# NDA/NA

## National Defence Academy/Naval Academy

# SOLVED PAPER 2018 (I)

**1.** If $n \in N$, then $121^n - 25^n + 190^n - (-4)^n$ is divisible by which one of the following?
(a) 1904 (b) 2000 (c) 2002 (d) 2006

⟩ **(b)** We have,
$$121^n - 25^n + 1900^n - (-4)^n$$
On putting $n = 1$, we get
$$(121)^1 - (25)^1 + (1900)^1 - (-4)^1$$
$$= 121 - 25 + 1900 + 4$$
$$= 2000$$
Which is divisible by 2000.

**2.** If $n = (2017)!$, then what is
$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n}$$
$$+ \ldots + \frac{1}{\log_{2017} n} \text{equal to ?}$$
(a) 0 (b) 1 (c) $\frac{n}{2}$ (d) $n$

⟩ **(b)** We have,
$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{2017} n}$$
$$= \log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 2017$$
$$\left[ \because \log_b a = \frac{1}{\log_a b} \right]$$
$$= \log_n (2 \cdot 3 \cdot 4 \ldots \ldots 2017)$$
$$[\because \log a + \log b = \log ab]$$
$$= \log_n (1 \cdot 2 \cdot 3 \cdot 4 \ldots \ldots 2017)$$
$$= \log_n (2017)!$$
$$[\because n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1 = n!]$$
$$= \log_{(2017)!} (2017)! \qquad [\because n = 2017!]$$
$$= 1 \qquad [\because \log_a a = 1]$$

**3.** In the expansion of $(1 + x)^{43}$, if the coefficients of $(2r + 1)$th and $(r + 2)$th terms are equal, then what is the values of $r(r \ne 1)$?
(a) 5 (b) 14 (c) 21 (d) 22

⟩ **(b)** We have, $(1 + x)^{43}$
∴ General term, $T_{r+1} = {}^{43}C_r x^r$
Now, $T_{2r+1} = {}^{43}C_{2r} x^{2r}$
and $T_{r+2} = {}^{43}C_{r+1} x^{r+1}$
Now, according to the question
Coefficients of $(2r + 1)$th and $(r + 2)$th terms are equal
∴ $\qquad {}^{43}C_{2r} = {}^{43}C_{r+1}$
$\Rightarrow \qquad 2r + r + 1 = 43$
$\qquad [\because \text{if } {}^nC_x = {}^nC_y \Rightarrow x + y = n]$
$\Rightarrow \qquad 3r = 42$
$\Rightarrow \qquad r = 14$

**4.** What is the principal argument of $(-1 - i)$, where $i = \sqrt{-1}$?
(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $-\frac{3\pi}{4}$ (d) $\frac{3\pi}{4}$

⟩ **(c)** Let $z - 1 - i$
Now, $\tan\alpha = \frac{|b|}{|a|} = \frac{|-1|}{|-1|}$
$$[\because a = -1, b = -1]$$
∴ $\qquad \alpha = \tan^{-1}(1) = \frac{\pi}{4}$
Since $a$, $b$ both are negative,
∴ $\arg(z) = \alpha - \pi$
$$= \frac{\pi}{4} - \pi = \frac{-3\pi}{4}$$

**5.** Let $\alpha$ and $\beta$ be real number and $z$ be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with $\mathrm{Re}\,(z) = 1$, then it is necessary that
(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

⟩ **(c)** Let $z = x + iy$
Now, we have
$$z^2 + \alpha z + \beta = 0$$
$$\Rightarrow (x + iy)^2 + \alpha(x + iy) + \beta = 0$$
$$\Rightarrow x^2 - y^2 + 2ixy + \alpha x + i\alpha y + \beta = 0$$
$$[\because i^2 = -1]$$
$$\Rightarrow (x^2 - y^2 + \alpha x + \beta) + (2xy + \alpha y)i = 0$$
On comparing,
$$x^2 - y^2 + \alpha x + \beta = 0 \text{ and } 2xy + \alpha y = 0$$
$$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0 \text{ and } (2x + \alpha)y = 0$$
$$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0 \text{ and } 2x + \alpha = 0$$
$$[\because y \ne 0]$$
$$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0 \text{ and } \alpha = -2x$$
$$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0 \text{ and } \alpha = -2$$
$$[\because \mathrm{Re}(z) = 1 = x]$$
$$\Rightarrow 1 - y^2 - 2 + \beta = 0 \quad [\because x = 1, \alpha = -2]$$
$$\Rightarrow \qquad \beta = y^2 + 1$$
$$\Rightarrow \qquad \beta \in (1, \infty)$$
$$[\because y^2 \ge 0 \Rightarrow y^2 + 1 \ge 1]$$

**6.** Let $A$ and $B$ be subsets of $X$ and $C = (A \cap B') \cup (A' \cap B)$, where $A'$ and $B'$ are complements of $A$ and $B$ respectively in $X$. What is $C$ equal to ?
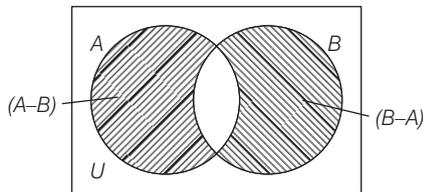(a) $(A \cup B') - (A \cap B')$
(b) $(A' \cup B) - (A' \cap B)$
(c) $(A \cup B) - (A \cap B)$
(d) $(A' \cup B') - (A' \cap B')$

**(c)** We have, $C = (A \cap B') \cup (A' \cap B)$

$= (A - B) \cup (B - A)$
$[\because X \cap Y' = x - y]$

$= (A \cup B) - (A \cap B)$
[from venn diagram]



**7.** How many numbers between 100 and 1000 can be formed with the digits 5, 6, 7, 8, 9, if the repetition of digits is not allowed?
(a) $3^5$      (b) $5^3$
(c) 120      (d) 60

**(d)** Number lying between 100 and 1000 are of three digit. Since the numbers are to be formed with 5, 6, 7, 8, 9 and repetition is not allowed, so total number of numbers

$= 5 \times 4 \times 3 = 60$

**8.** The number of non-zero integral solution of the equation $|1 - 2i|^x = 5^x$ is

(a) Zero (no solution)
(b) One
(c) Two
(d) Three

**(a)** We have,

$$|1 - 2i|^x = 5^x$$

$\Rightarrow \quad (\sqrt{(1)^2 + (-2)^2})^x = 5^x$

$[\because |a + ib| = \sqrt{a^2 + b^2}]$

$\Rightarrow \quad (\sqrt{1 + 4})^x = 5^x$

$\Rightarrow \quad (\sqrt{5})^x = 5^x$

$\Rightarrow \quad 5^{x/2} = 5^x$

$\Rightarrow \quad \dfrac{x}{2} = x$

$[\because a^m = a^n \Rightarrow m = n]$

$\Rightarrow \quad x - \dfrac{x}{2} = 0$

$\Rightarrow \quad \dfrac{x}{2} = 0$

$\Rightarrow \quad x = 0$

But $x$ is non-zero integral.
$\therefore$ Given equation has no solution.

**9.** If the ratio of AM of GM of two positive numbers $a$ and $b$ is $5 : 3$, then $a : b$ is equal to
(a) $3 : 5$      (b) $2 : 9$
(c) $9 : 1$      (d) $5 : 3$

**(c)** Let $a$ and $b$ be two numbers.

According to the question,

$$\dfrac{\dfrac{a + b}{2}}{\sqrt{ab}} = \dfrac{5}{3}$$

$$\left[\because A : G = 5 : 3, A = \dfrac{a + b}{2}, G = \sqrt{ab}\right]$$

$\Rightarrow \quad \dfrac{a + b}{\sqrt{ab}} = \dfrac{10}{3}$

$\Rightarrow \quad \dfrac{(a + b)^2}{ab} = \left(\dfrac{10}{3}\right)^2$

$\Rightarrow \quad \dfrac{a^2 + b^2 + 2ab}{ab} = \dfrac{100}{9}$

$\Rightarrow \quad \dfrac{a}{b} + \dfrac{b}{a} + 2 = \dfrac{100}{9}$

$\Rightarrow \quad t + \dfrac{1}{t} + 2 = \dfrac{100}{9} \quad \left[\because \dfrac{a}{b} = t\right]$

$\Rightarrow \quad \dfrac{t^2 + 1 + 2t}{t} = \dfrac{100}{9}$

$\Rightarrow \quad 9t^2 - 82t + 9 = 0$

$\Rightarrow \quad (t - 9)(9t - 1) = 0$

$\Rightarrow \quad t = 9, \dfrac{1}{9}$

$\therefore \quad \dfrac{a}{b} = 9 \text{ or } \dfrac{a}{b} = \dfrac{1}{9} \quad \left[\because t = \dfrac{a}{b}\right]$

$\Rightarrow \quad a : b = 9 : 1 \text{ or } 1 : 9$

**10.** If the coefficients of $a^m$ and $a^n$ is the expansion of $(1 + a)^{m+n}$ are $\alpha$ and $\beta$, then which one of the following is correct?
(a) $\alpha = 2\beta$      (b) $\alpha = \beta$
(c) $2\alpha = \beta$      (d) $\alpha = (m + n)\beta$

**(b)** We have

$(1 + a)^{m+n}$

$\therefore \quad T_{r+1} = {}^{m+n}C_r a^r$

$\therefore$ Coefficient of $a^m = {}^{m+n}C_m \quad [\because r = m]$

and coefficient of $a^n = {}^{m+n}C_n \quad [\because r = n]$

$\therefore \quad \alpha = {}^{m+n}C_m$

and $\quad \beta = {}^{m+n}C_n$

$= {}^{m+n}C_{m+n-n}[\because {}^nC_r = {}^nC_{n-r}]$

$= {}^{m+n}C_m = \alpha$

$\therefore \quad \alpha = \beta$

**11.** If $x + \log_{15}(1 + 3^x) = x\log_{15} 5 + \log_{15} 12$, where $x$ is an integer, then what is $x$ equal to?
(a) $-3$    (b) 2    (c) 1    (d) 3

**(c)** We have,

$x + \log_{15}(1 + 3^x) = x\log_{15} 5 + \log_{15} 12$

$\Rightarrow \log_{15} 15^x + \log_{15}(1 + 3^x)$

$\qquad\qquad = \log_{15} 5^x + \log_{15} 12$

$[\because \log_a a = 1 \text{ and } \log_b a^m = m\log_b a]$

$\Rightarrow \log_{15}[15^x (1 + 3^x)] = \log_{15}(5^x \times 12)$

$[\because \log a + \log b = \log ab]$

$\Rightarrow \quad 15^x (1 + 3^x) = 12 \cdot 5^x$

$\Rightarrow \quad 3^x(1 + 3^x) = 12$

$\Rightarrow \quad y(1 + y) = 12 \qquad [\text{where } y = 3^x]$

$\Rightarrow \quad y^2 + y - 12 = 0$

$\Rightarrow (y + 4)(y - 3) = 0$

$\Rightarrow \qquad\qquad y = -4, 3$

$\Rightarrow \qquad\qquad 3^x = -4, 3$

$\Rightarrow \qquad\qquad 3^x = 3 \qquad [\because 3^x \neq -4]$

$\Rightarrow \qquad\qquad x = 1$

**12.** How many four-digit numbers divisible by 10 can be formed using 1, 5, 0, 6, 7 without repetition of digits?
(a) 24      (b) 36
(c) 44      (d) 64

**(a)** We have to form four digit numbers which are divisible by 10 and using 1, 5, 0, 6, 7. Since numbers must be divisible by 10, so unit place must be zero.

$\therefore$ Total number of such numbers = Permutations of three digits using 1, 5, 6, 7

$= {}^4P_3 = \dfrac{4!}{(4 - 3)!} = 4! = 24$

**Directions** (Q. Nos. 13-14) *Consider the information given below and answer the two items that follow*

In a class, 54 students are good in Hindi only, 63 students are good in Mathematics only and 41 students are good in English only. There are 18 students who are good in both Hindi and Mathematics. 10 students are good in all three subjects.
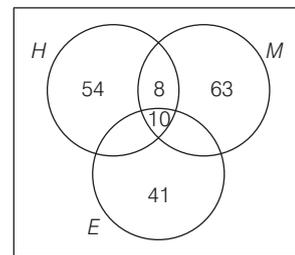
**13.** What is the number of students who are good in either Hindi or Mathematics but not English?
(a) 99      (b) 107
(c) 125      (d) 130

**14.** What is the number of students who are good in Hindi and Mathematics but not in English?
(a) 18    (b) 12    (c) 10    (d) 8

**Solution (Q. Nos. 13-14)**



**(c)** From given Venn-diagram

$n(H \cup M \cup E') = 54 + 8 + 63 = 125$

**(d)** From given Venn-diagram

$n(H \cap M \cap E') = 18 - 10 = 8$

**15.** If $\alpha$ and $\beta$ are different complex numbers with $|\alpha| = 1$, then what is $\left|\dfrac{\alpha - \beta}{1 - \overline{\alpha}\beta}\right|$ equal to ?

(a) $|\beta|$  (b) 2  (c) 1  (d) 0

⊘ **(c)** We have,

$$\left|\frac{\alpha - \beta}{1 - \overline{\alpha}\beta}\right| = \left|\frac{\alpha - \beta}{\alpha\overline{\alpha} - \overline{\alpha}\beta}\right|$$

$$[\because |\alpha| = 1 \Rightarrow |\alpha|^2 = 1 \Rightarrow \alpha \cdot \overline{\alpha} = 1]$$

$$= \left|\frac{\alpha - \beta}{\alpha(\overline{\alpha} - \overline{\beta})}\right|$$

$$= \frac{1}{|\alpha|}\left|\frac{\alpha - \beta}{\overline{\alpha} - \overline{\beta}}\right|$$

$$= \frac{|\alpha - \beta|}{|\alpha||\overline{\alpha} - \overline{\beta}|}$$

$$= \frac{|\alpha - \beta|}{|\alpha||\alpha - \beta|} \qquad [\because |\overline{z}| = |z|]$$

$$= \frac{1}{|\alpha|} = 1$$

$$[\because |\alpha| = 1]$$

**16.** The equation $|1 - x| + x^2 = 5$ has

(a) a rational root and an irrational root
(b) two rational roots
(c) two irrational roots
(d) no real roots

⊘ **(a)** We have,

$$|1 - x| + x^2 = 5$$

**Case I** When $x < 1$

$$1 - x + x^2 = 5$$

$$[\because x < 1 \Rightarrow |1 - x| = 1 - x]$$

$$\Rightarrow x^2 - x - 4 = 0$$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 - 4(1)(-4)}}{2}$$

$$= \frac{1 \pm \sqrt{17}}{2}$$

$$\Rightarrow \quad x = \frac{1 - \sqrt{17}}{2} \qquad [\because x < 1]$$

**Case II** When $x \geq 1$

$$-(1 - x) + x^2 = 5$$

$$[\because x \geq 1 \Rightarrow |1 - x| = -(1 - x)]$$

$$\Rightarrow \quad -1 + x + x^2 = 5$$

$$\Rightarrow \quad x^2 + x - 6 = 0$$

$$\Rightarrow (x + 3)(x - 2) = 0$$

$$\Rightarrow \qquad x = -3, 2$$

$$\Rightarrow \qquad x = 2 \qquad [\because x \geq 1]$$

∴ Given equation has a rational root and an irrational root.

**17.** The binary number expression of the decimal number 31 is

(a) 1111  (b) 10111
(c) 11011  (d) 11111

⊘ **(d)**

| 2 | 31 | |
|---|----|---|
| 2 | 15 | 1 |
| 2 | 7 | 1 |
| 2 | 3 | 1 |
| 2 | 1 | 1 |
| | 0 | 1 |

$$\therefore (31)_{10} = (11111)_2$$

**18.** What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where $i = \sqrt{-1}$) ?

(a) 0  (b) $i$  (c) $-i$  (d) 1

⊘ **(a)** We have,

$$i^{1000} + i^{1001} + i^{1002} + i^{1003}$$

$$= i^{1000}[1 + i + i^2 + i^3]$$

$$= i^{1000}[1 + i - 1 - i]$$

$$[\because i^2 = -1, i^3 = -i]$$

$$= 0$$

**19.** What is

$$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + \dots$$

$$+ \frac{1}{\log_{100} N} \text{ equal to } (N \neq 1) ?$$

(a) $\dfrac{1}{\log_{100!} N}$  (b) $\dfrac{1}{\log_{99!} N}$

(c) $\dfrac{99}{\log_{100!} N}$  (d) $\dfrac{99}{\log_{99!} N}$

⊘ **(a)** We have,

$$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + \dots + \frac{1}{\log_{100} N}$$

$$= \log_N 2 + \log_N 3 + \log_N 4 + \dots + \log_N 100$$

$$\left[\because \log_b a = \frac{1}{\log_a b}\right]$$

$$= \log_N 2 \cdot 3 \cdot 4 \dots 100$$

$$[\because \log a + \log b = \log ab]$$

$$= \log_N 1 \cdot 2 \cdot 3 \cdot 4 \dots 100$$

$$= \log_N (100)!$$

$$[\because n! = n(n-1)(n-2)\dots 2 \cdot 1]$$

$$= \frac{1}{\log_{(100)!} N}$$

**20.** The modulus-amplitude form of $\sqrt{3} + i$, where $i = \sqrt{-1}$ is

(a) $2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$

(b) $2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

(c) $4\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$

(d) $4\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

⊘ **(b)** Let $z = \sqrt{3} + i$

$$\therefore \quad |z| = \sqrt{(\sqrt{3})^2 + (1)^2}$$

$$[\because z = a + ib \Rightarrow |z| = \sqrt{a^2 + b^2}]$$

$$= \sqrt{3 + 1} = \sqrt{4} = 2$$

Now, $\text{amp}(z) = \tan^{-1}\left(\dfrac{b}{a}\right)$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\left[\because \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}\right]$$

$$\therefore \qquad z = r(\cos\theta + i\sin\theta)$$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$[\because r = |z| = 2 \text{ and } \theta = \text{amp}(z) = \frac{\pi}{6}]$$

**21.** What is the number of non-zero terms in the expansion of $(1 + 2\sqrt{3}x)^{11} + (1 - 2\sqrt{3}x)^{11}$ (after simplification)?

(a) 4  (b) 5
(c) 6  (d) 11

⊘ **(c)** In $(a + b)^n + (a - b)^n$, number of terms

$$= \begin{cases} \dfrac{n + 2}{2}, & \text{if } n \text{ is even} \\ \dfrac{n + 1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

∴ Number of terms in

$$(1 + 2\sqrt{3}x)^{11} + (1 - 2\sqrt{3}x)^{11}$$

$$= \frac{11 + 1}{2} \qquad [\because n = 11, \text{ is odd}]$$

$$= \frac{12}{2} = 6$$

**22.** What is the greatest integer among the following, by which the number $5^5 + 7^5$ is divisible?

(a) 6  (b) 8  (c) 11  (d) 12

⊘ **(d)** We know that when $m$ is odd then $(x^m + y^m)$ is divisible by $(x + y)$.

∴ $5^5 + 7^5$ is divisible by $5 + 7 = 12$ as $m = 5$ is odd.

**23.** If $x = 1 - y + y^2 - y^3 \dots$ up to infinite terms, where $|y| < 1$, then which one of the following is correct?

(a) $x = \dfrac{1}{1 + y}$  (b) $x = \dfrac{1}{1 - y}$

(c) $x = \dfrac{y}{1 + y}$  (d) $x = \dfrac{y}{1 - y}$

⊘ **(a)** We have,

$$x = 1 - y + y^2 - y^3 + \dots \infty, |y| < 1$$

$$= \frac{1}{1 - (-y)}$$

$$\left[\because a + ar + ar^2 + \dots \infty = \frac{a}{1 - r}, r < 1\right]$$

$$= \frac{1}{1 + y}$$

**24.** What is the inverse of the matrix

$$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} ?$$

(a) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

⟩ **(a)** We have,

$$A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$|A| = 1 [\cos^2\theta - (-\sin^2\theta)] = 1 \neq 0$

$C_{11} = \begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} = \cos\theta$

$C_{12} = -\begin{vmatrix} -\sin\theta & 0 \\ 0 & 1 \end{vmatrix} = \sin\theta$

$C_{13} = \begin{vmatrix} -\sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix} = 0$

$C_{21} = -\begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix} = -\sin\theta$

$C_{22} = \begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} = \cos\theta$

$C_{23} = -\begin{vmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{vmatrix} = 0$

$C_{31} = \begin{vmatrix} \sin\theta & 0 \\ \cos\theta & 0 \end{vmatrix} = 0$

$C_{32} = -\begin{vmatrix} \cos\theta & 0 \\ -\sin\theta & 0 \end{vmatrix} = 0$

$C_{33} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$

$\therefore \text{adj} A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{|A|} \text{adj}$

$A = \dfrac{1}{1} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**25.** If $A$ is a $2 \times 3$ matrix and $AB$ is a $2 \times 5$ matrix, then $B$ must be a

(a) $3 \times 5$ matrix    (b) $5 \times 3$ matrix
(c) $3 \times 2$ matrix    (d) $5 \times 2$ matrix

⟩ **(a)** Let order of $B$ is $m \times n$.

Now, according to the question

$A_{2 \times 3} \times B_{m \times n} = (AB)_{2 \times 5}$

$\therefore m = 3$ and $n = 5$

$\therefore$ Order of $B$ is $3 \times 5$.

**26.** If $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and $A^2 - kA - I_2 = O$,

where $I_2$ is the $2 \times 2$ identity matrix, then what is the value of $k$?

(a) 4    (b) $-4$
(c) 8    (d) $-8$

⟩ **(a)** We have,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \cdot 1 + 2 \cdot 2 & 1 \cdot 2 + 2 \cdot 3 \\ 2 \cdot 1 + 3 \cdot 2 & 2 \cdot 2 + 3 \cdot 3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

Now, it is given that,

$A^2 - kA - I_2 = 0$

$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 4 & 8 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix}$

$\Rightarrow k = 4$

**27.** What is the number of triangles that can be formed by choosing the vertices from a set of 12 points in a plane, seven of which lie on the same straight line?

(a) 185    (b) 175
(c) 115    (d) 105

⟩ **(a)** Required number of triangle

$= {}^{12}C_3 - {}^{7}C_3$

$= \dfrac{12 \times 11 \times 10}{3 \times 2 \times 1} - \dfrac{7 \times 6 \times 5}{3 \times 2 \times 1}$

$= 220 - 35 = 185$

**28.** What is

$C(n, r) + 2C(n, r - 1) + C(n, r - 2)$

equal to?

(a) $C(n + 1, r)$
(b) $C(n - 1, r + 1)$
(c) $C(n, r + 1)$
(d) $C(n + 2, r)$

⟩ **(d)** We have,

$C(n, r) + 2C(n, r - 1) + C(n, r - 2)$

$= {}^{n}C_r + 2 \cdot {}^{n}C_{r-1} + {}^{n}C_{r-2}$

$= ({}^{n}C_r + {}^{n}C_{r-1}) + ({}^{n}C_{r-1} + {}^{n}C_{r-2})$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1}$

$[\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r]$

$= {}^{n+1+1}C_r$

$= {}^{n+2}C_r$

$= C(n + 2, r)$

**29.** Let $|x|$ denote the greatest integer function. What is the number of solutions of the equation $x^2 - 4x + [x] = 0$ in the interval $[0, 2]$?

