WBJEE - 2025

DATE : 27/04/2025

SUB : MATHEMATICS

CATEGORY - 1 (Q:1 to Q50)

(Carry 1 mark each. Only one option is correct. Negative marks: – ¼)

1. Let $f(x)$ be continuous on [0, 5] and differentiable in (0, 5). If $f(0) = 0$ and $|f'(x)| \le \frac{1}{5}$ for all x in (0, 5), then $\forall x$ in [0, 5].

(A) $|f(x)| \le 1$ (B) $|f(x)| \le \frac{1}{5}$ (C) $f(x) = \frac{x}{5}$ (D) $|f(x)| \ge 1$

Ans : (A)

Hint : $|f'(x)| \le \frac{1}{5}$, $\Rightarrow \left|\frac{f(x)-f(0)}{x-0}\right| \le \frac{1}{5}$

$\Rightarrow \left|\frac{f(x)}{x}\right| \le \frac{1}{5}$, $|f(x)| \le \frac{|x|}{5}$ $\Rightarrow |f(x)| \le 1$ $\left(0 \le x \le 5 \Rightarrow 0 \le \frac{x}{5} \le 1\right)$

2. If $g(f(x)) = |\sin x|$ and $f(g(x)) = \left(\sin\sqrt{x}\right)^2$, then

(A) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ (B) $f(x) = \sin x$, $g(x) = |x|$

(C) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$ (D) $f(x) = |x|$, $g(x) = \sin x$

Ans : (A)

Hint : $g(x) = \sqrt{x}$, $f(x) = \sin^2 x$

3. Let $f_n(x) = \tan\frac{x}{2}(1+\sec x)(1+\sec 2x)\ldots(1+\sec 2^n x)$, then

(A) $f_5\left(\frac{\pi}{16}\right) = 1$ (B) $f_4\left(\frac{\pi}{16}\right) = 1$ (C) $f_3\left(\frac{\pi}{16}\right) = 1$ (D) $f_2\left(\frac{\pi}{16}\right) = 1$

Ans : (D)

Hint : $f_n(x) = \tan 2^n x$, $f_5\left(\frac{\pi}{16}\right) = \tan\left(32.\frac{\pi}{16}\right) = 0$

$f_4\left(\frac{\pi}{16}\right) = \tan\left(16.\frac{\pi}{16}\right) = 0$

$f_3\left(\frac{\pi}{16}\right) = \tan\left(8.\frac{\pi}{16}\right) = \text{undefined}$

$f_2\left(\frac{\pi}{16}\right) = \tan\left(4.\frac{\pi}{16}\right) = 1$

4. If for a matrix A, |A| = 6 and adj $A = \begin{bmatrix} 1 & -2, & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix}$, then k is equal to

(A) –1 (B) 1 (C) 2 (D) 0

Ans : (C)

Hint : A.(adj A) = |A| I

$\Rightarrow |A||\text{adj} A| = |A|^3 \quad \Rightarrow |\text{adj} A| = |A|^2$

– 1((–2 – 4) – K (1 – 16) = 36 $\Rightarrow$ 6 – K + 16K = 36

$\Rightarrow$ 15 K = 30 $\Rightarrow$ K = 2

5. If x = –1 and x = 2 are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$, $(x \neq 0)$, then

(A) $\alpha = -6, \beta = \frac{1}{2}$ (B) $\alpha = -6, \beta = -\frac{1}{2}$ (C) $\alpha = 2, \beta = -\frac{1}{2}$ (D) $\alpha = 2, \beta = \frac{1}{2}$

Ans : (C)

Hint : $f'(x) = \frac{\alpha}{|x|}.\frac{|x|}{x} + 2\beta x + 1$, f(–1) = f(2) = 0

So, $-\alpha - 2\beta + 1 = 0 \Rightarrow \alpha + 2\beta - 1 = 0$ ....(i)

& $\frac{\alpha}{2} + 4\beta + 1 = 0 \Rightarrow \alpha + 8\beta + 2 = 0$ .... (ii)

(ii) – (i) $\Rightarrow 6\beta + 3 = 0 \Rightarrow \beta = -1/2$

(i) $\Rightarrow \alpha - 1 - 1 = 0 \Rightarrow \alpha = 2$

6. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors of equal magnitude such that the angle between $\vec{a}$ and $\vec{b}$ is α, $\vec{b}$ and $\vec{c}$ is β and $\vec{c}$ and $\vec{a}$ is γ. Then the minimum value of $\cos\alpha + \cos\beta + \cos\gamma$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

Ans : (D)

Hint : Let $|a| = |\bar{b}| = |\bar{c}| = \lambda$

We have, $\bar{a}.\bar{b} = \lambda^2\cos\alpha$, $\bar{b}.\bar{c} = \lambda^2\cos\beta$, $\bar{c}.\bar{a} = \lambda^2\cos\gamma$

Now, $\left|\bar{a}+\bar{b}+\bar{c}\right| \geq 0$

$\Rightarrow 3\lambda^2 + 2\lambda^2(\cos\alpha + \cos\beta + \cos\gamma) \geq 0$

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma \geq -3/2$

7. Let $p(x)$ be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p′(0) is equal to

(A) 8 (B) 9 (C) 3 (D) 6

Ans : (B)

Hint : Degree 3

$P'(x) = a(x-1)(x-3)$

$\Rightarrow P'(x) = a(x^2 - 4x + 3) \Rightarrow P(x) = a\left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right) + b$

$P(1) = 6 \Rightarrow a\left(\frac{1}{3} - 2 + 3\right) + b = 6 \Rightarrow \frac{4a}{3} + b = 6$

$\Rightarrow 4a + 3b = 18$ ....(i)

$\Rightarrow P(3) = 2 \Rightarrow a(9 - 18 + 9) + b = 2$

$\Rightarrow b = 2$ ....(ii)

Put in (i) $\Rightarrow 4a + 6 = 18 \Rightarrow a = 3$

So, $p'(x) = 3(x-1)(x-3) \Rightarrow P'(0) = 9$

8. The expression $2^{4n} - 15n - 1$, where $n \in \mathbb{N}$ (the set of natural numbers) is divisible by

(A) 125 (B) 225 (C) 325 (D) 425

Ans : (B)

Hint : $2^{4n} - 15n - 1 = (1+15)^n - 15n - 1 = \left({}^nC_0 + {}^nC_1.15 + {}^nC_2 15^2 + ... + {}^nC_n.15^n\right) - 15n - 1$

$= 15^2\left({}^nC_2 + {}^nC_3.15 .... + {}^nC_n.15^{n-2}\right)$

$= 225\,K,\ K \in \mathbb{N}$

9. Let $\phi(x) = f(x) + f(2a - x), x \in [0, 2a]$ and $f''(x) > 0$ for all $x \in [0, a]$. Then $\phi(x)$ is

(A) increasing on [0, a] (B) decreasing on [0, a] (C) increasing on [0, 2a] (D) decreasing on [0, 2a]

Ans : (B)

Hint : $\phi'(x) = f'(x) - f'(2a - x) \quad \because f''(x) > 0$

$\therefore f'(x)$ is increasing

So, $\phi(x)$ is decreasing in (0, a) as $f'(x) < f'(2a - x)$ in (0, a)

10. If E and F are two independent events with P(E) = 0.3 and $P(E \cup F) = 0.5$, then *P(E/F) – P(F/E)* equals

(A) $\frac{2}{7}$ (B) $\frac{3}{35}$ (C) $\frac{1}{70}$ (D) $\frac{1}{7}$

Ans : (C)

Hint : $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow 0.5 = 0.3 + P(F) - P(E).P(F)$

$\Rightarrow 0.2 = P(F) - 0.3\,P(F)$

$\Rightarrow P(F) = \frac{2}{7}$

So, $P\left(E/F\right) - P\left(F/E\right) = P(E) - P(F) = \frac{3}{10} - \frac{2}{7} = \frac{21-20}{70} = \frac{1}{70}$

11. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is

(A) ${}^{52}C_3$ (B) ${}^{51}C_4$ (C) ${}^{52}C_4$ (D) ${}^{51}C_3$

Ans : (C)

Hint : ${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 \quad \left(\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$

$= {}^{52}C_4$

12. A function $f : \mathbb{R} \to \mathbb{R}$, satisfies $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)+f(0)}{3}$ for all, x, y $\in \mathbb{R}$. If the function 'f' is differentiable at x = 0, then f is

(A) linear (B) quadratic (C) cubic (D) biquadratic

Ans : (A)

Hint : Put x = 0, y = 0 $\Rightarrow$ f(0) = 0, $\Rightarrow$ f(x) = ax (linear)

13. Suppose, α, β, γ are the roots of the equation $x^3 + qx + r = 0$ (with $r \neq 0$) and they are in A.P. Then the rank of the matrix

$\begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{pmatrix}$ is

(A) 3 (B) 2 (C) 0 (D) 1

Ans : (Data inadequate)

Hint : Here, $\alpha + \beta + \gamma = 0$

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = -\left(\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma\right) = 0 \; (\because \alpha + \beta + \gamma = 0)$$

Given, $2\beta = \alpha + \gamma \Rightarrow 3\beta = \alpha + \beta + \gamma \Rightarrow \beta = 0 \Rightarrow r = 0$ (But given $r \neq 0$)

14. An n × n matrix is formed using 0, 1 and –1 as its elements. The number of such matrices which are skew symmetric is

(A) $\frac{n(n-1)}{2}$ (B) $(n-1)^2$ (C) $2^{n(n-1)/2}$ (D) $3^{n(n-1)/2}$

Ans : (D)

Hint : $3^{\frac{n(n-1)}{2}}$

Each diagonal entry must be '0' and sum of conjugate elements will be '0'. So, we need to select elements of one side of the diagonal, where each entry has three options.

15. If $x=\int_0^y \frac{1}{\sqrt{1+9t^2}}dt$ and $\frac{d^2y}{dx^2}=ay$, then a is equal to

(A) 3 (B) 6 (C) 9 (D) 1

Ans : (C)

Hint : $\Rightarrow \frac{dy}{dx}=\sqrt{1+9y^2} \Rightarrow \frac{d^2y}{dx^2}=9y$

16. If the sum of 'n' terms of an A.P. is $3n^2 + 5n$ and its mth term is164, then the value of m is

(A) 26 (B) 27 (C) 28 (D) 29

Ans : (B)

Hint : $t_1 = S_1 = 8$

$t_1 + t_2 = S_2 = 22$ So, $t_2 = 14 \Rightarrow d=6$

Given, $a+(m-1)d = 164 \Rightarrow 8 + (m-1)\,6 = 164 \Rightarrow m = 27$

17. The line parallel to the x-axis passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a =$ where $(a, b)\neq(0, 0)$ is

(A) above x-axis at a distance $\frac{3}{2}$ from it (B) above x-axis at a distance $\frac{2}{3}$ from it

(C) below x-axis at a distance $\frac{3}{2}$ from it (D) below x-axis at a distance $\frac{2}{3}$ from it

Ans : (C)

Hint : Let the line be $(ax + 2by + 3b) + \lambda(bx - 2ay - 3a)=0$

$\because$ It is parallel to x-axis.

$\therefore \frac{a+b\lambda}{2a\lambda-2b}=0 \Rightarrow \lambda=-\frac{a}{b}$

So, equation is $ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a)=0$

$\Rightarrow ax+2by+3b-ax+\frac{2a^2y}{b}+\frac{3a^2}{b}=0 \Rightarrow \left(2b+\frac{2a^2}{b}\right)y=-\left(\frac{3a^2}{b}+3b\right)$

$\Rightarrow y=-\frac{3}{2}$

18. If ${}^9P_5 + 5.{}^9P_4 = {}^{10}P_r$, then the value 'r' is

(A) 4 (B) 8 (C) 5 (D) 7

Ans : (C)

Hint : ${}^9P_5 + 5.{}^9P_4 = \frac{9!}{4!} + 5.\frac{9!}{5!}$

$= \frac{9!}{4!} + \frac{9!}{4!} = 2.\frac{9!}{4!}$

$= 2.\frac{10.9!}{10.4!} = \frac{10!}{5!} = {}^{10}P_r$

$\Rightarrow r = 5$

19. Consider three points P(cos α, sin β), Q(sin α, cos β), R (0, 0), where $0 < \alpha, \beta < \frac{\pi}{4}$. Then

(A) P lies on the line segment RQ (B) Q lies on the line segment PR

(C) R lies on the line segment PQ (D) P, Q, R are non-collinear

Ans : (D)

20. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors. Suppose $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Then $\vec{a}$ is

(A) $\vec{b} \times \vec{c}$ (B) $\vec{c} \times \vec{b}$ (C) $\vec{b} + \vec{c}$ (D) $\pm 2\left(\vec{b} \times \vec{c}\right)$

Ans : (D)

Hint : $\overline{a} \perp \overline{b}$ & $\overline{a} \perp \overline{c}$

Let $\overline{a} = \pm \lambda(\overline{b} \times \overline{c})$

$\Rightarrow |\overline{a}| = \pm\lambda\, |\overline{b}|\,|\overline{c}|\, \sin \pi/6$

$\Rightarrow \lambda = \pm 2$

21. The number of reflexive relations on a set A of n elements is equal to

(A) 2^{n^2} (B) n^2 (C) $2^{n(n-1)}$ (D) $n^2 - n$

Ans : (C)

Hint : Total number of ordered pair = n^2

Number of pair that can be included or excluded = $n^2 - n$

Each of the remaining $n^2 - n$ pairs we have two choices

Include it in the relation or not include it

So total no. of reflexive relation is $2^{n(n-1)}$

22. If the matrix $\begin{pmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{pmatrix}$ is orthogonal, then the values of a, b, c are

(A) $a=\pm\frac{1}{\sqrt{3}}, b=\pm\frac{1}{\sqrt{6}}, c=\pm\frac{1}{\sqrt{2}}$

(B) $a=\pm\frac{1}{\sqrt{2}}, b=\pm\frac{1}{\sqrt{6}}, c=\pm\frac{1}{\sqrt{3}}$

(C) $a=-\frac{1}{\sqrt{2}}, b=-\frac{1}{\sqrt{6}}, c=-\frac{1}{\sqrt{3}}$

(D) $a=\frac{1}{\sqrt{3}}, b=\frac{1}{\sqrt{6}}, c=\frac{1}{\sqrt{3}}$

Ans : (B)

Hint : $\vec{r}_1 = 0\hat{i} + a\hat{j} + a\hat{k}$ $\quad$ $\vec{r}_1.\vec{r}_1 = 1 \Rightarrow 2a^2 = 1$

$\vec{r}_2 = 2b\hat{i} + b\hat{j} - b\hat{k}$ $\quad$ $\vec{r}_2.\vec{r}_2 = 1 \Rightarrow 6b^2 = 1$

$\vec{r}_3 = c\hat{i} - c\hat{j} + c\hat{k}$ $\quad$ $\vec{r}_3.\vec{r}_3 = 1 \Rightarrow 3c^2 = 1$

$a=\pm\frac{1}{\sqrt{2}}, b=\pm\frac{1}{\sqrt{6}}, c=\pm\frac{1}{\sqrt{3}}$

23. For what value of 'a', the sum of the squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ will have the least value?

(A) 2 (B) 0 (C) 3 (D) 1

Ans : (D)

Hint : $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-2)^2 - 2(1-a)$

$= a^2 - 4a + 4 - 2 + 2a$

$= a^2 - 2a + 2$

$= (a-1)^2 + 1$

24. The value of the integral $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

(A) 2 (B) $\frac{3}{4}$ (C) 0 (D) –2

Ans : (C)

Hint : $I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

Applying King's property

$I = \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$

$$2I = \int_0^{\pi/2} \log\left(\left(\frac{4+3\sin x}{4+3\cos x}\right)\left(\frac{4+3\cos x}{4+3\sin x}\right)\right)dx = 0$$

$I = 0$

25. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A|^2 = 25$, then $|\alpha|$ equals to

(A) 5^2 (B) 1 (C) $\frac{1}{5}$ (D) 5

Ans : (C)

Hint : $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$|A| = 5(5\alpha) - 5\alpha(0) + \alpha(0)$

$|A| = 25\alpha \quad \Rightarrow |A|^2 = 625\alpha^2 = 25 \quad \Rightarrow \alpha^2 = \frac{1}{25} \quad \Rightarrow |\alpha| = \frac{1}{5}$

26. The sum of the first four terms of an arithmetic progression is 56. The sum of the last four terms is 112. If its first term is 11, then the number of terms is

(A) 10 (B) 11 (C) 12 (D) 13

Ans : (B)

Hint : $a + a + d + a + 2d + a + 3d = 56$

$4a + 6d = 56$

$6d = 56 - 44 = 12$

$\Rightarrow d = 2$

$t_1 + t_2 + t_3 + t_4 = 56$

$t_{n-3} + t_{n-2} + t_{n-1} + t_n = 112$

$t_1 = 11$

$t_1 + t_n = t_2 + t_{n-1} = t_3 + t_{n-2} = t_4 + t_{n-3} = k$

$t_1 + t_2 + t_3 + t_4 + t_n + t_{n-1} + t_{n-2} + t_{n-3} = 4k$

$\Rightarrow k = 42$

$t_1 + t_n = 42$

$\Rightarrow t_n = 42 - 11 = 31$

$t_n = t_1 + (n-1)d$

$\Rightarrow 31 - 11 = (n-1)d \quad \Rightarrow (n-1)d = 20$

$\Rightarrow n = 11$

27. If 'f' is the inverse function of 'g' and $g'(x) = \frac{1}{1+x^n}$, then the value of $f'(x)$ is

(A) $1 + \{f(x)\}^n$ (B) $1 - \{f(x)\}^n$ (C) $\{1 + f(x)\}^n$ (D) $\{f(x)\}^n$

Ans : (A)

Hint : $g^{-1}(x) = f(x)$

$$g'(x) = \frac{1}{1+x^n} \qquad \Rightarrow g'\left(g^{-1}(x)\right) = \frac{1}{1+\left(g^{-1}(x)\right)^n} = \frac{1}{1+\left(f(x)\right)^n}$$

$g(g^{-1}(x)) = x$

$g'(g^{-1}(x).(g^{-1}(x))' = 1$

$$(g^{-1}(x))' = \frac{1}{g'\left(g^{-1}(x)\right)} = 1+\left(f(x)\right)^n$$

28. $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}dx$ is equal to

(A) log 2 (B) 2 log 2 (C) $\frac{1}{2}\log 2$ (D) 4 log 2

Ans : (B)

Hint : $I = \int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}dx$

Applying King's property

$$I = \int_{-1}^{1} \frac{-x^3+|x|+1}{x^2+2|x|+1}dx$$

Applying $I = \int_{-1}^{1} \frac{|x|+1}{x^2+2|x|+1}dx$ being even function

$$I = 2\int_{0}^{1} \frac{x+1}{(x+1)^2}dx = 2\left[\ln(x+1)\right]_0^1 = 2\ln 2$$

29. A function f is defined by $f(x) = 2 + (x-1)^{2/3}$ on [0, 2]. Which of the following statements is incorrect?

(A) f is not derivable in (0, 2) (B) f is continuous in [0, 2]

(C) f(0) = f(2) (D) Rolle's theorem is applicable on [0, 2]

Ans : (D)

Hint : $f(x) = 2 + (x-1)^{2/3}$

f(x) is not differentiable at x = 1

Hence Rolle's theorem is not applicable on [0, 2]

30. $\int_{0}^{1.5}\left[x^2\right]dx$ is equal to

(A) 2 (B) $2-\sqrt{2}$ (C) $2+\sqrt{2}$ (D) $\sqrt{2}$

Ans : (B)

Hint : $\int_{0}^{1.5}\left[x^2\right]dx$

$$=\int_{0}^{1}\left[x^2\right]dx+\int_{1}^{\sqrt{2}}\left[x^2\right]dx+\int_{\sqrt{2}}^{3/2}\left[x^2\right]dx$$

$$=0+1\left(\sqrt{2}-1\right)+2\left(1.5-\sqrt{2}\right)$$

$$=2-\sqrt{2}$$

31. The value of the integral $\int_{3}^{6}\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) 1

Ans : (B)

Hint : $I=\int_{3}^{6}\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

Applying King's property

$$I=\int_{3}^{6}\frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}}dx$$

Adding $2I=\int_{3}^{6}dx=3 \quad \Rightarrow I=\frac{3}{2}$

32. If a, b, c are positive real numbers each distinct from unity, then the value of the determinant

$$\begin{vmatrix} 1 & \log_a b & \log_a c \\ \log_b a & 1 & \log_b c \\ \log_c a & \log_c b & 1 \end{vmatrix} \text{ is}$$

(A) 0 (B) 1 (C) $\log_e(abc)$ (D) $\log_e a \log_e b \log_e c$

Ans : (A)

Hint : Expanding through first row, $\log_b a=\frac{1}{\log_a b}$ (by property)

33. The function $f(x) = 2x^3 - 3x^2 - 12x + 4, x \in \mathbb{R}$ has

(A) two points of local maximum (B) two points of local minimum

(C) one local maximum and one local minimum (D) neither maximum nor minimum

Ans : (C)

Hint : $f'(x) = 0 \quad \Rightarrow x = -1, 2$

$x = -1$ is local maxima

$x = 2$ is local minima

34. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + ... + a_{12}x^{12}$, then the value of $a_2 + a_4 + a_6 + ... + a_{12}$ is

(A) 21 (B) 31 (C) 32 (D) 64

Ans : (B)

Hint : $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + ... + a_{12}x^{12}$

$x = 1 \Rightarrow 0 = 1 + a_1 + a_2 + ... + a_{12}$

$x = -1 \Rightarrow 64 = 1 - a_1 + a_2 + ... + a_{12}$

Adding $64 = 2(1 + a_2 + a_4 + ... + a_{12})$

$a_2 + a_4 + ... + a_{12} = 31$

35. Let f be a function which is differentiable for all real x. If $f(2) = -4$ and $f'(x) \geq 6$ for all $x \in [2,4]$

(A) $f(4) < 8$ (B) $f(4) \geq 12$ (C) $f(4) \geq 8$ (D) $f(4) < 12$

Ans : (C)

Hint : By LMVT

$$f'(x) = \frac{f(4) - f(2)}{4 - 2}$$

as $f'(x) \geq 6 \Rightarrow f(4) \geq 8$

36. If z_1, z_2 are complex numbers such that $\frac{2z_1}{3z_2}$ is a purely imaginary number, then the value of $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (A)

Hint : Let $\frac{2z_1}{3z_2} = \lambda i \quad (\lambda \in R - \{0\}) \quad \Rightarrow \frac{z_1}{z_2} = \frac{3\lambda i}{2}$

37. The set of points of discontinuity of the function $f(x) = x - [x]$, $x \in \mathbb{R}$ is

(A) Q (B) $\mathbb{R}$ (C) $\mathbb{N}$ (D) $\mathbb{Z}$

Ans : (D)

Hint : Discontinuous at integral point.

38. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ is equal to

(A) 0 (B) 1 (C) 6 (D) 12

Ans : (C)

Hint : $\alpha = \beta = \gamma = -1$

39. If $\vec{\alpha} = 3\vec{i} - \vec{k}, |\vec{\beta}| = \sqrt{5}$ and $\vec{\alpha} \cdot \vec{\beta} = 3$, then the area of the parallelogram for which $\vec{\alpha}$ and $\vec{\beta}$ are adjacent sides is

(A) $\sqrt{17}$ (B) $\sqrt{14}$ (C) $\sqrt{7}$ (D) $\sqrt{41}$

Ans : (D)

Hint : $|\vec{\alpha} \times \vec{\beta}| = |\vec{\alpha}||\vec{\beta}|\sin\theta$, $\cos\theta = \frac{3}{\sqrt{50}}$

$$= \sqrt{50} \times \frac{\sqrt{41}}{\sqrt{50}} = \sqrt{41}$$

40. If 'θ' is the angle between two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 7, |\vec{b}| = 1$ and $|\vec{a} \times \vec{b}|^2 = k^2 - \left(\vec{a} \cdot \vec{b}\right)^2$, then the values of k and θ are

(A) k = 1, θ = 45° (B) k = 7, θ = 60° (C) k = 49, θ = 90° (D) k = 7 and θ is arbitrary

Ans : (D)

Hint : $k^2 = |\vec{a} \times \vec{b}|^2 + \left(\vec{\alpha} \cdot \vec{\beta}\right)^2$

$$= |\vec{a}|^2 |\vec{b}|^2 \left(\cos^2\theta + \sin^2\theta\right)$$

$$= 49$$

41. $\lim\limits_{x \to 0} \dfrac{\tan\left(\left[-\pi^2\right]x^2\right) - x^2 \tan\left(\left[-\pi^2\right]\right)}{\sin^2 x}$ equals

(A) 0 (B) tan 10 – 10 (C) tan 9 – 9 (D) 1

Ans : (B)

Hint : $\underset{x \to 0}{\text{lt}} \dfrac{-\tan 10x^2 + x^2 \tan 10}{\sin^2 x}$ (Applying series)

$$\underset{x \to 0}{\text{lt}} \frac{-\left(10x^2 + \frac{10^3 x^6}{3}\right) + x^2 \tan 10}{x^2\left(1 - \frac{x^2}{6}\right)^2} = \tan 10 - 10$$

42. Let f(x) = |1 – 2x|, then

(A) f(x) is continuous but not differentiable at $x = \frac{1}{2}$ (B) f(x) is differentiable but not continuous at $x = \frac{1}{2}$

(C) f(x) is both continuous and differentiable at $x = \frac{1}{2}$ (D) f(x) is neither differentiable nor continuous at $x = \frac{1}{2}$

Ans : (A)

43. If $f(x)=\begin{cases}x^2+3x+a, & x\le 1\\ bx+2, & x>1\end{cases}$, $x\in\mathbb{R}$, is everywhere differentiable, then

(A) a = 3, b = 5 (B) a = 0, b = 5 (C) a = 0, b = 3 (D) a = b = 3

Ans : (A)

Hint : LHL = RHL $\Rightarrow$ 4 + a = b + 2

LHD = RHD $\Rightarrow$ b = 5 $\Rightarrow$ a = 3

44. Let f(x) be a second degree polynomial. If f(1) = f(–1) and p, q, r are in A.P., then $f'(p), f'(q), f'(r)$ are

(A) in A.P. (B) in G.P.

(C) in H.P. (D) neither in A.P. or G.P. or H.P.

Ans : (A)

Hint : $f(-1)=f(1)\Rightarrow b=0$

$f(x)=ax^2+c \quad \Rightarrow f'(x)=2ax$

45. If adj B = A, |P| = |Q| = 1, then adj($Q^{-1}BP^{-1}$) =

(A) PQ (B) QAP (C) PAQ (D) $PA^{-1}Q$

Ans : (C)

Hint : adj($Q^{-1}BP^{-1}$)

= $|Q^{-1}BP^{-1}|\,(Q^{-1}BP^{-1})^{-1}$

= $|Q^{-1}|\,|B|\,|P^{-1}|\,PB^{-1}Q$

= $|B|\;P\dfrac{(\text{adj }B)}{|B|}Q$

= PAQ

46. The line $y-\sqrt{3}x+3=0$ cuts the parabola $y^2 = x + 2$ at the points P and Q. If the co-ordinates of the point X are $\left(\sqrt{3},0\right)$, then the value of $XP\cdot XQ$ is

(A) $\dfrac{4\left(2+\sqrt{3}\right)}{3}$ (B) $\dfrac{4\left(2-\sqrt{3}\right)}{2}$ (C) $\dfrac{5\left(2+\sqrt{3}\right)}{3}$ (D) $\dfrac{5\left(2-\sqrt{3}\right)}{3}$

Ans : (A)

Hint : Arbitrary point on the line $\left(\sqrt{3}+\dfrac{r}{2},\dfrac{r\sqrt{3}}{2}\right)$ satisfying in the parabola $\Rightarrow 3r^2-2r-\left(4\sqrt{3}+8\right)=0$

$\because$ roots r_1 & r_2

$\therefore$ product $r_1\,r_2 = -\dfrac{4\sqrt{3}+8}{3}$

$\therefore$ $XP\cdot XQ = |r_1\,r_2| = \dfrac{4}{3}\left(\sqrt{3}+2\right)$

47. Let $\omega(\neq 1)$ be a cubic root of unity. Then the minimum value of the set $\{|a + b\omega + c\omega^2|^2$; a, b, c are distinct non-zero integers$\}$ equals

(A) 15 (B) 5 (C) 3 (D) 4

Ans : (C)

Hint : $|a + b\omega + c\omega^2|^2 = a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

$\therefore$ Min $= \frac{1+1+4}{2} = 3$ (a, b, c $\in \{1, 2, 3\}$)

48. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is

(A) parallel to the x-axis
(B) parallel to the y-axis
(C) parallel to the z-axis
(D) perpendicular to the z-axis

Ans : (D)

Hint :

49. If the sum of the squares of the roots of the equation $x^2 - (a-2)x - (a+1) = 0$ is least for an appropriate value of the variable parameter a, then the value of 'a' will be

(A) 3 (B) 2 (C) 1 (D) 0

Ans : (C)

Hint : $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-2)^2 + 2(a+1)$

$= a^2 - 2a + 6 = (a-1)^2 + 5$

50. $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for

(A) no value of λ
(B) all except one value of λ
(C) all except two values of λ
(D) all values of λ

Ans : (C)

Hint : for co-planar : $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda-1 \end{vmatrix} = 0 \Rightarrow \lambda = 0, \frac{1}{2}$

CATEGORY - 2 (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : –½)

51. If $|Z_1| = |Z_2| = |Z_3| = 1$ and $Z_1 + Z_2 + Z_3 = 0$, then the area of the triangle whose vertices are Z_1, Z_2, Z_3 is

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4}$ (C) 1 (D) 2

Ans : (A)

Hint : Considering cube root of unity, implies equilateral triangle.

52. Let $f(x) = |x - \alpha| + |x - \beta|$, where α, β are the roots of the equation $x^2 - 3x + 2 = 0$. Then the number of points in $[\alpha, \beta]$ at which f is not differentiable is

(A) 2 (B) 0 (C) 1 (D) infinite

Ans : (B)

Hint : $\because$ Two distinct real roots 1, 2

$f(x) = |x - 1| + |x - 2|$

$\therefore$ Differentiable in [1,2]

53. If $f(x) = \dfrac{3x-4}{2x-3}$, then $f(f(f(x)))$ will be

(A) x (B) 2x (C) $\dfrac{2x-3}{3x-4}$ (D) $\dfrac{3x-4}{2x-3}$

Ans : (D)

Hint : $f(x) = \dfrac{3x-4}{2x-3}$

$$f(f(f(x))) = f\left(f\left(\frac{3x-4}{2x-3}\right)\right)$$

$$= f\left(\frac{3\left(\frac{3x-4}{2x-3}\right)-4}{2\left(\frac{3x-4}{2x-3}\right)-3}\right) = f(x)$$

54. If f(x) and g(x) are two polynomials such that $\phi(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then

(A) $\phi(x)$ is divisible by $(x - 1)$

(B) none of f(x) and g(x) is divisible by $(x - 1)$

(C) g(x) is divisible by $(x - 1)$ but f(x) is not divisible by $(x - 1)$

(D) f(x) is divisible by $(x - 1)$ but g(x) is not divisible by $(x - 1)$

Ans : (A)

Hint : $\because$ By remainder theorom

$\phi(\omega) = 0$ & $\phi(\omega^2) = 0$ (ω, ω^2 are complex cube root of unity)

$\Rightarrow f(1) + \omega g(1) = 0$ & $f(1) + \omega^2 g(1) = 0$

$\Rightarrow f(1) = g(1) = 0$

$\therefore \phi(1) = f(1) + g(1) = 0$

55. Let a_n denote the term independent of x in the expansion of $\left[x + \dfrac{\sin\left(\frac{1}{n}\right)}{x^2}\right]^{3n}$, then $\lim\limits_{n\to\infty} \dfrac{(a_n)n!}{{}^{3n}P_n}$ equals

(A) 0 (B) 1 (C) e (D) $\dfrac{e}{\sqrt{3}}$

Ans : (A)

Hint : $\left[x+\frac{\sin\left(\frac{1}{n}\right)}{x^2}\right]^{3n}$

$T_{r+1} = {}^{3n}C_r(x)^{3n-r}\left(\frac{\sin\left(\frac{1}{n}\right)}{x^2}\right)^r$

$= {}^{3n}C_r(x)^{3n-3r}\left(\sin\left(\frac{1}{n}\right)\right)^r$

$r = n$

$a_n = {}^{3n}C_n\left(\sin\left(\frac{1}{n}\right)\right)^n$

$$\lim_{n\to\infty}\frac{{}^{3n}C_n\left(\sin\left(\frac{1}{n}\right)\right)^n\cdot n!}{{}^{3n}P_n} = 0$$

56. The maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Ans : (C)

Hint : $y^2 = 4ax$ and $x^2 = 4by$

Eq. of normal to $y^2 = 4ax$ and $x^2 = 4by$

are $y = mx - 2am - am^3$ and $y = mx + 2b + \frac{b}{m^2}$

For common normal; $2b+\frac{b}{m^2}+2am+am^3 = 0$

$\Rightarrow am^5 + 2am^3 + 2bm^2 + b = 0$

Max. 5 common normal

57. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$, $x^2 + y^2 + 6x + 18y + 26 = 0$ is

(A) 2 (B) 3 (C) 4 (D) 5

Ans : (B)

Hint : $(x-2)^2 + (y-3)^2 - 12 - 13 = 0$

$(x-2)^2 + (y-3)^2 = 5^2$

$(x+3)^2 + (y+9)^2 = -26 + 9 + 81 = 64$

$C_1(2, 3)$ $r_1 = 5$

$C_2(-3, -9)$ $r_2 = 8$

$C_1C_2 = \sqrt{25+144} = 13$

$C_1C_2 = r_1 + r_2$

$\Rightarrow$ 3 common tangents.

58. The probability that a non-leap year selected at random will have 53 Sundays or 53 Saturdays is

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) 1 (D) $\frac{2}{365}$

Ans : (B)

Hint : No. days = 365 = 52 weeks + 1 extra day

{S, M, T, W, Th, F, Sat}

P((53 S) U (53 Sat)) = P(Sat) + P(S)

$= \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

59. Let $u + v + w = 3$, $u, v, w \in \mathbb{R}$ and $f(x) = ux^2 + vx + w$ be such that $f(x + y) = f(x) + f(y) + xy$, $\forall\, x, y \in \mathbb{R}$. Then f(1) is equal to

(A) $\frac{5}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 3

Ans : (D)

Hint : $u + v + w = 3$

$f(x) = ux^2 + vx + w$

$f(x + y) = f(x) + f(y) + xy$

$f(1) = 3$

60. The number of solutions of $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$ is

(A) 0 (B) 1 (C) 2 (D) 4

Ans : (C)

Hint : $\sin^{-1}x + \sin^{-1}(1 - x) = \cos^{-1}x$

$\Rightarrow 1 - x = \sqrt{1-x}\sqrt{1-x} - x.x$

$\Rightarrow x = 0$ or ½

61. Let $f(x) = \max\{x + |x|, x - [x]\}$, where [x] stands for the greatest integer not greater than x. Then $\int_{-3}^{3} f(x)\,dx$ has the value

(A) $\frac{51}{2}$ (B) $\frac{21}{2}$ (C) 1 (D) 0

Ans : (B)

Hint : $f(x) = \max\cdot\{x + |x|,\ x - [x]\} = \max\{x + |x|, \{x\}\}$

$\int_{-3}^{3} f(x)\,dx$

$= \int_{-3}^{0} \{x\}\,dx + \int_{0}^{3} 2x\,dx$

$= \frac{3}{2} + 9 = \frac{21}{2}$

62. Let $f(\theta)=\begin{vmatrix}1 & \cos\theta & -1\\ -\sin\theta & 1 & -\cos\theta\\ -1 & \sin\theta & 1\end{vmatrix}$. Suppose A and B are respectively maximum and minimum values of f(θ). Then (A, B) is eaual to

(A) (2, 1) (B) (2, 0) (C) $(\sqrt{2},1)$ (D) $\left(2,\frac{1}{\sqrt{2}}\right)$

Ans : (B)

Hint : $f(\theta)=\begin{vmatrix}1 & \cos\theta & -1\\ -\sin\theta & 1 & -\cos\theta\\ -1 & \sin\theta & 1\end{vmatrix}$ (after expansion)

$f(\theta) = 1 + \sin 2\theta$

A = 2, B = 0 (A, B) = (2, 0)

63. If $\cos(\theta+\phi)=\frac{3}{5}$ and $\sin(\theta-\phi)=\frac{5}{13}$, $0<\theta,\phi<\frac{\pi}{4}$, then cot(2θ) has the value

(A) $\frac{16}{63}$ (B) $\frac{63}{16}$ (C) $\frac{3}{13}$ (D) $\frac{13}{3}$

Ans : (A)

Hint : $\tan 2\theta=\tan\left((\theta+\phi)+(\theta-\phi)\right)$ $\left[\because \tan(\theta+\phi)=\frac{4}{3},\ \tan(\theta-\phi)=\frac{5}{12}\right]$

$$=\frac{\tan(\theta+\phi)+\tan(\theta-\phi)}{1-\tan(\theta+\phi)\tan(\theta-\phi)}$$

$$=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}=\frac{16+5}{36-20}=\frac{21\times 3}{16}=\frac{63}{16}$$

$$\cot(2\theta)=\frac{16}{63}$$

64. If a, b, c are in A.P. and if the equations $(b-c)x^2+(c-a)x+(a-b)=0$ and $2(c+a)x^2+(b+c)x=0$ have a common root, then

(A) a^2, b^2, c^2 are in A.P. (B) a^2, c^2, b^2 are in A.P. (C) c^2, a^2, b^2 are in A.P. (D) a^2, b^2, c^2 are in G.P.

Ans : (B)

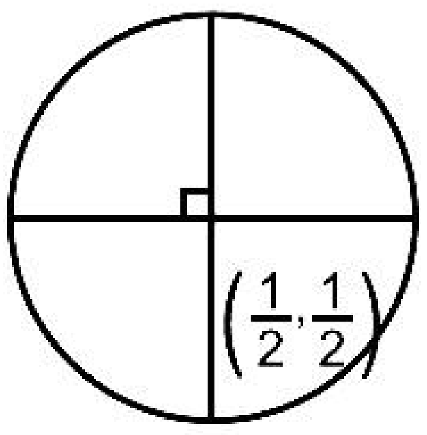
Hint : as x = 1 is common root

65. Let x – y = 0 and x + y = 1 be two perpendicular diameters of a circle of radius R. The circle will pass through the origin if R is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{3}$

Ans : (B)

Hint :



$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=R^2$$

$$\Rightarrow R^2=\frac{1}{2};\ R=\frac{1}{\sqrt{2}}$$

CATEGORY - 3 (Q66 to Q75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. If P is a non-singular matrix of order 5×5 and the sum of the elements of each row is 1, then the sum of the elements of each row in P^{-1} is

(A) 0 (B) 1 (C) $\frac{1}{8}$ (D) 8

Ans : (B)

Hint : Given, $P\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}\Rightarrow P^{-1}P\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}=P^{-1}\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}\Rightarrow I_5\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}=P^{-1}\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$ (I_5 is identify Matrix)

67. Three numbers are chosen at random without replacement from {1, 2, ... 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7, is

(A) $\frac{5}{40}$ (B) $\frac{3}{40}$ (C) $\frac{11}{40}$ (D) $\frac{9}{40}$

Ans : (C)

Hint : $\frac{^7C_2+^6C_2-^3C_1}{^{10}C_3}=\frac{33}{120}=\frac{11}{40}$

68. The population p(t) at time t of a certain mouse species follows the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is

(A) log 9 (B) $\frac{1}{2}\log 18$ (C) log 18 (D) 2 log 18

Ans : (D)

Hint : $\frac{dp}{dt} = 0.5P - 450$

or, $\int_{850}^{0} \frac{dp}{P-900} = \int_{0}^{t} \frac{dt}{2}$

or, $[\ln|P-900|]_{850}^{0} = \frac{t}{2}$

or, $\frac{t}{2} = \ln\left|\frac{900}{50}\right| = \ln|18|$

or, $t = 2\ln|18|$

69. The value of $\int_{-100}^{100} \frac{(x+x^3+x^5)}{(1+x^2+x^4+x^6)} dx$ is

(A) 100 (B) 1000 (C) 0 (D) 10

Ans : (C)

Hint : $\int_{-100}^{100} \frac{\text{odd function}}{\text{Even function}} = \int_{-100}^{100} \text{odd function} = 0$

70. Let f : [0, 1] → $\mathbb{R}$ and g : [0, 1] → $\mathbb{R}$ be defined as follows :

$\left.\begin{aligned} f(x) &= 1 \text{ if } x \text{ is rational} \\ &= 0 \text{ if } x \text{ is irrational} \end{aligned}\right]$ and

$\left.\begin{aligned} g(x) &= 0 \text{ if } x \text{ is rational} \\ &= 1 \text{ if } x \text{ is irrational} \end{aligned}\right]$ then

(A) f and g are continuous at the point x = $\frac{1}{2}$

(B) f+g is continuous at the point x = $\frac{2}{3}$ but f and g are discontinuous at x = $\frac{2}{3}$

(C) f(x).g(x) > 0 for some points x $\in$ (0, 1)

(D) f+g is not differentiable at the point x = $\frac{3}{4}$

Ans : (B)

Hint : Graphically

71. If $f(x)=\int_0^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt$ and $g(x)=\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$, then the value of f(x) + g(x) is

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\sin^2x + \sin x + x$

Ans : (B)

Hint : $f'(x) + g'(x) = 0$

$\Rightarrow f(x) + g(x) =$ constant (say c)

putting $x=\frac{\pi}{4}$

$\therefore c=\int_0^{1/2}\left(\sin^{-1}\sqrt{t}+\cos^{-1}\sqrt{t}\right)dt = \frac{\pi}{4}$

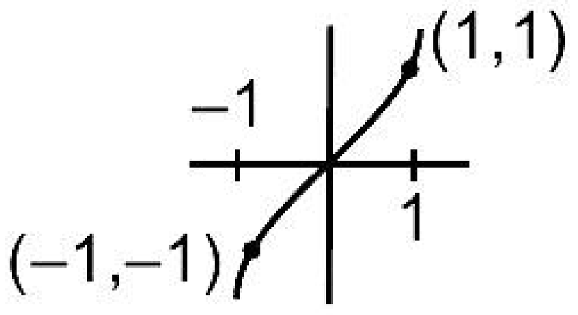
72. Let $f(x)=x^3$, $x\in[-1, 1]$. Then which of the following are correct ?

(A) 'f' has a minimum at x = 0 (B) 'f' has a maximum at x = 1

(C) 'f' is continuous on [–1, 1] (D) 'f' is bounded on [–1, 1]

Ans : (B, C & D)

Hint :



73. If $0 \le a, b \le 3$ and the equation $x^2+4+3\cos(ax+b)=2x$ has real solution, then the value of (a+b) is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 2π

Ans : (C)

Hint : $(x^2-2x+1) +3 (\cos(ax+b) +1) = 0$

or, $(x-1)^2+3(\cos(ax+b)+1) = 0$

$a+b = \pi$

74. The solution set of the equation $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ $\tan(\pi\tan x) = \cot(\pi\cot x)$ is

(A) {0} (B) $\left\{\frac{\pi}{4}\right\}$ (C) ϕ (D) $\left\{\frac{\pi}{6}\right\}$

Ans : (C)

Hint : $\tan(\pi \tan x) = \cot(\pi \cot x)$

or, $\tan(\pi\tan x) = \tan\left(\frac{\pi}{2}-\pi\cot x\right)$

or, $\pi\tan x = \frac{\pi}{2}-\pi\cot x$

or, $\tan x = \frac{1}{2} - \cot x$

or, $\tan x + \cot x = \frac{1}{2}$

or, $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{2}$

or, $\frac{1}{\sin 2x} = \frac{1}{4}$

or, $\sin 2x = 4 \quad x \in \phi$

75. If the equation $\sin^4 x - (p+2)\sin^2 x - (p+3) = 0$ has a solution, the p must lie in the interval

(A) $[-3, -2]$ (B) $(-3, -2)$ (C) $(2, 3)$ (D) $[-5, -3]$

Ans : (A & B)

Hint : $\sin^2 x = \frac{(P+2) \pm \sqrt{(P+2)^2 + 4.1(P+3)}}{2.1}$

$= \frac{(P+2) \pm \sqrt{P^2 + 4p + 4 + 4p + 12}}{2}$

$= \frac{(P+2) \pm \sqrt{(P+4)^2}}{2}$

$= \frac{2p+6}{2} = P+3$

$= \sin^2 x \in [0,1]$

$0 \le P + 3 \le 1$

$\underline{-3 \le P \le -2}$

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WBJEE - 2025

DATE : 27/04/2025

SUB : PHYSICS & CHEMISTRY

PHYSICS

CATEGORY - 1 (Q1 to Q30)

(Carry 1 mark each. Only one option is correct. Negative mark : – ¼)

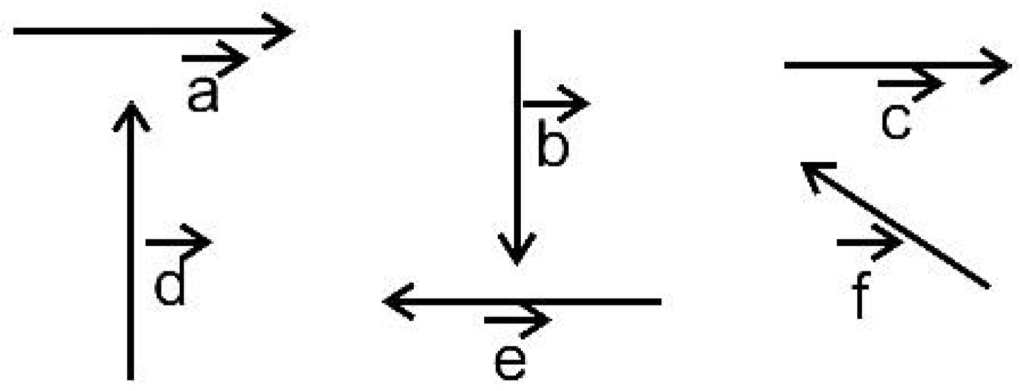
1. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is the length, ΔV is a potential difference and Δt is a time interval. The dimension of X is same as that of

(A) Resistance (B) Charge (C) Voltage (D) Current

Ans : (D)

Hint : $X = \varepsilon_0 . \frac{\text{Volt} \times \text{m}}{\text{sec}} = \varepsilon_0 \frac{d\phi}{dt} = i_d$ = current

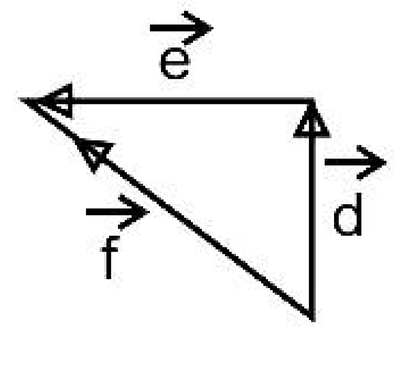
2. Six vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}$ and $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true ?



