i) and (ii)]

$\Rightarrow \dfrac{d^2y}{dx^2} - \dfrac{2dy}{dx} + 2y = 0$ is the required differential equation

73. (d) Given $f(x) = ax - b$ ...(i)

and $g(x) = cx + d$ .... (ii)

Now $f(g(x)) = g(f(x))$

$\Rightarrow a(g(x)) - b = c(f(x)) + d$

$\Rightarrow a(cx + d) - b = c(ax - b) + d$ [From (i) and (ii)]

$\Rightarrow acx + ad - b = acx - bc + d$

$ad - b = d - bc \Rightarrow ad - b + bc = d$

$\Rightarrow \therefore f(d) + g(b) = 2d$ [f(d) = ad - b & g(b) = bc + d]

74. (b) Let $I = \int_{-1}^{1}\left(3\sin x - \sin 3x\right)\cos^2 x\, dx$

$I = \int_{-1}^{1}\left(3\sin x - 3\sin x + 4\sin^3 x\right)\cos^2 x\, dx$

[$\sin 3x = 3\sin x - 4\sin^3 x$]

$I = \int_{-1}^{1} 4\sin^3 x \cos^2 x\, dx = \int_{-1}^{1} f(x)\, dx$

Since, $f(x) = 4\sin^3 x \cos^2 x$ is an odd function as $f(x) = -f(x)$.

$\Rightarrow I = \int_{-1}^{1} 4\sin^3 x \cos^2 x\, dx = 0$

$\left[\int_{-a}^{a} f(x)\, dx = 0, \text{if } f(x) = -f(x)\right]$

75. (d) Given : $\left\{2 - \left(\dfrac{dy}{dx}\right)^2\right\}^{0.6} = \dfrac{d^2y}{dx^2}$

$\left\{2 - \left(\dfrac{dy}{dx}\right)^2\right\}^{\frac{3}{5}} = \dfrac{d^2y}{dx^2}$

Raise power of 5 both sides

$$\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^3 = \left(\frac{d^2y}{dx^2}\right)^5$$

Now differential equation is free from decimal or fraction power.

Order = 2, Degree = 5.

76. (a) Given,

$$\frac{dy}{dx} = 2e^x y^3 \implies \frac{dy}{y^3} = 2e^x dx$$

Integrating both sides wrt x

$$\frac{-1}{2y^2} = 2e^x + C \qquad ...(i)$$

Using $y(0) = \frac{1}{2}$, in equation (i)

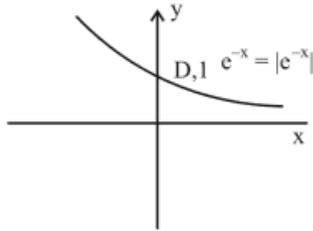$$-2 = 2 + c \implies c = -4$$

Now, $\frac{-1}{2y^2} = 2e^x - 4$

$$\implies 4y^2(e^x) - 8y^2 = -1 \implies 4y^2(2 - e^x) = 1$$

77. (d) Given,

$$P = \int_a^b f(x)\,dx \text{ and } q = \int_a^b |f(x)|\,dx$$

As $f(x) = e^{-x}$ is always +ve $\forall x \in R$

$$\implies |f(x)| = |e^{-x}| = e^{-x} = f(x)$$

$$\implies \int_a^b f(x)\,dx = \int_a^b |f(x)|\,dx$$

Hence, p = q

78. (a) Let $I = \int_0^{\frac{\pi}{2}} \frac{a + \sin x}{2a + \sin x + \cos x}\,dx$ ...(i)

$$\implies I = \int_0^{\frac{\pi}{2}} \frac{a + \cos x\,dx}{2a + \cos x + \sin x} \qquad ...(ii)$$

$$\left[\int_o^a f(x)\,dx = \int_o^a f(a-x)\,dx\right]$$

Adding equation (i) and (ii)

$$\implies 2I\int_0^{\frac{\pi}{2}} \frac{2a + \sin x + \cos x}{2a + \sin x + \cos x}\,dx = \int_0^{\frac{\pi}{2}}dx = [x]_0^{\frac{\pi}{2}} \implies I = \frac{\pi}{4}$$

79. (d) Given

$$f(x) = \frac{16x^3}{3} - 4bx^2 + x$$

$$f'(x) = 16x^2 - 8bx + 1$$

Since f(x) is neither maximum nor minimum, then $f'(x) \ne 0$

$\implies$ f'(x) > 0 or f'(x) < 0 (but here a > 0 for f'(x) so not possible) for f'(x) > 0

Discriminant of f'(x) is D < 0 and a > 0

D = $64b^2 - 64 < 0$

$b^2 - 1 < 0$ and b is non-negative is $b \ge 0$

$b \in (-1, 1)$ and $b \ge 0 \implies 0 \le b < 1$.

80. (a)
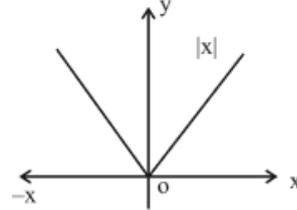
$f(x) = \frac{1}{\sqrt{|x| - x}}$ for domain of f(x) = |x| - x > 0

ie $|x| > x \implies x \in (-\infty, 0)$

$g(x) = \frac{1}{\sqrt{x - |x|}}$, for domain of g(x), x - |x| > 0

$\implies |x| < x$ (Not possible $\because |x| \ge x$)

Sol (81 - 82)

Let $I = \int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x}\,dx$

Let $3\cos x + 4\sin x = A(2\cos x + 5\sin x) +$

$$B\left(\frac{d}{dx}(2\cos x + 5\sin x)\right)$$

$3\cos x + 4\sin x = A(2\cos x + 5\sin x) + B(-2\sin x + 5\cos x)$

Comparing coefficient of sinx and cosx

$3 = 2A + 5B$ ...(i)

