NDA SOLVED PAPER 2020-I

MATHEMATICS

1. What is the modulus of the complex number $\dfrac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$, where $i = \sqrt{-1}$?

   (a) $\dfrac{1}{2}$　(b) 1　(c) $\dfrac{3}{2}$　(d) 2

2. Consider the proper subsets of $\{1, 2, 3, 4\}$. How many of these proper subsets are superset of the set $\{3\}$?

   (a) 5　(b) 6　(c) 7　(d) 8

3. Let p, q and r be three distinct positive real numbers. If

   $$D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix},$$ then which one of the following is correct?

   (a) $D < 0$

   (b) $D \leq 0$

   (c) $D > 0$

   (d) $D \geq 0$

4. What is the sum of the last five coefficients in the expansion of $(1 + x)^9$ when it is expanded in ascending powers of x?

   (a) 256　(b) 512　(c) 1024　(d) 2048

5. Consider the following in respect of a non-singular matrix of order 3 :
   1. $A(\text{adj } A) = (\text{adj } A) A$
   2. $|\text{adj } A| = |A|$

   Which of the above statements is/are correct?

   (a) 1 only

   (b) 2 only

   (c) Both 1 and 2

   (d) Neither 1 nor 2

6. The center of the circle

   $(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$ is

   (a) $(2a, 2c)$

   (b) $(2b, 2d)$

   (c) $(a + b, c + d)$

   (d) $(a - b, c - d)$

7. The point $(1, -1)$ is one of the vertices of a square. If $3x + 2y = 5$ is the equation of one diagonal of the square, then what is the equation of the other diagonal?

   (a) $3x - 2y = 5$

   (b) $2x - 3y = 1$

   (c) $2x - 3y = 5$

   (d) $2x + 3y = -1$

8. Let $P(x, y)$ be any point on the ellipse $25x^2 + 16y^2 = 400$. If $Q(0, 3)$ and $R(0, -3)$ are two points, then what is $(PQ + PR)$ equal to ?

   (a) 12　(b) 10　(c) 8　(d) 6

9. If the circumcentre of the triangle formed by the lines $x + 2 = 0, y + 2 = 0$ and $kx + y + 2 = 0$ is $(-1, -1)$, then what is the value of k ?

   (a) -1

   (b) -2

   (c) 1

   (d) 2

10. In the parabola, $y^2 = x$, what is the length of the chord passing through the vertex and inclined to the x-axis at an angle θ?

    (a) $\sin\theta \cdot \sec^2\theta$

    (b) $\cos\theta \cdot \operatorname{cosec}^2\theta$

    (c) $\cot\theta \cdot \sec^2\theta$

    (d) $2\tan\theta \cdot \operatorname{cosec}^2\theta$

11. Under which condition, are the points (a, b), (c, d) and $(a - c, b - d)$ collinear?

    (a) $ab = cd$

    (b) $ac = bd$

    (c) $ad = bc$

    (d) $abc = d$

12. Let ABC be a triangle. If $D(2, 5)$ and $E(5, 9)$ are the mid-points of the sides AB and AC respectively, then what is the length of the side BC?

    (a) 8

    (b) 10

    (c) 12

    (d) 14

13. If the foot of the perpendicular drawn from the point $(0, k)$ to the line $3x - 4y - 5 = 0$ is $(3, 1)$, then what is the value of k?

    (a) 3

    (b) 4

    (c) 5

    (d) 6

14. What is the obtuse angle between the lines whose slopes are $2 - \sqrt{3}$ and $2 + \sqrt{3}$?

    (a) $105°$

    (b) $120°$

    (c) $135°$

    (d) $150°$

15. If $3x - 4y - 5 = 0$ and $3x - 4y + 15 = 0$ are the equations of a pair of opposite sides of a square, then what is the area of the square?

    (a) 4 square units

    (b) 9 square units

    (c) 16 square units

    (d) 25 square units

DIRECTIONS (Qs. 16-18) : *Read the following information and answer the three items that follow:*

Let $a\sin^2 x + b\cos^2 x = c$; $b\sin^2 y + a\cos^2 y = d$ and $p\tan x = q\tan y$.

16. What is $\tan^2 x$ equal to ?

    (a) $\dfrac{c - b}{a - c}$

    (b) $\dfrac{a - c}{c - b}$

    (c) $\dfrac{c - a}{c - b}$

    (d) $\dfrac{c - b}{c - a}$

17. What is $\dfrac{d-a}{b-d}$ equal to?

    (a) $\sin^2 y$ (b) $\cos^2 y$

    (c) $\tan^2 y$ (d) $\cot^2 y$

18. What is $\dfrac{p^2}{q^2}$ equal to?

    (a) $\dfrac{(b-c)(b-d)}{(a-d)(a-c)}$ (b) $\dfrac{(a-d)(c-a)}{(b-c)(d-b)}$

    (c) $\dfrac{(d-a)(c-a)}{(b-c)(d-b)}$ (d) $\dfrac{(b-c)(b-d)}{(c-a)(a-d)}$

DIRECTIONS (Qs. 19-21) : *Read the following information and answer the three items that follow:*

Let $t_n = \sin^n \theta + \cos^n \theta$.

19. What is $\dfrac{t_3 - t_5}{t_5 - t_7}$ equal to?

    (a) $\dfrac{t_1}{t_3}$ (b) $\dfrac{t_3}{t_5}$ (c) $\dfrac{t_5}{t_7}$ (d) $\dfrac{t_1}{t_7}$

20. What is $t_1^2 - t_2$ equal to?

    (a) $\cos 2\theta$ (b) $\sin 2\theta$

    (c) $2\cos\theta$ (d) $2\sin\theta$

21. What is the value of t_{10} where $\theta = 45°$?

    (a) 1 (b) $\dfrac{1}{4}$

    (c) $\dfrac{1}{16}$ (d) $\dfrac{1}{32}$

DIRECTIONS (Qs. 22-24) : *Read the following information and answer the three items that follow:*

Let $\alpha = \beta = 15°$.

22. What is the value of $\sin\alpha + \cos\beta$?

    (a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2\sqrt{2}}$

    (c) $\dfrac{\sqrt{3}}{2\sqrt{2}}$ (d) $\dfrac{\sqrt{3}}{\sqrt{2}}$

23. What is the value of $\sin 7\alpha - \cos 7\beta$?

    (a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2\sqrt{2}}$

    (c) $\dfrac{\sqrt{3}}{2\sqrt{2}}$ (d) $\dfrac{\sqrt{3}}{\sqrt{2}}$

24. What is $\sin(\alpha+1°) + \cos(\beta+1°)$ equal to?

    (a) $\sqrt{3}\cos 1° + \sin 1°$

    (b) $\sqrt{3}\cos 1° - \dfrac{1}{2}\sin 1°$

    (c) $\dfrac{1}{\sqrt{2}}(\sqrt{3}\cos 1° - \sin 1°)$

    (d) $\dfrac{1}{2}(\sqrt{3}\cos 1° + \sin 1°)$

25. If $\sin x + \sin y = \cos y - \cos x$, where $0 < y < x < \dfrac{\pi}{2}$,

    then what is $\tan\left(\dfrac{x-y}{2}\right)$ equal to?

    (a) 0 (b) $\dfrac{1}{2}$

    (c) 1 (d) 2

26. If A is a matrix of order 3×5 and B is a matrix of order 5×3, then the order of AB and BA will respectively be

    (a) 3×3 and 3×3 (b) 3×5 and 5×3

    (c) 3×3 and 5×5 (d) 5×3 and 3×5

27. If p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP, then which of the following is/are correct?

    1. p, q and r are in GP.

    2. $\ln p, \ln q$ and $\ln r$ are in AP.

    Select the correct answer using the code given below :

    (a) 1 only   (b) 2 only

    (c) Both 1 and 2   (d) Neither 1 nor 2

28. If $\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 - 3x + 2 = 0$, then what is $\cot(\alpha+\beta)$ equal to ?

    (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$ (c) 2 (d) 3

29. The roots α and β of a quadratic equation, satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is the number of such quadratic equations ?
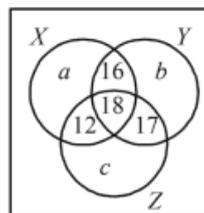
    (a) 0 (b) 2 (c) 3 (d) 4

30. What is the argument of the complex number $\dfrac{1-i\sqrt{3}}{1+i\sqrt{3}}$,

    where $i = \sqrt{-1}$?

    (a) $240°$ (b) $210°$ (c) $120°$ (d) $60°$

DIRECTIONS (Qs. 31-33) : *Read the following information and answer the three items that follow:*

Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by $n(Z)$ which is equal to 90.

31. If the number of elements in Y and Z are in the ratio $4 : 5$, then what is the value of b?
    (a) 18 (b) 19
    (c) 21 (d) 23
32. What is the value of
    $$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z)$$
    $$+ n(X \cap Y \cap Z)?$$
    (a) $a + b + 43$ (b) $a + b + 63$
    (c) $a + b + 96$ (d) $a + b + 106$
33. If the number of elements belonging to neither X nor Y, nor Z is equal to p, then what is the number of elements in the complement of X?
    (a) $p + b + 60$ (b) $p + b + 40$
    (c) $p + a + 60$ (d) $p + a + 40$

DIRECTIONS (Qs. 34-35) : *Read the following information and answer the two items that follow:*

Let $\dfrac{\tan 3A}{\tan A} = K$, where $\tan A \neq 0$ and $K \neq \dfrac{1}{3}$.

34. What is $\tan^2 A$ equal to?
    (a) $\dfrac{K+3}{3K-1}$ (b) $\dfrac{K-3}{3K-1}$

    (c) $\dfrac{3K-3}{K-3}$ (d) $\dfrac{K+3}{3K+1}$

35. For real values of $\tan A$, K **cannot** lie between
    (a) $\dfrac{1}{3}$ and 3 (b) $\dfrac{1}{2}$ and 2

    (c) $\dfrac{1}{5}$ and 5 (d) $\dfrac{1}{7}$ and 7

DIRECTIONS (Qs. 36-37) : *Read the following information and answer the two items that follow:*

$ABCD$ is a trapezium such that AB and CD are parallel and BC is perpendicular to them. Let $\angle ADB = \theta$, $\angle ABD = \alpha$, $BC = p$ and $CD = q$ and $CD = q$.

36. Consider the following :
    1. $AD \sin \theta = AB \sin \alpha$
    2. $BD \sin \theta = AB \sin(\theta + \alpha)$
    Which of the above is/are correct?
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2

37. What is AB equal to?
    (a) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\dfrac{(p^2 - q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

    (c) $\dfrac{(p^2 + q^2)\sin\theta}{q\cos\theta + p\sin\theta}$ (d) $\dfrac{(p^2 - q^2)\cos\theta}{q\cos\theta + p\sin\theta}$

38. If $\tan\theta = \dfrac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$, then what is the value of θ?
    (a) $0°$ (b) $28°$
    (c) $38°$ (d) $52°$

39. A and B are positive acute angles such that $\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$. What is the value of $(A + 2B)$?
    (a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$

    (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

40. What is
    $$\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$$
    equal to ?
    (a) 0 (b) 1
    (c) $2\sin 2x$ (d) $4\cos 4x$

41. The value of ordinate of the graph of $y = 2 + \cos x$ lies in the interval
    (a) $[0, 1]$ (b) $[0, 3]$
    (c) $[-1, 1]$ (d) $[1, 3]$

42. What is the value of $8\cos 10° \cdot \cos 20° \cdot \cos 40°$?
    (a) $\tan 10°$ (b) $\cot 10°$
    (c) $\operatorname{cosec} 10°$ (d) $\sec 10°$

43. What is the value of $\cos 48° - \cos 12°$?
    (a) $\dfrac{\sqrt{5} - 1}{4}$ (b) $\dfrac{1 - \sqrt{5}}{4}$

    (c) $\dfrac{\sqrt{5} + 1}{2}$ (d) $\dfrac{1 - \sqrt{5}}{8}$

44. Consider the following statements :
    1. If ABC is a right-angled triangle, right-angled at A and if $\sin B = \dfrac{1}{3}$, then $\operatorname{cosec} C = 3$.
    2. If $b \cos B = c \cos C$ and if the triangle ABC is not right-angled, then ABC must be isosceles.
    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2
45. Consider the following statements :
    1. If in a triangle ABC, $A = 2B$ and $b = c$, then it must be an obtuse-angled triangle.
    2. There exists no triangle ABC with $A = 40°$, $B = 65°$ and $\dfrac{a}{c} = \sin 40° \operatorname{cosec} 15°$.
    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

46. If matrix $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$ where $i = \sqrt{-1}$, then which one of the following is correct?
   (a) A is hermitian
   (b) A is skew-hermitian
   (c) $(\bar{A})^T + A$ is hermitian
   (d) $(\bar{A})^T + A$ is skew-hermitian

47. The term independent of x in the binomial expansion of $\left(\dfrac{2}{x^2} - \sqrt{x} \right)^{10}$ is equal to
   (a) 180   (b) 120
   (c) 90   (d) 72

48. If $(1 + 2x - x^2)^6 = a_0 + a_1 x + a_2 x^2 + ... + a_{12} x^{12}$, then what is $a_0 - a_1 + a_2 - a_3 + a_4 - .... + a_{12}$ equal to?
   (a) 32   (b) 64
   (c) 2048   (d) 4096

49. If $C(20, n+2) = C(20, n-2)$, then what is n equal to?
   (a) 18   (b) 25
   (c) 10   (d) 12

50. For how many values of k, is the matrix $\begin{bmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{bmatrix}$ singular?
   (a) Only one   (b) Only two
   (c) Only four   (d) Infinite

51. The number $(1101101 + 1011011)_2$ can be written in decimal system as
   (a) $(198)_{10}$ (b) $(199)_{10}$
   (c) $(200)_{10}$ (d) $(201)_{10}$

52. What is the value of
   $\dfrac{1}{10} \log_5 1024 - \log_5 10 + \dfrac{1}{5} \log_5 3125$?
   (a) 0   (b) 1
   (c) 2   (d) 3

53. If $x = \log_c(ab)$, $y = \log_a(bc)$, $z = \log_b(ca)$, then which of the following is correct?
   (a) $xyz = 1$
   (b) $x + y + z = 1$
   (c) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = 1$
   (d) $(1+x)^{-2} + (1+y)^{-2} + (1+z)^{-2} = 1$

54. Let $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. If $AB = C$, then what is the value of the determinant of the matrix A?
   (a) –10   (b) –14
   (c) –24   (d) –34

55. If $1.5 \le x \le 4.5$, then which one of the following is correct?
   (a) $(2x-3)(2x-9) > 0$
   (b) $(2x-3)(2x-9) < 0$
   (c) $(2x-3)(2x-9) \ge 0$
   (d) $(2x-3)(2x-9) \le 0$

56. Let $S = \{1, 2, 3, ...\}$. A relation R on $S \times S$ is defined by xRy if $\log_a x > \log_a y$ when $a = \dfrac{1}{2}$. Then the relation is
   (a) reflexive only
   (b) symmetric only
   (c) transitive only
   (d) both symmetric and transitive

57. What is the value of the determinant $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$ where $i = \sqrt{-1}$?
   (a) 0   (b) –2
   (c) $4i$ (d) $-4i$

58. Let $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then what is AB equal to?
   (a) $\begin{bmatrix} ax+hy+gz \\ y \\ z \end{bmatrix}$
   (b) $\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ z \end{bmatrix}$
   (c) $\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$
   (d) $\begin{bmatrix} ax+hy+gz & hx+by+fz & gx+fy+cz \end{bmatrix}$

59. What is the number of ways in which the letters of the word 'ABLE' can be arranged so that the vowels occupy even places?
   (a) 2   (b) 4
   (c) 6   (d) 8

60. What is the maximum number of points of intersection of 5 non-overlapping circles?
   (a) 10   (b) 15
   (c) 20   (d) 25

Marks	Number of students	
	Physics	Mathematics
10 - 20	8	10
20 - 30	11	21
30 - 40	30	38
40 - 50	26	15
50 - 60	15	10
60 - 70	10	6

61. The difference between number of students under Physics and Mathematics is largest for the interval
(a) 20 - 30
(b) 30 - 40
(c) 40 - 50
(d) 50 - 60

62. Consider the following statements :
1. Modal value of the marks in Physics lies in the interval 30 - 40.
2. Median of the marks in Physics is less than that of marks in Mathematics.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

63. What is the mean of marks in Physics?
(a) 38.4
(b) 39.4
(c) 40.9
(d) 41.6

64. What is the standard deviation of the observations
$-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}$?
(a) $\sqrt{2}$
(b) 2
(c) $2\sqrt{2}$
(d) 4

65. If $\Sigma x_i = 20$, $\Sigma x_i^2 = 200$ and $n = 10$ for an observed variable x, then what is the coefficient of variation?
(a) 80
(b) 100
(c) 150
(d) 200

66. What is the probability that February of a leap year selected at random, will have five Sundays?
(a) $\frac{1}{5}$
(b) $\frac{1}{7}$
(c) $\frac{2}{7}$
(d) 1

67. The arithmetic mean of 100 observations is 40. Later, it was found that an observation '53' was wrongly read as '83'. What is the correct arithmetic mean?
(a) 39.8
(b) 39.7
(c) 39.6
(d) 39.5

68. A husband and wife appear in an interview for two vacancies for the same post. The probability of the husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. If the events are independent, then the probability of which one of the following is $\frac{11}{35}$?
(a) At least one of them will be selected
(b) Only one of them will be selected
(c) None of them will be selected
(d) Both of them will be selected

69. A dealer has a stock of 15 gold coins out of which 6 are counterfeits. A person randomly picks 4 of the 15 gold coins. What is the probability that all the coins picked will be counterfeits?
(a) $\frac{1}{91}$
(b) $\frac{4}{91}$
(c) $\frac{6}{91}$
(d) $\frac{15}{91}$

70. A committee of 3 is to be formed from a group of 2 boys and 2 girls. What is the probability that the committee consists of 2 boys and 1 girl?
(a) $\frac{2}{3}$
(b) $\frac{1}{4}$
(c) $\frac{3}{4}$
(d) $\frac{1}{2}$

71. In a lottery of 10 tickets numbered 1 to 10, two tickets are drawn simultaneously. What is the probability that both the tickets drawn have prime numbers?
(a) $\frac{1}{15}$
(b) $\frac{1}{2}$
(c) $\frac{2}{15}$
(d) $\frac{1}{5}$

72. Let X and Y represent prices (in ₹) of a commodity in Kolkata and Mumbai respectively. It is given that $X = 65$, $Y = 67$, $\sigma_X = 2.5$, $\sigma_Y = 3.5$ and $r(X, Y) = 0.8$. What is the equation of regression of Y on X?
(a) $Y = 0.175X - 5$
(b) $Y = 1.12X - 5.8$
(c) $Y = 1.12X - 5$
(d) $Y = 0.17X + 5.8$

73. Consider a random variable X which follows Binomial distribution with parameters $n = 10$ and $p = \frac{1}{5}$. Then $Y = 10 - X$ follows Binomial distribution with parameters n and p respectively given by
(a) $5, \frac{1}{5}$
(b) $5, \frac{2}{5}$
(c) $10, \frac{3}{5}$
(d) $10, \frac{4}{5}$

74. If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, then consider the following statements :

1. $P(\overline{A} \cup B) = 0.9$.

2. $P(\overline{B} \mid \overline{A}) = 0.6$.

Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

75. Three cooks X, Y and Z bake a special kind of cake, and with respective probabilities 0.02, 0.03 and 0.05, it fails to rise. In the restaurant where they work, X bakes 50%, Y bakes 30% and Z bakes 20% of cakes. What is the proportion of failures caused by X?