(A) $\vec{b} + \vec{e} = \vec{f}$ (B) $\vec{b} + \vec{c} = \vec{f}$ (C) $\vec{d} + \vec{c} = \vec{f}$ (D) $\vec{d} + \vec{e} = \vec{f}$

Ans : (D)

Hint :



3. The minimum force required to start pushing a body up a rough (having co-efficient of friction μ) inclined plane is $\vec{F_1}$ while the minimum force needed to prevent it from sliding is $\vec{F_2}$. If the inclined plane makes an angle θ with the horizontal such that $\tan\theta = 2\mu$, then the ratio F_1/F_2 is

(A) 4 (B) 1 (C) 2 (D) 3

Ans : (D)

Hint : When body is being prevented from sliding down

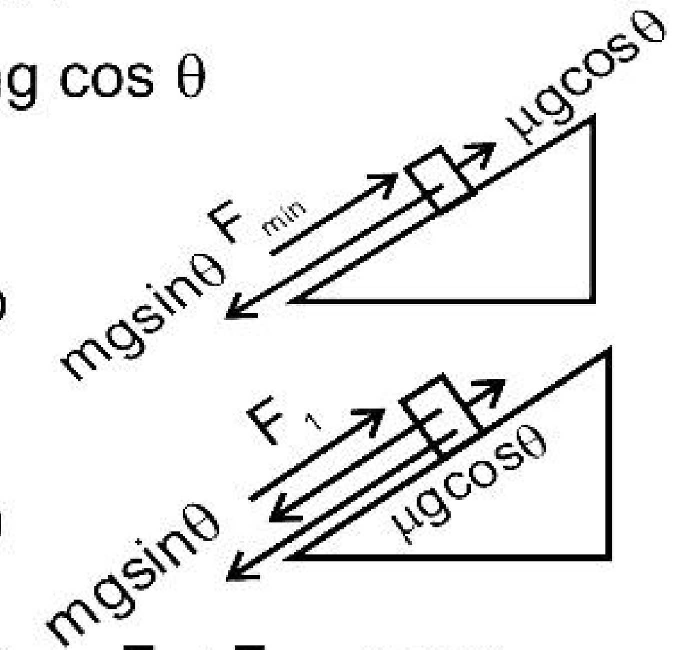
$F_{min} = F_2 = mg\sin\theta - \mu mg\cos\theta$

$F_2 = mg\sin\theta - \mu mg\cos\theta$

When body is pushed up

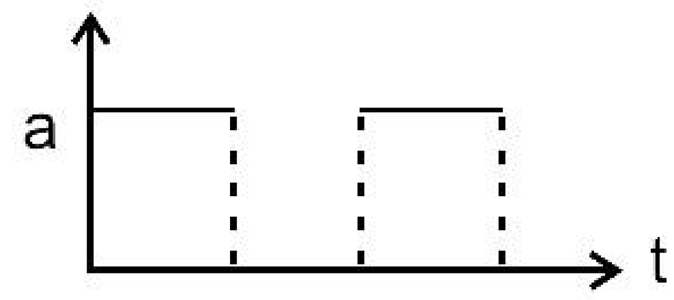
friction acts downward

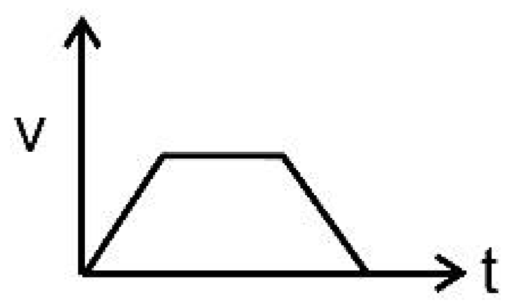
$F_1 = mg\sin\theta + \mu mg\cos\theta$

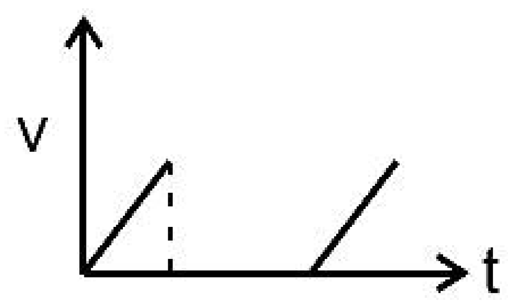


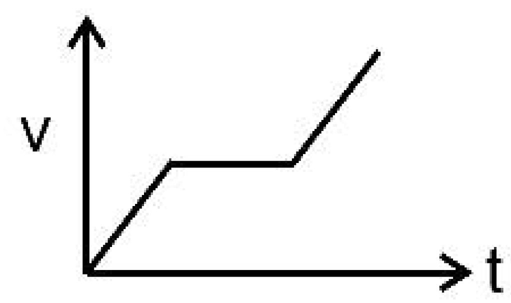
$$\left.\begin{matrix} F_1 - F_2 = 2\,\mu mg\cos\theta \\ F_1 + F_2 = 2\,mg\sin\theta \end{matrix}\right\} \quad \frac{F_1+F_2}{F_1-F_2} = \frac{\tan\theta}{\mu} = 2:1 \quad \Rightarrow \frac{2F_1}{2F_2} = \frac{2+1}{2-1} \text{ or, } \frac{F_1}{F_2} = 3:1$$

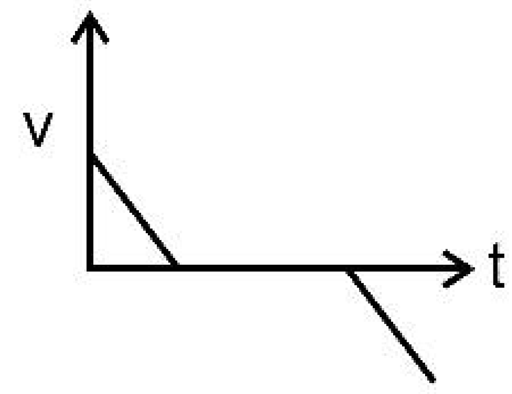
4. Acceleration - time (a–t) graph of a body is shown in the figure, Corresponding velocity-time (v–t) graph is



(A) 

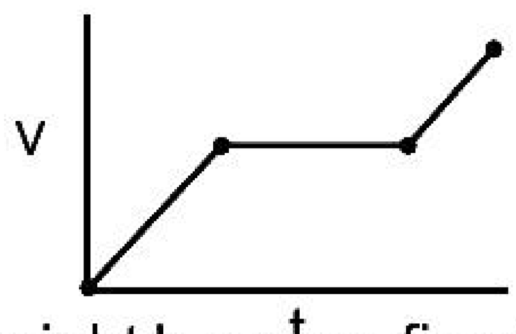
(B) 

(C) 

(D) 

Ans : (C)

Hint : $a = \tan\theta$



5. A ball falls from a height h upon a fixed horizontal floor. The co-efficient of restitution for the collision between the ball and the floor is 'e'. The total distance covered by the ball before coming to rest is [neglect the air resistance]

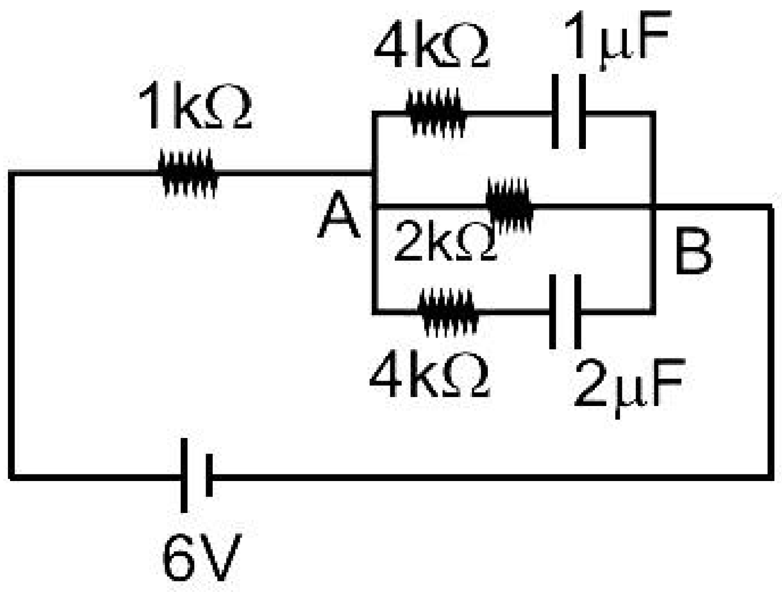
(A) $\frac{1-e^2}{1+e^2}h$ (B) $\frac{1+e^2}{1-e^2}h$ (C) $\frac{1-2e^2}{1+e^2}h$ (D) $\frac{1+2e^2}{1-e^2}h$

Ans : (B)

Hint : Dist = $h + 2e^2h + 2e^4h + \text{-----}$

$$d = \left[\frac{1+e^2}{1-e^2}\right]h$$

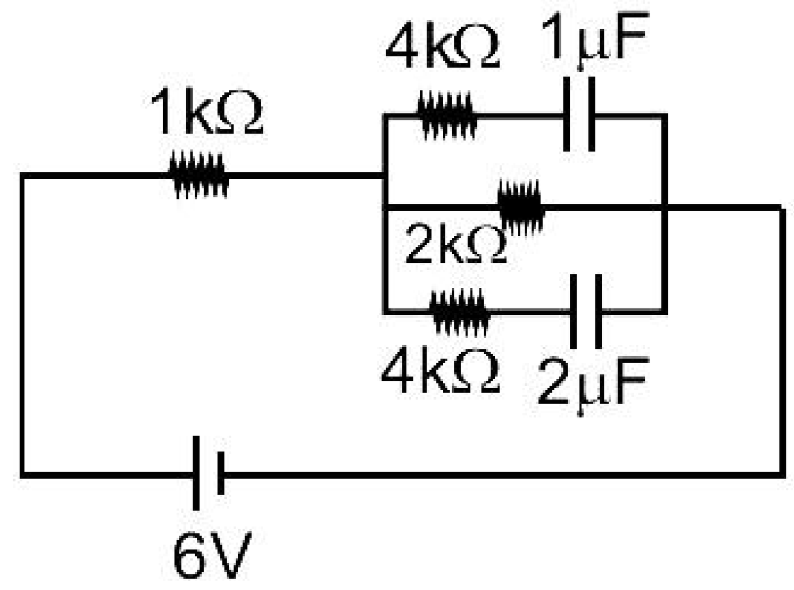
6. What are the charges stored in the 1μF and 2μF capacitors in the circuit as shown in figure once the current (I) become steady ?



(A) 8 μC and 4 μC (B) 4 μC and 8 μC (C) 3 μC and 6 μC (D) 6 μC and 3 μC

Ans : (B)

Hint :

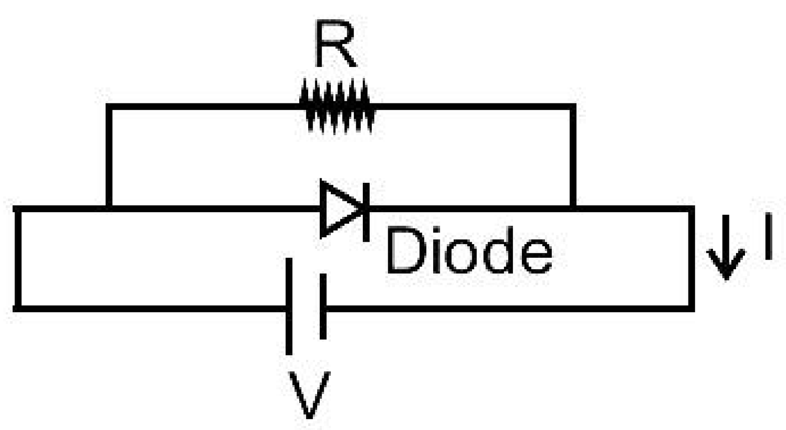


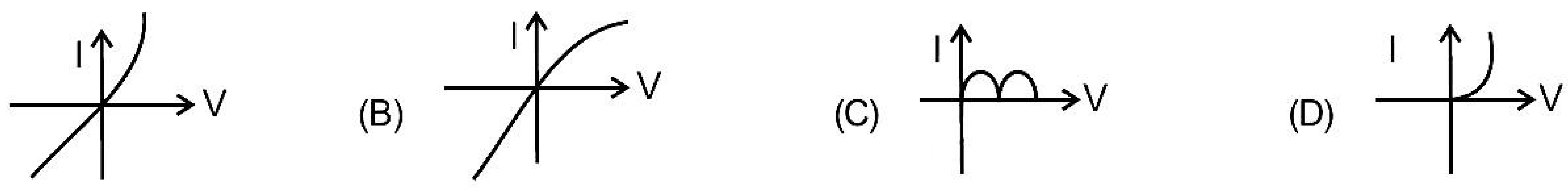
$$I = \frac{6}{(1+2)\times 10^3} = 2 \times 10^{-3}$$

$V = IR = 2\times10^{-3} \times 2\times10^{3} = 4V$

$q_1 = 1\times4 = 4\ \mu C,\ q_2 = 2\times4 = 8\ \mu C$

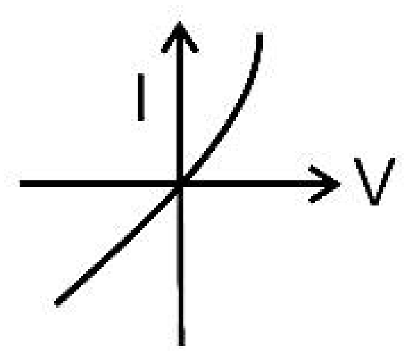
7. A diode is connected in parallel with a resistance as shown in figure. The most probable current (I) –voltage (V) characteristic is





Ans : (A)

Hint : In reverse bias only resistance R is considered, so Ohm's law is valid $V \propto I$



In forward bias initially current flows through resistance, then after knee voltage, it flows through p–n diode

8. Ruma reached the metro station and found that the escalator was not working. She walked up the stationary escalator with velocity v_1 in time t_1. On other day if she remains stationary on the escalator moving with velocity v_2, then escalator takes her up in time t_2. The time taken by her to walk up with velocity v_1 on the moving escalator will be

(A) $\frac{t_1t_2}{t_2 - t_1}$ (B) $\frac{t_1t_2}{t_2 + t_1}$ (C) $\frac{t_1 - t_2}{t_1 + t_2}$ (D) $\frac{t_1 + t_2}{2(t_1 - t_2)}$

Ans : (B)

Hint : D = Distance to be covered

for stationary escalator, $V_1t_1 = D$ ------ (1)

for moving escalator, $V_2t_2 = D$ ----- (2)

for ruma moving on moving escalator, $(V_1+V_2)\,t_{req} = D$ ----- (3)

from (1), (2) & (3), we get

$$\frac{D}{t_1} + \frac{D}{t_2} = \frac{D}{t_{req}} \Rightarrow t_{req} = \frac{t_1t_2}{t_1 + t_2}$$

9. The variation of displacement with time of a simple harmonic motion (SHM) for a particle of mass m is represented by $y = 2\sin\left(\frac{\pi t}{2} + \phi\right)$ cm. The maximum acceleration of the particle is

(A) $\frac{\pi}{2}$ cm/sec^2 (B) $\frac{\pi}{2m}$ cm/sec^2 (C) $\frac{\pi^2}{2m}$ cm/sec^2 (D) $\frac{\pi^2}{2}$ cm/sec^2

Ans : (D)

Hint : Given y = 2 sin $\left(\frac{\pi t}{2} + \phi\right)$cm

$a_{max} = \omega^2 A$

$= \left(\frac{\pi}{2}\right)^2 .2 = \frac{\pi^2}{2}$ cm/sec^2

10. A particle of charge 'q' and mass 'm' moves in a circular orbit of radius 'r' with angular speed 'ω'. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(A) ω and q (B) ω, q and m (C) q and m (D) ω and m

Ans : (C)

Hint : Gyromagnetic ratio, $\frac{M}{L} = \frac{q}{2m}$

11. The de-Broglie wavelength of a moving bus with speed is v is λ. Some passengers left the bus at a stoppage. Now when the bus moves with twice of its initial speed. Its kinetic energy is found to be twice of its value. What is the de-Broglie wavelength of the bus now?

(A) λ (B) 2λ (C) $\frac{\lambda}{2}$ (D) $\frac{\lambda}{4}$

Ans : (A)

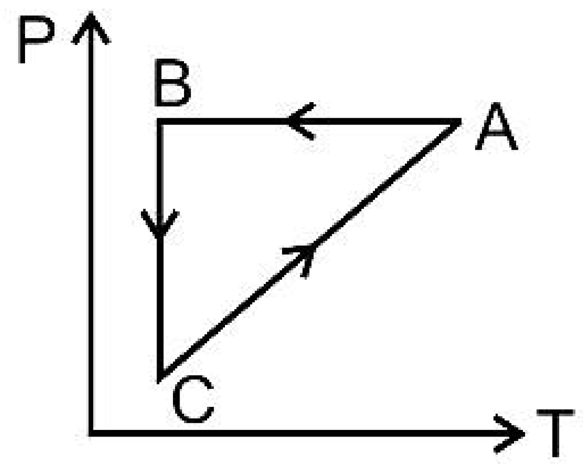
Hint : Initially $\lambda = \frac{h}{\sqrt{2mE}}$

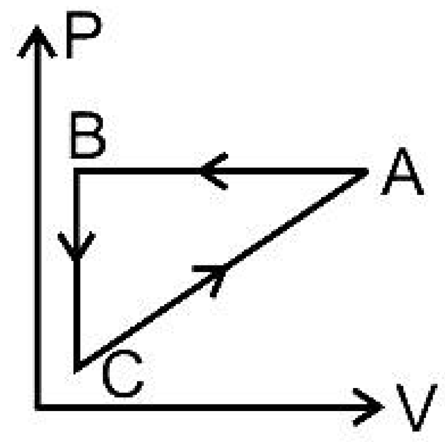
Now acc. to question, V′ = 2V, E′ = 2E

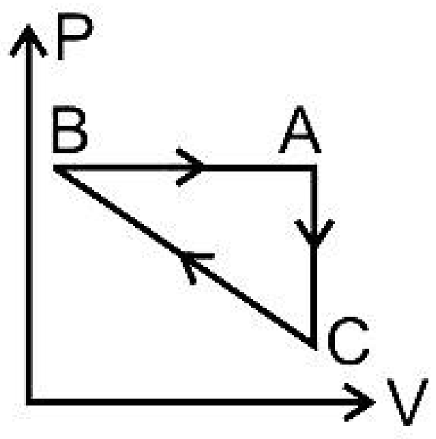
$\Rightarrow \frac{1}{2}.m'(2V)^2 = 2.\frac{1}{2}mv^2 = m' = \frac{m}{2}$

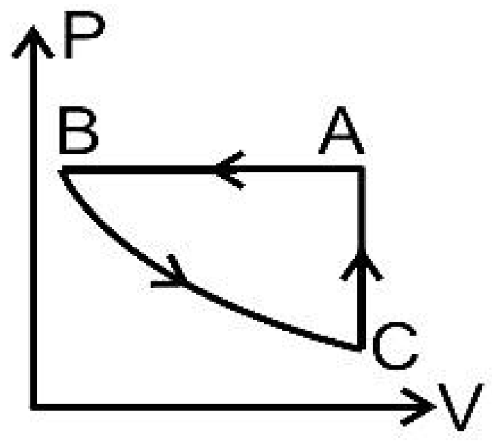
$\therefore \quad \lambda' = \frac{h}{\sqrt{2\frac{m}{2}.2E}} = \lambda$

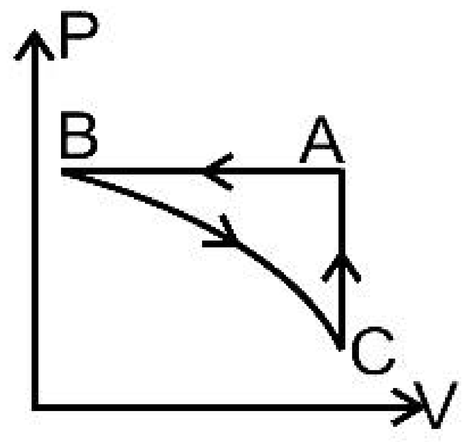
12. For an ideal gas, a cyclic process ABCA as shown in P–T diagram, when presented in P–V plot, it would be



(A) 

(B) 

(C) 

(D) 

Ans : (C)

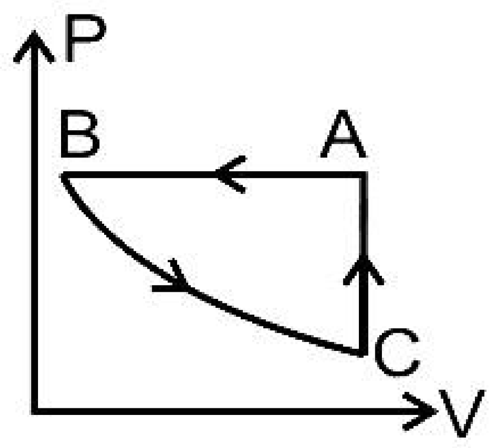
Hint :

AB $\rightarrow$ Isobaric compression

BC $\rightarrow$ Isothermal expansion

CA $\rightarrow$ Isochoric process

$\therefore$ Correct P-V diagram is



13. An electron in Hydrogen atom jumps from the second Bohr orbit to the ground state and the difference between the energies of the two states is radiated in the form of a photon. This photon strikes a material. If the work function of the material is 4.2 eV, then the stopping potential is (Energy of electron in n-th orbit = $-\frac{13.6}{n^2}$eV)

(A) 2 V (B) 4 V (C) 6 V (D) 8 V

Ans : (C)

Hint : Energy of photon emitted from hydrogen atom

$$= -13.6\left[\frac{1}{2^2}-\frac{1}{1^2}\right] = 10.2 \text{ eV}$$

Now using Einstein's photoelectric equation.

10.2 eV = 4.2 eV + eV_s

$\therefore V_s$ = 6 Volt.