(a) Zero (no solution)    (b) One
(c) Two    (d) Three

⟩ **(b)** We have,

$x^2 - 4x + [x] = 0$

**Case I** $x \in [0, 1]$

$\therefore \quad x^2 - 4x + 0 = 0$

$[\because x \in [0, 1] \Rightarrow [x] = 0]$

$\Rightarrow \quad x^2 - 4x = 0$

$\Rightarrow \quad x(x - 4) = 0$

$\Rightarrow \quad x = 0, 4$

$\Rightarrow \quad x = 0$

$[\because x \in [0, 1]]$

**Case II** $x \in [1, 2]$

$\therefore \quad x^2 - 4x + 1 = 0$

$[\because x \in [1, 2] \Rightarrow x = 1]$

$\Rightarrow \quad x = \dfrac{4 \pm \sqrt{16 - 4}}{2}$

$\Rightarrow \quad x = \dfrac{4 \pm 2\sqrt{3}}{2}$

$\Rightarrow \quad x = 2 \pm \sqrt{3}$

$\Rightarrow \quad x = 0.268, 3.732$

No solution    $[\because x \in [1,2]]$

$\therefore$ Given equation has only one solution i.e. $x = 0$.

**30.** A survey of 850 students in a University yields that 680 students like music and 215 like dance. What is the least number of students who like both music and dance?

(a) 40    (b) 45
(c) 50    (d) 55

⟩ **(b)** Let $A$ be the set of students who like music and $B$ be the set of students whose like dance.

$\therefore n(A) = 680, n(B) = 215$ and $n(U) = 850$

We know that,

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow$

$n(A \cap B)_{min} = n(A) + n(B) - n(A \cup B)_{max}$

$\Rightarrow n(A \cap B)_{min} = 680 + 215 - 850$

$\qquad [\because n(A \cup B)_{max} = n(\cup)]$

$\qquad = 45$

**31.** What is the sum of all two-digit numbers, which when divided by 3 leave 2 as the remainder ?

(a) 1565      (b) 1585
(c) 1635      (d) 1655

⊙ **(c)** Required numbers are 11, 14, 17, ... 98 which is an AP.

We know that,

$\qquad a_n = a + (n-1)d$
$\qquad 98 = 11 + (n-1)(3)$
$\Rightarrow \quad 98 = 11 + 3n - 3$
$\Rightarrow \quad 98 = 3n + 8$
$\Rightarrow \quad 90 = 3n$
$\Rightarrow \quad n = 30$

∴ Sum = 11 + 14 + 17 + .... + 98

$\qquad = \dfrac{30}{2}[11 + 98] \left[\because S_n = \dfrac{n}{2}(a + l)\right]$

$\qquad = 15 \times 109$
$\qquad = 1635$

**32.** If $0 > a < 1$, the value of $\log_{10} a$ is negative. This is justified by

(a) Negative power of 10 is less than 1
(b) Negative power of 10 is between 0 and 1
(c) Negative power of 10 is positive
(d) Negative power of 10 is negative

⊙ **(b)** Let $\log_{10} a = x$

$\Rightarrow \qquad a = 10^x$

It is given that

$\qquad 0 < a < 1$
$\Rightarrow \quad 0 < 10^x < 1$

$\Rightarrow x$ must be negative
∴ If $0 < a < 1$, the value of $\log_{10} a$ is negative implies that negative power of 10 is between 0 and 1.

**33.** The third term of a GP is 3. What is the product of the first five terms?

(a) 216
(b) 226
(c) 243
(d) Cannot be determined due to insufficient data

⊙ **(c)** Let $a$ and $r$ be the first term and common ratio of of the GP.

∴ $\qquad a_3 = 3$
$\Rightarrow \quad ar^2 = 3 \qquad [\because a_n = ar^{n-1}] \dots (i)$

Required product $= a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5$
$\qquad = (a)(ar)(ar^2)(ar^3)(ar^4)$
$\qquad = a^5 r^{10} = (ar^2)^5$
$\qquad = (3)^5 \qquad\qquad$ [from Eq. (i)]
$\qquad = 243$

**34.** If $x, \dfrac{3}{2}, z$ are in AP; $x, 3, z$ are in GP; then which one of the following will be in HP?

(a) $x$, 6, $z$      (b) $x$, 4, $z$
(c) $x$, 2, $z$      (d) $x$, 1, $z$

⊙ **(a)** We have,

$x, \dfrac{3}{2}, z$ are in AP.

$\Rightarrow \qquad \dfrac{x + z}{2} = \dfrac{3}{2}$

$\Rightarrow \qquad x + z = 3 \qquad\qquad \dots (i)$

Also, $x, 3, z$ are in GP

$\Rightarrow \qquad xz = 3^2$

$\Rightarrow \qquad xz = 9 \qquad\qquad \dots (ii)$

Now, from Eqs. (i) and (ii), we have

$\qquad \dfrac{2xz}{x + y} = \dfrac{2 \times 9}{3}$

$\Rightarrow \qquad \dfrac{2xz}{x + z} = 6$

$\Rightarrow x$, 6, $z$ are in HP.

**35.** What is the value of the sum $\displaystyle\sum_{n=2}^{11}(i^n + i^{n+1})$, where $i = \sqrt{-1}$ ?

(a) $i$    (b) $2i$    (c) $-2i$    (d) $1 + i$

⊙ **(c)** We have,

$\displaystyle\sum_{n=2}^{11}(i^n + i^{n+1}) = \sum_{n=2}^{11} i^n (1 + i)$

$\qquad = (1 + i) \displaystyle\sum_{n=2}^{11} i^n$

$\qquad = (1 + i)[i^2 + i^3 + i^4 + \dots + i^{11}]$

$\qquad = (1 + i) i^2 \left[\dfrac{i^{10} - 1}{i - 1}\right]$

$\qquad \left[\because a + ar + ar^2 + \dots + ar^{n-1} = a\left[\dfrac{r^n - 1}{r - 1}\right]\right]$

$\qquad = \dfrac{(1 + i) i^2 (i^{2 \times 4 + 2} - 1)}{(i - 1)}$

$\qquad = \dfrac{-(1 + i)(i^2 - 1)}{(i - 1)} \qquad [\because i^2 = -1]$

$\qquad = \dfrac{-(1 + i)(-1 - 1)}{(i - 1)} = \dfrac{2(1 + i)}{(i - 1)}$

$\qquad = \dfrac{2(1 + i)}{(i - 1)} \times \dfrac{i + 1}{(i + 1)}$

$\qquad = \dfrac{2(i + 1 + i^2 + i)}{i^2 - 1}$

$\qquad = \dfrac{2(i + 1 - 1 + i)}{-1 - 1}$

$\qquad = -2i$

**36.** If $\sin x = \dfrac{1}{\sqrt{5}}$, $\sin y = \dfrac{1}{\sqrt{10}}$, where $0 < x < \dfrac{\pi}{2}$, $0 < y < \dfrac{\pi}{2}$, then what is $(x + y)$ equal to ?

(a) $\pi$    (b) $\dfrac{\pi}{2}$    (c) $\dfrac{\pi}{4}$    (d) 0

⊙ **(c)** We have, $\sin x = \dfrac{1}{\sqrt{5}}$ and $\sin y = \dfrac{1}{\sqrt{10}}$

$\Rightarrow x = \sin^{-1}\dfrac{1}{\sqrt{5}}$ and $y = \sin^{-1}\dfrac{1}{\sqrt{10}}$

Now, $x + y = \sin^{-1}\dfrac{1}{\sqrt{5}} + \sin^{-1}\dfrac{1}{\sqrt{10}}$

$= \sin^{-1}\left[\dfrac{1}{\sqrt{5}}\sqrt{1 - \left(\dfrac{1}{\sqrt{10}}\right)^2} + \dfrac{1}{\sqrt{10}}\sqrt{1 - \left(\dfrac{1}{\sqrt{5}}\right)^2}\right]$

$\quad [\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1 - y^2} + y\sqrt{1 - x^2}]$

$= \sin^{-1}\left[\dfrac{1}{\sqrt{5}}\sqrt{1 - \dfrac{1}{10}} + \dfrac{1}{\sqrt{10}}\sqrt{1 - \dfrac{1}{5}}\right]$

$= \sin^{-1}\left[\dfrac{1}{\sqrt{5}} \times \dfrac{3}{\sqrt{10}} + \dfrac{1}{\sqrt{10}} \times \dfrac{2}{\sqrt{5}}\right]$

$= \sin^{-1}\left[\dfrac{5}{\sqrt{5} \times \sqrt{10}}\right]$

$= \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{\pi}{4} \qquad\qquad \left[\because \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}\right]$

**37.** What is $\dfrac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$ equal to ?

(a) $\sin x$      (b) $\cos x$
(c) $\tan x$      (d) $\cot x$

⊙ **(c)** Given, $\dfrac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$

$= \dfrac{2\cos\dfrac{5x + 3x}{2} \cdot \sin\dfrac{5x - 3x}{2}}{2\cos\dfrac{5x + 3x}{2} \cdot \cos\dfrac{5x - 3x}{2}}$

$\left[\because \sin C - \sin D = 2\cos\left(\dfrac{C + D}{2}\right) \cdot \sin\left(\dfrac{C - D}{2}\right) \text{ and } \cos C + \cos D = 2\cos\left(\dfrac{C + D}{2}\right)\cos\left(\dfrac{C - D}{2}\right)\right]$

$= \dfrac{2\cos 4x \sin x}{2\cos 4x \cos x} = \dfrac{\sin x}{\cos x} = \tan x$

**38.** What is $\sin 105° + \cos 105°$ equal to ?

(a) $\sin 50°$      (b) $\cos 50°$
(c) $\dfrac{1}{\sqrt{2}}$      (d) 0

⊙ **(c)** We have, $\sin 105° + \cos 105°$

$= \sin(90° + 15°) + \cos 105°$
$= \cos 15° + \cos 105°$

$\qquad\qquad [\because \sin(90 + \theta) = \cos\theta]$

$= 2\cos\left(\dfrac{105° + 15°}{2}\right)\cos\left(\dfrac{105° - 15°}{2}\right)$

$\left[\because \cos C + \cos D = 2\cos\left(\dfrac{C + D}{2}\right)\cos\left(\dfrac{C - D}{2}\right)\right]$

$$= 2\cos 60° \cos 45° = 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$\left[\because \cos 60° = \frac{1}{2}, \cos 45° = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}}$$

**39.** In a $\triangle ABC$, if $a = 2, b = 3$

and $\sin A = \frac{2}{3}$, then what is $\angle B$ equal

to ?

(a) $\frac{\pi}{4}$    (b) $\frac{\pi}{2}$    (c) $\frac{\pi}{3}$    (d) $\frac{\pi}{6}$

⊙ **(b)** We have, $a = 2, b = 3$ and $\sin A = \frac{2}{3}$

Now, from sine formula

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\Rightarrow \quad \frac{\frac{2}{3}}{2} = \frac{\sin B}{b}$$

$$\Rightarrow \quad \sin B = 1$$

$$\Rightarrow \quad B = \frac{\pi}{2} \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$$

**40.** What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ ?

(a) $\frac{\pi}{4}$    (b) $\frac{\pi}{2}$    (c) $\frac{\pi}{3}$    (d) $\frac{2\pi}{3}$

⊙ **(c)** We have, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$$

$$= \sin^{-1}\sin\frac{\pi}{3} \quad [\because \sin(\pi - \theta) = \sin\theta]$$

$$= \frac{\pi}{3}$$

$$\left[\because \sin^{-1}\sin\theta = \theta, \text{ if } \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

**41.** If $x, x - y$ and $x + y$ are the angles of a triangle (not an equilateral triangle) such that $\tan(x - y), \tan x$ and $\tan(x + y)$ are in GP, then what is $x$ equal to ?

(a) $\frac{\pi}{4}$      (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$      (d) $\frac{\pi}{2}$

⊙ **(b)** We have,

$x, x - y, x + y$ are the angles of a triangle. Since, sum of angles of a triangle $= \pi$
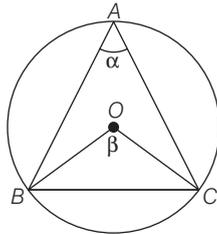
$$\therefore \quad x + x - y + x + y = \pi$$

$$\Rightarrow \quad 3x = \pi$$

$$\Rightarrow \quad x = \frac{\pi}{3}$$

**42.** $ABC$ is a triangle inscribed in a circle with centre $O$. Let $\alpha = \angle BAC$, where $45° < \alpha < 90°$. Let $\beta = \angle BOC$. Which one of the following is correct?

(a) $\cos\beta = \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$

(b) $\cos\beta = \dfrac{1 + \tan^2\alpha}{1 - \tan^2\alpha}$

(c) $\cos\beta = \dfrac{2\tan\alpha}{1 + \tan^2\alpha}$

(d) $\sin\beta = 2\sin^2\alpha$

⊙ **(a)** We know that angle subtended by a chord at centre is always double the angle subtended by it at any other part of the circle.



$$\therefore \qquad \beta = 2\alpha$$

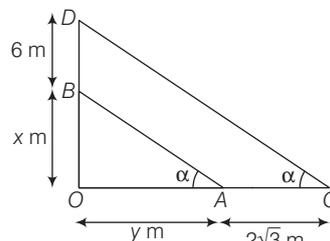$$\Rightarrow \quad \cos\beta = \cos 2\alpha$$

$$\Rightarrow \quad \cos\beta = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$

$$\left[\because \cos 2\theta = \frac{1 + \tan^2\theta}{1 + \tan^2\theta}\right]$$

**43.** If a flage-staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then what is the angle that the sun makes with the ground ?

(a) 60°      (b) 45°
(c) 30°      (d) 15°

⊙ **(a)** Let $OB$ and $BD$ be the tower and flag-staff respectively. $OA$ and $AC$ be the shadow of tower and flag-staff respectively.



Again let $\alpha$ be the angle that sun makes with the ground.

$$\therefore \qquad \angle OAB = \angle OCD = \alpha$$

Now, in $\triangle OAB$

$$\tan\alpha = \frac{x}{y} \qquad \qquad \dots \text{(i)}$$

and in $\triangle OCD$

$$\tan\alpha = \frac{x + 6}{y + 2\sqrt{3}} \qquad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{x}{y} = \frac{x + 6}{y + 2\sqrt{3}}$$

$$\Rightarrow \quad xy + 2\sqrt{3}x = xy + 6x$$

$$\Rightarrow \quad \frac{x}{y} = \sqrt{3}$$

$$\Rightarrow \quad \tan\alpha = \sqrt{3} \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad \alpha = 60°$$

**44.** What is $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$

equal to ?

(a) 0      (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$      (d) $\frac{\pi}{2}$

⊙ **(b)** We have

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$$

$$= \tan^{-1}\left[\frac{\frac{1}{4} + \frac{3}{5}}{1 - \frac{1}{4} \times \frac{3}{5}}\right]$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x + y}{1 - xy},\right.$$
$$\left. xy < 1\right]$$

$$= \tan^{-1}\left[\frac{\frac{5 + 12}{20}}{\frac{20 - 3}{20}}\right]$$

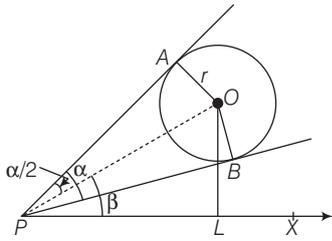$$= \tan^{-1}\left(\frac{17}{17}\right) \qquad = \tan^{-1}1$$

$$= \frac{\pi}{4} \qquad \left[\because \tan^{-1}1 = \frac{\pi}{4}\right]$$

**45.** A spherical balloon of radius $r$ subtends an angle $\alpha$ at the eye of an observer, while the angle of elevation of its centre is $\beta$. What is the height of the centre of the balloon (neglecting the height of the observer)?

(a) $\dfrac{r\sin\beta}{\sin\left(\frac{\alpha}{2}\right)}$      (b) $\dfrac{r\sin\beta}{\sin\left(\frac{\alpha}{4}\right)}$

(c) $\dfrac{r\sin\left(\frac{\beta}{2}\right)}{\sin\alpha}$      (d) $\dfrac{r\sin\alpha}{\sin\left(\frac{\beta}{2}\right)}$

⊙ **(a)** Let $O$ bet the centre of the balloon, $P$ be the eye of the observer and $\angle APB$ be the angle subtendd by the balloon at the eye of the observer. $\angle APB = \alpha$

$$\therefore \quad \angle APO = \angle BPO = \frac{\alpha}{2}$$

In $\triangle OAP$

$$\sin\frac{\alpha}{2} = \frac{OA}{OP}$$

$$\Rightarrow \quad \sin\frac{\alpha}{2} = \frac{r}{OP} \Rightarrow OP = r \ \text{cosec} \ \frac{\alpha}{z} \quad \text{... (i)}$$

In $\triangle OPL$,

$$\sin\beta = \frac{OL}{OP}$$

$$\Rightarrow \quad OL = OP\sin\beta$$

$$\Rightarrow \quad OL = r \ \text{cosec} \ \frac{\alpha}{2} \cdot \sin\beta$$

[from Eqs. (i)]

$$\therefore \quad OL = \frac{r \ \sin\beta}{\sin\left(\dfrac{\alpha}{2}\right)}$$

**46.** If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, then what is

$\dfrac{\tan x}{\tan y}$ equal to ?

(a) $\dfrac{a}{b}$        (b) $\dfrac{b}{a}$

(c) $\dfrac{a+b}{a-b}$    (d) $\dfrac{a-b}{a+b}$

&#9655; **(a)** We have,

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

On using componendo and dividendo rule, we get

$$\frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\Rightarrow$$

$$\frac{2\sin\left(\dfrac{x+y+x-y}{2}\right)\cos\left(\dfrac{x+y-x+y}{2}\right)}{2\cos\left(\dfrac{x+y+x-y}{2}\right)\sin\left(\dfrac{x+y-x+y}{2}\right)}$$

$$= \frac{2a}{2b}$$

$$\Rightarrow \quad \frac{\sin x \cos y}{\cos x \sin y} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{\tan x}{\tan y} = \frac{a}{b}$$

**47.** If $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$, where $0 < \beta < \alpha < 2\pi$, then which one of the following is correct?

(a) $\alpha = \pi - \beta$      (b) $\alpha = \pi + \beta$

(c) $\alpha = 2\pi - \beta$     (d) $2\alpha = \pi + 2\beta$

&#9655; **(b)** We have,

$$\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$$

$$\therefore (\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = 0$$

$$\Rightarrow \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha$$

$$+ \cos^2\beta + 2\cos\alpha\cos\beta = 0$$

$$\Rightarrow (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta)$$

$$+ 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 0$$

$$\Rightarrow \quad 1 + 1 + 2\cos(\alpha - \beta) = 0$$

$$\Rightarrow \quad 2\cos(\alpha - \beta) = -2$$

$$\Rightarrow \quad \cos(\alpha - \beta) = -1$$

$$\Rightarrow \quad \alpha - \beta = \pi$$

$$\Rightarrow \quad \alpha = \beta + \pi$$

**48.** Suppose $\cos A$ is given. If only one value of $\cos\left(\dfrac{A}{2}\right)$ is possible, then $A$ must be

(a) An odd multiple of $90°$
(b) A multiple of $90°$
(c) An odd multiple of $180°$
(d) A multiple of $180°$

&#9655; **(c)** We know that,

$$\cos A = 2\cos^2\frac{A}{2} - 1$$

Since, $\cos A$ is given and $\cos\dfrac{A}{2}$ has only one solution. So, $A$ must be odd multiple of $180°$.

**49.** If $\cos\alpha + \cos\beta + \cos\gamma = 0$, where $0 < \alpha \leq \dfrac{\pi}{2}, 0 < \beta \leq \dfrac{\pi}{2}, 0 < \gamma \leq \dfrac{\pi}{2}$, then what is the value of $\sin\alpha + \sin\beta = \sin\gamma$ ?

(a) 0          (b) 3

(c) $\dfrac{5\sqrt{2}}{2}$     (d) $\dfrac{3\sqrt{2}}{2}$

&#9655; **(b)** We have,

$$\cos\alpha + \cos\beta + \cos\gamma = 0,$$

$$0 < \alpha \leq \frac{\pi}{2}, 0 < \beta \leq \frac{\pi}{2}, 0 < \gamma \leq \frac{\pi}{2}$$

$$\cos\alpha + \cos\beta + \cos\gamma = 0$$

$$\Rightarrow \quad \alpha = \beta = \gamma = \frac{\pi}{2}$$

$$\therefore \quad \sin\alpha + \sin\beta + \sin\gamma$$

$$= \sin\frac{\pi}{2} + \sin\frac{\pi}{2} + \sin\frac{\pi}{2}$$

$$\left[\because \text{if} \ \theta \in \left[0, \frac{\pi}{2}\right] \cos\frac{\pi}{2} = 0\right]$$

$$= 1 + 1 + 1 = 3$$

**50.** The maximum value of $\sin\left(x + \dfrac{\pi}{5}\right) + \cos\left(x + \dfrac{\pi}{5}\right)$, where $x \in \left(0, \dfrac{\pi}{2}\right)$, is attained at

(a) $\dfrac{\pi}{20}$   (b) $\dfrac{\pi}{15}$   (c) $\dfrac{\pi}{10}$   (d) $\dfrac{\pi}{2}$

&#9655; **(a)** Let $f(x) = \sin\left(x + \dfrac{\pi}{5}\right) + \cos\left(x + \dfrac{\pi}{5}\right)$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\left(x + \frac{\pi}{5}\right) + \frac{1}{\sqrt{2}}\cos\left(x + \frac{\pi}{5}\right)\right]$$

$$= \sqrt{2}\left[\begin{array}{l}\sin\left(x + \dfrac{\pi}{5}\right)\cos\dfrac{\pi}{4} + \\ \cos\left(x + \dfrac{\pi}{5}\right)\sin\dfrac{\pi}{4}\end{array}\right]$$

$$= \sqrt{2}\left[\sin\left(x + \frac{\pi}{5} + \frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\sin\left(x + \frac{\pi}{5} + \frac{\pi}{4}\right)$$

$f(x)$ attains maximum value, when

$$x + \frac{\pi}{5} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\Rightarrow \qquad x = \frac{\pi}{20}$$

**51.** What is the distance between the points which divide the line segment joining $(4, 3)$ and $(5, 7)$ internally and externally in the ratio $2 : 3$ ?

(a) $\dfrac{12\sqrt{17}}{5}$       (b) $\dfrac{13\sqrt{17}}{5}$

(c) $\dfrac{\sqrt{17}}{5}$        (d) $\dfrac{6\sqrt{17}}{5}$

&#9655; **(a)** Let $P$ and $Q$ be the points which divides $A(4, 3)$ and $B(5, 7)$ internally and externally in the ratio $2 : 3$ respectively.

$$\therefore P = \left(\frac{2 \times 5 + 3 \times 4}{2 + 3}, \frac{2 \times 7 + 3 \times 3}{2 + 3}\right)$$



$$= \left(\frac{22}{5}, \frac{23}{5}\right)$$

$$\text{and} \ Q = \left(\frac{2 \times 5 - 3 \times 4}{2 + 3}, \frac{2 \times 7 - 3 \times 3}{2 - 3}\right)$$



$$= (2, -5)$$

$$\therefore \text{Required distance} = PQ$$

$$= \sqrt{\left(2 - \frac{22}{5}\right)^2 + \left(-5 - \frac{23}{5}\right)^2} = \frac{12}{5}\sqrt{17}$$

**52.** What is the angle between the straight lines $(m^2 - mn)y = (mn + n^2)x + n^3$ and $(mn + m^2)y = (mn - n^2)x + m^3$, where $m > n$?