$4 = 5A - 2B$ ...(ii)

Solving (i) and (ii) we get

$$A = \frac{26}{29}, B = \frac{7}{29}$$

Now $I = \int \frac{\frac{26}{29}(2\cos x + 5\sin x) + \frac{7}{29}(-2\sin x + 5\cos x)}{2\cos x + 5\sin x}\,dx$

$$\implies I = \int \frac{26}{29}\,dx + \frac{7}{29}\int \frac{(-2\sin x + 5\cos x)}{2\cos x + 5\sin x}\,dx$$

$$\implies I = \frac{26}{29}x + \frac{7}{29}\ln|2\cos x + 5\sin x| + c$$

$\alpha = 26, \beta = 7$ (on comparing)

81. (d) $\alpha = 26$

82. (a) $\beta = 7$,

83. (d) Given

$$f(x) = \frac{x}{\ln(x)}(x > 1)$$

Differentiate w.r.t x

$$f'(x) = \frac{\ln x - 1}{(\ln(x))^2}$$

$$\overset{\text{—ve} \qquad\qquad \text{+ve}}{\underset{1 \qquad e}{\vdash\!\!\!\!+\!\!\!\!+\!\!\!\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-}}$$

f'(x) is the when x ∈(e, ∞) ie f(x) is increasing in interval (e, ∞) ⇒ (A) is correct.

f'(x) is –ve, when x ∈(1,e) ie f(x) is decreasing in the interval (1,e) ⇒ (B) is correct.

Since lnx is an increasing function.

$7^9 > 9^7$

$\ln 7^9 > \ln 9^7$

9 ln 7 > 7 ln 9 ⇒ statement (C) is correct

Hence, statement (A), (B), and (C) are correct.

84. (d) Again differentiating w.r.t x

$$f''(x) = \frac{(\ln x)^2 \times \frac{1}{x} - (\ln x - 1) \times 2\frac{\ln x}{x}}{(\ln x)^4}$$

$$= \frac{(\ln x)^2 - 2(\ln x)^2 + 2\ln x}{x(\ln x)^4}$$

$$f''(e) = \frac{1 - 2 + 2}{e \times 1} = \frac{1}{e} \ (>0) \Rightarrow \text{statement (A) is correct}$$

As f'' (e) > 0 (+ve)

So, f(x) attains local minima at x = e ⇒ statement (B) is correct

A local minimum value occurs at x = e,

$$f(x) \text{ at } x = e, \text{ is } f(e) = \frac{e}{\ln e} = e \Rightarrow \text{statement (C) is correct}$$

Hence, statement (A), (B) and (C) are correct.

Sol (85 - 86)

Given :

$$g(x) = x - \frac{1}{x} \text{ and } \log(x) = x^3 - \frac{1}{x^3}$$

$$f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3} \Rightarrow f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

85. (a) $f(x) = x^3 + 3x$ ...(i)

So, $g[f(x) - 3x] = f(x) - 3x - \dfrac{1}{f(x) - 3x}$

$$g[f(x) - 3x] = x^3 + 3x - 3x - \frac{1}{x^3 + 3x - 3x} = x^3 - \frac{1}{x^3}$$

$$g\left[f(x) - 3x\right] = x^3 - \frac{1}{x^3}$$

86. (d) $f'\square(x) = 3x^2 + 3$

f' (x) = 6x

Sol (87 - 88)

Given f(x) = |x| + 1 and g(x) = [x] – 1,

$$h(x) = \frac{f(x)}{g(x)} = \frac{|x| + 1}{[x] - 1}$$

87. (a) (A). $\dfrac{\text{for } x < 0}{f(x) = -x + 1}$

so f' (x) = – 1 then f(x) is differentiable ∀ x < 0

⇒ statement (A) is correct

(B) at x = .0001

g(x) is continuous every where except for integer value as [x] is continuous ∀ x ∈ R except integers.

So g(x) is continuous at x = .0001, ⇒ statement (B) is correct

(C) g(x) = [x] – 1   [x] ⇒ Always a integer

$$g'(x) = \frac{d}{dx}[x] - 0$$

g' (x) = 0 – 0 = 0 ⇒ statement (C) is not correct.

∴   (A), and (B) only correct.

88. (a) Consider $\lim\limits_{x \to 0^-} h(x) + \lim\limits_{x \to 0^+} h(x)$

$$= \lim_{x \to 0^-} \frac{|x| + 1}{[x] - 1} + \lim_{x \to 0^+} \frac{|x| + 1}{[x] - 1}$$

$$= \frac{0 + 1}{-1 - 1} + \frac{0 + 1}{0 - 1} \qquad \left\{ \begin{matrix} [0-h] = -1 \\ [0+h] = 0 \end{matrix} \right\}$$

$$= -\frac{1}{2} - 1 = \frac{-3}{2}$$

89. (d) Given

$$\phi(a) = \int\limits_a^{a+100\pi} |\sin x|\, dx$$

Since period of |sinx| = π and |sinx| = sinx in the interval 0 to π.

$$\phi(a) = \int_0^{100\pi} |\sin x|\, dx = 100\int_0^{\pi} |\sin x|\, dx = 100\int_0^{\pi} \sin x\, dx$$

$$\phi(a) = 100(-\cos x)_0^{\pi} = 100 \times 2 = 200$$

90. (a) $\phi(a) = 200$

Differentiating w.r.t a

$\phi'(a) = 0$

Sol (91 - 92)

Given, y = 2 f(x) + ax – b   ...(i)

Differentiating w.r.t. 'x' we get,

$\Rightarrow \dfrac{dy}{dx} = 2f'(x) + a(1)$ ...(ii)

Again differentiating w.r.t. x, we get,

$\Rightarrow \dfrac{d^2y}{dx^2} = 2f''(x)$ ...(iii)

91. (c) Since $f(x)$ has a local maximum at $x = 0$. Then
$f'(0) = 0$ and $f''(0) < 0$ ...(iv)
So, option (c) is correct.

