NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2021 (I)

> PAPER I : Mathematics

1. The smallest positive integer n for which

$$\left(\frac{1-i}{1+i}\right)^{n^2} = 1$$

where $i = \sqrt{-1}$, is

(a) 2   (b) 4   (c) 6   (d) 8

⊙ **(a)** $\left(\dfrac{1-i}{1+i}\right)^{n^2} = 1$, where $i = \sqrt{-1}$

$\left(\dfrac{1-i}{1+i} \times \dfrac{1-i}{1-i}\right)^{n^2} = 1$

$\left(\dfrac{1 + i^2 - 2i}{1 - i^2}\right)^{n^2} = 1$

$\left(\dfrac{1 - 1 - 2i}{1 + 1}\right)^{n^2} = 1$

$\Rightarrow \qquad (-i)^{n^2} = (-i)^4$

$\Rightarrow \qquad\qquad n^2 = 4$

$\qquad\qquad\qquad n = 2$

Hence, option (a) is correct.

2. The value of x, satisfying the equation $\log_{\cos x} \sin x = 1$, where $0 < x < \dfrac{\pi}{2}$, is

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{6}$

⊙ **(c)** $\log_{\cos x} \sin x = 1$, where $0 < x < \dfrac{\pi}{2}$

$\Rightarrow (\cos x)^1 = \sin x \Rightarrow \cos x = \sin x$

$\Rightarrow \tan x = 1 \Rightarrow \tan x = \tan \pi/4$

$\Rightarrow x = \pi/4$

Hence, option (c) is correct.

3. If Δ is the value of the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

then what is the value of the following determinant?

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

$(p \neq 0$ or $1, q \neq 0$ or $1)$

(a) $p\Delta$ (b) $q\Delta$

(c) $(p + q)\Delta$ (d) $pq\Delta$

⊙ **(d)** Given, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta$

$\therefore \quad \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$

$= p \cdot q \cdot \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$= pq\Delta$

Hence, option (d) is correct.

4. If $C_0, C_1, C_2, ..., C_n$ are the coefficients in the expansion of $(1 + x)^n$, then what is the value of $C_1 + C_2 + C_3 + ... + C_n$?

(a) 2^n (b) $2^n - 1$

(c) 2^{n-1} (d) $2^n - 2$

⊙ **(b)** $\because (1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n$

and we know that

${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

$\therefore {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n - {}^nC_0$

$\qquad\qquad\qquad\qquad = 2^n - 1$

Hence, option (b) is correct.

5. If $a + b + c = 4$ and $ab + bc + ca = 0$, then what is the value of the following determinant?

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(a) 32   (b) -64

(c) -128 (d) 64

⊙ **(b)** Let

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$

(by $C_1 \to C_1 + C_2 + C_3$)

$= (a + b + c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$

(To take common $a + b + c$ from C_1)

$= (a + b + c) \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix}$

(by $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$)

$= (a + b + c)\, [(b - c)(a - b) - (c - a)^2]$

$= (a + b + c)\, (ab - b^2 - ca + bc$

$$-c^2 - a^2 + 2ca)$$

$$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= -(a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$$

$$= -(4)[16 - 0] = -64.$$

6. The number of integer values of k, for which the equation $2\sin x = 2k + 1$ has a solution, is

(a) zero   (b) one

(c) two   (d) four

⊘ **(c)** Given,

$$2\sin x = 2k + 1$$

$$\because \quad -1 \le \sin x \le 1 \Rightarrow -2 \le 2\sin x \le 2$$

$$-2 - 1 \le 2\sin x - 1 \le 2 - 1$$

$$-3 \le 2k \le 1$$

$$\frac{-3}{2} \le k \le \frac{1}{2} \Rightarrow -1\cdot5 \le k \le 0\cdot5$$

$\therefore$ Integer values of $k = -1, 0$

Hence, option (c) is correct.

7. If $a_1, a_2, a_3, ..., a_9$ are in GP, then what is the value of the following determinant?

$$\begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$$

(a) 0   (b) 1

(c) 2   (d) 4

⊘ **(a)** Let first term and common ratio of GP are a and r respectively.

$$\therefore \begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix} = \begin{vmatrix} \log a & \log ar & \log ar^2 \\ \log ar^3 & \log ar^4 & \log ar^5 \\ \log ar^6 & \log ar^7 & \log ar^8 \end{vmatrix}$$

$$= \begin{vmatrix} \log a & \log a + \log r & \log a + 2\log r \\ \log a + 3\log r & \log a + 4\log r & \log a + 5\log r \\ \log a + 6\log r & \log a + 7\log r & \log a + 8\log r \end{vmatrix}$$

$$[\because \log mn = \log m + \log n]$$

$$= \begin{vmatrix} \log a & \log r & \log r \\ \log a + 3\log r & \log r & \log r \\ \log a + 6\log r & \log r & \log r \end{vmatrix}$$

$$(\text{by } C_2 \to C_2 - C_1 \text{ and } C_3 \to C_3 - C_2)$$

$$= 0 \qquad\qquad\qquad [\because C_2 = C_3]$$

8. If the roots of the quadratic equation $x^2 + 2x + k = 0$ are real, then

(a) $k < 0$ (b) $k \le 0$

(c) $k < 1$ (d) $k \le 1$

⊘ **(d)** Given quadratic equation,

$$x^2 + 2x + k = 0 \qquad\qquad ... \text{(i)}$$

Since, roots are real

$$\Rightarrow \quad D \ge 0 \Rightarrow b^2 - 4ac \ge 0$$

$$(2)^2 - 4(1)(k) \ge 0 \Rightarrow 4 \ge 4k \Rightarrow \quad k \le 1$$

Hence, option (d) is correct.

9. If $n = 100!$, then what is the value of the following?

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + ... + \frac{1}{\log_{100} n}$$

(a) 0   (b) 1   (c) 2   (d) 3

⊘ **(b)** $$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + ... + \frac{1}{\log_{100} n}$$

$$= \log_n 2 + \log_n 3 + \log_n 4 + ... + \log_n 100$$

$$= \log_n (2 \cdot 3 \cdot 4 \cdot 5 ... 100)$$

$$= \log_{100!} (100!) \qquad\qquad [\because n = 100!]$$

$$= 1 \qquad\qquad\qquad\qquad [\because \log_a^a = 1]$$

Hence, option (b) is correct.

10. If $z = 1 + i$, where $i = \sqrt{-1}$, then what is the modulus of $z + \dfrac{2}{z}$?

(a) 1   (b) 2   (c) 3   (d) 4

⊘ **(b)** $z = 1 + i$, where $i = \sqrt{-1}$

$$\left| z + \frac{2}{z} \right| = \left| (1 + i) + \frac{2}{(1 + i)} \right| = \left| (1 + i) + \frac{2}{(1 + i)} \times \frac{(1 - i)}{(1 - i)} \right|$$

$$= \left| (1 + i) + \frac{2(1 - i)}{2} \right| = | 1 + i + 1 - i | = | 2 | = 2$$

Hence, option (b) is correct.

11. If A and B are two matrices such that AB is of order $n \times n$, then which one of the following is correct?

(a) A and B should be square matrices of same order.

(b) Either A or B should be a square matrix.

(c) Both A and B should be of same order.

(d) Orders of A and B need not be the same.

⊘ **(d)** Given that, order of matrix $AB = n \times n$

If we take $A_{n \times p}$ and $B_{p \times n}$, then AB will be of order $n \times n$.

So, orders of A and B need not be the same, is correct.

Hence, option (d) is correct.

12. How many matrices of different orders are possible with elements comprising all prime numbers less than 30?

(a) 2   (b) 3   (c) 4   (d) 6

⊘ **(c)** $\because$ Prime numbers less than $30 = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$\Rightarrow$ Number of elements $= 10$

$\therefore$ Possible order of matrices with 10 elements $= 10 \times 1, 1 \times 10, 2 \times 5, 5 \times 2$

$\therefore$ Number of matrices of different order $= 4$

Hence, option (c) is correct.

13. Let, $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$ where p, q, r and s are any four different prime numbers less than 20. What is the maximum value of the determinant?

(a) 215   (b) 311   (c) 317   (d) 323

⊘ **(c)** $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$, prime numbers less than 20

$$= \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow A = ps - rq$$

For maximum values of A, p and s must be maximum and r and q must be minimum.

Then, $p = 17, s = 19, r = 2, q = 3$

$$\therefore \quad A = 17 \times 19 - 2 \times 3$$

$$= 323 - 6 = 317$$

Hence, option (c) is correct.

14. If A and B are square matrices of order 2 such that $\det(AB) = \det(BA)$, then which one of the following is correct?

(a) A must be a unit matrix
(b) B must be a unit matrix
(c) Both A and B must be unit matrices
(d) A and B need not be unit matrices

⊘ **(d)** $A_{2 \times 2}$ and $B_{2 \times 2}$ are two matrices

and $|AB| = |BA| \Rightarrow |A||B| = |B||A|$

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$ then, $|AB| = |BA|$

Hence, we can say A and B need not be the unit matrices.

Hence, option (d) is correct.

15. What is $\cot 2x \cot 4x - \cot 4x \cot 6x - \cot 6x \cot 2x$ equal to?

(a) -1 (b) 0   (c) 1   (d) 2

⊘ **(c)** $\because \cot 6x = \cot(2x + 4x)$

$\cot 6x = \dfrac{\cot 2x \cdot \cot 4x - 1}{\cot 2x + \cot 4x}$ $\left[\because \cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}\right]$

$\Rightarrow \cot 6x \cdot \cot 2x + \cot 6x \cdot \cot 4x = \cot 2x \cdot \cot 4x - 1$

$\therefore \cot 2x \cdot \cot 4x - \cot 4x \cdot \cot 6x - \cot 6x \cdot \cot 2x = 1$

Hence, option (c) is correct.

16. If $\tan x = -\dfrac{3}{4}$ and x is in the second quadrant, then what is the value of $\sin x \cdot \cos x$?

(a) $\dfrac{6}{25}$ (b) $\dfrac{12}{25}$ (c) $-\dfrac{6}{25}$ (d) $-\dfrac{12}{25}$

⊘ **(d)** Given,

$\tan x = \dfrac{-3}{4}$ and x is in the 2nd quadrant.

Let perpendicular be $3k$ and base be $4k$, then

Hypotenuse $= \sqrt{(3k)^2 + (4k)^2} = 5k$

$\sin x = \dfrac{3}{5}$ and $\cos x = \dfrac{-4}{5}$

$\therefore \sin x \cdot \cos x = \dfrac{3}{5} \times \left(\dfrac{-4}{5}\right) = \dfrac{-12}{25}$

Hence, option (d) is correct.

17. What is the value of the following?

$\operatorname{cosec}\left(\dfrac{7\pi}{6}\right)\sec\left(\dfrac{5\pi}{3}\right)$

(a) $\dfrac{4}{3}$ (b) 4   (c) -4 (d) $-\dfrac{4}{\sqrt{3}}$

⊘ **(c)** $\operatorname{cosec}\left(\dfrac{7\pi}{6}\right) \cdot \sec\left(\dfrac{5\pi}{3}\right) = \operatorname{cosec}\left(\pi + \dfrac{\pi}{6}\right) \cdot \sec\left(2\pi - \dfrac{\pi}{3}\right)$

$= -\operatorname{cosec}\dfrac{\pi}{6} \cdot \sec\dfrac{\pi}{3} = -2 \times 2 = -4$

Hence, option (c) is correct.

18. If the determinant

$\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$

then what is x equal to?

(a) -2 or 2   (b) -3 or 3   (c) -1 or 1   (d) 3 or 4

⊘ **(c)** Given, $\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$ ... (i)

$-1(x^2 - 1) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1$

$\Rightarrow \quad x = \pm 1 \Rightarrow 1 - x^2 = 0$

$\therefore \qquad x^2 = 1$

$x = +1, -1$

Hence, option (c) is correct.

19. What is the value of the following?

$\tan 31° \tan 33° \tan 35° ... \tan 57° \tan 59°$

(a) -1 (b) 0   (c) 1   (d) 2

⊘ **(c)** $\tan 31° \cdot \tan 33° \cdot \tan 35° ... \tan 57° \cdot \tan 59°$

$= \tan 31° \cdot \tan 33° \cdot \tan 35° ... x \tan 45° x ... \tan(90° - 33°)$ $\cdot \tan(90° - 31°)$

$= \tan 31° \cdot \tan 33° \cdot \tan 35° ... \cot 35° \cdot \cot 33° \cdot \cot 31°$

$= (\tan 31° \cdot \cot 31°) \cdot (\tan 33° \cdot \cot 33°) \cdot (\tan 35° \cdot \cot 35°) ...$

$= 1 \cdot 1 \cdot 1 ... = 1$

Hence, option (c) is correct.

20. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

then what is $f(-1) + f(0) + f(1)$ equal to?

(a) 0   (b) 1
(c) 100   (d) -100

⊘ **(a)** $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

$f(-1) = \begin{vmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ 6 & 12 & 0 \end{vmatrix} = 0 \Rightarrow f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix} = 0$

$f(1) = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$\therefore f(-1) + f(0) + f(1) = 0 + 0 + 0 = 0$

Hence, option (a) is correct.

21. The equation $\sin^{-1} x - \cos^{-1} x = \dfrac{\pi}{6}$ has

(a) no solution
(b) unique solution
(c) two solutions
(d) infinite number of solutions

⊘ **(b)** $\because \sin^{-1} x - \cos^{-1} x = \dfrac{\pi}{6}$ ... (i)

and we know that $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$ ... (ii)

Adding Eqs. (i) and (ii), we get

$2\sin^{-1} x = \dfrac{\pi}{6} + \dfrac{\pi}{2} \Rightarrow 2\sin^{-1} x = \dfrac{2\pi}{3}$

$\Rightarrow \quad \sin^{-1} x = \dfrac{\pi}{3}$

$x = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

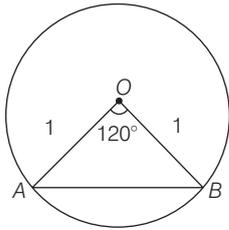Hence, the given equation has a unique solution.

Hence, option (b) is correct.

22. What is the value of the following?

$(\sin 24° + \cos 66°)(\sin 24° - \cos 66°)$
(a) − 1   (b) 0   (c) 1   (d) 2

⊘ **(b)** $(\sin 24° + \cos 66°)(\sin 24° - \cos 66°)$

$= (\sin 24° + \cos 66°)$
$\{\sin(90° - 66°) - \cos 66°\}$
$\qquad\qquad [\because \sin(90° - \theta) = \cos\theta]$
$= (\sin 24° + \cos 66°)(\cos 66° - \cos 66°)$
$= (\sin 24° + \cos 66°)(0) = 0$

Hence, option (b) is correct.

23. A chord subtends an angle 120° at the centre of a unit circle. What is the length of the chord?
(a) $\sqrt{2} - 1$ units   (b) $\sqrt{3} - 1$ units
(c) $\sqrt{2}$ units   (d) $\sqrt{3}$ units

⊘ **(d)** Given, radius of the circle = 1 unit

$\angle AOB = 120°$
By using cosine rule,

$\cos 120° = \dfrac{OA^2 + OB^2 - AB^2}{2 \cdot OA \cdot OB}$ … (i)

Let $AB = x$ unit, $OA = 1$ unit, $OB = 1$ unit
From Eq. (i),

$\dfrac{-1}{2} = \dfrac{1 + 1 - x^2}{2 \cdot 1 \cdot 1} \Rightarrow -1 = 2 - x^2$

$\Rightarrow \quad x^2 = 3 \Rightarrow x = \sqrt{3}$ unit

Hence, option (d) is correct.

24. What is $(1 + \cot\theta - \mathrm{cosec}\,\theta)$
$(1 + \tan\theta + \sec\theta)$ equal to?
(a) 1   (b) 2   (c) 3   (d) 4

⊘ **(b)** $(1 + \cot\theta - \mathrm{cosec}\,\theta)$

$\qquad\qquad (1 + \tan\theta + \sec\theta)$

$= \left(1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)\left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)$

$= \left(\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\dfrac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)$

$= \dfrac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta \cdot \cos\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$= \dfrac{1 + 2\sin\theta \cdot \cos\theta - 1}{\sin\theta \cdot \cos\theta} = 2$

Hence, option (b) is correct.

25. What is $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2$

equal to?

(a) 0   (b) 1
(c) $2\tan\theta$ (d) $2\cot\theta$

⊘ **(a)** $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2$

$= \dfrac{1 + \tan^2\theta}{1 + \dfrac{1}{\tan^2\theta}} - \left(\dfrac{1 - \tan\theta}{1 - \dfrac{1}{\tan\theta}}\right)^2$

$= \tan^2\theta\left(\dfrac{1 + \tan^2\theta}{\tan^2\theta + 1}\right) - \left(\dfrac{\tan\theta(1 - \tan\theta)}{\tan\theta - 1}\right)^2$

$= \tan^2\theta - \tan^2\theta = 0$

Hence, option (a) is correct.

26. What is the interior angle of a regular octagon of side length 2 cm?
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{3\pi}{5}$ (d) $\dfrac{3\pi}{8}$

⊘ **(b)** Given, length of side of regular octagon = 2 cm

$\because$ Sum of interior angles of octagon
$\qquad = (8 - 2) \times 180°$
$\qquad = 6 \times 180°$
$\qquad [\because$ sum of interior angles of polygon
$\qquad\qquad = (n - 2) \times 180°]$

$\therefore$ Interior angle $= \dfrac{6 \times 180°}{8}$

$\qquad = 135° = \dfrac{3\pi}{4}$

Hence, option (b) is correct.

27. If $7\sin\theta + 24\cos\theta = 25$, then what is the value of $(\sin\theta + \cos\theta)$?
(a) 1   (b) $\dfrac{26}{25}$ (c) $\dfrac{6}{5}$ (d) $\dfrac{31}{25}$

⊘ **(d)** Given, $7\sin\theta + 24\cos\theta = 25$

Since, we know that if
$\qquad a\sin\theta + b\cos\theta = c$
then $b\sin\theta - a\cos\theta = \sqrt{a^2 + b^2 - c^2}$

$\because \qquad 7\sin\theta + 24\cos\theta = 25$ … (i)
$\therefore 24\sin\theta - 7\cos\theta = \sqrt{7^2 + 24^2 - 25^2}$
$\quad 24\sin\theta - 7\cos\theta = 0$ … (ii)
Eq. (i) × 7 + Eq. (ii) × 24
$\quad 49\sin\theta + 168\cos\theta = 175$
$\quad \underline{576\sin\theta - 168\cos\theta = 0}$
$\qquad\qquad 625\sin\theta = 175$

$\qquad\qquad \sin\theta = \dfrac{175}{625} = \dfrac{7}{25}$

$\therefore \cos\theta = \sqrt{1 - \left(\dfrac{7}{25}\right)^2} = \dfrac{24}{25}$

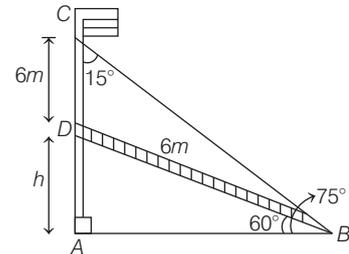$\therefore \sin\theta + \cos\theta = \dfrac{7}{25} + \dfrac{24}{25} = \dfrac{31}{25}$

Hence, option (d) is correct.