(a) $\tan^{-1}\left(\dfrac{2mn}{m^2 + n^2}\right)$ (b) $\tan^{-1}\left(\dfrac{4m^2n^2}{m^2 - n^2}\right)$

(c) $\tan^{-1}\left(\dfrac{4m^2n^2}{m^4 + n^4}\right)$ (d) $45°$

**(c)** Given equations of lines are

$$(m^2 - mn)\, y = (mn + n^2)\, x + n^3$$

and $(mn + m^2)y = (mn - n^2)\, x + m^3$

Given equation of lines can be written as

$$y = \frac{mn + n^2}{m^2 - mn}\, x + \frac{n^3}{m^2 - mn}$$

and $y = \dfrac{mn - n^2}{mn + m^2}\, x + \dfrac{m^3}{mn + m^2}$

Let $m_1$ and $m_2$ be the slopes of given lines.

$$\therefore \quad m_1 = \frac{mn + n^2}{m^2 - mn}$$

and $m_2 = \dfrac{mn - n^2}{mn + m^2}$

If $\theta$ is the angle between these lines, then

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\dfrac{mn + n^2}{m^2 - mn} - \dfrac{mn - n^2}{mn + m^2}}{1 + \dfrac{mn + n^2}{m^2 - mn} \cdot \dfrac{mn - n^2}{mn + n^2}}$$

$$= \frac{(mn+n^2)(mn+m^2)-(mn-n^2)(m^2-mn)}{(m^2-mn)(mn+m^2)+(mn+n^2)(mn-n^2)}$$

$$= \frac{\begin{array}{l}m^2n^2+m^3n+mn^3+m^2n^2-m^3n+m^2n^2\\ \quad +\,m^2n^2-mn^3\end{array}}{\begin{array}{l}m^3n+m^4-m^2n^2-m^3n^2-m^2n^2-mn^3\\ \quad +\,mn^3-n^4\end{array}}$$
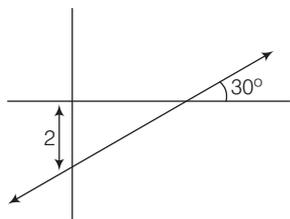
$$= \frac{4m^2n^2}{m^4 - n^4}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{4m^2n^2}{m^4 - n^4}\right)$$

**53.** What is the equation of the straight line cutting-off an intercept 2 from the negative direction of $Y$-axis and inclined at $30°$ with the positive direction of $X$-axis ?
(a) $x - 2\sqrt{3}y - 3\sqrt{2} = 0$
(b) $x + 2\sqrt{3}y - 3\sqrt{2} = 0$
(c) $x + \sqrt{3}y - 2\sqrt{3} = 0$
(d) $x - \sqrt{3}y - 2\sqrt{3} = 0$

**(d)** From the given figure, it is clear that



Slope of line $= \tan 30° = \dfrac{1}{\sqrt{3}}$ and line passes through the point $(0, -2)$.

$\therefore$ Equation of line is

$$y - (-2) = \frac{1}{\sqrt{3}}\, (x - 0)$$

$$\Rightarrow \qquad y + 2 = \frac{1}{\sqrt{3}}\, x$$

$$\Rightarrow \quad \sqrt{3}\, y + 2\sqrt{3} = x$$
$$\Rightarrow \quad x - \sqrt{3}y - 2\sqrt{3} = 0$$

**54.** What is the equation of the line passing through the point of intersection of the lines $x + 2y - 3 = 0$ and $2x - y + 5 = 0$ and parallel to the line $y - x + 10 = 0$ ?
(a) $7x - 7y + 18 = 0$
(b) $5x - 7y + 18 = 0$
(c) $5x - 5y + 18 = 0$
(d) $x - y + 5 = 0$

**(c)** Equation of line passing through intersection point of lines $x + 2y - 3 = 0$ and $2x - y + 5 = 0$ is

$$x + 2y - 3 + \lambda\,(2x - y + 5) = 0 \quad \dots \text{(i)}$$
$$\Rightarrow (1 + 2\lambda)x + (2 - \lambda)y + 5\lambda - 3 = 0$$

$\therefore$ Slope of above line $= -\dfrac{(1 + 2\lambda)}{(2 - \lambda)}$

Since line is parallel to $y - x + 10 = 0$

$$= \frac{-(1 + 2\lambda)}{(2 - \lambda)} = \frac{-(-1)}{1}$$

$$\Rightarrow \quad -(1 + 2\lambda) = 2 - \lambda$$
$$\Rightarrow \quad -1 - 2\lambda = 2 - \lambda$$
$$\Rightarrow \qquad -\lambda = 3$$
$$\Rightarrow \qquad \lambda = -3$$

Putting $\lambda = -3$ in Eq. (i), we get

$$x + 2y - 3 - 3\,(2x - y + 5) = 0$$
$$\Rightarrow \qquad 5x - 5y + 18 = 0$$

Which is equation of required line.

**55.** Consider the following statements
I. The length $p$ of the perpendicular from the origin to the line $ax + by = c$ satisfies the relation

$$p^2 = \frac{c^2}{a^2 + b^2}.$$

II. The length $p$ of the perpendicular from the origin to the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ satisfied the relation

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

III. The length $p$ of the perpendicular from the origin to the line $y = mx + c$ satisfies the relation

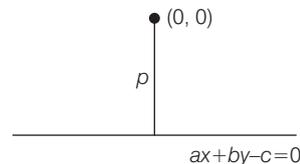$$\frac{1}{p^2} = \frac{1 + m^2 + c^2}{c^2}.$$

*Which of the above is/are correct?*
(a) I, II and III
(b) I only
(c) I and II
(d) II only

**(c)** We know that $x_1$ distance of a point $(x_1\ y_1)$ from the line $Ax + By + C = 0$ is given as

$$\text{Distance} = \left[\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right]$$

**Statement I**



$$\therefore \quad p = \left|\frac{a \cdot 0 + b \cdot 0 - c}{\sqrt{a^2 + b^2}}\right|$$

$$\Rightarrow \quad p = \frac{c}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \quad p^2 = \frac{c^2}{a^2 + b^2}$$

It is true.

**Statement II**
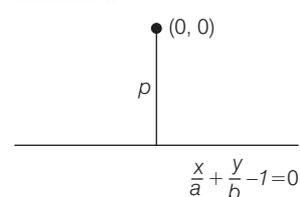


$$\therefore \quad p = \left|\frac{\dfrac{0}{a} + \dfrac{0}{b} - 1}{\sqrt{\left(\dfrac{1}{a}\right)^2 + \left(\dfrac{1}{b}\right)^2}}\right|$$

$$\Rightarrow \quad p = \frac{1}{\sqrt{\dfrac{1}{a^2} + \dfrac{1}{b^2}}}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

It is true.

**Statement III**
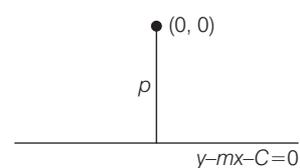


$$\therefore \quad p = \left|\frac{0 - m \times 0 - c}{\sqrt{(-m)^2 + (1)^2}}\right|$$

$$\Rightarrow \quad p = \frac{c}{\sqrt{m^2 + 1}}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1 + m^2}{c^2}$$

It is false.

**56.** What is the equation of the ellipse whose vertices are $(\pm 5, 0)$ and foci are at $(\pm 4, 0)$?

(a) $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

(b) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

(c) $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

(d) $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

⊘ **(a)** We have,

Vertices = $(\pm 5, 0)$ and Foci = $(\pm 4, 0)$

∴ $a = 5$ and $ae = 4$

[∵ vertex = $(\pm a, 0)$ and focus $(\pm ae, 0)$]

$\Rightarrow e = \dfrac{4}{5}$

Now, $e = \sqrt{1 - \dfrac{b^2}{a^2}}$

$\Rightarrow \left(\dfrac{4}{5}\right)^2 = 1 - \dfrac{b^2}{(5)^2}$    [∵ $a = 5$]

$\Rightarrow \dfrac{16}{25} = 1 - \dfrac{b^2}{25}$

$\Rightarrow 16 = 25 - b^2$

$\Rightarrow b^2 = 9$

$\Rightarrow b = 3$

∴ Equation of ellipse is

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

**57.** What is the equation of the straight line passing through the point $(2, 3)$ and making an intercept on the positive $Y$-axis equal to twice its intercept on the positive $X$-axis?

(a) $2x + y = 5$    (b) $2x + y = 7$

(c) $x + 2y = 7$    (d) $2x - y = 1$

⊘ **(b)** Let the equation of line be

$\dfrac{x}{a} + \dfrac{y}{b} = 1$

It is given that, $b = 2a$ and line passes through the point $(2, 3)$.

∴ $\dfrac{2}{a} + \dfrac{3}{2a} = 1$

$\Rightarrow \dfrac{4 + 3}{2a} = 1$

$\Rightarrow 7 = 2a$

$\Rightarrow a = \dfrac{7}{2}$

$\Rightarrow b = 2a = 2 \times \dfrac{7}{2} = 7$

∴ Equation of line is $\dfrac{x}{7/2} + \dfrac{y}{7} = 1$

$\Rightarrow 2x + y = 7$

**58.** Let the coordinates of the points $A, B, C$ be $(1, 8, 4), (0, -11, 4)$ and $(2, -3, 1)$ respectively. What are the coordinates of the point $D$ which is the foot of the perpendicular from $A$ on $BC$?
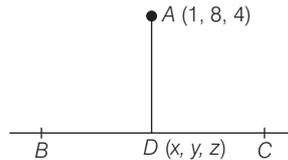
(a) $(3, 4, -2)$    (b) $(4, -2, 5)$

(c) $(4, 5, -2)$    (d) $(2, 4, 5)$

⊘ **(c)** We have,

$A$ $(1, 8, 4), B$ $(0, -11, 4)$ and $C$ $(2, -3, 1)$

∴ Equation of $BC$ is

$\dfrac{x - 0}{2 - 0} = \dfrac{y + 11}{-3 + 11} = \dfrac{z - 4}{1 - 4}$

$\Rightarrow \dfrac{x}{2} = \dfrac{y + 11}{8} = \dfrac{z - 4}{-3} = \lambda$   [say]

$\Rightarrow x = 2\lambda, y = 8\lambda - 11, z = -3\lambda + 4$



Now, DR's of

$AD = <2\lambda - 1, 8\lambda - 11 - 8, -3\lambda + 4 - 4>$

$= <2\lambda - 1, 8\lambda - 19, -3\lambda>$

Since, $AD \perp BC$

∴ $2(2\lambda - 1) + 8(8\lambda - 19) - 3(-3\lambda) = 0$

$\Rightarrow 4\lambda - 2 + 64\lambda - 152 + 9\lambda = 0$

$\Rightarrow 77\lambda = 154$

$\Rightarrow \lambda = 2$

∴ Coordinates of

$D = (2 \times 2, 8 \times 2 - 11, -3 \times 2 + 4)$

$= (4, 5, -2)$

**59.** What is the equation of the plane passing through the points $(-2, 6, -6)$, $(-3, 10, -9)$ and $(-5, 0, -6)$?

(a) $2x - y - 2z = 2$

(b) $2x + y + 3z = 3$

(c) $x + y + z = 6$

(d) $x - y - z = 3$

⊘ **(a)** Equation of the plane passing through three points $(x_1, y_1, z_1),$ $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ is

$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

Equation of plane is

$\begin{vmatrix} x - (-2) & y - 6 & z - (-6) \\ -3 - (-2) & 10 - 6 & -9 - (-6) \\ -5 - (-2) & 0 - 6 & -6 - (-6) \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x + 2 & y - 6 & z + 6 \\ -1 & 4 & -3 \\ -3 & -6 & 0 \end{vmatrix} = 0$

$\Rightarrow (x + 2)[4 \times 0 - (-6)(-3)] - (y - 6)$

$[(-1)(0) - (-3)(-3)] + (z + 6)[(-1)(-6)$

$- (-3)(4)] = 0$

$\Rightarrow (x + 2)(-18) - (y - 6)(-9) +$

$(z + 6)(18) = 0$

$\Rightarrow 2(x + 2) - (y - 6) - 2(z + 6) = 0$

$\Rightarrow 2x - y - 2z - 2 = 0$

$\Rightarrow 2x - y - 2z = 2$

**60.** A sphere of constant radius $r$ through the origin intersects the coordinate axes in $A$, $B$ and $C$. What is the locus of the centroid of the $\Delta$ $ABC$?

(a) $x^2 + y^2 + z^2 = r^2$

(b) $x^2 + y^2 + z^2 = 4r^2$

(c) $9(x^2 + y^2 + z^2) = 4r^2$

(d) $3(x^2 + y^2 + z^2) = 2r^2$

⊘ **(c)** Let $A$ $(a, 0, 0), B$ $(0, b, 0)$ and $C$ $(0, 0, c)$

∴ Equation of sphere passing through $A, B, C$ and origin is

$x^2 + y^2 + z^2 - ax - by - cz = 0$

∴ Radius $= r = \sqrt{\dfrac{a^2}{4} + \dfrac{b^2}{4} + \dfrac{c^2}{4}}$

$\Rightarrow 4r^2 = a^2 + b^2 + c^2$    ... (i)

Let $(\alpha, \beta, \gamma)$ be the centroid of triangle.

∴ $\alpha = \dfrac{a + 0 + 0}{3}, \beta = \dfrac{0 + b + 0}{3},$

$\gamma = \dfrac{0 + 0 + c}{3}$

$\Rightarrow a = 3\alpha, \beta = 3\beta, c + 3\gamma$    ... (ii)

From Eqs. (i) and (ii), we have

$(3\alpha)^2 + (3\beta)^2 + (3\gamma)^2 = 4r^2$

$\Rightarrow 9(\alpha^2 + \beta^2 + \gamma^2) = 4r^2$

∴ Locus of the centroid of $\Delta ABC$ is

$9(x^2 + y^2 + z^2) = 4r^2 z$

**61.** The coordinates of the vertices $P, Q$ and $R$ of a triangle $PQR$ are $(1, -1, 1), (3, -2, 2)$ and $(0, 2, 6)$ respectively. If $\angle RQP = 9$, then what is $\angle PRQ$ equal to?

(a) $30° + \theta$    (b) $45° - \theta$

(c) $60° - \theta$    (d) $90° - \theta$

⊘ **(d)**
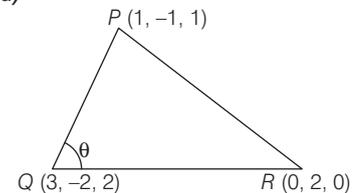


DR's of

$PQ = <3 - 1, -2 - (-1), 2 - 1>$

$<a_1, b_1, c_1> = <2, -1, 1>$

and DR's of

$PR = <0 - 1, 2 - (-1), 6 - 1>$

$<a_2, b_2, c_2> = <-1, 3, 5>$

Now, $a_1 a_2 + b_1 b_2 + c_1 c_2 =$

$2 \times (-1) + (-1) \times 3 + 1 \times 5$

$= -2 - 3 + 5 = 0$

$\therefore$ $PQ \perp PR$

$\Rightarrow$ $\angle QPR = 90°$

Now, by angle sum property

$\angle PQR + \angle QPR + \angle PRQ = 180°$

$\Rightarrow$ $\theta + 90° + \angle PRQ = 180°$

$\Rightarrow$ $\angle PRQ = 90° - \theta$

**62.** The perpendiculars that fall from any point of the straight line $2x + 11y = 5$ upon the two straight lines $24x + 7y = 20$ and $4x - 3y = 2$ are

(a) 12 and 4 respectively
(b) 11 and 5 respectively
(c) Equal to each other
(d) Not equal to each other

⊗ **(c)** Let $(-3, 1)$ be a point on $2x + 11y = 5$

Now, perpendicular from $(-3, 1)$ on $24x + 7y = 20$

$= \left| \dfrac{24(-3) + 7(1) - 20}{\sqrt{(24)^2 + (7)^2}} \right|$

$= \left| \dfrac{-72 + 7 - 20}{\sqrt{576 + 49}} \right|$

$= \left| \dfrac{-85}{25} \right| = \dfrac{17}{5}$

Again, perpendicular from $(-3, 1)$ on $4x - 3y = 2$

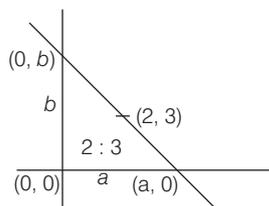$= \left| \dfrac{4(-3) - 3(1) - 2}{\sqrt{4^2 + (-3)^2}} \right|$

$= \left| \dfrac{-12 - 3 - 2}{\sqrt{16 + 9}} \right| = \dfrac{17}{5}$

$\therefore$ Both perpendicular are equal to each other.

**63.** The equation of the line, when the portion of it intercepted between the axes is divided by the point (2, 3) in the ratio of $3 : 2$, is

(a) Either $x + y = 4$ or $9x + y = 12$
(b) Either $x + y = 5$ or $4x + 9y = 30$
(c) Either $x + y = 4$ or $x + 9y = 12$
(d) Either $x + y = 5$ or $9x + 4y = 30$

⊗ **(d) Case I**



From above figure,

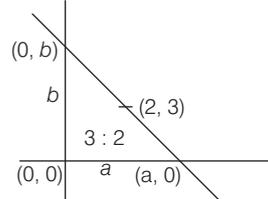$\dfrac{2a + 3 \times 0}{2 + 3} = 2$ and $\dfrac{2 \times 0 + 3b}{2 + 3} = 3$

$\Rightarrow$ $2a = 10$ and $3b = 15$

$\Rightarrow$ $a = 5$ and $b = 5$

$\therefore$ Equation of line is $\dfrac{x}{5} + \dfrac{y}{5} = 1$

$\Rightarrow$ $x + y = 5$

*Case* **II**



From above figure,

$\dfrac{3a + 2 \times 0}{3 + 2} = 2$ and $\dfrac{3 \times 0 + 2b}{3 + 2} = 3$

$\Rightarrow$ $3a = 10$ and $2b = 15$

$\Rightarrow$ $a = \dfrac{10}{3}$ and $b = \dfrac{15}{2}$

$\therefore$ Equation of line is

$\dfrac{x}{10/3} + \dfrac{y}{15/2} = 1$

$\Rightarrow$ $9x + 4y = 30$

**64.** What is the distance between the straight lines $3x + 4y = 9$ and $6x + 8y = 15$ ?

(a) $\dfrac{3}{2}$      (b) $\dfrac{3}{10}$

(c) 6      (d) 5

⊗ **(b)** Given equation of straight lines are

$3x + 4y = 9$     … (i)

and $6x + 8y = 15$

$\Rightarrow$ $3x + 4y = \dfrac{15}{2}$     … (ii)

$\therefore$ Required distance $= \dfrac{\left| 9 - \dfrac{15}{2} \right|}{\sqrt{3^2 + 4^2}}$

[$\because$ distance between two lines $ax + by = c_1$ and $ax + by = c_2$ is given

by $\left| \dfrac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right|$]

$= \dfrac{\dfrac{3}{2}}{5} = \dfrac{3}{10}$

**65.** What is the equation of the sphere whose centre is at $(-2, 3, 4)$ and radius is 6 units?

(a) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$
(b) $x^2 + y^2 + z^2 + 6x - 4y - 8z = 7$
(c) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 4$
(d) $x^2 + y^2 + z^2 + 4x + 6y + 8z = 4$

⊗ **(a)** Given, centre $= (-2, 3, 4)$ and radius $= 6$ units

Equation of the sphere having centre at $(\alpha, \beta, \gamma)$ and radius $r$ is $(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 = r^2$

So, equation of sphere

$\Rightarrow$ $\{x - (-2)\}^2 + (y - 3)^2 + (z - 4)^2 = 6^2$

$\Rightarrow (x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 36$

$\Rightarrow$ $x^2 + 4x + 4 + y^2 + 9 - 6y + z^2$
$+ 16 - 8z = 36$

$\Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z + 29 - 36 = 0$

$\Rightarrow$ $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$

**66.** If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2,$ $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k},$ then what is the acute angle between **a** and **b** ?

(a) 30°    (b) 45°    (c) 60°    (d) 90°

⊗ **(a)** Given, $|\vec{a}| = 2$

$|\vec{b}| = 7$

and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\because$ $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow$ $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

$= \dfrac{|3\hat{i} + 2\hat{j} + 6\hat{k}|}{2 \times 7}$

$= \dfrac{\sqrt{3^2 + 2^2 + 6^2}}{14} = \dfrac{\sqrt{49}}{14}$

$\Rightarrow \sin\theta = \dfrac{7}{14} = \dfrac{1}{2}$

$\Rightarrow \sin\theta = \sin 30°$

$\Rightarrow$ $\theta = 30°$

**67.** Let $\vec{p}$ and $\vec{q}$ be the position vectors of the points $P$ and $Q$ respectively with respect to origin O. The points $R$ and $S$ divide $PQ$ internally and externally respectively in the ratio $2 : 3$. If $\overrightarrow{OR}$ and $\overrightarrow{OS}$ are perpendicular, then which one of the following is correct?

(a) $9p^2 = 4q^2$    (b) $4p^2 = 9q^2$
(c) $9p = 4q$    (d) $4p = 9q$

⊗ **(a)** The points $R$ and $S$ divider s $PQ$ internally and externally respectively in the ratio 2 : 3. The position vectors of $R$ and $S$ are

$\dfrac{3\vec{p} + 2\vec{q}}{5}$ and $3\vec{p} - 2\vec{q}$ respectively.

$\overrightarrow{OR} = \dfrac{3\vec{p} - 2\vec{q}}{5}$

$\overrightarrow{OS} = 3\vec{p} - 2\vec{q}$

Now, $\overrightarrow{OR} \perp \overrightarrow{OS}$

$\Rightarrow$ $\overrightarrow{OR} \cdot \overrightarrow{OS} = 0$

$\Rightarrow$ $\left( \dfrac{3\vec{p} + 2\vec{q}}{5} \right) \cdot (3\vec{p} - 2\vec{q}) = 0$

$\Rightarrow$ $(3\vec{p} + 2\vec{q}) \cdot (3\vec{p} - 2\vec{q}) = 0$

$\Rightarrow \quad 9\vec{\mathbf{p}}.\vec{\mathbf{p}} - 6\vec{\mathbf{p}}.\vec{\mathbf{q}} + 6\vec{\mathbf{q}}.\vec{\mathbf{p}} - 4\vec{\mathbf{q}}.\vec{\mathbf{q}} = 0$

$\Rightarrow \quad 9|\vec{\mathbf{p}}|^2 - 4|\vec{\mathbf{q}}|^2 = 0$

$[\because \vec{\mathbf{a}}.\vec{\mathbf{a}} = |\vec{\mathbf{a}}|^2 \text{ and } \vec{\mathbf{a}}.\vec{\mathbf{b}} = \vec{\mathbf{b}}.\vec{\mathbf{a}}]$

$\Rightarrow \quad 9|\vec{\mathbf{p}}|^2 = 4|\vec{\mathbf{q}}|^2$

$\Rightarrow \quad 9p^2 = 4q^2$

**68.** What is the moment about the point $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ of a force represented by $3\hat{\mathbf{i}} + \hat{\mathbf{k}}$ acting through the point $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ ?

(a) $-3\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

(c) $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

◈ **(a)** Given that,

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

and $\vec{\mathbf{F}} = 3\hat{\mathbf{i}} + \hat{\mathbf{k}}$

∴ Moment $\vec{\tau} = \vec{\mathbf{r}} + \vec{\mathbf{F}}$

$= (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} + \hat{\mathbf{k}})$

$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 4 \\ 3 & 0 & 1 \end{vmatrix}$

$= \hat{\mathbf{i}}(-3 - 0) - \hat{\mathbf{j}}(1 - 12) + \hat{\mathbf{k}}(0 + 9)$

$= -3\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

**69.** If $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 3\vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} = \lambda\ (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$, then what is the value of $\lambda$ ?

