NDA SOLVED PAPER 2020-I

MATHEMATICS

1. What is the modulus of the complex number $\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$, where $i = \sqrt{-1}$?

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

2. Consider the proper subsets of $\{1, 2, 3, 4\}$. How many of these proper subsets are superset of the set $\{3\}$?

(a) 5 (b) 6 (c) 7 (d) 8

3. Let p, q and r be three distinct positive real numbers. If $D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$, then which one of the following is correct?

(a) $D < 0$ (b) $D \leq 0$
(c) $D > 0$ (d) $D \geq 0$

4. What is the sum of the last five coefficients in the expansion of $(1 + x)^9$ when it is expanded in ascending powers of x?

(a) 256 (b) 512 (c) 1024 (d) 2048

5. Consider the following in respect of a non-singular matrix of order 3 :

1. A (adj A) = (adj A) A
2. | adj A | = | A |

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

6. The center of the circle $(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$ is

(a) $(2a, 2c)$ (b) $(2b, 2d)$
(c) $(a + b, c + d)$ (d) $(a - b, c - d)$

7. The point $(1, -1)$ is one of the vertices of a square. If $3x + 2y = 5$ is the equation of one diagonal of the square, then what is the equation of the other diagonal?

(a) $3x - 2y = 5$ (b) $2x - 3y = 1$
(c) $2x - 3y = 5$ (d) $2x + 3y = -1$

8. Let $P(x, y)$ be any point on the ellipse $25x^2 + 16y^2 = 400$. If $Q(0, 3)$ and $R(0, -3)$ are two points, then what is $(PQ + PR)$ equal to ?

(a) 12 (b) 10 (c) 8 (d) 6

9. If the circumcentre of the triangle formed by the lines $x + 2 = 0$, $y + 2 = 0$ and $kx + y + 2 = 0$ is $(-1, -1)$, then what is the value of k ?

(a) -1 (b) -2
(c) 1 (d) 2

10. In the parabola, $y^2 = x$, what is the length of the chord passing through the vertex and inclined to the x-axis at an angle θ?

(a) $\sin\theta \cdot \sec^2\theta$ (b) $\cos\theta \cdot \text{cosec}^2\theta$
(c) $\cot\theta \cdot \sec^2\theta$ (d) $2\tan\theta \cdot \text{cosec}^2\theta$

11. Under which condition, are the points (a, b), (c, d) and $(a - c, b - d)$ collinear?

(a) $ab = cd$ (b) $ac = bd$
(c) $ad = bc$ (d) $abc = d$

12. Let ABC be a triangle. If $D(2, 5)$ and $E(5, 9)$ are the mid-points of the sides AB and AC respectively, then what is the length of the side BC?

(a) 8 (b) 10
(c) 12 (d) 14

13. If the foot of the perpendicular drawn from the point $(0, k)$ to the line $3x - 4y - 5 = 0$ is $(3, 1)$, then what is the value of k?

(a) 3 (b) 4
(c) 5 (d) 6

14. What is the obtuse angle between the lines whose slopes are $2 - \sqrt{3}$ and $2 + \sqrt{3}$?

(a) 105° (b) 120°
(c) 135° (d) 150°

15. If $3x - 4y - 5 = 0$ and $3x - 4y + 15 = 0$ are the equations of a pair of opposite sides of a square, then what is the area of the square?

(a) 4 square units (b) 9 square units
(c) 16 square units (d) 25 square units

DIRECTIONS (Qs. 16-18) : *Read the following information and answer the three items that follow:*

Let $a\sin^2 x + b\cos^2 x = c$; $b\sin^2 y + a\cos^2 y = d$ and $p\tan x = q\tan y$.

16. What is $\tan^2 x$ equal to ?

(a) $\frac{c-b}{a-c}$ (b) $\frac{a-c}{c-b}$
(c) $\frac{c-a}{c-b}$ (d) $\frac{c-b}{c-a}$

17. What is $\frac{d-a}{b-d}$ equal to?

(a) $\sin^2 y$ (b) $\cos^2 y$
(c) $\tan^2 y$ (d) $\cot^2 y$

18. What is $\frac{p^2}{q^2}$ equal to?

(a) $\frac{(b-c)(b-d)}{(a-d)(a-c)}$ (b) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$

(c) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$ (d) $\frac{(b-c)(b-d)}{(c-a)(a-d)}$

DIRECTIONS (Qs. 19-21) : *Read the following information and answer the three items that follow:*

Let $t_n = \sin^n \theta + \cos^n \theta$.

19. What is $\frac{t_3 - t_5}{t_5 - t_7}$ equal to?

(a) $\frac{t_1}{t_3}$ (b) $\frac{t_3}{t_5}$ (c) $\frac{t_5}{t_7}$ (d) $\frac{t_1}{t_7}$

20. What is $t_1^2 - t_2$ equal to?

(a) $\cos 2\theta$ (b) $\sin 2\theta$
(c) $2\cos\theta$ (d) $2\sin\theta$

21. What is the value of t_{10} where $\theta = 45°$?

(a) 1 (b) $\frac{1}{4}$

(c) $\frac{1}{16}$ (d) $\frac{1}{32}$

DIRECTIONS (Qs. 22-24) : *Read the following information and answer the three items that follow:*

Let $\alpha = \beta = 15°$.

22. What is the value of $\sin\alpha + \cos\beta$?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

23. What is the value of $\sin 7\alpha - \cos 7\beta$?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

24. What is $\sin(\alpha + 1°) + \cos(\beta + 1°)$ equal to?

(a) $\sqrt{3}\cos 1° + \sin 1°$

(b) $\sqrt{3}\cos 1° - \frac{1}{2}\sin 1°$

(c) $\frac{1}{\sqrt{2}}(\sqrt{3}\cos 1° - \sin 1°)$

(d) $\frac{1}{2}(\sqrt{3}\cos 1° + \sin 1°)$

25. If $\sin x + \sin y = \cos y - \cos x$, where $0 < y < x < \frac{\pi}{2}$, then what is $\tan\left(\frac{x-y}{2}\right)$ equal to?

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

26. If A is a matrix of order 3×5 and B is a matrix of order 5×3, then the order of AB and BA will respectively be

(a) 3×3 and 3×3 (b) 3×5 and 5×3
(c) 3×3 and 5×5 (d) 5×3 and 3×5

27. If p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP, then which of the following is/are correct?

1. p, q and r are in GP.
2. $\ln p$, $\ln q$ and $\ln r$ are in AP.

Select the correct answer using the code given below :

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

28. If $\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 - 3x + 2 = 0$, then what is $\cot(\alpha + \beta)$ equal to ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3

29. The roots α and β of a quadratic equation, satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is the number of such quadratic equations ?

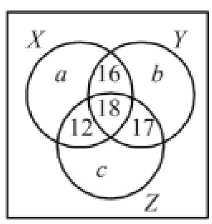
(a) 0 (b) 2 (c) 3 (d) 4

30. What is the argument of the complex number $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$, where $i = \sqrt{-1}$?

(a) 240° (b) 210° (c) 120° (d) 60°

DIRECTIONS (Qs. 31-33) : *Read the following information and answer the three items that follow:*

Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by $n(Z)$ which is equal to 90.



31. If the number of elements in Y and Z are in the ratio 4 : 5, then what is the value of b?
(a) 18 (b) 19
(c) 21 (d) 23

32. What is the value of
$$n(X)+n(Y)+n(Z)-n(X\cap Y)-n(Y\cap Z)-n(X\cap Z)+n(X\cap Y\cap Z)?$$
(a) $a+b+43$ (b) $a+b+63$
(c) $a+b+96$ (d) $a+b+106$

33. If the number of elements belonging to neither X nor Y, nor Z is equal to p, then what is the number of elements in the complement of X?
(a) $p+b+60$ (b) $p+b+40$
(c) $p+a+60$ (d) $p+a+40$

DIRECTIONS (Qs. 34-35) : *Read the following information and answer the two items that follow:*

Let $\frac{\tan 3A}{\tan A}=K$, where $\tan A\neq 0$ and $K\neq\frac{1}{3}$.

34. What is $\tan^2 A$ equal to?
(a) $\frac{K+3}{3K-1}$ (b) $\frac{K-3}{3K-1}$
(c) $\frac{3K-3}{K-3}$ (d) $\frac{K+3}{3K+1}$

35. For real values of $\tan A$, K **cannot** lie between
(a) $\frac{1}{3}$ and 3 (b) $\frac{1}{2}$ and 2
(c) $\frac{1}{5}$ and 5 (d) $\frac{1}{7}$ and 7

DIRECTIONS (Qs. 36-37) : *Read the following information and answer the two items that follow:*

$ABCD$ is a trapezium such that AB and CD are parallel and BC is perpendicular to them. Let $\angle ADB=\theta$, $\angle ABD=\alpha$, $BC=p$ and $CD=q$ and $CD=q$.

36. Consider the following :
1. $AD\sin\theta = AB\sin\alpha$
2. $BD\sin\theta = AB\sin(\theta+\alpha)$

Which of the above is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

37. What is AB equal to?
(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (b) $\frac{(p^2-q^2)\sin\theta}{p\cos\theta+q\sin\theta}$
(c) $\frac{(p^2+q^2)\sin\theta}{q\cos\theta+p\sin\theta}$ (d) $\frac{(p^2-q^2)\cos\theta}{q\cos\theta+p\sin\theta}$

38. If $\tan\theta=\frac{\cos 17°-\sin 17°}{\cos 17°+\sin 17°}$, then what is the value of θ?
(a) 0° (b) 28°
(c) 38° (d) 52°

39. A and B are positive acute angles such that $\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$. What is the value of $(A+2B)$?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

40. What is
$\sin 3x+\cos 3x+4\sin^3 x-3\sin x+3\cos x-4\cos^3 x$
equal to ?
(a) 0 (b) 1
(c) $2\sin 2x$ (d) $4\cos 4x$

41. The value of ordinate of the graph of $y = 2 + \cos x$ lies in the interval
(a) [0, 1] (b) [0, 3]
(c) [–1, 1] (d) [1, 3]

42. What is the value of $8\cos 10°\cdot\cos 20°\cdot\cos 40°$?
(a) tan 10° (b) cot 10°
(c) cosec 10° (d) sec 10°

43. What is the value of cos 48° – cos 12°?
(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{1-\sqrt{5}}{4}$
(c) $\frac{\sqrt{5}+1}{2}$ (d) $\frac{1-\sqrt{5}}{8}$

44. Consider the following statements :
1. If ABC is a right-angled triangle, right-angled at A and if $\sin B=\frac{1}{3}$, then cosec $C = 3$.
2. If $b\cos B = c\cos C$ and if the triangle ABC is not right-angled, then ABC must be isosceles.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

45. Consider the following statements :
1. If in a triangle ABC, $A = 2B$ and $b = c$, then it must be an obtuse-angled triangle.
2. There exists no triangle ABC with $A = 40°$, $B = 65°$ and $\frac{a}{c}=\sin 40°\ \text{cosec}\ 15°$.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

46. If matrix $A=\begin{bmatrix}1-i & i\\ -i & 1-i\end{bmatrix}$ where $i=\sqrt{-1}$, then which one of the following is correct?
(a) A is hermitian
(b) A is skew-hermitian
(c) $(\bar{A})^T + A$ is hermitian
(d) $(\bar{A})^T + A$ is skew-hermitian

47. The term independent of x in the binomial expansion of $\left(\frac{2}{x^2}-\sqrt{x}\right)^{10}$ is equal to
(a) 180 (b) 120
(c) 90 (d) 72

48. If $(1+2x-x^2)^6 = a_0 + a_1x + a_2x^2 + ... + a_{12}x^{12}$, then what is $a_0 - a_1 + a_2 - a_3 + a_4 - .... + a_{12}$ equal to?
(a) 32 (b) 64
(c) 2048 (d) 4096