14. The resistance $R = \frac{V}{I}$ where $V = (25 \pm 0.4)$ Volt and $I = (200 \pm 3)$ Ampere. The percentage error in 'R' is

(A) 1.5% (B) 1.6% (C) 3.1% (D) 0.1%

Ans : (C)

Hint :

$$R = \frac{V}{I} \Rightarrow \left(\frac{\Delta R}{R}\right)_{max} \times 100\% \quad = \left(\frac{\Delta V}{V}+\frac{\Delta I}{I}\right)\times 100\ \%$$

$$= \left(\frac{0.4}{25}+\frac{3}{200}\right)\times 100\ \% = 3.1\%$$

15. The minimum wavelength of Lyman series lines is P, then the maximum wavelength of these lines is

(A) $\frac{4P}{3}$ (B) 2 P (C) $\frac{2P}{3}$ (D) ∞

Ans : (A)

Hint :

$\lambda_{min} = P$

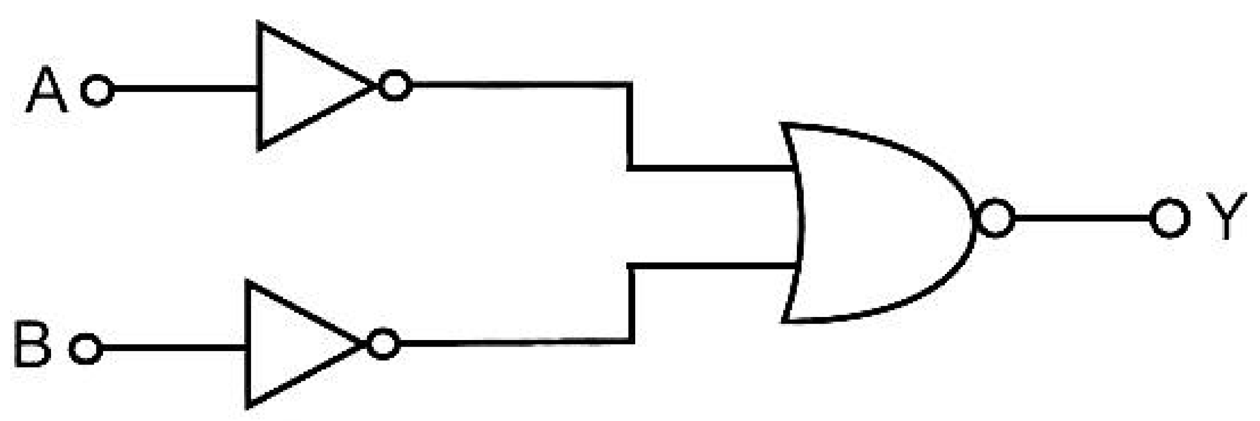
$\frac{hc}{\lambda_{min}} = c_1\left(\frac{1}{1} - \frac{1}{\infty}\right)$

$\lambda_{min} = \frac{hc}{c_1}$

$\frac{hc}{\lambda_{max}} = c_1\left(1 - \frac{1}{4}\right) = c_1\frac{3}{4}$

$\lambda_{max} = \frac{4hc}{3c_1} = \frac{4}{3}P$

16. Which logic gate is represented by the following combinations of logic gates ?



(A) NAND (B) AND (C) NOR (D) OR

Ans : (B)

Hint :

$\overline{\overline{A} + \overline{B}} = A \, . \, B \rightarrow$ AND gate

17. A force $\vec{F} = a\hat{i} + b\hat{j} + c\hat{k}$ is acting on a body of mass m. The body was initially at rest at the origin. The co-ordinates of the body after time 't' will be

(A) $\frac{at^2}{2m}, \frac{bt^2}{2m}, \frac{ct^2}{2m}$ (B) $\frac{at^2}{2m}, \frac{bt^2}{m}, \frac{ct^2}{2m}$ (C) $\frac{at^2}{m}, \frac{bt^2}{2m}, \frac{ct^2}{2m}$ (D) $\frac{at^2}{2m}, \frac{bt^2}{2m}, \frac{ct^2}{m}$

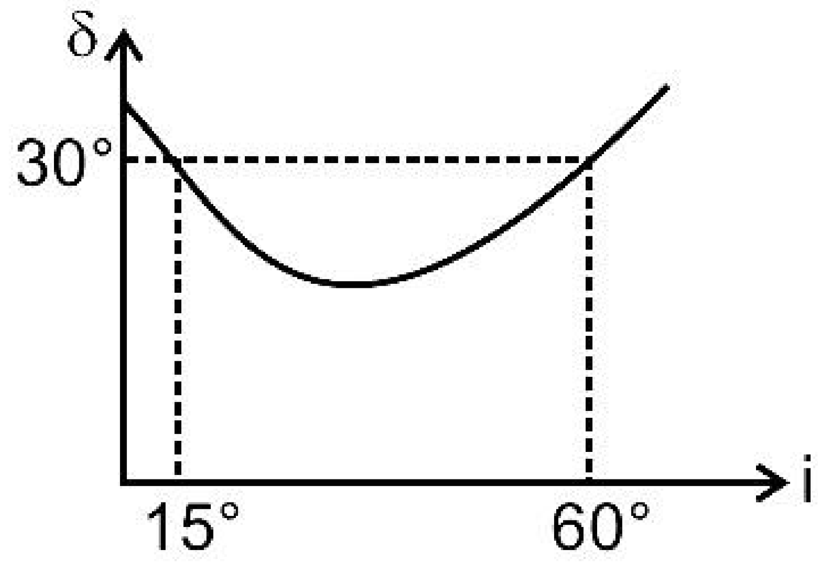
Ans : (A)

Hint :

$\vec{F} = a\hat{i} + b\hat{j} + c\hat{k}$

$x = \frac{1}{2}\left(\frac{a}{m}\right)t^2, \; y = \frac{1}{2}\left(\frac{b}{m}\right)t^2, \; z = \frac{1}{2}\left(\frac{c}{m}\right)t^2$

18. Figure shows the graph of angle of deviation δ versus angle of incidence i for a light ray striking a prism. The prism angle is



(A) 30° (B) 45° (C) 60° (D) 75°

Ans : (B)

Hint :

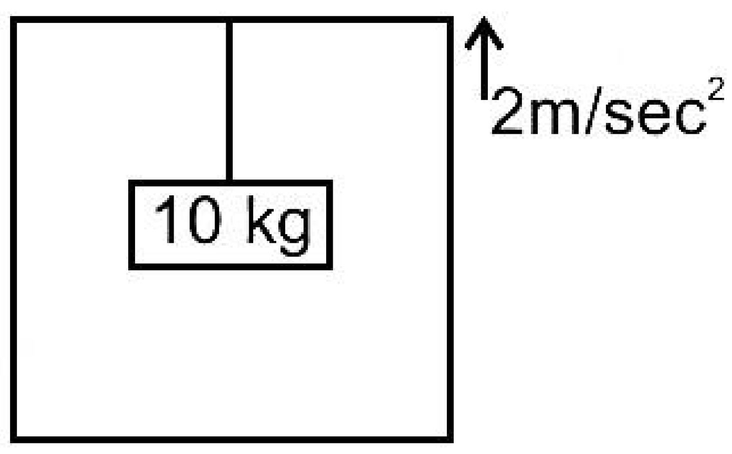
$= i + e - A$

$A = i + e - \delta$

$= 60 + 15 - 30$

$= 75 - 30 = 45^\circ$

19. One end of a steel wire is fixed to the ceiling of an elevator moving up with an acceleration $2 m/s^2$ and a load of 10 kg hangs from the other end. If the cross section of the wire is $2\ cm^2$ then the longitudinal strain in the wire will be ($g = 10 m/s^2$ and $Y = 2.0 \times 10^{11}\ N/m^2$)



(A) 4×10^{11} (B) 3×10^{-6} (C) 8×10^{-6} (D) 2×10^{-6}

Ans : (B)

Hint :

$T - mg = ma$

$T = m(g + a)$

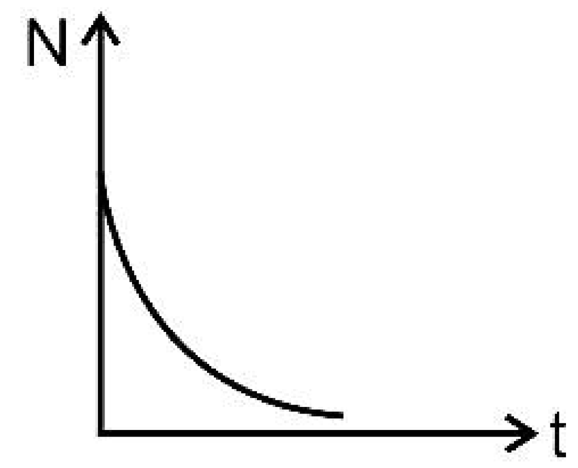
$= 10(10 + 2) = 120\ N$

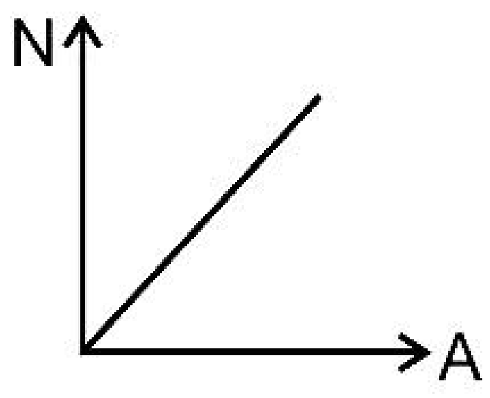
$$y = \frac{TL}{A\Delta\ell}$$

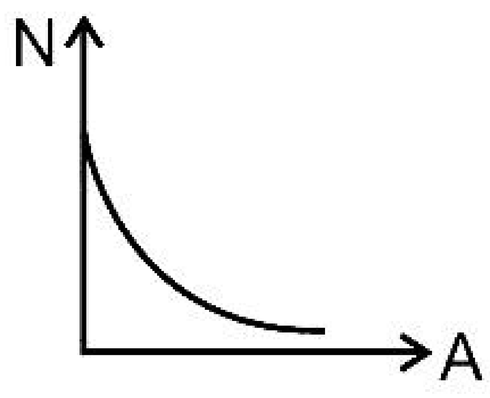
$$\frac{\Delta\ell}{L} = \frac{T}{YA} = \frac{\cancel{120}\,30}{\cancel{2} \times 10^{11} \times \cancel{2} \times 10^{-4}}$$

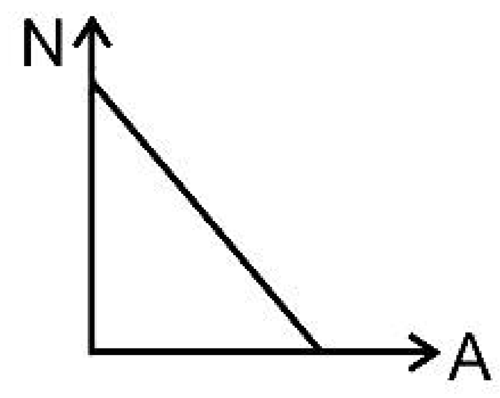
$= 30 \times 10^{-7} = 3 \times 10^{-6}$

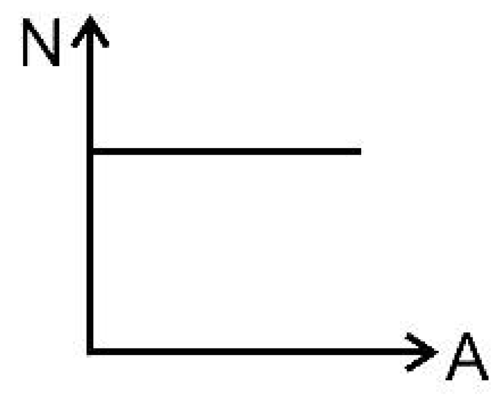
20. The number of undecayed nuclei N in a sample of radioactive material as a function of time (t) is shown in the figure. Which of the following graphs correctly show the relationship between N and the activity 'A' ?



(A) 

(B) 

(C) 

(D) 

Ans : (A)

Hint :

$N = N_0 e^{-\lambda t}$

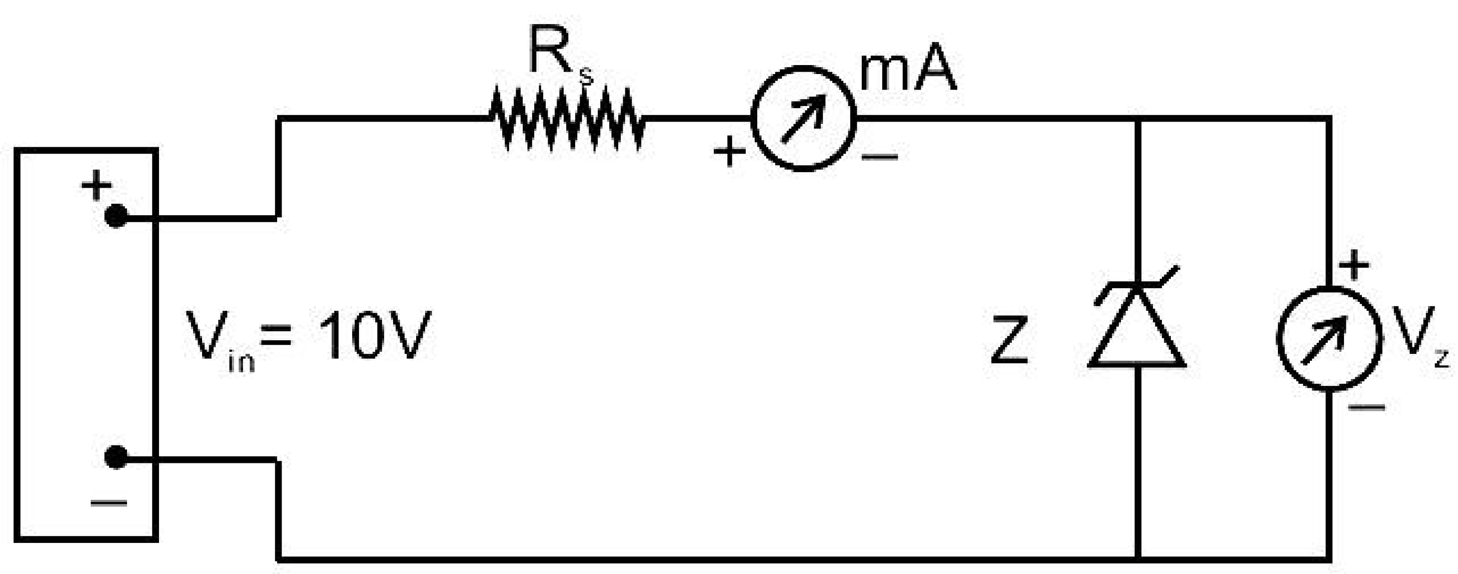
$-\frac{dN}{dt} = -N_0(-\lambda)e^{-\lambda t}$

$A = +N\lambda$

Straight line passing through origin.

21. Manufacturers supply a zener diode with zener voltage V_z = 5.6V and maximum power dissipation $P_{z\,max} = \frac{1}{4}W$.

This zener diode is used in the following circuit. Calculate the minimum value of the resistance R_s in the circuit so that the zener diode will not burn when the input voltage is V_{in} = 10 V.



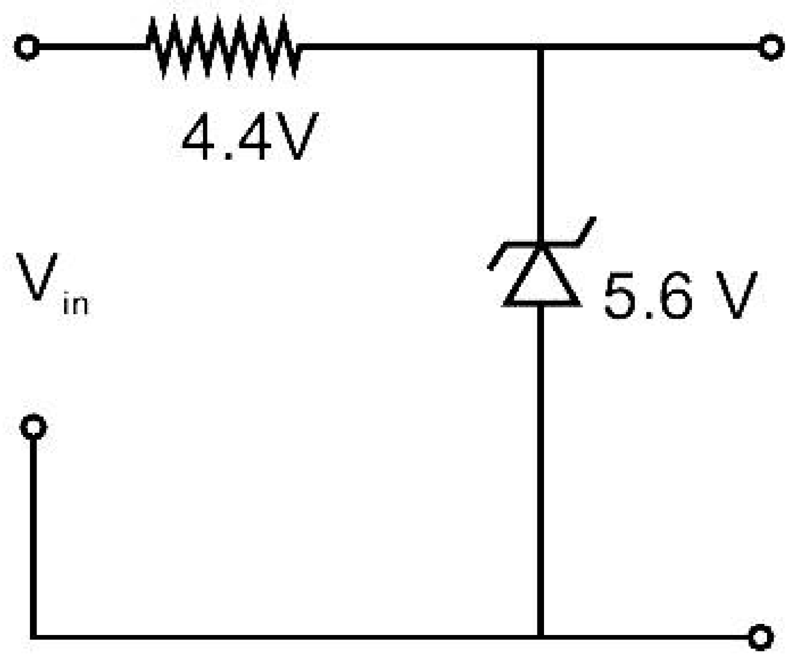
(A) 98.56 Ω (B) 170.52 Ω (C) 306.21 Ω (D) 412.37 Ω

Ans : (A)

Hint :

V_z = 5.6V $P_{z\,max} = \frac{1}{4}W$

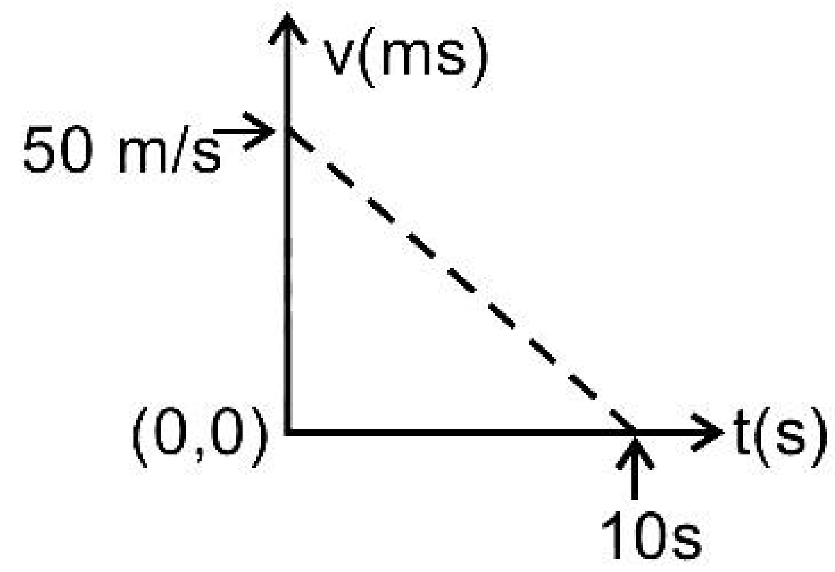
V_{in} = 10V.



$P = V_z I_z \Rightarrow \frac{1}{4} = 5.6 \times I_z \qquad I_Z = \frac{1}{4 \times 5.6} = I_s$

Now, $4.4 = I_s R_s \Rightarrow R_s = 4.4 \times 4 \times 5.6 = 98.56\ \Omega$

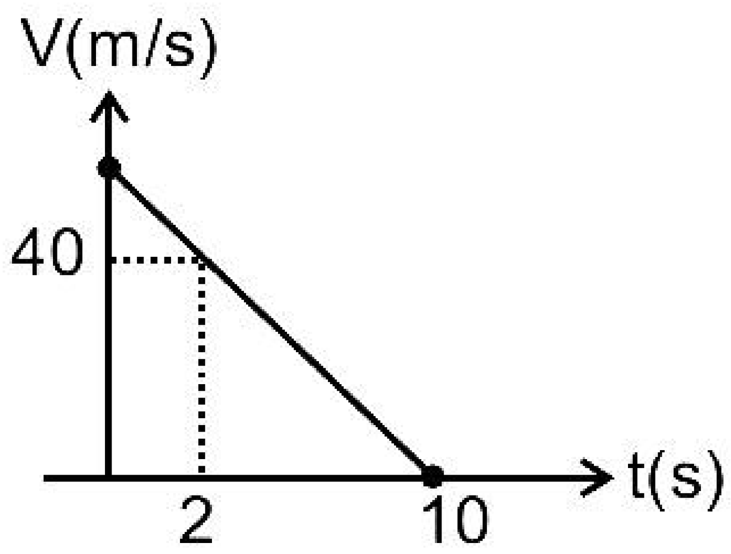
22. The velocity-time graph for a body of mass 10 kg is shown in the figure. Work done on the body in the first two seconds of motion is



(A) –9300 J (B) 12000 J (C) –4500 J (D) – 1200 J

Ans : (C)

Hint :



$V = -\frac{50}{10}t + 50$

$V = 50 - 5t$

at t = 2s

V = 50 – 10 = 40 m/s

$W = k_f - k_i$

$\frac{1}{2} \times 10\left(40^2 - 50^2\right] = -\frac{1}{2} \times 10[90 \times 10]$

= – 4500 J

23. A simple pendulum is taken at a place where its distance from the earth's surface is equal to the radius of the earth. Calculate the time period of small oscillations if the length of the string is 4.0 m. (Take $g = \pi^2\ ms^{-1}$ at the surface of the earth.)

(A) 4 s (B) 6 s (C) 8 s (D) 2 s

Ans : (C)

Hint : $T = 2\pi\sqrt{\ell/g}$, $\quad g_{h=R} = \frac{gR^2}{r^2} = g/4$, $T = 2\pi\sqrt{\frac{4}{\pi^2/4}} = \frac{2\pi \times 4}{\pi} = 8\,m/s^2$

24. For a domestic AC supply of 220 V at 50 cycles per sec, the potential difference between the terminals of a two-pin electric outlet in a room is given by

(A) $V(t) = 220\sqrt{2}\ \text{Cos}(100\pi t)$ (B) $V(t) = 220\ \text{Sin}(50t)$

(C) $V(t) = 220\ \text{Cos}(100\pi t)$ (D) $V(t) = 220\sqrt{2}\ \text{Cos}(50t)$

Ans : (A)

Hint : $V_{rms} = 220 \quad \therefore V_0 = 220\sqrt{2}$

$\therefore V = 220\sqrt{2}\cos 100\pi t \quad \omega = 2\pi f = 2\pi \times 50 = 100\pi$

25. A single slit diffraction pattern is obtained using a beam of red light. If red light is replaced by blue light then

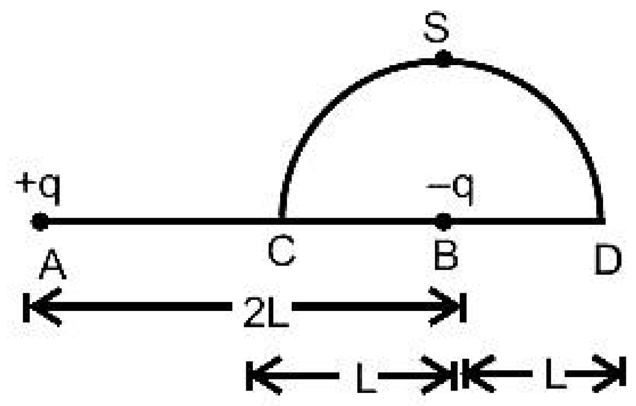
(A) the diffraction pattern will disappear (B) fringes will become narrower and crowded together

(C) fringes will become broader and will be further apart (D) there is no change in the diffraction pattern

Ans : (B)

Hint : Fringe width $\propto \lambda$ $\therefore$ Fringes will be narrower and crowed.

26. Two charges +q and –q are placed at points A and B respectively which are at a disetance 2*L* apart. C is the mid point of A and B. The workdone in moving a charge + Q along the semicircle CSD(W_1) and along the line CBD (W_2) are



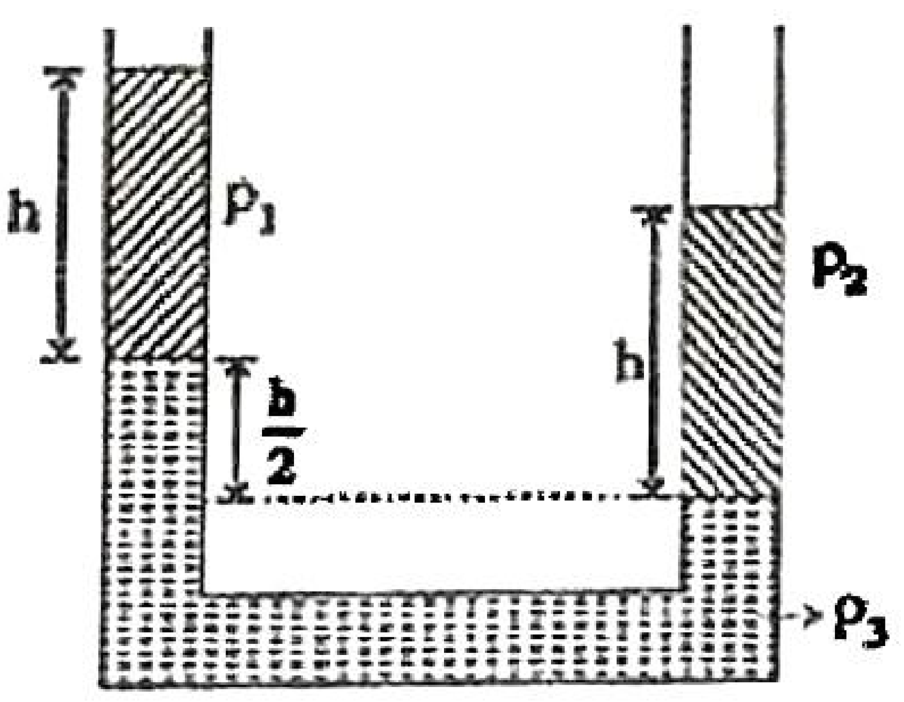
(A) $\frac{-Qq}{6\pi \epsilon_0 L}, \frac{-Qq}{6\pi \epsilon_0 L}$ (B) $\frac{qQ}{4\pi \epsilon_0 L}, \frac{qQ}{4\pi \epsilon_0 L}$ (C) $\frac{-Qq}{6\pi \epsilon_0 L}, \frac{-Qq}{12\pi \epsilon_0 L}$ (D) $\frac{qQ}{4\pi \epsilon_0 L}, 0$

Ans : (A)

Hint : $W_{CSD} = Q(V_{DA} - V_{CA}) = = \frac{Qq}{4\pi \epsilon_0 L}\left[\frac{1}{3} - 1\right] = = \frac{-2Qq}{12\pi \epsilon_0 L} = \frac{-Qq}{6\pi \epsilon_0 L} = W_{CBD}$

$W_{CBD} = W_{CSD}$ path independent

27. Three different liquids are filled in a U-tube as shown in figure. Their densities are ρ_1, ρ_2 and ρ_3 respectively. From the figure we may conclude that

