NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2019 (I)

PAPER I : Mathematics

1. What is the nth term of the sequence $25, -125, 625, -3125, \ldots$?

(a) $(-5)^{2n-1}$ (b) $(-1)^{2n} 5^{n+1}$

(c) $(-1)^{2n-1} 5^{n+1}$ (d) $(-1)^{n-1} 5^{n+1}$

(*d*) Given, sequence $25, -125, 625, -3125$

Here, $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots\ldots\ldots\ldots$

So, this sequence in GP whose common ratio is -5.

then $a = 25$, $r = -5$

$\therefore n$th term of sequence $= ar^{n-1}$

$= 25(-5)^{n-1}$

$= (-1)^{n-1} 5^2 \times 5^{n-1} = (-1)^{n-1} 5^{n+1}$

2. Suppose $X = \{1, 2, 3, 4\}$ and R is a relation on X. If $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2,3), (3, 2)\}$, then which one of the following is correct?

(a) R is reflexive and symmetric, but not transitive

(b) R is symmetric and transitive, but not reflexive

(c) R is reflexive and transitive, but not symmetric

(d) R is neither reflexive nor transitive, but symmetric

(*d*) We have, $X = \{1, 2, 3, 4\}$

$R = \{(1,1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

Since, $(4, 4) \notin R$,

Hence, R is not reflexive.

Since, $(1, 2) \in R$, $(2, 3) \in R$ but $(1, 3) \notin R$, R is not transitive.

$(1, 2), (2, 3) \in R$

and also $(2, 1), (3, 2) \in R$

$\therefore R$ is symmetric.

Hence, R is neither reflexive nor transitive but symmetric.

3. A relation R is defined on the set N of natural numbers as $xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$. Then, which one of the following is correct?

(a) R is reflexive and symmetric, but not transitive

(b) R is reflexive and transitive, but not symmetric

(c) R is reflexive, symmetric and transitive

(d) R is reflexive, but neither symmetric nor transitive

(*d*) Given, $xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$

For reflexive

$xRx \Rightarrow x^2 - 4x^2 + 3x^2 = 0$

So, $(x, x) \in R, \forall x \in N$

Hence, R is reflexive.

For symmetric

$xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$

$\therefore \quad yRx \Rightarrow y^2 - 4xy + 3x^2$

It is not clear, that $y^2 - 4xy + 3x^2$ is equal to zero or not.

i.e. $(x, y) \in R$ but $(y, x) \notin R \cdot \forall x, y \in N$

Hence, R is not symmetric.

For transitive

$xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$

$yRz \Rightarrow y^2 - 4yz + 3z^2 = 0$ (let)

$xRz \Rightarrow x^2 - 4xz + 3z^2$

It is not clear, that $x^2 - 4xz + 3z^2$ is equal to zero or not.

So, $(x, y) \in R, (y, z) \in R$

$\Rightarrow (x, z) \notin R \;\forall\; x, y, z \in N$

Hence, R is not transitive.

4. If $A = \{x \in Z : x^3 - 1 = 0\}$ and $B = \{x \in Z : x^2 + x + 1 = 0\}$, where, Z is set of complex numbers, then what is $A \cap B$ equal to?

(a) Null set

(b) $\left[\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right]$

(c) $\left[\frac{-1+\sqrt{3}i}{4}, \frac{-1-\sqrt{3}i}{4}\right]$

(d) $\left[\frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}\right]$

(*b*) We have, $A = \{x \in Z : x^3 - 1 = 0\}$

and $B = \{x \in Z : x^2 + x + 1 = 0\}$

$A = \left\{1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

$B = \left\{\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

$A \cap B = \left\{\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

5. Consider the following statements for the two non-empty sets A and B.

1. $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup B$
2. $(A \cup (\overline{A} \cap \overline{B})) = A \cup B$

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

(*a*) We have,

1. $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup B$

LHS $\equiv (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B)$

$= \{A \cap (B \cup \overline{B})\} \cup (\overline{A} \cap B)$

[by distributive property]

$= (A \cap U) \cup (\bar{A} \cap B)$

$[\because B \cup \bar{B} = U]$

$= A \cup (\bar{A} \cap B)$

$= (A \cup \bar{A}) \cap (A \cup B)$

$= U \cap (A \cup B) = A \cup B =$ RHS

Hence, 1 is correct.

2. $A \cup (\bar{A} \cap \bar{B}) = A \cup B$

LHS $\equiv A \cup (\bar{A} \cap \bar{B})$

$= (A \cup \bar{A}) \cap (A \cup \bar{B})$

$= U \cap (A \cup \bar{B})$

$= A \cup \bar{B} \neq A \cup B$

Hence, 2 is false.

$\therefore$ Only 1 is correct.

6. Let X be a non-empty set and let A, B, C be subsets of X. Consider the following statements.

1. $A \subset C \Rightarrow (A \cap B) \subset (C \cap B)$, $(A \cup B) \subset (C \cup B)$
2. $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$
3. $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Which of the above statements are correct?

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) 1, 2 and 3

(***d***) Let $X = \{1, 2, 3, 4\}$

$A = \{1, 2\}$, $B = \{2, 3, 4\}$, $C = \{1, 2, 3\}$

$A \subset C$

$A \cap B = \{2\}$, $C \cap B = \{2, 3\}$

Clearly, $(A \cap B) \subset (C \cap B)$

$A \cup B = \{1, 2, 3, 4\}$, $(C \cup B) = \{1, 2, 3, 4\}$

$(A \cup B) \subset (C \cap B)$

Hence, Statement 1 is correct.

2. $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$

Hence, Statement 2 is also correct.

3. $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Hence, Statement 3 is also correct.

7. If $B = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, then what is adjoint of B equal to?

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (d) It does not exist

(***a***) We have, $B = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Co-factor of B,

$B_{11} = 0, B_{12} = 0, B_{13} = -2$

$B_{21} = 0, B_{22} = 0, B_{23} = -1$

$B_{31} = 0, B_{32} = 0, B_{33} = 8$

$$\text{adj } B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}'$$

$$= \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & 8 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 8 \end{bmatrix}$$

8. What are the roots of the equation $|x^2 - x - 6| = x + 2$?

(a) −2, 1, 4 (b) 0, 2, 4
(c) 0, 1, 4 (d) −2, 2, 4

(***d***) We have,

$|x^2 - x - 6| = x + 2$

$\Rightarrow$ $|(x-3)(x-2)| = x + 2$

Case I $x < 2$

$x^2 - x - 6 = x + 2$

$x^2 - 2x - 8 = 0$

$\begin{bmatrix} x^2 - 4x + 2x - 8 = 0 \\ x(x-4) + 2(x-4) = 0 \end{bmatrix}$

$(x-4)(x+2) = 0$

$x = -2$ but $x \neq 4$ $[\because x < 2]$

Case II $2 \le x < 3$

$x^2 - x - 6 = -(x+2)$

$x^2 - x - 6 + x + 2 = 0$

$x^2 - 4 = 0$

$x = \pm 2$

$x = 2$ but $x \neq -2$ $[\because x \in (2, 3)]$

Case III $x \ge 3$

$x^2 - x - 6 = x + 2$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = 4$ but $x \neq -2$ $[\because x \ge 3]$

$\therefore$ $x = -2, 2, 4$

9. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the matrix A is a/an

(a) singular matrix
(b) involutory matrix
(c) nilpotent matrix
(d) idempotent matrix

(***b***) We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|A| = -1$

Since, $|A| \neq 0$

Hence, A is not singular.

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A^2 = I$

Hence, A is involutory matrix.

10. If $\begin{vmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{vmatrix} = 6 + 11i$, then what are the values of x and y respectively?

(a) −3, 4 (b) 3, 4
(c) 3, − 4 (d) −3, −4

(***a***) We have, $\begin{vmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{vmatrix} = 6 + 11i$

$\Rightarrow$ $x(-i+2) - y(-3-2i) = 6 + 11i$

$\Rightarrow$ $2x + 3y + (-x + 2y)i = 6 + 11i$

On equating real and imaginary parts, on both sides,

we get $2x + 3y = 6$ ...(i)

and $-x + 2y = 11$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = -3$ and $y = 4$

11. The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{2017} + z^{2018} + 1 = 0$ are

(a) $-1, \omega$ (b) $1, \omega^2$
(c) $-1, \omega^2$ (d) ω, ω^2

(***d***) We have, $z^3 + 2z^2 + 2z + 1 = 0$

$(z+1)(z^2 + z + 1) = 0$

$\Rightarrow$ $z + 1 = 0$ or $z^2 + z + 1 = 0$

$z = -1$

or $z = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$= \dfrac{-1 + i\sqrt{3}}{2}, \dfrac{-1 - i\sqrt{3}}{2} = \omega, \omega^2$

Now, $z^{2017} + z^{2018} + 1 = 0$

Put $z = -1$,

LHS $= (-1)^{2017} + (-1)^{2018} + 1$

$= -1 + 1 + 1$

$= 1 \neq 0$ (RHS)

$\therefore$ $z = -1$ is not a root of equation.

Put $z = \omega$,

LHS $= (\omega)^{2017} + (\omega)^{2018} + 1$

$= (\omega^3)^{672} . \omega + (\omega^3)^{672} . \omega^2 + 1$

$= \omega + \omega^2 + 1$ $[\because \omega^3 = 1]$

$[\because 1 + \omega + \omega^2 = 0]$

$= 0 =$ RHS

$\therefore$ $z = \omega$ is a root of equation.

put $z = \omega^2$,

LHS $= (\omega^2)^{2017} + (\omega^2)^{2018} + 1$

$= \omega^{4034} + \omega^{4036} + 1$

$= (\omega^3)^{1344} . \omega^2 + (\omega^3)^{1345} . \omega + 1$

$= \omega^2 + \omega + 1 = 0$ RHS

$\therefore$ $z = \omega^2$ is a root of equation.

Hence, ω, ω^2 are the common roots of these equations.

12. If $C(20, n+2) = C(20, n-2)$, then what is n equal to

(a) 8 (b) 10
(c) 12 (d) 16

(**b**) We have, $C(20, n+2) = C(20, n-2)$

$\Rightarrow \quad {}^{20}C_{n+2} = {}^{20}C_{n-2}$

$\Rightarrow \quad n + 2 + n - 2 = 20$

$[\because {}^{n}C_x = {}^{n}C_y \Rightarrow x + y = n]$

$\therefore \quad n = 10$

13. There are 10 points in a plane. No three of these points are in a straight line. What is the total number of straight lines which can be formed by joining the points?

