WB JEE
Engineering Entrance Exam
Solved Paper 2019

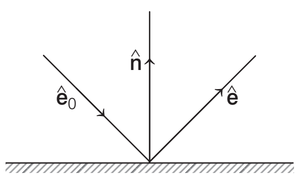
Physics

Category I (Q. Nos. 1 to 30)

Carry 1 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

1. A ray of light is reflected by a plane mirror. $\hat{\mathbf{e}}_0$, $\hat{\mathbf{e}}$ and $\hat{\mathbf{n}}$ be the unit vectors along the incident ray, reflected ray and the normal to the reflecting surface respectively.

Which of the following gives an expression for $\hat{\mathbf{e}}$?



(a) $\hat{e}_0 + 2(\hat{e}_0 \cdot \hat{n})\hat{n}$ (b) $\hat{e}_0 - 2(\hat{e}_0 \cdot \hat{n})\hat{n}$
(c) $\hat{e}_0 - (\hat{e}_0 \cdot \hat{n})\hat{n}$ (d) $\hat{e}_0 + (\hat{e}_0 \cdot \hat{n})\hat{n}$

2. A parent nucleus X undergoes α-decay with a half-life of 75000 yrs. The daughter nucleus Y undergoes β-decay with a half-life of 9 months. In a particular sample, it is found that the rate of emission of β-particles is nearly constant (over several months) at 10^7/h. What will be the number of α-particles emitted in an hour?

(a) 10^2 (b) 10^7 (c) 10^{12} (d) 10^{14}

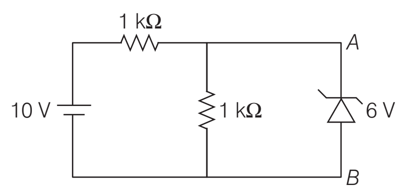
3. A proton and an electron initially at rest are accelerated by the same potential difference. Assuming that a proton is 2000 times heavier than an electron, what will be the relation between the de Broglie wavelength of the proton (λ_p) and that of electron (λ_e)?

(a) $\lambda_p = 2000\lambda_e$ (b) $\lambda_p = \dfrac{\lambda_e}{2000}$
(c) $\lambda_p = 20\sqrt{5}\,\lambda_e$ (d) $\lambda_p = \dfrac{\lambda_e}{20\sqrt{5}}$

4. To which of the following the angular velocity of the electron in the n-th Bohr orbit is proportional?

(a) n^2 (b) $\dfrac{1}{n^2}$ (c) $\dfrac{1}{n^{3/2}}$ (d) $\dfrac{1}{n^3}$

5. In the circuit shown, what will be the current through the 6V zener?



(a) 6 mA, from A to B (b) 2 mA, from A to B
(c) 2 mA, from B to A (d) Zero

6. Each of the two inputs A and B can assume values either 0 or 1. Then which of the following will be equal to $\overline{A} \cdot \overline{B}$?

(a) $A + B$ (b) $\overline{A + B}$
(c) $\overline{A \cdot B}$ (d) $\overline{A} + \overline{B}$

7. The correct dimensional formula for impulse is given by

(a) ML^2T^{-2} (b) MLT^{-1} (c) ML^2T^{-1} (d) MLT^{-2}

8. The density of the material of a cube can be estimated by measuring its mass and the length of one of its sides. If the maximum error in the measurement of mass and length are 0.3% and 0.2% respectively, the maximum error in the estimation of the density of the cube is approximately.

(a) 1.1% (b) 0.5% (c) 0.9% (d) 0.7%

9. Two weights of the mass m_1 and $m_2 (> m_1)$ are joined by an inextensible string of negligible mass passing over a fixed frictionless pulley. The magnitude of the acceleration of the loads is

(a) g (b) $\frac{m_2 - m_1}{m_2} g$

(c) $\frac{m_1}{m_2 + m_1} g$ (d) $\frac{m_2 - m_1}{m_2 + m_1} g$

10. A body starts from rest, under the action of an engine working at a constant power and moves along a straight line. The displacement s is given as a function of time (t) as

(a) $s = at + bt^2$, a and b are constants

(b) $s = bt^2$, b is a constant

(c) $s = at^{3/2}$, a is a constant

(d) $s = at$, a is a constant

11. Two particles are simultaneously projected in the horizontal direction from a point P at a certain height. The initial velocities of the particles are oppositely directed to each other and have magnitude v each. The separation between the particles at a time when their position vectors (drawn from the point P) are mutually perpendicular, is

(a) $\frac{v^2}{2g}$ (b) $\frac{v^2}{g}$

(c) $\frac{4v^2}{g}$ (d) $\frac{2v^2}{g}$

12. Assume that the earth moves around the sun in a circular orbit of radius R and there exists a planet which also move around the sun in a circular orbit with an angular speed twice as large as that of the earth. The radius of the orbit of the planet is

(a) $2^{-2/3}R$ (b) $2^{2/3}R$ (c) $2^{-1/3}R$ (d) $\frac{R}{\sqrt{2}}$

13. A compressive force is applied to a uniform rod of rectangular cross-section so that its length decreases by 1%. If the Poisson's ratio for the material of the rod be 0.2, which of the following statements is correct?

"The volume approximately ...."

(a) decreases by 1% (b) decreases by 0.8%

(c) decreases by 0.6% (d) increases by 0.2%

14. A small spherical body of radius r and density ρ moves with the terminal velocity v in a fluid of coefficient of viscosity η and density σ. What will be the net force on the body?

(a) $\frac{4\pi}{3} r^3(\rho - \sigma) g$ (b) $6\pi\eta rv$

(c) Zero (d) Infinity

15. Two black bodies A and B have equal surface areas and are maintained at temperatures 27°C and 177°C respectively. What will be the ratio of the thermal energy radiated per second by A to that by B?

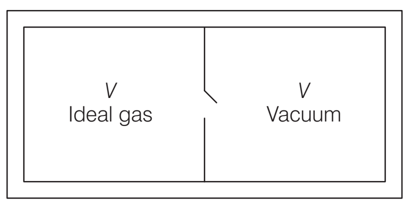
(a) 4 : 9 (b) 2 : 3 (c) 16 : 81 (d) 27 : 177

16. What will be the molar specific heat at constant volume of an ideal gas consisting of rigid diatomic molecules?

(a) $\frac{3}{2}R$ (b) $\frac{5}{2}R$

(c) R (d) $3R$

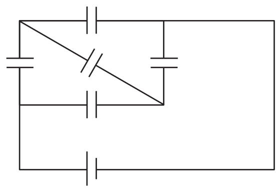
17. Consider the given diagram. An ideal gas is contained in a chamber (left) of volume V and is at an absolute temperature T. It is allowed to rush freely into the right chamber of volume V which is initially vacuum. The whole system is thermally isolated. What will be the final temperature of the equilibrium has been attained?



(a) T (b) $\frac{T}{2}$

(c) $2T$ (d) $\frac{T}{4}$

18. Five identical capacitors, of capacitance $20\mu F$ each, are connected to a battery of 150 V, in a combination as shown in the diagram. What is the total amount of charge stored?



(a) $15 \times 10^{-3}C$
(b) $12 \times 10^{-3}C$
(c) $10 \times 10^{-3}C$
(d) $3 \times 10^{-3}C$

19. Eleven equal point charges, all of them having a charge $+Q$, are placed at all the hour positions of a circular clock of radius r, except at the 10 h position. What is the electric field strength at the centre of the clock?

(a) $\frac{Q}{4\pi\varepsilon_0 r^2}$ from the centre towards the mark 10
(b) $\frac{Q}{4\pi\varepsilon_0 r^2}$ from the mark 10 towards the centre
(c) $\frac{Q}{4\pi\varepsilon_0 r^2}$ from the centre towards the mark 6
(d) Zero.

20. A negative charge is placed at the midpoint between two fixed equal positive charges, separated by a distance $2d$. If the negative charge is given a small displacement $x (x << d)$ perpendicular to the line joining the positive charges, how the force (F) developed on it will approximately depend on x?

(a) $F \propto x$ (b) $F \propto \frac{1}{x}$
(c) $F \propto x^2$ (d) $F \propto \frac{1}{x^2}$

21. To which of the following quantities, the radius of the circular path of a charged particle moving at right angles to a uniform magnetic field is directly proportional?

(a) energy of the particle
(b) magnetic field
(c) charge of the particle
(d) momentum of the particle

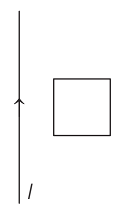
22. An electric current 'I' enters and leaves a uniform circular wire of radius r through diametrically opposite points. A particle carrying a charge q moves along the axis of the circular wire with speed v. What is the magnetic force experienced by the particle when it passes through the centre of the circle?

(a) $qv\frac{\mu_0 i}{a}$ (b) $qv\frac{\mu_0 i}{2a}$
(c) $qv\frac{\mu_0 i}{2\pi a}$ (d) Zero

23. A current 'I' is flowing along an infinite, straight wire, in the positive Z-direction and the same current is flowing along a similar parallel wire 5 m apart, in the negative Z-direction. A point P is at a perpendicular distance 3m from the first wire and 4m from the second. What will be magnitude of the magnetic field **B** of P?

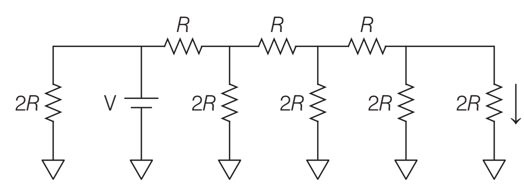
(a) $\frac{5}{12}(\mu_0 I)$ (b) $\frac{7}{24}(\mu_0 I)$
(c) $\frac{5}{24}(\mu_0 I)$ (d) $\frac{25}{288}(\mu_0 I)$

24. A square conducting loop is placed near an infinitely long current carrying wire with one edge parallel to the wire as shown in the figure. If the current in the straight wire is suddenly halved, which of the following statements will be true?



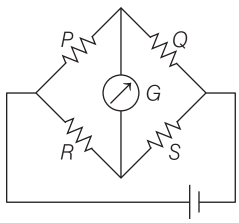
"The loop will ....".
(a) stay stationary
(b) move towards the wire
(c) move away from the wire
(d) move parallel to the wire

25. What is the current I shown in the given circuit?



(a) $\frac{V}{2R}$ (b) $\frac{V}{R}$
(c) $\frac{V}{16R}$ (d) $\frac{V}{8R}$

26. When the value of R in the balanced Wheatstone bridge, shown in the figure, is increased from 5Ω to 7Ω, the value of S has to be increased by 3Ω in order to maintain the balance. What is the initial values of S?



(a) 2.5 Ω (b) 3 Ω
(c) 5 Ω (d) 7.5 Ω

27. When a 60 mH inductor and a resistor are connected in series with an AC voltage source, the voltage leads the current by 60°. If the inductor is replaced by a 0.5 μF capacitor, the voltage lags behind the current by 30°. What is the frequency of the AC supply?

(a) $\frac{1}{2\pi}\times 10^4$ Hz (b) $\frac{1}{\pi}\times 10^4$ Hz
(c) $\frac{3}{2\pi}\times 10^4$ Hz (d) $\frac{1}{2\pi}\times 10^8$ Hz

28. A point object is placed on the axis of a thin convex lens of focal length 0.05 m at a distance of 0.2 m from the lens and its image is formed on the axis. If the object is now made to oscillate along the axis with a small amplitude of A cm, then what is the amplitude of oscillation of the image?

$$\left[\text{you may assume, } \frac{1}{1+x} \approx 1 - x, \text{ where } x << 1\right]$$

(a) $\frac{4A}{9}\times 10^{-2}$ m (b) $\frac{5A}{9}\times 10^{-2}$ m
(c) $\frac{A}{3}\times 10^{-2}$ m (d) $\frac{A}{9}\times 10^{-2}$ m

29. In Young's experiment for the interference of light, the separation between the slits is d and the distance of the screen from the slits is D. If D is increased by 0.5% and d is decreased by 0.3% then for the light of a given wavelength, which one of the following is true?

"The fringe width ....

(a) increases by 0.8% (b) decreases by 0.8%
(c) increases by 0.2% (d) decreases by 0.2%

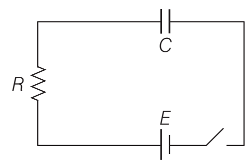
30. When the frequency of the light used is changed from $4\times 10^{14}\,s^{-1}$ to $5\times 10^{14}\,s^{-1}$, the angular width of the principal (central) maximum in a single slit Fraunhoffer diffraction pattern changes by 0.6 radian. What is the width of the slit (assume that the experiment is performed in vacuum)?

(a) 1.5×10^{-7} m (b) 3×10^{-7} m
(c) 5×10^{-7} m (d) 6×10^{-7} m

Category II (Q. Nos. 31 to 35)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

31. A capacitor of capacitance C is connected in series with a resistance R and DC source of emf E through a key. The capacitor starts charging when the key is closed. By the time the capacitor has been fully charged, what amount of energy is dissipated in the resistance R?



(a) $\frac{1}{2}CE^2$ (b) 0 (c) CE^2 (d) $\frac{E^2}{R}$

32. A horizontal fire hose with a nozzle of cross-sectional area $\frac{5}{\sqrt{21}}\times 10^{-3}$ m^2 delivers a cubic metre of water in 10s. What will be the maximum possible increase in the temperature of water while it hits a rigid wall (neglecting the effect of gravity)?

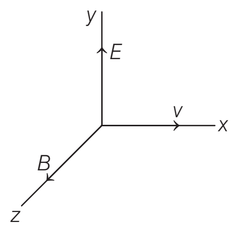
(a) 1°C (b) 0.1°C (c) 10°C (d) 0.01°C

33. Two identical blocks of ice move in opposite directions with equal speed and collide with each other. What will be the minimum speed required to make both the blocks melt completely, if the initial temperatures of the blocks were – 8°C each?

(Specific heat of ice is 2100 $Jkg^{-1}K^{-1}$ and latent heat of fusion of ice is 3.36×10^5 Jkg^{-1})

(a) 840 ms^{-1} (b) 420 ms^{-1}
(c) 8.4 ms^{-1} (d) 84 ms^{-1}

34. A particle with charge q moves with a velocity v in a direction perpendicular to the directions of uniform electric and magnetic fields, E and B respectively, which are mutually perpendicular to each other. Which one of the following gives the condition for which the particle moves undeflected in its original trajectory?



(a) $v = \frac{E}{B}$ (b) $v = \frac{B}{E}$ (c) $v = \sqrt{\frac{E}{B}}$ (d) $v = q\frac{B}{E}$

35. A parallel plate capacitor in series with a resistance of 100 Ω, an inductor of 20 mH and an AC voltage source of variable frequency shows resonance at a frequency of $\frac{1250}{\pi}$ Hz.

If this capacitor is charged by a DC voltage source to a voltage 25 V, what amount of charge will be stored in each plate of the capacitor?

(a) 0.2 μC (b) 2 mC (c) 0.2 mC (d) 0.2 C

Category III (Q. Nos. 36 to 40)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero marks will be awarded.

36. Electrons are emitted with kinetic energy T from a metal plate by an irradiation of light of intensity J and frequency ν. Then, which of the following will be true?

(a) $T \propto J$
(b) T linearly increasing with ν
(c) $T \propto$ time of irradiation
(d) Number of electrons emitted $\propto J$

37. The initial pressure and volume of a given mass of an ideal gas $\left(\text{with } \frac{C_p}{C_V} = \gamma\right)$, taken in a cylinder fitted with a piston, are p_0 and V_0 respectively. At this stage the gas has the same temperature as that of the surrounding medium which is T_0. It is adiabatically compressed to a volume equal to $\frac{V_0}{2}$.

Subsequently the gas is allowed to come to thermal equilibrium with the surroundings. What is the heat released to the surrounding?

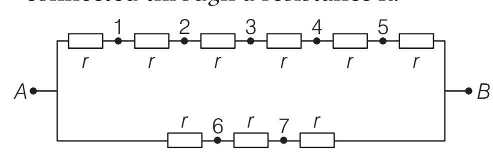
(a) 0 (b) $(2^{\gamma-1} - 1)\frac{p_0V_0}{\gamma - 1}$

(c) $\gamma p_0V_0 \ln 2$ (d) $\frac{p_0V_0}{2(\gamma - 1)}$

38. A projectile thrown with an initial velocity of 10 ms^{-1} at an angle α with the horizontal, has a range of 5 m. Taking $g = 10$ ms^{-2} and neglecting air resistance, what will be the estimated value of α?

(a) 15° (b) 30° (c) 45° (d) 75°

39. In the circuit shown in the figure all the resistance are identical and each has the value r Ω. The equivalent resistance of the combination between the points A and B will remain unchanged even when the following pairs of points marked in the figure are connected through a resistance R.