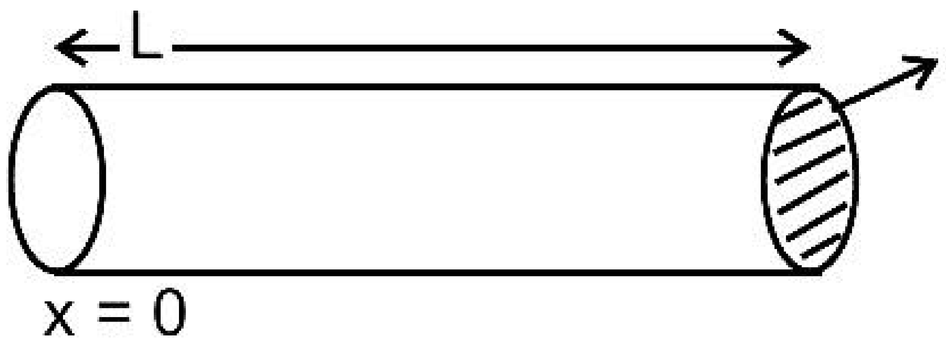


(A) $\rho_3 = 4(\rho_2 - \rho_1)$ (B) $\rho_3 = 4(\rho_1 - \rho_2)$ (C) $\rho_3 = 2(\rho_2 - \rho_1)$ (D) $\rho_3 = \frac{\rho_1 + \rho_2}{2}$

Ans : (C)

Hint : $h\rho_1 + \frac{h}{2}\rho_3 = h\rho_2, \quad \rho_1 + \frac{\rho_3}{2} = \rho_2 \qquad \rho_3 = 2(\rho_2 - \rho_1)$

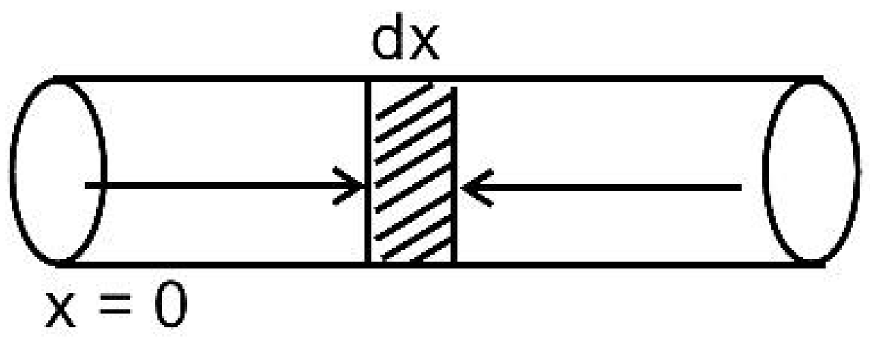
28. The variation of density of a solid cylindrical rod of cross sectional area α and length L is $\rho = \rho_0 \frac{x^2}{L^2}$, where x is the distance from one end of the rod. The position of its centre of mass from one end (x = 0) is



(A) $\frac{2L}{3}$ (B) $\frac{L}{2}$ (C) $\frac{L}{3}$ (D) $\frac{3L}{4}$

Ans : (D)

Hint : $\rho = \rho_0 \frac{x^2}{L^2}$,



$X = \frac{\int x dm}{\int dm}$ $\quad dm = \alpha dx \rho = \frac{\alpha \rho_0 x^2}{L^2} dx$

$= \frac{\rho_0 \int_0^L x^3 dx}{\frac{\rho_0}{L^2} \int_0^L x^2 dx} = \frac{\left[x^4/4\right]_0^L}{\left[x^3/3\right]_0^L} = 3/4 L$

29. Consider a particle of mass 1 gm and charge 1.0 Coulomb is at rest. Now the particle is subjected to an electric field $E(t) = E_0 \sin\omega t$ in the x-direction, where E_0 = 2 Newton/Coulomb and ω = 1000 rad/sec. The maximum speed attained by the particle is

(A) 2 m/sec (B) 4 m/sec (C) 6 m/sec (D) 8 m/sec

Ans : (B)

Hint : $V_{max} = \frac{1}{m}\int_0^{\frac{T}{2}} F dt$, $\quad = \frac{1}{m}\int_0^{\frac{T}{2}} qE_0 \sin(\omega t) dt$

$= \frac{2qE_0}{m\omega} = 4 \text{ m/s}$

30. A radioactive nucleus decays as follows :

$X \xrightarrow{\alpha} X_1 \xrightarrow{\beta} X_2 \xrightarrow{a} X_3 \xrightarrow{\gamma} X_4$

If the mass number and atomic number of 'X_4' are 172 and 69 respectively, then the atomic number and mass number of 'X' are

(A) 72, 180 (B) 69, 170 (C) 68, 172 (D) 70, 172

Ans : (A)

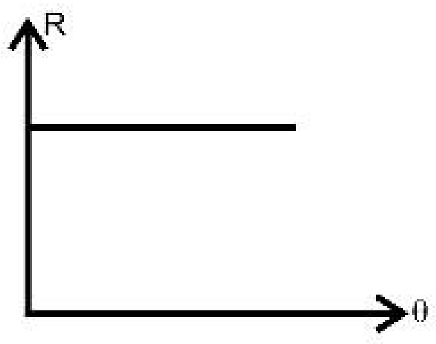
Hint : ${}_Z X^A$ $\quad Z - 2 + 1 - 2 = 69$ $\quad A - 4 - 4 = 172$

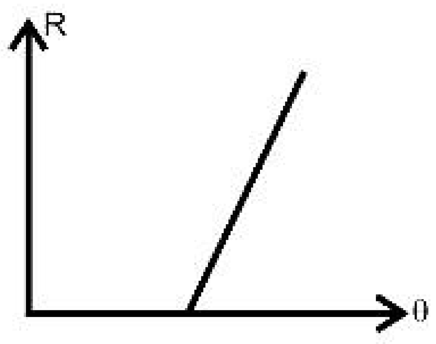
$Z = 72$ $\quad A = 180$

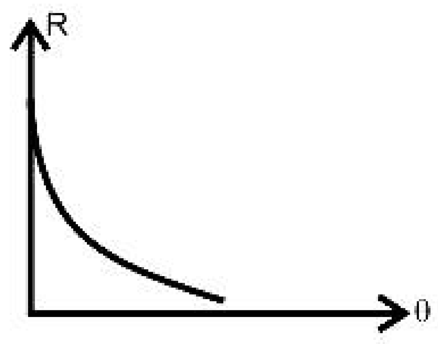
Category 2 (Q. 31 to 35)

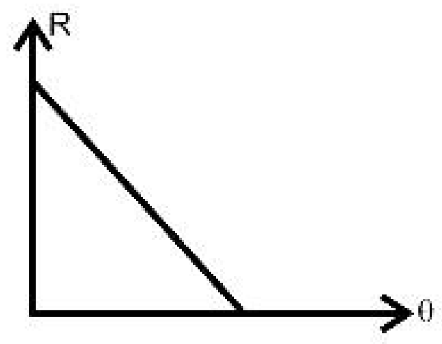
(Carry 2 marks each. Only one option is correct. Negative marks – ½)

31. Temperature of a body θ is slightly more than the temperature of the surrounding θ_0. Its rate of cooling (R) versus temperature of the body (θ) is plotted. Its shape would be

(A) 

(B) 

(C) 

(D) 

Ans : (B)

Hint : $-\frac{d\theta}{dt} = k(\theta - \theta_0)$

32. The equation of stationary wave along a stretched string is given by $y = 5\sin\frac{\pi x}{3}\cos 40\pi t$.

Here x and y are in cm and t in second. The separation between two adjacent nodes is

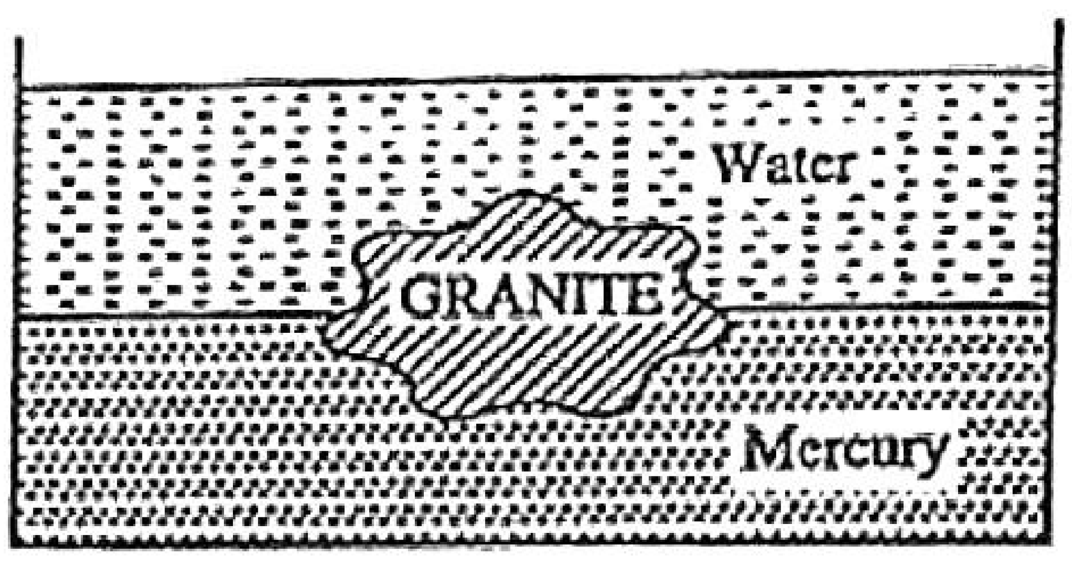
(A) 1.5 cm (B) 3 cm (C) 6 cm (D) 14 cm

Ans : (B)

Hint : $y = 5\sin\left(\frac{\pi}{3}x\right)\cos(40\pi t)$, $\omega = 40\pi$ $k = 3\text{cm}$, $\lambda = \frac{2\pi}{k} = 6\text{cm}$

Separation between nodes = $\frac{\lambda}{2} = 3\text{cm}$

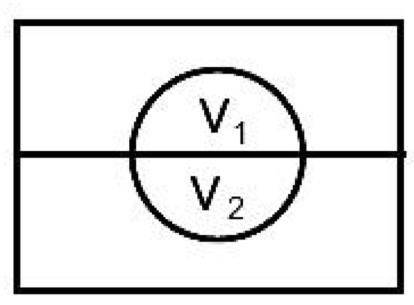
33. A piece of granite floats at the interface of mercury and water contained in a beaker as in figure. If the densities of granite, water and mercury are ρ, ρ_1 and ρ_2 respectively, the ratio of the volume of granite in water to the volume of granite in mercury is



(A) $\frac{\rho_2 - \rho}{\rho - \rho_1}$ (B) $\frac{\rho_2 + \rho}{\rho_1 + \rho}$ (C) $\frac{\rho_1 \rho_2}{\rho}$ (D) $\frac{\rho_1}{\rho_2}$

Ans : (A)

Hint : $V_1\rho_1 g + V_2\rho_2 g = V\rho g$



$\Rightarrow V_1\rho_1 g + V_2\rho_2 g = (V_1 + V_2)\rho g$

$\Rightarrow V_1\rho_1 g - V_1\rho g = V_2\rho g - V_2\rho_2 g$, $\Rightarrow V_1(\rho - \rho_1) = V_2(\rho_2 - \rho)$

$\Rightarrow \boxed{\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}}$

34. 10^{20} photons of wavelength 660 nm are emitted per second from a lamp. The wattage of the lamp is (Planck's constant

$= 6.6 \times 10^{-34}$ Js)

(A) 30 W (B) 60 W (C) 100 W (D) 500 W

Ans : (A)

Hint : $P = N\left(\frac{hc}{\lambda}\right)$

$P = 10^{20} \times \frac{6.6\times10^{-34}\times3\times10^{8}}{660\times10^{-9}} = 30\,W$

35. The apparent coefficient of expansion of a liquid, when heated in a copper vessel is C and when heated in silver vessel is S. If A is linear coefficient of expansion of copper, then linear coefficient of expansion of silver is

(A) $\frac{C-S-3A}{3}$ (B) $\frac{C+3A-S}{3}$ (C) $\frac{S+3A-C}{3}$ (D) $\frac{C+S+3A}{3}$

Ans : (B)

Hint : $C = \gamma_L - \gamma_C \Rightarrow C = \gamma_L - 3A \Rightarrow \gamma_L = C + 3A$

$S = \gamma_L - \gamma_S \Rightarrow \gamma_S = C + 3A - S$

$\Rightarrow \alpha_S = \frac{C+3A-S}{3}$

Category 3 (Q36 to 40)

(Carry 2 marks each. One or more options are correct. No negative marks)

36. Let the binding energy per nucleon of nucleus is denoted by 'E_{bn}' and radius of the nucleus is denoted by'r'. If mass number of nuclei A and B are 64 and 125 respectively, then

(A) $r_A < r_B$ (B) $r_A > r_B$ (C) $E_{bnA} > E_{bnB}$ (D) $E_{bnA} < E_{bnB}$

Ans : (A, C)

Hint : $r \propto A^{\frac{1}{3}}$

$r_A < r_B$

$E_{bnA} > E_{bnB} \rightarrow \left(\frac{B.E}{Nucleon}\right)$ for Fe is maximum and mass number of Nucleus A is closer to the mass number of Fe

37. Let $\bar{V}, V_{rms}, V_P$ denotes the mean speed , root mean square speed and most probable speed of the molecules each of mass m in an ideal monoatomic gas at absolute temperature T Kelvin. Which statement(s) is/are correct?

(A) No molecules can have speed greater than $\sqrt{2}\,V_{rms}$ (B) No molecules can have speed less than $V_p/\sqrt{2}$

(C) $V_p < \bar{V} < V_{rms}$ (D) Average kinetic energy of a molecule is $\frac{3}{4}mV_p^2$

Ans : (C, D)

Hint : $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$

$V_{mp} = \sqrt{\frac{2RT}{M}}$, $V_p < \bar{V} < V_{rms}$

K.E =½ $mV_{rms}^2 = \frac{1}{2}m\frac{3RT}{M} = \frac{3}{4}mV_p^2$

38. A wave disturbance in a medium is described by $y(x, t) = 0.02 \cos\left(50\pi t + \frac{\pi}{2}\right) \cos(10\pi x)$ where x, y are in meters and t is in second. Which statement(s) is/are correct?

(A) A node occurs at x = 0.15 m
(B) An antinode occurs at x = 0.3 m
(C) The speed of the wave is 4 m/sec
(D) The wavelength of the wave is 0.2 m

Ans : (A, B, D)

Hint : $\cos 10\pi x = 0 \rightarrow$ Node

$10\pi x = n\pi + \frac{\pi}{2}$

$10\pi x = n\pi + \frac{1}{2}$

$x = 0, \frac{1}{20}m, \frac{3}{20}m$

$\cos \pi x = \pm 1 \rightarrow$ Antinode

$10\pi x = n\pi$, $x = 0, \frac{1}{10}m, \frac{2}{10}m, \frac{3}{10}m$

wave speed (v) = $\frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s, $k = 10\pi$, $\frac{2\pi}{\lambda} = 10\pi$, $\lambda = \frac{2}{10} = 0.2$ m

39. If the dimensions of length are expressed as $G^xC^yh^z$, where G, C and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

(A) $x = \frac{1}{2}, y = \frac{1}{2}$
(B) $x = \frac{1}{2}, z = \frac{1}{2}$
(C) $y = \frac{1}{2}, z = \frac{3}{2}$
(D) $y = -\frac{3}{2}, z = \frac{1}{2}$

Ans : (B, D)

Hint : $L = [M^{-1} L^3 T^{-2}]^x [LT^{-1}]^y [ML^2T^{-1}]^z$

$M^0LT^0 = M^{-x+z} L^{3x+y+2z} T^{-2x-y-z}$

$-x + z = 0$; $\quad 3x + y + 2z = 1$; $\quad 2x + y + z = 0$

$x = z$; $\quad 2z = 1$ $\quad 3x + y = 0$

$z = ½$

$x = \frac{1}{2}$

and $y = -\frac{3}{2}$

40. Two spheres S_1 and S_2 of masses m_1 and m_2 respectively collide with each other. Initially S_1 is at rest and S_2 is moving

with velocity v along x-axis. After collision S_2 has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The sphere S_1 moves after collision

(A) with a velocity of magnitude $\frac{m_2}{m_1} v \frac{\sqrt{5}}{2}$

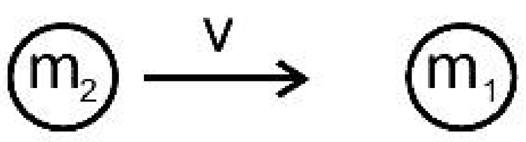
(B) with a velocity in the direction $\theta = \tan^{-1}\left(-\frac{1}{3}\right)$ to the x-axis

(C) with a velocity whose direction makes an angle θ with x-axis such that $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ or $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$

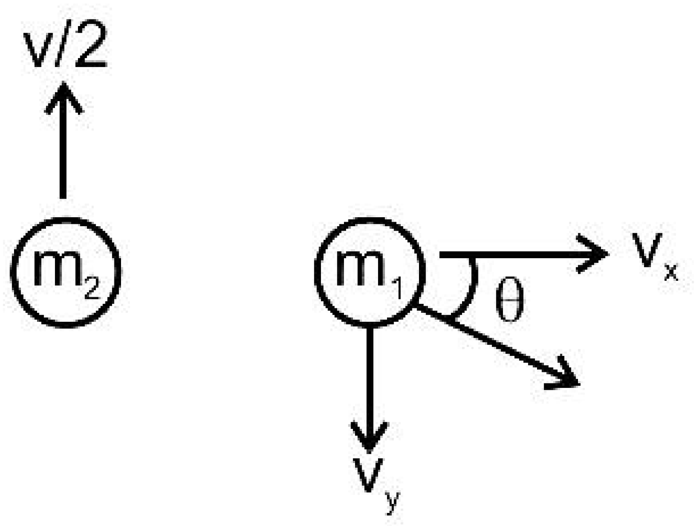
(D) with a velocity of magnitude $\frac{m_1}{2m_2} v\sqrt{5}$

Ans : (A, C)

Hint : before collision



After collision



$\rightarrow v_x = \frac{m_2 v}{m_1}$ $\qquad v_y = \frac{m_2 v}{2m_1}$

$$v = \frac{\sqrt{5}}{2} v\left(\frac{m_2}{m_1}\right)$$

$\rightarrow \tan\theta = \frac{v_y}{v_x}\left(\frac{1}{2} \text{or} -\frac{1}{2}\right)$ (As per choice of original direction)

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \text{ or } \tan^{-1}\left(-\frac{1}{2}\right)$$

CHEMISTRY

CATEGORY - 1 (Q 41 to 70)

(Carry 1 mark each. Only one option is correct. Negative marks: –¼)

41. Increasing order of solubility of AgCl in (i) H_2O, (ii) 1M NaCl (aq.), (iii) 1M $CaCl_2$(aq.) and (iv) 1M $NaNO_3$ (aq.) solution

(A) $CaCl_2 < NaNO_3 < NaCl < H_2O$
(B) $CaCl_2 > H_2O > NaCl > NaNO_3$
(C) $CaCl_2 > NaCl > H_2O > NaNO_3$
(D) $CaCl_2 < NaCl < H_2O < NaNO_3$

Ans : (D)

Hint : In H_2O solubility = $\sqrt{K_{sp}}$

In 1M NaCl and in 1 M $CaCl_2$ more the [Cl^-] less is the solubility

Hence $CaCl_2 < NaCl < H_2O < NaNO_3$

42. The bond order of HeH^+ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (A)

Hint : $\sigma_{1s^2}\sigma^*_{1s^0}$ B.O = $\frac{1}{2}[2-0] = 1$

43. If three elements A, B, C crystalise in a cubic solid lattice with B atoms at the cubic centres, C atoms at the centre of edges and A atoms at the corners, then formula of the compound is

(A) AB_3C (B) A_3BC (C) ABC_3 (D) ABC

Ans : (C)

Hint : Number of A atoms = $\frac{1}{8}\times 8 = 1$

Number of C atoms = $\frac{1}{4}\times 12 = 3$

Number of B atom = 1

Hence ABC_3

44. An LPG (Liquified Petroleum Gas) cylinder weighs 15.0 kg when empty. When full, it weighs 30.0 kg and shows a pressure of 3.0 atm. In the course of usage at 27°C, the mass of the full cylinder is reduced to 24.2 kg. The volume of the used gas in cubic metre at the normal usage condition (1 atm and 27°C) is (assume LPG to be normal butane and it behaves ideally)

(A) $24.6\ m^3$ (B) $246\ m^3$ (C) $0.246\ m^3$ (D) $2.46\ m^3$

Ans : (D)

Hint : Weight of gas = 15 kg

Weight of gas in the cylinder after use = 24.2 – 15.0 = 9.2

$\therefore$ Gas used = 15 – 9.2 = 5.8 kg

$$\therefore V = \frac{w}{M}\times\frac{RT}{P} = \frac{5.8\times10^3\times0.0821\times300}{58\times1}L$$

$$= \frac{100\times.0821\times300}{10^3}m^3$$

$$=2.463\ m^3$$

45. P and Q combines to form two compounds PQ_2 and PQ_3. If 1 g PQ_2 is dissolved in 51 g benzene the depression of freezing point becomes 0.8°C. On the other hand if 1 g PQ_3 is dissolved in 51 g of benzene, the depression of freezing point becomes 0.625°C. The atomic mass of P and Q are (K_f of benzene = 5.1 K kg mol^{-1})

(A) 35, 55 (B) 45, 45 (C) 55, 45 (D) 55, 35

Ans : (D)

Hint : $0.8 = \frac{5.1 \times 1000 \times 1}{51 \times M_{PQ_2}} \Rightarrow M_{PQ_2} = 125$

$0.625 = \frac{5.1 \times 1000 \times 1}{51 \times M_{PQ_3}} \Rightarrow M_{PQ_3} = 160$

$\therefore$ At wt of P = 55 and Q = 35

46. For a chemical reaction, half-life period $\left(t_{\frac{1}{2}}\right)$ is 10 minutes. How much reactant will be left after 20 minutes if one starts with 100 moles of reactant and the order of the reaction be (i) zero, (ii) one and (iii) two?

