NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2021 (I)

PAPER I : Mathematics

1. The smallest positive integer n for which

$$\left(\frac{1-i}{1+i}\right)^{n^2} = 1$$

where $i = \sqrt{-1}$, is

(a) 2 (b) 4 (c) 6 (d) 8

(a) $\left(\frac{1-i}{1+i}\right)^{n^2} = 1$, where $i = \sqrt{-1}$

$$\left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{n^2} = 1$$

$$\left(\frac{1+i^2-2i}{1-i^2}\right)^{n^2} = 1$$

$$\left(\frac{1-1-2i}{1+1}\right)^{n^2} = 1$$

$\Rightarrow \quad (-i)^{n^2} = (-i)^4$

$\Rightarrow \quad n^2 = 4$

$n = 2$

Hence, option (a) is correct.

2. The value of x, satisfying the equation $\log_{\cos x} \sin x = 1$, where $0 < x < \frac{\pi}{2}$, is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(c) $\log_{\cos x} \sin x = 1$, where $0 < x < \frac{\pi}{2}$

$\Rightarrow (\cos x)^1 = \sin x \Rightarrow \cos x = \sin x$

$\Rightarrow \tan x = 1 \Rightarrow \tan x = \tan \pi/4$

$\Rightarrow x = \pi/4$

Hence, option (c) is correct.

3. If Δ is the value of the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

then what is the value of the following determinant?

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

$(p \neq 0 \text{ or } 1, q \neq 0 \text{ or } 1)$

(a) $p\Delta$ (b) $q\Delta$

(c) $(p+q)\Delta$ (d) $pq\Delta$

(d) Given, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta$

$$\therefore \quad \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

$$= p \cdot q \cdot \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$= pq\Delta$

Hence, option (d) is correct.

4. If $C_0, C_1, C_2, \ldots, C_n$ are the coefficients in the expansion of $(1+x)^n$, then what is the value of $C_1 + C_2 + C_3 + \ldots + C_n$?

(a) 2^n (b) $2^n - 1$

(c) 2^{n-1} (d) $2^n - 2$

(b) $\because (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

and we know that

${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

$\therefore {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - {}^nC_0$

$= 2^n - 1$

Hence, option (b) is correct.

5. If $a + b + c = 4$ and $ab + bc + ca = 0$, then what is the value of the following determinant?

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(a) 32 (b) -64

(c) -128 (d) 64

(b) Let

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

(by $C_1 \to C_1 + C_2 + C_3$)

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

(To take common $a + b + c$ from C_1)

$$= (a+b+c)\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix}$$

(by $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$)

$= (a+b+c)\,[(b-c)(a-b) - (c-a)^2]$

$= (a+b+c)\,(ab - b^2 - ca + bc$

$-c^2-a^2+2ca)$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -(a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$

$= -(4)[16-0] = -64.$

6. The number of integer values of k, for which the equation $2\sin x = 2k+1$ has a solution, is

(a) zero (b) one
(c) two (d) four

(c) Given,

$2\sin x = 2k+1$

$\because \quad -1 \le \sin x \le 1 \Rightarrow -2 \le 2\sin x \le 2$

$-2-1 \le 2\sin x - 1 \le 2-1$

$-3 \le 2k \le 1$

$\frac{-3}{2} \le k \le \frac{1}{2} \Rightarrow -1\cdot5 \le k \le 0\cdot5$

$\therefore$ Integer values of $k = -1, 0$

Hence, option (c) is correct.

7. If $a_1, a_2, a_3, \ldots, a_9$ are in GP, then what is the value of the following determinant?

$$\begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$$

(a) 0 (b) 1
(c) 2 (d) 4

(a) Let first term and common ratio of GP are a and r respectively.

$$\therefore \begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix} = \begin{vmatrix} \log a & \log ar & \log ar^2 \\ \log ar^3 & \log ar^4 & \log ar^5 \\ \log ar^6 & \log ar^7 & \log ar^8 \end{vmatrix}$$

$$= \begin{vmatrix} \log a & \log a + \log r & \log a + 2\log r \\ \log a + 3\log r & \log a + 4\log r & \log a + 5\log r \\ \log a + 6\log r & \log a + 7\log r & \log a + 8\log r \end{vmatrix}$$

$[\because \log mn = \log m + \log n]$

$$= \begin{vmatrix} \log a & \log r & \log r \\ \log a + 3\log r & \log r & \log r \\ \log a + 6\log r & \log r & \log r \end{vmatrix}$$

(by $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$)

$= 0 \qquad [\because C_2 = C_3]$

8. If the roots of the quadratic equation $x^2+2x+k=0$ are real, then

(a) $k<0$ (b) $k \le 0$
(c) $k<1$ (d) $k \le 1$

(d) Given quadratic equation,

$x^2+2x+k=0 \qquad \ldots \text{(i)}$

Since, roots are real

$\Rightarrow \quad D \ge 0 \Rightarrow b^2-4ac \ge 0$

$(2)^2 - 4(1)(k) \ge 0 \Rightarrow 4 \ge 4k \Rightarrow \quad k \le 1$

Hence, option (d) is correct.

9. If $n = 100!$, then what is the value of the following?

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{100} n}$$

(a) 0 (b) 1 (c) 2 (d) 3

(b) $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{100} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 100$

$= \log_n (2\cdot3\cdot4\cdot5\ldots100)$

$= \log_{100!}(100!) \qquad [\because n = 100!]$

$= 1 \qquad [\because \log_a^a = 1]$

Hence, option (b) is correct.

10. If $z = 1+i$, where $i = \sqrt{-1}$, then what is the modulus of $z + \frac{2}{z}$?

(a) 1 (b) 2 (c) 3 (d) 4

(b) $z = 1+i$, where $i = \sqrt{-1}$

$$\left| z + \frac{2}{z} \right| = \left| (1+i) + \frac{2}{(1+i)} \right| = \left| (1+i) + \frac{2}{(1+i)} \times \frac{(1-i)}{(1-i)} \right|$$

$$= \left| (1+i) + \frac{2(1-i)}{2} \right| = |1+i+1-i| = |2| = 2$$

Hence, option (b) is correct.

11. If A and B are two matrices such that AB is of order $n \times n$, then which one of the following is correct?

(a) A and B should be square matrices of same order.
(b) Either A or B should be a square matrix.
(c) Both A and B should be of same order.
(d) Orders of A and B need not be the same.

(d) Given that, order of matrix $AB = n \times n$

If we take $A_{n \times p}$ and $B_{p \times n}$, then AB will be of order $n \times n$.

So, orders of A and B need not be the same, is correct.

Hence, option (d) is correct.

12. How many matrices of different orders are possible with elements comprising all prime numbers less than 30?

(a) 2 (b) 3 (c) 4 (d) 6

(c) $\because$ Prime numbers less than 30 = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

$\Rightarrow$ Number of elements = 10

$\therefore$ Possible order of matrices with 10 elements = $10 \times 1, 1 \times 10, 2 \times 5, 5 \times 2$

$\therefore$ Nubmer of matrices of different order = 4

Hence, option (c) is correct.

13. Let, $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$

where p, q, r and s are any four different prime numbers less than 20. What is the maximum value of the determinant?

(a) 215 (b) 311 (c) 317 (d) 323

(c) $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$, prime numbers less than 20

$= \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow A = ps - rq$

For maximum values of A, p and s must be maximum and r and q must be minimum.

Then, $p = 17, s = 19, r = 2, q = 3$

$\therefore \quad A = 17 \times 19 - 2 \times 3$

$= 323 - 6 = 317$

Hence, option (c) is correct.

14. If A and B are square matrices of order 2 such that $\det(AB) = \det(BA)$, then which one of the following is correct?
(a) A must be a unit matrix
(b) B must be a unit matrix
(c) Both A and B must be unit matrices
(d) A and B need not be unit matrices

(d) $A_{2\times 2}$ and $B_{2\times 2}$ are two matrices

and $|AB| = |BA| \Rightarrow |A||B| = |B||A|$

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$ then, $|AB| = |BA|$

Hence, we can say A and B need not be the unit matrices.
Hence, option (d) is correct.

15. What is $\cot 2x \cot 4x - \cot 4x \cot 6x - \cot 6x \cot 2x$ equal to?
(a) – 1 (b) 0 (c) 1 (d) 2

(c) $\because \cot 6x = \cot(2x + 4x)$

$$\cot 6x = \frac{\cot 2x \cdot \cot 4x - 1}{\cot 2x + \cot 4x} \quad \left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

$\Rightarrow \cot 6x \cdot \cot 2x + \cot 6x \cdot \cot 4x = \cot 2x \cdot \cot 4x - 1$
$\therefore \cot 2x \cdot \cot 4x - \cot 4x \cdot \cot 6x - \cot 6x \cdot \cot 2x = 1$
Hence, option (c) is correct.

16. If $\tan x = -\frac{3}{4}$ and x is in the second quadrant, then what is the value of $\sin x \cdot \cos x$?
(a) $\frac{6}{25}$ (b) $\frac{12}{25}$ (c) $-\frac{6}{25}$ (d) $-\frac{12}{25}$

(d) Given,
$\tan x = \frac{-3}{4}$ and x is in the 2nd quadrant.

Let perpendicular be $3k$ and base be $4k$, then

Hypotenuse $= \sqrt{(3k)^2 + (4k)^2} = 5k$

$\sin x = \frac{3}{5}$ and $\cos x = \frac{-4}{5}$

$\therefore \sin x \cdot \cos x = \frac{3}{5} \times \left(\frac{-4}{5}\right) = \frac{-12}{25}$

Hence, option (d) is correct.

17. What is the value of the following?
$\operatorname{cosec}\left(\frac{7\pi}{6}\right)\sec\left(\frac{5\pi}{3}\right)$
(a) $\frac{4}{3}$ (b) 4 (c) – 4 (d) $-\frac{4}{\sqrt{3}}$

(c) $\operatorname{cosec}\left(\frac{7\pi}{6}\right)\cdot\sec\left(\frac{5\pi}{3}\right) = \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right)\cdot\sec\left(2\pi - \frac{\pi}{3}\right)$

$= -\operatorname{cosec}\frac{\pi}{6}\cdot\sec\frac{\pi}{3} = -2 \times 2 = -4$

Hence, option (c) is correct.

18. If the determinant

$$\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$$

then what is x equal to?
(a) – 2 or 2 (b) – 3 or 3 (c) – 1 or 1 (d) 3 or 4

(c) Given, $\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$ … (i)

$-1(x^2 - 1) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1$
$\Rightarrow x = \pm 1 \Rightarrow 1 - x^2 = 0$
$\therefore x^2 = 1$
$x = +1, -1$
Hence, option (c) is correct.

19. What is the value of the following?
$\tan 31° \tan 33° \tan 35° \ldots \tan 57° \tan 59°$
(a) – 1 (b) 0 (c) 1 (d) 2

(c) $\tan 31° \cdot \tan 33° \cdot \tan 35° \ldots \tan 57° \cdot \tan 59°$
$= \tan 31° \cdot \tan 33° \cdot \tan 35° \ldots x \tan 45° x \ldots \tan(90° - 33°) \cdot \tan(90° - 31°)$
$= \tan 31° \cdot \tan 33° \cdot \tan 35° \ldots \cot 35° \cdot \cot 33° \cdot \cot 31°$
$= (\tan 31° \cdot \cot 31°) \cdot (\tan 33° \cdot \cot 33°) \cdot (\tan 35° \cdot \cot 35°) \ldots$
$= 1 \cdot 1 \cdot 1 \ldots = 1$
Hence, option (c) is correct.

20. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

then what is $f(-1) + f(0) + f(1)$ equal to?
(a) 0 (b) 1
(c) 100 (d) – 100

(a) $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

$f(-1) = \begin{vmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ 6 & 12 & 0 \end{vmatrix} = 0 \Rightarrow f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix} = 0$

$f(1) = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$\therefore f(-1) + f(0) + f(1) = 0 + 0 + 0 = 0$
Hence, option (a) is correct.

21. The equation $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ has
(a) no solution (b) unique solution
(c) two solutions (d) infinite number of solutions

(b) $\because \sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ … (i)

and we know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ … (ii)

Adding Eqs. (i) and (ii), we get

$2\sin^{-1} x = \frac{\pi}{6} + \frac{\pi}{2} \Rightarrow 2\sin^{-1} x = \frac{2\pi}{3}$

$\Rightarrow \sin^{-1} x = \frac{\pi}{3}$

$x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

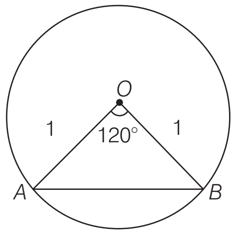
Hence, the given equation has a unique solution.
Hence, option (b) is correct.

22. What is the value of the following?
$(\sin 24° + \cos 66°)(\sin 24° - \cos 66°)$
(a) – 1 (b) 0 (c) 1 (d) 2

(b) $(\sin 24° + \cos 66°)(\sin 24° - \cos 66°)$
$= (\sin 24° + \cos 66°)$
$\{\sin(90° - 66°) - \cos 66°\}$
$[\because \sin(90° - \theta) = \cos\theta]$
$= (\sin 24° + \cos 66°)(\cos 66° - \cos 66°)$
$= (\sin 24° + \cos 66°)(0) = 0$
Hence, option (b) is correct.

23. A chord subtends an angle 120° at the centre of a unit circle. What is the length of the chord?
(a) $\sqrt{2} - 1$ units (b) $\sqrt{3} - 1$ units
(c) $\sqrt{2}$ units (d) $\sqrt{3}$ units

(d) Given, radius of the circle = 1 unit



$\angle AOB = 120°$
By using cosine rule,
$$\cos 120° = \frac{OA^2 + OB^2 - AB^2}{2 \cdot OA \cdot OB} \quad \dots \text{(i)}$$
Let $AB = x$ unit, $OA = 1$ unit, $OB = 1$ unit
From Eq. (i),
$$\frac{-1}{2} = \frac{1 + 1 - x^2}{2 \cdot 1 \cdot 1} \Rightarrow -1 = 2 - x^2$$
$\Rightarrow \quad x^2 = 3 \Rightarrow x = \sqrt{3}$ unit
Hence, option (d) is correct.

24. What is $(1 + \cot\theta - \operatorname{cosec}\theta)$ $(1 + \tan\theta + \sec\theta)$ equal to?
(a) 1 (b) 2 (c) 3 (d) 4

(b) $(1 + \cot\theta - \operatorname{cosec}\theta)$ $(1 + \tan\theta + \sec\theta)$
$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$
$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)$$
$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta \cdot \cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$
$$= \frac{1 + 2\sin\theta \cdot \cos\theta - 1}{\sin\theta \cdot \cos\theta} = 2$$
Hence, option (b) is correct.

25. What is $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2$ equal to?
(a) 0 (b) 1
(c) $2\tan\theta$ (d) $2\cot\theta$

(a) $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2$
$$= \frac{1 + \tan^2\theta}{1 + \dfrac{1}{\tan^2\theta}} - \left(\frac{1 - \tan\theta}{1 - \dfrac{1}{\tan\theta}}\right)^2$$
$$= \tan^2\theta\left(\frac{1 + \tan^2\theta}{\tan^2\theta + 1}\right) - \left(\frac{\tan\theta(1 - \tan\theta)}{\tan\theta - 1}\right)^2$$
$= \tan^2\theta - \tan^2\theta = 0$
Hence, option (a) is correct.

26. What is the interior angle of a regular octagon of side length 2 cm?
(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{3\pi}{5}$ (d) $\frac{3\pi}{8}$

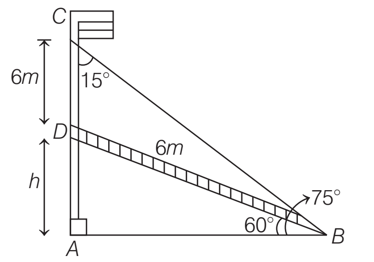
(b) Given, length of side of regular octagon = 2 cm
$\because$ Sum of interior angles of octagon
$= (8 - 2) \times 180°$
$= 6 \times 180°$
[$\because$ sum of interior angles of polygon $= (n - 2) \times 180°$]
$$\therefore \text{Interior angle} = \frac{6 \times 180°}{8}$$
$$= 135° = \frac{3\pi}{4}$$
Hence, option (b) is correct.

27. If $7\sin\theta + 24\cos\theta = 25$, then what is the value of $(\sin\theta + \cos\theta)$?
(a) 1 (b) $\frac{26}{25}$ (c) $\frac{6}{5}$ (d) $\frac{31}{25}$

(d) Given, $7\sin\theta + 24\cos\theta = 25$
Since, we know that if
$a\sin\theta + b\cos\theta = c$
then $b\sin\theta - a\cos\theta = \sqrt{a^2 + b^2 - c^2}$
$\because \quad 7\sin\theta + 24\cos\theta = 25 \quad \dots$ (i)
$\therefore 24\sin\theta - 7\cos\theta = \sqrt{7^2 + 24^2 - 25^2}$
$24\sin\theta - 7\cos\theta = 0 \quad \dots$ (ii)
Eq. (i) × 7 + Eq. (ii) × 24
$49\sin\theta + 168\cos\theta = 175$
$576\sin\theta - 168\cos\theta = 0$
$625\sin\theta = 175$
$$\sin\theta = \frac{175}{625} = \frac{7}{25}$$
$$\therefore \cos\theta = \sqrt{1 - \left(\frac{7}{25}\right)^2} = \frac{24}{25}$$
$$\therefore \sin\theta + \cos\theta = \frac{7}{25} + \frac{24}{25} = \frac{31}{25}$$
Hence, option (d) is correct.

28. A ladder 6 m long reaches a point 6 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the top of the flagstaff is 75°. What is the height of the flagstaff ?
(a) 12 m (b) 9 m
(c) $(6 + \sqrt{3})$ m (d) $(6 + 3\sqrt{3})$ m

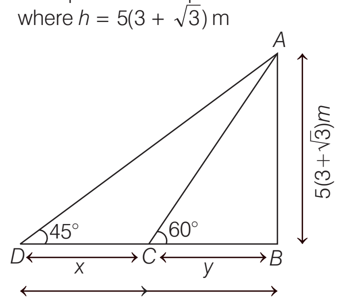
(d) Let AC be a vertical flagstaff.
$\therefore \quad CD = 6$ m, $BD = 6$ m
$\angle CBD = 75°$



Let $AD = h$ meter
In ΔABC
$90 + 75 + \angle C = 180°$ [$\because$ sum of interior angle of triangle is 180°]
$\angle C = 15°$
In ΔBCD,
$BD = CD \Rightarrow \angle BCD = \angle CBD = 15°$
$\therefore \angle ABD = 75° - 15° = 60°$
In ΔABD, $\sin 60° = \dfrac{h}{6} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{h}{6}$
$h = 3\sqrt{3}$ m
$\therefore$ Height of the flagstaff $= (h + 6)$ m
$= (3\sqrt{3} + 6)$ m
Hence, option (d) is correct.

29. The shadow of a tower is found to be x metre longer, when the angle of elevation of the sun changes from 60° to 45°. If the height of the tower is $5(3 + \sqrt{3})$ m, then what is x equal to?
(a) 8 m (b) 10 m
(c) 12 m (d) 15 m

(b) In the given diagram,
AB represents the position of tower, where $h = 5(3 + \sqrt{3})$ m



$CD = x$ m
In ΔABC,
$$\tan 60° = \frac{5(3 + \sqrt{3})}{BC} \Rightarrow \sqrt{3} = \frac{5(3 + \sqrt{3})}{BC}$$

$\therefore \quad BC = 5(\sqrt{3} + 1)\,m$

In ΔABD,

$$\tan 45^\circ = \frac{5(3+\sqrt{3})}{BD}$$

$$\Rightarrow \quad 1 = \frac{5(3+\sqrt{3})}{BD}$$

$\therefore \quad BD = 5(3 + \sqrt{3})\,m$

Since, $x = BD - BC$

$x = 5(3 + \sqrt{3}) - 5(\sqrt{3} + 1)$

$x = 5(3 + \sqrt{3} - \sqrt{3} - 1)$

$x = 10\,m$

Hence, option (b) is correct.

30. If $3\cos\theta = 4\sin\theta$, then what is the value of $\tan(45^\circ + \theta)$?

(a) 10 (b) 7 (c) $\frac{7}{2}$ (d) $\frac{7}{4}$

(b) If $3\cos\theta = 4\sin\theta$

$$\Rightarrow \quad \frac{3}{4} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \quad \tan\theta = \frac{3}{4}$$

$$\therefore \tan(45^\circ + \theta) = \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ \cdot \tan\theta}$$

$$= \frac{1 + \frac{3}{4}}{1 - 1 \times \frac{3}{4}} = \frac{4+3}{4-3} = 7$$

Hence, option (b) is correct.

31. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ holds, when

(a) $x \in R$

(b) $x \in R - (-1, 1)$ only

(c) $x \in R - \{0\}$ only

(d) $x \in R - [-1, 1]$ only

(a) Since, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

for all $x \in R$.

Hence, option (a) is correct.

32. If $\tan A = \frac{1}{7}$, then what is $\cos 2A$ equal to?

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{12}{25}$ (d) $\frac{6}{25}$

(a) $\tan A = \frac{1}{7}$

$$\therefore \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - (1/7)^2}{1 + (1/7)^2}$$

$$= \frac{49 - 1}{49 + 1} = \frac{48}{50}$$

$$\cos 2A = \frac{24}{25}$$

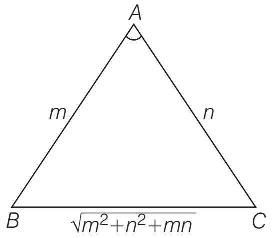
Hence, option (a) is correct.

33. The sides of a triangle are m, n and $\sqrt{m^2 + n^2 + mn}$. What is the sum of the acute angles of the triangle?

(a) 45° (b) 60° (c) 75° (d) 90°

(b) Let $AB = m$, $AC = n$

$$BC = \sqrt{m^2 + n^2 + mn}$$



By using cosine rule,

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2AB \cdot AC}$$

$$\Rightarrow \cos A = \frac{m^2 + n^2 - m^2 - n^2 - mn}{2mn}$$

$$\Rightarrow \cos A = \frac{-1}{2} \quad \Rightarrow \quad A = 120^\circ$$

$\therefore \angle B + \angle C = 180 - \angle A$

[$\because$ sum of interior angle is 180°]

$= 180^\circ - 120^\circ$

$\angle B + \angle C = 60^\circ$

Hence, option (b) is correct.