49. If $C(20, n+2) = C(20, n-2)$, then what is n equal to?
(a) 18 (b) 25
(c) 10 (d) 12

50. For how many values of k, is the matrix $\begin{bmatrix}0 & k & 4\\ -k & 0 & -5\\ -k & k & -1\end{bmatrix}$ singular?
(a) Only one (b) Only two
(c) Only four (d) Infinite

51. The number $(1101101 + 1011011)_2$ can be written in decimal system as
(a) $(198)_{10}$ (b) $(199)_{10}$
(c) $(200)_{10}$ (d) $(201)_{10}$

52. What is the value of
$\frac{1}{10}\log_5 1024 - \log_5 10 + \frac{1}{5}\log_5 3125$?
(a) 0 (b) 1
(c) 2 (d) 3

53 If $x = \log_c(ab)$, $y = \log_a(bc)$, $z = \log_b(ca)$, then which of the following is correct?
(a) $xyz = 1$
(b) $x + y + z = 1$
(c) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = 1$
(d) $(1+x)^{-2} + (1+y)^{-2} + (1+z)^{-2} = 1$

54. Let $A=\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}$, $B=\begin{bmatrix}2\\ -1\end{bmatrix}$ and $C=\begin{bmatrix}3\\ 2\end{bmatrix}$. If $AB = C$, then what is the value of the determinant of the matrix A?
(a) –10 (b) –14
(c) –24 (d) –34

55. If $1.5 \le x \le 4.5$, then which one of the following is correct?
(a) $(2x-3)(2x-9) > 0$
(b) $(2x-3)(2x-9) < 0$
(c) $(2x-3)(2x-9) \ge 0$
(d) $(2x-3)(2x-9) \le 0$

56. Let $S = \{1, 2, 3, ...\}$. A relation R on $S \times S$ is defined by xRy if $\log_a x > \log_a y$ when $a = \frac{1}{2}$. Then the relation is
(a) reflexive only
(b) symmetric only
(c) transitive only
(d) both symmetric and transitive

57. What is the value of the determinant $\begin{vmatrix}i & i^2 & i^3\\ i^4 & i^6 & i^8\\ i^9 & i^{12} & i^{15}\end{vmatrix}$ where $i=\sqrt{-1}$?
(a) 0 (b) –2
(c) $4i$ (d) $-4i$

58. Let $A=\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}$ and $B=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, then what is AB equal to?
(a) $\begin{bmatrix}ax+hy+gz\\ y\\ z\end{bmatrix}$
(b) $\begin{bmatrix}ax+hy+gz\\ hx+by+fz\\ z\end{bmatrix}$
(c) $\begin{bmatrix}ax+hy+gz\\ hx+by+fz\\ gx+fy+cz\end{bmatrix}$
(d) $\begin{bmatrix}ax+hy+gz & hx+by+fz & gx+fy+cz\end{bmatrix}$

59. What is the number of ways in which the letters of the word 'ABLE' can be arranged so that the vowels occupy even places?
(a) 2 (b) 4
(c) 6 (d) 8

60. What is the maximum number of points of intersection of 5 non-overlapping circles?
(a) 10 (b) 15
(c) 20 (d) 25

DIRECTIONS (Qs. 61-63) : *Read the following information and answer the three items that follow:*

Marks	Number of students	
	Physics	Mathematics
10 - 20	8	10
20 - 30	11	21
30 - 40	30	38
40 - 50	26	15
50 - 60	15	10
60 - 70	10	6

61. The difference between number of students under Physics and Mathematics is largest for the interval
(a) 20 - 30 (b) 30 - 40
(c) 40 - 50 (d) 50 - 60

62. Consider the following statements :
1. Modal value of the marks in Physics lies in the interval 30 - 40.
2. Median of the marks in Physics is less than that of marks in Mathematics.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

63. What is the mean of marks in Physics?
(a) 38.4 (b) 39.4
(c) 40.9 (d) 41.6

64. What is the standard deviation of the observations
$-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}$?
(a) $\sqrt{2}$ (b) 2
(c) $2\sqrt{2}$ (d) 4

65. If $\Sigma x_i = 20$, $\Sigma x_i^2 = 200$ and $n = 10$ for an observed variable x, then what is the coefficient of variation?
(a) 80 (b) 100
(c) 150 (d) 200

66. What is the probability that February of a leap year selected at random, will have five Sundays?
(a) $\frac{1}{5}$ (b) $\frac{1}{7}$
(c) $\frac{2}{7}$ (d) 1

67. The arithmetic mean of 100 observations is 40. Later, it was found that an observation '53' was wrongly read as '83'. What is the correct arithmetic mean?
(a) 39.8 (b) 39.7
(c) 39.6 (d) 39.5

68. A husband and wife appear in an interview for two vacancies for the same post. The probability of the husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. If the events are independent, then the probability of which one of the following is $\frac{11}{35}$?
(a) At least one of them will be selected
(b) Only one of them will be selected
(c) None of them will be selected
(d) Both of them will be selected

69. A dealer has a stock of 15 gold coins out of which 6 are counterfeits. A person randomly picks 4 of the 15 gold coins. What is the probability that all the coins picked will be counterfeits?
(a) $\frac{1}{91}$ (b) $\frac{4}{91}$
(c) $\frac{6}{91}$ (d) $\frac{15}{91}$

70. A committee of 3 is to be formed from a group of 2 boys and 2 girls. What is the probability that the committee consists of 2 boys and 1 girl?
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$
(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

71. In a lottery of 10 tickets numbered 1 to 10, two tickets are drawn simultaneously. What is the probability that both the tickets drawn have prime numbers?
(a) $\frac{1}{15}$ (b) $\frac{1}{2}$
(c) $\frac{2}{15}$ (d) $\frac{1}{5}$

72. Let X and Y represent prices (in ₹) of a commodity in Kolkata and Mumbai respectively. It is given that $X = 65$, $Y = 67$, $\sigma_X = 2.5$, $\sigma_Y = 3.5$ and $r(X, Y) = 0.8$. What is the equation of regression of Y on X?
(a) $Y = 0.175X - 5$ (b) $Y = 1.12X - 5.8$
(c) $Y = 1.12X - 5$ (d) $Y = 0.17X + 5.8$

73. Consider a random variable X which follows Binomial distribution with parameters n = 10 and $p = \frac{1}{5}$. Then $Y = 10 - X$ follows Binomial distribution with parameters n and p respectively given by
(a) $5, \frac{1}{5}$ (b) $5, \frac{2}{5}$
(c) $10, \frac{3}{5}$ (d) $10, \frac{4}{5}$

74. If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, then consider the following statements :

1. $P(\overline{A} \cup B) = 0.9$.
2. $P(\overline{B} \mid \overline{A}) = 0.6$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

75. Three cooks X, Y and Z bake a special kind of cake, and with respective probabilities 0.02, 0.03 and 0.05, it fails to rise. In the restaurant where they work, X bakes 50%, Y bakes 30% and Z bakes 20% of cakes. What is the proportion of failures caused by X?

(a) $\frac{9}{29}$ (b) $\frac{10}{29}$

(c) $\frac{19}{29}$ (d) $\frac{28}{29}$

76. Consider the following statements for $f(x) = e^{-|x|}$:

1. The function is continuous at $x = 0$.
2. The function is differentiable at $x = 0$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

77. What is the maximum value of $\sin x \cdot \cos x$?

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $2\sqrt{2}$

78. What is $\lim_{x \to 0} \frac{3^x + 3^{-x} - 2}{x}$ equal to?

(a) 0 (b) -1

(c) 1 (d) Limit does not exist

79. What is the derivative of $\tan^{-1}x$ with respect to $\cot^{-1}x$?

(a) -1 (b) 1

(c) $\frac{1}{x^2+1}$ (d) $\frac{x}{x^2+1}$

80. The function $u(x, y) = c$ which satisfies the differential equation $x(dx - dy) + y(dy - dx) = 0$, is

(a) $x^2 + y^2 = xy + c$ (b) $x^2 + y^2 = 2xy + c$

(c) $x^2 - y^2 = xy + c$ (d) $x^2 - y^2 = 2xy + c$

81. What is the minimum value of $3\cos\left(A + \frac{\pi}{3}\right)$ where $A \in R$?

(a) -3 (b) -1

(c) 0 (d) 3

82. Consider the following statements :

1. The function $f(x) = \ln x$ increases in the interval $(0, \infty)$.
2. The function $f(x) = \tan x$ increases in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

83. Which one of the following is correct in respect of the graph of $y = \frac{1}{x-1}$?

(a) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of reals.

(b) The domain is $\{x \in R \mid x \neq 1\}$, the range is $\{y \in R \mid y \neq 0\}$ and the graph intersects y-axis at $(0, -1)$.

(c) The domain is the set of reals and the range is the singleton set $\{0\}$.

(d) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of points on the y-axis.

84. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) = x$?

(a) $y = e^x + c$ (b) $y = e^{-x} + c$

(c) $y = \ln x + c$ (d) $y = 2\ln x + c$

85. Let l be the length and b be the breadth of a rectangle such that $l + b = k$. What is the maximum area of the rectangle?

(a) $2k^2$ (b) k^2

(c) $\frac{k^2}{2}$ (d) $\frac{k^2}{4}$

86. The numbers 4 and 9 have frequencies x and $(x - 1)$ respectively. If their arithmetic mean is 6, then what is the value of x?

(a) 2 (b) 3

(c) 4 (d) 5

87. If three dice are rolled under the condition that no two dice show the same face, then what is the probability that one of the faces is having the number 6?

(a) $\frac{5}{6}$ (b) $\frac{5}{9}$

(c) $\frac{1}{2}$ (d) $\frac{5}{12}$

88. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\text{not } A) = \frac{1}{2}$, then which one of the following is **not** correct?

(a) $P(B) = \frac{2}{3}$

(b) $P(A \cap B) = P(A)P(B)$

(c) $P(A \cup B) > P(A) + P(B)$

(d) $P(\text{not } A \text{ and not } B) = P(\text{not } A)\, P(\text{not } B)$

89. The sum of deviations of n number of observations measured from 2.5 is 50. The sum of deviations of the same set of observations measured from 3.5 is –50. What is the value of n?

(a) 50 (b) 60
(c) 80 (d) 100

90. A data set of n observations has mean $2M$, while another data set of $2n$ observations has mean M. What is the mean of the combined data sets?

(a) M (b) $\frac{3M}{2}$

(c) $\frac{2M}{3}$ (d) $\frac{4M}{3}$

91. If $f(x) = 3x^2 - 5x + p$ and $f(0)$ and $f(1)$ are opposite in sign, then which of the following is correct?

(a) $-2 < p < 0$ (b) $-2 < p < 2$
(c) $0 < p < 2$ (d) $3 < p < 5$

92. If $e^{\theta\varphi} = c + 4\theta\varphi$, where c is an arbitrary constant and φ is a function of θ, then what is $\varphi\, d\theta$ equal to?

