NDA/NA SOLVED PAPER 2024-I

MATHEMATICS

1. Let A and B be matrices of order 3 × 3. If $|A| = \frac{1}{2\sqrt{2}}$ and $|B| = \frac{1}{729}$, then what is the value of $|2B(adj(3A)|$?
(a) 27 (b) $\frac{27}{2\sqrt{2}}$
(c) $\frac{27}{2}$ (d) 1

2. If z is any complex number and $iz^3 + z^2 - z + i = 0$, where $i = \sqrt{-1}$, then what is the value of $(|z|+1)^2$?
(a) 1 (b) 4
(c) 81 (d) 121

3. What is the sum of all four digit numbers formed by using all digits 0, 1, 4, 5 without repetition of digits?
(a) 44440 (b) 46460
(c) 46440 (d) 64440

4. If x, y and z are the cube roots of unity, then what is the value of $xy + yz + zx$?
(a) 0 (b) 1
(c) 2 (d) 3

5. A man has 7 relatives (4 women and 3 men). His wife also has 7 relatives (3 women and 4 men). In how many ways can they invite 3 women and 3 men so that 3 of them are man's relatives and 3 of them are his wife's relatives?
(a) 340 (b) 484
(c) 485 (d) 469

6. A triangle PQR is such that 3 points lie on the side PQ, 4 points ·on QR and 5 points on RP respectively. Triangles are constructed using these points as vertices. What is the number of triangles so formed ?
(a) 205 (b) 206
(c) 215 (d) 220

7. If $\log_b a = p, \log_d c = 2p$ and $\log_f e = 3p$, then what is $(ace)^{\frac{1}{p}}$ equal to?
(a) bd^2f^3 (b) bdf
(c) b^3d^2f (d) $b^2d^2f^2$

8. If $-\sqrt{2}$ and $\sqrt{3}$ are roots of the equation $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$ where a_0, a_1, a_2, a_3 are integers, then which one of the following is correct?
(a) $a_2 = a_3 = 0$ (b) $a_2 = 0$ and $a_3 = -5$
(c) $a_0 = 6, a_3 = 0$ (d) $a_1 = 0$ and $a_2 = 5$

9. Let z_1 and z_2 be two complex numbers such that $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1$, then what is $\text{Re}\left(\frac{z_1}{z_2}\right) + 1$ equal to?
(a) –1 (b) 0
(c) 1 (d) 5

10. If $26! = n8^k$, where k and n are positive integers, then what is the maximum value of k?
(a) 6 (b) 7
(c) 8 (d) 9

11. Consider the following statements in respect of two non-singular matrices A and B of the same order n:
A. $adj(AB) = (adjA)(adjB)$
B. $adj(AB) = adj(BA)$
C. $(AB)adj(AB) - |AB|I_n$ is a null matrix of order n
How many of the above statements are correct?
(a) None (b) Only one statement
(c) Only two statements (d) All three statements

12. Consider the following statements in respect of a non-singular matrix A of order n:
A. $A(adjA^T) = A(adjA)^T$
B. If $A^2 = A$, then A is identity matrix of order n
C. If $A^3 = A$, then A is identity matrix of order n
Which of the statements given above are correct?
(a) A and B only (b) B and C only
(c) A and C only (d) A, B and C

13. How many four-digit natural numbers are there such that all of the digits are even?
(a) 625 (b) 500
(c) 400 (d) 256

14. If $\omega \neq 1$ is a cube root of unity, then what are the solutions of $(z - 100)^3 + 1000 = 0$?
(a) $10(1 - \omega), 10(10 - \omega^2), 100$
(b) $10(10 - \omega), 10(10 - \omega^2), 90$
(c) $10(1 - \omega), 10(10 - \omega^2), 1000$
(d) $(1 + \omega), (10 + \omega^2), -1$

15. What is $(1 + i)^4 + (1 - i)^4$ equal to, where $i = \sqrt{-1}$?
(a) 4 (b) 0
(c) –4 (d) –8

16. Consider the following statements in respect of a skew-symmetric matrix A of order 3 :
A. All diagonal elements are zero.
B. The sum of all the diagonal elements of the matrix is zero.
C. A is orthogonal matrix.
Which of the statements given above are correct ?
(a) A and B only (b) B and C only
(c) A and C only (d) A, B and C

17. Four digit numbers are formed by using the digits 1, 2, 3, 5 without repetition of digits. How many of them are divisible by 4 ?

(a) 120 (b) 24
(c) 12 (d) 6

18. What is the remainder when 2^{120} is divided by 7 ?

(a) 1 (b) 3
(c) 5 (d) 6

19. For what value of n is the determinant

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix} = 0$$

for every $m > n$?

(a) 4 (b) 5
(c) 6 (d) 7

20. If ABC is a triangle, then what is the value of the determinant

$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix} ?$$

(a) –1 (b) 0
(c) 1 (d) 3

21. What is the number of different matrices, each having 4-entries that can be formed using 1, 2, 3, 4 (repetition is allowed)?

(a) 72 (b) 216
(c) 254 (d) 768

22. Let $A = \{x \in R : -1 < x < 1\}$. Which of the following is/are bijective functions from A to itself?

(A) $f(x) = x\,|x|$
(B) $g(x) = \cos(\pi x)$

Select the correct answer using the code given below:

(a) A only (b) B only
(c) Both A and B (d) Neither A nor B

23. Let R be a relation on the open interval (–1, 1) and is given by $R = \{(x, y) : |x + y| < 2\}$. Then which of the following is correct?

(a) R is reflexive but neither symmetric nor transitive
(b) R is reflexive and symmetric but not transitive
(c) R is reflexive and transitive but not symmetric
(d) R is an equivalence relation

24. For any three non-empty sets A, B, C, what is $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$ is equal to?

(a) Null set (b) A
(c) B (d) $(A \cup B) - (A \cap B)$

25. If a, b, c are the sides of triangle ABC, then what is

$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$ equal to?

(a) Zero (b) Area of triangle
(c) Perimeter of triangle (d) $a^2 + b^2 + c^2$

26. If a, b, c are in AP; b, c, d are in GP; c, d, e are in HP, then which of the following is/ are correct?

(A) a, c and e are in GP
(B) $\frac{1}{a}, \frac{1}{c}, \frac{1}{e}$ are in GP

Select the correct answer using the code given below:

(a) A only (b) B only
(c) Both A and B (d) Neither A nor B

27. What is the number of solutions of $\log_4(x-1) = \log_2(x-3)$?

(a) Zero (b) One
(c) Two (d) Three

28. For $x \geq y > 1$,

let $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right) = k$,

then the value of k can never be equal to

(a) –1 (b) $-\frac{1}{2}$
(c) 0 (d) 1

29. If $A = \begin{vmatrix} \sin 2\theta & 2\sin^2\theta - 1 & 0 \\ \cos 2\theta & 2\sin\theta\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$, then which of the following statements is/ are correct?

(A) $A^{-1} = adjA$
(B) A is skew-symmetric matrix
(C) $A^{-1} = A^T$

Select the correct answer using the code given below:

(a) A only (b) A and B only
(c) A and C (c) B and C

30. What is the coefficient of x^{10} in the expansion of $(1-x^2)^{20}\left(2 - x^2 - \frac{1}{x^2}\right)^{-5}$?

(a) –1 (b) 1
(c) 10
(d) Coefficient of x^{10} does not exist

31. If the 4th term in the expansion of $\left(mx + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, then what is the value of mn?

(a) –3 (b) 3
(c) 6 (d) 12

32. If a, b and $c(a > 0,\ c > 0)$ are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$:

(A) The equation has imaginary roots.
(B) The ratio of the roots of the equation is 1 : ω where ω is a cube root of unity.
(C) The product of roots of the equation is $\left(\frac{b^2}{a^2}\right)$.

Which of the statements given above are correct?

(a) (A) and (B) only (b) (B) and (C) only

(c) (A) and (C) only (d) (A), (B) and (C)

33. If $x^2 + mx + n$ is an integer for all integral values of x, then which of the following is/ are correct?

(A) m must be an integer

(B) n must be an integer

Select the correct answer using the code given below:

(a) A only (b) B only

(c) Both (A) and (B) (d) Neither (A) nor B

34. In a binomial expansion of $(x+y)^{2n+1}(x-y)^{2n+1}$, the sum of middle terms is zero. What is the value of $\left(\frac{x^2}{y^2}\right)$?

(a) 1 (b) 2

(c) 4 (d) 8

35. Let A= {1, 2, 3, 4, 5} and B = {6, 7}. What is the number of onto functions from A to B?

(a) 10 (b) 20

(c) 30 (d) 32

36. What is $\frac{\sqrt{3}\cos 10° - \sin 10°}{\sin 25° \cos 25°}$ equal to ?

(a) 1 (b) $\sqrt{3}$

(c) 2 (d) 4

37. What is (sin 9° – cos 9°) is equal to?

(a) $-\frac{\sqrt{5-\sqrt{5}}}{2}$ (b) $-\frac{\sqrt{5-\sqrt{3}}}{2}$

(c) $\frac{\sqrt{5-\sqrt{5}}}{2}$ (d) $\frac{\sqrt{5-\sqrt{5}}}{4}$

38. If in a triangle ABC, $\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \sin B \sin C$, then what is the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$; where a, b, c are sides of the triangle?

(a) $a + b + c$ (b) $ab + bc + ca$

(c) $(a + b)(b + c)(c + a)$ (d) 0

39. If $\cos^{-1}x = \sin^{-1}x$, then which one of the following is correct?

(a) $x = 1$ (b) $x = \frac{1}{2}$

(c) $x = \frac{1}{\sqrt{2}}$ (d) $x = \frac{1}{\sqrt{3}}$

40. What is the number of solutions of $(\sin \theta - \cos \theta)^2 = 2$ where $-\pi < \theta < \pi$?

(a) Only one (b) Only two

(c) Four (d) No solution

41. ABC is a triangle such that angle C = 60°, then what is $\frac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)}$ equal to?

(a) 2 (b) $\sqrt{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

42. What is $\sqrt{15 + \cot^2\left(\frac{\pi}{4} - 2\cot^{-1} 3\right)}$ equal to?

(a) 1 (b) 7

(c) 8 (d) 16

43. What is the value of sin 10°·sin 50° + sin 50°·sin 250° + sin 250°·sin 10° equal to?

(a) $-\frac{1}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{3\sin 10°}{4}$ (d) $-\frac{3\cos 10°}{4}$

44. What is $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$ eqaul to

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\tan^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$ (d) $\tan^{-1}\left(\frac{2ab}{a^2+b^2}\right)$

45. Under which one of the following conditions does the equation $(\cos \beta - 1)x^2 + (\cos \beta)x + \sin \beta = 0$ in x have a real root for $\beta \in [0, \pi]$?

(a) $1 - \cos \beta < 0$ (b) $1 - \cos \beta \le 0$

(c) $1 - \cos \beta > 0$ (d) $1 - \cos \beta \ge 0$

46. In a triangle ABC, AB = 16 cm, BC = 63 cm and AC= 65 cm. What is the value of cos 2A + cos 2B + cos 2C?

(a) –1 (b) 0

(c) 1 (d) $\frac{76}{65}$

47. If $f(\theta) = \frac{1}{1+\tan\theta}$ and $\alpha + \beta = \frac{5\pi}{4}$, then what is the value of $f(\alpha) f(\beta)$?

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

48. If tan α and tan β are the roots of the equation $x^2 - 6x + 8 = 0$, then what is the value of cos(2α + 2β) ?

(a) $\frac{13}{75}$ (b) $\frac{13}{85}$

(c) $\frac{17}{85}$ (d) $\frac{19}{85}$

49. What is the value of

$$\tan 65° + 2\tan 45° - 2\tan 40° - \tan 25° ?$$

(a) 0 (b) 1

(c) 2 (d) 4

50. Consider the following statements :

A. In a triangle ABC, if $\cot A \cdot \cot B \cdot \cot C > 0$, then the triangle is an acute angled triangle.

B. In a triangle ABC, if $\tan A . \tan B . \tan C > 0$, then the triangle is an obtuse angled triangle.

Which of the statements given above is/are correct ?