(A) 0, 25, 33.33 (B) 25, 0, 33.33 (C) 33.33, 25, 0 (D) 25, 33.33, 0

Ans : (A)

Hint : For zero order, $k = \frac{x}{t}$

$t_{\frac{1}{2}} = \frac{a}{2K}$

$10 = \frac{100}{2 \times K} \Rightarrow K = 5$

$5 = \frac{x}{20} \Rightarrow x = 100 \therefore (a - x) = 0$

For 1st order, No. of half lives $= \frac{20}{10} = 2$

$\therefore (a - x) = \frac{100}{2^2} = 25$

For 2nd order, $k = \frac{x}{at(a - x)}$

$\therefore t_{\frac{1}{2}} = \frac{1}{ak}$

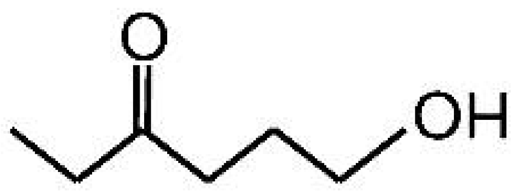
$10 = \frac{1}{100 \times k} \Rightarrow k = 10^{-3}$

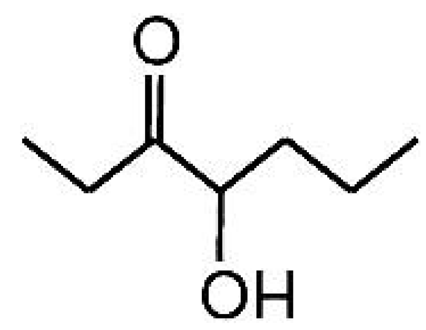
$\therefore 10^{-3} = \frac{x}{100 \times 20 \times (100 - x)}$

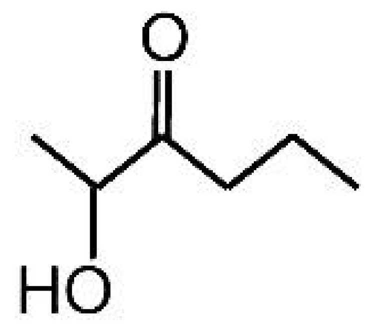
$\Rightarrow$ x = 66.66

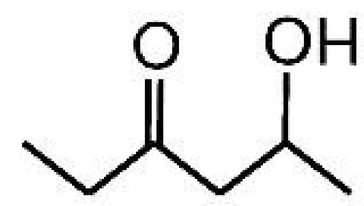
$\therefore$ (a – x) = 33.33

47. Which one among the following compounds will most readily be dehydrated under acidic condition?

(A) 

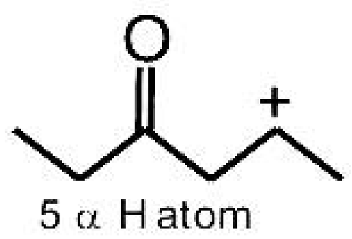
(B) 

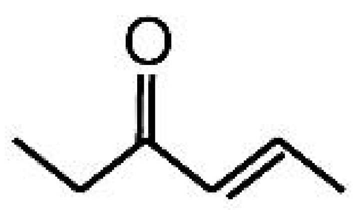
(C) 

(D) 

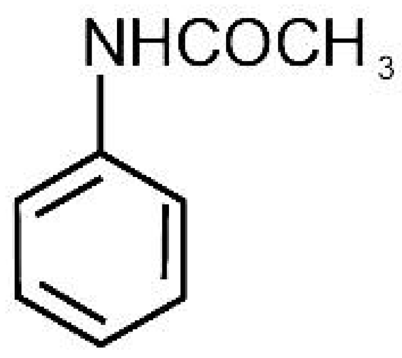
Ans : (D)

Hint : Ease of dehydration α Carbocation Stability and Product Stability



 ⇒ Product Resonance Stabilised

48. Kjeldahl's method cannot be used for the estimation of nitrogen in which compound?

(A) 

(B) $CH_2(NH_2)COOH$

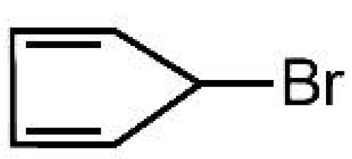
(C) $C_6H_5 - N = N - C_6H_5$

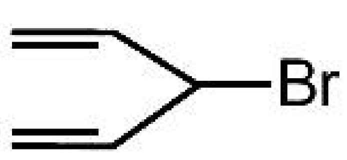
(D) $CH_3CH(NH_2)COOH$

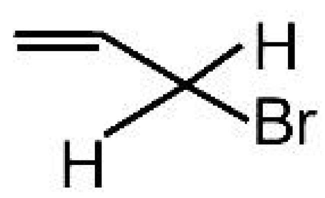
Ans : (C)

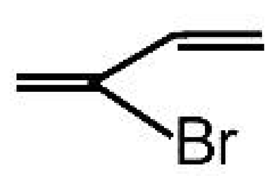
Hint : Azo compound does not respond to Kjeldahl's method.

49. Which of the following compounds is most reactive in S_N1 reaction?

(A) 

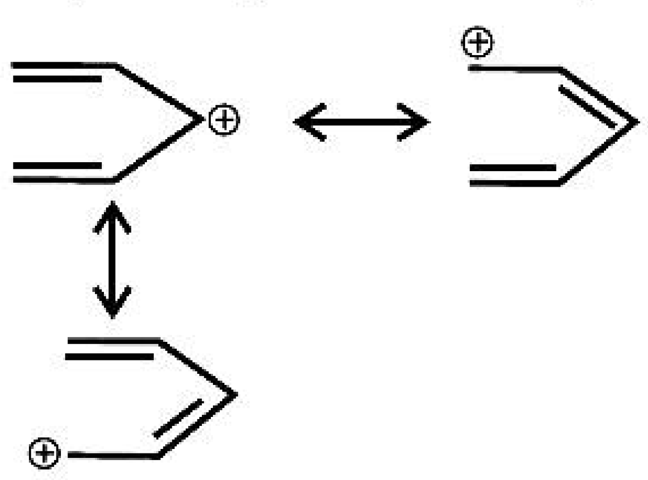
(B) 

(C) 

(D) 

Ans : (B)

Hint : Reactivity of S_N1 α stability of Carbocation



50. Equal volume of two solutions A and B of a strong acid having pH = 6.0 and pH = 4.0 respectively are mixed together to form a new solution. The pH of the new solution will be in the range

(A) between 5 and 6 (B) between 6 and 7 (C) between 4 and 5 (D) between 3 and 4

Ans : (C)

Hint : pH = 6 $\therefore [H^+] = 10^{-6}$ M

Let volume is 1L

1L 10^{-6}M SA contains 10^{-6} moles H^+

pH = 4 $\therefore [H^+] = 10^{-4}$ M

1L 10^{-4} M SA contains 10^{-4} moles H^+

$\therefore$ Total $H^+ = 10^{-6} + 10^{-4} = 1.01 \times 10^{-4}$ moles

$$\therefore \left[H^+\right] = \frac{1.01\times10^{-4}}{2}(M) = 5.05\times10^{-5}$$

pH = – log(5.05×10^{-5}) = 4.29

51. Adiabatic free expansion of ideal gas must be

(A) Isobaric (B) Isochoric (C) Isothermal (D) Isoentropic

Ans : (C)

Hint : According to first law of thermodynamics

Q = dU – W

$\Rightarrow Q = dU - \left[-P_{external}\left(V_f - V_i\right)\right]$

[In Adiabatic Process Q = 0, In Free expansion $P_{external} = 0$]

$\therefore$ Putting the values, we get $\Rightarrow dU = 0$ $\therefore$ U = constant

As internal energy remains unchanged; temperature must be constant.

$\therefore$ Isothermal Process

52. ${}_5B^{10} + {}_2He^4 \rightarrow X + {}_0n^1$

In the above nuclear reaction 'X' will be

(A) ${}_7N^{14}$ (B) ${}_7N^{13}$ (C) ${}_6C^{12}$ (D) ${}_7N^{14}$

Ans : (B)

Hint : ${}_5B^{10} + {}_2He^4 \longrightarrow {}_b^a x + {}_0n^1$

Balancing atomic numbers, we have

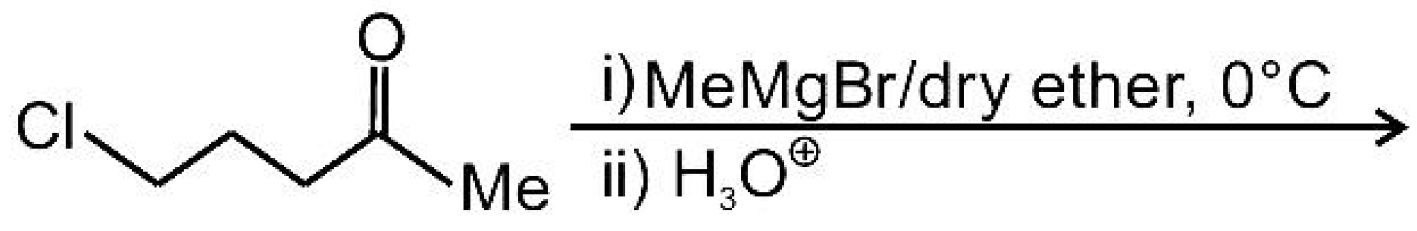
5 + 2 = b + 0

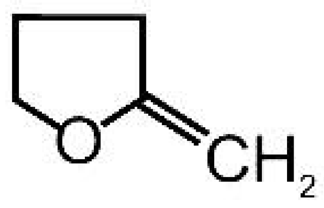
$\therefore$ b = 7

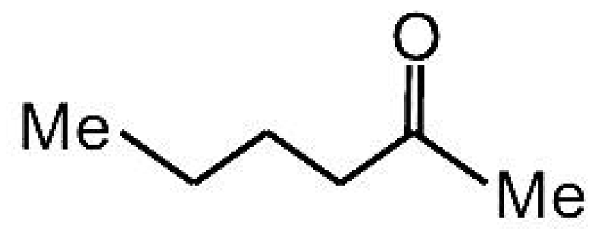
Balancing mass numbers; we have

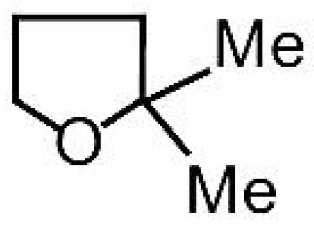
10 + 4 = a + 1

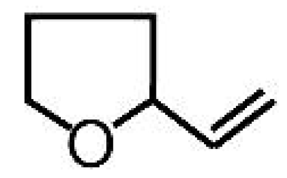
$\therefore$ a = 13

53. In the following reaction, the major product (H) is 

(A) 

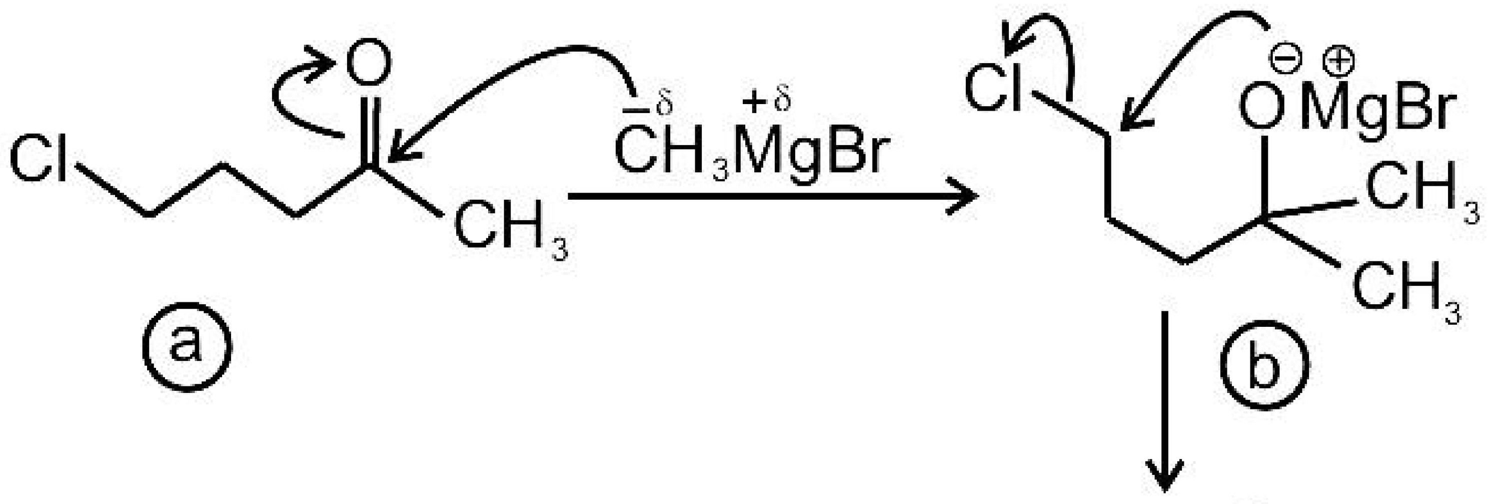
(B) 

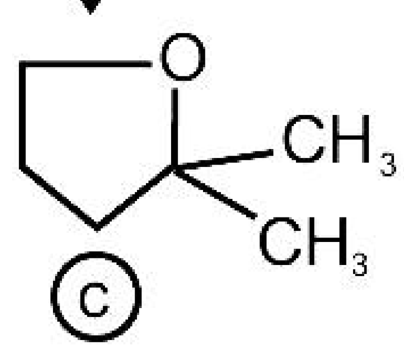
(C) 

(D) 

Ans : (C)

Hint :





a ⟶ Nucleophilic attack by CH_3MgBr

b ⟶ Internal nucleophilic substitution by $\overset{\ominus}{O}$ results in ring closure.

54. Which of the following hydrogen bonds is likely to be the weakest?

(A) $C-H\cdots O$ (B) $N-H\cdots O$ (C) $O-H\cdots O$ (D) $O-H\cdots F$

Ans : (A)

Hint : For $x - \overset{\delta+}{H} \cdots\cdots \overset{\delta-}{O}$ hydrogen bond; lower the electronegativity of x; lower is the residual $\delta+$ change induced on H & hence a lesser electrostatic attraction is expected between H & O; thus diminishing bond strength.

Hence, chosen option is the atom of lowest electronegativity, i.e. C.

55. How many oxygen atoms are present in 0.36 g of a drop of water at STP?

(A) 6.023×10^{22} (B) 1.205×10^{22} (C) 6.023×10^{23} (D) 1.205×10^{23}

Ans : (B)

Hint : 0.36 g water drop has $\frac{0.36}{18}$ moles

$\frac{0.36}{18}$ moles of water has $\frac{0.36}{18} \times 6.023 \times 10^{23}$ molecules of water

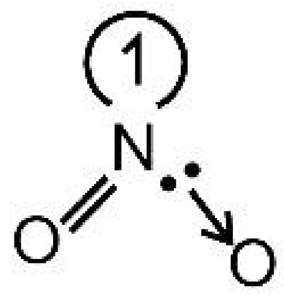
1 molecule of water has 1 oxygen atom.

$\therefore$ Number of O atoms = $\frac{0.36}{18} \times 6.023 \times 10^{23} = 1.205 \times 10^{22}$

56. Which of the following oxides is paramagnetic?

(A) SO_2 (B) NO_2 (C) SiO_2 (D) CO_2

Ans : (B)

Hint : 

Presence of unpaired electron in NO_2 results in paramagnetic character.

57. 360 cm^3 of a hydrocabon diffuses in 30 minutes, while under the same conditions 360 cm^3 of SO_2 gas diffuses in one hour. The molecular formula of the hydrocabon is

(A) CH_4 (B) C_2H_6 (C) C_2H_4 (D) C_2H_2

Ans : (A)

Hint : According to Graham's Law: (MW = Molecular Weight)

$$\frac{R_d(\text{hydrocarbon})}{R_d(SO_2)} = \sqrt{\frac{MW_{SO_2}}{MW_{\text{hydrocarbon}}}}$$

$$\Rightarrow \frac{360/30}{360/60} = \sqrt{\frac{64}{MW_{\text{hydrocarbon}}}}$$

$$\Rightarrow 4 = 64/MW_{\text{hydrocabon}}$$

$$\Rightarrow MW_{\text{hydrocarbon}} = \frac{64}{4} = 16$$

CH_4 is the hydrocarbon with possible molecular weight 16.

58. The molar conductances of $Ba(OH)_2$, $BaCl_2$ and NH_4Cl at infinite dilution are 523.28, 280.0 and 129.8 S cm^2 mol^{-1} respectively. The molar conductance of NH_4OH at infinite dilution will be

(A) 125.72 S cm^2 mol^{-1} (B) 251.44 S cm^2 mol^{-1} (C) 502.88 S cm^2 mol^{-1} (D) 754.32 S cm^2 mol^{-1}

Ans : (B)

Hint : $2\lambda^{\infty}_{m[NH_4OH]} = 2\lambda^{\infty}_{m[NH_4Cl]} - \lambda^{\infty}_{m[BaCl_2]} + \lambda^{\infty}_{m[Ba(OH)_2]}$

$$2\lambda^{\infty}_{m(NH_4OH)} = 2 \times (129.8) - 280 + 523.28$$

$$2\lambda^{\infty}_{m(NH_4OH)} = 502.88$$

$$\lambda^{\infty}_{m(NH_4OH)} = 251.44 \text{ S cm}^2 \text{ mol}^{-1}$$

59. What is the four-electron reduced form of O_2?

(A) Superoxide (B) Peroxide (C) Oxide (D) Ozone

Ans : (C)

Hint : $O_2 + 4e \rightarrow 2O^{2(-)}$

Each oxygen atom takes $2e^-$ to form oxide ion.

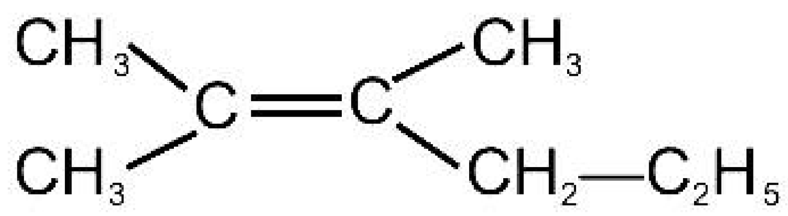
60. How many electrons are needed to reduce N_2 to NH_3?

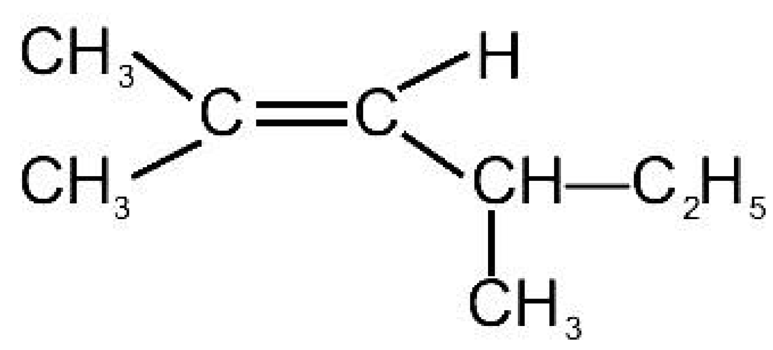
(A) 3 (B) 4 (C) 5 (D) 6

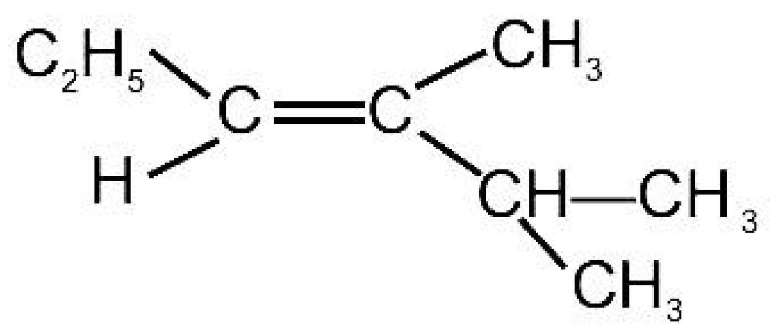
Ans : (D)

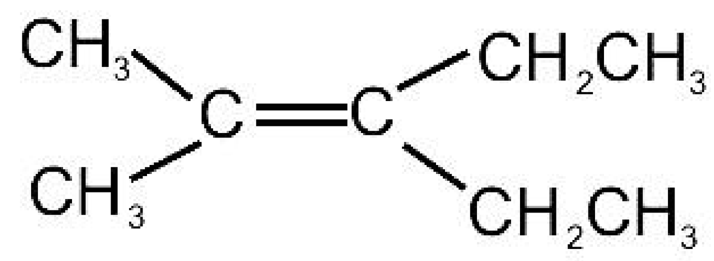
Hint : $N_2 + 6e^- + 6H^+ \rightarrow 2NH_3$

61. In optically active alkene having molecular formula C_8H_{16} gives acetone as one of the products on ozonolysis. The structure of the alkene is

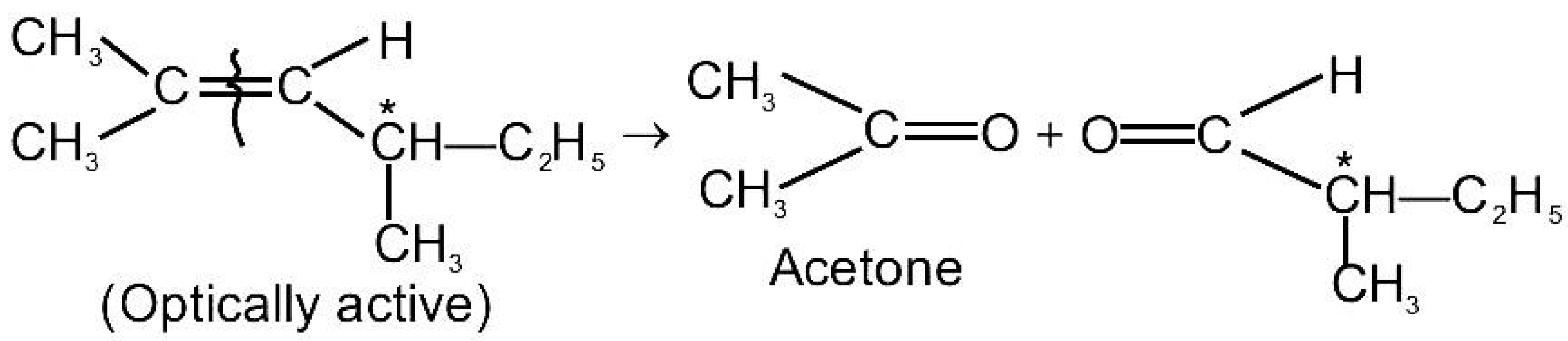
(A) 

(B) 

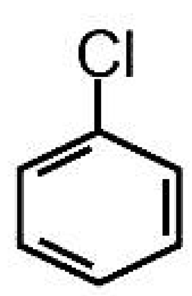
(C) 

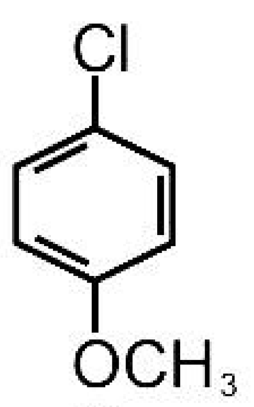
(D) 

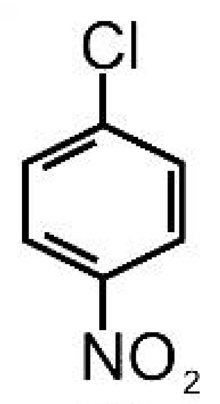
Ans : (B)

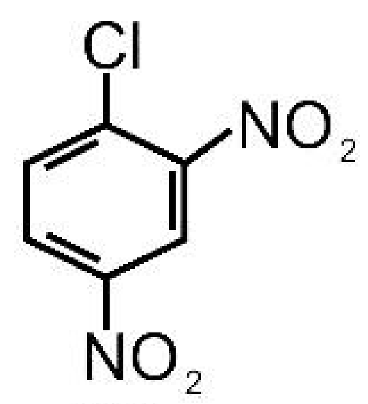
Hint : 

62. Increasing order of the nucleophilic substitution of following compounds is


I


II


III


IV

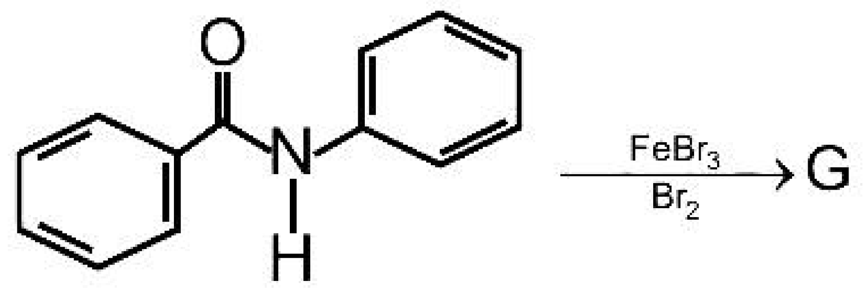
(A) $I < III < II < IV$ (B) $II < I < III < IV$ (C) $II < III < I < IV$ (D) $IV < III < II < I$

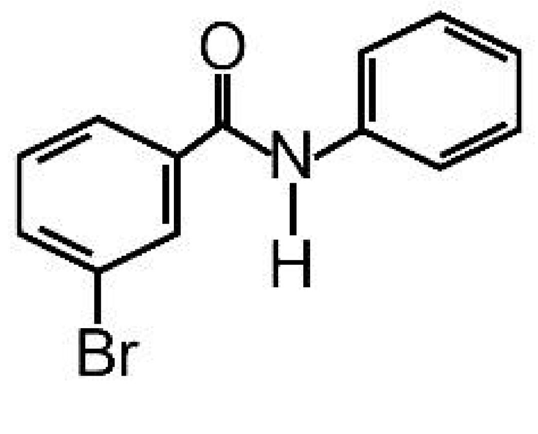
Ans : (B)

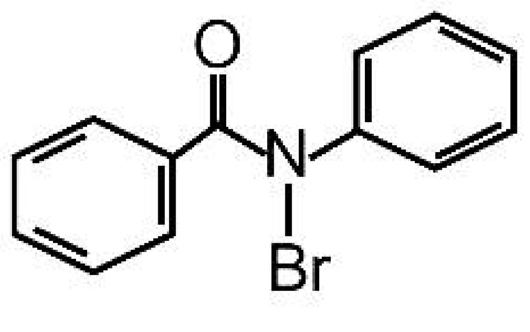
Hint : $II < I < III < IV$

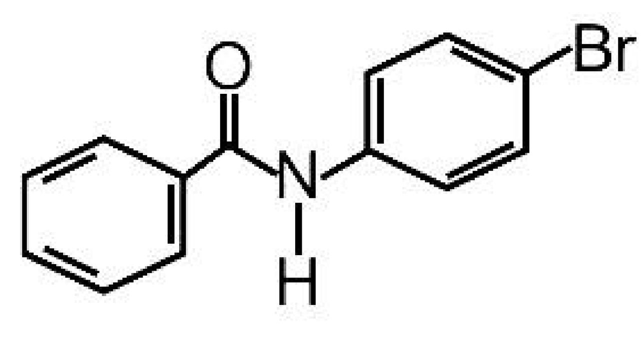
Electron withdrawing group decreases the electrondensity and favours nucleophilic substitution.