28. A ladder 6 m long reaches a point 6 m below the top of a vertical flagstaff. From the foot of the

ladder, the elevation of the top of the flagstaff is 75°. What is the height of the flagstaff ?
(a) 12 m   (b) 9 m
(c) $(6 + \sqrt{3})$ m   (d) $(6 + 3\sqrt{3})$ m

⊘ **(d)** Let AC be a vertical flagstaff.
$\therefore \quad CD = 6$ m, $BD = 6$ m
$\angle CBD = 75°$

Let $AD = h$ meter
In $\triangle ABC$
$90 + 75 + \angle C = 180°$ [$\because$ sum of interior angle of triangle is 180°]
$\angle C = 15°$
In $\triangle BCD$,
$\quad BD = CD \Rightarrow \angle BCD = \angle CBD = 15°$
$\therefore \angle ABD = 75° - 15° = 60°$

In $\triangle ABD$, $\sin 60° = \dfrac{h}{6} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{h}{6}$

$\qquad\qquad h = 3\sqrt{3}$ m
$\therefore$ Height of the flagstaff $= (h + 6)$ m
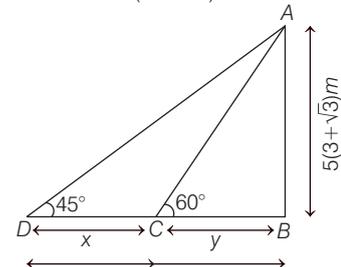$\qquad\qquad = (3\sqrt{3} + 6)$ m

Hence, option (d) is correct.

29. The shadow of a tower is found to be x metre longer, when the angle of elevation of the sun changes from 60° to 45°. If the height of the tower is $5(3 + \sqrt{3})$ m, then what is x equal to?
(a) 8 m   (b) 10 m
(c) 12 m   (d) 15 m

⊘ **(b)** In the given diagram,
AB represents the position of tower,
where $h = 5(3 + \sqrt{3})$ m

$CD = x$ m
In $\triangle ABC$,

$\tan 60° = \dfrac{5(3 + \sqrt{3})}{BC} \Rightarrow \sqrt{3} = \dfrac{5(3 + \sqrt{3})}{BC}$

$$\therefore \qquad BC = 5(\sqrt{3} + 1)\,m$$

In $\triangle ABD$,

$$\tan 45^\circ = \frac{5(3 + \sqrt{3})}{BD}$$

$$\Rightarrow \qquad 1 = \frac{5(3 + \sqrt{3})}{BD}$$

$$\therefore \qquad BD = 5(3 + \sqrt{3})\,m$$

Since, $x = BD - BC$

$$x = 5(3 + \sqrt{3}) - 5(\sqrt{3} + 1)$$
$$x = 5(3 + \sqrt{3} - \sqrt{3} - 1)$$
$$x = 10\,m$$

Hence, option (b) is correct.

30. If $3\cos\theta = 4\sin\theta$, then what is the value of $\tan(45^\circ + \theta)$?

(a) 10   (b) 7   (c) $\frac{7}{2}$ (d) $\frac{7}{4}$

⊘ **(b)** If $3\cos\theta = 4\sin\theta$

$$\Rightarrow \qquad \frac{3}{4} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \qquad \tan\theta = \frac{3}{4}$$

$$\therefore \tan(45^\circ + \theta) = \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ \cdot \tan\theta}$$

$$= \frac{1 + \frac{3}{4}}{1 - 1 \times \frac{3}{4}} = \frac{4 + 3}{4 - 3} = 7$$

Hence, option (b) is correct.

31. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ holds, when

(a) $x \in R$
(b) $x \in R - (-1, 1)$ only
(c) $x \in R - \{0\}$ only
(d) $x \in R - [-1, 1]$ only

⊘ **(a)** Since, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

for all $x \in R$.
Hence, option (a) is correct.

32. If $\tan A = \frac{1}{7}$, then what is $\cos 2A$ equal to?

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{12}{25}$ (d) $\frac{6}{25}$

⊘ **(a)** $\tan A = \frac{1}{7}$

$$\therefore \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - (1/7)^2}{1 + (1/7)^2}$$

$$= \frac{49 - 1}{49 + 1} = \frac{48}{50}$$

$$\cos 2A = \frac{24}{25}$$

Hence, option (a) is correct.
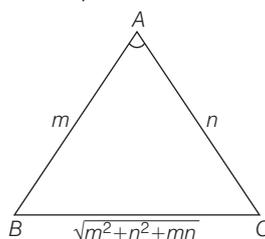
33. The sides of a triangle are m, n and $\sqrt{m^2 + n^2 + mn}$. What is the sum of the acute angles of the triangle?

(a) 45°   (b) 60°   (c) 75°   (d) 90°

⊘ **(b)** Let $AB = m$, $AC = n$

$$BC = \sqrt{m^2 + n^2 + mn}$$

By using cosine rule,

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2\,AB \cdot AC}$$

$$\Rightarrow \cos A = \frac{m^2 + n^2 - m^2 - n^2 - mn}{2mn}$$

$$\Rightarrow \cos A = \frac{-1}{2} \quad \Rightarrow \quad A = 120^\circ$$

$$\therefore \angle B + \angle C = 180 - \angle A$$

[∵ sum of interior angle is 180°]

$$= 180^\circ - 120^\circ$$

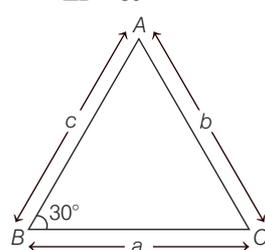$$\angle B + \angle C = 60^\circ$$

Hence, option (b) is correct.

34. What is the area of the triangle ABC with sides $a = 10$cm, $c = 4$cm and angle $B = 30°$?

(a) 16 cm²   (b) 12 cm²
(c) 10 cm²   (d) 8 cm²

⊘ **(c)** Given, $a = 10$ cm

$$c = 4 \text{ cm}$$
$$\angle B = 30^\circ$$

∵ Area of triangle $= \frac{1}{2} ac \sin(\angle B)$

$$= \frac{1}{2} \times 10 \times 4 \times \sin 30^\circ = \frac{1}{2} \times 40 \times \frac{1}{2}$$

$$= 10 \text{ sq cm}$$

Hence, option (c) is correct.

35. Consider the following statements
1. $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$ are equivalent sets.
2. $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\}$ are equal sets

Which of the above statements is/are correct?

(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊘ **(c)** $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$

Since, number of elements are same in both the sets.
⇒ A and B are equivalent sets.
If $A = \{1, 5, 9\}$, $B = \{1, 5, 5, 9, 9\}$
Which is nothing but $B = \{1, 5, 9\}$
Since, elements are same in A and B
⇒ A and B are equal sets
Hence, option (c) is correct.

36. Consider the following statements
1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct?

(a) 1 and 2 only   (b) 2 and 3 only
(c) 1 and 3 only   (d) 1, 2 and 3

⊘ **(d)** Since we know that null set is a subset of every set and every set is a subset of itself.

If $n(A) = 10$

$$\therefore n(P(A)) = 2^{10} = 1024$$

∴ all the given statements are true.
Hence, option (d) is correct.

37. Let R be a relation defined as xRy if and only if $2x + 3y = 20$, where $x, y \in N$. How many elements of the form (x, y) are there in R?

(a) 2   (b) 3
(c) 4   (d) 6

⊘ **(b)** ∵ $xRy \Leftrightarrow 2x + 3y = 20$

where, $x, y \in N$

$$\therefore \qquad y = \frac{20 - 2x}{3}$$

All ordered pair which satisfies the given relations are (1, 6), (4, 4), (7, 2).

$$\therefore \qquad R = \{(1, 6), (4, 4), (7, 2)\}$$

$$\therefore \qquad n(R) = 3$$

Hence, option (b) is correct.

38. Consider the following statements
1. A function $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(x) = x + 1$, is one-one as well as onto.
2. A function $f : \mathbb{N} \to \mathbb{N}$, defined by $f(x) = x + 1$, is one-one but not onto.

Which of the above statement(s) is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Statement I

$$f : \mathbb{Z} \to z$$
$$f(x) = x + 1$$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one in $\mathbb{Z}$.

and every element of co-domain has its pre-image in domain.

$\Rightarrow f$ is onto.

Statement II

$$f : \mathbb{N} \to \mathbb{N}$$
$$f(x) = x + 1$$

Let $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one in $\mathbb{N}$.

But there is no element in $\mathbb{N}$ such that

$$f(x) = 1$$

Hence, f is not onto on $\mathbb{N}$.

Given statements are correct.

Hence, option (c) is correct.

39. Consider the following in respect of a complex number z.

1. $\overline{(z^{-1})} = (\overline{z})^{-1}$
2. $zz^{-1} = |z|^2$

Which of the above is/are correct?

(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

(a) Let $z = x + iy$

$$\overline{z} = x - iy$$

$$(\overline{z})^{-1} = \frac{1}{x - iy} = \frac{x + iy}{x^2 + y^2}$$

Also, $z^{-1} = \frac{1}{x + iy} = \frac{x - iy}{x^2 + y^2}$

$$\overline{(z^{-1})} = \frac{x + iy}{x^2 + y^2} = (\overline{z})^{-1}$$

$\therefore$ Statement 1 is correct.

$$|z| = \sqrt{x^2 + y^2}$$

$$\Rightarrow |z|^2 = x^2 + y^2$$

But $zz^{-1} = (x + iy)\frac{(x - iy)}{x^2 + y^2}$

$$= \frac{x^2 + y^2}{x^2 + y^2} = 1 \neq |z|^2$$

$\therefore$ Statement 2 is wrong.

Hence, option (a) is correct.

40. Consider the following statements in respect of an arbitrary complex number z.

1. The difference of z and its conjugate is an imaginary number.

2. The sum of z and its conjugate is a real number.

Which of the above statement(s) is/are correct?

(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

(c) Let $z = x + iy$

$$\overline{z} = x - iy$$

$\therefore z - \overline{z} = x + iy - x + iy = 2iy$ which is an imaginary number.

$\Rightarrow$ Statement-1 is correct.

Also, $z + \overline{z} = x + iy + x - iy = 2x$ which is real.

$\Rightarrow$ Statement-2 is correct.

Hence, option (c) is correct.

41. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$, where $n \in N$ and $i = \sqrt{-1}$?

(a) -1 (b) 1   (c) $\sqrt{2}$ (d) 2

(b) Let $z = i^{2n+1}(-i)^{2n-1}$, where $n \in \mathbb{N}$

$$= (i)^{2n}(i)(-i)^{2n}(-i)^{-1}$$

$$= (i^{2n})(-1)^{2n} \cdot (i^{2n})\left(\frac{i}{-i}\right)$$

$$= (i^{4n})(-1) = (i^4)^n \cdot (-1)$$

$$= -1 = -1 + 0i$$

$\therefore \quad |z| = 1$

Hence, option (b) is correct.

42. If α and β are the roots of the equation $4x^2 + 2x - 1 = 0$, then which one of the following is correct?

(a) $\beta = -2\alpha^2 - 2\alpha$ (b) $\beta = 4\alpha^2 - 3\alpha$
(c) $\beta = \alpha^2 - 3\alpha$ (d) $\beta = -2\alpha^2 + 2\alpha$

(a) Given quadratic equation

$$4x^2 + 2x - 1 = 0 \qquad \dots \text{(i)}$$

If α, β are the roots of Eq. (i), then these value will satisfy the given equation.

$$4\alpha^2 + 2\alpha - 1 = 0 \qquad \dots \text{(ii)}$$

and $\quad 4\beta^2 + 2\beta - 1 = 0 \qquad \dots \text{(iii)}$

From Eq. (i),

Sum of roots $= \dfrac{-2}{4}$

$$\alpha + \beta = \frac{-1}{2}$$

$$\beta = \frac{-1}{2} - \alpha$$

On putting the value of β in Eq. (iii),

$$4\left(\frac{-1}{2} - \alpha\right)^2 + 2\beta - 1 = 0$$

$$4\left(\frac{1}{4} + \alpha^2 + \alpha\right) - 1 = -2\beta$$

$$1 + 4\alpha^2 + 4\alpha - 1 = -2\beta$$

$$\Rightarrow \quad \beta = \frac{4(\alpha^2 + \alpha)}{-2}$$

$$\beta = -2\alpha^2 - 2\alpha$$

Hence, option (a) is correct.

43. If one root of $5x^2 + 26x + k = 0$ is reciprocal of the other, then what is the value of k?

(a) 2   (b) 3   (c) 5   (d) 8

(c) Given quadratic equation

$$5x^2 + 26x + k = 0 \qquad \dots \text{(i)}$$

Let α and β be the roots.

According to question, $\beta = \dfrac{1}{\alpha}$

$\therefore$ Product of roots $= \dfrac{k}{5}$

$$\alpha \cdot \beta = \frac{k}{5}$$

$$\Rightarrow \quad \alpha \cdot \frac{1}{\alpha} = \frac{k}{5} \Rightarrow 1 = \frac{k}{5}$$

$$\Rightarrow \quad k = 5$$

Hence, option (c) is correct.

44. In how many ways can a team of 5 players be selected from 8 players so as not to include a particular player?

(a) 42   (b) 35
(c) 21   (d) 20

(c) Given that there are 8 players among which one particular player is there.

Hence, number of ways to select 5 players $= {}^{8-1}C_5$

$$= {}^7C_5 = \frac{7 \times 6}{1 \times 2} = 21$$

Hence, option (c) is correct.

45. What is the coefficient of the middle term in the expansion of $(1 + 4x + 4x^2)^5$?

(a) 8064   (b) 4032
(c) 2016   (d) 1008

(a) $(1 + 4x + 4x^2)^5$

$$= \{(1 + 2x)^2\}^5 = (1 + 2x)^{10}$$

$\therefore$ Total number of term in the expansion of $(1 + 2x)^{10} = 10 + 1 = 11$

$\therefore$ Middle term $= \left(\dfrac{11 + 1}{2}\right)$th term

$$= 6\text{th term}$$

$$T_6 = T_{5+1} = {}^{10}C_5 (2x)^5$$

$$= {}^{10}C_5 \times 2^5 \times x^5$$

$\therefore$ Coefficient of middle term $= {}^{10}C_5 \cdot 2^5$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} \times 2^5 = 8064$$

Hence, option (a) is correct.

46. What is
$C(n, 1) + C(n, 2) + ... + C(n, n)$ equal to?
(a) $2 + 2^2 + 2^3 + ... + 2^n$
(b) $1 + 2 + 2^2 + 2^3 + ... + 2^n$
(c) $1 + 2 + 2^2 + 2^3 + ... + 2^{n-1}$
(d) $2 + 2^2 + 2^3 + ... + 2^{n-1}$

(c) $C(n, 1) + C(n, 2) + ... + C(n, n)$

$= {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n$

$\{\because {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n\}$

$= 2^n - {}^nC_0 = 2^n - 1$

Now, we shall solve the option to check whether sum is $2^n - 1$ or not.

Let's take
$S = 1 + 2 + 2^2 + 2^3 + ... + 2^{n-1}$ which forms a GP.

where $a = 1$

$r = \dfrac{2}{1} = 2 > 1$

$\therefore \quad S = \dfrac{a(r^n - 1)}{r - 1}$

$\therefore \quad S = \dfrac{1(2^n - 1)}{2 - 1} = 2^n - 1$

Hence, $2^n - 1 = {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$\therefore$ Option (c) is correct.

47. What is the sum of the coefficients of first and last terms in the expansion of $(1 + x)^{2n}$, where n is a natural number?
(a) 1   (b) 2
(c) n (d) $2n$

(b) Expand $(1 + x)^{2n}$ by using binomial expansion
$= {}^{2n}C_0 x^0 + {}^{2n}C_1 x^1 + {}^{2n}C_2 x^2$
$\qquad\qquad + ... + {}^{2n}C_{2n} x^{2n}$

$\therefore$ The coefficient of first and last term of the expansion
$= {}^{2n}C_0 + {}^{2n}C_{2n}$
$= 1 + 1 = 2$

Hence, option (b) is correct.

48. If the first term of an AP is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms, then what is the sum of the first ten terms?
(a) – 500   (b) – 250
(c) 500   (d) 250

(b) Given, first term of an AP $(a) = 2$

and $a_1 + a_2 + a_3 + a_4 + a_5 = \dfrac{1}{4}$

$(a_6 + a_7 + a_8 + a_9 + a_{10})$, where
$a_n = a + (n - 1)d$

$\Rightarrow \dfrac{5}{2}[2a + (5 - 1)d]$

$= \dfrac{1}{4} \times \dfrac{5}{2} [2a_6 + (5 - 1)d]$

$[\because$ sum of n terms of AP,

$S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$4(2 \times 2 + 4d) = 2a_6 + 4d$

$16 + 16d = 2a_6 + 4d$

$16 + 16d = 2(a + 5d) + 4d$

$16 + 16d = 2a + 14d$

$16 + 16d = 2 \times 2 + 14d$

$2d = -12 \Rightarrow d = -6$

$\therefore \quad S_{10} = \dfrac{10}{2}[2a + (10 - 1)d]$

$= 5[2 \times 2 + 9(-6)]$

$= 5[4 - 54]$

$S_{10} = -250$

Hence, option (b) is correct.

49. Consider the following statements
1. If each term of a GP is multiplied by same non-zero number, then the resulting sequence is also a GP.
2. If each term of a GP is divided by same non-zero number, then the resulting sequence is also a GP.

Which of the above statements is/are correct?
(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

(c) Let us take a GP.
$a, ar, ar^2, ...$ is in GP.

$\Rightarrow ak, akr, akr^2, ...$ will also be in GP
where, k is non-zero number.

If $k = \dfrac{1}{m}, m \neq 0$

$\Rightarrow \dfrac{a}{m}, \dfrac{a}{m}r, \dfrac{a}{m}r^2, ...$ will also be in GP.

Hence, both statements are correct.

50. How many 5-digit prime numbers can be formed using the digits 1, 2, 3, 4, 5 if the repetition of digits is not allowed?
(a) 5   (b) 4   (c) 3   (d) 0

(d) Given digits are 1, 2, 3, 4, 5
Since, the sum of digits
$= 1 + 2 + 3 + 4 + 5 = 15$ is divisible by 3.
$\Rightarrow$ Every 5 digit number formed by the given digits will be divisible by 3.
$\Rightarrow$ There is no prime number.
Hence, option (d) is correct.

51. If $f(x + 1) = x^2 - 3x + 2$, then what is $f(x)$ equal to?
(a) $x^2 - 5x + 4$ (b) $x^2 - 5x + 6$
(c) $x^2 + 3x + 3$ (d) $x^2 - 3x + 1$

(b) If $f(x + 1) = x^2 - 3x + 2$

Let $x + 1 = y$
$\Rightarrow \qquad x = y - 1$ or $x \to x - 1$
$\therefore f(x) = (x - 1)^2 - 3(x - 1) + 2$
$= x^2 + 1 - 2x - 3x + 3 + 2$
$= x^2 - 5x + 6$

Hence, option (b) is correct.

52. If $x^2, x, -8$ are in AP, then which one of the following is correct?
(a) $x \in \{-2\}$ (b) $x \in \{4\}$
(c) $x \in \{-2, 4\}$ (d) $x \in \{-4, 2\}$

(c) If $x^2, x, -8$ are in AP, then
$2x = x^2 - 8$
$\Rightarrow \quad x^2 - 2x - 8 = 0$
$\Rightarrow (x - 4)(x + 2) = 0$
$x \in \{-2, 4\}$
Hence, option (c) is correct.