(a) $\dfrac{9}{29}$
(b) $\dfrac{10}{29}$

(c) $\dfrac{19}{29}$
(d) $\dfrac{28}{29}$

76. Consider the following statements for $f(x) = e^{-|x|}$:

1. The function is continuous at $x = 0$.
2. The function is differentiable at $x = 0$.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

77. What is the maximum value of $\sin x \cdot \cos x$?

(a) 2
(b) 1

(c) $\dfrac{1}{2}$
(d) $2\sqrt{2}$

78. What is $\lim\limits_{x \to 0} \dfrac{3^x + 3^{-x} - 2}{x}$ equal to?

(a) 0
(b) -1
(c) 1
(d) Limit does not exist

79. What is the derivative of $\tan^{-1}x$ with respect to $\cot^{-1}x$?

(a) -1
(b) 1

(c) $\dfrac{1}{x^2 + 1}$
(d) $\dfrac{x}{x^2 + 1}$

80. The function $u(x, y) = c$ which satisfies the differential equation $x(dx - dy) + y(dy - dx) = 0$, is

(a) $x^2 + y^2 = xy + c$
(b) $x^2 + y^2 = 2xy + c$

(c) $x^2 - y^2 = xy + c$
(d) $x^2 - y^2 = 2xy + c$

81. What is the minimum value of $3\cos\left(A + \dfrac{\pi}{3}\right)$ where $A \in R$?
(a) -3
(b) -1
(c) 0
(d) 3

82. Consider the following statements :
1. The function $f(x) = \ln x$ increases in the interval $(0, \infty)$.

2. The function $f(x) = \tan x$ increases in the interval $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

83. Which one of the following is correct in respect of the graph of $y = \dfrac{1}{x-1}$?

(a) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of reals.

(b) The domain is $\{x \in R \mid x \neq 1\}$, the range is $\{y \in R \mid y \neq 0\}$ and the graph intersects y-axis at $(0, -1)$.

(c) The domain is the set of reals and the range is the singleton set $\{0\}$.

(d) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of points on the y-axis.

84. What is the solution of the differential equation $\ln\left(\dfrac{dy}{dx}\right) = x$?

(a) $y = e^x + c$
(b) $y = e^{-x} + c$

(c) $y = \ln x + c$
(d) $y = 2\ln x + c$

85. Let l be the length and b be the breadth of a rectangle such that $l + b = k$. What is the maximum area of the rectangle?

(a) $2k^2$
(b) k^2

(c) $\dfrac{k^2}{2}$
(d) $\dfrac{k^2}{4}$

86. The numbers 4 and 9 have frequencies x and $(x - 1)$ respectively. If their arithmetic mean is 6, then what is the value of x?
(a) 2
(b) 3
(c) 4
(d) 5

87. If three dice are rolled under the condition that no two dice show the same face, then what is the probability that one of the faces is having the number 6?

(a) $\dfrac{5}{6}$
(b) $\dfrac{5}{9}$

(c) $\dfrac{1}{2}$
(d) $\dfrac{5}{12}$

88. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\text{not } A) = \frac{1}{2}$, then which one of the following is **not** correct?

(a) $P(B) = \frac{2}{3}$

(b) $P(A \cap B) = P(A)P(B)$

(c) $P(A \cup B) > P(A) + P(B)$

(d) $P(\text{not } A \text{ and not } B) = P(\text{not } A) P(\text{not } B)$

89. The sum of deviations of n number of observations measured from 2.5 is 50. The sum of deviations of the same set of observations measured from 3.5 is –50. What is the value of n?

(a) 50   (b) 60

(c) 80   (d) 100

90. A data set of n observations has mean $2M$, while another data set of $2n$ observations has mean M. What is the mean of the combined data sets?

(a) M

(b) $\frac{3M}{2}$

(c) $\frac{2M}{3}$

(d) $\frac{4M}{3}$

91. If $f(x) = 3x^2 - 5x + p$ and $f(0)$ and $f(1)$ are opposite in sign, then which of the following is correct?

(a) $-2 < p < 0$ (b) $-2 < p < 2$

(c) $0 < p < 2$ (d) $3 < p < 5$

92. If $e^{\theta \varphi} = c + 4\theta\varphi$, where c is an arbitrary constant and φ is a function of θ, then what is $\varphi \, d\theta$ equal to?

(a) $\theta \, d\varphi$ (b) $-\theta \, d\varphi$

(c) $4\theta \, d\varphi$ (d) $-4\theta \, d\varphi$

93. If $p(x) = (4e)^{2x}$, then what is $\int p(x) \, dx$ equal to?

(a) $\frac{p(x)}{1 + 2\ln 2} + c$

(b) $\frac{p(x)}{2(1 + 2\ln 2)} + c$

(c) $\frac{2p(x)}{1 + \ln 4} + c$

(d) $\frac{p(x)}{1 + \ln 2} + c$

94. What is the value of $\int_0^{\pi/4} (\tan^3 x + \tan x) dx$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

95. Let $y = 3x^2 + 2$. If x changes from 10 to 10.1, then what is the total change in y?

(a) 4.71   (b) 5.23

(c) 6.03   (d) 8.01

96. If $f(x) = \frac{\sin x}{x}$, where $x \in R$, is to be continuous at $x = 0$, then the value of the function at $x = 0$

(a) should be 0   (b) should be 1

(c) should be 2   (d) cannot be determined

97. The solution of the differential equation $dy = (1 + y^2) dx$ is

(a) $y = \tan x + c$ (b) $y = \tan(x + c)$

(c) $\tan^{-1}(y + c) = x$ (d) $\tan^{-1}(y + c) = 2x$

98. What is $\int (e^{\log x} + \sin x) \cos x \, dx$ equal to?

(a) $\sin x + x\cos x + \frac{\sin^2 x}{2} + c$

(b) $\sin x - x\cos x + \frac{\sin^2 x}{2} + c$

(c) $x\sin x + \cos x + \frac{\sin^2 x}{2} + c$

(d) $x\sin x - x\cos x + \frac{\sin^2 x}{2} + c$

99. What is the domain of the function $f(x) = \cos^{-1}(x - 2)$?

(a) $[-1, 1]$ (b) $[1, 3]$

(c) $[0, 5]$ (d) $[-2, 1]$

100. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{2}{3}$ (d) 2

101. If $f(x) = 2x - x^2$, then what is the value of $f(x + 2) + f(x - 2)$ when $x = 0$?

(a) –8   (b) –4

(c) 8   (d) 4

102. If $x^m y^n = a^{m+n}$, then what is $\frac{dy}{dx}$ equal to?

(a) $\frac{my}{nx}$ (b) $-\frac{my}{nx}$

(c) $\frac{mx}{ny}$ (d) $-\frac{ny}{mx}$

103. What is $\int \dfrac{dx}{x(x^n+1)}$ equal to?

(a) $\dfrac{1}{n}\ln\left(\dfrac{x^n}{x^n+1}\right)+c$

(b) $\ln\left(\dfrac{x^n+1}{x^n}\right)+c$

(c) $\ln\left(\dfrac{x^n}{x^n+1}\right)+c$

(d) $\dfrac{1}{n}\ln\left(\dfrac{x^n+1}{x^n}\right)+c$

104. What is the minimum value of $|x-1|$, where $x \in R$?

(a) 0   (b) 1

(c) 2   (d) –1

105. What is the value of k such that integration of $\dfrac{3x^2+8-4k}{x}$ with respect to x, may be a rational function?

(a) 0   (b) 1

(c) 2   (d) –2

106. What is the length of the diameter of the sphere whose centre is at $(1, -2, 3)$ and which touches the plane $6x-3y+2z-4=0$?

(a) 1 unit   (b) 2 units

(c) 3 units   (d) 4 units

107. What is the perpendicular distance from the point $(2, 3, 4)$ to the line $\dfrac{x-0}{1}=\dfrac{y-0}{0}=\dfrac{z-0}{0}$?

(a) 6 units   (b) 5 units

(c) 3 units   (d) 2 units

108. If a line has direction ratios $<a+b,\ b+c,\ c+a>$, then what is the sum of the squares of its direction cosines?

(a) $(a+b+c)^2$ (b) $2(a+b+c)$

(c) 3   (d) 1

109. Into how many compartments do the coordinate planes divide the space?

(a) 2   (b) 4

(c) 8   (d) 16

110. What is the equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane?

(a) $x+y=5$ (b) $z=5$

(c) $z=0$ (d) $x+y+z=5$

111. If $\hat{a}$ is a unit vector in the xy-plane making an angle $30°$ with the positive x-axis, then what is $\hat{a}$ equal to?

(a) $\dfrac{\sqrt{3}\hat{i}+\hat{j}}{2}$

(b) $\dfrac{\sqrt{3}\hat{i}-\hat{j}}{2}$

(c) $\dfrac{\hat{i}+\sqrt{3}\hat{j}}{2}$

(d) $\dfrac{\hat{i}-\sqrt{3}\hat{j}}{2}$

112. Let A be a point in space such that $|\overrightarrow{OA}|=12$, where O is the origin. If $\overrightarrow{OA}$ is inclined at angles $45°$ and $60°$ with x-axis and y-axis respectively, then what is $\overrightarrow{OA}$ equal to ?

(a) $6\hat{i}+6\hat{j}\pm\sqrt{2}\hat{k}$

(b) $6\hat{i}+6\sqrt{2}\hat{j}\pm6\hat{k}$

(c) $6\sqrt{2}\hat{i}+6\hat{j}\pm6\hat{k}$

(d) $3\sqrt{2}\hat{i}+3\hat{j}\pm6\hat{k}$

113. Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. What is the magnitude of dot product of vectors which represent its diagonals?

(a) 21   (b) 25   (c) 31   (d) 36

114. If $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$, then what is $|\vec{b}|$ equal to ?

(a) 3   (b) 4   (c) 6   (d) 8

115. If the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+p\hat{k}$ are coplanar, then what is the value of p?

(a) 1   (b) –1   (c) 5   (d) –5

116. What is $\lim\limits_{x\to1}\dfrac{x+x^2+x^3-3}{x-1}$ equal to ?

(a) 1   (b) 2   (c) 3   (d) 6

117. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

(a) 4.4 cm/sec   (b) 8.4 cm/sec

(c) 8.8 cm/sec   (d) 15.4 cm/sec

118. If $\lim\limits_{x\to1}\dfrac{x^4-1}{x-1}=\lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$, where $k\neq0$, then what is the value of k?

(a) $\dfrac{2}{3}$ (b) $\dfrac{4}{3}$

(c) $\dfrac{8}{3}$ (d) 4

119. The order and degree of the differential equation $k\dfrac{dy}{dx}=\int\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{2}{3}}dx$ are respectively

(a) 1 and 1   (b) 2 and 3

(c) 2 and 4   (d) 1 and 4

120. What is $\lim\limits_{x\to0}\dfrac{\sin x\,\log(1-x)}{x^2}$ equal to?

(a) –1   (b) Zero

(c) $-e$ (d) $-\dfrac{1}{e}$

PART- A : ENGLISH

ANTONYMS

DIRECTIONS (Qs. 1-10) : *Each item in this section consists of a sentence with an underlined word followed by four words. Select the option that is **opposite in meaning** to the underlined word and mark your response on your Answer Sheet accordingly.*

1. He nodded <u>absently</u> throughout the meeting.
   (a) capably   (b) alertly
   (c) agitatedly   (d) dreamily

2. I fullly believe that the cornerstone of good policy is an electorate that is <u>educated</u> on the national issues.
   (a) cerebral   (b) enlightened
   (c) ignorant   (d) erudite

3. For <u>important</u> medical decisions, even finding a doctor you trust is not enough.
   (a) significant   (b) trivial
   (c) basic   (d) probable

4. Planets <u>move</u> in their orbits.
   (a) push   (b) rotate
   (c) stall   (d) flow

5. Temperature is a measure of internal energy of an object and is frequently <u>expressed</u> by physicists in units of Kelvin.
   (a) found   (b) told
   (c) distributed   (d) concealed

6. People argue about why Venus is so much <u>warmer</u> than the Earth.
   (a) friendlier   (b) colder
   (c) wilder   (d) heavier

7. Scientists are <u>concerned</u> whether the oceans and land biosphere will take up as much carbon in the future as they presently do.
   (a) worried   (b) indifferent
   (c) curious   (d) puzzled

8. The biggest <u>debate</u> among scientists today is about *cloud feedback*.
   (a) contend   (b) moot
   (c) wrangle   (d) agreement

9. The Earth's climate <u>sensitivity</u> is conventionally defined as the equilibrium temperature increase caused by a doubling of carbon dioxide.
   (a) imperviousness   (b) willingness
   (c) responsiveness   (d) closeness

10. The oceans carry a <u>huge</u> amount of heat from the tropics to the high latitudes.
    (a) significant   (b) major
    (c) tiny   (d) dormant

ORDERING OF WORDS IN A SENTENCE

DIRECTIONS (Qs. 11-20): *Each of the following items in this section consists of a sentences(s), the parts of which have been jumbled. These parts have been labelled as P, Q, R and S. Given below each sentence are four sequences namely (a), (b), (c) and (d). You are required to rearrange the jumbled parts of the sentence and mark your response accordingly.*

11. P. several years ago,
    Q. course on climate change at Texas A & M University
    R. Professor Andrew Dessler created an introductory
    S. for freshmen and sophomores
    (a) P R Q S   (b) Q R P S
    (c) S Q R P   (d) P Q R S

12. P. I realize that solving the climate change problem
    Q. than solving
    R. will be much harder
    S. the ozone depletion problem
    (a) P R Q S   (b) Q R P S
    (c) S Q R P   (d) P Q R S

13. P. although the temperature of this layer of the
    Q. when directly comparing the satellite
    R. measurements of temperature
    S. atmoshphere should generally track the surface temperature, we must be careful
    (a) P R Q S   (b) P S Q R
    (c) S Q R P   (d) P Q R S

14. P. the element heats up,
    Q. eventually reaching high temperatures
    R. glowing like a dark orange that radiates
    S. the visible range,
    (a) P Q R S   (b) Q P S R
    (c) S Q R P   (d) P R S Q

15. P. two people argue about why Venus is so much warmer
    Q. to the Sun, so it absorbs more solar energy. The second argues
    R. that it's because Venus has a think, greenhouse-gas rich atmosphere
    S. than the Earth. The first argues that it's because Venus is closer
    (a) P S Q R   (b) Q R P S
    (c) S Q R P   (d) P Q R S

16. P. are now used by more than a thousand firms
    Q. and are growing in popularity
    R. in the United States and Europe
    S. gain sharing-plans
    (a) P R Q S   (b) Q R P S
    (c) P R S Q   (d) S P R Q

17. P. adversity without succumbing
    Q. to the clouds of doubt and jealousy
    R. between friends which is subjected to both prosperity and
    S. the essay 'Of Friendship' by Francis Bacon celebrated the intimacy
    (a) P R Q S   (b) Q R P S
    (c) S R P Q   (d) P Q R S

18. P. friends without which the world is
    Q. make friends and a person wills to want true
    R. it is miserable solitude that compels a person to
    S. nothing other than a place of wilderness
    (a) P R Q S   (b) Q R P S
    (c) R Q P S   (d) P Q R S
19. P. takes the longest   Q. that never started
    R. the job   S. to finish
    (a) P R Q S   (b) R Q P S
    (c) S Q R P   (d) P Q R S
20. P. to what you can create
    Q. control, shift your energy
    R. intead of worrying about
    S. what you cannot
    (a) P R S Q   (b) Q R S P
    (c) S Q R P   (d) R S Q P

SYNONYMS

DIRECTIONS (Qs. 21-30): *Each item in this section consists of a sentence with an underlined word/words followed by four words. Select the option that is **nearest in meaning** to the underlined word and mark your response on your Answer Sheet accordingly.*

21. She got the divorce within no time.
    (a) detachment   (b) breaking down
    (c) annulment   (d) punishment
22. He was known for his gentle disposition.
    (a) harmful   (b) amiable
    (c) cunning   (d) adjusting
23. The harder we kick, the better the ball bounces back.
    (a) recoils   (b) deflates
    (c) inflates   (d) ascends
24. I plan and execute.
    (a) desire   (b) debate
    (c) accomplish   (d) discard
25. He listened to me with equanimity.
    (a) carelessly   (b) excitedly
    (c) patiently   (d) half-heartedly
26. The whole thing was a futile exercise.
    (a) costly   (b) pointless
    (c) indecisive   (d) successful
27. He was too cunning for her.
    (a) capable   (b) fit
    (c) intelligent   (d) dodgy
28. This decision is crucial for me.
    (a) momentous   (b) natural
    (c) primitive   (d) inept
29. The media went into a frenzy about the case.
    (a) silent   (b) creative
    (c) berserk   (d) wrong
30. He has been yearning to go home.
    (a) declining   (b) demanding
    (c) begging   (d) hankering

PREPOSITIONS

DIRECTIONS (Qs. 31-40): *Each item in this section has a sentence with a missing preposition. Select the correct preposition from the given options and mark your response on your Answer Sheet accordingly.*

31. Simulations of the 20th century by climate models that exclude the observed increase _____ greenhouse gases fail to simulate the increase in temperature over the second half of the 20th century.
    (a) of   (b) in
    (c) by   (d) to
32. In extremely poor societies, children can be put to work _____ a young age and are therefore a source of income.
    (a) in   (b) on
    (c) by   (d) at
33. People who are averse _____ hard work, generally do not succeed in life.
    (a) to   (b) about
    (c) at   (d) on
34. I have known her _____ a long time.
    (a) since   (b) for
    (c) at   (d) before
35. I accepted the offer _____ certain conditions.
    (a) on   (b) in
    (c) by   (d) within
36. She is a woman _____ humble origin.
    (a) off   (b) of
    (c) from   (d) within
37. There is no cure _____ the common cold.
    (a) for   (b) of
    (c) to   (d) on
38. I ran _____ John yesterday, and it was a pleasant surprise.
    (a) by   (b) off
    (c) beside   (d) into
39. My grandmother enjoyed boating _____ the lovely lake.
    (a) in   (b) on
    (c) beside   (d) within
40. He visits the needy to relieve them _____ their sufferings and poverty.
    (a) from   (b) off
    (c) of   (d) on

SPOTTING ERRORS

DIRECTIONS (Qs. 41-50): *Each item in this section has a sentence with three underlined parts labelled as (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part and indicate your response on the Answer Sheet against the corresponding letter i.e., (a) or (b) or (c). If you find no error, your response should be indicated as (d).*

41. This building (a) / comprises of six houses,(b) / three parking lots and one basement. (c) / No error. (d)
42. I look forward (a) / to meet (b) / you. (c) / No error. (d)

43. I cannot (a) / cope up (b) / with this pressure. (c) / No error. (d)
44. I came (a) / to this place (b) / by walk. (c) / No error.(d)
45. What is (a) / the time (b) / in your watch? (c) / No error. (d)
46. The price of my dress (a) / is higher than that of (b) / yours. (c) / No error. (d)
47. My friend's (a) / daughter-in-laws (b) / have come. (c) / No error. (d)
48. I prefer (a) / seafood (b) / than Mediterranean food. (c) / No error. (d)
49. We were (a) / discussing about (b) / the issue. (c) / No error. (d)
50. Dispose off (a) / the garbage (b) / from the shelves. (c) / No error. (d)

PART- B : GENERAL KNOWLEDGE

51. New Zealand is considered part of which one of the following island groups ?
    (a) Micronesia
    (b) Melanesia
    (c) Polynesia
    (d) Hawaii Island chain
52. Which of the following statements with regard to *Pradhan Mantri Krishi Sinchayee Yojana* (PMKSY) is/ are correct?
    1. PMKSY was launched during 2015-16
    2. The basic aim of PMKSY is to enhance physical access of water on farm
    3. One of the main objectives of PMKSY is to achieve convergence of investments in irrigation at the field level
    Select the correct answer using the code given below:
    (a) 1 only
    (b) 1 and 2 only
    (c) 2 and 3 only
    (d) 1, 2 and 3
53. According to Article 79 of the Constitution of India, which of the following is/are described as a part of Parliament of India?
    1. The House of the People
    2. The Council of States
    3. The President of India
    Select the correct answer using the code given below:
    (a) 1 only
    (b) 1 and 2 only
    (c) 2 and 3 only
    (d) 1, 2 and 3
54. The Five Year Plan was first launched in
    (a) China   (b) USSR
    (c) India   (d) Bhutan
55. The call for *Garibi Hatao* was incorporated in
    (a) Fourth Five Year Plan
    (b) Fifth Five Year Plan
    (c) Sixth Five Year Plan
    (d) Seventh Five Year Plan

56. Match List I with List II and select the correct answer using the code given below the Lists :

List I (Major Objective)	List II (Five Year Plan)
A. Faster and more inclusive growth	1. First
B. Faster, more inclusive and sustainable growth	2. Fifth
C. Correction of disequilibrium caused by Second World War	3. Eleventh
D. Attaining self-reliance	4. Twelfth

Codes :

