# NDA/NA SOLVED PAPER 2023-I

## PAPER -I: MATHEMATICS

1. If $\omega$ is a non -real cube root of 1, then what is the value of $\left|\frac{1-\omega}{\omega+\omega^2}\right|$?

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) 1 (d) $\frac{4}{\sqrt{3}}$

2. What is the number of 6 -digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6?

(a) 96 (b) 120
(c) 192 (d) 312

3. What is the binary number equivalent to decimal number 1011?

(a) 1011 (b) 111011
(c) 11111001 (d) 111110011

4. Let A be a matri x of order 3 × 3 and |A| = 4. If $|2\,\text{adj}(3A)| = 2^{\alpha} 3^{\beta}$, then what is the value of ( $\alpha+\beta$)?

(a) 12 (b) 13
(c) 17 (d) 24

5. If $\alpha$ and $\beta$ are the distinct roots of equation $x^2 - x + 1 = 0$, then wha t is the value of $\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right|$?

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) 1 (d) $\frac{1}{\sqrt{3}}$

6. Let A and B be symmetric matrices of same order, then which one of the following is correct regarding (AB–BA)?

1. Its diagonal entries are equal but nonzero
2. The sum of its non -diagonal entries is zero

Select the correct answer using the code given below:

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

7. Consider the following stateme nts in respect of square matrices A,B,C each of same order then :

1. $AB = AC \Rightarrow B = C$ if A is non singular
2. If $BX = CX$ for every column matrix X having n rows then $B = C$

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

8. The system of linear equations $x + 2y + z = 4, 2x + 4y + 2z = 8$ and $3x + 6y + 3z = 10$ has

(a) a unique solution (b) infinite many solutions
(c) no solution (d) exactly three solutions

9. Let $AX = B$ be a system of 3 linear equations with 3 - unknowns. Let $X_1$ and $X_2$ be its two distinct solutions. If the combination $aX_1 + bX_2$ is a solution of AX=B; where a, b are real numbers, then which one of the following is correct?

(a) $a = b$ (b) $a + b = 1$
(c) $a + b = 0$ (d) $a - b = 1$

10. What is the sum of the roots of the equation $\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0$?

(a) $a + b + c$ (b) $a - b + c$
(c) $a + b - c$ (d) $a - b - c$

11. If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of $(a + b)$?

(a) –11 (b) –3
(c) 0 (d) 3

12. If $z=\frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ where $i=\sqrt{-1}$, then what is the argument of $z$?

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{6}$

13. If a, b, c are in AP, then what is

$$\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+3 \end{vmatrix}$$ equal to?

(a) –1 (b) 0
(c) 1 (d) 2

14. If $\log_x a, a^x$ and $\log_b x$ are in GP, then what is x equal to?

(a) $\log_a(\log_b a)$ (b) $\log_b(\log_b b)$
(c) $\frac{\log_a(\log_b a)}{2}$ (d) $\frac{\log_b(\log_a b)}{2}$

15. If $2^{\frac{1}{c}}, 2^{\frac{b}{ac}}, 2^{\frac{1}{a}}$ are in GP, then which one of the following is correct?

(a) a, b, c are in AP (b) a, b, c are in GP
(c) a, b, c are in HP (d) ab, bc, ca are in AP

16. The first and the second terms of an AP are $\frac{5}{2}$ and $\frac{23}{12}$ respectively. If $n^{th}$ term is the largest negative term, what is the value of n?

(a) 5 (b) 6
(c) 7 (d) n cannot be determined

17. For how many integral values of k, the equation $x^2-4x+k=0$, where k is an integer has real roots and both of them lie in the interval (0, 5)?

(a) 3 (b) 4
(c) 5 (d) 6

18. In an AP, the first term is x and the sum of the first n terms is zero. What is the sum of next m terms?

(a) $\frac{mx(m+n)}{n-1}$ (b) $\frac{mx(m-n)}{1-n}$
(c) $\frac{nx(m+n)}{m-1}$ (d) $\frac{nx(m+n)}{1-m}$

19. Consider the following s tatements:

1. (25)! +1 is divisible by 26
2. (6)! +1 is divisible by 7

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

20. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) 1 (d) 2

21. How many real numbers satisfy the equation $|x-4|+|x-7|=15$ ?

(a) Only one (b) Only two
(c) Only three (d) Infinitely many

22. A mapping $f: A \to B$ defined as $f(x)=\frac{2x+3}{3x+5}$, $x \in A$. If f is to be onto, then what are A and B equal to?

(a) $A = R \setminus \left\{-\frac{5}{3}\right\}$ and $B = R \setminus \left\{-\frac{2}{3}\right\}$

(b) $A = R$ and $B = R \setminus \left\{-\frac{5}{3}\right\}$

(c) $A = R \setminus \left\{-\frac{3}{2}\right\}$ and $B = R \setminus \{0\}$

(d) $A = R \setminus \left\{-\frac{5}{3}\right\}$ and $B = R \setminus \left\{\frac{2}{3.}\right\}$

23. $\alpha$ and $\beta$ are distinct real roots of the quadratic equation $x^2+ax+b=0$. Which of the following statements is/are sufficient to find $\alpha$?

1. $\alpha+\beta=0, \alpha^2+\beta^2=2$
2. $\alpha\beta^2=-1, \alpha=0$

Select the correct answer using the code given be low:

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

24. If the sixth term in the binomial expansion of $\left(x^{\frac{-8}{3}}+x^2\log_{10}x\right)^8$ is 5600, then what is the value of x?

(a) 6 (b) 8
(c) 9 (d) 10

25. How many terms are there in the expan sion of $(3x-y)^4 (x+3y)^4$?

(a) 9 (b) 12

(c) 15 (d) 17

26. p, q, r and s and in AP such that $p+s=8$ and $qr=15$. What is the difference between largest and smallest numbers?

(a) 6 (b) 5

(c) 4 (d) 3

27. Consider the following statements for a fixed natural number n;

1. C(n, r) is greatest if n =2r

2. C(n, r) is greatest if n =2r–1 and n =2r-1 and n=2r+1

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

28. m parallel lines cut n parallel lines giving rise to 60 parallelograms. What is the value of ( m + n)?

(a) 6 (b) 7

(c) 8 (d) 9

29. Let x be the number of permutations of the word 'PERMUTATIONS' and y be the number of permutations of the word 'COMBINATIONS'. Which one of the following is correct?

(a) $x = y$ (b) $y = 2x$

(c) $x = 4y$ (d) $y = 4x$

30. 5-digit numbers are formed using the digits 0,1,2,4,5 without repetition. What is the percentage of numbers which are greater than 50,000?

(a) 20% (b) 25%

(c) $\frac{100}{3}\%$ (d) $\frac{110}{3}\%$

Directions for (31 to 32): Consider the following for the next two (02) items that follow:

Let $\sin\beta$ be the GM of $\sin\alpha$ and $\cos\alpha$; **tan$\gamma$ be the AM of** $\sin\alpha$ and $\cos\alpha$.

31. What is $\cos 2\alpha$ equal to?

(a) $(\cos\alpha - \sin\alpha)^2$ (b) $(\cos\alpha + \sin\alpha)^2$

(c) $(\cos\alpha + \sin\alpha)^3$ (d) $\frac{(\cos\alpha - \sin\alpha)^2}{2}$

32. What is the value of $\sec 2\gamma$?

(a) $\frac{3-\sin 2\alpha}{5+2\sin 2\alpha}$ (b) $\frac{5-\sin 2\alpha}{3-\sin 2\alpha}$

(c) $\frac{3-2\sin 2\alpha}{4+\sin 2\alpha}$ (d) $\frac{3-\sin 2\alpha}{4+3\sin 2\alpha}$

Directions for (33 to 34): Consider the following for the next two (02) items that follow:

A flagstaff 20 m long standing on a pillar 10 m high subtends an angle $\tan^{-1}(0.5)$ at a point P on the ground. Let $\theta$ be the angle subtended by the pillar at this point P.

33. If x is the distance of P form bottom of the pillar, then consider the following statements:

1. x can take two values which are in the ratio 1 : 3

2. x can be equal to h eight of the flagstaff

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

34. What is a possible value of $\tan\theta$?

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Directions for (35 to 36): Consider the following for the next two (02) items that follow:

The perimeter of a triangle ABC is 6 times the AM of sine of angles of the triangle.

Further $BC = 3$ and $CA = 1$.

35. What is the perim eter of the triangle?

(a) $\sqrt{3}+1$ (b) $\sqrt{3}+2$

(c) $\sqrt{3}+3$ (d) $2\sqrt{3}+1$

36. Consider the following statements:

1. ABC is right angled triangle

2. The triangle of the triangle are in AP

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 or 2

Directions for (37 to 38): Consider the following for the next two (02) items that follow:

Let $x = \frac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \le \frac{\pi}{2}$

37. What is the minimum value of x ?

(a) 1 (b) 2

(c) 3 (d) 4

38. At what value of A does x attain the minimum value?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Directions for (39 to 40): Consider the following for the next two (02) items that follow:

In the triangle ABC $a^2 + b^2 + c^2 = ac + \sqrt{3}bc$

39. What is the nature of the triangle?
 (a) Equilateral
 (b) Isosceles
 (c) Right angled triangle
 (d) Scalene but not right angled

40. If c = 8, what is the area of the triangle?
 (a) $4\sqrt{3}$ (b) $6\sqrt{3}$ (c) $8\sqrt{3}$ (d) $12\sqrt{3}$

Directions for (41 to 42): Consider the following for the next two (02) items that follow: Consider the function $f(x) = |x-2| + |3-x| + |4-x|$ where $x \in R$

41. At what value x does the function attain minimum value ?
 (a) 2 (b) 3 (c) 4 (d) 0

42. What is the minimum value of the function ?
 (a) 2 (b) 3
 (c) 4 (d) 0

Directions for (43 to 44): Consider the following for the next two (02) items that follow: Consider the sum $S = 0! + 1! + 2! + 3! + 4! + ... + 100!$

43. If the sum S is divided by 8, what is the remainder?
 (a) 0 (b) 1
 (c) 2 (d) Cannot be determined

44. If the sum S is divided by 60, what is the remainder?
 (a) 1 (b) 3 (c) 17 (d) 34

Directions for (45 to 46): Consider the following for the next two (02) items that follow:

In a triangle PQR, P is the largest angle and $\cos p = \frac{1}{3}$.

Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively such that the lengths PN, QL and RM are n, n + 2, n + 4 respectively where n is an integer.

45. What is the value of n?
 (a) 4 (b) 6 (c) 8 (d) 10

46. What is the length of the smallest side ?
 (a) 12 (b) 14 (c) 16 (d) 18

Directions for (47 to 48): Consider the following for the next two (02) items that follow:

Given that $\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

47. The given equation can be reduced to
 (a) $\sin^2 2x - 44 \sin 2x + 36 = 0$
 (b) $\sin^2 2x + 44 \sin 2x - 36 = 0$
 (c) $\sin^2 2x - 22 \sin 2x + 18 = 0$
 (d) $\sin^2 2x + 22 \sin 2x - 18 = 0$

48. If $\sin 2x = a - b\sqrt{c}$, where a and b are natural number, and c is prime number, then what is the value of a − b+2c ?
 (a) 0 (b) 14 (c) 21 (d) 28

Directions for (49 to 50): Consider the following for the next two (02) items that follow:

A quadratic equation is given by

$\left(3+2\sqrt{2}\right)x^2 - \left(4+2\sqrt{3}\right)x + \left(8+4\sqrt{3}\right) = 0$

49. What is the HM of the roots of the equation?
 (a) 2 (b) 4
 (c) $2\sqrt{2}$ (d) $2\sqrt{3}$

50. What is the GM of the roots of the equation?
 (a) $\sqrt{2}\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
 (b) $\sqrt{2}\left(\sqrt{6} + \sqrt{3} - \sqrt{2} - 1\right)$
 (c) $\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
 (d) $\left(\sqrt{6} + \sqrt{3} + \sqrt{2} - 1\right)$

Directions for (51 to 52): Consider the following for the next two (02) items that follow:

Let $\Delta(a, b, c, \alpha) = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$

51. if $\Delta(a,b,c,\alpha) = 0$ for every $\alpha > 0$, then which one of the following is correct?
 (a) a, b, c are in AP (b) a, b, c are in GP
 (c) a, 2b, c are in AP (d) a, 2b, c are in GP

52. If $\Delta(7,4,2,\alpha) = 0$, then $\alpha$ is a root of which one of the following equations?
 (a) $7x^2 + 4x + 2 = 0$ (b) $7x^2 - 4x + 2 = 0$
 (c) $7x^2 + 8x + 2 = 0$ (d) $7x^2 - 8x + 2 = 0$

Directions for (53 to 54): Consider the following for the next two (02) items that follow:

Given that $m(\theta) = \cot^2\theta + n^2 \tan^2\theta + 2n$, where n is a fixed positive real number.

53. What is the least value of $m(\theta)$?
 (a) n (b) 2n
 (c) 3n (d) 4n

54. Under what condition does m attain the least value?

(a) $n = \tan^2\theta$ (b) $n = \cot^2\theta$

(c) $n = \sin^2\theta$ (d) $n = \cos^2\theta$

Directions for ( 55 to 56): Consider the following for the next two (02) items that follow:

A quadrilateral is formed by the lies $x = 0, y = 0, x + y = 1$ and $6x + y = 3$.

55. What is the equation of diagonal through origin?

(a) $3x + y = 0$ (b) $2x + 3y = 0$

(c) $3x - 2y = 0$ (d) $3x + 2y = 0$

56. What is the equation of other diagonal?

(a) $x + 2y - 1 = 0$ (b) $x - 2y - 1 = 0$

(c) $2x + y + 1 = 0$ (d) $2x + y - 1 = 0$

Directions for ( 57 to 58): Consider the following for the next two (02) items that follow:

$P(x, y)$ is any point on the ellipse $x^2 + 4y^2 = 1$. Let E, F be the foci of the ellipse.

57. What is PE + PF equal to?

(a) 1 (b) 2

(c) 3 (d) 4

58. Consider the following points:

1. $\left(\frac{\sqrt{3}}{2}, 0\right)$ 2. $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$ 3. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

Which of the above points lie on latus rectum of ellipse?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

Directions for ( 59 to 60): Consider the following for the next two (02) items that follow:

The line $y = x$ partitions the circle $(x - a)^2 + y^2 = a^2$ in t wo segments.

59. What is the area of minor segment?

(a) $\frac{(\pi - 2)a^2}{4}$ (b) $\frac{(\pi - 1)a^2}{4}$

(c) $\frac{(\pi - 2)a^2}{2}$ (d) $\frac{(\pi - 1)a^2}{2}$

60. What is the area of major segment?

(a) $\frac{(3\pi - 2)a^2}{4}$ (b) $\frac{(3\pi + 2)a^2}{4}$

(c) $\frac{(3\pi - 2)a^2}{4}$ (d) $\frac{(3\pi + 2)a^2}{2}$

Directions for ( 61 to 62): Consider the following for the next two (02) items that follow:

Let A(1,–1,2) and B(2,1,–1) be the end points of the diameter of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$.

61. What is u + v + w equal to?

(a) –2 (b) –1 (c) 1 (d) 2

62. If P(x, y,z) is any point on the sphere, then what is $PA^2 + PB^2$ equal t o?

(a) 15 (b) 14 (c) 13 (d) 6.5

Directions for ( 63 to 64): Consider the following for the next two (02) items that follow:

Consider two lines whose direction ratios are (2, –1, 2) and (k, 3, 5). They are inclined at an angle $\frac{\pi}{4}$.

63. What is the value of k ?

(a) 4 (b) 2

(c) 1 (d) –1

64. What are the direction ratios of a line which is perpendicular to both the lines?

(a) (1, 2, 10) (b) (–1, –2, 10)

(c) (11, 12, –10) (d) (11, 2, –10)

Directions for ( 65 to 66): Consider the following for the next two (02) items that follow:

Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a} \cdot \vec{b} = 27$ and $\vec{a} \times \vec{b} = 9\vec{c}$

65. What is $\vec{b}$ equal to?

(a) $3\hat{i} + 4\hat{j} + 2\hat{k}$ (b) $5\hat{i} + 2\hat{j} + 2\hat{k}$

(c) $5\hat{i} - 2\hat{j} + 6\hat{k}$ (d) $3\hat{i} + 3\hat{j} + 4\hat{k}$

66. What is the angle between $(\vec{a} + \vec{b})$ and $\vec{c}$ ?

(a) $\frac{\pi}{2}$ b. $\frac{\pi}{3}$

c. $\frac{\pi}{4}$ d. $\frac{\pi}{6}$

Directions for ( 67 to 68): Consider the following for the next two (02) items that follow:

Let a vector $\vec{a} = 4\hat{i} - 8\hat{j} + \hat{k}$ make angles $\alpha, \beta, \gamma$ with positive directions of x,y,z axes respectively.

67. What is cos $\alpha$ equal to?

(a) $\frac{1}{3}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{2}{3}$

68. What is $\cos 2\beta + \cos 2\gamma$ equal to?

(a) $-\frac{32}{81}$ (b) $-\frac{16}{81}$ (c) $\frac{16}{81}$ (d) $\frac{32}{81}$

Directions for ( 69 to 70): Consider the following for the next two (02) items that follow:

The position vectors of two points A and B are $\hat{i} - \hat{j}$ and $\hat{j} + \hat{k}$ respectively.

69. Consider the following points:

1. (–1, –3, 1) 2. (–1, 3, 2) 3. (–2, 5, 3)

Which of the above points lie on the line joining A and B?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

70. What is the magnitude of $\overrightarrow{AB}$?

(a) 2 (b) 3 (c) $\sqrt{6}$ (d) $\sqrt{3}$

Directions for ( 71 to 73): Consider the following for the next three (03) items that follow: Let $f(x) = Pe^{x} + Qe^{2x} + Re^{3x}$, where P, Q, R are real numbers. Further $f(0) = 6$, $f'(\ln 3) = 282$ and $\int_0^{\ln 2} f(x)dx = 11$

71. What is the value of Q?

(a) 1 (b) 2
(c) 3 (d) 4

72. What is the value of R?

(a) 1 (b) 2
(c) 3 (d) 4

73. What is $f'(0)$ equal to?

(a) 18 (c) 16
(c) 15 (d) 14

Directions for ( 74 to 75): Consider the following for the next two (02) items that follow:

Suppose E is the differential equation representing family of curves $y^2 = 2cx + 2c\sqrt{c}$ where c is a positive parameter.

74. What is the order of the differential equation?

(a) 1 (b) 2
(c) 3 (d) 4

75. What is the degree of the differential equation?

(a) 2 (b) 3
(c) 4 (d) Degree does not exist

Directions for ( 76 to 78): Consider the following for the next three (03) items that follow:

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

76. What is $f(0)$ equal to?

(a) –1 (b) 0
(c) 1 (d) 2

77. What is $\lim_{x\to\infty} \frac{f(x)}{x}$ equal to?

(a) –1 (b) 0
(c) 1 (d) 2

78. What is $\lim_{x\to\infty} \frac{f(x)}{x^2}$ equal to?

(a) –1 (b) 0 (c) 1 (d) 2

Directions for ( 79 to 80): Consider the following for the next two (02) items that follow:

Let $f(x) = \sin[\pi^3]x + \cos[-\pi^2]x$ where [.] is a greatest integer function.