92. (b) Since y has a relative maxima at $x = 0$.
Then
$$\left(\frac{dy}{dx}\right)_{x=0} = 0$$
$\Rightarrow \quad 2f'(0) + a = 0$ {from (ii)}
$\Rightarrow \quad 2(0) + a = 0$ {from (iv)}
$\Rightarrow \quad a = 0$
also $\left(\dfrac{d^2 y}{dx^2}\right)_{x=0} < 0 \Rightarrow 2f''(0) < 0$
$\Rightarrow \quad f''(0) < 0$
So, y has a relative maxima for $a = 0$ and all value of 'b'

Sol (93 - 94); We have given
$f(x) = |x - 1|$ ...(i)
$g(x) = [x]$ ...(ii)
and $h(x) = f(x) \cdot g(x)$
$\Rightarrow \quad h(x) = |x - 1| \cdot [x]$ ...(iii)

93. (a) Now
$$\int_{-1}^{0} h(x)dx = \int_{-1}^{0} f(x).g(x).dx = \int_{-1}^{0} |x-1|.[x]dx$$
$$= \int_{-1}^{0}(1-x).(-1)\,dx = \int_{-1}^{0}(x-1)dx$$
$$= \left[\frac{x^2}{2} - x\right]_{-1}^{0} = \left[0 - \left(\frac{1}{2} + 1\right)\right] = -\frac{3}{2}$$

94. (d) Now
$$\int_{0}^{2} h(x)dx = \int_{0}^{1}|x-1|[x]dx + \int_{1}^{2}|x-1|[x]dx$$
$$= \int_{0}^{1}(1-x).(0)\,dx + \int_{1}^{2}(x-1)(1)dx$$
$$= \int_{1}^{2}(x-1)dx = \left[\frac{x^2}{2} - x\right]_{1}^{2}$$
$$= \left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right) = \frac{1}{2}$$

Sol. (95 - 96) We have given,
$$\int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + C \quad ...(i)$$
Let $I = \int \dfrac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \int \dfrac{(\sqrt{x+1} + \sqrt{x-1})dx}{(\sqrt{x+1} - \sqrt{x-1})(\sqrt{x+1} + \sqrt{x-1})}$
$\Rightarrow I = \frac{1}{2}\int \sqrt{(x+1)}\,dx + \frac{1}{2}\int \sqrt{x-1}\,dx$

$\Rightarrow I = \frac{1}{2} \cdot \dfrac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{1}{2}\dfrac{(x-1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$

$\Rightarrow I = \frac{1}{3}(x+1)^{\frac{3}{2}} + \frac{1}{3}(x-1)^{\frac{3}{2}} + C$

$\Rightarrow \int \dfrac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \frac{1}{3}(x+1)^{-} + \frac{1}{3}(x-1)^{-} + C \quad ...(ii)$
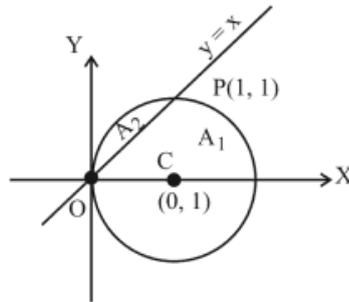
95. (a) Comparing (i) and (ii) we get
$$\alpha = \frac{1}{3}$$

96. (c) Comparing (i) & (ii) we get
$$\beta = \frac{1}{3}$$

Sol (97 - 98)

Equation of circle
$x^2 + y^2 - 2x = 0$
$\Rightarrow (x - 1)^2 + y^2 = 1$
$\Rightarrow y^2 = 1 - (x - 1)^2$

Area of minor segment $(A_2) = \int_{0}^{1}\left(\sqrt{1-(x-1)^2} - x\right)dx$

$\Rightarrow A_2 = \left[\dfrac{(x-1)}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1) - \dfrac{x^2}{2}\right]_{0}^{1}$

$\Rightarrow A_2 = \left[\left(0 + \frac{1}{2}(0) - \frac{1}{2}\right) - \left(0 + \frac{1}{2}\sin^{-1}(-1) - 0\right)\right]$

$\Rightarrow A_2 = \dfrac{\pi - 2}{4}$

97. (d) Since Area of given circle $= \pi(1)^2 = \pi$
$\Rightarrow A_1 + A_2 = \pi$
$\Rightarrow A_1 = \pi - \dfrac{\pi - 2}{4} = \dfrac{3\pi + 2}{4}$

98. (a) $\dfrac{2(A_1 + A_2)}{A_1 - 3A_2} = \dfrac{2(\pi)}{\left(\dfrac{3\pi + 2}{4}\right) - \dfrac{3(\pi - 2)}{4}} = \pi$

Sol (99 - 100)

99. (d) We have given, $3f(x) + f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 1$ ...(i)

Replacing (x) by $\left(\dfrac{1}{x}\right)$, we get,

$3f\left(\dfrac{1}{x}\right) + f(x) = x + 1$ ...(ii)

Now, on $3 \times$ [Eq. (i)] - Eq. (ii), we get

$8f(x) = \dfrac{3}{x} + 3 - x - 1$

$\Rightarrow f(x) = \dfrac{3}{8x} - \dfrac{x}{8} + \dfrac{1}{4}$ ...(iii)

100. (a) $8\displaystyle\int_1^2 f(x)dx = 8\int_1^2 \left(\dfrac{3}{8x} - \dfrac{x}{8} + \dfrac{1}{4}\right)dx$

$= \displaystyle\int_1^2 \left(\dfrac{3}{x} - x + 2\right)dx = \left[3\ln x - \dfrac{x^2}{2} + 2x\right]_1^2$