34. What is the area of the triangle ABC with sides $a = 10$cm, $c = 4$cm and angle $B = 30°$?

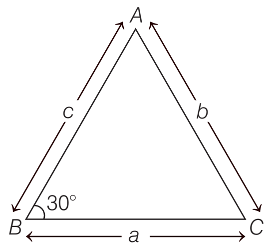
(a) 16 cm² (b) 12 cm²

(c) 10 cm² (d) 8 cm²

(c) Given, $a = 10$ cm

$c = 4$ cm

$\angle B = 30^\circ$



$\because$ Area of triangle $= \frac{1}{2} ac \sin(\angle B)$

$$= \frac{1}{2} \times 10 \times 4 \times \sin 30^\circ = \frac{1}{2} \times 40 \times \frac{1}{2}$$

$= 10$ sq cm

Hence, option (c) is correct.

35. Consider the following statements

1. $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$ are equivalent sets.
2. $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\}$ are equal sets

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

(c) $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$

Since, number of elements are same in both the sets.

$\Rightarrow A$ and B are equivalent sets.

If $A = \{1, 5, 9\}$, $B = \{1, 5, 5, 9, 9\}$

Which is nothing but $B = \{1, 5, 9\}$

Since, elements are same in A and B

$\Rightarrow A$ and B are equal sets

Hence, option (c) is correct.

36. Consider the following statements

1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

(d) Since we know that null set is a subset of every set and every set is a subset of itself.

If $n(A) = 10$

$\therefore n(P(A)) = 2^{10} = 1024$

$\therefore$ all the given statements are true.

Hence, option (d) is correct.

37. Let R be a relation defined as xRy if and only if $2x + 3y = 20$, where $x, y \in N$. How many elements of the form (x, y) are there in R?

(a) 2 (b) 3

(c) 4 (d) 6

(b) $\because xRy \Leftrightarrow 2x + 3y = 20$

where, $x, y \in \mathbb{N}$

$$\because \quad y = \frac{20 - 2x}{3}$$

All ordered pair which satisfies the given relations are (1, 6), (4, 4), (7, 2).

$\therefore \quad R = \{(1, 6), (4, 4), (7, 2)\}$

$\therefore \quad n(R) = 3$

Hence, option (b) is correct.

38. Consider the following statements

1. A function $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(x) = x + 1$, is one-one as well as onto.
2. A function $f : \mathbb{N} \to \mathbb{N}$, defined by $f(x) = x + 1$, is one-one but not onto.

Which of the above statement(s) is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

(c) **Statement I**

$f : \mathbb{Z} \to z$

$f(x) = x + 1$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one in $\mathbb{Z}$.

and every element of co-domain has its pre-image in domain.

$\Rightarrow f$ is onto.

Statement II

$f : \mathbb{N} \to \mathbb{N}$

$f(x) = x + 1$

Let $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one in $\mathbb{N}$.

But there is no element in $\mathbb{N}$ such that

$f(x) = 1$

Hence, f is not onto on $\mathbb{N}$.

Given statements are correct.

Hence, option (c) is correct.

39. Consider the following in respect of a complex number z.

1. $\overline{(z^{-1})} = (\bar{z})^{-1}$
2. $zz^{-1} = |z|^2$

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Let $z = x + iy$

$\bar{z} = x - iy$

$(\bar{z})^{-1} = \dfrac{1}{x - iy} = \dfrac{x + iy}{x^2 + y^2}$

Also, $z^{-1} = \dfrac{1}{x + iy} = \dfrac{x - iy}{x^2 + y^2}$

$\overline{(z^{-1})} = \dfrac{x + iy}{x^2 + y^2} = (\bar{z})^{-1}$

$\therefore$ Statement 1 is correct.

$|z| = \sqrt{x^2 + y^2}$

$\Rightarrow |z|^2 = x^2 + y^2$

But $zz^{-1} = (x + iy)\dfrac{(x - iy)}{x^2 + y^2}$

$= \dfrac{x^2 + y^2}{x^2 + y^2} = 1 \neq |z|^2$

$\therefore$ Statement 2 is wrong.

Hence, option (a) is correct.

40. Consider the following statements in respect of an arbitrary complex number z.

1. The difference of z and its conjugate is an imaginary number.
2. The sum of z and its conjugate is a real number.

Which of the above statement(s) is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Let $z = x + iy$

$\bar{z} = x - iy$

$\therefore z - \bar{z} = x + iy - x + iy = 2iy$ which is an imaginary number.

$\Rightarrow$ Statement-1 is correct.

Also, $z + \bar{z} = x + iy + x - iy = 2x$ which is real.

$\Rightarrow$ Statement-2 is correct.

Hence, option (c) is correct.

41. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$, where $n \in N$ and $i = \sqrt{-1}$?

(a) -1 (b) 1 (c) $\sqrt{2}$ (d) 2

(b) Let $z = i^{2n+1}(-i)^{2n-1}$, where $n \in \mathbb{N}$

$= (i)^{2n}(i)(-i)^{2n}(-i)^{-1}$

$= (i^{2n})(-1)^{2n}.(i^{2n})\left(\dfrac{i}{-i}\right)$

$= (i^{4n})(-1) = (i^4)^n.(-1)$

$= -1 = -1 + 0i$

$\therefore |z| = 1$

Hence, option (b) is correct.

42. If α and β are the roots of the equation $4x^2 + 2x - 1 = 0$, then which one of the following is correct?

(a) $\beta = -2\alpha^2 - 2\alpha$ (b) $\beta = 4\alpha^2 - 3\alpha$
(c) $\beta = \alpha^2 - 3\alpha$ (d) $\beta = -2\alpha^2 + 2\alpha$

(a) Given quadratic equation

$4x^2 + 2x - 1 = 0$ ... (i)

If α, β are the roots of Eq. (i), then these value will satisfy the given equation.

$4\alpha^2 + 2\alpha - 1 = 0$ ... (ii)

and $4\beta^2 + 2\beta - 1 = 0$ ... (iii)

From Eq. (i),

Sum of roots $= \dfrac{-2}{4}$

$\alpha + \beta = \dfrac{-1}{2}$

$\beta = \dfrac{-1}{2} - \alpha$

On putting the value of β in Eq. (iii),

$4\left(\dfrac{-1}{2} - \alpha\right)^2 + 2\beta - 1 = 0$

$4\left(\dfrac{1}{4} + \alpha^2 + \alpha\right) - 1 = -2\beta$

$1 + 4\alpha^2 + 4\alpha - 1 = -2\beta$

$\Rightarrow \beta = \dfrac{4(\alpha^2 + \alpha)}{-2}$

$\beta = -2\alpha^2 - 2\alpha$

Hence, option (a) is correct.

43. If one root of $5x^2 + 26x + k = 0$ is reciprocal of the other, then what is the value of k?

(a) 2 (b) 3 (c) 5 (d) 8

(c) Given quadratic equation

$5x^2 + 26x + k = 0$ ... (i)

Let α and β be the roots.

According to question, $\beta = \dfrac{1}{\alpha}$

$\because$ Product of roots $= \dfrac{k}{5}$

$\alpha \cdot \beta = \dfrac{k}{5}$

$\Rightarrow \alpha \cdot \dfrac{1}{\alpha} = \dfrac{k}{5} \Rightarrow 1 = \dfrac{k}{5}$

$\Rightarrow k = 5$

Hence, option (c) is correct.

44. In how many ways can a team of 5 players be selected from 8 players so as not to include a particular player?

(a) 42 (b) 35
(c) 21 (d) 20

(c) Given that there are 8 players among which one particular player is there.

Hence, number of ways to select 5 players $= {}^{8-1}C_5$

$= {}^{7}C_5 = \dfrac{7 \times 6}{1 \times 2} = 21$

Hence, option (c) is correct.

45. What is the coefficient of the middle term in the expansion of $(1 + 4x + 4x^2)^5$?

(a) 8064 (b) 4032
(c) 2016 (d) 1008

(a) $(1 + 4x + 4x^2)^5$

$= \{(1 + 2x)^2\}^5 = (1 + 2x)^{10}$

$\therefore$ Total number of term in the expansion of $(1 + 2x)^{10} = 10 + 1 = 11$

$\therefore$ Middle term $= \left(\dfrac{11 + 1}{2}\right)$th term

$= 6$ th term

$T_6 = T_{5+1} = {}^{10}C_5 (2x)^5$

$= {}^{10}C_5 \times 2^5 \times x^5$

$\therefore$ Coefficient of middle term $= {}^{10}C_5 \cdot 2^5$

$= \dfrac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} \times 2^5 = 8064$

Hence, option (a) is correct.

46. What is $C(n,1) + C(n,2) + ... + C(n,n)$ equal to?

(a) $2 + 2^2 + 2^3 + ... + 2^n$

(b) $1 + 2 + 2^2 + 2^3 + ... + 2^n$

(c) $1 + 2 + 2^2 + 2^3 + ... + 2^{n-1}$

(d) $2 + 2^2 + 2^3 + ... + 2^{n-1}$

(c) $C(n,1) + C(n,2) + \dots + C(n,n)$

$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$

$\{\because {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n\}$

$= 2^n - {}^nC_0 = 2^n - 1$

Now, we shall solve the option to check whether sum is $2^n - 1$ or not.

Let's take

$S = 1 + 2 + 2^2 + 2^3 + \dots + 2^{n-1}$ which forms a GP.

where $a = 1$

$r = \frac{2}{1} = 2 > 1$

$\because \quad S = \frac{a(r^n - 1)}{r - 1}$

$\therefore \quad S = \frac{1(2^n - 1)}{2 - 1} = 2^n - 1$

Hence, $2^n - 1 = {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$

$\therefore$ Option (c) is correct.

47. What is the sum of the coefficients of first and last terms in the expansion of $(1 + x)^{2n}$, where n is a natural number?

(a) 1 (b) 2

(c) n (d) $2n$

(b) Expand $(1 + x)^{2n}$ by using binomial expansion

$= {}^{2n}C_0x^0 + {}^{2n}C_1x^1 + {}^{2n}C_2x^2 + \dots + {}^{2n}C_{2n}x^{2n}$

$\therefore$ The coefficient of first and last term of the expansion

$= {}^{2n}C_0 + {}^{2n}C_{2n}$

$= 1 + 1 = 2$

Hence, option (b) is correct.

48. If the first term of an AP is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms, then what is the sum of the first ten terms?

(a) -500 (b) -250

(c) 500 (d) 250

(b) Given, first term of an AP $(a) = 2$

and $a_1 + a_2 + a_3 + a_4 + a_5 = \frac{1}{4}(a_6 + a_7 + a_8 + a_9 + a_{10})$, where $a_n = a + (n-1)d$

$\Rightarrow \frac{5}{2}[2a + (5-1)d]$

$= \frac{1}{4} \times \frac{5}{2}[2a_6 + (5-1)d]$

$\left[\because \text{sum of } n \text{ terms of AP, } S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$4(2 \times 2 + 4d) = 2a_6 + 4d$

$16 + 16d = 2a_6 + 4d$

$16 + 16d = 2(a + 5d) + 4d$

$16 + 16d = 2a + 14d$

$16 + 16d = 2 \times 2 + 14d$

$2d = -12 \Rightarrow d = -6$

$\therefore \quad S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 5[2 \times 2 + 9(-6)]$

$= 5[4 - 54]$

$S_{10} = -250$

Hence, option (b) is correct.

49. Consider the following statements

1. If each term of a GP is multiplied by same non-zero number, then the resulting sequence is also a GP.
2. If each term of a GP is divided by same non-zero number, then the resulting sequence is also a GP.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Let us take a GP.

$a, ar, ar^2, \dots$ is in GP.

$\Rightarrow ak, akr, akr^2, \dots$ will also be in GP where, k is non-zero number.

