NDA/NA SOLVED PAPER 2022-I

MATHEMATICS

1. If $\Delta_1 = \begin{vmatrix} 1 & p & q \\ 1 & q & r \\ 1 & r & p \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix}$ where $p \neq q \neq r$, then $\Delta_1 + \Delta_2$ is
   (a) 0
   (b) always positive
   (c) always negative
   (d) positive if p, q, r are positive else negative

2. If $(a-b)(b-c)(c-a) = 2$ and $abc = 6$, then what is the value of $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$?
   (a) 3 (b) 12
   (c) 14 (d) 15

3. Under which of the following conditions does the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ vanish?
   1. $a + b + c = 0$
   2. $a^3 + b^3 + c^3 = 3abc$
   3. $a^2 + b^2 + c^2 - ab - bc - ca = 0$

   Select the correct answer using the code given below :
   (a) 1 and 2 only (b) 2 and 3 only
   (c) 1 and 3 only (d) 1, 2 and 3

4. Consider the following in respect of the matrices:
   $A = [m\ n]$, $B = [-n\ -m]$ and $C = \begin{bmatrix} m \\ -m \end{bmatrix}$
   1. $CA = CB$
   2. $AC = BC$
   3. $C(A + B) = CA + CB$

   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) 2 and 3 (d) 1 and 2

5. If $A = \begin{bmatrix} 2\sin\theta & \cos\theta & 0 \\ -2\cos\theta & \sin\theta & 0 \\ -1 & 1 & 1 \end{bmatrix}$, then what is $A(adj\,A)$ equal to?
   (a) Null matrix (b) $-I$
   (c) I (d) $2I$
   where I is the identify matrix.

6. For what value of k is the matrix
   $\begin{bmatrix} 2\cos 2\theta & 2\cos 2\theta & 6 \\ 1 - 2\sin^2\theta & 2\cos^2\theta - 1 & 3 \\ k & 2k & 1 \end{bmatrix}$ singular?
   (a) 0 only (b) 1 only
   (c) 2 only (d) Any real value

7. Let A be a non-singular matrix and $B = adj\,A$. Which of the following statements is/are correct?
   1. $AB = BA$
   2. AB is a scalar matrix
   3. AB can be a null matrix

   Select the correct answer using the code given below :
   (a) 1 only (b) 1 and 2 only
   (c) 2 only (d) 1, 2 and 3

8. Consider the following statements in respect of square matrices A and B of same order:
   1. If AB is a null matrix, then at last one of A and B is a null matrix.
   2. If AB is an identity matrix, then $BA = AB$.

   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

9. If A is the identity matrix of order 3 and B is its transpose, then what is the value of the determinant of the matrix $C = A + B$?
   (a) 1 (b) 2
   (c) 4 (d) 8

10. Let A and B be non-singular matrices of the same order such that $AB = A$ and $BA = B$. Which of the following statements is/are correct?
    1. $A^2 = A$
    2. $AB^2 = A^2B$

    Select the correct answer using the code given below :
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

11. How many terms are there in the expansion of
    $\left(1+\frac{2}{x}\right)^9 \left(1-\frac{2}{x}\right)^9$?
    (a) 9 (b) 10
    (c) 19 (d) 20

12. Consider the following statements in respect of the expansion of $(x + y)^{10}$:

1. Among all the coefficients of the terms, the coefficient of the 6th term has the highest value
2. The coefficient of the 3rd term is equal to coefficient of the 9th term

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

13. If $C(3n, 2n) = C(3n, 2n - 7)$, then what is the value of $C(n, n-5)$?

(a) 42 (b) 35
(c) 28 (d) 21

14. What is the value of $C(51, 21) - C(51, 22) + C(51, 23) - C(51, 24) + C(51, 25) - C(51, 26) + C(51, 27) - C(51, 28) + C(51, 29) - C(51, 30)$?

(a) $C(51, 25)$ (b) $C(51, 27)$
(c) $C(51, 51) - C(51, 0)$ (d) $C(51, 25) - C(51, 27)$

15. How many odd numbers between 300 and 400 are there in which none of the digits is repeated?

(a) 32 (b) 36
(c) 40 (d) 45

16. How many permutations are there of the letters of the word 'TIGER' in which the vowels should not occupy the even positions?

(a) 72 (b) 36
(c) 18 (d) 12

17. Let α and β be the roots of the equation $x^2 + px + q = 0$. If α^3 and β^3 are the roots of the equation $x^2 + mx + n = 0$, then what is the value of $m + n$?

(a) $p^3 + q^3 + pq$ (b) $p^3 + q^3 - pq$
(c) $p^3 + q^3 + 3pq$ (d) $p^3 + q^3 - 3pq$

18. Let α and β be the roots of the equation $x^2 - ax - bx + ab - c = 0$. What is the quadratic equation whose roots are a and b?

(a) $x^2 - \alpha x - \beta x + \alpha\beta + c = 0$
(b) $x^2 - \alpha x - \beta x + \alpha\beta - c = 0$
(c) $x^2 + \alpha x + \beta x + \alpha\beta + c = 0$
(d) $x^2 + \alpha x + \beta x + \alpha\beta - c = 0$

19. If the roots of the equation $x^2 - ax - bx - cx + bc + ca = 0$ are equal, then which one of the following is correct?

(a) $a + b + c = 0$ (b) $a - b + c = 0$
(c) $a + b - c = 0$ (d) $-a + b + c = 0$

20. Let α and β $(\alpha > \beta)$ be the roots of the equation $x^2 - 8x + q = 0$. If $\alpha^2 - \beta^2 = 16$, then what is the value of q?

(a) –15 (b) –10
(c) 10 (d) 15

21. What is the maximum value of n such that 5^n divides $(30! + 35!)$, where n is a natural number?

(a) 4 (b) 6
(c) 7 (d) 8

22. What is the value of $2(2 \times 1) + 3(3 \times 2 \times 1) + 4(4 \times 3 \times 2 \times 1) + 5(5 \times 4 \times 3 \times 2 \times 1) + \ldots\ldots\ldots + 9(9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 2$?

(a) 11! (b) 10!
(c) 10 + 10! (d) 11 + 10!

23. If $A = \{(1, 2, 3)\}$, then how many elements are there in the power set of A?

(a) 1 (b) 2
(c) 4 (d) 8

24. If a, b, c are in GP where $a > 0, b > 0, c > 0$, then which of the following are correct?

1. a^2, b^2, c^2 are in GP
2. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in GP
3. $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in GP

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

25. If $\frac{a+b}{2}, b, \frac{b+c}{2}$ are in HP, then which one of the following is correct?

(a) a, b, c are in AP
(b) a, b, c are in GP
(c) $a + b, b + c, c + a$ are in GP
(d) $a + b, b + c, c + a$ are in AP

26. What is value of $\cot^2 15° + \tan^2 15°$?

(a) 12 (b) 14
(c) $8\sqrt{3}$ (d) 4

27. In a triangle ABC,

$$\sin A - \cos B - \cos C = 0.$$

What is angle B equal to?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

28. If $\alpha + \beta = \frac{\pi}{4}$ and $2\tan\alpha = 1$, then what is $\tan 2\beta$ equal to?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) $\frac{3}{5}$

29. If $\tan(45° + \theta) = 1 + \sin 2\theta$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, then what is the value of $\cos 2\theta$?

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

30. Let $\sin 2\theta = \cos 3\theta$, where θ is acute angle. What is the value of $1 + 4\sin\theta$?

$\left(\text{given that } \sin 18° = \frac{\sqrt{5}-1}{4}\right)$

(a) $\sqrt{3}$ (b) 2

(c) $\sqrt{5}$ (d) 3

31. If $\tan\theta = -\frac{5}{12}$, then what can be the value of $\sin\theta$?

(a) $\frac{5}{13}$ but cannot be $-\frac{5}{13}$

(b) $-\frac{5}{13}$ but cannot be $\frac{5}{13}$

(c) $\frac{5}{13}$ or $-\frac{5}{13}$

(d) None of the above

32. What is the value of $\cos^4\frac{7\pi}{8} + \cos^4\frac{5\pi}{8}$?

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$

(c) $\frac{3}{8}$ (d) $\frac{3}{16}$

33. What is $\sin^2\left(\frac{\pi}{4}+\theta\right) - \sin^2\left(\frac{\pi}{4}-\theta\right)$ equal to?

(a) $\sin 2\theta$ (b) $\cos 2\theta$

(c) $2\sin\theta$ (d) $2\cos\theta$

34. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane the angles of elevation of the bottom and top of the flagstaff are θ and 2θ respectively. What is the height of the tower?

(a) $h\cos\theta$ (b) $h\sin\theta$

(c) $h\cos 2\theta$ (d) $h\sin 2\theta$

35. The shadow of a tower becomes x metre longer, when the angle of elevation of sun changes from 60° to θ. If the height of the tower is $\sqrt{3}x$ metre, then which one of the following is correct?

(a) $0 < \theta < 30°$ (b) $30° < \theta < 45°$

(c) $45° < \theta < 60°$ (d) $60° < \theta < 90°$

36. If $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4}$, where $0 < x < 6$, then what is x equal to?

(a) 1 (b) 2

(c) 3 (d) 5

37. If $3\sin^{-1}x + \cos^{-1}x = \pi$, then what is x equal to?

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

38. If $\tan\alpha + \tan\beta = 1 - \tan\alpha \cdot \tan\beta$, where then which of the following is one of the values of $(\alpha + \beta)$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

39. If $(1 + \tan\theta)(1 + \tan 9\theta) = 2$, then what is the value of $\tan(10\theta)$?

(a) 0 (b) 1

(c) 2 (d) Infinite

40. What is the value of
$\sin 0° + \sin 10° + \sin 20° + \sin 30° + ... + \sin 360°$?

(a) –1 (b) 0

(c) 1 (d) 2

41. Consider all the subsets of the set $A = \{1, 2, 3, 4\}$. How many of them are supersets of the set $\{4\}$?

(a) 6 (b) 7

(c) 8 (d) 9

42. Consider the following statements in respect of two non-empty sets A and B :

1. $x \notin (A \cup B) \Rightarrow x \notin A$ or $x \notin B$
2. $x \notin (A \cap B) \Rightarrow x \notin A$ and $x \notin B$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

43. Consider the following statements in respect of two non-empty sets A and B :

1. $A \cup B = A \cap B$ iff $A = B$
2. $A \Delta B = \varphi$ iff $A = B$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

44. Consider the following statements in respect of the relation R in the set IN of natural numbers defined by xRy if $x^2 - 5xy + 4y^2 = 0$:

1. R is reflexive
2. R is symmetric
3. R is transitive

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) 1 and 2 only (d) 1, 2 and 3

45. Consider the following statements in respect of any relation R on a set A :

1. If R is reflexive, then R^{-1} is also reflexive
2. If R is symmetric, then R^{-1} is also symmetric
3. If R is transitive, then R^{-1} is also transitive

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

46. What is the principal argument of $\frac{1}{1+i}$ where $i = \sqrt{-1}$?

(a) $-\frac{3\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

47. What is the modulus of $\left(\frac{\sqrt{-3}}{2}-\frac{1}{2}\right)^{200}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2^{200}

48. Consider the following statements :

1. $\frac{n!}{3!}$ is divisible by 6, where $n > 3$
2. $\frac{n!}{3!}+3$ is divisible by 7, where $n > 3$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

49. In how many ways can a team of 5 players be selected out of 9 players so as to exclude two particular players?

(a) 14 (b) 21
(c) 35 (d) 42

50. In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, what is the $(n + 1)$th term from the end (when arranged in descending powers of x)?