	A	B	C	D		A	B	C	D
(a)	3	1	4	2	(b)	3	4	1	2
(c)	2	4	1	3	(d)	2	1	4	3

57. Which one of the following Directive Principles was inserted by the Constitution (42nd Amendment) Act, 1976?
    (a) The State shall minimise inequalities in income
    (b) Equal justice and free legal aid
    (c) Promotion of co-operative societies
    (d) Provision for early childhood care
58. A common High Court for two or more states can be established by
    (a) a law passed by the Parliament
    (b) an order of the Supreme Court of India
    (c) an order of the President of India
    (d) an amendment to the Constitution of India
59. In the year 1928, a committee of Congress leaders drafted a Constitution for India. The Committee was headed by
    (a) Mahatma Gandhi
    (b) T. B. Sapru
    (c) Motilal Nehru
    (d) Jawaharlal Nehru
60. Which one of the following statements about Jaipal Singh is NOT correct?
    (a) He was a member of the Constituent Assembly
    (b) He founded the Adivasi Maha Sabha
    (c) He was the captain of the first Indian national hockey team
    (d) He campaigned for a separate state of Chhattisgarh
61. What is 'Tikki Mausi' in the context of Malnutrition?
    (a) A specially packaged food item
    (b) A mascot
    (c) Name of a scheme
    (d) Name given to the healthcare providers
62. Which one of the following was the theme of the World Tourism: Day, 2019?
    (a) Sustainable tourism
    (b) Tourism and the digital transformation
    (c) Tourism responding to the challenge of climate change
    (d) Tourism and jobs : a better future for all

63. Which one of the following countries has hosted the Army Exercise TSENTR, 2019?
    (a) Russia   (b) China
    (c) Kazakhstan   (d) Kyrgyzstan

64. Which one of the following countries is called the 'country of winds' ?
    (a) India   (b) China
    (c) Denmark   (d) Germany

65. Which one of the following is the oldest scientific department of Government of India ?
    (a) Department of Biotechnology
    (b) Survey of India
    (c) India Meteorological Department
    (d) DRDO

66. 'Naseem-Al-Bahr' is a bilateral naval exercise between India and
    (a) United Arab Emirates (b) Iran
    (c) Saudi Arabia   (d) Oman

67. Koneru Humpy excels in which one of the following sports?
    (a) Boxing   (b) Table Tennis
    (c) Chess   (d) Billiards

68. Which one of the following was the official mascot of Khelo India Youth Games, 2020?
    (a) Vijaya   (b) Yaya
    (c) Rongmon   (d) Ammu

69. In January 2020, a passenger aircraft crashed in Iran soon after taking off from Tehran's Imam Khomeini airport killing about 170 people onboard. The airplane belongs to
    (a) Qatar Airways
    (b) Ukraine International Airlines
    (c) Singapore Airlines
    (d) Cathay Pacific

70. MILAN, a multilateral naval exercise, 2020 was hosted by which one of the following cities?
    (a) Port Blair   (b) Kochi
    (c) Visakhapatnam   (d) Panaji

71. Chalk and marble are different forms of
    (a) Calcium hydrogen carbonate
    (b) Calcium carbonate
    (c) Calcium acetate
    (d) Sodium carbonate

72. The number of maximum electrons in N Shell is
    (a) 2   (b) 8   (c) 18   (d) 32

73. Vinegar is also known as
    (a) ethanoic acid   (b) nitric acid
    (c) sulphuric acid   (d) tartaric acid

74. A liquid is kept in a glass beaker. Which one of the following statements is correct regarding the pressure exerted by the liquid column at the base of the beaker ?

(a) The pressure depends on the area of the base of the beaker
(b) The pressure depends on the height of liquid column
(c) The pressure does not depend on the density of the liquid
(d) The pressure neither depends on the area of the base of the beaker nor on the height of liquid column

75. Which of the following statements is NOT correct regarding the travel of sound waves?
    (a) Sound waves can travel through water
    (b) Sound waves can travel through air
    (c) Sound waves can travel through steel
    (d) Sound waves can travel through vacuum

76. Deendayal Port is located at
    (a) Kerala   (b) Gujarat
    (c) Maharashtra   (d) Goa

77. Which one of the following cities is associated with Biosafety Protocol to the Convention on Biological Diversity (2000)?
    (a) Geneva   (b) Nairobi
    (c) Cartagena   (d) Rio de Janeiro

78. Krishna Raja Sagara Dam / Reservoir is developed on
    (a) river Krishna   (b) river Tungabhadra
    (c) river Godavari   (d) river Kavery

79. Which one of the following Indian states does NOT share international border with two or more countries?
    (a) Arunachal Pradesh (b) Assam
    (c) Mizoram   (d) Tripura

80. According to Census of India, 2011, which one among the following is the least populated state in India?
    (a) Maharashtra   (b) Madhya Pradesh
    (c) Odisha   (d) Punjab

81. The speakers of major Indian languages belong to how many language families?
    (a) Two   (b) Three   (c) Four   (d) Six

82. Where did Gandhiji initially forge the techniques of *Satyagraha*?
    (a) England   (b) South Africa
    (c) North Africa   (d) India

83. Who was the Viceroy of India at the time of Gandhiji's Dandi march?
    (a) Lord Irwin   (b) Lord Linlithgow
    (c) Lord Reading   (d) Lord Willingdon

84. Which one of the following American newsmagazines was highly sceptical of Gandhiji's Dandi march initially but within a week completely changed its opinion and saluted him as a Saint and Statesman?
    (a) Saturday Evening Post (b) Readers Digest
    (c) Time   (d) Life

85. In which of the following years were passengers trains introduced in England?
(a) 1823   (b) 1825
(c) 1848   (d) 1861

86. The correct relation between the radius of curvature R and focal length f of a spherical mirror is
(a) $R = f$ (b) $R = 2f$
(c) $R = 3f$ (d) $R = 4f$

87. A lemon kept in water in a glass tumbler appears to be larger than its actual size. It is because of
(a) reflection of light   (b) scattering of light
(c) refraction of light   (d) polarization of light

88. Light enters the eye through a thin mambrane called
(a) retina   (b) cornea
(c) pupil   (d) iris

89. Name the scientist who first used a glass prism to obtain the spectrum of sunlight
(a) C. V. Raman   (b) Lord Rayleigh
(c) Isaac Newton   (d) S. Chandrasekhar

90. The cost of energy to operate an industrial refrigerator that consumes 5 kW power working 10 hours per day for 30 days will be
(Given that the charge per kW.h of energy = ₹ 4)
(a) ₹ 600   (b) ₹ 6,000
(c) ₹ 1,200   (d) ₹ 1,500

91. Which one of the following statements regarding magnetic field is NOT correct?
(a) Magnetic field is a quantity that has direction and magnitude
(b) Magnetic field lines are closed curves
(c) Magnetic field lines are open curves
(d) No two magnetic field lines are found to cross each other

92. Which one of the following statements is NOT correct?
(a) Buckminsterfullerene is an allotrope of carbon
(b) Diamond is a good conductor of electricity
(c) Graphite is a good conductor of electircity
(d) In graphite, each carbon atom is linked to three other carbon atoms

93. How many covalent bonds are present in a Chloropropane molecule having molecular formula, C_3H_7Cl?
(a) 6   (b) 8   (c) 9   (d) 10

94. Which one of the following is the most fundamental characteristic of an element?
(a) Melting point   (b) Atomic number
(c) Colour   (d) Atomic weight

95. Neutrons were discovered by
(a) James Chadwick   (b) Ernest Rutherford
(c) J. J. Thomson   (d) John Dalton

96. Atomic mass of an element is equal to the sum of number of
(a) electrons and protons only
(b) protons and neutrons only
(c) electrons and neutrons only
(d) electrons, protons and neutrons

97. Which one of the following element's isotope is used in the treatment of cancer?
(a) Uranium   (b) Cobalt
(c) Sodium   (d) Iodine

98. Which one of the following cell organelles may play a role in expelling excess water and wastes in case of unicellular organisms?
(a) Lysosome   (b) Vacuole
(c) Golgi body   (d) Endoplasmic reticulum

99. Which one of the following terms describes the practice of growing two or more crops simultaneously on the same piece of land?
(a) Crop rotation   (b) Mixed cropping
(c) Intercropping   (d) Mixed farming

100. Which one of the following statements is correct about effects of antibiotics on viruses?
(a) Viruses are "non-living" entities but it can interact with antibiotics
(b) Taking antibiotics cures viral infections
(c) Viruses do not possess metabolic pathways on which antibiotics can function, whereas bacteria have such pathways
(d) Viruses are resistant to antibiotics

101. The Panama Canal opened in 1914, links
(a) Red Sea and Mediterranean Sea
(b) Atlantic Ocean and Pacific Ocean
(c) Indian Ocean and Pacific Ocean
(d) Adriatic Sea and Black Sea

102. The United Nations Charter was signed by 51 original members of the United Nations in 1945 at the
(a) Hague Conference
(b) London Conference
(c) San Francisco Conference
(d) Berlin Conference

103. Land Revenue Records maintained in India have categorized land-use into
(a) 6 categories
(b) 9 categories
(c) 15 categories
(d) 21 categories

104. Dry land farming in India is largely confined to areas with rainfall less than
(a) 100 cm   (b) 85 cm
(c) 80 cm   (d) 75 cm

105. For an area to be excluded from the drought-prone category, what percentage of its gross cropped area should be under irrigation?
    (a) 10 per cent or more   (b) 20 per cent or more
    (c) 25 per cent or more   (d) 30 per cent or more

106. Which one of the following is NOT a current of Pacific Ocean?
    (a) Oyashio current   (b) Alaska current
    (c) Agulhas current   (d) California current

107. The duration of monsoon in India extends for an average period of
    (a) 80 - 140 days   (b) 100 - 120 days
    (c) 90 - 130 days   (d) 100 - 140 days

108. Which one of the following is the natural vegetation of South east China?
    (a) Subtropical broadleaf evergreen forest
    (b) Tropical broadleaf evergreen forest
    (c) Tropical deciduous forest
    (d) Temperate evergreen forest

109. Which one of the following is a cold local wind?
    (a) Santa Ana   (b) Chinook
    (c) Mistral   (d) Loo

110. What was the consequence of Permanent Settlement on rural society in Bengal?
    (a) The zamindars invested capital and enterprise to improve agriculture along lines of British yeoman farmers
    (b) A group of rich peasants known as jotedars succeeded in consolidating their position in the villages
    (c) The ryots prospered as a result of fixed revenue levy imposed on them
    (d) The system of Collectorate introduced by the Company for exercising supervisory control on zamindars failed to take off

111. What was *Damin-i Koh* in Rajmahal area?
    (a) A large area of land demarcated and declared to be the land of the Santhals
    (b) The land of the Paharias cultivated exclusively for paddy
    (c) The British territory marked for their military camp
    (d) The land earmarked for locating settled agriculturists

112. Which one of the following statements about the Revolt of 1857 is correct?
    (a) It was a Revolt carefully organised and planned by the Rajas, Nawabs and Taluqdars
    (b) Rumours and prophecies did not play any role in its outbreak and spread
    (c) The rebel proclamations in 1857 repeatedly appealed to all sections of the population irrespective of their caste and creed.
    (d) The British succeeded in quickly and easily controlling the rebels

113. Which one among the following was NOT a *Panch Sheel* principle?
    (a) Peaceful Co-existence
    (b) Mutual respect for territorial integrity
    (c) Nuclear deterrence
    (d) Non-interference in internal affairs

114. Who among the following had organised, in 1904, a secret society of revolutionaries named *Abhinav Bharat*?
    (a) Khudiram Bose
    (b) Shyamji Krishna Verma
    (c) Har Dayal
    (d) V D Savarkar

115. The principle that the framing of the new Constitution for independent India should be primarily (though not solely) the responsibility of Indians themselves, was for the first time conceded in the
    (a) Government of India Act, 1935
    (b) August Offer of Viceroy Linlithgow
    (c) Cripps Proposals
    (d) Cabinet Mission

116. Which one of the following is a major environmental issue in eastern Canada?
    (a) Acid precipitation
    (b) Groundwater depletion
    (c) Land degradation
    (d) Desertification

117. Which one of the following mountains lies in between Caspian Sea and Black Sea?
    (a) Caucasus   (b) Carpathians
    (c) Apennine   (d) Elburz

118. Bagalihar, Dulhasti and Salal hydro power projects have been developed on which of the following rivers?
    (a) Chenab and Jhelum   (b) Chenab and Indus
    (c) Ravi   (d) Chenab only

119. Which of the following statements with regard to Biosphere Reserve is/are correct?
    1. The idea of Biosphere Reserve was initiated by UNESCO in 1973-74
    2. There are 18 designated Biosphere Reserves in India
    3. All Biosphere Reserves in India have been included in world network of Biosphere Reserves of UNESCO
    Select the correct answer using the code given below:
    (a) 1 only   (b) 2 only
    (c) 1 and 2 only   (d) 1, 2 and 3

120. Which one of the following countries does NOT have direct access to the sea/ocean?
    (a) Syria   (b) Jordan
    (c) Azerbaijan   (d) Armenia

121. In an incandescent electric bulb, the filament of the bulb is made up of which metal ?

    (a) Aluminium   (b) Copper

    (c) Tungsten   (d) Silver

122. Two equal resistors R are connected in parallel, and a battery of 12 V is connected across this combination. A dc current of 100 mA flows through the circuit as shown below:
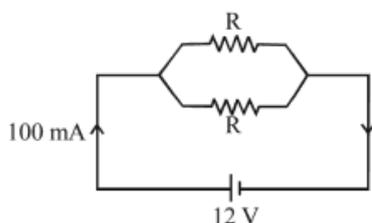
The value of R is

    (a) $120\,\Omega$ (b) $240\,\Omega$

    (c) $60\,\Omega$ (d) $100\,\Omega$

123. Which one of the following is NOT the unit of energy ?

    (a) Joule   (b) Watt-hr

    (c) Newton-metre   (d) kg-metre/sec^2

124. Which one of the following statements is NOT correct?

    (a) Biomass is a renewable source of energy

    (b) Gobar gas is produced when cowdung, crop residues, vegetable waste and sewage are allowed to decompose in the absence of oxygen

    (c) Biogas generation reduces soil and water pollution

    (d) Heating capacity of biogas is very low

125. In prokaryotic organisms, nuclear region is not surrounded by a membrane. This undefined nuclear region is known as

    (a) Nucleic acid   (b) Nucleoid

    (c) Nucleolus   (d) Nucleosome

126. Which one of the following plant tissues has dead cells?

    (a) Epidermis   (b) Parenchyma

    (c) Collenchyma   (d) Sclerenchyma

127. Cartilage is NOT found in

    (a) larynx   (b) nose

    (c) ear   (d) urinary bladder

128. Two planets orbit the Sun in circular orbits, with their radius of orbit as $R_1 = R$ and $R_2 = 4R$. Ratio of their periods (T_1/T_2) around the Sun will be

    (a) 1 / 16   (b) 1 / 8

    (c) 1 / 4   (d) 1 / 2

129. A metallic wire having resistance of $20\,\Omega$ is cut into two equal parts in length. These parts are then connected in parallel. The resistance of this parallel combination is equal to

    (a) $20\,\Omega$ (b) $10\,\Omega$

    (c) $5\,\Omega$ (d) $15\,\Omega$

130. Light of uniform intensity impinges perpendicularly on a totally reflecting surface. If the area of the surface is halved, the radiation force on it will become

    (a) double   (b) half

    (c) four times   (d) one fourth

131. The part of the human eye on which the image is formed is

    (a) pupil   (b) cornea

    (c) retina   (d) iris

132. Consider the following image :
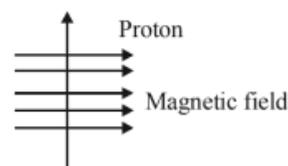
A proton enters a magnetic field at right angles to it, as shown above. The direction of force acting on the proton will be

    (a) to the right   (b) to the left

    (c) out of the page   (d) into the page

133. Which one of the following statements about sound is NOT correct?

    (a) Sound travels at a speed slower than the speed of light

    (b) Sound waves are transverse waves

    (c) Sound waves are longitudinal waves

    (d) Sound travels faster in moist air than in dry air

134. When the short circuit condition occurs, the current in the circuit

    (a) becomes zero

    (b) remains constant

    (c) increases substantially

    (d) keeps on changing randomly

135. Which one of the following is NOT a component of human male reproductive system ?

    (a) Cervix   (b) Urethra

    (c) Seminal vesicle   (d) Vas deferens

136. Which one of the following is NOT a reason of decrease in biodiversity ?

    (a) Large scale deforestation

    (b) Exploitation of forest produce

    (c) Maintaining sacred groves

    (d) Encroachment in forest areas

137. Which one of the following is NOT a cause of depletion in groundwater?

    (a) Afforestation

    (b) Loss of forests

    (c) Excessive pumping of groundwater

    (d) Construction of large scale concrete buildings

138. Which one of the following types of radiations has the smallest wavelength?
    (a) Microwaves
    (b) Infra-red
    (c) Visible light
    (d) X-rays

139. The instrument used for detecting the presence of electric current in a circuit is
    (a) Refractometer
    (b) Galvanometer
    (c) Viscometer
    (d) Diffractometer

140. Which one of the following is the largest composition in biogas ?
    (a) Carbon dioxide
    (b) Methane
    (c) Hydrogen
    (d) Hydrogen sulphide

141. The Sun appears reddish during sunrise and sunset. The phenomenon in optics which is responsible for this appearance of the Sun is
    (a) Reflection
    (b) Total internal reflection
    (c) Scattering
    (d) Interference

142. A lens has a power of +2.0 Dioptre. Which one of the following statements about the lens is true?
    (a) The lens is concave and has a focal length of 0.5 metre
    (b) The lens is convex and has a focal length of 2.0 metre
    (c) The lens is convex and has a focal length of 0.5 metre
    (d) The lens is concave and has a focal length of 2.0 metre

143. At nearly 70°C, sodium bicarbonate shows the property of gradually decomposing, which makes it usable in bakery products. The product of decomposition responsible for this use of sodium bicarbonate is
    (a) Carbon dioxide
    (b) Hydrogen
    (c) Water vapour
    (d) Oxygen

144. Number of molecules of water of crystallization in copper sulphate, sodium carbonate and Gypsum are
    (a) 5, 10 and 2 respectively
    (b) 10, 2 and 5 respectively
    (c) 5, 2 and 10 respectively
    (d) 2, 5 and 10 respectively

145. Which one of the following is the correct sequence of change in colours when a turmeric stain on white clothes is scrubbed by soap and then washed with water ?
    (a) Yellow - pink - blue
    (b) Yellow - reddish brown- yellow
    (c) Yellow- reddish brown - blue
    (d) Yellow - blue - pink

146. Which one of the following statements regarding Bleaching powder and D.D.T. is correct ?
    (a) Both are inorganic compounds
    (b) Both are organic compounds
    (c) Both contain chlorine
    (d) Both contain calcium

147. Which one of the following is the best example of desiccant?
    (a) Silica gel
    (b) Polystyrene
    (c) Sodium chloride
    (d) Sodium carbonate

148. Which one of the following was the first mineral acid discovered?
    (a) Sulphuric acid
    (b) Hydrochloric acid
    (c) Nitric acid
    (d) Phosphoric ácid

149. The refractive index of fused quartz is 1.46 and that of sapphire is 1.77. If v_q is the speed of light in quartz and v_s is the speed of light in sapphire, then which one of the following relations is correct?
    (a) $v_q > v_s$ (b) $v_s > v_q$ (c) $v_s = v_q$ (d) $v_s = \dfrac{v_q}{2}$

150. In case of a concave mirror, if an object is kept between principal focus F and pole P of the mirror, then which one of the following statements about the image is NOT correct?
    (a) The image will be virtual
    (b) The image will be enlarged or magnified
    (c) The image will be formed at infinity
    (d) The image will be erect

HINTS & SOLUTIONS

MATHEMATICS

1. (b) $Z = \dfrac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$

   $= \dfrac{(\cos\theta + i\sin\theta)}{(\cos\theta - i\sin\theta)} \times \dfrac{(\cos\theta + i\sin\theta)}{(\cos\theta + i\sin\theta)}$

   $= \dfrac{(\cos\theta + i\sin\theta)^2}{\cos^2\theta - (i\sin\theta)^2}$

   $= \dfrac{\cos 2\theta + i\sin 2\theta}{\cos^2\theta + \sin^2\theta} = \cos 2\theta + i\sin 2\theta.$

   Modulus of $Z = |Z| = \sqrt{\cos^2(2\theta) + \sin^2(2\theta)} = 1$

2. (c) Number of proper subset of any set of n elements $= 2^n - 1$
   Here given set $= \{1, 2, 3, 4\}$
   Number of proper subset $= 2^4 - 1 = 16 - 1 = 15$.
   Proper subset $= \{(1), (2), (3), (4), (1, 2), (1, 3), (1, 4),$
   $(2, 3), (2, 4), (3, 4), (1, 2, 3),$
   $(1, 2, 4), (1, 3, 4), (2, 3, 4)(\phi)\}$
   Now, A is superset of B, if B is proper set of A, but B is not proper set of A.
   i.e. $B \le A$ but $A \not\subset B$. Then $A \ge B$.
   So, superset of $\{3\}$ are $\{(3), (1, 3), (2, 3), (3, 4), (1, 2, 3), (1, 3, 4),$
   $(2, 3, 4)\}$
   Hence, number of superset of $\{3\} = 7$.