(a) 2      (b) 3

(c) 4      (d) 6

◈ **(d)** Given that,

$\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 3\vec{\mathbf{c}} = 0$

$\Rightarrow \quad \vec{\mathbf{a}} + 2\vec{\mathbf{b}} = -3\vec{\mathbf{c}}$

$\Rightarrow \quad (\vec{\mathbf{a}} + 2\vec{\mathbf{b}}) \times \vec{\mathbf{b}} = -3\vec{\mathbf{c}} \times \vec{\mathbf{b}}$

$\Rightarrow \quad \vec{\mathbf{a}} \times \vec{\mathbf{b}} + 2\vec{\mathbf{b}} \times \vec{\mathbf{b}} = 3 (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$[\because \vec{\mathbf{c}} \times \vec{\mathbf{b}} = -\vec{\mathbf{b}} \times \vec{\mathbf{c}}]$

$\Rightarrow \quad \vec{\mathbf{a}} \times \vec{\mathbf{b}} = 3 (\vec{\mathbf{b}} \times \vec{\mathbf{c}})\ [\vec{\mathbf{b}} \times \vec{\mathbf{b}} = 0]$ ... (i)

Again $\quad 3\vec{\mathbf{c}} + \vec{\mathbf{a}} = -2\vec{\mathbf{b}}$

$\Rightarrow \quad (3\vec{\mathbf{c}} + \vec{\mathbf{a}}) \times \vec{\mathbf{a}} = -2\vec{\mathbf{b}} \times \vec{\mathbf{a}}$

$\Rightarrow \quad 3\vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{a}} = 2 (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$\Rightarrow \quad 3(\vec{\mathbf{c}} \times \vec{\mathbf{a}}) = 2 (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$\Rightarrow \quad 3(\vec{\mathbf{c}} \times \vec{\mathbf{a}}) = 6 (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

[from Eq. (i)]

$\Rightarrow \quad \vec{\mathbf{c}} \times \vec{\mathbf{a}} = 2 (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ ... (ii)

---

Now, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} = \lambda\ (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= 3\ (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + 2\ (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= 6(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ [from Eqs. (i) and (ii)]

on comparing, we get

$\lambda = 6$

**70.** If the vectors $\vec{\mathbf{K}}$ and $\vec{\mathbf{A}}$ are parallel to each other, then what is $k\vec{\mathbf{K}} \times \vec{\mathbf{A}}$ equal to ?

(a) $k^2\vec{\mathbf{A}}$    (b) $\vec{\mathbf{0}}$    (c) $-k^2\vec{\mathbf{A}}$   (d) $\vec{\mathbf{A}}$

◈ **(b)** Since, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = 0$, if $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are parallel. So, $k\vec{\mathbf{K}} \times \vec{\mathbf{A}} = 0$ if $\vec{\mathbf{K}}$ and $\vec{\mathbf{A}}$ are parallel to each other.

**71.** When one of the following is correct in respect of the function $f : \mathbf{R} \to \mathbf{R}^+$ defined as $f(x) = |x + 1|$ ?

(a) $f(x^2) = [f(x)]^2$

(b) $f(|x|) = |f(x)|$

(c) $f(x + y) = f(x) + f(y)$

(d) None of the above

◈ **(d)** Given, $f(x) = |x + 1|$

By checking the options, we get

(a) $\qquad f(x^2) = |x^2 + 1|$

$\{f(x)\}^2 = (x + 1)^2$

Which implies that $f(x^2) \ne \{f(x)\}^2$

(b) $\qquad f(|x|) = ||x| + 1|$

$|f(x)| = ||x + 1| = |x + 1|$

which implies that $f(|x|) \ne |f(x)|$

(c) $\qquad f(x + y) = |x + y + 1|$

$f(x) + f(y) = |x + 1| + |y + 1|$

which implies that $f(x + y) \ne f(x) + f(y)$

So, option (d) is correct.

**72.** Suppose $f : \mathbf{R} \to \mathbf{R}$ is defined by $f(x) = \dfrac{x^2}{1 + x^2}$. What is the range of the function?

(a) [0, 1)   (b) [0, 1]   (c) (0, 1]   (d) (0, 1)

◈ **(a)** Let $f(x) = y$

Then, $y \ge 0$ and $f(x) = y$

∴ $\qquad \dfrac{x^2}{x^2 + 1} = y$

$\Rightarrow \quad \dfrac{x^2 + 1}{x^2} = \dfrac{1}{y}$ for $y > 0$

$\Rightarrow \quad \dfrac{1}{x^2} = \dfrac{1 - y}{y}$

$\Rightarrow \quad x = \sqrt{\dfrac{y}{1 - y}}$

Now, $\sqrt{\dfrac{y}{1 - y}}$ is real $\Rightarrow \dfrac{y}{1 - y} \ge 0$

$\Rightarrow 0 \le y < 1$

So, Range of $f(x)$ is [0, 1).

---

**73.** If $f(x) = |x| + |x - 1|$, then which one of the following is correct?

(a) $f(x)$ is continuous at $x = 0$ at $x = 1$

(b) $f(x)$ is continuous at $x = 0$ but not at $x = 1$

(c) $f(x)$ is continuous at $x = 1$ but not at $x = 0$

(d) $f(x)$ is neither continuous at $x = 0$ nor at $x = 1$

◈ **(a)** We have,

$f(x) = |x| + |x - 1|$

$\Rightarrow \quad f(x) = \begin{cases} -2x + 1, & x < 0 \\ x - x + 1, & 0 \le x < 1 \\ x + x - 1, & x \ge 1 \end{cases}$

$\Rightarrow \quad f(x) = \begin{cases} -2x + 1, & x < 0 \\ 1, & 0 \le x < 1 \\ 2x - 1, & x \ge 1 \end{cases}$

Clearly, $\lim\limits_{x \to 0^{-1}} f(x) = 1 = \lim\limits_{x \to 0^+} f(x)$

and $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x - 1^+} f(x)$.

So, $f(x)$ is continuous at $x = 0, 1$.

**74.** Consider the function

$f(x) = \begin{cases} x^2\ \text{In}\,|x| & x \ne 0 \\ 0 & x = 0 \end{cases}$ What is $f'(0)$ equal to ?

(a) 0             (b) 1

(c) $-1$        (d) It does not exist

◈ **(a)** Given function is

$f(x) = \begin{cases} x^2\ln|x|, & x \ne 0 \\ 0, & x = 0 \end{cases}$

∴ $f'(0) = \lim\limits_{h \to 0} \dfrac{f(h) - f(0)}{h}$

$= \lim\limits_{h \to 0} \dfrac{h^2 \log h}{h}$

$= \lim\limits_{h \to 0} h \log h = 0$

**75.** What is the area of the region bounded by the parabolas $y^2 = 6(x - 1)$ and $y^2 = 3x$ ?

(a) $\dfrac{\sqrt{6}}{3}$   (b) $\dfrac{2\sqrt{6}}{3}$   (c) $\dfrac{4\sqrt{6}}{3}$   (d) $\dfrac{5\sqrt{6}}{3}$
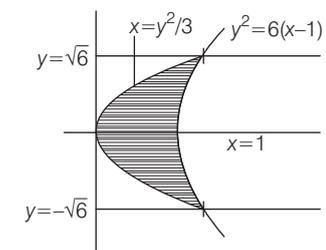
◈ **(c)** Given,

$y^2 = 6(x - 1)$      ... (i)

and $y^2 = 3x$      ... (ii)

on solving Eqs. (i) and (ii), we get

$x = 2$ and $y = \pm \sqrt{6}$

∴ Required area $= \int_{-\sqrt{6}}^{\sqrt{6}} \left(1 + \frac{y^2}{6} - \frac{y^2}{3}\right) dy$

$= 2 \int_{0}^{\sqrt{6}} \left(1 - \frac{y^2}{6}\right) dy$

$= 2 \left[y - \frac{y^3}{18}\right]_{0}^{\sqrt{6}}$

$= 2 \times \left[\frac{18y - y^3}{18}\right]_{0}^{\sqrt{6}}$

$= 2 \times \left[\frac{18\sqrt{6} - 6\sqrt{6}}{18}\right]$

$= \frac{12\sqrt{6}}{9} = \frac{4\sqrt{6}}{3}$

**Directions** (Q. Nos. 76-78) *Consider the following information for the next three items that follow Three sides of a trapezium are each equal to 6 cm. Let* $\alpha \varepsilon \left(0, \frac{\pi}{2}\right)$ *be the angle between a pair of adjacent sides.*

**76.** If the area of the trapezium is the maximum possible, then what is α equal to ?

(a) $\frac{\pi}{6}$  (b) $\frac{\pi}{4}$  (c) $\frac{\pi}{3}$  (d) $\frac{2\pi}{5}$

⊛ **(c)**
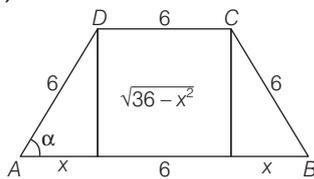


∴ Area $= A = \frac{1}{2}(6 + 6 + 2x) \sqrt{36 - x^2}$

$= (6 + x) \sqrt{36 - x^2}$

∴ $\frac{d(A)}{dx} = \frac{d}{dx}[(6 + x) \sqrt{36 - x^2}]$

$= (6 + x)\left[\frac{-2x}{2\sqrt{36 - x^2}}\right] + \sqrt{36 - x^2}$

$= \sqrt{36 - x^2} - \frac{x(6 + x)}{\sqrt{36 - x^2}}$

$= \frac{36 - 6x - 2x^2}{\sqrt{36 - x^2}}$

For maximum area,

$$\frac{dA}{dx} = 0$$

$\Rightarrow \quad 36 - 6x - 2x^2 = 0$

$\Rightarrow \quad 2x^2 + 6x - 36 = 0$

$\Rightarrow \quad x^2 + 3x - 18 = 0$

$\Rightarrow \quad x^2 + 6x - 3x - 18 = 0$

$\Rightarrow \quad x(x + 6) - 3(x + 6) = 0$

$\Rightarrow \quad (x + 6)(x - 3) = 0$

$\Rightarrow \quad x = 3, -6$

Again, on differentiating it

$\frac{d^2 A}{dx^2} = \frac{(-6 - 4x)\sqrt{36 - x^2} - \frac{(36 - 6x - 2x^2)(-2x)}{2\sqrt{36 - x^2}}}{(36 - x^2)}$

At $x = 3, \frac{d^2 A}{dx^2} = -6 - 12 = -18$

∴ $\frac{d^2 A}{dx^2} < 0$

So, at $x = 3$ is maximum.

Now, $\cos\alpha = \frac{x}{6} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \quad \cos\alpha = \cos\frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{3}$

**77.** If the area of the trapezium is maximum, what is the length of the fourth side?

(a) 8 cm  (b) 9 cm
(c) 10 cm  (d) 12 cm

⊛ **(d)** So, fourth side $= x + 6 + x$

$= 3 + 6 + 3 = 12$

**78.** What is the maximum area of the trapezium?

(a) $36\sqrt{3}$ cm$^2$  (b) $30\sqrt{3}$ cm$^2$
(c) $27\sqrt{3}$ cm$^2$  (d) $24\sqrt{3}$ cm$^2$

⊛ **(c)** Maximum area $= (6 + x) \sqrt{36 - x^2}$

$= (6 + 3) \sqrt{36 - 9}$

$= 9 \times \sqrt{27} = 9 \times \sqrt{27}$

$= 9 \times 3\sqrt{3} = 27\sqrt{3}$ cm$^2$

**79.** What is $\int_{0}^{\pi} e^x \sin x\, dx$ equal to ?

(a) $\frac{e^{\pi} + 1}{2}$  (b) $\frac{e^{\pi} - 1}{2}$

(c) $e^{\pi} + 1$  (d) $\frac{e^{\pi} + 1}{4}$

⊛ **(a)** Let $I = \int_{0}^{\pi} e^x \sin x\, dx$

$= [\sin x \cdot e^x]_{0}^{\pi} - \int_{0}^{\pi} \left[\frac{d}{dx}\{\sin x\} \cdot e^x\right] dx$

$= [\sin x \cdot e^x]_{0}^{\pi} - \int_{0}^{\pi} \cos x \cdot e^x dx$

$= 0 - \left\{[\cos x \cdot e^x]_{0}^{\pi} + \int_{0}^{\pi} \sin x \cdot e^x dx\right\}$

$\Rightarrow \quad I = -[-e^{\pi} - 1] - I$

$\Rightarrow \quad I + I = e^{\pi} + 1$

$\Rightarrow \quad 2I = e^{\pi} + 1$

$\Rightarrow \quad I = \frac{e^{\pi} + 1}{2}$

**80.** If $f(x) = \frac{x - 9}{x^2 - 2x - 3}, \quad x \neq 3$ is continuous at $x = 3$, then which one of the following is correct?

(a) $f(3) = 0$  (b) $f(3) = 1.5$
(c) $f(3) = 3$  (d) $f(3) = -1.5$

⊛ **(b)** Since, $f(x)$ is continuous at $x = 3$

Therefore, $f(3) = \lim_{x \to 3} \frac{x^2 - 9}{x^2 - 2x - 3}$

Applying L' Hospital rule

$f(3) = \lim_{x \to 3} \frac{\frac{d}{dx}(x^2 - 9)}{\frac{d}{dx}(x^2 - 2x - 3)}$

$= \lim_{x \to 3} \frac{2x}{2x - 2}$

$= \frac{2 \cdot 3}{2 \cdot 3 - 2} = \frac{6}{4} = 1.5$

**81.** What is $\int_{1}^{e} x \ln x\, dx$ equal to ?

(a) $\frac{e + 1}{4}$  (b) $\frac{e^2 + 1}{4}$

(c) $\frac{e - 1}{4}$  (d) $\frac{e^2 - 1}{4}$

⊛ **(b)** Let $1 = \int_{1}^{e} x \log x\, dx$

$= [\log x \cdot \int x\, dx]_{1}^{e} - \int_{1}^{e} \left[\frac{d}{dx}\{\log x\} \cdot \int x\, dx\right] dx$

$= \left[\log x \cdot \frac{x^2}{2}\right]_{1}^{e} - \int_{1}^{e} \frac{1}{x} \cdot \frac{x^2}{2} dx$

$= \frac{e^2}{2} - \frac{1}{2} \times \frac{1}{2} [x^2]_{1}^{e}$

$= \frac{e^2}{2} - \frac{[e^2 - 1]}{4}$

$= \frac{2e^2 - e^2 + 1}{4} = \frac{e^2 + 1}{4}$

**82.** What is $\int_{0}^{\sqrt{2}} [x^2]\, dx$ equal to (where [.] is the greatest integer function) ?

(a) $\sqrt{2} - 1$  (b) $1 - \sqrt{2}$
(c) $2(\sqrt{2} - 1)$  (d) $\sqrt{3} - 1$

⊛ **(a)** Let $I = \int_{0}^{\sqrt{2}} [x^2]\, dx$

$= \int_{0}^{1} [x^2]\, dx + \int_{1}^{\sqrt{2}} [x^2]\, dx$

$= \int_{0}^{1} 0\, dx + \int_{1}^{\sqrt{2}} 1\, dx$

$\left[\because [x] = \begin{cases} 0 & 0 \leq x < 1 \\ 1 & 1 \leq x < \sqrt{2} \end{cases}\right]$

$= 0 + [x]_{1}^{\sqrt{2}} = \sqrt{2} - 1$

**83.** What is the maximum value of $16\sin\theta - 12\sin^2\theta$ ?

(a) $\dfrac{3}{4}$  (b) $\dfrac{4}{3}$  (c) $\dfrac{16}{3}$  (d) 4

**(c)** Let $f(x) = 16\sin\theta - 12\sin^2\theta$

$$= -12\left[\sin^2\theta - \frac{16}{12}\sin\theta\right]$$

$$= -12\left[\sin^2\theta - \frac{4}{3}\sin\theta\right]$$

$$= -12\left[\left(\sin\theta - \frac{2}{3}\right)^2 - \frac{4}{9}\right]$$

$$= -12\left(\sin\theta - \frac{2}{3}\right)^2 + \frac{16}{3}$$

$$\therefore \quad f(x) \le \frac{16}{3}$$

$\therefore$ Maximum value of $f(x) = \dfrac{16}{3}$

**84.** If $f : \mathbf{R} \to S$ defined by $f(x) = 4\sin x - 3\cos x + 1$ is onto, then what is $S$ equal to ?

(a) $[-5, 5]$  (b) $(-5, 5)$
(c) $(-4, 6)$  (d) $[-4, 6]$

**(d)** We have

$$f(x) = 4\sin x - 3\cos x + 1$$

We know that,

$$-\sqrt{4^2 + (-3)^2} \le 4\sin x - 3\cos x$$
$$\le \sqrt{4^2 + (-3)^2}$$

$[\because -\sqrt{a^2 + b^2} \le a\sin x + b\cos x$

$$\le \sqrt{a^2 + b^2}]$$

$\Rightarrow -5 \le 4\sin x - 3\cos x \le 5$
$\Rightarrow -5 + 1 \le 4\sin x - 3\cos x + 1 \le 5 + 1$
$\Rightarrow -4 \le f(x) \le 6$
$\therefore \quad f(x) \in [-4, 6]$

since, $f(x)$ is onto.
$\therefore \quad S = \text{Range of } f = [-4, 6]$

**85.** For $f$ to be a function, what is the domain of $f$, if $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ ?

(a) $(-\infty, 0)$  (b) $(0, \infty)$
(c) $(-\infty, \infty)$  (d) $(-\infty, 0)$

**(a)** We have,

$$f(x) = \frac{1}{\sqrt{|x| - x}}$$

$f(x)$ is defined, if

$|x| - x > 0$
$\Rightarrow \qquad |x| > x$
**Case I** $x > 0$
$\therefore \qquad x > x \, [\because |x| = x, x > 0]$
which is not possible
**Case II** $\quad x < 0$
$\therefore \quad -x > x \, [\because |x| = -x, x < 0]$
$\Rightarrow \qquad\qquad 0 > 2x$
$\Rightarrow \qquad\qquad x < 0$

Which is possible
$\therefore$ Domain of $f(x) = (-\infty, 0)$

**86.** What is the solution of the differential equation $x\,dy - y\,dx = 0$ ?

(a) $xy = c$
(b) $y = cx$
(c) $x + y = c$
(d) $x - y = c$

**(b)** Given differentiation equation

$$xdy - ydx = 0$$
$$\Rightarrow \qquad\qquad xdy = y\,dx$$

Variable seprate on both sides

$$\Rightarrow \qquad\qquad \frac{dy}{y} = \frac{dx}{x}$$

On integration both sides, we get

$$\Rightarrow \qquad \int\frac{dy}{y} = \int\frac{dx}{x}$$

$\Rightarrow \log y = \log x + \log c$

[where $\log c$ is integrating constant]

$\Rightarrow \qquad y = xc$
$\Rightarrow y = cx$

**87.** What is the derivative of the function

$$f(x) = e^{\tan x} + \ln(\sec x) - e^{\ln x} \quad \text{at}$$

$$x = \frac{\pi}{4} \; ?$$

(a) $\dfrac{e}{2}$  (b) $e$  (c) $2e$  (d) $4e$

**(c)** We have,

$$F(x) = e^{\tan x} + \log(\sec x) - e^{\log x}$$

$$f(x) = e^{\tan x} + \log(\sec x) - x$$

$$[\because a\log_a b = b]$$

On differentiating with respect to $x$ both the sides, we get

$$f'(x) = e^{\tan x}\cdot\sec^2 x + \frac{1}{\sec x}\cdot\sec x\tan x - 1$$

$$= \sec^2 x e^{\tan x} + \tan x - 1$$

$$\therefore [f'(x)]_{x = \frac{\pi}{4}} = e^{\tan\frac{\pi}{4}}\cdot\sec^2\cdot\frac{\pi}{4} + \tan\frac{\pi}{4} - 1$$

$$= e^1(\sqrt{2})^2 + 1 - 1$$

$$= e\cdot 2 + 1 - 1 = 2e$$

**88.** Which one of the following differential equations has a periodic solution?
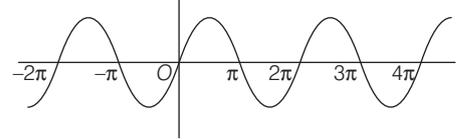
(a) $\dfrac{d^2x}{dt^2} + \mu x = 0$  (b) $\dfrac{d^2x}{dt^2} - \mu x = 0$

(c) $x\dfrac{dx}{dt} + \mu t = 0$  (d) $\dfrac{dx}{dt} + \mu xt = 0$

**(a)** $\dfrac{d^2x}{dt^2} + \mu x = 0$  is the differential equation of simple harmonic motion, which has a periodic solution.

**89.** What is the period of the the function $f(x) = \sin x$ ?

(a) $\dfrac{\pi}{4}$  (b) $\dfrac{\pi}{2}$
(c) $\pi$  (d) $2\pi$

**(d)** We have,
$$f(x) = \sin x$$



$$f(x + 2\pi) = \sin(x + 2\pi).$$
$$= \sin x$$
$$= f(x)$$
$\therefore$ Period of $f(x)$ is $2\pi$.

**90.** What is $\displaystyle\int\frac{dx}{2^x - 1}$ equal to ?

(a) $\ln(2^x - 1) + C$

(b) $\dfrac{\ln(1 - 2^{-x})}{\ln 2} + C$

(c) $\dfrac{\ln(2^{-x} - 1)}{2\ln 2} + C$

(d) $\dfrac{\ln(1 + 2^{-x})}{\ln 2} + C$

**(b)** Let $I = \displaystyle\int\frac{dx}{2^x - 1}$

$$= \int\frac{2^{-x}}{1 - 2^{-x}}\,dx$$

$$= \frac{1}{\log 2}\int\frac{2^{-x}\log 2}{1 - 2^{-x}}\,dx$$

Put $\qquad 1 - 2^{-x} = t$
$\Rightarrow \quad 2^{-x}\log 2\,dx = dt$

$$\therefore \qquad\qquad I = \frac{1}{\log 2}\int\frac{dt}{t}$$

$$= \frac{1}{\log 2}\cdot\log t + C$$

$$= \frac{\log(1 - 2^{-x})}{\log 2} + C$$

**91.** The order and degree of the differential equation $y^2 = 4a(x - a)$, where '$a$' is an arbitrary constant, are respectively

(a) 1, 2  (b) 2, 1
(c) 2, 2  (d) 1, 1

**(a)** We have,

$$y^2 = 4a(x - a) \qquad\qquad \dots\text{(i)}$$
$$\Rightarrow \qquad y^2 = 4ax - 4a^2$$

On differentiating both sides, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \qquad\qquad a = \frac{1}{2}y\frac{dy}{dx} \qquad\qquad \dots\text{(ii)}$$

On putting the values of $a$ from Eq. (ii) in Eq. (i), we get

$$y^2 = 4 \times \frac{1}{2} y \frac{dy}{dx} \left( x - \frac{1}{2} y \frac{dy}{dx} \right)$$

$$\Rightarrow \quad y^2 = 2xy \frac{dy}{dx} - y^2 \left( \frac{dy}{dx} \right)^2$$

$\therefore$ Order = 1 and degree = 2

**92.** What is the value of $\int\limits_{-\pi/4}^{\pi/4} ( \sin x - \tan x) \, dx$ ?

(a) $- \dfrac{1}{\sqrt{2}} + \ln\left( \dfrac{1}{\sqrt{2}} \right)$ (b) $\dfrac{1}{\sqrt{2}}$

(c) 0 (d) $\sqrt{2}$

$\circledcirc$ **(c)** Let $I = \int_{-\pi/4}^{\pi/4} (\sin x - \tan x) \, dx$

Let $f(x) = \sin x - \tan x$

$\therefore \quad f(-x) = \sin(-x) - \tan(-x)$

$\qquad = -\sin x + \tan x$

$[\because \sin(-\theta) = -\sin\theta, \ \tan(-\theta) = -\tan\theta]$

$\qquad = -(\sin x - \tan x)$

$\qquad = -f(x)$

$\therefore f(x)$ is odd function.