(a) A Only (b) B Only
(c) Both A and B (d) Neither A nor B

51. If (a, b) is the centre and c is the radius of the circle $x^2 + y^2 + 2x + 6y + 1 = 0$, then what is the value of $a^2 + b^2 + c^2$?

(a) 19 (b) 18
(c) 17 (d) 11

52. If (1, –1, 2) and (2, 1, –1) are the end points of a diameter of a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$, then what is $u + v + w$ equal to ?

(a) –2 (b) –1
(c) 1 (d) 2

53. The number of points represented by the equation $x = 5$ on the xy-plane is

(a) Zero (b) One
(c) Two (d) Infinitely many

54. If $< l, m, n >$ are the directions cosines of a normal to the plane $2x - 3y + 6z + 4 = 0$, then what is the value of $49(7l^2 + m^2 - n^2)$?

(a) 0 (b) 1
(c) 3 (d) 71

55. A line through (1, –1, 2) with directions ratios $< 3, 2, 2 >$ meets the plane $x + 2y + 3z = 18$. What is the point of intersection of line and plane ?

(a) (4, 4, 1) (b) (2, 4, 1)
(c) (4, 1, 4) (d) (3, 4, 7)

56. If p is the perpendicular distance from origin to the plane passing through (1, 0, 0), (0, 1, 0) and (0, 0, 1), then what is $3p^2$ equal to ?

(a) 4 (b) 3
(c) 2 (d) 1

57. If the directions cosines $< l, m, n >$ of a line are connected by relation $l + 2m + n = 0$, $2l - 2m + 3n = 0$, then what is the value of $l^2 + m^2 - n^2$?

(a) $\frac{1}{101}$ (b) $\frac{29}{101}$
(c) $\frac{41}{101}$ (d) $\frac{92}{101}$

58. If a variable line passes through the point of intersection of the lines $x + 2y - 1 = 0$ and $2x - y - 1 = 0$ and meets the coordinate axes in A and B, then what is the locus of the mid-point of AB?

(a) $3x + y = 10xy$ (b) $x + 3y = 10xy$
(c) $3x + y = 10$ (d) $x + 3y = 10$

59. What is the equation to the straight line passing through the point $(-\sin\theta, \cos\theta)$ and perpendicular to the line $x\cos\theta + y\sin\theta = 9$?

(a) $x\sin\theta - y\cos\theta - 1 = 0$ (b) $x\sin\theta - y\cos\theta + 1 = 0$
(c) $x\sin\theta - y\cos\theta = 0$ (d) $x\cos\theta - y\sin\theta + 1 = 0$

60. Two points P and Q lie on line $y = 2x + 3$. These two points P and Q are at a distance 2 units from another point R(1, 5). What are the coordinates of the points P and Q?

(a) $\left(1+\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right)\left(1-\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

(b) $\left(3+\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(-1-\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

(c) $\left(1-\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(1+\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

(d) $\left(3-\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(-1+\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

61. If two sides of a square lie on the lines $2x + y - 3 = 0$ and $4x + 2y + 5 = 0$, then what is the area of the square in square units?

(a) 6.05 (b) 6.15
(c) 6.25 (d) 6.35

62. ABC is a triangle with $A(3, 5)$. The mid-points of sides AB, AC are at (–1, 2), (6, 4) respectively. What are the coordinates of centroid of the triangle ABC ?

(a) $\left(\frac{8}{3}, \frac{11}{3}\right)$ (b) $\left(\frac{7}{3}, \frac{7}{3}\right)$
(c) $\left(2, \frac{8}{3}\right)$ (d) $\left(\frac{8}{3}, 2\right)$

63. ABC is an acute angled isosceles triangle. Two equal sides AB and AC lie on the lines $7x - y - 3 = 0$ and $x + y - 5 = 0$. If θ is one of the equal angles, then what is $\cot\theta$ equal to ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) 2

64. In the parabola $y^2 = 8x$, the focal distance ·of a point P lying on it is 8 units. Which of the following statements is/are correct?

(A) The coordinates of P can be $\left(6, 4\sqrt{3}\right)$

(B) The perpendicular distance of P from the directrix of parabola is 8 units.

Select the correct answer using the code given below :

(a) A only
(b) B only
(c) Both A and B
(d) Neither A nor B

65. What is the eccentricity of the ellipse if the angle between the straight lines joining the foci to an extremity of the minor axis is 90° ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

66. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$.

If $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$, then what is the value of $\alpha + \beta + \gamma$?

(a) 8 (b) 7
(c) 6 (d) 1

67. If a vector of magnitude 2 units makes an angle $\frac{\pi}{3}$ with $2\hat{i}$, $\frac{\pi}{4}$ with $3\hat{j}$ and an acute angle θ with $4\hat{k}$ then what are the components of the vector?

(a) $(1, \sqrt{2}, 1)$ (b) $(1, -\sqrt{2}, 1)$
(c) $(1, -\sqrt{2}, -1)$ (d) $(1, \sqrt{2}, -1)$

68. Consider the following in respect of moment of a force :
(A) The moment of force about a point is independent of point of application of force.
(B) The moment of a force about a line is a vector quantity.
Which of the statements given above is/are correct ?
(a) A only (b) B only
(c) Both A and B (d) Neither A nor B

69. For any vector $\vec{r}$, what is

$(\vec{r}\cdot\hat{i})(\vec{r}\times\hat{i}) + (\vec{r}\cdot\hat{j})(\vec{r}\times\hat{j}) + (\vec{r}\cdot\hat{k})(\vec{r}\times\hat{k})$ equal to

(a) $\vec{0}$ (b) $\vec{r}$
(c) $2\vec{r}$ (d) $3\vec{r}$

70. Let $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 4 inclined at an angle $\frac{\pi}{3}$, then what is the angle between $\vec{a}$ and $\vec{a} - \vec{b}$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

71. Let $y_1(x)$ and $y_2(x)$ be two solutions of the differential equation $\frac{dy}{dx} = x$. If $y_1(0) = 0$ and $y_2(0) = 4$, then what is the number of points of intersection of the curves $y_1(x)$ and $y_2(x)$?
(a) No point (b) One point
(c) Two points (d) More than two points

72. The differential equation, representing the curve $y = e^x(a \cos.x + b \sin x)$ where a and b are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} + 2y = 0$ (b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

(c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (d) $\frac{d^2y}{dx^2} + y = 0$

73. If $f(x) = ax - b$ and $g(x) = cx + d$ are such that $f(g(x)) = g(f(x))$, then which one of the following holds?
(a) $f(d) = g(b)$
(b) $f(b) + g(d) = 0$
(c) $f(a) + g(c) = 2a$
(d) $f(d) + g(b) = 2d$

74. What is $\int_{-1}^{1} (3\sin x - \sin 3x)\cos^2 x dx$ equal to?

(a) $-\frac{1}{4}$ (b) 0

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

75. What are the order and degree respectively of the differential equation

$$\left\{2 - \left(\frac{dy}{dx}\right)^2\right\}^{0.6} = \frac{d^2y}{dx^2}?$$

(a) 2, 2 (b) 2, 3
(c) 5, 2 (d) 2, 5

76. If $\frac{dy}{dx} = 2e^x y^3$, $y(0) = \frac{1}{2}$ then what is $4y^2(2 - e^x)$ equal to?
(a) 1 (b) 2
(c) 3 (d) 4

77. Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|dx$. If $f(x) = e^{-x}$, then which one of the following is correct?
(a) $p = 2q$ (b) $p = -q$
(c) $4p = q$ (d) $p = q$

78. What is $\int_0^{\frac{\pi}{2}} \frac{a + \sin x}{2a + \sin x + \cos x} dx$ equal to?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) 1 (d) 0

79. The non-negative values of b for which the function $\frac{16x^3}{3} - 4bx^2 + x$ has neither maximum nor minimum in the range $x > 0$ is
(a) $0 < b < 1$ (b) $1 < b < 2$
(c) $b > 2$ (d) $0 \le b < 1$

80. Which one of the following is correct in respect of $f(x) = \frac{1}{\sqrt{|x| - x}}$ and $g(x) = \frac{1}{\sqrt{x - |x|}}$?
(a) $f(x)$ has some domain and $g(x)$ has no domain
(b) $f(x)$ has no domain and $g(x)$ has some domain
(c) $f(x)$ and $g(x)$ have the same domain
(d) $f(x)$ and $g(x)$ do not have any domain

DIRECTIONS (Qs. 81-82): *Consider the following for the next two (02) items that follow:*

Given that $\int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x}dx =$

$$\frac{\alpha x}{29} + \frac{\beta}{29}\ln|2\cos x + 5\sin x| + c$$

81. What is the value of α?
(a) 7 (b) 13
(c) 17 (d) 26

82. What is the value of β?
(a) 7 (b) 13
(c) 17 (d) 26

Consider the following for the next two (02) items that follow:

Let $f(x) = \frac{x}{\ln x}; (x > 1)$

83. Consider the following statements:
(A) $f(x)$ is increasing in the interval (e, ∞)
(B) $f(x)$ is decreasing in the interval $(1, e)$
(C) $9ln7 > 7ln9$
Which of the statements given above are correct?
(a) (A) and (B) only (b) (B) and (C) only
(c) (A) and (C) only (d) (A), (B) and (C)

84. Consider the following statements:
(A) $f''(e) = \frac{1}{e}$
(B) $f(x)$ attain local minimum value at $x = e$
(C) A local minimum value of $f(x)$ is e
Which of the statements given above are correct?
(a) (A) and (B) only
(b) (B) and (C) only
(c) (A) and (C) only
(d) (A), (B), and (C)

Consider the following for the next two (02) items that follow:
Let f(x) and g(x) be two functions such that

$g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$.

85. What is $g[f(x) - 3x]$ equal to ?
(a) $x^3 - \frac{1}{x^3}$ (b) $x^3 + \frac{1}{x^3}$
(c) $x^2 - \frac{1}{x^2}$ (d) $x^2 + \frac{1}{x^2}$

86. What is $f''(x)$ equal to?
(a) $-\frac{2}{x^3}$ (b) $2x + \frac{2}{x^3}$
(c) $6x + 3$ (d) $6x$

Consider the following for the next two (02) items that follow:

Let $f(x) = |x| + 1$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

Let $h(x) = \frac{f(x)}{g(x)}$

87. Consider the following statements:
(A) $f(x)$ is differentiable for all $x < 0$
(B) $g(x)$ is continuous at $x = 0.0001$
(C) The derivative of $g(x)$ at $x = 2.5$ is 1
Which of the statements given above are correct?
(a) A and B only (b) B and C only
(c) A and C only (d) A, B and C

88. What is $\lim_{x \to 0^-} h(x) + \lim_{x \to 0^+} h(x)$ equal to?
(a) $-\frac{3}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

Consider the following for the next two (02) items that follow:

Let $\varphi(a) = \int_a^{a+100\pi} |\sin x| dx$

89. What is $\varphi(a)$ equal to?
(a) 0 (b) a
(c) 100 a (d) 200

90. What is $\varphi'(a)$ equal to?
(a) 0 (b) π
(c) 100 (d) 200

DIRECTIONS (Qs. 91-92): *Consider th following for the next two(02) items that follow:*

A differentiable function $f(x)$ has a local maximum at $x = 0$. Let $y = 2f(x) + ax - b$.

91. Which of the following is/are correct?
(A) $f'(0) = 0$
(B) $f''(0) < 0$
Select the correct answer using the code given below:
(a) A only (b) B only
(c) Both A and B (d) Neither A nor B

92. The function y has a relative maxima at $x = 0$ for
(a) $a > 0, b = 0$ (b) for all b and $a = 0$
(c) for all $b > 0$ only (d) for all a and $b = 0$

DIRECTIONS (Qs. 93-94): *Consider the following for the next two (02) items that follow:*

Let $f(x) = |x - 1|$, $g(x) = [x]$ and $h(x) = f(x)g(x)$ where [.] is greatest integer function.