$= \left(3\ln 2 - \dfrac{4}{2} + 4\right) - \left(3\ln 1 - \dfrac{1}{2} + 2\right)$

$= \dfrac{1}{2} + 3\ln 2 = \dfrac{1}{2}\ln e + \ln 2^3$

$= \ln\sqrt{e} + \ln 8 = \ln\left(8\sqrt{e}\right)$

101. (c) No. of ways to choose two black from the bag $= {}^5C_2$
No. of ways to choose two white ball from the bag $= {}^4C_2$
Number of ways to choose both of the ball of same colour
$= {}^5C_2 + {}^4C_2$
So, the required probability is

$P(E) = \dfrac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \dfrac{10 + 6}{36}$

$P(E) = \dfrac{16}{36} = \dfrac{4}{9}$

102. (c) Given, Mean = 5 and Variance = 4
$5^{23} P(X = 3) = \lambda 4^\lambda$ ....(i)
$\because$ Mean $= 5 \Rightarrow np = 5$
Variance $= 4 \Rightarrow npq = 4$

$\Rightarrow 5q = 4 \Rightarrow q = \dfrac{4}{5}$

Now, $p = 1 - q = 1 - \dfrac{4}{5} = \dfrac{1}{5}$

$\because np = 5 \Rightarrow n = 25$
Now, $5^{23}P(X = 3) = 5^{23}\left[{}^nC_3 \, p^3 \, q^{n-3}\right]$

$= 5^{23}\left[{}^{25}C_3\left(\dfrac{1}{5}\right)^3\left(\dfrac{4}{5}\right)^{22}\right]$

$= 5^{23}\left[\dfrac{25 \times 24 \times 23}{6} \times \dfrac{1}{5^3} \times \dfrac{4^{22}}{5^{22}}\right] = 4 \times 23 \times 4^{22}$

$\Rightarrow 5^{23} P(X = 3) = 23 \times 4^{23}$ ...(ii)
From equation (i) & (ii) :-
$\lambda = 23$

103. (b) Given data

x	y	xy	x^2
− 4	1	−4	16
− 1	2	− 2	1
2	7	14	4
3	1	3	9

$\Sigma x = -4 - 1 + 2 + 3 = 0$
$\Sigma y = 1 + 2 + 7 + 1 = 11$
$\Sigma xy = -4 - 2 + 14 + 3 = 11$
$\Sigma x^2 = 16 + 1 + 4 + 9 = 30$

$\bar{x} = \dfrac{\Sigma x}{4} = \dfrac{0}{4} = 0$

$\bar{y} = \dfrac{\Sigma y}{4} = \dfrac{11}{4}$

$b_{yx} = \dfrac{\Sigma xy - \dfrac{\Sigma x \, \Sigma y}{4}}{\Sigma x^2 - \dfrac{(\Sigma x)^2}{4}} = \dfrac{11 - 0}{30 - 0} = \dfrac{11}{30}$

Equation of regression line of y on x will be
$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$

$\Rightarrow y - \dfrac{11}{4} = \dfrac{11}{30}(x - 0) \Rightarrow y = \dfrac{11}{30}x + \dfrac{11}{4}$

$\therefore a = \dfrac{11}{4}$ and $b = \dfrac{11}{30}$

$2a + 15b = 2 \times \dfrac{11}{4} + 15 \times \dfrac{11}{30} = 11$

104. (a) Slope of line $x + 2y + 1 = 0 \Rightarrow m_1 = -1/2$
Slope of line $2x + 3y + 4 = 0 \Rightarrow m_2 = -2/3$

Then, $\tan\theta = \left|\dfrac{-\dfrac{1}{2} + \dfrac{2}{3}}{1 + \dfrac{1}{2} \times \dfrac{2}{3}}\right| = \dfrac{1}{8}$

And, $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$= \dfrac{\dfrac{3}{8} - \dfrac{1}{8^3}}{1 - \dfrac{3}{64}} = \dfrac{192 - 1}{8^3} \times \dfrac{8^2}{61} = \dfrac{191}{8 \times 61}$

Therefore,

$488\tan 3\theta = 488 \times \dfrac{191}{8 \times 61} = 191$

Sol. (117-118)

117. (b) Given

$$\Sigma x_i f_i = 1.1 + \frac{2}{2} + \frac{3}{2^2} + ... + \frac{n}{2^{(n-1)}} \quad ...(i)$$

and $\frac{1}{2}\Sigma x_i f_i = \frac{1}{2} + \frac{2}{2^2} + ... + \frac{n-1}{2^{n-1}} + \frac{n}{2^n}$ ...(ii)

Subtracting (ii) from (i), we get

$$\Rightarrow \left(1 - \frac{1}{2}\right)\Sigma x_i f_i = \left[1.1 + \frac{2-1}{2} + \frac{3-2}{2^3} + \frac{4-3}{2^4} + ... + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \left[1 + \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^4} + ... + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} - \frac{1}{2^n} = \frac{2\left(2^n - 1\right)}{2^n} - \frac{n}{2^n}$$

$$\Rightarrow \Sigma x_i f_i = \frac{2^{n+1} - n - 2}{2^{n-1}}$$

118. (c) $\Sigma f_i = 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{(n-1)}}$

$$\Sigma f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = \frac{2^n - 1}{2^{n-1}}$$

Calculating mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2^{n+1} - n - 2}{2^{n-1}} \cdot \frac{2^{n-1}}{2^n - 1}$

$$= \frac{2^{n+1} - n - 2}{2^n - 1}$$

Sol. (119 - 120);

Ascending data is, 15, 18, 19, 21, 24, 32, 35, 41, 47, 50

Largest 5 observation is, 32, 35, 41, 47, 50

Mean $\mu = \frac{32 + 35 + 41 + 47 + 50}{5} = \frac{205}{5} \Rightarrow \mu = 41$

119. (c) Mean deviation $= \frac{\Sigma|x_i - \mu|}{n}$

$$= \frac{|32 - 41| + |35 - 41| + |141 - 41| + |47 - 41| + |50 - 41|}{5}$$

$$= \frac{9 + 6 + 0 + 6 + 9}{5} = 6$$

120. (d) $\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{5} = \frac{9^2 + 6^2 + 0^2 + 6^2 + 9^2}{5}$

$$= \frac{234}{5} = 46.8$$

GENERAL ABILITY

PART- A: ENGLISH

1. (d) The meaning of the word, 'deplorable' is morally bad or deserving disapproval. The antonym for deplorable will be commendable meaning deserving praise.