$\therefore \quad I = \int_{-\pi/4}^{\pi/4} (\sin x - \tan x) \, dx$

$\qquad = 0$

$\qquad [\because \int_{-a}^{a} f(x) \, dx = 0, \text{ if } f(x) \text{ is odd}]$

**93.** If $\int\limits_{a}^{b} x^3 \, dx = 0$ and $\int\limits_{a}^{b} x^2 \, dx = \dfrac{2}{3}$, then what are the values of $a$ and $b$ respectively?

(a) $-1, 1$ (b) 1, 1
(c) 0, 0 (d) 2, $-2$

$\circledcirc$ **(a)** We have,

$$\int\limits_{a}^{b} x^3 dx = 0$$

$$\Rightarrow \quad \left[ \frac{x^4}{4} \right]_a^b = 0$$

$$\Rightarrow \quad \frac{b^4 - a^4}{4} = 0$$

$$\Rightarrow \quad b^4 = a^4$$

$$\Rightarrow \quad b = \pm a \qquad \dots \text{(i)}$$

But $\int\limits_{a}^{b} x^3 \, dx = 0$ and $x^3$ is an odd function.

$$\therefore \qquad a = -b \qquad \dots \text{(ii)}$$

Again, $\int\limits_{a}^{b} x^2 dx = \dfrac{2}{3}$

$$\Rightarrow \quad \left[ \frac{x^3}{3} \right]_a^b = \frac{2}{3}$$

$$\Rightarrow \quad \frac{b^3 - a^3}{3} = \frac{2}{3}$$

$$\Rightarrow \quad b^3 - a^3 = 2$$

$\Rightarrow b^3 - (-b)^3 = 2 \qquad [\because \text{ from Eq. (ii)}]$

$\Rightarrow \qquad 2b^3 = 2$

$\Rightarrow \qquad b^3 = 1$

$\Rightarrow \qquad b = 1$

$\Rightarrow \qquad a = -1 \qquad \text{[from Eq. (ii)]}$

**94.** What is $\int\limits_{0}^{1} x \, (1 - x)^9 \, dx$ equal to ?

(a) $\dfrac{1}{110}$ (b) $\dfrac{1}{132}$

(c) $\dfrac{1}{148}$ (d) $\dfrac{1}{240}$

$\circledcirc$ **(a)** Let $I = \int\limits_{0}^{1} x(1 - x)^9 \, dx$

$$= \int\limits_{0}^{1} (1 - x) \, x^9 \, dx$$

$$\left[ \because \int\limits_{0}^{a} f(x) \, dx = \int\limits_{0}^{a} f(a - x) \, dx \right]$$

$$= \int\limits_{0}^{1} (x^9 - x^{10}) \, dx = \left[ \frac{x^{10}}{10} - \frac{x^{11}}{11} \right]_0^1$$

$$= \left( \frac{1}{10} - \frac{1}{11} \right) = \frac{1}{110}$$

**95.** What is $\lim\limits_{x \to 0} \dfrac{\tan x}{\sin 2x}$ equal to

(a) $\dfrac{1}{2}$

(b) 1

(c) 2

(d) Limit does not exist

$\circledcirc$ **(a)** We have,

$$\lim\limits_{x \to 0} \frac{\tan x}{\sin 2x}$$

By using L' Hospital rule, we have

$$= \lim\limits_{x \to 0} \frac{\sec^2 x}{2\cos 2x}$$

$$= \lim\limits_{x \to 0} \frac{1}{2\cos 2x \cdot \cos^2 x}$$

$$= \frac{1}{2\cos 0° \cos^2 0°} = \frac{1}{2 \times 1 \times 1}$$

$$= \frac{1}{2}$$

**96.** What is $\lim\limits_{x \to 0} \dfrac{\sqrt{2x + 3h} - \sqrt{2x}}{2h}$ equal to ?

(a) $\dfrac{1}{2\sqrt{2x}}$ (b) $\dfrac{1}{\sqrt{2x}}$

(c) $\dfrac{3}{2\sqrt{2x}}$ (d) $\dfrac{3}{4\sqrt{2x}}$

$\circledcirc$ **(d)** $\lim\limits_{x \to 0} \dfrac{\sqrt{2x + 3h} - \sqrt{2x}}{2h}$

By using, L' Hospital rule, we get

$$= \lim\limits_{h \to 0} \frac{\dfrac{1}{2\sqrt{2x + 3h}} \cdot 3 - 0}{2}$$

$$= \lim\limits_{h \to 0} \frac{3}{4\sqrt{2x + 3h}}$$

$$= \frac{3}{4} \cdot \frac{1}{\sqrt{2x + 0}} = \frac{3}{4\sqrt{2x}}$$

**97.** If $f(x)$ is an even function, where $f(x) \neq 0$, then which one of the following is correct?
(a) $f'(x)$ is an even function
(b) $f'(x)$ is an odd function
(c) $f'(x)$ may be an even or odd function depending on the type of function
(d) $f'(x)$ is a constant function

$\circledcirc$ **(b)** We have, $f(x)$ is an even function.

$\therefore \qquad f(-x) = f(x)$

On differentiating both the sides, we have

$\qquad -f'(-x) = f'(x)$

$\Rightarrow \qquad f'(-x) = -f'(x)$

$\therefore f'(x)$ is an odd function.

**98.** If $y = e^{x^2} \sin 2x$, then what is $\dfrac{dy}{dx}$ at $x = \pi$ equal to ?

(a) $(1 + \pi) e^{\pi^2}$ (b) $2\pi e^{\pi^2}$
(c) $2e^{\pi^2}$ (d) $e^{\pi^2}$

$\circledcirc$ **(c)** We have, $y = e^{x^2} \sin 2x$

On differentiating both the sides, we get

$$\frac{dy}{dx} = 2\cos 2x \, e^{x^2} + 2x e^{x^2} \sin 2x$$

$$\therefore \left( \frac{dy}{dx} \right)_{x = \pi} = 2\cos 2\pi e^{\pi^2} + 2\pi e^{\pi^2} \sin 2\pi$$

$$= 2(1) e^{\pi^2} + 2\pi e^{\pi^2} (0)$$

$$= 2e^{\pi^2}$$

**99.** What is the solution of $(1 + 2x)dy - (1 - 2y)dx = 0$ ?
(a) $x - y - 2xy = c$
(b) $y - x - 2xy = c$
(c) $y + x - 2xy = c$
(d) $x + y + 2xy = c$

$\circledcirc$ **(a)** We have,

$\qquad (1 + 2x) \, dy - (1 - 2y) \, dx = 0$

$\Rightarrow \quad (1 + 2x) \, dy = (1 - 2y) \, dx$

$\Rightarrow \qquad \dfrac{dy}{1 - 2y} = \dfrac{dx}{1 + 2x}$

On integrating both the sides, we get

$$\int \frac{dy}{1 - 2y} = \int \frac{dy}{1 + 2x}$$

$\Rightarrow \ -\dfrac{1}{2}\log(1 - 2y) = \dfrac{1}{2}\log(1 + 2x) + C'$

$\Rightarrow \ -\log(1 - 2y) = \log(1 + 2x) + 2C'$

$\Rightarrow \ \log(1 + 2x) + \log(1 - 2y) = -2C'$

$\Rightarrow \qquad \log(1 + 2x)(1 - 2y) = -2C'$

$\Rightarrow \qquad (1 + 2x)(1 - 2y) = e^{-2C'}$

$\Rightarrow \qquad 1 - 2y + 2x - 4xy = e^{-2C'}$

$\Rightarrow \qquad 2x - 2y - 4xy = e^{-2C'} - 1$

$$\Rightarrow x - y - 2xy = \frac{1}{2}(e^{-2C'} - 1)$$
$$\Rightarrow x - y - 2xy = C$$
$$[\text{where } C = \frac{1}{2}(e^{-2C'} - 1)]$$

**100.** What are the order and degree, respectively, of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^2 = y^4 + \left(\dfrac{dy}{dx}\right)^5$ ?

(a) 4, 5      (b) 2, 3
(c) 3, 2      (d) 5, 4

&#9655; **(c)** Given differential equation is
$$\left(\frac{d^3y}{dx^3}\right)^2 = y^4 + \left(\frac{dy}{dx}\right)^5$$

Here, highest order derivative is $\dfrac{d^3y}{dx^3}$.

So, order = 3, and degree = 2

**101.** In a Binomial distribution, the mean is three times its variance. What is the probability of exactly 3 successes out of 5 trials ?

(a) $\dfrac{80}{243}$      (b) $\dfrac{40}{243}$
(c) $\dfrac{20}{243}$      (d) $\dfrac{10}{243}$

&#9655; **(a)** According to the question,

Mean = 3 (Variance)
$$\Rightarrow np = 3npq$$
$$[\text{where } n = \text{number, of trials}]$$
$$\Rightarrow q = \frac{1}{3}$$
$$\because p + q = 1$$
$$\Rightarrow p + \frac{1}{3} = 1$$
$$\Rightarrow p = \frac{2}{3}$$
$$\therefore p(X = 3) = {}^5C_3 \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^2$$
$$= \frac{5!}{3!\,2!} \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^2 = \frac{80}{243}$$

**102.** Consider the following statements
I. $P(\overline{A} \cup B) = P(\overline{A})$
$$= P(B) - P(\overline{A} \cap B)$$
II. $P(A \cap \overline{B}) = P(B) - P(A \cap B)$
III. $P(A \cap B) = P(B)\,P(A \mid B)$

Which of the above statements are correct?
(a) I and II      (b) I and III
(c) II and III      (d) I, II and III

&#9655; **(b)** Here,
**Statement I**
$P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$   is correct.

**Statement II**
$P(A \cap \overline{B}) = P(B) - P(A \cap B)$ is wrong as
$P(A \cap \overline{B}) = P(A) - P(A \cap B)$.

**Statement III**
$P(A \cap B) = P(B) \times P\left(\dfrac{A}{B}\right)$ is correct. [by conditional theorem]

Hence, Statements I and III are correct.

**103.** If the correlation coefficient between $x$ and $y$ is 0.6, covariance is 27 and variance of $y$ is 25, then what is the variance of $x$?

(a) $\dfrac{9}{5}$      (b) $\dfrac{81}{25}$
(c) 9      (d) 81

&#9655; **(d)** Given, $\sigma^2(y) = 25 \Rightarrow \sigma(y) = 5$

Correlation coefficient $= \dfrac{C \cdot V}{\sigma_x \cdot \sigma_y}$

$$0.6 = \frac{27}{\sigma_x \times \sqrt{5}}$$
$$\Rightarrow \sigma_x = \frac{27}{0.6 \times 5} = \frac{27}{3} = 9$$

$\therefore$ Variance of $x = \sigma^2(x) = (9)^2 = 81$

**104.** The probabilities that a student will solve Question $A$ and Questions $B$ are 0.4 and 0.5 respectively. What is the probability that he solves atleast one of the two questions?

(a) 0.6      (b) 0.7
(c) 0.8      (d) 0.9

&#9655; **(b)** Given that,
$$P(A) = 0.4$$
and $P(B) = 0.5$
$$\because P(A \cup B) = 1 - P(A' \cap B')$$
$$= 1 - [(1 - 0.4) \times (1 - 0.5)]$$
$$= 1 - (0.6)(0.5)$$
$$= 1 - (0.3) = 0.7$$

**105.** Let $\bar{x}$ be the mean of $x_1, x_2, x_3, \dots x_n$. If $x_i = a + cy_i$ for some constants $a$ and $c$, then what will be the mean of $y_1, y_2, y_3, \dots, y_n$ ?

(a) $a + c\bar{x}$      (b) $a - \dfrac{1}{c}\bar{x}$
(c) $\dfrac{1}{c}\bar{x} - a$      (d) $\dfrac{\bar{x} - a}{c}$

&#9655; **(d)** Given that,

Mean of $x_1, x_2, x_3, \dots x_n$ i.e. $x = \bar{x}$
Now, we have
$$x_i = a + cy_i$$
$$\Rightarrow y_i = \frac{1}{c}(x - a)$$
$$\Rightarrow \bar{y} = \frac{1}{c}(\bar{x} - a)$$

**106.** Consider the following statements
I. If the correlation coefficient $r_{xy} = 0$, then the two lines of regression are parallel to each other.
II. If the correlation coefficient $r_{xy} = 1$, then the two lines of regression are perpendicular to each other.

*Which of the above statements is/are correct?*
(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

&#9655; **(d)** According to correlation condition,

If correlation coefficient $r_{r \cdot y} = 0$, then lines of regression are parpendicular

And if $r_{ry} = 1$, then lines of regression are parallel.
So, both statements are wrong.

**107.** If $4x - 5y + 33 = 0$ and $20x - 9y = 107$ are two lines of regression, then what are the values of $\bar{x}$ and $\bar{y}$ respectively?

(a) 12 and 18      (b) 18 and 12
(c) 13 and 17      (d) 17 and 13

&#9655; **(c)** Given lines of regression are
$$4x - 5y + 33 = 0 \qquad \dots \text{(i)}$$
and $20x + 9y - 107 = 0 \qquad \dots \text{(ii)}$
on multiplying Eq. (i) by 5 and subtract Eq. (ii) from it, we get
$$20x - 25y + 165 = 0$$
$$20x - 9y - 107 = 0$$
$$\underline{\quad - \quad\quad + \quad\quad + \quad}$$
$$-16y = -272$$
$$y = 17$$
on putting the value of $y$ in Eq. (i), we get
$$4x - 85 + 33 = 0$$
$$\Rightarrow 4x = 52 \Rightarrow x = 13$$

The mean of two regression lines are the solution set at given regression lines,
Here, $\bar{X} = 13$ and $\bar{Y} = 17$

**108.** Consider the following statements
I. Mean is independent of change in scale and change in origin.
II. Variance is independent of change in scale but not in origin.

*Which of the above statements is/are correct?*
(a) I only      (b) II only
(c) Both I and II      (d) Neither I nor II

&#9655; **(d)** Since, mean changes with changes in origin. So, Statement I is wrong.

And variance is independent to the choice of origin. So, Statement II is also wrong. Hence, both statements are wrong.

**109.** Consider the following statements
I. The sum of deviations from mean is always zero.
II. The sum of absolute deviations is minimum when taken around median.

*Which of the above statements is/are correct.*
(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

**(c)** By the property of deviation both statement are correct.

**110.** What is the median of the numbers 4.6, 0, 9.3, −4.8, 7.6 2.3, 12.7, 3.5, 8.2, 6.1, 3.9, 5.2 ?
(a) 3.8
(b) 4.9
(c) 5.7
(d) 6.0

**(b)** On arranging the given number is ascending order, we have

− 4.8, 0, 2.3, 3.5, 3.9, 4.6, 5.2, 6.1, 7.6, 8.2, 9.3, 12.7

Here, $n = 12$

So, median

$$= \frac{\text{Value of}\left(\frac{12}{2}\right)\text{th number} + \text{Value of}\left(\frac{12}{2}+1\right)\text{th number}}{2}$$

$$= \frac{\text{Value of 6th number} + \text{Value 7th number}}{2}$$

$$= \frac{4.6 + 5.2}{2}$$

$$= 4.9$$

**111.** In a test in Mathematics, 20% of the students obtained "first class". If the data are represented by a pie chart, what is the central angle corresponding to "first class"?
(a) 20°
(b) 36°
(c) 72°
(d) 144°

**(c)** Pie chart contains total angle equal to 360°.

So, central angle corresponding to "First class"

$$= 20\% \text{ of } 360°$$

$$= \frac{20}{100} \times 360°$$

$$= 72°$$

**112.** The mean and standard deviation of a set of values are 5 and 2 respectively. If 5 is added to each value, then what is the coefficient of variation for the new set of values?
(a) 10
(b) 20
(c) 40
(d) 70

**(b)** Given, mean = 5

and standard deviation $(\sigma) = 2$

Since, 5 is added to each value.

So, new mean = 5 + 5

= 10

But standard deviation will remain same.

Hence, coefficient of variation

$$= \frac{\sigma}{\text{mean}} \times 100$$

$$= \frac{2}{10} \times 100 = 20$$

**113.** A train covers the first 5 km of its journey at a speed of 30 km/h and the next 15 km at a speed of 45 km/h. What is the average speed of the train?
(a) 35 km/h
(b) 37.5 km/h
(c) 39.5 km/h
(d) 40 km/h

**(d)** Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$$= \frac{5 + 15}{\frac{5}{30} + \frac{15}{45}} = \frac{20}{\frac{1}{6} + \frac{1}{3}}$$

$$= \frac{20}{\frac{1+2}{6}}$$

$$= \frac{20 \times 6}{3} = 40 \text{ km/h}$$

**114.** Two fair dice are rolled. What is the probability of getting a sum of 7?
(a) $\dfrac{1}{36}$
(b) $\dfrac{1}{6}$
(c) $\dfrac{7}{12}$
(d) $\dfrac{5}{12}$

**(b)** Here, $n(S) = 36$

and $E$ be the event of getting a sum of 7 on two fair dice.

$= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\therefore n(E) = 6$

So, required probability $= \dfrac{n(E)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

**115.** If $A$ and $B$ are two events such that $2P(A) = 3P(B)$, where $0 < P(A) < P(B) < 1$, then which one of the following is correct?
(a) $P(A|B) < P(B|A) < P(A \cap B)$
(b) $P(A \cap B) < P(B|A) < P(A|B)$
(c) $P(B|A) < P(A|B) < P(A \cap B)$
(d) $P(A \cap B) < P(A|B) < P(B|A)$

**(b)** Given that, $2P(A) = 3P(B)$

$$\Rightarrow 2\frac{P(A)}{P(A \cap B)} = \frac{3P(B)}{P(A \cap B)}$$

[dividing both sides by $P(A \cup B)$]

$$\Rightarrow \frac{1}{2} \times \frac{P(A \cap B)}{P(A)} = \frac{1}{3} \times \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{1}{2} \times P\left(\frac{B}{A}\right) = \frac{1}{3}P\left(\frac{A}{B}\right)$$

$$\Rightarrow P\left(\frac{B}{A}\right) < P\left(\frac{A}{B}\right)$$

**116.** A box has ten chits numbered 0, 1, 2, 3, ..., 9. First, one chit is drawn at random and kept aside. From the remaining, a second chit is drawn at random. What is the probability that the second chit drawn is "9" ?
(a) $\dfrac{1}{10}$
(b) $\dfrac{1}{9}$
(c) $\dfrac{1}{90}$
(d) None of these

**(a)** Let $E_1$ be the event at drawing a chit which is not 9 and $E_2$ be the event of drawing second chit bearing number 9.

$$\therefore \quad P(E_1) = \frac{{}^9C_1}{{}^{10}C_1} = \frac{9}{10}$$

$$\text{and} \quad P(E_2) = \frac{{}^1C_1}{{}^9C_1} = \frac{1}{9}$$

$\therefore$ Required probability $= P(E_1) \cdot P(E_2)$

$$= \frac{9}{10} \times \frac{1}{9} = \frac{1}{10}$$

**117.** One bag contains 3 white and 2 black balls, another bag contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the change that it is white?
(a) $\dfrac{3}{8}$
(b) $\dfrac{49}{80}$
(c) $\dfrac{8}{13}$
(d) $\dfrac{1}{2}$

**(b)** Let $E_1$ be the event of selecting the first bag and $E_2$ be the event of selecting the second bag. Let $A$ be the event of drawing white ball.

So, by theorem at total probability.

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{{}^3C_1}{{}^5C_1} + \frac{1}{2} \times \frac{{}^5C_1}{{}^8C_1}$$

$$= \frac{1}{2}\left[\frac{3}{5} + \frac{5}{8}\right] = \frac{1}{2} \times \frac{24 + 25}{40}$$

$$= \frac{1}{2} \times \frac{49}{40} = \frac{49}{80}$$

**118.** Consider the following in respect of two events $A$ and $B$
I. $P(A$ occurs but not $B) = P(A) − P(B)$ if $B \subset A$
II. $P(A$ alone or $B$ alone occurs) $= P(A) = P(B) − P(A \cap B)$
III. $P(A \cup B) = P(A) + P(B)$ if $A$ and $B$ are mutually exclusive

*Which of the above is/are correct?*
(a) I only
(b) I and III
(c) II and III
(d) I and II

**(b)** If $B \subset A$, then $P(A − B)$

$$= P(A) − P(A \cap B) = P(A) − P(B)$$

$$[\because B \subset A \Rightarrow A \cap B = B]$$

So, Statement I is correct.

$P$ ($A$ alone or $B$ alone)
$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= P(A) + P(B) - 2\, P(A \cap B)$

So, Statement II is wrong.

If $A$ and $B$ are mutually exclusive,

then $P(A \cap B) = 0$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B)$

So, Statement III is correct.

Here, Statement I and III are correct.

**119.** A committee of three has to be chosen from a group of 4 men and 5 women. If the selection is made at random, what is the probability that exactly two members are men?

(a) $\dfrac{5}{14}$      (b) $\dfrac{1}{21}$

(c) $\dfrac{3}{14}$      (d) $\dfrac{8}{21}$

⟩ **(a)** Total number of selecting three members $= {}^{9}C_3$

Favourable numbers of selecting two members as men
$$= {}^{4}C_2 \times {}^{5}C_1$$

So, required probability $= \dfrac{{}^{4}C_2 \times {}^{5}C_1}{{}^{9}C_3}$

$$= \dfrac{\dfrac{4 \times 3}{2 \times 1} \times \dfrac{5}{1}}{\dfrac{9 \times 8 \times 7}{3 \times 2 \times 1}}$$

$$= \dfrac{2 \times 3 \times 5}{3 \times 4 \times 7} = \dfrac{5}{14}$$

**120.** The standard deviation $\sigma$ of the first N natural numbers can be obtained using which one of the following formulae?

(a) $\sigma = \dfrac{N^2 - 1}{12}$

(b) $\sigma = \sqrt{\dfrac{N^2 - 1}{12}}$

(c) $\sigma = \sqrt{\dfrac{N - 1}{12}}$

(d) $\sigma = \sqrt{\dfrac{N^2 - 1}{6N}}$

⟩ **(b)** $\because \sigma^2 = \dfrac{1}{N} \Sigma X_i^2 - (\bar{X})^2$

$$= \dfrac{1}{N}(1^2 + 2^2 + \dots + N^2)$$
$$- \left[\dfrac{1}{N}(1 + 2 + 3 + \dots + N)\right]^2$$

$$= \dfrac{1}{N} \times \dfrac{N(N+1)(2N+1)}{6} - \left[\dfrac{(N+1)}{2}\right]^2$$

$$= \dfrac{N^2 - 1}{12}$$

$$\sigma = \sqrt{\dfrac{N^2 - 1}{12}}$$

# PAPER II   English Language and General Studies

## Part A (English Language)

**Directions** (Q. Nos. 1-10) *Each item in this section consists of a sentence with an underlined word/words followed by four words. Select the option that is nearest in meaning to the underlined word/words and mark your response in your answer sheet accordingly.*

**1.** I do not want you to lead a life of <u>sycophancy</u> as you did during the foreign rule.
(a) admiration    (b) love
(c) appreciation    (d) flattery

⟩ **(d)** A sycophant is one who praises people in authority in a way that is not sincere, usually in order to get some advantage of them. So, 'flattery' is the nearest in meaning to sycophancy.

**2.** In India, it has become easy to <u>attack</u> cultural artefacts these days.
(a) beckon    (b) assault
(c) belch    (d) appreciate

⟩ **(b)** assault is the correct synonym of attack. Other synonyms are beat, strike at, knock down, etc.

**3.** A local court <u>granted</u> bail to the criminal on Thursday.
(a) confessed    (b) donated
(c) allowed    (d) yielded

⟩ **(c)** is the correct answer as allowed is the synonym of granted.