3. (a) $D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$

   $= p(rq - p^2) + q(pr - q^2) + r(pq - r^2)$
   $= 3pqr - p^3 - q^3 - r^3$
   $= -(p^3 + q^3 + r^3 - 3pqr).$
   For real and distinct positive real value of p, q and r.
   $p^3 + q^3 + r^3 - 3pqr > 0$
   $\therefore D < 0.$

4. (a) Expansion of
   $(1+x)^9 = {}^9C_0 + {}^9C_1 x + {}^9C_2 x^2 + {}^9C_3 x^3 + {}^9C_4 x^4$
   $\qquad\qquad + {}^9C_5 x^5 + {}^9C_6 x^6 + {}^9C_7 x^7 + {}^9C_8 x^8 + {}^9C_9 x^9$
   Sum of last five co-efficient
   $= {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$
   $= 126 + 84 + 36 + 9 + 1 = 256.$

5. (a)
   (1) We know that,
   $\qquad A \cdot (\mathrm{adj}\, A) = (\mathrm{adj}\, A) \cdot A = |A| \cdot I$
   Hence, statement (1) is correct.

   (2) $|\mathrm{adj}(A)| = |A^{-1} \cdot |A\|$
   Hence, statement (2) is not correct.

6. (c) $(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$
   $x^2 - 2(a+b)x + 4ab + y^2 - 2(c+d)y + 4cd = 0$
   $x^2 + y^2 - 2(a+b)x - 2(c+d)y + 4(ab+cd) = 0$
   From general equation of circle,
   $x^2 + y^2 + 2gx + 2fy + c = 0$
   Center $= (-g, -f)$ and radius $= \sqrt{(-g)^2 + (-f)^2 + C}$
   $\therefore$ Here $g = -(a+b)$ and $f = -(c+d)$.
   Hence, center $= (-g, -f) = ((a+b), (c+d))$.

7. (c) We know that diagonal of a square bisect each other perpendicularly.
   Equation of a diagonal : $3x + 2y = 5$ (given).
   Now, equation of other diagonal that is perpendicular to the given diagonal $= 2x - 3y = K$.
   As vertex point $(1, -1)$ does not lies on $3x + 2y = 5$
   $\qquad\qquad\qquad\qquad \{\because 3(1) + 2(-1) \ne 5\}$
   Then, point $(1, -1)$, must be on the diagonal
   $2x - 3y = K$
   Then, $2(1) - 3(-1) = K$.
   $\therefore K = 5.$
   Hence, equation of other diagonal : $2x - 3y = 5$.

8. (b) Equation of the Ellipse : $25x^2 + 16y^2 = 400$.
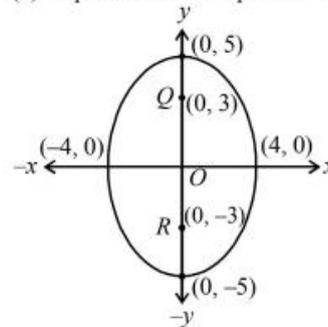
   $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$
   Here $a^2 = 16$, $b^2 = 25$

   Focci points $= (0, \pm be) = (0, \pm\sqrt{b^2 - a^2})$
   $\qquad\qquad = (0, \pm\sqrt{25 - 16}) = (0, \pm 3)$
   Point $Q = (0, 3)$, and $R = (0, -3)$.
   Now, from question point $P(x, y)$ lies on the ellipse, then sum of its distance from two fixed points is always constant, equal to length of its major axis.
   Thus, $(PQ + PR) = 2 \times 5 = 10$.

9. (c) Equation of two sides of the triangle are $x + 2 = 0$ and $y + 2 = 0$.
   They intersect at right angle.

Thus, triangle formed by them is a right angle triangle.
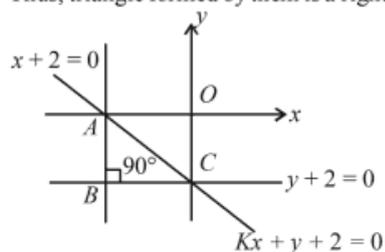
Circumcentre of the right triangle lies on its hypotaneous.
So, circumcentre $(-1, -1)$ must lies on the line
$Kx + y + 2 = 0$
$\therefore K(-1) + (-1) + 2 = 0 \Rightarrow K = 1.$

10. (b) Given parabola : $y^2 = x.$

From standard equation of parabola $y^2 = 4ax$
Focus $= (a, 0)$

Here $a = \dfrac{1}{4}$
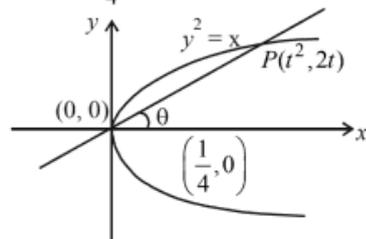
$\therefore$ Focus of parabola $y^2 = x = \left(\dfrac{1}{4}, 0\right)$

Vertex of the parabola $= (0, 0)$

Let $P\left(\dfrac{t^2}{4}, \dfrac{t}{2}\right)$ lies on the parabola

then, its slope $= \tan(\theta)$ {given}

$\therefore \dfrac{\dfrac{t}{2}}{\dfrac{t^2}{4}} = \tan\theta \Rightarrow t = 2\cot\theta$

So, point $P\left(\dfrac{t^2}{4}, \dfrac{t}{2}\right) = (\cot^2\theta, \cot\theta)$

Its distance from vertex $(0, 0)$

$= \sqrt{(\cot^2\theta - 0)^2 + (\cot\theta - 0)^2}$

$= \cot\theta\sqrt{\cot^2\theta + 1} = \cot\theta \cdot \csc\theta$

$= \cos\theta \cdot \csc^2\theta$

11. (c) Given points $(a, b), (c, d)$ and $(a-c, b-d)$ are collinear, if

$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$

$a(d - b + d) + b(a - c - c) + 1(c(b - d) - d(a - c)) = 0$
$2ad - ab + ab - 2bc + bc - ad = 0$
$\Rightarrow ad - bc = 0$
$\therefore ad = bc.$

12. (b) As point $D(2, 5)$ and point $E(5, 9)$ are mid point of side AB and AC, then
$2 \times DE = BC$
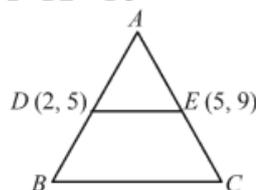
Now,
$BC = 2 \times \sqrt{(5-2)^2 + (9-5)^2}$
$= 2 \times \sqrt{3^2 + 4^2} = 2 \times 5 = 10.$

13. (c) Slope of the line, $3x - 4y - 5 = 0$ is $m = \dfrac{3}{4}$

Slope of any line perpendicular to $3x - 4y - 5 = 0$ is $m' = -\dfrac{4}{3}$.
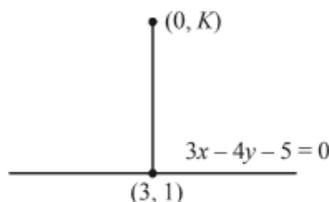
Now, required line passes through the points $(0, K)$ and $(3, 1)$.
$\therefore \dfrac{K - 1}{0 - 3} = -\dfrac{4}{3} \Rightarrow K = 5.$

14. (b) Here, $m_1 = (2 - \sqrt{3})$ and $m_2 = (2 + \sqrt{3})$.
Obtuse angle between them,
$\theta = \tan^{-1}\left(\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}\right)$

$= \tan^{-1}\left(\dfrac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}\right)$

$= \tan^{-1}\left(\dfrac{-2\sqrt{3}}{2}\right) = \tan^{-1}(-\sqrt{3}) = 120°.$

15. (c) We know that pair of opposite sides of my square are parallel.
So, distance between two parallel sides
= Side length of the square
$\therefore$ Side length of the square

$= \left|\dfrac{15 - (-5)}{\sqrt{3^2 + 4^2}}\right| = \dfrac{20}{5} = 4$ units

Area of the square $= 4 \times 4 = 16$ units square.

16. (a) From $a\sin^2 x + b\cos^2 x = c$

$a\sin^2 x + b\cos^2 x = c(\sin^2 x + \cos^2 x)$

$(a-c)\sin^2 x = (c-b)\cos^2 x$

$\Rightarrow \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{(c-b)}{(a-c)}$

$\Rightarrow \tan^2 x = \dfrac{(c-b)}{(a-c)}$

17. (c) From $b\sin^2 y + a\cos^2 y = d$

$b\sin^2 y + a\cos^2 y = d(\sin^2 y + \cos^2 y)$

$(b-d)\sin^2 y = (d-a)\cos^2 y$

$\Rightarrow \dfrac{\sin^2 y}{\cos^2 y} = \dfrac{(d-a)}{(b-d)} \Rightarrow \tan^2 y = \dfrac{(d-a)}{(b-d)}$

18. (b) From, $p\cdot\tan x = q\cdot\tan y$

$p^2\cdot\tan^2 x = q^2\cdot\tan^2 y$

$\Rightarrow \dfrac{p^2}{q^2} = \dfrac{\tan^2 y}{\tan^2 x} = \dfrac{(d-a)}{(b-d)}\cdot\dfrac{(a-c)}{(c-b)}$

$= \dfrac{(a-d)(c-a)}{(b-c)(d-b)}.$

19. (a) $t_n = \sin^n\theta + \cos^n\theta$

Now, $\dfrac{t_3 - t_5}{t_5 - t_7} = \dfrac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}$

$= \dfrac{\sin^3\theta(1-\sin^2\theta) + \cos^3\theta(1-\cos^2\theta)}{\sin^5\theta(1-\sin^2\theta) + \cos^7\theta(1-\cos^2\theta)}$

$= \dfrac{\sin^3\theta\cdot\cos^2\theta + \cos^3\theta\cdot\sin^2\theta}{\sin^5\theta\cdot\cos^2\theta + \cos^7\theta\cdot\sin^2\theta}$

$= \dfrac{\sin^2\theta\cdot\cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta\cdot\cos^2\theta(\sin^3\theta + \cos^3\theta)}$

$= \dfrac{(\sin\theta + \cos\theta)}{(\sin^3\theta + \cos^3\theta)} = \dfrac{t_1}{t_3}$

20. (b) $t_1^2 - t_2 = (\sin\theta + \cos\theta)^2 - (\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta - (\sin^2\theta + \cos^2\theta)$

$= 1 + \sin 2\theta - 1 = \sin 2\theta.$

21. (c) $t_{10} = \sin^{10}(\theta) + \cos^{10}(\theta)$

When $\theta = 45°$,

$t_{10} = \sin^{10}(45°) + \cos^{10}(45°)$

$= \left(\dfrac{1}{\sqrt{2}}\right)^{10} + \left(\dfrac{1}{\sqrt{2}}\right)^{10} = 2\cdot\left(\dfrac{1}{\sqrt{2}}\right)^{10}$

$= 2\times\dfrac{1}{32} = \dfrac{1}{16}.$

22. (d) $\alpha = \beta = 15°$ \qquad (given)

Now, $\sin\alpha + \cos\beta$

$= \sqrt{2}\cdot\left(\dfrac{1}{\sqrt{2}}\cdot\sin\alpha + \dfrac{1}{\sqrt{2}}\cdot\cos\alpha\right)$ \qquad $\{\because \alpha = \beta\}$

$= \sqrt{2}(\sin\alpha\cdot\cos 45° + \cos\alpha\cdot\sin 45°)$

$= \sqrt{2}(\sin(\alpha + 45°)) = \sqrt{2}\cdot\sin(15° + 45°)$

$= \sqrt{2}\cdot\sin 60° = \sqrt{2}\times\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{\sqrt{2}}.$

23. (d) $\sin 7\alpha - \cos 7\beta$

$= \sin 7(15°) - \cos 7(15°) = \sin(105°) - \cos(105°)$

$= \sin(90° + 15°) - \cos(90° + 15°)$

$= \cos 15° - (-\sin(15°)) = \cos 15° + \sin 15° = \dfrac{\sqrt{3}}{\sqrt{2}}$

24. (None) $\sin(\alpha + 1°) + \cos(\beta + 1°)$

$= \sqrt{2}\left\{\dfrac{1}{\sqrt{2}}\cdot\sin(\alpha + 1°) + \dfrac{1}{\sqrt{2}}\cdot\cos(\beta + 1°)\right\}$

$= \sqrt{2}\left\{\cos 45°\cdot\sin(\alpha + 1) + \sin 45°\cdot\cos(\alpha + 1)\right\}$

\qquad\qquad $(\because \alpha = \beta)$

$= \sqrt{2}\{\sin(\alpha + 1 + 45°)\} = \sqrt{2}\cdot\sin(15° + 1° + 45°)$

$= \sqrt{2}\cdot\sin 61°$ \qquad\qquad (i)

Again

$\dfrac{1}{\sqrt{2}}(\sqrt{3}\cdot\cos 1° + \sin 1°)$

$= \dfrac{2}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}\cdot\cos 1° + \dfrac{1}{2}\sin 1°\right)$

$= \sqrt{2}\cdot(\sin 60°\cdot\cos 1° + \cos 60°\cdot\sin 1°)$

$= \sqrt{2}(\sin 61°).$ \qquad\qquad (ii)

Thus (i) = (ii)

But none of the option have $\dfrac{1}{\sqrt{2}}(\sqrt{3}\cdot\cos 1° + \sin 1°).$

25. (c) $\sin x + \sin y = \cos y - \cos x$

$\dfrac{\sin x + \sin y}{\cos y - \cos x} = 1$

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}=1$$

$$\cot\left(\frac{x-y}{2}\right)=1\Rightarrow\tan\left(\frac{x-y}{2}\right)=1.$$

26. (c) $A=[i\times j]_{3\times5}$, $B=[i\times j]_{5\times3}$

Now, $AB=[i\times j]_{3\times3}$ and $BA=[j\times i]_{5\times5}$

27. (c)
(1) p^2, q^2 and r^2 in G.P.

Then, $(q^2)^2=p^2\cdot r^2\Rightarrow q^2=(p^2\cdot r^2)^{\frac{1}{2}}$

$\Rightarrow q^2=p\cdot r$

Hence, p, q and r in G.P.

(2) As, $q^2=p\cdot r$

Taking log on both sides, we have

$\ln(q^2)=\ln(p\cdot r)$

$2\ln(q)=\ln p+\ln r$

Hence, $\ln p, \ln q$ and $\ln r$ are in A.P.

28. (b) Given equation : $x^2-3x+2=0$

Sum of roots, $\cot\alpha+\cot\beta=-(-3)=3$ ...(i)

Product of roots, $\cot\alpha\cdot\cot\beta=2$ ...(ii)

Now, $\cot(\alpha+\beta)=\dfrac{\cot\alpha\cdot\cot\beta-1}{\cot\alpha+\cot\beta}=\dfrac{2-1}{3}=\dfrac{1}{3}.$

29. (b) $\alpha\beta=\alpha^2\beta^2\Rightarrow\alpha\beta(1-\alpha\beta)=0$

$\therefore\alpha\beta=0$ or 1

When $\alpha\beta=1$ then $\alpha=\dfrac{1}{\beta}$

Again from $\alpha+\beta=\alpha^2+\beta^2$

$\Rightarrow\dfrac{1}{\beta}+\beta=\dfrac{1}{\beta^2}+\beta^2\Rightarrow\beta^2-\beta=\dfrac{1}{\beta}-\dfrac{1}{\beta^2}$

$\Rightarrow\beta(\beta-1)=\dfrac{(\beta-1)}{\beta^2}\Rightarrow(\beta-1)\left(\beta-\dfrac{1}{\beta^2}\right)=0$

$\Rightarrow(\beta-1)(\beta^3-1)=0\Rightarrow(\beta-1)^2(\beta^2+\beta+1)=0$

$\therefore\beta=1$ and $\beta=\dfrac{-1\pm\sqrt{3}i}{2}$

Again, when $\beta=1$, then $\alpha=\dfrac{1}{\beta}=1$, roots $(\alpha,\beta)=(1,1)$

When $\beta=\dfrac{-1+\sqrt{3}i}{2}$, then $\alpha=\dfrac{-1-\sqrt{3}i}{2}$

Roots $(\alpha,\beta)=\left(\dfrac{-1\pm\sqrt{3}i}{2},\dfrac{-1\mp\sqrt{3}i}{2}\right)$

Thus, number of different quadratic equations $=2$.

30. (a) Let $x+iy=\dfrac{1-i\sqrt{3}}{1+i\sqrt{3}}$

$=\dfrac{(1-i\sqrt{3})(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}=\dfrac{(1-i\sqrt{3})^2}{1^2-(i\sqrt{3})^2}$

$=\dfrac{1-3-i2\sqrt{3}}{1+3}=-\left(\dfrac{1+i\sqrt{3}}{2}\right)$

$=-\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}$

$\therefore x=-\dfrac{1}{2}, y=-\dfrac{\sqrt{3}}{2}$

Argument $=\tan^{-1}\left(\dfrac{y}{x}\right)=\tan^{-1}\left(\dfrac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$=\pi+\tan^{-1}(\sqrt{3})=180°+60°=240°.$

31. (c) $n(z)=90$

$12+18+17+C=90\Rightarrow C=43.$

From question, $\dfrac{n(y)}{n(z)}=\dfrac{4}{5}$

$\dfrac{16+18+17+b}{90}=\dfrac{4}{5}$

$b=72-51=21.$

32. (d) $n(X)+n(Y)+n(Z)-n(X\cap Y)$

$-n(Y\cap Z)-n(X\cap Z)+n(X\cap Y\cap Z)$

$=n(X\cup Y\cup Z)$

$=a+b+90+16$

$=a+b+106.$

33. (a) $n(X\cup Y\cup Z)^1=P$

and $n(Z)=90$ (given)

$\therefore n(X)^1=P+90-12-18+b=p+b+60.$

34. (b) $\dfrac{\tan 3A}{\tan A}=K$

$\dfrac{3\tan A-\tan^3 A}{\tan A\cdot(1-3\tan^2 A)}=K$

$\dfrac{(3-\tan^2 A)}{(1-3\tan^2 A)}=K$

$\Rightarrow 3 - \tan^2 A = K - 3K \tan^2 A$

$(3K-1)\tan^2 A = K - 3 \Rightarrow \tan^2 A = \dfrac{K-3}{(3K-1)}$

35. (a, b) $\tan A = \sqrt{\dfrac{(K-3)}{(3K-1)}}$

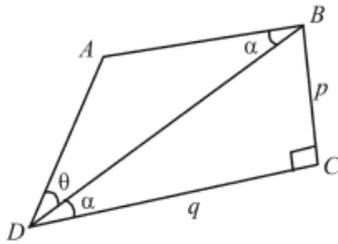For real value of $\tan A$, $\left(\dfrac{K-3}{3K-1}\right) > 0$.

∴ For, $\dfrac{1}{3} < K < 3$.

$\tan A$ is not real.

Also, for $\dfrac{1}{2} < k < 2$, $\tan A$ is not real.

36. (c)

(1) Applying Sine ryke in $\triangle ABD$

$\dfrac{AD}{\sin \alpha} = \dfrac{AB}{\sin \theta}$

$\Rightarrow AD \cdot \sin \theta = AB \cdot \sin \alpha$

Hence, (1) is correct.