(a) 2 and 6 (b) 3 and 6
(c) 4 and 7 (d) 4 and 6

40. A metallic loop is placed in a uniform magnetic field **B** with the plane of the loop perpendicular to **B**. Under which condition(s) given an emf will be induced in the loop?

"If the loop is ...."

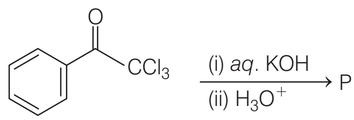
(a) moved along the direction of B
(b) squeezed to a smaller area
(c) rotated about its axis
(d) rotated about one of its diameters

Chemistry

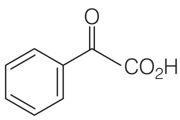
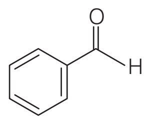
Category I (Q. Nos. 41 to 70)

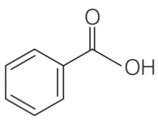
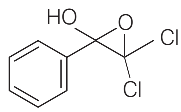
Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

41. One of the products of the following reaction is P.

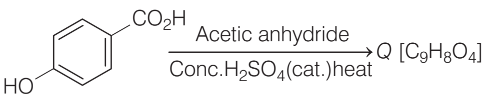


Structure of P is

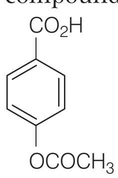
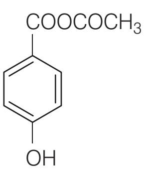
(a)  (b) 

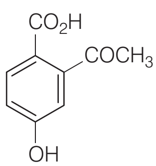
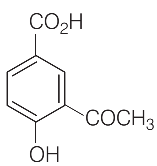
(c)  (d) 

42. For the reaction below, the product is Q.



The compound Q is

(a)  (b) 

(c)  (d) 

43. Cyclopentanol on reaction with NaH followed by CS_2 and CH_3I produces a/an

(a) ketone (b) alkene
(c) ether (d) xanthate

44. The compound, which evolves carbon dioxide on treatment with aqueous solution of sodium bicarbonate at 25°C, is

(a) C_6H_5OH (b) CH_3COCl
(c) CH_3CONH_2 (d) $CH_3COOC_2H_5$

45. The indicated atom is **not** a nucleophilic site in

(a) $\underset{\uparrow}{B}H_4^-$ (b) $\underset{\uparrow}{C}H_3MgI$
(c) $CH_3\underset{\uparrow}{O}H$ (d) $CH_3\underset{\uparrow}{N}H_2$

46. The charge carried by 1 millimole of M^{n+} ions is 193 coulombs. The value of n is

(a) 1 (b) 2 (c) 3 (d) 4

47. Which of the following mixtures will have the lowest pH at 298 K?

(a) 10 mL 0.05 N CH_3COOH + 5 mL 0.1 N NH_4OH
(b) 5 mL 0.2 N NH_4Cl + 5 mL 0.2 N NH_4OH
(c) 5 mL 0.1N CH_3COOH +10 mL 0.05 N CH_3COONa
(d) 5 mL 0.1 N CH_3COOH + 5 mL 0.1 N NaOH

48. Consider the following two first order reactions occurring at 298 K with same initial concentration of A :

(1) $A \rightarrow B$; rate constant, $k = 0.693\ \text{min}^{-1}$

(2) $A \rightarrow C$; half-life, $t_{1/2} = 0.693$ min

Choose the correct option.

(a) Reaction (1) is faster than reaction (2).
(b) Reaction (1) is slower than reaction (2).
(c) Both reactions proceed at the same rate.
(d) Since two different products are formed, rates cannot be compared.

49. For the equilibrium, $H_2O(l) \rightleftharpoons H_2O(v)$, which of the following is correct?

(a) $\Delta G = 0, \Delta H < 0, \Delta S < 0$
(b) $\Delta G < 0, \Delta H > 0, \Delta S > 0$
(c) $\Delta G > 0, \Delta H = 0, \Delta S > 0$
(d) $\Delta G = 0, \Delta H > 0, \Delta S > 0$

50. For a van der Waals' gas, the term $\left(\frac{ab}{V^2}\right)$ represents some

(a) pressure (b) energy
(c) critical density (d) molar mass

51. In the equilibrium, $H_2 + I_2 \rightleftharpoons 2HI$, if at a given temperature the concentrations of the reactants are increased, the value of the equilibrium constant, K_C, will

(a) increase
(b) decrease
(c) remain the same
(d) cannot be predicted with certainty

52. If electrolysis of aqueous $CuSO_4$ solution is carried out using Cu-electrodes, the reaction taking place at the anode is

(a) $H^+ + e^- \rightarrow H$
(b) $Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$
(c) $SO_4^{2-}(aq) - 2e^- \rightarrow SO_4$
(d) $Cu(s) - 2e^- \rightarrow Cu^{2+}(aq)$

53. Which one of the following electronic arrangements is absurd?

(a) $n = 3, l = 1, m = -1$ (b) $n = 3, l = 0, m = 0$
(c) $n = 2, l = 0, m = -1$ (d) $n = 2, l = 1, m = 0$

54. The quantity $h\nu / K_B$ corresponds to

(a) wavelength (b) velocity
(c) temperature (d) angular momentum

55. In the crystalline solid $MSO_4 \cdot nH_2O$ of molar mass 250 g mol^{-1}, the percentage of anhydrous salt is 64 by weight.
The value of n is

(a) 2 (b) 3 (c) 5 (d) 7

56. At S.T.P. the volume of 7.5 g of a gas is 5.6 L. The gas is

(a) NO (b) N_2O
(c) CO (d) CO_2

57. The half-life period of $_{53}I^{125}$ is 60 days. The radioactivity after 180 days will be

(a) 25% (b) 12.5%
(c) 33.3% (d) 3.0%

58. Consider, the radioactive disintegration

$$_{82}A^{210} \rightarrow B \rightarrow C \rightarrow {}_{82}D^{206}$$

The sequence of emission can be

(a) β, β, β (b) α, α, β
(c) β, β, γ (d) β, β, α

59. The second ionisation energy of the following elements follows the order

(a) Zn > Cd < Hg (b) Zn > Cd > Hg
(c) Cd > Hg < Zn (d) Zn < Cd < Hg

60. The melting points of (i) $BeCl_2$ (ii) $CaCl_2$ and (iii) $HgCl_2$ follows the order

(a) i < ii < iii (b) iii < i < ii
(c) i < iii < ii (d) ii < i < iii

61. Which of these species will have non-zero magnetic moment?

(a) Na^+ (b) Mg (c) F^- (d) Ar^+

62. The first electron affinity of C, N and O will be of the order

(a) C < N < O (b) N < C < O
(c) C < O < N (d) O < N < C

63. The H—N—H angle in ammonia is 107.6°, while the H—P—H angle in phosphine is 93.5°. Relative to phosphine, the p-character of the lone-pair on ammonia is expected to be

(a) Less (b) More
(c) Same (d) Cannot be predicted

64. The reactive species in chlorine bleach is

(a) Cl_2O (b) OCl^-
(c) ClO_2 (d) HCl

65. The conductivity measurement of a coordination compound of cobalt (III) shows that it dissociates into 3 ions in solution. The compound is

(a) hexaamminecobalt(III) chloride
(b) pentaamminesulphatocobalt(III) chloride
(c) pentaamminechloridocobalt(III) sulphate
(d) pentaamminechloridocobalt(III) chloride

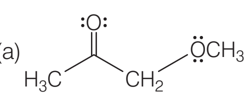
66. In the Bayer's process, the leaching of alumina is done by using

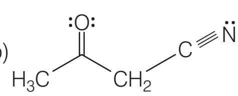
(a) Na_2CO_3 (b) NaOH (c) SiO_2 (d) CaO

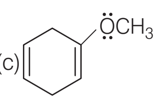
67. Which atomic species cannot be used as a nuclear fuel?

(a) $^{233}_{92}U$ (b) $^{235}_{92}U$ (c) $^{239}_{94}Pu$ (d) $^{238}_{92}U$

68. The molecule/molecules that has/have delocalised lone pair(s) of electrons is/are

(a) 

(b) 

(c) 

(d) $CH_3CH{=}CHCH_2\ddot{N}HCH_3$

(a) I, II and III (b) I, II and IV
(c) I and III (d) only III

69. The conformations of *n*-butane, commonly known as eclipsed, gauche and anti-conformations can be interconverted by

(a) rotation around C—H bond of a methyl group
(b) rotation around C—H bond of a methylene group
(c) rotation around C1-C2 linkage
(d) rotation around C2-C3 linkage

70. The correct order of the addition reaction rates of halogen acids with ethylene is

(a) hydrogen chloride > hydrogen bromide > hydrogen iodide
(b) hydrogen iodide > hydrogen bromide > hydrogen chloride
(c) hydrogen bromide > hydrogen chloride > hydrogen iodide
(d) hydrogen iodide > hydrogen chloride > hydrogen bromide

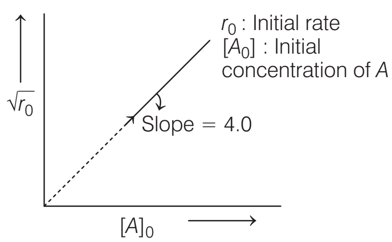
Category II (Q. Nos. 71 to 75)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

71. The total number of isomeric linear dipeptides which can be synthesised from racemic alanine is

(a) 1 (b) 2 (c) 3 (d) 4

72. The kinetic study of a reaction like $vA \rightarrow P$ at 300 K provides the following curve, where concentration is taken in mol dm^{-3} and time in min.



Identify the correct order (n) and rate constant (k)

(a) $n = 0$, $k = 4.0$ mol dm^{-3} min^{-1}
(b) $n = 1/2$, $k = 2.0$ $mol^{1/2}$ $dm^{-3/2}$ min^{-1}
(c) $n = 1$, $k = 8.0$ min^{-1}
(d) $n = 2$, $k = 16.0$ dm^3 mol^{-1} min^{-1}

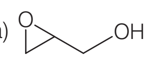
73. At constant pressure, the heat of formation of a compound is not dependent on temperature, when

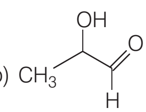
(a) $\Delta C_p = 0$ (b) $\Delta C_v = 0$
(c) $\Delta C_p > 0$ (d) $\Delta C_p < 0$

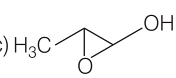
74. A copper coin was electroplated with Zn and then heated at high temperature until there is a change in colour. What will be the resulting colour?

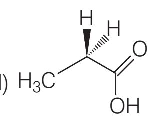
(a) white (b) black
(c) silver (d) golden

75. Oxidation of allyl alcohol with a peracid gives a compound of molecular formula $C_3H_6O_2$, which contains an asymmetric carbon atom. The structure of the compound is

(a) 

(b) 

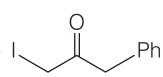
(c) 

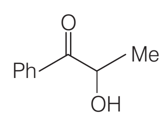
(d) 

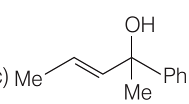
Category III (Q.Nos. 76 to 80)

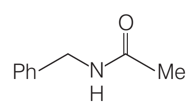
Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero mark will be awarded.

76. Haloform reaction with I_2 and KOH will be responded by

(a) 

(b) 

(c) 

(d) 

77. Identify the correct statement(s):

(a) The oxidation number of Cr in CrO_5 is + 6.

(b) $\Delta H > \Delta U$ for the reaction $N_2O_4(g) \rightarrow 2NO_2(g)$, provided both gases behave ideally.

(c) pH of 0.1 N H_2SO_4 is less than that of 0.1 N HCl at 25°C.

(d) $\left(\frac{RT}{F}\right) = 0.0591$ volt at 25°C.

78. Compounds with spin-only magnetic moment equivalent to five unpaired electrons are

(a) $K_4[Mn(CN)_6]$ (b) $[Fe(H_2O)_6]Cl_3$

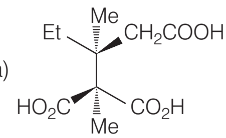
(c) $K_3[FeF_6]$ (d) $K_4[MnF_6]$

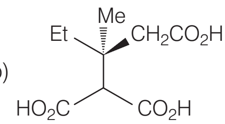
79. Which of the following chemicals may be used to identify three unlabelled beakers containing conc.NaOH, conc.H_2SO_4 and water?

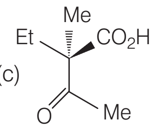
(a) NH_4NO_3 (b) NaCl

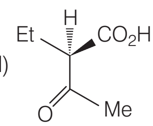
(c) $(NH_4)_2CO_3$ (d) HCOONa

80. The compound(s), capable of producing achiral compound on heating at 100°C is/are

(a)


(b)


(c)


(d)


Mathematics

Category I (Q. Nos. 1 to 50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

1. $\lim_{x \to 0^+} (x^n \ln x),\ n > 0$

(a) does not exist (b) exists and is zero

(c) exists and is 1 (d) exists and is e^{-1}

2. If $\int \cos x \log\left(\tan\frac{x}{2}\right) dx$

$= \sin x \log\left(\tan\frac{x}{2}\right) + f(x)$, then $f(x)$ is equal to

(assuming c is a arbitrary real constant)

(a) c (b) $c - x$ (c) $c + x$ (d) $2x + c$

3. $y = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\} dx$ is an equation of a family of

(a) straight lines (b) circles

(c) ellipses (d) parabolas

4. The value of the integration

$$\int_{-\pi/4}^{\pi/4} \left(\lambda |\sin x| + \frac{\mu \sin x}{1 + \cos x} + \gamma\right) dx$$

(a) is independent of λ only

(b) is independent of μ only

(c) is independent of γ only

(d) depends on λ, μ and γ

5. The value of $\lim_{x \to 0} \frac{1}{x}\left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt\right]$ is equal to

(a) $e^{\sin^2 y}$ (b) $e^{2\sin y}$ (c) $e^{|\sin y|}$ (d) $e^{\operatorname{cosec}^2 y}$

6. If $\int 2^{2^x} \cdot 2^x\, dx = A \cdot 2^{2^x} + C$, then A is equal to

(a) $\frac{1}{\log 2}$ (b) log2 (c) $(\log 2)^2$ (d) $\frac{1}{(\log 2)^2}$

7. The value of the integral $\int_{-1}^{1} \left\{\frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} + \frac{1}{e^{|x|}}\right\} dx$ is equal to

(a) 0 (b) $1 - e^{-1}$ (c) $2e^{-1}$ (d) $2(1 - e^{-1})$

8. $\lim_{n\to\infty}\frac{3}{n}\left\{1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+\ldots+\sqrt{\frac{n}{n+3(n-1)}}\right\}$

(a) does not exist (b) is 1
(c) is 2 (d) is 3

9. The general solution of the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{x/y}\,dy=0$ is (C is an arbitrary constant)

(a) $x-ye^{\frac{x}{y}}=C$ (b) $y-xe^{\frac{x}{y}}=C$
(c) $x+ye^{\frac{x}{y}}=C$ (d) $y+xe^{\frac{x}{y}}=C$

10. General solution of $(x+y)^2\frac{dy}{dx}=a^2$, $a\neq 0$ is (C is an arbitrary constant)

(a) $\frac{x}{a}=\tan\frac{y}{a}+C$ (b) $\tan xy=C$
(c) $\tan(x+y)=C$ (d) $\tan\frac{y+C}{a}=\frac{x+y}{a}$

11. Let $P(4,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at P intersects the X-axis at $(16, 0)$, then the eccentricity of the hyperbola is

(a) $\frac{\sqrt{5}}{2}$ (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{3}$

12. If the radius of a spherical balloon increases by 0.1%, then its volume increases approximately by

(a) 0.2% (b) 0.3% (c) 0.4% (d) 0.05%

13. The three sides of a right angled triangle are in GP (geometric progression). If the two acute angles be α and β, then $\tan\alpha$ and $\tan\beta$ are

(a) $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$ (b) $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$
(c) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

14. If $\log_2^6+\frac{1}{2x}=\log_2\left(2^{\frac{1}{x}}+8\right)$, then the values of x are

(a) $\frac{1}{4},\frac{1}{3}$ (b) $\frac{1}{4},\frac{1}{2}$ (c) $-\frac{1}{4},\frac{1}{2}$ (d) $\frac{1}{3},-\frac{1}{2}$

15. Let z be a complex number such that the principal value of argument, $\arg z>0$. Then, $\arg z-\arg(-z)$ is

(a) $\frac{\pi}{2}$ (b) $\pm\pi$ (c) π (d) $-\pi$

16. The general value of the real angle θ, which satisfies the equation, $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)\ldots(\cos n\theta+i\sin n\theta)=1$ is given by, (assuming k is an integer)

(a) $\frac{2k\pi}{n+2}$ (b) $\frac{4k\pi}{n(n+1)}$
(c) $\frac{4k\pi}{n+1}$ (d) $\frac{6k\pi}{n(n+1)}$

17. Let a, b, c be real numbers such that $a+b+c<0$ and the quadratic equation $ax^2+bx+c=0$ has imaginary roots. Then,

(a) $a>0, c>0$ (b) $a>0, c<0$
(c) $a<0, c>0$ (d) $a<0, c<0$

18. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B, each containing 6 questions and he/she is not permitted to attempt more than 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions?