**4.** The judge told that he would analyse the evidence and then <u>deliver</u> the verdict.
(a) liberate    (b) surrender
(c) transfer    (d) pronounce

⟩ **(d)** pronounce is the correct synonym of deliver.

Some other synonyms are accepted, acknowledged, assumed, etc.

**5.** The growth and development of the peasant movement was <u>indissolubly</u> linked with the national struggle for freedom.
(a) firmly    (b) vaguely
(c) individually    (d) steadily

⟩ **(a)** In the given sentence, indissolubly means incapable of being broken. So, 'firmly' is the nearest in meaning to it.

**6.** Weather conditions have been <u>improving</u> over the past few days.
(a) mending    (b) amending
(c) becoming better (d) advancing

⟩ **(c)** becoming better is the perfect synonym of improving. Some other synonyms are correcting, elaborating, fixing, etc.

**7.** The confusion on the <u>interlocutor's</u> face was gratifying.
(a) government officer (b) party worker
(c) dialogist    (d) revolutionary

⟩ **(c)** dialogist is the correct answer. An interliocutor is the person who engages himself in dialogues. Some other synonyms are interviewer, speaker, etc.

**8.** He spends his money <u>lavishly</u>.
(a) hesitatingly
(b) generously
(c) foolishly
(d) carefully

⟩ **(b)** generously is the correct answer. Other options do not match here. Lavishly may also mean to spend money extravagantly.

**9.** The government's new policies will come into force from the next <u>fiscal</u> year.
(a) calendar    (b) academic
(c) financial    (d) leap

⟩ Fiscal means relating to money and finances. So, option (c) financial is nearest in meaning to it.

**10.** <u>Abundant</u> food was· available for the soldiers in the mess.
(a) little
(b) plentiful
(c) delicious
(d) wholesome

⟩ **(b)** plentiful is the correct answer for the word abundant which means in large quantity. Some other synonyms are plenty, profuse, bounteous, etc.

**11.** The country's economy must be <u>geared to</u> wartime requirements.
  (a) subordinated to
  (b) related to
  (c) adjusted to
  (d) unlinked to

  ⊙ *(d)* unlinked to is correct antonym of word geared which means to adjust or adapt.

**12.** Why does fire <u>attract</u> insects?
  (a) discharge       (b) destroy
  (c) repel           (d) remove

  ⊙ *(c)* repel is opposite in meaning to attract as it means to force something or someone to move away or stop attacking you.

**13.** The party was excellent, and I would like to thank all the people <u>concerned</u>.
  (a) cared           (b) attentive
  (c) dependable      (d) uninvolved

  ⊙ *(d)* uninvolved is correct antonym of concerned which means interested or involved.

**14.** He is very <u>serious</u> by temperament.
  (a) grave           (b) trivial
  (c) sober           (d) stupid

  ⊙ *(b)* trivial is the only correct antonym from the given options. Grave and sober are synonyms of serious. Stupid does not match. Trivial means of small value or unserious matter.

**15.** There are a few <u>miscellaneous</u> items to discuss in this meeting.
  (a) pure            (b) mixed
  (c) homogenous      (d) discordant

  ⊙ *(c)* homogenous is correct antonym of miscellaneous. Miscellaneous means of mixed type whereas homogenous means of the same kind.

**16.** Due to the postal strike, the <u>outgoing</u> mail got delayed.
  (a) urgent          (b) incoming
  (c) ordinary        (d) speedy

  ⊙ *(b)* incoming is the correct antonym of outgoing. In and out are perfect antonyms.

**17.** He had a <u>fine</u> ear for music.
  (a) small           (b) close
  (c) coarse          (d) smooth

  ⊙ *(c)* coarse, from the given options is correct. Other antonyms could be unsatisfactory, ill, etc.

**18.** There is no <u>likeness</u> between him and his brother.
  (a) unlikeliness    (b) unlikelihood
  (c) dissimilarity   (d) disaffinity

  ⊙ *(c)* dissimilarity is the correct antonym as likeness means resemblance and dissimilarity means when there is nothing similar or resembling with the other.

**19.** Cultural <u>diversity</u> in the working place is good for business.
  (a) uniformity      (b) conformity
  (c) identity        (d) similarity

  ⊙ *(a)* diversity means the condition of having or being composed of different types of people or things. So, its opposite will be uniformity.

**20.** The company was <u>liquidated</u> within five years.
  (a) bankrupt        (b) closed down
  (c) flourishing     (d) privatised

  ⊙ *(c)* liquidate means to cause a business to close, so that its assets can be sold to pay its debts. So, its opposite would be 'flourishing'.

**Directions** (Q. Nos. 21-25) *Each item in this section has a sentence with three underlined parts labelled (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part and indicate your response in the Answer Sheet against the corresponding letter i.e., (a) or (b) or (c). If you find no error, your response should be indicated as (d).*

**21.** <u>The politician lost face in his constituency</u> (a)/<u>when he broke the pre-election promises</u> (b)/<u>he made to his people</u>. (c)/<u>No error</u>. (d)

  ⊙ *(c)* has error. As sentence is in simple past tense, here we must use past perfect tense 'he had made to the people' should be used here.

**22.** At the <u>request of the Defence Attorney</u>, (a)/<u>the jury were called</u> (b)/<u>and their individual verdicts were recorded</u>. (c)/<u>No error</u>. (d)

  ⊙ *(b)* has error of correct use of verb with jury which is taken as singular noun, verb 'was' will be used here.

**23.** Frank Lloyd Wright <u>has been acclaimed</u> (a)/<u>by colleagues</u> (b)/<u>as the greater of all modern architects</u>. (c)/ <u>No error</u>. (d)

  ⊙ *(c)* has error of correct degree of adjective. Here, we will use superlative degree of greater i.e. 'greatest' as we are comparing one with many people.

**24.** <u>In my younger days</u> (a)/<u>I could run four miles</u> (b)/<u>at a stretch</u>. (c)/<u>No error</u>. (d)

  ⊙ *(a)* has error of correct syntax 'When I was young' is better construction.

**25.** <u>The owner</u> (a)/<u>as well as his servants</u> (b)/<u>is honest</u>. (c)/<u>No error</u>. (d)

  ⊙ *(d)* The sentence is correct.

**Directions** (Q. Nos. 26-30) *Given below are some idioms/phrases followed by four alternative meanings to each. Choose the response (a), (b), (c) or (d) which is the most appropriate expression.*

**26.** <u>Cry over spilt milk</u>
  (a) Complaining about a loss in the past
  (b) Too much inquisitive about something
  (c) When something is done badly to save money
  (d) Dealing with a problem only in an emergency situation

  ⊙ *(a)* Complaining about a loss in the past is the correct meaning of the idiom 'Cry over spilt milk.'

**27.** <u>Cut the mustard</u>
  (a) Prepare spices out of mustard seeds
  (b) To come up to expectations
  (c) Making absurd expectations
  (d) Very enthusiastic

  ⊙ *(b)* To come up to the expectations is the correct meaning of the idiom 'cut the mustard'.

**28.** <u>Devil's advocate</u>
  (a) A dangerous person
  (b) To present a counter argument
  (c) Very argumentative person
  (d) Creating an unpleasant situation

  ⊙ *(c)* A devil's advocate is a person who expresses an unpopular opinion in order to provoke debate. So, out of the given options 'very argumentative person' best expresses its meaning.

**29.** <u>Don't count your chickens before the eggs have hatched</u>
  (a) If you are not good at something, better to avoid that
  (b) Don't make plans for something that might not happen
  (c) Not to come up to expectations
  (d) Don't put all your resources in one possibility

  ⊙ *(b)* This idiom means to not make plans for something that might not happen.

**30.** <u>Give the benefit of doubt</u>
- (a) To be partial to someone
- (b) To be judgemental
- (c) Regard someone as innocent until proven otherwise
- (d) Say something exactly right

⊙ **(c)** Benefit of doubt means to regard someone as innocent until proven otherwise.

**Directions** (Q. Nos. 31-35) *In this section each item consists of six sentences of a passage. The first and sixth sentences are given in the beginning as S1 and S6. The middle four sentences in each have been jumbled up and labelled P, Q, R and S. You are required to find the proper sequence of the four sentences and mark your response accordingly on the Answer Sheet.*

**31.** S1 : The Subsidiary Alliance system was extremely advantageous to the British.

S 6 : They controlled the defence and the foreign relations of the protected ally.

P : They could now maintain a large army at the cost of Indian states.

Q : if many war occurred in the territories

R : either of the British ally or of the Britishers

S : This enabled them to fight wars far away from their own territories

*The proper sequence should be*
- (a) P Q R S
- (b) P S Q R
- (c) Q R P S
- (d) S R P Q

⊙ **(b)** PSQR is the correct sequence.

**32.** S1 : In reality, by signing a Subsidiary Alliance, an Indian state virtually signed away its independence.

S6 : In fact, the Indian ruler lost all vestiges of sovereignty in external matters.

P : of maintaining diplomatic relations

Q : It lost the right of self defence

R : with its neighbours

S : and of settling its disputes

*The proper sequence should be*
- (a) P Q R S
- (b) R S P Q
- (c) Q P S R
- (d) Q S R P

⊙ **(c)** QPSR is the correct sequence.

**33.** S1 : A mighty popular Revolt broke out in Northern and Central India in 1857.

S 6 : Millions of peasants, artisans and soldiers fought heroically and wrote a glorious chapter.

P : Sepoys, or the Indian soldiers of the Company's army

Q : but soon engulfed wide regions and involved the masses

R : and nearly swept away the British rule

S : It began with a mutiny of the

*The proper sequence should be*
- (a) R S P Q
- (b) P Q R S
- (c) S R P Q
- (d) Q R P S

⊙ **(a)** RSPQ is the correct sequence.

**34.** S1: The Indian Civil Service gradually developed into one of the most efficient and powerful civil services in the world.

S6 : though these qualities obviously served. British, and not Indian interests.

P : and often participated in the making of policy

Q : independence, integrity and hard work

R : They developed certain traditions of

S : Its members exercised vast power

*The proper sequence should be*
- (a) P Q R S
- (b) Q R S P
- (c) R S Q P
- (d) S P R Q

⊙ **(d)** SPRQ is the correct sequence.

**35.** S1 : The ruin of India handicrafts was reflected in the ruin of the towns and cities which were famous for their manufactures.

S6 : Centres were developed and laid waste.

P : Dacca, Surat, Murshidabad and many other rising industrial

Q : ravages of war and plunder, failed to

R : survive British conquest

S : Cities which had withstood the

*The proper sequence should be*
- (a) P Q R S
- (b) S Q R P
- (c) S R P Q
- (d) Q R S P

⊙ **(b)** SQRP is the correct sequence.

**Directions** (Q. Nos. 36-40) *In this section you have few short passages. After each passage, you will find some items based on the passage. First, read a passage and answer the items based on it. You are required to select your answers based on the contents of the passage and opinion of the author only.*

## Passage 1

The rule of the road means that in order that the liberties of all may be preserved, the liberties of everybody must be curtailed. When the policeman, say, at a road-crossing steps into the middle of the road and puts out his hand, he is the symbol not of tyranny but of liberty. You have submitted to a curtailment of private liberty in order that you may enjoy a social order which makes your liberty a reality. We 'have both liberties to preserve — our Individual liberty and our social liberty. That is, we must have a judicious mixture of both. I shall not permit any authority to say that my child must go to this school or that, shall specialise in science or arts. These things are personal. But if I say that my child shall have no education at all, then society will firmly tell me that my child must have education whether I like it or not.

**36.** According to the author, the ''rule of the road'' implies
- (a) the rule regulating the traffic on the road
- (b) the principle on which a road is constructed to ensure safe traffic
- (c) unrestricted freedom for all to lead a happy life
- (d) restricted individual freedom to ensure freedom for all

⊙ **(d)** Rule of the road according to the passage means restricted individual freedom to ensure freedom for all.

**37.** The author thinks that when a policeman signals you to stop on a road-crossing, he is
- (a) behaving in a whimsical manner
- (b) interfering with your freedom to use the road
- (c) protecting the liberty of all to use the road
- (d) mischievously creating hurdles in your way from some personal motive

⊙ **(c)** The policeman is protecting the liberty of all to use the road.

**38.** The author is of the view that we should
- (a) have absolute individual liberty without any restrictions imposed by the society
- (b) have everything, controlled by the society without any kind of individual liberty

(c) try to strike a sensible balance between our individual liberty and our social liberty

(d) have more of social liberty than individual liberty

⊘ **(c)** The author is of the view that we should try to strike a sensible balance between our individual liberty and our social liberty.

**39.** The author holds that

(a) educating or not educating his child is a matter of personal liberty

(b) educating or not educating his child is also a matter of social liberty

(c) choosing the school for his child is a matter of social liberty

(d) choosing the subject of study for his child is a matter of social liberty

⊘ **(b)** The author is of the opinion that educating or not educating his child is also a matter of social liberty.

**40.** The most suitable title of the passage would be

(a) The Policeman at a Road Crossing

(b) The Laws of the Road

(c) Importance of Liberty

(d) Education of Children

⊘ The most suitable title of the passage would be option (c). 'Importance of Liberty'.

## Passage 2

My most interesting visitor comes at night, when the lights are still burning — a tiny bat who prefers to fly in through the open door, and will use the window only if there is no alternative. His object in entering the house is to snap up the moths that cluster around the lamps. All the bats I have seen fly fairly high, keeping near the ceiling; but this particular bat flies in low, like a dive-bomber, zooming in and out of chair legs and under tables. Once, he passed straight between my legs. Has his radar gone wrong, I wondered, or is he just plain crazy?

**41.** Consider the following statements

1. The tiny bat flew in low like a dive bomber.

2. The tiny bat like all bats keeps near the ceiling.

3. It has lost direction because its radar has gone wrong.

4. It wants to entertain the author with its skill in flying.

*Which of the above statements may be assumed to be true from the information given in the passage?*

(a) 1 only          (b) 1 and 3

(c) 2 and 4          (d) 3 and 4

⊘ **(b)** From the given information statements 1 and 3 are true.

**42.** The bat entered the room

(a) because there was no alternative

(b) to eat the moths round the lamps

(c) as it had gone mad

(d) as it preferred to fly in through the open door

⊘ **(b)** The bat entered the room to eat the moths round the lamps.

**43.** After comparing the habits of the tiny bat with those of other bats, the author was

(a) sure that this bat had lost its direction

(b) not sure of its preferences

(c) surprised to find that it was an expert flier

(d) unable to give the correct explanation for its behaviour

⊘ **(d)** The author was confused and hence was unable to give the correct explanation for the behaviour of the bat.

**44.** The author calls the tiny bat an ''interesting visitor''. This means

(a) the bat visits him at night

(b) the bat is interested in the moths

(c) this bat has peculiar qualities

(d) this bat surprises him by zooming in and out like a dive-bomber

⊘ **(c)** The tiny bat is called an interesting visitor by the author as it has peculiar qualities.

**45.** What, according to you, can be the most suitable title for the passage?

(a) Someone visits me

(b) Night of Mysteries

(c) My Nocturnal Visitor

(d) A Funny Visitor

⊘ **(c)** My Nocturnal Visitor is the most suitable title of the passage from the given options.

**Directions** (Q. Nos. 46-50) *Each of the following sentences in this section has a blank space and four words or group of words given after the sentence. Select the word or group of words you consider most appropriate for the blank space and indicate your response on the Answer Sheet accordingly.*

**46.** The tired traveller .............. on in the hope of finding some resting place.

(a) strolled          (b) scurried

(c) paraded          (d) plodded

⊘ **(d)** The word 'plodded' means to walk with slow heavy steps. So, from the given options it is most suitable to fill the given blank.

**47.** The car was damaged beyond repair in the ............ accident.

(a) outrageous          (b) ghastly

(c) nasty          (d) heinous

⊘ **(b)** Ghastly means unpleasant and shocking. So, it should be used to fill the given blank.

**48.** They gave a ............ dinner to celebrate the occasion, which impressed every guest.

(a) austere          (b) public

(c) sumptuous          (d) summary

⊘ **(c)** Sumptuous means of high quality and often expensive. So, from the given options it is the most suitable word to fill the given blank.

**49.** Once the ............ manuscript is received by the publishers, it is typed in double space.

(a) total          (b) full

(c) complete          (d) filled

⊘ **(c)** complete is the correct answer.

**50.** I am used to ............ in queues.

(a) stand          (b) standing

(c) stand up          (d) standing still

⊘ **(b)** starding is the correct answer.

# Part B (General Studies)

**51.** Which one of the following statements is correct?
- (a) Any energy transfer that does not involve temperature difference in some way is not heat.
- (b) Any energy transfer always requires a temperature difference
- (c) On heating the length and volume of the object remain exactly the same
- (d) Whenever there is a temperature difference, heat is the only way of energy transfer

⊘ **(a)** Practically, all the bodies around us either release or absorb energy often in the form of heat. The form of energy which is exchanged among various bodies or system on account of temperature difference is defined as heat. On heating the matter, the energy of atoms increases and hence the average distance between them increases. This results in thermal expansion. Solids can expand in one dimension, two dimensions and three dimensions while liquid and gases usually expand in volume.

**52.** If T is the time period of an oscillating pendulum, which one of the following statements is NOT correct?
- (a) The motion repeats after time T only once
- (b) $T$ is the least time after which motion repeats itself
- (c) The motion repeats itself after $nT$, where $n$ is a positive integer
- (d) T remains the same only for small angular displacements

⊘ **(a)** Time-period ($T$) is the time taken by the particle to complete one oscillation. Time-period of the simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is clear that the time period is independent of the mass of the pendulum. This formula is valid only when radius of point mass of pendulum is negligible compared to length of pendulum. Here, motion of pendulum repeats itself after a fixed interval i.e it remains same only for small angular displacements (Q). Hence, it is incorrect to say that this motion repeats after time $T$ only once.

**53.** If an object moves with constant velocity then which one of the following statements is NOT correct?
- (a) Its motion is along a straight line
- (b) Its speed changes with time
- (c) Its acceleration is zero
- (d) Its displacement increases linearly with time

⊘ **(d)** Whenever an object moves with constant velocity, the motion is always in form of straight line, which is parallel to the x-axis. As velocity remains same, hence acceleration of the object is zero. Also the speed of the object changes due to change in displacement so than in distance. Hence, option (d) is incorrect because it is not necessary displacement increases linearly. It decreases linearly for constant velocity.

**54.** An object is moving, with uniform acceleration $a$. Its initial velocity is $u$ and after time $t$ its velocity is $v$. The equation of its motion is $v = u + at$. The velocity (along $y$-axis) time (along $x$-axis) graph shall be a straight line
- (a) passing through origin
- (b) with $x$-intercept $u$
- (c) with $y$-intercept $u$
- (d) with slope $u$

⊘ **(c)** From the graph for the equation of its motion.

$$v = (u + at)$$
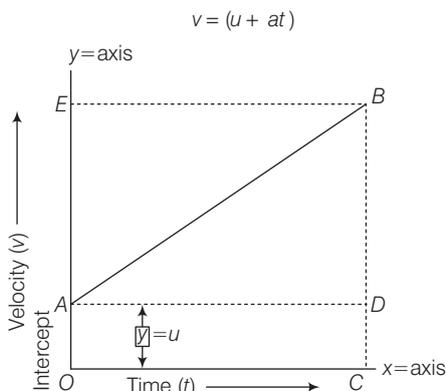


The object has an initial velocity $u$ at point $A$ and then its velocity changes at a uniform rate from $A$ to $B$ in time $t$. In other words, there is a uniform acceleration '$a$' from $A$ to $B$ and after time $t$ its final velocity becomes '$v$' equal to $B$ in the graph. Hence, the slope of a velocity-time graph is equal to acceleration. Thus, we can say that the velocity time graph for the equation, $v = u + at$ will be straight line having slope equal to acceleration $a$ with $y$ intercept $u$.

∴ Option (c) is correct.

**55.** What is the net force experienced by a bar magnet placed in a uniform magnetic field?
- (a) Zero
- (b) Depends upon length of the magnet
- (c) Never zero
- (d) Depends upon temperature

⊘ **(a)** The net force experienced by a bar magnet placed in a uniform magnetic field is zero. A magnetic field is said to be uniform over a region if its magnetic field induction ($\vec{B}$) has the same magnitude and direction at all point in the region. Thus, net force experienced by a bar magnet in a uniform magnetic field is zero.

**56.** Which one of the following has maximum inertia ?
- (a) An atom
- (b) A molecule
- (c) A one-rupee coin
- (d) A cricket ball

⊘ **(d)** Inertia means resistance or opposition offered by the body to change its state. Mass of a body is the measurement of its inertia. A body with greater mass shows greater inertia, i.e., it is more difficult to change its state of rest or uniform motion as compared to that of a body having small mass. Hence, in the given option a cricket ball has maximum mass, so it has maximum inertia.

**57.** Which one of the following is the value of 1 kWh of energy converted into joules?
- (a) $1.8 \times 10^6$ J
- (b) $3.6 \times 10^6$ J
- (c) $6.0 \times 10^6$ J
- (d) $7.2 \times 10^6$ J

⊘ **(b)** 1 kilowatt hour is the unit of electrical power.

1 kilowatt hour = 1000 watt × 3600s
$$= 3.6 \times 10^6 \text{ Joule}$$

(∵ 1 Joule = 1 watt × 1 sec)

**58.** Which one of the following statements about gravitational force is NOT correct?
- (a) It is experienced by all bodies in the universe
- (b) It is .a dominant force between celestial bodies
- (c) It is a negligible force for atoms
- (d) It is same for all pairs of bodies in our universe

⊘ **(d)** Gravitational force is defined as a non-contact force of attraction between any two bodies in universe (no matter how far the bodies are). It is experienced by all bodies in the universe. The celestial bodies experienced these forces, but it is not same for all pairs of bodies in our universe.

**59.** Whether an object will float or sink in a liquid, depends on

(a) mass of the object only

(b) mass of the object and density of liquid only

(c) difference in the densities of the object and liquid

(d) mass and shape of the object only

◉ **(c)** When an object is floating then

$$= \frac{\text{Volume of sinking part of the object}}{\text{Total volume of the body}}$$

$$= \frac{\text{Density of the object}}{\text{Density of the liquid}}$$

Volume of sinking part of the body

$$= \frac{\text{Mass of object}}{\text{Density of liquid}}$$

In case of sinking, if the buoyant force or upthrust exerted by the liquid is less than the weight of the object, the object will sink in liquid. Therefore, it is clear that whether an object will float or sink in a liquid depends on difference in the densities of the object and liquid.

**60.** Which one of the, following devices is non-ohmic?

(a) Conducting copper coil

(b) Electric heating coil

(c) Semi conductor diode

(d) Rheostat

◉ **(c)** According to Ohm's law, the electric current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the physical condition (such as temperature) remain unchanged. Hence, the device which follows Ohm's law for all voltage across it is called as an ohmic device.

For example, a wire rheostat conducting copper coil, electric heating coil, etc. The device that does not follow Ohm's law is known as non-ohmic device.

For example, Semi conductor diode.

**61.** Which one of the following is the natural phenomenon based on which a simple periscope works?

(a) Reflection of light

(b) Refraction of light

(c) Dispersion of light

(d) Total internal reflection of light

◉ **(a)** Periscope is an optical instrument which is based on the phenomenon of reflection of light. Light is reflected from the first mirror into the second one at 45° angle which is then reflected to the eye of the viewer.

**62.** Which one of the following frequency range is sensitive to human ears?

(a) 0 – 200 Hz

(b) 20 – 20,000 Hz

(c) 200 – 20,000 Hz only

(d) 2,000 – 20,000 Hz only

◉ **(b)** 20-20,000 Hz, human ear can respond to minute pressure variations in air. The audible frequency range to which ears can respond is 20-20,000 Hz. The sound frequency beyond this limit may damage ear drums and cause hearing disabilities in humans.