53. The third term of a GP is 3. What is the product of its first five terms?
(a) 81
(b) 243
(c) 729
(d) Cannot be determined due to insufficient data

(b) Given
$a_3 = 3$
$\because \quad a_3 = ar^2$ in GP $[\because a_n = ar^{n-1}$ in GP$]$
$ar^2 = 3$ ... (i)

To find $a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5$
$= a(ar)(ar^2)(ar^3)(ar^4)$
$= a^5 r^{10} = (ar^2)^5 = 3^5 = 243$

Hence, option (b) is correct.

54. The element in the ith row and the j th column of a determinant of third order is equal to $2(i + j)$. What is the value of the determinant?
(a) 0   (b) 2   (c) 4   (d) 6

(a) Given,
$a_{ij} = 2(i + j)$
$\therefore a_{11} = 2(1 + 1) = 4, a_{21} = 2(2 + 1) = 6$
$a_{12} = 2(1 + 2) = 6, a_{22} = 2(2 + 2) = 8$
$a_{13} = 2(1 + 3) = 8, a_{23} = 2(2 + 3) = 10$
$a_{31} = 2(3 + 1) = 8, a_{32} = 2(3 + 2) = 10,$
$a_{33} = 2(3 + 3) = 12$

$\Delta = \begin{vmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{vmatrix} = 2 \cdot 2 \cdot 2 \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$

$= 8[2(24 - 25) - 3(18 - 20) + 4(15 - 16)]$
$\Delta = 8[-2 + 6 - 4]$
$\Delta = 0$
Hence, option (a) is correct.

55. With the numbers 2, 4, 6, 8, all the possible determinants with these four different elements are constructed. What is the sum of the values of all such determinants?
(a) 128   (b) 64   (c) 32   (d) 0

⊙ **(d)** Given numbers are 2, 4, 6, 8.
∴ We can form determinant of order 2.
Number of determinats
$$= 4 \times 3 \times 2 \times 1 = 24$$
Let's observe some determinants
$$\begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 6 & 2 \\ 4 & 8 \end{vmatrix} = 40$$
$$\begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 6 & 4 \\ 2 & 8 \end{vmatrix} = 40$$
$$\begin{vmatrix} 4 & 8 \\ 6 & 2 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 8 & 4 \\ 2 & 6 \end{vmatrix} = 40$$
$$\begin{vmatrix} 4 & 6 \\ 8 & 2 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = 40$$

Hence, we can see that we are getting a pattern where each determinant value will be neutralised by other value.
Hence, sum of the values of all determinants = 0
Hence, option (d) is correct.

56. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?
(a) 14 units   (b) 10.5 units
(c) 7 units   (d) 3.5 units

⊙ **(d)** Given equation of circle
$$4x^2 + 4y^2 - 20x + 12y - 15 = 0$$
$$\Rightarrow x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$$
On comparing with
$$x^2 + y^2 + 2gx + 2fy + c = 0$$
$$g = \frac{-5}{2}, f = \frac{3}{2}, c = \frac{-15}{4}$$
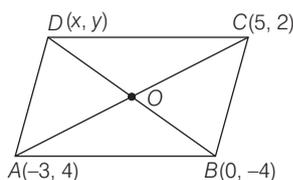$$\therefore \text{Radius} = \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{\frac{25}{4} + \frac{9}{4} + \frac{15}{4}} = \frac{7}{2} = 3.5 \text{ unit}$$
Hence, option (d) is correct.

57. A parallelogram has three consecutive vertices $(-3, 4), (0, -4)$ and $(5, 2)$. The fourth vertex is
(a) (2, 10)   (b) (2, 9)
(c) (3, 9)   (d) (4, 10)

⊙ **(a)**

Let the fourth vertex be D (x, y)

Diagonals of a parallelogram bisect each other.
O is mid-point of AC.
$$\Rightarrow \text{Coordinate of } O \left(\frac{-3 + 5}{2}, \frac{4 + 2}{2}\right) \text{ or}$$
$(1, 3)$
O is mid-point of BD.
$$\Rightarrow \text{Coordinate of } O \text{ is} \left(\frac{x + 0}{2}, \frac{y - 4}{2}\right)$$
$$\text{or} \left(\frac{x}{2}, \frac{y - 4}{2}\right)$$
Therefore, compare the coordinate of O
$$\Rightarrow \quad \frac{x}{2} = 1 \Rightarrow x = 2$$
$$\text{and} \quad \frac{y - 4}{2} = 3 \Rightarrow y = 10$$
Hence, the fourth vertex is $(2, 10)$.

58. If the lines $y + px = 1$ and $y - qx = 2$ are perpendicular, then which one of the following is correct?
(a) $pq + 1 = 0$ (b) $p + q + 1 = 0$
(c) $pq - 1 = 0$ (d) $p - q + 1 = 0$

⊙ **(c)** Given $y + px = 1$ … (i)
$$y - qx = 2 \quad … (ii)$$
Eqs. (i) and (ii) are perpendicular
$\Rightarrow m_1 \cdot m_2 = -1$ where m_1 and m_2 are the slope of Eqs. (i) and (ii)
$$\text{and} \quad m = \frac{-\text{ coefficient of } x}{\text{coefficient of } y}$$
$$\Rightarrow \frac{-p}{1} \times \frac{-(-q)}{1} = -1$$
$$\Rightarrow \quad -pq = -1$$
$$\Rightarrow \quad pq - 1 = 0$$
Hence, option (c) is correct.

59. If A, B and C are in AP, then the straight line $Ax + 2By + C = 0$ will always pass through a fixed point. The fixed point is
(a) (0, 0)   (b) (- 1, 1)
(c) (1, - 2)   (d) (1, - 1)

⊙ **(d)** Given A, B, C are in AP.
$$\Rightarrow \quad 2B = A + C$$
$$\Rightarrow \quad A - 2B + C = 0 \quad … (i)$$
On comparing $A - 2B + C = 0$
with the given line $Ax + 2By + C = 0$,
we get $x = 1, y = -1$
Hence, line $Ax + 2By + C = 0$ will pass through $(1, -1)$
Hence, option (d) is correct.

60. If the image of the point $(-4, 2)$ by a line mirror is $(4, -2)$, then what is the equation of the line mirror?
(a) $y = x$ (b) $y = 2x$
(c) $4y = x$ (d) $y = 4x$
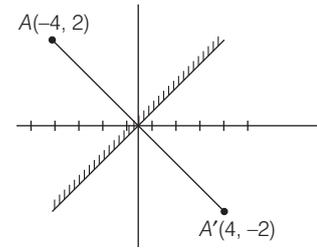
⊙ **(b)** Let $A = (-4, 2)$

image point
$A' = (4, -2)$
∴ Mid-point of
$$AA' = \left(\frac{-4 + 4}{2}, \frac{2 + (-2)}{2}\right) = (0, 0)$$
$$\text{Slope of } AA' = \frac{-2 - 2}{4 - (-4)}$$
$$= \frac{-4}{8} = \frac{-1}{2}$$
Since, AA' and mirror line are perpendicular.
Slope of line mirror
$$= \frac{-1}{\text{Slope of } AA'} = \frac{-1}{-1/2} = 2$$
Equation of a line is $y - y_1 = m(x - x_1)$
∴ Equation of a line mirror is
$$y - 0 = 2(x - 0)$$
$$\Rightarrow \quad y = 2x$$
Hence, option (b) is correct.

61. Consider the following statements in respect of the points $(p, p - 3)$, $(q + 3, q)$ and $(6, 3)$
  1. The points lie on a straight line.
  2. The points always lie in the first quadrant only for any value of p and q.
Which of the above statement(s) is/are correct?
(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊙ **(a)** Given points are $A(p, p - 3)$,
$B(q + 3, q)$ and $C(6, 3)$
As, Points lies on a straight line,
so slope of AB = slope of BC
$$\frac{q - p + 3}{q + 3 - p} = \frac{3 - q}{6 - q - 3}$$
$$\left[\because \text{slope of a line} = \frac{y_2 - y_1}{x_2 - x_1}\right]$$
$$1 = 1$$
∴ Statement 1 is correct.
But it's not necessary that the collinear points lie in the first quadrant only.
∴ Statement 2 is wrong.
Hence, option (a) is correct.

62. What is the acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x - y - 2 = 0$?

(a) $0°$ (b) $30°$ (c) $45°$ (d) $60°$

(b) $l_1 : x - 2 = 0$

$l_2 : \sqrt{3}x - y - 2 = 0$

∴ Slope of line l_1, $m_1 = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } y}$

$= \dfrac{-1}{0} = \infty$

The line l_1 is parallel to Y-axis or perpendicular to X-axis.

∴ Slope of line, l_2, $m_2 = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$

The line l_2 makes an angle $60°$ from positive X-axis.

∴ Angle between l_1 and

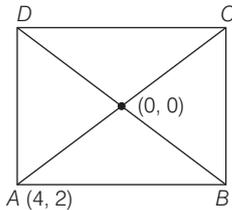$l_2 = 90° - 60° = 30°$

Hence, option (b) is correct.

63. The point of intersection of diagonals of a square $ABCD$ is at the origin and one of its vertices is at $A(4, 2)$. What is the equation of the diagonal BD?

(a) $2x + y = 0$ (b) $2x - y = 0$
(c) $x + 2y = 0$ (d) $x - 2y = 0$

(a) Since, diagonal BD passes through the origin $O(0, 0)$.

∵ Slope of $OA = \dfrac{0-2}{0-4} = \dfrac{1}{2}$

∵ OA and OB are perpendicular to each other

∴ slope of $OB = \dfrac{-1}{\text{slope of } OA} = \dfrac{-1}{1/2}$

$= -2$

∴ Eqs. of BD having slope -2 and passes through $(0, 0)$

$y - 0 = -2 [x - 0]$

[∵ Equation of a line $\Rightarrow y - y_1 = m(x - x_1)$]

$\Rightarrow \quad 2x + y = 0$

Hence, option (a) is correct.

64. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$, then what is the eccentricity of the hyperbola?

(a) $\dfrac{3}{2}$ (b) $\dfrac{5}{2}$ (c) $\dfrac{\sqrt{11}}{2}$ (d) $\dfrac{\sqrt{13}}{2}$

(d) Given point is $(3\tan\theta, 2\sec\theta)$

$\Rightarrow x = 3\tan\theta, y = 2\sec\theta$

$\dfrac{x}{3} = \tan\theta, \dfrac{y}{2} = \sec\theta$

∵ $\sec^2\theta - \tan^2\theta = 1$

$\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$ which represents conjugate Hyperbola.

$\Rightarrow a^2 = 9, b^2 = 4$

∴ $\quad e = \sqrt{1 + \dfrac{a^2}{b^2}}$

$= \sqrt{1 + \dfrac{9}{4}} = \sqrt{\dfrac{13}{4}}$

$e = \dfrac{\sqrt{13}}{2}$

Hence, option (d) is correct.

65. Consider the following with regard to eccentricity (e) of a conic section

1. $e = 0$ for circle
2. $e = 1$ for parabola
3. $e < 1$ for ellipse

Which of the above are correct?

(a) 1 and 2   (b) 2 and 3
(c) 1 and 3   (d) 1, 2 and 3

(d) Since, we know that circle has eccentricity 0

and parabola has eccentricity 1

and ellipse has eccentricity $e < 1$

and hyperbola has eccentricity $e > 1$.

Hence, option (d) is correct.

66. What is the angle between the two lines having direction ratios $\langle 6, 3, 6 \rangle$ and $\langle 3, 3, 0 \rangle$?

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

(b) Direction ratios of line $l_1 = \langle 6, 3, 6 \rangle$

$a_1 = 6, b_1 = 3, c_1 = 6$

Direction ratios of line $l_2 = \langle 3, 3, 0 \rangle$

$\Rightarrow a_2 = 3, b_2 = 3, c_2 = 0$

∵ $\cos\theta = \dfrac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \dfrac{6 \times 3 + 3 \times 3 + 6 \times 0}{\sqrt{6^2 + 3^2 + 6^2} \cdot \sqrt{3^2 + 3^2 + 0}}$

$\Rightarrow \cos\theta = \dfrac{27}{9 \times 3\sqrt{2}}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}} = \cos\dfrac{\pi}{4}$

∴ $\quad \theta = \dfrac{\pi}{4}$

Hence, option (b) is correct.

67. If l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to?

(a) 1   (b) $\dfrac{11}{27}$ (c) $\dfrac{13}{27}$ (d) 4

(b) Given line is

$x - 1 = 2(y + 3) = 1 - z$

$\Rightarrow \dfrac{x-1}{2} = \dfrac{y+3}{1} = \dfrac{1-z}{2}$

$\Rightarrow \dfrac{x-1}{2} = \dfrac{y-(-3)}{1} = \dfrac{z-1}{-2}$

∴ Direction ratios are $<2, 1, -2>$

∴ Direction cosines are

$< \dfrac{2}{\sqrt{2^2 + 1^2 + (-2)^2}}, \dfrac{1}{\sqrt{2^2 + 1^2 + (-2)^2}},$

$\dfrac{-2}{\sqrt{2^2 + 1^2 + (-2)^2}}$

∴ $l = \dfrac{2}{3}, m = \dfrac{1}{3}, n = \dfrac{-2}{3}$

∴ $l^4 + m^4 + n^4 = \left(\dfrac{2}{3}\right)^4 + \left(\dfrac{1}{3}\right)^4 + \left(\dfrac{-2}{3}\right)^4$

$= \dfrac{16 + 1 + 16}{81} = \dfrac{33}{81} = \dfrac{11}{27}$

Hence, option (b) is correct.

68. What is the projection of the line segment joining $A(1, 7, -5)$ and $B(-3, 4, -2)$ on Y-axis?

(a) 5   (b) 4   (c) 3   (d) 2

(c) $A = (1, 7, -5)$ and $B = (-3, 4, -2)$

∴ Direction ratios of

$AB = < (-3 - 1), (4 - 7), (-2 + 5) >$

$= < -4, -3, 3 >$

$\Rightarrow a = -4, b = -3, c = 3$

Direction cosines of Y-axis $= < 0, 1, 0 >$

$l = 0, m = 1, n = 0$

∴ Projection of AB on Y-axis

$= | al + bm + cn |$

$= | -4 \times 0 + (-3) \times 1 + 3 \times 0 | = 3$

Hence, option (c) is correct.

69. What is the number of possible values of k for which the line joining the points $(k, 1, 3)$ and $(1, -2, k + 1)$ also passes through the point $(15, 2, -4)$?

(a) Zero   (b) One   (c) Two   (d) Infinite

(c) Let $A = (k, 1, 3), B = (1, -2, k + 1)$

and $C = (15, 2, -4)$

Since, line AB passes through C also.

Hence, points A, B and C are collinear.

∴ $\begin{vmatrix} k & 1 & 3 \\ 1 & -2 & k+1 \\ 15 & 2 & -4 \end{vmatrix} = 0$

$k(8 - 2k - 2) - 1(-4 - 15k - 15)$
$+ 3(2 + 30) = 0$

$6k - 2k^2 + 19 + 15k + 96 = 0$
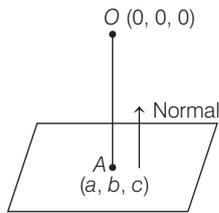
$2k^2 - 21k - 115 = 0$ which is quadratic equation.

$\Rightarrow k$ has two values.

Hence, option (c) is correct.

70. The foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ is
(a) (0, 1, 2)
(b) (0, 0, 3)
(c) (1, 1, 1)
(d) (– 1, 1, 3)

⊘ *(c)* Let the foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ be (a, b, c).

$$O \ (0, 0, 0)$$

↑ Normal

A●
(a, b, c)

Direction ratios of the plane $= < 1, 1, 1 >$

∴ Direction ratios of OA and normal will be in the same ratio.

∴ $\dfrac{a - 0}{1} = \dfrac{b - 0}{1} = \dfrac{c - 0}{1}$
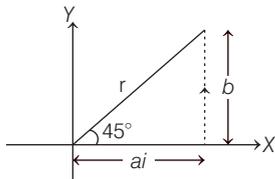
$\Rightarrow a = 1, b = 1, c = 1$

∴ $A = (1, 1, 1)$

Hence, option (c) is correct.

71. A vector $\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ is equally inclined to both x and y axes. If the magnitude of the vector is 2 units, then what are the values of a and b respectively?
(a) $\dfrac{1}{2}, \dfrac{1}{2}$
(b) $\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$
(c) $\sqrt{2}, \sqrt{2}$
(d) 2, 2

⊘ *(c)* $\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

$|\mathbf{r}| = \sqrt{a^2 + b^2} = 2$

Y

r ↕ b

45°

ai → X

Since, r is equally inclined from X-axis and Y-axis.

Hence, r makes 45° from the X-axis.

∴ $a = |\mathbf{r}| \cos 45°$ and $b = |\mathbf{r}| \sin 45°$

$a = 2 \times \dfrac{1}{\sqrt{2}}$, and $b = 2 \times \dfrac{1}{\sqrt{2}}$

$a = \sqrt{2}$ and $b = \sqrt{2}$

Hence, option (c) is correct.

72. Consider the following statements in respect of a vector $\mathbf{c} = \mathbf{a} + \mathbf{b}$, where $|\mathbf{a}| = |\mathbf{b}| \neq 0$
1. $\mathbf{c}$ is perpendicular to $(\mathbf{a} - \mathbf{b})$.
2. $\mathbf{c}$ is perpendicular to $(\mathbf{a} \times \mathbf{b})$.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

⊘ *(c)* $c = a + b$ where $a = b \neq 0$

Consider, $c \cdot (a - b) = (a + b) \cdot (a - b)$
$= |a|^2 - |b|^2 = |b|^2 - |b|^2 = 0$

$\Rightarrow c$ is perpendicular to $(a - b)$.

Also, $c \cdot (a \times b) = (a + b) \cdot (a \times b)$
$= a \cdot (a \times b) + b \cdot (a \times b)$
$= 0 + 0 = 0$

$\Rightarrow c$ is perpendicular to $(a \times b)$

Hence, option (c) is correct.

73. If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}| = 4$, then which one of the following is correct?
(a) a and b must be unit vectors
(b) a must be parallel to b
(c) a must be perpendicular to b
(d) a must be equal to b

⊘ *(c)* Given, $|a + b| = |a - b| = 4$

$\Rightarrow |a + b|^2 = |a - b|^2$

$|a|^2 + |b|^2 + 2a \cdot b$
$= |a|^2 + |b|^2 - 2a \cdot b$

$\Rightarrow 4 a \cdot b = 0 \Rightarrow a \cdot b = 0$

$\Rightarrow a$ must be perpendicular to b.

Hence, option (c) is correct.

74. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar, then what is $(2\mathbf{a} \times 3\mathbf{b}) \cdot 4\mathbf{c} + (5\mathbf{b} \times 3\mathbf{c}) \cdot 6\mathbf{a}$ equal to?
(a) 114
(b) 66
(c) 0
(d) – 66

⊘ *(c)* Given that, a, b and c are coplanar

$\Rightarrow \qquad [a \ b \ c] = 0 \qquad \ldots$ (i)

∴ $(2a \times 3b) \cdot 4c + (5b \times 3c) \cdot 6a$

$= 2 \cdot 3 \cdot 4 [a \ b \ c] + 5 \cdot 3 \cdot 6 [b \ c \ a]$

$= 24 [a \ b \ c] + 90 [a \ b \ c] \{ \because [a \ b \ c]$
$= [b \ c \ a] \}$

$= 24 \times 0 + 90 \times 0 = 0$

Hence, option (c) is correct.

75. Consider the following statements
1. The cross product of two unit vectors is always a unit vector.
2. The dot product of two unit vectors is always unity.
3. The magnitude of sum of two unit vectors is always greater than the magnitude of their difference.