(a) 850 (b) 800 (c) 750 (d) 700

19. There are 7 greeting cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is,

(a) ${}^7C_3$ (b) $2.{}^7C_3$ (c) $3!^4C_4$ (d) $3!^7C_3{}^4C_3$

20. $7^{2n}+16n-1\ (n\in N)$ is divisible by

(a) 65 (b) 63 (c) 61 (d) 64

21. The number of irrational terms in the expansion of $\left(3^{\frac{1}{8}}+5^{\frac{1}{4}}\right)^{84}$ is

(a) 73 (b) 74 (c) 75 (d) 76

22. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then, the matrix $A-3I_3$ is

(a) invertible (b) orthogonal
(c) non-invertible
(d) real Skew Symmetric matrix

23. If M is any square matrix of order 3 over $\mathbb{R}$ and if M' be the transpose of M, then $\text{adj}(M')-(\text{adj } M)'$ is equal to

(a) M (b) M'
(c) null matrix (d) identity matrix

24. If $A=\begin{pmatrix}5 & 5x & x\\ 0 & x & 5x\\ 0 & 0 & 5\end{pmatrix}$ and $|A^2|=25$, then $|x|$ is equal to

(a) $\frac{1}{5}$ (b) 5 (c) 5^2 (d) 1

25. Let A and B be two square matrices of order 3 and $AB=O_3$, where O_3 denotes the null matrix of order 3. Then,

(a) must be $A=O_3$, $B=O_3$
(b) if $A\neq O_3$, must be $B\neq O_3$
(c) if $A=O_3$, must be $B\neq O_3$
(d) may be $A\neq O_3$, $B\neq O_3$

26. Let P and T be the subsets of k,y-plane defined by

$P=\{(x,y):x>0,\ y>0 \text{ and } x^2+y^2=1\}$

$T=\{(x,y):x>0,\ y>0 \text{ and } x^8+y^8<1\}$

Then, $P\cap T$ is

(a) the void set ϕ (b) P
(c) T (d) $P-T^C$

27. Let $f:R\to R$ be defined by $f(x)=x^2-\frac{x^2}{1+x^2}$ for all $x\in R$. Then,

(a) f is one-one but not onto mapping
(b) f is onto but not one-one mapping
(c) f is both one-one and onto
(d) f is neither one-one nor onto

28. Let the relation ρ be defined on R as $a\rho b$ if $1+ab>0$. Then,

(a) ρ is reflexive only.
(b) ρ is equivalence relation.
(c) ρ is reflexive and transitive but not symmetric
(d) ρ is reflexive and symmetric but not transitive.

29. A problem in mathematics is given to 4 students whose chances of solving individually are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ and $\frac{1}{5}$. The probability that the problem will be solved at least by one student is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$
(c) $\frac{4}{5}$ (d) $\frac{3}{4}$

30. If X is a random variable such that $\sigma(X)=2.6$, then $\sigma(1-4X)$ is equal to

(a) 7.8 (b) – 10.4 (c) 13 (d) 10.4

31. If $e^{\sin x}-e^{-\sin x}-4=0$, then the number of real values of x is

(a) 0 (b) 1
(c) 2 (d) 3

32. The angles of a triangle are in the ratio 2 : 3 : 7 and the radius of the circumscribed circle is 10 cm. The length of the smallest side is

(a) 2 cm (b) 5 cm
(c) 7 cm (d) 10 cm

33. A variable line passes through a fixed point (x_1, y_1) and meets the axes at A and B. If the rectangle $OAPB$ be completed, the locus of P is, (O being the origin of the system of axes).

(a) $(y-y_1)^2=4(x-x_1)$ (b) $\frac{x_1}{x}+\frac{y_1}{y}=1$

(c) $x^2+y^2=x_1^2+y_1^2$ (d) $\frac{x^2}{2x_1^2}+\frac{y^2}{y_1^2}=1$

34. A straight line through the point $(3,-2)$ is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If it intersects the X-axis, then its equation will be

(a) $y+x\sqrt{3}+2+3\sqrt{3}=0$
(b) $y-x\sqrt{3}+2+3\sqrt{3}=0$
(c) $y-x\sqrt{3}-2-2\sqrt{3}=0$
(d) $x-x\sqrt{3}+2-3\sqrt{3}=0$

35. A variable line passes through the fixed point (α, β). The locus of the foot of the perpendicular from the origin on the line is

(a) $x^2 + y^2 - \alpha x - \beta y = 0$
(b) $x^2 - y^2 + 2\alpha x + 2\beta y = 0$
(c) $\alpha x + \beta y \pm \sqrt{(\alpha^2 + \beta^2)} = 0$
(d) $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

36. If the point of intersection of the lines $2ax + 4ay + c = 0$ and $7bx + 3by - d = 0$ lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then $ad : bc$ equals to

(a) 2 : 3 (b) 2 : 1 (c) 1 : 1 (d) 3 : 2

37. A variable circle passes through the fixed point $A(p, q)$ and touches X-axis. The locus of the other end of the diameter through A is

(a) $(x - p)^2 = 4qy$ (b) $(x - q)^2 = 4py$
(c) $(y - p)^2 = 4qx$ (d) $(y - q)^2 = 4px$

38. If $P(0, 0)$, $Q(1, 0)$ and $R\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for which the lines PQ, QR and RP are the tangents is

(a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$ (d) $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

39. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains fixed when α varies?

(a) directrix (b) vertices
(c) foci (d) eccentricity

40. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

41. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

(a) $x = 3 \pm \sqrt{\frac{13}{6}}$ (b) $x = 3 \pm \sqrt{\frac{6}{13}}$
(c) $x = 6 \pm \sqrt{\frac{13}{3}}$ (d) $x = 6 \pm \sqrt{\frac{3}{13}}$

42. P is the extremity of the latusrectum of ellipse $3x^2 + 4y^2 = 48$ in the first quadrant. The eccentric angle of P is

(a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

43. The direction ratios of the normal to the plane passing through the points $(1, 2, -3)$, $(-1, -2, 1)$ and parallel to $\frac{x - 2}{2} = \frac{y + 1}{3} = \frac{z}{4}$ is

(a) (2, 3, 4) (b) (14, – 8, – 1)
(c) (– 2, 0, – 3) (d) (1, – 2, – 3)

44. The equation of the plane, which bisects the line joining the points (1, 2, 3) and (3, 4, 5) at right angles is

(a) $x + y + z = 0$ (b) $x + y - z = 9$
(c) $x + y + z = 9$ (d) $x + y - z + 9 = 0$

45. The limit of the interior angle of a regular polygon of n sides as $n \to \infty$ is

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) $\frac{2\pi}{3}$

46. Let $f(x) > 0$ for all x and $f'(x)$ exists for all x. If f is the inverse function of h and $h'(x) = \frac{1}{1 + \log x}$. Then, $f'(x)$ will be

(a) $1 + \log(f(x))$ (b) $1 + f(x)$
(c) $1 - \log(f(x))$ (d) $\log f(x)$

47. Consider the function $f(x) = \cos x^2$. Then,

(a) f is of period 2π (b) f is of period $\sqrt{2\pi}$
(c) f is not periodic (d) f is of period π

48. $\lim_{x \to 0^+} (e^x + x)^{1/x}$

(a) Does not exist finitely (b) is 1
(c) is e^2 (d) is 2

49. Let $f(x)$ be a derivable function, $f'(x) > f(x)$ and $f(0) = 0$. Then,

(a) $f(x) > 0$ for all $x > 0$
(b) $f(x) < 0$ for all $x > 0$
(c) no sign of $f(x)$ can be ascertained
(d) $f(x)$ is a constant function

50. Let $f : [1, 3] \to R$ be a continuous function that is differentiable in (1, 3) an $f'(x) = |f(x)|^2 + 4$ for all $x \in (1, 3)$. Then,

(a) $f(3) - f(1) = 5$ is true
(b) $f(3) - f(1) = 5$ is false
(c) $f(3) - f(1) = 7$ is false
(d) $f(3) - f(1) < 0$ only at one point of (1, 3)

Category II (Q.Nos. 51 to 65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

51. Let $a = \min\{x^2 + 2x + 3 : x \in R\}$ and $b = \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$. Then $\sum_{r=0}^{n} a^r b^{n-r}$ is

(a) $\frac{2^{n+1} - 1}{3 \cdot 2^n}$ (b) $\frac{2^{n+1} + 1}{3 \cdot 2^n}$

(c) $\frac{4^{n+1} - 1}{3 \cdot 2^n}$ (d) $\frac{1}{2}(2^n - 1)$

52. Let $a > b > 0$ and $I(n) = a^{1/n} - b^{1/n}$, $J(n) = (a - b)^{1/n}$ for all $n \geq 2$, Then

(a) $I(n) < J(n)$ (b) $I(n) > J(n)$

(c) $I(n) = J(n)$ (d) $I(n) + J(n) = 0$

53. Let $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ be three unit vectors such that $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$ where $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

54. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} - \lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, the value of λ is

(a) 0 (b) 1 (c) 2 (d) – 4

55. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1, then it moves $\frac{1}{2}$ unit vertically up and reaches P_2, then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3, then it moves $\frac{1}{8}$ unit vertically down and reaches P_4, then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on. Let $P_n = (x_n, y_n)$ and $\lim_{n \to \infty} x_n = \alpha$ and $\lim_{n \to \infty} y_n = \beta$. Then, (α, β) is

(a) (2, 3) (b) $\left(\frac{4}{3}, \frac{2}{5}\right)$ (c) $\left(\frac{2}{5}, 1\right)$ (d) $\left(\frac{4}{3}, 3\right)$

56. For any non-zero complex number z, the minimum value of $|z| + |z - 1|$ is

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) $\frac{3}{2}$

57. The system of equations

$$\lambda x + y + 3z = 0$$
$$2x + \mu y - z = 0$$
$$5x + 7y + z = 0$$

has infinitely many solutions in R. Then,

(a) $\lambda = 2, \mu = 3$ (b) $\lambda = 1, \mu = 2$

(c) $\lambda = 1, \mu = 3$ (d) $\lambda = 3, \mu = 1$

58. Let $f : X \to Y$ and A, B are non-void subsets of Y, then (where the symbols have their usual interpretation)

(a) $f^{-1}(A) - f^{-1}(B) \supset f^{-1}(A - B)$ but the opposite does not hold.

(b) $f^{-1}(A) - f^{-1}(B) \subset f^{-1}(A - B)$ but the opposite does not hold.

(c) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$

(d) $f^{-1}(A - B) = f^{-1}(A) \cup f^{-1}(B)$

59. Let S, T, U be three non-void sets and $f : S \to T$, $g : T \to U$ be so that $gof : s \to U$ is surjective. Then,

(a) g and f are both surjective

(b) g is surjective, f may not be so

(c) f is surjective, g may not be so

(d) f and g both may not be surjective

60. The polar coordinate of a point P is $\left(2, -\frac{\pi}{4}\right)$.

The polar coordinate of the point Q which is such that line joining PQ is bisected perpendicularly by the initial line, is

(a) $\left(2, \frac{\pi}{4}\right)$ (b) $\left(2, \frac{\pi}{6}\right)$ (c) $\left(-2, \frac{\pi}{4}\right)$ (d) $\left(-2, \frac{\pi}{6}\right)$

61. The length of conjugate axis of a hyperbola is greater than the length of transverse axis. Then, the eccentricity e is

(a) $= \sqrt{2}$ (b) $> \sqrt{2}$ (c) $< \sqrt{2}$ (d) $< \frac{1}{\sqrt{2}}$

62. The value of $\lim_{x \to 0^+} \frac{x}{p}\left[\frac{q}{x}\right]$ is

(a) $\frac{[q]}{p}$ (b) 0 (c) 1 (d) ∞

63. Let $f(x) = x^4 - 4x^3 + 4x^2 + c, c \in R$. Then,

(a) $f(x)$ has infinitely many zeros in (1, 2) for all c
(b) $f(x)$ has exactly one zero in (1, 2) if $-1 < c < 0$
(c) $f(x)$ has double zeros in (1, 2) if $-1 < c < 0$
(d) whatever be the value of c, $f(x)$ has no zero in (1, 2)

64. The graphs of the polynomial $x^2 - 1$ and $\cos x$ intersect

(a) at exactly two points
(b) at exactly 3 points
(c) at least 4 but at finitely many points.
(d) at infinitely many points.

65. A point is in motion along a hyperbola $y = \frac{10}{x}$ so that its abscissa x increases uniformly at a rate of 1 unit per second. Then, the rate of change of its ordinate when the point passes through (5, 2)

(a) increases at the rate of $\frac{1}{2}$ unit per second
(b) decreases at the rate of $\frac{1}{2}$ unit per second
(c) decreases at the rate of $\frac{2}{5}$ unit per second
(d) increases at the rate of $\frac{2}{5}$ unit per second

Category III (Q. Nos. 66 to 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score $= 2 \times$ number of correct answers marked $\div$ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then

(a) a_1, a_2, a_3 are in GP
(b) b_1, b_2, b_3 are in AP
(c) c_1, c_2, c_3 are in HP
(d) a_1, a_2, a_3 are in AP

67. Two particles A and B move from rest along a straight line with constant accelerations f and h, respectively. If A takes m seconds more than B and describes n units more than that of B acquiring the same speed, then

(a) $(f + h) m^2 = fhn$
(b) $(f - fh) m^2 = fhn$
(c) $(h - f)n = \frac{1}{2} fhm^2$
(d) $\frac{1}{2}(f + h) n = fhm^2$

68. The area bounded by $y = x + 1$ and $y = \cos x$ and the X-axis, is

(a) 1 sq unit
(b) $\frac{3}{2}$ sq unit
(c) $\frac{1}{4}$ sq unit
(d) $\frac{1}{8}$ sq unit

69. Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$.

If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in GP, then ab equals

(a) $\frac{24}{5}$
(b) 64
(c) 16
(d) 8

70. If $\theta \in \mathbb{R}$ and $\frac{1 - i\cos\theta}{1 + 2i\cos\theta}$ is real number, then θ will be (when I : Set of integers)

(a) $(2n + 1)\frac{\pi}{2}, n \in I$
(b) $\frac{3n\pi}{2}, n \in I$
(c) $n\pi, n \in I$
(d) $2n\pi, n \in I$

71. Let $A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. Then, the roots of the equation $\det(A - \lambda I_3) = 0$ (where I_3 is the identity matrix of order 3) are

(a) 3, 0, 3
(b) 0, 3, 6
(c) 1, 0, – 6
(d) 3, 3, 6

72. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that $AB = AC$. The equation of line AC passing through (2,– 7) is

(a) $x - y - 9 = 0$
(b) $23x + 7y + 3 = 0$
(c) $2x - y - 11 = 0$
(d) $7x - 6y - 56 = 0$

73. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point $(2, 8)$ is

(a) $3x - y + 2 = 0$ (b) $3x + y - 14 = 0$
(c) $23x - 3y - 22 = 0$ (d) $3x - 23y + 178 = 0$

74. Let f and g be differentiable on the interval I and let $a, b \in I$, $a < b$. Then,

(a) If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ is solvable in (a, b)

(b) If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ may not be solvable in (a, b).

(c) If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ is solvable in (a, b), $k \in \mathbb{R}$

(d) If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ may not be solvable in (a, b), $k \in \mathbb{R}$.

75. Consider the function $f(x) = \frac{x^3}{4} - \sin \pi x + 3$

(a) $f(x)$ does not attain value within the interval $[-2, 2]$

(b) $f(x)$ takes on the value $2\frac{1}{3}$ in the interval $[-2, 2]$

(c) $f(x)$ takes on the value $3\frac{1}{4}$ in the interval $[-2, 2]$

(d) $f(x)$ takes no value p, $1 < p < 5$ in the interval $[-2, 2]$.