**63.** The accidental touch of Nettle leaves creates a burning sensation, which is due to inject of

(a) Hydrochloric acid (b) Methanoic acid

(c) Citric acid

(d) Sulphuric acid

◉ **(b)** The accidental touch of Nettle Leaves create a burning sensation. This is due to the methanoic acid secreted by them. It is a Herbaceous plant that grows in the wild. The leaf of the dock plant, which often grows beside the nettle, is the traditional remedy for it.

**64.** Which of the following properties is true for a tooth paste?

(a) It is acidic (b) It is neutral

(c) It is basic

(d) It is made up of Calcium phosphate, the material of tooth enamel

◉ **(c)** Tooth pastes are derived from different variety of components namely abrasives, flouride and detergents. Abrasives includes particles of calcium hydrogen phosphates, aluminium hydroxide, etc. Tooth pastes are basic in nature and prevents tooth decay by neutralising the excess of acid. They can neither be acidic nor normal.

**65.** Which one of the following gives the highest amount of hydrogen ions ($H^+$) ?

(a) Sodium hydroxide solution

(b) Milk of magnesia

(c) Lemon juice

(d) Gastric juice

◉ **(d)** The amount of $H^+$ ions produced depends upon the strength of an acid which further depends on the corresponding pH values. Relation between pH and concentration of $H^+$ ions is given by

$$pH = -\log [H^+] = \log \frac{1}{[H^+]}$$

As the pH increases, the acidic nature decreases and thus the amount of $H^+$ ions. The order of pH values for the above given options are as follows:

Gastric Juice (1.3 – 3.0) < Lemon Juice (2.2-2.4) < Milk of Magnesia (10.5) < Sodium hydroxide ($\approx$ 14)

Thus, Gastric juice gives the highest amount of $H^+$ ions.

**66.** Brine is an aqueous solution of

(a) NaCl          (b) NaOH

(c) $NaHCO_3$          (d) $Na_2CO_3$

◉ **(a)** Brine is an aqueous solution of sodium chloride (NaCl). Electrolysis of brine produces Sodium Hydroxide (NaOH), Chlorine ($Cl_2$) and Hydrogen ($H_2$). This forms the basis of the chlor-alkali industry.

**67.** Which one of the following is the chemical formula of Washing Soda ?

(a) $NaHCO_3$          (b) $Na_2CO_3 \cdot 10H_2O$

(c) $Na_2CO_3 \cdot 5H_2O$   (d) NaOH

◉ **(b)** $2NaCl(aq) + 2H_2OCl \rightarrow 2NaOH(aq)$ $+ Cl_2(g) + H_2(g)$ chemically washing soda is sodium carbonate decahydrate with formula $Na_2CO_3 \cdot 10H_2O$. Its anhydrous form, i.e. $Na_2CO_3$ is called soda ash.

$$Na_2CO_3 + 10H_2O \longrightarrow \underset{\text{Washing Soda}}{Na_2CO_3 \cdot 10H_2O}$$
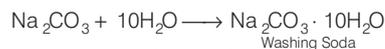
**68.** Which one of the following is NOT true for bleaching powder?

(a) It is used as a reducing agent in chemical industries

(b) It is used for bleaching wood pulp in paper factories

(c) It is used for disinfecting drinking water

(d) It is used for bleaching linen in textile industry

◉ **(a)** Bleaching powder ($CaOCl_2$) is produced by the action of chlorine on dry slaked lime [$Ca(OH)_2$]. The following are the uses of bleaching powder:

(i) It is used for bleaching purposes in textile industries, paper industry and in laundry.

(ii) It is also used as a disinfectant for water to make it free of germs.

(iii) It is used as an oxidising agent in many chemical industries.

Thus among the given, option (a) is incorrect.

**69.** AIDS is caused by a virus whose genetic material is

(a) single stranded circular DNA

(b) double stranded DNA

(c) single stranded RNA

(d) double stranded RNA

◉ **(c)** AIDS is caused by Human Immunodeficiency Virus (HIV). It belongs to family retroviridae and genus lentivirus. HIV genome consists of two identical copies of the SSRNA genome, reverse transcriptase enzyme and other proteins to code for viral proteins that are essential during its replication. All these components are: enclosed within the core of viral particle.

**70.** Which one of the following organelle is NOT found in prokaryotic cells?
(a) Cell wall        (b) Mitochondria
(c) Plasma membrane    (d) Ribosome

⊙ *(b)* Prokaryotic cells are simple and primitive type of cells e.g. bacteria, methanogens, blue-green algae, etc.

These cell do not contain membrane bound nucleus and other cellular organelles like other than mitochondria, prokaryotes also lack. Mitochondria is a double membraned organelle, which is responsible for the production of ATP molecules. It is only found in eukaryotic cells.

**71.** Which one of the following parts of body does NOT take part in the process of breathing?
(a) Bronchi
(b) Bowman's capsule
(c) Diaphragm
(d) Trachea

⊙ *(b)* Bowman's capsule is a part of human excretory system. It is closed cup-shaped and double-walled structure of Malpighian body of a kidney.

The human respiratory system includes nose, larynx, trachea, bronchi and bronchioles, alveoli and lungs.

Diaphragm is a dome-shaped sheet that separate the thoracic cavity from abdominal cavity. It also serves major role during breathing.

Trachea or wind pipe conduct air to lungs and bronchi are the branches of wind pipe inside lungs.

**72.** Which one of the following statements about classification of plants is correct?
(a) Thallophytes have well differentiated body design
(b) Funaria is a fungus
(c) All Pteridophytes are Phanerogams
(d) Vascular system is not found among Bryophytes

⊙ *(d)* Bryophytes are very small, photosynthetic and first terrestrial plants. These are non-vascular plants too.

Thyllophyta is a group of thalloid plants in which body is not differentiated into true roots, stems and leaves. e.g. Algae.

*Funaria* (Moss) is a genus of approximately 2010 species of class Bryopsida of Bryophyta.

Pteridophytes have hidden reproductive organs, so, these are called cryptogams. While all gymnosperms and angiosperms are phanerogams.

**73.** Which one of the following is the correct sequence of levels of hierarchy of classification of organisms from higher to lower?
(a) Phylum - Class - Order - Family - Genus
(b) Phylum - Class - Family - Order - Genus
(c) Family - Order - Class - Species ' - Genus
(d) Class - Family - Order - Species - Genus

⊙ *(a)* A hierarchical system is used for taxonomic classification of organisms. Each level in this classification includes organisms with similar characteristics. According to Linnean hierarchy, proposed by Carlous Linnaeus (Father of systematic Botany), the hierarchy involves seven obligate categories as shown below

Kingdom → Phylum (in Animals)/Division (in plants) → Class → Order → Family → Genus → Species

**74.** Which one of the following statements about meristematic tissues in plants is correct ?
(a) These are dead tissues and form wood
(b) They provide flexibility to plant due to their thickened walls
(c) These are present in the bark of a tree only
(d) Growth occurs in plants due to division of cells of these tissues

⊙ *(d)* Growth occurs in plants due to the division of meristematic cells. Meristematic tissues are a group of undifferentiated immature cells that are either preparing to divide or are in continuous state of division. These tissues contribute towards the growth of the plants as they are present in the actively growing regions of plants.

Due to meristematic tissues, a plant body retains the capacity of unlimited growth throughout their life.

**75.** Which one of the following Union Territories has the highest female literacy rate?
(a) Chandigarh
(b) Lakshadweep
(c) Andaman and Nicobar Islands
(d) Puducherry

⊙ *(b)* Lakshadweep has a sex ratio of 946 females for every 1000 males and a literacy rate of 92.28%. The female literacy of lakshwadweep is 87.95 percent which the highest among all the union territories in India. Daman and Diu comes next with 79.59% in female literacy.

**76.** Consider the following statements about Roaring Forties
1. They are strong Westerly winds found in the oceans of Southern Hemisphere.
2. The strong east to west air currents are caused by the combination of air being displaced from the Equator towards the South Pole and the earth's rotation and abundance of landmasses to serve as wind breaks.

*Which of the statements given above is/ are correct?*
(a) 1 only        (b) 2 only
(c) Both 1 and 2    (d) Neither 1 nor 2

⊙ *(c)* The Roaring Forties are strong westerly winds found in the Southern Hemisphere, generally between the latitudes of 40 and 50 degrees.The strong west-to-east air currents are caused by the combination of air being displaced from the Equator towards the South Pole and the Earth's rotation, and there are few landmasses to serve as windbreaks. Hence, both the given statements are correct.

**77.** Consider the following statements
1. The doldrums is a low pressure area around Equator where the prevailing winds are calm.
2. Chinook is a hot and dry wind that blows in winter and therefore raises the temperature in a short time.

*Which of the statements given above is/ are correct?*
(a) 1 only        (b) 2 only
(c) Both 1 and 2    (d) Neither 1 nor 2

⊙ *(c)* Doldrums, (also called equatorial calms) are equatorial regions of light ocean currents and winds within the Inter Tropical Convergence Zone (ITCZ), a belt of converging winds and rising air encircling Earth near the Equator. It is a low pressure area around the equator where the prevailing winds are calm with no movement.

Chinook are local winds that blow over the Rocky mountains in USA. They are essentially hot and dry wind that blow in the winter season and raises the temperature in a short time thus removing the frost from the crops.

**78.** Which one of the following is the driest desert of the world ?
(a) Atacama       (b) Gobi
(c) Sahara        (d) Kalahari

**(a)** Atacama desert located in the Andes mountain is the driest desert in the world, as well as the only true desert to receive less precipitation than the polar deserts. Officially, it the driest place on Earth. It receives almost no rainfall. This is due to the fact that the desert lies on the leeward side of the Andes that blocks any winds from reaching here.

**79.** Which of the following statements about latent heat for a given substance is/are correct?

1. It is fixed at a given temperature.
2. It depends upon the temperature and volume.
3. It is independent of temperature and volume.
4. It depends on the temperature but independent of volume.

*Select the correct 'answer using the codes given below*

(a) 2　(b) 1 and 3　(c) 4 only　(d) 1 and 4

**(b)** The amount of heat per unit mass transferred during change of state of the substance is called Latent heat of the substance. It is always fixed at a given temperature and does not changes with temperature and volume of the substance. e.g. latent heat of ice is $3.34 \times 105$J.

**80.** Which one of the following statements about the mass of a body is correct?

(a) It changes from one place to another
(b) It is same everywhere
(c) It depends on its shape
(d) It does not depend on its temperature

**(b)** Mass is a quantity of matter present inside the body which measures the inertia of a body. The mass of the body cannot be zero. Irrespective of the position of body in the universe, mass of the body always remains constant everywhere, which is independent on its temperature.

**81.** A ball balanced on a vertical rod is an example of

(a) stable equilibrium
(b) unstable equilibrium
(c) neutral equilibrium
(d) perfect equilibrium

**(b)** When the centre of gravity of a body lies below point of suspension or support, the body is said to be in stable equilibrium. When the centre of gravity of a body lies above the point of suspension or support, the body is said to be in unstable equilibrium. A balanced ball on a vertical rod is an example of unstable equilibrium because when the rod is slightly disturbed its centre of gravity is lowered. The line of action of its weight lies outside the base of the rod. Then the torque due to weight of the toppled it down.

When the centre of gravity of a body lies at the point of suspension, the body is said to be inventral equilibrium.

**82.** Which of the following statements about a fluid at rest in a cup is/are correct?

1. Pressure is same-at all the points in the fluid.
2. Pressure is exerted on the walls.
3. Pressure exists everywhere in the fluid.

*Select the correct answer 'using the codes given below*

(a) 1 and 2 only　(b) 2 and 3 only
(c) 1only　(d) 1, 2 and 3

**(d)** When a fluid is at rest in a cup then pressure is same at all the points in the fluid, pressure is exerted on the walls and pressure exists everywhere in the fluid. The pressure at a point within a volume of fluid means that the point to be surrounded by a small container by the area of the container.

**83.** Which one of the following devices is used to measure atmospheric pressure?

(a) Ammeter　(b) Barometer
(c) Potentiometer　(d) Lactometer

**(b)** Barometer is used to measure the atmospheric pressure. The function of a barometer is to know whether or not atmospheric pressure is rising or falling. Ammeter is a device which is used to measure electric current in amperes. Potentiometer is used to measure the emf of a cell and. Loctometer is used to measure the purity of milk.

**84.** Which one of the following is the number of water molecules that share with two formula unit $CaSO_4$ in Plaster of Paris?

(a) One　(b) Two
(c) Five　(d) Ten

**(a)** Plaster of Paris is chemically known as calcium sulphate hemihydrate $\left( CaSO_4 \cdot \frac{1}{2} H_2O \right)$. Thus, with two formula unit of $CaSO_4$, one molecule of water is shared.

It is prepared by heating gypsum at 393 K.

$$2\left( \underset{\text{Gypsum}}{CaSO_4 \cdot 2H_2O} \right) \xrightarrow{393\ K} 2(CaSO_4)_2 \cdot H_2O$$
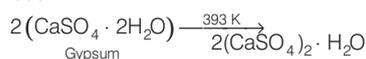
**85.** How is carbon black obtained?

(a) By heating wood at high temperature in absence of air
(b) By heating coal at high temperature in absence of air
(c) By burning hydrocarbons in a limited supply of air
(d) By heating coal at high temperature in presence of air

**(c)** Carbon black is obtained by burning hydrocarbons in a limited supply of air.

$$\underset{\substack{\text{(limited}\\\text{supply)}}}{CH_4} +\ O_2 \longrightarrow \underset{\substack{\text{Carbon}\\\text{black}}}{C}\ + 2H_2O$$
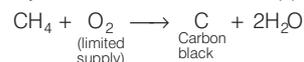
It is used as a black pigment in black ink and as filler in automobile tyres. Which charcol and coke are obtained by heating wood or coal respectively at high temperature.

**86.** Which one of the following properties is NOT true for graphite?

(a) Hybridisation of each carbon atom is $sp^3$
(b) Hybridisation of each carbon atom is $sp^2$
(c) Electrons are delocalised over the whole sheet of atoms
(d) Each layer is composed of hexagonal rings

**(a)** A graphite crystal consists of layers of carbon atoms or sheets of carbon atoms. Each carbon atom in a graphite layer is joined to other three carbon atoms by strong covalent bonds to form flat hexagonal rings. In this case, only 3 of the 4 valence electrons of C participate in bonding. Thus, hybridisation is $sp^2$ having covalent bonds with other 3 C-atoms in same plane.

The electrons are delocalised over the whole sheet of atoms which makes it good conductor of electricity. Thus, among the given options, option 'a' is incorrect.

**87.** Which one of the following is the purest form of Carbon?

(a) Charcoal
(b) Coke
(c) Fullerene
(d) Carbon black

**(d) Carbon black** (lamp black) is the most purest form of carbon. It contains about 98-99% of carbon. It is used for making printer ink, black paint, varnishes and carbon papers.

**Charcoal** is the impure form of carbon because of its porous structure and large surface area.

**Coke** contains 80-85% carbon. It is used as a fuel and as a reducing agent in metallurgy.

**88.** The Poisonous nature of Carbon monoxide (CO) is due to its
(a) insolubility in water
(b) ability to form a complex with haemoglobin
(c) ability to reduce some metal oxides
(d) property of having one sigma bond

⊙ *(b)* Carbon monoxide (CO) is highly poisonous to living beings because it has an ability to form more stable carboxyhaemoglobin. It is 300 times more stable than oxygen-haemoglobin complex. In blood, when the concentration of carbonyhaemoglobin reaches about 3-4 per cent. The oxygen carrying capacity of blood is greatly reduced. The oxygen deficiency results into headache, weak eyesight, nervousness, etc.

**89.** Which one of the following elements is needed in the human body to transfer electrical signals by nerve cells?
(a) Lithium      (b) Sodium
(c) Rubidium     (d) Caesium

⊙ *(b)* Sodium in the form of ions are found in large proportions in biological fluids. It plays an important role in the transmission of nerve signals. It also regulates the flow of water across cell membranes and in transport of sugars and amino acids into the cells.

**90.** Who among the following first discovered cell ?
(a) Robert Brown
(b) Robert Hooke
(c) Leeuwenhoek
(d) Rudolf Virchow

⊙ *(b)* Robert Hooke was the first who studied and discovered cell from a thin section of dead cork. He observed small honeycomb-like structures and named them as cells.

Robert Brown reported the discovery of nucleus. Leeuwenhock was the first person to observe living cells under microscope. Rudolf Virchow proposed that all cells arise from pre-existing cells.

**91.** Which one of the following group of organisms forms a food chain?
(a) Grass, human and fish
(b) Grass, goat and human
(c) Tree, tree cutter and tiger
(d) Goat, cow and human

⊙ *(b)* The series of organisms in which one organism is being eaten by other is called food chain. The organisms given in the option (b) form the following food chain:

Grass (Producers) → Goat (Primary Consumer) → Human (Secondary Consumer)

**92.** Which one of the following types of tissues will have contractile proteins?
(a) Nervous tissue   (b) Muscle tissue
(c) Bone tissue      (d) Blood tissue

⊙ *(b)* Contractile proteins are found in muscles. These proteins are actin and myosin and they are, the main components of thin and thick filaments respectively. Muscle contraction occurs when these filaments slide over one another in a series of repetitive events.

**93.** If by an unknown accident the acid secreting cells of the stomach wall of an individual are damaged. Digestion of which one of the following biomolecule will be affected to a greater extent?
(a) Protein only
(b) Lipid
(c) Carbohydrate only
(d) Protein and Carbohydrate

⊙ *(a)* Pepsinogen is an inactive form of pepsin which is secreted by oxyntic cells. It is converted into active form called Pepsin in the presence of HCl. Pepsin converts protein molecules into proteases, peptones and ultimately into amino acids. Hence, proteins will not be digested if acid secreting cells of stomach wall of an individual get damaged.

**94.** In which one of the following places, Headquarters of a Railway Zone is located?
(a) Kanpur       (b) Lucknow
(c) Hajipur      (d) New Jalpaiguri

⊙ *(c)* The East Central Railway (ECR) is one of the 17 railway zones in India. It is headquartered at Hajipur (Bihar) and comprises Sonpur, Samastipur, Danapur, Mughalsarai and Dhanbad divisions.

**95.** Which of the following statements about Indian Academy of Highway Engineers is/ are correct?
1. It is a registered society.
2. It is a collaborative body of both Central Government and State Governments.

*Select the correct answer using the codes given below*
(a) 1 only        (b) 2 only
(c) Both 1 and 2  (d) Neither 1 nor 2

⊙ *(d)* Indian Academy of Highway Engineers is the apex training institute set-up to address the training needs of Highway and Bridge Engineers in the country. It was set-up as an Institute in the year 1983 with the primary objective to fulfill the need for training of highway engineers at the entry level and during the service. It is neither a registered society nor is a collaboration between the central and the State Governments.

**96.** Which one of the following is NOT a tributary of Indus River?
(a) Beas         (b) Ravi
(c) Chenab       (d) Tawi

⊙ *(d)* The Tributaries of Indus include Beas, Chenab, Gar, Gilgit, Gomal, Hunza, Jhelum, Kabul, Kunar, Kurram, Panjnad, Ravi, Shyok, Soan, Suru, Satluj, Swat, Zanskar, Zhob. Tawi is not one of the tributaries of Indus river. It is one of the left bank tributary of the Chenab River.

**97.** Which one among the following is the largest tiger reserve of India in terms of area of the core/critical tiger habitat?
(a) Manas
(b) Pakke
(c) Nagarjunasagar Srisailam
(d) Periyar

⊙ *(c)* Nagarjunsagar-Srisailam Tiger Reserve is the largest tiger reserve in India in terms of area of core and critical habitat. The reserve spreads over five districts, Kurnool district, Prakasam district, Guntur district, Nalgonda district and Mahbubnagar district. The total area of the tiger reserve is 3,568 km2 (1,378 sq mi).

**98.** Which one of the, following is NOT a coastal depositional feature ?
(a) Tombolo      (b) Sand bar
(c) Stack        (d) Spit

⊙ *(c)* A stack or sea stack is a geological landform consisting of a steep and often vertical column or columns of rock in the sea near a coast, formed by wave erosion. Stacks are formed over time by wind and water, processes of coastal geomorphology. All the other options are forms of coastal depositional landform.

**99.** Which of the following is/are coastal erosional feature( s) ?
1. Notch        2. Sea Arch
3. Cliff        4. Hook

*Select the correct answer using the codes given below*
(a) 1, 2 and 3   (b) 2, 3 and 4
(c) 2 and 3 only (d) 1 only

⊙ *(a)* A notch or small hollow is an indenture or a concave cut into a surface or edge (as in a coastline). It is not an erosional feature Sea Arch and Cliff are examples of Coastline erosional features. A sea arch is a natural opening eroded out of a cliff face by marine processes.

Some arches appear to have developed from surge channels, which are created by wave refraction causing the focussing of wave fronts on the side of a headland. A cliff is a high area of rock with a very steep side, often on a coast.

**100.** Chemical weathering of rocks is much greater in a place with
(a) cold and dry climate
(b) hot and humid climate
(c) hot and dry climate
(d) cold and humid climate

⊘ **(b)** Chemical Weathering results from chemical reactions between minerals in rocks and external agents like air or water. Oxygen oxidizes minerals to alteration products whereas water can convert minerals to clays or dissolve minerals completely. It occurs majorly in areas with hot and humid climate.

**101.** Which of the following statements about specific heat of a body is/are correct?
1. It depends upon mass and shape of the body
2. It is independent of mass and shape of the body
3. It depends only upon the temperature of the body

*Select the correct answer using the codes given below*
(a) 1 only
(b) 2 and 3
(c) 1 and 3
(d) 2 only

⊘ **(a)** Every substance has a unique value for the amount of heat absorbed or rejected to change the temperature of unit mass of it by one unit. This quantity is referred as the specific heat capacity of a body. It depends on the nature as well as mass of the substance and its change in temperature. Hence, option (a) is correct.

**102.** Which one of the, following is an example of the force of gravity of the earth acting on a vibrating pendulum bob?
(a) Applied force
(b) Frictional force
(c) Restoring force
(d) VIrtual force

⊘ **(c)** An object that is vibrating is acted upon by a restoring force. The restoring force causes the vibrating object to slow down as it moves away from the equilibrium position and to speed up as it approaches the equilibrium position. There is the force of gravity that acts downward upon the bob. And the tension force results from string pulling upon the bob of the pendulum.

**103.** Which one of the following statements about the refractive index of a material medium with respect to air is correct?
(a) It can be either positive or negative
(b) It can have zero value
(c) It is unity for all materials
(d) It is always greater than one

⊘ **(d)** Refractive index of a material is given by the ratio of speed of light in vacuum to the speed of light in any medium. Refractive index is always greater than one because by the definition of refractive index, $n$ of a medium (e.g. glass) is the ratio of the speed of light in a vacuum $C$, to the speed in the medium v and speed of light is maximum in vaccum, which gives the value of n greater than one.

**104.** Which one of the following statements about magnetic field lines is NOT correct?
(a) They can emanate from a point
(b) They do not cross each other
(c) Field lines between. two poles cannot be precisely straight lines at the ends
(d) There are no field lines within a bar magnet

⊘ **(c)** The magnetic field lines is defined as the path along which the compass needless are aligned. They cannot intersect i.e., do not cross each other. Tangent drawn at any point gives the direction of magnetic field. Outside a magnet, they are directed from north to south pole and inside a magnet they are directed from south to north. These lines are always normal to the surface, which are precisely straight at the ends. Hence, statement (c) is not correct.