Which of the above statements are not correct?
(a) 1 and 2
(b) 2 and 3
(c) 1 and 3
(d) 1, 2 and 3

⊘ *(d)* **Statement I**

Let a and b are unit vectors

i.e. $|a| = |b| = 1$

$a \times b = |a| |b| \sin \theta \ \hat{n}$

$|a \times b| = |a| |b| \sin \theta$

$= \sin \theta \in [-1, 1]$

Therefore, statement I is incorrect.

Statement II

Let a and b are unit vectors

i.e. $a \cdot b = |a| |b| \cos \theta$

$= \cos \theta \in [-1, 1]$

Therefore, statement II is also incorrect.

Statement III

Let $a = \hat{i}$ and $b = \hat{j}$

$\Rightarrow |a| = 1, |b| = 1$

$|a + b| = |\hat{i} + \hat{j}| = \sqrt{2}$

$|a - b| = |\hat{i} - \hat{j}| = \sqrt{2}$

76. If $\lim\limits_{x \to a} \dfrac{a^x - x^a}{x^a - a^a} = -1$

then what is the value of a?
(a) – 1
(b) 0
(c) 1
(d) 2

⊘ *(c)* $\lim\limits_{x \to a} \dfrac{a^x - x^a}{x^a - a^a} = -1$

$\Rightarrow \quad \lim\limits_{x \to a} \dfrac{a^x - x^a}{x^a - a^a} = -1 \left(\dfrac{0}{0} \ \text{form} \right)$

By using L' Hospital rule,

$\lim\limits_{x \to a} \dfrac{a^x \log_e a - ax^{a-1}}{ax^{a-1} - 0} = -1$

$\Rightarrow \dfrac{a^a \log_e a - a \cdot a^{a-1}}{a \cdot a^{a-1}} = -1$

$\Rightarrow \dfrac{a^a (\log_e a - 1)}{a^a} = -1$

$\Rightarrow \qquad \log_e a = -1 + 1$

$\Rightarrow \qquad \log_e a = 0$

∴ $\qquad a = e^0 = 1$

∴ $\qquad a = 1$

Hence, option (c) is correct.

77. A particle starts from origin with a velocity (in m/s) given by the equation $\dfrac{dx}{dt} = x + 1$. The time (in second) taken by the particle to traverse a distance of 24 m is
(a) ln 24
(b) ln 5
(c) 2 ln 5
(d) 2 ln 4

⊘ *(c)* $\dfrac{dx}{dt} = x + 1$

$\Rightarrow \qquad \dfrac{dx}{x + 1} = dt$

Column 1:

On integrating both sides

$$\int \frac{dx}{x+1} = \int dt$$

$$\ln(x+1) = t + c \qquad \ldots \text{(i)}$$

Since, at $t = 0$, distance $(x) = 0$

$$\therefore \ln(0+1) = 0 + c$$

$$\boxed{0 = c}$$

$$\therefore \ln(x+1) = t$$

At $x = 24$ m

$$t = \ln(24+1) = \ln 25 = \ln 5^2$$

$$t = 2\ln 5$$

Hence, option (c) is correct.

78. What is $\int_0^a \dfrac{f(a-x)}{f(x)+f(a-x)}\, dx$ equal to?

(a) a (b) $2a$

(c) 0 (d) $\dfrac{a}{2}$

⊛ **(d)** Let $I = \int_0^a \dfrac{f(a-x)}{f(x)+f(a-x)}\, dx \qquad \ldots \text{(i)}$

$$x \to a - x$$

$$I = \int_0^a \frac{f(a-a+x)}{f(a-x)+f(a-a+x)}\, dx$$

$$I = \int_0^a \frac{f(x)}{f(a-x)+f(x)}\, dx \qquad \ldots \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_0^a \frac{f(a-x)+f(x)}{f(a-x)+f(x)}\cdot dx$$

$$2I = \int_0^a 1 \cdot dx$$

$$2I = x \big|_0^a$$

$$2I = a - 0$$

$$I = \frac{a}{2}$$

Hence, option (d) is correct.

79. What is $\lim\limits_{x \to -1} \dfrac{x^3 + x^2}{x^2 + 3x + 2}$ equal to?

(a) 0 (b) 1 (c) 2 (d) 3

⊛ **(b)** Given, $\lim\limits_{x \to -1} = \dfrac{x^3 + x^2}{x^2 + 3x + 2}\left(\dfrac{0}{0}\text{ from}\right)$

By using L' Hospital rule

$$\lim_{x \to -1} = \frac{3x^2 + 2x}{2x + 3}$$

$$= \frac{3(-1)^2 + 2(-1)}{2(-1)+3} = \frac{3-2}{-2+3} = 1$$

Hence, option (b) is correct.

80. If $\int_0^a [f(x) + f(-x)]dx = \int_{-a}^a g(x)dx$ then what is $g(x)$ equal to?

(a) $f(x)$ (b) $f(-x) + f(x)$

(c) $-f(x)$ (d) None of these

⊛ **(a)** Given that,

$$\int_0^a [f(x) + f(-x)]dx = \int_{-a}^a g(x)dx$$

If $g(x) = f(x)$

Column 2:

R.H.S. Let $I = \int_{-a}^a g(x)\, dx$

$$\Rightarrow \qquad I = \int_{-a}^a f(x)\, dx \qquad \ldots \text{(i)}$$

$$\left[\because \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx\right]$$

$$I = \int_{-a}^a f(-x)\, dx \qquad \ldots \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_{-a}^a [f(x) + f(-x)]\, dx \quad \text{(even function)}$$

$$\Rightarrow 2I = 2\int_0^a [f(x) + f(-x)]\, dx$$

$$I = \int_0^a [f(x) + f(-x)]dx = \text{L.H.S}$$

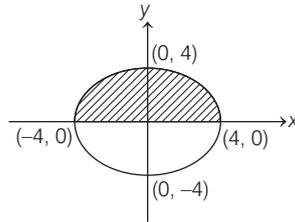$$\Rightarrow g(x) = f(x) \text{ and}$$

Hence, option (a) is correct.

81. What is the area bounded by $y = \sqrt{16 - x^2}$, $y \geq 0$ and the X-axis?

(a) 16π sq. units   (b) 8π sq. units

(c) 4π sq. units   (d) 2π sq. units

⊛ **(b)** Shaded portion in the diagram represents the area bounded by $y = \sqrt{16 - x^2}$, $y \geq 0$ and X-axis.

Put $y = 0$, then $16 - x^2 = 0$

$$\Rightarrow x = \pm 4$$

$$\therefore \text{ Required area} = \int_{-4}^4 \sqrt{16 - x^2}\, dx$$

$$= 2\int_0^4 \sqrt{16 - x^2}\, dx$$

$$= 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= 2[0 + 8\sin^{-1} 1] = 2 \times \frac{8\pi}{2}$$

$$= 8\pi \text{ sq units}$$

Hence, option (b) is correct.

82. The curve $y = -x^3 + 3x^2 + 2x - 27$ has the maximum slope at

(a) $x = -1$ (b) $x = 0$

(c) $x = 1$ d) $x = 2$

⊛ **(c)** Given that, $y = -x^3 + 3x^2 + 2x - 27$

$$\text{Slope} = \frac{dy}{dx} = -3x^2 + 6x + 2$$

$$\therefore f'(x) = -3x^2 + 6x + 2$$

For maxima/minima of $f'(x)$.

$$\frac{d}{dx}[f'(x)] = -6x + 6 = 0$$

$$\Rightarrow 6x = 6 \Rightarrow x = 1$$

Column 3:

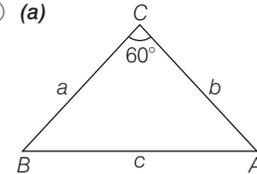At $x = 1$, $\dfrac{d^2}{dx^2} f'(x) = -6 < 0$

$\therefore$ At $x = 1$, $f'(x)$ is maximum.

Hence, option (c) is correct.

83. A 24 cm long wire is bent to form a triangle with one of the angles as 60°. What is the altitude of the triangle having the greatest possible area?

(a) $4\sqrt{3}$ cm   (b) $2\sqrt{3}$ cm

(c) 6 cm   (d) 3 cm

⊛ **(a)**

Given, $a + b + c = 24$

$$\Rightarrow \qquad c = 24 - (a+b)$$

Again $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$$\Rightarrow \cos 60° = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \qquad \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \qquad ab = a^2 + b^2 - c^2$$

$$\Rightarrow \qquad ab = a^2 + b^2 - [24 - (a+b)]^2$$

$$\Rightarrow \qquad ab = a^2 + b^2 - 576$$

$$- (a+b)^2 + 48(a+b)$$

$$\Rightarrow ab = a^2 + b^2 - 576 - a^2 - b^2$$

$$- 2ab + 48(a+b)$$

$$\Rightarrow 3ab - 48(a+b) = -576$$

$$\Rightarrow ab - 16(a+b) = -192$$

$$\Rightarrow \qquad ab - 16a = 16b - 192$$

$$\Rightarrow \qquad a(b - 16) = 16(b - 12)$$

$$\Rightarrow \qquad a = \frac{16(b - 12)}{b - 16}$$

Again $ar(\triangle ABC)$, $A = \dfrac{1}{2}ab\sin C$

$$= \frac{1}{2} \times \frac{16(b-12)b}{b-16} \times \sin 60°$$

$$= \frac{1}{2} \times \frac{16(b-12)b}{b-16} \times \frac{\sqrt{3}}{2}$$

$$= \frac{4\sqrt{3}(b^2 - 12b)}{b - 16}$$

$$\therefore \frac{dA}{db} = 4\sqrt{3}$$

$$\left[\frac{(2b-12)(b-16) - (b^2 - 12b)\cdot 1}{b - 16}\right]$$

Maximum value for A

$$\frac{dA}{db} = 0$$

$$\Rightarrow \frac{4\sqrt{3}}{b - 16}[2b^2 - 32b - 12b$$

$$+192 - b^2 + 12b] = 0$$

$\Rightarrow b^2 - 32b + 192 = 0$

$\Rightarrow (b - 24)(b - 8) = 0$

$\Rightarrow \qquad b = 24, 8$

when, $b = 24$

$a = \dfrac{16(24 - 12)}{24 - 16} = \dfrac{16 \times 12}{8} = 24$

and $c = 24 - (24 + 24) = -24$

It is impossible,

when, $b = 8$

$a = \dfrac{16(8 - 12)}{8 - 16} = \dfrac{16(-4)}{(-8)} = 8$

and $c = 24 - (8 + 8) = 8$

$\therefore$ So triangle will be equilateral

$\therefore$ Height $= \dfrac{\sqrt{3}}{2}$ (side)

$= \dfrac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}$ cm

84. If $f(x) = e^{|x|}$, then which one of the following is correct?

(a) $f'(0) = 1$ (b) $f'(0) = -1$
(c) $f'(0) = 0$ (d) $f'(0)$ does not exist

(d) Given that, $f(x) = e^{|x|}$

$\Rightarrow f(x) = \begin{cases} e^x & ; x \geq 0 \\ e^{-x} & ; x < 0 \end{cases}$

LHD at $x = 0$,

$f'(0^-) = \lim\limits_{h \to 0^-} \dfrac{f(0 + h) - f(0)}{h}$

$= \lim\limits_{h \to 0^-} \dfrac{e^{-h} - e^0}{h}$

(by using L' Hospital rule)

$= \lim\limits_{h \to 0^-} -\dfrac{e^{-h}}{1} = -1$

RHD at $x = 0$,

$f'(0^+) = \lim\limits_{h \to 0^+} \dfrac{f(0 + h) - f(0)}{h}$

$= \lim\limits_{h \to 0^+} \dfrac{e^h - e^0}{h}$

$= \lim\limits_{h \to 0^+} \dfrac{e^h}{1} = e^0 = 1$

$\because$ LHD $\neq$ RHD

$\therefore f'(x)$ does not exist at $x = 0$

Hence, option (d) is correct.

85. What is $\int \dfrac{dx}{\sec x + \tan x}$ equal to?

(a) $\ln(\sec x) + \ln|\sec x + \tan x| + c$
(b) $\ln(\sec x) - \ln|\sec x + \tan x| + c$
(c) $\sec x \tan x - \ln|\sec x - \tan x| + c$
(d) $\ln|\sec x + \tan x| - \ln|\sec x| + c$

(d) Let $I = \int \dfrac{dx}{\sec x + \tan x}$

$I = \int \dfrac{1}{(\sec x + \tan x)} \times \dfrac{(\sec x - \tan x)}{(\sec x - \tan x)} dx$

$I = \int \dfrac{(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$

$I = \int \dfrac{(\sec x - \tan x)}{1} dx$

$= \int \sec x \, dx - \int \tan x \, dx$

$I = \log|\sec x + \tan x| - \log|\sec x| + C$

Hence, option (d) is correct.

86. What is $\int \dfrac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$ (b) $\tan^{-1} x + c$
(c) $\sec^{-1} x + c$ (d) $\cos^{-1} x + c$

(b) Let $I = \int \dfrac{dx}{\sec^2(\tan^{-1} x)}$

$I = \int \dfrac{dx}{1 + \tan^2(\tan^{-1} x)}$

$[\because \sec^2 x = 1 + \tan^2 x]$

$= \int \dfrac{dx}{1 + x^2}$

$I = \tan^{-1} x + C$

Hence, option (b) is correct.

87. If $x + y = 20$ and $P = xy$, then what is the maximum value of P?

(a) 100   (b) 96
(c) 84   (d) 50

(a) Given, $x + y = 20$

$\Rightarrow y = 20 - x$

$P = xy$

$P = x(20 - x)$

$P = 20x - x^2$

$\therefore \dfrac{dP}{dx} = 20 - 2x$

For maxima/minima, $\dfrac{dP}{dx} = 0$

$20 - 2x = 0$

$\Rightarrow \qquad x = 10$

At $x = 10, \dfrac{d^2P}{dx^2} = -2 < 0$

$\therefore P$ is maximum at $x = 10$

$\Rightarrow y = 20 - 10 = 10$

$\therefore$ Maximum value of $P = xy$

$= 10 \times 10 = 100$

Hence, option (a) is correct.

88. What is the derivative of $\sin(\ln x) + \cos(\ln x)$ with respect to x at $x = e$?

(a) $\dfrac{\cos 1 - \sin 1}{e}$ (b) $\dfrac{\sin 1 - \cos 1}{e}$

(c) $\dfrac{\cos 1 + \sin 1}{e}$ (d) 0

(a) Let $y = \sin(\ln x) + \cos(\ln x)$

$\therefore \dfrac{dy}{dx} = \cos(\ln x) \cdot \dfrac{1}{x} + \left(-\sin(\ln x) \cdot \dfrac{1}{x}\right)$

$= \dfrac{1}{x}[\cos(\ln x) - \sin(\ln x)]$

At $x = e$,

$\dfrac{dy}{dx} = \dfrac{1}{e}[\cos(\ln e) - \sin(\ln e)]$

$= \dfrac{1}{e}[\cos 1 - \sin 1]$

$[\because \ln e = 1]$

Hence, option (a) is correct.

89. If $x = e^t \cos t$ and $y = e^t \sin t$, then what is $\dfrac{dx}{dy}$ at $t = 0$ equal to?

(a) 0   (b) 1   (c) $2e$ (d) -1

(b) Given that, $x = e^t \cos t$, $y = e^t \sin t$

$\therefore \quad \dfrac{dx}{dt} = e^t \dfrac{d}{dt} \cos t + \cos t \dfrac{d}{dt} e^t$

$= e^t(-\sin t) + \cos t \cdot e^t$

$\dfrac{dy}{dt} = e^t \dfrac{d}{dt} \sin t + \sin t \cdot \dfrac{d}{dt} e^t$

$\dfrac{dy}{dt} = e^t \cos t + e^t \sin t$

$\therefore \quad \dfrac{dx}{dy} = \dfrac{dx/dt}{dy/dt}$

$\dfrac{dx}{dy} = \dfrac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)}$

At $t = 0$,

$\therefore \dfrac{dx}{dy} = \dfrac{\cos 0° - \sin 0°}{\cos 0° + \sin 0°} = \dfrac{1 - 0}{1 + 0}$

$\left(\dfrac{dx}{dy}\right)_{t = 0} = 1$

Hence, option (b) is correct.

90. What is the maximum value of $\sin 2x \cdot \cos 2x$?

(a) $\dfrac{1}{2}$ (b) 1   (c) 2   (d) 4

(a) Let $y = \sin 2x \cdot \cos 2x$

$y = \dfrac{1}{2}[2 \sin 2x \cdot \cos 2x]$

$y = \dfrac{1}{2} \sin 4x$

Since, we know that

$-1 \leq \sin 4x \leq 1 \Rightarrow \dfrac{-1}{2} \leq \dfrac{1}{2} \sin 4x \leq \dfrac{1}{2}$

$\therefore$ Maximum value $= \dfrac{1}{2}$

Hence, option (a) is correct.

91. What is the derivative of e^x with respect to x^e?

(a) $\dfrac{xe^x}{ex^e}$ (b) $\dfrac{e^x}{x^e}$ (c) $\dfrac{xe^x}{x^e}$ (d) $\dfrac{e^x}{ex^e}$

(a) Let $y_1 = e^x$ and $y_2 = x^e$

$\therefore \quad \dfrac{dy_1}{dx} = e^x, \dfrac{dy_2}{dx} = ex^{e-1}$

$\therefore \quad \dfrac{dy_1}{dy_2} = \dfrac{e^x}{e x^{e-1}} = \dfrac{xe^x}{e x^e}$

Hence, option (a) is correct.

92. If a differentiable function $f(x)$ satisfies $\lim\limits_{x \to -1} \dfrac{f(x)+1}{x^2-1} = -\dfrac{3}{2}$, then what is $\lim\limits_{x \to -1} f(x)$ equal to?

(a) $-\dfrac{3}{2}$ 　　　　(b) -1

(c) 0 　　　　(d) 1

◉ **(b)** Given, $\lim\limits_{x \to -1} \dfrac{f(x)+1}{x^2-1} = \dfrac{-3}{2}$

$\therefore \lim\limits_{x \to -1} \dfrac{f(x)+1}{x^2-1}$ has denominator 0 at $x = -1$

$\Rightarrow \quad \lim\limits_{x \to -1} f(x) + 1 = 0$

$\Rightarrow \quad \lim\limits_{x \to -1} f(x) = -1$

Hence, option (b) is correct.

93. If the function

$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$

is continuous, then what is the value of $(a + b)$?

(a) 5 　　　　(b) 10
(c) 15 　　　　(d) 20

◉ **(a)** Given that, $f(x) = \begin{cases} a + bx & ; & x < 1 \\ 5 & ; & x = 1 \\ b - ax & ; & x > 1 \end{cases}$

$\because f(x)$ is continuous.

$\Rightarrow f(x)$ will be continuous at $x = 1$

$\lim\limits_{x \to 1^-} f(x) = f(1) = \lim\limits_{x \to 1^+} f(x)$

$\lim\limits_{x \to 1^-} (a + bx) = 5 = \lim\limits_{x \to 1^+} (b - ax)$

$a + b = 5 = b - a$

$\Rightarrow \quad a + b = 5$

Hence, option (a) is correct.

94. Consider the following statements in respect of the function $f(x) = \sin x$

1. $f(x)$ increases in the interval $(0, \pi)$.