2. (a) The meaning of the word, 'counteract' is to reduce the effect of something by acting against it. The antonym for counteract will be 'exacerbate' meaning to make something worse, could be a disease or problem.

3. (a) The meaning of the word, 'persevere' is to continue trying to achieve something that is difficult. The antonym for persevere will be 'to give up' meaning not wanting to work on a difficult situation after a certain period of time.

4. (b) The meaning of the word, 'hideous' is very ugly or unpleasant. The antonym for hideous will be beautiful.

5. (d) The meaning of the word, 'squander' is to waste time or money. The antonym for squander will be manage.

6. (b) Continually: The mentioned word will be appropriately placed in the blank since the sentence talks about climate which is something that keeps changing continuously.

7. (a) The mentioned word will follow the first statement, i.e., changes have already been stated previously, hence, it is a continuation of it.

8. (c) Since the activities listed ahead are carried out by the humankind, hence the appropriate word is 'human'.

9. (a) The word 'assimilate' means to be taken in or absorbed. The sentence will be: 'emissions from industry and transport, etc. which have led to the assimilation of gases...'

10. (c) Article, 'the' will be the correct word to fill in the blank since the sentence is a stated fact.

11. (a) The greenhouse effect allows the greenhouse gases to trap the heat from the earth's surface and raise air temperatures.

12. (d) The greenhouse gases act on the surface of the earth, i.e., the planet in order to protect it.

13. (a) The preposition 'for' will appropriately be placed in the sentence here since it talks about assessment of climate change.

14. (d) have initiated. This will correctly fit into the sentence since the research has initiated.

15. (b) The preposition 'at' will complete the sentence since the activity takes places at regular intervals.

16. (b) The complete sentence will be – "He paused for a few moments." The remaining options will be grammatically incorrect if used.

17. (a) In relation to vehicles, they 'break down' when something goes wrong in their functioning.

18. (b) In the sentence, it states that "all countries signed the treaty", therefore, only the remaining two will when mentioned will have 'except' before them.

19. (a) In the case of electricity, the lights are always 'turned off' or 'switched off'.

20. (c) When you refer to an individual, you use 'who'. It refers to 'first person' in this context.

21. (a) When referring to months, the preposition 'at' or 'in' will be used. It can be used in multiple contexts such as telling time or location.

22. (b) The determiner 'that' is used to modify or introduce a noun in any sentence.

23. (b) In this sentence, the grammatically correct sentence will be- "The thieves denied stealing the money."

24. (b) The correct sentence will be- "Did you drop in to see Sunita on your way home?" The phrase 'drop in' refers to a casual visit one makes.

25. (b) "Would you like to meet her?" will be the grammatically correct sentence.

26. (c) The idiom, 'be bad news' refers to 'someone who is considered undesirable'.

27. (c) The idiom, 'back to the drawing board' refers to starting planning again because the previous plan has failed.

28. (d) The idiom, 'Be in the eye of the storm' refers to be in the midst of a controversy which affects or interests a lot of people.

29. (d) The idiom, 'Life in the fast lane' refers to an exciting and eventful lifestyle, especially one which brings wealth and success.

30. (b) The idiom, 'To pass the buck' refers to shifting the responsibility of something to someone else.

31. (a) The correct sentence formed will be- "The fundamental rights are basic rights and include basic freedoms guaranteed to the individual."

32. (a) The correct sentence formed will be- "The Indian nation is the product of a historical process that has been in the making for many millennia."

33. (a) The correct sentence formed will be- "India's Independence represented for its people the start of an epoch that was imbued with a new vision."

34. (c) The correct sentence formed will be- "All students have been instructed to be present before the principal for a discussion on the forthcoming science and mathematics exhibition."

35. (c) The correct sentence formed will be- "With the selection of another player to the new team, the team now has two bowlers and two batsmen respectively."

36. (a) In this sentence, the word 'abode' is a noun. It refers to a residence, dwelling or habitat.

37. (d) The word, 'beyond' is used as an adverb in this sentence.

38. (d) In the above mentioned sentence, 'except' acts as a conjunction.

39. (b) In the given sentence, 'nicely' is used as an adverb.

40. (a) In the given sentence, 'this' has been used as a demonstrative determiner.

41. (b) Human belongs to abiotic community.

42. (b) The nature of biotic community largely depends on abiotic components.

43. (c) Algae is a living organism, so it does not belong to abiotic community.

44. (b) Aquatic community refers to the biotic community living in water.

45. (c) Species are defined on the basis of both similarities as well as differences.

46. (a) Deported is a synonym for banished.

47. (a) Brilliant is a synonym for dazzling.

48. (b) Rejected is a synonym for rebuffed.

49. (b) Abandoned is a synonym for disowned.

50. (c) Doubtful is a synonym for skeptical.