If $k = \frac{1}{m}, m \neq 0$

$\Rightarrow \frac{a}{m}, \frac{a}{m}r, \frac{a}{m}r^2, \dots$ will also be in GP.

Hence, both statements are correct.

50. How many 5-digit prime numbers can be formed using the digits 1, 2, 3, 4, 5 if the repetition of digits is not allowed?

(a) 5 (b) 4 (c) 3 (d) 0

(d) Given digits are 1, 2, 3, 4, 5

Since, the sum of digits $= 1 + 2 + 3 + 4 + 5 = 15$ is divisible by 3.

$\Rightarrow$ Every 5 digit number formed by the given digits will be divisible by 3.

$\Rightarrow$ There is no prime number.

Hence, option (d) is correct.

51. If $f(x+1) = x^2 - 3x + 2$, then what is $f(x)$ equal to?

(a) $x^2 - 5x + 4$ (b) $x^2 - 5x + 6$

(c) $x^2 + 3x + 3$ (d) $x^2 - 3x + 1$

(b) If $f(x+1) = x^2 - 3x + 2$

Let $x + 1 = y$

$\Rightarrow \quad x = y - 1$ or $x \to x - 1$

$\therefore f(x) = (x-1)^2 - 3(x-1) + 2$

$= x^2 + 1 - 2x - 3x + 3 + 2$

$= x^2 - 5x + 6$

Hence, option (b) is correct.

52. If $x^2, x, -8$ are in AP, then which one of the following is correct?

(a) $x \in \{-2\}$ (b) $x \in \{4\}$

(c) $x \in \{-2, 4\}$ (d) $x \in \{-4, 2\}$

(c) If $x^2, x, -8$ are in AP, then

$2x = x^2 - 8$

$\Rightarrow \quad x^2 - 2x - 8 = 0$

$\Rightarrow (x-4)(x+2) = 0$

$x \in \{-2, 4\}$

Hence, option (c) is correct.

53. The third term of a GP is 3. What is the product of its first five terms?

(a) 81

(b) 243

(c) 729

(d) Cannot be determined due to insufficient data

(b) Given

$a_3 = 3$

$\because \quad a_3 = ar^2$ in GP $\quad [\because a_n = ar^{n-1}$ in GP$]$

$ar^2 = 3 \quad \dots$ (i)

To find $a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5$

$= a(ar)(ar^2)(ar^3)(ar^4)$

$= a^5r^{10} = (ar^2)^5 = 3^5 = 243$

Hence, option (b) is correct.

54. The element in the ith row and the jth column of a determinant of third order is equal to $2(i + j)$. What is the value of the determinant?

(a) 0 (b) 2 (c) 4 (d) 6

(a) Given,

$a_{ij} = 2(i + j)$

$\therefore a_{11} = 2(1+1) = 4, a_{21} = 2(2+1) = 6$

$a_{12} = 2(1+2) = 6, a_{22} = 2(2+2) = 8$

$a_{13} = 2(1+3) = 8, a_{23} = 2(2+3) = 10$

$a_{31} = 2(3+1) = 8, a_{32} = 2(3+2) = 10,$

$a_{33} = 2(3+3) = 12$

$$\Delta = \begin{vmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{vmatrix} = 2 \cdot 2 \cdot 2 \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

$= 8[2(24 - 25) - 3(18 - 20) + 4(15 - 16)]$

$\Delta = 8[-2 + 6 - 4]$

$\Delta = 0$

Hence, option (a) is correct.

55. With the numbers 2, 4, 6, 8, all the possible determinants with these four different elements are constructed. What is the sum of the values of all such determinants?

(a) 128 (b) 64 (c) 32 (d) 0

(d) Given numbers are 2, 4, 6, 8.

$\therefore$ We can form determinant of order 2.

Number of determinats

$$= 4 \times 3 \times 2 \times 1 = 24$$

Let's observe some determinants

$$\begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 6 & 2 \\ 4 & 8 \end{vmatrix} = 40$$

$$\begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 6 & 4 \\ 2 & 8 \end{vmatrix} = 40$$

$$\begin{vmatrix} 4 & 8 \\ 6 & 2 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 8 & 4 \\ 2 & 6 \end{vmatrix} = 40$$

$$\begin{vmatrix} 4 & 6 \\ 8 & 2 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = 40$$

Hence, we can see that we are getting a pattern where each determinant value will be neutralised by other value.

Hence, sum of the values of all determinants = 0

Hence, option (d) is correct.

56. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?

(a) 14 units (b) 10.5 units

(c) 7 units (d) 3.5 units

(d) Given equation of circle

$4x^2 + 4y^2 - 20x + 12y - 15 = 0$

$\Rightarrow x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$g = \frac{-5}{2}, f = \frac{3}{2}, c = \frac{-15}{4}$

$\therefore$ Radius $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{\frac{25}{4} + \frac{9}{4} + \frac{15}{4}} = \frac{7}{2} = 3.5 \text{ unit}$$

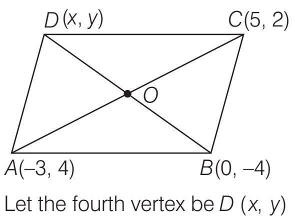
Hence, option (d) is correct.

57. A parallelogram has three consecutive vertices $(-3, 4), (0, -4)$ and $(5, 2)$. The fourth vertex is

(a) (2, 10) (b) (2, 9)

(c) (3, 9) (d) (4, 10)

(a)



Let the fourth vertex be $D(x, y)$

Diagonals of a parallelogram bisect each other.

O is mid-point of AC.

$\Rightarrow$ Coordinate of $O \left(\frac{-3+5}{2}, \frac{4+2}{2} \right)$ or (1, 3)

O is mid-point of BD.

$\Rightarrow$ Coordinate of O is $\left(\frac{x+0}{2}, \frac{y-4}{2} \right)$

or $\left(\frac{x}{2}, \frac{y-4}{2} \right)$

Therefore, compare the coordinate of O

$\Rightarrow \quad \frac{x}{2} = 1 \Rightarrow x = 2$

and $\quad \frac{y-4}{2} = 3 \Rightarrow y = 10$

Hence, the fourth vertex is (2, 10).

58. If the lines $y + px = 1$ and $y - qx = 2$ are perpendicular, then which one of the following is correct?

(a) $pq + 1 = 0$ (b) $p + q + 1 = 0$

(c) $pq - 1 = 0$ (d) $p - q + 1 = 0$

(c) Given $\quad y + px = 1 \quad \ldots$ (i)

$y - qx = 2 \quad \ldots$ (ii)

Eqs. (i) and (ii) are perpendicular

$\Rightarrow m_1 \cdot m_2 = -1$ where m_1 and m_2 are the slope of Eqs. (i) and (ii)

and $\quad m = \frac{-\text{ coefficient of } x}{\text{coefficient of } y}$

$\Rightarrow \frac{-p}{1} \times \frac{-(-q)}{1} = -1$

$\Rightarrow \quad -pq = -1$

$\Rightarrow \quad pq - 1 = 0$

Hence, option (c) is correct.

59. If A, B and C are in AP, then the straight line $Ax + 2By + C = 0$ will always pass through a fixed point. The fixed point is

(a) (0, 0) (b) (– 1, 1)

(c) (1, – 2) (d) (1, – 1)

(d) Given A, B, C are in AP.

$\Rightarrow \quad 2B = A + C$

$\Rightarrow \quad A - 2B + C = 0 \quad \ldots$ (i)

On comparing $A - 2B + C = 0$

with the given line $Ax + 2By + C = 0$,

we get $x = 1, y = -1$

Hence, line $Ax + 2By + C = 0$ will pass through (1, – 1)

Hence, option (d) is correct.

60. If the image of the point $(-4, 2)$ by a line mirror is $(4, -2)$, then what is the equation of the line mirror?

(a) $y = x$ (b) $y = 2x$

(c) $4y = x$ (d) $y = 4x$

(b) Let $A = (-4, 2)$

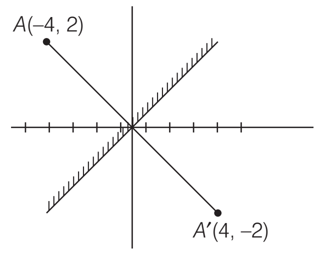


image point

$A' = (4, -2)$

$\therefore$ Mid-point of

$AA' = \left(\frac{-4+4}{2}, \frac{2+(-2)}{2} \right) = (0, 0)$

Slope of $AA' = \frac{-2-2}{4-(-4)}$

$= \frac{-4}{8} = \frac{-1}{2}$

Since, AA' and mirror line are perpendicular.

Slope of line mirror

$= \frac{-1}{\text{Slope of } AA'} = \frac{-1}{-1/2} = 2$

Equation of a line is $y - y_1 = m(x - x_1)$

$\therefore$ Equation of a line mirror is

$y - 0 = 2(x - 0)$

$\Rightarrow \quad y = 2x$

Hence, option (b) is correct.

61. Consider the following statements in respect of the points $(p, p - 3)$, $(q + 3, q)$ and (6, 3)

1. The points lie on a straight line.
2. The points always lie in the first quadrant only for any value of p and q.

Which of the above statement(s) is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Given points are $A(p, p-3)$, $B(q+3, q)$ and $C(6, 3)$

As, Points lies on a straight line, so slope of AB = slope of BC

$$\frac{q - p + 3}{q + 3 - p} = \frac{3 - q}{6 - q - 3}$$

$$\left[\because \text{slope of a line} = \frac{y_2 - y_1}{x_2 - x_1} \right]$$

$$1 = 1$$

$\therefore$ Statement 1 is correct.

But it's not necessary that the collinear points lie in the first quadrant only.

$\therefore$ Statement 2 is wrong.

Hence, option (a) is correct.

62. What is the acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x - y - 2 = 0$?

(a) 0° (b) 30° (c) 45° (d) 60°

(b) $l_1 : x - 2 = 0$

$l_2 : \sqrt{3}x - y - 2 = 0$

$\therefore$ Slope of line l_1, $m_1 = \dfrac{-\text{ Coefficient of } x}{\text{Coefficient of } y}$

$= \dfrac{-1}{0} = \infty$

The line l_1 is parallel to Y-axis or perpendicular to X-axis.

$\therefore$ Slope of line, l_2, $m_2 = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$

The line l_2 makes an angle 60° from positive X-axis.

$\therefore$ Angle between l_1 and

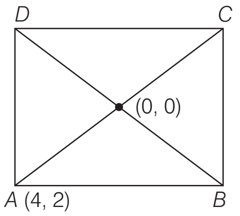
$l_2 = 90° - 60° = 30°$

Hence, option (b) is correct.

63. The point of intersection of diagonals of a square $ABCD$ is at the origin and one of its vertices is at $A(4, 2)$. What is the equation of the diagonal BD?

(a) $2x + y = 0$ (b) $2x - y = 0$
(c) $x + 2y = 0$ (d) $x - 2y = 0$

(a) Since, diagonal BD passes through the origin $O(0, 0)$.



$\because$ Slope of $OA = \dfrac{0-2}{0-4} = \dfrac{1}{2}$

$\because OA$ and OB are perpendicular to each other

$\therefore$ slope of $OB = \dfrac{-1}{\text{slope of } OA} = \dfrac{-1}{1/2}$

$= -2$

$\therefore$ Eqs. of BD having slope -2 and passes through (0, 0)

$y - 0 = -2\,[x - 0]$

[$\because$ Equation of a line $\Rightarrow y - y_1 = m(x - x_1)$]

$\Rightarrow \quad 2x + y = 0$

Hence, option (a) is correct.

64. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$, then what is the eccentricity of the hyperbola?

(a) $\dfrac{3}{2}$ (b) $\dfrac{5}{2}$ (c) $\dfrac{\sqrt{11}}{2}$ (d) $\dfrac{\sqrt{13}}{2}$

(d) Given point is $(3\tan\theta, 2\sec\theta)$

$\Rightarrow x = 3\tan\theta,\ y = 2\sec\theta$

$\dfrac{x}{3} = \tan\theta,\ \dfrac{y}{2} = \sec\theta$

$\because \sec^2\theta - \tan^2\theta = 1$

$\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$ which represents conjugate Hyperbola.

$\Rightarrow a^2 = 9,\ b^2 = 4$

$\therefore \quad e = \sqrt{1 + \dfrac{a^2}{b^2}}$

$= \sqrt{1 + \dfrac{9}{4}} = \sqrt{\dfrac{13}{4}}$

$e = \dfrac{\sqrt{13}}{2}$

Hence, option (d) is correct.

65. Consider the following with regard to eccentricity (e) of a conic section

1. $e = 0$ for circle
2. $e = 1$ for parabola
3. $e < 1$ for ellipse

Which of the above are correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(d) Since, we know that circle has eccentricity 0

and parabola has eccentricity 1

and ellipse has eccentricity $e < 1$

and hyperbola has eccentricity $e > 1$.

Hence, option (d) is correct.

66. What is the angle between the two lines having direction ratios $\langle 6, 3, 6 \rangle$ and $\langle 3, 3, 0 \rangle$?

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

(b) Direction ratios of line $l_1 = < 6, 3, 6 >$

$a_1 = 6, b_1 = 3, c_1 = 6$

Direction ratios of line $l_2 = < 3, 3, 0 >$

$\Rightarrow a_2 = 3, b_2 = 3, c_2 = 0$

$\because \cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \dfrac{6 \times 3 + 3 \times 3 + 6 \times 0}{\sqrt{6^2 + 3^2 + 6^2} \cdot \sqrt{3^2 + 3^2 + 0}}$

$\Rightarrow \cos\theta = \dfrac{27}{9 \times 3\sqrt{2}}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}} = \cos\dfrac{\pi}{4}$

$\therefore \quad \theta = \dfrac{\pi}{4}$

Hence, option (b) is correct.

67. If l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to?

(a) 1 (b) $\dfrac{11}{27}$ (c) $\dfrac{13}{27}$ (d) 4

(b) Given line is

$x - 1 = 2(y + 3) = 1 - z$

$\Rightarrow \dfrac{x-1}{2} = \dfrac{y+3}{1} = \dfrac{1-z}{2}$

$\Rightarrow \dfrac{x-1}{2} = \dfrac{y-(-3)}{1} = \dfrac{z-1}{-2}$

$\therefore$ Direction ratios are $<2, 1, -2>$

$\therefore$ Direction cosines are

$< \dfrac{2}{\sqrt{2^2 + 1^2 + (-2)^2}}, \dfrac{1}{\sqrt{2^2 + 1^2 + (-2)^2}}, \dfrac{-2}{\sqrt{2^2 + 1^2 + (-2)^2}}$

$\therefore l = \dfrac{2}{3}, m = \dfrac{1}{3}, n = \dfrac{-2}{3}$

$\therefore l^4 + m^4 + n^4 = \left(\dfrac{2}{3}\right)^4 + \left(\dfrac{1}{3}\right)^4 + \left(\dfrac{-2}{3}\right)^4$

$= \dfrac{16 + 1 + 16}{81} = \dfrac{33}{81} = \dfrac{11}{27}$

Hence, option (b) is correct.

68. What is the projection of the line segment joining $A(1, 7, -5)$ and $B(-3, 4, -2)$ on Y-axis?

(a) 5 (b) 4 (c) 3 (d) 2

(c) $A = (1, 7, -5)$ and $B = (-3, 4, -2)$

$\therefore$ Direction ratios of

$AB = < (-3-1), (4-7), (-2+5) >$

$= < -4, -3, 3 >$

$\Rightarrow a = -4, b = -3, c = 3$

Direction cosines of Y-axis $= < 0, 1, 0 >$

$l = 0, m = 1, n = 0$

$\therefore$ Projection of AB on Y-axis

$= | al + bm + cn |$

$= | -4 \times 0 + (-3) \times 1 + 3 \times 0 | = 3$

Hence, option (c) is correct.

69. What is the number of possible values of k for which the line joining the points $(k, 1, 3)$ and $(1, -2, k + 1)$ also passes through the point $(15, 2, -4)$?

(a) Zero (b) One (c) Two (d) Infinite

(c) Let $A = (k, 1, 3)$, $B = (1, -2, k + 1)$ and $C = (15, 2, -4)$

Since, line AB passes through C also.

Hence, points A, B and C are collinear.

$\therefore \quad \begin{vmatrix} k & 1 & 3 \\ 1 & -2 & k+1 \\ 15 & 2 & -4 \end{vmatrix} = 0$

$k(8 - 2k - 2) - 1(-4 - 15k - 15) + 3(2 + 30) = 0$

$6k - 2k^2 + 19 + 15k + 96 = 0$

$2k^2 - 21k - 115 = 0$ which is quadratic equation.

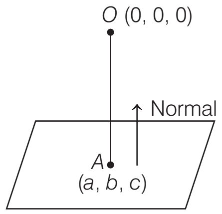
$\Rightarrow k$ has two values.

Hence, option (c) is correct.

70. The foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ is

(a) (0, 1, 2)
(b) (0, 0, 3)
(c) (1, 1, 1)
(d) (– 1, 1, 3)

(c) Let the foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ be (a, b, c).



Direction ratios of the plane = < 1, 1, 1 >

$\therefore$ Direction ratios of OA and normal will be in the same ratio.

$\therefore \frac{a-0}{1} = \frac{b-0}{1} = \frac{c-0}{1}$

$\Rightarrow a = 1, b = 1, c = 1$

$\therefore A = (1, 1, 1)$

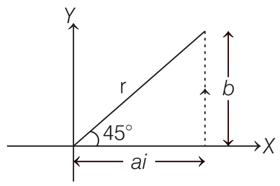
Hence, option (c) is correct.

71. A vector $\mathbf{r} = a\hat{i} + b\hat{j}$ is equally inclined to both x and y axes. If the magnitude of the vector is 2 units, then what are the values of a and b respectively?

(a) $\frac{1}{2}, \frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(c) $\sqrt{2}, \sqrt{2}$ (d) 2, 2

(c) $\mathbf{r} = a\hat{i} + b\hat{j}$

$|\mathbf{r}| = \sqrt{a^2 + b^2} = 2$



Since, r is equally inclined from X-axis and Y-axis.

Hence, r makes 45° from the X-axis.

$\therefore a = |\mathbf{r}| \cos 45^\circ$ and $b = |\mathbf{r}| \sin 45^\circ$

$a = 2 \times \frac{1}{\sqrt{2}}$, and $b = 2 \times \frac{1}{\sqrt{2}}$

$a = \sqrt{2}$ and $b = \sqrt{2}$

Hence, option (c) is correct.

72. Consider the following statements in respect of a vector $\mathbf{c} = \mathbf{a} + \mathbf{b}$, where $|\mathbf{a}| = |\mathbf{b}| \neq 0$

1. $\mathbf{c}$ is perpendicular to $(\mathbf{a} - \mathbf{b})$.
2. $\mathbf{c}$ is perpendicular to $(\mathbf{a} \times \mathbf{b})$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) $c = a + b$ where $a = b \neq 0$

Consider, $c \cdot (a - b) = (a + b) \cdot (a - b)$

$= |a|^2 - |b|^2 = |b|^2 - |b|^2 = 0$

$\Rightarrow c$ is perpendicular to $(a - b)$.

Also, $c \cdot (a \times b) = (a + b) \cdot (a \times b)$

$= a \cdot (a \times b) + b \cdot (a \times b)$

$= 0 + 0 = 0$

$\Rightarrow c$ is perpendicular to $(a \times b)$

Hence, option (c) is correct.

73. If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}| = 4$, then which one of the following is correct?

(a) a and b must be unit vectors
(b) a must be parallel to b
(c) a must be perpendicular to b
(d) a must be equal to b

(c) Given, $|a + b| = |a - b| = 4$

$\Rightarrow |a + b|^2 = |a - b|^2$

$|a|^2 + |b|^2 + 2a \cdot b$

$= |a|^2 + |b|^2 - 2a \cdot b$

$\Rightarrow 4a \cdot b = 0 \Rightarrow a \cdot b = 0$

$\Rightarrow a$ must be perpendicular to b.

Hence, option (c) is correct.

74. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar, then what is $(2\mathbf{a} \times 3\mathbf{b}) \cdot 4\mathbf{c} + (5\mathbf{b} \times 3\mathbf{c}) \cdot 6\mathbf{a}$ equal to?

(a) 114 (b) 66 (c) 0 (d) – 66

(c) Given that, a, b and c are coplanar

$\Rightarrow [a\ b\ c] = 0$ ... (i)

$\therefore (2a \times 3b) \cdot 4c + (5b \times 3c) \cdot 6a$

$= 2 \cdot 3 \cdot 4[a\ b\ c] + 5 \cdot 3 \cdot 6[b\ c\ a]$

$= 24[a\ b\ c] + 90[a\ b\ c]$ $\{\because [a\ b\ c] = [b\ c\ a]\}$

$= 24 \times 0 + 90 \times 0 = 0$

Hence, option (c) is correct.

75. Consider the following statements

1. The cross product of two unit vectors is always a unit vector.
2. The dot product of two unit vectors is always unity.
3. The magnitude of sum of two unit vectors is always greater than the magnitude of their difference.

Which of the above statements are not correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(d) Statement I

Let a and b are unit vectors

i.e. $|a| = |b| = 1$

$a \times b = |a||b| \sin\theta\, \hat{n}$

$|a \times b| = |a||b| \sin\theta$

$= \sin\theta \in [-1, 1]$

Therefore, statement I is incorrect.

Statement II

Let a and b are unit vectors

i.e. $a \cdot b = |a||b| \cos\theta$

$= \cos\theta \in [-1, 1]$

Therefore, statement II is also incorrect.

Statement III

Let $a = \hat{i}$ and $b = \hat{j}$

$\Rightarrow |a| = 1, |b| = 1$

$|a + b| = |\hat{i} + \hat{j}| = \sqrt{2}$

$|a - b| = |\hat{i} - \hat{j}| = \sqrt{2}$

76. If $\lim_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1$ then what is the value of a?

(a) – 1 (b) 0 (c) 1 (d) 2

(c) $\lim_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1$

$\Rightarrow \lim_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1 \left(\frac{0}{0} \text{ form}\right)$

By using L' Hospital rule,

$\lim_{x \to a} \frac{a^x \log_e a - ax^{a-1}}{ax^{a-1} - 0} = -1$

$\Rightarrow \frac{a^a \log_e a - a \cdot a^{a-1}}{a \cdot a^{a-1}} = -1$

$\Rightarrow \frac{a^a(\log_e a - 1)}{a^a} = -1$

$\Rightarrow \log_e a = -1 + 1$

$\Rightarrow \log_e a = 0$

$\therefore a = e^0 = 1$

$\therefore a = 1$

Hence, option (c) is correct.