**105.** Two convex lenses with power 2 dioptre are kept in contact with each other. The focal length of the combined lens system is
(a) 0.10 m
(b) 2m
(c) 4m
(d) 0·25m

⊘ **(d)** Here, $P_1 = P_2 = 2$ ∴ dioptre ($D$)
Using the formula total power, $P = P_1 + P_2$
$= 2 + 2 = 4D, f = \dfrac{1}{P} = \dfrac{1}{4m} = 0.25\,m$

**106.** Which one of the following alkali metals has lowest melting point?
(a) Sodium
(b) Potassium
(c) Rubidium
(d) Caesium

⊘ **(d)** The melting point of alkali metals decreases on moving down the group due to decrease in the extent of metallic bonding. Thus, the sequence of melting point among the given options is

Na > K > Rb > Cs

Therefore, Caesium (Cs) have the least melting point.

**107.** Which one of the following metals is alloyed with sodium to transfer heat in a nuclear reactor?
(a) Potassium
(b) Calcium
(c) Magnesium
(d) Strontium

⊘ **(a)** Sodium-potassium alloy (NaK) is an alloy of two alkali metals sodium and potassium. It is used as a heat-transfer. Coolant in fast-breeder nuclear reactors and experimentally in gas-turbine power plants.

**108.** Which one of the following metals is used in the filaments of photo-electric cells that convert light energy into electric energy?
(a) Tungsten
(b) Copper
(c) Rubidium
(d) Aluminium

⊘ **(a)** Photo electric cells is a device which convert light-energy directly into electric energy. Main type of these cells are photo voltaic cell, photo emissive cell, photo conductive cell. The filament of photo electric cells are made up of tungsten. Electrons are obtained from the filament by thermionic emission and are accelerated to the anode by a potential difference (~ 100 kV).

**109.** Which of the following statements about Ring of Fire is/are correct?
1. It is a zone of convergence plate boundaries
2. It is an active seismic and volcanic zone
3. It is associated with deep trench

*Select the correct answer using the codes given below*
(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 only
(d) 1, 2 and 3

⊘ **(d)** The Ring of Fire is a ring of volcanoes around the Pacific Ocean that result from subduction of oceanic plates beneath lighter continental plates. This is also where the Earth's deep ocean trenches are and where the Earth's deep earthquakes are. The trenches form because the downgoing plate is bent downward as it subducts. It is the zone of convergence plate boundaries.

**110.** Which one of the following Himalayan vegetation species is found between the altitudes of 1800 to 2600 metres?
(a) Saal
(b) Chir
(c) Spruce
(d) Deodar

⊘ **(d)** Deodar tree is also known as Himalayan Cedar. These evergreen coniferous trees are native of Pakistan and Himalayas as they thrive at altitudes of 1,500-3,200 meters above sea level.

Deodar grows to a height of 40–60 meters (131–164 feet) with a trunk diameter of 3 meters (10 feet). The timber of Deodar tree is used as construction material for its durability.

**111.** Which one of the following rivers is NOT a tributary of river Cauvery?
(a) Hemavati        (b) Arkavati
(c) Indravati       (d) Amravati

◎ *(c)* The tributaries of Kaveri include Amaravati, Arkavathy, Bhavani, Chinnar, Hemavati, Honnuhole, Kabini, Kannika, Kollidam, Lakshmana Tirtha, Lokapavani, Noyyal, Pambar, Shimsha, Sujyothi. Indravati River is a stream of the river Godavari and is its tributary.

**112.** Which of the following conditions is/are essential for tea cultivation?
1. Tropical and sub-tropical climate
2. Heavy rainfall ranging from 150 cm to 250 cm
3. Soil should contain good amount of lime.

*Select the correct answer using the codes given below*
(a) 1, 2 and 3        (b) 1 and 2 only
(c) 2 and 3 only      (d) 1 only

◎ *(b)* Tea is an evergreen plant. It grows both in tropical and subtropical regions. The tea plant grows well in hot and humid climate. Actually, there is an intimate relationship between climate, yield per hectare and the quality of tea.
  (i) The ideal temperature for growing tea is between 20 °C and 30 °C.
  (ii) It needs heavy rainfall between 150 and 250 cm. It should be well-distributed throughout the year and should be in the form of frequent showers.

However it does not require soils with lime for its cultivation.

**113.** Bharatmala Project is related to
(a) Improving road connectivity
(b) Interlinking ports and railways
(c) Interlinking of rivers
(d) Interlinking major cities with gas pipelines

◎ *(a)* Bharatmala project is the second largest highways construction project in the country. It aims to build 50,000 km of highway roads targeted across the country. Bharatmala will look to improve connectivity particularly on economic corridors, border areas and far flung areas with an aim of quicker movement of cargo and boosting exports.

**114.** Which one of the following is a local wind that blows out from Siberia ?
(a) Bora        (b) Purga
(c) Mistral     (d) Blizzard

◎ *(d)* Blizzards are the local winds of Siberia which blows out from the Siberian region to parts of south Asia. Blizzards are dangerous winter storms that are a combination of blowing snow and wind resulting in very low visibilities.

**115.** Which one of the following centres is NOT known for iron and steel industry ?
(a) Bhadravati        (b) Salem
(c) Vishakhapatnarn (d) Renukoot

◎ *(d)* Mysore Iron and Steel Company (MISCO) is located at Bhadravati on the banks of river Bhadravati in Shimoga district of Karnataka. Salem steel plant is located in the Salem district of Tamil Nadu. The plant has the advantage of rich iron ore and limestone, which is readily available in the adjoining areas. It also enjoys the facilities of cheap power, charcoal and vast market. Hindalco's plant is located in Renukoot which is involved in the smelting of Aluminium.

**116.** Which of the following are essential prerequisites for establishment of a thermal power station?
1. Availability of fossil fuels
2. Proximity to a river, lake or sea
3. Good transport network
4. Proximity to an urban centre

*Select the correct answer using the codes given below*
(a) 1, 2 and 3        (b) 2 and 4
(c) 2 and 3 only      (d) 1 and 3 only

◎ *(a)* The essential prerequisites of a thermal power plant include that it must be in proximity to the fossil fuel and proximity to a water source like river, lake or sea. It should also have a good transport network for the transportation of raw materials and the finished products. However proximity to an urban centre is not one of the essential conditions for establishment of a power plant.

**117.** Which of the following statements about 'Aadi Mahotsav' held recently in New Delhi is/are correct?
1. The theme of the festival was 'A Celebration of the Spirit of Tribal Culture, Cuisine and Commerce'
2. The festival was organised by the Ministry of Tourism, Government of India

*Select the correct answer using the codes given below*
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

◎ *(a)* The Aadi Mahotsav, a celebration of the spirit of Tribal Culture, Craft, Cuisine and Commerce was held at Dilli Haat, New Delhi, on the of 30th November, 2017. The theme of this festival was 'A Celebration of the Spirit of Tribal Culture, Cuisine and Commerce'. It was organised by the Tribal secretary and not ministry of tourism.

**118.** Light year is a unit for measurement of
(a) age of universe
(b) very large distance
(c) very small time intervals
(d) very high temperature

◎ *(b)* Light year is a unit of measure of distance travelled by light in one year. It is used for measuring very large distance.
1 light year = $9.46 \times 10^{15}$ m.

**119.** Which of the following statements about electromagnetic waves, sound waves and water waves is/are'' correct?
1. They exhibit reflection
2. They carry energy
3. They exert pressure
4. They can travel in vacuum

*Select the correct answer using the codes given below*
(a) 1, 2 and 3        (b) 2 and 4
(c) 1 and 3 only      (d) 1 only

◎ *(a)* Electromagnetic waves, sound waves and water waves exhibit reflection, carry energy (EM waves), exert pressure (water waves), but sound waves cannot travel in vaccum.

**120.** Thermal capacity of a body depends on the
(a) mass of the body only
(b) mass and shape of the body only
(c) density. of the body
(d) mass, shape and temperature of the body

◎ *(d)* Thermal capacity of a body can be defined as the amount of heat required to change the temperature of an object by a certain degree. It depends upon the mass, shape and temperature of the body.

**121.** Who among the following first used the term 'Industrial Revolution' in English to describe, the changes that occurred in British industrial development between 1760 and 1820?
(a) Karl Marx        (b) Georges Michelet
(c) Arnold Toynbee  (d) Friedrich Engels

◎ *(c)* The term Industrial Revolution was first popularised by the English economic historian Arnold Toynbee to describe Britain's economic development from 1760 to 1820.

**122.** Which one of the following statements about the Olympe de Gouges (1748- 1793) is correct?
(a) She declared that although citizens should have equal rights, they are not entitled to the same honours by the State
(b) She was a supporter of the Jacobin government
(c) She was jailed for treason by the National Assembly
(d) She declared that the nation is the union of woman and man

⊚ *(a)* Only the first statement (a) is correct. She declared that although citizens should have equal rights they are not entitled to the same honours by the state.

**123.** Who among the following built a model steam engine in 1698 called ''Miner's Friend'' to drain mines?
(a) Thomas Savery
(b) Thomas Newcomen
(c) James Watt
(d) Richard Arkwright

⊚ *(a)* Thomas Savery was an English inventor and engineer, born in England. He invented the first commercially used steam powered device in 1698, a steam pump which is often referred to as an 'engine'. Savery's 'engine' was a revolutionary method of pumping water, which solved the problem of mine drainage and made widespread public water supply practical.

**124.** Which one of the following statements about Renaissance Humanist culture is NOT true?
(a) It slackened the control of religion over human life
(b) It believed that human nature was many-sided
(c) It was concerned with good manners
(d) It criticised material wealth, power and glory

⊚ *(c)* The term Renaissance Humanist was derived from the program of studies called the 'Studia Humanitatis'. However, the idea of calling this 'Humanism' only really arose in the nineteenth century. It majorly advocated that human outlook should be based on a modern plan and not blindly follow a religious plan. It never spoke about good manners.

**125.** Who gifted the *Badshah Nama* to King George in 1799?
(a) Abul Fazl
(b) Abdul Hamid Lahori
(c) Nawab of Awadh
(d) William Jones

⊚ *(c)* In 1799, the Nawab (provincial governor) of Awadh in northern India sent the Padshahnama, to King George III of England. Badshahnama is a genre of works written as the official visual history of Mughal emperor, Shah Jahan's reign.

**126.** What is the name of the award given to meritorious men in the Mughal Court in the form of a robe of honour that was once worn by the Emperor?
(a) Sarapa    (b) Patka
(c) Padma murassa  (d) Khilat

⊚ *(c) Padma murassa* i.e., a lotus blossom set with jewel was considered a special gift. It was gifted by the emperor only in exceptional circumstances. The emperor would receive several gifts from the courtiers. The courtiers would not go to emperor empty-handed.

**127.** Who among the following social reformer started a society for the encouragement of widow remarriage in 1866 in Maharashtra ?
(a) Bal Gangadhar Tilak
(b) Jyotirao Phule
(c) Vishnushastri Pandit
(d) Pandita Ramabai

⊚ *(b)* Jyotirao Govindrao Phule was an Indian social activist, a thinker and anti-caste social reformer from Maharashtra. He worked for widow remarriage and started a home for pregnant Brahmin widows to give birth in a safe and secure place in 1863.

**128.** Name the first major voluntary association representing primarily Indian landlord interests that was set up in Calcutta in 1851 ?
(a) British Indian Association
(b) Landholder's Society
(c) Madras Native Association
(d) Bombay Association

⊚ *(a)* British Indian Association was created after amalgamating the "Landholders Society" and "British India Society" on 31 October 1851. This was the first political organisation that brought the Indian landlords together. The President of the first committee of this organisation was Raja Radhakanta Deb.

**129.** Who among the following introduced the Permanent Settlement of Bengal in 1793 ?
(a) Lord Cornwallis
(b) Lord Ripon
(c) Robert Clive
(d) John Adam

⊚ *(a)* The Permanent Settlement was introduced by Lord Cornwallis in 1793. It was an agreement between the British East India Company and the Landlords of Bengal to settle the Land Revenue to be raised. In this agreement it was agreed that the landlords would have perpetual and hereditary rights over the land, so long as they pay the fixed revenue to the British Government.

**130.** Name the rebel who fought against the British in the battle of Chinhat in the course of the 1857 Revolt?
(a) Ahmadullah Shah (b) Shah Mal
(c) Mangal Pandey  (d) Kunwar Singh

⊚ (*) The Battle of Chinhat between the British forces and Indian rebels, at Ismailganj, near Chinhat. The British were led by The Chief Commissioner of Oude, Sir Henry Lawrence. The insurgent force, which consisted of mutineers from the East India Company's army and retainers of local landowners was led by Barkat Ahmad, a mutineer officer of the Company's army.

**131.** Who among the following are the two civil servants who assisted the Constituent Assembly in framing the Constitution of India ?
(a) B. N. Rau and K. M. Munshi
(b) S. N. Mukherjee and Alladi Krishnaswamy Aiyar
(c) B. N. Rau and S. N. Mukherjee
(d) K. M. Munshi and Alladi Krishnaswamy Aiyar

⊚ *(c)* Sir Benegal Narsing Rao was an Indian civil servant, jurist, diplomat and statesman known for his key role in drafting the Constitution of India. S.N Mukherjee was another civil servant who assisted in the framing of the Constitution of India.

**132.** Which member of 'the Constituent Assembly proposed the resolution that the National Flag of India be a ''horizontal tricolour of saffron, white and dark green in equal proportion'', with a wheel in navy, blue at the centre?
(a) Jawaharlal Nehru
(b) B. R. Ambedkar
(c) Rajendra.Prasad
(d) Sardar Vallabhbhai Patel

⊚ *(c)* In 1947, when India gained freedom from the British, a committee headed by Dr. Rajendra Prasad decided to adopt the flag of the Congress as the national flag of India with a few modifications. With this in mind, the flag of 1931 was adopted as the national flag of India, but the charkha in the middle was replaced with the Ashoka Chakra. Thus, the Indian national flag was born. The saffron colour on top represents sacrifice, white represents peace and purity and green signifies the law of dharma (righteousness).

**133.** Which of the following is/are NOT historical biography/biographies?

1. Dipavamsa
2. Harshacharita
3. Vikramankadevacharita
4. Prithvirajavijaya

*Select the correct answer from the codes given below*

(a) 1 only          (b) 2 and 3 only
(c) 2, 3 and 4 only   (d) 1, 2, 3 and 4

⊘ *(a)* Dipavamsa is one of the most important works in Pali Literature. It details the tooth relic and Bodhi Tree's arrival in Sri Lanka. It also deals with the arrival of Buddha's teaching and preachers in Sri Lanka. It mentions that Buddha visited Kelaniya and Dighavapi in Sri Lanka. All the other options are biographies about kings.

**134.** Which of the following pairs are correctly matched ?

| | Traveller | Country from |
|---|---|---|
| 1. | Marco Polo | Italy |
| 2. | Ibn Battuta | Morocco |
| 3. | Nikitin | Russia |
| 4. | Seydi Ali Reis | Turkey |

*Select the correct answer using the codes given below*

(a) 1, 2 and 3 only   (b) 2 and 3 only
(c) 1, 2, 3 and 4     (d) 1 and 4 only

⊘ *(c)* Marco Polo was an Italian merchant, explorer and writer, born in the Republic of Venice. Ibn Battuta was a Moroccan scholar who widely travelled the medieval world. Afanasy Nikitin was a Russian merchant of Tver and one of the first Europeans to travel and to document his visit to India.  Seydi Ali Reis was an Ottoman admiral and navigator from Turkey. He commanded the left wing of the Ottoman fleet at the naval Battle of Preveza in 1538. So all the given options are correct.

**135.** Which of the following clans are included in the Agnikula Rajputs ?

1. Pratiharas     2. Chaulukyas
3. Paramaras      4. Chahamanas

*Select the correct answer from the codes given below*

(a) 1 and 3 only    (b) 1, 3 and 4 only
(c) 1, 2, 3 and 4   (d) 2 and 4 only

⊘ *(c)* The four Rajput clans from Agnikunda were Chauhans, Chalukyas, Parmaras and Pratiharas. This theory comes from the Prithvirajraso of Chandarbardai. The Rajputs descended from Agni and this theory is based upon the Agnikula Legend of Bhavishyapurana .

**136.** Who among the following was the author of Humayun Nama?

(a) Roshanara Begum
(b) Ruquaiya Sultan Begum
(c) Gulbadan Begum
(d) Gauhara Begum

⊘ *(c)* Gulbadan begum is best known as the author of Humayun-Nama, the account of the life of her half-brother, Emperor Humayun, which she wrote on the request of her nephew, Emperor Akbar.

**137.** Which one of the following about the Parliament of India is NOT correct?

(a) The Parliament consists of the President, the Lok Sabha and the Rajya Sabha
(b) There are no nominated members in the Lok Sabha
(c) The Rajya Sabha cannot be dissolved
(d) Some members of the Rajya Sabha are nominated by the President

⊘ *(c)* Anglo-Indians are the only community that has its own representatives nominated to the Lok Sabha (Lower House) in India's Parliament. It is done only if the President of India feels that the Anglo Indian community has not been adequately represented in the Lok Sabha.

**138.** Which one of the following statements with regard to the Comptroller and Auditor General (CAG) of India is NOT correct?

(a) He is appointed by the President of India
(b) He can be removed from office in the same way as the judge of the Supreme Court of India
(c) The CAG is eligible for further office under the Government of India after he has ceased to hold his office
(d) The salary of the CAG is charged upon the Consolidated Fund of India

⊘ *(c)* The CAG is not eligible for further office either under the Government of India or under the Government of any State after he has ceased to hold his office. These provisions are in order to ensure the independence of CAG. All the other provisions mentioned are correct.

**139.** The Superintendence, direction and control of elections in India is vested in

(a) The Supreme Court of India
(b) The Parliament of India
(c) The Election Commission of India
(d) The Chief Election Commissioner

⊘ *(c)* According to Article 324 of the Constitution of India the superintendence, direction and control of the preparation of the electoral rolls for,

and the conduct of, all elections shall be vested in the Election Commission. Therefore it is the sole authority in India that takes charge of elections in India.

**140.** Which of the following provision(s) of the Constitution of India became effective from .26th November, 1949?

1. Elections
2. Citizenship
3. Emergency provisions
4. Appointment of the Judges

*Select the correct answer using the codes given below*

(a) 1 only          (b) 1 and 2 only
(c) 1, 2 and 3      (d) 2 and 4

⊘ *(b)* The provisions of elections and citizenship became effective from 26th November 1949. Rest of the provisions including the Emergency provisions and the appointment of the judges  were added later in the Constitution.

**141.** Which of the following statements regarding construction of Rohtang tunnel is NOT correct?

(a) It is located at an altitude of 5,000 feet
(b) It will provide all-year connectivity to Lahaul and Spiti Valley
(c) The tunnel is being built by the Border Roads Organisation
(d) It will reduce the length of the Leh-Manali highway by approximately 50 km

⊘ *(a)* The Rohtang tunnel is located at 3,100 metres (10,171 ft). It is being built by the Border Roads organisation to provide all year connectivity to Lahaul and Spiti valley. It also aims at reducing the distance of Manali Leh Highway by Approx 50 km.

**142.** Who among the following recently became the first woman pilot in Indian Navy?

(a) Astha Segal    (b) Roopa A
(c) Sakthi Maya S (d) Shubhangi Swaroop

⊘ *(d)* Shubhangi Swaroop has made history by becoming the first woman to be inducted into the Indian Navy as a pilot.

**143.** Who among the following Indians did NOT hold the title of Miss World ?

(a) Reita Faria
(b) Sushmita Sen
(c) Diana Hayden
(d) Yukta Mookhey

⊘ *(b)* Sushmita Sen is the winner of the Miss Universe pageant of 1994. Sen was the first Indian woman to win the crown. All the other options are former Miss World.

**144.** Which one of the following countries has failed to qualify for the first time in 60 years for the FIFA World Cup to be held in Russia in the year 2018 ?
(a) Mexico     (b) Iran
(c) Saudi Arabia     (d) Italy

⊘ *(d)* Italy has failed to qualify for first World Cup in 60 years after play-off defeat to Sweden.

**145.** The Defence Technology and Trade Initiative (DTTI) is a forum for dialogue on defence partnership between India and
(a) Russia
(b) United States of America
(c) Israel
(d) France

⊘ *(b)* Defence Technology and Trade Initiative (DTTI) originally called the 'Carter Initiative' after the name of U.S. Deputy Secretary of Defence Ashton Carter. Its main objective is to strengthen defence cooperation by facilitating the Indian companies to collaborate with US partners in defense co-production, where the US provides technology and guidance for building modern weapon systems.

**146.** As per the policy applicable in 2017, how much Foreign Direct Investment (FDI) is permitted in the defence sector in India?
(a) 49 per cent through the automatic route
(b) 26 per cent through the government route
(c) 26 per cent through the automatic route and beyond that up to 49 per cent through the government route
(d) 75 per cent through the automatic route

⊘ *(a)* In the defence sector foreign investment up to 49% is permitted under the automatic route, foreign investment beyond 49% and upto 100% is permitted through Government approval, wherever it is likely to result in access to modern technology or for other reasons to be recorded.

**147.** Which one of the following countries did NOT participate in the 21st edition of Exercise Malabar?
(a) United States of America
(b) Japan
(c) India
(d) Australia

⊘ *(d)* The 2017 Malabar exercise was the 21st edition of the exercise and conducted from 10 to 17 July 2017. This edition involved navies from India, USA and Japan. The exercise included a harbor phase at Chennai. Australia was not a participant in this exercise.

**148.** Justice Dalveer Bhandari of India was recently re-elected to the International Court of Justice after Christopher Greenwood pulled out before 12th round of voting. Christopher Greenwood was a nominee of
(a) Canada     (b) Russia
(c) Britain     (d) USA

⊘ *(c)* Sir Christopher John Greenwood is a British judge at the International Court of Justice, to which he was elected on 6th November 2008. He recently pulled out of before the 12th round of voting.

**149.** In order to review the Income Tax Act, 1961 and to draft a new Direct Tax Law in consonance with economic needs of the country, the Government of India in November 2017 has constituted a Task Force. Who among the following is made the convenor of it ?
(a) Shri Arvind Subramanian
(b) Shri Arbind Modi
(c) Shri Amitabh Kant
(d) Dr. Bibek Debroy

⊘ *(b)* The Union Government has constituted a task force to review the Income Tax Act 1961 and draft a new direct tax law in consonance with current economic needs. Central Board of Direct Taxes (CBDT) member Arbind Modi will be the convener of task force while Chief Economic Adviser Dr. Arvind Subramanian will be a permanent Special Invitee. The task force will draft an appropriate direct tax legislation keeping in view the direct tax system in various countries, international best practices and economic needs of the country. It will submit its report within six months.

**150.** The 5th Global Conference on Cyber Space (GCCS) was held in New Delhi in November, 2017. Which of the following statements about GCCS is/are correct?

1. The 4th version of GCCS was held in London.

2. The main theme of GCCS 2017 is 'Cyber4All : A Secure and Inclusive Cyberspace for Sustainable Development'.

3. 'Bindu' is the logo of GCCS 2017.

*Select the correct answer using the codes given below*
(a) 1 only
(b) 2 only
(c) 2 and 3 only
(d) 1, 2 and 3

⊘ *(b)* The Global Conference on CyberSpace (GCCS) was started in 2011 in London, second GCCS was held in 2012 in Budapest with focus on relationship between internet rights and internet security. The third edition of GCCS was held in 2013 in Seoul . The fourth version GCCS 2015 was held on April 16-17, 2015 in The Hague, Netherlands, which saw participation from 97 countries. Hence the 1st statement is incorrect. Bindu is not the logo for this conference hence the third statement is also incorrect.