(a) 90 (b) 45
(c) 40 (d) 30

(**b**) Given, 10 points in a plane where no three of these points are in straight line.

Total number of straight line formed from 10 points is

$${}^{10}C_2 = \frac{10!}{2!8!} = \frac{10 \times 9}{2} = 45$$

14. The equation $px^2 + qx + r = 0$ (where p, q, r, all are positive) has distinct real roots a and b. Which one of the following is correct?

(a) $a > 0, b > 0$
(b) $a < 0, b < 0$
(c) $a > 0, b < 0$
(d) $a < 0, b > 0$

(**b**) Given, $px^2 + qx + r = 0$, where $p, q, r > 0$ and a and b are distinct roots.

$\therefore \quad a + b = \frac{-q}{p}$ and $ab = r$

Now, $r > 0$

$\therefore \quad ab > 0$

$\Rightarrow \quad a > 0, b > 0 \quad$ ...(i)

or $\quad a < 0, b < 0 \quad$ ...(ii)

Now, $\frac{-q}{p} < 0 \; q, p > 0$

$\therefore \quad a + b < 0$

$a < 0, b < 0 \quad$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$\therefore \quad a < 0$ and $b < 0$

15. If $A = \{\lambda, \{\lambda, \mu\}\}$, then the power set of A is

(a) $\{\phi, \{\phi\}, \{\lambda\}, \{\lambda, \mu\}\}$
(b) $\{\phi, \{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$
(c) $\{\phi, \{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$
(d) $\{\{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$

(**b**) We have, $A = \{\lambda, \{\lambda, \mu\}\}$

$P(A) = \{\phi, \{\lambda\}, \{\{\lambda, \mu\}\}, \{\lambda, \{\lambda, \mu\}\}\}$

Directions (Q. Nos. 16 and 17) *Read the information carefully and answer the given questions.*

In a school, all the students play atleast one of three indoor games– chess, carrom and table tennis. 60 play chess, 50 play table tennis, 48 play carrom, 12 play chess and carrom, 15 play carrom and table tennis, 20 play table tennis and chess.

16. What can be the minimum number of students in the school?

(a) 123 (b) 111 (c) 95 (d) 63

(**b**) Let

A = Student play chess

B = Student play table tennis

C = Student play carrom

Given, $n(A) = 60, n(B) = 50, n(C) = 48$

$n(A \cap B) = 20, n(B \cap C) = 15$

$n(A \cap C) = 12$

For minimum number of students in school

$n(A \cap B \cap C)$ must be zero.

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 60 + 50 + 48 - 20 - 15 - 12 + 0 = 111$

17. What can be the maximum number of students in the school?

(a) 111 (b) 123
(c) 125 (d) 135

(**b**) For maximum number of students in school $n(A \cap B \cap C)$ must be 12.

$\therefore \quad n(A \cup B \cup C)$

$= 60 + 50 + 48 - 20 - 15 - 12 + 12$

$= 123$

18. If A is an identity matrix of order 3, then its inverse (A^{-1})

(a) is equal to null matrix
(b) is equal to A
(c) is equal to $3A$ (d) does not exist

(**b**) Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$

19. A is a square matrix of order 3 such that its determinant is 4. What is the determinant of its transpose?

(a) 64 (b) 36
(c) 32 (d) 4

(**d**) Given, $|A| = 4$

$\therefore \quad |A'| = 4 \quad [\because |A| = |A'|]$

20. From 6 programmers and 4 typists, an office wants to recruit 5 people. What is the number of ways this can be done so as to recruit atleast one typist?

(a) 209 (b) 210
(c) 246 (d) 242

(**c**) We have,

6 programmers and 4 typists

Number of ways of 5 recruit people such that atleast one typist

$= {}^4C_1 \, {}^6C_4 + {}^4C_2 \, {}^6C_3 + {}^4C_3 \, {}^6C_2 + {}^4C_4 \, {}^6C_1$

$= 4 \times 15 + 6 \times 20 + 4 \times 15 + 1 \times 6$

$= 60 + 120 + 60 + 6 = 246$

21. What is the number of terms in the expansion of $[(2x-3y)^2(2x+3y)^2]^2$?

(a) 4 (b) 5
(c) 8 (d) 16

(**b**) Given, $[(2x - 3y)^2 (2x + 3y)^2]^2$

$= [4x^2 - 9y^2]^4$

$\therefore$ Total number of terms $= 4 + 1 = 5$

22. In the expansion of $(1 + ax)^n$, the first three terms are respectively $1, 12x$ and $64x^2$. What is n equal to?

(a) 6 (b) 9
(c) 10 (d) 12

(**b**) Given, first three terms of expansion $(1 + ax)^n$ is $1, 12x, 64x^2$,

Now,

$$(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2} a^2x^2 + \dots$$

On equating first three terms, we get

$$na = 12 \text{ and } \frac{n(n-1)}{2} a^2 = 64$$

On putting the value of a in $\frac{n(n-1)}{2} a^2 = 64$, we get

$$\frac{n(n-1)}{2} \left(\frac{12}{n}\right)^2 = 64$$

$$\Rightarrow \quad \frac{144(n-1)}{2n} = 64$$

$\therefore \quad n = 9$

23. The numbers 1, 5 and 25 can be three terms (not necessarily consecutive)of

(a) only one AP
(b) more than one but finite numbers of APs
(c) infinite number of APs
(d) finite number of GPs

(**d**) We have, 1, 5, 25 be three terms.

Clearly, 1, 5, 25 are finite number of GPs.

24. The sum of $(p+q)$th and $(p-q)$th terms of an AP is equal to

(a) $(2p)$ th term (b) $(2q)$ th term
(c) twice the p th term
(d) twice the q th term

(**c**) Let a is first term and d is common difference of AP.

$a_{p+q} = a + (p+q-1)d$

and $a_{p-q} = a + (p-q-1)d$

Sum of $(p+q)$ th and $(p-q)$ th terms

$= a_{p+q} + a_{p-q} = 2a + (2p-2)d$
$= 2(a + (p-1)d) = 2a_p$
= twice of p th term

25. If A is a square matrix of order $n > 1$, then which one of the following is correct?

(a) $\det(-A) = \det A$
(b) $\det(-A) = (-1)^n \det A$
(c) $\det(-A) = -\det A$
(d) $\det(-A) = n \det A$

Sol. (b) We know that if A is a square matrix of order $n > 1$, then $\det(-A) = (-1)^n \det A$

For example If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$,

then $-A = \begin{bmatrix} -2 & -3 \\ -4 & -5 \end{bmatrix}$

$\therefore \det A = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2$ ...(i)

and $\det(-A) = \begin{vmatrix} -2 & -3 \\ -4 & -5 \end{vmatrix} = 10 - 12 = -2$

$= (-1)^2(-2)$ [$\because$ here $n = 2$]

$= (-1)^2 \det A$ [from Eq. (i)]

if $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 4 & 3 & -2 \end{bmatrix}$

Then, $-A = \begin{bmatrix} -1 & -2 & -3 \\ -3 & -1 & 0 \\ -4 & -3 & 2 \end{bmatrix}$

$\therefore \det A = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 4 & 3 & -2 \end{vmatrix}$

$= 1(-2-0) - 2(-6-0) + 3(9-4)$
$= -2 + 12 + 15 = 25$

and $\det(-A) = \begin{vmatrix} -1 & -2 & -3 \\ -3 & -1 & 0 \\ -4 & -3 & 2 \end{vmatrix}$

$= -1(-2-0) + 2(-6-0) - 3(9-4)$
$= 2 - 12 - 15 = -25$
$= (-1)^3 25$ [here $n = 3$]
$= (-1)^3 \det A$ [from Eq. (i)]

26. What is the least value of $25\ \text{cosec}^2 x + 36 \sec^2 x$?

(a) 1
(b) 11
(c) 120
(d) 121

(**d**) Given, $25\ \text{cosec}^2 x + 36 \sec^2 x$

$= 25(1 + \cot^2 x) + 36(1 + \tan^2 x)$

$= 25 + 25\cot^2 x + 36 + 36 \tan^2 x$

$= 25 + 36 + 25 \cot^2 x + 36 \tan^2 x$

$= 61 + (5 \cot x - 6 \tan x)^2 + 2 \times 5 \times 6$

$\geq 61 + 60 = 121$ [$\because$ minimum value of $(5 \cot x - 6 \tan x)^2 = 0$]

$\therefore$ Minimum value of

$25\ \text{cosec}^2 x + 36 \sec^2 x = 121$

Directions (Q. Nos. 27 and 28) *Read the information carefully and answer the given questions.*

Let A and B be 3×3 matrices with $\det A = 4$ and $\det B = 3$.

27. What is $\det(2AB)$ equal to?

(a) 96
(b) 72
(c) 48
(d) 36

(**a**) A and B be (3×3) matrices with $\det A = 4$ and $\det B = 3$

We know that,

$\det(KAB) = K^n \det(A) \times \det(B)$

where, n is the order or A and B, K is a real number.

$\therefore \det(2AB) = (2)^3 \det A \times \det B$

[$\because n = 3$ and $k = 2$]

$= 8 \times 4 \times 3$
$= 96$

28. What is $\det(3AB^{-1})$ equal to?

(a) 12 (b) 18
(c) 36 (d) 48

(**c**) A and B be (3×3) matrices with $\det A = 4$ and $\det B = 3$

We know that,

$\det(KAB^{-1}) = K^n \det(A) \times \dfrac{1}{\det(B)}$,

where n is the order of A and B, K is a real number]

$\therefore \det(3AB^{-1}) = (3)^3 \det(A) \times \dfrac{1}{\det B}$

$= 27 \times 4 \times \dfrac{1}{3}$

$= 36$

Directions (Q. Nos. 29 and 30) *Read the information carefully and answer the given questions.*

A complex number is given by

$z = \dfrac{1 + 2i}{1 - (1 - i)^2}$.

29. What is the modulus of z?

(a) 4 (b) 2 (c) 1 (d) $\dfrac{1}{2}$

(**c**) We have, $z = \dfrac{1+2i}{1-(1-i)^2}$

$z = \dfrac{1+2i}{1-(1-1-2i)}$

$= \dfrac{1+2i}{1+2i} = 1$

$\therefore |z| = 1$

30. What is the principal argument of z?

(a) 0 (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) π

(**a**) $\arg(z) = \tan^{-1}\left(\dfrac{\text{Im}(z)}{\text{Re}(z)}\right)$

$= \tan^{-1}\left(\dfrac{0}{1}\right) = \tan^{-1} 0 = 0$

31. What is the value of

$\dfrac{\sin 34° \cos 236° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$?