Answers

Physics

1. (b)	2. (b)	3. (d)	4. (d)	5. (d)	6. (b)	7. (b)	8. (c)	9. (d)	10. (c)
11. (c)	12. (a)	13. (c)	14. (c)	15. (c)	16. (b)	17. (a)	18. (d)	19. (a)	20. (a)
21. (d)	22. (d)	23. (*)	24. (c)	25. (c)	26. (d)	27. (a)	28. (d)	29. (c)	30. (c)
31. (a)	32. (a)	33. (a)	34. (a)	35. (c)	36. (b, d)	37. (b)	38. (a, d)	39. (a, c)	40. (b, d)

Chemistry

41. (c)	42. (a)	43. (d)	44. (b)	45. (a)	46. (b)	47. (c)	48. (b)	49. (d)	50. (b)
51. (c)	52. (d)	53. (c)	54. (c)	55. (c)	56. (a)	57. (b)	58. (d)	59. (a)	60. (b)
61. (d)	62. (b)	63. (a)	64. (b)	65. (d)	66. (b)	67. (d)	68. (d)	69. (d)	70. (b)
71. (d)	72. (d)	73. (a)	74. (d)	75. (a)	76. (a, b)	77. (a, b)	78. (b, c, d)	79. (a, c)	80. (d)

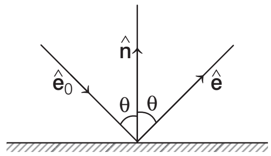
Mathematics

1. (b)	2. (b)	3. (d)	4. (b)	5. (a)	6. (d)	7. (d)	8. (c)	9. (c)	10. (d)
11. (b)	12. (b)	13. (b)	14. (b)	15. (c)	16. (b)	17. (d)	18. (a)	19. (b)	20. (d)
21. (b)	22. (c)	23. (c)	24. (a)	25. (d)	26. (a)	27. (d)	28. (d)	29. (c)	30. (d)
31. (a)	32. (d)	33. (b)	34. (b)	35. (a)	36. (b)	37. (a)	38. (c)	39. (c)	40. (c)
41. (a)	42. (c)	43. (b)	44. (c)	45. (a)	46. (a)	47. (c)	48. (c)	49. (a)	50. (b, c)
51. (c)	52. (a)	53. (d)	54. (d)	55. (b)	56. (a)	57. (c)	58. (c)	59. (b)	60. (a)
61. (b)	62. (a)	63. (b)	64. (a)	65. (c)	66. (b, d)	67. (c)	68. (b)	69. (b)	70. (a)
71. (b)	72. (a, b)	73. (a, c)	74. (a, c)	75. (d)					

Answer *with* Explanations

Physics

1. *(b)* According to the question,



We know that incident ray, reflected ray and normal lie in the same plane.

and angle of incidence = angle of reflection

Therefore $\hat{\mathbf{n}}$ will be along the angle bisector of $\hat{\mathbf{e}}$ and $-\hat{\mathbf{e}}_0$,

i.e. $$\hat{\mathbf{n}} = \frac{\hat{\mathbf{e}} + (-\hat{\mathbf{e}}_0)}{|\hat{\mathbf{e}} - \hat{\mathbf{e}}_0|} \quad \text{...(i)}$$

[∵ Bisector will along a vector dividing in same ratio as the ratio of sides forming that angle]

But $\hat{\mathbf{n}}$ is a unit vector where $|\hat{\mathbf{e}} - \hat{\mathbf{e}}_0| = OC$

$$= 2OP = 2|\hat{\mathbf{e}}|$$

$$\cos\theta = 2\cos\theta$$

Substituting this value in Eq. (i), we get

$$\hat{\mathbf{n}} = \frac{\hat{\mathbf{e}} - \hat{\mathbf{e}}_0}{2\cos\theta}$$

$$\hat{\mathbf{e}} = \hat{\mathbf{e}}_0 + (2\cos\theta)\,\hat{\mathbf{n}}$$

$$\hat{\mathbf{e}} = \hat{\mathbf{e}}_0 - 2(\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_0)\hat{\mathbf{n}} \quad [\because \hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_0 = -\cos\theta]$$

2. *(b)* According to the question,

$$X \xrightarrow{\alpha\text{-particle}} Y \xrightarrow{\beta\text{-particle}} Z(\text{say})$$

According to Rutherford Soddy law of radioactive decay, the rate of decay of radioactive along at any instant is proportional to the number of atoms present at that instant

$$\frac{dN}{dt} = -\lambda N$$

Since, λ = constant (decay constant) and

N = number of atoms

Rate of decay of Y particle is given as,

$$\lambda_X N_X - \lambda_Y N_Y = \frac{dN_Y}{dt} = 0$$

[∵ Decay rate for β-particle become constant after some time]

Given, rate of emission of β-particle = 10^7/h

$$\therefore \lambda_X N_X = \lambda_Y N_Y = 10^7/\text{h}$$

3. *(d)* As we know that de-Broglie wavelength is given as $\lambda = \frac{h}{p} = \frac{h}{mv}$ ...(i)

where, h = Planck constant

p = momentum of particle

v = velocity of particle

and m = mass of the particle.

Eq. (i) can be written as,

$$\lambda = \frac{h}{2m(\text{KE})} = \frac{h}{\sqrt{2mqv}} \quad [\because \text{KE} = qv]$$

where, KE = Kinetic energy of particle

Hence, $\lambda \propto \frac{1}{\sqrt{m}}$

Now, $$\frac{\lambda_{\text{proton}}}{\lambda_{\text{electron}}} = \sqrt{\frac{m_{\text{electron}}}{m_{\text{proton}}}}$$

Given, mass of proton, $m_{\text{proton}} = 2000\, m_{\text{electron}}$

$$\frac{\lambda_{\text{proton}}}{\lambda_{\text{electron}}} = \sqrt{\frac{1}{2000}}$$

$$\frac{\lambda_{\text{proton}}}{\lambda_{\text{electron}}} = \frac{1}{20\sqrt{5}} \Rightarrow \lambda_p = \frac{\lambda_e}{20\sqrt{5}}$$

4. *(d)* According to the Bohr's atomic model,

Angular momentum, $L = mvr = \frac{nh}{2\pi}$ ...(i)

Since, angular velocity $\omega = \frac{v}{r}$

$$\Rightarrow \quad v = r\omega$$

From Eq. (i), we get

$$m(r\omega)\, r = \frac{nh}{2\pi}$$

$$m\omega r^2 = \frac{nh}{2\pi}$$

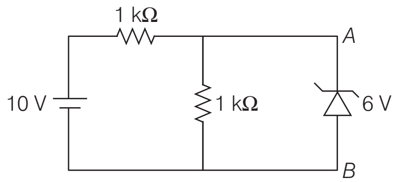
$$\omega = \frac{nh}{2\pi m r^2} \quad \text{...(ii)}$$

Since, the radius of the electron in n^{th} orbit of Bohr's atomic model is given as, $r = \frac{n^2 h^2 \varepsilon_0}{\pi m z e^2}$ ...(iii)

Squaring the Eq. (iii) and substituting its value in Eq. (ii), we get

$$\omega = \frac{nh(\pi m z e)^2}{2\pi m (n^4 h^4 \varepsilon_0^2)} \Rightarrow \omega \propto \frac{1}{n^3}$$

5. *(d)* According to the question,



Input voltage $V_S = 10\text{V}$

Source resistance $R_S = 1\text{k}\Omega$

Zener diode voltage $V_Z = 6\text{V}$

$\because$ Breakdown voltage of zener diode is = 6V, and the potential difference across the zener diode 5V.

$\therefore$ Current flow in zener diode $I_Z = 0$

6. *(b)* By using the de-Morgan's law

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Hence, option (b) is the correct.

7. *(b)* As we know that,

Impulse, I = force (F) × small time interval

$$I = ma \times t \qquad [\because F = ma]$$

$$I = [\text{M}]\,[\text{LT}^{-2}] \times [\text{T}]$$

$$I = [\text{M L T}^{-1}]$$

Hence, the correct dimensional formula for impulse is given by $[\text{M L T}^{-1}]$.

8. *(c)* Given,

Maximum error in the measurement of mass = 0.3%

Maximum error in the measurement of length = 0.2%

We know that,

Error in density is given as,

Density, $\rho = \dfrac{\text{mass }(m)}{\text{volume }(V)} = \dfrac{m}{L^3}$

where, L = side of cube

Error in density is given as,

$$\left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta m}{m} + \frac{3\Delta L}{L}$$

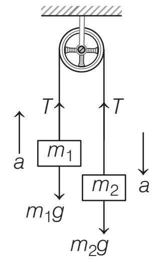
or $\left(\dfrac{\Delta\rho}{\rho}\right) \times 100 = \left(\dfrac{\Delta m}{m} + \dfrac{3\Delta L}{L}\right) \times 100$

Substituting the given values, we get

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = (0.3\% + 3(0.2)\%) = 0.3\% + 0.6\%$$

$\therefore$ Maximum percentage error measurement of density $\left(\dfrac{\Delta\rho}{\rho}\right)_{\max} = 0.9\%$.

9. *(d)* According to the question, we can draw the following diagram



Here, T is the tension in the string.

In equilibrium condition,

For mass m_1, $\quad T - m_1 g = m_1 a \qquad \dots$(i)

For mass m_2, $\quad m_2 g - T = m_2 a \qquad \dots$(ii)

After adding Eqs. (i) and (ii), we get

$$m_2 g - m_1 g = m_1 a + m_2 a$$

$$(m_2 - m_1) g = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} g$$

10. *(c)* Given, Power (P) = constant

Kinetic Energy (KE) $= \dfrac{1}{2} mv^2$

We know that, $\quad P = \dfrac{\text{KE}}{\Delta t} \Rightarrow P = \dfrac{mv^2}{2}$

$\because P$ = constant,

Hence, velocity of the body $v \propto \sqrt{t} \qquad \dots$(i)

As, Velocity $v = \dfrac{ds}{dt} \qquad \dots$(ii)

From Eqs. (i) and (ii), we get

So, $\quad \dfrac{ds}{dt} \propto \sqrt{t}$

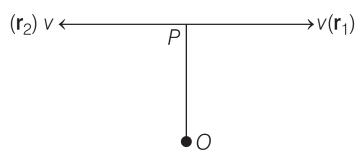
Integrating the above equation w.r.t. time (t),

$$\int \frac{ds}{dt} \propto \int \sqrt{t}$$

we get, displacement of the body $s \propto t^{3/2}$

$\because$ Displacement $s = at^{3/2}$, where a is constant.

11. *(c)* According to the question,



Representation of position vectors of two particles (drawn from the point P)

In two dimension, the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ represented as

$$r_1 = vt\,\hat{i} - \frac{1}{2}gt^2\hat{j} \quad \ldots(i)$$

$$r_2 = vt(-\hat{i}) - \frac{1}{2}gt^2(\hat{j}) \quad \ldots(ii)$$

$\because$ We know that, when the two vectors are mutually perpendicular, i.e.

$$\theta = 90^\circ$$

So, $r_1 \cdot r_2 = r_1 r_2 \cos 90^\circ$

$$r_1 \cdot r_2 = 0$$

Substituting the values r_1 and r_2 in the above relation, we get

$$((vt)\hat{i} - \frac{1}{2}gt^2\hat{j}) \cdot (vt(-\hat{i}) - \frac{1}{2}gt^2(\hat{j})) = 0$$

$$-v^2t^2 + \frac{1}{4}4g^2t^4 = 0 \text{ (where, } \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1)$$

$$v^2t^2 = \frac{1}{4}g^2t^4$$

$$\Rightarrow \quad v^2 = \frac{1}{4}g^2t^2$$

$\therefore$ Magnitude of velocity of the particles, $v = \frac{1}{2}gt$

We know that, separation distance between particles at a time t

$$\Delta x = 2vt$$

$$\Delta x = 2 \times v \times \frac{2v}{g} \Rightarrow \Delta x = \frac{4v^2}{g}$$

12. *(a)* According to the Kepler's third law

$$T^2 \propto r^3$$

where, T = time period of revolution

r = radius

$$\text{Now, } \left(\frac{T_E}{T_P}\right)^2 = \left(\frac{r_E}{r_P}\right)^3 = \frac{r_E}{r_P} = \left(\frac{T_E}{T_P}\right)^{2/3}$$

$$\frac{r_E}{r_P} = \left(\frac{2\pi/\omega_E}{2\pi/\omega_P}\right)^{2/3} \quad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\frac{r_E}{r_P} = \left(\frac{\omega_P}{\omega_E}\right)^{2/3}$$

According to the question, $\omega_P = 2\omega_E$ and $r_E = R$

$$\Rightarrow \quad \frac{R}{r_P} = \left(\frac{2\omega_E}{\omega_E}\right)^{2/3}$$

$$\frac{R}{r_P} = (2)^{2/3}$$

$$r_P = \frac{R}{(2)^{2/3}} = R(2)^{-2/3}$$

13. *(c)* Given, Decrement in the length = 1%.

Poisson's ratio for material of the rod, $\sigma = 0.2$

As we know that,

Volume, $v = \pi r^2 l$, [where, l is the length of the rod]

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l} \quad \ldots(i)$$

$$\text{Since, Poisson's ratio, } \sigma = \frac{-\Delta D/D}{\Delta L/L} = \frac{-\Delta r/r}{\Delta L/L}$$

So, Eq. (i) can be written as,

$$\frac{\Delta V}{V} = \frac{\Delta l}{l}(1 - 2\sigma)$$

$$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} \times 100\right) \times (1 - 2\sigma)$$

$$= -1 \times [1 - 2 \times (0.2)]$$

$$= -1 \times [1 - 0.4]$$

$$= -0.6\%$$

Here, negative sign shows the decrement in the volume.

14. *(c)* When the spherical body falls with constant velocity, i.e. terminal velocity then the net force becomes zero, i.e the weight of body is equal to the buoyancy force.

Hence, $F_{net} = 0$

15. *(c)* According to the question,

Area of both bodies A and $B = A$

Temperature of body $A = 27^\circ C = 27 + 273\,K$

Temperature of body $B = 177^\circ C = 177 + 273\,K$

Now, by Stefan-Boltzmann law, thermal energy radiated per second by a body

$$Q = \sigma A T^4,$$

where, A = Area

T = temperature

and σ = Stefan-Boltzmam's constant

So, the ratio of thermal energy radiated per second by A to that by B is

$$\frac{Q_1}{Q_2} = \frac{\sigma A}{\sigma A}\left(\frac{T_1}{T_2}\right)^4$$

$$\text{Now, } \frac{Q_1}{Q_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\frac{Q_1}{Q_2} = \left(\frac{273 + 27}{273 + 177}\right)^4$$

$$\frac{Q_1}{Q_2} = \left(\frac{300}{450}\right)^4$$

$$\frac{Q_1}{Q_2} = \left(\frac{2}{3}\right)^4 = \left(\frac{16}{81}\right)$$

Ratio of thermal energy radiated per second

$$Q_1 : Q_2 = 16 : 81$$

16. *(b)* For a gas at temperature T, the internal energy,

$$U = \frac{f}{2}\mu RT$$

where, $f =$ degree of freedom

$\Rightarrow$ Change in energy, $\Delta U = \frac{f}{2}\mu R\,\Delta T$ ...(i)

Also, as we know for any gas heat supplied at constant volume

$$(\Delta Q_V) = \mu C_V \Delta T = \Delta U \quad \text{...(ii)}$$

where, $C_V =$ molar specific heat at constant volume

From Eqs. (i) and (ii), we get

$$C_V = \frac{1}{2} fR$$

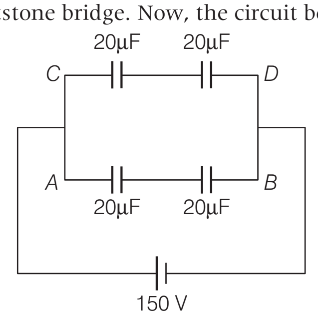
For diatomic gas, degree of freedom, $f = 5$

$$C_V = \frac{5}{2}R$$

17. *(a)* Since, the whole system is isolated, this means, there is no transfer of heat between the system and the surrounding. Also, the right side container is initially vacuum, so the gas could easily rush there without any resistive force. As the right side container has no temperature. Thus, the temperature of the ideal gas would remain same even if it enters right side chamber.

Therefore, final temperature attained at the equilibrium will be T.

18. *(d)* The given circuit shows a balanced Wheatstone bridge. Now, the circuit becomes



In the branch AB, both capacitor are arranged in the series combination. Hence, its equivalent capacitance is given by

$$\frac{1}{C_{AB}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} = \frac{1}{10}$$

$$C_{AB} = 10\,\mu\text{F}$$

Similarly, in branch CD,

$$C_{CD} = 10\,\mu\text{F}$$

Now, C_{AB} and C_{CD} are connected in parallel combination. Hence, the equivalent capacitance of the circuit,

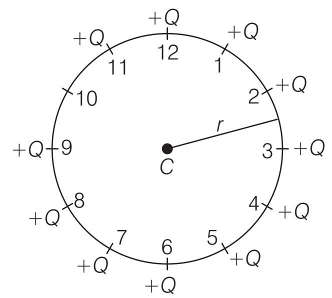
$$C_{eq} = 10 + 10 = 20\,\mu\text{F}$$

We know that, charge $Q = C_{eq}V$

$$Q = 20 \times 10^{-6} \times 150\text{ V}$$

Amount of charge stored $Q = 3 \times 10^{-3}$C

19. *(a)* The clock diagram is as shown below



In the above diagram, charge $+Q$ is not placed at 10 h position.