(2) $\dfrac{AB}{\sin \theta} = \dfrac{BD}{\sin(180° - (\alpha + \theta))}$

$\dfrac{AB}{\sin \theta} = \dfrac{BD}{\sin(\alpha + \theta)}$

$AB \cdot \sin(\theta + \alpha) = BD \cdot \sin \theta$

Hence, (2) is correct.

37. (a) From $\triangle ABD$,

$\dfrac{AB}{\sin \theta} = \dfrac{BD}{\sin(180° - (\alpha + \theta))}$

$\Rightarrow AB = \dfrac{BD \cdot \sin \theta}{\sin(\alpha + \theta)} = \dfrac{BD \cdot \sin \theta}{\sin \alpha \cdot \cos \theta + \cos \alpha \cdot \sin \theta}$

Again from $\triangle BCD$,

$BD = \sqrt{(BC)^2 + (CD)^2} = \sqrt{p^2 + q^2}$

$\sin \alpha = \dfrac{BC}{BD} = \dfrac{p}{\sqrt{p^2 + q^2}}$

$\cos \alpha = \dfrac{CD}{BD} = \dfrac{q}{\sqrt{p^2 + q^2}}$

∴ $AB = \dfrac{\sqrt{p^2 + q^2} \cdot \sin \theta}{\dfrac{p \cdot \cos \theta}{\sqrt{p^2 + q^2}} + \dfrac{q \cdot \sin \theta}{\sqrt{p^2 + q^2}}}$

$= \dfrac{(p^2 + q^2) \cdot \sin \theta}{p \cdot \cos \theta + q \cdot \sin \theta}$

38. (b) $\tan \theta = \dfrac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$

$= \dfrac{1 - \dfrac{\sin 17°}{\cos 17°}}{1 + \dfrac{\sin 17°}{\cos 17°}} = \dfrac{1 - \tan 17°}{1 + \tan 17°}$

$\tan \theta = \dfrac{\tan 45° - \tan 17°}{1 + \tan 45° \cdot \tan 17°}$

$\tan \theta = \tan(45° - 17°)$

$\tan \theta = \tan(28°) \Rightarrow \theta = 28°$.

39. (d) From question, we have

$\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$

Now,

$\cos(A + 2B) = \cos A \cdot \cos 2B - \sin A \cdot \sin 2B$

$= \cos A \cdot 3\sin^2 A - \sin A \cdot \dfrac{3}{2} \sin 2A$

$= 3\cos A \cdot \sin^2 A - \dfrac{3}{2} \cdot \sin A \cdot (2\sin A - \cos A)$

$= 3\cos A \cdot \sin^2 A - 3\cos A \cdot \sin^2 A = 0$.

∴ $\cos(A + 2B) = \cos\left(\dfrac{\pi}{2}\right)$

∴ $A + 2B = \dfrac{\pi}{2}$.

40. (a) $\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$

$= \sin 3x + \cos 3x - \sin 3x - \cos 3x = 0$.

41. (d) $y = 2 + \cos x$

Range of $\cos x = [-1, 1]$

$y_{min} = 2 - 1 = 1$.

$y_{max} = 2 + 1 = 3$

Thus, ordinate of the graph $= [1, 3]$.

42. (b) $8 \cdot \cos 10° \cdot \cos 20° \cdot \cos 40°$

$= \dfrac{4}{\sin 10°} [2 \cdot \cos 10° \cdot \sin 10°] \cdot \cos 20° \cdot \cos 40°$

$= \dfrac{4}{\sin 10°} [\sin 20°] \cdot \cos 20° \cdot \cos 40°$

$= \dfrac{2}{\sin 10°} [2\sin 20° \cdot \cos 20°] \cdot \cos 40°$

$$= \frac{2}{\sin 10°}[\sin 40°] \cdot \cos 40°$$

$$= \frac{1}{\sin 10°}[2\sin 40° \cdot \cos 40°] = \frac{\sin 80°}{\sin 10°}$$

$$= \frac{\sin(90° - 10°)}{\sin 10°} = \frac{\cos 10°}{\sin 10°} = \cot 10°.$$

43. (b) $\cos 48° - \cos 12°$

$$= 2 \cdot \sin\left(\frac{48° + 12°}{2}\right) \cdot \sin\left(\frac{12° - 48°}{2}\right)$$

$$= 2 \cdot \sin 30° \cdot \sin(-18°) = -\sin 18°$$

$$= -\left(\frac{\sqrt{5} - 1}{4}\right) = \left(\frac{1 - \sqrt{5}}{4}\right).$$

44. (b)
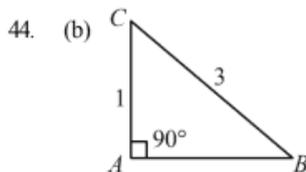
(1) $\triangle ABC$ is a right angled triangle.

$$\sin B = \frac{AC}{BC} = \frac{1}{3}$$

$$\therefore AC = 1,\ BC = 3.$$

Now,

$$AB = \sqrt{(BC)^2 - (AC)^2} = \sqrt{(3)^2 - (1)^2} = 2\sqrt{2}.$$

Now, $\sin C = \dfrac{AB}{BC} = \dfrac{2\sqrt{2}}{3}$

$$\therefore \csc C = \frac{3}{2\sqrt{2}}.$$

Hence, (1) is not correct.

(2) $b \cdot \cos B = c \cdot \cos C$

$$b \cdot \frac{(a^2 + c^2 - b^2)}{2ac} = c \cdot \frac{(a^2 + b^2 - c^2)}{2ab}$$

$$b^2(a^2 + c^2 - b^2) = c^2(a^2 + b^2 - c^2)$$

$$a^2 b^2 - b^4 - a^2 c^2 + c^4 = 0$$

$$a^2(b^2 - c^2) - (b^4 - c^4) = 0$$

$$(b^2 - c^2)(a^2 - b^2 - c^2) = 0$$

Either $b^2 + c^2 - a^2 = 0$ or $(b^2 - c^2) = 0$

When, $b^2 + c^2 - a^2 = 0$

$$b^2 + c^2 = a^2$$

Hence, $\triangle ABC$ is a right angle triangle.

And when, $b^2 - c^2 = 0 \Rightarrow b = c$
Hence, $\triangle ABC$ is an isosceles triangle.
From question $\triangle ABC$ is not right angle triangle.
Hence, $\triangle ABC$ must be an isosceles triangle.

45. (b)
(1) Consider a right angle triangle ABC, right angle at A and $B = C = 45°$.
Then, $b = c$ is also true.
Hence, for the given condition, $\triangle ABC$ must not be an obtuse-angled triangle.

(2)
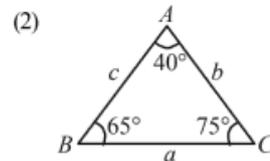
In $\triangle ABC$, $\angle A = 40°$, $\angle B = 65°$

$\therefore \angle C = 180° - 40° - 65° = 75°$.
From sine rule,

$$\frac{a}{\sin 40°} = \frac{c}{\sin 75°}$$

$$\therefore \frac{a}{c} = \sin 40° \cdot \csc 75°$$

So, $\dfrac{a}{c} \neq \sin 40° \cdot \csc 15°$

Hence, $\triangle ABC$ is not possible.
Thus, statement (2) is correct.

46. (c) A square M matrix is said to be Hermitian (or self-adjoint) if it is equal to its. Own Hermitian conjugate, i.e.

$$(\overline{M})^T = M$$

Given Matrix $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$

$$(\overline{A})^T = \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix}$$

Now, $A + (\overline{A})^T = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix} + \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix}$

$$= \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

Conjugate transpose of $(A + (\overline{A})^T) = 2\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$

Hence, $(A + (\overline{A})^T)$ is hermitian.

47. (a) Let $(r+1)^{\text{th}}$ term is independent of x.

$$(r+1)^{\text{th}}\text{ term} = {}^{10}C_r \cdot \left(\frac{2}{x^2}\right)^{10-r} \cdot (-\sqrt{x})^r$$

Exponent of x in independent term $= 0$

i.e. $\dfrac{r}{2} - 2(10 - r) = 0$

$r - 4(10 - r) = 0 \Rightarrow 5r - 40 = 0 \Rightarrow r = 8$.
Thus, 9th term is independent of x.

9th term $= {}^{10}C_8(2)^{10-8} \cdot (-1)^8 = {}^{10}C_8(2)^2$

$\qquad = \dfrac{10 \times 9}{2} \times 4 = 180$.

48. (b) $(1 + 2x - x^2)^6 = a_0 + a_1 \cdot x + a_2 \cdot x^2 + ... + a_{12} \cdot x^{12}$
Putting $x = -1$,

$a_0 - a_1 + a_2 + ... + a_{12} = (1 + 2(-1) - (-1)^2)^6$

$\qquad\qquad = (1 - 2 - 1)^{12} = (-2)^6 = 64$.

49. (c) ${}^{20}C_{(n+2)} = {}^{20}C_{(n-2)}$

$\Rightarrow \dfrac{20!}{(n+2)!(20-n-2)!} = \dfrac{20!}{(n-2)!(20-n+2)!}$

$\Rightarrow \dfrac{(22-n)!}{(18-n)!} = \dfrac{(n+2)!}{(n-2)!}$

$\Rightarrow (22-n)(21-n)(20-n)(19-n)$
$\qquad\qquad\qquad = (n+2)(n+1) \cdot n \cdot (n-1)$

For $n = 10$

$(22-10)(21-10)(20-10)(19-10)$
$\qquad\qquad\qquad = (10+2)(10+1) \cdot 10 \cdot (10-1)$

$\Rightarrow 12 \cdot 11 \cdot 10 \cdot 9 = 12 \cdot 11 \cdot 10 \cdot 9$
Hence, $n = 10$.

50. (d) For singular matrix,

$\begin{bmatrix} 0 & K & 4 \\ -K & 0 & -5 \\ -K & K & -1 \end{bmatrix} = 0$

$K(5K - K) + 4(-K^2) = 0$

$4K^2 - 4K^2 = 0$
Hence, for all values of K, the given matrix is singular matrix.

51. (c) $(1101101)_2 + (1011011)_2$

$= (1 \times 2^6 + 1 \times 2^5 + 0 + 1 \times 2^3 + 1 \times 2^2 + 0 + 1 \times 2^0)_{10}$

$\quad + (1 \times 2^6 + 0 + 1 \times 2^4 + 1 \times 2^3 + 0 + 1 \times 2^1 + 1 \times 2^0)_{10}$

$= (64 + 32 + 8 + 4 + 1)_{10} + (64 + 0 + 16 + 8 + 2 + 1)_{10}$

$= (109 + 91)_{10} = (200)_{10}$.

52. (a) $\dfrac{1}{10} \log_5 1024 - \log_5 10 + \dfrac{1}{5} \log_5 3125$

$= \log_5 (1025)^{1/10} - \log_5 10 + \log_5 (3125)^{1/5}$

$= \log_5 (2^{10})^{\frac{1}{10}} - \log_5 10 + \log_5 (5^5)^{\frac{1}{5}}$

$= \log_5 (2) - \log_5 10 + \log_5 5$

$= \log_5 \left(\dfrac{2 \times 5}{10} \right) = \log_5 1 = 0$.

53. (c) $1 + x = \log_c(ab) + 1$
$\qquad = \log_c(ab) + \log_c c = \log_c(abc)$

$(1 + x)^{-1} = \dfrac{1}{\log_c(abc)} = \log_{(abc)} c$

Similarly, $(1 + y)^{-1} = \log_{(abc)} a$

and $(1 + z)^{-1} = \log_{(abc)} b$

Now, $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1}$

$= \log_{(abc)} c + \log_{(abc)} a + \log_{(abc)} b$

$= \log_{(abc)}(cab) = 1$.

54. (b) $AB = \begin{bmatrix} x + y & y \\ 2x & x - y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 2(x + y) - y \\ 4x - x + y \end{bmatrix} = \begin{bmatrix} 2x + y \\ 3x + y \end{bmatrix}$

As $AB = C$

$\therefore \begin{bmatrix} 2x + y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$2x + y = 3$ \qquad ...(i)

and $3x + y = 2$ \qquad ...(ii)
From equation (i) and (ii), we get
$x = -1, y = 5$.

$\therefore A = \begin{bmatrix} -1 + 5 & 5 \\ -2 & -1 - 5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

$= 4(-6) - 5(-2) = -14$.

55. (d) $1.5 \le x \le 4.5$

$\dfrac{3}{2} \le x \le \dfrac{9}{2} \Rightarrow 3 \le 2x \le 9$

$\therefore (2x - 3) \ge 0$ and $(2x - 9) \le 0$

Hence, $(2x - 3)(2x - 9) \le 0$

56. (c) Give set $S = \{1, 2, 3, ...\}$
For xRy, $\log_a x > \log_a y$

$\Rightarrow x > y$

As xRx, $\log_a x > \log_a x$ is not valid.
Hence, relation is not reflexive.

For xRy, $\log_a x > \log_a y \Rightarrow x > y$

yRx, $\log_a y > \log_a x \Rightarrow y > x$

This is also not valid. Hence, relation is not symmetric also.

For xRy, $\log_a x > \log_a y \Rightarrow x > y$

For yRz, $\log_a y > \log_a z \Rightarrow y > z$

So, xRz, $\log_a x > \log_a z \Rightarrow x > z$

This is a valid relation. Hence, relation is only transitive.

57. (d) $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix} = \begin{vmatrix} i & -1 & -i \\ 1 & -1 & 1 \\ i & 1 & -i \end{vmatrix}$

$= i(i-1) - 1(i-(-i)) - i(1+i)$

$= i^2 - i - 2i - i - i^2 = -4i.$

58. (c) $AB = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$

59. (b) 2nd and 4th place are even place, so vowel 'A' and 'E' arrange either 2nd or 4th place in $2! = 2 \times 1 = 2$ ways
Consonent letter, 'B' and 'L' arrange at 1st and 3rd place in $2! = 2 \times 1 = 2$ ways.
Total number of arrangement $= 2 \times 2 = 4$.

60. (c) Maximum number of points of intersection of 2 non-overlapping circles $= 2$.
So, maximum number of points of intersection of 5 non-overlapping circles $= 8 + 6 + 4 + 2 = 20$.

61. (c)

Number of students		Marks Range	Difference of number of students
Physics	Maths		
11	21	20 – 30	21 – 11 = 10
30	38	30 – 40	38 – 30 = 8
26	15	40 – 50	26 – 15 = 11 ←Maximum
15	10	50 – 60	15 – 10 = 5

Hence, difference is largest for the interval (40-50).

62. (a)
(1) Modal value of the marks of Physics is the interval in which maximum number of students got his marks. In the marks interval of (30–40), number of students in Physics is 30, which is largest number of students in any interval. Hence, modal values of marks in Physics is (30–40). Statement (1) is correct.

(2) Median class is given by $\left(\dfrac{N}{2}\right)^{th}$ item i.e. $\left(\dfrac{100}{2}\right)^{th}$ item

which is 50th item. This corresponds to the class interval of (40–50) for Physics and (30–40) for Mathematics.

Marks	Number of Physics students	Cumulative Frequency	Number of Maths student	Cumulative Frequency
10-20	8	8	10	10
20-30	11	19	21	31
30-40	30	49	38	69
40-50	26	75	15	84
50-60	15	90	10	94
60-70	10	100	6	100

$\text{Medium} = l_1 + \dfrac{\dfrac{N}{2} - C.f.}{f} \times i$

$\therefore \text{Median for Physics} = 40 + \dfrac{\dfrac{100}{2} - 49}{26} \times 10$

$= 40 + \dfrac{50 - 49}{26} \times 10 = 40.385$

$\text{Median for Maths} = 30 + \dfrac{\dfrac{100}{2} - 31}{38} \times 10$

$= 30 + \dfrac{50 - 31}{38} \times 10 = 35$

Thus, median of the marks in Physics is more than median of the marks in Mathematics.
Hence, statement (2) is not correct.

63. (c) For physics

Marks	c_i	f_i	$c_i f_i$
10–20	15	8	120
20–30	25	11	275
30–40	35	30	1050
40–50	45	26	1170
50–60	55	15	825
60–70	65	10	650
		$\Sigma_i f_i$ 070	$\Sigma_i f_i$ 070

$\text{Mean of marks of physics} = \dfrac{4090}{100} = 40.9.$

64. (b) $\text{Standard deviation} = \sqrt{\dfrac{(x-\bar{x})^2}{N}}$

Given data: $-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}$.

$\bar{x} = \dfrac{\text{Sum of datas}}{\text{Number of data}} = 0$

$(x-\bar{x})^2 = (-\sqrt{6}-0)^2 + (-\sqrt{5}-0)^2 + (-\sqrt{4}-0)^2 +$

$(-1-0)^2 + (1-0)^2 + (\sqrt{4}-0)^2 + (\sqrt{5}-0)^2 + (\sqrt{6}-0)^2$

$= 6 + 5 + 4 + 1 + 1 + 4 + 5 + 6 = 32$

$\text{S.D.} = \sqrt{\dfrac{(x-\bar{x})^2}{N}} = \sqrt{\dfrac{32}{8}} = \sqrt{4} = 2$

65. (d) Coefficient of Variation (C.V.)

$$= \frac{\text{Standard Deviation } (\sigma)}{\text{Mean } (\mu)} \times 100$$

Now, standard deviation $(\sigma) = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

$$= \sqrt{\frac{200}{10} - \left(\frac{20}{10}\right)^2} = 4$$

Mean $(\mu) = \frac{\Sigma x_i}{n} = \frac{20}{10} = 2.$

∴ Co-efficient of variation (C.V.) $= \frac{4}{2} \times 100 = 200.$

66. (b) Number of days in February, when year is a leap year
= 29 days = 4 weeks + 1 odd day
4 weeks have 4 sundays.

Probability that 1 odd day is sunday $= \frac{1}{7}.$

67. (b) Correct arithmetic mean

$$= 40 + \frac{(-83 + 53)}{100} = 40 + \frac{(-30)}{100} = 39.7.$$

68. (a) Probability for Husband's selection $P(H) = \frac{1}{7}$

Probability when Husband is not selection

$$P(H') = 1 - \frac{1}{7} = \frac{6}{7}$$

Probability for wife's selection $P(W) = \frac{1}{5}$

Probability when Wife is not selected $P(W') = 1 - \frac{1}{5} = \frac{4}{5}$

As both are independent event.
So, probability for atleast one of them will be selected
$$= P(H)P(W') + P(H')P(W)$$
$$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{11}{35}.$$

69. (a) Number of ways of picking 4 counterfeits gold coins out of 6 counterfeits coins = $^6C_4$
Number of ways of picking 4 coins out of 15 gold coins = $^{15}C_4$

∴ Required probability $= \frac{^6C_4}{^{15}C_4} = \frac{15}{15 \times 7 \times 13} = \frac{1}{91}.$

70. (d) Number of ways of selecting 2 boys out of 2 boys
$= {}^2C_2 = 1.$
Number of ways of selecting 1 girl out of 2 girls $= {}^2C_1 = 2.$
Number of ways of selecting 3 out of 4 persons $= {}^4C_3 = 4.$

∴ Required probability $= \frac{^2C_1 \times {}^2C_2}{^4C_3} = \frac{2 \times 1}{4} = \frac{1}{2}.$

71. (c) Prime number between 1 to 10 are 2, 3, 5, 7.
Now, number of ways of selecting 2 prime number out of 4
prime number = $^4C_2 = 6.$
Number of ways of selecting 2 numbers out of 10 numbers
$= {}^{10}C_2 = 45.$

∴ Required probability $= \frac{6}{45} = \frac{2}{15}.$

72. (b) Regression of Y on X :

$$Y = \left\{\frac{\sigma_y}{\sigma_x} \times r(x, y)\right\} X - a \quad \text{(where } a = \text{constant term)}$$

$$\Rightarrow Y = \left(\frac{3.5}{2.5} \times 0.8\right) X - 5.8$$

$$\therefore Y = 1.12X - 5.8.$$

73. (d) From question $P = \frac{1}{5}$

Now, $P + q = 1$

$$\Rightarrow \frac{1}{5} + q = 1 \Rightarrow q = 1 - \frac{1}{5} = \frac{4}{5}.$$

and $n = 10.$

74. (d) $P(\bar{A}) = 1 - P(A) = 1 - 0.6 = 0.4.$

$P(\bar{B}) = 1 - P(B) = 1 - 0.5 = 0.5.$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$$= 0.6 + 0.5 - 0.4 = 0.7.$$

(1) Now $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$
$$= P(\bar{A}) + P(B) - \{P(B) - P(A \cap B)\}$$
$$= 0.4 + 0.5 - \{0.5 - 0.4\} = 0.8.$$

Hence, statement (1) is not correct.