(a) –2 (b) –1 (c) 2 (d) 1

(**a**) We have,

$\dfrac{\sin 34° \cos 236° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$

$= \dfrac{\sin 34° \cos(180° + 56°) - \sin 56° \sin(90° + 34°)}{\cos 28° \cos 88° + \cos(90° + 88°) \sin(180° + 28°)}$

$= \dfrac{-\sin 34° \cos 56° - \sin 56° \cos 34°}{\cos 28° \cos 88° + \sin 88° \sin 28°}$

$= \dfrac{-\sin(56° + 34°)}{\cos(88° - 28°)} = \dfrac{-\sin 90°}{\cos 60°}$

$= \dfrac{-1}{\frac{1}{2}} = -2$

32. $\tan 54°$ can be expressed as

(a) $\dfrac{\sin 9° + \cos 9°}{\sin 9° - \cos 9°}$ (b) $\dfrac{\sin 9° - \cos 9°}{\sin 9° + \cos 9°}$

(c) $\dfrac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$ (d) $\dfrac{\sin 36°}{\cos 36°}$

(**c**) We have, $\tan 54° = \tan(45° + 9°)$

$= \dfrac{\tan 45° + \tan 9°}{1 - \tan 45° \tan 9°} = \dfrac{1 + \tan 9°}{1 - \tan 9°}$

$= \dfrac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$

Directions (Q. Nos. 33-35) *Read the given information carefully and answer the given questions.*

If $p = X\cos\theta - Y\sin\theta$,

$q = X\sin\theta + Y\cos\theta$ and

$p^2 + 4pq + q^2 = AX^2 + BY^2$,

$0 \le \theta \le \frac{\pi}{2}$.

33. What is the value of θ?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(c) We have,

$p = X\cos\theta - Y\sin\theta$ ...(i)

$q = X\sin\theta + Y\cos\theta$ ...(ii)

and $p^2 + 4pq + q^2 = AX^2 + BY^2$ ...(iii)

From Eqs. (i) and (ii), we get

$p^2 + q^2 = (X\cos\theta - Y\sin\theta)^2 + (X\sin\theta + Y\cos\theta)^2$

$\Rightarrow p^2 + q^2 = X^2 + Y^2$

and $pq = (X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta)$

$\Rightarrow pq = (X^2 - Y^2)\sin\theta\cos\theta + XY\cos 2\theta$

$\therefore p^2 + 4pq + q^2 = X^2 + Y^2 + 2(X^2 - Y^2)\sin 2\theta + 4XY\cos 2\theta$

Given, $p^2 + 4pq + q^2 = AX^2 + BY^2$

$\therefore X^2 + Y^2 + 2(X^2 - Y^2)\sin 2\theta + 4XY\cos 2\theta = AX^2 + 2BY^2$

Coefficient of $XY = 0$

$\therefore \cos 2\theta = 0$

$\Rightarrow 2\theta = \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{4}$

34. What is the value of A?

(a) 4

(b) 3

(c) 2

(d) 1

(b) $X^2 + Y^2 + 2(X^2 - Y^2)\sin\frac{\pi}{2} = AX^2 + BY^2$

$\Rightarrow X^2 + Y^2 + 2X^2 - 2Y^2 = AX^2 + BY^2$

$\Rightarrow 3X^2 - Y^2 = AX^2 + BY^2$

$\therefore A = 3, B = -1$

35. What is the value of B?

(a) –1 (b) 0

(c) 1 (d) 2

(a) $B = -1$

Directions (Q. Nos. 36 and 37) *Read the given information carefully and answer the given questions.*

It is given that $\cos(\theta - \alpha) = a$, $\cos(\theta - \beta) = b$.

36. What is $\cos(\alpha - \beta)$ equal to ?

(a) $ab + \sqrt{1-a^2}\sqrt{1-b^2}$

(b) $ab - \sqrt{1-a^2}\sqrt{1-b^2}$

(c) $a\sqrt{1-a^2} - b\sqrt{1-a^2}$

(d) $a\sqrt{1-b^2} + b\sqrt{1-a^2}$

(a) Given $\cos(\theta - \alpha) = a$

$\cos(\theta - \beta) = b$

$\cos(\alpha - \beta) = \cos\{(\theta - \beta) - (\theta - \alpha)\}$

$= \cos(\theta - \beta)\cos(\theta - \alpha) + \sin(\theta - \beta)\sin(\theta - \alpha)$

$= ab + \sqrt{1-a^2}\sqrt{1-b^2}$

37. What is $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$ equal to?

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $b^2 - a^2$ (d) $-(a^2 + b^2)$

(a) $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$

$= 1 - \cos^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$

$= 1 - (ab + \sqrt{1-a^2}\sqrt{1-b^2})^2 + 2ab(ab + \sqrt{1-a^2}\sqrt{1-b^2})$

$= 1 - [a^2b^2 + (1-a^2)(1-b^2) + 2ab\sqrt{1-a^2}\sqrt{1-b^2}] + 2a^2b^2 + 2ab(\sqrt{1-a^2})(\sqrt{(1-b^2})$

$= 1 - a^2b^2 - 1 + a^2 + b^2 - a^2b^2 - 2ab\sqrt{1-a^2}\sqrt{1-b^2} + 2a^2b^2 + 2ab\sqrt{1-a^2}\sqrt{1-b^2}$

$= a^2 + b^2$

38. If $\sin\alpha + \cos\alpha = p$, then what is $\cos^2(2\alpha)$ equal to?

(a) p^2 (b) $p^2 - 1$

(c) $p^2(2 - p^2)$ (d) $p^2 + 1$

(c) We have, $\sin\alpha + \cos\alpha = p$

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = p^2$

$\Rightarrow 1 + \sin 2\alpha = p^2$

$\Rightarrow \sin 2\alpha = p^2 - 1$

$\Rightarrow \sin^2 2\alpha = (p^2 - 1)^2$

$\Rightarrow 1 - \cos^2 2\alpha = p^4 - 2p^2 + 1$

$\Rightarrow \cos^2 2\alpha = 2p^2 - p^4$

$\therefore \cos^2 2\alpha = p^2(2 - p^2)$

39. What is the value of $\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{4} - \frac{\pi}{2}$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) 0

(d) We have,

$\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{4} - \frac{\pi}{2}$

$= \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} - \frac{\pi}{2}$ $\left[\because \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$

$= \frac{\pi}{2} - \frac{\pi}{2} = 0$ $\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

40. If $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-q^2}{1+q^2} = \tan^{-1}\frac{2x}{1-x^2}$, then what is x equal to?

(a) $\frac{p+q}{1+pq}$ (b) $\frac{p-q}{1+pq}$

(c) $\frac{pq}{1+pq}$ (d) $\frac{p+q}{1-pq}$

(b) Given,

$\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}p - 2\tan^{-1}q = 2\tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}x$

$\therefore x = \frac{p-q}{1+pq}$

41. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then what is the value of $(\theta + \phi)$?

(a) 0 (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(c) Given, $\tan\theta = \frac{1}{2}$, $\tan\phi = \frac{1}{3}$

$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

$\Rightarrow \tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}$

$\Rightarrow \tan(\theta + \phi) = \frac{3+2}{6-1} = \frac{5}{5} = 1$

$\Rightarrow \tan(\theta + \phi) = 1$

$\Rightarrow \theta + \phi = \tan^{-1}1 = \frac{\pi}{4}$

42. If $\cos A = \frac{3}{4}$, then what is the value of $\sin\left(\frac{A}{2}\right)\sin\left(\frac{3A}{2}\right)$?

(a) $\frac{5}{8}$ (b) $\frac{5}{16}$ (c) $\frac{5}{24}$ (d) $\frac{7}{32}$

(*b*) Given, $\cos A = \frac{3}{4}$

Now, $\sin\frac{A}{2}\sin\frac{3A}{2}$

$= \frac{1}{2}\left(2\sin\frac{A}{2}\sin\frac{3A}{2}\right)$

$= \frac{1}{2}\left[\cos\left(\frac{A}{2}-\frac{3A}{2}\right) - \cos\left(\frac{A}{2}+\frac{3A}{2}\right)\right]$

$= \frac{1}{2}[\cos A - \cos 2A]$

$= \frac{1}{2}(\cos A - 2\cos^2 A + 1)$

$= \frac{1}{2}\left[\frac{3}{4} - 2\times\frac{9}{16} + 1\right]$

$= \frac{1}{2}\left[\frac{3}{4} - \frac{9}{8} + 1\right] = \frac{1}{2}\left[\frac{6-9+8}{8}\right] = \frac{5}{16}$

43. What is the value of $\tan 75° + \cot 75°$?

(a) 2 (b) 4
(c) $2\sqrt{3}$ (d) $4\sqrt{3}$

(*b*) We have, $\tan 75° + \cot 75°$

$= \tan(90° - 15°) + \cot(90° - 15°)$

$= \cot 15° + \tan 15°$

$= \frac{\cos 15°}{\sin 15°} + \frac{\sin 15°}{\cos 15°}$

$= \frac{\cos^2 15° + \sin^2 15°}{\sin 15° \cos 15°}$

$= \frac{1}{\sin 15° \cos 15°}$

$= \frac{2}{2\sin 15° \cos 15°}$

$= \frac{2}{\sin 30°} = \frac{2}{\frac{1}{2}} = 4$

44. What is the value of $\cos 46° \cos 47° \cos 48° \cos 49° \cos 50° \ldots\ldots \cos 135°$?

(a) –1 (b) 0
(c) 1 (d) Greater than 1

(*b*) We have,
$\cos 46° \cos 47° \cos 48° \cos 49° \cos 50° \ldots \cos 90° \ldots \cos 135°$
$= 0$ $[\because \cos 90° = 0]$

45. If $\sin 2\theta = \cos 3\theta$, where $0 < \theta < \frac{\pi}{2}$, then what is $\sin\theta$ equal to ?

(a) $\frac{\sqrt{5}+1}{4}$ (b) $\frac{\sqrt{5}-1}{4}$
(c) $\frac{\sqrt{5}+1}{16}$ (d) $\frac{\sqrt{5}-1}{16}$

(*b*) Given, $\sin 2\theta = \cos 3\theta$

$\Rightarrow 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 2\sin\theta = 4\cos^2\theta - 3$ $[\because \cos\theta \neq 0]$

$\Rightarrow 2\sin\theta = 4(1-\sin^2\theta) - 3$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-2 \pm \sqrt{4+16}}{2\times 4}$

$\Rightarrow \sin\theta = \frac{-2 \pm 2\sqrt{5}}{2\times 4} = \frac{-1 \pm \sqrt{5}}{4}$

$\therefore \sin\theta = \frac{\sqrt{5}-1}{4}$ $\left[\because \theta \in \left[0, \frac{\pi}{2}\right]\right]$

46. If the roots of the equation $x^2 + px + q = 0$ are $\tan 19°$ and $\tan 26°$, then which one of the following is correct ?