(a) $\theta\, d\varphi$ (b) $-\theta\, d\varphi$
(c) $4\theta\, d\varphi$ (d) $-4\theta\, d\varphi$

93. If $p(x) = (4e)^{2x}$, then what is $\int p(x)\,dx$ equal to ?

(a) $\frac{p(x)}{1+2\ln 2} + c$ (b) $\frac{p(x)}{2(1+2\ln 2)} + c$

(c) $\frac{2p(x)}{1+\ln 4} + c$ (d) $\frac{p(x)}{1+\ln 2} + c$

94. What is the value of $\int_0^{\pi/4} (\tan^3 x + \tan x)dx$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

95. Let $y = 3x^2 + 2$. If x changes from 10 to 10.1, then what is the total change in y?

(a) 4.71 (b) 5.23
(c) 6.03 (d) 8.01

96. If $f(x) = \frac{\sin x}{x}$, where $x \in R$, is to be continuous at $x = 0$, then the value of the function at $x = 0$

(a) should be 0 (b) should be 1
(c) should be 2 (d) cannot be determined

97. The solution of the differential equation $dy = (1+y^2)dx$ is

(a) $y = \tan x + c$ (b) $y = \tan(x + c)$

(c) $\tan^{-1}(y + c) = x$ (d) $\tan^{-1}(y + c) = 2x$

98. What is $\int (e^{\log x} + \sin x)\cos x\, dx$ equal to ?

(a) $\sin x + x\cos x + \frac{\sin^2 x}{2} + c$

(b) $\sin x - x\cos x + \frac{\sin^2 x}{2} + c$

(c) $x\sin x + \cos x + \frac{\sin^2 x}{2} + c$

(d) $x\sin x - x\cos x + \frac{\sin^2 x}{2} + c$

99. What is the domain of the function $f(x) = \cos^{-1}(x - 2)$?

(a) $[-1, 1]$ (b) $[1, 3]$
(c) $[0, 5]$ (d) $[-2, 1]$

100. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{2}{3}$ (d) 2

101. If $f(x) = 2x - x^2$, then what is the value of $f(x+2) + f(x-2)$ when $x = 0$?

(a) –8 (b) –4
(c) 8 (d) 4

102. If $x^m y^n = a^{m+n}$, then what is $\frac{dy}{dx}$ equal to ?

(a) $\frac{my}{nx}$ (b) $-\frac{my}{nx}$

(c) $\frac{mx}{ny}$ (d) $-\frac{ny}{mx}$

103. What is $\int \frac{dx}{x(x^n+1)}$ equal to?

(a) $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right)+c$ (b) $\ln\left(\frac{x^n+1}{x^n}\right)+c$

(c) $\ln\left(\frac{x^n}{x^n+1}\right)+c$ (d) $\frac{1}{n}\ln\left(\frac{x^n+1}{x^n}\right)+c$

104. What is the minimum value of $|x-1|$, where $x \in R$?

(a) 0 (b) 1
(c) 2 (d) –1

105. What is the value of k such that integration of $\frac{3x^2+8-4k}{x}$ with respect to x, may be a rational function?

(a) 0 (b) 1
(c) 2 (d) –2

106. What is the length of the diameter of the sphere whose centre is at (1, –2, 3) and which touches the plane $6x-3y+2z-4=0$?

(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

107. What is the perpendicular distance from the point (2, 3, 4) to the line $\frac{x-0}{1}=\frac{y-0}{0}=\frac{z-0}{0}$?

(a) 6 units (b) 5 units
(c) 3 units (d) 2 units

108. If a line has direction ratios $<a+b, b+c, c+a>$, then what is the sum of the squares of its direction cosines?

(a) $(a+b+c)^2$ (b) $2(a+b+c)$
(c) 3 (d) 1

109. Into how many compartments do the coordinate planes divide the space?

(a) 2 (b) 4
(c) 8 (d) 16

110. What is the equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane?

(a) $x+y=5$ (b) $z=5$
(c) $z=0$ (d) $x+y+z=5$

111. If $\hat{a}$ is a unit vector in the xy-plane making an angle 30° with the positive x-axis, then what is $\hat{a}$ equal to ?

(a) $\frac{\sqrt{3}\hat{i}+\hat{j}}{2}$ (b) $\frac{\sqrt{3}\hat{i}-\hat{j}}{2}$

(c) $\frac{\hat{i}+\sqrt{3}\hat{j}}{2}$ (d) $\frac{\hat{i}-\sqrt{3}\hat{j}}{2}$

112. Let A be a point in space such that $|\overrightarrow{OA}|=12$, where O is the origin. If $\overrightarrow{OA}$ is inclined at angles 45° and 60° with x-axis and y-axis respectively, then what is $\overrightarrow{OA}$ equal to ?

(a) $6\hat{i}+6\hat{j}\pm\sqrt{2}\hat{k}$ (b) $6\hat{i}+6\sqrt{2}\hat{j}\pm 6\hat{k}$

(c) $6\sqrt{2}\hat{i}+6\hat{j}\pm 6\hat{k}$ (d) $3\sqrt{2}\hat{i}+3\hat{j}\pm 6\hat{k}$

113. Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. What is the magnitude of dot product of vectors which represent its diagonals?

(a) 21 (b) 25 (c) 31 (d) 36

114. If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$, then what is $|\vec{b}|$ equal to ?

(a) 3 (b) 4 (c) 6 (d) 8

115. If the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+p\hat{k}$ are coplanar, then what is the value of p?

(a) 1 (b) –1 (c) 5 (d) –5

116. What is $\lim_{x\to 1}\frac{x+x^2+x^3-3}{x-1}$ equal to ?

(a) 1 (b) 2 (c) 3 (d) 6

117. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

(a) 4.4 cm/sec (b) 8.4 cm/sec
(c) 8.8 cm/sec (d) 15.4 cm/sec

118. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, where $k\neq 0$, then what is the value of k?

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$

(c) $\frac{8}{3}$ (d) 4

119. The order and degree of the differential equation $k\frac{dy}{dx}=\int\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{2}{3}}dx$ are respectively

(a) 1 and 1 (b) 2 and 3
(c) 2 and 4 (d) 1 and 4

120. What is $\lim_{x\to 0}\frac{\sin x\log(1-x)}{x^2}$ equal to?

(a) –1 (b) Zero

(c) $-e$ (d) $-\frac{1}{e}$

HINTS & SOLUTIONS

MATHEMATICS

1. (b) $Z = \dfrac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$

$= \dfrac{(\cos\theta + i\sin\theta)}{(\cos\theta - i\sin\theta)} \times \dfrac{(\cos\theta + i\sin\theta)}{(\cos\theta + i\sin\theta)}$

$= \dfrac{(\cos\theta + i\sin\theta)^2}{\cos^2\theta - (i\sin\theta)^2}$

$= \dfrac{\cos 2\theta + i\sin 2\theta}{\cos^2\theta + \sin^2\theta} = \cos 2\theta + i\sin 2\theta.$

Modulus of $Z = |Z| = \sqrt{\cos^2(2\theta) + \sin^2(2\theta)} = 1$

2. (c) Number of proper subset of any set of n elements $= 2^n - 1$

Here given set = {1, 2, 3, 4}

Number of proper subset $= 2^4 - 1 = 16 - 1 = 15$.

Proper subset = {(1), (2), (3), (4), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4) (ϕ)}

Now, A is superset of B, if B is proper set of A, but B is not proper set of A.

i.e. $B \le A$ but $A \not\subset B$. Then $A \ge B$.

So, superset of {3} are {(3), (1, 3), (2, 3), (3, 4), (1, 2, 3), (1, 3, 4), (2, 3, 4)}

Hence, number of superset of {3} = 7.

3. (a) $D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$

$= p(rq - p^2) + q(pr - q^2) + r(pq - r^2)$

$= 3pqr - p^3 - q^3 - r^3$

$= -(p^3 + q^3 + r^3 - 3pqr).$

For real and distinct positive real value of p, q and r.

$p^3 + q^3 + r^3 - 3pqr > 0$

$\therefore D < 0$.

4. (a) Expansion of

$(1+x)^9 = {}^9C_0 + {}^9C_1x + {}^9C_2x^2 + {}^9C_3x^3 + {}^9C_4x^4 + {}^9C_5x^5 + {}^9C_6x^6 + {}^9C_7x^7 + {}^9C_8x^8 + {}^9C_9x^9$

Sum of last five co-efficient

$= {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$

$= 126 + 84 + 36 + 9 + 1 = 256.$

5. (a)

(1) We know that,

$A \cdot (\text{adj} A) = (\text{adj} A) \cdot A = |A| \cdot I$

Hence, statement (1) is correct.

(2) $|\text{adj}(A)| = |A^{-1} \cdot |A||$

Hence, statement (2) is not correct.

6. (c) $(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$

$x^2 - 2(a+b)x + 4ab + y^2 - 2(c+d)y + 4cd = 0$

$x^2 + y^2 - 2(a+b)x - 2(c+d)y + 4(ab + cd) = 0$

From general equation of circle,

$x^2 + y^2 + 2gx + 2fy + c = 0$

Center = $(-g, -f)$ and radius $= \sqrt{(-g)^2 + (-f)^2 + C}$

$\therefore$ Here $g = -(a+b)$ and $f = -(c+d)$.

Hence, center $= (-g, -f) = ((a+b), (c+d))$.

7. (c) We know that diagonal of a square bisect each other perpendicularly.

Equation of a diagonal : $3x + 2y = 5$ (given).

Now, equation of other diagonal that is perpendicular to the given diagonal $= 2x - 3y = K$.

As vertex point $(1, -1)$ does not lies on $3x + 2y = 5$

$\{\because 3(1) + 2(-1) \ne 5\}$

Then, point $(1, -1)$, must be on the diagonal

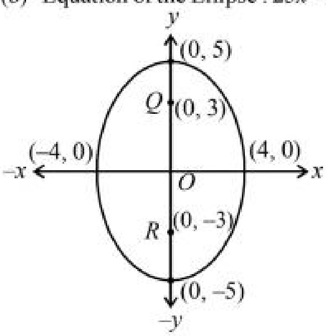
$2x - 3y = K$

Then, $2(1) - 3(-1) = K$.

$\therefore K = 5$.

Hence, equation of other diagonal : $2x - 3y = 5$.

8. (b) Equation of the Ellipse : $25x^2 + 16y^2 = 400$.



$\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$

Here $a^2 = 16$, $b^2 = 25$

Focci points $= (0, \pm be) = (0, \pm\sqrt{b^2 - a^2})$

$= (0, \pm\sqrt{25 - 16}) = (0, \pm 3)$

Point $Q = (0, 3)$, and $R = (0, -3)$.

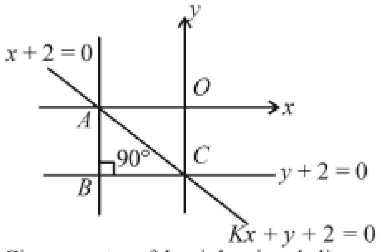
Now, from question point $P(x, y)$ lies on the ellipse, then sum of its distance from two fixed points is always constant, equal to length of its major axis.

Thus, $(PQ + PR) = 2 \times 5 = 10$.

9. (c) Equation of two sides of the triangle are $x + 2 = 0$ and $y + 2 = 0$.

They intersect at right angle.

Thus, triangle formed by them is a right angle triangle.



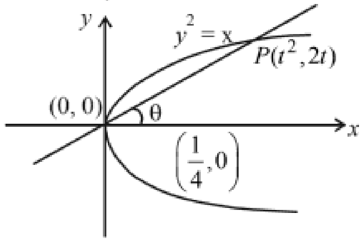
Circumcentre of the right triangle lies on its hypotaneous.
So, circumcentre (–1, –1) must lies on the line
$Kx + y + 2 = 0$
$\therefore K(-1) + (-1) + 2 = 0 \Rightarrow K = 1.$

10. (b) Given parabola : $y^2 = x$.