(2) $P(\bar{B} \mid \bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = \frac{P(B \cup A)'}{P(\bar{A})} = \frac{1 - P(A \cup B)}{P(\bar{A})}$

$$= \frac{1 - 0.7}{0.4} = 0.75.$$

Hence, statement (2) is not correct.

75. (b) From question, we have
$P(F \mid X) = 0.02, P(F \mid Y) = 0.03, P(F \mid Z) = 0.05$
and
$$P(F) = P(X) \cdot P(F \mid X) + P(Y) \cdot P(F \mid Y)$$
$$+ P(Z) \cdot P(F \mid Z)$$
$$= 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.05$$
$$= 0.01 + 0.009 + 0.01 = 0.029.$$
Now,
$$P(X \cap F) = P(X) \cdot P(F \mid X) = 0.5 \times 0.02 = 0.01.$$

$$\therefore P(X \mid F) = \frac{P(X \cap F)}{P(F)} = \frac{0.01}{0.029} = \frac{10}{29}.$$

76. (a)
(1) $f(x) = e^{-|x|}, f(0) = e^{-|0|} = 1.$

$f(x) = e^{-x}$, for $x \geq 0$
$\quad = e^x$, for $x < 0.$

$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} e^x = 1$

$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} e^{-x} = 1$

As, $f(0) = \text{LHL} = \text{RHL}$

Thus $f(x)$ is continuous at $x = 0$.

(2) $\text{LHD} = \lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} e^x = e^0 = 1$

$\text{RHD} = \lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} -e^{-x} = -e^0 = -1$

As $\text{LHD} \neq \text{RHD}$

Hence, $f(x)$ is not differentiable at $x = 0$.

77. (c) $y = \sin x \cdot \cos x$

$= \dfrac{2\sin x \cdot \cos x}{2} = \dfrac{\sin 2x}{2}$

Now, maximum value of $\sin 2x = 1$.

$\therefore y_{\max} = \dfrac{1}{2}$.

78. (a) $\lim_{x \to 0} \dfrac{3^x + 3^{-x} - 2}{x} = \lim_{x \to 0} \dfrac{3^x \cdot \log 3 - 3^{-x} \cdot \log 3}{1}$

(By L'Hospital rule)

$= \dfrac{3^0 \cdot \log 3 - 3^0 \cdot \log 3}{1} = 0$

79. (a) Let $y = \tan^{-1} x$; and $z = \cot^{-1} x$

Then, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ and $\dfrac{dz}{dx} = -\dfrac{1}{1+x^2}$

Now, $\dfrac{dy}{dz} = \dfrac{\frac{dy}{dx}}{\frac{dz}{dx}} = \dfrac{\frac{1}{1+x^2}}{-\left(\frac{1}{1+x^2}\right)} = -1$.

80. (b) $x(dx - dy) + y(dy - dx) = 0$

On integrating both sides, we have

$\int x(dx - dy) + \int y(dy - dx) = C$ (where C = constant)

$\dfrac{x^2}{2} - xy + \dfrac{y^2}{2} - xy = C$

$x^2 + y^2 - 2xy = C$

Or, $x^2 + y^2 = 2xy + C$.

81. (a) Let $y = 3 \cdot \cos\left(A + \dfrac{\pi}{3}\right)$, $y' = -3 \cdot \sin\left(A + \dfrac{\pi}{3}\right)$
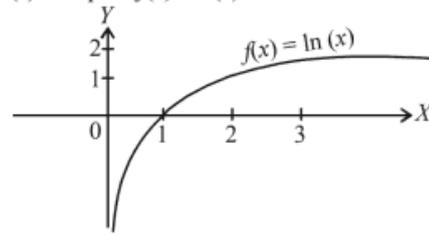
For extremum value, $y' = 0$

$\sin\left(A + \dfrac{\pi}{3}\right) = 0 \Rightarrow A + \dfrac{\pi}{3} = 0 \Rightarrow A = -\dfrac{\pi}{3}$

$y'' = -3 \cdot \cos\left(A + \dfrac{\pi}{3}\right)$
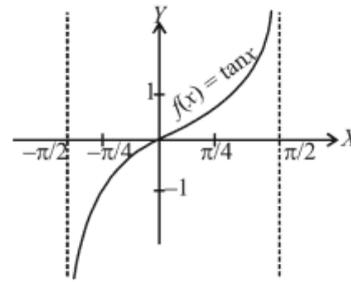
$= -3\cos\left(-\dfrac{\pi}{3} + \dfrac{\pi}{3}\right) = -3\cos(0) = -3$.

82. (c)

(1) Graph of $f(x) = \ln(x)$

This is increasing function in the interval $(0, \infty)$.

(2) Graph of $f(x) = \tan x \cdot$

This is also increasing in $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.
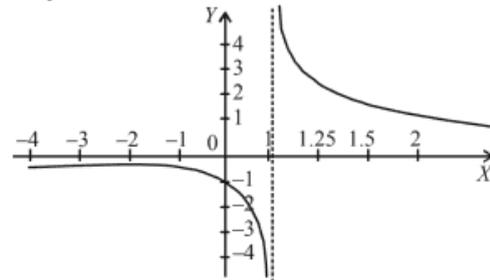
83. (b) Given graph $Y = \dfrac{1}{x-1}$

This is defined for all real x, except $x = 1$.

Range of the graph is $y \in R \mid y \neq 0$

Table for the graph :

x	−4	−3	−1	0	0.25	0.5	0.75	1.25	1.5	2
y	−0.2	−0.25	−0.5	−1	−1.33	−2	−4	4	2	1

Graph of function -

Thus, graph intersect y-axis at $(0, -1)$.

84. (a) $\ln\left(\dfrac{dy}{dx}\right) = x$

$\dfrac{dy}{dx} = e^x$

Integrating both sides, we get

$\int dy = \int e^x \, dx$

$y = e^x + C$, where C = integration constant.

85. (d) $l + b = K$, where l = length, b = breadth.

Area of the rectangle $A = l \cdot b$.

$A = l(K - l) = lK - l^2$

For maximum area, $\dfrac{dA}{dl} = 0$

$\dfrac{d}{dl}(lk - l^2) = 0$

$K - 2l = 0 \Rightarrow l = \dfrac{K}{2}$

Now, $\dfrac{K}{2} + b = K \Rightarrow b = K - \dfrac{K}{2} = \dfrac{K}{2}$

Area, $A = l \cdot b = \dfrac{K}{2} \cdot \dfrac{K}{2} = \dfrac{K^2}{4}$

86. (b) Arithmetic mean $= \dfrac{\text{Sum of data}}{\text{Number of data}}$

$6 = \dfrac{4 \times x + 9(x - 1)}{(x + x - 1)}$

$6(2x - 1) = 4x + 9x - 9$

$12x - 6 = 13x - 9$

$x = 9 - 6 = 3$.

87. (c) Number of ways in which one of the face having the number 6 and no two dice show the same number.

$(1, 2, 6), (1, 3, 6), (1, 4, 6), (1, 5, 6), (2, 3, 6), (2, 4, 6), (2, 5, 6), (3, 4, 6), (3, 5, 6) \ldots\ldots$

Total favourable case $= 20 + 20 + 20 = 60$.

Number of total output when three top faces of three dice shows different number $= 6 \times 5 \times 4 = 120$.

$\therefore$ Required probability $= \dfrac{60}{120} = \dfrac{1}{2}$.

88. (c) $P(A \cup B) = \dfrac{5}{6}$, $P(A \cap B) = \dfrac{1}{3}$, $P(A') = \dfrac{1}{2}$

$P(A) = 1 - P(A') = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\dfrac{5}{6} = \dfrac{1}{2} + P(B) - \dfrac{1}{3}$

(a) $P(B) = \dfrac{5}{6} - \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5 - 3 + 2}{6} = \dfrac{4}{6} = \dfrac{2}{3}$

(b) $P(A \cap B) = \dfrac{1}{3} = \dfrac{1}{2} \cdot \dfrac{2}{3} = P(A) \cdot P(B)$

$\therefore P(A \cap B) = P(A) \cdot P(B)$.

(c) $P(A \cup B) = \dfrac{5}{6}$

$P(A) + P(B) = \dfrac{1}{2} + \dfrac{2}{3} = \dfrac{7}{6}$

$\therefore P(A \cup B) < P(A) + P(B)$

Hence, option (c) is not correct.

(d) $P(A' \cap B') = P(A') \cdot P(B')$

$1 - P(A \cup B) = 1 - \dfrac{5}{6} = \dfrac{1}{6}$

and $P(A') \cdot P(B') = (1 - P(A)) \cdot (1 - P(B))$

$= \left(1 - \dfrac{1}{2}\right) \cdot \left(1 - \dfrac{2}{3}\right) = \dfrac{1}{2} \cdot \dfrac{1}{3} = \dfrac{1}{6}$.

Hence, $P(A' \cap B') = P(A') \cdot P(B')$.

89. (d) Number of data set $= n$.

Mean $= 2.5$

Sum of deviations $= 50$

Again, sum of deviations, when mean $= 3.5$ is -50.

So, $(3.5 - 2.5) \times n = 50 - (-50)$

$\therefore n = 100$

90. (d) Sum of n observation $= 2M \times n$

Sum of $2n$ observation $= M \times 2n$

Mean of combined data sets $= \dfrac{2Mn + 2Mn}{(n + 2n)} = \dfrac{4}{3}M$.

91. (c) $f(0) = 3(0)^2 - 5(0) + P = P$

$f(1) = 3(1)^2 - 5(1) + P = P - 2$

Clearly, $f(0)$ and $f(1)$ are opposite in sign.

$\therefore P > 0$ and $P - 2 < 0$.

$\Rightarrow 0 < P < 2$.

92. (b) $e^{\theta\phi} = C + 4\theta \cdot \phi$

$(\theta \cdot \phi) \ln e = \ln(C + 4\theta \cdot \phi)$

$\theta \cdot \phi = \ln(C + 4\theta \cdot \phi)$

$\Rightarrow \theta \cdot g\phi = 4\{\ln(\theta) + \ln(\phi)\} + \ln C$

Differentiating both sides with respect to 'θ'.

$\phi + \theta \cdot \dfrac{d\phi}{d\theta} = 4\left(\dfrac{1}{\theta} + \dfrac{1}{\phi} \cdot \dfrac{d\phi}{d\theta}\right)$

$\phi \cdot d\theta + \theta \cdot d\phi = 4\phi \cdot d\theta + 4 \cdot \theta \cdot d\phi$

$\Rightarrow 3\theta \cdot d\phi = -3\phi \cdot d\theta$

$\therefore \phi \cdot d\theta = -\theta \cdot d\phi$

93. (b) $\int P(x) \cdot dx = \int (4 \cdot e)^{2x} \cdot dx = \int e^{2x(\ln e + \ln 4)} \cdot dx$

$= \int e^{2x(1 + \ln 4)} \cdot dx = \dfrac{e^{2x(1 + \ln 4)}}{2(1 + \ln 4)} + C$ where C = constant

$= \dfrac{(4e)^{2x}}{2(1 + 2\ln 2)} + C$

$\therefore \int P(x) \cdot dx = \dfrac{P(x)}{2(1 + 2\ln(2))} + C$.

94. (b) $I = \int\limits_{0}^{\frac{\pi}{4}} (\tan^3 x + \tan x) \cdot dx$

$= \int\limits_{0}^{\frac{\pi}{4}} \tan^3 x \cdot dx + \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot \tan^2 x \cdot dx + \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot (\sec^2 x - 1) \cdot dx + \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot \sec^2 x \cdot dx - \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot dx + \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int\limits_{0}^{\frac{\pi}{4}} \tan x \cdot \sec^2 x \cdot dx$

Let $\tan x = z, dz = \sec^2 x \cdot dx$

$\therefore \int \tan x \cdot \sec^2 x \cdot dx = \int z \cdot dz = \frac{z^2}{2} + C$

where $C = $ constant

$\therefore I = \left[\frac{\tan^2 x}{2} \right]_{0}^{\frac{\pi}{4}} = \frac{1}{2} - 0 = \frac{1}{2}.$

95. (c) From question, $dx = 10.1 - 10 = 0.1$
At, $x = 10$

$y = (3)(10)^2 + 2 = 302.$

At $x = 10.1,\ y = 3(10.1)^2 + 2 = 308.03.$

Total change in $y = 308.03 - 302 = 6.03.$

96. (b) As the given function is continuous at $x = 0$, then
LHL = RHL

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \frac{\sin x}{x} = \lim\limits_{x \to 0^-} \left[\frac{\cos x}{1} \right] = 1$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \frac{\sin x}{x} = \lim\limits_{x \to 0^+} \left[\frac{\cos x}{1} \right] = 1$

$\therefore \frac{\sin x}{x}$ at $x = 0$, should be 1.

97. (b) $dy = (1 + y^2) \cdot dx$

$\frac{1}{(1 + y^2)} \cdot dy = dx$

Integrating both sides, we get

$\int \frac{dy}{(1 + y^2)} = \int dx$

$\tan^{-1}(y) = x + C,$ where $C = $ constant.

$\therefore y = \tan(x + C).$

98. (c) $\int (e^{\log x} + \sin x) \cdot \cos x \cdot dx = \int (x + \sin x) \cdot \cos x \cdot dx$

$= \int (x \cdot \cos x + \sin x \cdot \cos x) \cdot dx$

$= \int x \cdot \cos x \cdot dx + \int \sin x \cdot \cos x \cdot dx$

$= x \cdot \int \cos x \cdot dx - \int \left(\frac{dx}{dx} \cdot \int (\cos x) \cdot dx \right) \cdot dx$

$+ \int \sin x \cdot \cos x \cdot dx$

$= x \cdot \sin x - \int \sin x \cdot dx + \int \sin x \cdot \cos x \cdot dx$

$= x \cdot \sin x - (-\cos x) + \int \sin x \cdot \cos x \cdot dx$

$= x \sin x + \cos x + \int \sin x \cdot \cos x \cdot dx$

Now, $\int \sin x \cdot \cos x \cdot dx;$

Let $\sin x = z,\ dz = \cos x \cdot dx$

$\int \sin x \cdot \cos x \, dx = \int z \cdot dz = \frac{z^2}{2} = \frac{\sin^2 x}{2}$

$\therefore I = x \cdot \sin x + \cos x + \frac{\sin^2 x}{2} + C.$

99. (b) $f(x) = \cos^{-1}(x - 2)$

Domain of $\cos^{-1}(x - 2) = [-1, 1]$

$\therefore -1 \le x - 2 \le 1$

$-1 + 2 \le x \le 1 + 2$

$1 \le x \le 3$

$\therefore$ Domain $= [1, 3]$

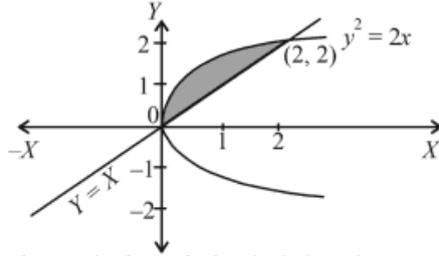100. (c) Two given graphs are $y^2 = 2x$ and $y = x.$
Point of intersections are :

$x^2 = 2x \Rightarrow x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ and 2 & $y = 0$ and $2.$

So, points of intersections are $(0, 0)$ and $(2, 2).$

The graph are

The required area is the shaded portion

$$A = \left| \int_0^2 (y_1 - y_2) \cdot dx \right|$$

$$= \left| \int_0^2 (\sqrt{2x} - x) \cdot dx \right| = \left[\frac{2\sqrt{2}(x)^{3/2}}{3} - \frac{x^2}{2} \right]_0^2$$

$$= \frac{2\sqrt{2}}{3}(2)^{\frac{3}{2}} - 2 = \frac{8}{3} - 2 = \frac{2}{3}$$

101. (a) $f(x) = 2x - x^2$

$f(x+2) + f(x-2)$

$= 2(x+2) - (x+2)^2 + 2(x-2) - (x-2)^2$

$= 4 + 4x - x^2 - 4 - 4x - x^2 - 4 + 4x - 4$

$= 8x - 2x^2 - 8$

When $x = 0$, then, $f(x+2) + f(x-2) = -8$

102. (b) $x^m y^n = a^{m+n}$

$$\frac{x^m}{a^m} = \frac{a^n}{y^n} \qquad \qquad ...(i)$$

Differentiating equation (i) w.r.t. x, we get

$$\frac{m \cdot x^{(m-1)}}{a^m} = \frac{a^n \cdot (-n)}{y^{n+1}} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{m \cdot y^{(n+1)} \cdot (x)^{(m-1)}}{-n \cdot a^m \cdot a^n}$$

$$= -\frac{my}{n \cdot x} \cdot \left(\frac{x^m \cdot y^n}{a^{m+n}} \right) = -\frac{my}{nx}(1) = -\frac{my}{nx}$$

103. (a) $\int \dfrac{dx}{x(x^n + 1)} = \int \dfrac{x^{n-1}}{x^n(x^n + 1)} \cdot dx$

Let $z = x^n \Rightarrow dz = nx^{n-1}dx$

$$\therefore \int \frac{dx}{x(x^n + 1)} = \int \frac{x^{n-1}}{x^n(x^n + 1)} \cdot dx = \int \frac{1}{z(z+1)} \cdot \frac{dz}{n}$$

$$= \frac{1}{n} \left[\int \frac{1}{z} \cdot dz - \int \frac{dz}{z+1} \right]$$

$$= \frac{1}{n} \left[\ln z - \ln(z+1) \right] + C \qquad \text{where } C = \text{constant}$$

$$= \frac{1}{n} \cdot \ln \left(\frac{z}{z+1} \right) + C = \frac{1}{n} \ln \left(\frac{x^n}{x^n + 1} \right) + C$$

104. (a) Minimum value of any modulus is 0.

105. (c) $I = \int \left(\dfrac{3x^2 + 8 - 4k}{x} \right) \cdot dx$

$$= \int 3x \, dx + \int \frac{8 - 4k}{x} \cdot dx$$

$$= \frac{3}{2}x^2 + (8 - 4k) \cdot \ln(x) + C \qquad \text{where } C = \text{constant}$$

To get integration as rational function,

$$(8 - 4k) \cdot \ln(x) = 0 \Rightarrow 8 - 4k = 0 \Rightarrow k = \frac{8}{4} = 2$$

106. (d) Radius of the sphere

$$= \frac{6(1) - 3(-2) + 2(3) - 4}{\sqrt{(6)^2 + (-3)^2 + (2)^2}} = \frac{6+6+6-4}{7} = \frac{14}{7} = 2$$

Diameter of the sphere $= 2 \times 2 = 4$ units

107. (b) Perpendicular distance $= \sqrt{(4)^2 + (3)^2} = 5$ units.

108. (d) Direction ratios are $<a + b, b + c, c + a>$
Then, direction cosine,

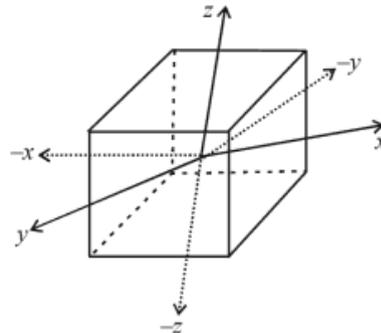$$l = \frac{(a+b)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$m = \frac{(b+c)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$n = \frac{(c+a)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

Sum of squares of direction cosines

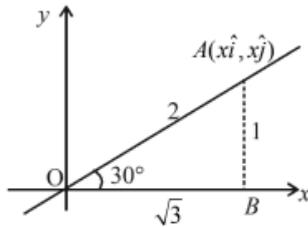$$l^2 + m^2 + n^2 = \frac{(a+b)^2 + (b+c)^2 + (c+a)^2}{(a+b)^2 + (b+c)^2 + (c+a)^2} = 1.$$

109. (c)

Co-ordinate plane divide the space into 8 octanes.

110. (b) Equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane, is
$z = 5, y = 0, x = 0$.

111. (a)

Let $A(x\hat{i}, y\hat{j})$ is a point in the xy-plane.