77. A particle starts from origin with a velocity (in m/s) given by the equation $\frac{dx}{dt} = x + 1$. The time (in second) taken by the particle to traverse a distance of 24 m is

(a) ln24 (b) ln5
(c) 2ln5 (d) 2ln4

(c) $\frac{dx}{dt} = x + 1$

$\Rightarrow \frac{dx}{x+1} = dt$

On integrating both sides

$$\int \frac{dx}{x+1} = \int dt$$

$$\ln(x+1) = t + c \quad \ldots \text{(i)}$$

Since, at $t = 0$, distance $(x) = 0$

$\therefore \ln(0+1) = 0 + c$

$\boxed{0 = c}$

$\therefore \ln(x+1) = t$

At $x = 24$ m

$t = \ln(24+1) = \ln 25 = \ln 5^2$

$t = 2\ln 5$

Hence, option (c) is correct.

78. What is $\int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx$ equal to?

(a) a (b) $2a$

(c) 0 (d) $\frac{a}{2}$

(d) Let $I = \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx \quad \ldots \text{(i)}$

$x \to a - x$

$$I = \int_0^a \frac{f(a-a+x)}{f(a-x)+f(a-a+x)}\,dx$$

$$I = \int_0^a \frac{f(x)}{f(a-x)+f(x)}\,dx \quad \ldots \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_0^a \frac{f(a-x)+f(x)}{f(a-x)+f(x)} \cdot dx$$

$$2I = \int_0^a 1 \cdot dx$$

$$2I = x|_0^a$$

$$2I = a - 0$$

$$I = \frac{a}{2}$$

Hence, option (d) is correct.

79. What is $\lim_{x \to -1} \frac{x^3+x^2}{x^2+3x+2}$ equal to?

(a) 0 (b) 1 (c) 2 (d) 3

(b) Given, $\lim_{x \to -1} = \frac{x^3+x^2}{x^2+3x+2}\left(\frac{0}{0} \text{ from}\right)$

By using L' Hospital rule

$$\lim_{x \to -1} = \frac{3x^2+2x}{2x+3}$$

$$= \frac{3(-1)^2+2(-1)}{2(-1)+3} = \frac{3-2}{-2+3} = 1$$

Hence, option (b) is correct.

80. If $\int_0^a [f(x)+f(-x)]dx = \int_{-a}^a g(x)dx$ then what is $g(x)$ equal to?

(a) $f(x)$ (b) $f(-x)+f(x)$

(c) $-f(x)$ (d) None of these

(a) Given that,

$$\int_0^a [f(x)+f(-x)]\,dx = \int_{-a}^a g(x)\,dx$$

If $g(x) = f(x)$

R.H.S. Let $I = \int_{-a}^a g(x)\,dx$

$$\Rightarrow \quad I = \int_{-a}^a f(x)\,dx \quad \ldots \text{(i)}$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$I = \int_{-a}^a f(-x)\,dx \quad \ldots \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$2I = \int_{-a}^a [f(x)+f(-x)]\,dx$ (even function)

$\Rightarrow 2I = 2\int_0^a [f(x)+f(-x)]\,dx$

$I = \int_0^a [f(x)+f(-x)]dx =$ L.H.S

$\Rightarrow g(x) = f(x)$ and

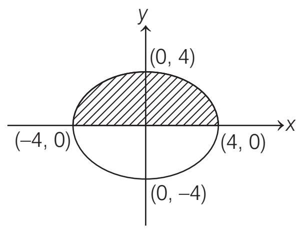
Hence, option (a) is correct.

81. What is the area bounded by $y = \sqrt{16-x^2}$, $y \geq 0$ and the X-axis?

(a) 16π sq. units (b) 8π sq. units

(c) 4π sq. units (d) 2π sq. units

(b) Shaded portion in the diagram represents the area bounded by $y = \sqrt{16-x^2}$, $y \geq 0$ and X-axis.



Put $y = 0$, then $16 - x^2 = 0$

$\Rightarrow x = \pm 4$

$\therefore$ Required area $= \int_{-4}^4 \sqrt{16-x^2}\,dx$

$= 2\int_0^4 \sqrt{16-x^2}\,dx$

$= 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$= 2\,[0 + 8\sin^{-1} 1] = 2 \times \frac{8\pi}{2}$

$= 8\pi$ sq units

Hence, option (b) is correct.

82. The curve $y = -x^3+3x^2+2x-27$ has the maximum slope at

(a) $x = -1$ (b) $x = 0$

(c) $x = 1$ d) $x = 2$

(c) Given that, $y = -x^3+3x^2+2x-27$

Slope $= \frac{dy}{dx} = -3x^2+6x+2$

$\therefore f'(x) = -3x^2+6x+2$

For maxima/minima of $f'(x)$.

$\frac{d}{dx}[f'(x)] = -6x+6 = 0$

$\Rightarrow 6x = 6 \Rightarrow x = 1$

At $x = 1$, $\frac{d^2}{dx^2} f'(x) = -6 < 0$

$\therefore$ At $x = 1$, $f'(x)$ is maximum.

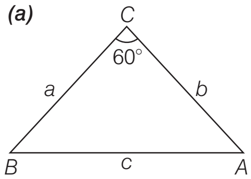
Hence, option (c) is correct.

83. A 24 cm long wire is bent to form a triangle with one of the angles as 60°. What is the altitude of the triangle having the greatest possible area?

(a) $4\sqrt{3}$ cm (b) $2\sqrt{3}$ cm

(c) 6 cm (d) 3 cm

(a)



Given, $a + b + c = 24$

$\Rightarrow \quad c = 24 - (a+b)$

Again $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos 60° = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \quad \frac{1}{2} = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \quad ab = a^2+b^2-c^2$

$\Rightarrow \quad ab = a^2+b^2-[24-(a+b)]^2$

$\Rightarrow \quad ab = a^2+b^2-576 - (a+b)^2 + 48(a+b)$

$\Rightarrow ab = a^2+b^2-576-a^2-b^2 - 2ab + 48(a+b)$

$\Rightarrow 3ab - 48(a+b) = -576$

$\Rightarrow \quad ab - 16(a+b) = -192$

$\Rightarrow \quad ab - 16a = 16b - 192$

$\Rightarrow \quad a(b-16) = 16(b-12)$

$\Rightarrow \quad a = \frac{16(b-12)}{b-16}$

Again $ar(\Delta ABC)$, $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times \frac{16(b-12)b}{b-16} \times \sin 60°$

$= \frac{1}{2} \times \frac{16(b-12)b}{b-16} \times \frac{\sqrt{3}}{2}$

$= \frac{4\sqrt{3}(b^2-12b)}{b-16}$

$\therefore \frac{dA}{db} = 4\sqrt{3}\left[\frac{(2b-12)(b-16)-(b^2-12b)\cdot 1}{b-16}\right]$

Maximum value for A

$$\frac{dA}{db} = 0$$

$\Rightarrow \frac{4\sqrt{3}}{b-16}[2b^2-32b-12b+192-b^2+12b] = 0$

$\Rightarrow b^2 - 32b + 192 = 0$

$\Rightarrow (b-24)(b-8) = 0$

$\Rightarrow b = 24, 8$

when, $b = 24$

$$a = \frac{16(24-12)}{24-16} = \frac{16\times 12}{8} = 24$$

and $c = 24-(24+24) = -24$

It is impossible,

when, $b = 8$

$$a = \frac{16(8-12)}{8-16} = \frac{16(-4)}{(-8)} = 8$$

and $c = 24-(8+8) = 8$

$\therefore$ So triangle will be equilateral

$\therefore$ Height $= \frac{\sqrt{3}}{2}$ (side)

$= \frac{\sqrt{3}}{2}\times 8 = 4\sqrt{3}$ cm

84. If $f(x) = e^{|x|}$, then which one of the following is correct?

(a) $f'(0) = 1$ (b) $f'(0) = -1$

(c) $f'(0) = 0$ (d) $f'(0)$ does not exist

(d) Given that, $f(x) = e^{|x|}$

$$\Rightarrow f(x) = \begin{cases} e^x & ; x \ge 0 \\ e^{-x} & ; x < 0 \end{cases}$$

LHD at $x = 0$,

$$f'(0^-) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{e^{-h}-e^0}{h}$$

(by using L' Hospital rule)

$$= \lim_{h\to 0^-} -\frac{e^{-h}}{1} = -1$$

RHD at $x = 0$,

$$f'(0^+) = \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{e^h - e^0}{h}$$

$$= \lim_{h\to 0^+} \frac{e^h}{1} = e^0 = 1$$

$\because$ LHD $\ne$ RHD

$\therefore f'(x)$ does not exist at $x = 0$

Hence, option (d) is correct.

85. What is $\int \frac{dx}{\sec x + \tan x}$ equal to?

(a) $\ln(\sec x) + \ln|\sec x + \tan x| + c$

(b) $\ln(\sec x) - \ln|\sec x + \tan x| + c$

(c) $\sec x \tan x - \ln|\sec x - \tan x| + c$

(d) $\ln|\sec x + \tan x| - \ln|\sec x| + c$

(d) Let $I = \int \frac{dx}{\sec x + \tan x}$

$$I = \int \frac{1}{(\sec x + \tan x)} \times \frac{(\sec x - \tan x)}{(\sec x - \tan x)} dx$$

$$I = \int \frac{(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$$

$$I = \int \frac{(\sec x - \tan x)}{1} dx$$

$$= \int \sec x\, dx - \int \tan x\, dx$$

$I = \log|\sec x + \tan x| - \log|\sec x| + C$

Hence, option (d) is correct.

86. What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$ (b) $\tan^{-1} x + c$

(c) $\sec^{-1} x + c$ (d) $\cos^{-1} x + c$

(b) Let $I = \int \frac{dx}{\sec^2(\tan^{-1} x)}$

$$I = \int \frac{dx}{1+\tan^2(\tan^{-1} x)}$$

$[\because \sec^2 x = 1 + \tan^2 x]$

$$= \int \frac{dx}{1+x^2}$$

$I = \tan^{-1} x + C$

Hence, option (b) is correct.

87. If $x + y = 20$ and $P = xy$, then what is the maximum value of P?

(a) 100 (b) 96

(c) 84 (d) 50

(a) Given, $x + y = 20$

$\Rightarrow y = 20 - x$

$P = xy$

$P = x(20 - x)$

$P = 20x - x^2$

$\therefore \frac{dP}{dx} = 20 - 2x$

For maxima/minima, $\frac{dP}{dx} = 0$

$20 - 2x = 0$

$\Rightarrow x = 10$

At $x = 10$, $\frac{d^2P}{dx^2} = -2 < 0$

$\therefore P$ is maximum at $x = 10$

$\Rightarrow y = 20 - 10 = 10$

$\therefore$ Maximum value of $P = xy$

$= 10 \times 10 = 100$

Hence, option (a) is correct.

88. What is the derivative of $\sin(\ln x) + \cos(\ln x)$ with respect to x at $x = e$?

(a) $\frac{\cos 1 - \sin 1}{e}$ (b) $\frac{\sin 1 - \cos 1}{e}$

(c) $\frac{\cos 1 + \sin 1}{e}$ (d) 0

(a) Let $y = \sin(\ln x) + \cos(\ln x)$

$$\therefore \frac{dy}{dx} = \cos(\ln x)\cdot\frac{1}{x} + \left(-\sin(\ln x)\cdot\frac{1}{x}\right)$$

$$= \frac{1}{x}[\cos(\ln x) - \sin(\ln x)]$$

At $x = e$,

$$\frac{dy}{dx} = \frac{1}{e}[\cos(\ln e) - \sin(\ln e)]$$

$$= \frac{1}{e}[\cos 1 - \sin 1]$$

$[\because \ln e = 1]$

Hence, option (a) is correct.