From standard equation of parabola $y^2 = 4ax$

Focus = $(a, 0)$

Here $a = \frac{1}{4}$



$\therefore$ Focus of parabola $y^2 = x = \left(\frac{1}{4}, 0\right)$

Vertex of the parabola = (0, 0)

Let $P\left(\frac{t^2}{4}, \frac{t}{2}\right)$ lies on the parabola

then, its slope = tan (θ) {given}

$$\therefore \frac{\frac{t}{2}}{\frac{t^2}{4}} = \tan\theta \Rightarrow t = 2\cot\theta$$

So, point $P\left(\frac{t^2}{4}, \frac{t}{2}\right) = (\cot^2\theta, \cot\theta)$

Its distance from vertex (0, 0)

$= \sqrt{(\cot^2\theta - 0)^2 + (\cot\theta - 0)^2}$

$= \cot\theta\sqrt{\cot^2\theta + 1} = \cot\theta \cdot \text{cosec}\,\theta$

$= \cos\theta \cdot \text{cosec}^2\theta$

11. (c) Given points (a, b), (c, d) and $(a - c, b - d)$ are collinear, if

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

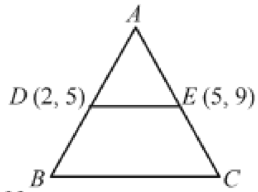
$a(d - b + d) + b(a - c - c) + 1(c(b - d) - d(a - c)) = 0$

$2ad - ab + ab - 2bc + bc - ad = 0$

$\Rightarrow ad - bc = 0$

$\therefore ad = bc.$

12. (b) As point $D(2, 5)$ and point $E(5, 9)$ are mid point of side AB and AC, then
$2 \times DE = BC$



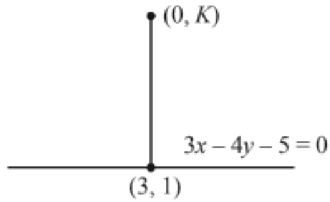
Now,

$BC = 2 \times \sqrt{(5-2)^2 + (9-5)^2}$

$= 2 \times \sqrt{3^2 + 4^2} = 2 \times 5 = 10.$

13. (c) Slope of the line, $3x - 4y - 5 = 0$ is $m = \frac{3}{4}$

Slope of any line perpendicular to $3x - 4y - 5 = 0$ is $m' = -\frac{4}{3}$.



Now, required line passes through the points $(0, K)$ and $(3, 1)$.

$\therefore \frac{K-1}{0-3} = -\frac{4}{3} \Rightarrow K = 5.$

14. (b) Here, $m_1 = (2 - \sqrt{3})$ and $m_2 = (2 + \sqrt{3})$.

Obtuse angle between them,

$\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right)$

$= \tan^{-1}\left(\frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}\right)$

$= \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \tan^{-1}(-\sqrt{3}) = 120°.$

15. (c) We know that pair of opposite sides of my square are parallel.
So, distance between two parallel sides
= Side length of the square
$\therefore$ Side length of the square

$= \left|\frac{15 - (-5)}{\sqrt{3^2 + 4^2}}\right| = \frac{20}{5} = 4$ units

Area of the square = 4 × 4 = 16 units square.

16. (a) From $a\sin^2 x + b\cos^2 x = c$

$a\sin^2 x + b\cos^2 x = c(\sin^2 x + \cos^2 x)$

$(a-c)\sin^2 x = (c-b)\cos^2 x$

$\Rightarrow \frac{\sin^2 x}{\cos^2 x} = \frac{(c-b)}{(a-c)}$

$\Rightarrow \tan^2 x = \frac{(c-b)}{(a-c)}$

17. (c) From $b\sin^2 y + a\cos^2 y = d$

$b\sin^2 y + a\cos^2 y = d(\sin^2 y + \cos^2 y)$

$(b-d)\sin^2 y = (d-a)\cos^2 y$

$\Rightarrow \frac{\sin^2 y}{\cos^2 y} = \frac{(d-a)}{(b-d)} \Rightarrow \tan^2 y = \frac{(d-a)}{(b-d)}$

18. (b) From, $p \cdot \tan x = q \cdot \tan y$

$p^2 \cdot \tan^2 x = q^2 \cdot \tan^2 y$

$\Rightarrow \frac{p^2}{q^2} = \frac{\tan^2 y}{\tan^2 x} = \frac{(d-a)}{(b-d)} \cdot \frac{(a-c)}{(c-b)}$

$= \frac{(a-d)(c-a)}{(b-c)(d-b)}.$

19. (a) $t_n = \sin^n \theta + \cos^n \theta$

Now, $\frac{t_3 - t_5}{t_5 - t_7} = \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}$

$= \frac{\sin^3\theta(1-\sin^2\theta) + \cos^3\theta(1-\cos^2\theta)}{\sin^5\theta(1-\sin^2\theta) + \cos^7\theta(1-\cos^2\theta)}$

$= \frac{\sin^3\theta \cdot \cos^2\theta + \cos^3\theta \cdot \sin^2\theta}{\sin^5\theta \cdot \cos^2\theta + \cos^7\theta \cdot \sin^2\theta}$

$= \frac{\sin^2\theta \cdot \cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta \cdot \cos^2\theta(\sin^3\theta + \cos^3\theta)}$

$= \frac{(\sin\theta + \cos\theta)}{(\sin^3\theta + \cos^3\theta)} = \frac{t_1}{t_3}$

20. (b) $t_1^2 - t_2 = (\sin\theta + \cos\theta)^2 - (\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta - (\sin^2\theta + \cos^2\theta)$

$= 1 + \sin 2\theta - 1 = \sin 2\theta.$

21. (c) $t_{10} = \sin^{10}(\theta) + \cos^{10}(\theta)$

When $\theta = 45°$,

$t_{10} = \sin^{10}(45°) + \cos^{10}(45°)$

$= \left(\frac{1}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}\right)^{10} = 2 \cdot \left(\frac{1}{\sqrt{2}}\right)^{10}$

$= 2 \times \frac{1}{32} = \frac{1}{16}.$

22. (d) $\alpha = \beta = 15°$ (given)

Now, $\sin\alpha + \cos\beta$

$= \sqrt{2}.\left(\frac{1}{\sqrt{2}} \cdot \sin\alpha + \frac{1}{\sqrt{2}} \cdot \cos\alpha\right)$ $\{\because \alpha = \beta\}$

$= \sqrt{2}(\sin\alpha \cdot \cos 45° + \cos\alpha \cdot \sin 45°)$

$= \sqrt{2}(\sin(\alpha + 45°)) = \sqrt{2} \cdot \sin(15° + 45°)$

$= \sqrt{2} \cdot \sin 60° = \sqrt{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}.$

23. (d) $\sin 7\alpha - \cos 7\beta$

$= \sin 7(15°) - \cos 7(15°) = \sin(105°) - \cos(105°)$

$= \sin(90° + 15°) - \cos(90° + 15°)$

$= \cos 15° - (-\sin(15°)) = \cos 15° + \sin 15° = \frac{\sqrt{3}}{\sqrt{2}}$

24. (None) $\sin(\alpha + 1°) + \cos(\beta + 1°)$

$= \sqrt{2}\left\{\frac{1}{\sqrt{2}} \cdot \sin(\alpha + 1°) + \frac{1}{\sqrt{2}} \cdot \cos(\beta + 1°)\right\}$

$= \sqrt{2}\{\cos 45° \cdot \sin(\alpha + 1) + \sin 45° \cdot \cos(\alpha + 1)\}$

$(\because \alpha = \beta)$

$= \sqrt{2}\{\sin(\alpha + 1 + 45°)\} = \sqrt{2} \cdot \sin(15° + 1° + 45°)$

$= \sqrt{2} \cdot \sin 61°$ (i)

Again

$\frac{1}{\sqrt{2}}(\sqrt{3} \cdot \cos 1° + \sin 1°)$

$= \frac{2}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} \cdot \cos 1° + \frac{1}{2}\sin 1°\right)$

$= \sqrt{2} \cdot (\sin 60° \cdot \cos 1° + \cos 60° \cdot \sin 1°)$

$= \sqrt{2}(\sin 61°).$ (ii)

Thus (i) = (ii)

But none of the option have $\frac{1}{\sqrt{2}}\left(\sqrt{3} \cdot \cos 1° + \sin 1°\right).$

25. (c) $\sin x + \sin y = \cos y - \cos x$

$\frac{\sin x + \sin y}{\cos y - \cos x} = 1$

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}=1$$

$$\cot\left(\frac{x-y}{2}\right)=1\Rightarrow\tan\left(\frac{x-y}{2}\right)=1.$$

26. (c) $A=[i\times j]_{3\times5}$, $B=[i\times j]_{5\times3}$

Now, $AB=[i\times j]_{3\times3}$ and $BA=[j\times i]_{5\times5}$

27. (c)

(1) p^2, q^2 and r^2 in G.P.

Then, $(q^2)^2=p^2\cdot r^2\Rightarrow q^2=(p^2\cdot r^2)^{\frac{1}{2}}$

$\Rightarrow q^2=p\cdot r$

Hence, p, q and r in G.P.

(2) As, $q^2=p\cdot r$

Taking log on both sides, we have

$\ln(q^2)=\ln(p\cdot r)$

$2\ln(q)=\ln p+\ln r$

Hence, $\ln p$, $\ln q$ and $\ln r$ are in A.P.

28. (b) Given equation : $x^2-3x+2=0$

Sum of roots, $\cot\alpha+\cot\beta=-(-3)=3$ ...(i)

Product of roots, $\cot\alpha\cdot\cot\beta=2$ ...(ii)

Now, $\cot(\alpha+\beta)=\frac{\cot\alpha\cdot\cot\beta-1}{\cot\alpha+\cot\beta}=\frac{2-1}{3}=\frac{1}{3}.$

29. (b) $\alpha\beta=\alpha^2\beta^2\Rightarrow\alpha\beta(1-\alpha\beta)=0$

$\therefore\alpha\beta=0$ or 1

When $\alpha\beta=1$ then $\alpha=\frac{1}{\beta}$

Again from $\alpha+\beta=\alpha^2+\beta^2$

$\Rightarrow\frac{1}{\beta}+\beta=\frac{1}{\beta^2}+\beta^2\Rightarrow\beta^2-\beta=\frac{1}{\beta}-\frac{1}{\beta^2}$

$\Rightarrow\beta(\beta-1)=\frac{(\beta-1)}{\beta^2}\Rightarrow(\beta-1)\left(\beta-\frac{1}{\beta^2}\right)=0$

$\Rightarrow(\beta-1)(\beta^3-1)=0\Rightarrow(\beta-1)^2(\beta^2+\beta+1)=0$

$\therefore\beta=1$ and $\beta=\frac{-1\pm\sqrt{3}i}{2}$

Again, when $\beta=1$, then $\alpha=\frac{1}{\beta}=1$, roots $(\alpha,\beta)=(1,1)$

When $\beta=\frac{-1+\sqrt{3}i}{2}$, then $\alpha=\frac{-1-\sqrt{3}i}{2}$

Roots $(\alpha,\beta)=\left(\frac{-1\pm\sqrt{3}i}{2},\frac{-1\mp\sqrt{3}i}{2}\right)$

Thus, number of different quadratic equations = 2.