(a) $C(2n, n)x$ (b) $C(2n, n-1)x$
(c) $C(2n, n)$ (d) $C(2n, n-1)$

51. If the sum of the first 9 terms of an AP is equal to sum of the first 11 terms, then what is the sum of the first 20 terms?

(a) 20 (b) 10
(c) 2 (d) 0

52. If the 5th term of an AP is $\frac{1}{10}$ and its 10th term is $\frac{1}{5}$, then what is the sum of first 50 terms?

(a) 25 (b) 25.5
(c) 26 (d) 26.5

53. What is $(1110011)_2 \div (10111)_2$ equal to?

(a) $(101)_2$ (b) $(1001)_2$
(c) $(111)_2$ (d) $(1011)_2$

54. If $x^3 + y^3 = (100010111)_2$ and $x + y = (11111)_2$, then what is $(x - y)^2 + xy$ equal to?

(a) $(1101)_2$ (b) $(1001)_2$
(c) $(1011)_2$ (d) $(1111)_2$

55. Consider the inequations $5x - 4y + 12 < 0, x + y < 2, x < 0$ and $y > 0$. Which one of the following points lies in the common region?

(a) (0, 0) (b) (–2, 4)
(c) (–1, 4) (d) (–1, 2)

56. Consider the following statements in respect of the function $y = [x],\ x \in (-1, 1)$ where [.] is the greatest integer function:

1. Its derivative is 0 at $x = 0.5$
2. It is continuous at $x = 0$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

57. What is the degree of the differential equation

$$1+\left(\frac{dy}{dx}\right)^2=\left(\frac{d^2y}{dx^2}\right)^{\frac{4}{3}} ?$$

(a) $\frac{4}{3}$ (b) 2
(c) 3 (d) 4

58. A radioactive substance decays at a rate proportional to the amount of substance present. If half of the substance decays in 100 years, then what is the decay constant (proportionality constant)?

(a) $\frac{\ell n 2}{100}$ (b) $\frac{\ell n 5}{100}$
(c) $\frac{\ell n 10}{100}$ (d) $\frac{2\ell n 2}{100}$

59. What is the domain of the function $f(x)=\sqrt{1-(x-1)^2}$?

(a) (0, 1) (b) [–1, 1]
(c) (0, 2) (d) [0, 2]

60. The area of the region bounded by the parabola $y^2 = 4kx$, where $k > 0$ and its latus rectum is 24 square units. What is the value of k?

(a) 1 (b) 2
(c) 3 (d) 4

61. What is $\int_0^{\frac{\pi}{4}} \frac{dx}{(\sin x+\cos x)^2}$ equal to?

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) $\frac{3}{2}$

62. What is $\int (\sin x)^{-1/2} (\cos x)^{-3/2} dx$ equal to?

(a) $\sqrt{\tan x} + c$ (b) $2\sqrt{\tan x} + c$

(c) $\sqrt{\cot x} + c$ (d) $\sqrt{2 \tan x} + c$

63. If $I_1 = \int \frac{e^x dx}{e^x + e^{-x}}$ and $I_2 = \int \frac{dx}{e^{2x}+1}$, then what is $I_1 + I_2$ equal to?

(a) $\frac{x}{2} + c$ (b) $x + c$

(c) $\ell n(e^x + e^{-x}) + c$ (d) $\ell n(e^x - e^{-x}) + c$

64. What is $\int_{-2}^{-1} \frac{x}{|x|} dx$ equal to?

(a) –2 (b) –1

(c) 1 (d) 2

65. How many extreme values does $\sin 4x + 2x$, where $0 < x < \frac{\pi}{2}$ have?

(a) 1 (b) 2

(c) 4 (d) 8

66. What is the maximum value of the function

$f(x) = \frac{1}{\tan x + \cot x}$, where $0 < x < \frac{\pi}{2}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

67. If $4f(x) - f\left(\frac{1}{x}\right) = \left(2x + \frac{1}{x}\right)\left(2x - \frac{1}{x}\right)$, then what is $f(2)$ equal to?

(a) 0 (b) 1

(c) 2 (d) 4

68. If $f(x) = 4x + 3$, then what is $f \circ f \circ f(-1)$ equal to?

(a) –1 (b) 0

(c) 1 (d) 2

69. If $x^y y^x = 1$, then what is $\frac{dy}{dx}$ at (1, 1) equal to?

(a) –1 (b) 0

(c) 1 (d) 4

70. If $y = (x^x)^x$, then what is the value of $\frac{dy}{dx}$ at $x = 1$?

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 4

71. Let $y = [x + 1]$, $-4 < x < -3$ where [.] is the greatest integer function. What is the derivative of y with respect to x at $x = -3.5$?

(a) –4 (b) –3.5

(c) –3 (d) 0

72. If $\frac{dy}{dx} = (\ell n 5) y$ with $y(0) = \ln 5$, then what is $y(1)$ equal to?

(a) 0 (b) 5

(c) $2 \ell n 5$ (d) $5 \ell n 5$

73. Consider the following in respect of the function $f(x) = 10^x$:

1. Its domain is $(-\infty, \infty)$
2. It is a continuous function
3. It is differentiable at $x = 0$

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

74. What is $\lim_{x \to 0} x^3 (\operatorname{cosec} x)^2$ equal to?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) Limit does not exist

75. What is $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$ equal to?

(a) 0 (b) 3

(c) 6 (d) Limit does not exist

76. In which one of the following intervals is the function $f(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 5$ decreasing?

(a) $(-\infty, 1)$ only (b) $(1, 6)$

(c) $(6, \infty)$ only (d) $(-\infty, 1) \cup (6, \infty)$

77. If the derivative of the function $f(x) = \frac{m}{x} + 2nx + 1$ vanishes at $x = 2$, then what is the value of $m + 8n$?

(a) –2

(b) 0

(c) 2

(d) Cannot be determined due to insufficient data

78. What is the area included in the first quadrant between the curves $y = x$ and $y = x^3$?

(a) $\frac{1}{8}$ square unit (b) $\frac{1}{4}$ square unit

(c) $\frac{1}{2}$ square unit (d) 1 square unit

79. If $xy = 4225$ where x, y are natural numbers, then what is the minimum value of $x + y$?

(a) 130 (b) 260

(c) 2113 (d) 4226

80. What does the equation $x\frac{dy}{dx} - 2y = 0$ represent?

(a) A family of straight lines

(b) A family of circles

(c) A family of parabolas

(d) A family of ellipses

81. If the points with coordinates $(-5, 0)$, $(5p^2, 10p)$ and $(5q^2, 10q)$ are collinear, then what is the value of pq where $p \neq q$?

(a) -2 (b) -1

(c) 1 (d) 2

82. What is the equation of the straight line which passes through the point $(1, -2)$ and cuts off equal intercepts from the axes?

(a) $x + y - 1 = 0$ (b) $x - y - 1 = 0$

(c) $x + y + 1 = 0$ (d) $x - y - 2 = 0$

83. What is the equation of the circle which touches both the axes in the first quadrant and the line $y - 2 = 0$?

(a) $x^2 + y^2 - 2x - 2y - 1 = 0$

(b) $x^2 + y^2 + 2x + 2y + 1 = 0$

(c) $x^2 + y^2 - 2x - 2y + 1 = 0$

(d) $x^2 + y^2 - 4x - 4y + 4 = 0$

84. What is the equation of the parabola with focus $(-3, 0)$ and directrix $x - 3 = 0$?

(a) $y^2 = 3x$ (b) $x^2 = 12y$

(c) $y^2 = 12x$ (d) $y^2 = -12x$

85. What is the distance between the foci of the ellipse $x^2 + 2y^2 = 1$?

(a) 1 (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

86. Let a, b, c be the lengths of sides BC, CA, AB respectively of a triangle ABC. If p is the perimeter and q is the area of the triangle, then what is $p(p-2a)\tan\left(\frac{A}{2}\right)$ equal to?

(a) q (b) $2q$

(c) $3q$ (d) $4q$

87. A straight line passes through the point of intersection of $x + 2y + 2 = 0$ and $2x - 3y - 3 = 0$. It cuts equal intercepts in the fourth quadrant. What is the sum of the absolute values of the intercepts?

(a) 2 (b) 3

(c) 4 (d) 6

88. Under which one of the following conditions are the lines $ax + by + c = 0$ and $bx + ay + c = 0$ parallel $(a \neq 0,\ b \neq 0)$?

(a) $a - b = 0$ only (b) $a + b = 0$ only

(c) $a^2 - b^2 = 0$ (d) $ab + 1 = 0$

89. What is the equation of the locus of the mid-point of the line segment obtained by cutting the line $x + y = p$, (where p is a real number) by the coordinate axes?

(a) $x - y = 0$ (b) $x + y = 0$

(c) $x - y = p$ (d) $x + y = p$

90. If the point (x, y) is equidistant from the points $(2a, 0)$ and $(0, 3a)$ where $a > 0$, then which one of the following is correct?

(a) $2x - 3y = 0$ (b) $3x - 2y = 0$

(c) $4x - 6y + 5a = 0$ (d) $4x - 6y - 5a = 0$

Consider the following for the next ***three*** (03) items that follow:

The plane $6x + ky + 3z - 12 = 0$ where meets the coordinate axes at A, B and C respectively. The equation of the sphere passing through the origin and A, B, C is $x^2 + y^2 + z^2 - 2x - 3y - 4z = 0$.