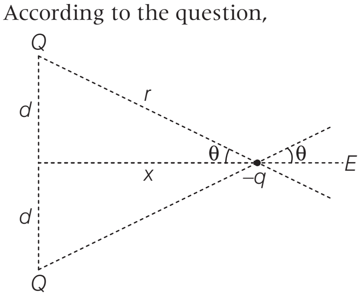
So, net electric field strength at centre C,

$$E_{net} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7 + E_8 + E_9 + E_{11} + E_{12}$$

$$\Rightarrow E_{net} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + (-E_1 - E_2 - E_3 - E_5 - E_6)$$

$\Rightarrow E_{net} = E_4 = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, from centre towards the mark 10.

20. *(a)* According to the question,



Force experienced by the charge $-q$ due to charge Q

$$F = -\frac{2kQq}{r^2}\cos\theta \quad \text{... (i)} \left[\text{where, } k = \frac{1}{4\pi\varepsilon_0}\right]$$

From diagram, $\cos\theta = \dfrac{x}{r}$

By substituting the value of $\cos\theta$ in Eq. (i)

$$F = -\frac{2kQq}{r^2} \cdot \frac{x}{r}$$

or $$F = -\frac{2kQx}{r^3}$$

or $$F = -\frac{2kQx}{(x^2 + d^2)^{3/2}} \quad \left[\begin{matrix} \because r^2 = x^2 + d^2 \\ r = \sqrt{x^2 + d^2} \end{matrix}\right]$$

For, $x << d$, so x^2 can be neglected

$$F = -\frac{2kQx}{d^3}$$

So, the force developed by negative charge $(-q)$ due to the system of the charges as shown in the figure is,

$$F = \frac{-4kQqx}{d^3}$$

$\Rightarrow$ $F \propto x$

So, the forced developed by negative charge is directly proportional to the distance x.

21. *(d)* If velocity of particle, **v** is perpendicular to magnetic field, **B** i.e. $\theta = 90^o$, then particle will experience maximum magnetic force, i.e. $F_{max} = qvB$. This force acts in a direction perpendicular to the motion of charged particle. Therefore the trajectory of the particle is a circle.

In this case path of charged particle is circular and magnetic force provides the necessary centripetal force,

i.e. $$qvB = \frac{mv^2}{r}$$

$\Rightarrow$ Radius of path, $r = \frac{mv}{qB}$

$$r = \frac{p}{qB} \quad [\because p = mv]$$

where, p = momentum of the particle

$\therefore$ $r \propto$ momentum

Hence, option (d) is correct.

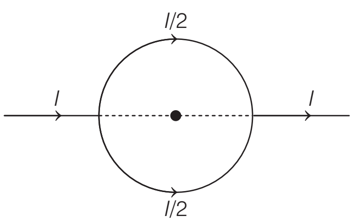
22. *(d)* Given, electric current in circular wire = I

Radius of wire = r

Charge on particle = q

Speed of the particle = v

The given loop can be as shown in the figure below.

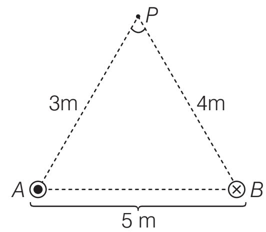


$\because$ Net magnetic field at the centre = 0

So, magnetic force $F_{magnetic} = q(v \times B) = 0$

So, the magnetic force experienced by the particle when it passes through the centre is $F = 0$

23. *(c)* According to the question,



Magnetic field due to first wire (A) is $B_1 = \frac{\mu_0 \times I}{2\pi \times 3}$

Magnetic field due to second wire (B) is $B_2 = \frac{\mu_0 \times I}{2\pi \times 4}$

Net magnitude of magnetic field $B = \sqrt{B_1^2 + B_2^2}$

$$B = \frac{\mu_0 I}{2\pi}\sqrt{\frac{1}{9} + \frac{1}{16}} = \frac{\mu_0 I \times 5}{2\pi \times 3 \times 4}$$

Magnetic field $B = \frac{5}{24} \times \frac{\mu_0 I}{\pi}$

24. *(c)* As we know that, magnetic field due to a long wire at a distance x from it is given by

$B = \frac{\mu_0 I}{2\pi x}$, where, I is the current flowing through the wire $\beta \propto I$

$\therefore$ Magnetic flux associated with the square loop,

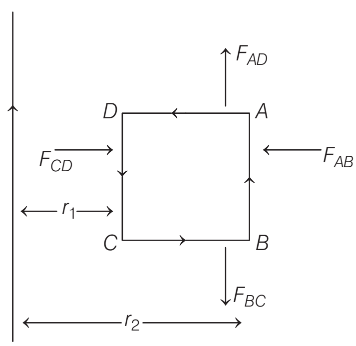
$$\phi \propto \beta \propto I$$

Now, if the current increase, then ϕ also increases.

Direction of long wire will be $\otimes$.

This means, magnetic field due to induced current will be opposite to the existing magnetic field, i.e. according to Lenz's law,

The induced current in the loop will be in the anti-clockwise direction. Now,



Since, wires attract each other, if current flowing through them is in same direction and repel each other, if currents are in opposite direction.

$\therefore$ Part CD of the loop will experience a force of repulsion, whereas part AB will experience attraction. Parts BC and AD will not experience any force. Thus, the overall force will be a force of repulsion because AB is closer to straight. The force between two current carrying conductors is inversely proportional to the distance between them

$$F \propto \frac{1}{r}$$

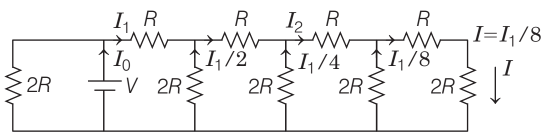
$\because$ $r_1 < r_2$

So, $F_{CD} > F_{AB}$

$$F_{net} = F_{CD} - F_{AB}$$

Hence, the loop will moves away from the wire.

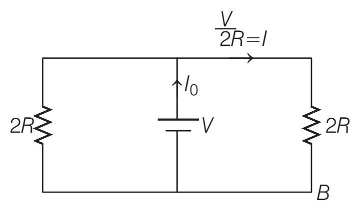
25. *(c)* According to the question,



Total resistance of the given circuit

$$R_{eq} = 2R$$

Now, circuit



$\therefore$ $I = \frac{V/2R}{8}$ $\quad (\because I_1 = V/2R)$

So, current I in the circuit $I = \frac{V}{16R}$ $\quad \left(\because I = \frac{I_1}{8}\right)$

26. *(d)* According to the balanced condition of Wheatstone bridge,

In the first case,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\frac{P}{Q} = \frac{5}{S} \quad \ldots(i)$$

In the second case

$$\frac{P}{Q} = \frac{7}{S+3} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{5}{S} = \frac{7}{S+3}$$

$$5(S+3) = 7S$$

$$5S + 15 = 7S$$

$$2S = 15$$

$$S = 7.5\,\Omega.$$

27. *(a)* Given, inductance of inductor, $L = 60$ mH

$$= 60 \times 10^{-3} \text{ H}$$

Phase difference between voltage and current in L-R circuit, $\theta_1 = 60^\circ$

Capacitance of capacitor, $C = 0.5\,\mu\text{F} = 0.5 \times 10^{-6}$F

Phase difference between voltage and current in R-C circuit, $\theta_2 = 30^\circ$

For L-R circuit,

$$\tan\theta_1 = \frac{X_L}{R}$$

$$\tan\theta_1 = \frac{\omega L}{R} \quad \ldots(i)\ [\because X_L = \omega L]$$

Similarly, for R-C circuit,

$$\tan\theta_2 = \frac{X_C}{R}$$

$$\tan\theta_2 = \frac{\frac{1}{\omega C}}{R} \quad \ldots(ii) \left[\because X_C = \frac{1}{\omega C}\right]$$

From Eqs. (i) and (ii), we get

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\omega L}{R \times \frac{1}{\omega CR}} = \omega^2 LC$$

$$\frac{\tan 60^\circ}{\tan 30^\circ} = \omega^2 LC$$

$$\frac{\sqrt{3}}{1/\sqrt{3}} = \omega^2 LC$$

$$\omega^2 LC = 3$$

$$\omega^2 = \frac{3}{LC}$$

$$\omega^2 = \frac{3}{60 \times 10^{-3} \times 0.5 \times 10^{-6}}$$

$$\omega^2 = \frac{3}{30 \times 10^{-9}}$$

$$\omega = \sqrt{10^8}$$

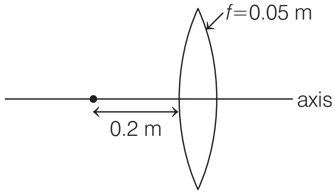
$$\omega = 10^4$$

As we know that,

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{10^4}{2\pi}\text{ Hz} = \frac{1}{2\pi} \times 10^4 \text{ Hz}$$

28. *(d)* According to the question, we can draw the following diagram



Given, $u = -0.2\text{ m}$ and $f = 0.05\text{ m}$

As we know that,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \ldots\text{(i)}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{0.05} - \frac{1}{0.2}$$

$$\frac{1}{v} = \frac{100}{5} - \frac{10}{2}$$

$$\frac{1}{v} = 20 - 5 \Rightarrow v = \frac{1}{15}\text{ m}$$

Now, differentiating eq. (i), we get

$$-\frac{dv}{v^2} = -\frac{du}{u^2} \quad \therefore \quad dv = du\,\frac{v^2}{u^2}$$

$$A_{\max} = A \times \left(\frac{1}{15}\right)^2 \times \left(\frac{1}{(-0.2)}\right)^2$$

$$A_{\max} = A \times \frac{1}{225} \times 25$$

$$A_{\max} = \frac{A}{9}$$

Here A is in cm.

Hence, $A_{\max} = \frac{A}{9} \times 10^{-2}\text{ m}$

29. *(c)* As we know that,

fringe width, $\beta = \frac{\lambda D}{d}$

where, λ = wavelength of light

D = Distance of the screen from the slits

d = separation between the slits

Now, $\frac{\Delta\beta}{\beta} \times 100 = \frac{\Delta D}{D} \times 100 + \frac{\Delta d}{d} \times 100$

According to the question,

$$\frac{\Delta\beta}{\beta} \times 100 = 0.5 + (-0.3) = 0.5 - 0.3 = 0.2\%$$

Hence, the fringe width increases by 0.2%.

30. *(c)* Given, initial frequency of light,

$$f_1 = 4 \times 10^{14}\text{ s}^{-1}$$

Final frequency of light, $f_2 = 5 \times 10^{14}\text{ s}^{-1}$

Change in wavelength, $\Delta\lambda = \lambda_1 - \lambda_2$

or $$\Delta\lambda = \frac{c}{f_1} - \frac{c}{f_2}$$

$$= \frac{3 \times 10^8}{4 \times 10^{14}} - \frac{3 \times 10^8}{5 \times 10^{14}}$$

$$= \frac{3 \times 10^8}{10^{14}}\left(\frac{1}{4} - \frac{1}{5}\right)$$

$$= \frac{3 \times 10^8}{10^{14}} \times \frac{1}{20}$$

$$= 1.5 \times 10^{-7}\text{ m}$$

Now, we know that,

Angular width of central maxima,

$$\theta = \frac{2\lambda}{d} \text{ or } \Delta\theta = \frac{2\Delta\lambda}{d}$$

$$d = \frac{2\Delta\lambda}{\Delta\theta}$$

Here, $\Delta\theta = 0.6$ radian

$$d = \frac{2 \times 1.5 \times 10^{-7}}{0.6} = 5 \times 10^{-7}\text{ m}$$

31. *(a)* We know that, energy stored in the capacitor

$$= \frac{1}{2}CE^2$$

and energy supplied by the source of emf

$$= CE^2$$

$\therefore$ Energy dissipated in resistance R

= Energy supplied by the source of emf E – Energy stored in the capacitor

$$= CE^2 - \frac{1}{2}CE^2$$

$$= \left(1 - \frac{1}{2}\right)CE^2 = \frac{1}{2}CE^2$$

32. *(a)* Given, cross sectional area of nozzle

$$A = \frac{5}{\sqrt{21}} \times 10^{-3}\text{m}^2$$

and rate of heat transfer $Q = \frac{1}{10} = 10^{-1}\text{ m}^3/\text{s}$

$\because$ Heat transfer $Q = AV$

$$\therefore \quad V = \frac{Q}{A} = \frac{10^{-1}\text{ m}^3/s}{\frac{5}{\sqrt{21}} \times 10^{-3}\text{m}^2} = 2\sqrt{21} \times 10\text{ m/s}$$

When it hits a rigid wall then maximum possible increase in temperature of water can be expressed as

$$\frac{1}{2}mv^2 = ms\Delta T \quad \ldots(i)$$

where, m = mass, s = specific heat of water

$$= 1 \text{ cal/g} = 4.2 \times 10^3 \text{ J/kg}$$

and ΔT = increase in temperature

From Eq. (i), $\Delta T = \frac{1}{2}\frac{v^2}{s}$

$$= \frac{(2\sqrt{21} \times 10^1)^2}{2 \times 4.2 \times 10^3}$$

$$= \frac{84 \times 10^2}{8.4 \times 10^3}$$

$$= 1°\text{C}$$

33. *(a)* Maximum loss in $K.E$ = Total $K.E$ energy before collision of two ice blocks

$$K.E_{\text{max loss}} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

This maximum loss in kinetic energy will be equal to loss in heat to melt two ice block.

$$\Rightarrow \quad mv^2 = (ms\,\Delta\theta + mL)2 \quad \ldots(i)$$

According to question,

$$L = 3.36 \times 10^5 \text{J kg}^{-1}$$

$$S = 2100 \text{ J kg}^{-1}\text{K}^{-1}$$

$$\Delta\theta = 8^\circ C$$

From Eq. (i) $v = \sqrt{2(s\,\Delta\theta + L)}$

$$= \sqrt{2(2100 \times 8 + 3.36 \times 10^5)}$$

$$v = 840 \text{ ms}^{-1}$$

34. *(a)* According to the question,

Charge on particle is = q

Velocity of particle = v

Due to uniform electric field,

electric force on particle

$$F_{\text{electric}} = qE \quad \ldots(i)$$

Due to a uniform magnetic field,

magnetic force on particle is given by,

$$F_{\text{magnetic}} = q(v \times B) \quad \ldots(ii)$$

When v is perpendicular to E and B, which are mutually perpendicular to each other,

$$F_{\text{electric}} = F_{\text{magnetic}}$$

From Eqs. (i) and (ii)

$$qE = qvB$$

$$\therefore \quad v = \frac{E}{B}$$

So, $v = \frac{E}{B}$ is the condition for which the particle moves undeflected in its original trajectory.

35. *(c)* In given, Series R-L-C circuit

Resistance $R = 100\ \Omega$

Inductance of Inductor, $L = 20 \text{ mH} = 20 \times 10^{-3}\text{H}$

Resonance frequency $f = \frac{1250}{\pi}$ Hz

Source voltage $V_{DC} = 25\text{V}$

According to resonant frequency,

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

or $$(2\pi f_0)^2 = \frac{1}{LC}$$

or $$4\pi^2 \frac{1250 \times 1250}{\pi \times \pi} = \frac{1}{LC}$$

or $$C = \frac{1000}{1250 \times 1250 \times 4 \times 20}$$

($\because$ By substituting $L = 20 \times 10^{-3}$)

or $$C = 8 \times 10^{-6}\text{F}$$

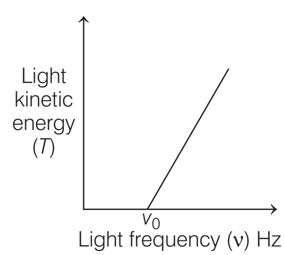
We know that, Charge $Q_S = CV$

$$Q_S = 8 \times 10^{-6} \times 25 = 0.2 \text{ mC}$$

So, the amount of charge stored in each plate of capacitor is 0.2 mC.

36. *(b,d)* In photoelectric effect, if the incident light had a frequency less than a minimum frequency ν_0, then no electrons are ejected regardless of the light's amplitude. This minimum frequency is also called the threshold frequency, and the value of ν_0 depends on the metal. For frequency greater than ν_0, electrons would be ejected from the metal. Furthermore, the kinetic energy of the photoelectrons is proportional to the light frequency (ν).

The relationship between photoelectron kinetic energy T and light frequency ν is shown in graph below



Graph shows that kinetic energy T is linearly increasing with light frequency (ν).

However, for a given photosensitive material and frequency of incident light, number of photoelectrons emitted per second is directly proportional to the intensity of incident light i.e., Number of electrons emitted $\propto J$.

Also, photoelectric emission is an instantaneous process.

37. ***(b)*** The equation of state for an ideal gas undergoing adiabatic process is given as,

$$TV^{\gamma-1} = \text{constant}$$

Let the temperature after adiabatic compression as given in the question be T then,

$$T_0 V^{\gamma-1} = T\left(\frac{V_0}{2}\right)^{\gamma-1}$$

$$\Rightarrow \quad T = T_0 2^{\gamma-1}$$

Now, heat released at volume $\frac{V_0}{2}$ to achieve temperature T_0.