93. What is $\int_{-1}^{0} h(x)dx$ equal to?
(a) $-\frac{3}{2}$ (b) -1
(c) 0 (d) $\frac{1}{2}$

94. What is $\int_0^2 h(x)dx$ equal to?

(a) $-\frac{3}{2}$ (b) -1

(c) 0 (d) $\frac{1}{2}$

DIRECTIONS (Qs. 95-96): *Consider the following for the next two (02) items that follow:*

Let $\int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + c$

95. What is the value of α ?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

96. What is the value of β?

(a) $-\frac{2}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

DIRECTIONS (Qs. 97-98): *Consider the following for the next two (02) items that follow:*

The circle $x^2 + y^2 - 2x = 0$ is partitioned by line $y = x$ in two segments. Let A_1, A_2 be the areas of major and minor segments respectively.

97. What is the value of A_1?

(a) $\frac{\pi-2}{4}$ (b) $\frac{\pi+2}{4}$

(c) $\frac{3\pi-2}{4}$ (d) $\frac{3\pi+2}{4}$

98. What is the value of $\frac{2(A_1+A_2)}{A_1-3A_2}$?

(a) π (b) 1

(c) -1 (d) $-\pi$

DIRECTIONS (Qs. 99-100): *Consider the following for the next two (02) items that follow:*

Let $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$

99. What is $f(x)$ equal to?

(a) $\frac{1}{8x} - \frac{x}{8} + \frac{1}{4}$ (b) $\frac{3}{8x} + \frac{x}{8} + \frac{3}{4}$

(c) $\frac{3}{8x} + \frac{x}{8} + \frac{1}{4}$ (d) $\frac{3}{8x} - \frac{x}{8} + \frac{1}{4}$

100. What is $8\int_1^2 f(x)dx$ equal to?

(a) $\ln(8\sqrt{e})$ (b) $\ln(4\sqrt{e})$

(c) $\ln 2$ (d) $\ln 2 - 1$

101. A bag contains 5 black and 4 white balls. A man selects two balls at random. What is the probability that both of these are of the same colour?

(a) $\frac{1}{6}$ (b) $\frac{5}{108}$

(c) $\frac{4}{9}$ (d) $\frac{5}{18}$

102. If a random variable (x) follows binomial distribution with mean 5 and variance 4, $5^{23}P(X=3) = \lambda 4^{\lambda}$, then what is the value of λ?

(a) 3 (b) 5

(c) 23 (d) 25

103. From data (–4, 1), (–1, 2), (2, 7) and (3, 1), the regression line of y on x is obtained as $y = a + bx$, then what is the value of $2a + 15b$?

(a) 6 (b) 11

(c) 17 (d) 21

104. Let $x + 2y + 1 = 0$ and $2x + 3y + 4 = 0$ are two lines of regression computed from some bivariate data. If θ is the acute angle between them, then what is the value of 488 tan 3θ?

(a) 191 (b) 161

(c) 131 (d) 121

105. If two random variables X and Y are connected by relation $\frac{2X-3Y}{5X+4Y} = 4$ and X follows Binomial distribution with parameters $n = 10$ and $p = \frac{1}{2}$, then what is the variance of Y?

(a) $\frac{810}{361}$ (b) $\frac{9}{19}$

(c) $\frac{21}{361}$ (d) $\frac{121}{361}$

106. If a, b, c are in HP, then what is $\frac{1}{b-a} + \frac{1}{b-c}$ equal to?

(A) $\frac{2}{b}$

(B) $\frac{1}{a} + \frac{1}{c}$

(C) $\frac{1}{2}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

Select the correct answer using the code given below:

(a) A only (b) B only

(c) C only (d) A, B and C

107. An edible oil is sold at the rates 150, 200, 250, 300 rupees per litre in four consecutive years. Assuming that an equal amount of money is spent on oil by a family in every year during these years, what is the average price of oil in rupees (approximately) per litre?

(a) 210 (b) 220
(c) 230 (d) 240

108. If the letters of the word "TIRUPATI" are written down at random, then what is the probability that both Ts are always consecutive?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{7}$ (d) $\frac{1}{14}$

109. Let $m = 77^n$. The index n is given a positive integral value at random. What is the probability that the value of m will have 1 in the units place?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{n}$

110. Three different numbers are selected at random from the first 15 natural numbers. What is the probability that the product of two of the numbers is equal to third number?

(a) $\frac{1}{91}$ (b) $\frac{2}{455}$
(c) $\frac{1}{65}$ (d) $\frac{6}{455}$

DIRECTIONS (Qs. 111-112): *Consider the following for the next two (02) items that follow:*

Let A and B be two events such that $P(A \cup B) \geq 0.75$ and $0.125 \leq P(A \cap B) \leq 0.375$.

111. What is the minimum value of $P(A) + P(B)$?

(a) 0.625 (b) 0.750
(c) 0.825 (d) 0.875

112. What is the maximum value of $P(A) + P(B)$?

(a) 0.75 (b) 1.125
(c) 1.375 (d) 1.625

DIRECTIONS (Qs. 113-114): *Consider the following for the next two (02) items that follow:*

A, B and C are three events such that P(A) = 0.6, P(B) = 0.4, P(C) = 0.5, $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$ and $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$.

113. What is the minimum value of $P(B \cap C)$?

(a) 0.1 (b) 0.2
(c) 0.35 (d) 0.45

114. What is the maximum value of $P(B \cap C)$?

(a) 0.1 (b) 0.2
(c) 0.35 (d) 0.45

DIRECTIONS (Qs. 115-116): *Consider the following for the next two (02) items that follow:*

An unbiased coin in tossed n times. The probability of getting at least one tail is p and the probability of at least two tails is q and $p - q = \frac{5}{32}$.

115. What is the value of n?

(a) 4 (b) 5
(c) 6 (d) 7

116. What is the value of $p + q$?

(a) $\frac{57}{32}$ (b) $\frac{53}{32}$
(c) $\frac{51}{32}$ (d) 1

DIRECTIONS (Qs. 117-118): *Consider the following for the next two (02) items that follow:*

x_i	1	2	3	...	n
f_i	1	2^{-1}	2^{-2}	...	$2^{-(n-1)}$

117. What is $\sum_i^n x_i f_i$ equal to?

(a) $\frac{2^{n+1}-n+2}{2^{n-1}}$ (b) $\frac{2^{n+1}-n-2}{2^{n-1}}$
(c) $\frac{2^{n+1}+n+2}{2^{n-1}}$ (d) $\frac{2^{n+1}-n-2}{2^{n}}$

118. What is the mean of the distribution?

(a) $\frac{2^{n+1}-n+2}{2^{n}-1}$
(b) $\frac{2^{n+1}-n-2}{2^{n-1}}$
(c) $\frac{2^{n+1}-n-2}{2^{n}-1}$
(d) $\frac{2^{n+1}-n+2}{2^{n}}$

DIRECTIONS (Qs. 119-120): *Consider the following for th next two (02) items that follow:*

The marks obtained by 10 students in a Statistics test are 24, 47, 18, 32, 19, 15, 21, 35, 50 and 41.

119. What is the mean deviation of the largest five observations?

(a) 4.8 (b) 5.5
(c) 6 (d) 7.5

120. What is the variance of the largest five observations?

(a) 14.6
(b) 21.8
(c) 25.2
(d) 46.8

HINTS & SOLUTIONS

MATHEMATICS

1. (d) Given $|A| = \frac{1}{2\sqrt{2}}, |B| = \frac{1}{729}$

$|2B\ (adj\ (3A)| = |2B|\ |adj\ (3A)|$

We know that $|adj\ A| = |A|^{n-1}$ and $|kA| = k^n\ |A|$

$= 2^3\ |B|\ |3A|^{3-1} = 8\ |B|\ (3^3|A|)^2$

$= 8|B|3^6\ |A|^2 = 8.\frac{1}{729} \times 3^6 \left(\frac{1}{2\sqrt{2}}\right)^2 \quad = 8 \times \frac{1}{8} = 1$

2. (b) Since that $iz^3 + z^2 - z + i = 0$ put $z = i$

$i^4 + i^2 - i + i = 0 \Rightarrow 1 - 1 - i + i = 0$

Satisfied then, $(|z| + 1)^2 = [|i| + 1]^2 = 4$

3. (d) Number of four digit numbers begins with 1.

$= 3 \times 2 \times 1 = 6$ and repeatation of digit $= \frac{6}{3} = 2$

Digit total for each place $= (0 + 4 + 5) \times 2 = 18$

Total of all four-digit numbers beginning with 1

$= 1 \times 6 \times 1000 + 18 \times 100 + 18 \times 10 + 18 = 7998$

Number of four-digit numbers begins with 4.

$= 3 \times 2 \times 1 = 6$ and repeatation of digit $= \frac{6}{3} = 2$

Digit total for each place $= (0 + 1 + 5) \times 2 = 12$

Total of all four-digit numbers beginning with 4.

$= 4 \times 6 \times 1000 + 12 \times 100 + 12 \times 10 + 12 = 25332.$

Number of four-digit numbers begins with 5

$= 3 \times 2 \times 1 = 6$ and repetition of digit $= \frac{6}{3} = 2$

Digit total for each place $= (0 + 1 + 4) \times 2 = 10$

Total of all four-digit numbers beginning with 5

$= 6 \times 5 \times 1000 + 10 \times 100 + 10 \times 10 + 10 = 31110$

Required sum $= 7998 + 25332 + 31110 = 64440.$

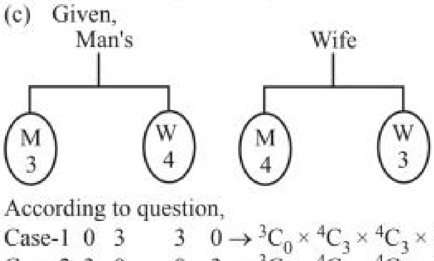
4. (a) Considering that - x, y and z are the cube roots of unity

Assume $x = 1, y = \omega$ and $z = \omega^2$

So, $xy + yz + zx = \omega + \omega^3 + \omega^2$

$= \omega + 1 + \omega^2 = 0\ (\because \omega^3 = 1)$

5. (c) Given,



According to question,

Case-1 0 3 3 0 $\rightarrow {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0 = 16$

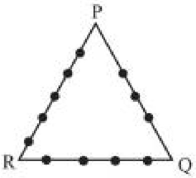
Case-2 3 0 0 3 $\rightarrow {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3 = 1$

Case-3 1 2 2 1 $\rightarrow {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 = 324$

Case-4 2 1 1 2 $\rightarrow {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 = \underline{114}$

485

6. (a)



Required number of triangles

$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 220 - (1 + 4 + 10) = 205$

7. (a) Given, $\log_b a = p \Rightarrow a = b^p$

also, $\log_b c = 2p \Rightarrow c = d^{2p}$

also, $\log_f e = 3p \Rightarrow e = f^{3p}$

Now, $(ace)^{1/p} = (b^p\ d^{2p}\ f^{3p})^{1/p} = bd^2\ f^3$

8. (c) As irrational roots occurs in pair and $-\sqrt{2}$ and $\sqrt{3}$ are roots of the given equation. So, $\sqrt{2}$ and $-\sqrt{3}$ are also roots of the given equation.