91. What is the value of k?

(a) 3 (b) 4

(c) 6 (d) 12

92. If p is the perpendicular distance from the centre of the sphere to the plane, then which one of the following is correct?

(a) $0 < p < 0.5$ (b) $0.5 < p < 1$

(c) $1 < p < 1.5$ (d) $p > 1.5$

93. What is the equation of the line through the origin and the centre of the sphere?

(a) $x = y = z$ (b) $2x = 3y = 4z$

(c) $6x = 3y = 4z$ (d) $6x = 4y = 3z$

Consider the following for the next ***two*** (02) items that follow:

Let the plane $\frac{2x}{k} + \frac{2y}{3} + \frac{z}{3} = 2$ pass through the point $(2, 3, -6)$.

94. What are the direction ratios of a normal to the plane?

(a) <3, 2, 1> (b) <2, 3, 6>

(c) <6, 3, 2> (d) <1, 2, 3>

95. If p, q and r are the intercepts made by the plane on the coordinate axes respectively, then what is $(p + q + r)$ equal to?

(a) 10 (b) 11

(c) 12 (d) 13

96. If $4\hat{i} + \hat{j} - 3\hat{k}$ and $p\hat{i} + q\hat{j} - 2\hat{k}$ are collinear vectors, then what are the possible values of p and q respectively?

(a) 4, 1 (b) 1, 4

(c) $\frac{8}{3}, \frac{2}{3}$ (d) $\frac{2}{3}, \frac{8}{3}$

97. If are the position vectors of the vertices A, B, C respectively of a triangle ABC and G is the centroid of the triangle, then what is equal to?

(a) $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$ (b) $\frac{2\vec{a} - \vec{b} - \vec{c}}{3}$

(c) $\frac{\vec{b} + \vec{c} - 2\vec{a}}{3}$ (d) $\frac{\vec{a} - 2\vec{b} - 2\vec{c}}{3}$

98. Consider the following statements :

1. Dot product over vector addition is distributive
2. Cross product over vector addition is distributive
3. Cross product of vectors is associative

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) 1 and 2 only (d) 1, 2 and 3

99. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors such that $\vec{a}\times\vec{b}=\vec{c}$. Consider the following statements :

1. $\vec{a}$ is unique if $\vec{b}$ and $\vec{c}$ are given
2. $\vec{c}$ is unique if $\vec{a}$ and $\vec{b}$ are given

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

100. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a}-\vec{b}|<2$. If 2θ is the angle between $\vec{a}$ and $\vec{b}$, then which one of the following is correct?

(a) $0<\sin\theta<1$ only (b) $-\frac{1}{2}<\sin\theta<\frac{1}{2}$ only
(c) $-1<\sin\theta<0$ only (d) $-1<\sin\theta<1$

101. Two digits out of 1, 2, 3, 4, 5 are chosen at random and multiplied together. What is the probability that the last digit in the product appears as 0?

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$
(c) $\frac{2}{5}$ (d) $\frac{4}{5}$

102. The frequency curve (assuming unimodal) corresponding to the data obtained in an experiment is skewed to the left. What conclusion can be drawn from the curve?

(a) Mean > Median > Mode
(b) Mean > Mode > Median
(c) Median > Mean > Mode
(d) Mode > Median > Mean

103. The variance of five positive observations is 3.6. If four of the observations are 2, 2, 4, 5 then what is the remaining observation?

(a) 4 (b) 5
(c) 7 (d) 9

104. What is the arithmetic mean of 50 terms of an AP with first term 4 and common difference 4?

(a) 50 (b) 51
(c) 100 (d) 102

105. What is the coefficient of mean deviation of 21, 34, 23, 39, 26, 37, 40, 20, 33, 27 (taken from mean)?

(a) 0.11 (b) 0.22
(c) 0.33 (d) 0.44

Consider the following for the next ***three*** (03) items that follow:

The algebraic sum of the deviations of a set of values x_1, x_2, x_3, ... x_n measured from 100 is –20 and the algebriac sum of the deviations of the same set of values measured from 92 is 140.

106. What is the mean of the values?

(a) 91 (b) 96
(c) 98 (d) 99

107. What is the algebraic sum of the deviations of the same set of values measured from 99?

(a) 0 (b) 10
(c) 20 (d) 40

108. If the algebraic sum of the deviations of the same set of values measured from y is 180, then what is the value of y?

(a) 80 (b) 85
(c) 90 (d) 95

Consider the following data for the next ***three*** (03) items that follow:

The marks obtained by 51 students in a class are in AP with its first term 4 and common difference 3.

109. What is the mean of the marks?

(a) 67 (b) 71
(c) 75 (d) 79

110. What is the median of the marks?

(a) 79.5 (b) 79
(c) 78.5 (d) 77

111. What is the sum of the deviations measured from the median?

(a) –1 (b) 0
(c) 1 (d) 2

Consider the following for the next ***three*** (03) items that follow:

There are 90 applicants for a job. Some of them are graduates. Some of them have less than three years experience.

	Number of graduates	*Number of non-graduates*
At least 3 years experience	18	9
Less than 3 years experience	36	27

Let G be the event that the first applicant interviewed is a graduate and T be the event that first applicant interviewed has at least 3 years experience.

112. What is $P(G\cap\overline{T})$ equal to?

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

113. What is $P(G\,|\,\overline{T})$ equal to?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$
(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

114. What is $P(\overline{T} \mid \overline{G})$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{3}{5}$ (d) $\frac{3}{4}$

Consider the following for the next ***three*** (03) items that follow:

The incidence of suffering from a disease among workers in an industry has a chance of $33\frac{1}{3}\%$.

115. What is the probability that exactly 3 out of 6 workers suffer from a disease?

(a) $\frac{80}{729}$ (b) $\frac{10}{81}$

(c) $\frac{10}{243}$ (d) $\frac{160}{729}$

116. What is the probability that no one out of 6 workers suffers from a disease?

(a) $\frac{665}{729}$ (b) $\frac{64}{729}$

(c) $\frac{4}{243}$ (d) $\frac{1}{729}$

117. What is the probability that at least one out of 6 workers suffer from a disease?

(a) $\frac{728}{729}$ (b) $\frac{665}{729}$

(c) $\frac{653}{729}$ (d) $\frac{596}{729}$

Consider the following frequency distribution for the next ***three*** (03) items that follow:

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	$p+q$	32	$p-3q$	19

The total frequency is 120. The mean is 50.

118. What is the value of p?

(a) 25 (b) 26

(c) 27 (d) 28

119. What is the value of q?

(a) 1 (b) 2

(c) 3 (d) 4

120. If the frequency of each class is doubled, then what would be the mean?

(a) 25 (b) 50

(c) 75 (d) 100

HINTS & SOLUTIONS

MATHEMATICS

1. (c) $\because\ \Delta_1^1 = \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ q & r & p \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix} \begin{bmatrix} \text{Applying} \\ c_1 \longleftrightarrow c_2 \\ c_2 \longleftrightarrow c_3 \end{bmatrix}$

$\therefore \quad \Delta_1^1 = \Delta_1 = \Delta_2$

$\therefore \quad \Delta_1 + \Delta_2 = 2\,\Delta_2 = 2\begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix}$

[Applying $c_2 \to c_2 - c_1$ and $c_3 \to c_3 - c_1$]

$= 2\begin{vmatrix} 1 & 0 & 0 \\ q & r-q & p-q \\ r & p-r & q-r \end{vmatrix}$

$= -2[p^2 + q^2 + r^2 - pq - qr - pr]$

$= -[(p-q)^2 + (q-r)^2 + (r-p)^2] < 0.$

Always negative.

2. (b) Let $\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

$= abc\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Applying $c_1 \to c_1 - c_2$ and $c_2 \to c_2 - c_3$

$= abc\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \end{vmatrix}$

$= abc\,(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & (b+c) & c^2 \end{vmatrix}$

$= abc\,(a-b)(b-c)\,(c-a)$

$= 6 \times 2 = 12$

3. (d) Let $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

Applying $c_2 \to c_2 - c_1$ and $c_3 \to c_3 - c_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & c-b & a-b \\ c & a-c & b-c \end{vmatrix} = 0$$

$= (a+b+c)\,[-a^2 - b^2 - c^2 + ab + bc + ca] = 0$
$= -(a+b+1)\,[a^2 + b^2 + c^2 - ab - bc - ca] = 0$
$= -(a^3 + b^3 + c^3 - 3abc) = 0$
$\Rightarrow a + b + c = 0$ or $a^2 + b^2 + c^2 - ab - bc - ca = 0$
or $a^3 + b^3 + c^3 = 3abc$
Hence all statement 1, 2 and 3 are correct.

4. (c) $CA = \begin{bmatrix} m \\ -m \end{bmatrix}[m\ n] = \begin{bmatrix} m^2 & mn \\ -m^2 & -mn \end{bmatrix}$

$CB = \begin{bmatrix} m \\ -m \end{bmatrix}[-n\ -m] = \begin{bmatrix} -mn & -m^2 \\ mn & m^2 \end{bmatrix}$

$\therefore \quad CA \neq CB$
So, statement 1 is not correct.

$AC = [m \quad n]\begin{bmatrix} m \\ -m \end{bmatrix} = [m^2 - mn]$

$BC = [-n \quad -m]\begin{bmatrix} m \\ -m \end{bmatrix} = [-mn + m^2]$

$\therefore \quad AC = BC$
So, statement 2 is correct.

$C(A+B) = \begin{bmatrix} m \\ -m \end{bmatrix}[m-n \quad n-m]$

$= \begin{bmatrix} m^2 - mn & mn - m^2 \\ -m^2 + mn & -mn + m^2 \end{bmatrix}$

$CA + CB = \begin{bmatrix} m^2 - mn & mn - m^2 \\ -m^2 + mn & -mn + m^2 \end{bmatrix} = C(A+B)$

So, statement 3 is correct.

5. (d) $|A| = \begin{vmatrix} 2\sin\theta & \cos\theta & 0 \\ -2\cos\theta & \sin\theta & 0 \\ -1 & 1 & 1 \end{vmatrix} = 2$

We know that
$A(\text{adj } A) = |A|\, I = 2I$

6. (d) $\begin{vmatrix} 2\cos 2\theta & 2\cos 2\theta & 6 \\ 1 - 2\sin^2\theta & 2\cos^2\theta - 1 & 3 \\ k & 2k & 1 \end{vmatrix}$

$= \begin{vmatrix} 2\cos^2\theta & 2\cos^2\theta & 6 \\ \cos^2\theta & \cos^2\theta & 3 \\ k & 2k & 1 \end{vmatrix} = 0$

($\because R_1$ and R_2 are identical)
Hence, for any real value of k the given matrix is singular.