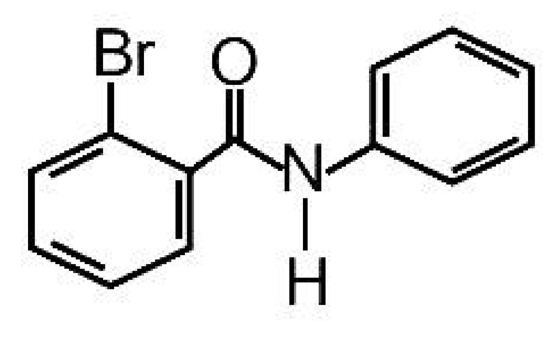
63. Identify the major product (G) in the following reaction



(A) 

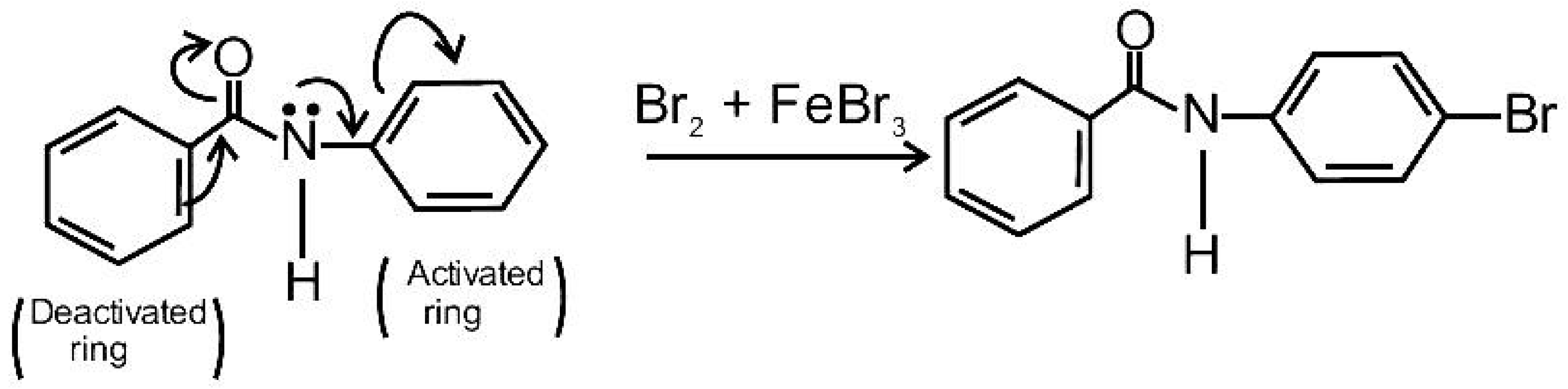
(B) 

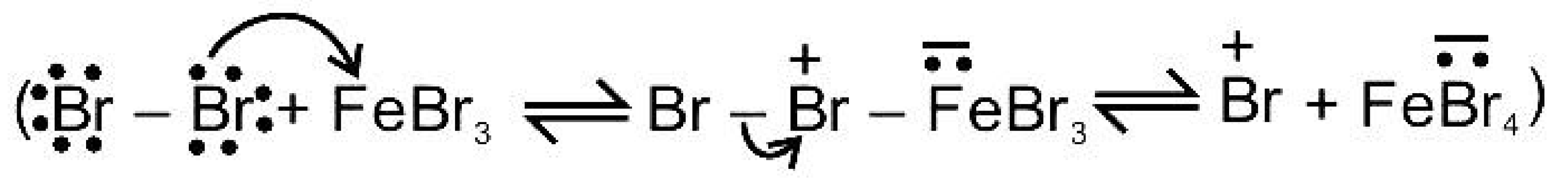
(C) 

(D) 

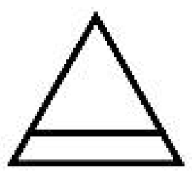
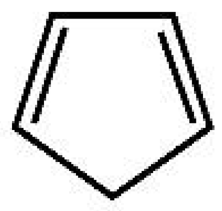
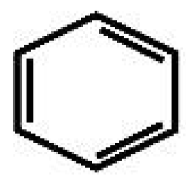
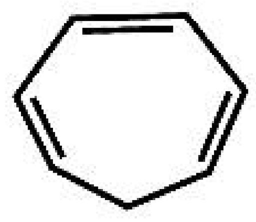
Ans : (C)

Hint :



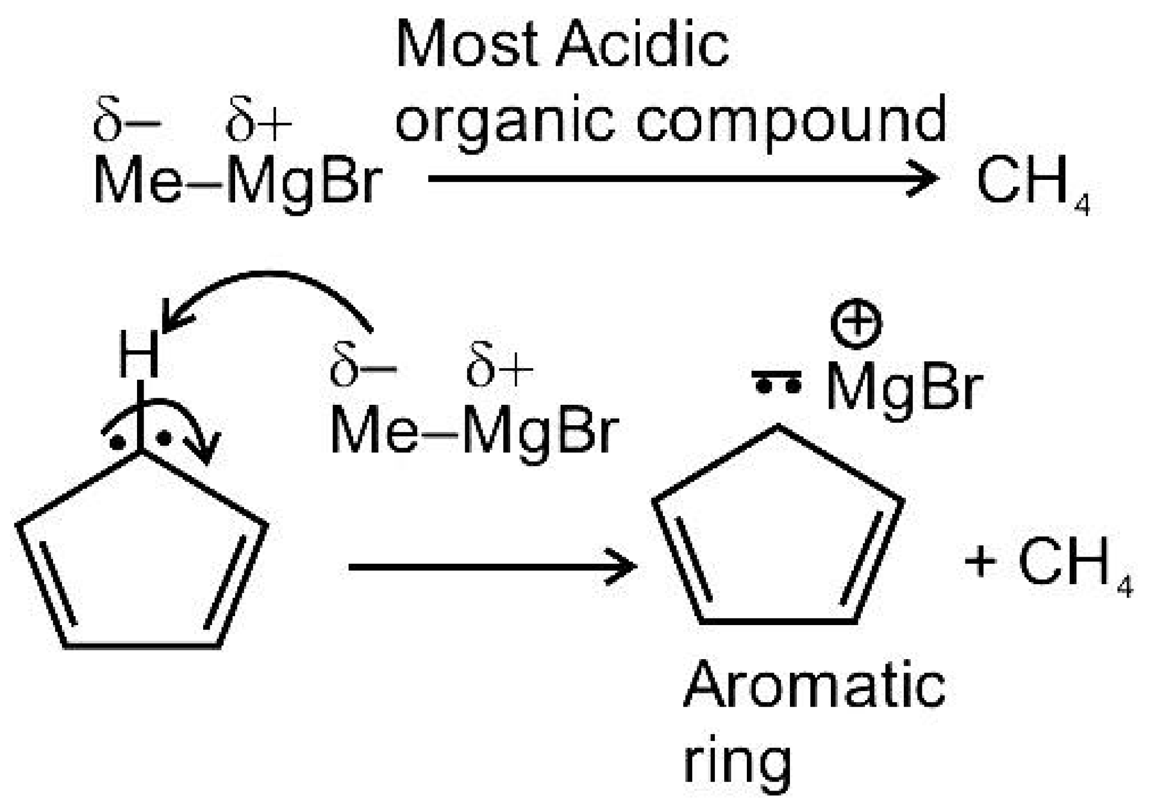


64. Which of the following hydrocarbons reacts easily with MeMgBr to give methane?

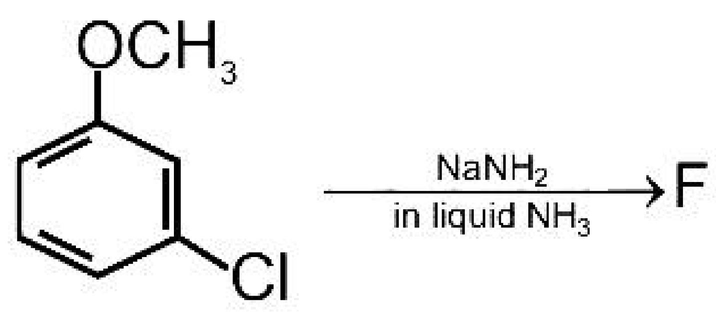
(A) 
(B) 
(C) 
(D) 

Ans : (B)

Hint :



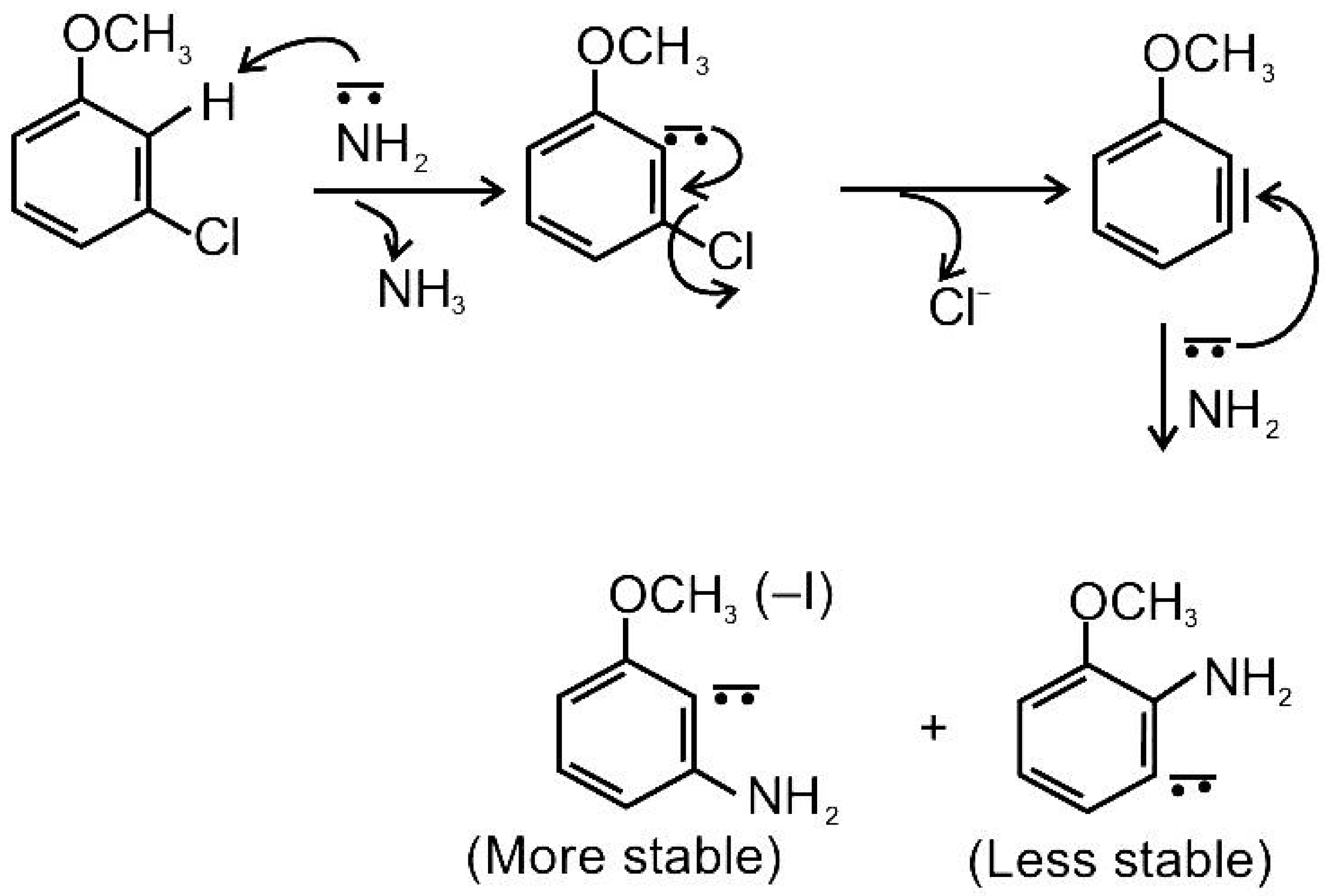
65. The major product (F) in the following reaction is

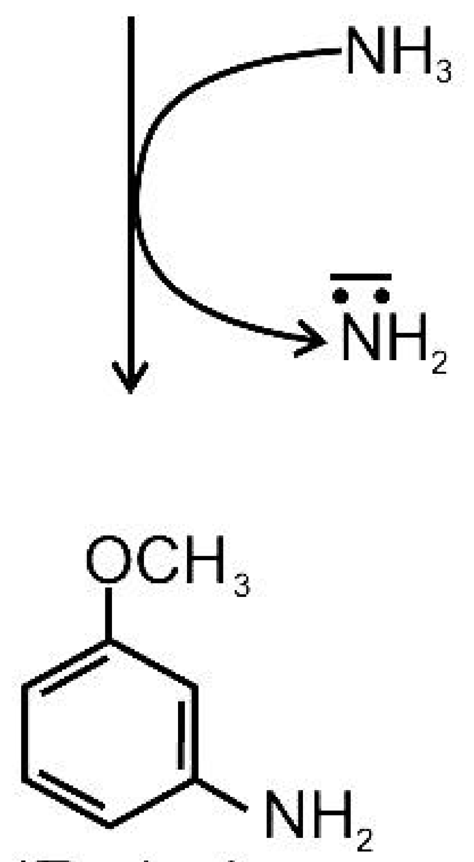


(A) o-Anisidine (B) m-Anisidine (C) p-Anisidine (D) p-Chloro aniline

Ans : (B)

Hint :





(Exclusive Product)

66. The common stable oxidation states of Eu and Gd are respectively

(A) + 3 and + 3 (B) + 3 and + 2 (C) + 2 and + 3 (D) + 2 and + 2

Ans : (A)

Hint : Fact

67. Arrange the following compounds in order of their increasing acid strength

I	II	III	IV
$HO_2C(CH_2)_4COOH$	$CH_2(COOH)_2$	COOH–COOH	$CO_2H(CH_2)_2CO_2H$

(A) I < II < III < IV (B) IV < III < II < I (C) I < IV < II < III (D) II < I < III < IV

Ans : (C)

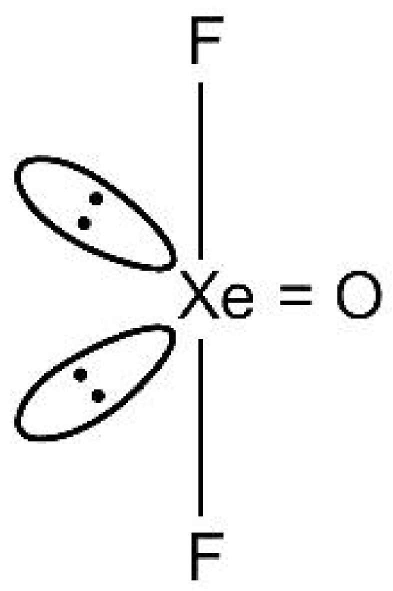
Hint : As distance decreases –I effect increases, thus acidic strength increases.

68. The number of lone pair of electrons and the hybridization of Xenon (Xe) in $XeOF_2$ are

(A) 1, sp^3 (B) 1, dsp^2 (C) 3, dsp^3 (D) 2, sp^3d

Ans : (D)

Hint :



(sp^3d)

69. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for As_2S_3 sol increases in the order

(A) $Al^{3+} < Ba^{2+} < Na^+$ (B) $Na^+ < Ba^{2+} < Al^{3+}$ (C) $Ba^{2+} < Na^+ < Al^{3+}$ (D) $Al^{3+} < Na^+ < Ba^{2+}$

Ans : (B)

Hint : As_2S_3 is a negatively charged sol, coagulating power α charge of ion.

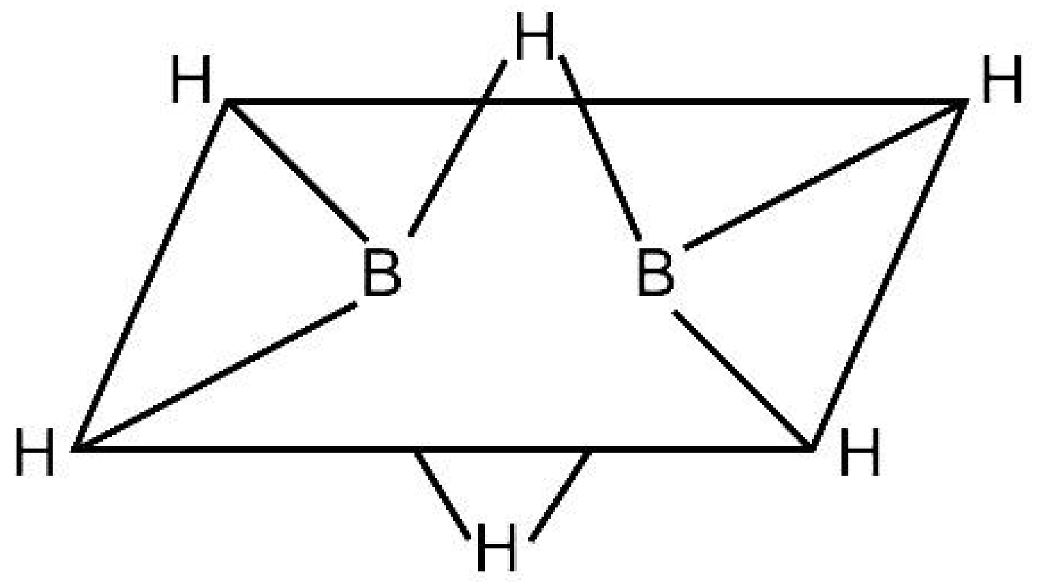
$Na^+ < Mg^{2+} < Al^{3+}$

70. The number of terminal and bridging hydrogens in B_2H_6 are respectively

(A) 4 and 2 (B) 2 and 4 (C) 2 and 2 (D) 4 and 4

Ans : (A)

Hint :



Category 2 (Q71 to Q 75)

(Carry 2 marks each. Only one option is correct. Negative marks :– ½)

71. Consider the following gas phase dissociation, $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ with equilibrium constant K_P at a particular temperature and at pressure P. The degree of dissociation (α) for $PCl_5(g)$ is

(A) $\alpha = \left(\frac{K_P}{K_P + P}\right)^{1/3}$ (B) $\alpha = \left(\frac{K_P}{K_P + P}\right)$ (C) $\alpha = \left(\frac{K_P}{K_P + P}\right)^{1/2}$ (D) $\alpha = \left(\frac{K_P}{K_P + P}\right)^{2}$

Ans : (C)

Hint : $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

	PCl_5	PCl_3	Cl_2
initially moles taken	1	0	0
moles of equilibrium	$1-\alpha$	α	α

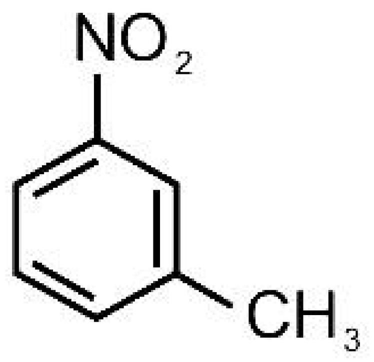
$$K_P = \frac{P_{PCl_3} \cdot P_{Cl_2}}{P_{PCl_5}} \qquad \because P = X_{gas} \cdot P_T$$

$$K_P = \frac{\left(\frac{\alpha}{1+\alpha} \cdot P\right)^2}{\left(\frac{1-\alpha}{1+\alpha} \cdot P\right)} \qquad X_{PCl_3} = X_{Cl_2} = \frac{\alpha}{1+\alpha}$$

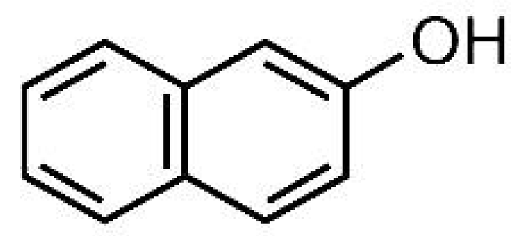
$$K_P = \frac{\alpha^2 P}{1-\alpha^2} \qquad X_{PCl_5} = \frac{1-\alpha}{1+\alpha}$$

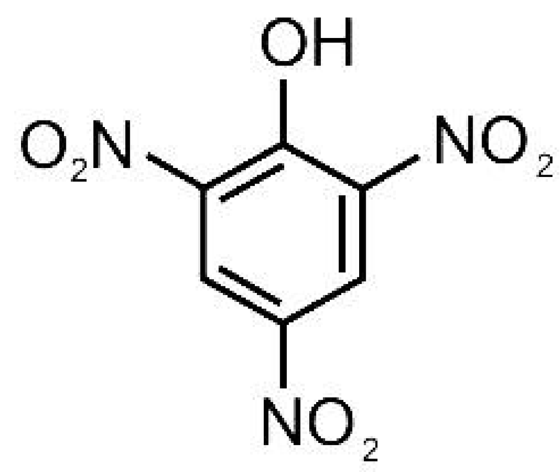
$$\alpha = \left(\frac{K_P}{K_P + P}\right)^{1/2}$$

72. Compound given below will produce effervescence when mixed with aqueous sodium bicarbonate solution

(A) 

(B) CH_3COCH_3

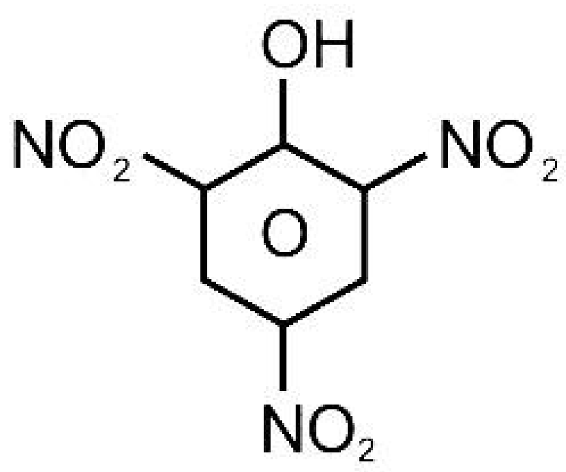
(C) 

(D) 

Ans : (D)

Hint : Acids having pK_a lower than H_2CO_3 are soluble in $NaHCO_3$

$\therefore$ pK_a (Picric acid) < pK_a (H_2CO_3)

$\therefore$  soluble in $NaHCO_3$

(Picric acid)

73. As per the following equation, 0.217g of HgO (molecular mass = 217g mol^{-1}) reacts with excess iodide. On titration of the resulting solution, how many mL of 0.01 M HCl is required to reach the equivalence point?

$$HgO + 4I^- + H_2O \longrightarrow HgI_4^{2-} + 2OH^-$$

(A) 50 mL (B) 200 mL (C) 10 mL (D) 5 mL

Ans : (B)

Hint : $HgO + 4I^- + H_2O \rightarrow HgI_4^{-2} + 2OH^-$

(limiting reagent) (excess reagent)

$2H^+ (HC\ell) + 2OH^- \rightarrow 2H_2O(\ell)$

$$\therefore n_{HgO} = \frac{1}{2}n_{H^+} = \frac{1}{2}n_{HC\ell}$$

$$\frac{0.217}{217} = \frac{1}{2}\times M\times\frac{V(mL)}{1000} = \frac{1}{2}\times 0.01\times\frac{V(mL)}{1000}$$