30. (a) Let $x+iy=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$

$=\frac{(1-i\sqrt{3})(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}=\frac{(1-i\sqrt{3})^2}{1^2-(i\sqrt{3})^2}$

$=\frac{1-3-i2\sqrt{3}}{1+3}=-\left(\frac{1+i\sqrt{3}}{2}\right)$

$=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

$\therefore x=-\frac{1}{2},\ y=-\frac{\sqrt{3}}{2}$

Argument $=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$=\pi+\tan^{-1}(\sqrt{3})=180°+60°=240°.$

31. (c) $n(z)=90$

$12+18+17+C=90\Rightarrow C=43.$

From question, $\frac{n(y)}{n(z)}=\frac{4}{5}$

$\frac{16+18+17+b}{90}=\frac{4}{5}$

$b=72-51=21.$

32. (d) $n(X)+n(Y)+n(Z)-n(X\cap Y)$

$-n(Y\cap Z)-n(X\cap Z)+n(X\cap Y\cap Z)$

$=n(X\cup Y\cup Z)$

$=a+b+90+16$

$=a+b+106.$

33. (a) $n(X\cup Y\cup Z)^1=P$

and $n(Z)=90$ (given)

$\therefore n(X)^1=P+90-12-18+b=p+b+60.$

34. (b) $\frac{\tan 3A}{\tan A}=K$

$\frac{3\tan A-\tan^3 A}{\tan A\cdot(1-3\tan^2 A)}=K$

$\frac{(3-\tan^2 A)}{(1-3\tan^2 A)}=K$

$\Rightarrow 3-\tan^2 A = K - 3K\tan^2 A$

$(3K-1)\tan^2 A = K-3 \Rightarrow \tan^2 A = \dfrac{K-3}{(3K-1)}$

35. (a, b) $\tan A = \sqrt{\dfrac{(K-3)}{(3K-1)}}$

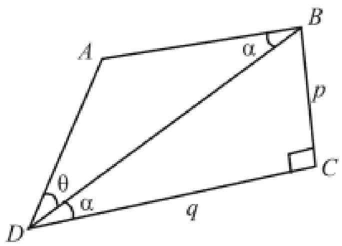
For real value of tan$A$, $\left(\dfrac{K-3}{3K-1}\right) > 0.$

$\therefore$ For, $\dfrac{1}{3} < K < 3.$

tan A is not real.

Also, for $\dfrac{1}{2} < k < 2$, tan A is not real.

36. (c)



(1) Applying Sine ryke in ΔABD

$\dfrac{AD}{\sin\alpha} = \dfrac{AB}{\sin\theta}$

$\Rightarrow AD\cdot\sin\theta = AB\cdot\sin\alpha$

Hence, (1) is correct.

(2) $\dfrac{AB}{\sin\theta} = \dfrac{BD}{\sin(180° - (\alpha+\theta))}$

$\dfrac{AB}{\sin\theta} = \dfrac{BD}{\sin(\alpha+\theta)}$

$AB\cdot\sin(\theta+\alpha) = BD\cdot\sin\theta$

Hence, (2) is correct.

37. (a) From ΔABD,

$\dfrac{AB}{\sin\theta} = \dfrac{BD}{\sin(180° - (\alpha+\theta))}$

$\Rightarrow AB = \dfrac{BD\cdot\sin\theta}{\sin(\alpha+\theta)} = \dfrac{BD\cdot\sin\theta}{\sin\alpha\cdot\cos\theta + \cos\alpha\cdot\sin\theta}$

Again from ΔBCD,

$BD = \sqrt{(BC)^2+(CD)^2} = \sqrt{p^2+q^2}$

$\sin\alpha = \dfrac{BC}{BD} = \dfrac{p}{\sqrt{p^2+q^2}}$

$\cos\alpha = \dfrac{CD}{BD} = \dfrac{q}{\sqrt{p^2+q^2}}$

$\therefore AB = \dfrac{\sqrt{p^2+q^2}\cdot\sin\theta}{\dfrac{p\cdot\cos\theta}{\sqrt{p^2+q^2}} + \dfrac{q\cdot\sin\theta}{\sqrt{p^2+q^2}}}$

$= \dfrac{(p^2+q^2)\cdot\sin\theta}{p\cdot\cos\theta + q\cdot\sin\theta}$

38. (b) $\tan\theta = \dfrac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$

$= \dfrac{1-\dfrac{\sin 17°}{\cos 17°}}{1+\dfrac{\sin 17°}{\cos 17°}} = \dfrac{1-\tan 17°}{1+\tan 17°}$

$\tan\theta = \dfrac{\tan 45° - \tan 17°}{1+\tan 45°\cdot\tan 17°}$

$\tan\theta = \tan(45° - 17°)$

$\tan\theta = \tan(28°) \Rightarrow \theta = 28°.$

39. (d) From question, we have

$\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$

Now,

$\cos(A+2B) = \cos A\cdot\cos 2B - \sin A\cdot\sin 2B$

$= \cos A\cdot 3\sin^2 A - \sin A\cdot\dfrac{3}{2}\sin 2A$

$= 3\cos A\cdot\sin^2 A - \dfrac{3}{2}\cdot\sin A\cdot(2\sin A - \cos A)$

$= 3\cos A\cdot\sin^2 A - 3\cos A\cdot\sin^2 A = 0.$

$\therefore \cos(A+2B) = \cos\left(\dfrac{\pi}{2}\right)$

$\therefore A+2B = \dfrac{\pi}{2}.$

40. (a) $\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$

$= \sin 3x + \cos 3x - \sin 3x - \cos 3x = 0.$

41. (d) $y = 2+\cos x$

Range of $\cos x = [-1, 1]$

$y_{\min} = 2-1 = 1.$

$y_{\max} = 2+1 = 3$

Thus, ordinate of the graph = [1, 3].

42. (b) $8\cdot\cos 10°\cdot\cos 20°\cdot\cos 40°$

$= \dfrac{4}{\sin 10°}[2\cdot\cos 10°\cdot\sin 10°]\cdot\cos 20°\cdot\cos 40°$

$= \dfrac{4}{\sin 10°}[\sin 20°]\cdot\cos 20°\cdot\cos 40°$

$= \dfrac{2}{\sin 10°}[2\sin 20°\cdot\cos 20°]\cdot\cos 40°$

$$= \frac{2}{\sin 10^\circ}[\sin 40^\circ] \cdot \cos 40^\circ$$

$$= \frac{1}{\sin 10^\circ}[2\sin 40^\circ \cdot \cos 40^\circ] = \frac{\sin 80^\circ}{\sin 10^\circ}$$

$$= \frac{\sin(90^\circ - 10^\circ)}{\sin 10^\circ} = \frac{\cos 10^\circ}{\sin 10^\circ} = \cot 10^\circ.$$

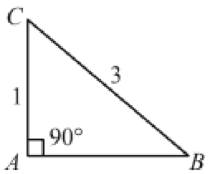
43. (b) $\cos 48^\circ - \cos 12^\circ$

$$= 2 \cdot \sin\left(\frac{48^\circ + 12^\circ}{2}\right) \cdot \sin\left(\frac{12^\circ - 48^\circ}{2}\right)$$

$$= 2 \cdot \sin 30^\circ \cdot \sin(-18^\circ) = -\sin 18^\circ$$

$$= -\left(\frac{\sqrt{5}-1}{4}\right) = \left(\frac{1-\sqrt{5}}{4}\right).$$

44. (b)



(1) ΔABC is a right angled triangle.

$$\sin B = \frac{AC}{BC} = \frac{1}{3}$$

$\therefore AC = 1,\ BC = 3.$

Now,

$$AB = \sqrt{(BC)^2 - (AC)^2} = \sqrt{(3)^2 - (1)^2} = 2\sqrt{2}.$$

$$\text{Now, } \sin C = \frac{AB}{BC} = \frac{2\sqrt{2}}{3}$$

$$\therefore \text{cosec } C = \frac{3}{2\sqrt{2}}.$$

Hence, (1) is not correct.

(2) $b \cdot \cos B = c \cdot \cos C$

$$b \cdot \frac{(a^2 + c^2 - b^2)}{2ac} = c \cdot \frac{(a^2 + b^2 - c^2)}{2ab}$$

$$b^2(a^2 + c^2 - b^2) = c^2(a^2 + b^2 - c^2)$$

$$a^2b^2 - b^4 - a^2c^2 + c^4 = 0$$

$$a^2(b^2 - c^2) - (b^4 - c^4) = 0$$

$$(b^2 - c^2)(a^2 - b^2 - c^2) = 0$$

Either $b^2 + c^2 - a^2 = 0$ or $(b^2 - c^2) = 0$

When, $b^2 + c^2 - a^2 = 0$

$b^2 + c^2 = a^2$

Hence, ΔABC is a right angle triangle.

And when, $b^2 - c^2 = 0 \Rightarrow b = c$

Hence, ΔABC is an isosceles triangle.

From question ΔABC is not right angle triangle.

Hence, ΔABC must be an isosceles triangle.

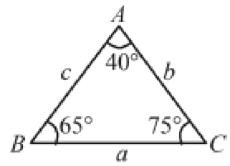
45. (b)

(1) Consider a right angle triangle ABC, right angle at A and $B = C = 45^\circ$.

Then, $b = c$ is also true.

Hence, for the given condition, ΔABC must not be an obtuse-angled triangle.

(2)



In $\Delta ABC,\ \angle A = 40^\circ,\ \angle B = 65^\circ$

$\therefore \angle C = 180^\circ - 40^\circ - 65^\circ = 75^\circ.$

From sine rule,

$$\frac{a}{\sin 40^\circ} = \frac{c}{\sin 75^\circ}$$

$$\therefore \frac{a}{c} = \sin 40^\circ \cdot \text{cosec} 75^\circ$$

So, $\frac{a}{c} \neq \sin 40^\circ \cdot \text{cosec} 15^\circ$

Hence, ΔABC is not possible.

Thus, statement (2) is correct.

46. (c) A square M matrix is said to be Hermitian (or self-adjoint) if it is equal to its. Own Hermitian conjugate, i.e.

$$(\bar{M})^T = M$$

Given Matrix $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$

$$(\bar{A})^T = \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix}$$

Now, $A + (\bar{A})^T = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix} + \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix}$

$$= \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

Conjugate transpose of $(A + (\bar{A})^T) = 2\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$

Hence, $(A + (\bar{A})^T)$ is hermitian.

47. (a) Let $(r+1)^{\text{th}}$ term is independent of x.

$$(r+1)^{\text{th}} \text{ term} = {}^{10}C_r \cdot \left(\frac{2}{x^2}\right)^{10-r} \cdot (-\sqrt{x})^r$$

Exponent of x in independent term = 0

i.e. $\frac{r}{2}-2(10-r)=0$

$r-4(10-r)=0 \Rightarrow 5r-40=0 \Rightarrow r=8.$

Thus, 9th term is independent of x.

9th term $= {}^{10}C_8(2)^{10-8}\cdot(-1)^8 = {}^{10}C_8(2)^2$

$= \frac{10\times 9}{2}\times 4 = 180.$

48. (b) $(1+2x-x^2)^6 = a_0 + a_1\cdot x + a_2\cdot x^2 + ... + a_{12}\cdot x^{12}$

Putting $x=-1$,

$a_0 - a_1 + a_2 + ... + a_{12} = (1+2(-1)-(-1)^2)^6$

$= (1-2-1)^{12} = (-2)^6 = 64.$

49. (c) ${}^{20}C_{(n+2)} = {}^{20}C_{(n-2)}$

$\Rightarrow \frac{20!}{(n+2)!(20-n-2)!} = \frac{20!}{(n-2)!(20-n+2)!}$

$\Rightarrow \frac{(22-n)!}{(18-n)!} = \frac{(n+2)!}{(n-2)!}$

$\Rightarrow (22-n)(21-n)(20-n)(19-n)$

$= (n+2)(n+1)\cdot n\cdot(n-1)$

For $n=10$

$(22-10)(21-10)(20-10)(19-10)$

$= (10+2)(10+1)\cdot 10\cdot(10-1)$

$\Rightarrow 12\cdot 11\cdot 10\cdot 9 = 12\cdot 11\cdot 10\cdot 9$

Hence, $n=10$.

50. (d) For singular matrix,

$$\begin{bmatrix} 0 & K & 4 \\ -K & 0 & -5 \\ -K & K & -1 \end{bmatrix} = 0$$

$K(5K-K)+4(-K^2)=0$

$4K^2-4K^2=0$

Hence, for all values of K, the given matrix is singular matrix.