7. (b) Given that $|A| \neq 0$ and $B = \text{adj } A$
$AB = A(\text{adj } A) = |A|I$ ...(i)
$BA = [\text{adj } (A)]\, A = |A|I$
$\therefore \quad AB = BA$
So, statement 1 is correct
It is clear from (i) AB is a scalar matrix not null matrix
So statement 2 is correct and 3 is not correct.

8. (b) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

But $A \neq 0$ and $B \neq 0$
So, statement I is not correct
$\because \quad AB = \text{I} \Rightarrow \quad B = A^{-1}$
We know that $AA^{-1} = A^{-1}A = I$
$\therefore \quad BA = I$
So, statement 2 is correct.

9. (d) Given that $A = I$
and $B = A^T = I^T = \text{I}$
Now $\text{C} = \text{A} + \text{B} = \text{I} + \text{I} = 2\text{I}$
$\therefore \quad |C| = (2)^3\,|I| = 8.1 = 8$

10. (b) Given that $|A| \neq 0$, $|B| \neq 0$
$AB = A \Rightarrow A^{-1}AB = A^{-1}A \Rightarrow B = I$
and $BA = B \Rightarrow B^{-1}BA = B^{-1}B$
$A = I$
$\therefore \quad A^2 = I^2 = I = A$
So, statement 1 is correct
Now, $\quad AB^2 = I^3 = I$
$\qquad A^2B = I^3 = I$
$\therefore \quad AB^2 = A^2B$
So, statement 2 is correct

11. (b) $\left(1 + \dfrac{2}{x}\right)^9 \left(1 - \dfrac{2}{x}\right)^9 = \left(1 - \dfrac{4}{x^2}\right)^9$

Number of terms $= 9 + 1 = 10$

12. (c) Middle term of $(x+y)^{10}$ is $\left(\dfrac{10+2}{2}\right)^{th}$ term $= 6^{th}$ term

We know that coefficient of middle term is highest value.
So, statement 1 is correct
Since total number of term $= 10 + 1 = 11$
Coefficient of 3rd term = coefficient of $(11 - 3 + 1)^{th}$ term
= Coefficient of 9^{th} term
So, statement 2 is also correct.

13. (d) Given that

$${}^{3n}C_{2n} = {}^{3n}C_{2n-7}$$

$\Rightarrow$ $2An + 2n - 7 = 3n$

$\Rightarrow$ $n = 7$

$\therefore$ $^nC_{n-5} = {}^7C_2 = \dfrac{7.6}{2.1} = 21$

14. (c) $^{51}C_{21} - {}^{51}C_{22} + {}^{51}C_{23} - {}^{51}C_{24} + {}^{51}C_{25} - {}^{51}C_{26} + {}^{51}C_{27} - {}^{51}C_{28} + {}^{51}C_{29} - {}^{51}C_{30}$

$= {}^{51}C_{51-21} - {}^{51}C_{51-22} + {}^{51}C_{51-23} - {}^{51}C_{51-24} + {}^{51}C_{51-25} -$

$^{51}C_{26} + {}^{51}C_{27} - {}^{51}C_{28} + {}^{51}C_{29} - {}^{51}C_{30}$ $[\because\ {}^nC_r = {}^nC_{n-r}]$

$= {}^{51}C_{30} - {}^{51}C_{29} - {}^{51}C_{28} - {}^{51}C_{27} + {}^{51}C_{26} - {}^{51}C_{26} + {}^{51}C_{27} - {}^{51}C_{28} + {}^{51}C_{29} - {}^{51}C_{30}$

$= 0$

Now, $^{51}C_{51} - {}^{51}C_0 = 1 - 1 = 0$

15. (a)

3	.	.
1	8	4

3 is fixed in hundred place, so no. of choice = 1

Number of choice for unit place {1, 5, 7, 9} = 4

Number of choice for tens place = 8

$\therefore$ Number of odd number between 300 and 400

$= 1 \times 4 \times 8 = 32$

16. (b) Vowels = {I, E}

1	2	3	4	5
v		v		v

$\therefore$ Number of words in which vowels not occupy the even positions $= {}^3C_2 \cdot 2! \cdot 3! = 36$

17. (d) Given that α, β are roots of equation $x^2 + px + q = 0$

$\therefore$ $\alpha + \beta = -p$ and $\alpha . \beta = q$

and α^3 and β^3 are roots of equation $x^2 + mn + n = 0$

$\therefore$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -m$

$\Rightarrow$ $-p^3 + 3pq = -m \Rightarrow m = p^3 - 3pq$ ...(*i*)

$\alpha^3 . \beta^3 = (\alpha\beta)^3 = q^3 = n$ ...(*ii*)

$\therefore$ $m + n = p^3 + q^3 - 3pq$ [from (*i*) and (*ii*)]

18. (a) Given that α and β are roots of the equation

$x^2 - ax - bx + ab - c = 0$

i.e $x^2 - (a + b)x + ab - c = 0$

$\therefore$ $\alpha + \beta = a + b$ $\alpha.\beta = ab - c$

$\Rightarrow$ $ab = \alpha\beta + c$

The quadratic equation whose roots are a and b is

$x^2 - (a + b)x + ab = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

$x^2 - \alpha x - \beta x + \alpha\beta + c = 0$

19. (c) Since $x^2 - (a + b + c)x + bc + ca = 0$ has equal roots

$\therefore$ $[-(a + b + c)]^2 - 4.1\,(bc + ca) = 0$

$\Rightarrow$ $[(a + b) + c]^2 - 4c\,(a + b) = 0$

$\Rightarrow$ $(a + b)^2 + c^2 + 2c\,(a + b) - 4c\,(a + b) = 0$

$\Rightarrow$ $(a + b - c)^2 = 0$

$\Rightarrow$ $a + b - c = 0$

20. (d) Given that α and β $(\alpha > \beta)$ are roots of the equation $x^2 - 8x + q = 0$

$\therefore$ $\alpha + \beta = 8$ and $\alpha.\beta = q$

$\therefore$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 64 - 4q$

$(\alpha - \beta) = \sqrt{64 - 4q}$ $(\because \alpha > \beta)$

Now $\alpha^2 - \beta^2 = 16$

$\Rightarrow$ $(\alpha + \beta) - (\alpha - \beta) = 16$

$\Rightarrow$ $8\sqrt{64 - 4q} = 16$ $\Rightarrow$ $\sqrt{64 - 4q} = 2$

$\Rightarrow$ $64 - 4q = 4$ $\Rightarrow$ $q = 15$

21. (c) $30! + 35! = 30! + 35.34.33.32.31.30!$

$30!\,(1 + 35.34.33.32.31)$

$\because$ $1 + 35!\ .34.33.32.31$ is not divisible by 5

So, maximum value of n such that 5^n divides 30!

$= \left[\dfrac{30}{5}\right] + \left[\dfrac{30}{5^2}\right] + \left[\dfrac{30}{5^3}\right] + ...$

$= 6 + 1 + 0 + 0 ... = 7.$

22. (b) $2(2 \times 1) + 3\,(3 \times 2 \times 1) + 4\,(4 \times 3 \times 2 \times 1) + 5\,(5 \times 4 \times 3 \times 2 \times 1) + ... + 9(9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 2$

$= 2 + 2.2! + 3.3! + 4.4! + 5.5! + ... + 9.9!$

$= 2! + (2 + 1) + 3.3! + 4.4! + 5.5! + ... + 9.9!$

$= 3! + 3.3! + 4.4! + 5.5! + ... + 9.9!$

Proceed further we get

$= 9! + 9.9! = 10!$

23. (b) $\because$ A = {(1, 2, 3)}

$\therefore$ n(A) = 1

Number of element in P(A) $= 2^1 = 2$

24. (d) $\because$ a, b, c are in G.P

$\therefore$ $b^2 = ac$ ...(i)

Squaring both sides

$(b^2)^2 = a^2 c^2$

$\therefore$ a^2, b^2, c^2 are in G.P

From (i)

$\left(\dfrac{1}{b}\right)^2 = \dfrac{1}{a} \times \dfrac{1}{c}$

$\therefore$ $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ *are in* G.P

From (i)

$b = \sqrt{ac} \Rightarrow \left(\sqrt{b}\right)^2 = \sqrt{a}\sqrt{b}$

$[\because$ a > 0, b > 0 and c > 0]

$\therefore$ $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

25. (b) $\because$ $\dfrac{a + b}{2}, b, \dfrac{b + c}{2}$ are in H.P

$\therefore$ $\dfrac{2}{b} = \dfrac{2}{a + b} + \dfrac{2}{b + c} = \dfrac{2(a + 2b + c)}{(a + b)(b + c)}$

$\Rightarrow$ $(a + b)\,(b + c) = b(a + 2b + c)$

$\Rightarrow$ $ab + ca + b^2 + bc = ab + 2b^2 + bc$

$\Rightarrow$ $b^2 = ca$

$\therefore$ a, b, c are in G.P.