$\therefore$ V(mL) = 200

74. An egg takes 4.0 minutes to boil at sea level where the boiling point of water is T_1K, where as it takes 8.0 minutes to boil on a mountain top where the boiling point of water is T_2K. The activation energy for the reaction that takes place during the boiling of egg is

(A) $0.693\frac{T_1 - T_2}{T_1T_2}$ (B) $0.693\frac{T_2 - T_1}{T_1T_2}$ (C) $0.693R\frac{T_1T_2}{T_2 - T_1}$ (D) $0.693R\frac{T_1T_2}{T_1 - T_2}$

Ans : (D)

Hint : As per Arrhenius Equation

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$$

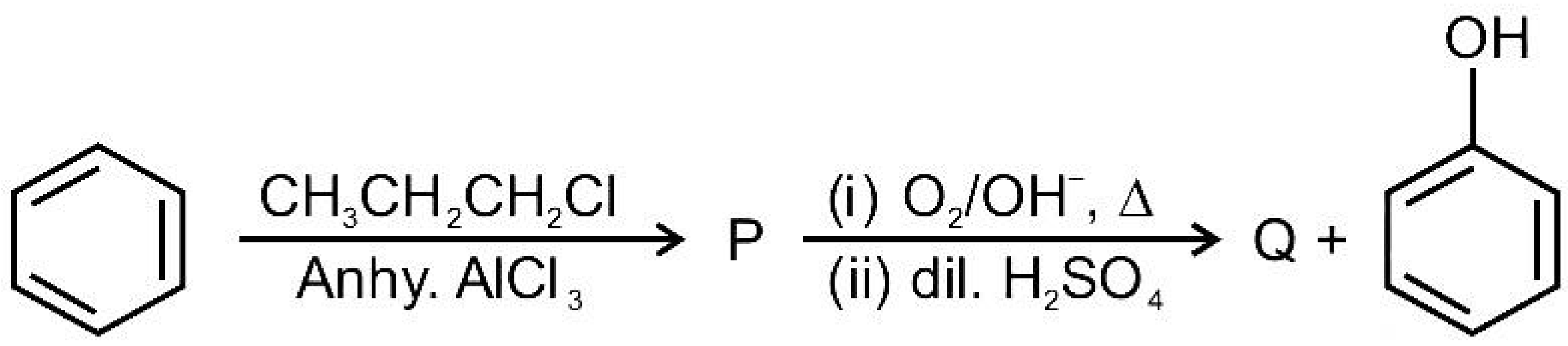
According to question

$$\ln\left(\frac{1/8}{1/4}\right) = \frac{E_a}{R}\left[\frac{T_1 - T_2}{T_1T_2}\right]$$

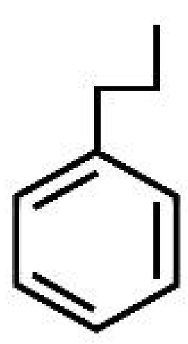
$$\ln 2 = \frac{E_a}{R}\left[\frac{T_1 - T_2}{T_1T_2}\right]$$

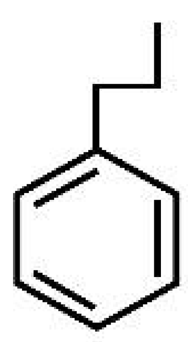
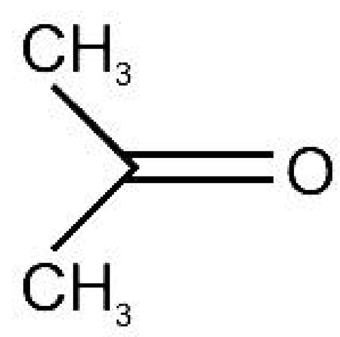
$$0.693 = \frac{E_a}{R}\left[\frac{T_1 - T_2}{T_1T_2}\right]$$

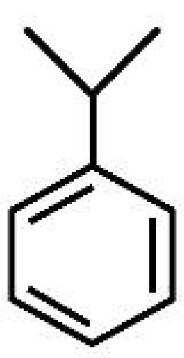
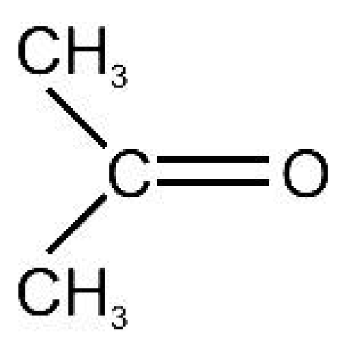
$$\therefore\ E_a = 0.693R\frac{T_1T_2}{T_1 - T_2}$$

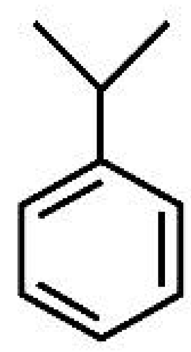
75. 

The major product 'P' and 'Q' in the above reactions are

(A)  and CH_3CH_2CHO

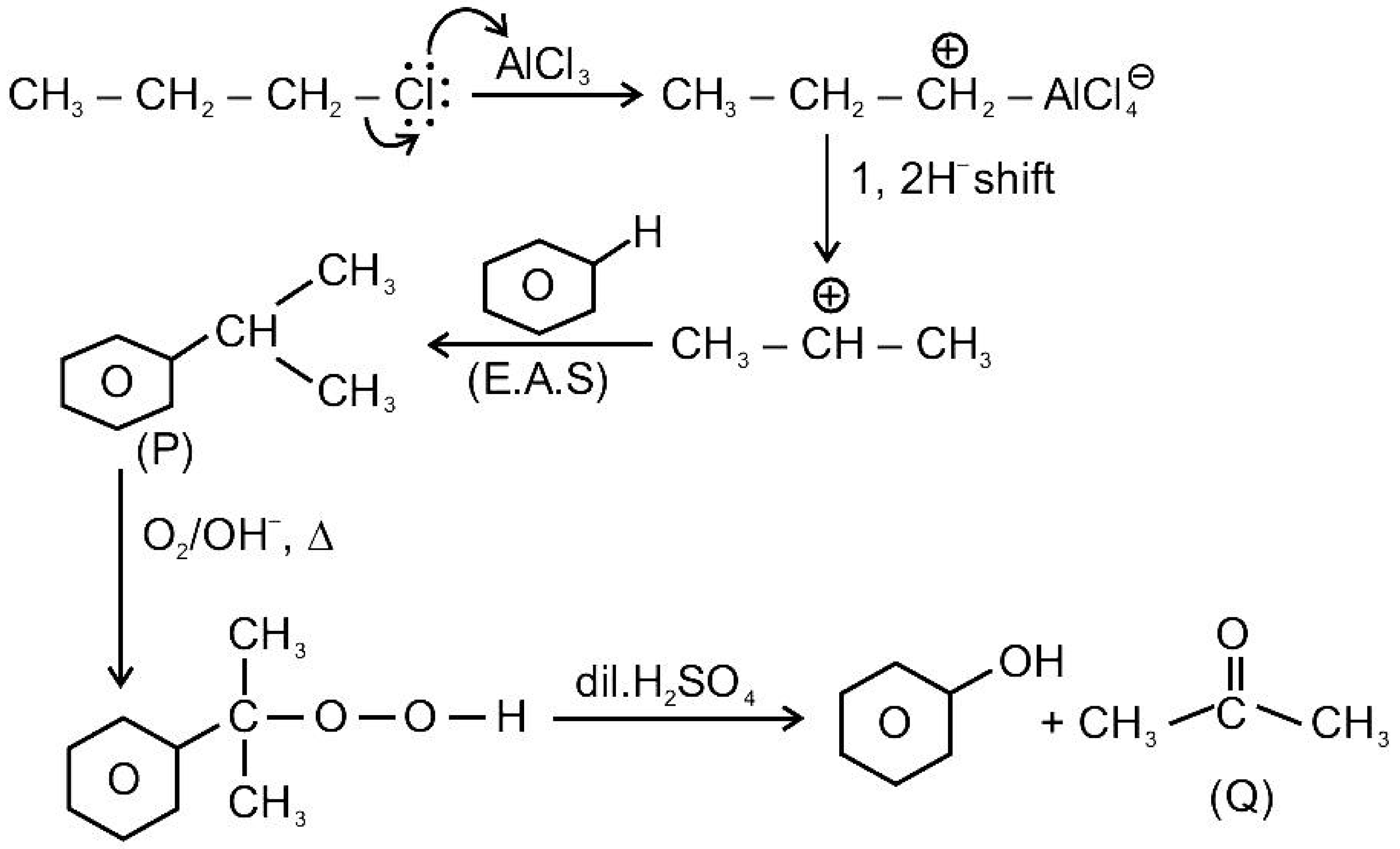
(B)  and 

(C)  and 

(D)  and CH_3CH_2CHO

Ans : (C)

Hint :



Category 3 (Q76 to Q80)

(Carry 2 marks each. One or more options are correct. No negative marks)

76. Which pair of ions among the following can be separated by precipitation method?

(A) Eu(II) and Dy(III) (B) Gd(III) and Dy(III) (C) Eu(II) and Yb(II) (D) Eu(II) and Gd(II)

Ans : (A, D)

Hint : A $\rightarrow$ Eu^{+2} and yb^{2+} have unique property & show similarity with Sr^{+2}. Hence $EuSO_4$ is insoluble like $SrSO_4$ & other lanthanide ions remains unaffected in solution.

D $\rightarrow$ fact

77. Identify 'P' and 'Q' in the following reaction

$$[Cd(NH_3)_4](NO_3)_2 + KCN + H_2O \longrightarrow P + KNO_3 + NH_4OH$$

$$P \xrightarrow{H_2S} Q$$

(A) P = $K_2[Cd(CN)_4]$, Q = CdS

(B) P = CdS, Q = $K_2[Cd(CN)_4]$

(C) P = $Cd(NO_3)_2$, Q = $CdSO_4$

(D) P = $(Cd(OH_2)_4](NO_3)_2$, Q = $[Cd(NO_3)_4](NO_3)_2$

Ans : (A)

Hint : $[Cd(NH_3)_4](NO_3)_2 + KCN + H_2O$

$$\downarrow$$

$K_2[Cd(CN)_4] + KNO_3 + NH_4OH$

(P)

$$\downarrow H_2S$$

CdS + KCN + HCN

(Q)

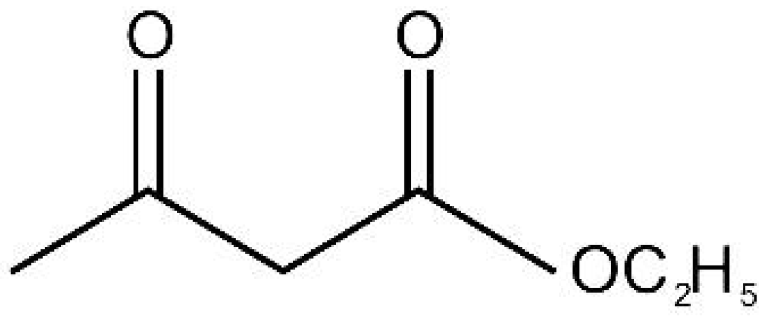
78. X is an extensive property and x is an intensive property of a thermodynamic system. Which of the following statement(s) is (are) correct?

(A) xX is extensive (B) $\frac{x}{X}$ is intensive (C) $\frac{X}{x}$ is extensive (D) $\frac{dX}{dx}$ is intensive

Ans : (A,C)

Hint : Fact

79. Which of the following statement(s) is/are correct about the given compound?



(A) It exhibits tautomerism

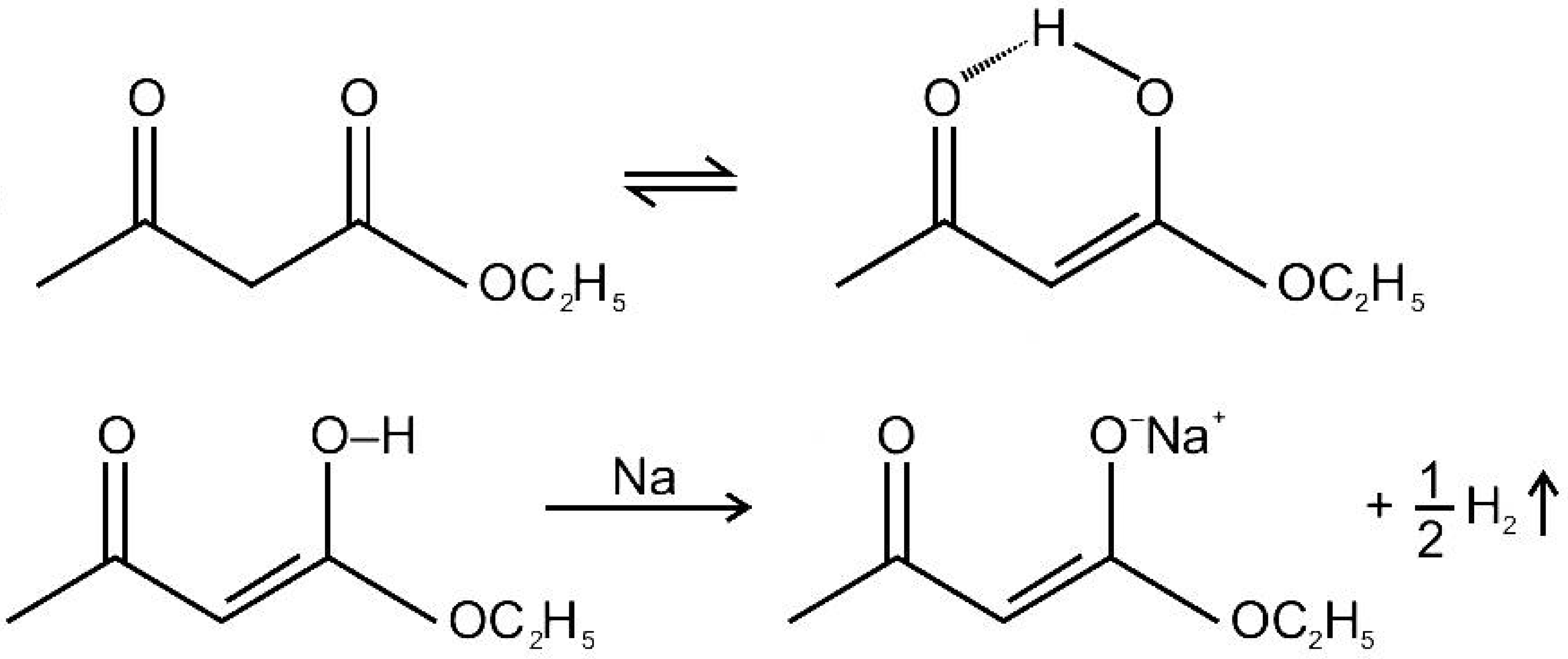
(B) It does not react with metallic sodium

(C) It gives reddish-violet coloration with $FeCl_3$ solution

(D) It gives precipitate with 2,4-dinitrophenyl hydrazine solution

Ans : (A,C,D)

Hint : (A)

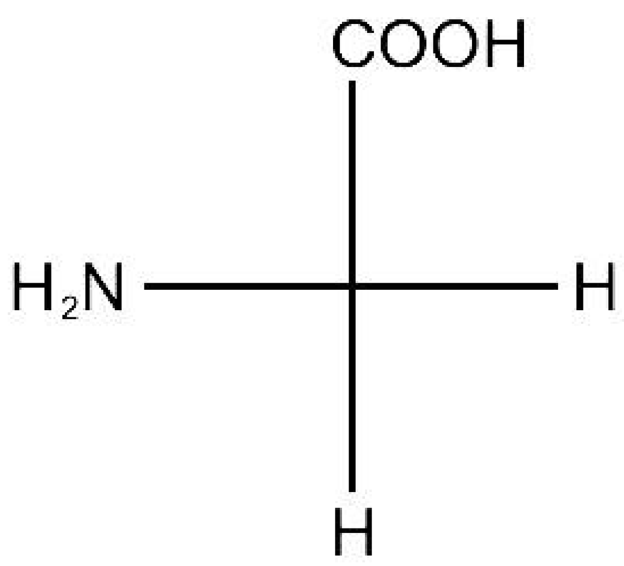


(C) It gives reddish violet coloration with $FeCl_3$ solution due to enol content.

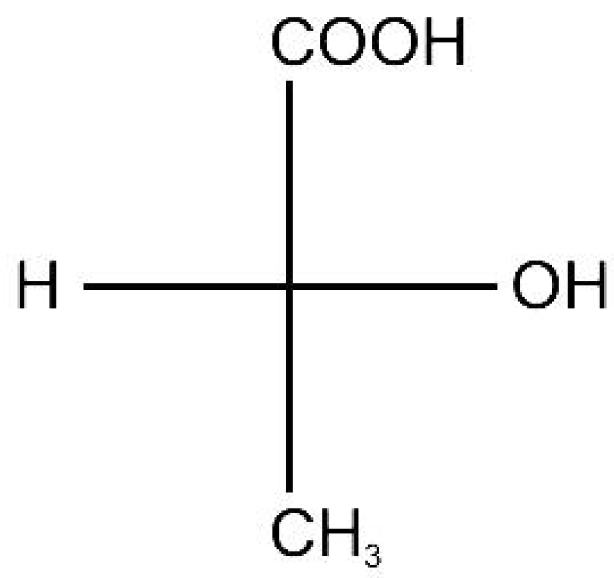
(D) It gives DNP Test due to presence of Keto group.

80. The compound(s) showing optical activity is/are

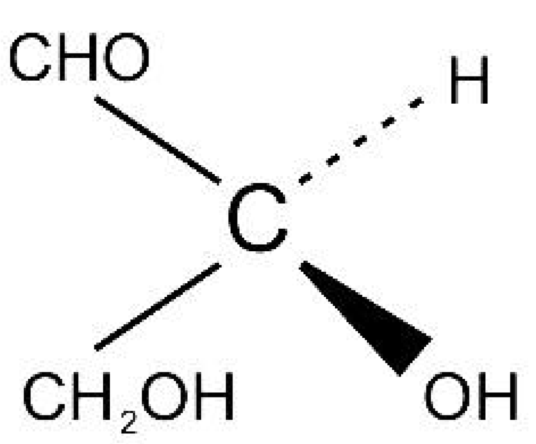
(A)



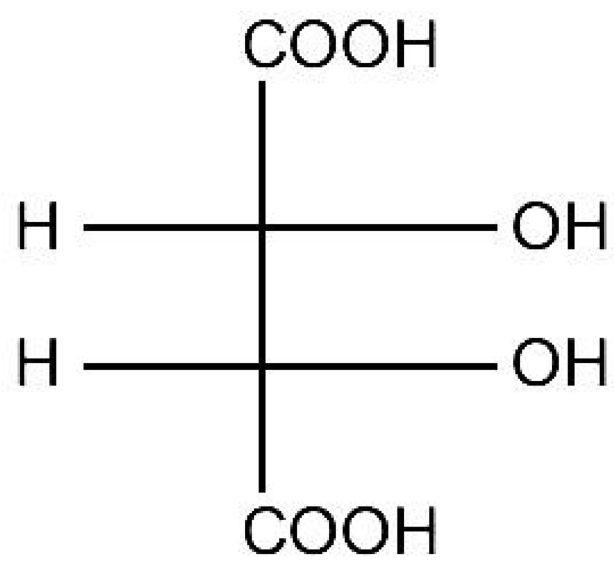
(B)



(C)

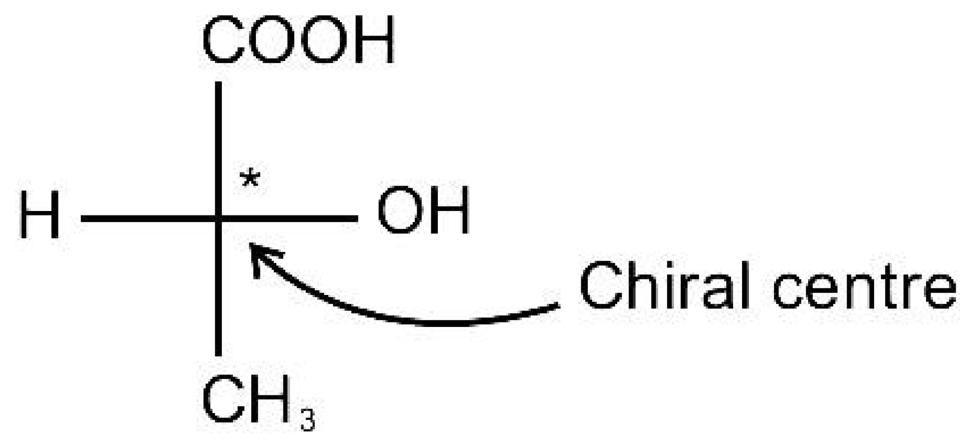


(D)

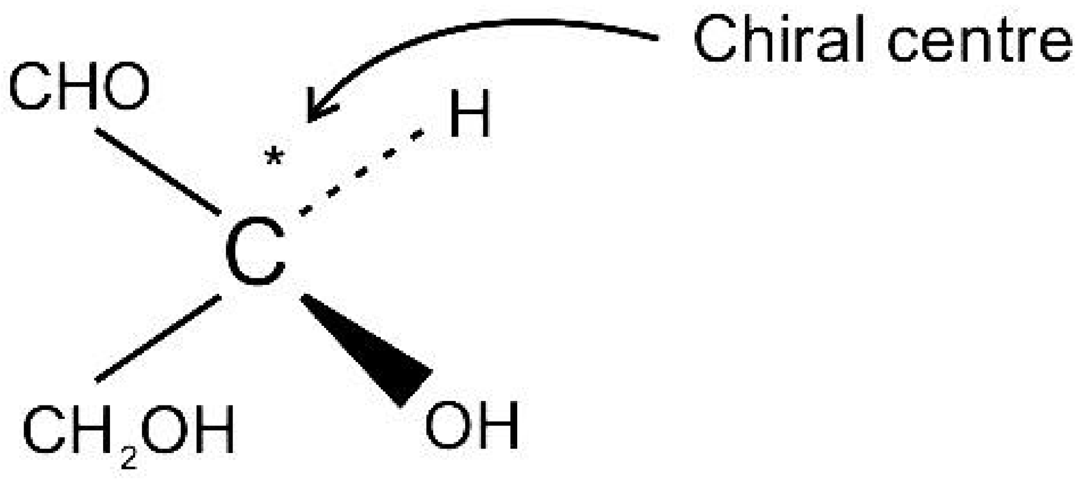


Ans : (B, C)

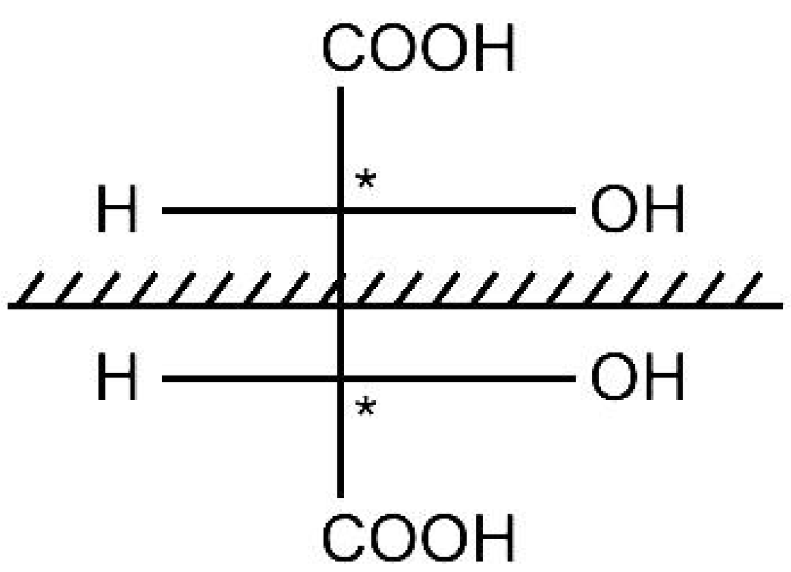
Hint : (A) No chiral centre.

(B) 

(Opticaly Active)

(C) 

(Opticaly Active)

(D) 

Meso form, So Opticaly Inactive

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