51. (c) $(1101101)_2 + (1011011)_2$

$= (1\times 2^6 + 1\times 2^5 + 0 + 1\times 2^3 + 1\times 2^2 + 0 + 1\times 2^0)_{10}$

$+(1\times 2^6 + 0 + 1\times 2^4 + 1\times 2^3 + 0 + 1\times 2^1 + 1\times 2^0)_{10}$

$= (64+32+8+4+1)_{10} + (64+0+16+8+2+1)_{10}$

$= (109+91)_{10} = (200)_{10}.$

52. (a) $\frac{1}{10}\log_5 1024 - \log_5 10 + \frac{1}{5}\log_5 3125$

$= \log_5(1025)^{1/10} - \log_5 10 + \log_5(3125)^{1/5}$

$= \log_5(2^{10})^{\frac{1}{10}} - \log_5 10 + \log_5(5^5)^{\frac{1}{5}}$

$= \log_5(2) - \log_5 10 + \log_5 5$

$= \log_5\left(\frac{2\times 5}{10}\right) = \log_5 1 = 0.$

53 (c) $1+x = \log_c(ab)+1$

$= \log_c(ab) + \log_c c = \log_c(abc)$

$(1+x)^{-1} = \frac{1}{\log_c(abc)} = \log_{(abc)} c$

Similarly, $(1+y)^{-1} = \log_{(abc)} a$

and $(1+z)^{-1} = \log_{(abc)} b$

Now, $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

$= \log_{(abc)} c + \log_{(abc)} a + \log_{(abc)} b$

$= \log_{(abc)}(cab) = 1.$

54. (b) $AB = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 2(x+y)-y \\ 4x-x+y \end{bmatrix} = \begin{bmatrix} 2x+y \\ 3x+y \end{bmatrix}$

As $AB = C$

$\therefore \begin{bmatrix} 2x+y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$2x+y=3$ ...(i)

and $3x+y=2$ ...(ii)

From equation (i) and (ii), we get

$x=-1, y=5.$

$\therefore A = \begin{bmatrix} -1+5 & 5 \\ -2 & -1-5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

$= 4(-6)-5(-2) = -14.$

55. (d) $1.5 \le x \le 4.5$

$\frac{3}{2} \le x \le \frac{9}{2} \Rightarrow 3 \le 2x \le 9$

$\therefore (2x-3) \ge 0$ and $(2x-9) \le 0$

Hence, $(2x-3)(2x-9) \le 0$

56. (c) Give set $S = \{1, 2, 3, ...\}$

For xRy, $\log_a x > \log_a y$

$\Rightarrow x > y$

As xRx, $\log_a x > \log_a x$ is not valid.

Hence, relation is not reflexive.

For xRy, $\log_a x > \log_a y \Rightarrow x > y$

$yRx,\ \log_a y > \log_a x \Rightarrow y > x$

This is also not valid. Hence, relation is not symmetric also.

For $xRy,\ \log_a x > \log_a y \Rightarrow x > y$

For $yRz,\ \log_a y > \log_a z \Rightarrow y > z$

So, $xRz,\ \log_a x > \log_a z \Rightarrow x > z$

This is a valid relation. Hence, relation is only transitive.

57. (d) $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix} = \begin{vmatrix} i & -1 & -i \\ 1 & -1 & 1 \\ i & 1 & -i \end{vmatrix}$

$= i(i-1) - 1(i-(-i)) - i(1+i)$

$= i^2 - i - 2i - i - i^2 = -4i.$

58. (c) $AB = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$

59. (b) 2nd and 4th place are even place, so vowel '*A*' and '*E*' arrange either 2nd or 4th place in 2! = 2 × 1 = 2 ways

Consonent letter, '*B*' and '*L*' arrange at 1st and 3rd place in 2! = 2 × 1 = 2 ways.

Total number of arrangement = 2 × 2 = 4.

60. (c) Maximum number of points of intersection of 2 non-overlapping circles = 2.

So, maximum number of points of intersection of 5 non-overlapping circles = 8 + 6 + 4 + 2 = 20.

61. (c)

Number of students		Marks Range	Difference of number of students	
Physics	Maths			
11	21	20 – 30	21 – 11 = 10	
30	38	30 – 40	38 – 30 = 8	
26	15	40 – 50	26 – 15 = 11	←Maximum
15	10	50 – 60	15 – 10 = 5	

Hence, difference is largest for the interval (40-50).

62. (a)

(1) Modal value of the marks of Physics is the interval in which maximum number of students got his marks. In the marks interval of (30–40), number of students in Physics is 30, which is largest number of students in any interval. Hence, modal values of marks in Physics is (30–40). Statement (1) is correct.

(2) Median class is given by $\left(\frac{N}{2}\right)^{\text{th}}$ item i.e. $\left(\frac{100}{2}\right)^{\text{th}}$ item which is 50th item. This corresponds to the class interval of (40–50) for Physics and (30–40) for Mathematics.

Marks	Number of Physics students	Cumulative Frequency	Number of Maths student	Cumulative Frequency
10-20	8	8	10	10
20-30	11	19	21	31
30-40	30	49	38	69
40-50	26	75	15	84
50-60	15	90	10	94
60-70	10	100	6	100

Medium $= l_1 + \dfrac{\frac{N}{2} - C.f.}{f} \times i$

$\therefore$ Median for Physics $= 40 + \dfrac{\frac{100}{2} - 49}{26} \times 10$

$= 40 + \dfrac{50-49}{26} \times 10 = 40.385$

Median for Maths $= 30 + \dfrac{\frac{100}{2} - 31}{38} \times 10$

$= 30 + \dfrac{50-31}{38} \times 10 = 35$

Thus, median of the marks in Physics is more than median of the marks in Mathematics.

Hence, statement (2) is not correct.

63. (c) For physics

Marks	c_i	f_i	$c_i f_i$
10–20	15	8	120
20–30	25	11	275
30–40	35	30	1050
40–50	45	26	1170
50–60	55	15	825
60–70	65	10	650
		Σ [illegible]	Σ [illegible]

Mean of marks of physics $= \dfrac{4090}{100} = 40.9.$

64. (b) Standard deviation $= \sqrt{\dfrac{(x-\bar{x})^2}{N}}$

Given data: $-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}.$

$\bar{x} = \dfrac{\text{Sum of datas}}{\text{Number of data}} = 0$

$(x-\bar{x})^2 = \left(-\sqrt{6}-0\right)^2 + \left(-\sqrt{5}-0\right)^2 + \left(-\sqrt{4}-0\right)^2 +$

$(-1-0)^2 + (1-0)^2 + \left(\sqrt{4}-0\right)^2 + \left(\sqrt{5}-0\right)^2 + \left(\sqrt{6}-0\right)^2$

$= 6 + 5 + 4 + 1 + 1 + 4 + 5 + 6 = 32$

S.D. $= \sqrt{\dfrac{(x-\bar{x})^2}{N}} = \sqrt{\dfrac{32}{8}} = \sqrt{4} = 2$

65. (d) Coefficient of Variation (C.V.)

$$= \frac{\text{Standard Deviation } (\sigma)}{\text{Mean } (\mu)} \times 100$$

Now, standard deviation $(\sigma) = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

$$= \sqrt{\frac{200}{10} - \left(\frac{20}{10}\right)^2} = 4$$

Mean $(\mu) = \frac{\Sigma x_i}{n} = \frac{20}{10} = 2.$

$\therefore$ Co-efficient of variation (C.V.) $= \frac{4}{2} \times 100 = 200.$

66. (b) Number of days in February, when year is a leap year = 29 days = 4 weeks + 1 odd day
4 weeks have 4 sundays.

Probability that 1 odd day is sunday $= \frac{1}{7}.$

67. (b) Correct arithmetic mean

$$= 40 + \frac{(-83+53)}{100} = 40 + \frac{(-30)}{100} = 39.7.$$

68. (a) Probability for Husband's selection $P(H) = \frac{1}{7}$

Probability when Husband is not selection

$$P(H') = 1 - \frac{1}{7} = \frac{6}{7}$$

Probability for wife's selection $P(W) = \frac{1}{5}$

Probability when Wife is not selected $P(W') = 1 - \frac{1}{5} = \frac{4}{5}$

As both are independent event.
So, probability for atleast one of them will be selected

$$= P(H)P(W') + P(H')P(W)$$

$$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{11}{35}.$$

69. (a) Number of ways of picking 4 counterfeits gold coins out of 6 counterfeits coins = ${}^6C_4$
Number of ways of picking 4 coins out of 15 gold coins = ${}^{15}C_4$

$\therefore$ Required probability $= \frac{{}^6C_4}{{}^{15}C_4} = \frac{15}{15 \times 7 \times 13} = \frac{1}{91}.$

70. (d) Number of ways of selecting 2 boys out of 2 boys = ${}^2C_2 = 1.$
Number of ways of selecting 1 girl out of 2 girls = ${}^2C_1 = 2.$
Number of ways of selecting 3 out of 4 persons = ${}^4C_3 = 4.$

$\therefore$ Required probability $= \frac{{}^2C_1 \times {}^2C_2}{{}^4C_3} = \frac{2 \times 1}{4} = \frac{1}{2}.$

71. (c) Prime number between 1 to 10 are 2, 3, 5, 7.
Now, number of ways of selecting 2 prime number out of 4 prime number = ${}^4C_2 = 6.$
Number of ways of selecting 2 numbers out of 10 numbers = ${}^{10}C_2 = 45.$

$\therefore$ Required probability $= \frac{6}{45} = \frac{2}{15}.$

72. (b) Regression of Y on X:

$Y = \left\{\frac{\sigma_y}{\sigma_x} \times r(x, y)\right\} X - a$ (where a = constant term)

$$\Rightarrow Y = \left(\frac{3.5}{2.5} \times 0.8\right) X - 5.8$$

$$\therefore Y = 1.12X - 5.8.$$

73. (d) From question $P = \frac{1}{5}$

Now, $P + q = 1$

$$\Rightarrow \frac{1}{5} + q = 1 \Rightarrow q = 1 - \frac{1}{5} = \frac{4}{5}.$$

and $n = 10.$

74. (d) $P(\bar{A}) = 1 - P(A) = 1 - 0.6 = 0.4.$

$P(\bar{B}) = 1 - P(B) = 1 - 0.5 = 0.5.$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.5 - 0.4 = 0.7.$

(1) Now $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$

$= P(\bar{A}) + P(B) - \{P(B) - P(A \cap B)\}$
$= 0.4 + 0.5 - \{0.5 - 0.4\} = 0.8.$

Hence, statement (1) is not correct.

(2) $P(\bar{B} \mid \bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = \frac{P(B \cup A)'}{P(\bar{A})} = \frac{1 - P(A \cup B)}{P(\bar{A})}$

$= \frac{1 - 0.7}{0.4} = 0.75.$

Hence, statement (2) is not correct.

75. (b) From question, we have

$P(F \mid X) = 0.02,\ P(F \mid Y) = 0.03,\ P(F \mid Z) = 0.05$

and

$$P(F) = P(X) \cdot P(F \mid X) + P(Y) \cdot P(F \mid Y) + P(Z) \cdot P(F \mid Z)$$

$= 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.05$
$= 0.01 + 0.009 + 0.01 = 0.029.$

Now,

$P(X \cap F) = P(X) \cdot P(F \mid X) = 0.5 \times 0.02 = 0.01.$

$\therefore P(X \mid F) = \frac{P(X \cap F)}{P(F)} = \frac{0.01}{0.029} = \frac{10}{29}$

76. (a)

(1) $f(x) = e^{-|x|},\ f(0) = e^{-|0|} = 1.$

$f(x) = e^{-x}$, for $x \geq 0$
$= e^{x}$, for $x < 0.$

LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^x = 1$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{-x} = 1$

As, $f(0) = \text{LHL} = \text{RHL}$

Thus $f(x)$ is continuous at $x = 0$.