89. If $x = e^t \cos t$ and $y = e^t \sin t$, then what is $\frac{dx}{dy}$ at $t = 0$ equal to?

(a) 0 (b) 1 (c) $2e$ (d) -1

(b) Given that, $x = e^t \cos t$, $y = e^t \sin t$

$$\therefore \frac{dx}{dt} = e^t \frac{d}{dt}\cos t + \cos t \frac{d}{dt} e^t$$

$$= e^t(-\sin t) + \cos t \cdot e^t$$

$$\frac{dy}{dt} = e^t \frac{d}{dt}\sin t + \sin t \cdot \frac{d}{dt} e^t$$

$$\frac{dy}{dt} = e^t \cos t + e^t \sin t$$

$$\therefore \frac{dx}{dy} = \frac{dx/dt}{dy/dt}$$

$$\frac{dx}{dy} = \frac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)}$$

At $t = 0$,

$$\therefore \frac{dx}{dy} = \frac{\cos 0^\circ - \sin 0^\circ}{\cos 0^\circ + \sin 0^\circ} = \frac{1-0}{1+0}$$

$$\left(\frac{dx}{dy}\right)_{t=0} = 1$$

Hence, option (b) is correct.

90. What is the maximum value of $\sin 2x \cdot \cos 2x$?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

(a) Let $y = \sin 2x \cdot \cos 2x$

$$y = \frac{1}{2}[2\sin 2x \cdot \cos 2x]$$

$$y = \frac{1}{2}\sin 4x$$

Since, we know that

$$-1 \le \sin 4x \le 1 \Rightarrow \frac{-1}{2} \le \frac{1}{2}\sin 4x \le \frac{1}{2}$$

$\therefore$ Maximum value $= \frac{1}{2}$

Hence, option (a) is correct.

91. What is the derivative of e^x with respect to x^e?

(a) $\frac{xe^x}{ex^e}$ (b) $\frac{e^x}{x^e}$ (c) $\frac{xe^x}{x^e}$ (d) $\frac{e^x}{ex^e}$

(a) Let $y_1 = e^x$ and $y_2 = x^e$

$$\therefore \frac{dy_1}{dx} = e^x, \frac{dy_2}{dx} = ex^{e-1}$$

$\therefore \quad \dfrac{dy_1}{dy_2} = \dfrac{e^x}{ex^{e-1}} = \dfrac{xe^x}{ex^e}$

Hence, option (a) is correct.

92. If a differentiable function $f(x)$ satisfies $\lim_{x \to -1} \dfrac{f(x)+1}{x^2-1} = -\dfrac{3}{2}$, then what is $\lim_{x \to -1} f(x)$ equal to?

(a) $-\dfrac{3}{2}$ (b) -1

(c) 0 (d) 1

(b) Given, $\lim_{x \to -1} \dfrac{f(x)+1}{x^2-1} = \dfrac{-3}{2}$

$\because \lim_{x \to -1} \dfrac{f(x)+1}{x^2-1}$ has denominator 0 at $x = -1$

$\Rightarrow \lim_{x \to -1} f(x) + 1 = 0$

$\Rightarrow \lim_{x \to -1} f(x) = -1$

Hence, option (b) is correct.

93. If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a + b)$?

(a) 5 (b) 10

(c) 15 (d) 20

(a) Given that, $f(x) = \begin{cases} a + bx & ; \ x < 1 \\ 5 & ; \ x = 1 \\ b - ax & ; \ x > 1 \end{cases}$

$\because f(x)$ is continuous.

$\Rightarrow f(x)$ will be continuous at $x = 1$

$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$

$\lim_{x \to 1^-} (a + bx) = 5 = \lim_{x \to 1^+} (b - ax)$

$a + b = 5 = b - a$

$\Rightarrow \quad a + b = 5$

Hence, option (a) is correct.

94. Consider the following statements in respect of the function $f(x) = \sin x$

1. $f(x)$ increases in the interval $(0, \pi)$.
2. $f(x)$ decreases in the interval $\left(\dfrac{5\pi}{2}, 3\pi\right)$.

Which of the above statement is/are correct?

(a) 1 only

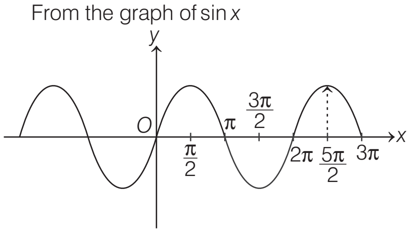
(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

(b) Given, $f(x) = \sin x$

From the graph of $\sin x$



We can see that $f(x)$ increases in $\left[0, \dfrac{\pi}{2}\right]$ and decreases in $\left[\dfrac{\pi}{2}, \pi\right]$ and $\left(\dfrac{5\pi}{2}, 3\pi\right)$.

$\Rightarrow$ Statement-1 is wrong and Statement-2 is correct.

Hence, option (b) is correct.

95. What is the domain of the function $f(x) = 3^x$?

(a) $(-\infty, \infty)$ (b) $(0, \infty)$

(c) $[0, \infty)$ (d) $(-\infty, \infty) - \{0\}$

(a) Given, $f(x) = 3^x$

$\because$ We know that, domain of exponential function is $(-\infty, \infty)$.

$\therefore$ Domain of $3^x = (-\infty, \infty)$

Hence, option (a) is correct.

96. If the general solution of a differential equation is $y^2 + 2cy - cx + c^2 = 0$, where c is an arbitrary constant, then what is the order of the differential equation?

(a) 1 (b) 2 (c) 3 (d) 4

(a) Given that, $y^2 + 2cy - cx + c^2 = 0$

Since, the above equation contains only one variable constant.

Hence, order of the differential equation = 1

Hence, option (a) is correct.

97. What is the degree of the following differential equation?

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

(a) 1

(b) 2

(c) 3

(d) Degree is not defined

(a) Let $x = \sqrt{1 + \dfrac{d^2y}{dx^2}}$

$\Rightarrow x^2 = 1 + \dfrac{d^2y}{dx^2} \Rightarrow \left(\dfrac{d^2y}{dx^2}\right)^1 = x^2 - 1$

$\therefore$ Degree = exponent of highest order derivative = 1

Hence, option (a) is correct.

98. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$?

(a) $\dfrac{d^2y}{dx^2} + y = 0$ (b) $\dfrac{d^2y}{dx^2} - y = 0$

(c) $\dfrac{d^2y}{dx^2} + y = 1$ (d) $\dfrac{dy}{dx} - y = 0$

(b) Given, $y = ae^x + be^{-x}$

$\therefore \quad \dfrac{dy}{dx} = ae^x - be^{-x}$

$\dfrac{d^2y}{dx^2} = ae^x + be^{-x} = y$

$\Rightarrow \dfrac{d^2y}{dx^2} - y = 0$

Hence, option (b) is correct.

99. What is the solution of the following differential equation?

$$\ln\left(\frac{dy}{dx}\right) + y = x$$

(a) $e^x + e^y = c$ (b) $e^{x+y} = c$

(c) $e^x - e^y = c$ (d) $e^{x-y} = c$

(c) Given, $\ln\left(\dfrac{dy}{dx}\right) + y = x$

$\Rightarrow \quad \ln\left(\dfrac{dy}{dx}\right) = x - y$

$\Rightarrow \quad \dfrac{dy}{dx} = e^{x-y}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{e^x}{e^y}$

$\Rightarrow \quad e^y dy = e^x dx$

On integrating both sides,

$\int e^y dy = \int e^x dx$

$e^y + c = e^x \Rightarrow e^x - e^y = c$

Hence, option (c) is correct.

100. What is $\int e^{(2 \ln x + \ln x^2)} dx$ equal to?

(a) $\dfrac{x^4}{4} + C$ (b) $\dfrac{x^3}{3} + C$

(c) $\dfrac{2x^5}{5} + C$ (d) $\dfrac{x^5}{5} + C$

(d) Let $I = \int e^{(2 \ln x + \ln x^2)} dx$

$= \int e^{(\ln x^2 + \ln x^2)} dx$

$= \int e^{2 \ln x^2} dx = \int e^{\ln (x^2)^2} dx = \int x^4 dx$

$I = \dfrac{x^5}{5} + C$

Hence, option (d) is correct.

101. Consider the following measures of central tendency for a set of N numbers

1. Arithmetic mean
2. Geometric mean

Which of the above uses/use all the data?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Since, we know that the measures of central tendency are Mean, Median and Mode.

Where Arithmetic Mean and Geometric mean are the type mean.

Hence, option (c) is correct.

102. The numbers of Science, Arts and Commerce graduates working in a company are 30, 70 and 50 respectively. If these figures are represented by a pie chart, then what is the angle corresponding to Science graduates?

(a) 36° (b) 72°
(c) 120° (d) 168°

(b) The ratio of Science, Arts and Commerce graduates

$= 30 : 70 : 50 = 3 : 7 : 5$

$\therefore$ Angle corresponding to Science graduates $= \dfrac{3}{3+7+5} \times 360^\circ$

$= \dfrac{3}{15} \times 360^\circ = 72^\circ$

Hence, option (b) is correct.

103. For a histogram based on a frequency distribution with unequal class intervals, the frequency of a class should be proportional to

(a) the height of the rectangle
(b) the area of the rectangle
(c) the width of the rectangle
(d) the perimeter of the rectangle

(b) Since, we know that for a histogram, based on a frequency distribution with equal intervals, the frequency of a class is proportional to height of the rectangle and for a histogram based on frequency distribution with unequal intervals, the frequency of a class is proportional to Area of the rectangle.

Hence, option (b) is correct.

104. The coefficient of correlation is independent of

(a) change of scale only
(b) change of origin only
(c) both change of scale and change of origin
(d) neither change of scale nor change of origin

(c) Since, we know that coefficient of correlation is independent of both change of scale and change of origin.

Hence, option (c) is correct.

105. The following table gives the frequency distribution of number of peas per pea pod of 198 pods

Number of peas	1	2	3	4	5	6	7
Frequency	4	33	76	50	26	8	1

What is the median of this distribution?

(a) 3 (b) 4 (c) 5 (d) 6

(a)

Number of Peas	Frequency	Cumulative frequency
1	4	4
2	33	37
3	76	113
4	50	163
5	26	189
6	8	197
7	1	198
	$\Sigma f = 198$	

$\because N = 198, \dfrac{N}{2} = \dfrac{198}{2} = 99$

$$\text{Median} = \frac{\frac{N}{2}\text{th term} + \left(\frac{N}{2}+1\right)\text{th term}}{2}$$

$= \dfrac{99\text{th term} + 100\text{th term}}{2}$

$= \dfrac{3+3}{2} = 3$

$\therefore$ Median = 3

Hence, option (a) is correct.

106. If M is the mean of n observations $x_1 - k, x_2 - k, x_3 - k, \ldots, x_n - k$, where k is any real number, then what is the mean of $x_1, x_2, x_3, \ldots, x_n$?

(a) M (b) $M + k$
(c) $M - k$ (d) kM

(b) Given that,

Mean of $x_1 - k, x_2 - k, x_3 - k, \ldots, x_n - k$

$\therefore M = \dfrac{(x_1 - k) + (x_2 - k) + \ldots + (x_n - k)}{n}$

$M = \dfrac{(x_1 + x_2 + \ldots + x_n)}{n} - \dfrac{nk}{n}$

$M + k = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$

$\therefore$ Mean of $x_1, x_2, x_3, \ldots, x_n = M + k$

Hence, option (b) is correct.