Thus, $x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

$= \left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$

$= (x^2 - 2)\ (x^2 - 3) = x^4 - 5x^2 + 6$

On comparing $a_3 = 0, a_2 = -5, a_1 = 0$ and $a_0 = 6$

9. (c) Consider $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Now, $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1 \Rightarrow |z_1 + z_2| = |z_1 - z_2|$

$\Rightarrow \quad |(x_1 + x_2) + i(y_1 + y_2)| = |(x_1 - x_2) + i\ (y_1 - y_2)|$

$\Rightarrow \quad (x_1 + x_2)^2 + (y_1 + y_2)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$\Rightarrow \quad x_1^2 + x_2^2 + 2x_1x_2 + y_1^2 + y_2^2 + 2y_1y_2 = x_1^2 + x_2^2 - 2x_1x_2 + y_1^2 + y_2^2 - 2y_1y_2$

$\Rightarrow \quad x_1\ x_2 + y_1\ y_2 = 0 \qquad ...(i)$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2} = \frac{(x_1x_2 + y_1y_2) + i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2}$$

$$= 0 + \frac{i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2} \qquad \therefore\ \text{Re}\left(\frac{z_1}{z_2}\right) = 0$$

Required answer $\text{Re}\left(\frac{z_1}{z_2}\right) + 1 = 1$

10. (b) Given, $26! = n\ 8^k = n\ 2^{3k}$

maximum power of 2

$= \left[\frac{26}{2}\right] + \left[\frac{26}{4}\right] + \left[\frac{26}{8}\right] + \left[\frac{26}{16}\right] = 13 + 6 + 3 + 1 = 23$

As power of 2 is multiple of 3

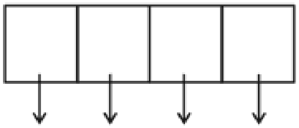
So, maximum value of $3k = 21$

$k = 7.$

11. (b) (1) We know that adj (AB) = (adj B) . (adj A)
Hence, statement 1 is not correct.
(2) We know that $AB \neq BA$
$\Rightarrow$ adj (AB) $\neq$ adj (BA)
Hence, statement 2 is not correct.
(3) We know that A . adj A = |A| I
(AB) adj (AB) – $|AB|\ I_n$
$= |AB|\ I_n - |AB|\ I_n$ = Null matrix
Hence, statement 3 is correct.

12. (d) As, adj $A^T = (\text{Adj } A)^T \Rightarrow A(\text{adj } A^T) = A(\text{Adj } A)^T$
Statement 1 is correct
We know that, If $A^n = A$ then A is identify matrix.
$\therefore$ Statements 2 and 3 are correct
Hence, all statement are correct.

13. (b) All even digit are 0, 2, 4, 6, 8



Choice$\rightarrow$ 4 5 5 5
$\therefore$ Total required numbers = $4 \times 5 \times 5 \times 5 = 500$

14. (b) Given, $(z - 100)^3 + 1000 = 0 \Rightarrow (z - 1)^3 = (-10)^3$
$\therefore$ $z - 100 = -10\ (\omega, \omega^2, 1)$ $\therefore$ $z - 100 = -10 \Rightarrow z = 90$
$z - 100 = -10w \Rightarrow z = 100 - 10\omega$
$z - 100 = -10w^2 \Rightarrow z = 100 - 10\omega^2$

15. (d) $(1 + i)^4 + (1 - i)^4 = [(1 + i)^2]^2 + [(1 - i)^2]^2$
$= (1 - 1 + 2i)^2 + (1 - 1 - 2i)^2 = (2i)^2 + (-2i)^2$
$= 4i^2 + 4i^2 = -4 - 4 = -8$

16. (a) All the diagonal elements of skew-symmetric matrix are zero.
Hence, statements 1 and 2 are correct we know that if $A A^T = I$ then A is called orthogonal matrix.
But $A A^T = A(-A) = -A^2$ $[A^T = -A]$
Hence, Statement 3 is not correct.

17. (d) We know that a number is divisible by 4 if its last 2 digits is divisible by 4.
Total number divisible by 4
$= 0 \quad 0 \quad 0 \quad 2\ 1 \times 2 \times 3 \times 1 = 1 \times 2 \times 3 \times 1 = 6$

18. (a) $2^{120} = (2^3)^{40} = 8^{40} = (1 + 7)^{40}$
$= 1 + {}^{40}C_1\ 7 + {}^{40}C_2\ 7^2 + \ldots\ldots\ {}^{40}C_{40}\ 7^{40}$
$= 1 + 7\ [{}^{40}C_1 + {}^{40}C_2\ 7 + \ldots\ldots + {}^{40}C_{40}\ 7^{39}]$
$\Rightarrow$ Remainder = 1 [using division algorithm]

19. (c) Given,

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix} = 0$$

Applying $C_3 \rightarrow C_1 + C_2 - C_3$

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,4) - C(10,n-2) \\ C(11,6) & C(11,5) & C(12,6) - (12,n) \\ C(m,7) & C(m,6) & C(m+1,7) - C(m+1,n+1) \end{vmatrix} = 0$$

Since determinant value is zero. $\therefore$ $C_3 = 0$
So, $n - 2 = 4 \Rightarrow n = 6$

20. (b) Consider

$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix}$$

Expanding along C_1
$= \cos C\ (-\sin B\ .\ \tan\ (B + C)) - \tan A\ (\sin B \cos C)$

$$= -\sin B.\cos C\left[\frac{\sin(B+C)}{\cos(B+C)} + \frac{\sin A}{\cos A}\right]$$

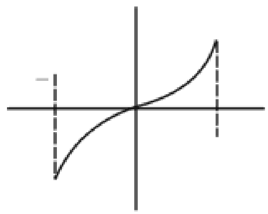
$$= -\sin B.\cos C\left[\frac{\sin(B+C).\cos A + \sin A \cos(B+C)}{\cos A.\cos(B+C)}\right]$$

$$= -\sin B \cos C \frac{\sin(A+B+C)}{\cos A.\cos(B+C)}$$

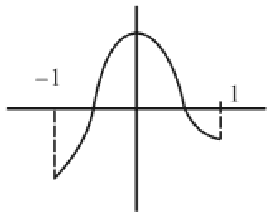
$$= \frac{-\sin B \cos C \sin(\pi)}{\cos A.\cos(B+C)} = 0$$

21. (d) Possible order of matrices with 4 entries
1×4, 2×2, 4×1
$\therefore$ Total number of matrices = $4 \times 4 \times 4 \times 4 \times 3 = 768$

22. (a)



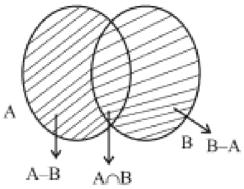
$f(x) = x|x|$
is one-one and onto



$g(x) = \cos(\pi x)$ is not one-one as any horizontal line passes g(x) twice.

23. (d) Given $xRy \Rightarrow |x + y| < 2$ in $(-1, 1)$
for reflexive
$|x + x| < 2 \Rightarrow |x| < 1$ true $\Rightarrow$ R is reflexive
for symmetric
Let $xRy \Rightarrow |x + y| < 2$
$\Rightarrow$ $|y + x| < 2 \Rightarrow yRx$ $\Rightarrow$ R is symmetric
for transitive $\because -1 < x < 1$
Let $|x + y| < 2$ and $|y + 2| < 2$
then $|x + z| < 2$
$\Rightarrow$ R is transitive.

24. (a) $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B\}$
From the venn diagram
$(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$



$\Rightarrow \quad (A \cup B) - (A \cup B) = \phi$ (Null set)

25. (a) $\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$

Expanding along R_1

$= a^2(1 - \cos^2 A) - b\sin A(b\sin A - c\cos A\sin A) + c\sin A$ $(b\sin A \cdot \cos A - c\sin A)$

$= a^2\sin^2 A - b^2\sin^2 A + bc\sin^2 A \cdot \cos A + bc\sin^2 A \cdot \cos A - c^2\sin^2 A$

$= a^2\sin^2 A - b^2\sin^2 A - c^2\sin^2 A + 2bc\sin^2 A \cdot \cos A$

$= \sin^2 A(a^2 - b^2 - c^2 + 2bc\cos A) = \sin^2 A \times 0 = 0$

26. (a) Given, a, b, c are in AP $\Rightarrow 2b = a + c$ ...(i)

also, b, c, d are in G.P $\Rightarrow c^2 = bd$ ...(ii)

also, c, d, e are in H.P $\Rightarrow \dfrac{2}{d} = \dfrac{1}{c} + \dfrac{1}{e}$ ... (iii)

from (i) and (ii)

$2c^2 = (a + c)\,d \Rightarrow \dfrac{2}{d} = \dfrac{a+c}{c^2} \Rightarrow \dfrac{1}{c} + \dfrac{1}{e} = \dfrac{a}{c^2} + \dfrac{1}{c}$ from (iii)

$\dfrac{1}{e} = \dfrac{a}{c^2} \Rightarrow c^2 = ae$

So, a, c, e are in G.P.

27. (b) Given, $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \dfrac{1}{2}\log_2(x - 1) = \log_2(x - 3)$

$\Rightarrow \log_2(x - 1) = \log_2(x - 3)^2$

$\Rightarrow x - 1 = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow x^2 - 5x - 2x + 10 = 0 \Rightarrow (x - 5)(x - 2) = 0$

$x = 2, 5$, but $x - 3 > 0 \Rightarrow x > 3$

$\Rightarrow x = 5$ only solution

28. (d) Given, $\log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right) = k$

$\Rightarrow \log_x x - \log_x y + \log_y y - \log_y x = k$

$= 1 - \log_x y + 1 - \dfrac{1}{\log_x y} = k \; [\because \log_a a = 1]$

Let $\log_x y = t$

$\therefore \quad 2 - t - \dfrac{1}{t} = k \Rightarrow 2t - t^2 - 1 = kt \Rightarrow t^2 + (k-2)t + 1 = 0$

For solution $D = b^2 - 4ac \ge 0$

$(k - 2)^2 - 4 \ge 0 \Rightarrow (k - 2)^2 \ge 4 \Rightarrow k - 2 \ge 2$ or $k - 2 \le -2$

$k \ge 4$ or $k \le 0 \Rightarrow k \ne 1$

29. (c) Given

$A = \begin{vmatrix} \sin 2\theta & -\cos 2\theta & 0 \\ \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

We know that if $|A| = \pm 1$ then A is orthogonal matrix

So $A^{-1} = A^T$ and $A^{-1} = \text{adj } A$

30. (a) $\left(1 - x^2\right)^{20}\left(-\left(\dfrac{1}{x^2} + x^2 - 2\right)\right)^{-5}$

$= -\left(1 - x^2\right)^{20}\left(x - \dfrac{1}{x}\right)^{-10}$

$= -(1 - x^2)^{20}(x^2 - 1)^{-10} \cdot x^{10} = -(1 - x^2)^{10} \cdot x^{10}$

General term $T_{r+1} = -\,{}^{10}c_r\,(1)^{10-r}(-x^2)^{10-r} \cdot x^{10}$

$= (-1)^{11-r} \cdot {}^{10}c_r\, x^{30-2r}$

$\because \quad 30 - 2r = 10 \Rightarrow r = 10$

So, coefficient $x^{10} = (-1)\,{}^{10}C_{10} = -1$.

31. (b) Given,

$T_4 = T_{3+1} = {}^nC_3\,(mx)^{n-3}\left(\dfrac{1}{x}\right)^3 = \dfrac{5}{2}$

${}^nC_3\,(m)^{n-3}\,x^{n-6} = \dfrac{5}{2}x^0$

On comparing $n - 6 = 0 \Rightarrow n = 6$

and ${}^6C_3\,m^3 = \dfrac{5}{2}$

$\Rightarrow \dfrac{6.5.4}{3.2}m^3 = \dfrac{5}{2} \Rightarrow m^3 = \dfrac{1}{8} \Rightarrow m = \dfrac{1}{2}$

$\therefore \quad mn = 6 \times \dfrac{1}{2} = 3$

32. (d) Since a, b, c are in G.P $\Rightarrow b^2 = ac$

Now, for equal $ax^2 + bx + c = 0$

$D = b^2 - 4ac = ac - 4ac = -3ac < 0$

$\therefore$ Roots are imaginary

Let $a = 2, b = 4, c = 8$

$\therefore \quad x^2 + 2x + 4 = 0 \Rightarrow x = \dfrac{-2 \pm \sqrt{4 - 16}}{2} = -1 \pm \sqrt{3}$

Ratio of roots $= \dfrac{-1 + \sqrt{3}}{-1 - \sqrt{3}} = \dfrac{\omega}{\omega^2} = \dfrac{1}{\omega}$

Product of roots $= \dfrac{\left(-1 + \sqrt{3}\right)}{2}\dfrac{\left(-1 - \sqrt{3}\right)}{2} \times 4 = \omega.\omega^2.4$

$= 4 = \dfrac{b^2}{a^2}$

33. (c) $\therefore f(x) = x^2 + mx + n \in I, \forall\, x \in I$

$f(0) = 0 + 0 + n \in I \Rightarrow x \in I$

$f(1) = 1 + m + n \in I \Rightarrow m + n \in I$

$\Rightarrow m \in I \quad (\therefore n \in I)$

So, both m and n are integer.