79. What is $f\left(\frac{\pi}{2}\right)$ equal to?

(a) –1 (b) 0
(c) 1 (d) 2

80. What is $f\left(\frac{\pi}{4}\right)$ equal to?

(a) $-\frac{1}{\sqrt{2}}$ (b) –1
(c) 1 (d) $\frac{1}{\sqrt{2}}$

Directions for ( 81 to 83): Consider the following for the next three (03) items that follow:

Let $I_1 = \int_0^{\pi} \frac{x}{1+\cos^2 x}dx$ and $I_2 = \int_0^{\pi} \frac{1}{1+\sin^2 x}dx$

81. What is the value of $\frac{I_1 + I_2}{I_1 - I_2}$?

(a)1 (b) π
(c) $\pi^2$ (d) $\frac{\pi+1}{\pi-1}$

82. What is the value of $8I_1^2$?

(a)π (b) $\pi^2$
(c) $\pi^3$ (d) $\pi^4$

83. What is the value of $I_2$?

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2\sqrt{2}}$

(c) $\frac{3\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

Directions for ( 84 to 85): Consider the following for the next two (02) items that follow:

Let $I = \int_a^b \frac{|x|}{x} dx,\ a < b$

84. What is I equal to when $a < 0 < b$ ?

(a) $a + b$ (b) $a - b$

(c) $b - a$ (d) $\frac{(a+b)}{2}$

85. What is I equal to when $a < b < 0$ ?

(a) $a + b$ (b) $a - b$

(c) $b - a$ (d) $\frac{(a+b)}{2}$

Directions for (86 to 88): Consider the following for the next three (03) items that follow:

Let $f(x) = |\ln x|,\ x \neq 1$

86. What is the derivative of $f(x)$ at $x = 0\cdot5$ ?

(a) –2 (b) –1

(c) 1 (d) 2

87. What is the derivative of $f(x)$ at $x = 2$?

(a) $-\frac{1}{2}$ (b) –1

(c) $\frac{1}{2}$ (d) 2

88. What is the derivative of $f \circ f(x)$, where $1 < x < 2$?

(a) $\frac{1}{\ln x}$ (b) $\frac{1}{x \ln x}$ (c) $\frac{1}{\ln x}$ (d) $-\frac{1}{x \ln x}$

Directions for (89 to 90): Consider the following for the next two (02) items that follow:

Let $f(x) = \begin{cases} x + 6, x \le 1 \\ px + q, 1 < x < 2 \\ 5x, x \ge 2 \end{cases}$ and $f(x)$ is continuous

89. What is the value of $p$?

(a) 2 (b) 3

(c) 4 (d) 5

90. What is the value of $q$?

(a) 2 (b) 3 (c) 4 (d) 5

91. Consider the following statements:

1. $f(x) = \ln x$ is increasing in $(0, \infty)$

2. $g(x) = e^x + e^{\frac{1}{x}}$ is decreasing in $(0, \infty)$

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

92. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$?

(a) –1 (b) 1 (c) sin2x (d) cos2x

93. For what value of $m$ with $m < 0$, is the area bounded by the lines $y = x$, $y = mx$ and $x = 2$ equal to 3?

(a) $-\frac{1}{2}$ (b) – 1 (c) $-\frac{3}{2}$ (d) –2

94. What is the derivative of $\text{cosec}(x^\circ)$?

(a) $-\text{cosec}(x^\circ)\cot(x^\circ)$ (b) $-\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$

(c) $\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$ (d) $-\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$

95. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$ is

(a) $y = 2x$ (b) $y = 2x + 4$

(c) $y = x^2 - 1$ (d) $y = \frac{(x^2 - 2)}{2}$

96. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to?

(a) 3 (b) 1

(c) –2 (d) –3

97. What is the range of the function $f(x) = x + |x|$

(a) $(0, \infty)$ (b) $[0, \infty)$

(c) $(-\infty, \infty)$ (d) $[1, \infty)$

98. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to?

(a) $-\sin 3\theta$ (b) $-\cos 3\theta$ (c) $\sin 3\theta$ (d) $-\sin 4\theta$

99. What is $\lim_{x \to 5} \frac{5 - x}{|x - 5|}$ equal to?

(a) –1 (b) 0

(c) 1 (d) Limit does not exist

100. What is $\lim_{x \to 1} \frac{x^9 - 1}{x^3 - 1}$ equal to?

(a) –1 (b) –3

(c) 3 (d) Limit does not exist

101. The mean and variance of five observation are 14 and 13.2 respectively. Three of the five observations are 11, 16 and 29. What are the other two obs ervations?

(a) 8 and 15 (b) 9 and 14
(c) 10 and 13 (d) 11 and 12

102. Let A and B be two independent events such that $P(\bar{A}) = 0.7, P(\bar{B}) = k, P(A \cup B) = 0.8$
What is the value of k?

(a) $\frac{5}{7}$ (b) $\frac{4}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{7}$

103. A biased coin with the probability of getting head equal to $\frac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head?

(a) $\frac{81}{512}$ (b) $\frac{81}{1024}$ (c) $\frac{81}{256}$ (d) $\frac{27}{1024}$

104. A coin is biased so that heads comes up thrice as likely as tails. In four independent tosses of the coin, what is probabi lity of getting exactly three heads?

(a) $\frac{81}{256}$ (b) $\frac{27}{64}$ (c) $\frac{27}{256}$ (d) $\frac{9}{256}$

105. Let X and Y be two random variables such that X + Y = 100. If X follows Binomial distribution with parameters n = 100 and $p = \frac{4}{5}$, what is the variance of Y?

(a) 1 (b) $\frac{1}{2}$
(c) 16 (d) $\frac{1}{16}$

106. If two lines of regression are $x + 4y + 1 = 0$ and $4x + 9y + 7 = 0$, then what is the value of x when $y = -3$?

(a) –13 (b) –5
(c) 5 (d) 7

107. The central angles p, q, r and s (in degrees) of four sectors in a Pie Chart satisfy the relation $9p = 3q = 2r = 6s$. What is the value of $4p - q$?

(a) 12 (b) 24
(c) 30 (d) 36

108. The observations 4, 1, 4, 3, 6, 2, 1, 3, 4, 5, 1, 6 are outputs of 12 dices thrown simultaneously. If m and M are means of lowest 8 observations and highest 4 observations respectively, then what is (2m + M) equal to?

(a) 10 (b) 12
(c) 17 (d) 21

109. A bivariate data set contains only two points (–1,1) and (3, 2). What will be the line of regression of y on x ?

(a) $x - 4y + 5 = 0$ (b) $3x + 2y - 1 = 0$
(c) $x + 4y + 1 = 0$ (d) $5x - 4y + 1 = 0$

110. A die is thrown 10 times and obtained the following outputs: 1, 2, 1, 1, 2, 1, 4, 6, 5, 4
What will be the mode of data so obtained?

(a) 6 (b) 4
(c) 2 (d) 1

111. Consider the following frequency distribution:

| | | | | |
|---|---|---|---|---|
| x | 1 | 2 | 3 | 6 |
| f | 4 | 6 | 9 | 7 |

What is the value of median of the distribution?

(a) 1 (b) 2
(c) 3 (d) 3.5

112. For data –1, 1, 4, 3, 8, 12, 17, 19, 9, 11; if M is the median of first 5 observations and N is the median of last five observation, then what is the value of 4 M –N?

(a) 7 (b) 4 (c) 1 (d) 0

113. Let P, Q, R represent mean, median and mode. If for some distribution are $5P = 4Q = \frac{R}{2}$, then what is $\frac{P + Q}{2P + 0.7R}$ equal to?

(a) $\frac{1}{12}$ (b) $\frac{1}{7}$ (c) $\frac{2}{9}$ (d) $\frac{1}{4}$

114. If G is the geometric mean of numbers $1, 2, 2^2, 2^3, \ldots, 2^{n-1}$, then what is the value of $1 + 2\log_2 G$?

(a) 1 (b) 4 (c) n –1 (d) n

115. If H is the harmonic mean of numbers $1, 2, 2^2, 2^3, \ldots, 2^{n-1}$ then what is n/H equal to?

(a) $2 - \frac{1}{2^{n+1}}$ (b) $2 - \frac{1}{2^{n-1}}$ (c) $2 + \frac{1}{2^{n-1}}$ (d) $2 - \frac{1}{2^n}$

116. Let P be the median, Q be the mean and R be the mode of observations $x_1, x_2, x_3, \ldots x_n$. Let $S = \sum_{i=1}^{n} (2x_i - a)^2$. S takes minimum value, when a is equal to

(a) P (b) $\frac{Q}{2}$ (c) 2Q (d) R

117. One bag contains 3 white a nd 2 black balls, another bag contains 2 white and 3 black balls. Two balls are drawn from the first bag and put it into the second bag and then a ball is drawn from the second bag. What is the probability that it is white?

(a) $\frac{2}{7}$ (b) $\frac{33}{70}$ (c) $\frac{3}{10}$ (d) $\frac{1}{70}$

118. Three dice are thrown. What is the probability that each face shows only multiples of 3?

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$ (c) $\frac{1}{27}$ (d) $\frac{1}{3}$

119. What is the probability that the month of December has 5 Sundays?

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$

120. A natur al number n is chosen from the first 50 natural numbers. What is the probability that $n + \frac{50}{n} < 50$?

(a) $\frac{23}{25}$ (b) $\frac{47}{50}$ (c) $\frac{24}{25}$ (d) $\frac{49}{50}$

# Answers with Explanations

## PAPER -I: MATHEMATICS

1. (a) $\sqrt{3}$

$$\left|\frac{1-(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)}{-1}\right| = \frac{\left|\frac{3}{2}+\frac{\sqrt{3}}{2}i\right|}{1^{-11}} = \frac{\sqrt{\frac{9}{4}+\frac{3}{4}}}{1}$$

$$= \sqrt{\frac{12}{4}} = \sqrt{3}$$

2. (d) 120: Divisible by 6 means by 2 and 3 must divisible 6 digit possible number

| | | | | | |
|---|---|---|---|---|---|
| . | . | . | . | | 0,2,4 |

For that last 2 digit must be 0, 2, 4

1 2 3 4 5 3

$\Rightarrow$ $1 \times 2 \times 3 \times 4 \times 5 = 360$

5 digit number must be avoided

| | | | | | |
|---|---|---|---|---|---|
| 0 | | | | | 2,4 |

1 2 3 4 2 □

$\Rightarrow$ $1 \times 2 \times 3 \times 4 \times 2 = 48$

Therefore $360 - 48 = 312$

3. (d) 111110011

| | | |
|---|---|---|
| 2 | 1011 | 1 |
| 2 | 505 | 1 |
| 2 | 252 | 0 |
| 2 | 125 | 0 |
| 2 | 63 | 1 |
| 2 | 35 | 1 |
| 2 | 15 | 1 |
| 2 | 7 | 1 |
| 2 | 3 | 1 |
| | 1 | |