26. (b) $\cot^2 15° + \tan^2 15°$

$= \text{cosec}^2 15° - 1 + \sec^2 15° - 1$

$= \text{cosec}^2 15° + \sec^2 15° - 2$

$$= \frac{1}{\sin^2 15^\circ} + \frac{1}{\cos^2 15^\circ} - 2$$

$$= \frac{\sin^2 15^\circ + \cos^2 15^\circ}{\frac{1}{4}(4\sin^2 15^\circ . \cos^2 15^\circ)} - 2$$

$$= \frac{4}{\sin^2 30^\circ} - 2 = \frac{4}{\frac{1}{4}} - 2 = 16 - 2 = 14$$

27. (d) In ΔABC

$A + B + C = \pi$

$$\sin A = \cos B + \cos C = 2\cos\frac{B+C}{2}.\cos\frac{B-C}{2}$$

$$\Rightarrow \sin A = 2\cos\left(\frac{\pi}{2} - \frac{A}{2}\right).\cos\left(\frac{B-C}{2}\right)$$

$[\because B + C = \pi - A]$

$$\Rightarrow 2\sin\frac{A}{2}.\cos\frac{A}{2} = 2\sin\frac{A}{2}.\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \cos\frac{A}{2} = \cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \frac{A}{2} = \frac{B-C}{2} \Rightarrow A = B - C$$

$\Rightarrow A + C = B \Rightarrow \pi - B = B$

$$\Rightarrow B = \frac{\pi}{2}$$

28. (c) $\because \quad 2\tan\alpha = 1 \Rightarrow 2\tan\alpha = \frac{1}{2}$

$$\alpha + \beta = \frac{\pi}{4} \Rightarrow \tan(\alpha+\beta) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = 1.$$

$\Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha . \tan\beta$

$$\Rightarrow \frac{1}{2} + \tan\beta = 1 - \frac{1}{2}.\tan\beta$$

$\Rightarrow 1 + 2\tan\beta = 2 - \tan\beta$

$$\tan\beta = \frac{1}{3}$$

$$\text{Now, } \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}} = \frac{2}{3}\times\frac{9}{8} = \frac{3}{4}$$

29. (c) $\tan(45^\circ + \theta) = 1 + \sin 2\theta$

$$\Rightarrow \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ.\tan\theta} - 1 = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta} - 1 = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan\theta} = \frac{2\tan\theta}{1+\tan^2\theta}$$

$\Rightarrow 2\tan\theta\,(1 + \tan^2\theta) = 2\tan\theta\,(1 - \tan\theta)$

$\Rightarrow \tan\theta\,[1 + \tan^2\theta - 1 + \tan\theta] = 0$

$\Rightarrow \tan^2\theta\,(\tan\theta + 1) = 0$

$\therefore \tan\theta = 0$ or $\tan\theta = -1$ (Not possible)

$$\Rightarrow \theta = 0 \qquad \left[\therefore \frac{-\pi}{4} < \theta < \frac{\pi}{4}\right]$$

Now, $\cos 2\theta = \cos\theta = 1$

30. (a) $\sin 2\theta = \cos 3\theta = \sin(90^\circ - 3\theta)$

$\Rightarrow 2\theta = 90^\circ - 3\theta \Rightarrow \theta = 18^\circ$

Now, $1 + 4\sin\theta = 1 + 4\sin 18^\circ$

$$= 1 + 4.\left(\frac{\sqrt{3}-1}{4}\right) = \sqrt{3}$$

31. (c) $\tan\theta = \frac{-5}{12} = \frac{p}{b}$; Let $p = 5k \quad b = 12k$

$$\therefore \quad h = \sqrt{p^2 + b^2} = \sqrt{25k^2 + 144k^2} = 13k$$

Since value of $\tan\theta$ is –ve therefore θ lies in 2nd and 3rd quadrant.

$$\therefore \quad \sin\theta = \frac{p}{h} = \pm\frac{5}{13}$$

32. (b) $\cos^4\frac{7\pi}{8} + \cos^4\frac{5\pi}{8} = \left(\cos^2\frac{7\pi}{8}\right)^2 + \left(\cos^2\frac{5\pi}{8}\right)^2$

$$= \left(\cos^2\frac{7\pi}{8} - \cos^2\frac{5\pi}{8}\right)^2 + 2\cos^2\frac{7\pi}{8}.\cos^2\frac{5\pi}{8}$$

$$= \left[(-1)\sin\left(\frac{7\pi}{8}+\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}-\frac{5\pi}{8}\right)\right]^2 + \frac{1}{2}\left[2\cos\frac{7\pi}{8}.\cos\frac{5\pi}{8}\right]^2$$

$$= \left[\sin\frac{3\pi}{2}.\sin\frac{\pi}{4}\right]^2 + \frac{1}{2}\left[\cos\frac{3\pi}{2} + \cos\frac{\pi}{4}\right]^2$$

$$= \left[(-1).\frac{1}{\sqrt{2}}\right]^2 + \frac{1}{2}\left[0 + \frac{1}{\sqrt{2}}\right]^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

33. (a) $\sin^2\left(\frac{\pi}{4}+\theta\right) - \sin^2\left(\frac{\pi}{4}-\theta\right)$

$$= \sin\left[\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right]\sin\left[\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right]$$

$[\because \quad \sin^2 x - \sin^2 y = \sin(x+y).\sin(x-y)]$

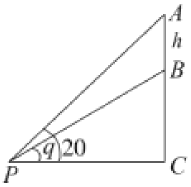
$$= \sin\frac{\pi}{2}.\sin 2\theta = \sin 2\theta$$

34. (c) In Δ PBC

$$\tan\theta = \frac{BC}{PC}$$

$\Rightarrow$ $PC = BC\cot\theta$ ...(i)

In $\Delta\, PAC$



$$\tan 2\theta = \frac{h+BC}{PC}$$

$\Rightarrow$ $PC\tan 2\theta = h + BC$

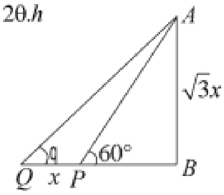
$\Rightarrow$ $BC\,.\,\cot\theta\,.\,\dfrac{2\tan\theta}{1-\tan^2\theta} = h + BC$

$\Rightarrow$ $\dfrac{2BC}{1-\tan^2\theta} - BC = h$

$\Rightarrow$ $\dfrac{1+\tan^2\theta}{1-\tan^2\theta}\,BC = h$

$\Rightarrow$ $BC = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}\,h = \cos 2\theta . h$

35. (b) In Δ APB



$$\tan 60° = \frac{\sqrt{3}x}{BP}$$

$\Rightarrow$ $\sqrt{3} = \dfrac{\sqrt{3}x}{BP}$

$\Rightarrow$ $BP = x$ ...(*i*)

In Δ AQB

$\tan\theta = \dfrac{\sqrt{3}\,x}{x+PB}$ $\Rightarrow$ $\dfrac{\sqrt{3}\,x}{x+x} = \dfrac{\sqrt{3}}{2}$

$\because$ $\dfrac{1}{\sqrt{3}} < \dfrac{\sqrt{3}}{2} < 1$

$\therefore$ $30° < \theta < 45°$

36. (a) $\because$ $\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{x}{3}\right) = \dfrac{\pi}{4}$

$\Rightarrow$ $\tan^{-1}\left(\dfrac{x}{3}\right) = \tan^{-1}1 - \tan^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow$ $\tan^{-1}\dfrac{x}{3} = \tan^{-1}\left(\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$

$\Rightarrow$ $\tan^{-1}\dfrac{x}{3} = \tan^{-1}\left(\dfrac{1}{3}\right)$

$\Rightarrow$ $\dfrac{x}{3} = \dfrac{1}{3}$ $\Rightarrow$ $x = 1$

37. (c) $3\sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow$ $2\sin^{-1}x + \sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow$ $2\sin^{-1}x + \dfrac{\pi}{2} = \pi$

$\Rightarrow$ $2\sin^{-1}x = \dfrac{\pi}{2}$ $\Rightarrow$ $\sin^{-1}x = \dfrac{\pi}{4}$

$\Rightarrow$ $x = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

38. (b) $\tan\alpha + \tan\beta = 1 - \tan\alpha\,.\,\tan\beta$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta} = 1$$

$\Rightarrow$ $\tan(\alpha+\beta) = \tan\dfrac{\pi}{4}$

$\Rightarrow$ $\alpha+\beta = \dfrac{\pi}{4}$

39. (b) $\because$ $(1+\tan\theta)(1+\tan 9\theta) = 2$

$\Rightarrow$ $1 + \tan 9\theta + \tan\theta + \tan\theta\,.\,\tan 9\theta = 2$

$\Rightarrow$ $\tan\theta + \tan 9\theta = 1 - \tan\theta\,.\,\tan 9\theta$

$\Rightarrow$ $\dfrac{\tan\theta + \tan 9\theta}{1-\tan\theta.\tan 9\theta} = 1$

$\Rightarrow$ $\tan(\theta + 9\theta) = 1$

$\Rightarrow$ $\tan 10\theta = 1$

40. (b) $\sin 0° + \sin 10° + \sin 20° + \sin 30° + ... + \sin 360°$
$= (\sin 0° + \sin 360°) + (\sin 10° + \sin 350°) + (\sin 20° + \sin 340°) + ... + \sin 180°$
$= 2\sin 180°\,.\,\cos 180° + 2\sin 180°\,.\,\cos 170° + 2\sin 180°\,.\,\cos 160°) + ... + \sin 180°$
$= 0 + 0 + 0 .... + 0$ $[\because \sin 180° = 0]$
$= 0$

41. (c) The subsets of the set $\{4\}$
$= \{4\}, \{1,4\}, \{2,4\}, \{3,4\}, \{2.3,4\}, \{1,2,4\}$
$\{1,3,4\}, \{1,2,3,4\}$
number of subsets of the set $\{4\} = 8$

42. (d) $\because$ $x \in$ C AUB $\Rightarrow n \notin A$ and $n \notin B$
So, statement 1 is not correct
Let $A = \{1,2\}, B = \{2,3\} \Rightarrow A \cap B = \{2\}$
$\because 1 \notin A \cap B$ But $1 \in A$
So, Statement 2 is not correct

43. (c) Given that A and B are non empty set statement-1
Let $x \in A \cup B = x \in A$ or $x \in B$ ...(i)
$X \in A \cap B = x \in A$ and $x \in B$ ....(ii)
from (i) and (ii)
$A = B$
Let $A = B$ then $A \cup B = A \cap B$
So, statement 1 is correct
Statement 2
Given that $A\,\Delta\,B = Q$
$\therefore$ $A\Delta B = (A \cup B) - (B \cap A) = Q$
$\Rightarrow A \cup B = B \cap A \Rightarrow A = B$
So, statement 2 is correct

44. (a) Given that

$R = \{(x, y) : x^2 - 5xy + 4y^2 = 0, x, y \in N\}$

$x^2 - 5xy + 4y^2 = 0$

$\Rightarrow x^2 - 4xy - xy + 4y^2 = 0$

$\Rightarrow x(x-4y) - y(x-4y) = 0$

$\Rightarrow (x-y)(x-4y) = 0$

For reflexive.