34. (a) Given,

$(x + y)^{2n+1} \cdot (x - y)^{2n+1} = (x^2 - y^2)^{2x+1}$

Middle term $\dfrac{2n+2}{2}, \dfrac{2n+4}{2} = (x + 1)^{th}$ term, $(n + 2)^{th}$ term

$$T_{n+1} = {}^{2n+1}C_n \left(x^2\right)^{n+1}\left(y^2\right)^n$$

$$T_{n+2} = {}^{2n+1}C_{n+1}\left(x^2\right)^n\left(y^2\right)^{x+1}$$

$${}^{2n+1}C_n\left(x^2\right)^{n+1}\left(y^2\right)^n = {}^{2n+1}C_{n+1}\left(x^2\right)^n\left(y^2\right)^{n+1}$$

$$\frac{{}^{2n+1}C_n}{{}^{2n+1}C_{n+1}} = \frac{x^{2n}y^{2n+2}}{x^{2n+2}y^{2n}} \Rightarrow \frac{(2n+1)!}{(n+1)!n!}.\frac{n!(n+1)!}{(2n+1)!} = \frac{y^2}{x^2}$$

$$\Rightarrow \frac{y^2}{x^2} = 1$$

35. (c) Given, n (A) = 5 and n (B) = 2
Number of onto function from A to B = $2^5 - {}^2C_1\,(2-1)^5$
= 32 – 2 = 30.

36. (d)

$$\frac{\sqrt{3}\cos 10° - \sin 10°}{\sin 25°.\cos 25°} = \frac{2\left(\frac{\sqrt{3}}{2}.\cos 10° - \frac{1}{2}\sin 10°\right)}{\frac{1}{2}\left(2\sin 25°.\cos 25°\right)}$$

$$= \frac{4.\sin\left(60° - 10°\right)}{\sin 50°} = \frac{4\sin 50°}{\sin 50°} = 4$$

37. (c) $\sin 9° - \cos 9° = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 9° - \frac{1}{\sqrt{2}}\cos 9°\right)$

$$= \sqrt{2}\sin\left(45° - 9°\right) = \sqrt{2}\sin 36°$$

$$= \sqrt{2}\sqrt{1-\cos^2 36°} \qquad \left[\because \cos 36° = \frac{\sqrt{5}+1}{4}\right]$$

$$= \sqrt{2}\sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)2}$$

$$= \sqrt{2}.\frac{\sqrt{10-2\sqrt{5}}}{4} = \frac{\sqrt{5-\sqrt{5}}}{2}$$

38. (d) Given
$\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A . \sin B. \text{Sin } C$
$\Rightarrow$ sin A + sin B + sin C = 0
$\Rightarrow$ a k + k b + k c = 0 $\Rightarrow$ a + b + c = 0

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ b+c+a & c & a \\ c+a+b & a & b \end{vmatrix} \left(\text{Applying } C_1 \to C_1 + C_2 + C_3\right)$$

$$= \begin{vmatrix} 0 & b & c \\ 0 & c & a \\ 0 & a & b \end{vmatrix} = 0$$

39. (c) Given $\cos^{-1}x = \sin^{-1}x \Rightarrow \frac{\pi}{2} - \sin^{-1}x = \sin^{-1}x$

$$\Rightarrow 2\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}x = \frac{\pi}{4}$$

$$\Rightarrow x = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

40. (b) Given, $(\sin\theta - \cos\theta)^2 = 2$
$\Rightarrow$ $\sin^2 + \cos^2\theta - 2\sin\theta.\cos\theta = 2$
$\Rightarrow$ $\sin 2\theta = -1$
$\Rightarrow$ Two solution possible in $(-\pi, \pi)$

41. (c) $\dfrac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)} = \dfrac{2\cos\frac{A+B}{2}.\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)}$

$$= 2\cos\left(\frac{\pi - C}{2}\right) = 2\cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = 2\sin\frac{C}{2}$$

$$= 2\sin 30° = 2 \times \frac{1}{2} = 1$$

42. (c) $\sqrt{15 + \cot^2\left(\frac{\pi}{4} - 2\cot^{-1}3\right)}$

Consider, $2\cot^{-1}3 = 2\tan^{-1}\frac{1}{3} = \tan\dfrac{\frac{2}{3}}{1-\frac{1}{9}}$

$$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right) = \tan^{-1}\frac{3}{4}$$

So, $\sqrt{15 + \cot^2\left(\tan^{-1}1 - \tan^{-1}\frac{3}{4}\right)}$

$$= \sqrt{15 + \cot^2\left(\tan^{-1}\left(\frac{1-\frac{3}{4}}{1+\frac{3}{4}}\right)\right)}$$

$$= \sqrt{15 + \cot^2\left(\tan^{-1}\frac{1}{7}\right)} = \sqrt{15 + \cot^2\left(\cot^{-1}7\right)}$$

$$= \sqrt{15 + 49} = 8$$

43. (b) sin10° . sin 50° + sin 50° . sin 250° + sin 250 . sin 10°

$= \frac{1}{2}$ (cos 40° – cos 60° + cos 200° – cos 300° + cos 240° – cos 260°)

$= \frac{1}{2}$ [cos40° – $\frac{1}{2}$ + cos (180 + 20)° – cos (360 – 60)° + cos (180 + 60)° – cos (180 + 80)°]

$= \frac{1}{2}$ [cos 40° – $\frac{1}{2}$ – cos 20° – cos 60° – cos 60° + cos 80°]

$$= \frac{1}{2}\left[\cos 40° - \cos 20° + \cos 80° - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$$

$$= \frac{1}{2} [-2 \sin 30°. \sin 10° + \cos(90° - 10°) - \frac{3}{2}]$$

$$= \frac{1}{2}\left[-\sin 10° + \sin 10° - \frac{3}{2}\right] = -\frac{3}{4}$$

44. (b) $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$

$$= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{\frac{a}{b}-1}{1+\frac{a}{b}.1}\right)$$

$$= \tan^{-1}\frac{a}{b} - \tan^{-1}\frac{a}{b} + \tan^{-1} 1 = \tan^{-1}(1) = \frac{\pi}{4}$$

45. (d) For real roots, discriminant ≥ 0

$\Rightarrow$ $(\cos B)^2 - 4 \sin B (\cos B - 1) \geq 0$

$(\cos B)^2 + 4\sin B (1 - \cos B) \geq 0$

As $-1 \leq \cos B \leq 1$ and $\sin B \geq 0$ for B $[-0, \pi]$

So, it is only possible when $1 - \cos B \geq 0$

46. (c) $\cos 2A + \cos 2B + \cos 2C$ $[\because \cos 2x = 1 - 2\sin^2 x]$

$= 1 - 2\sin^2 A + 1 - 2\sin^2 B + 1 - 2\sin^2 C$

$= 3 - 2 (\sin^2 A + \sin^2 B + \sin^2 C)$

$$= 3 - 2\left(\left(\frac{16}{65}\right)^2 + \left(\frac{63}{65}\right)^2 + 0\right) \text{ [since, } 16^2 + 63^2 = 65^2$$

$\Rightarrow [\angle B = 90°] = 3 - 2 = 1.$

47. (b) Given $\alpha + \beta = \frac{5\pi}{4}$ $\tan(\alpha + \beta) = \tan\frac{5\pi}{4}$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = 1$$

$\tan\alpha + \tan\beta + \tan\alpha . \tan\alpha = 1$

Also, $f(\theta) = \frac{1}{1 + \tan\theta}$

so, $f(\alpha).f(\beta) = \frac{1}{1+\tan\alpha} \times \frac{1}{1+\tan\beta}$

$$= \frac{1}{1 + \tan\alpha + \tan\beta + \tan\alpha.\tan\beta} = \frac{1}{1+1} = \frac{1}{2}$$

48. (b) $\tan\alpha + \tan\beta = 6$

$\tan\alpha, \tan\beta = 8$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \frac{6}{1-8} = \frac{6}{-7}$$

$\cos(2\alpha + 2\beta) = \cos 2(\alpha + \beta)$

$$= \frac{1 - \tan^2(\alpha+\beta)}{1 + \tan^2(\alpha+\beta)} = \frac{1 - \frac{36}{49}}{1 + \frac{36}{49}} = \frac{13}{85}$$

49. (c) $\tan(90 - 65)° + 2 - 2\tan 40° - \tan 25°$

$= \cot 25° - \tan 25° - 2\tan 40° + 2$

$$= \frac{\cos^2 25 - \sin^2 25}{\sin 25.\cos 25} - 2\tan 40° + 2$$

$$= \frac{2\cos 50°}{\sin 50°} - 2\tan 40° + 2$$

$= 2\cot 50° - 2\tan 40° + 2 = 2\tan 40° - 2\tan 40° + 2 = 2$

50. (a) 1. $\cot A \cot B \cot C > 0$

$\therefore$ $\cot A > 0, \cot B > 0, \cot C > 0$

$\therefore$ $0 < A < \frac{\pi}{2}, 0 < B < \frac{\pi}{2}, 0 < C < \frac{\pi}{2}$

$\therefore$ ΔABC is an acute angled triangle.

Hence, statement 1 is correct.

2. $\tan A \tan B \tan C > 0$

If ΔABC is obtuse angled triangle then two of tan A, tan B, tan C < 0

$\Rightarrow$ Two of angles are obtuse but it in not possible

Hence, statement 2 is not correct.

51. (a) $x^2 + y^2 + 2x + 6y + 1 = 0$

To obtain the center (a,b) and the radius c of a circle equation, rewrite it as $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the circle's center and r is its radius.

Consider, $x^2 + y^2 + 2x + 6y + 1 = 0$

To obtain the equation in the desired form use completing the square

$(x^2 + 2x + 1) - 1 + (y^2 + 6y + 9) - 9 + 1 = 0$

$\Rightarrow$ $(x + 1)^2 + (y + 3)^2 = 9$

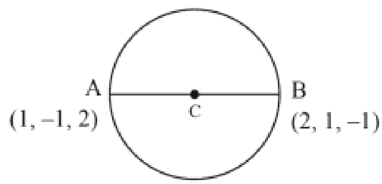
$\Rightarrow$ $(x - (-1))^2 + (y - (-3))^2 = (3)^2$

Center of the circle is $(-1, -3)$ and radius is 3.