The net heat released can be determined by the equation.

$$\Delta Q = \mu C_V \Delta T$$

$$= \mu \times \frac{R}{\gamma-1} \qquad (T_0 2^{\gamma-1} - T_0)$$

$$= \frac{\mu R T_0}{\gamma-1}(2^{\gamma-1} - 1) \; (\because \mu T_0 R = p_0 V_0)$$

$$\therefore \text{ Heat released} = \frac{p_0 V_0}{\gamma-1}(2^{\gamma-1} - 1)$$

38. ***(a, d)*** Given, initial velocity of particle,

$$u = 10 \text{ ms}^{-1}$$

Range, $R = 5$ m

Gravitational acceleration,

$$g = 10 \text{ ms}^{-2}$$

As we known that,

Range of projectile, $R = \frac{u^2 \sin 2\theta}{g}$

$$5 = \frac{(10)^2 \sin 2\alpha}{10}$$

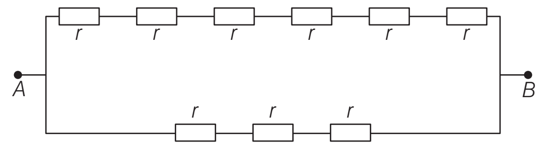
$$\sin 2\alpha = \frac{5}{10}$$

$$\sin 2\alpha = \frac{1}{2}$$

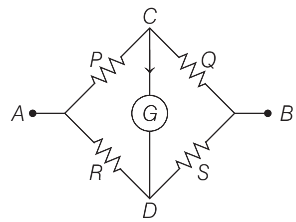
$$\alpha = 15^\circ$$

or $\qquad (90^\circ - 15^\circ = 75^\circ)$

39. ***(a, c)*** The circuit diagram is as shown below,



According to question, when given pair of point in options are connected through a resistance R, then equivalent resistance between point A and B (R_{AB}) remains unchanged. It is only possible when current does not flows through resistance R and circuit become Wheatstone bridge as shown in the figure below,

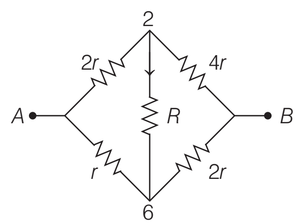


At the balanced condition, $\frac{P}{Q} = \frac{R}{S}$

The current flow in CD branch will be zero.

Now by checking each option,

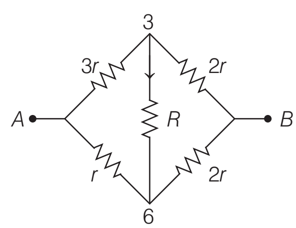
from option (a), circuit is,



Now, $\because \frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{2r}{4r} = \frac{r}{2r} \Rightarrow \frac{1}{2} = \frac{1}{2}$

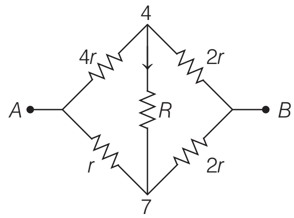
Hence, option (a) is correct.

From option (b), circuit is



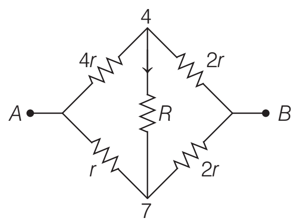
Now, $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{3r}{3r} \neq \frac{r}{2r}$

So, option (b) is also incorrect.
From option (c) circuit is,



Now, $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{4r}{2r} = \frac{2r}{r} \Rightarrow \frac{2}{1} = \frac{2}{1}$

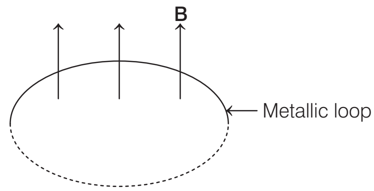
So, option (c) is also correct.
From option (d), circuit is



Now, $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{4r}{2r} \neq \frac{r}{2r}$

So, option (d) is also incorrect.
Hence, option (a) and (c) are correct.

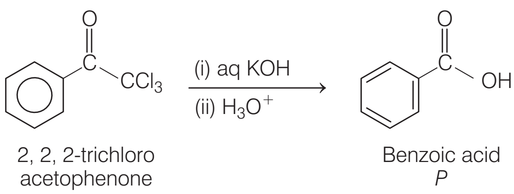
40. *(b, d)* According to the question,



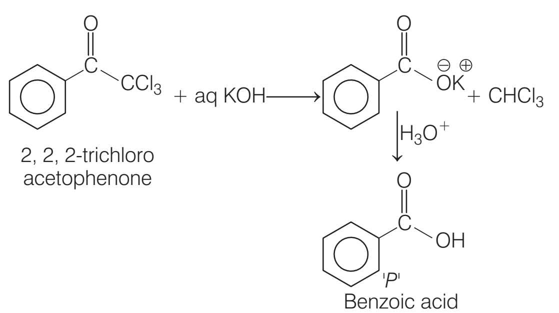
Given, A metallic loop is placed in uniform magnetic field B. Magnetic field B and metallic loop are perpendicular to each other. Then, if metallic loop is moved along B or rotated about its own axis, the net flux associated with it remains constant. Thus, no emf will be induced in these cases. However, when the loop is squeezed to a smaller area or rotated about one of its diameters. Then its flux changes. Thus, emf is induced. So, option (b) and (d) both are correct.

Chemistry

41. *(c)* 2, 2, 2-trichloroacetophenone reacts with aqueous KOH to form potassium Salt of benzoic acid, which undergoes hydrolysis to form benzoic acid as the final product.

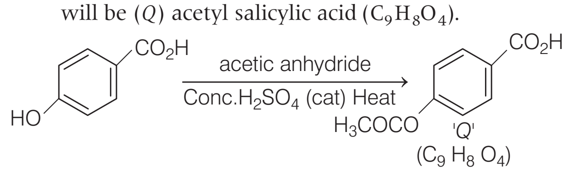


Mechanism

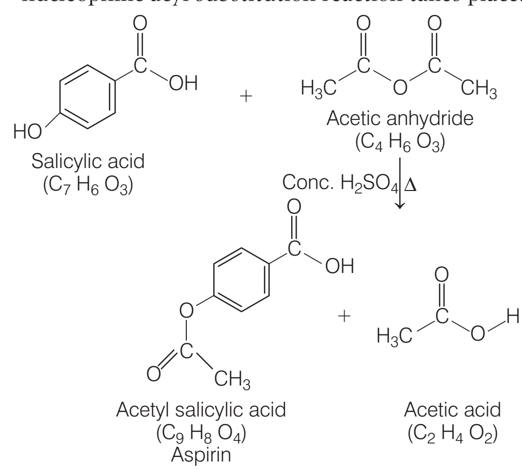


The product of the above reaction P is benzoic acid.
Thus, option (c) is correct answer.

42. *(a)* When salicylic acid reacts with acetic anhydride in the presence of conc.H_2SO_4 and heat, the product will be (Q) acetyl salicylic acid ($C_9H_8O_4$).

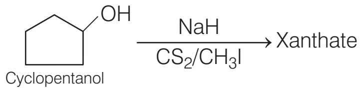


Mechanism In presence of acid anhydride, nucleophilic acyl substitution reaction takes place.

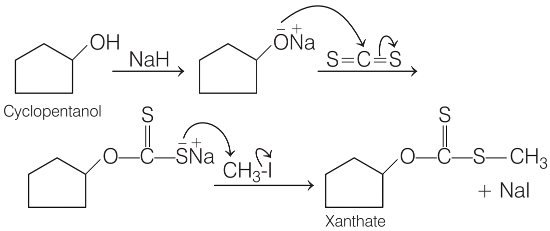


So, the option (a) is correct answer.

43. *(d)* When cyclopentanol on reaction with NaH followed by CS_2 and CH_3I produces a xanthate.



Mechanism



So, the option (d) is correct answer.

44. *(b)* $CH_3-\underset{\underset{O}{\|}}{C}-Cl$ hydrolyses to form CH_3COOH even at 25°C, which subsequently reacts with $NaHCO_3$ (sodium bicarbonate) present in the same medium to form carbon dioxide (CO_2).

The reaction involved is given below.

$$CH_3COCl + NaHCO_3 \longrightarrow CH_3COONa + HCl + CO_2$$

Thus, the option (b) is correct answer.

45. *(a)* In $\underset{\uparrow}{B}H_4^-$ indicated atom is not a nucleophilic site because there is no lone-pair on 'B' atom.

Hence, the option (a) is correct answr.

46. *(b)* One millimole = 0.001 mol

i.e. 1×10^{-3} mol.

$\because$ 96500 C is required for 1 mol of electrons.

$\therefore$ 193 C give $\frac{193 \times 1}{96500} = 0.002$ mol, i.e. 2×10^{-3} mol of electrons.

Thus, value of n in M^{n+} is $= \frac{0.002}{0.001} = 2$

Hence, option (b) is the correct answer.

47. *(c)* **Key Point**

For buffer solutions,

$$pH = pK_a + \log \frac{[Salt]}{[Acid]/[Base]}$$

Moles of CH_3COOH = 5mL × 0.01 m mol = 0.5 m mol

Moles of CH_3COONa = 10 mL × 0.05 m mol

= 0.5 m mol

Total volume of solution = 5 + 10 = 15 mL

$$\therefore \quad pH = pK_a + \log \frac{0.5/15}{0.5/15} \Rightarrow pH = pK_a$$

The value of pK_a is the lowest for the mixture of CH_3COOH and CH_3COONa (acidic buffer). Therefore, it has lowest pH.

Hence, pH value for option (c) is the lowest.

48. *(b)* Given, for two first order reactions.

$A \rightarrow B$, $k = 0.693 \text{ min}^{-1}$ ...(1)

and $A \rightarrow C$, $t_{1/2} = 0.693$ min ...(2)

For first order reaction

$$t_{1/2} = \frac{0.693}{k},$$

Thus, for the reaction $A \rightarrow B$

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.693} = 1.0 \text{ min}$$

Also, for first order reaction, lower the $t_{1/2}$ value, faster is the reaction.

$$\because \quad t_{1/2} \propto \frac{1}{k}$$

and k for $A \rightarrow C$ is $\frac{0.693}{0.693} = 1 \text{ min}^{-1}$

Thus, reaction (2) is faster than reaction (1) or reaction (1) is slower than reaction (2).

Hence, option (b) is the correct answer.

49. *(d)* For the given relation $H_2O(l) \rightleftharpoons H_2O(v)$.

(i) At equilibrium $\Delta G = 0$

(ii) Entropy change, i.e. ΔS is the measurement of randomness. It increases in vapour-state, i.e. $\Delta S = (+)$ ve or $\Delta S > 0$

(iii) When liquid H_2O changes into vapour H_2O, it requires heat energy. Thus, $\Delta H > 0$

Hence, correct set is

$$\Delta G = 0, \Delta H > 0 \text{ and } \Delta S > 0$$

So, option (d) is the correct answer.

50. *(b)* Dimensions of $\left(\frac{ab}{V^2}\right)$ are shown below.

$$\frac{a}{V^2} \times b = \text{pressure} \times \text{volume}$$

$$= \frac{\text{force}}{\text{area}} \times \text{volume}$$

$$= \frac{MLT^{-2}}{L^2} \times L^3 = ML^2T^{-2}$$

The units of energy (joule) is,

$$1 \text{ J} = 1 \text{ kg m}^2\text{s}^{-2} = ML^2T^{-2}$$

Therefore, the dimensions of $\left(\frac{ab}{V^2}\right)$ is same as that of energy. So, the option (b) is correct.

51. *(c)* In the equilibrium $H_2 + I_2 \rightleftharpoons 2HI$, if at a given temperature, the concentrations of the reactants are increased, the value of the equilibrium constant K_C will remain the same because equilibrium constant does not depend on the molar concentration of reactants.

The option (c) is correct answer.

52. *(d)* Electrolysis of copper sulphate solution using Cu-electrodes,

Ions present : Cu^{2+}, H^+, SO_4^{2-}, OH^-

At cathode $Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$

At anode $Cu(s) - 2e^- \rightarrow Cu^{2+}(aq)$

So, the option (d) is correct answer.

53. *(c)* The three quantum numbers n, l and m that describe an orbital has integer values of 0, 1, 2, 3 and so on.

Name	Symbol	Range of Values
Principal quantum number	n	$1 \le n$
Azimuthal quantum number	l	$0 \le l \le n-1$
Magnetic quantum number	m	$-l \le m \le l$

The electronic arrangement of option (c) is absurd because

$$n = 2$$
$$l = 0$$
$$m = 0$$

To sum up, where $n = 2$, $l = 0$, $m = 0$

Hence, for $l = 0$, value of m should be zero.

54. *(c)* The quantity hv / K_B corresponds to temperature. $hv = 3/2\, K_B T$

where, T = temperature

K_B = Boltzmann constant

The percentage of H_2O in the solid is

$$= (100 - 64) = 36\%$$

$$\frac{hv}{K_B} = \frac{3}{2}T$$

Since, $\frac{3}{2}$ is a constant, the value of $\frac{hv}{K_B}$ corresponds to temperature. Therefore option (c) is correct.

55. *(c)* Mass of $H_2O = \frac{36}{100} \times 250 = 90$ g

Moles of $H_2O = \frac{90}{18} = 5$ mol

In the crystalline solid $MSO_4 \cdot nH_2O$, the value of n is 5.

Hence, option (c) is correct answer.

56. *(a)* Given, mass of gas (W) = 7.5 g

Volume of gas at STP (V) = 5.6 L

$\because$ Moles of gas at STP $= \frac{V(L)}{22.4(L)} = \frac{W}{M}$

Where M = molar mass of gas.

$$\therefore \quad M = \frac{W \times 22.4}{V}$$

$$= \frac{7.5 \times 22.4}{5.6} = 30.00 \text{ g mol}^{-1}$$

Among the given options

Molar mass(M) of

(a) $NO = 14 + 16 = 30.00$ g mol^{-1}

(b) $N_2O = 28 + 16 = 44.00$ g mol^{-1}

(c) $CO = 12 + 16 = 28.00$ g mol^{-1}

(d) $CO_2 = 12 + 32 = 44.00$ g mol^{-1}

$\because$ Molar mass of NO = 30 g mol^{-1}.

Thus, the given gas is NO.

Hence, (a) is the correct option.

57. *(b)* Let, initial concentration $(a) = 100$

Given, half-life $(t_{1/2})$ = 60 days

To find, radioactivity,

i.e. $(a - x)$ after time T (180 days)

$$\because \quad T = n \times t_{1/2}$$

where, n = no. of half-lives

$$180 = n \times 60$$

$$n = \frac{180}{60} = 3$$

and $(a - x) = \frac{a}{2^n} = \frac{100}{2^3} = \frac{100}{8}$

$$(a - x) = 12.5\%$$

Hence, (b) is the correct answer.

58. *(d)* Given, ${}_{82}A^{210} \longrightarrow B \longrightarrow C \longrightarrow {}_{82}D^{206}$

i.e. difference in mass no. between A and D is of 4 units i.e., mass no. is decreased by 4 units while atomic no. remains the same, i.e. 82 for A and D.

Also,

(i) On emission of each β-particle, mass no. remains the same but atomic no. is increased by one unit.

(ii) On emission of each α-particle, mass no. is decreased by 4 units, while atomic no. is decreased by 2 units.

Thus, on emission of 2-β and 1-α particle, we get ${}_{82}D^{206}$,

i.e. $${}_{82}A^{210} \xrightarrow{-\beta} {}_{83}A^{210} \xrightarrow{-\beta} {}_{84}A^{210} \xrightarrow{-\alpha} {}_{82}A^{206}$$

Thus, option (d) is the correct answer.

59. *(a)* **Key Point** *For similar electronic configuration of outer most shell, size will decide the value of ionisation energy. More the size, lesser is the value of ionisation energy.*

Electronic configuration	IInd I.E
Zn^+ ion ($Z = 30$) = [Ar] $3d^{10} 4s^1$,	(1734 kJ/mol).
Cd^+ ion ($Z = 48$) = [Kr] $4d^{10} 5s^1$,	(1631 kJ/mol)
Hg^+ ion ($Z = 80$) = [Xe] $4f^{14} \cdot 5d^{10}\ 6s^1$	(1809 kJ/mol)

∵ Size of $Cd^+ > Zn^+$, thus it has lower IInd ionisation energy while due to lanthanoid effect (i.e., poor screening by $4f$ and $5d$ electrons, Hg^+ has higher IInd ionisation-energy.

Hence, correct order is,

Zn > Cd < Hg and option (a) is the correct answer.

60. *(b)* **Key Point** *A compound having more ionic character, has more melting point.*

Be and Ca belong to the same group and ionic character increases down the group.

Thus, $BeCl_2$ is less ionic than $CaCl_2$. This means that melting point of $CaCl_2$ is higher than that of $BeCl_2$.