$\Rightarrow$ 111110011

4. (b) We know $|\lambda A| = \lambda^n |A|$

$\text{adi}(\lambda A) = \lambda^{n-1} \text{adi } A$

$(\text{adi } A) = |A|^{n-1}$

By using above properties

$|2\,\text{adi}(3A)| = 2^3 |\text{adi}(3A)|$

$= 2^3 |3^2 |\text{adi } A||$

$= 2^3 9^3 |\text{adi } A|$

$= 2^3 3^6 |\text{adi } A|$

$= 2^3 3^6 4^{3-1} = 2^3 3^6 4^2 = 2^3 3^6 2^4$

$= 2^7 3^6 \Rightarrow \alpha = 7, \beta = 6$

$\therefore$ $\alpha + \beta = 7 + 6 = 13$

5. (d) $\frac{1}{\sqrt{3}}$

$x^2 - x + 1 = 0$

Equation roots are $\alpha = -\omega$ and $\beta = -\omega^2$

$$\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right| \Rightarrow \left|\frac{(-\omega)^{100}+(-\omega^2)^{100}}{(-\omega)^{100}-(-\omega^2)^{100}}\right|$$

$$\left|\frac{\omega^{100}+\omega^{200}}{\omega^{100}-\omega^{200}}\right|$$

$$\left|\frac{(\omega^3)^{33}\omega+(\omega)^{66}\omega^2}{(\omega^3)^{33}\omega-(\omega^3)^{66}\omega^2}\right| = \left|\frac{\omega+\omega^2}{\omega-\omega^2}\right|$$

We know $1+\omega+\omega^2=0$ and $\omega^3=1$

$\Rightarrow \quad \left|\frac{-1}{\omega-(-1-\omega)}\right|=\left|\frac{-1}{1+2\omega}\right|=\frac{1}{\sqrt{3}}$

6. (b) Given, A and B are symmetric matrices, therefore, we have: $A'=A$ and $B'=B$...(i)

Consider $(AB-BA)'=(AB)'-(BA)'$ [$\because (A-B)'=A'-B'$]

$=B'A'-A'B'$, [$\because (AB)'=B'A'$]

$=BA-AB$ by (i)

$=-(AB-BA)$

$\therefore \quad (AB-BA)'=(AB-BA)$

Thus, $(AB-BA)$ is a skew-symmetric matrix.

7. (a) 1 only

For statement 1,

$AB=AC$

Multiplying by $A^{-1}$ both sides

$A^{-1}AB=A^{-1}AC$

$\Rightarrow B=C$ Invertible

Similarly for statement 2 $\Rightarrow B=C$

$Bx=Cx$ not satisfied for column matrix $X$.

8. (c) no solution

$$\Delta=\begin{vmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 3 & 6 & 3\end{vmatrix}=0$$

Similarly calculate $\Delta_1$ and $\Delta_2=0$ we get no solution.

9. (b) Let $X_1$ and $X_2$ be its two distinct solutions. This implies that 'AX = B' has infinitely many solutions, as any linear combination of '$X_1$' and '$X_2$' (with 'a' and 'b' being real numbers) will also satisfy the system of linear equations. If the combination $aX_1+bX_2$ is a solution of AX=B; where a, b are real numbers, then $a+b=1$.

10. (b) $a-b+c$

$$\begin{vmatrix}0 & x-a & x-b\\ 0 & 0 & x-c\\ x+b & x+c & 1\end{vmatrix}=0$$

After expending along $C_1$, we obtain

$(x+b)(x-a)(x-c)=0$

$(x+b)x^2-(a+c)x+ac)=0$

$x^3-(a+c)x^2+acx+bx^2-ba(c+x)+abc=0$

$x^3-(a+c-b)x^2+(ac-ba-bc)x+abc=0$

11. (d) $\alpha=2-i\sqrt{3}, \qquad \beta=2+i\sqrt{3}$

$\alpha+\beta=-4=a \qquad \Rightarrow a=-4$

$\alpha\beta=4+3=b \qquad \Rightarrow b=7$

$a+b=7-4=3$

12. (b) $\frac{2\pi}{3} \qquad z=\frac{2\left(\sqrt{3}i-1\right)}{4}$

$z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$\theta=\text{Argument}(z)=\tan^{-1}\left(\frac{y}{2}\right)$

$\theta=\tan^{-1}\left(-\sqrt{3}\right)$

$\Rightarrow \quad \theta=\frac{2\pi}{3}$

13. (b) $\begin{vmatrix}x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\\ x+a & x+b & x+c\end{vmatrix}=\begin{vmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ a & b & c\end{vmatrix}$

Applying $c_2\to c_2-2c_1$ and $c_3\to c_3-3c_1$

$$=\begin{vmatrix}1 & 0 & 0\\ 2 & -1 & -2\\ a & b-2a & c-3a\end{vmatrix}$$

$=-(c-3a)+2(b-2a)$

$=-c+3a+2b-4a$

$=-c-a+2b$

For AP $b=\frac{a+c}{2}$

Therefore the value of determinate must be zero.

14. (c) $\frac{\log_a(\log_b a)}{2}$

In G. P. $\left(a^x\right)^2=\log x^a\times\log b^x$

$a^{2x}=\log b^a$

$2x=\log a \qquad \log b^a$

$x=\frac{\log a \qquad \log b^a}{2}$

15. (a) a, b, c are in AP

In G. P.

$\left(2^{\frac{b}{ac}}\right)^2=\left(2^{\frac{1}{c}}\right)\left(2^{\frac{1}{a}}\right)$

$2^{\frac{2b}{ac}}=2^{\frac{1}{c}+\frac{1}{a}}$

$\therefore \quad \frac{2b}{ac} = \frac{a+c}{ac} \qquad 2b = a+c$

$b = \frac{a+c}{2}$

$\therefore$ a,b andc are inAP .

16. (b) $a = \frac{5}{2}, \quad a+d = \frac{23}{12}$

$d = \frac{23}{12} - \frac{5}{2} = \frac{23-30}{12} = \frac{-7}{12}$

$a + (n-1)d = \frac{5}{2} + (n-1)\left(\frac{-7}{12}\right) < 0$

$\frac{5}{2} = (n-1)\frac{7}{12}$

$n-1 = \frac{12}{7} \times \frac{5}{2} = \frac{30}{7}$

$n = \frac{30}{7} + 1 = \frac{37}{7} \simeq 5\frac{2}{7}$

Means n = 6

17. (b) For real roots $b^2 - 4ac \geq 0$

$(4)^2 - 4k \geq 0$

$k = 4$

18. (b) $\frac{mx(m+n)}{1-n}$

$a = x$

$Sn = \frac{n}{2}[2a + (n-1)d] = 0$

or $2x + (n-1)d = 0$

$d = -\frac{2x}{n-1} = \frac{22}{1-n}$

$Sm = \frac{m}{2}[2a + (m-1)d]$

$Sm = \frac{m}{2}\left[2x + \left(\frac{m-1}{n-1}\right)2x\right]$

$= mx\left[\frac{1-n+m-1}{1-n}\right] = \frac{mx(m-n)}{1-n}$

19. (b) 2 only

1. $\lfloor 25 + 1$ is divisible by 26

$\lfloor 25$ last digit wi ll be 0 therefore By adding one last digit will be 1 not be divisible by 26

2. $\lfloor 6 + 1 = 721$ will be divisible by 7.

20. (c) $\frac{z+1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy}$

$= \frac{\{(x-1)+iy\}\{(x+1)-iy\}}{(x+1)^2 + y^2}$

$= \frac{(x^2+1)iy(x-1)+iy(x+1)+y^2}{(x+1)^2 + y^2}$

$= \frac{(x^2+y^2-1)+iy(x-1-x-1)}{(x+1)^2 + y^2}$

$= \frac{(x^2+y^2-1)+2iy}{(x+1)^2 + y^2}$

For purely imaginary $x^2 = y^2 - 1 = 0 \qquad = x^2 + y^2 = 1$

$\therefore |z| = 1$

21. (b) Only two

$|x-4| + |x-7| = 15$

Critical points are x = 4 and x = 7

Case I $\qquad x<4$

$(4-x) + (7-x) = 15$

$-2x = 4 \Rightarrow x = -2$

$x = -2$ Satisfying the equation

Case II $\qquad 4 < x < 7$

$x - 4 + 7 - 2 = 15$ Not possible

Case III $\qquad x \geq 7$

$x - 4 + x - 7 = 15$

$2x = 26$

$x = 13$

$x = -2, 13$ Two solutions.

22. (d) $f(x) = \frac{2x+3}{3x+5}$

Domain $A \to x \Rightarrow 3x+5 \neq 0$

$\therefore \quad x \neq -\frac{5}{3}$

In onto function range = co-domain

$\frac{2x+3}{3x+5} = y$ or $x = \frac{3-5y}{3y-2}$

Means $3y - 2 \neq 0$

$y \neq \frac{2}{3}$

Therefore $A = R\left|\left\{-\frac{5}{3}\right\}\right.$ and $B = R\left|\left\{\frac{2}{3}\right\}\right.$

23. (c) Both 1 and 2

$x^2+ax+b=0$

$\alpha+\beta=-a$

$\alpha+\beta=0\Rightarrow -a=0$

$a=0$

$\alpha^2+\beta^2=2$

$(\alpha+\beta)^2-2\alpha\beta=2$

$(-a)^2-2b=2$

$a^2-2b=2$

$(\alpha\beta)\beta=-1, \alpha=0$

$b\beta=-1$

$\beta=-\frac{1}{b}$

$\Rightarrow \quad \alpha=-\frac{1}{\beta^2}=-\frac{1}{\left(-\frac{1}{b}\right)^2}=-b^2$

$\alpha=-b^2=0$

Individual statements are not sufficient to find α both together needed to get $\alpha$ value.

$\propto=\pm 1, \beta=\pm 1$

$\alpha\beta^2=-1\Rightarrow \alpha$ must be $-1$

24. (d) $T_6=5600$

$T_{r+1}={}^nC_r x^{n-r}a^r$

$T_6={}^8C_5\,(x^{\frac{-8}{3}})^{8-5}(x^2\log_{10}x)^5=5600$

$\frac{8\times 7\times 6}{1\times 2\times 3}x^{-8}x^{10}(\log_{10}x)^5=5600$

$x^2(\log_{10}x)^5=100$ Satisfied for $x=10$

25. (a) $(3x-y)^4(x+3y)^4$

$=[(3x-y)(x+3y)]^4$

$=[3x^2+9xy-xy-3y^2]^4$

$=[3x^2=8xy-3y^2]^4$

In expansion we obtain 9 terms

26. (a) p,q,r and s arein AP.