$\therefore$ $a = -1, b = -3$ and $c = 3$

$\therefore$ $a^2 + b^2 + c^2 = (-1)^2 + (-3)^2 + (3)^2 = 1 + 9 + 9 = 19.$

52. (a) Given: Equation of sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$ with AB as its diameter and $A \equiv (1, -1, 2)$ and $B \equiv (2, 1, -1)$ So, C will be midpoint of AB.



$$C = \left(\frac{1+2}{2}, \frac{1-1}{2}, \frac{-1+2}{2}\right) = \left(\frac{3}{2}, 0, \frac{1}{2}\right)$$

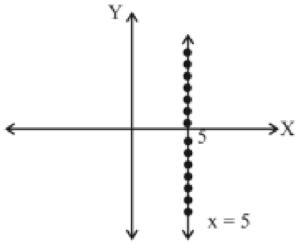
Since centre of the sphere is $(-u, -v, -w)$

On comparing we get

$$u = \frac{-3}{2}, v = 0, w = \frac{-1}{2}$$

Hence, $u + v + w = \frac{-3}{2} + 0 - \frac{1}{2} = \frac{-4}{2} = -2$

53. (d)



x = 5 represents a line parallel to y-axis so there are infinite points on x y – plane

54. (b) Equation of plane is ax + by + cz = d

Given the plane equation is 2x – 3y + 6z + 4 = 0

On comparing we get a = 2, b = – 3, c = 6, d = – 4

Here we need to find the direction cosine of a normal to the plane i.e < l, m, n >

Direction cosines of normal to plane

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{2}{\sqrt{49}} = \frac{2}{7}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-3}{\sqrt{49}} = \frac{-3}{7}$$

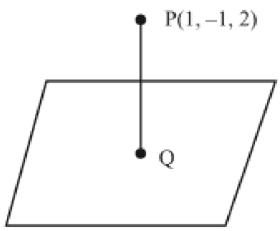
$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{6}{\sqrt{49}} = \frac{6}{7}$$

$$\therefore \quad <l, m, n> = <\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}>$$

$$\therefore \quad 49(7l^2 + m^2 - n^2) = 49\left(7\left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2 - \left(\frac{6}{7}\right)^2\right)$$

$$= 49\left(\frac{28}{49} + \frac{9}{49} - \frac{36}{49}\right) = 1$$

55. (c)



Assume the line from (1, –1, 2) meet the plane at Q

Direction ratios of the line from the point (1, –1, 2) to the given plane is <3, 2, 2>

So the equation of the line passing through P and with direction ratios will be:

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{2} = \lambda$$

x = 3λ + 1, y = 2λ – 1, z = 2λ + 2

Now, since Q lies on the plane so it must satisfy the equation of the plane.

i.e x + 2y + 3z = 18 $\therefore$ 3λ + 1 + 4λ – 2 + 6λ + 6 = 18

13λ + 5 = 18 ⇒ λ = 1

Therefore, coordinates of Q are

(3 + 1, 2 – 1, 2 + 2) = (4, 1, 4)

56. (d) Let Plane ax + by + cz + d = 0 is passing through (1, 0, 0) (0, 1, 0) & (0, 0, 1)

We will find the equation of plane

Hence a + b = 0, b + d = 0, c + d = 0

Solving above three equations, we get a = b = c = –d

Now Let a = b = c = –d = λ

So, equation of plane will be x + y + z – 1 = 0

Perpendicular distance from origin to the plane

$$(p) = \frac{|-1|}{\sqrt{(1)^2+(1)^2+(1)^2}} = \frac{1}{\sqrt{3}} \text{ unit}$$

Hence, $3p^2 = 3 \times \frac{1}{3} = 1$

57. (b) Given l + 2m + n = 0 ...(i)

and 2l – 2m + 3n = 0 ...(ii)

From (i)

l = – 2m – n ...(iii)

Substitute (iii) in (ii)

2 (–2m – n) – 2m + 3n = 0

– 4m – 2n – 2m + 3n = 0 ⇒ – 6m + n = 0 ⇒ n = 6m

From (iii) we get, l = – 2m – n = – 2m – 6m = – 8m

$$\frac{l}{-8m} = \frac{m}{m} = \frac{n}{6m}$$

$$\therefore \quad \frac{l}{-8} = \frac{m}{1} = \frac{n}{6} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(-8)^2+(1)^2+(6)^2}} = \frac{1}{\sqrt{101}}$$

$$\therefore \quad l = -\frac{8}{\sqrt{101}}, n = \frac{1}{\sqrt{101}}, n = \frac{6}{\sqrt{101}}$$

$$\text{Hence, } l^2 + m^2 - n^2 = \frac{64}{101} + \frac{1}{101} - \frac{36}{101} = \frac{29}{101}$$

58. (b)

59. (b) Rewrite the equation as follows:

y sin θ = 9 – x cos θ

$$\Rightarrow \quad y = \frac{9}{\sin\theta} - x\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad y = -x\frac{\cos\theta}{\sin\theta} + \frac{9}{\sin\theta} \qquad ...(i)$$

$\because$ The general equation of line is.

y = mx + c ...(ii)

On comparing (i) and (ii), we get

$$m = -\frac{\cos\theta}{\sin\theta}$$

Since, the slope of perpendicular line are negative inverse of each other.

So, the slope m_1 of the required line can be

$$m_1 = -\left(\frac{1}{-\frac{\cos\theta}{\sin\theta}}\right) \Rightarrow m_1 = \frac{\sin\theta}{\cos\theta}$$

Also, the line passes through the point ($-\sin\theta, \cos\theta$)
So, the equation of the required line is

$$y - \cos\theta = \frac{\sin\theta}{\cos\theta}(x + \sin\theta)$$

$\Rightarrow \quad \cos\theta\, y - \cos^2\theta = x\sin\theta + \sin^2\theta)$
$\Rightarrow \quad x\sin\theta - y\cos\theta + \sin^2\theta + \cos^2\theta = 0$
$\Rightarrow \quad x\sin\theta - y\cos\theta + 1 = 0$

60. (a) Given that P and Q lie on line $y = 2x + 3$
For P put $x = a$ then $y = 2a + 3$
For Q put $x = b$ then $y = 2b + 3$
Coordinates $P = (a, 2a + 3)$ and $Q = (b, 2b + 3)$
From $R = (1, 5)$ using distance formula, we have

$$PR = \sqrt{(a-1)^2 + (2a+3-5)^2} = 2 \quad ...(i)$$

$$QR = \sqrt{(b-1)^2 + (2b+3-5)^2} = 2 \quad ...(ii)$$

From (i) we get
$(a - 1)^2 + (2a - 2)^2 = 4$
$a^2 - 2a + 1 + 4a^2 + 4 - 8a = 4$
$5a^2 - 10a + 1 = 0$
From (ii), we get
$5b^2 - 10b + 1 = 0$
By using quadratic formula

$$(a,b) = \frac{10 \pm \sqrt{100-80}}{10} \Rightarrow a = 1 \pm \frac{2}{\sqrt{5}}$$

If $a = 1 + \frac{2}{\sqrt{5}}$ then
Coordinate of $P = (a, 2a + 3) = \left(1 + \frac{2}{\sqrt{5}}, 2 + \frac{4}{\sqrt{5}} + 3\right)$

$$= \left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

If $a = 1 - \frac{2}{\sqrt{5}}$ then
Coordinate of $P = (a, 2a + 3) = \left(1 - \frac{2}{\sqrt{5}}, 2 - \frac{4}{\sqrt{5}} + 3\right)$

$$= \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

Coordinate of $Q = \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$ or $\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$

Required coordinates of the point P and Q are

$$\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

61. (a) Given that two side of a square lie on the lines
$2x + y - 3 = 0$ ...(i)
$4x + 2y + 5 = 0$ ...(ii)
Divide (ii) by 2 we get, $2x + y + \frac{5}{2} = 0$ ...(iii)

Here $2x + y - 3 = 0$ and $2x + y + \frac{5}{2} = 0$ are parallel lines
So, length of the side of the square = Distance between parallel side $= \frac{|C_1 - C_2|}{\sqrt{a_1^2 + b_1^2}}$

where $C_1 = -3$ and $C_2 = \frac{5}{2}$
$(a_1, b_1) = (2, 1)$

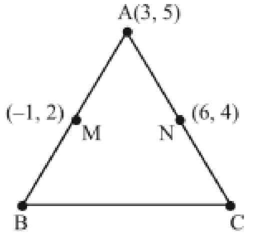
$$\Rightarrow \frac{\left|-3 - \frac{5}{2}\right|}{\sqrt{(2)^2 + (1)^2}} = \frac{\left|-\frac{11}{2}\right|}{\sqrt{5}} = \frac{11}{2\sqrt{5}}$$

So, length of the side of square $= \frac{11}{2\sqrt{5}}$ units

Required area of the square = $(\text{side})^2 = \left(\frac{11}{2\sqrt{5}}\right)^2$

= 6.05 square units.

62. (b) Given, $A \equiv (3, 5)$ and mid-points of sides AB and AC are $(-1, 2)$ and $(6, 4)$ respectively



Consider $B = (x_1, y_1)$ and $C = (x_2, y_2)$
By using mid point formula

$$M = (-1, 2) = \left(\frac{x_1 + 3}{2}, \frac{y_1 + 5}{2}\right)$$

$$N = (6, 4) = \left(\frac{x_2 + 3}{2}, \frac{y_2 + 5}{2}\right)$$

By comparing we get

$$\frac{x_1 + 3}{2} = -1, \frac{y_1 + 5}{2} = 2 \Rightarrow x_1 = -5, y_1 = -1$$

and $\frac{x_2 + 3}{2} = 6, \ \frac{y_2 + 5}{2} = 4$
$\Rightarrow \quad x_2 = 9, y_2 = 3$
So, $B = (-5, -1)$ and $C\ (9, 3)$

Centroid of $\Delta ABC = \left(\frac{3 - 5 + 9}{3}, \frac{5 - 1 + 3}{3}\right)$

$$= \left(\frac{7}{3}, \frac{7}{3}\right)$$

63. (b)
64. (c) Equation of parabola is $y^2 = 8x$
$\therefore \quad a = 2$

Since focal distance of point $P_{(x_1, y_1)}$ is $x_1 + a$

So, focal distance from point $\left(6, 4\sqrt{3}\right)$ is 6 + 2 = 8

Hence, Statement 1 is correct.

Now, distance of point P $\left(6, 4\sqrt{3}\right)$ from direction

$$PF = \sqrt{(6-2)^2 + \left(4\sqrt{3} - 0\right)^2} = \sqrt{16 + 48} = 8$$

Hence, statement 2 is also correct.

65. (d)

66. (Bonus) Given,

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Using vector triple product

$\vec{a} \times \left(\vec{b} \times \vec{a}\right) = \left(\vec{a}.\vec{a}\right)\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{a}$

$= \left(\left(\hat{i} - \hat{j} + \hat{k}\right).\left(\hat{i} - \hat{j} + \hat{k}\right)\right)\vec{b} - \left(\left(\hat{i} - \hat{j} + \hat{k}\right).\left(\hat{i} + 2\hat{j} - \hat{k}\right)\right)\vec{a}$

$= (1+1+1)\vec{b} - (1-2-1)\vec{a}$

$= 3\left(\hat{i} + 2\hat{j} - \hat{k}\right) + 2\left(\hat{i} - \hat{j} + \hat{k}\right)$

$= 5\hat{i} + 4\hat{j} - \hat{k}$ ...(i)

Given $\vec{a} \times \left(\vec{b} \times \vec{a}\right) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$

Compare with (i), we get

$\alpha = 5, \beta = -4, \gamma = -1$

$\Rightarrow \alpha + \beta + \gamma = 5 - 4 - 1 = 0$

67. (a)

68. (b) Statement 1 is not correct

We know that $t = \vec{r} \times \vec{F}$

The moment of force about a point is dependent of application of force.

Statement 2 is correct.

The moment of force about a line is a vector quantity. Because gross product of two vector is again a vector

$\vec{t} = \vec{r} \times \vec{F}$

↓ ↓ ↓

Vec Vec Vec

69. (a) Let $\vec{I} = \left(\vec{r}.\hat{i}\right)\left(\vec{r} \times \hat{i}\right) + \left(\vec{r}.\hat{j}\right) + \left(\vec{r} \times \hat{j}\right) + \left(\vec{r}.\hat{k}\right)\left(\vec{r} \times \hat{k}\right)$ ...(i)

Let $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

$\left(\vec{r}.\hat{i}\right) = \left(a\hat{i} + b\hat{j} + c\hat{k}\right).\hat{i} = a$

$\left(\vec{r}.\hat{j}\right) = b \Rightarrow \left(\vec{r}.\hat{k}\right) = c$

and $\vec{r} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 1 & 0 & 0 \end{vmatrix} = -\hat{j}(-c) - b\hat{k} = c\hat{j} - b\hat{k}$

Similarly, $\vec{r} \times \hat{j} = -c\hat{i} + a\hat{k}$ and $\vec{r} \times \hat{k} = b\hat{i} - a\hat{j}$

Now substitute in equation (i), we get

$\vec{I} = a\left(c\hat{j} - b\hat{k}\right) + b\left(-c\hat{i} + a\hat{k}\right) + c\left(b\hat{i} - a\hat{j}\right)$

$= ac\hat{j} - ab\hat{k} - cb\hat{i} + ab\hat{k} + bc\hat{i} - ac\hat{j} = \vec{O}$

70. (b)

71. (a)

72. (c) We have,

$y = e^x (a \cos x + b \sin x)$ ...(i)

$\Rightarrow \dfrac{dy}{dx} = e^x (-a \sin x + b \cos x) + (a \cos x + b \sin x) e^x$

$\Rightarrow \dfrac{dy}{dx} = e^x (-a \sin x + b \cos x) + y$ (From (i)) ....(ii)

Again differentiate w.r.t x.