(2) LHD = $\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} e^x = e^0 = 1$

RHD = $\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} -e^{-x} = -e^0 = -1$

As LHD $\neq$ RHD

Hence, $f(x)$ is not differentiable at $x = 0$.

77. (c) $y = \sin x \cdot \cos x$

$$= \frac{2\sin x \cdot \cos x}{2} = \frac{\sin 2x}{2}$$

Now, maximum value of $\sin 2x = 1$.

$\therefore y_{\max} = \frac{1}{2}$.

78. (a) $\lim_{x\to 0} \frac{3^x + 3^{-x} - 2}{x} = \lim_{x\to 0} \frac{3^x \cdot \log 3 - 3^{-x} \cdot \log 3}{1}$

(By L'Hospital rule)

$$= \frac{3^0 \cdot \log 3 - 3^0 \cdot \log 3}{1} = 0$$

79. (a) Let $y = \tan^{-1} x$; and $z = \cot^{-1} x$

Then, $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $\frac{dz}{dx} = -\frac{1}{1+x^2}$

Now, $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{\frac{1}{1+x^2}}{-\left(\frac{1}{1+x^2}\right)} = -1.$

80. (b) $x(dx - dy) + y(dy - dx) = 0$

On integrating both sides, we have

$\int x(dx - dy) + \int y(dy - dx) = C$ (where C = constant)

$\frac{x^2}{2} - xy + \frac{y^2}{2} - xy = C$

$x^2 + y^2 - 2xy = C$

Or, $x^2 + y^2 = 2xy + C$.

81. (a) Let $y = 3 \cdot \cos\left(A + \frac{\pi}{3}\right)$, $y' = -3 \cdot \sin\left(A + \frac{\pi}{3}\right)$

For extremum value, $y' = 0$

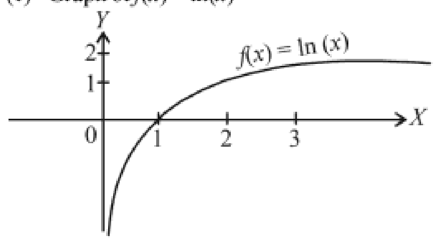
$\sin\left(A + \frac{\pi}{3}\right) = 0 \Rightarrow A + \frac{\pi}{3} = 0 \Rightarrow A = -\frac{\pi}{3}$

$y'' = -3 \cdot \cos\left(A + \frac{\pi}{3}\right)$

$= -3\cos\left(-\frac{\pi}{3} + \frac{\pi}{3}\right) = -3\cos(0) = -3.$

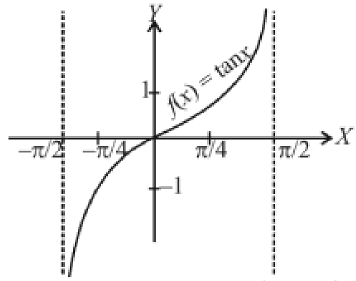
82. (c)

(1) Graph of $f(x) = \ln(x)$



This is increasing function in the interval $(0, \infty)$.

(2) Graph of $f(x) = \tan x$ ·



This is also increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

83. (b) Given graph $Y = \frac{1}{x-1}$

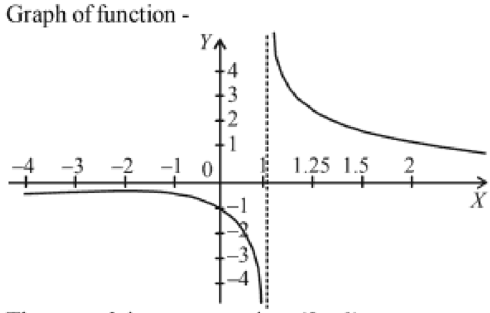
This is defined for all real x, except $x = 1$.

Range of the graph is $y \in R \mid y \neq 0$

Table for the graph :

x	–4	–3	–1	0	0.25	0.5	0.75	1.25	1.5	2
y	–0.2	–0.25	–0.5	–1	–1.33	–2	–4	4	2	1

Graph of function -



Thus, graph intersect y-axis at $(0, -1)$.

84. (a) $\ln\left(\frac{dy}{dx}\right) = x$

$\frac{dy}{dx} = e^x$

Integrating both sides, we get

$\int dy = \int e^x . dx$

$y = e^x + C$, where C = integration constant.

85. (d) $l+b=K$, where l = length, b = breadth.

Area of the rectangle $A=l\cdot b$.

$A=l(K-l)=lK-l^2$

For maximum area, $\frac{dA}{dl}=0$

$\frac{d}{dl}(lk-l^2)=0$

$K-2l=0\Rightarrow l=\frac{K}{2}$

Now, $\frac{K}{2}+b=K\Rightarrow b=K-\frac{K}{2}=\frac{K}{2}$

Area, $A=l\cdot b=\frac{K}{2}\cdot\frac{K}{2}=\frac{K^2}{4}$

86. (b) Arithmetic mean = $\frac{\text{Sum of data}}{\text{Number of data}}$

$6=\frac{4\times x+9(x-1)}{(x+x-1)}$

$6(2x-1)=4x+9x-9$

$12x-6=13x-9$

$x=9-6=3.$

87. (c) Number of ways in which one of the face having the number 6 and no two dice show the same number.

(1,2,6), (1,3,6), (1,4,6), (1,5,6), (2,3,6), (2,4,6), (2,5,6), (3,4,6), (3,5,6)......

Total favourable case = 20 + 20 + 20 = 60.

Number of total output when three top faces of three dice shows different number = 6 × 5 × 4 = 120.

$\therefore$ Required probability $=\frac{60}{120}=\frac{1}{2}$.

88. (c) $P(A\cup B)=\frac{5}{6},\ P(A\cap B)=\frac{1}{3},\ P(A')=\frac{1}{2}$

$P(A)=1-P(A')=1-\frac{1}{2}=\frac{1}{2}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{5}{6}=\frac{1}{2}+P(B)-\frac{1}{3}$

(a) $P(B)=\frac{5}{6}-\frac{1}{2}+\frac{1}{3}=\frac{5-3+2}{6}=\frac{4}{6}=\frac{2}{3}$

(b) $P(A\cap B)=\frac{1}{3}=\frac{1}{2}\cdot\frac{2}{3}=P(A)\cdot P(B)$

$\therefore P(A\cap B)=P(A)\cdot P(B)$.

(c) $P(A\cup B)=\frac{5}{6}$

$P(A)+P(B)=\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$

$\therefore P(A\cup B)<P(A)+P(B)$

Hence, option (c) is not correct.

(d) $P(A'\cap B')=P(A')\cdot P(B')$

$1-P(A\cup B)=1-\frac{5}{6}=\frac{1}{6}$

and $P(A')\cdot P(B')=(1-P(A))\cdot(1-P(B))$

$=\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{2}{3}\right)=\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$.

Hence, $P(A'\cap B')=P(A')\cdot P(B')$.

89. (d) Number of data set = n.

Mean = 2.5

Sum of deviations = 50

Again, sum of deviations, when mean = 3.5 is –50.

So, (3.5 – 2.5) × n = 50 – (–50)

$\therefore n=100$

90. (d) Sum of n observation = $2M\times n$

Sum of $2n$ observation = $M\times 2n$

Mean of combined data sets $=\frac{2Mn+2Mn}{(n+2n)}=\frac{4}{3}M$.

91. (c) $f(0)=3(0)^2-5(0)+P=P$

$f(1)=3(1)^2-5(1)+P=P-2$

Clearly, $f(0)$ and $f(1)$ are opposite in sign.

$\therefore P>0$ and $P-2<0$.

$\Rightarrow 0<P<2$.

92. (b) $e^{\theta\phi}=C+4\theta\cdot\phi$

$(\theta\cdot\phi)\ln e=\ln(C+4\theta\cdot\phi)$

$\theta\cdot\phi=\ln(C+4\theta\cdot\phi)$

$\Rightarrow\theta\cdot g\phi=4\{\ln(\theta)+\ln(\phi)\}+\ln C$

Differentiating both sides with respect to 'θ'.

$\phi+\theta\cdot\frac{d\phi}{d\theta}=4\left(\frac{1}{\theta}+\frac{1}{\phi}\cdot\frac{d\phi}{d\theta}\right)$

$\phi\cdot d\theta+\theta\cdot d\phi=4\phi\cdot d\theta+4\cdot\theta\cdot d\phi$

$\Rightarrow 3\theta\cdot d\phi=-3\phi\cdot d\theta$

$\therefore\phi\cdot d\theta=-\theta\cdot d\phi$

93. (b) $\int P(x)\cdot dx=\int(4\cdot e)^{2x}\cdot dx=\int e^{2x(\ln e+\ln 4)}\cdot dx$

$=\int e^{2x(1+\ln 4)}\cdot dx=\frac{e^{2x(1+\ln 4)}}{2(1+\ln 4)}+C$ where C = constant

$=\frac{(4e)^{2x}}{2(1+2\ln 2)}+C$

$\therefore\int P(x)\cdot dx=\frac{P(x)}{2(1+2\ln(2))}+C.$

94. (b) $I = \int_0^{\frac{\pi}{4}} (\tan^3 x + \tan x) \cdot dx$

$= \int_0^{\frac{\pi}{4}} \tan^3 x \cdot dx + \int_0^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int_0^{\frac{\pi}{4}} \tan x \cdot \tan^2 x \cdot dx + \int_0^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int_0^{\frac{\pi}{4}} \tan x \cdot (\sec^2 x - 1) \cdot dx + \int_0^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x \cdot dx - \int_0^{\frac{\pi}{4}} \tan x \cdot dx + \int_0^{\frac{\pi}{4}} \tan x \cdot dx$

$= \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x \cdot dx$

Let $\tan x = z$, $dz = \sec^2 x \cdot dx$

$\therefore \int \tan x \cdot \sec^2 x \cdot dx = \int z \cdot dz = \frac{z^2}{2} + C$

where C = constant

$\therefore I = \left[\frac{\tan^2 x}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2} - 0 = \frac{1}{2}.$

95. (c) From question, $dx = 10.1 - 10 = 0.1$
At, $x = 10$

$y = (3)(10)^2 + 2 = 302.$

At $x = 10.1$, $y = 3(10.1)^2 + 2 = 308.03.$

Total change in $y = 308.03 - 302 = 6.03.$

96. (b) As the given function is continuous at $x = 0$, then LHL = RHL

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin x}{x} = \lim_{x \to 0^-} \left[\frac{\cos x}{1}\right] = 1$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin x}{x} = \lim_{x \to 0^+} \left[\frac{\cos x}{1}\right] = 1$

$\therefore \frac{\sin x}{x}$ at $x = 0$, should be 1.

97. (b) $dy = (1 + y^2) \cdot dx$

$\frac{1}{(1+y^2)} \cdot dy = dx$

Integrating both sides, we get

$\int \frac{dy}{(1+y^2)} = \int dx$

$\tan^{-1}(y) = x + C$, where C = constant.