PART- B: GENERAL KNOWLEDGE

51. (b) The equivalent circuit diagram is
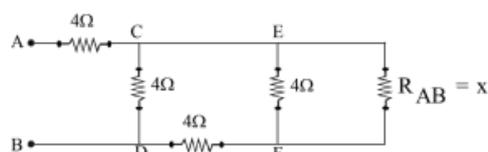
∴ The net resistance between points E & F is

$$R_{EF} = \frac{4x}{4+x}$$

The net resistance between points C & D is

$$R_{CD} = \frac{4\left(4 + \dfrac{4x}{4+x}\right)}{4 + 4 + \dfrac{4x}{4+x}} = 4\left(\frac{4+2x}{8+3x}\right)$$
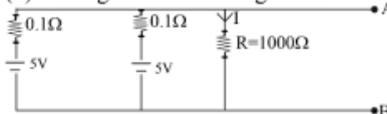
The net reistance between points A & B is

$$R_{AB} = 4 + R_{CD} = 4\left(1 + \frac{4+2x}{8+3x}\right)$$

$$\Rightarrow x = 4\left(\frac{12+5x}{8+3x}\right) \Rightarrow x^2 - 4x - 16 = 0$$

$$\therefore \quad x = \frac{(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-16)}}{2 \times 1}$$

$$= \left(2 \pm 2\sqrt{5}\right)\Omega$$

52. (d) The given circuit diagram is

The potential difference across points A & B is

$$V = \frac{\dfrac{5}{0.1} + \dfrac{5}{0.1}}{\dfrac{1}{0.1} + \dfrac{1}{0.1} + \dfrac{1}{1000}} = \frac{10^5}{20001} V$$

∴ Current through resistor R is

$$I = \frac{V_{AB}}{R} = \frac{10^5}{20001 \times 1000} \approx 5mA$$

53. (d) For electrical machine,
I = 15A, t = 10 min = 10 × 60s
∴ Current flow, Q = If = 15×10×60 = 9000C

54. (a) The lightening conductor is made of metal rod with a sharp pointed edge on its top in order to provide low resistance for conducting path. The electric discharge from the lightening smoothly fed into earth.

55. (d) There are four fundamental forces in nature:
    (i) Gravitational forces
    (ii) Electromagnetic forces
    (iii) Strong nuclear forces
       (iv) Weak nuclear forces

56. (b) For the large sphere,

$$\rho_1 = \rho,\ \rho_2 = \frac{\rho}{2},$$

$$V_1 = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{1}{8}\left(\frac{4}{3}\pi R^3\right)\alpha\frac{1}{8}$$

$$V_2 = \frac{4}{3}\pi\left(R^3\left(\frac{R}{2}\right)^3\right) = \frac{7}{8}\left(\frac{4}{3}\pi R^3\right)\alpha\frac{7}{8}$$

∴ Equivalent density is

$$\rho_e = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} = \frac{\rho\left(\frac{1}{8}\right) + \frac{\rho}{2}\left(\frac{7}{8}\right)}{\frac{1}{8} + \frac{7}{8}} = \frac{9\rho}{16}$$

57. (d) The pupil of human eye acts as aperture and its size changes with the intensity of light. Also, the eye can focus the image of both far-off and near objects by changing the foal length of eye lens. So, human eye has a lens with variable focal length and variable aperture size.

58. (c) Article 371A states that **no act of Parliament shall apply to Nagaland in respect** of the **religious or social practices** of the Nagas, the Naga **customary law** and procedure, the **administration of civil and criminal justice** involving decisions according to the Naga customary law, and the **ownership and transfer of land** and its resources, unless the Nagaland Legislative Assembly decides otherwise by a resolution.

59. (c) **Article 191. Disqualifications for membership of the the person.**
    A person shall be disqualified for being chosen as, and for being, a member of the Legislative Assembly or Legislative Council of a State
    • if he holds any office of profit under the Government of India or the Government of any State specified in the First Schedule, other than an office declared by the Legislature of the State by law not to disqualify its holder.

60. (a)

Convention	Adopted on
The Universal Declaration of Human Rights	10 December 1948
The Convention Relating to the Status of Refugees / Refugee Convention or the Geneva Convention	28 July 1951
The International Covenant on Economic, Social and Cultural Rights	16 December 1966.
The Convention on the Elimination of all Forms of Discrimination Against Women	1979

61. (c) The main objectives of the Second Plan are an increase of 25 per cent in real national income over the five-year period 1956–57 to 1960–61; a large expansion of employment opportunities; rapid industrialization; and reduction of economic inequalities.

62. (b) Second urbanization means the emergence of sixteen Mahajanapadas from Janapadas during ancient India in the period of 500 to 600 BC in the Indian subcontinent.

63. (a) The earliest deciphered inscriptions in India are the Edicts of Ashoka, written in Prakrit using the Brahmi script in the 3rd century BCE.

64. (a) Let the % abundance of B – 10 = x
    % Abundance of B – 11= (100 – x)
    $10.8 = 10.013x + (100 - x)11./100$
    $\Rightarrow 10.8 = 10.013x + 1100.9 - 11.009x \Rightarrow x = 19.00\%$
    ∴ Abundance of B – 10 = 20.98%, Abundance of B-11 = 100 – 19.00 = 81%

65. (b) Portland cement consists essentially of compounds of lime (calcium oxide, CaO) mixed with silica (silicon dioxide, SiO_2) and alumina (aluminium oxide, $Al2O_3$). The lime is obtained from a calcareous (lime-containing) raw material, and the other oxides are derived from an argillaceous (clayey) material.

66. (b) Litmus is a natural indicator derived from lichens, specifically species like Roccella tinctorum and Lecanora tartarea. It's a mixture of organic compounds that turns red in acidic solutions and blue in alkaline solutions.

67. (d) $V_2O_5 = 0$
    $2 \times V + 5 \times - 2 = 0$
    $2V - 10 = 0$
    $V = 10/2$
    $V = 5$

68. (a) Graphite is soft and slippery material.
    Diamond is the hardest natural substance.
    Fullerene is very light and strong material.
    Graphene is thinest and strongest material.