From question,

$\angle AOB = 30°$ and $\overrightarrow{OA} = 1$

$x = |\overrightarrow{OA}| \cdot \cos 30° = 1 \times \dfrac{\sqrt{3}}{2}\hat{i} = \dfrac{\sqrt{3}}{2}\hat{i}$

$y = |\overrightarrow{OA}| \cdot \sin 30° = 1 \times \dfrac{1}{2}\hat{j} = \dfrac{\hat{j}}{2}$

$\therefore \vec{a} = \dfrac{\sqrt{3}}{2}\hat{i} + \dfrac{1}{2}\hat{j} = \dfrac{\sqrt{3}\hat{i} + 1\hat{j}}{2}$

112. (c) Let $A = (x, y, z)$

Then, $|\overrightarrow{OA}| = \sqrt{x^2 + y^2 + z^2} = 12$

Also, $x = |\overrightarrow{OA}| \cdot \cos 45° = 12 \cdot \dfrac{1}{\sqrt{2}} = 6\sqrt{2}\hat{i}$.

$y = |OA| \cdot \cos 60° = 12 \cdot \dfrac{1}{2} = 6\hat{j}$.

Hence, $\overrightarrow{OA} = 6\sqrt{2}\hat{i} + 6\hat{j} \pm 6\hat{k}$

113. (c) Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Two diagonals of the parallelograms are given by

$(\vec{a} + \vec{b}) = (2+1)\hat{i} - (4+2)\hat{j} + (5-3)\hat{k}$

$\qquad = 3\hat{i} - 6\hat{j} + 2\hat{k}$

and $(\vec{a} - \vec{b}) = (2-1)\hat{i} - (4-2)\hat{j} + (5-(-3))\hat{k}$

$\qquad = \hat{i} - 2\hat{j} + 8\hat{k}$

Dot products of the diagonals

$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot (\hat{i} - 2\hat{j} + 8\hat{k})$

$\qquad = (3 + 12 + 16) = 31$ units

114. (a) $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$

$\{|\vec{a}| \cdot |\vec{b}| \cdot \sin\theta\}^2 + \{|\vec{a}| \cdot |\vec{b}| \cdot \cos\theta\}^2 = 144$

$(|\vec{a}| \cdot |\vec{b}|)^2 (\sin^2\theta + \cos^2\theta) = 144$

$|\vec{a}| \cdot |\vec{b}| = \sqrt{144} = 12$

$4 \cdot |\vec{b}| = 12 \Rightarrow |\vec{b}| = \dfrac{12}{4} = 3$.

115. (b) As the given vectors are coplanar, then

$\vec{c} \times \vec{a} \times \vec{b} = 0 \Rightarrow \begin{vmatrix} 0 & 1 & p \\ 2 & -3 & 1 \\ 1 & 2 & -3 \end{vmatrix} = 0$

$\Rightarrow 0 + 1(1+6) + p(4+3) = 0$

$\Rightarrow 7 + 7p = 0 \Rightarrow p = -1$.

116. (d) $\displaystyle\lim_{x \to 1} \dfrac{x + x^2 + x^3 - 3}{x - 1}$

This is in $\dfrac{0}{0}$ form, so we can use L' Hospital rule.

$\displaystyle\lim_{x \to 1} \dfrac{x + x^2 + x^3 - 3}{x - 1} = \lim_{x \to 1} \dfrac{1 + 2x + 3x^2}{1} = \dfrac{1 + 2 + 3}{1} = 6$

117. (a) Circumference of a circle $C = 2\pi r$

Differentiating both sides w.r.t. time (t).

$\dfrac{dc}{dt} = 2\pi \cdot \dfrac{dr}{dt} = 2\pi \cdot (0.7) = 1.4\pi = 1.4 \times \dfrac{22}{7} = 4.4$ cm/sec.

118. (c) $\displaystyle\lim_{x \to 1} \dfrac{x^4 - 1}{x - 1} = \lim_{x \to 1} \dfrac{(x-1)(x+1)(x^2+1)}{(x-1)}$

$\qquad = \displaystyle\lim_{x \to 1}(x+1)(x^2+1) = (1+1)(1+1) = 4$.

Now, $\displaystyle\lim_{x \to k} \dfrac{x^3 - k^3}{x^2 - k^2} = 4$

$\displaystyle\lim_{x \to k} \dfrac{(x-k)(x^2 + k^2 + xk)}{(x-k)(x+k)} = 4$

$\displaystyle\lim_{x \to k} \dfrac{(x^2 + k^2 + xk)}{(x+k)} = 4$

$\dfrac{k^2 + k^2 + k \cdot k}{k + k} = 4 \Rightarrow \dfrac{3}{2}k = 4 \Rightarrow k = \dfrac{8}{3}$

119. (b) Given differential equation is

$k \cdot \dfrac{dy}{dx} = \int \left[1 + \left(\dfrac{dy}{dx}\right)^2 \right]^{2/3} \cdot dx$

On differentiating both sides, we get

$k \cdot \dfrac{d^2 y}{dx^2} = \left[1 + \left(\dfrac{dy}{dx}\right)^2 \right]^{2/3}$

$\Rightarrow \left(k \cdot \dfrac{d^2 y}{dx^2} \right)^3 = \left[1 + \left(\dfrac{dy}{dx}\right)^2 \right]^2$

Now, order of the D.E. = 2
Degree of the D.E. = 3.

120. (a) $\lim_{x \to 0} \dfrac{\sin x \cdot \log(1-x)}{x^2}$

This is in $\dfrac{0}{0}$ form, by using L' Hospital rule.

$\lim_{x \to 0} \dfrac{\sin x \cdot \log(1-x)}{x^2}$

$= \lim_{x \to 0} \dfrac{\cos x \cdot \log(1-x) + \sin x \cdot \dfrac{1}{(1-x)}(-1)}{2x}$

$= \lim_{x \to 0} \dfrac{\log(1-x) \cdot (-\sin x) - \dfrac{\cos x}{(1-x)} - \dfrac{1}{(1-x)} \cdot \cos x - \dfrac{\sin x}{(1-x)^2}}{2}$

$= \dfrac{0 - 1 - 1 - 0}{2} = \dfrac{-2}{2} = -1.$

GENERAL ABILITY

PART- A: ENGLISH

1. (b) Absently means in an absentminded or preoccupied manner; its antonyms must be alertly which means in mentally perceptive and responsive way. Other options capably, agitatedly and dreamily mean in a competent capable manner, in a physically disturbed manner and in a dreamy manner respectively.

2. (c) Educated means possessing an education (especially having more than average knowledge) while ignorant means lacking knowledge or sophistication. Other options cerebral, enlightened and erudite mean involving intelligence rather than emotions or instinct, having knowledge and spiritual insight and having or showing profound knowledge respectively.

3. (b) Important means of great significance or value; trivial means small and of little importance.

4. (c) Move means move so as to change position, perform a non-translational motion; its antonym stall means come to a stop.

5. (d) Express means serve as a means for expressing something; its antonym conceal means prevent from being seen or discovered.

6. (b) Warmer means having or producing a comfortable and agreeable degree of heat or imparting or maintaining heat; its antonym is colder.

7. (b) Concerned means feeling or showing worry or solicitude; its antonym indifferent means marked by a lack of interest.

8. (d) Debate means a discussion in which reasons are advanced for and against some proposition or proposal; agreement means harmony of people's opinions, actions or characters.

9. (a) Sensitivity means the ability to respond to physical stimuli or to register small physical amounts or differences; imperviousness means the quality of being impenetrable (by people, light or missiles etc.).

10. (c) Huge means unusually great in size or amount or degree or especially extent or scope; tiny means very small.

11. (a) PRQS. The sentence must start with (P) several years ago, then the subject (R) professor Andrew Dessler then after introductory, there should be 'course' (Q), the last remains (S).

12. (a) PRQS is the correct sequence.

13. (b) At least three options start with P, hence starting with P is final which also meaningful. P ends with layer of the...hence connecting part must be S, then come Q and R.

14. (d) PRSQ is the correct sequence which forms a meaningful sentence.

15. (a) PSQR. P ends with the comparative 'warmer' so it must connect with the 'than' which is (S). Again S ends with 'closer' which connects the preposition 'to' Q, R is the last.

16. (d) Only S part is independent and should be the beginning of sentence. Then most suitable connecting part must be P, then come R and Q respectively.

17. (c) SRPQ is the correct sequence which forms a meaningful sentence.

18. (c) Only RQPS makes a substantive expression.

19. (b) RQPS. The sentence must start with R, the job. Then come Q, P and S respectively which form a correct sentence.

20. (d) RSQP. The sentence must start with R i.e. instead of worrying about...then comes S part, after model cannot, there must be a verb starting with and that is control in Q. P is the last part.

21. (c) Divorce means the legal dissolution of a marriage while annulment means a formal termination (of a relationship or a judicial proceeding etc) which is synonym in meaning.

22. (b) Gentle means kind; not harsh, stern or severe; its synonym amiable means diffusing warmth and friendliness.

23. (a) Bounce back means to return quickly to a normal condition after a difficult situation or event; recoil means come back to the originator of an action with an undesired effect which expresses similar meaning.

24. (c) Execute means carry out an action; its synonym accomplish means cause to happen; complete successfully.

25. (c) With equanimity means with steadiness of mind under stress; patiently means with patience; in a patient manner.

26. (b) Futile means producing no result or effect; pointless also means the same i.e. serving no useful purpose; having no excuse for being.

27. (d) Cunning means marked by skill in deception; dodgy means of doubtful quality or legality which is similar in meaning.

28. (a) Crucial means of the greatest importance; momentous means of very great significance.

29. (c) Frenzy means state of violent mental agitation; berserk also means frenzied, out of control.

30. (d) Yearning means have a desire for something or someone who is not present; hankering means a yearning for something or to do something.

31. (b) Here the increase in greenhouse gases is the perfect expression.

32. (d) With the noun age in this context, the preposition 'at' is at its best. Other options do not fit the bill.

33. (a) If you are averse to something that means you are strongly opposed to that thing. Averse is usually followed by 'to'. Other options do not fit in the sentence.

34. (b) The choice is between since and for. For is used with a period of time; since is used to refer to a specific point in time.

35. (a) In the given context, the preposition 'on' is the only option. E.g. He spoke on condition that he not be identified. Agreement was reached to repay part of the debt, on condition that the remaining debt be cancelled.

36. (b) If we want to say that a man or woman is of something, then we use 'of'- a man of letter, a woman of substance, etc.

37. (a) The noun cure usually takes the preposition for. There's still no cure for cancer. The best cure for boredom is hard work!

38. (d) The phrase 'run into somebody' means to meet someone you know when you are not expecting to. Rajesh ran into Supriya at the theatre.

39. (a) Commonly we use boating in the lake. On the lake means 'only on the surface of lake.'

40. (c) Relieve someone of something means take (a burden) from someone. He relieved her of her baggage. Doctor relieved him of that abominable pain.

41. (b) The construction 'comprises of' is incorrect. It should be either 'comprises' or 'is comprised of' (in passive).

42. (b) The phrase 'look forward to verb +ing' is the correct construction. I'm looking forward to seeing you.

43. (b) The correct phrase 'cope with something' and not 'cope up with something'.

44. (c) 'Walking' is the correct replacement of 'by walk.'

45. (c) The correct phrase/usage is "by your watch". Time is an abstract concept and it does not have a body. So, it cannot be contained "in" a watch or anything else.

46. (d) The sentence is correct.

47. (b) Daughter-in-law is used to indicate one or single female married into a family. Its plural is daughters-in-law and not daughter-in-laws.

48. (c) When comparing two things with prefer, use 'to' and not 'than'. Some people prefer camping to staying in hotels.

49. (b) With discuss, the preposition 'about' is redundant.

50. (a) The correct phrase is dispose of and not dispose off. Shyama will dispose of her old clothes. The old clothes have been disposed of.

PART- B : GENERAL KNOWLEDGE

51. (c) Polynesia is considered as a sub-region of Oceania comprises of thousands of islands scattered over the central and southern part of Pacific Ocean. At consists of New Zealand, Tuvalu, Tonga, Samoa and many other small islands.

52. (d) Pradhan Mantri Krishi Sinchayee Yojana (PMKSY) was launched during 2015-16. The main objectives of the PMKSY are, to ensure access for means of irrigation to all agricultural farms in the country, to enhance the physical access of water on the farm, to achieve convergence of investments in irrigation at the field level, to enhance integrated development of rain-fed areas etc.

53. (d) Article 79 of the Constitution of India deals with composition of Parliament of India. The president of India, and both the lower house and the upper house that is Lok sabha (House of the People) and Rajya sabha (council of states) constitute the Parliament.

54. (b) The Five Year Plan was first launched in USSR. The first five-year plan in India was launched in 1951 and since then. It was implemented by then USSR premier Joseph Stalin in 1928. India had launched twelve Five Year Plans and it is no longer exists.

55. (b) The call for *Garibi Hatao* was incorporated in Fifth Five-Year Plan of India. The main emphasis of the fifth five-year plan was on poverty alleviation, employment generation, Self reliance in agriculture production and justice. The period of this five year plan was between 1974 to 1978 but in 1978 the plan was rejected by then government and replaced with new plan.

56. (b) Faster and more inclusive growth is related to Eleventh Five-Year Plan, Faster, more Inclusive and sustainable growth is related to Twelfth Five-Year Plan. Correction of disequilibrium caused by Second World War was the objective of first five-year plan. Attaining self-reliance in agriculture production, poverty alleviation and employment generation was the objective of fifth five-year plan.

57. (b) Equal justice and free legal aid was inserted by the Constitution (42nd Amendment) Act, 1976. It defines- the State shall secure that the operation of the legal system promotes justice, on a basis of equal opportunity, and shall, in particular, provide free legal aid, by suitable legislation or schemes or in any other way, to ensure that opportunities for securing justice are not denied to any citizen by reason of economic or other disabilities.

58. (a) A common High Court for two or more states or two or more States and a Union territory can be established by a law passed by the Parliament of India. Calcutta high court serves as the high court of Anadaman & Nicobar islands and West Bengal. Similarly, Bombay high court is the high court for Maharashtra, Goa, Dadra and Nagar Haveli and Daman and Diu.

59. (c) In 1928 a committee of Congress headed by Motilal Nehru, drafted a Constitution for India. It is known as Nehru Report. The Committee worked on the principles of the Constitution of India along with the problem of communalism and issue of dominion status and submitted its report in 1929.

60. (d) Jaipal Singh was a prolific sportsperson, a politician and a tribal activist. He was member of the Constituent Assembly. He was the captain of Indian hockey team that clinched Gold in the 1928 Summer Olympics in Amsterdam. He also worked for the rights of tribals and creation of a separate homeland in Chhota nagpur region.

61. (b) Tikki Mausi is a mascot which was created to spread awareness about nutrition of children and women. The mascot was created by *Department of Women & Child Development*, Government of Odisha in joint collaboration with United Nations International Children's Emergency Fund (UNICEF).

62. (d) Tourism and jobs: a better future for all was the theme of World Tourism Day 2019. The theme was selected to for the empowerment of people and communities for Progress, harmony and peace through tourism related activities.

63. (a) Army Exercise TSENTR, 2019 was hosted by Russia. It is one of the strategic-level military exercises conducted by Russia and participation from various countries like India, China, Kazakhstan, Kyrgyzstan, Pakistan, Tajikistan and Uzbekistan. More than 1lakh of Army personnel and 20000 equipments were involved during this multi-nation military exercise.

64. (c) Denmark is called the country of winds. The country uses more than 25% of country's energy needs through wind power.

65. (b) The oldest scientific department of Government of India is Survey of India. It was established in the year 1967. Its main role is mapping and surveying of country's territories.

66. (d) Naseem- Al-Bahr is a naval exercise between India and Oman. This bilateral exercise is conducted since 1993 between two countries. 12th edition of this bilateral maritime exercise was conducted in Mormugao Port, Goa.

67. (c) Koneru Humpy is a famous Chess player of India. She became youngest Grandmaster in 2002 and winner of women Rapid chess Champioship. She is recipient of Arjuna Award and Padma Shri.

68. (a) The mascot of Khleo India Youth Games 2020 was Vijaya. The Khleo India Youth Games 2020 was held in January 2020 in Guwahati, Assam. Maharashtra wins the highest number of medals in the event. Haryana and Delhi were at the second and third place.

69. (b) In January 2020, the plane belonging to Ukraine International Airlines crashed minutes after take-off from the Imam Khomeini International Airport in Iran and burst into flames. The Kiev-bound Ukraine International Airlines Boeing 737 plane crashed after takeoff killing about 170 people onboard.

70. (c) The multilateral naval exercise MILAN 2020 was scheduled to be hosted by Visakhapatnam. MILAN is a multilateral naval exercise hosted by the Indian Navy and participated by more than 15 countries in earlier edition of this naval exercise.

71. (b) Chalk and marbles are different forms of Calcium carbonate. Chemical formula of both Marble and Chalk is $CaCO_3$. Marble is metamorphosed limestone; Chalk is also a form of limestone consists of soft, porous rocks.

72. (d) The number of maximum electrons in N Shell is 32. The maximum number of electrons that can fit in any given shell is given by the formula $2n^2$.
This means in N shell the number of maximum electrons can be given by $2 \times 4^2 = 32$ electrons.

73. (a) Vinegar is also known as ethanoic acid. It is a colourless liquid with the chemical formula CH_3COOH. It contains 5% to 20% ethanoic acid by volume and characterised by its pungent smell.

74. (b) The pressure of liquid kept in a glass beaker depends upon the height of liquid column inside the beaker. Pressure at different heights from the surface of liquid is given by formula, $P = P_A + \rho gh$
Where, P is total pressure, ρ = density of liquid, g is acceleration due to gravity and h is height of the liquid from the surface.

75. (d) Sound Wave travels only through a medium. It can travel through solid, liquid, gas but cannot travel through vacuum where there is no medium available for its travel.

76. (b) Deendayal Port is located at Gujarat. It is present in Kutch district of Gujarat. It is also known as Kandla port. It is located in the Gulf of Kutch at the western coast of India.

77. (c) Cartagena is associated with Biosafety Protocol to the Convention on Biological Diversity (2000). The Cartagena Protocol on Biosafety to the Convention on Biological Diversity is an international agreement which aims to ensure the safe handling, transport and use of living modified organisms resulting from modern biotechnology that may have adverse effects on biological diversity.

78. (d) Krishna Raja Sagara Dam/Reservoir is developed on river Kavery. M. Visvesvaraya was the chief architect who designed the conceptualised the idea of this dam. The main purpose of this dam for irrigation, regular water supply and power generation in the drought prone Mysore region. The dam was completed in 1938.

79. (d) Tripura does NOT share international border with two or more countries. It only international border with Bangladesh. Arunachal Pradesh shares border with Bhutan, Myanmar and China, Assam shares border with, Bangladesh and Bhutan, Mizoram shares border with Bangladesh and Myanmar.

80. (d) According to Census of India, 2011 among the states of Maharashtra, Madhya Pradesh, Odisha and Punjab, the least populated was Punjab. The population of Punjab was 27,743,338 in comparison to 112,374,333 of Maharashtra, 72,626,809 of Madhya Pradesh and 41,974,218 of Odisha.

81. (c) The speakers of major Indian languages belong to 4 major language families. The major language families are Indo-Aryan languages, Dravidian languages, Austroasiatic and Sino-Tibetan.

82. (b) Mahatma Gandhi initially forge the techniques of Satyagraha in South Africa. He initially advocated the doctrine of Satyagraha or non-violent protest against racial discrimination practiced in South Africa.

83. (a) Lord Irwin was the Viceroy of India at the time of Gandhiji's Dandi march in 1930. This march is also known as salt satyagraha and was an act of non-violent civil disobedience as a campaign of tax resistance and non-violent protest against the British salt monopoly.

84. (c) The Time magazine was highly sceptical of Gandhiji initially but within a week completely changed its opinion and saluted him as a Saint and Statesman following Dandi March. Mahatma Gandhi was declared Time Person of the year in 1930.

85. (b) In 1825 passenger trains were introduced for the first time in England. This was powered by steam.

86. (b) The correct relation between the radius of curvature R and focal length f of a spherical mirror is given by formula R=2f as the focal length of a spherical mirror is half of its radius of curvature.

87. (c) Due to phenomenon of refraction of light a lemon or other things kept in water in a glass tumbler appears to be larger than its actual size. When light travels from dense medium to rare medium as it passes from water into air, due to refraction water acts as convex lens and lemon and other things appeared bigger than their actual size.