$(x-x)(x-4x) = 0$

$\therefore (x, x) \in R$

So it is reflexive

For symmetric

Let $(x, y) \in R$

$x^2 - 5xy + 4y^2 = 0$

but

$y^2 - 5xy + 4x^2$

may be equal to zero

So it is not symmetric

For transitive

Let $(x, y) \in R$

$x^2 - 5xy + 4y^2 = 0$ ....(i)

and $(y, z) \in R$

$y^2 - 5y^2 + 4z^2 = 0$ ....(ii)

from (i) and (ii)

$x^2 - 5xy + 4y^2 = y^2 - 5yz + 4z^2$

$x^2 + 3y^2 - 4z^2 - 5xy + 5yz = 0$

$(x^2 - 5xz + 4z^2) + (3y^2 - 4z^2 + 5xz - 5xy + 5yz) = 0$

$\therefore 3y^2 - 4z^2 + 5xz - 5xy + 5yz \neq 0$

$\therefore x^2 - 5xz + 4z^2 \neq 0$

$\therefore (x, z) \notin R$

So, it is not transitive

45. (d) We know that

If R {x,y} : $x \in A$ and $y \in B$ then

$R^{-1} = \{(y, x) : y \in B \text{ and } x \in A\}$

Statement-1

Let R is reflexive

$\therefore (x, x) \in R \Rightarrow (x, x) \in R^{-1}$

So R^{-1} is also reflexive

Statement 2

Let R is symmetric

and $(x, y) \in R \Rightarrow (y, x) \in R$ ...(i)

Let $(y, x) \in R^{-1} \Rightarrow (x, y) \in R^{-1}$

So R^{-1} is also symmetric

Statement 3

Let R is transitive

So, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, 2) \in R$

$\therefore (x, y) \in R \Rightarrow (y, x) \in R$

$(y, z) \in R \Rightarrow (z, y) \in R$

and $(x, z) \in R \Rightarrow (z, x) \in R$

$\therefore (z, y) \in R$ and $(z, y) \in R \Rightarrow (z, x) \in R$

So R^{-1} is also transitive

46. (b) Let $Z = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1+1} = \frac{1}{2} - \frac{1}{2}i$

$$\tan\theta = \left|\frac{-\frac{1}{2}}{\frac{1}{2}}\right| = 1 = \tan\frac{\pi}{4}$$

$\therefore \theta = \frac{\pi}{4}$ But z lies in 4^{th} quadrant

$\therefore$ Principal argument $= \frac{-\pi}{4}$

47. (c) $\left|\left(\frac{\sqrt{-3}}{2} - \frac{1}{2}\right)^{200}\right| = \left|\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{200}\right|$

$$= \left|-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right|^{200} \quad [\because |Z^n| = |Z|^n]$$

$$= \left|\sqrt{\frac{1}{4} + \frac{3}{4}}\right|^{200} = 1$$

48. (d) Statement 1

Let n = 4

$\therefore \frac{n!}{3!} = \frac{4!}{3!} = 4$ is not divisible by 6

Statement 2.

Let n = 5

$\therefore \frac{n!}{3!} + 3 = \frac{5!}{3!} + 3 = 5.4 + 3$

= 23 is not divisible by 7.

49. (b) Selecting 5 players out of 5 players (exclude two particular players).

$= {}^7C_5 = \frac{7.6}{2.1} = 21$

50. (c) $(x + 1)^{th}$ term from the end of expression

$\left(x + \frac{1}{x}\right)^{2n} = (n+1)^{th}$ term

from begining of expression $\left(\frac{1}{x} + x\right)^{2n}$

$\therefore (n + 1)^{th}$ term of $\left(\frac{1}{x} + x\right)^{2n}$ is

$$T_{n+1} = {}^{2n}C_n \left(\frac{1}{x}\right)^n x^n = {}^{2n}C_n$$

51. (d) According to questions

$S_9 = S_{11}$

$$\frac{9}{2}(2a+8d) = \frac{11}{2}(2a+10d)$$

$\Rightarrow 18a + 72d = 22a + 110\,d$

$\Rightarrow 4a + 38d = 0 \Rightarrow 2a + 19d = 0$

$$\therefore\ S_{20} = \frac{20}{2}\left(2a+19d\right) = 0$$

52. (b) $a_5 = \frac{1}{10} \Rightarrow a + 4d = \frac{1}{10}$ ...(i)

$\Rightarrow$ $a_{10} = \frac{1}{5} \Rightarrow a + 9d = \frac{1}{5}$ .... (ii)

from (i) and (ii), we get

$a = \frac{1}{50}$ and $d = \frac{1}{50}$.

$$\therefore\ S_{50} = \frac{50}{2}\left[\frac{2}{50}+\frac{49}{50}\right] = \frac{50}{2}\times\frac{51}{50} = 25.5$$

53. (a) $(1110011)_2 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 2^1 + 2^0$

$= 64 + 32 + 16 + 2 + 1 = 115$

$(10111)_2 = 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 16 + 4 + 2 + 1 = 23$

$\therefore (1110011)_2 \div (10111)_2 = 115 \div 23 = 5$

$\therefore$

2	5	1
2	2	0
	1	

54. (b) $x^3 + y^3 = 2^8 + 2^4 + 2^2 + 2^1 + 2^0$

$= 256 + 16 + 4 + 2 + 1 = 279$

$x + y = 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

$= 16 + 8 + 4 + 2 + 1 = 31$

Now, $(x - y)^2 + xy = \dfrac{(x+y)[x^2 - xy + y^2]}{x+y}$

$$= \frac{x^3 - y^3}{x+y} = \frac{279}{31} = 9$$

$\therefore$

2	9	
2	4	1
2	2	0
	1	0

$\therefore 9 = [1001]_2$

55. (d) (a) $\because$ $x > 0$ and $y > 0$

So (0, 0) not lies in region

(b) $-2 + 4 = 2$ is not less than 2

So, (–2, 4) not lies in region

(c) $-1 + 4 = 3 > 2$

So, (–1, 4) not lies in region

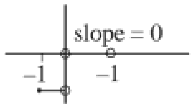
(d) $-1 + 2 = 1 < 2$

and $5(-1) - 4(2) + 12 = -5 - 8 + 12 = -1 < 0$

Since, satisfy all inequation

So, (–1, 2) lies in region

56. (a)



Slope of tangent at x = 0 is 0.

So, desirative is 0

It is clear from graph function is discontinuous at x = 0.

57. (d) $1+\left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^{4/3}$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^4$$

$\therefore$ Degree = 4

58. (a) $\frac{dP}{dt} = -kP$

$$\int \frac{1}{p}dp = -k\int dt$$

$\Rightarrow \ell n\, p = -kt + C$

When $\ell = 0$, P = 8P

$\ell np = C =$

put t = 100 and C = ℓnp

$\ell n\,(p/2) = -100\,k + \ell np$

$\Rightarrow \ell np - \ell n2 = -100\,k = \ell np$

$$\Rightarrow k = \frac{\ell n\, 2}{100}$$

59. (d) $\because f(x) = \sqrt{1-(x-1)^2}$

$\therefore\ 1 - (x-1)^2 \geq 0 \Rightarrow 1 - x^2 + 2x - 1 \geq 0$

$\Rightarrow -x(x-2) \geq 0 \Rightarrow x(x-2) \leq 0$

$-\infty$ + 0 – 2 + ∞

$\therefore x \in [0, 2]$

60. (c) Required area

$$24. = 2\int_0^k 2\sqrt{k}\sqrt{x}\,dx$$

$$\Rightarrow 2\sqrt{k}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^k = 12$$

$$\Rightarrow k^{\frac{1}{2}} \times \frac{2}{3}\cdot k^{\frac{3}{2}} = 6$$

$$\Rightarrow k^2 = 6\times\frac{3}{2} = 9$$

$\Rightarrow k = 3\ (\because k > 0)$

61. (b) $\int_0^{\frac{\pi}{4}} \frac{dx}{(\sin x+\cos x)^2} = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{4}} \frac{dx}{\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos\right)^2}$

$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{4}} \frac{dx}{(cas^2\left(\frac{\pi}{4} - x\right)} = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{4}} \sec^2\left(\frac{\pi}{4}x\right)dx$

$= \frac{1}{\sqrt{2}}\left[-\tan\left(\frac{\pi}{4} - x\right)\right]_0^{\frac{\pi}{4}} = \frac{-1}{\sqrt{2}}\left[\tan 0 - \tan\frac{\pi}{4}\right]$

$= \frac{-1}{\sqrt{2}}\left[0 - \frac{1}{\sqrt{2}}\right] = \frac{1}{2}$

62. (b) Let $I = \int (\sin x)^{-\frac{1}{2}} (\cos x)^{-\frac{3}{2}} dx$

$= \int \frac{1}{(\sin x)^{\frac{1}{2}} (\cos x)^{\frac{3}{2}}} dx = \int \frac{dx}{\cos^2 x (\tan x)^{\frac{1}{2}}}$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

Let $\tan x = t^2 \Rightarrow \sec^2 x\, dx = 2t dt$

$= \int \frac{1}{t} \times 2t dt = 2\int 1\, dt = 2t + c$

$= 2\sqrt{\tan x} + c$

63. (b) $I_1 = \int \frac{e^x}{e^x + e^{-x}}\, dx = \int \frac{e^x}{e^x + \frac{1}{e^x}} dx$

$= \int \frac{e^{2x}}{e^{2x+1}}\, dx$

$\therefore\ I_1 + I_2 = \int \frac{e^{2x}+1}{e^{2x}+1}\, dx = \int 1 dx$

$= x + C$

64. (b) $\int_{-2}^{-1} \frac{x}{|x|} dx$

$= \int_{-2}^{-1} \frac{x}{-x} dx$ $\quad [\because |x| = -x, x < 0]$

$= -\int_{-2}^{-1} dx = -[x]_{-2}^{-1}$

$= -\{(-1) - (-2)\} = -1.$

65. (b) Let $f(x) = \sin 4x + 2x$

$f'(x) = 4\cos 4x + 2 = 0$

$\Rightarrow \cos 4x = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow 4x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = n\frac{\pi}{2} \pm \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{\pi}{3}\left[\because 0 < x < \frac{\pi}{2}\right]$

66. (b) $f(x) = \frac{1}{\tan x + \cot x} = \frac{\sin x - \cos x}{\sin^2 x + \cos^2 x}$

$= \frac{1}{2}\sin 2x$

$f'(x) = \cos 2x = 0$

$\therefore x = \frac{\pi}{4}$

$f'(x) = -2\sin 2x = -2\sin\frac{\pi}{4} = -2 < 0$

$\therefore$ f (x) is maximum at $x = \frac{\pi}{4}$

$\therefore$ Maximum value is

$f\left(\frac{\pi}{4}\right) = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) = \frac{1}{2}$

67. (d) $4f(x) - f\left(\frac{1}{x}\right) = \left(2x + \frac{1}{x}\right)\left(2x - \frac{1}{x}\right)$

$4f(x) - f\left(\frac{1}{x}\right) = 4x^2 - \frac{1}{x^2}$ ....(i)