$\dfrac{d^2y}{dx^2} = -e^x\left(a\cos x + b\sin x\right) + \left(-a\sin x + b\cos x\right)e^x + \dfrac{dy}{dx}$

Eliminating arbitrary constants

$\Rightarrow \dfrac{d^2y}{dx^2} = -y + \dfrac{dy}{dx} - y + \dfrac{dy}{dx}$ [From (i) and (ii)]

$\Rightarrow \dfrac{d^2y}{dx^2} - \dfrac{2dy}{dx} + 2y = 0$ is the required differential equation

73. (d) Given f(x) = ax – b ...(i)

and g(x) = cx + d .... (ii)

Now f(g(x)) = g(f(x))

$\Rightarrow$ a(g(x)) – b = c (f(x)) + d

$\Rightarrow$ a(cx + d) – b = c(ax – b) + d [From (i) and (ii)]

$\Rightarrow$ acx + ad – b = acx – bc + d

ad – b = d – bc $\Rightarrow$ ad – b + bc = d

$\Rightarrow \therefore$ f(d) + g(b) = 2d [f(d) = ad – b & g(b) = bc + d)]

74. (b) Let $I = \int_{-1}^{1}\left(3\sin x - \sin 3x\right)\cos^2 x\, dx$

$I = \int_{-1}^{1}\left(3\sin x - 3\sin x + 4\sin^3 x\right)\cos^2 x\, dx$

[$\sin 3x = 3\sin x - 4\sin^3 x$]

$I = \int_{-1}^{1} 4\sin^3 x \cos^2 x\, dx = \int_{-1}^{1} f(x)\, dx$

Since, $f(x) = 4\sin^3 x\cos^2 x$ is an odd function as $f(x) = -f(x)$.

$\Rightarrow I = \int_{-1}^{1} 4\sin^3 x \cos^2 x\, dx = 0$

$\left[\int_{-a}^{a} f(x)\, dx = 0, \text{ if } f(x) = -f(x)\right]$

75. (d) Given : $\left\{2 - \left(\dfrac{dy}{dx}\right)^2\right\}^{0.6} = \dfrac{d^2y}{dx^2}$

$\left\{2 - \left(\dfrac{dy}{dx}\right)^2\right\}^{\frac{3}{5}} = \dfrac{d^2y}{dx^2}$

Raise power of 5 both sides

$$\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^3=\left(\frac{d^2y}{dx^2}\right)^5$$

Now differential equation is free from decimal or fraction power.

Order = 2, Degree = 5.

76. (a) Given,

$$\frac{dy}{dx}=2e^x y^3 \Rightarrow \frac{dy}{y^3}=2e^x dx$$

Integrating both sides wrt x

$$\frac{-1}{2y^2}=2e^x+C \qquad ...(i)$$

Using $y(0)=\frac{1}{2}$, in equation (i)

$-2=2+c \Rightarrow c=-4$

Now, $\frac{-1}{2y^2}=2e^x-4$

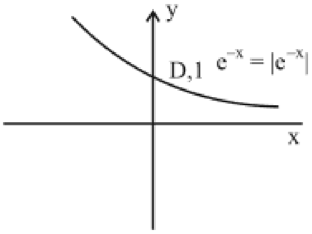
$\Rightarrow$ $4y^2(e^x)-8y^2=-1 \Rightarrow 4y^2(2-e^x)=1$

77. (d) Given,

$$P=\int_a^b f(x)\,dx \text{ and } q=\int_a^b |f(x)|\,dx$$

As $f(x)=e^{-x}$ is always +ve $\forall x \in R$

$\Rightarrow$ $|f(x)|=|e^{-x}|=e^{-x}=f(x)$



$$\Rightarrow \int_a^b f(x)\,dx=\int_a^b |f(x)|\,dx$$

Hence, p = q

78. (a) Let $I=\int_0^{\frac{\pi}{2}}\frac{a+\sin x}{2a+\sin x+\cos x}dx \qquad ...(i)$

$$\Rightarrow \quad I=\int_0^{\frac{\pi}{2}}\frac{a+\cos x\,dx}{2a+\cos x+\sin x} \qquad ...(ii)$$

$$\left[\int_o^a f(x)\,dx=\int_o^a f(a-x)\,dx\right]$$

Adding equation (i) and (ii)

$$\Rightarrow \quad 2I\int_0^{\frac{\pi}{2}}\frac{2a+\sin x+\cos x}{2a+\sin x+\cos x}dx=\int_0^{\frac{\pi}{2}}dx=[x]_0^{\frac{\pi}{2}} \Rightarrow I=\frac{\pi}{4}$$

79. (d) Given

$$f(x)=\frac{16x^3}{3}-4bx^2+x$$

$f'(x)=16x^2-8bx+1$

Since f(x) is neither maximum nor minimum, then $f'(x)\neq 0$

$\Rightarrow$ $f'(x)>0$ or $f'(x)<0$ (but here a > 0 for f'(x) so not possible) for $f'(x)>0$

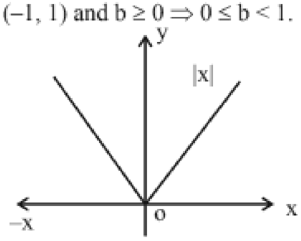
Discriminant of f'(x) is D < 0 and a > 0

$D=64b^2-64<0$

$b^2-1<0$ and b is non-negative is $b\geq 0$

$b\in(-1,1)$ and $b\geq 0 \Rightarrow 0\leq b<1$.

80. (a)



$f(x)=\frac{1}{\sqrt{|x|-x}}$ for domain of $f(x)=|x|-x>0$

ie $|x|>x \Rightarrow x\in(-\infty,0)$

$g(x)=\frac{1}{\sqrt{x-|x|}}$, for domain of g(x), $x-|x|>0$

$\Rightarrow$ $|x|<x$ (Not possible $\because$ $|x|\geq x$)

Sol (81 - 82)

Let $I=\int\frac{3\cos x+4\sin x}{2\cos x+5\sin x}dx$

Let $3\cos x+4\sin x=A(2\cos x+5\sin x)+B\left(\frac{d}{dx}(2\cos x+5\sin x)\right)$

$3\cos x+4\sin x=A(2\cos x+5\sin x)+B(-2\sin x+5\cos x)$

Comparing coefficient of sinx and cosx

$3=2A+5B \qquad ...(i)$

$4=5A-2B \qquad ...(ii)$

Solving (i) and (ii) we get

$A=\frac{26}{29}, B=\frac{7}{29}$

$$\text{Now } I=\int\frac{\frac{26}{29}(2\cos x+5\sin x)+\frac{7}{29}(-2\sin x+5\cos x)}{2\cos x+5\sin x}dx$$

$$\Rightarrow \quad I=\int\frac{26}{29}dx+\frac{7}{29}\int\frac{(-2\sin x+5\cos x)}{2\cos x+5\sin x}dx$$

$$\Rightarrow \quad I=\frac{26}{29}x+\frac{7}{29}\ln|2\cos x+5\sin x|+c$$

$\alpha=26, \beta=7$ (on comparing)

81. (d) $\alpha=26$

82. (a) $\beta=7$,

83. (d) Given

$$f(x)=\frac{x}{\ln(x)}(x>1)$$

Differentiate w.r.t x

$$f'(x)=\frac{\ln x-1}{(\ln(x))^2}$$

−ve +ve
1 e

f'(x) is the when $x \in (e, \infty)$ ie f(x) is increasing in interval $(e, \infty) \Rightarrow$ (A) is correct.

f'(x) is −ve, when $x \in (1,e)$ ie f(x) is decreasing in the interval $(1,e) \Rightarrow$ (B) is correct.

Since lnx is an increasing function.

$7^9 > 9^7$

$\ln 7^9 > \ln 9^7$

$9 \ln 7 > 7 \ln 9 \Rightarrow$ statement (C) is correct

Hence, statement (A), (B), and (C) are correct.

84. (d) Again differentiating w.r.t x

$$f''(x)=\frac{(\ln x)^2\times\frac{1}{x}-(\ln x-1)\times 2\frac{\ln x}{x}}{(\ln x)^4}$$

$$=\frac{(\ln x)^2-2(\ln x)^2+2\ln x}{x(\ln x)^4}$$

$f''(e)=\frac{1-2+2}{e\times 1}=\frac{1}{e}$ $(>0) \Rightarrow$ statement (A) is correct

As $f''(e) > 0$ (+ve)

So, f(x) attains local minima at $x = e \Rightarrow$ statement (B) is correct

A local minimum value occurs at x = e,

f(x) at x = e, is $f(e)=\frac{e}{\ln e}=e \Rightarrow$ statement (C) is correct

Hence, statement (A), (B) and (C) are correct.

Sol (85 - 86)

Given :

$$g(x)=x-\frac{1}{x} \text{ and } \log(x)=x^3-\frac{1}{x^3}$$

$$f\left(x-\frac{1}{x}\right)=x^3-\frac{1}{x^3}\Rightarrow f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$$

85. (a) $f(x) = x^3 + 3x$ ...(i)

So, $g[f(x) - 3x] = f(x) - 3x - \frac{1}{f(x)-3x}$

$$g[f(x)-3x]=x^3+3x-3x-\frac{1}{x^3+3x-3x}=x^3-\frac{1}{x^3}$$

$$g[f(x)-3x]=x^3-\frac{1}{x^3}$$

86. (d) f'□(x) = $3x^2 + 3$

f' (x) = 6x

Sol (87 - 88)

Given f(x) = |x| + 1 and g(x) = [x] − 1,

$$h(x)=\frac{f(x)}{g(x)}=\frac{|x|+1}{[x]-1}$$

87. (a) (A). $\frac{\text{for } x<0}{f(x)=-x+1}$

so f' (x) = − 1 then f(x) is differentiable $\forall$ x < 0

$\Rightarrow$ statement (A) is correct

(B) at x = .0001

g(x) is continuous every where except for integer value as [x] is continuous $\forall x \in R$ except integers.

So g(x) is continuous at x = .0001, $\Rightarrow$ statement (B) is correct

(C) g(x) = [x] − 1 [x] $\Rightarrow$ Always a integer

$$g'(x)=\frac{d}{dx}[x]-0$$

g′ (x) = 0 − 0 = 0 $\Rightarrow$ statement (C) is not correct.

$\therefore$ (A), and (B) only correct.