2. $f(x)$ decreases in the interval $\left(\dfrac{5\pi}{2}, 3\pi\right)$.
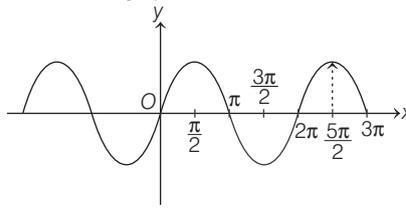
Which of the above statement is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

◉ **(b)** Given, $f(x) = \sin x$

From the graph of $\sin x$

We can see that $f(x)$ increases in $\left[0, \dfrac{\pi}{2}\right]$

and decreases in $\left[\dfrac{\pi}{2}, \pi\right]$ and $\left(\dfrac{5\pi}{2}, 3\pi\right)$.

$\Rightarrow$ Statement-1 is wrong and Statement-2 is correct.

Hence, option (b) is correct.

95. What is the domain of the function $f(x) = 3^x$?

(a) $(-\infty, \infty)$ 　　(b) $(0, \infty)$
(c) $[0, \infty)$ 　　(d) $(-\infty, \infty) - \{0\}$

◉ **(a)** Given, $f(x) = 3^x$

$\because$ We know that, domain of exponential function is $(-\infty, \infty)$.

$\therefore$ Domain of $3^x = (-\infty, \infty)$

Hence, option (a) is correct.

96. If the general solution of a differential equation is $y^2 + 2cy - cx + c^2 = 0$, where c is an arbitrary constant, then what is the order of the differential equation?

(a) 1 　　(b) 2 　　(c) 3 　　(d) 4

◉ **(a)** Given that, $y^2 + 2cy - cx + c^2 = 0$

Since, the above equation contains only one variable constant.

Hence, order of the differential equation $= 1$

Hence, option (a) is correct.

97. What is the degree of the following differential equation?

$$x = \sqrt{1 + \dfrac{d^2y}{dx^2}}$$

(a) 1
(b) 2
(c) 3
(d) Degree is not defined

◉ **(a)** Let $x = \sqrt{1 + \dfrac{d^2y}{dx^2}}$

$\Rightarrow x^2 = 1 + \dfrac{d^2y}{dx^2} \Rightarrow \left(\dfrac{d^2y}{dx^2}\right)^1 = x^2 - 1$

$\therefore$ Degree = exponent of highest order derivative = 1

Hence, option (a) is correct.

98. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$?

(a) $\dfrac{d^2y}{dx^2} + y = 0$ 　(b) $\dfrac{d^2y}{dx^2} - y = 0$

(c) $\dfrac{d^2y}{dx^2} + y = 1$ 　(d) $\dfrac{dy}{dx} - y = 0$

◉ **(b)** Given, $y = ae^x + be^{-x}$

$\therefore \qquad \dfrac{dy}{dx} = ae^x - be^{-x}$

$\dfrac{d^2y}{dx^2} = ae^x + be^{-x} = y$

$\Rightarrow \dfrac{d^2y}{dx^2} - y = 0$

Hence, option (b) is correct.

99. What is the solution of the following differential equation?

$$\ln\left(\dfrac{dy}{dx}\right) + y = x$$

(a) $e^x + e^y = c$ 　(b) $e^{x+y} = c$
(c) $e^x - e^y = c$ 　(d) $e^{x-y} = c$

◉ **(c)** Given, $\ln\left(\dfrac{dy}{dx}\right) + y = x$

$\Rightarrow \qquad \ln\left(\dfrac{dy}{dx}\right) = x - y$

$\Rightarrow \qquad \dfrac{dy}{dx} = e^{x-y}$

$\Rightarrow \qquad \dfrac{dy}{dx} = \dfrac{e^x}{e^y}$

$\Rightarrow \qquad e^y dy = e^x dx$

On integrating both sides,

$\int e^y dy = \int e^x dx$

$e^y + c = e^x \Rightarrow e^x - e^y = c$

Hence, option (c) is correct.

100. What is $\int e^{(2\ln x + \ln x^2)} dx$ equal to?

(a) $\dfrac{x^4}{4} + C$ 　　(b) $\dfrac{x^3}{3} + C$

(c) $\dfrac{2x^5}{5} + C$ 　　(d) $\dfrac{x^5}{5} + C$

◉ **(d)** Let $I = \int e^{(2\ln x + \ln x^2)} dx$

$= \int e^{(\ln x^2 + \ln x^2)} dx$

$= \int e^{2\ln x^2} dx = \int e^{\ln(x^2)^2} dx = \int x^4 dx$

$I = \dfrac{x^5}{5} + C$

Hence, option (d) is correct.

101. Consider the following measures of central tendency for a set of N numbers

1. Arithmetic mean

2. Geometric mean

Which of the above uses/use all the data?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Since, we know that the measures of central tendency are Mean, Median and Mode.
Where Arithmetic Mean and Geometric mean are the type mean.
Hence, option (c) is correct.

102. The numbers of Science, Arts and Commerce graduates working in a company are 30, 70 and 50 respectively. If these figures are represented by a pie chart, then what is the angle corresponding to Science graduates?
(a) 36°
(b) 72°
(c) 120°
(d) 168°

(b) The ratio of Science, Arts and Commerce graduates
$$= 30 : 70 : 50 = 3 : 7 : 5$$
∴ Angle corresponding to Science
$$\text{graduates} = \frac{3}{3 + 7 + 5} \times 360°$$
$$= \frac{3}{15} \times 360° = 72°$$
Hence, option (b) is correct.

103. For a histogram based on a frequency distribution with unequal class intervals, the frequency of a class should be proportional to
(a) the height of the rectangle
(b) the area of the rectangle
(c) the width of the rectangle
(d) the perimeter of the rectangle

(b) Since, we know that for a histogram, based on a frequency distribution with equal intervals, the frequency of a class is proportional to height of the rectangle and for a histogram based on frequency distribution with unequal intervals, the frequency of a class is proportional to Area of the rectangle.
Hence, option (b) is correct.

104. The coefficient of correlation is independent of
(a) change of scale only
(b) change of origin only
(c) both change of scale and change of origin
(d) neither change of scale nor change of origin

(c) Since, we know that coefficient of correlation is independent of both change of scale and change of origin.
Hence, option (c) is correct.

105. The following table gives the frequency distribution of number of peas per pea pod of 198 pods

Number of peas	1	2	3	4	5	6	7
Frequency	4	33	76	50	26	8	1

What is the median of this distribution?
(a) 3
(b) 4
(c) 5
(d) 6

(a)

Number of Peas	Frequency	Cumulative frequency
1	4	4
2	33	37
3	76	113
4	50	163
5	26	189
6	8	197
7	1	198
$\Sigma f = 198$		

$$\because N = 198, \frac{N}{2} = \frac{198}{2} = 99$$

$$\text{Median} = \frac{\frac{N}{2}\text{th term} + \left(\frac{N}{2} + 1\right)\text{th term}}{2}$$

$$= \frac{99\text{th term} + 100\text{th term}}{2}$$

$$= \frac{3 + 3}{2} = 3$$

∴ Median = 3
Hence, option (a) is correct.

106. If M is the mean of n observations $x_1 - k, x_2 - k, x_3 - k, ..., x_n - k$, where k is any real number, then what is the mean of $x_1, x_2, x_3, ..., x_n$?
(a) M
(b) $M + k$
(c) $M - k$
(d) kM

(b) Given that,
Mean of $x_1 - k, x_2 - k, x_3 - k, ... , x_n - k$
$$\therefore M = \frac{(x_1 - k) + (x_2 - k) + ... + (x_n - k)}{n}$$
$$M = \frac{(x_1 + x_2 + ... + x_n)}{n} - \frac{nk}{n}$$
$$M + k = \frac{x_1 + x_2 + ... + x_n}{n}$$
∴ Mean of $x_1, x_2, x_3, ... , x_n = M + k$
Hence, option (b) is correct.

107. What is the sum of deviations of the variate values 73, 85, 92, 105, 120 from their mean?
(a) – 2
(b) – 1
(c) 0
(d) 5

(c) Mean of 73, 85, 92, 105, 120
$$\bar{x} = \frac{73 + 85 + 92 + 105 + 120}{5}$$
$$= \frac{475}{5}$$
$$\bar{x} = 95$$
∴ Sum of deviations from their mean
$$= (73 - 95) + (85 - 95)$$
$$+ (92 - 95) + (105 - 95) + (120 - 95)$$
$$= -22 - 10 - 3 + 10 + 25 = 0$$
Hence, option (c) is correct.

108. Let x be the HM and y be the GM of two positive numbers m and n. If $5x = 4y$, then which one of the following is correct?
(a) $5m = 4n$
(b) $2m = n$
(c) $4m = 5n$
(d) $m = 4n$

(d) Given, two positive numbers are m and n.
$$\therefore \text{H.M. of } m \text{ and } n = \frac{2mn}{m + n}$$
$$x = \frac{2mn}{m + n} \qquad \text{... (i)}$$
G.M. of m and $n = \sqrt{mn}$
$$y = \sqrt{mn} \qquad \text{... (ii)}$$
$\because 5x = 4y$
$$5\left(\frac{2mn}{m + n}\right) = 4\sqrt{mn}$$
Squaring both sides, we get
$$\left(\frac{5mn}{m + n}\right)^2 = (2\sqrt{mn})^2$$
$$\Rightarrow \quad \frac{25m^2n^2}{m^2 + n^2 + 2mn} = 4mn$$
$$\Rightarrow \quad 25mn = 4m^2 + 4n^2 + 8mn$$
$$[\because m \neq 0, n \neq 0]$$
$$\Rightarrow \quad 4m^2 + 4n^2 - 17mn = 0$$
$$\Rightarrow \quad 4m^2 - 16mn - mn + 4n^2 = 0$$
$$\Rightarrow \quad 4m(m - 4n) - n(m - 4n) = 0$$
$$\Rightarrow \quad (m - 4n)(4m - n) = 0$$
$$\Rightarrow \quad m = 4n \text{ or } n = 4m$$
Hence, option (d) is correct.

109. If the mean of a frequency distribution is 100 and the coefficient of variation is 45%, then what is the value of the variance?
(a) 2025
(b) 450
(c) 45
(d) 4.5

(a) Since, we know that
Coefficient of variation
$$(CV) = \frac{\sigma}{x} \times 100 \qquad \text{... (i)}$$
Where σ is standard deviation and $\bar{x}$ is mean.

Given, $\bar{x} = 100$ and $CV = 45\%$

From Eqs. (i), $45 = \dfrac{\sigma}{100} \times 100$

$\Rightarrow \qquad \sigma = 45$

$\therefore$ Variance $= \sigma^2 = (45)^2 = 2025$

Hence, option (a) is correct.

110. Let two events A and B be such that $P(A) = L$ and $P(B) = M$. Which one of the following is correct?

(a) $P(A|B) < \dfrac{L + M - 1}{M}$

(b) $P(A|B) > \dfrac{L + M - 1}{M}$

(c) $P(A|B) \geq \dfrac{L + M - 1}{M}$

(d) $P(A|B) = \dfrac{L + M - 1}{M}$

⊚ **(c)** Given, $P(A) = L$, $P(B) = M$

$\because P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \mid B) = \dfrac{P(A) + P(B) - P(A \cup B)}{P(B)}$

$P(A \mid B) = \dfrac{L + M - P(A \cup B)}{P(B)}$

$\therefore P(A \cup B) = L + M - P(B)P(A \mid B)$

$\because 0 \leq P(A \cup B) \leq 1$

$\Rightarrow L + M - P(B) \cdot P(A \mid B) \leq 1$

$\Rightarrow P(B) \cdot P(A \mid B) \geq L + M - 1$

$P(A \mid B) \geq \dfrac{L + M - 1}{M} \qquad [\because P(B) = M]$

Hence, option (c) is correct.

111. For which of the following sets of numbers do the mean, median and mode have the same value?

(a) 12, 12, 12, 12, 24

(b) 6, 18, 18, 18, 30

(c) 6, 6, 12, 30, 36

(d) 6, 6, 6, 12, 30

⊚ **(b)** For option (a), Mean

$= \dfrac{12 + 12 + 12 + 12 + 24}{5}$

$= 14 \cdot 4 \neq$ mode (12)

For option (b), Mean

$= \dfrac{6 + 18 + 18 + 18 + 30}{5} = 18$

Mode $= 18$, Median $= 18$

Hence, for the data 6, 18, 18, 18, 30,

Mean = Mode = Median = 18

Hence, option (b) is correct.

112. The mean of 12 observations is 75. If two observations are discarded, then the mean of the remaining

observations is 65. What is the mean of the discarded observations?

(a) 250

(b) 125

(c) 120

(d) Cannot be determined due to insufficient data

⊚ **(b)** Given, mean of 12 observations = 75 and M.

$\therefore \qquad \bar{x} = \dfrac{\sum\limits_{i=1}^{12} x_i}{12} \Rightarrow 75 = \dfrac{\sum\limits_{i=1}^{12} x_i}{12}$

$\Rightarrow \qquad \sum\limits_{i=1}^{12} x_i = 900 \qquad \ldots \text{(i)}$

Let observations x_{11} and x_{12} is discarded

then mean $= \dfrac{\sum\limits_{i=1}^{10} x_i}{10} = 65$

$\therefore \qquad \sum\limits_{i=1}^{10} x_i = 10 \times 65 = 650 \qquad \ldots \text{(ii)}$

From Eqs. (i)

$\sum\limits_{i=1}^{12} x_i = 900$

$\Rightarrow \sum\limits_{i=1}^{10} x_i + x_{11} + x_{12} = 900$

$\Rightarrow 650 + x_{11} + x_{12} = 900$

$\Rightarrow \qquad x_{11} + x_{12} = 250$

$\therefore$ Mean of x_{11} and $x_{12} = \dfrac{250}{2} = 125$

Hence, option (b) is correct.

113. If k is one of the roots of the equation $x(x + 1) + 1 = 0$, then what is its other root?

(a) 1   (b) $-k$

(c) k^2 (d) $-k^2$

⊚ **(c)** Given, quadratic equation

$x(x + 1) + 1 = 0$

$x^2 + x + 1 = 0 \qquad \ldots \text{(i)}$

Since, we know that ω, ω^2 are the roots of Equation when

$\omega = \dfrac{-1 + \sqrt{3}i}{2}$ and $\omega^2 = \dfrac{-1 - \sqrt{3}i}{2}$

$\Rightarrow$ If one of the roots of Eqs. (i) is k, then other root will be k^2.

Hence, option (c) is correct.

114. The geometric mean of a set of observations is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is

(a) 810   (b) 900

(c) 30000   (d) 81000

⊚ **(c)** Given that, geometric mean of a set of observations = 10

Since, we know that if

Geometric mean of $x_1, x_2, x_3, \ldots x_n$ is G.

$\Rightarrow$ Geometric mean of $x_1^2, x_2^2, x_3^2, \ldots x_n^2$ is G^2

Geometric mean of $fx_1^2, fx_2^2, \ldots, fx_n^2$ is fG^2.

$\therefore$ Required geometric mean $= 3(10)^4$

$= 3 \times 10000 = 30000$

Hence, option (c) is correct.

115. If $P(A \cup B) = \dfrac{5}{6}$, $P(A \cap B) = \dfrac{1}{3}$ and $P(\overline{A}) = \dfrac{1}{2}$, then which of the following is/are correct?

1. A and B are independent events.
2. A and B are mutually exclusive events.

Select the correct answer using the code given below.

(a) 1 only   (b) 2 only

(c) Both 1 and 2   (d) Neither 1 nor 2

⊚ **(a)** Given, $P(A \cup B) = \dfrac{5}{6}$, $P(A \cap B) = \dfrac{1}{3}$

$P(\overline{A}) = \dfrac{1}{2}$

$\Rightarrow \qquad P(A) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

$\therefore P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= \dfrac{5}{6} - \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{4}{6} = \dfrac{2}{3}$

$\because$ If A and B are independents, then

$P(A \cap B) = P(A) \cdot P(B)$

$\because P(A \cap B) = \dfrac{1}{3}$

and $P(A) \cdot P(B) = \dfrac{1}{2} \times \dfrac{2}{3} = \dfrac{1}{3}$

$\Rightarrow$ Statement-1 is correct.

If A and B are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$

$\because \qquad P(A \cup B) = \dfrac{5}{6}$

$\therefore \qquad P(A) + P(B) = \dfrac{1}{2} + \dfrac{2}{3} = \dfrac{7}{6}$

$\Rightarrow$ Statement-2 is wrong.

Hence, option (a) is correct.

116. The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is

(a) reduced by $\dfrac{1}{3}$ (b) increased by $\dfrac{10}{3}$

(c) reduced by $\dfrac{10}{3}$ (d) reduced by 50

⊚ **(c)** Let unit digit for wrongly recorded observation $= b$

When tens digit is 8, then number

$= 10 \times 8 + b = 80 + b$

When tens digit is 3, then number

$= 10 \times 3 + b = 30 + b$

$\Rightarrow$ One observation is recorded $((80 + b) - (30 + b))$ more while calculating average.

Hence, after correcting the observation, the average will be reduced by

$$= \frac{\{(80 + b) - (30 + b)\}}{15}$$

$$= \frac{50}{15} = \frac{10}{3}$$

Hence, option (c) is correct.

117. A coin is tossed twice. If E and F denote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

⊙ **(c)** Given that, a coin is tossed twice.

$\therefore$ $S = \{HH, HT, TH, TT\}$

Given, E be the event of occurrence of head on first toss and F be the event of occurrence of head on second toss.

$\therefore$ $E = \{HH, HT\}$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$F = \{TH, HH\}, P(F) = \frac{2}{4} = \frac{1}{2}$

$\therefore$ $E \cap F = \{HH\}$

$\therefore P(E \cap F) = \frac{1}{4}$

$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Hence, option (c) is correct.

118. In a binomial distribution, the mean is $\frac{2}{3}$ and variance is $\frac{5}{9}$. What is the probability that random variable $X = 2$?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$

(c) $\frac{25}{54}$ (d) $\frac{25}{216}$

⊙ **(d)** For a binomial distribution mean $= np$ and variance $= npq$

Where p is probability of success and q is the probability of unsuccess and n is number of observations.

$\therefore$ Given, $np = \frac{2}{3}$, $npq = \frac{5}{9}$

$\Rightarrow$ $\frac{2}{3} \cdot q = \frac{5}{9}$

$q = \frac{5}{6}$

$\therefore$ $p = 1 - q = 1 - \frac{5}{6}$

$= \frac{1}{6}$

$\therefore$ $np = \frac{2}{3}$

$\Rightarrow$ $n\left(\frac{1}{6}\right) = \frac{2}{3}$

$\Rightarrow$ $n = 4$

$\because p(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$

$\therefore P(X = 2) = {}^{4}C_{2} p^{2} q^{4-2}$

$= 6 \times \left(\frac{1}{6}\right)^{2} \times \left(\frac{5}{6}\right)^{2} = \frac{25}{216}$

Hence, option (d) is correct.

119. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x?

(a) 10   (b) 12
(c) 13   (d) 15

⊙ **(d)** Given, observations are

10, 12, 13, 15, 15, 13, 12, 10, x

$\because$ Mode $= 15$

Scores	Frequency
10	2
12	2
13	2
15	2

Since, frequency of all other numbers is same as frequency of 15.

But mode is the number of highest frequency.

$\therefore$ x should be 15

Hence, option (d) is correct.

120. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then consider the following statements

1. The minimum value of $P(A \cup B)$ is $\frac{3}{4}$.

2. The maximum value of $P(A \cap B)$ is $\frac{5}{8}$.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ **(c)** Given, $P(A) = \frac{3}{4}$

$P(B) = \frac{5}{8}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Here $P(A \cup B)$ will be minimum if $P(A \cap B)$ is maximum and *vice-versa*.