$q-p=s-r=r-q=d$

$p+s=8, qr=15$

Let four terms

$p=(a-3d), q=(a-d), r=(a+d), s=(a+3d)$

$p+s\Rightarrow 2a=8\Rightarrow a=4$

$qr=(a-d)(a+d)=15$

$a^2-d^2=15$

$16-d^2=15\Rightarrow d=\pm 1$

First term $p=(4-3)=1$,

Last term $s=4+3=7$

$s-p=7-1=6$

27. (d) Neither 1 nor 2

${}^nC_r$ if n is even …(i)

Maximum value is ${}^nC_{n/2}\Rightarrow \frac{n}{2}=r, n=2r$

If n is odd …(ii)

${}^nC_r$ means $={}^nC_{\frac{n-1}{2}}$ or $\frac{n+1}{2}$

means $\frac{n-1}{2}=r,\ n=2r+1$

$\frac{n+1}{2}=r, n=2r-1$

28. (d) ${}^mC_2\times{}^nC_2=60$

${}^5C_2\times{}^4C_2=60$

$\Rightarrow \quad m+n=5+4=9$

29. (c) $x=4y$

PERMUTATIONS in 12 letter t is repeated

$x=\frac{12!}{2!}$

Combinations again 12 letters 0, 1, n are repeated

$y=\frac{12!}{2!2!2!}=\frac{x}{4}$

$\Rightarrow \quad x=4y$

30. (a) 20%

5 digit number __ __ __ __ __

4× 4× 3× 2 × 1= 96 ways

Fixed 5 __ __ __ __

4 × 3 × 2 × 1 = 24 ways

$\frac{24}{96}\times 100=25\%$

31. (a) $(\cos\alpha-\sin\alpha)^2$

$\sin^2\beta=\sin\alpha\cos\alpha$ (i)

$\tan\gamma = \dfrac{\sin\alpha+\cos\alpha}{2}$ ...(ii)

From (i)

$2\sin^2\beta = 2\sin\alpha\cos\alpha$

$\cos 2\beta = 1 - 2\sin 2\beta$

$\Rightarrow 1-\cos 2\beta = 2\sin\alpha\cos\alpha$

$\cos 2\beta = 2 - 2\sin\alpha\cos\alpha$

$\cos 2\beta = \sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha$

$\cos 2\beta = (\cos\alpha - \sin\alpha)^2$

32. (b) $\dfrac{5-\sin 2\alpha}{3-\sin 2\alpha}$

$\tan^2\gamma = \dfrac{\sin 2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha}{4}$

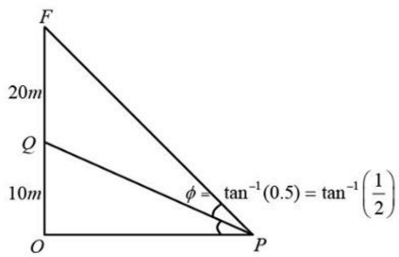
$\tan^2\gamma = \dfrac{1+\sin 2\alpha}{4}$

$\sec 2\gamma = \dfrac{-+\tan^2\gamma}{1-\tan^2\gamma}$

$\sec 2\gamma = \dfrac{1+\dfrac{1+\sin 2\alpha}{4}}{1-\dfrac{1+\sin 2\alpha}{4}}$

$\sec 2\gamma = \dfrac{5+\sin 2\alpha}{3-\sin 2\alpha}$

33. (a) 1 only




$\tan\theta = \dfrac{10}{x}$ ...(i)

$\tan(\theta+\phi) = \dfrac{30}{x}$ ...(ii)

Where $\phi = \tan^{-1}\left(\dfrac{1}{2}\right) \Rightarrow \tan\phi = \dfrac{1}{2}$

$\tan(\theta+\phi) = \dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

$= \dfrac{\tan\theta + \dfrac{1}{2}}{1-\dfrac{\tan\theta}{2}} = \dfrac{2\tan\theta+1}{2-\tan\theta}$

$\Rightarrow \dfrac{\tan\theta+1}{2-\tan\theta} = \dfrac{30}{x}$ ...(iii)

$\dfrac{\text{(iii)}}{\text{(i)}} \quad = \dfrac{\dfrac{2\tan\theta+1}{2-\tan\theta}}{\tan\theta} = \dfrac{\dfrac{30}{x}}{\dfrac{10}{x}}$

$\dfrac{2\tan\theta+1}{(2-\tan\theta)\tan\theta} = 3$

$\dfrac{2\tan\theta+1}{(2-\tan\theta)\tan\theta} = 3$

$2\tan\theta+1 = 6\tan\theta - 3\tan^2\theta$

$3\tan^2\theta - 4\tan\theta + 1 = 0$

$3\tan^2\theta - 3\tan\theta - \tan\theta + 1 = 0$

$3\tan\theta(\tan\theta-1) - 1(\tan\theta - 1) = 0$

$(\tan\theta-1)(3\tan\theta-1) = 0$

$\Rightarrow \tan\theta = 1 \text{ or } \tan\theta = \dfrac{1}{3}$ ...(iv)

Therefore sum equation (i) and equation (iv)

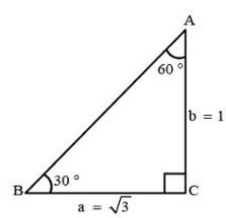
$x = \dfrac{10}{\tan\theta} \Rightarrow x = 10, 30$

$x \neq 20$

34. (c) $\dfrac{1}{3}$

$\tan\theta = \dfrac{1}{3}$

35. (c) $\sqrt{3}+3$




$\dfrac{a}{b} = \dfrac{\sqrt{3}}{1} = \dfrac{\dfrac{\sqrt{3}}{2}}{\dfrac{1}{2}} = \dfrac{\sin 60°}{\sin 30°} = \dfrac{\sin A}{\sin B}$

$AB = \sqrt{(\sqrt{3})^2+1} = 2$

Perimeter of $ABC\Delta = 2+1+\sqrt{3}$

$= 3+\sqrt{3}$

36. (c) Both 1 and 2
As C = < 90° Right angled triangle
Angles are 30°, 60°, 90° are in AP

37. (c) $x = \frac{\sin^2 A = \sin A + 1}{\sin A}$

$x = \sin A + \frac{1}{\sin A} + 1$

$x = \sin A + \operatorname{cosec} A + 1$

$\sin A + \operatorname{cosec} A$ minimum value is 2

$\Rightarrow$ $x = 3$

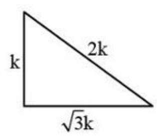
38. (d) $\frac{\pi}{2}$; $A = \frac{\pi}{2}$ for minimum value

39. (c) $a^2 + b^2 + c^2 = ac + \sqrt{3}bc$
Let angles are 30°, 60°, 90°

$\sin\theta \Rightarrow \frac{1}{2}, \frac{\sqrt{3}}{2}, 1 = 1 : \sqrt{3} : 2$

This satisfied right angle triangle.

40. (c) $8\sqrt{3}$




Area of $\Delta = \frac{1}{2} \times \left(\sqrt{3}k\right) \times k = \frac{\sqrt{3}}{2}k^2$

$k = 4$

$\Delta = \frac{\sqrt{3}}{2} \times 16 = 8\sqrt{3}$

41. (b) $f(2) = 3$
$f(3) = 2$
$f(4) = 3$
$f(0) = 9$
Means minimum value is f(3) = 2

42. (a) $f(3) = 2$
2 is minimum value of function

43. (c) 4! onward multiple of 8 till 100! remainder will be zero.

$\Rightarrow$ $S = \frac{0! + 1! + 2! + 3!}{8} = S = \frac{1 + 1 + 2 + 6}{8} = \frac{10}{8} = 1\frac{2}{8}$

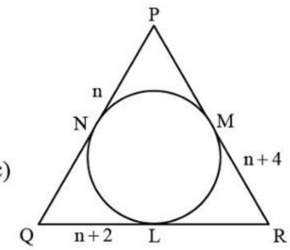
2 is the remainder

44. (d) Now divided by 60
From 5! onward divisible by 60

Therefore $S = \frac{1 + 1 + 2 + 6 + 24}{60} = \frac{34}{60}$

Remainder is 34

45. (c)




$\cos P = \frac{1}{3}$

$PN = PM = n$
$QL = QN = n + 2$
$RM = RL = n + 4$
$PQ = 2n + 2,\ QR = 2n + 6,\ PR = 2n + 4$

$\frac{PQ^2 + PR^2 - QR)^2}{2(PQ \times PR)} = \frac{1}{3}$

$\frac{(2n+2)^2 + (2n+4)^2 - (2n+6)^2}{2(2n+2)(2n+4)} = \frac{1}{3}$

After so solving we obtain n = 8

46. (d) Smallest side is PQ = 2n+2
= 2(8)+2=18

47. (a) $\sin^2 2x - 44\sin 2x + 36 = 0$

$\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

$\sin z + \cos x + \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\cos x} + \frac{1}{\sin x} = 7$

$\sin x + \cos x + \left[\frac{\sin^2 x + \cos^2 x + \cos x}{\sin^2 x}\right] = 7$

$\left(\sin x + \cos x\right) + 2\left[\frac{1 + \sin x + \cos x}{\sin 2x}\right] = 7$

$\left(\sin x + \cos x\right)\left(1 + \frac{2}{\sin 2x}\right) = 7 - \frac{2}{\sin 2x}$

$\Rightarrow$ $\sin^2 2x - 44\sin 2x + 36 = 0$

48. (d) Let $x = 7\frac{1}{2}$

$\sin 15^0 = a - b\sqrt{c}$

$\frac{\sqrt{3} - 1}{2\sqrt{2}} = a - b\sqrt{c}$

The possible value of a – b + 2c = 28

49. (b) H. M. $= \dfrac{2\alpha\beta}{\alpha+\beta} = \dfrac{\frac{2c}{a}}{\frac{-b}{a}} = \dfrac{2c}{b} = \dfrac{2(8+4\sqrt{3})}{(4+2\sqrt{3})} = 4$

50. (a) $\sqrt{2}\left(\sqrt{6}-\sqrt{3}+\sqrt{2}-1\right)$

G.M $\sqrt{\alpha\beta}$

$= \sqrt{\dfrac{c}{a}} = \sqrt{\dfrac{8+5\sqrt{3}}{3+2\sqrt{2}}}$

$= \sqrt{\dfrac{8+4\sqrt{3}}{3+2\sqrt{2}} \times \dfrac{3-2\sqrt{2}}{3-2\sqrt{2}}}$

$= \sqrt{\dfrac{(8+4\sqrt{3})(3-2\sqrt{2})}{9-8}}$

$= \sqrt{24+12\sqrt{3}-16\sqrt{2}-8\sqrt{6}}$

$\Rightarrow \quad \sqrt{2}(\sqrt{6}-\sqrt{3}+\sqrt{2}-1)$

51. (b) Let $\alpha = 1$

$$\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix} = 0$$

$R_3 \to R_3 - (R_1 + R_2)$

$$\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ c & 0 & -(a+2b+c) \end{vmatrix} = 0$$

$\Rightarrow \quad -(a+2b+c)(ac-b^2) = 0$

From $(ac-b^2) = 0$

a, b, c are in G.P.