Replace x by $\frac{1}{x}$ we get

$4f\left(\frac{1}{x}\right) - f(x) = \frac{4}{x^2} - x$ ...(ii)

from $4 \times (i) + (ii)$

$16f(x) - 4f\left(\frac{1}{x}\right) = 16x^2 - \frac{4}{x^2}$

$4f\left(\frac{1}{x}\right) - f(x) = \frac{4}{x^2} - x$

$15f(x) = 15x^2$

$\therefore f(x) = x^2$

Now, $f(2) = 2^2 = 4.$

68. (a) Given that $f(x) = 4x + 3$

$\therefore$ fofo $f(-1) = f(f(f(-1)))$

$= f(f(4(-1)+3)) = f(f(-1))$

$= f(4(-1)+3) = f(-1) = 4(-1) + 3 = -1$

69. (a) $x^y y^x = 1$

Taking log both sides

$y \log x + x \log y = 0$

$$y' \log x + \frac{y}{x} + \log y + \frac{x}{y} y' = 0$$

put x = 1, y = 1

$y' \log (1) + 1 + \log (1) + y' = 0$

$0 + 1 + 0 + y' = 0 \Rightarrow y' = -1.$

70. (b) $y = \left(x^x\right)^x = (x)^{x^2}$

Taking log both side

$\log y = x^2 \log x$

$$\frac{1}{y}\frac{dy}{dx} = 2x.\log x + x^2.\frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = 2x \log x + x$$

put x = 1, y = 1

$$\frac{dy}{dx} = 2\log(1) + 1 = 1$$

71. (d) $\because -4 < x < -3$

$-3 < x + 1 < -2$

$\Rightarrow [x + 1] = -3$

$\therefore y = -3$

$$\frac{dy}{dx} = 0$$

72. (d) $\frac{dy}{dx} = (\ell n 5) y \Rightarrow \int \frac{1}{y} dy = (\ell n 5)\int dx$

$\ell n\, y = (\ell n\, 5)\, x + c$

put x = 0, $y = \ell n\, 5$

$\ell n\, (\ell n\, 5) = 0 + c \Rightarrow c = \ell n\, (\ell n\, 5)$

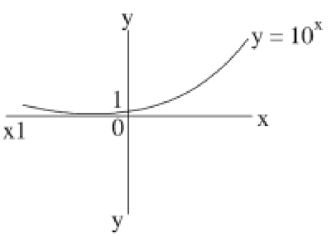
$\therefore \ell n\, y = (\ell n\, 5)x + \ell n\, (\ell n\, 5)$

put x = 1

$\ell n\, y = \ell n\, 5 + \ell n\, (\ell n\, 5) = \ell n\, (5\, \ell n\, 5)$

$y = 5\, \ell n\, 5$

73. (d)



It is clear from function that domain $= (-\infty, \infty)$ and it is continuous function

unique tangant can be drawn at x = 0 so, it is differentiable at x = 0

74. (a) $\lim_{x \to 0} x^3 (\text{cosec } x)^2$

$$= \lim_{x \to 0} \frac{x^3}{\sin^2 x} = \lim_{x \to 0}\left(\frac{x}{\sin x}\right)^2 .x$$

$= 1.0 = 0$

75. (c) $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{\left(x^3 - 1\right)\left(\sqrt{x} + 1\right)}{x - 1}$

$$= \lim_{x \to 1} \frac{(x - 1)(x^2 + x + 1)(\sqrt{x} + 1)}{x - 1}$$

$= (1 + 1 + 1)\ (1 + 1) = 6$

76. (b) $f(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 5$

$f'(x) = x^2 - 7x + 6 = 0$

$\therefore x = 1, 6$

+ − +

$-\infty$ 1 6 ∞

$\therefore$ f(x) is decreasing in (1, 6).

77. (d) $F(x) = \frac{m}{x} + 2nx + 1$

$$\therefore f'(x) = \frac{-m}{x^2} + 2n$$

given that $f'(2) = 0$

$$\therefore \frac{-m}{(2)^2} + 2n = 0$$

$-m + 8n = 0$

$\Rightarrow 8n - m = 0$ ...... (i)

from equation we can't find value of m + 8n.

78. (b) y = x ......(i)

$y = x^3$ ......(ii)

Solving equations (i) and (ii) we get

$\therefore$ Required area

$$= \int_0^1 x\,dx - \int_0^1 x^3 dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

79. (a) $xy = 4225 \Rightarrow y = \frac{4225}{x}$

Let $S = x + y = x + \frac{4225}{x}$

$$\frac{ds}{dx} = 1 - \frac{4225}{x^2} = 0$$

$\Rightarrow x = 65$

$$\frac{d^2s}{dx^2} = \frac{2.4225}{x^3}$$

$$\left(\frac{d^2s}{dx^2}\right)_{(x=65)} = \frac{2.4225}{(65)^3} > 0$$

$\therefore$ x + y is minimum at x = 65 and y = 65

$\therefore$ Minimum value of x + y = 65 + 65 = 130

80. (c) $x\frac{dy}{dx} - 2y = 0$

$$x\frac{dy}{dx} = 2y$$

$$\int \frac{1}{y}dy = 2\int \frac{1}{x}dx$$

$\ell ny = 2\ \ell nx + \ \ell nc$

$\ell ny = \ell nx^2 + \ell n\ c$

$\ell ny = \ell n\ (cx^2)$

$y = cx^2$

It represent the equation of parabolas.

81. (c) Since (–5, 0), $(5p^2, 10p)$ and $(5q^2, 10q)$ are collinear.

$$\therefore \begin{vmatrix} -5 & 0 & 1 \\ 5p^2 & 10p & 1 \\ 5q^2 & 10q & 1 \end{vmatrix} = 0$$

$$\Rightarrow 5 \times 10 \begin{vmatrix} -1 & 0 & 1 \\ p^2 & p & 1 \\ q^2 & q & 1 \end{vmatrix} = 0$$

Applying $c_1 \to c_1 + c_3$

$$\Rightarrow 50 \begin{vmatrix} 0 & 0 & 1 \\ p^2+1 & p & 1 \\ q^2+1 & q & 1 \end{vmatrix} = 0$$

$\Rightarrow q(p^2+1) - p(q^2+1) = 0$

$\Rightarrow p^2q + q - pq^2 - p = 0$

$\Rightarrow p^2q - pq^2 + q - p = 0$

$\Rightarrow pq\ (p-q) - 1\ (p-q) = 0$

$\Rightarrow (p-q)\ (pq-1) = 0$

$\Rightarrow pq - 1 = 0 \Rightarrow pq = 1 \qquad (\because\ p \neq q)$

82. (c) Let line cuts off equal intercepts 'a' from axes

$\therefore$ Equation of line

$$\frac{x}{a} + \frac{y}{a} = 1 \quad \Rightarrow x + y = a$$

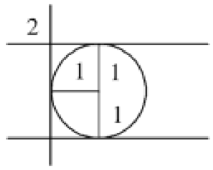
Since it passes through (1, –2)

$\therefore$ $1 - 2 = a \quad \Rightarrow a = -1$

$\therefore$ Required equation is

$x + y + 1 = 0$

83 (c) Since circle touches both the axes in the first quadrant and the line y – 2 = 0



$\therefore$ centre (1,1)

radius = 1

$\therefore$ Equation of circle is

$(x-1)^2 + (y-1)^2 = 1$

$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

84. (d) Focus of parabola = (–3, 0) = (9, 0)

$\Rightarrow a = -3$

Equation of direction x – 3 = 0

$\therefore$ axis is x-axis

So, equation of parabola

$y^2 = 4ax \Rightarrow y^2 = 4(-3)x$

$\Rightarrow y^2 = -12x.$

85. (b) Given equation of ellipse

$$x^2 + 2y^2 = 1 \Rightarrow \frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1$$

$$\Rightarrow a^2 = 1 \text{ and } b^2 = \frac{1}{2}$$

$\because b^2 = a^2 - c^2$

$$c^2 = a^2 - b^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$c = \frac{1}{\sqrt{2}}$$

The distance between foci of ellipse

$$= 2C = \frac{2}{\sqrt{2}} = \sqrt{2}$$

86. (d) Given that sides of triangle be a, b, c,

perimeter = P = a + b + c

$$\therefore S = \frac{P}{2}$$

Area of triangle $\Delta = q$

we know that

$$\tan\frac{A}{2} = \frac{\Delta}{S(S-a)}$$

$$\Rightarrow S\ (S-a)\tan\frac{A}{2} = \Delta$$

$$\Rightarrow \frac{P}{2}\left(\frac{P}{2} - a\right)\tan\frac{A}{2} = q$$

$$P(P-2a)\tan\frac{A}{2} = 42q$$

87. (a) Equation of line passing through intersection of two given lines is:

$x + 2y + 2 + \lambda\ (2x - 3y - 3) = 0$

$(1 + 2\lambda)\ x + (2 - 3\lambda)y + (2 - 3\lambda) = 0$

x-intercept (a) $= -\dfrac{(2-3\lambda)}{1+2\lambda}$

y-intercept (b) $= -\dfrac{(2-3\lambda)}{2+3\lambda} = -1$

since line cuts equal intercepts in the fourth quadrant

$\therefore$ If y-intercept is –1 then x-intercept is 1

Now $|a| + |b| = |1| + |-1| = 2$

88. (c) Since ax + by + c = 0 and bx + ay + c = 0 are parallel

$$\therefore \frac{a}{b} = \frac{b}{a} \Rightarrow a^2 - b^2 = 0$$

89. (a) Given equation of line is

$x + y = p$

$$\therefore \frac{x}{p} + \frac{y}{p} = 1$$

$\therefore$ line has equal intercept P with coordinate axes.