Since, minimum value of $P(A \cap B)$ is zero and maximum value of $P(A \cap B)$ is minimum $(P(A), P(B))$.

$\Rightarrow$ maximum$P(A \cap B)$

$= \text{minimum} \left(\frac{3}{4}, \frac{5}{8}\right)$

$= \frac{5}{8}$

$\Rightarrow$ Statement-2 is correct.

Also, minimum value of $P(A \cup B)$ is maximum $(P(A), P(B))$

$\therefore$ Minimum value of

$P(A \cup B) = \text{maximum} \left(\frac{3}{4}, \frac{5}{8}\right)$

$= \frac{3}{4}$

$\Rightarrow$ Statement-1 is correct.

Hence, correct option is (c).

PAPER II English Language and General Studies

Part A (English Language)

1. Her smile was <u>contagious</u>.
   (a) arrogant (b) disrespectful
   (c) sarcastic (d) catching

   ⊘ *(d)* 'Contagious' means infectious. Hence, 'catching' is its correct synonym.

2. Her <u>dynamic</u> nature impressed everyone.
   (a) enduring (b) attentive
   (c) evolutionary (d) jealous

   ⊘ *(c)* 'Dynamic' refers to process or system characterised by constant change, activity or progress. Hence, 'evolutionary' is nearest in meaning to dynamic.

3. She was <u>lamenting</u> her destiny.
   (a) celebrating (b) bemoaning
   (c) blaming (d) making

   ⊘ *(b)* 'Lamenting' means to express regret or disappointment about something. Hence, 'bemoaning' is its correct synonym.

4. Under his leadership the company grew in an <u>organic</u> manner.
   (a) natural (b) speedy
   (c) unusual (d) disciplined

   ⊘ *(a)* 'Organic' means characterised by gradual or natural development. Hence, 'natural' is its correct synonym.

5. His charm lies in his <u>oratory</u>.
   (a) deceit
   (b) eloquence
   (c) looks
   (d) nobility

   ⊘ *(b)* 'Oratory' means the art or practice of formal speaking in public. Hence, 'eloquence' is its correct synonym.

6. She is a <u>patron</u> of art and culture.
   (a) entrepreneur
   (b) admirer
   (c) critique
   (d) backer

   ⊘ *(d)* 'Patron' refers to a person who gives financial or other support to a person, organisation or cause. Hence, 'backer' is its correct synonym.

7. <u>Arrogance</u> is a hallmark of his nature.
   (a) concern
   (b) unpretentiousness
   (c) conceit
   (d) simplicity

   ⊘ *(c)* 'Arrogance' describes to someone who has an exaggerated sense of their own importance. Hence, 'conceit' meaning excessive pride in oneself is its correct synonym.

8. She undertook a <u>reconnaissance</u> of the entire issue.
   (a) revaluation (b) ratification
   (c) investigation (d) regularisation

   ⊘ *(c)* 'Reconnaissance' refers to military observation of a region to locate an enemy or ascertain strategic features. Hence, 'investigation' is its correct synonym.

9. My daughter is my <u>replica</u>.
   (a) pride (b) clone
   (c) love (d) original

   ⊘ *(b)* 'Replica' means an exact copy or model of something. Hence, 'clone' is its correct synonym.

10. The <u>sardonic</u> nature of her stories made her stand out among the contemporary writers.
   (a) compassionate (b) insightful
   (c) mocking (d) comic

   ⊘ *(c)* 'Sardonic' means grimly mocking or cynical.

11. He found her extremely <u>attractive</u> and charming.
   (a) unnatural
   (b) modern
   (c) repulsive
   (d) disapproving

   ⊘ *(c)* 'Attractive' means pleasing or agreeable. Hence, 'repulsive' meaning arousing intense distaste is its correct antonym.

12. The sky is <u>boundless</u>.
   (a) high (b) vast
   (c) expansive (d) finite

   ⊘ *(d)* 'Boundless' means unlimited or immense. Hence, 'finite' meaning limited in extent or size is its correct antonym.

13. The sky is <u>clear</u> today.
   (a) bright (b) opaque
   (c) cloudless (d) blue

   ⊘ *(b)* The antonym of clear is 'opaque', which means unclear or hard to be seen through.

14. I have a fascination for <u>deep</u> waters.
   (a) dark (b) light
   (c) dangerous (d) shallow

   ⊘ *(d)* The antonym of deep is 'shallow', which means of little or no depth.

15. My boss has been too <u>generous</u>.
   (a) stingy (b) rough
   (c) evil (d) hostile

   ⊘ *(a)* The antonym of generous is 'stingy', which means miserly.

16. Spring is a time of <u>plenty</u>.
   (a) ugliness (b) scarcity
   (c) roughness (d) dryness

   ⊘ *(b)* The antonym of plenty is 'scarcity', which means shortage or lack of something.

17. He is an <u>industrious</u> workman.
   (a) active (b) productive
   (c) lazy (d) disloyal

   ⊘ *(c)* 'Industrious' means hard-working and diligent. Hence, 'lazy' is its correct antonym.

18. Plants <u>grow</u> in abundance here.
   (a) shrivel (b) stretch
   (c) spread (d) enlarge

   ⊘ *(a)* The antonym of grow is 'shrivel', which means to wrinkle or shrink.

19. She <u>rarely</u> comes here.
   (a) seldom (b) never
   (c) always (d) frequently

   ⊘ *(d)* 'Rarely' means not often or seldom. Hence, 'frequently' meaning regularly or habitually is its correct antonym.

20. She is a rather <u>crooked</u> woman.
   (a) polite
   (b) generous
   (c) straightforward
   (d) happy

(c) 'Crooked' means dishonest or insincere. Hence, 'straightforward' meaning honest and frank is its correct antonym.

Directions (Q. Nos. 21-30) *Given below are some idioms/phrases followed by four alternative meanings to each. Choose the response (a), (b), (c) or (d), whichever is the most appropriate meaning and mark your answer.*

21. Overstep the mark
  (a) To tell people how successful you are
  (b) To step into someone else's areas of expertise
  (c) To upset someone by doing/saying more than you should
  (d) To do something in an excited way

⊘ **(c)** The idiom 'overstep the mark' means to upset someone by going beyond what was allowed. Hence, option (c) is the correct answer.

22. Palsy-walsy friends
  (a) Good friends
  (b) Friends who help each other in difficult situations
  (c) Friends by choice and not by chance
  (d) Unfriendly

⊘ **(a)** The phrase 'palsy-walsy friends' means good friends.

23. Open a Pandora's box
  (a) To do something that causes a lot of new problems that you did not expect
  (b) To do something out of compulsion
  (c) To do something beyond expectation
  (d) To do something out of the box, that brings award and ceremonies for you

⊘ **(a)** The idiom 'open a Pandora box' means to begin or introduce something that leads to many other problems. Hence, option (a) is the correct answer.

24. Pull your socks up
  (a) To get well-dressed for the occasion
  (b) Improve your work or behaviour
  (c) To speak in an honest way without hesitation
  (d) To be in control of an organisation, often secretly

⊘ **(b)** The idiom 'pull your socks up' means to improve your behaviour or work to achieve something.

25. To get under somebody's skin
  (a) To deceive someone
  (b) To admire someone
  (c) To annoy someone
  (d) To support someone

⊘ **(c)** The idiom 'to get under somebody's skin' means to irritate or upset someone.

26. Turn topsy-turvy
  (a) To completely change something
  (b) To completely evaluate something
  (c) To enjoy yourself greatly
  (d) To exhaust yourself completely

⊘ **(a)** The phrase 'turn topsy-turvy' means to be in utter confusion or disorder. Hence, 'to completely change something' best expresses the meaning of the phrase.

27. A clarion call
  (a) A trumpet call
  (b) An intimidating voice
  (c) A strong request
  (d) An urgent order

⊘ **(c)** The idiom 'a clarion call' means a strong request for something to happen.

28. Fire in the belly
  (a) Fear and hatred
  (b) Powerful ambition
  (c) Love and dedication
  (d) Lethargy and indifference

⊘ **(b)** The idiom 'fire in the belly' means a strong determination to succeed. Hence, option (b) is the correct answer.

29. A hunky-dory situation
  (a) There are serious issues among people
  (b) There are no problems and people are happy
  (c) There is war and bloodshed all over
  (d) There is no work, only enjoyment

⊘ **(b)** The idiom 'a hunky-dory situation' means a very satisfactory and problem free situation. Hence, option (b) is the correct answer.

30. Give somebody a leg up
  (a) To pull someone down
  (b) To deceive and betray someone
  (c) To help someone for their livelihood
  (d) To help someone to be successful

⊘ **(d)** The idiom 'give somebody a leg up' means to help someone/achieve something and become successful.

Directions (Q. Nos. 31-40) *Each question in this section has a sentence with three underlined parts labelled (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part. If you find no error, your response should be indicated as (d).*

31. You don't have
        (a)
a monopoly on suffering;
        (b)
other people don't have problems too.
        (c)
No error
  (d)

⊘ **(c)** The use of 'too' in the sentence shows that a further point with a similar meaning is being added. Hence, remove 'don't' to make the sentence grammatically correct.

32. If you say that someone
        (a)
you admire has feet of clay,
        (b)
you mean that they have hidden faults.
        (c)
No error
  (d)

⊘ **(d)** The given sentence is grammatically correct.

33. He refused to change
        (a)
his decision;
        (b)
he refused it point out.
        (c)
No error
  (d)

⊘ **(c)** 'Point out' is used incorrectly in the sentence. It should be replaced by 'point blank', which means in a blunt and direct manner.

34. The importance of trade in Mughal times reinforced
              (a)
the cultural definition of wealth as something
              (b)
comprising of movable property.
              (c)
No error
  (d)

⊘ **(c)** The use of 'of' after 'comprising' is invalid, as 'comprise' means consist of. Hence, remove 'of' to make the sentence grammatically correct.

35. In the nineteenth century,
        (a)
most traditional scholars
        (b)
tried to stay clear from the imperial Government.
              (c)
No error
  (d)

(c) The preposition 'from' is used incorrectly. It should be replaced by 'of' to make the sentence correct. To 'stay clear of' means to avoid someone or something.

36. <u>He began his discussion by pointing</u>
<u>over</u>
                    (a)
<u>that men and women</u>
           (b)
<u>had different biological functions.</u>
                      (c)
<u>No error</u>
   (d)

⊘ **(a)** 'Pointing over' is used incorrectly in the sentence. It should be replaced by 'pointing out', which means to say something to make someone aware of a situation.

37. <u>Though he is poor,</u> <u>but he is</u>
           (a)   (b)
<u>honest.</u> <u>No error</u>
   (c)   (d)

⊘ **(b)** 'But' should not be used with 'though'. Hence, remove 'but' to make the sentence grammatically correct.

38. <u>My mother</u> <u>has been doing</u>
       (a)   (b)
<u>everything for the family.</u>
                  (c)
<u>No error</u>
   (d)

⊘ **(b)** Replace 'has been doing' with 'does' to make the sentence error free and grammatically correct.

39. <u>When learning to dive</u> <u>it is important</u>
           (a)   (b)
<u>to relax in between and take breaks.</u>
                        (c)
<u>No error</u>
   (d)

(a) Replace 'when' with 'while' to make the sentence error free and grammatically correct.

40. <u>I have the opportunity</u> <u>to study</u>
           (a)   (b)
<u>in American next year.</u> <u>No error</u>
           (c)   (d)

⊘ **(d)** The given sentence is grammatically correct.

Directions (Q. Nos. 41-50) *Each of the following sentences in this section has a blank space and four words or group of words given after the sentence. Select the word or group of words you consider most appropriate for the blank space and mark your answer.*

41. There was a time when West Germany was a distinct ....
(a) policy   (b) polity
(c) abstract   (d) hierarchy

⊘ **(b)** 'Polity' means an organised society; a state as a political entity. Hence, it is appropriate to fill the blank.

42. I was .... with the film; I had expected it to be better.
(a) disappointed   (b) disappointing
(c) annoying   (d) prejudiced

⊘ **(a)** Disappointed

43. It was a .... experience. Everybody was shocked.
(a) terrified   (b) horrified
(c) terrifying   (d) denouncing

⊘ **(c)** Terrifying

44. Elephants .... when they perceive danger.
(a) trumpet   (b) frolic
(c) whine   (d) sing

⊘ **(a)** The sound that elephants make is called 'trumpet'. Hence, it is appropriate to fill the blank.

45. The first film on Tagore was such a success that now they are going to make a ....
(a) serial
(b) sequence
(c) sequel
(d) sequential

⊘ **(c)** Sequel

46. The United Nations had .... 2020 as the International Year of Plant Health.
(a) ruled
(b) ordered
(c) foretold
(d) declared

⊘ **(d)** Declared

47. My brother is .... punctual, but he is late today.
(a) normatively   (b) primarily
(c) normally   (d) basically

⊘ **(c)** Normally

48. My son is very ....; he trusts everyone.
(a) fallible   (b) gullible
(c) sensible   (d) credible

⊘ **(b)** 'Gullible' means easily persuaded to believe something. Hence, it is appropriate to fill the blank.

49. Mahatma Gandhi was a lover of humanity and a .... believer in the goodness of human nature.
(a) staunch   (b) powerful
(c) cheerful   (d) hopeful

⊘ **(a)** Staunch

50. I wish I .... her before we met.
(a) know
(b) have known
(c) knew
(d) known

⊘ **(c)** Knew

Part B (General Studies)

51. Consider the following statements about light year.
1. Light year is a unit for measurement of very large distances.
2. Light year is a unit for measurement of very large time intervals.
3. Light year is a unit for measurement of intensity of light.
Which of the statements given above is/are correct?

(a) 1, 2 and 3   (b) 2 and 3
(c) 1 and 2   (d) Only 1

⊘ **(d)** Light year is a unit for measurement of very large distances. It is used to express the distances of celestial bodies.

52. Which one of the following regarding density of water at atmospheric pressure is correct?
(a) Density of water at 4°C is 1000 kg/m^3.
(b) Density of water at 0°C is 1000 kg/m^3.
(c) Density of water at 0°C is 100 kg/m^3.
(d) Density of water at 4°C is 10 kg/m^3.

⊘ **(a)** Density of water at 4°C is 1000kg/m^3 or 10^3 kg/m^3. It is the maximum density of water. The ratio of mass and volume of substance is known as density.

53. Which of the following pairs of physical phenomena and the discoverer is/are correctly matched?

1. James Chadwick	:	Photoelectric effect
2. Albert Einstein	:	Neutron
3. Marie Curie	:	Radium

Select the correct answer using the codes given below.
(a) 1, 2 and 3
(b) 1 and 2
(c) 2 and 3
(d) Only 3

> **(d) Photoelectric effect by Albert Einstein** According to Einstein, light is made up of little packets, at first called quanta and later photons. When a photon hits the electrons, it gives electrons enough energy to escape from the surface of the metal. This explains the behaviour of light striking the metal.
> - **Neutron by James Chadwick** The neutron is a subatomic particle, symbol n, which has a neutral charge, and a mass slightly greater than that of a proton.
> - **Radium by Marie Curie** Radium is a chemical element with the symbol Ra and atomic number 88, also known as the alkaline earth metals. Pure radium is silvery-white, but it readily reacts with nitrogen on exposure to air, forming a black surface layer of radium nitride.

54. LED (a semiconductor device) is an abbreviation that stands for
(a) Licence for Energy Detector
(b) Light Energy Device
(c) Light Emitting Diode
(d) Lost Energy Detector

> **(c)** LED is an abbreviation that stands for Light Emitting Diode. It is a semiconductor light source that emits light when current flows through it.

55. The statement "friction force is a contact force while magnetic force is a non-contact force" is
(a) always true
(b) true only at 0°C
(c) a false statement
(d) Either true or false depending upon the temperature of the surroundings

> **(a)** Friction force is a contact force because it works between the contact surface of two bodies and magnetic force is a non-contact force.

56. Which one of the following is the chemical formula of hypobromous acid?
(a) $HBrO_4$
(b) $HOBr$
(c) HBr
(d) $HBrO_3$

> **(b)** Hypobromous acid is a weak, unstable acid with chemical formula HOBr, where the bromine atom is in the +1 oxidation state. It is also called 'bromanol' or 'hydroxidobromine'.

57. The composition of gases in exosphere is
(a) helium and hydrogen
(b) neon and oxygen
(c) neon and hydrogen
(d) helium and neon

> **(a)** The exosphere is mainly composed of extremely low densities of hydrogen and helium; and the heavy molecules nitrogen, oxygen and carbon dioxide.

58. Which one of the following is not used as a raw material in the manufacture of glass?
(a) Soda
(b) Alumina
(c) Borax
(d) Gypsum

> **(d)** The correct answer is gypsum. Glass is made from natural and abundant raw materials (soda, alumina and borax, etc.) that are melted at very high temperature to form a new material glass.

59. In electrolytic refining of copper, the electrolyte is a solution of
(a) acidified copper chloride
(b) acidified copper sulphate
(c) potassium chloride
(d) sodium sulphate

> **(b)** In the electrolytic refining of copper (electrolyte is $CuSO_4$ solution + dil. H_2SO_4) or acidified copper sulphate. Pure Cu cathode and impure Cu anode. Metallic impurities having lower oxidation potential than Cu are deposited as sludge.

60. Solder is an alloy of
(a) Cu and Sn
(b) Fe and Zn
(c) Pb and Sn
(d) Ag and Zn

> **(c)** Solder is a mixture of lead (Pb) and tin (Sn). Tin-lead solder is the most popular in solder paste printing and it is mainly composed of tin and lead, with same trace metal element like antimony (Sb).

61. Which one of the following statements about dihydrogen (H_2) is not correct?
(a) H_2 is lighter than air and insoluble in water.
(b) H_2 is inert at room temperature due to high H—H bond dissociation enthalpy.
(c) H_2 reacts with alkali metals at high temperature to yield metal hydrides.
(d) A mixture of NO_2 and H_2 is known as Syngas.

> **(d)** Statement (d) is incorrect because syngas or synthesis gas is a fuel gas mixture consisting primary of hydrogen, carbon monoxides.

62. Which of the following sets of elements has the same valency?
(a) Na, Mg, Ca
(b) Na, Mg, Al
(c) Mg, Ca, K
(d) Mg, Ca, Ba

> **(d)** Mg, Ca, Ba sets of elements has the same valency. The valency of an element is determined by the number of valence electron in the outermost shell of an atom of an element.

63. Which one of the following is the lowest possible temperature?
(a) 0° celsius
(b) −73° celsius
(c) −173° celsius
(d) −273° celsius

> **(d)** The lowest possible temperature is the absolute zero. At this temperature, no more heat can be remove from a body. Absolute zero is precisely equivalent to −273.15 degrees celsius on celsius scale.

64. Numerically two thermometers, one in Fahrenheit scale and another in Celsius scale shall read same at
(a) − 40°
(b) 0°
(c) −273°
(d) 100°

> **(a)** The relation between the Fahrenheit and Celsius scale is given by the equation. Therefore, Celsius and Fahrenheit scale coincide at − 40°.
>
> $$\frac{(°F - 32)}{180} = \frac{°C}{100}$$
>
> Now, let us assume that Fahrenheit and Celsius scale coincide at $x°$, therefore
>
> $$\Rightarrow \quad \frac{(°x - 32)}{180} = \frac{°x}{100}$$
>
> On solving, we get
>
> $$\Rightarrow \quad \frac{5}{9}(°x - 32) = °x$$
>
> Therefore, $x = −40°$

65. The image we see in plane mirror is
(a) real and thus can be photographed
(b) virtual and nearer than the object
(c) virtual and is laterally inverted
(d) The real but cannot be photographed

> **(c)** The image in a plane mirror is virtual, erect same size and laterally inverted. For a plane mirror, the size of the image is equal to the size of the object.