52. (c) Given $a = 7, b = 4, c = 2$

$$\Delta(7,4,2,\alpha) = \begin{vmatrix} 7 & 4 & 7\alpha+4 \\ 4 & 2 & 4\alpha+2 \\ 7\alpha+4 & 4\alpha+2 & 0 \end{vmatrix}$$

$\Rightarrow \quad -7(4\alpha+2)^2 + 8(7\alpha+4)(4\alpha+2) - 2(7\alpha+4)^2 = 0$

$\Rightarrow \quad -7(16\alpha^2+16\alpha+4) + 8(28\alpha^2+14\alpha+16\alpha+8)$

$-2(49\alpha^2+56\alpha+16) = 0$

$\Rightarrow \quad -112\alpha^2 - 112\alpha - 28 + 224\alpha^2 + 240\alpha + 64$

$-98\alpha^2 - 112\alpha - 32 = 0$

$\Rightarrow \quad 14\alpha^2 + 16\alpha + 4 = 0$

$\Rightarrow \quad 2(7\alpha^2 + 8\alpha + 2) = 0$

For $\alpha = x$ $7x^2 + 8x + 2 = 0$

53. (d) 4n

$m(\theta) = \cot^2\theta + n^2\tan^2\theta + 2n$

$m'(\theta) = -2\cot\theta(\operatorname{cosec}^2\theta)$

$2n^2\tan\theta(\sec^2\theta)$

$= 0$

$n^2 = \dfrac{\cot\theta \operatorname{cosec}^2\theta}{\tan\theta\sec^2\theta}$

$n^2 = \dfrac{\cos\theta}{\sin\theta}\dfrac{\cos\theta}{\sin\theta} \times \dfrac{\cos^2\theta}{\sin^2\theta}$

$n^2 = \cot^4\theta$

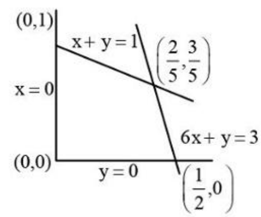
$\Rightarrow \quad \cot^2\theta = n \qquad \ldots(i)$

therefore $m(\theta) = n + n^2\dfrac{1}{n} + 2n$ Minimum = 4

54. (b) $n = \cot^2\theta$

$\cot^2\theta = n$ from equation (i) does m attain the least value.

55. (c) $3x - 2y = 0$




$y - 0 = \dfrac{\left(\frac{3}{5}-0\right)}{\left(\frac{2}{5}-0\right)}(x-0)$

$y = \dfrac{3}{2}x$

$2y = 3x \Rightarrow 3x - 2y = 0$

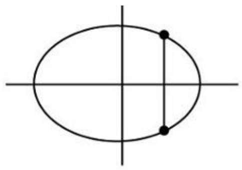
56. (d) $x + 2y - 1 = 0$

$y - 1 = \dfrac{0-1}{\frac{1}{2}-0}(x-0)$

$y - 1 = -2x$

$2x + y - 1 = 0$

57. (b) $\dfrac{x^2}{1} + \dfrac{y^2}{\left(\frac{1}{2}\right)^2} = 1$

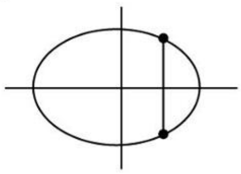


$a = 1, b = \frac{1}{2}$ $\quad PE + PF = 2a = 2$

58. (d) 1, 2 and 3

Point on latus rectum

$\left(c, \pm\frac{b^2}{a}\right)$

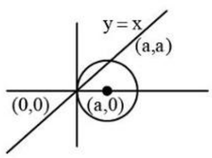


$\frac{b^2}{a} = \frac{1}{4}$

$c = \frac{\sqrt{3}}{2},$

Therefore points are $\left(\frac{\sqrt{3}}{2}, 0\right)$, $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$ and $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

59. (a) $\frac{(\pi - 2)a^2}{4}$




$(x-a)^2 + y^2 = a^2$

$y^2 = 2ax - x^2$

Line $y = x$ Area $= \frac{1}{4}\int_0^a \sqrt{2ax - x^2}\,dx - \int x\,dx$

Of minor segment $= \frac{(\pi - 2)a^2}{4}$

60. (b) $\frac{(3\pi + 2)a^2}{4}$

Area of major segment $= \pi a^2 - \frac{(\pi - 2)a^2}{4}$

$\frac{4\pi a^2 - \pi a^2 + 2a^2}{4} = \frac{3\pi a^2 + 2a^2}{4}$

$= \frac{(3\pi + 2)a^2}{4}$

61. (a) Sphere diameter from

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0$

$(x-1)(x-2) + (y+1)(y-1) + (z-2)(z+1) = 0$

$x^2 + y^2 + z^2 - 3x - z + 2 - 1 - 2 = 0$

$x^2 + y^2 + z^2 - 3x - z - 1 = 0$ ...(i)

Given $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$ ...(ii)

Comparing equation (i) and equation (ii)

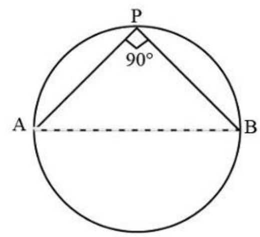
$2u = -3 \Rightarrow u = -\frac{3}{2}$

$2v = 0 \Rightarrow v = 0$

$2w = -1 \Rightarrow w = -\frac{1}{2}$

$u + v + w = -\frac{3}{2} + 0 - \frac{1}{2} = \frac{-3-1}{2} = -2$

62. (b) 14




$PA^2 + PB^2 = AB^2$

$AB^2 = (2-1)^2 + (1+1)^2 + (-1-2)^2$

$1 + 4 + 9 = 14$

63. (a) Direction ratios are (2,–1,2) and (k, 3,5)

$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\frac{\pi}{4} = \frac{2k - 3 + 10}{\sqrt{4+1+4}\sqrt{k^2 + 9 + 25}}$

$\frac{1}{\sqrt{2}} = \frac{2k + 7}{3\sqrt{k^2 + 34}}$

$9(k^2 + 34) = 2(4k^2 + 28k + 49)$

$9k^2 + 306 = 8k^2 + 56k + 98$

$k^2 - 56k + 208 = 0$

$\Rightarrow \quad k = 4$

64. (d) (11, 2, –10)

Now direction ratios are (2,–1, 2) and (4,3,5)

For perpendicular $\perp \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

(11,2,–10) satisfied both

Consider the following for the next two (02) items that follow:

Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a}\cdot\vec{b} = 27$ and $\vec{a}\times\vec{b} = 9\vec{c}$

65. (b) $5\hat{i}+2\hat{j}+2\hat{k}$

Let $\vec{b}=x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{a}\cdot\vec{b}=27$

$\Rightarrow (3\hat{i}+3\hat{j}+3\hat{k})\cdot(x\hat{i}+y\hat{i}+z\hat{i})=27$

$3x+3y+3z=27$

$\Rightarrow x+y+z=9 \quad (i)$

$\vec{a}\times\vec{b}=9\vec{c}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 3 & 3\\ x & y & z\end{vmatrix}=9\left(\hat{j}-\hat{k}\right)$$

$\hat{i}(3z-3y)-\hat{j}(3z-3x)+\hat{k}(3y-3x)=9\hat{j}-9\hat{k}$

$\Rightarrow 3z-3y=0 \qquad \Rightarrow y=z$

$3z-3x=-9 \qquad \Rightarrow z-x=-3 \text{ or } y-x=-3$

$3y-3x=-9 \qquad \Rightarrow y-x=-3 \quad ...(ii)$

Using equation (ii) and equation (i) we get

$x-2y=9 \qquad (iii)$

Adding equation (ii) and equation (iii) we obtain

$3y=6\Rightarrow y=2$

$\Rightarrow z=2 \text{ and } x=5$

Therefore $\vec{b}=5\hat{i}+2\hat{j}+2\hat{k}$

66. (a) $\frac{\pi}{2}$

$\vec{a}+\vec{b}=(3\hat{i}+3\hat{j}+3\hat{k})+(5\hat{i}-2\hat{j}+2\hat{k})$

$\vec{a}+\vec{b}=8\hat{i}-5\hat{j}+5\hat{k}$ and $\vec{c}=\hat{j}-\hat{k}$.

$(\vec{a}+\vec{b})\cdot\vec{c}=|\vec{a}+\vec{b}||\vec{c}|\cos\theta$

$$\cos\theta=\frac{\left(8\hat{i}+5\hat{j}+5\hat{k}\right)\cdot\left(\hat{j}-\hat{k}\right)}{\sqrt{64+25+25}\sqrt{1+1}}$$

$$\cos\theta=\frac{5-5}{\sqrt{114}\sqrt{2}}=0$$

$\Rightarrow \theta=90^\circ=\frac{\pi}{2}$

67. (b) $\frac{4}{9}$

$$\cos\alpha=\frac{Q}{\sqrt{a^2+b^2+c^2}}$$

$$\cos\alpha=\frac{4}{\sqrt{16+64+1}}=\frac{4}{\sqrt{81}}=\frac{4}{Q}$$

$\cos\alpha=\frac{4}{9}$

68. (a) $-\frac{32}{81}$

$\cos 2\beta+\cos 2\gamma$

$=2\cos^2\beta-1+2\cos^2\gamma-1$

we have $\cos\beta=\frac{8}{9},\cos\gamma=\frac{1}{9}$

$=2\left(\frac{8}{9}\right)^2-1+2\left(\frac{1}{9}\right)^2-1$

$\frac{128-81+2-81}{81}=\frac{-32}{81}$

69. (b) b. 2 and 3 only

The position vectors of two points A and B are

$\vec{A}=\hat{i}-\hat{j}\,;\hat{B}=\hat{j}+\hat{k}$

The position vectors of A B is

$\overrightarrow{AB}=\hat{i}+2\hat{j}+\hat{k}$

Equation of line (1,-1,0) ; (0,1,1)