Also, Hg is a transition metal, its compound is less ionic than $BeCl_2$. Therefore, order of melting point will be

$$CaCl_2 > BeCl_2 > HgCl_2$$

or (ii) > (i) > (iii)

Hence, option (b) is the correct answer.

61. *(d)* **Key point** *The species which has one or more unpaired electrons have non-zero magnetic moment.*

Electronic configuration of

$Na^+ (Z = 11) = 1s^2\ 2s^2\ 2p^6$

$Mg\ (Z = 12) = 1s^2\ 2s^2\ 2p^6\ 3s^2$

$F^- (Z = 9) = 1s^2\ 2s^2\ 2p^6$

$Ar^+ (Z = 18) = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^5$

As, Ar^+ has an unpaired electron, it has non-zero magnetic moment. Hence, (d) is the correct option.

62. *(b)* **Key Point**

(i) *Half-filled/fully-filled configuration have* (–) ve *or low electron affinity i.e.,* (+) ve *or high electron gain enthalpy.*

(ii) *Smaller the size, more is value of electron affinity, i.e. more* (–) ve *will be the electron gain enthalpy.*

On moving across a period, the size of atoms decreases due to increase in nuclear charge.

(i) Electron configuration of the elements are shown below.

$C(Z = 6) = 1s^2\ 2s^2\ 2p^2$

$N(Z = 7) = 1s^2\ 2s^2\ 2p^3$

$O(Z = 8) = 1s^2\ 2s^2\ 2p^4$

(ii) Due to half-filled electronic configuration of nitrogen (i.e. 3 electrons in $3p$ orbitals), it is highly stable and has (+) ve value of electron gain enthalpy (i.e. about 30.9 kJ/mol).

(iii) As size of 'O', is smaller than that of 'C', 'O' atom has higher (–) ve electron gain enthalpy (about – 141.1 kJ / mol) than that of carbon (C-atom) [about – 122.3 kJ / mol].

Hence, correct order is O > C > N and (b) is the correct option.

63. *(a)* **Key point** *As the percentage of s character in a bond increases, the bond angle also increases. Therefore, as the bond angle increases, the percentage of p-character decreases.*

In the given options,

Ammonia (NH_3) has bond angle = 107.6°

and Phosphine (PH_3) has bond angle = 93.5°

∵ Bond angle $\propto \dfrac{1}{p\text{ - character}}$ and $\propto s$-character

Bond angle of $NH_3 > PH_3$ due to lone-pair lone-pair repulsion.

Thus, the lone pair on NH_3 has less p-character.

Hence, option (a) is the correct answer.

64. *(b)* Chlorine bleach is NaOCl. The hypochlorite ion in chlorine bleach dissociates to give nascent oxygen as shown below.

$$OCl^- \longrightarrow [O] + Cl^-$$

Therefore, the reactive species in chlorine bleach is OCl^-.

So, the option (b) is correct.

65. *(d)* Co-ordinate compound of Co (III) dissociates into 3 ions in the solution means it has two ionisable-ions out side the coordination sphere with oxidation state of cobalt (Co) = + 3

In option (a)

Name : Hexaammine cobalt (III) chloride

Formula : $[Co(NH_3)_6]Cl_3$

∵ Oxidation state of Co = (+) 3

In option (b)

Name : Pentaammine sulphatocobalt (III) chloride

Formula : $[Co(NH_3)_5(SO_4)]Cl$

$\because$ Oxidation state of $Co = +3$

In option (c)

Name : Pentaamminechloridocobalt (III) sulphate

Formula : $[Co(NH_3)_5Cl]SO_4$

$\because$ Oxidation state of $Co = +3$

In option (d)

Name : Pentaamminechloridocobalt (III) chloride

Formula : $[Co(NH_3)_5Cl]Cl_2$

$\because$ Oxidation state of $Co = +3$

Dissociation of this compound is shown below:

$$[CO(NH_3)_5Cl]Cl_2 \rightarrow \underbrace{CO(NH_3)_5Cl]^{2+} + 2Cl^-}_{\text{3 ions}}$$

This compound can give total 3-ions, as follows.

one $[Co(NH_3)_5Cl]^{2+}$ and two Cl^- ions.

Hence, option (d) is the correct answer.

66. *(b)* In the Bayer's process, the leaching of alumina is done by using NaOH.

$$\underset{\text{Bauxite ore}}{Al_2O_3(s)} + 2NaOH(aq) \xrightarrow{150^\circ C,\ 35\ atm} \underset{\substack{\text{Sodium}\\\text{aluminate}\\\text{(soluble in water)}}}{2Na[Al(OH)_4]} + 3H_2O$$

$$2Na[Al(OH)_4] + CO_2 \rightarrow Al_2O_3 \cdot xH_2O + 2NaHCO_3$$

$$Al_2O_3 \cdot xH_2O(s) \xrightarrow{1470\ K} \underset{\text{Pure alumina}}{Al_2O_3(s)} + xH_2O(g)$$

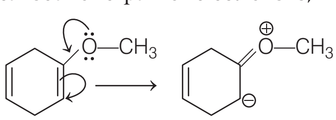
So, the option (b) is correct answer.

67. *(d)* Among the given options,

$_{92}U^{238}$ is an isotope of uranium but cannot be used as a nuclear-fuel. U-238 is non-fissile, i.e., it does not undergo fission by thermal neutrons. The energy released when U-238 absorbs a neutron is insufficient to carry out nuclear fission.

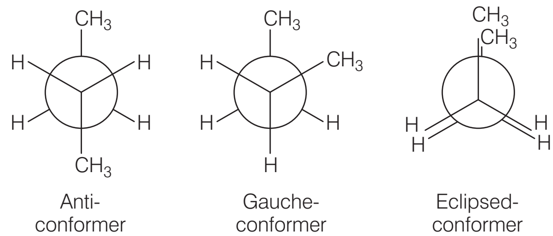
Hence, (d) is the correct option.

68. *(d)* The molecule in which a lone pair of electrons undergoes conjugation with an alternate double bond is said to be delocalised. The molecule that has delocalised lone-pair of electrons is,



So, option (d) is correct.

69. *(d)* The conformation of *n*-butane, commonly known as eclipsed, gauche and anti-conformations can be interconverted by rotation around C2-C3 linkage.



Hence, the option (d) is correct answer.

70. *(b)* In halogen acids, as the size of halogen atom increases, the bond between halogen and hydrogen atom weakens. Therefore, the case with which the bond can be broken increases.

The correct order of the addition reaction rates of halogen acids with ethylene is hydrogen iodide (HI) > Hydrogen bromide (HBr)

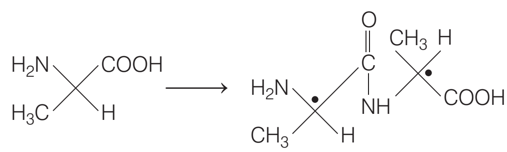
> Hydrogen chloride (HCl)

So, the option (b) is correct answer.

71. *(d)* Total number of isomeric linear dipeptides which can be synthesised from racemic mixture of alanine are four (4), by (*R*) type and (*S*) type.

($\because$ Has two chiral centres)

i.e.



The possible isomers are

(*RR*), (*SR*), (*RS*) and (*SS*).

Hence, total (4) isomeric linear dipeptides for alanine can be synthesised and option (d) is the correct answer.

72. *(d)* From the given graph

$$\text{Slope} = \frac{(\text{rate})^{1/2}}{[A]} = 4$$

$\because$ rate $(r) = k[A]^n$

Where, k = rate constant

$[A]$ = concentration at time t (in min).

n = order.

According to the graph $(n = 2)$

$$\therefore \quad k = \frac{\text{rate}}{[A]^n} = [4]^2 = 16$$

Hence, $n = 2$ and

$k = 16.0\ dm^3\ mol^{-1}\ min^{-1}$

and option (d) is the correct answer.

73. *(a)* **Key Point**

According to Kirchhoff's equation

$$\Delta H_{(f)} = \Delta H_{(i)} + C_P(\Delta T)$$

At constant pressure

$$\Delta C_p = O$$

Therefore, heat of formation (ΔH_f) does not depend on temperature and value of ΔC_p,

i.e. when $\Delta C_p = 0$

Hence, option (a) is the correct answer.

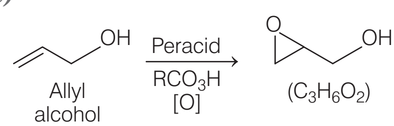
74. *(d)* A copper coin was electroplated with zinc (Zn) and then heated at high temperature until there is a change in colour. The resulting colour will be golden.

The golden colour is due to the zinc migrating through the copper to form the alpha-form of brass alloy (percentage of Cu > 65% and that of Zn < 35%).

$$\text{Zn} + \text{Cu} \xrightarrow{\Delta} \text{Brass}$$

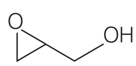
Thus, the option (d) is correct answer.

75. *(a)*



Oxidation of allyl alcohol with a peracid gives a compound of molecular formula $C_3H_6O_2$, which contains an asymmetric carbon atom.

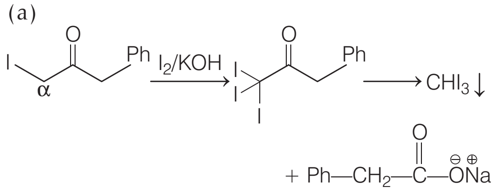
The structure of the compound is



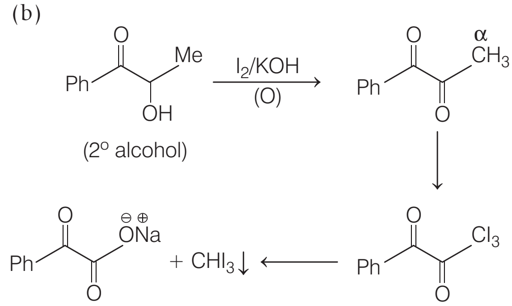
So, the option (a) is correct answer.

76. *(a, b)* Haloform reaction with I_2 and KOH

(a)



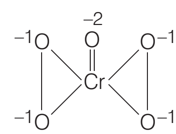
(b)



Thus, options (a, b) are correct.

77. *(a, b)*

(a) In CrO_5 oxidation no. of Cr is + 6.



(b) For the reaction (in ideal case)

$$N_2O_4(g) \longrightarrow 2NO_2(g)$$

$$\Delta H = \Delta U + p.\Delta V$$

$\because \quad \Delta H = \Delta U + \Delta n_g RT$

($\because p\Delta V = \Delta n_g RT$)

Where, Δn_g = (gaseous moles of product) − (gaseous moles of reactant)

$$\Delta n_g = 2 - 1 = 1$$

Thus, $\Delta H > \Delta U$

(c) pH of 0.1 N H_2SO_4 is less than of 0.1 HCl at 25° is a wrong statement.

pH of H_2SO_4

$\because \quad pH = -\log[H^+] = -\log[10^{-2}]$

$\because \quad (N = C \times Z)$ and $(Z = 2$ for $H_2SO_4)$

pH = 2 and pH of HCl

$pH = -\log[10^{-1}] = 1$

Thus, pH of H_2SO_4 > pH of HCl.

(d) $\frac{RT}{F} = 0.0591$ at 25°C is a wrong statement.

Where, R = Gas constant = 8.314

T = Temperature = 273 + 25 = 298 K

F = Charge over one mole of electrons = 96500C

Thus, $\frac{RT}{F} = \frac{8.314 \times 298}{96500} = 0.02567$

The correct value is $\frac{2.303\,RT}{F} = 0.0591$

Hence, only option (a) and (b) are correct.

78. *(b,c,d)* Among the given species

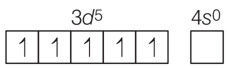
(a) H_2O and F are weak field ligands and do not cause pairing of electrons in the central metal atom. As CN is a strong field ligand, it causes pairing of electrons in the central metal atom.

Now,

Oxidation state of :

(b) Fe in $[Fe(H_2O)_6]\,Cl_3 = (+)\ 3$ and electronic configuration of Fe^{3+} ions is $3d^5 4s^0$

$Fe^{3+} =$



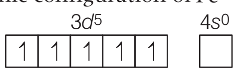
i.e. have 5 unpaired electrons

(c) Fe in $K_3[FeF_6]$

Oxidation no. of Fe = (+) 3

electronic configuration of Fe^{3+} is $3d^5 4s^0$

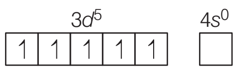
$Fe^{3+} =$



i.e. have 5 unpaired electrons.

(d) Oxidation state of Mn in $K_4[MnF_6] = (+)\ 2$ and electronic configuration of Mn^{2+} is $3d^5 4s^0$

$Mn^{2+} =$



i.e., have 5 unpaired electrons.

Hence, options (b) (c) and (d) are the correct options.

79. *(a, c)*

(a) NH_4NO_3 will evolve NH_3 pungent smelling gas with NaOH.

- NO_2 brown gas with con.H_2SO_4
- No reaction with water

Thus, NH_4NO_3 can be used to label all three beakers.

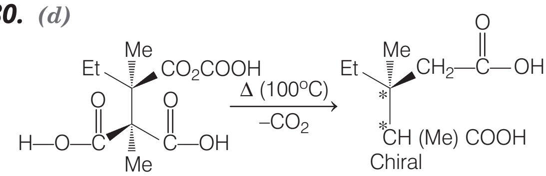
(c) $(NH_4)_2CO_3$ will evolve pungent smelling gas NH_3 with NaOH.

- Effervescence of CO_2 gas with con.H_2SO_4
- No reaction with water.

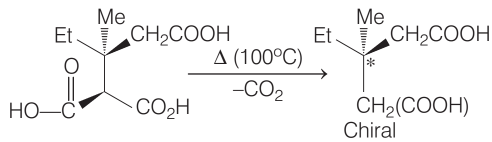
Thus, $(NH_4)_2CO_3$ can be used to label all three beakers.

Thus, options (a, c) are correct.

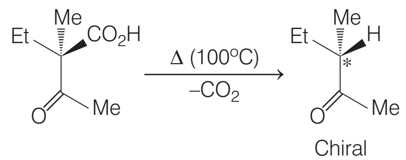
80. *(d)*



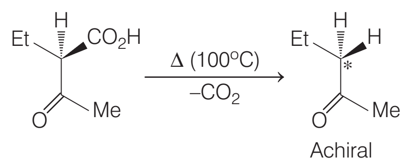
(b)



(c)



(d)



Thus, compound (d), capable of producing achiral compound on heating at 100°C.

Mathematics

1. *(b)* We have,

$$\lim_{x \to 0^+} x^n \ln x$$

$$= \lim_{x \to 0^+} \frac{\ln x}{x^{-n}}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-nx^{-n-1}} \quad \text{[using L'Hospital's rule]}$$

$$= \lim_{x \to 0^+} \frac{-1}{nx^{-n}} = 0 \quad \left[\because \frac{1}{x-n} = 0, \text{ when } x = 0\right]$$

2. *(b)* Let $I = \int \cos x \log\left(\tan\frac{x}{2}\right) dx$

$$= \log\left(\tan\frac{x}{2}\right)\cdot \sin x - \int \sin x \cdot \frac{1}{\tan\frac{x}{2}} \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2}\, dx$$

$$= \sin x \log\left(\tan\frac{x}{2}\right) - \int \sin x \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx$$

$$= \sin x \cdot \log\left(\tan\frac{x}{2}\right) - \int \frac{\sin x}{\sin x} dx$$

$$= \sin x \cdot \log\left(\tan\frac{x}{2}\right) - \int 1\, dx$$

$$= \sin x \log\left(\tan\frac{x}{2}\right) - x + c$$

$\therefore f(x) = c - x$

3. *(d)* Let $I = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\} dx$

Put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta\, d\theta$

$$\therefore \quad I = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}(-2\sin 2\theta)\, d\theta$$

$$= -2\int \cos\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\}\sin 2\theta\, d\theta$$

$$= -2\int \cos\{2\tan^{-1}(\tan\theta)\}\sin 2\theta\, d\theta$$

$$= -2\int \cos 2\theta \sin 2\theta\, d\theta$$

$$= \int \cos 2\theta\, d(\cos 2\theta)$$

$$= \frac{\cos^2 2\theta}{2} + C$$

$$= \frac{x^2}{2} + C$$

Which is an equation of parabola.

4. *(b)* Let $I = \int_{-\frac{\pi}{4}}^{\pi/4}\left(\lambda|\sin x| + \frac{\mu \sin x}{1+\cos x} + \gamma\right) dx$

$$= \int_{-\pi/4}^{\pi/4}(\lambda|\sin x| + \gamma)\, dx + \mu\int_{-\frac{\pi}{4}}^{\pi/4}\frac{\sin x}{1+\cos x} dx$$

Let $\quad f(x) = \frac{\sin x}{1+\cos x}$

$$\Rightarrow f(-x) = \frac{\sin(-x)}{1+\cos(-x)} = \frac{-\sin x}{1+\cos x} = -f(x)$$

$\therefore$ $f(x)$ is an odd function.