(a) $q - p = 1$ (b) $p - q = 1$
(c) $p + q = 2$ (d) $p + q = 3$

(*a*) Given, $\tan 19°$ and $\tan 26°$ are roots of $x^2 + px + q = 0$.

$\therefore \tan 19° + \tan 26° = -p$

$\tan 19° \cdot \tan 26° = q$

$\tan(19° + 26°) = \frac{\tan 19° + \tan 26°}{1 - \tan 19° \tan 26°}$

$\Rightarrow \tan 45° = \frac{-p}{1-q} \Rightarrow 1 = \frac{-p}{1-q}$

$\Rightarrow 1 - q = -p$

$\therefore q - p = 1$

47. What is the fourth term of an AP of n terms whose sum is $n(n+1)$?

(a) 6 (b) 8
(c) 12 (d) 20

(*b*) Given,
Sum of n terms of an AP
i.e. $S_n = n(n+1)$
$a_4 = S_4 - S_3$ $[\because a_n = S_n - S_{n-1}]$
$a_4 = 4(4+1) - 3(3+1)$
$a_4 = 20 - 12 = 8$
$\therefore$ Fourth term of an AP = 8

48. What is $-\sec^2\alpha\sec^2\beta$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

(*a*) We have,
$(1 + \tan\alpha\tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha\sec^2\beta$

$= 1 + \tan^2\alpha\tan^2\beta + 2\tan\alpha\tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha\tan\beta - \sec^2\alpha\sec^2\beta$

$= 1 + \tan^2\alpha\tan^2\beta + \tan^2\alpha + \tan^2\beta - \sec^2\alpha\sec^2\beta$

$= (1 + \tan^2\alpha)(1 + \tan^2\beta) - \sec^2\alpha\sec^2\beta$

$= \sec^2\alpha\sec^2\beta - \sec^2\alpha\sec^2\beta = 0$

49. If $p = \text{cosec}\,\theta - \cot\theta$ and $q = (\text{cosec}\,\theta + \cot\theta)^{-1}$, then which one of the following is correct?

(a) $pq = 1$ (b) $p = q$
(c) $p + q = 1$ (d) $p + q = 0$

(*b*) Given, $p = \text{cosec}\,\theta - \cot\theta$

$q = (\text{cosec}\,\theta + \cot\theta)^{-1}$

$\Rightarrow q = \left(\frac{1}{\text{cosec}\,\theta + \cot\theta}\right)\left(\frac{\text{cosec}\,\theta - \cot\theta}{\text{cosec}\,\theta - \cot\theta}\right)$

$\Rightarrow q = \text{cosec}\,\theta - \cot\theta$

$\therefore q = p$

50. If the angles of a triangle ABC are in the ratio 1 : 2 : 3, then the corresponding sides are in the ratio

(a) 1 : 2 : 3 (b) 3 : 2 : 1
(c) $1 : \sqrt{3} : 2$ (d) $1 : \sqrt{3} : \sqrt{2}$

(*c*) We have, angle of triangle ABC are in the ratio 1 : 2 : 3

$\therefore x + 2x + 3x = 180°$

$\Rightarrow x = 30°$

$\therefore$ Angles of triangle are 30°, 60°, 90°.

We know that, sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{c}{\sin 90°}$

$\Rightarrow \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1}$

$\therefore a : b : c = 1 : \sqrt{3} : 2$

51. Consider the following statements

1. For an equation of a line, $x\cos\theta + y\sin\theta = p$, in normal form, the length of the perpendicular from the point (α, β) to the line is $|\alpha\cos\theta + \beta\sin\theta + p|$.

The length of the perpendicular from the point (α, β) to the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\left|\frac{a\alpha + b\beta - ab}{\sqrt{a^2 + b^2}}\right|$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) **1.** Equation of line $x\cos\theta + y\sin\theta = p$

Perpendicular distance from (α, β) to the given line is

$\left|\frac{\alpha\cos\theta + \beta\sin\theta - p}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$= (\alpha\cos\theta + \beta\sin\theta - p)$

Hence, statement 1 is incorrect.

2. Length of the perpendicular from the point (α, β) to the line $\frac{x}{a} + \frac{y}{b} = 1$ is

$$\left| \frac{\frac{\alpha}{a} + \frac{\beta}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| = \left| \frac{\alpha b + a\beta - ab}{\sqrt{a^2 + b^2}} \right|$$

Hence, statement 2 is incorrect.

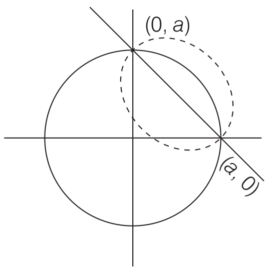
$\therefore$ Neither 1 nor 2.

52. A circle is drawn on the chord of a circle $x^2 + y^2 = a^2$ as diameter. The chord lies on the line $x + y = a$. What is the equation of the circle?

(a) $x^2 + y^2 - ax - ay + a^2 = 0$
(b) $x^2 + y^2 - ax - ay = 0$
(c) $x^2 + y^2 + ax + ay = 0$
(d) $x^2 + y^2 + ax + ay - 2a^2 = 0$

(**b**) Given, equation of circle is $x^2 + y^2 = a^2$.

$x + y = a$ is chord of a circle.



$\therefore$ End points of diameter of required circle is $(a, 0)$ and $(0, a)$.

$\therefore$ Equation of circle is

$x(x - a) + y(y - a) = 0$

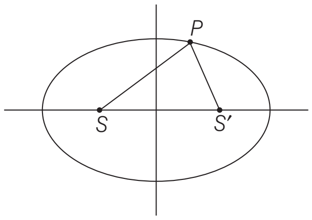
$\Rightarrow x^2 + y^2 - ax - ay = 0$

53. The sum of the focal distances of a point on an ellipse is constant and equal to

(a) length of minor axis
(b) length of major axis
(c) length of latusrectum
(d) sum of the lengths of semi major and semi minor axes

(**b**) The sum of the focal distance of a point on a ellipse is constant and equal to the length of major axis.

We know that, $PS + PS' = 2a$



54. The equation $2x^2 - 3y^2 - 6 = 0$ represents

(a) a circle
(b) a parabola
(c) an ellipse
(d) a hyperbola

(**d**) Given, $2x^2 - 3y^2 - 6 = 0$

$\Rightarrow \quad 2x^2 - 3y^2 = 6$

$\Rightarrow \quad \frac{x^2}{3} - \frac{y^2}{2} = 1$

Which represents the equation of a hyperbola.

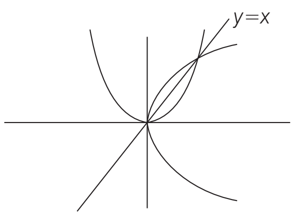
55. The two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ intersect

(a) at two points on the line $y = x$
(b) only at the origin
(c) at three points one of which lies on $y + x = 0$
(d) only at $(4a, 4a)$

(**a**) Given, $y^2 = 4ax$

and $\quad x^2 = 4ay$

The graph of given curve is clearly from graph the given curve is intersect at two points on the line $y = x$



56. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. What is the value of c?

(a) 2 (b) –2
(c) 4 (d) –4

(**d**) The points (1, 3) and (5, 1) are two opposite vertex of rectangle. The other two vertices lie on the line $y = 2x + c$.

$\therefore$ The mid point of vertices lie on the line i.e. $\left(\frac{1+5}{2}, \frac{3+1}{2}\right) \equiv (3, 2)$ lie on the line

$y = 2x + c$

$\therefore \quad 2 = 2(3) + c$

$\Rightarrow \quad c = -4$

57. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?

(a) 1 (b) 3
(c) 6 (d) $\frac{1}{2}$

(**c**) The lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent.

$$\therefore \quad \begin{vmatrix} 3 & 4 & -1 \\ 1 & -1 & -5 \\ 5 & b & -3 \end{vmatrix} = 0$$

$\Rightarrow 3(3 + 5b) - 4(-3 + 25) - 1(b + 5) = 0$

$\Rightarrow \quad 9 + 15b + 12 - 100 - b - 5 = 0$

$\Rightarrow \quad 14b = 84$

$\therefore \quad b = 6$

58. What is the equation of the straight line which is perpendicular to $y = x$ and passes through (3, 2)?

(a) $x - y = 5$
(b) $x + y = 5$
(c) $x + y = 1$
(d) $x - y = 1$

(**b**) Equation of line perpendicular to $y = x$ is $x + y = \lambda$.

Since, this line is passes through (3, 2)

$\therefore \quad 3 + 2 = \lambda \Rightarrow \lambda = 5$

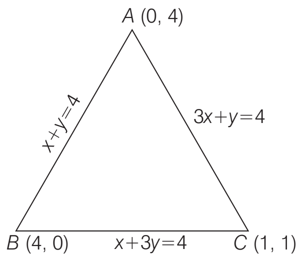
Hence, equation of required line is $x + y = 5$.

59. The straight lines $x + y - 4 = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle, which is

(a) isosceles (b) right angled
(c) equilateral (d) scalene

(**a**) Given, equation of line

$x + y = 4 \quad$ ...(i)
$3x + y = 4 \quad$ ...(ii)
$x + 3y = 4 \quad$ ...(iii)



On solving Eqs. (i) and (ii), we get

$x = 0, y = 4, A = (0, 4)$

On solving Eqs. (i) and (iii), we get

$x = 4, y = 0, B = (4, 0)$

On solving Eqs. (ii) and (iii), we get

$x = 1, y = 1, C = (1, 1)$

Clearly, $AC = BC$

$\therefore$ Triangle is an isosceles.