$\therefore y = \tan(x + C).$

98. (c) $\int (e^{\log x} + \sin x) \cdot \cos x \cdot dx = \int (x + \sin x) \cdot \cos x \cdot dx$

$= \int (x \cdot \cos x + \sin x \cdot \cos x) \cdot dx$

$= \int x \cdot \cos x \cdot dx + \int \sin x \cdot \cos x \cdot dx$

$= x \cdot \int \cos x \cdot dx - \int \left(\frac{dx}{dx} \cdot \int (\cos x) \cdot dx\right) \cdot dx$

$+ \int \sin x \cdot \cos x \cdot dx$

$= x \cdot \sin x - \int \sin x \cdot dx + \int \sin x \cdot \cos x \cdot dx$

$= x \cdot \sin x - (-\cos x) + \int \sin x \cdot \cos x \cdot dx$

$= x \sin x + \cos x + \int \sin x \cdot \cos x \cdot dx$

Now, $\int \sin x \cdot \cos x \cdot dx$;

Let $\sin x = z$, $dz = \cos x \cdot dx$

$\int \sin x \cdot \cos x \, dx = \int z \cdot dz = \frac{z^2}{2} = \frac{\sin^2 x}{2}$

$\therefore I = x \cdot \sin x + \cos x + \frac{\sin^2 x}{2} + C.$

99. (b) $f(x) = \cos^{-1}(x - 2)$

Domain of $\cos^{-1}(x-2) = [-1, 1]$

$\therefore -1 \le x - 2 \le 1$

$-1 + 2 \le x \le 1 + 2$

$1 \le x \le 3$

$\therefore$ Domain = [1, 3]

100. (c) Two given graphs are $y^2 = 2x$ and $y = x$.
Point of intersections are :

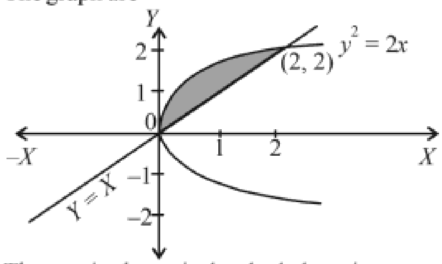
$x^2 = 2x \Rightarrow x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ and 2 & $y = 0$ and 2.

So, points of intersections are (0, 0) and (2, 2).

The graph are



The required area is the shaded portion

$$A = \left|\int_0^2 (y_1 - y_2)\cdot dx\right|$$

$$= \left|\int_0^2 (\sqrt{2x} - x)\cdot dx\right| = \left[\frac{2\sqrt{2}(x)^{3/2}}{3} - \frac{x^2}{2}\right]_0^2$$

$$= \frac{2\sqrt{2}}{3}(2)^{\frac{3}{2}} - 2 = \frac{8}{3} - 2 = \frac{2}{3}$$

101. (a) $f(x) = 2x - x^2$

$f(x+2) + f(x-2)$

$= 2(x+2) - (x+2)^2 + 2(x-2) - (x-2)^2$

$= 4 + 4x - x^2 - 4 - 4x - x^2 - 4 + 4x - 4$

$= 8x - 2x^2 - 8$

When $x = 0$, then, $f(x+2) + f(x-2) = -8$

102. (b) $x^m y^n = a^{m+n}$

$$\frac{x^m}{a^m} = \frac{a^n}{y^n} \quad \text{...(i)}$$

Differentiating equation (i) w.r.t. x, we get

$$\frac{m\cdot x^{(m-1)}}{a^m} = \frac{a^n\cdot(-n)}{y^{n+1}}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{m\cdot y^{(n+1)}\cdot(x)^{(m-1)}}{-n\cdot a^m\cdot a^n}$$

$$= -\frac{my}{n\cdot x}\cdot\left(\frac{x^m\cdot y^n}{a^{m+n}}\right) = -\frac{my}{nx}(1) = -\frac{my}{nx}$$

103. (a) $\int \frac{dx}{x(x^n+1)} = \int \frac{x^{n-1}}{x^n(x^n+1)}\cdot dx$

Let $z = x^n \Rightarrow dz = nx^{n-1}dx$

$$\therefore \int \frac{dx}{x(x^n+1)} = \int \frac{x^{n-1}}{x^n(x^n+1)}\cdot dx = \int \frac{1}{z(z+1)}\cdot\frac{dz}{n}$$

$$= \frac{1}{n}\left[\int \frac{1}{z}\cdot dz - \int \frac{dz}{z+1}\right]$$

$$= \frac{1}{n}[\ln z - \ln(z+1)] + C \quad \text{where } C = \text{constant}$$

$$= \frac{1}{n}\cdot\ln\left(\frac{z}{z+1}\right) + C = \frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right) + C$$

104. (a) Minimum value of any modulus is 0.

105. (c) $I = \int\left(\frac{3x^2+8-4k}{x}\right)\cdot dx$

$$= \int 3x\,dx + \int \frac{8-4k}{x}\cdot dx$$

$$= \frac{3}{2}x^2 + (8-4k)\cdot\ln(x) + C \quad \text{where } C = \text{constant}$$

To get integration as rational function,

$$(8-4k)\cdot\ln(x) = 0 \Rightarrow 8 - 4k = 0 \Rightarrow k = \frac{8}{4} = 2$$

106. (d) Radius of the sphere

$$= \frac{6(1) - 3(-2) + 2(3) - 4}{\sqrt{(6)^2 + (-3)^2 + (2)^2}} = \frac{6+6+6-4}{7} = \frac{14}{7} = 2$$

Diameter of the sphere = 2 × 2 = 4 units

107. (b) Perpendicular distance $= \sqrt{(4)^2 + (3)^2} = 5$ units.

108. (d) Direction ratios are $\langle a+b, b+c, c+a \rangle$

Then, direction cosine,

$$l = \frac{(a+b)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

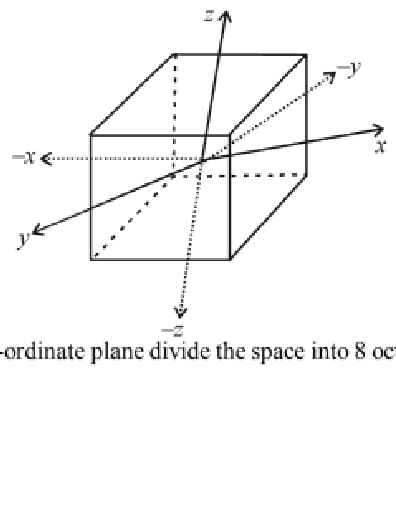
$$m = \frac{(b+c)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$n = \frac{(c+a)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

Sum of squares of direction cosines

$$l^2 + m^2 + n^2 = \frac{(a+b)^2 + (b+c)^2 + (c+a)^2}{(a+b)^2 + (b+c)^2 + (c+a)^2} = 1.$$

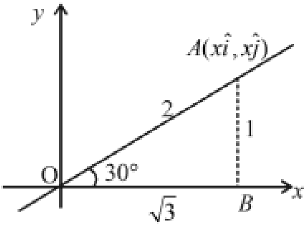
109. (c)



Co-ordinate plane divide the space into 8 octanes.

110. (b) Equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane, is
$z=5, y=0, x=0$.

111. (a)



Let $A(x\hat{i},\ y\hat{j})$ is a point in the xy-plane.

From question,

$\angle AOB = 30°$ and $\overrightarrow{OA} = 1$

$$x = |\overrightarrow{OA}| \cdot \cos 30° = 1 \times \frac{\sqrt{3}}{2}\hat{i} = \frac{\sqrt{3}}{2}\hat{i}$$

$$y = |\overrightarrow{OA}| \cdot \sin 30° = 1 \times \frac{1}{2}\hat{j} = \frac{\hat{j}}{2}$$

$$\therefore \vec{a} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j} = \frac{\sqrt{3}\hat{i} + 1\hat{j}}{2}$$

112. (c) Let $A = (x, y, z)$

Then, $|\overrightarrow{OA}| = \sqrt{x^2+y^2+z^2} = 12$

Also, $x = |\overrightarrow{OA}| \cdot \cos 45° = 12 \cdot \frac{1}{\sqrt{2}} = 6\sqrt{2}\hat{i}$.

$y = |OA| \cdot \cos 60° = 12 \cdot \frac{1}{2} = 6\hat{j}$.

Hence, $\overrightarrow{OA} = 6\sqrt{2}\hat{i} + 6\hat{j} \pm 6\hat{k}$

113. (c) Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Two diagonals of the parallelograms are given by

$(\vec{a}+\vec{b}) = (2+1)\hat{i} - (4+2)\hat{j} + (5-3)\hat{k}$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

and $(\vec{a}-\vec{b}) = (2-1)\hat{i} - (4-2)\hat{j} + (5-(-3))\hat{k}$

$= \hat{i} - 2\hat{j} + 8\hat{k}$

Dot products of the diagonals

$(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot (\hat{i} - 2\hat{j} + 8\hat{k})$

$= (3+12+16) = 31$ units

114. (a) $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$

$\{|\vec{a}| \cdot |\vec{b}| \cdot \sin\theta\}^2 + \{|\vec{a}| \cdot |\vec{b}| \cdot \cos\theta\}^2 = 144$

$(|\vec{a}| \cdot |\vec{b}|)^2 (\sin^2\theta + \cos^2\theta) = 144$

$|\vec{a}| \cdot |\vec{b}| = \sqrt{144} = 12$

$4 \cdot |\vec{b}| = 12 \Rightarrow |\vec{b}| = \frac{12}{4} = 3$.

115. (b) As the given vectors are coplanar, then

$$\vec{c} \times \vec{a} \times \vec{b} = 0 \Rightarrow \begin{vmatrix} 0 & 1 & p \\ 2 & -3 & 1 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$\Rightarrow 0 + 1(1+6) + p(4+3) = 0$

$\Rightarrow 7 + 7p = 0 \Rightarrow p = -1$.

116. (d) $\lim_{x\to 1} \frac{x + x^2 + x^3 - 3}{x-1}$

This is in $\frac{0}{0}$ form, so we can use L' Hospital rule.

$$\lim_{x\to 1} \frac{x+x^2+x^3-3}{x-1} = \lim_{x\to 1} \frac{1+2x+3x^2}{1} = \frac{1+2+3}{1} = 6$$

117. (a) Circumference of a circle $C = 2\pi r$

Differentiating both sides w.r.t. time (t).

$$\frac{dc}{dt} = 2\pi \cdot \frac{dr}{dt} = 2\pi \cdot (0.7) = 1.4\pi = 1.4 \times \frac{22}{7} = 4.4 \text{ cm/ sec.}$$

118. (c) $\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)(x^2+1)}{(x-1)}$

$= \lim_{x\to 1} (x+1)(x^2+1) = (1+1)(1+1) = 4$.

Now, $\lim_{x\to k} \frac{x^3-k^3}{x^2-k^2} = 4$

$$\lim_{x\to k} \frac{(x-k)(x^2+k^2+xk)}{(x-k)(x+k)} = 4$$

$$\lim_{x\to k} \frac{(x^2+k^2+xk)}{(x+k)} = 4$$

$$\frac{k^2+k^2+k\cdot k}{k+k} = 4 \Rightarrow \frac{3}{2}k = 4 \Rightarrow k = \frac{8}{3}$$

119. (b) Given differential equation is

$$k \cdot \frac{dy}{dx} = \int \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3} \cdot dx$$

On differentiating both sides, we get

$$k \cdot \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3}$$

$$\Rightarrow \left(k \cdot \frac{d^2y}{dx^2}\right)^3 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^2$$

Now, order of the D.E. = 2

Degree of the D.E. = 3.

120. (a) $\lim_{x\to 0}\frac{\sin x\cdot\log(1-x)}{x^2}$

This is in $\frac{0}{0}$ form, by using L' Hospital rule.

$$\lim_{x\to 0}\frac{\sin x\cdot\log(1-x)}{x^2}$$

$$=\lim_{x\to 0}\frac{\cos x\cdot\log(1-x)+\sin x\cdot\frac{1}{(1-x)}(-1)}{2x}$$

$$=\lim_{x\to 0}\frac{\log(1-x)\cdot(-\sin x)-\frac{\cos x}{(1-x)}-\frac{1}{(1-x)}\cdot\cos x-\frac{\sin x}{(1-x)^2}}{2}$$

$$=\frac{0-1-1-0}{2}=\frac{-2}{2}=-1.$$