88. (b) The cornea is the transparent part of the eye and covers the front portion of the eye. Light enters the eye through cornea. Cornea covers the pupil, iris and anterior chamber. The main function of cornea is to refract the light as it enters the eye.

89. (c) Issac Newton was the first scientist who used a glass prism to obtain the spectrum of Sunlight. In 1666, Issac Newton performed this experiment which is famously known as Newton's prism experiment. White light was scatte/sed into seven different colours as it exits from the prism.

90. (b) The total energy consumption in a month is given by, $5 \times 10 \times 30 = 1500$ Kwh. As the charge per kWh of energy is ₹4, hence for 1500 Kwh it will be $1500 \times 4 = ₹6000$.

91. (c) Magnetic field has both direction and magnitude, the magnetic field lines are closed curves and no two magnetic field lines are found to cross each other.

92. (b) Diamond is bad conductor of electricity. Electric current is due to flow of free electrons but in diamond four other carbon atoms are covalently bonded hence no free electrons are available to conduct electric current.

93. (d) Chloropropane molecule with molecular formula C_3H_7Cl contains ten covalent bonds. In covalent bonds electron pairs are shared between atoms.

94. (b) Atomic number is the most fundamental characteristic of an element. It defines number of protons in that element. Elements are arranged in order of atomic number in the periodic table.

95. (a) Neutrons are subatomic particles that reside inside the nucleus of an atom along with proton. It contains neutral charge and 1 a.m.u. . Neutrons were discovered by James Chadwick in 1932.

96. (*) Mass number is equal to the sum of number of protons and neutrons in it. Atomic mass is the mass of that element. Atomic mass may be very close to its mass number, but both are not the same and are different.

97. (b) Isotope of cobalt is used in the treatment of cancer. Cobalt-60 is used in radiotherapy machines produce gamma rays. These gamma rays are focussed on patient's body to target tumor tissues.

98. (b) Vacuole plays an important role in expelling of excess water and wastes in case of unicellular organisms. Vacuole in multi-cellular organisms also play additional role of storage, ingestion, excretion etc.

99. (c) Practice of growing two or more crops simultaneously on the same piece of land is known as Intercropping. The main purpose of intercropping is greater yield and maximum utilization of available resources in the field.

100. (c) Antibiotics work by changing or altering any important metabolic pathway in bacteria. Different types of antibiotics act on different pathways to kill or inhibit bacterial growth. Viruses on the other hand do not possess metabolic pathways on which antibiotics can act. These antibiotics thus are of no use for viral infection.

101. (b) Since its opening in 1914, the Panama Canal has linked the Atlantic and Pacific oceans. A fifty-mile waterway, connecting canals, rivers, and lakes with locks, was built through the narrowest part of Panama. For, at last, the Atlantic and Pacific oceans were linked by a waterway. Now, ships could use the canal to shorten travel from New York to San Francisco and from Europe to the ports of Asia.

102. (c) The Charter of the United Nations was signed on 26 June 1945, in San Francisco, at the conclusion of the United Nations Conference on International Organization, and came into force on 24 October 1945. The Statute of the International Court of Justice is an integral part of the Charter. The United Nations is an international organization founded in 1945. It is currently made up of 193 Member States. The mission and work of the United Nations are guided by the purposes and principles contained in its founding Charter.

103. (b) The land use survey is conducted annually and is based on a 9-fold classification including
i.   forests;
ii.   Area under non-agricultural uses;
iii. Barren and uncultivable land;
iv.   Pastures and other grazing lands;
V.   Land under miscellaneous tree crops;
vi.   Culturable waste land;
vii. Fallow land other than current fallows;
viii. Current fallows; and
ix.   Net area sown area.
The "Land Use and Land Cover" (LULC) data is collected at village level and is later aggregated to higher hierarchical units such as districts and states in India.

104. (d) In India, the dryland farming is largely confined to the regions having annual rainfall less than 75 cm. These regions grow hardy and drought resistant crops such as ragi, bajra, moong, gram and guar (fodder crops) and practise various measures of soil moisture conservation and rain water harvesting.

105. (d) Meteorological drought is classified based on rainfall deficiency w.r.t. long term average - 25% or less is normal, 26-50% is moderate and more than 50% is severe.
Agricultural drought is identified by 4 consecutive weeks of meteorological drought, weekly rainfall is 50 mm from 15/5/ to 15/10, 6 such consecutive weeks rest of the year and crop planted is 80% in kharif season.
In India, around 68% of the country is prone to drought in varying degrees. 35% which receives rainfall between 750 mm and 1125 mm is considered drought prone while 33% receiving less than 750 mm is chronically drought prone.

106. (c) The Agulhas Current is current of the south western Indian Ocean. It flows down the east coast of Africa from 27°S to 40°S.

Pacific Ocean Currents includes various cold and warm current which moves clockwise circulation in Northen Pacific Ocean and Anticlockwise circulation in South Pacific Ocean thereby influencing the climatic pattern in the coastal regions. The Pacific Ocean currents includes;
- North Equatorial Current (Warm)
- South Equatorial Current (Warm)
- Counter Equatorial Current (Warm)
- Kuroshio System (Warm)
- North Pacific Drift (Warm)
- Oyashio Current (Cold)
- California Current (Cold)
- Peruvian or Humboldt Current (Cold)
- East Australia Current (Warm)
- Alaska Current

107. (b) According to the Indian Meteorological Department (IMD) reports, the southwest monsoon have an extended stay of eight days this year. This means that the normal duration of the entire monsoon in India, particularly over Mumbai across 100 to 120 days.

108. (a) China covers a large spectrum of vegetation types, ranging from tropical rain forests and subtropical evergreen broadleaf forests, through temperate deciduous broadleaf forests to boreal forests, and temperate and cold steppes and deserts.

109. (c) Mistral is a cold wind blows in Rhone valley of France. Santa Anas are dry, warm (often hot) winds that blow westward through Southern California toward the coast. They're usually seasonal, and typically occur between October and March and peak in December.

Chinook winds - also known as Foehn winds in other parts of the world - are a type of warm, dry wind that occur on the downward slope of a mountain when warm air has lost its moisture.

The Loo is a strong, dusty, gusty, hot and dry summer wind from the west which blows over the western Indo-Gangetic Plain region of North India and Pakistan. It is especially strong in the months of May and June.

110. (b) The social outcome of permanent settlement in Bengal was that the society was divided into two mutually hostile classes of Zamindars and Tenants. While Zamindars were favourite children of British Imperialism, they were few in numbers in comparison to the other class of tenants.

111. (a) Damin-i-koh was the land of Santhals situated in Rajmahal hills. Rajmahal hills are located in the Santhal Pargana division of Jharkhand. Damin-i-koh (or sometimes referred to simply as Damin) was the name given to the forested hilly areas of Rajmahal hills broadly in the area of present Sahebganj, Pakur and Godda districts in the Indian state of Jharkhand.

112. (d) The revolt of 1857 or Indian Mutiny or Sepoy Mutiny or First War of Independence was widespread but unsuccessful rebellion against British rule in India in 1857-59. It was begun in Meerut by Indian troops who were in the service of the British East India Company, it spread to Delhi, Agra, Kanpur, and Lucknow. The British succeeded in quickly and easily controlling the rebels.

113. (c) Panchsheel, or the Five Principles of Peaceful Co-existence, were first formally enunciated in the Agreement on Trade and Intercourse between the Tibet region of China and India signed on April 29, 1954, which stated, in its preamble, that the two Governments "have resolved to enter into the present Agreement based on the following five principles:-
i. Mutual respect for each other's territorial integrity and sovereignty,
ii. Mutual non-aggression,
iii. Mutual non-interference,
iv. Equality and mutual benefit, and
v. Peaceful co-existence.

Hence Nuclear deterrence is not among five principles of Panch *sheel*.

114. (d) Abhinav Bharat Society was a secret society founded by Vinayak Damodar Savarkar and his brother Ganesh Damodar Savarkar in 1904. The original organization believed in armed revolution, and was responsible for the assassinations of some officers of the ruling British government before being disbanded in 1952.

115. (b) The Viceroy Linlithgow made a set of proposals called the 'August offer'. For the first time, the right of Indians to frame their own constitution was acknowledged. As per the terms of the August Offer, a representative Indian body would be framed after the war to frame a constitution for India. Dominion status was the objective for India.

116. (a) A large amount of the Canadian population lives in urban areas and cities are notorious for their poor air quality. Environment Canada has singled out air pollution as a major concern as it affects wildlife, vegetation, soil and water. The government agency has said air pollution from urban areas causes acid rain and contributes to climate change.

117. (a) The mountain range that lies between the Black and Caspian Seas is called the Caucasus Mountain Range, or just the Caucasus Mountains. The great historic barrier of the Caucasus Mountains rises up across the wide isthmus separating the Black and Caspian seas in the region where Europe and Asia converge. Trending generally from northwest to southeast, the mountains consist of two ranges-the Greater Caucasus (Russian: Bolshoy Kavkaz) in the north and the Lesser Caucasus (Maly Kavkaz) in the south. Mount Elbrus in the Greater Caucasus range, at 18,510 feet (5,642 metres), is the highest peak.

118. (d) Bagalihar, Dulhasti and Salal hydro power projects have been developed on Chenab river. Baglihar Dam is built on Chenab River in the Doda district of Jammu & Kashmir. Dulhasti hydro power is located in Kishtwar district where as Salal power station is located in Reasi district of Union Territory of Jammu & Kashmir.

119. (b) Biosphere reserves are sites established by countries and recognized under UNESCO's Man and the Biosphere (MAB) Programme to promote sustainable development based on local community efforts and sound science. Presently, there are 18 notified biosphere reserves in India where as there are total 11 biosphere reserves of India which have been recognized internationally under Man and Biosphere Reserve program. The programme of Biosphere Reserve was initiated by UNESCO in 1971.

120. (d) Asia has 12 landlocked countries: Afghanistan, Armenia, Azerbaijan, Bhutan, Laos, Kazakhstan, Kyrgyzstan, Mongolia, Nepal, Tajikistan, Turkmenistan, and Uzbekistan. Syria is located in Western Asia, north of the Arabian Peninsula, at the eastern end of the Mediterranean Sea. Jordan is almost entirely land-locked and only has a small (27 km) marine coast to the Red Sea, cantered on the port of Aqaba. Azerbaijan has a coastline of 713 km on the Caspian Sea.

121. (c) The filament is the part of the light bulb that produces light. Filaments in incandescent light bulbs are made of tungsten. Whenever an electric current goes through the filament, the filament glows. The reason tungsten is used is because it has the highest melting temperature of any element at 3600K. For a typical incandescent light bulb, the Tungsten filament operates at approximately 2500 Celsius. Hence, the filaments are made of tungsten.

122. (b) Let R' be the equivalent resistance of two resistors. As they are connected in parallel

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R}$$

$$R' = \frac{R}{2}$$ ....(i)

Using ohm's law
$V = IR'$

$$R' = \frac{V}{I} = \frac{12}{100 \times 10^{-3}}$$

$\Rightarrow$ R' = 120Ω

But, $R' = \frac{R}{2}$

$\therefore$ R = 2 × R' = 2 × 120 = 240Ω

123. (d) A watt-hour (Wh) is a unit of energy; it's a way to measure the amount of work performed or generated. Household appliances and other electrical devices perform "work" and that requires energy in the form of electricity. The joule (symbol J) is the SI unit of energy-a measure of the capacity to do work or generate heat. Newton-Meter (N-m) is a unit of energy. One joule equals the work done (or energy expended) by a force of one newton (N) acting over a distance of one meter (m).
SI unit for force is Newton (N).
$1N = 1$ kg.m/s^2
Thus, kg.m/s^2 is the unit of Force not energy.

124. (d) Calorific values of commonly used fuels

Commonly used fuels	Calorific values in Kilo calories	Thermal efficiency
Bio-gas	4713/M^3	60%
Dung cake	2093/Kg	11%
Firewood	4978/Kg	17.30%
Diesel (HSD)	10550/Kg	66%
Kerosene	10850/Kg	50%
Petrol	11100/Kg	---

Hence Heating capacity of biogas is not very low

125. (b) The unidentified nuclear region of the cell is called **nucleoid**. The prokaryotic cell lacks chromosomes and nucleolus or nuclear membrane and nucleoplasm remains undifferentiated from the cytoplasm. The prokaryotic cells lack true nucleus and a circular DNA lies naked in the cytoplasm.

126. (d) Vascular plants have up to three types of supporting tissue: The collenchyma, a tissue of living cells, the sclerenchyma, a tissue of nearly always dead cells, and. the vascular tissue consisting of both living and dead cells.

127. (d) As cartilage is a structural component of our body act as connective tissue which is not required in the urinary bladder. But nose, ear, and larynx have many bones which need these connecting tissues because they are composed of many bones.

128. (b) According to Kepler's law of planetary motion,
$T^2 \propto R^3$
Therefore, the ratio of the time period is given by,

$$\frac{T_1^2}{T_2^2} = \frac{R^3}{(4R)^3} = \frac{1}{64}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1}{8}$$

i.e., $T_1 : T_2$ is 1 : 8

129. (c) The resistance of a wire depends directly on the length of a wire. Thus if we reduce the length of a wire to half the resistance too will drop to half.
Thus now two pieces will have 10 ohm resistance each
Thus let equivaent resistance in parallel connection be R_q:
Thus $1/R_q = 1/10 + 1/10$
$R_q = 10/2 = 5$ ohm

130. (b) We know that

force, $F = \frac{\Delta p}{\Delta t} = \frac{2IA}{C}$

Here, I = Intensity of light falling on the surface
C = Speed of light
A = Area of surface.
If A is halved, then

$$F = \frac{2I\frac{A}{2}}{C} = \frac{IA}{C}$$

Thus, Force becomes half.

131. (c) When the light enters the human eye through the cornea; it then passes through the aqueous humor, the lens and the vitreous humor. Then the image is formed on the retina of the eye. The eye is considered by most neuroscientists as actually part of the brain. It consists of a small spherical globe of about 2cm in diameter, which is free to rotate under the control of 6 extrinsic muscles. Light enters the eye through the transparent cornea, passes through *the* aqueous humor, the *lens*, and the vitreous humor, where it finally forms an image on the retina.

132. (d) We know that both the directions are perpendicular, thus for force direction = ?

Using Fleming's left hand rule, Direction of force is perpendicular to the direction of magnetic field and current. Thus direction of force is opposite to electron motion into the page at 90°.

133. (b) The sensation felt by our ears is called sound. It is a form of energy which makes us hear. Sound travels in the form of wave.

Longitudinal Waves: A wave in which the particles of the medium vibrate back and forth in the 'same direction' in which the wave is moving. Medium can be solid, liquid or gases. Therefore, sound waves are longitudinal waves. Sound is a longitudinal wave which consists of compressions and rarefactions travelling through a medium.

Transverse Waves: A wave in which the particles of the medium vibrate up and down 'at right angles' to the direction in which the wave is moving. These waves are produced only in a solids and liquids but not in gases.

134. (c) Normal Short Circuit is when a hot wire carrying current touches a neutral wire. When that happens, the resistance will go down instantly and a large volume of current will flow through an unexpected pathway.

135. (a) The male reproductive system includes the penis, scrotum, testes, epididymis, vas deferens, prostate, and seminal vesicles.
   - The penis and the urethra are part of the urinary and reproductive systems.
   - The scrotum, testes (testicles), epididymis, vas deferens, seminal vesicles, and prostate comprise the rest of the reproductive system.

The cervix is a cylinder-shaped neck of tissue that connects the vagina and uterus. Located at the lowermost portion of the uterus, the cervix is composed primarily of fibromuscular tissue. Thus Cervix is not a part of male reproductive system.

136. (c) The main cause of the loss of biodiversity can be attributed to the influence of human beings on the world's ecosystem, In fact human beings have deeply altered the environment, and have modified the territory, exploiting the species directly, for example by fishing and hunting, changing the biogeochemical cycles and transferring species from one area to another of the Planet. Alteration and loss of the habitats; Pollution; Climate change and Overexploitation of resources are main threats of environment.

137. (a) The two main causes of Depletion of Water Table are Deforestation and Over-pumping of groundwater. Constructions of large scale concrete buildings are also one of the causes of Depletion of Water.

138. (d) X- Rays have smaller wavelengths and therefore higher energy than ultraviolet waves. Microwaves are the highest frequency of radio waves. Their wavelength is only a few centimetres long. Infrared lies between the visible and microwave portions of the electromagnetic spectrum. Shorter, near infrared waves are not hot at all - in fact you cannot even feel them. Visible waves are the only electromagnetic waves we can see. We see these waves as the colors of the rainbow. Each color has a different wavelength. Red has the longest wavelength and violet has the shortest wavelength. When all the waves are seen together, they make white light.

139. (b) A galvanometer is an electromechanical instrument used for detecting and indicating an electric current. A galvanometer works as an actuator, by producing a rotary deflection (of a "pointer"), in response to electric current flowing through a coil in a constant magnetic field.

140. (b) Biogas is produced by anaerobic digestion of fermentation of biodegradable materials such as biomass, manure, sewage, municipal waste, green waste, plant material and energy crops. This type of biogas comprises primarily methane.

Composition of biogas

Constituent	Formula	Concentration (v/v)
Methane	CH_4	40-75%
Carbon dioxide	CO_2	15-60%
Moisture	H_2O	1-5%
Nitrogen	N_2	0-5%
Hydrogen	H_2	Traces
Hydrogen sulfide	H_2S	0-5000 ppm
Oxygen	O_2	<2%
Trace gases	–	<2%
Ammonia	–	0-500 ppm

141. (c) It is due to scattering of light. During sunrise and sunset, the sun is at the longest position. Here blue light gets scattered off and due to larger wavelength of the red light it can travel a longer distance and hence reach our eyes. This we see the sun red. But at noon, the sun is at the nearest point to earth. Hence all the colors get scattered almost equally and this you see white light.

142. (c) A dioptre, is a unit of measurement of the optical power of a lens, which is equal to the reciprocal of the focal length measured in meters (that is, 1/f metres). It is thus a unit of reciprocal length.

Hence, $P = 1/f \Rightarrow 2 = 1/f \Rightarrow f = 0.5$ meter.

As the value is positive, it is a positive lens, that is, convex. Hence, the option (c) is true.

143. (a) Above 70°C, sodium bicarbonate gradually decomposes into sodium carbonate, water and carbon dioxide as follows - $2 NaHCO_3 - Na_2CO_3 + H_2O + CO_2$, In bakery, it reacts with other components or decomposes at higher temperature to release carbon dioxide, which causes dough to rise.

144. (a) Copper Sulphate: $CuSO_4.5H_2O$
Sodium Carbonate: $NA_2CO_3.10H_2O$
Gypsum: $CaSO_4.2H_2O$
The number of molecules of water of crystallization in copper sulphate is 5, Sodium carbonate is 10 while in Gypsum it is 2.

145. (b) Soaps are basic in nature therefore when rubbed on turmeric stain on white cloths. Yellow colour of vturmeric changes to redish brown. The correct sequence of change in colour is;
Yellow—Redish brown—Yellow

146. (c) Dichlorodiphenyl trichloroethane (DOT) (organic compound)
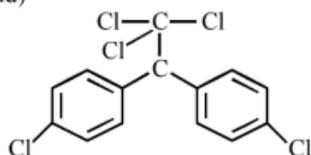
Bleaching Rowder $CaOCl_2$ calcium hypochlorite (Inorganic compound)
Both contains chlorine as it is evident from formula/structure.

147. (a) Desiccants are compounds or agents, such as Montmorillonite Clay or Silica Gel, used in facilitating low humidity environments by absorbing moisture content from the air. Typically used in transport, storage or maintenance of materials and products, desiccants are used to keep everything from gym shoes to military munitions dry. The most common desiccant is silica gel.

148. (a) Geber, a Spanish the pseudonym of a fourteenth-century alchemist, credited with the discovery of sulfuric acid, whose preparation he described along with that of other strong acids. These acids were capable of dissolving many metals—a property that helped to spur interest in alchemy throughout Europe. Today, these compounds are among the most important industrial chemicals.

149. (a) Refractive index of medium

$$\mu_{med} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

$\mu_{sapp} > \mu_{quartz}$ $\therefore$ $Vq > Vs$

150. (c)