$\therefore$ Midpoint of line segment

$$= \left(\frac{P}{2}, \frac{P}{2}\right)$$

Let (h, k) be the locus of the mid-point of the line segment

$$\therefore h = \frac{p}{2} \text{ and } k = \frac{p}{2}$$

$\therefore h = k$

So, equation of locus is x = y

i.e. $x - y = 0$

90. (c) Let p(x, y), A(2a, 0) and B (0, 3a)

According to question

$PA = PB \Rightarrow PA^2 = PB^2$

$(x-2a)^2 + (y-0)^2 = (x-0)^2 + (y-3a)^2$

$\Rightarrow x^2 - 4ax + 4a^2 + y^2 = x^2 + y^2 - 6ay + 9a^2$

$\Rightarrow 4ax - 6ay + 5a^2 = 0$

$\Rightarrow 4x - 6y + 5a = 0$

For questions 91 to 93

$6x + ky + 3z - 12 = 0 \ (k \neq 0)$

$\Rightarrow 6x + ky + 3z = 12$

$$\Rightarrow \frac{x}{2} + \frac{y}{\frac{12}{k}} + \frac{z}{4} = 1$$

$$\therefore A(2,0,0), B\left(0, \frac{12}{k}, 0\right) C(0,0,4)$$

Given equation of sphere is

$x^2 + y^2 + z^2 - 2x - 3y - 4z = 0$

Compare with general equation of sphere

$x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + c = 0$

$$g = -1, f = -\frac{3}{2}, h = -2, c = 0$$

$\therefore$ centre of sphere = (–g, –f, –h)

$$= \left(1, \frac{3}{2}, 2\right).$$

91. (b) Since $B\left(0, \dfrac{12}{k}, 0\right)$ lies on sphere

$$\therefore 0 + \left(\frac{12}{k}\right)^2 + 0 - 0 - 3\left(\frac{12}{k}\right)0 = 0$$

$$\Rightarrow \left(\frac{12}{k}\right)\left[\frac{12}{k} - 3\right] = 0$$

$$\Rightarrow \frac{12}{k} - 3 = 0 \ [\because k \neq 0]$$

$\Rightarrow k = 4$

92. (b) Perpendicular distance from centre $\left(1, \dfrac{3}{2}, 2\right)$ to the plane

$6x + 4y + 3z - 12 = 0$

$$P = \frac{6 + 4\left(\frac{3}{2}\right) + 3(2) - 12}{\sqrt{36 + 16 + 9}}$$

$$= \frac{6+6+6-12}{\sqrt{61}} = \frac{6}{\sqrt{61}} = 0.74$$

93. (d) Equation of line passing through (0, 0, 0)

and $\left(1, \dfrac{3}{2}, 2\right)$ is

$$\frac{x-0}{1} = \frac{y-0}{\frac{3}{2}} = \frac{z-0}{2}$$

$6x = 4y = 3z$

For question 94 and 95

Since point (2, 3, – 6) lies plane

$$\frac{2x}{k} + \frac{2y}{3} + \frac{z}{3} = 2$$

$$\therefore \quad \frac{4}{k} + \frac{6}{3} - \frac{6}{3} = 2$$

$$\Rightarrow \frac{4}{k} = 2 \Rightarrow k = 2$$

So, equation of plane is

$$x + \frac{2y}{3} + \frac{z}{3} = 2$$

$\Rightarrow 3x + 2y + z = 6$

94. (a) Direction ratios of a normal are <3, 2, 1>

95. (b) $3x + 2y + z = 6$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$$

$\therefore$ Intercept on axes are 2, 3, 6 respectively

$\therefore$ P = 2, q = 3 and r = 6

$\therefore p + q + r = 2 + 3 + 6 = 11$

96. (c) $4\hat{i}+\hat{j}-3\hat{k}$ and $p\hat{i}+q\hat{j}-2\hat{k}$ are collinear vectors

$\therefore \frac{4}{p}=\frac{1}{q}=\frac{-3}{-2} \Rightarrow P=\frac{2}{3}\times 4=\frac{8}{3}$

and $\frac{1}{q}=\frac{3}{2} \Rightarrow q=\frac{2}{3}$

97. (c) Given that $\vec{a},\vec{b}$ and $\vec{c}$ are position vectors of the vertices A, B and C resp.

$\therefore$ position vector of centroid $G=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

Now, vector $\overrightarrow{AG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}-\vec{a}$

$=\frac{\vec{b}+\vec{c}-2\vec{a}}{3}$

98. (c) $\because \vec{a}.(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

Thus, the dot- product is distributive over addition

and $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

Thus the cross product is distributive over addition

$\because \vec{a}\times(\vec{b}+\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

and $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\vec{b}-(\vec{c}.\vec{b})\vec{a}$

$\therefore \vec{a}\times(b\times\vec{c})\neq(\vec{a}\times\vec{b})\times\vec{c}$

Thus cross product of vectors is not associative

99. (b) Given that $\vec{a}\times\vec{b}=\vec{c}$

$\Rightarrow a\perp\vec{c}$ and $\vec{b}\perp\vec{c}$

If $\vec{b}$ and $\vec{c}$ are given then any vector $\vec{a}$ coplanar with $\vec{b}$ is also perpendicular to $\vec{c}$.

So, $\vec{a}$ is not unique

If $\vec{a}$ and $\vec{b}$ is given then its cross product is unique

So $\vec{c}$ is unique.

100. (d) Given that $|\vec{a}|=|\vec{b}|=1$

and $|\vec{a}-\vec{b}|<2$

$\Rightarrow|\vec{a}-\vec{b}|^2<4$

$\Rightarrow|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos 2\theta<4$

$\Rightarrow 1+1-2\cos 2\theta<4$

$\Rightarrow \cos 2\theta>-1$

$\Rightarrow 1-2\sin^2\theta>-1$

$\Rightarrow \sin^2\theta<1$

$\Rightarrow -1<\sin\theta<1$

101. Since two digits are chosen

then n (s) = ${}^5C_2$

To get last digit O there are two cases = 2 × 5 = 10

or 4 × 5 = 10

Hence, Probability $=\frac{2}{{}^5C_2}=\frac{1}{5}$

102. (d) Since frequency curve has left skewed so, In this case mode is the greatest and mean is the lowest hence. Option (d) is correct.

103. (c) Since variance $=\frac{\sum x_i^2}{n}-(x)^2$

$$3.6=\frac{2^2+2^2+4^2+5^2+x^2}{n}-\left(\frac{2+2+4+5+x}{n}\right)^2$$

$$3.6=\frac{4+4+16+25+x^2}{5}-\left(\frac{13+x}{5}\right)^2$$

On solving, we get

$2x^2-13x-7=0$

$\Rightarrow x=7$

104. (d) Given a = 4, d = 4 and n = 50

hence $S_n=\frac{n}{2}\left[2a+(x-1)d\right]$

$=\frac{50}{2}\left[2\times 4+49\times 4\right]$

$=50\,[4+98]$

Now $\overline{X}=\frac{S_n}{h}=\frac{50\times 102}{50}=102$

105. (b) Here mean

$=\frac{21+34+23+39+26+37+40+20+33+27}{10}$

$=\frac{300}{10}\Rightarrow 30$

Mean deviation

$=\frac{|-9|+4+|-7|+9+|-4|+7+10+|-10|+3+|-3|}{10}$

$=\frac{66}{10}=6.6$

Coefficient of mean deviation $=\frac{\text{Mean Deviation}}{\text{Mean}}$

$=\frac{6.6}{30}=0.22$

For Q 106 to 108

Here, $(x_1-100)+(x_2-100)+.....+(x_n-100)=-20$

hence $(x_1+x_2+x_3+.....x_n)-100x=-20$

Or $x_1+x_2+x_3+...............x_n=100n-20$ .....(i)

and accoding to question for same set

$x_1+x_2+x_3+..........x_n=92n+140$ ......(ii)

from (i) and (ii)

106. (d) $8n=160\Rightarrow n=20$

from (i) $x_1+x_2+x_3.....+x_n=1980$

Mean $=\frac{100\times 20-20}{20}=99$

107. (a) Acc to question

$(x_1 + x_2 + x_3 + ... x_n) - 99n$

$\Rightarrow 1980 - 99 \times 20$

$= 0$

108. (c) Acc. to question.

$(x_1 + x_2 + x_3 + ... x_n) - 20y = 180$

$1980 - 20y = 180$

or $20y = 1800 \Rightarrow y = 90$

109. (d) $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{51}{2}[8 + 50 \times 3] = \frac{51 \times 158}{2} = 51 \times 79$

$\text{Mean} = = \frac{S_n}{n} = \frac{51 \times 79}{51} = 79$

110. Median = Middle term of A.P $= \left(\frac{51+1}{2}\right)^{th}$ term $= 26^{th}$ term

$\therefore T_{26} = 4 + 25 \times (3) = 79$

111. (b) Sum of deviations measured from the median

$= S_n - n \times \text{Median}$

$= 51 \times 79 - 51 \times 79 = 0$

112. (b) $\therefore n(G \cap \bar{T}) = P$ (Graduates less than 3 years experience)

$= 36$

$P(G \cap \bar{T}) = \frac{36}{90} = \frac{2}{5}$

113. (c) $\because P(G \cap \bar{T}) = \frac{2}{5}$

$P(\bar{T}) = \frac{63}{90} = \frac{7}{10}$

$\therefore P(G/\bar{T}) = \frac{P(G \cap \bar{T})}{P(\bar{T})} = \frac{\frac{2}{5}}{\frac{7}{10}} = \frac{4}{7}$

114. (d) $\because n(\bar{T} \cap \bar{G}) = 27$

$n(\bar{G}) = 9 + 27 = 36$

$\therefore P(\bar{T}/\bar{G}) = \frac{P(\bar{T} \cap \bar{G})}{P(\bar{G})}$

$\frac{\frac{27}{90}}{\frac{36}{90}} = \frac{27}{36} = \frac{3}{4}$

For questions 115 to 117

Let P = probability of workers which in suffering from disease.

$= \frac{1}{3}$

$\therefore q = 1 - \frac{1}{3} = \frac{2}{3}$

115. (d) $P(x = 3) = 6_{C_3}\left(\frac{1}{3}\right)^3\left(\frac{1}{3}\right)^3 = \frac{6.5.4}{3.2.1} \times \frac{8}{3^6} = \frac{160}{729}$

116. (b) $P(X = 0) = 6_{C_0}\left(\frac{1}{3}\right)^{\circ}\left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6} = \frac{64}{729}$

117. (b) $P(x \geq 1) = 1 - P(X = 0)$

$= 1 - \frac{64}{729} = \frac{665}{729}$

Sol. (118-120):

C.I	f_i	x_i	$f_i x_i$
0-20	17	10	170
20-40	P + q	30	30(P + 8)
40-60	32	50	1600
60-80	P-3q	70	70P-210q
80-100	19	90	1710

$68 + 2P - 2q$ $\qquad$ $3480 + 100p - 180q$

$\therefore 68 + 2P - 2q = 120$

$\Rightarrow p - q = 26$ ..... (i)

$50 = \frac{3480 + 100P - 180q}{120}$

$6000 = 3480 + 100P - 180q$

$\Rightarrow 5P - 9q = 126$ ....(ii)

On solving equation (i) and (ii) we get

$P = 27$ and $q = 1$

118. (c)

119. (a)

120. (b) If frequency of each class is doubled

$\therefore \text{New Mean} = \frac{\sum_{i=1}^{n} x_i 2fi}{\sum 2fi} = 2\frac{\sum_{i=1}^{n} x_i fi}{2\sum fi}$

$= \frac{\sum_{i=1}^{n} x_i fi}{\sum fi} = 50$