60. The circle $x^2 + y^2 + 4x - 7y + 12 = 0$, cuts an intercept on Y-axis equal to

(a) 1 (b) 3
(c) 4 (d) 7

(**a**) Given, $x^2 + y^2 + 4x - 7y + 12 = 0$

For intercept on Y-axis put $x = 0$, we get

$$y^2 - 7y + 12 = 0$$
$$(y-4)(y-3) = 0$$
$$y = 3, y = 4$$

Length of intercept on Y-axis

$$= |y_2 - y_1| = |3 - 4| = 1$$

61. The centroid of the triangle with vertices $A(2, -3, 3)$, $B(5, -3, -4)$ and $C(2, -3, -2)$ is the point

(a) $(-3, 3, -1)$
(b) $(3, -3, -1)$
(c) $(3, 1, -3)$
(d) $(-3, -1, -3)$

(**b**) Given vertices of triangle ABC are

$A(2, -3, 3)$, $B(5, -3, -4)$

and $C(2, -3, -2)$

$\therefore$ Centroid of ΔABC

$$= \left(\frac{2+5+2}{3}, \frac{-3-3-3}{3}, \frac{3-4-2}{3}\right)$$

$$= \left(\frac{9}{3}, \frac{-9}{3}, \frac{-3}{3}\right) = (3, -3, -1)$$

62. What is the radius of the sphere $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$?

(a) 5 (b) 2
(c) 7 (d) 3

(**c**) Given, equation of sphere

$x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$

On comparing with

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, we get

$2u = -6, 2v = 8, 2w = -10, d = 1$

$\Rightarrow u = -3, v = 4, w = -5, d = 1$

$\therefore$ Radius of sphere $= \sqrt{u^2 + v^2 + w^2 - d}$

$$= \sqrt{(-3)^2 + (4)^2 + (-5)^2 - 1}$$
$$= \sqrt{9 + 16 + 25 - 1} = \sqrt{49} = 7$$

63. The equation of the plane passing through the intersection of the planes $2x + y + 2z = 9$, $4x - 5y - 4z = 1$ and the point $(3, 2, 1)$ is

(a) $10x - 2y + 2z = 28$
(b) $10x + 2y + 2z = 28$
(c) $10x + 2y - 2z = 28$
(d) $10x - 2y - 2z = 24$

(**a**) Equation of the plane passing through the intersection of plane

$2x + y + 2z = 9$, $4x - 5y - 4z = 1$ is

$$(2x + y + 2z - 9) + \lambda(4x - 5y - 4z - 1) = 0 \quad \text{...(i)}$$

Since, plane (i) passes through the point (3, 2, 1)

$$\therefore (2 \times 3 + 2 + 2 \times 1 - 9) + \lambda(4 \times 3 - 5 \times 2 - 4 \times 1 - 1) = 0$$

$$\Rightarrow 1 + \lambda(-3) = 0$$
$$\Rightarrow \lambda = \frac{1}{3}$$

On putting $\lambda = \frac{1}{3}$ in Eq. (i), we get

$$(2x + y + 2z - 9) + \frac{1}{3}(4x - 5y - 4z - 1) = 0$$

$$\Rightarrow 6x + 3y + 6z - 27 + 4x - 5y - 4z - 1 = 0$$

$$\Rightarrow 10x - 2y + 2z - 28 = 0$$

$$\therefore 10x - 2y + 2z = 28$$

64. The distance between the parallel planes $4x - 2y + 4z + 9 = 0$ and $8x - 4y + 8z + 21 = 0$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{7}{4}$

(**a**) Given equation of planes

$$4x - 2y + 4z + 9 = 0 \quad \text{...(i)}$$

and $8x - 4y + 8z + 21 = 0$

$$\Rightarrow 4x - 2y + 4z + \frac{21}{2} = 0 \quad \text{...(ii)}$$

Distance between parallel planes (i) and (ii)

$$= \frac{\left|\frac{21}{2} - 9\right|}{\sqrt{(4)^2 + (-2)^2 + (4)^2}}$$

$$= \frac{\frac{3}{2}}{\sqrt{16 + 4 + 16}} = \frac{\frac{3}{2}}{6} = \frac{1}{4}$$

65. What are the direction cosines of Z-axis?

(a) <1, 1, 1> (b) <1, 0, 0>
(c) <0, 1, 0> (d) <0, 0, 1>

(**d**) Direction cosines of Z-axis are $< \cos 90°, \cos 90°, \cos 0° > < 0, 0, 1 >$

66. If $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, then what is $(\mathbf{b} - \mathbf{a}) \cdot (3\mathbf{a} + \mathbf{b})$ equal to ?

(a) 106 (b) –106
(c) 53 (d) –53

(**b**) We have, $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$$\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$\therefore \mathbf{b} - \mathbf{a} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$$

and $3\mathbf{a} + \mathbf{b} = 3(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

$$= 5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$$

$$\therefore (\mathbf{b} - \mathbf{a}) \cdot (3\mathbf{a} + \mathbf{b}) = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}) \cdot (5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$$

$$= 5 - 15 - 96 = -106$$

67. If the position vectors of points **A** and **B** are $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ respectively, then what is the length of **AB** ?

(a) $\sqrt{14}$ (b) $\sqrt{29}$
(c) $\sqrt{43}$ (d) $\sqrt{53}$

(**d**) We have , $\mathbf{OA} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\mathbf{OB} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

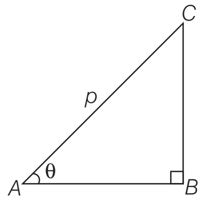
$\therefore |\mathbf{AB}| = |\mathbf{OB} - \mathbf{OA}|$

$$= |(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})|$$
$$= |(-\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|$$
$$= \sqrt{(-1)^2 + (6)^2 + (-4)^2}$$
$$= \sqrt{1 + 36 + 16} = \sqrt{53}$$

68. If in a right angled triangle ABC, hypotenuse $AC = p$, then what is $\mathbf{AB} \cdot \mathbf{AC} + \mathbf{BC} \cdot \mathbf{BA} + \mathbf{CA} \cdot \mathbf{CB}$ equal to?

(a) p^2 (b) $2p^2$
(c) $\frac{p^2}{2}$ (d) p

(**a**) In right angled ΔABC, we have $\angle ABC = 90°$



Let $\angle BAC = \theta$

Then, $\angle ACB = (90° - \theta)$

$\therefore \mathbf{AB} \cdot \mathbf{AC} + \mathbf{BC} \cdot \mathbf{BA} + \mathbf{CA} \cdot \mathbf{CB}$

$$= |\mathbf{AB}||\mathbf{AC}| \cos\theta + |\mathbf{BC}||\mathbf{BA}| \cos 90° + |\mathbf{CA}||\mathbf{CB}| \cos(90° - \theta)$$
$$= |\mathbf{AB}||\mathbf{AB}| + 0 + |\mathbf{CB}||\mathbf{CB}|$$
$$= |\mathbf{AB}|^2 + |\mathbf{CB}|^2$$
$$= |\mathbf{AC}|^2 = p^2$$

69. The sine of the angle between vectors

$\mathbf{a} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ is

(a) $\frac{1}{\sqrt{26}}$ (b) $\frac{5}{\sqrt{26}}$
(c) $\frac{5}{26}$ (d) $\frac{1}{26}$

(**b**) Let θ be the angle between vectors **a** and **b**

$$\therefore \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Since,

$$\mathbf{a} \cdot \mathbf{b} = (2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$$
$$= 8 - 18 + 3 = -7$$
$$|\mathbf{a}| = \sqrt{2^2 + (-6)^2 + (-3)^2}$$
$$= \sqrt{49} = 7$$

$$|\mathbf{b}| = \sqrt{4^2 + 3^2 + (-1)^2} = \sqrt{26}$$

$$\therefore \quad \cos\theta = \frac{-7}{7 \times \sqrt{26}} = -\frac{1}{\sqrt{26}}$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} = \frac{5}{\sqrt{26}}$$

70. What is the value of λ for which the vectors $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $-2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ are perpendicular?

(a) 1 (b) 2
(c) 3 (d) 4

(*d*) Since, given vectors are perpendicular.

$$\therefore (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 10\hat{\mathbf{k}}) = 0$$

$$\Rightarrow -6 + 4\lambda - 10 = 0$$

$$\Rightarrow \quad 4\lambda - 16 = 0$$

$$\therefore \quad \lambda = 4$$

71. What is the derivative of $\sec^2(\tan^{-1} x)$ with respect to x?

(a) $2x$ (b) $x^2 + 1$
(c) $x + 1$ (d) x^2

(*a*) Let $y = \sec^2(\tan^{-1} x)$

On differentiating both sides w.r.t x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\sec^2(\tan^{-1} x)$$

$$= 2\sec(\tan^{-1} x) \cdot \sec(\tan^{-1} x)$$

$$\tan(\tan^{-1} x)\frac{d}{dx}(\tan^{-1} x)$$

$$= 2\sec^2(\tan^{-1} x) \cdot x \cdot \frac{1}{1+x^2}$$

$$= 2(1+\tan^2(\tan^{-1} x)) \cdot \frac{x}{1+x^2}$$

$$= 2(1+x^2) \cdot \frac{x}{1+x^2} = 2x$$

72. If $f(x) = \log_{10}(1+x)$, then what is $4f(4) + 5f(1) - \log_{10} 2$ equal to?

(a) 0
(b) 1
(c) 2
(d) 4

(*d*) We have, $f(x) = \log_{10}(1+x)$

$$\therefore 4f(4) = 4\log_{10}(1+4) = 4\log_{10} 5$$

$$5f(1) = 5\log_{10}(1+1) = 5\log_{10} 2$$

$$\therefore 4f(4) + 5f(1) - \log_{10} 2$$

$$= 4\log_{10} 5 + 5\log_{10} 2 - \log_{10} 2$$

$$= 4\log_{10} 5 + 4\log_{10} 2$$

$$= 4(\log_{10} 5 \times \log_{10} 2)$$

$$= 4\log_{10}(5 \times 2)$$

$$= 4\log_{10} 10 = 4 \times 1 = 4$$

73. A function f defined by $f(x) = \ln(\sqrt{x^2+1} - x)$ is

(a) an even function
(b) an odd function
(c) both even and odd function
(d) neither even nor odd function

(*b*) We have, $f(x) = \log(\sqrt{x^2+1} - x)$

$$\therefore \quad f(-x) = \log(\sqrt{x^2+1} + x)$$

$$= \log\left(\frac{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}{\sqrt{x^2+1}-x}\right)$$

$$= \log\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}-x}\right)$$

$$= \log\left(\frac{1}{\sqrt{x^2+1}-x}\right)$$

$$= -\log(\sqrt{x^2+1} - x)$$

$$= -f(x)$$

74. The domain of the function f defined by $f(x) = \log_x 10$ is

(a) $x > 10$
(b) $x > 0$ excluding $x = 10$
(c) $x \geq 10$
(d) $x > 0$ excluding $x = 1$

(*d*) We have, $f(x) = \log_x 10$

$$= \frac{\log 10}{\log x} = \frac{1}{\log x}$$

$\therefore f(x)$ is define when $x > 0$ and $x \neq 1$.