66. Which one of the following colours may be obtained by combining green and red colours?
(a) Blue
(b) Magenta
(c) Pink
(d) Yellow

> **(d)** Yellow colour is obtained by combining green and red colours. It is a secondary colour.

67. Which of the following are the primary colours of light?
(a) Yellow, red and green
(b) Blue, red and green
(c) Violet, red and yellow
(d) Indigo, violet and green

⊙ **(b)** Primary colours of light are blue, red and green. This is the essential method used to create the precepting of a board range of colours.

68. According to the new cartesian sign convention, which one of the following is correct in respect of the formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, where symbols have their usual meanings?
(a) It applies only to spherical mirrors.
(b) It applies only to spherical lenses.
(c) It applies to spherical mirrors as well as spherical lenses.
(d) It is an invalid formula.

⊙ **(a)** For a spherical mirror, the formula showing the relationship between the distance of the object(u), the distance of the mirror (v) and the distance of the focus (f) is called the mirror formula, which is as follows
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

69. Movement of materials to different parts of cytoplasm and nucleus is generally carried out by
(a) Ribosomes
(b) Mitochondria
(c) Lysosomes
(d) Endoplasmic reticulum

⊙ **(d)** The movement of materials to different parts of the cytoplasm and nucleus is generally carried out Endoplasmic Reticulum (ER). although Golgi bodies have main role in the function of intracellular transporation. ER is the largest membrane-bound organelle in eukaryotic cells and performs a variety of essential cellular functions, including protein and lipid synthesis and biosynthesis of cell and nuclear membrane.

70. In mitochondria, ATP synthesising chemical reactions take place in the
(a) outer membrane
(b) matrix
(c) inner membrane
(d) DNA of mitochondria

⊙ **(c)** In eukaryotes, Adenosine Triphosphate (ATP) synthesis reaction occurs on the inner membrane of mitochondria which sides toward matrix compartment. It contains the ATP synthesis complex or oxisome or

F_1-particles which are the site of Electron Transportation System (ETS) of aerobic cellular espiration.

71. Squamous epithelial cells are found in the inner lining of
(a) oesophagus
(b) small intestine
(c) ducts of salivary gland
(d) kidney

⊙ **(a)** Squamous epithelial cells are found in the inner lining of the oesophagus. Simple squamous epithelium is a single layer of flat scale-shaped cells, which are helpful in transportation of food from buccal cavity to stomach.

72. Transformation of meristematic cells into specific permanent tissues occurs by the process of
(a) cell differentiation
(b) cell division
(c) cell multiplication
(d) cell regeneration

⊙ **(a)** Transformation of meristematic cells into specific permanent tissue occurs by the process of cell differentiation. In this process, cells formed by meristematic tissues take up a specific role and lose the ability to divide. As a result, they form a permanent tissue taking up a permanent shape, size and function.

73. The gaseous product of a process in plants is a requirement for another vital process that releases energy. Given below are four combinations of the process and product. Identify the correct answer.
(a) Respiration and nitric oxide
(b) Transpiration and water vapour
(c) Photosynthesis and oxygen
(d) Germination and carbon dioxide

⊙ **(c)** In plants, oxygen gas is produced as by-product of photosynthesis which is required by aerobic cellular respiration to produce energy (ATP). Photosynthesis occurs in chloroplast while aerobic cellular respiration occurs in mitochondria.

74. In a dicot pot herb, vaseline/vegetable oil was applied on the upper surface of one leaf (experimental leaf 1) and on the lower surface of another leaf (experimental leaf 2). Vaseline/Vegetable oil was not applied on the control leaf. The plant was deliberately not watered for several days. Which leaf will dry up last?

(a) Experimental leaf 1
(b) Experimental leaf 2
(c) Control leaf
(d) All the leaves will dry up simultaneously

⊙ **(b)** Experimental leaf 2 will dry up last, because in dicot plants, stomata are generally found in lower surface of leaves and applying vaseline/vegetable oil will close these stomata, thus preventing the water loss through transpiration (evaporation of water in the form of vapour form leaf surface) and drying process will take a lot more time compare to other leaves.

75. Which one of the following statements is not correct for light rays ?
(a) Light travels at different speeds in different media.
(b) Light travel at almost 300 million metres per second in air.
(c) Light speeds down as it leaves a water surface and enters the air.
(d) Light speeds up as it leaves a glass surface and enters the air.

⊙ The speed of light in air is 3×10^8 m/s and the speed of light is different in different media.

76. A glass prism splits white light into different colours. This phenomenon is called dispersion of light by prism. Which one of the following statements is correct?
(a) Red light will deviate the most and it is because of the reflection of light.
(b) Violet light will deviate the most and it is because of the refraction of light.
(c) Red light will deviate the most and it is because of the refraction of light.
(d) Violet light will deviate the most and it is because of the reflection of light.

⊙ VIBGYOR (increasing order of wavelength). (Frequency decreases in increasing order).
Refractive index is inversely proportional to wavelength. So, violet light will deviate the most.

77. A current of 1.0 A is drawn by a filament of an electric bulb for 10 minutes. The amount of electric charge that flows through the circuit is
(a) 0.1 C   (b) 10 C
(c) 600 C   (d) 800 C

⊙ **(c)** Given, $I = 1$A,
$t = 10$ min $= 10 \times 60$ s $= 600$ s
$\therefore \quad I = \frac{Q}{t}$
$\Rightarrow Q = It \quad \Rightarrow Q = 1 \times 600$
$\Rightarrow Q = 600$ C

78. Which one of the following formulae does not represent electrical power?
(a) I^2R
(b) IR^2
(c) VI
(d) V^2/R

⊙ **(b)** Electric power is the rate per unit time at which electrical energy is transferred by an electric circuit,

$$P = I^2R = \frac{V^2}{R} = VI$$

Its SI unit is watt.

79. The sound created in a big hall persists because of the repeated reflections. The phenomenon is called
(a) reverberation
(b) dispersion
(c) refraction
(d) diffraction

⊙ **(a)** The sound created in a big hall persists because of the repeated reflection. The phenomenon is called reverberation.

80. When light is scattered by a molecule and the frequency of the scattered light is changed, this phenomenon is called
(a) Rayleigh scattering
(b) Raman effect
(c) Photoelectric effect
(d) Rutherford scattering

⊙ **(b)** When light is scattered by a molecule and the frequency of the scattered light is changed, this phenomenon is called Raman effect.

81. Which one of the following statements about the cleansing action of soap is not true?
(a) The oil and dirt gets collected in the centre of the micelle.
(b) Soap micelle scatter light.
(c) Soaps are ammonium salts of long chain carboxylic acids.
(d) Soap forms insoluble precipitates with the calcium and magnesium ions in hard water.

⊙ **(c)** Detergents are ammonium or sulphonate salts of long chain carboxylic acids. Hence, option (c) is not true. Soap is a mixture of sodium salts of various naturally occurring fathy acids.

82. Hydrogenation of vegetable oils using nickel catalyst is an example of
(a) substitution reaction
(b) elimination reaction
(c) addition reaction
(d) free-radical polymerisation

⊙ **(c)** Hydrogenation of vegetable oils using nickel catalyst is an example of addition reaction. Hydrogenation is a process that uses hydrogen gas to change a liquid vegetable oil into a hard spreat. This process stabilises the oil and prevents spoilage from oxidation.

83. Which one of the following materials is not an allotrope of carbon?
(a) Diamond
(b) Graphene
(c) Fly ash
(d) Fullerene

⊙ **(c)** Diamond, graphene and fullerene are three allotropes of pure carbon. Fly ash is not a allotrope of carbon.
Hence, option (c) is correct.

84. Which one of the following reactions does not result in the evolution of hydrogen gas?
(a) Reaction of zinc metal with dilute sulphuric acid solution
(b) Mixing water to Plaster of Paris
(c) Heating zinc metal with sodium hydroxide solution
(d) Reaction of potassium metal with water

⊙ **(b)** When plaster of Paris is mixed with water, it gets converted into gypsum.
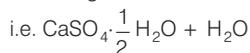
i.e. $CaSO_4 \cdot \frac{1}{2} H_2O + H_2O$
$$\longrightarrow CaSO_4 \cdot 2H_2O,$$
Hence, it doesn't give hydrogen gas.

85. Which one of the following acids is predominantly found in tomatoes?
(a) Acetic acid
(b) Tartaric acid
(c) Oxalic acid
(d) Lactic acid

⊙ **(c)** Oxalic acid a chemical compound that occur naturally in almost every plant to some degree including fruits, vegetables and grain plants. The amount of oxalic in tomatoes is about 50 mg per 100 g.

86. Which one of the following conclusions could not be derived from Rutherford's α-particle scattering experiment?
(a) Most of the space in the atom is empty.
(b) The radius of the atom is about 10^5 times the radius of the nucleus.
(c) Electrons move in a circular path of fixed energy called orbits.
(d) Nearly all the mass of the atom resides in the nucleus.

⊙ **(c)** The concept of electrons move in a circular path of fixed energy called orbits was put forward by Bohr and not derived from Rutherford's scattering experiment. Therefore, option (c) is incorrect.

87. Reaction of quick lime (CaO) with water to produce slaked lime ($Ca(OH)_2$) is an example of
(a) displacement reaction
(b) endothermic reaction
(c) decomposition reaction
(d) exothermic reaction

⊙ **(d)** The formation of slaked lime (calcium hydroxide) $Ca(OH)_2$ when water is added to lime (CaO) is exothermic.
$$CaO(s) + H_2O\,(l) \longrightarrow Ca(OH)_2\,(s)$$
[Evolution of heat]
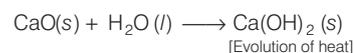
88. Which one of the following is not a bio-mass energy source?
(a) Wood
(b) Nuclear reactor
(c) Gobar gas
(d) Coal

⊙ **(b)** Nuclear reactors are sources of nuclear energy, not bio-mass energy. Wood, gobar gas and coal are the organic or bio-mass sources of energy.

89. Twinkling of stars is due to
(a) particular frequencies of the starlight.
(b) reflection of starlight from the oceanic surface.
(c) atmospheric refraction of starlight.
(d) magnetic field of earth.

⊙ **(c)** The twinkling of a star is due to atmospheric refraction of starlight. The starlight on entering the earth's atmosphere, undergoes refraction continuously before it reaches the earth. The atmospheric refraction occurs in the medium of gradually changing refractive index.

90. Which one of the following cannot be the unit of frequency of a sound wave?
(a) dB
(b) s^{-1}
(c) Hz
(d) min^{-1}

⊙ **(a)** We know that,
$$Frequency = \frac{1}{Time\ period}$$
$$= \frac{1}{Second\ or\ minute}$$
Apart from this, the SI unit of frequency is hertz (Hz).

91. 'Beats' is a phenomenon that occurs when frequencies of two harmonic waves are
(a) equal
(b) far apart
(c) multiples of each other
(d) nearly same

⊙ **(d)** When two sound waves of approximately equal frequency are produced simultaneously, the intensity of the resultant sound wave increases or decreases with time.
This change in the intensity of sound is called beats.

92. Light waves are incident on an air-glass boundary. Some of the light waves are reflected and some are refracted in the glass. Which one of the following properties is the same for the incident wave and the refracted wave?
(a) Speed
(b) Direction
(c) Brightness
(d) Frequency

⊙ *(d)* Frequency of refracted and reflected wave never changes because the frequency does not depends on the medium.

93. Which one of the following statements is true for a simple harmonic oscillator?
(a) Force acting is directly proportional to the displacement from the mean position and is in same direction.
(b) Force acting is directly proportional to the displacement from the mean position and is in opposite direction.
(c) Acceleration of the oscillator is constant.
(d) The velocity of the oscillator is not periodic.

⊙ $F = -k\,x$ (SHM equation)

where, x = displacement from mean position and F = acting force.

94. During seed germination, the part of the embryo which grows into root is
(a) radicle   (b) plumule
(c) cotyledon   (d) epicotyl

⊙ *(a)* During seed germination, the part of the embryo which grows into root is called radicle. It is the first part of a seedling (a growing plant embryo) to emerge from the seed during the process of germination to form embryonic root of the plant which grows downward in the soil.

95. In a typical flower, germinating pollen grains pass through several parts of the gynoecium before they reach the ovule. A list of the parts of gynoecium is given below in different combinations. Choose the combination that represents the correct sequence of pollen tube pathway/journey.
(a) Style, Stigma, Ovary
(b) Stigma, Style, Ovary
(c) Pistil, Stigma, Ovary
(d) Ovary, Pistil, Style

⊙ *(b)* Pollen grains gather on the surface of stigma of gynoecium through pollination and get germinated to

produce pollen tube. This pollen tube elongated in style and reaches ovary to release male gametes in embryo sac of ovule.

96. If human blood is placed in a 2% detergent solution, what will happen to the RBC?
(a) The RBC will shrink.
(b) The RBC will swell and become turgid.
(c) The RBC will swell and burst.
(d) The RBC will lyse.

⊙ *(a)* If human blood is placed in a 2% detergent solution, the RBC will shrink. Detergent solution is a hypertonic solution compared to red blood cells, hence RBCs are, if placed in it, the cell will shrink due to exosmosis, i.e. movement of water from its higher concentration (RBC) to lower concentration (Detergent solution) through cell membrane (SPM) of RBCs.

97. The major source of vitamins and minerals for vegetarians is
(a) black gram and wheat
(b) rice and mustard
(c) vegetables and fruits
(d) soya bean and milk

⊙ *(c)* Vegetables and fruits are major source of vitamins and minerals for vegetarians.

98. If a ray of light enters from a rarer medium to a denser medium at zero angle of incidence, it would
(a) reflect back
(b) go straight
(c) turn towards right
(d) bend at 45°

⊙ *(b)* When light falls normally on the interface of two medium, then light travels in straight line. So, the refraction angle is zero.

99. Mirage is an illustration of
(a) only dispersion of light
(b) only reflection of light
(c) only total internal reflection of light
(d) both refraction and total internal reflection of light

⊙ *(d)* A mirage is a naturally occurring optical phenomenon in which light rays bend *via* refraction to produce a displaced image of distant objects or the sky.

100. Common salt (NaCI) is not used as a raw material for perparation of which one of the following compounds ?
(a) Bleaching powder
(b) Baking soda
(c) Plaster of Paris
(d) Washing soda

⊙ *(c)* Preparation of plaster of Paris doesn't require the use of NaCl. It is simply obtained by heating gypsum ($CaSO_4 \cdot 2H_2O$) at 120°C in a rotary kiln.

101. Which one of the following Harappan sites was a specialied centre for making shell object ?
(a) Lothal   (b) Balakot
(c) Amri   (d) Kot Diji

⊙ *(b)* Among the given options, Harappan sites specialised centre for making shell object is Balakot and the other site is Nageshwar. Shell objects like bangles, ladles and inlay were made at these specialised centres.

102. Which one of the following was not a part of the dhamma of King Ashoka ?
(a) Honouring the king
(b) Tolerance of religions other than one's own
(c) Respecting Brahmanas
(d) Promoting the welfare of his subjects

⊙ *(a)* Honouring the king was not a part of the Dhamma of king Ashoka. Dhamma of king Ashoka established the idea of paternal kingship. He regarded all his subjects as his children and believed it the king's duty to look after the welfare of the subjects. Ashoka's philosophy state that everybody should serve parents, revere teachers, and practice ahimsa and truthfulness. He asked everyone to avoid animal slaughter and sacrifice. He advocated tolerance towards all religions and sought conquest through Dhamma and not war.

103. Which of the following statements about Saguna Bhakti traditions is/are correct?
1. Saguna Bhakti traditions focus on the worship of specific deities such as Vishnu or his avatars.
2. In Saguna Bhakti traditions, Gods and Goddesses are conceptualised in anthropomorphic forms.
Select the correct answer using the codes given below
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ *(c)* Both the statements about Saguna Bhakti are correct. Saguna Bhakti has form, attributes and quality. Saguna saints worshipped the anthropomorphic manifestations of the divine being, particularly Vishnu or his avatars. The

Bhakti saints emphasised on two ways of imaging the nature of the God viz. Nirguna and Saguna. Nirguna is the concept of a formless God, which has no attributes or quality. Saguna has form, attributes and quality.

104. At which one of the following places was a Shiva temple not constructed under the patronage of the Chola rulers?
(a) Chidambaram
(b) Thanjavur
(c) Gangaikonda Cholapuram
(d) Naneghat

⊙ *(d)* Shiva Temple was not constructed at Naneghat under the patronage of the Chola rulers. Naneghat, is a mountain pass in the Western Ghats range between the Konkan coast and the ancient town of Junnar in the Deccan plateau. Chola empire was not extended to this region therefore no temple was build in the region.

105. Which of the following statements about the Deccan Riots Commission is/are correct?
1. The Commission did not hold enquiries in the district which were not affected.
2. The Commission did record statements of ryots, sahukars eye- witnesses.
Select the correct answer using the codes given below.
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ *(c)* In 1875, peasants of Maharashtra in some parts of Pune and Ahmednagar districts revolted against increasing agrarian distress. The Deccan Riots of 1875 targeted conditions of debt peonage (kamiuti) to moneylenders. The rioters' specific purpose was to obtain and destroy the bonds, decrees, and other documents in the possession of the moneylenders. The movement also got support from the Poona Sarvajanik Sabha co-founded by M G Ranade. However, the Government of India pressurised Bombay to enquire into the matter for this the Deccan Riots Commission was set up which presented a report to the British Parliament in 1878. The commission did'nt hold inquiries in areas unaffected by it and also did'nt recorded the statements of royats, sahukars and eye-witnesses.

106. Which one of the following is the correct meaning of ziyarat ?
(a) Pilgrimage to the tombs of sufi saints.
(b) The practice of revenue farming
(c) The death anniversary of a sufi Shaikh
(d) A form of Islamic divorce

⊙ *(a)* In Islam, ziyara or ziyarat is a form of pilgrimage to sites associated with Muhammad, his family members and descendants, his companions and other venerated figures in Islam such as the prophets, Sufi auliya, and Islamic scholars.

107. Keppel Island is completely bleached mainly due to the expansion of
(a) Starfish
(b) Blue whale
(c) Octopus
(d) Sea horse

⊙ *(a)* Keppel Island is completely bleached mainly due to the expansion of starfish. The population of starfish is growing unconditionally due to which starfish are able to eat coral; a polyp that builds limestone reef on which the coral community lives. Keppel Island is located in the southern Great Barrier reef. Coral bleaching is the process when corals become white due to various stressors, such as changes in temperature, light or nutrients.

108. Which one of the following rivers is not a tributary of river Brahmaputra?
(a) River Manas
(b) River Kameng
(c) River Mahananda
(d) River Subansiri

⊙ (c) Among the given options, Mahananda river is not a tributary of river Brahmaputra. Tributaries of Brahmaputra: left-Lhasa River, Nyang River, Parlung Zangbo, Lohit River, Dhansiri River, Kolong River/ right-Kameng River, Manas River, Beki River, Raidak River, Jaldhaka River, Teesta River, Subansiri River.

The Mahananda River is a trans-boundary river that flows through the states of Bihar and West Bengal, and Bangladesh. It is an important tributary of the Ganges.