69. (a) IDL does not cut cross any country. The IDL is at 180 degrees longitude, not because it is just opposite to prime meridian but because it passes majorly through the oceans. The IDL is intentionally made so that it doesn't pass from any country's landmass, and the problem of two different dates in one country does not arise.

70. (d) 71. (d)

72. (b) The Black soil is also known as regur soil and is formed by the weathering of Igneous rocks that form the Deccan plateau. Black soil is developed over Deccan lava granite rocks under semi-arid conditions.

73. (b) • Clouds are indeed classified based on their altitude (height above the Earth's surface) and their form (shape and appearance).
    • Clouds are commonly classified into three altitude categories: high clouds (cirrus, cirrostratus, cirrocumulus), middle clouds (altostratus, altocumulus), and low clouds (stratus, stratocumulus, nimbostratus).

74. (d)

75. (c) • Alkali soil have high in pH (above 8.5), making it basic. These clay-rich soils often contain sodium carbonate/bicarbonate, leading to poor drainage and limited nutrient availability for plants.

- Krishna Deva Raya invaded the Raichur Doab and completely crushed the Adil Shahi of Bijapur. Then he attacked Gulbarga and captured the city of Raichur in c. 1520 CE. He restored the Bahmani Sultanate to Muhammad Shah, and took the title of Yavanarajya sthapanacharya.

98. (b) Water gas is a fuel gas that's primarily a mixture of carbon monoxide (CO) and hydrogen (H_2) in equal proportions.

99. (b) The human body functions within a pH range of 7.0 to 7.8, which is slightly alkaline.

100. (b) The latent heat of fusion of water is usually high.
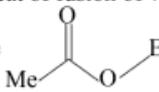
101. (c) Ethyl acetate $\text{Me}-\overset{\overset{\displaystyle O}{\|}}{C}-O-\text{Et}$ has a sweet and fruity smell.

102. (c) Coordination number of cation and Anion in Nacl lattice is 6,6.

103. (a) Methanoic acid, also called formic acid is present in the nettle leaves.

104. (d) Magnesium hydroxide is a chemical compound with the formula $Mg(OH)_2$, commonly known as milk of magnesia.

105. (d) A mixture of soap with water forms Lyotropic liquid crystal.

106. (b) Borax with the chemical formula $Na_2 H_{20} B_4 O_{17}$ (also written as $Na_2 H_{20} B_4 O_{17}. 10H_2O$)

107. (b) MgO has the highest melting point due to the strong ionic bonding between Mg^{2+} and O^{2-}

108. (c)   109. (b)

110. (c)
- Environmental temperature lapse rate is the rate at which the temperature of air decreases with altitude over a specific time and place. It's also known as the standard lapse rate. The lapse rate is positive when the temperature decreases with height, and negative when it increases. The standard rate is estimated to be 6.5 degrees Celsius per kilometer or 1000 meters.
- According to IMO Air temperature should be measured at a standard height of 1-2 meters (4-6 feet) above the ground surface. This is done to minimize the influence of local factors like ground heating or shading from buildings.

111. (c)

112. (b) **Cyclone**
- Cyclones are rapid inward air circulation around a low-pressure area. The air circulates in an anticlockwise direction in the Northern hemisphere and clockwise in the Southern hemisphere.
- Cyclones are usually accompanied by violent storms and bad weather.
- cyclones rotate counterclockwise in the Northern Hemisphere because of the Coriolis effect, which causes air currents to deflect to the right.

113. (a) **Mormugao Port**
Mormugao Port, located on India's western coast in the state of Goa, is one of the country's oldest ports, commissioned in 1885. It boasts a naturally protected open-harbor at the southern mouth of the Zuari River. This natural harbor is further secured by a breakwater and a mole, which extends from the breakwater and runs parallel to the wharfs.

114. (c) **Rourkela Steel plant**
The Rourkela Steel plant was set up in 1959 in the Sundargarh district of Orissa in collaboration with Germany. The plant was located on the basis of proximity to raw materials, thus, minimising the cost of transporting weight losing raw material. This plant has a unique locational advantage, as it receives coal from Jharia (Jharkhand) and iron ore from Sundargarh and Kendujhar. The Hirakud project supplies power for the electric furnaces and water is obtained from the Koel and Sankh rivers.

115. (b)

116. (b) • The Brahmaputra River, known as the Yarlung Tsangpo in Tibet, enters India through the Arunachal Pradesh region, where it is called the Dihang (or Siang) River before it merges with other rivers and becomes the Brahmaputra.
- The Godavari, with its sources from Trimbakeshwar, Maharashtra, is the river that has the second largest river basin in India.
- The Chambal and Betwa rivers are indeed important tributaries, but they join the Yamuna River, which is a major tributary of the Ganga. Therefore, they do not join the Ganga directly.

117. (b) • The Intertropical Convergence Zone (ITCZ) is a major contributor to the heavy rainfall in tropical wet climates. This belt of low pressure encircling Earth near the equator, where trade winds from the Northern and Southern Hemispheres converge, causes rising air, frequent thunderstorms, and heavy rain.
- The seasonal movement of the Intertropical Convergence Zone (ITCZ) and subtropical highs is a major cause of the distinct wet and dry seasons in tropical savanna climates.
- Tropical monsoon climates experience warm temperatures year-round, similar to tropical rainforests, but with abundant rainfall concentrated during the high-sun season.

118. (c)

119. (b) • The temperate climates (C-climates) are characterised by an average temperature of the coldest month between $-3°C$ and $18°C$, and an average temperature of the warmest month above $10°C$. They dominate in the mid-latitudes but may also occur at higher elevations in lower latitudes.
- These forests contain primarily deciduous trees - including maple, oak, hickory and beechwood.
- Raccoons, opossums, bats and squirrels are found in the trees.

120. (a) The chlorophyll present in green leaves absorbs blue and red light and reflects green light, hence it looks green.