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-1}{0-1}=\frac{y+1}{1+1}=\frac{z-0}{1-0}$$

$$\frac{x-1}{-1}=\frac{y+1}{2}=\frac{z}{1} \qquad (i)$$

Equation (i) will be satisfied by $(-1,3,2)$ and $(-2,5,3)$

70. (c) $\overrightarrow{AB}=(\hat{j}+\hat{k})-(\hat{i}-\hat{j})$

$\overrightarrow{AB}=\hat{i}+2\hat{j}+\hat{k}$

$|\overrightarrow{AB}|=\sqrt{1+4+1}=\sqrt{6}$

71. (b) $f(x)=Pe^{x}+Qe^{2x}+Re^{3x}$

$f(0)=6\Rightarrow P+Q+R=6 \qquad (i)$

$f'(x)=Pe^{x}+2Qe^{2x}+3Re^{3x}$

$f'(\ell n3)=3P+2Qe^{2\ell n3}+3Re^{3\ell n3}$

$=3P+18Q+81R=282$

$=P+6Q+27R=94 \qquad ...(ii)$

$$\int_0^{\ell n2}(Pe^{x}+Qe^{2x}+Re^{3x})dx$$

$$=\left[Pe^{x}+\frac{Qe^{2x}}{2}+\frac{Re^{3x}}{3}\right]_0^{\ell n2}$$

$$=\left[(2P+\frac{4}{2}Q+\frac{8}{3}R)-(P+\frac{Q}{2}+\frac{R}{3})\right]$$

$$=P+\frac{3}{2}Q+\frac{7}{3}R=11 \qquad ...(iii)$$

Using equation (i) then equation (ii) becomes

$$(6-Q-R)+6Q+27R=94$$

$$5Q+26R=88 \qquad ...(iv)$$

Using equation (i) then equation (iii) becomes

$$6-Q-R+\frac{3Q}{2}+\frac{7R}{3}=11$$

$$\frac{Q}{2}+\frac{4}{3}R=5\Rightarrow 6Q+8R=30 \qquad ...(v)$$

Solving equation (iv) and equation (v) we get

$Q=2$

72. (c) After solving equation (iv) and equation (v) for R

We get R = 3

73. (d) 14

$$f'(0)=P+2Q+3R$$

$$f'(0)=1+4+9=14$$

74. (a) $y^2=2cx+2c\sqrt{c}$

$$2y\frac{dy}{dx}=2c+0$$

$$y\frac{dy}{dx}=c \qquad ...(i)$$

The order of the differential is 1

75. (b) The degree of the differential formed by $y^2=2cx+2c\sqrt{c}$ is 3

76. (b) $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$ for x = 0

$$f(0)=\begin{vmatrix}1 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 1\end{vmatrix}$$

$$f(0)=0$$

77. (b) $f(x)=\cos x(x^2-2x^3)-x(2\sin x-2x\tan 2)$
$+1(2\sin 2-x^2\tan 2)$

$$\frac{f(x)}{x}=\frac{-x^2\cos x-2x(\sin x-\tan x)-(2\sin x-x\tan x)x}{x}$$

$$\lim_{x\to\infty}\frac{f(x)}{x}=0$$

78. (a) –1

$$\lim_{x\to\infty}\frac{f(x)}{x^2}=-1$$

79. (b) $f(x)=\sin[\pi^2]x+\cos[-\pi^2]x$

$$f\left(\frac{\pi}{2}\right)=\sin[\pi^2]\frac{\pi}{2}+\cos[-\pi^2]\frac{\pi}{2}$$

$\pi^2\simeq 9.85$ for greatest integer the values are

$[\pi^2]=9$ for positive value

$[-9.85]=-10$ for negative value

$$f(x)=\sin 9x+\cos 10x$$

$$f\left(\frac{\pi}{2}\right)=\sin\left(\frac{9\pi}{2}\right)+\cos\left(\frac{10\pi}{2}\right)=1-1=0$$

80. (d) $\frac{1}{\sqrt{2}}$

$$f(x)=\sin 9x+\cos 10x$$

$$f\left(\frac{\pi}{4}\right)=\sin\left(\frac{9\pi}{4}\right)+\cos\left(\frac{10\pi}{4}\right)$$

$$f\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}+0=\frac{1}{\sqrt{2}}$$

81. (d) $\frac{\pi+1}{\pi-1}$

$$I_1=\int_0^{\pi}\frac{x}{1+\cos^2 x}dx=\int_0^{\pi}\frac{\pi-x}{1+\{(\cos\pi-x)\}^2}dx$$

$$I_1=\int_0^{\pi}\frac{\pi}{1+\cos^2 x}-I_1$$

$$2I_1=\int_0^{\pi}\frac{\sec^2 xdx}{\sec^2+1}=\int_0^{\pi}\frac{\sec^2 xdx}{\tan^2 x+2}$$

Let $\tan x=t$ $\sec^2 xdx=dt$

After integrating we obtain

$$I_1=\frac{\pi^2}{2\sqrt{2}}$$

$$I_2=\int_0^{\pi}\frac{1dx}{1+\sin^2 x}=\int_0^{\pi}\frac{\operatorname{cosec}^2 xdx}{\operatorname{cosec}^2 x+1}=\frac{\pi}{2\sqrt{2}}$$

$$\frac{I_1+I_2}{I_1-I_2}=\frac{\frac{\pi^2}{2\sqrt{2}}+\frac{\pi}{2\sqrt{2}}}{\frac{\pi^2}{2\sqrt{2}}-\frac{\pi}{2\sqrt{2}}}=\frac{\pi+1}{\pi-1}$$

82. (d) $\pi^4$

We have $I_1=\frac{\pi^2}{2\sqrt{2}}$

Therefore $8I_1^2 = 8 \times \frac{\pi^2}{4\times 2} = \pi^4$

83. (b) $\frac{\pi}{2\sqrt{2}}$

$I_2 = \int_0^{\pi} \frac{1dx}{1+\sin^2 x} \quad = \frac{\pi}{2\sqrt{2}}$

84. (a) a + b

$\ell = \int_a^b \frac{|x|}{x} dx,$

$a < b$

$f(x) = |x| \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$

$\ell = \int_a^0 \frac{-x}{x} dx + \quad \int_0^b \frac{x}{x} dx$

$= -\int_a^0 dx \quad + \quad \int_0^b dx$

$= -[x]_a^0 \quad + \quad [x]_0^b$

$= -(0-a) + (b-0)$

$= a + b$

85. (b) a – b

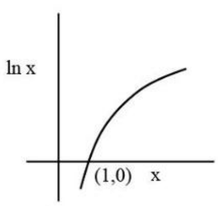
$a < b < 0$

Both negative $\ell = -\int_a^b dx = -[x]_a^b = -(b-a) = a - b$

86. (a) –2

$f(x) = |\ln x, x| x \neq 1$

$x = 0.5$




The derivative of $f(x)$

$f'(x) = -\frac{1}{x}$ as $x = 0.5 = -\frac{1}{0.5} = -2$

87. (a) $\frac{1}{2}$

The derivative of $f(x)$

$f(x) = |\ln x, x| x \neq 1$

$f'(x) = \frac{1}{x} = \frac{1}{2}$ As $x = 2$

88. (d) $-\frac{1}{x\ln x}$

$f \cdot f(x) \Rightarrow f \cdot (\ln x)$

$1 < x < 2$

$f(x)$ is +ive

Let $y = f \cdot f(x) = \ln \ln x$

$\frac{dy}{dx} = -\frac{1}{x\ln x}$

89. (b) 3

$f(x) = \begin{cases} x+6, x \leq 1 \\ Px+q, 1 < x < 2 \\ 5x, x \geq 2 \end{cases}$

for $f(x)$ continuous $f(i) \Rightarrow 1+6 = p+q$

$p + q = 7$ ...(i)

$f(ii) \Rightarrow 2p + q = 5(2)$

$2p + q = 10$ ...(ii)

Solving equation (i) and (ii) we get $p = 3$

90. (c) Using (i) $q = 7 - p$

$q = 7 - 3 = 4$

91. (a) 1 only

$f'(x) > 0$

$g'(x) < 0$

1. $f'(x) = \frac{1}{x} > 0$ is increasing in $(0, \infty)$

2. $g'(x) = e^x + e^{\frac{1}{x}}$

$g'(x) = e^x + e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$

$= e^x - \frac{e^{\frac{1}{x}}}{x^2} = \frac{e^x - e^{\frac{1}{x}}}{x^2}$

It is not the decreasing function.

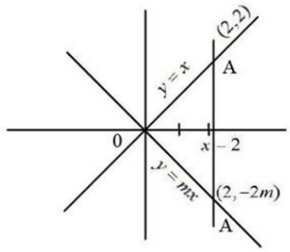
92. (a) –1

$y = \sin^2 x \qquad z = \cos^2 x$

$\frac{dy}{dx} = 2\sin x\cos x \qquad \frac{dz}{dx} = -2\cos x\sin x$

$$\frac{dy}{dz}=\frac{dy}{dx}\times\frac{dx}{dz}=\frac{2\sin x\cos x}{-2\cos x\sin x}=-1$$

93. (a) $-\frac{1}{2}$




Area of $OAB=\frac{1}{2}\times 2\times\sqrt{(2-2)^2+(2M+2)^2}$

$=2m+2=3 \quad m=\frac{1}{2}$

As slope of line is negative therefore $y=mx \therefore m=-\frac{1}{2}$

94. (b) $-\frac{\pi}{180}\operatorname{cosec}(x°)\cot(x°)$

$x=\frac{\pi}{180}x°$ $\qquad y=\operatorname{cosec}\left(\frac{\pi x}{180}\right)$

$\frac{dy}{dx}=-\frac{\pi}{180}\operatorname{cosec}x°\cot x°$

95. (d) $y=\frac{(x^2-2)}{2}$

$\left(\frac{dy}{dz}\right)^2-x\left(\frac{dy}{dx}\right)=0$

$y=2x\Rightarrow\frac{dy}{dx}=2$

$y=2x+4\Rightarrow\frac{dy}{dx}=2$

$y=2^2-1\Rightarrow\frac{dy}{dx}=\frac{1}{2}(2x)=x$

$y=\frac{(x^2-2)}{2}$ Satisfy the given differential equation.