109. Which one of the following is not a minor plate?
(a) Cocos plate
(b) Nazca plate
(c) Caroline plate
(d) Antarctic plate

⊙ *(d)* Antarctic plate is not a minor plate. There are 15 minor plates, these smaller

plates are often not shown on major plate maps. For purposes of this list, a minor plate is any plate with an area less than 20 million km^2 but greater than 1 million km^2. They are Somali Plate, Nazca Plate , Indian Plate , Amurian Plate , Sunda Plate, Philippine Sea Plate, Okhotsk Plate, Arabian Plate, Yangtze Plate, Caribbean Plate , Cocos Plate , Caroline Plate ,Scotia Plate, Burma Plate, New Hebrides Plate.

110. Advantage(s) of tectonic activity in Iceland include(s)
1. Source of natural geothermal energy
2. Creation of new land
3. Attraction of tourists
Select the correct answer using the codes given below.
(a) Only 1
(b) 2 and 3
(c) 1 and 3
(d) 1, 2 and 3

⊙ *(c)* The tectonic plates whose turbulent interactions formed Iceland, are the Eurasian tectonic plate and the North American tectonic plate. Spanning the Mid-Atlantic Ridge, Iceland emerged as a result of the divergent, spreading, boundary between these two plates and the activity of Iceland´s own hotspot or mantle plume.

111. The process whereby certain minerals absorb water, expand and change is called as
(a) Hydration
(b) Oxidation
(c) Hydrolysis
(d) Carbonation

⊙ *(a)* The process whereby certain minerals absorb water, expand and change is called as Hydration. A good example of hydration is the absorption of water by anhydrite, resulting in the formation of gypsum. Hydration expands volume and also results in rock deformation.

112. Which one of the following is the longest Latitude?
(a) 90 degree Latitude
(b) 23-5 degree Latitude
(c) 0·0 degree Latitude
(d) 66·5 degree Latitude

⊙ *(c)* The longest latitude is 0° latitude, known as the Equator which divides the Earth into two equal halves, the northern hemisphere and two southern hemisphere.

113. If it is 12.00 Noon in India, on which meridian will it be 7:00 am of the same day?
(a) 7.5 degree E. Longitude
(b) 7.5 degree W. Longitude
(c) 75 degree E. Longitude
(d) 75 degree W. Longitude

⊘ *(d)* If it is 12:00 Noon in India, on 75 degree, W. Longitude it will be 7:00 am of the same day.

Current time in India = 12:00 PM

Time at required meridian = 7:00 AM

Difference in time = 5 Hours, and this time at meridian (7:00 AM) is 5 hours behind. Hence, it is west direction.

As we know the Earth takes 24 hours, for one complete rotation, so

24 hours = 360°

1 hour = $\dfrac{360}{24}$ = 15°, thus the Earth rotates 15 degree in/hour

In 5 hours = 5 × 15° = 75°

Hence, at 75° West longitude, the time will be 7:00 AM.

114. Who among the following was considered to be the preceptor of Mirabai ?
(a) Dadu
(b) Raidas
(c) Ramanand
(d) Surdas

⊘ *(b)* Guru Raidas, a low caste leather worket was the preceptor of Mirabai. He was low class leather worker. One of the main principle of her philosophy was that one should abandon the comforts of life and devote fully to her God for attainment of peace and salvation.

115. Consider the following statements about the Mahanavami Dibba
1. It was the name of a giant box of sweets distributed at the Mahanavami festival.
2. It was the name of a massive platform with a base covered with relief carvings.
Which of the statements given above is/are correct?
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊘ *(b)* Mahanavami Dibba was the name of massive platform with a base covered with relief carvings was the King's palace in Vijayanagara though there is no definite evidence. The Mahanavami Dibba had a very impressive platform known as "the audience hall". It was surrounded by high double walls a street running between them.

116. Which one of the following statements about the Ain-i-Akbari is not correct?
(a) It was written by Abu'l Fazl.
(b) It is a part of a larger work called Akbar Nama.
(c) It describes the Mughal Empire as having a diverse population and a composite culture.
(d) It was later revised by Sadullah Khan on the orders of Shah Jahan.

⊘ *(d)* Abu'l Fazl worked on the 'Akbar Nama'. The Akbar Nama is divided into three books: The first book dealt with Akbar's ancestors, the second recorded the events of Akbar's reign ,the third is the Ain-i Akbari. Badshah Nama was written by Abul Hamid Lahori was later revised by Sadullah Khan on the orders of Shah Jahan.

117. Swami Dayanand Saraswati
1. was opposed to the worship of idols of Gods and Goddesses.
2. regarded the Vedas as infallible.
3. had met and had discussions with Ishwar Chandra Vidyasagar.
Which of the statements given above are correct?
(a) 1, 2 and 3
(b) 2 and 3
(c) 1and 3
(d) 1 and 2

⊘ *(a)* All the statements are correct.

Swami Dayanand Sarasawati was a philosopher, social leader and founder of Arya Samaj. He was opposed to the worship of idols of Gods and Goddesses as he believe that God has no shape and no form. Hence, statement 1 is correct. Swami Dayanand regarded vedas as eternal and infalliable. Hence, statement 2 is correct.

He met Ishwar Chandra Vidyasagar in Calcutta and had discussions with him. Hence, statement 3 is correct.

118. Consider the following statements.
1. Muhammadan Anglo-Oriental College was founded at Aligarh by Sayyid Ahmad Khan.
2. Sayyid Ahmad Khan was a great believer in religious toleration, and Hindus, Paris and Christians had contributed to the funds of his college.
Which of the statement(s) given above is/are correct?
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊘ *(c)* Muhammadan Anglo-Oriental College was founded in 1875 by Sir Syed Ahmad Khan, initially as a primary school, with the intention of taking it to a college level institution, known as Muhammadan Anglo Oriental Collegiate School. Sayyid Ahmad Khan was a great believer in religious toleration. He believed that all religions had a certain underlying unity which could be called practical morality. Hindus, Parsis, and Christians had contributed freely to the funds of his college whose doors were open for all the Indians.

119. Basket-of-eggs topography is related to
(a) Drumlins
(b) Eskers
(c) Cirques
(d) Moraines

⊘ *(a)* Basket of eggs topography is related to Drumlins. Drumlins which is a depositional landform formed by glaciers. They are forms of rounded hummocks resulting from the deposition of glacial till which look like inverted boat or spoon

120. Which one of the following planets has the highest density?
(a) Mercury
(b) Venus
(c) Jupiter
(d) Earth

⊘ *(d)* Among the given options, Earth has the highest density. Earth is the fourth smallest of the planets. Earth is the densest planet because it is made of dense material and it is heavy planet. The Average Density of Earth is 5.5 grams/cm^3.

121. Point of Origin of Earthquake wave is known as
(a) Epicentre
(b) Focus
(c) Photosphere
(d) Seismic Zone

⊘ *(b)* The focus is the place inside Earth's crust where an earthquake originates. The point on the Earth's surface directly above the focus is the epicenter.

122. The maximum depth of Lithosphere is found in the
(a) Pacific Ocean
(b) Siberian Plain
(c) Patagonian Desert
(d) Himalayan Mountains

⊘ *(a)* The maximum depth lithosphere is found in the pacific ocean. Earth's lithosphere, which constitutes the hard and rigid outer vertical layer of the Earth, includes the crust and the uppermost mantle. Mariana Trench is located in the western Pacific Ocean, it is the deepest oceanic trench on Earth and the deepest part of the Earth.

123. A large body of magmatic material that cools in the deeper depth of the Earth's crust and develops in the form of large domes is known as
(a) Batholiths
(b) Lacoliths
(c) Lopoliths
(d) Phacoliths

(a) A large body of magmatic material that cools in the deeper depth of the Earth's crust and develops in the form of large domes is known as Batholiths. Forms from cooled magma deep in Earth's crust. These areas are exposed to the surface through the process of erosion accelerated by continental uplift acting over many tens of millions to hundreds of millions of years.

124. Which one of the following Oil Refineries is not located in Assam?
(a) Tatipaka
(b) Numaligarh
(c) Bongaigaon
(d) Digboi

(a) Among the given options, Tatipaka Oil Refinery is not located in Assam, it is located in Andhra Pradesh. It was established in 2001. It is ONGC's first ever oil refinery to produce petro-products.

125. Where and when did Mahatma Gandhi make his first public appearance in India on coming back from South Africa after two decades?
(a) Champaran in 1917
(b) Lucknow in 1916
(c) Banaras Hindu University in 1916
(d) Ahmedabad in 1918

(c) Gandhi's first public appearance in India was on the occasion of the opening ceremony of the Banaras Hindu University in February 1916, which was distinguished by the presence of many magnets and princes and of the Viceroy himself. He also passed critical comments on the Princes, as a result many princes walked out.

126. When and where was the demand for "Purna Swaraj" or complete independence made by the Indian National Congress?
(a) Bombay, 1885
(b) Lahore, 1929
(c) Kheda, 1917
(d) Bombay, 1942

(b) The Indian National Congress, on 19th December, 1929, passed the 'Purna Swaraj' – (total independence) resolution – at its Lahore session. A public declaration was made on 26th January, 1930 – a day which the Congress Party urged Indians to celebrate as 'Independence Day'. The declaration was passed due to the breakdown of negotiations between leaders of the freedom movement and the British over the question of dominion status for India.

127. Which one of the following provides for the complete equality of men and women in India?
(a) Articles 14 and 15 of the Constitution of India
(b) Fifth Schedule of the Constitution of India
(c) The Indian Independence Act
(d) Article 20 of the Constitution of India

(a) **Article 14** The State shall not deny to any person equality before the law or the equal protection of the laws within the territory of India.

Article 15 secures the citizens from every sort of discrimination by the State, on the grounds of religion, race, caste, sex or place of birth or any of them. However, this Article does not prevent the State from making any special provisions for women or children.

128. Which of the following statements about the non-permanent member of the Security Council of the United Nation is/are correct?
1. Their total number is now 10 but was originally only 6.
2. They are elected for a term of two years only.
Select the correct answer using the codes given below.
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) The UNSC is composed of 15 members, 5 permanent and 10 non-permanent. Five permanent members: China, France, the Russian Federation, the United Kingdom, and the United States and Ten non-permanent members: Elected for two-year terms by the General Assembly. Each year, the General Assembly elects five non-permanent members (out of ten in total) for a two-year term. The ten non-permanent seats are distributed on a regional basis. As stipulated in rule 144 of the rules of procedure, a retiring member is not eligible for immediate re-election.

129. At which of the following was the American Declaration of Independence adopted on 4th July, 1776?
(a) Washington Conference
(b) San Francisco Confernee
(c) Second Continental Congress
(d) First Continental Congress

(c) The United States Declaration of Independence is the pronouncement adopted by the Second Continental Congress meeting in Philadelphia, Pennsylvania, on 4th July, 1776.

130. Who among the following was the head of the Government that was overthrown by the Bolsheviks in the 1917 Revolution?
(a) Alexander Kerensky
(b) Prince Lvov
(c) Grand Duke Sergei
(d) Tsar Nicholas II

(d) Tsar Nicolas II was the head of the Government that was overthrown by the Bolsheviks in the 1917 Revolution. The Russian Revolution was a period of political and social revolution that took place in the former Russian Empire and began during the First World War. The Russian Revolution was a series of two revolutions: the first of which overthrew the imperial government and the second placed the Bolsheviks in power. that Caused the Tsar of the Russian Empire, Nicholas II, to step down from his throne.

131. Which one of the following is not a form of non-permanent member of the Security condensation?
(a) Dew
(b) Fog
(c) Frost
(d) Sleet

(d) Condensation is the change of the state of matter from the gas phase into the liquid phase, and is the reverse of vaporisation. The word most often refers to the water cycle. Sleet is Rain and snow mixed is precipitation composed of rain and partially melted snow.

132. Which one of the following clouds is a rain-bearing cloud?
(a) Cumulus cloud
(b) Stratus cloud
(c) Nimbus cloud
(d) Cirrus cloud

(c) Nimbus cloud is a rain-bearing cloud. Nimbus clouds are dark clouds as they carry a substantial quantity of water droplets. Nimbus comes in union with other cloud forms that bring substantial precipitation.

133. Tn which one of the following countries is intensive subsistence agriculture not predominantly practised?
(a) India
(b) Japan
(c) Canada
(d) Indonesia

(c) Intensive subsistence agriculture is not predominantly practised in Canada. Intensive farming is the kind of farming in which farmers grow their crops, fruits, and vegetables on a small piece of land using simple tools.

134. The Headquarters of South-Eastern Railway is located at
(a) Bilaspur
(b) Secunderabad
(c) Kolkata
(d) Bhubaneswar

(c) The Headquarters of South-Eastern Railway is located at Kolkata (Gardern Reach). It is the administrative and purchase office of the entire zone.

135. Bharatmala Pariyojana is related to
(a) interlinking of Northern and Southern Indian rivers in a garland shape.
(b) networks of National Highways in India.
(c) interlinking of all cities of India through Railways.
(d) interlinking of all industrial regions of India through pipelines.

(b) Bharatmala Pariyojana is an umbrella program for the highways sector envisaged by the Ministry of Road Transport and Highways. To optimise the efficiency of freight and passenger movement across the country by bridging critical infrastructure gaps through effective interventions.

136. The natural vegetation which covers maximum geographical areas of India is
(a) Tropical deciduous forests
(b) Tropical thorn forests
(c) Montane forests
(d) Tropical evergreen forests

(a) Tropical deciduous are a variety of temperate forest 'dominated' by trees that lose their leaves each year. They are found in areas with warm moist summers and cool winters.

137. River Beas, flowing from Himachal Punjab, joins the river
(a) Indus
(b) Setluj
(c) Chenab
(d) Ravi

(b) River Bears, flowing from Himachal, Punjab, joint the river Settuj. The Beas River rises in the Himalayas in central Himachal Pradesh, and flows for some 470 km to the Setluj River in the Indian state of Punjab.

138. With regard to the Constitution of India, which one of the following statements is not correct?
(a) The words - Socialist and Secular, were not originally part of the Constitution.

(b) The Preamble states the objects of the Constitution of India,
(c) The Preamble is enforceable in a Court of Law.
(d) A Republic refers to the people as the source of all authority under the Constitution.

(c) Preamble is not enforceable in a court of law. A preamble is an introductory statement in a document that explains the document's philosophy and objectives. The preamble basically gives idea of the following things/objects: Source of the Constitution, Nature of Indian State, Statement of its objectives, Date of its adoption.

139. Which one of the following is not a power of the Speaker of the Lok Sabha ?
(a) Speaker shall preside over the House of the People.
(b) Speaker will cast vote in the first instance in the House.
(c) Speaker will have power to maintain order within the House of the People.
(d) Speaker can adjourn the House or suspend the meeting till there is a quorum.

(b) Speaker does not cast vote in the first instance in the house. Speaker can only cast his vote when both the government and the opposition get equal votes on a particular law or a motion.

140. Which one of the following is included in Article 51A (Part IV A) of the Constitution of India?
(a) Fundamental Duties
(b) Suspension of Fundamental Rights
(c) Special Powers of Governors
(d) Writs

(a) The idea of Fundamental Duties is inspired from the Constitution of Russia. These were incorporated in Part IV-A of the Constitution by the 42nd Constitutional Amendment Act, 1976 on the recommendations of Swaran Singh Committee. Listed in Article 51-A of the Constitution.

141. As per the data up to November, 2020, released by the Union Finance Ministry, which one of the following countries ranks 1 in terms of ODI (Outward Direct Investment) for the year 2020 - 21 ?
(a) USA
(b) Singapore
(c) Mauritius
(d) United Kingdom

(b) An Outward Direct Investment (ODI) is a business strategy in which a domestic firm expands its operations to a foreign country. For example, some companies will make a green field investment, which is when a parent company creates a subsidiary in a foreign country.

142. 'Exercise Desert Knight - 21' is a bilateral air exercise between the Indian Air Force and the Air Force of which one of the following countries?
(a) USA
(b) France
(c) Britain
(d) Israel

(b) Indian Air Force (IAF) and French Air and Space Force conducted a bilateral Air exercise, Ex Desert Knight-21 at Air Force Station Jodhpur from 20th to 24th January, 2021. The exercise is unique as it includes fielding of Rafale aircraft by both sides and is indicative of the growing interaction between the two premier Air Forces.

143. Tableau of which one of the following States/Union Territory was adjudged best in the Republic Day Parade, 2021 ?
(a) Ladakh
(b) Uttar Pradesh
(c) Tripura
(d) Uttarakhand

(b) Uttar Pradesh's tableau depicting the Ram Temple at 2021 Republic Day parade in New Delhi was adjudged the best and awarded by Union Minister Kiren Rijiju.

144. In the recently concluded elections in December 2020, Faustin-Archange Touadera has won a second term in office as President of one of the following countries. Identify the country.
(a) Central African Republic
(b) Republic of South Africa
(c) Republic of Ghana
(d) Republic of Mozambiqu.

(a) In recently concluded elections in December 2020, Faustin—Archange Touadera has now a second term in office as President of central African Republic.

145. Which one of the following statements is not correct in respect of the 'Legion of Merit' award?
(a) This award is conferred by the President of the United States of America.

(b) This award was conferred to the Prime Minister of India in December, 2020.

(c) This was also awarded to the Prime Minister of Australia, Scott Morrison.

(d) It is the highest civilian award of the United States of America.

⊙ *(d)* Legion of Merit award is a military award of the United States Armed forces that is given for exceptionally meritorious conduct in the performance of outstanding services and achievements. This award was confused to the Prime Minister of India in December, 2020. It was also awarded to the Prime Minister of Australia, Scott Morrison.

146. Which one of the following statements is most appropriate about 'Exercise Kavach' ?

(a) It is a military exercise of the Indian Army only.

(b) It is a joint military exercise involving the Indian Army and the Indian Navy only.

(c) It is a joint military exercise involving the Indian Army, the Indian Navy and the Indian Air Force only.

(d) It is a joint military exercise involving the Indian Army, the Indian Navy, the Indian Air Force and also the Indian Coast Guard.

⊙ *(d)* A large-scale Tri service Military 'Exercise Kavach' conducted in January 2021 under the aegis of the Andaman and Nicobar Command (ANC), the only Joint Forces Command of the country. The exercise involves synergised application of maritime surveillance assets, coordinated air and maritime strikes, air defence, submarine and landing operations.

147. Recently a state of emergency has been declared in which one of the following countries?

(a) Maldives   (b) Bhutan
(c) Nepal   (d) Myanmar

⊙ *(d)* State of emergency was declared in Myanmar. It was declared on 1st February, 2021. The state of emergency was declared for one year.

148. Which one of the following teams is the winner of the Syed Mushtaq Ali Trophy, 2021?

(a) Uttar Pradesh   (b) Punjab
(c) Tamil Nadu   (d) Baroda

⊙ *(c)* The Indian State of Tamil Nadu won the Syed Mushtaq Ali Trophy, 2021.

It is a domestic Twenty 20 cricket championship in India, organised by the Board of control for cricket in India, among the teams from the Ranji Trophy.

149. Which country has replaced Maldives to host the 2023 Indian Ocean Island Games?

(a) Madagascar   (b) Sri Lanka
(c) India   (d) Mauritius

⊙ *(a)* Madagascar replaced Maldives to host the 2023 Indian Ocean Island Games. It is a quadrennial multi sport event from Indian ocean Islands nations. The games was created by international Olympic Committee in 1977.

150. Identify the correct reason, out of the following, about Claire Polosak for being in the news.

(a) She has been honoured with the Pulitzer Prize.

(b) She recently won a Grand Slam championship.

(c) She became the first female match official to conduct a men's cricket test match.

(d) She has been conferred with the Gandhi Peace Prize.

⊙ *(c)* Claire Polosak was in the news because she became the first female match official to conduct a men's cricket test match.