75. $\lim_{x \to 0} \frac{1-\cos^3 4x}{x^2}$ is equal to

(a) 0 (b) 12
(c) 24 (d) 36

(*c*) $\lim_{x \to 0} \frac{1-\cos^3 4x}{x^2} \left[\frac{0}{0} \text{ form}\right]$

On apply L′ Hospital rule we get

$$\lim_{x \to 0} \frac{-3\cos^2(4x)(-\sin 4x)(4)}{2x}$$

$$= \lim_{x \to 0} \frac{12\cos^2 4x \sin 4x}{2x} \left[\frac{0}{0} \text{ form}\right]$$

Again, apply's L′ Hospital rule, we get

$$\lim_{x \to 0} \frac{12[2\cos(4x)(-\sin 4x)(4)\sin 4x + \cos^2 4x(\cos 4x)(4)]}{2}$$

$$= \lim_{x \to 0} \frac{12[-8\cos 4x \sin^2 4x + 4\cos^3 4x]}{2}$$

$$= 6(-8 \times 0 + 4)$$

$$= 24$$

76. For $r > 0$, $f(r)$ is the ratio of perimeter to area of a circle of radius r. Then, $f(1) + f(2)$ is equal to

(a) 1 (b) 2
(c) 3 (d) 4

(*c*) We have,

$$f(r) = \frac{\text{Perimeter of a circle with radius } r}{\text{Area of a circle with radius } r}$$

$$\Rightarrow \quad f(r) = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

$$\therefore \quad f(1) = \frac{2}{1} = 2 \Rightarrow f(2) = \frac{2}{2} = 1$$

$$\therefore \quad f(1) + f(2) = 2 + 1 = 3$$

77. If $f(x) = 3^{1+x}$, then $f(x)\,f(y)\,f(z)$ is equal to

(a) $f(x+y+z)$ (b) $f(x+y+z+1)$
(c) $f(x+y+z+2)$ (d) $f(x+y+z+3)$

(*c*) We have, $f(x) = 3^{1+x}$

Similarly, $f(y) = 3^{1+y}$

and $f(z) = 3^{1+z}$

$$\therefore f(x)\,f(y)\,f(z) = 3^{1+x+1+y+1+z}$$

$$= 3^{1+2+x+y+z}$$

$$= f(2+x+y+z)$$

78. The number of real roots for the equation $x^2 + 9|x| + 20 = 0$ is

(a) zero
(b) one
(c) two
(d) three

(*a*) Given, $x^2 + 9|x| + 20 = 0$

$$\Rightarrow \quad x^2 + 9x + 20 = 0$$

$$\text{or} \quad x^2 - 9x + 20 = 0$$

$$\Rightarrow \quad x^2 + 4x + 5x + 20 = 0$$

$$\text{or} \quad x^2 - 4x - 5x + 20 = 0$$

$$\Rightarrow \quad x(x+4) + 5(x+4) = 0$$

$$\text{or} \quad x(x-4) - 5(x-4) = 0$$

$$\Rightarrow \quad (x+4)(x+5) = 0$$

$$\text{or} \quad (x-4)(x-5) = 0$$

$$\Rightarrow \quad x = -4, -5, \text{ or } 4, 5$$

But these values of x does not satisfy the given equation.

Hence, number of real roots of the given equation is zero.

79. If $f(x) = \sin(\cos x)$, then $f'(x)$ is equal to

(a) $\cos(\cos x)$
(b) $\sin(-\sin x)$
(c) $(\sin x)\cos(\cos x)$
(d) $(-\sin x)\cos(\cos x)$

(*d*) Given, $f(x) = \sin(\cos x)$

$$\Rightarrow f'(x) = \cos(\cos x)(-\sin x)$$

80. The domain of the function $f(x)=\sqrt{(2-x)(x-3)}$ is

(a) $(0,\infty)$ (b) $[0,\infty)$
(c) $[2,3]$ (d) $(2,3)$

(*c*) We have, $f(x)=\sqrt{(2-x)(x-3)}$

$f(x)$ will be define if $(2-x)(x-3)\geq 0$

$\Rightarrow \quad (x-2)(x-3)\leq 0$

$\therefore \quad 2\leq x\leq 3$

81. The solution of the differential equation

$$\frac{dy}{dx}=\cos(y-x)+1 \text{ is}$$

(a) $e^x[\sec(y-x)-\tan(y-x)]=c$
(b) $e^x[\sec(y-x)+\tan(y-x)]=c$
(c) $e^x\sec(y-x)\tan(y-x)=c$
(d) $e^x=c\sec(y-x)\tan(y-x)$

(*a*) Given, $\frac{dy}{dx}=\cos(y-x)+1$ ...(i)

Let $y-x=t$

$\Rightarrow \quad \frac{dy}{dx}-1=\frac{dt}{dx}\Rightarrow\frac{dy}{dx}=1+\frac{dt}{dx}$

From Eq. (i), $1+\frac{dt}{dx}=\cos t+1$

$\Rightarrow \quad \frac{dt}{dx}=\cos t$

$\Rightarrow \quad \sec t\, dt=dx$

On integrating both sides, we get

$$\int\sec t\, dt=\int dx$$

$\log(\sec t+\tan t)=x+a$

$\Rightarrow \quad \sec t+\tan t=e^{x+a}$

$\Rightarrow \quad \sec t+\tan t=e^x\cdot e^a$

$\Rightarrow \quad \frac{e^x}{\sec t+\tan t}=e^{-a}$

$\Rightarrow \quad \frac{e^x(\sec t-\tan t)}{(\sec t+\tan t)(\sec t-\tan t)}=e^{-a}$

$\Rightarrow \quad \frac{e^x(\sec t-\tan t)}{\sec^2 t-\tan^2 t}=e^{-a}$

$\Rightarrow \quad e^x(\sec t-\tan t)=e^{-a}$

$\Rightarrow \quad e^x[\sec(y-x)-\tan(y-x)]=e^{-a}$

$\therefore \quad e^x[\sec(y-x)-\tan(y-x)]=c,$ [where, $c=e^{-a}$]

82. $\int_0^{\frac{\pi}{2}}|\sin x-\cos x|dx$ is equal to

(a) 0 (b) $2(\sqrt{2}-1)$
(c) $2\sqrt{2}$ (d) $2(\sqrt{2}+1)$

(*b*) $\int_0^{\frac{\pi}{2}}|\sin x-\cos x|\,dx$

$=\int_0^{\frac{\pi}{4}}(\cos x-\sin x)\,dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)\,dx$

$=[\sin x+\cos x]_0^{\frac{\pi}{4}}+[-\cos x-\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$=\left[\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-(0+1)\right]+\left[(-0-1)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)\right]$

$=\frac{2}{\sqrt{2}}-1-1+\frac{2}{\sqrt{2}}=\frac{4}{\sqrt{2}}-2$

$=2\sqrt{2}-2=2(\sqrt{2}-1)$

83. If $y=a\cos 2x+b\sin 2x$, then

(a) $\frac{d^2y}{dx^2}+y=0$ (b) $\frac{d^2y}{dx^2}+2y=0$
(c) $\frac{d^2y}{dx^2}-4y=0$ (d) $\frac{d^2y}{dx^2}+4y=0$

(*d*) Given, $y=a\cos 2x+b\sin 2x$ ...(i)

$\Rightarrow \quad \frac{dy}{dx}=-2a\sin 2x+2b\cos 2x$

$\Rightarrow \quad \frac{d^2y}{dx^2}=-4a\cos 2x-4b\sin 2x$

$\Rightarrow \quad \frac{d^2y}{dx^2}=-4(a\cos 2x+b\sin 2x)$

$\Rightarrow \quad \frac{d^2y}{dx^2}=-4y$ [using Eq. (i)]

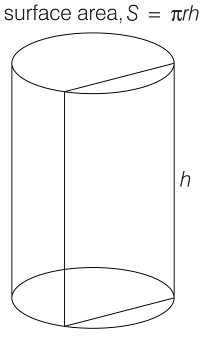
$\therefore \quad \frac{d^2y}{dx^2}+4y=0$

84. A given quantity of metal is to be cast into a half cylinder (i.e. with a rectangular base and semicircular ends). If the total surface area is to be minimum, then the ratio of the height of the half cylinder to the diameter of the semicircular ends is

(a) $\pi:(\pi+2)$ (b) $(\pi+2):\pi$
(c) 1 : 1 (d) None of these

(*a*) Let r be the radius and h be the height of the half cylinder,

Then, surface area, $S=\pi rh+\pi r^2+2rh$



$\therefore \quad \frac{dS}{dr}=\pi h+2\pi r+2h$

On putting $\frac{dS}{dr}=0$

$\Rightarrow \quad 2r=-\frac{(\pi h+2h)}{\pi}$

$\Rightarrow \quad 2r=\frac{-h(\pi+2)}{\pi}$

$\Rightarrow \quad \frac{2r}{h}=\frac{-(\pi+2)}{\pi}\Rightarrow\frac{h}{2r}=\frac{-\pi}{\pi+2}$

Neglecting – sign as r and h can not be negative.

$\therefore \quad \frac{h}{2r}=\frac{\pi}{\pi+2}$

85. $\int_0^{\frac{\pi}{2}}e^{\sin x}\cos x\,dx$ is equal to

(a) $e+1$ (b) $e-1$
(c) $e+2$ (d) e

(*b*) Let $I=\int_0^{\frac{\pi}{2}}e^{\sin x}\cos x\,dx$

Let $\sin x=t$

$\Rightarrow \quad \cos x\,dx=dt$

When $x=\frac{\pi}{2}, t=1$

$x=0, t=0$

$\therefore \quad I=\int_0^1 e^t\,dt=[e^t]_0^1$

$=e^1-e^0=e-1$

86. If $f(x)=\frac{x-2}{x+2}$, $x\neq -2$, then what is $f^{-1}(x)$ equal to ?

(a) $\frac{4(x+2)}{x-2}$ (b) $\frac{x+2}{4(x-2)}$
(c) $\frac{x+2}{x-2}$ (d) $\frac{2(1+x)}{1-x}$

(*d*) Given, $f(x)=\frac{x-2}{x+2}\Rightarrow y=\frac{x-2}{x+2}$

$\Rightarrow \quad x-2=xy+2y$

$\Rightarrow \quad x-xy=2y+2$

$\Rightarrow \quad x(1-y)=2y+2$

$\Rightarrow \quad x=\frac{2(y+1)}{1-y}$

$\Rightarrow \quad f^{-1}(y)=\frac{2(y+1)}{1-y}$

$\therefore \quad f^{-1}(x)=\frac{2(x+1)}{1-x}$

87. What is $\int\ln(x^2)dx$ equal to?

(a) $2x\ln(x)-2x+C$
(b) $\frac{2}{x}+C$
(c) $2x\ln(x)+C$
(d) $\frac{2\ln(x)}{x}-2x+C$

(*a*) Let $I=\int\ln(x^2)\,dx=\int 2\ln x\,dx$

$=\ln x\int 2\,dx-\int\left(\frac{d}{dx}(\ln x)\int(2\,dx)\right)dx$

$=\ln x\cdot 2x-\int\frac{1}{x}\cdot 2x\,dx$

$=2x\ln x-2x+C$

88. The minimum distance from the point (4, 2) to $y^2 = 8x$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) 2 (d) $3\sqrt{2}$

(**b**) Let (x, y) be any point on the curve $y^2 = 8x$.