96. (d) –3

$fg=(x^2+2)(2x-3)$

$fg=2x^3-3x^2+4x-6$

$fg(1)=2(1)^3-3(1)^2+4(1)-6$

$fg(1)=2-3+4-6=-3$

97. (b) $[0,\infty)$

$f(x)=x+|x|$

$[0,\infty]$ Satisfied the given function

98. (a) $-\sin 3\theta$

$f(\sin\theta)=\sin\theta(4\sin^2\theta-3)$

$=4\sin\theta-3\sin\theta$

$=-(3\sin\theta-4\sin^3\theta)$

$=-\sin 3\theta$

99. (d) Limit does not exist

$\lim_{x\to 5}\frac{5-x}{|x-5|}$

For $|x-5|=\begin{cases}x-5, x>5\\ 5-x, x<5\end{cases}$

LHL $=-1$; $\qquad$ RHL $=+1$

LHL is not equal to RHL therefore limit does not exist.

100. (c) 3

$\lim_{x\to 1}\frac{x^9-1}{x^3-1}$

By applying L hospital Rule

$\lim_{x\to 1}\frac{9x^8}{3x^2}=\frac{9}{3}=3$

101. (c) 10 and 13

$14=\frac{x+y+11+16+20}{5}$

$70=x+y+47$

$x+y=23$ ...(i)

Variance

$\sum_{i=1}^{5}\frac{(x_i-\bar{x})^2}{14}=13.2$

$(x-14)^2+(y-14)^2+(11-14)^2+(16-14)^2$

$+(20-14)^2=66$

$x^2-28x+196+y^2-28y+196+9+4+36=66$

$x^2+y^2-28(x+y)+375=0$ ...(ii)

use (i)

$x^2+y^2-28\times 23+375=0$

$x^2+y^2=269$ ...(iii)

Square equation (i) we get

$x^2 + y^2 + 2xy = 529$ ...(iv)

Using equation (iii) we obtain

$2xy = 260$

$xy = 130$

using equation (i)

$x(23 - x) = 130$

$x^2 - 23x + 130 = 0$

$(x - 13)(x - 10) = 0$

$x = 10, 13$

102. (c) $\frac{2}{7}$

$P(A \cap B) = P(A)P(B)$

$P(A) = 1 - P(\bar{A}) = 1 - 0.7 = 0.3$

$P(B) = 1 - k$

$P(A \cap B) = 0.3(1 - k)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.3 + 1 - k - (1 - k)0.3$

$= 0.5 - k - 0.3 + 0.3k$

$0 = 0.2 - 0.7k$

$k = \frac{2}{7}$

103. (b) $\frac{81}{1024}$

Biased Coin $P(H) = \frac{1}{4}, P(T) = \frac{3}{4}$

$P = TTTTH$

$P = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{8}{1024}$

104. (b) $\frac{27}{64}$

$P = {}^4C_3 P^3 Q^1$

Biased coin

$P(H) = \frac{3}{4}, P(T) = \frac{1}{4}$

$P = \frac{4 \times 3 \times 2}{1 \times 2 \times 3} \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)$

$P = \frac{4 \times 27}{64 \times 4} = \frac{27}{64}$

105. (c) 16

$X + Y = 100$, $n = 100$, $p = \frac{4}{5}$

$\Rightarrow q = 1 - \frac{4}{5} = \frac{1}{5}$

Variance is $= npq$

$= 100 \times \frac{4}{5} \times \frac{1}{5} = 16$

106. (c) $x + 4y + 1 = 0$ and $4x + 9y + 7 = 0$

x on y $\Rightarrow x = -4y - 1$

$x = \frac{-9y - 7}{4}$

$y = -3$

$x = \frac{27 - 7}{4} = 5$

107. (d) Let $9p = 3q = 2r = 6s = x$

$p = \frac{x}{9}, q = \frac{x}{3}, r = \frac{x}{2}, s = \frac{x}{6}$

$p + q + r + s = 360°$

$\frac{x}{9} + \frac{x}{3} + \frac{x}{2} + \frac{x}{6} = 360°$

$2x + 6x + 9x + 3x = 18 \times 360°$

$20x = 18 \times 360°$

$x = \frac{18 \times 360°}{20} = 18 \times 18$

$4p - q = \frac{4x}{9} - \frac{x}{3} = \frac{4x - 3x}{9} = \frac{x}{9}$

$\Rightarrow \frac{18 \times 18}{9} = 36°$

108. (a) 10, Rearranging in ascending order all the given data

1,1,1,2,3,3,4,4,4,5,6,6

Mean of lowest 8 observations

$m = \frac{194}{8}$

Mean of highest 4 observations

$M = \frac{29}{4}$

$2m + M = \frac{19}{8} \times 2 + \frac{21}{4}$

$= \frac{19 + 21}{4} = \frac{40}{4} = 10$

109. (a) $x-4y+5=0$

$(-1,1)$ and $(3,2)$

$y$ on $x \Rightarrow y = a + bx$

$$b=\frac{n\sum xy-\sum x\sum y}{n\sum x^2-(\sum x)^2}$$

$a=\bar{y}-a\bar{x}$

| | | | | |
|---|---|---|---|---|
| x | −1 | 3 | $\sum x$ | 2 |
| y | 1 | 2 | $\sum y$ | 3 |
| xy | −1 | 6 | $\sum xy$ | 5 |
| $x^2$ | 1 | 9 | $\sum x^2$ | 10 |

$\bar{x}=\frac{2}{2}=1, \qquad \bar{y}=\frac{3}{2}$

$$b=\frac{2\times5-2\times3}{2\times10-(2)^2}=\frac{10-6}{20-4}=\frac{4}{16}$$

$$b=\frac{1}{4}$$

$(y-\bar{y})=b(x-\bar{x})$

$\left(y-\frac{3}{2}\right)=\frac{1}{4}(x-1)$

$\frac{2y-3}{2}=\frac{x-1}{4}$

$4y-6=x-1$

$x-4y=5=0$

110. (d) 1

Mode is maximum happening x

| | | | | | |
|---|---|---|---|---|---|
| x | 1 | 2 | 4 | 5 | 6 |
| f | 4 | 2 | 2 | 1 | 1 |

Mode is 1

111. (c) 3

| x | f | cf |
|---|---|---|
| 1 | 4 | 4 |
| 2 | 6 | 10 |
| 3 | 9 | 19 |
| 5 | 7 | 26 |

$\sum f = 26$

For even N median $\frac{N}{2}=\frac{26}{2}=13$

13 corresponding value of x is 3 therefore median is 3

112. (d) Rearranging in ascending order all the given data

$-1,1,3,4,8,9,11,12,17,19$

Medians are $M=3$ of first 5 observations and $N=12$ of last five observation

$\Rightarrow \quad 4M-N=4(3)-12=0$

113. (d) $5P=4\theta=\frac{R}{2}$

$P=\frac{R}{10}$ and $Q=\frac{R}{8}$

$$\Rightarrow \quad \frac{P+Q}{2P+0.7R}=\frac{\frac{R}{10}+\frac{R}{8}}{\frac{2R}{10}+\frac{7R}{10}}$$

$$\Rightarrow \quad \frac{8R+10R}{80}\times\frac{10}{9R}$$

$$\Rightarrow \quad \frac{180R}{720R}=\frac{1}{4}$$

114. (d) n

The Geometric Mean

$$GM=\left(1\times2\times2^2\times\ldots\times2^{n-1}\right)^{\frac{1}{n}}$$

$$G=\left(2^{1+2+3\ldots+n-1}\right)^{\frac{1}{n}}$$

$$=\left[2^{\frac{(n-1)n}{2}}\right]^{\frac{1}{n}}=2^{\frac{n-1}{2}}$$

$$G=2\left(\frac{n-1}{2}\right)$$

$$\log_2 G=\frac{n-1}{2}\log_2{}^2$$

$$\log_2 G=\frac{n-1}{2}$$

$\Rightarrow \quad 2\log_2 G=n-1$

$\Rightarrow \quad 1+2\log_2 G=n$

115. (b) $2-\frac{1}{2^{n-1}}$

$$HM=\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}}$$

$$H=\frac{n}{\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}\ldots+\frac{1}{2^{n-1}}}$$

$$= \frac{n}{1\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right)}$$

$$H = \frac{n}{2\left(1-\frac{1}{2^n}\right)}$$

$$\Rightarrow \quad \frac{n}{H} = 2\left(1-\frac{1}{2^n}\right) = 2 - \frac{1}{2^{n-1}}$$

116. (c) 2Q

$$S = \sum_{i=1}^{n} (2x_i - a)^2$$

$$S = \sum_{i=1}^{n} (4x_i^2 - 4ax_i + a^2)$$

$$S = 4\sum_{i=1}^{n} x_i^2 - 4a\sum_{i=1}^{n} x_i + a^2$$

As P be the median, Q be the mean and R be the mode of observations are simplifying we get

$a = 2Q$

117. (a) Let's calculate the probability of drawing a white ball from the second bag after two balls are transferred from the first bag. First, let's calculate the probability of transferring two white balls from the first bag to the second bag. The probability of drawing a white ball from the first bag on the first draw is 3/5 (since there are 3 white balls out of 5 total balls).

After drawing one white ball from the first bag, there are 2 white balls left in the first bag and a total of 4 balls remaining. So the probability of drawing another white ball from the first bag on the second draw is 2/4, or 1/2. Therefore, the probability of transferring two white balls from the first bag to the second bag is $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

The total number of balls in the second bag after transferring two balls from the first bag is 7 (since there were originally 2 white balls + 3 black balls in the second bag, and two white balls were added from the first bag). The probab ility of drawing a white ball from the second bag is now $\frac{2}{7}$ (Since there are 2 white balls out of 7 total balls).

118. (c) Multiple of 3 are 3,6 Probability to get multiple of 3 in one dice $P(3,6) = \frac{2}{6} = \frac{1}{3}$

For three dice probability is $P = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

$$P = \frac{1}{27}$$

119. (c) In December we have 31 days means 4 weeks and 3 days extra from these 3 days probability of Sunday is

$$P = \frac{3}{7}$$

120. (b) $\frac{47}{50}$

$$n + \frac{50}{n} < 50$$

$$n^2 + 50 < 50n$$

$$n^2 - 50n + 50 < 0$$

$$n = \frac{50 \pm \sqrt{2500 - 100}}{2}$$

$$n = \frac{50 \pm \sqrt{2400}}{2}$$

$$n = \frac{50 \pm \sqrt{(400)6}}{2} = \frac{50 \pm 20\sqrt{6}}{2}$$

$$n = 25 \pm 10\sqrt{6}$$

$$n = 47$$

$$p = \frac{47}{50}$$