88. (a) Consider $\lim\limits_{x\to 0^-} h(x)+\lim\limits_{x\to 0^+} h(x)$

$$=\lim_{x\to 0^-}\frac{|x|+1}{[x]-1}+\lim_{x\to 0^+}\frac{|x|+1}{[x]-1}$$

$$=\frac{0+1}{-1-1}+\frac{0+1}{0-1} \quad \left\{\begin{matrix}[0-h]=-1\\ [0+h]=0\end{matrix}\right\}$$

$$=-\frac{1}{2}-1=\frac{-3}{2}$$

89. (d) Given

$$\phi(a)=\int_a^{a+100\pi}|\sin x|\,dx$$

Since period of |sinx| = π and |sinx| = sinx in the interval 0 to π.

$$\phi(a)=\int_0^{100\pi}|\sin x|dx=100\int_0^{\pi}|\sin x|dx=100\int_0^{\pi}\sin x\,dx$$

$$\phi(a)=100(-\cos x)_0^{\pi}=100\times 2=200$$

90. (a) $\phi(a) = 200$

Differentiating w.r.t a

$\phi'(a) = 0$

Sol (91 - 92)

Given, y = 2 f(x) + ax − b ...(i)

Differentiating w.r.t. 'x' we get,

$\Rightarrow \frac{dy}{dx} = 2f'(x) + a(1)$ ...(ii)

Again differentiating w.r.t. x, we get,

$\Rightarrow \frac{d^2y}{dx^2}=2f''(x)$ ...(iii)

91. (c) Since f(x) has a local maximum at x = 0. Then

$f'(0) = 0$ and $f''(0) < 0$ ...(iv)

So, option (c) is correct.

92. (b) Since y has a relative maxima at x = 0.

Then

$$\left(\frac{dy}{dx}\right)_{x=0} = 0$$

$\Rightarrow$ $2f'(0) + a = 0$ {from (ii)}

$\Rightarrow$ $2(0) + a = 0$ {from (iv)}

$\Rightarrow$ $a = 0$

also $\left(\frac{d^2y}{dx^2}\right)_{x=0} < 0 \Rightarrow 2f''(0) < 0$

$\Rightarrow$ $f''(0) < 0$

So, y has a relative maxima for a = 0 and all value of 'b'

Sol (93 - 94); We have given

$f(x) = |x - 1|$ ...(i)

$g(x) = [x]$ ...(ii)

and $h(x) = f(x) . g(x)$

$\Rightarrow$ $h(x) = |x - 1| . [x]$ ...(iii)

93. (a) Now

$$\int_{-1}^{0} h(x)dx = \int_{-1}^{0} f(x).g(x).dx = \int_{-1}^{0} |x-1|.[x]dx$$

$$= \int_{-1}^{0} (1-x).(-1)dx = \int_{-1}^{0} (x-1)dx$$

$$= \left[\frac{x^2}{2} - x\right]_{-1}^{0} = \left[0 - \left(\frac{1}{2}+1\right)\right] = -\frac{3}{2}$$

94. (d) Now

$$\int_{0}^{2} h(x)dx = \int_{0}^{1} |x-1|[x]dx + \int_{1}^{2} |x-1|[x]dx$$

$$= \int_{0}^{1} (1-x).(0)dx + \int_{1}^{2} (x-1)(1)dx$$

$$= \int_{1}^{2} (x-1)dx = \left[\frac{x^2}{2} - x\right]_{1}^{2}$$

$$= \left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right) = \frac{1}{2}$$

Sol. (95 - 96) We have given,

$$\int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + C \quad ...(i)$$

$$\text{Let } I = \int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \int \frac{\left(\sqrt{x+1} + \sqrt{x-1}\right)dx}{\left(\sqrt{x+1} - \sqrt{x-1}\right)\left(\sqrt{x+1} + \sqrt{x-1}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \sqrt{(x+1)}\,dx + \frac{1}{2}\int \sqrt{x-1}\,dx$$

$$\Rightarrow I = \frac{1}{2}.\frac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{1}{2}\frac{(x-1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$$

$$\Rightarrow I = \frac{1}{3}(x+1)^{\frac{3}{2}} + \frac{1}{3}(x-1)^{\frac{3}{2}} + C$$

$$\Rightarrow \int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \frac{1}{3}(x+1)^{-} + \frac{1}{3}(x-1)^{-} + C \quad ...(ii)$$

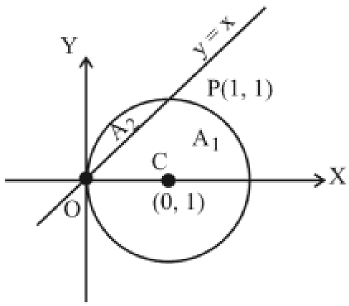
95. (a) Comparing (i) and (ii) we get

$$\alpha = \frac{1}{3}$$

96. (c) Comparing (i) & (ii) we get

$$\beta = \frac{1}{3}$$

Sol (97 - 98)



Equation of circle

$x^2 + y^2 - 2x = 0$

$\Rightarrow$ $(x - 1)^2 + y^2 = 1$

$\Rightarrow$ $y^2 = 1 - (x - 1)^2$

$$\text{Area of minor segment } (A_2) = \int_{0}^{1}\left(\sqrt{1-(x-1)^2} - x\right)dx$$

$$\Rightarrow A_2 = \left[\frac{(x-1)}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1) - \frac{x^2}{2}\right]_0^1$$

$$\Rightarrow A_2 = \left[\left(0 + \frac{1}{2}(0) - \frac{1}{2}\right) - \left(0 + \frac{1}{2}\sin^{-1}(-1) - 0\right)\right]$$

$$\Rightarrow A_2 = \frac{\pi - 2}{4}$$

97. (d) Since Area of given circle $= \pi(1)^2 = \pi$

$\Rightarrow$ $A_1 + A_2 = \pi$

$$\Rightarrow A_1 = \pi - \frac{\pi - 2}{4} = \frac{3\pi + 2}{4}$$

98. (a) $$\frac{2(A_1 + A_2)}{A_1 - 3A_2} = \frac{2(\pi)}{\left(\frac{3\pi+2}{4}\right) - \frac{3(\pi-2)}{4}} = \pi$$

Sol (99 - 100)

99. (d) We have given, $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$ ...(i)

Replacing (x) by $\left(\frac{1}{x}\right)$, we get,

$3f\left(\frac{1}{x}\right) + f(x) = x + 1$ ...(ii)

Now, on 3 × [Eq. (i)] - Eq. (ii), we get

$8f(x) = \frac{3}{x} + 3 - x - 1$

$\Rightarrow f(x) = \frac{3}{8x} - \frac{x}{8} + \frac{1}{4}$ ...(iii)

100. (a) $8\int_1^2 f(x)dx = 8\int_1^2 \left(\frac{3}{8x} - \frac{x}{8} + \frac{1}{4}\right)dx$

$= \int_1^2 \left(\frac{3}{x} - x + 2\right)dx = \left[3\ln x - \frac{x^2}{2} + 2x\right]_1^2$

$= \left(3\ln 2 - \frac{4}{2} + 4\right) - \left(3\ln 1 - \frac{1}{2} + 2\right)$

$= \frac{1}{2} + 3\ln 2 = \frac{1}{2}\ln e + \ln 2^3$

$= \ln\sqrt{e} + \ln 8 = \ln\left(8\sqrt{e}\right)$

101. (c) No. of ways to choose two black from the bag = $^5C_2$
No. of ways to choose two white ball from the bag = $^4C_2$
Number of ways to choose both of the ball of same colour = $^5C_2 + {}^4C_2$
So, the required probability is

$P(E) = \frac{^5C_2 + {}^4C_2}{^9C_2} = \frac{10+6}{36}$

$P(E) = \frac{16}{36} = \frac{4}{9}$

102. (c) Given, Mean = 5 and Variance = 4
5^{23} P(X = 3) = $\lambda 4^{\lambda}$ ....(i)
$\because$ Mean = 5 ⇒ np = 5
Variance = 4 ⇒ npq = 4

$\Rightarrow 5q = 4 \Rightarrow q = \frac{4}{5}$

Now, $p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5}$

$\because$ np = 5 ⇒ n = 25
Now, $5^{23}P(X = 3) = 5^{23}\left[{}^nC_3\, p^3\, q^{n-3}\right]$

$= 5^{23}\left[{}^{25}C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^{22}\right]$

$= 5^{23}\left[\frac{25\times24\times23}{6}\times\frac{1}{5^3}\times\frac{4^{22}}{5^{22}}\right] = 4 \times 23 \times 4^{22}$

$\Rightarrow 5^{23}P(X = 3) = 23 \times 4^{23}$ ...(ii)
From equation (i) & (ii) :-
$\lambda = 23$

103. (b) Given data

x	y	xy	x^2
– 4	1	–4	16
– 1	2	– 2	1
2	7	14	4
3	1	3	9

$\Sigma x = -4 - 1 + 2 + 3 = 0$
$\Sigma y = 1 + 2 + 7 + 1 = 11$
$\Sigma xy = -4 - 2 + 14 + 3 = 11$
$\Sigma x^2 = 16 + 1 + 4 + 9 = 30$

$\bar{x} = \frac{\Sigma x}{4} = \frac{0}{4} = 0$

$\bar{y} = \frac{\Sigma y}{4} = \frac{11}{4}$

$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x\,\Sigma y}{4}}{\Sigma x^2 - \frac{(\Sigma x)^2}{4}} = \frac{11-0}{30-0} = \frac{11}{30}$

Equation of regression line of y on x will be

$y - \bar{y} = b_{yx}(x - \bar{x})$

$\Rightarrow y - \frac{11}{4} = \frac{11}{30}(x - 0) \Rightarrow y = \frac{11}{30}x + \frac{11}{4}$

$\therefore a = \frac{11}{4}$ and $b = \frac{11}{30}$

$2a + 15b = 2\times\frac{11}{4} + 15\times\frac{11}{30} = 11$

104. (a) Slope of line x + 2y + 1 = 0 ⇒ $m_1 = -1/2$
Slope of line 2x + 3y + 4 = 0 ⇒ $m_2 = -2/3$

Then, $\tan\theta = \left|\frac{-\frac{1}{2} + \frac{2}{3}}{1 + \frac{1}{2}\times\frac{2}{3}}\right| = \frac{1}{8}$

And, $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$= \frac{\frac{3}{8} - \frac{1}{8^3}}{1 - \frac{3}{64}} = \frac{192-1}{8^3}\times\frac{8^2}{61} = \frac{191}{8\times61}$

Therefore,

$488\tan 3\theta = 488\times\frac{191}{8\times61} = 191$

Sol. (117-118)

117. (b) Given

$$\Sigma x_i f_i = 1.1 + \frac{2}{2} + \frac{3}{2^2} + ... + \frac{n}{2^{(n-1)}} \quad ...(i)$$

and $\frac{1}{2}\Sigma x_i f_i = \frac{1}{2} + \frac{2}{2^2} + ... + \frac{n-1}{2^{n-1}} + \frac{n}{2^n}$ ...(ii)

Subtracting (ii) from (i), we get

$$\Rightarrow \left(1 - \frac{1}{2}\right)\Sigma x_i f_i = \left[1.1 + \frac{2-1}{2} + \frac{3-2}{2^3} + \frac{4-3}{2^4} + ... + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \left[1 + \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^4} + ... + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} - \frac{1}{2^n} = \frac{2\left(2^n - 1\right)}{2^n} - \frac{n}{2^n}$$

$$\Rightarrow \Sigma x_i f_i = \frac{2^{n+1} - n - 2}{2^{n-1}}$$

118. (c) $\Sigma f_i = 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{(n-1)}}$

$$\Sigma f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = \frac{2^n - 1}{2^{n-1}}$$

Calculating mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2^{n+1} - n - 2}{2^{n-1}} . \frac{2^{n-1}}{2^n - 1}$

$$= \frac{2^{n+1} - n - 2}{2^n - 1}$$

Sol. (119 - 120);

Ascending data is, 15, 18, 19, 21, 24, 32, 35, 41, 47, 50

Largest 5 observation is, 32, 35, 41, 47, 50

Mean $\mu = \frac{32 + 35 + 41 + 47 + 50}{5} = \frac{205}{5} \Rightarrow \mu = 41$

119. (c) Mean deviation $= \frac{\Sigma|x_i - \mu|}{n}$

$$= \frac{|32 - 41| + |35 - 41| + |141 - 41| + |47 - 41| + |50 - 41|}{5}$$

$$= \frac{9 + 6 + 0 + 6 + 9}{5} = 6$$

120. (d) $\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{5} = \frac{9^2 + 6^2 + 0^2 + 6^2 + 9^2}{5}$

$$= \frac{234}{5} = 46.8$$