Then, the distance between (x, y) and $(4, 2)$ is

$$D^2 = (x-4)^2 + (y-2)^2$$

$$\Rightarrow \quad D^2 = \left(\frac{y^2}{8} - 4\right)^2 + (y-2)^2 \quad \text{...(i)}$$

$$[\because y^2 = 8x]$$

$$\Rightarrow \frac{dD^2}{dy} = 2\left(\frac{y^2}{8} - 4\right)\left(\frac{2y}{8}\right) + 2(y-2)$$

$$= 2\left(\frac{2y^3}{64} - 4 \times \frac{2y}{8}\right) + 2(y-2)$$

$$= \frac{y^3}{16} - 2y + 2y - 4 = \frac{y^3}{16} - 4$$

$$\Rightarrow \frac{d^2D^2}{d^2y} = \frac{3y^2}{16}$$

On putting $\frac{dD^2}{dy} = 0 \Rightarrow \frac{y^3}{16} - 4 = 0$

$\Rightarrow \quad y^3 = 64 \Rightarrow y = 4$

At $\quad y = 4, \frac{d^2D^2}{d^2y} > 0$

So, it is point of minima.

$\therefore$ Minimum $D = \sqrt{(2-4)^2 + (4-2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

89. The differential equation of the system of circles touching the Y-axis at the origin is

(a) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$

(b) $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

(c) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(d) $x^2 - y^2 - 2xy\frac{dy}{dx} = 0$

(**c**) The system of circles touching the Y-axis at the origin is

$$(x-a)^2 + y^2 = a^2$$

$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = a^2$

$\Rightarrow \quad x^2 - 2ax + y^2 = 0 \quad \text{...(i)}$

On differentiating Eq. (i) w.r.t. x, we get

$$2x - 2a + 2yy' = 0$$

$\Rightarrow \quad x + yy' = a$

Put value of a in Eq. (i), we get

$$x^2 - 2(x + yy')x + y^2 = 0$$

$\Rightarrow \quad x^2 - 2x^2 - 2xyy' + y^2 = 0$

$\Rightarrow \quad -x^2 - 2xyy' + y^2 = 0$

$\therefore \quad x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

90. Consider the following in respect of the differential equation :

$$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 9x = x$$

1. The degree of the differential equation is 1.
2. The order of the differential equation is 2.

Which of the above statements is/are correct ?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**c**) The order of highest order derivative occuring in the differential equation is 2 and its degree is 1.

91. What is the general solution of the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$?

(a) $x^2 + y^2 = C$ (b) $x^2 - y^2 = C$
(c) $x^2 + y^2 = Cxy$ (d) $x + y = C$

(**a**) Given differential equation,

$$\frac{dy}{dx} + \frac{x}{y} = 0$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y}$

$\Rightarrow \quad ydy = -xdx$

Integrating both sides, we get

$$\frac{y^2}{2} = \frac{-x^2}{2} + C_1$$

$\therefore \quad x^2 + y^2 = C \quad$ [where, $C = 2C_1$]

92. The value of k which makes $f(x) = \begin{cases} \sin x, & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at $x = 0$, is

(a) 2 (b) 1
(c) –1 (d) 0

(**d**) Given, $f(x)$ is continuous at $x = 0$.

$\therefore \quad \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad \lim_{x \to 0} \sin x = k$

$\therefore \quad k = 0$

93. What is the minimum value of $a^2x + b^2y$ where $xy = c^2$?

(a) abc (b) $2abc$
(c) $3abc$ (d) $4abc$

(**b**) Let $z = a^2x + b^2y \quad \text{...(i)}$

Since, $xy = c^2$

$\Rightarrow \quad y = \frac{c^2}{x}$

On putting $y = \frac{c^2}{x}$ in Eq. (i), we get

$\Rightarrow \quad z = a^2x + b^2\left(\frac{c^2}{x}\right) \quad \text{...(ii)}$

On differentiability Eq. (ii) both sides, we get

$$\frac{dz}{dx} = a^2 - \frac{b^2c^2}{x^2} \quad \text{...(iii)}$$

$\Rightarrow \quad \frac{d^2z}{dx^2} = \frac{2b^2c^2}{x^3} \quad \text{...(iv)}$

For maxima and minima, we put $\frac{dz}{dx} = 0$

$\therefore \quad a^2 - \frac{b^2c^2}{x^2} = 0$

$\Rightarrow \quad \frac{b^2c^2}{x^2} = a^2$

$\Rightarrow \quad x = \pm\frac{bc}{a}$

At $\quad x = \frac{bc}{a}, \frac{d^2z}{dx^2} = \frac{2a^3}{bc} > 0$

$\Rightarrow$ Gives minimum value

At $\quad x = \frac{-bc}{a}, \frac{d^2z}{dx^2} = -\frac{2a^3}{bc} < 0$

Gives maximum value

$\therefore$ Minimum value of z at $x = \frac{bc}{a}$ is $abc + abc = 2abc$.

94. What is $\int e^{x\ln(a)}dx$ equal to?

(a) $\frac{a^x}{\ln(a)} + C$ (b) $\frac{e^x}{\ln(a)} + C$
(c) $\frac{e^x}{\ln(ae)} + C$ (d) $\frac{ae^x}{\ln(a)} + C$

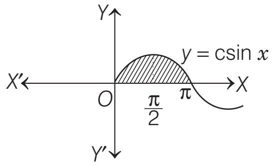
(**a**) Let $I = \int e^{x\ln a}dx = \int e^{\ln a^x}dx$

$$= \int a^x dx = \frac{a^x}{\ln a} + C$$

95. What is the area of one of the loops between the curve $y = c\sin x$ and X-axis ?

(a) c (b) $2c$
(c) $3c$ (d) $4c$

(**d**) $\therefore$ Required area $= 2\int_0^{\pi} c \sin x\, dx$



$= 2c\int_0^{\pi} \sin x\, dx = 2c[-\cos x]_0^{\pi}$

$= 2c\,[-(\cos\pi - \cos 0)]$

$= 2c\,(2) = 4c$ sq units

96. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then what is $(\cos\theta - \sin\theta)$ equal to ?

(a) $-\sqrt{2}\cos\theta$ (b) $-\sqrt{2}\sin\theta$
(c) $\sqrt{2}\sin\theta$ (d) $2\sin\theta$

(*c*) Given, $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \quad \sin\theta = (\sqrt{2}-1)\cos\theta$ ...(i)

Now, $\cos\theta - \sin\theta$

$= \cos\theta - (\sqrt{2}-1)\cos\theta$ [from Eq. (i)]

$= \cos\theta[1-(\sqrt{2}-1)] = \cos\theta\,[2-\sqrt{2}]$

$= \cos\theta \cdot \sqrt{2}\,(\sqrt{2}-1)$

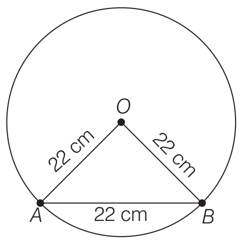
$= \sqrt{2}\sin\theta$ [using Eq. (i)]

97. In a circle of diameter 44 cm, the length of a chord is 22 cm. What is the length of minor arc of the chord?

(a) $\frac{484}{21}$ cm (b) $\frac{242}{21}$ cm

(c) $\frac{121}{21}$ cm (d) $\frac{44}{7}$ cm

(*a*) Given, diameter of a circle be 44 cm.



$\Rightarrow \quad 2r = 44$

$\Rightarrow \quad r = 22$

$\Rightarrow \Delta OAB$ is an equilateral triangle.

$\Rightarrow \quad \angle AOB = 60°$

$\therefore$ Length of minor arc

$$= \left(\frac{60°}{360°}\right) \times 2\pi \times 22$$

$$= \frac{1}{6} \times 2 \times \frac{22}{7} \times 22$$

$$= \frac{484}{21} \text{ cm}$$

98. If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$, then in which quadrant does θ lie?

(a) First
(b) Second
(c) Third
(d) Fourth

(*c*) We know that, if θ lies in third quadrant then, $\sin\theta < 0$ and $\tan\theta > 0$.

99. How many three digit even numbers can be formed using the digits 1, 2, 3, 4 and 5 when repetition of digits is not allowed?

(a) 36 (b) 30

(c) 24 (d) 12

(*c*) Here, unit digit can be filled by 2 or 4. so number of ways is 2. Since repetition is not allowed therefore hundred place and ten place can be fill in ${}^4C_2 \times 2$ ways

$\therefore$ Total number of three digits even number $= 4 \times 3 \times 2 = 24$

100. The angle of elevation of a tower of height h from a point A due South of it is x and from a point B due East of A is y. If $AB = z$, then which one of the following is correct ?

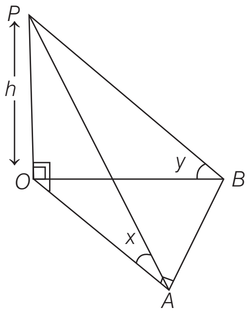
(a) $h^2(\cot^2 y - \cot^2 x) = z^2$

(b) $z^2(\cot^2 y - \cot^2 x) = h^2$

(c) $h^2(\tan^2 y - \tan^2 x) = z^2$

(d) $z^2(\tan^2 y - \tan^2 x) = h^2$

(*a*)



Here, OP be the tower,

$$OA = h\cot x$$

$$OB = h\cot y$$

In right-angled ΔOAB,

$$h^2\cot^2 y = z^2 + h^2\cot^2 x$$

$$\therefore \quad z^2 = h^2(\cot^2 y - \cot^2 x)$$

101. From a deck of cards, cards are taken out with replacement. What is the probability that the fourteenth card taken out is an ace?