107. What is the sum of deviations of the variate values 73, 85, 92, 105, 120 from their mean?

(a) – 2 (b) – 1
(c) 0 (d) 5

(c) Mean of 73, 85, 92, 105, 120

$\bar{x} = \dfrac{73 + 85 + 92 + 105 + 120}{5}$

$= \dfrac{475}{5}$

$\bar{x} = 95$

$\therefore$ Sum of deviations from their mean

$= (73 - 95) + (85 - 95) + (92 - 95) + (105 - 95) + (120 - 95)$

$= -22 - 10 - 3 + 10 + 25 = 0$

Hence, option (c) is correct.

108. Let x be the HM and y be the GM of two positive numbers m and n. If $5x = 4y$, then which one of the following is correct?

(a) $5m = 4n$ (b) $2m = n$
(c) $4m = 5n$ (d) $m = 4n$

(d) Given, two positive numbers are m and n.

$\therefore$ H.M. of m and $n = \dfrac{2mn}{m+n}$

$x = \dfrac{2mn}{m+n}$ … (i)

G.M. of m and $n = \sqrt{mn}$

$y = \sqrt{mn}$ … (ii)

$\because 5x = 4y$

$5\left(\dfrac{2mn}{m+n}\right) = 4\sqrt{mn}$

Squaring both sides, we get

$\left(\dfrac{5mn}{m+n}\right)^2 = (2\sqrt{mn})^2$

$\Rightarrow \dfrac{25m^2n^2}{m^2 + n^2 + 2mn} = 4mn$

$\Rightarrow 25mn = 4m^2 + 4n^2 + 8mn$ $[\because m \neq 0, n \neq 0]$

$\Rightarrow 4m^2 + 4n^2 - 17mn = 0$

$\Rightarrow 4m^2 - 16mn - mn + 4n^2 = 0$

$\Rightarrow 4m(m - 4n) - n(m - 4n) = 0$

$\Rightarrow (m - 4n)(4m - n) = 0$

$\Rightarrow m = 4n$ or $n = 4m$

Hence, option (d) is correct.

109. If the mean of a frequency distribution is 100 and the coefficient of variation is 45%, then what is the value of the variance?

(a) 2025 (b) 450
(c) 45 (d) 4.5

(a) Since, we know that

Coefficient of variation

$(CV) = \dfrac{\sigma}{x} \times 100$ … (i)

Where σ is standard deviation and $\bar{x}$ is mean.

Given, $\bar{x} = 100$ and $CV = 45\%$

From Eqs. (i), $45 = \frac{\sigma}{100} \times 100$

$\Rightarrow \quad \sigma = 45$

$\therefore$ Variance $= \sigma^2 = (45)^2 = 2025$

Hence, option (a) is correct.

110. Let two events A and B be such that $P(A) = L$ and $P(B) = M$. Which one of the following is correct?

(a) $P(A|B) < \frac{L + M - 1}{M}$

(b) $P(A|B) > \frac{L + M - 1}{M}$

(c) $P(A|B) \geq \frac{L + M - 1}{M}$

(d) $P(A|B) = \frac{L + M - 1}{M}$

(c) Given, $P(A) = L, P(B) = M$

$\because P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$P(A \mid B) = \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$

$P(A \mid B) = \frac{L + M - P(A \cup B)}{P(B)}$

$\therefore \; P(A \cup B) = L + M - P(B)P(A \mid B)$

$\because 0 \leq P(A \cup B) \leq 1$

$\Rightarrow L + M - P(B) \cdot P(A \mid B) \leq 1$

$\Rightarrow P(B) \cdot P(A \mid B) \geq L + M - 1$

$P(A \mid B) \geq \frac{L + M - 1}{M} \quad [\because P(B) = M]$

Hence, option (c) is correct.

111. For which of the following sets of numbers do the mean, median and mode have the same value?

(a) 12, 12, 12, 12, 24
(b) 6, 18, 18, 18, 30
(c) 6, 6, 12, 30, 36
(d) 6, 6, 6, 12, 30

(b) For option (a), Mean

$= \frac{12 + 12 + 12 + 12 + 24}{5}$

$= 14 \cdot 4 \neq$ mode (12)

For option (b), Mean

$= \frac{6 + 18 + 18 + 18 + 30}{5} = 18$

Mode = 18, Median = 18

Hence, for the data 6, 18, 18, 18, 30,

Mean = Mode = Median = 18

Hence, option (b) is correct.

112. The mean of 12 observations is 75. If two observations are discarded, then the mean of the remaining observations is 65. What is the mean of the discarded observations?

(a) 250
(b) 125
(c) 120
(d) Cannot be determined due to insufficient data

(b) Given, mean of 12 observations = 75 and M.

$\therefore \quad \bar{x} = \frac{\sum_{i=1}^{12} x_i}{12} \Rightarrow 75 = \frac{\sum_{i=1}^{12} x_i}{12}$

$\Rightarrow \quad \sum_{i=1}^{12} x_i = 900 \quad \ldots \text{(i)}$

Let observations x_{11} and x_{12} is discarded

then mean $= \frac{\sum_{i=1}^{10} x_i}{10} = 65$

$\therefore \quad \sum_{i=1}^{10} x_i = 10 \times 65 = 650 \quad \ldots \text{(ii)}$

From Eqs. (i)

$\sum_{i=1}^{12} x_i = 900$

$\Rightarrow \sum_{i=1}^{10} x_i + x_{11} + x_{12} = 900$

$\Rightarrow 650 + x_{11} + x_{12} = 900$

$\Rightarrow \quad x_{11} + x_{12} = 250$

$\therefore$ Mean of x_{11} and $x_{12} = \frac{250}{2} = 125$

Hence, option (b) is correct.

113. If k is one of the roots of the equation $x(x+1)+1=0$, then what is its other root?

(a) 1 (b) $-k$
(c) k^2 (d) $-k^2$

(c) Given, quadratic equation

$x(x + 1) + 1 = 0$

$x^2 + x + 1 = 0 \quad \ldots \text{(i)}$

Since, we know that ω, ω^2 are the roots of Equation when

$\omega = \frac{-1 + \sqrt{3}i}{2}$ and $\omega^2 = \frac{-1 - \sqrt{3}i}{2}$

$\Rightarrow$ If one of the roots of Eqs. (i) is k, then other root will be k^2.

Hence, option (c) is correct.

114. The geometric mean of a set of observations is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is

(a) 810 (b) 900
(c) 30000 (d) 81000

(c) Given that, geometric mean of a set of observations = 10

Since, we know that if

Geometric mean of $x_1, x_2, x_3, \ldots x_n$ is G.

$\Rightarrow$ Geometric mean of $x_1^2, x_2^2, x_3^2, \ldots x_n^2$ is G^2

Geometric mean of $fx_1^2, fx_2^2, \ldots, fx_n^2$ is fG^2.

$\therefore$ Required geometric mean $= 3(10)^4$

$= 3 \times 10000 = 30000$

Hence, option (c) is correct.

115. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$, then which of the following is/are correct?

1. A and B are independent events.
2. A and B are mutually exclusive events.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Given, $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$

$P(\bar{A}) = \frac{1}{2}$

$\Rightarrow \quad P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore \; P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= \frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{4}{6} = \frac{2}{3}$

$\because$ If A and B are independents, then $P(A \cap B) = P(A) \cdot P(B)$

$\because P(A \cap B) = \frac{1}{3}$

and $P(A) \cdot P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

$\Rightarrow$ Statement-1 is correct.

If A and B are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$

$\because \quad P(A \cup B) = \frac{5}{6}$

$\therefore \quad P(A) + P(B) = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

$\Rightarrow$ Statement-2 is wrong.

Hence, option (a) is correct.

116. The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is

(a) reduced by $\frac{1}{3}$ (b) increased by $\frac{10}{3}$
(c) reduced by $\frac{10}{3}$ (d) reduced by 50

(c) Let unit digit for wrongly recorded observation = b

When tens digit is 8, then number

$= 10 \times 8 + b = 80 + b$

When tens digit is 3, then number

$= 10 \times 3 + b = 30 + b$

$\Rightarrow$ One observation is recorded $((80 + b) - (30 + b))$ more while calculating average.

Hence, after correcting the observation, the average will be reduced by

$$= \frac{\{(80 + b) - (30 + b)\}}{15}$$

$$= \frac{50}{15} = \frac{10}{3}$$

Hence, option (c) is correct.

117. A coin is tossed twice. If E and F denote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

(c) Given that, a coin is tossed twice.

$\therefore \quad S = \{HH, HT, TH, TT\}$

Given, E be the event of occurrence of head on first toss and F be the event of occurrence of head on second toss.

$\therefore \quad E = \{HH, HT\}$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$F = \{TH, HH\}, P(F) = \frac{2}{4} = \frac{1}{2}$

$\because \quad E \cap F = \{HH\}$

$\therefore P(E \cap F) = \frac{1}{4}$

$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Hence, option (c) is correct.

118. In a binomial distribution, the mean is $\frac{2}{3}$ and variance is $\frac{5}{9}$. What is the probability that random variable $X = 2$?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$

(c) $\frac{25}{54}$ (d) $\frac{25}{216}$

(d) For a binomial distribution mean $= np$ and variance $= npq$

Where p is probability of success and q is the probability of unsuccess and n is number of observations.

$\therefore$ Given, $np = \frac{2}{3}$, $npq = \frac{5}{9}$

$\Rightarrow \quad \frac{2}{3} \cdot q = \frac{5}{9}$

$q = \frac{5}{6}$

$\therefore \quad p = 1 - q = 1 - \frac{5}{6}$

$= \frac{1}{6}$

$\because \quad np = \frac{2}{3}$

$\Rightarrow \quad n\left(\frac{1}{6}\right) = \frac{2}{3}$

$\Rightarrow \quad n = 4$

$\because p(X = x) = {}^nC_x p^x q^{n-x}$

$\therefore P(X = 2) = {}^4C_2 p^2 q^{4-2}$

$= 6 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = \frac{25}{216}$

Hence, option (d) is correct.

119. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x?

(a) 10 (b) 12

(c) 13 (d) 15

(d) Given, observations are

10, 12, 13, 15, 15, 13, 12, 10, x

$\because \quad$ Mode = 15

Scores	Frequency
10	2
12	2
13	2
15	2

Since, frequency of all other numbers is same as frequency of 15.

But mode is the number of highest frequency.

$\therefore \quad x$ should be 15

Hence, option (d) is correct.

120. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then consider the following statements

1. The minimum value of $P(A \cup B)$ is $\frac{3}{4}$.
2. The maximum value of $P(A \cap B)$ is $\frac{5}{8}$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

(c) Given, $P(A) = \frac{3}{4}$

$P(B) = \frac{5}{8}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Here $P(A \cup B)$ will be minimum if $P(A \cap B)$ is maximum and *vice-versa.*

Since, minimum value of $P(A \cap B)$ is zero and maximum value of $P(A \cap B)$ is minimum $(P(A), P(B))$.

$\Rightarrow$ maximum $P(A \cap B)$

$= \text{minimum}\left(\frac{3}{4}, \frac{5}{8}\right)$

$= \frac{5}{8}$

$\Rightarrow$ Statement-2 is correct.

Also, minimum value of $P(A \cup B)$ is maximum $(P(A), P(B))$

$\therefore$ Minimum value of

$P(A \cup B) = \text{maximum}\left(\frac{3}{4}, \frac{5}{8}\right)$

$= \frac{3}{4}$

$\Rightarrow$ Statement-1 is correct.

Hence, correct option is (c).