121. (d) The chromosome number in prokaryotic cell is 1.

122. (a) Bacterial DNA is referred to as naked as it is not associated with any scaffold or covered with membrane.

123. (a) Lymphocytes will produce antibodies against any foreign antigen.

124. (c) At equilibrium of the block P,
$\Sigma f_x = 0 \Rightarrow (2 + F + 15) - (10 + 15) = 0$
$\therefore F = 8N$

125. (c) The various components of a nuclear reactor are turbine, heat exchanger, reaction chamber etc. Nuclear reactor inherently do not produce CO_2 emissions during their operation.

126. (a) Potential difference across 10 Ω resistor

$\Delta V = I \times 0 = 0 \Rightarrow 10\,\Omega$, V = 10 V resistor

are short circuited. $\therefore R_{net} = 5 + 5 = 10\,\Omega$,

$\therefore$ Current = $I = \dfrac{V}{R_{net}} = \dfrac{10}{10} = 1A$

127. (a) A microscope may be a combination of two convex lenses. The lenses facing eye and object are called eyepiece and objective respectively.

128. (c) The eye lens has power of accomodation, due to which it maintain formation of image at the retina for different positions of the object. With increase of object distance, the focal length of eye lens increases and finally it becomes constant.

129. (b) For the formation of primary rainbow by water droplet, there are more than one refraction and only one internal reflection. The primary rainbow formed as inner bow, and its colours are visible in the sequence of red on the outer edge to violet on the inner edge.

130. (c) Niels Bohr mentioned that the electron in an atom revolves only in a certain fixed orbit in which angular momentum of electron is integral multiple of $\dfrac{h}{2\pi}$.

131. (d) The LIGO (Laser Interferometer Gravitational - Wave Observatory) experiment detects gravitational wave produced by accelerating masses like binary black holes, which confirms a major prediction of "General Theory of Relativity".

132. (b) For the block,
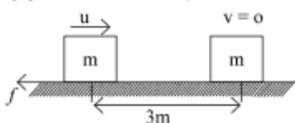
m = 2kg, u = 3m/s, v = 0

By work-energy theorem,

$W_f = \Delta K = 0 - \dfrac{1}{2}mu^2 = -\dfrac{1}{2} \times 2 \times (3)^2 = -9$

$\Rightarrow -f \times 3 = -9 \therefore$ Friction, f = 3N

133. (b) Metals have a lattice of positively charged ions surrounded by a 'sea' of free eletrons. When an electric field is applied across the ends of a metallic conductor, the free eletrons start drifting in the direction opposite to applied electric field, which contitutes conduction of electric current in the condutor.

134. (a)

135. (b) Bhanurnati is a commentary written by Cakrapanidatta on Susruta Samhita, which is also called Susrutatatparva- tika.

136. (d) • Tobacco cultivation in India was introduced by Portuguese in 1605.
• Initially tobacco was grown in Kaira and Mehsana districts of Gujarat and later spread to other areas of the country.
• Attempt to improve Indian tobacco has begun with the establishment of the Calcutta Botanical gardens in Howrah in 1787.

137. (c) **Women's Indian Association (WIA):** Founded in 1917 at Adyar, Madras by Annie Besant, Margaret Cousins, Dorothy Jinarajadasa, and others.

National Council of Women in India: Founded in 1925 by Sarala Devi Chaudhurani.

Bharat Stree Mahamandal: Established in 1910 by Lady Meherbai Tata.

138. (d) 139. (c)

140. (c) • The Ajanta cave paintings were created using a technique that involved preparing the rock surface with a mixture of clay, rice husk, and gum. This preparation was then coated with a layer of lime plaster to create a smooth surface suitable for painting.
• In the creation of Ajanta paintings, artists first sketched the outlines of the images on the prepared lime-coated surface. After the outlines were completed, the artists filled in the colors, which were derived from natural sources like minerals and plants.

141. (c)

142. (c) The two-week-long joint military exercise 'Desert Cyclone' between India and UAE held in the arid landscape of Rajasthan. The first edition of the India-UAE military exercise aims to enhance interoperability through exchanging knowledge and sharingbest practices in military operations.

143. (a) The **Government of India**, in adherence to **Article 280(1) of the Constitution**, has established the **Sixteenth Finance Commission,** appointing **Dr. Arvind Panagariya,** former Vice-Chairman of NITI Aayog and Professor at Columbia University, as its **Chairman.**

144. (d) In January 2024, the Ministry of Environment, Forest and Climate Change (MoEF&CC) nominated three cities in India for Wetland City Accreditation (WCA) under the Ramsar Convention on Wetlands: Indore, Bhopal and Udaipur.

145. (b) National Youth Day, also known as Vivekananda Jayanti, is celebrated on 12 January, being the birthday of a Hindu monk, Swami Vivekananda. In 1984, the Government of India declared this day as National Youth Day and since 1985 the event is celebrated in India every year.

146. (c)

147. (c) High-level Committee on Simultaneous Elections, constituted under the chairmanship of Shri Ram Nath Kovind, former President of India, has proposed simultaneous elections for Lok Sabha, state Assemblies, and local bodies in India.

148. (c) Raksha Mantri inaugurated first all-girls Sainik School at Vrindavan, Uttar Pradesh. This initiative is part of the plan to establish 100 new Sainik Schools in a partnership mode with NGOs/Public/State Government Schools in all states. The announcement was made on January 01, 2024

149. (a) Union Home Ministry on February 8 decided to scrap the Free Movement Regime (FMR) along Myanmar border to ensure the internal security of the country and to maintain the demographic structure of India's North Eastern States.

150. (b) Why Bharat Matters is a 2024 non-fictional book written by Indian politician and author S. Jaishankar , and published by Rupa Publications. Jaishankar was the incumbent of the Government of India in the Second Modi ministry at the time of publishing