(a) $\frac{1}{51}$ (b) $\frac{4}{51}$ (c) $\frac{1}{52}$ (d) $\frac{1}{13}$

(*d*) Total number of possible outcomes = 52

And number of favourable outcomes = 4

$\therefore$ Required probability $= \frac{4}{52} = \frac{1}{13}$

102. If A and B are two events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$, then what is $P(\overline{A \cup B})$ equal to ?

(a) 0.9 (b) 0.7

(c) 0.5 (d) 0.3

(*d*) $P(\overline{A \cup B}) = 1 - P(A \cup B)$

We have,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.6 - 0.4$$

$$= 1.1 - 0.4$$

$$= 0.7$$

$\therefore \quad P(\overline{A \cup B}) = 1 - 0.7 = 0.3$

103. A problem is given to three students A, B and C whose probabilities of solving the problem are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if they all solve the problem independently ?

(a) $\frac{29}{32}$ (b) $\frac{27}{32}$

(c) $\frac{25}{32}$ (d) $\frac{23}{32}$

(*a*) We have, $P(A) = \frac{1}{2}, P(\overline{A}) = \frac{1}{2}$

$P(B) = \frac{3}{4}, P(\overline{B}) = \frac{1}{4}$

and $P(C) = \frac{1}{4}, P(\overline{C}) = \frac{3}{4}$

$\therefore$ Required probability

$$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})$$

$$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{29}{32}$$

104. A pair of fair dice is rolled. What is the probability that the second dice lands on a higher value than does the first?

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{5}{12}$ (d) $\frac{5}{18}$

(*c*) Total number of possible outcomes = 36

Favourable outcomes
= (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 3), (2, 4), (2, 5), (2, 6) (3, 4),
(3, 5), (3, 6), (4, 5), (4, 6), (5, 6)

$\therefore$ Total number of favourable outcomes = 15

$\therefore$ Required probability $= \frac{15}{36} = \frac{5}{12}$

105. A fair coin is tossed and an unbiased dice is rolled together. What is the probability of getting a 2 or 4 or 6 along with head?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(*c*) Total number of possible outcomes $= 2 \times 6 = 12$

And favourable outcomes = (H, 2), (H, 4), (H, 6)

$\therefore$ Total number of possible outcomes = 3

$\therefore$ Required probability $= \frac{3}{12} = \frac{1}{4}$

106. If A, B and C are three events, then what is the probability that atleast two of these events occur together ?

(a) $P(A \cap B) + P(B \cap C) + P(C \cap A)$

(b) $P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$

(c) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

(d) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

(**c**) If A, B and C are three events, then atleast two events occur i.e.

$$(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C) \cup (A \cap B \cap C)$$

$\therefore$ Required probability

$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

107. If two variables X and Y are independent, then what is the correlation coefficient between them?

(a) 1 (b) –1

(c) 0 (d) None of these

(**c**) Correlation coefficient between two independent variables is zero.

108. Two independent events A and B are such that $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. If $P(B) < P(A)$, then what is $P(B)$ equal to ?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

(**b**) Given, $P(A \cup B) = \frac{2}{3}$

and $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{2}{3} = P(A) + P(B) - \frac{1}{6}$

$\Rightarrow P(A) + P(B) = \frac{2}{3} + \frac{1}{6}$

$\Rightarrow P(A) + P(B) = \frac{5}{6}$ ...(i)

And also, $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)\,P(B) = \frac{1}{6}$ ...(ii)

From Eqs. (i) and (ii), we get

$P(A)$ or $P(B) = \frac{1}{2}$ or $\frac{1}{3}$

Also, given $P(B) < P(A)$

$\therefore P(B) = \frac{1}{3}$

109. The mean of 100 observations is 50 and the standard deviation is 10. If 5 is subtracted from each observation and then it is divided by 4, then what will be the new mean and the new standard deviation respectively ?

(a) 45, 5

(b) 11.25, 1.25

(c) 11.25, 2.5

(d) 12.5, 2.5

(**c**) Given, mean $(\bar{x}) = 50$

The new mean $= \frac{50 - 5}{4}$

$= \frac{45}{4} = 11.25$

And standard deviation $(\sigma) = 10$

$\therefore$ The new standard deviation

$= \frac{10}{4}$

$= 2.5$

Since, addition and subtraction does not effect standard deviation.

110. If two fair dice are rolled, then what is the conditional probability that the first dice lands on 6, given that the sum of numbers on the dice is 8?

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) $\frac{1}{5}$

(d) $\frac{1}{6}$

(**c**) Let E_1 = Event of first dice on 6

E_2 = Event of the sum of numbers on dices 8

$\therefore$ Total number of sample space of two dices are rolled, $n(s) = 36$

Possible outcomes of E_1 (6, 2)

Possible outcomes of E_2 (2, 6) (3, 5) (4, 4) (5, 3) (6, 2)

$\therefore P(E_1 \cap E_2) = \frac{1}{36}$

and $P(E_2) = \frac{5}{36}$

$\therefore$ Required probability $= P\left(\frac{E_1}{E_2}\right)$

$= \frac{P(E_1 \cap E_2)}{P(E_2)}$, when $P(E_2 \neq 0)$

$= \frac{\frac{1}{36}}{5/36} = \frac{1}{5}$

111. Two symmetric dice flipped with each dice having two sides painted red, two painted black, one painted yellow and the other painted white. What is the probability that both land on the same colour ?

(a) $\frac{3}{18}$ (b) $\frac{2}{9}$

(c) $\frac{5}{18}$ (d) $\frac{1}{3}$

(**c**) P (two sides painted red) $= \frac{2}{6} \times \frac{2}{6}$

P (two sides painted black) $= \frac{2}{6} \times \frac{2}{6}$

P (one side painted yellow) $= \frac{1}{6} \times \frac{1}{6}$

and P (other side painted white) $= \frac{1}{6} \times \frac{1}{6}$

$\therefore$ Required probability that both land on the same colour

$= \frac{2}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$

$= \frac{4 + 4 + 1 + 1}{36}$

$= \frac{10}{36} = \frac{5}{18}$

112. There are n socks in a drawer, of which 3 socks are red. If 2 of the socks are chosen randomly and the probability that both selected socks are red is $\frac{1}{2}$, then what is the value of n ?

(a) 3

(b) 4

(c) 5

(d) 6

(**b**) Total number of socks $= n$

P (first socks is red) $= \frac{3}{n}$

P (second socks is red) $= \frac{2}{n - 1}$

According to the question,

$\frac{3}{n} \times \frac{2}{n - 1} = \frac{1}{2}$

$\Rightarrow n^2 - n = 12$

$\Rightarrow n^2 - n - 12 = 0$

$\Rightarrow n^2 - 4n + 3n - 12 = 0$

$\Rightarrow n(n - 4) + 3(n - 4) = 0$

$\Rightarrow (n - 4)(n + 3) = 0$

$\therefore n = 4, -3$

113. Two cards are chosen at random from a deck of 52 playing cards. What is the probability that both of them have the same value ?

(a) $\frac{1}{17}$ (b) $\frac{3}{17}$

(c) $\frac{5}{17}$ (d) $\frac{7}{17}$

(**a**) $\therefore$ Required probability $= \frac{{}^4C_2 \times 13}{{}^{52}C_2}$

$= \frac{4 \times 3 \times 13}{52 \times 51}$

$= \frac{1}{17}$

114. In eight throws of a die, 5 or 6 is considered a success. The mean and standard deviation of total number of successes is respectively given by

(a) $\frac{8}{3}, \frac{16}{9}$ (b) $\frac{8}{3}, \frac{4}{3}$

(c) $\frac{4}{3}, \frac{4}{3}$ (d) $\frac{4}{3}, \frac{16}{9}$

(**b**) We have, p (success) $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$\therefore \quad q = 1 - p = \frac{2}{3}$

Given, $n = 8$

$\therefore$ Mean $= np = 8 \times \frac{1}{3} = \frac{8}{3}$

Standard deviation $= \sqrt{npq}$

$= \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}}$

$= \sqrt{\frac{16}{9}} = \frac{4}{3}$

115. A and B are two events such that $\overline{A}$ and $\overline{B}$ are mutually exclusive. If $P(A) = 0.5$ and $P(B) = 0.6$, then what is the value of $P(A/B)$?

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$

(c) $\frac{2}{5}$ (d) $\frac{1}{3}$

(**b**) Given, $P(\overline{A} \cap \overline{B}) = 0$

$\Rightarrow \quad P(\overline{A \cup B}) = 0$

$\Rightarrow \quad 1 - P(A \cup B) = 0$

$\Rightarrow \quad P(A \cup B) = 1$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 1 = 0.5 + 0.6 - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.1$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.1}{0.6} = \frac{1}{6}$

116. Consider the following statements

1. The algebraic sum of deviations of a set of values from their arithmetic mean is always zero.
2. Arithmetic mean > Median > Mode for a symmetric distribution.

Which of the above statements is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**a**) We know that, the algebraic sum of deviations of a set of values from their arithmetic mean is always zero.

117. Let the correlation coefficient between X and Y be 0.6. Random variables Z and W are defined as $Z = X + 5$ and $W = \frac{Y}{3}$. What is the correlation coefficient between Z and W?

(a) 0.1 (b) 0.2
(c) 0.36 (d) 0.6

(**d**) Since, the correlation coefficient is independent of change of origin and scale. It is given that correlation coefficient between X and Y be 0.6.
So, correlation coefficient between Z and W be 0.6.

118. If all the natural numbers between 1 and 20 are multiplied by 3, then what is the variance of the resulting series?

(a) 99.75 (b) 199.75
(c) 299.25 (d) 399.25

(**c**) Variance of first n natural number

$= \frac{n^2 - 1}{12} = \frac{20^2 - 1}{12}$

$= \frac{399}{12} = 33.25$

If all the natural number between 1 and 20 multiplied by 3, then

$\therefore$ Required variance $= 9 \times 33.25$

$= 299.25$

119. What is the probability that an interior point in a circle is closer to the centre than to the circumference?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) It cannot be determined

(**a**) Let radius of circle be r, then the points closer to centre if circumference will lie within radius of $\frac{r}{2}$.

So, the favourable outcome would be the points inside the area of circle with radius $\frac{r}{2}$ whereas the total possible outcomes could be all the points inside the area of circle with radius r.

$\therefore$ Required probability $= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$

120. If A and B are two events, then what is the probability of occurrence of either event A or event B ?

(a) $P(A) + P(B)$ (b) $P(A \cup B)$
(c) $P(A \cap B)$ (d) $P(A)P(B)$

(**b**) If A and B are two events, then the probability of occurrence of either event A or event B is $P(A \cup B)$.