So, $\int_{-\frac{\pi}{4}}^{\pi/4}\left(\frac{\sin x}{1+\cos x}\right) dx = 0$

$$\therefore \quad I = \int_{-\frac{\pi}{4}}^{\pi/4}\lambda|\sin x| + \gamma)\, dx$$

$\therefore$ I is independent of μ.

5. *(a)* We have,

$$\lim_{x \to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}dt\right]$$

$$= \lim_{x \to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt + \int_a^{x+y} e^{\sin^2 t}dt\right]$$

$$= \lim_{x \to 0}\frac{1}{x}\int_y^{x+y} e^{\sin^2 t}\, dt$$

$$= \lim_{x \to 0}\frac{\int_y^{x+y} e^{\sin^2 t}dt}{x}$$

$$= \lim_{x \to 0}\frac{e^{\sin^2(x+y)}(1+0) - 0}{1} = e^{\sin^2 y}$$

6. *(d)* Let $I = \int 2^{2^x}\cdot 2^x\, dx$

Put $\quad t = 2^x$

$\Rightarrow \quad dt = 2^x \log 2\, dx$

$$\therefore \quad I = \int \frac{2^t}{\log 2} dt$$

$$= \frac{2^t}{(\log 2)^2} + C = \frac{2^{2^x}}{(\log 2)^2} + C$$

$$\therefore \quad A = \frac{1}{(\log 2)^2}$$

7. *(d)* Let $I = \int_{-1}^{1} \left\{ \frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} + \frac{1}{e^{|x|}} \right\} dx$

$$= \int_{-1}^{1} \frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} dx + \int_{-1}^{1} \frac{1}{e^{|x|}} dx$$

Let $f(x) = \frac{x^{2015}}{e^{|x|}(x^2 + \cos x)}$ and $g(x) = \frac{1}{e^{|x|}}$

Again, $f(-x) = \frac{(-x)^{2015}}{e^{|-x|}((-x)^2 + \cos(-x))}$

$$= \frac{-x^{2015}}{e^{|x|}(x^2 + \cos x)} = -f(x)$$

and $g(-x) = \frac{1}{e^{|-x|}} = \frac{1}{e^{|x|}} = g(x)$

$\because$ $f(x)$ is odd function and $g(x)$ is even function.

$\therefore$ $\int_{-1}^{1} f(x)\, dx = 0$ and $\int_{-1}^{1} g(x)\, dx = 2\int_{0}^{1} g(x)\, dx$

$\therefore$ $I = 2\int_0^1 \frac{1}{e^{|x|}} dx$

$$= 2\int_0^1 \frac{1}{e^x} dx = 2\int_0^1 e^{-x}\, dx$$

$$= -2[e^{-x}]_0^1$$

$$= -2[e^{-1} - e^0]$$

$$= -2(e^{-1} - 1)$$

$$= 2(1 - e^{-1})$$

8. *(c)* $\lim_{n \to \infty} \frac{3}{n}\left[1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}}\right]$

$$= \lim_{n \to \infty} \frac{3}{n}\left[\sqrt{\frac{n}{n+3\times 0}} + \sqrt{\frac{n}{n+3\times 1}} + \sqrt{\frac{n}{n+3\times 2}} + \sqrt{\frac{n}{n+3\times 3}} + \dots + \sqrt{\frac{n}{n+3(n-1)}}\right]$$

$$= \lim_{n \to \infty} \frac{3}{n}\left[\sum_{r=0}^{n-1} \sqrt{\frac{n}{n+3r}}\right]$$

$$= \lim_{n \to \infty} \frac{3}{n}\left[\sum_{r=0}^{n-1} \sqrt{\frac{1}{1 + 3\left(\frac{r}{n}\right)}}\right]$$

$$= 3\lim_{n \to \infty} \frac{1}{n} \sum_{r=0}^{n-1} \sqrt{\frac{1}{1 + 3\left(\frac{r}{n}\right)}}$$

$$= 3\int_0^1 \sqrt{\frac{1}{1+3x}}\, dx$$

$$= 3\int_0^1 (1+3x)^{-1/2}\, dx$$

$$= 3\left[\frac{2\sqrt{1+3x}}{3}\right]_0^1$$

$$= [4 - 2]$$

$$= 2$$

9. *(c)* $\left(1 + e^{\frac{x}{y}}\right) dx + \left(1 - \frac{x}{y}\right) e^{x/y}\, dy = 0$

$\Rightarrow$ $(1 + e^{x/y})dx = -e^{x/y}\left(1 - \frac{x}{y}\right) dy$

$\Rightarrow$ $\frac{dx}{dy} = \frac{-e^{x/y}(1 - x/y)}{(1 + e^{x/y})}$ ...(i)

This is homogeneous differential equation.

So, put $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy} \quad \text{...(ii)}$$

Therefore, $v + y\frac{dv}{dy} = \frac{-e^v(1-v)}{1+e^v}$

[from Eqs. (i) and (ii)]

$\Rightarrow$ $y\frac{dv}{dy} = \frac{-e^v + ve^v}{1+e^v} - v$

$$= \frac{-e^v + ve^v - v(1+e^v)}{1+e^v}$$

$$= -e^v + ve^v - v - ve^v$$

$\Rightarrow$ $y\frac{dv}{dy} = \frac{-(e^v + v)}{1+e^v}$

$\Rightarrow$ $\frac{1+e^v}{v+e^v} dv = -\frac{dy}{y}$ [variable separation]

$\Rightarrow$ $\int \frac{1+e^v}{v+e^v} dv = -\int \frac{dy}{y}$ [by integration] ...(iii)

Let $v + e^v = t \Rightarrow (1 + e^v)\, dv = dt$

$\therefore$ $\int \frac{1+e^v}{v+e^v} dv = \int \frac{dt}{t} = \log t = \log(v + e^v)$ ...(iv)

$\Rightarrow$ $\log(v + e^v) = -\log y + \log C$

[from Eqs. (iii) and (iv)]

$\Rightarrow$ $\log(v + e^v) + \log y = \log C$

$\Rightarrow$ $y(v + e^v) = C$

$\Rightarrow$ $y\left(\frac{x}{y} + e^{x/y}\right) = C$

$$\Rightarrow \quad y\left(\frac{x + ye^{x/y}}{y}\right) = C$$

$$\Rightarrow \quad x + ye^{x/y} = C$$

10. *(d)* $(x + y)^2 \frac{dy}{dx} = a^2, a \neq 0$

Let $x + y = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dt}{dx} - 1$$

Therefore, $t^2\left(\frac{dt}{dx} - 1\right) = a^2$

$$\Rightarrow \quad t^2 \frac{dt}{dx} - t^2 = a^2$$

$$\Rightarrow \quad t^2 \frac{dt}{dx} = a^2 + t^2$$

$$\Rightarrow \quad \frac{t^2}{a^2 + t^2} dt = dx \quad \text{[variable separation]}$$

$$\Rightarrow \quad \int \frac{t^2}{(t^2 + a^2)} dt = \int dx \quad \text{[integration]}$$

$$\Rightarrow \quad \int \frac{t^2 + a^2 - a^2}{(t^2 + a^2)} dt = x + C'$$

[doing a^2 adding and subtracting]

$$\Rightarrow \quad \int \left(1 - \frac{a^2}{t^2 + a^2}\right) dt = x + C'$$

$$\Rightarrow \quad \int dt - \int \frac{a^2}{t^2 + a^2} dt = x + C'$$

$$\Rightarrow \quad t - a^2 \int \frac{dt}{t^2 + a^2} = x + C'$$

$$\Rightarrow \quad t - a^2\left[\frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)\right] = x + C'$$

$$\Rightarrow \quad (x + y) - a\tan^{-1}\left(\frac{x + y}{a}\right)$$

$= x + C'$ [on putting the value of t]

$$\Rightarrow \quad \frac{y - C'}{a} = \tan^{-1}\left(\frac{x + y}{a}\right)$$

$$\Rightarrow \quad \frac{x + y}{a} = \tan\left(\frac{y - C'}{a}\right)$$

$$\Rightarrow \frac{x + y}{a} = \tan\left(\frac{y + C}{a}\right)$$

[where, $C = -C'$ is an arbitrary constant]

$$\Rightarrow \quad \tan\left(\frac{y + C}{a}\right) = \frac{x + y}{a}$$

11. *(b)* Given hyperbola equation

$$H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$P(4, 3)$ lie on H

$$\Rightarrow \quad \frac{16}{a^2} - \frac{9}{b^2} = 1$$

Normal equation at $P(4, 3)$ for H

$$a^2 y_1 (x - x_1) + b^2 x_1 (y - y_1) = 0$$

$$\Rightarrow \quad 3a^2(x - 4) + 4b^2(y - 3) = 0$$

Normal cuts the X-axis at $(16, 0)$

$$\Rightarrow \quad 3a^2(16 - 4) + 4b^2(0 - 3) = 0$$

$$\Rightarrow \quad 3a^2(12) + 4b^2(-3) = 0$$

$$\Rightarrow \quad 36a^2 - 12b^2 = 0$$

$$\Rightarrow \quad 3a^2 - b^2 = 0$$

$$\Rightarrow \quad 3a^2 = b^2$$

$\therefore$ Eccentricity of the hyperbola is

$$e = \frac{\sqrt{a^2 + b^2}}{a}$$

$$= \frac{\sqrt{a^2 + 3a^2}}{a} = \frac{\sqrt{4a^2}}{a} = \frac{2a}{a} = 2$$

12. *(b)* Let V be the volume of spherical ballon of radius r.

Then, $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \quad \log V = \log\left(\frac{4\pi}{3}\right) + 3\log r \Rightarrow \frac{1}{V}\frac{dV}{dr} = 0 + \frac{3}{r}$$

$$\Rightarrow \quad \frac{1}{V}\frac{dV}{dr} = \frac{3}{r} \Rightarrow \frac{1}{V}\frac{\Delta V}{\Delta r} = \frac{3}{r}$$

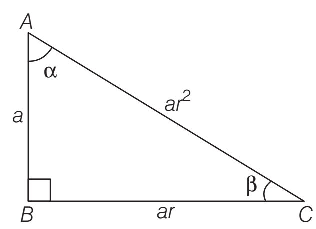
$$\Rightarrow \quad \frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$

$$\Rightarrow \quad \frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100$$

$$\Rightarrow \quad \frac{\Delta V}{V} \times 100 = 3 \times 0.1 = 0.3$$

$\therefore$ Percentage increase in volume is 0.3%.

13. *(b)* Let ΔABC be a right angled triangle at B. Let $\angle A$ and $\angle C$ be α and β



Since, sides are in GP so sides are a, ar, ar^2

Now, $AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (ar^2)^2 = a^2 + (a \cdot r)^2$

$\Rightarrow \quad a^2r^4 = a^2 + a^2r^2$

$\Rightarrow \quad r^4 - r^2 - 1 = 0$

$\Rightarrow \quad r^2 = \dfrac{1+\sqrt{5}}{2} \qquad [r^2 > 0]$

$\Rightarrow \quad r = \sqrt{\dfrac{\sqrt{5}+1}{2}}$

$\therefore \quad \tan\alpha = \dfrac{BC}{AB} = \dfrac{ar}{a}$

$\Rightarrow \quad r = \sqrt{\dfrac{\sqrt{5}+1}{2}}$

and $\tan\beta = \dfrac{AB}{BC} = \dfrac{a}{ar} = \dfrac{1}{r}$

$= \dfrac{1}{\sqrt{\dfrac{\sqrt{5}+1}{2}}} = \sqrt{\dfrac{\sqrt{5}-1}{2}}$

14. *(b)* We have,

$$\log_2^6 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$$

$\Rightarrow \quad 1 + \log_2^3 + \dfrac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$

$\Rightarrow \quad \dfrac{2^{\frac{1}{x}} + 8}{3} = 2^{1+\frac{1}{2x}}$

$\Rightarrow \quad 2^{\frac{1}{x}} + 8 = 3 \cdot 2 \cdot 2^{\frac{1}{2x}}$

$\Rightarrow \quad 2^{\frac{1}{x}} + 8 = 6 \cdot 2^{\frac{1}{2x}}$

Let $\quad y = 2^{\frac{1}{2x}}$

$\Rightarrow \quad y^2 + 8 = 6y$

$\Rightarrow \quad y^2 - 6y + 8 = 0$

$\Rightarrow \quad (y-4)(y-2) = 0$

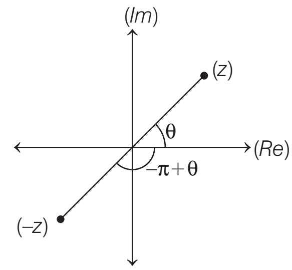
$\Rightarrow \quad y = 4, 2$

$\Rightarrow \quad 2^{\frac{1}{2x}} = 4$ and $2^{\frac{1}{2x}} = 2$

$\Rightarrow \quad \dfrac{1}{2x} = 2$ and $\dfrac{1}{2x} = 1$

$\Rightarrow \quad x = \dfrac{1}{4}, \dfrac{1}{2}$

15. *(c)*



$\therefore \quad \arg(z) - \arg(-z) = \theta - (-\pi + \theta)$

$= \pi$

16. *(b)* We have,

$$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\ldots(\cos n\theta + i\sin n\theta) = 1$$

$\Rightarrow \quad e^{i\theta} \cdot e^{i(2\theta)} \cdot e^{i(3\theta)} \cdot \ldots e^{i(n\theta)} = 1$

$\Rightarrow \quad e^{i\theta(1+2+3+\ldots+n)} = 1$

$\Rightarrow \quad e^{\frac{i\,n(n+1)\,\theta}{2}} = 1$

$\Rightarrow \cos\left(\dfrac{n(n+1)}{2}\theta\right) + i\sin\left(\dfrac{n(n+1)}{2}\right)\theta = 1 + 0i$

$\Rightarrow \quad \cos\left(\dfrac{n(n+1)}{2}\theta\right) = 1$

$\Rightarrow \quad \dfrac{n(n+1)}{2}\theta = 2k\pi$

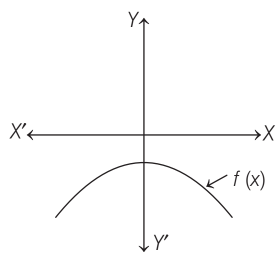
$\Rightarrow \quad \theta = \dfrac{4k}{n(n+1)}\pi$

17. *(d)* Let $f(x) = ax^2 + bx + c$

$\Rightarrow \quad f(1) = a + b + c < 0$

Again, $f(x)$ has imaginary zeros. So, $a < 0$.

Also, $f(0) = c$. Since $f(x)$ is downward parabola. So, $c < 0$.



18. *(a)*

A	B
4	2
3	3
2	4

$\therefore$ Total number of ways

$$= {}^6C_4 \times {}^6C_2 + {}^6C_3 \times {}^6C_3 + {}^6C_2 \times {}^6C_4$$

$$= \frac{6 \times 5}{2 \times 1} \times \frac{6 \times 5}{2 \times 1} + \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} + \frac{6 \times 5}{2 \times 1} \times \frac{6 \times 5}{2 \times 1}$$

$$= 15 \times 15 + 20 \times 20 + 15 \times 15$$

$$= 225 + 400 + 225$$

$$= 850$$

19. *(b)* Required number of ways

$$= {}^7C_4 \times D(3)$$

$$= {}^7C_3 \times \left[3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)\right]$$

$$\left[\because {}^nC_r = {}^nC_{n-r}, \text{ and } D(n) = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots\right]\right]$$

$$= {}^7C_3 \times 2 = 2({}^7C_3)$$

20. *(d)* We have,

$$7^{2n} + 16n - 1$$

Put $n = 1$, we get

$$7^2 + 16 - 1$$

$$= 49 + 15$$

$$= 64$$

which is divisible by 64.

21. *(b)* Given, $\left(3^{\frac{1}{8}} + 5^{\frac{1}{4}}\right)^{84}$

here, $T_{r+1} = {}^{84}C_r \left(3^{\frac{1}{8}}\right)^{84-r} \left(5^{\frac{1}{4}}\right)^r$

$$= {}^{84}C_r \left(3^{\frac{84-r}{8}}\right) \cdot 5^{\frac{r}{4}}$$

Since, T_{r+1} is rational for $r = 4, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84$

$\therefore$ no. of rational terms $= 11$

Here, total number of terms $= 85$

$\therefore$ Number of irrational terms $= 85 - 11$

$= 74$

22. *(c)* Given, that $A - 3I_3$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$= [-2(4-1) - 1(-2-1) + 1(1+2)]$$

$$= [-2(3) - 1(-3) + 1(3)]$$

$$= [-6 + 3 + 3] = 0$$

Now, det $= 0$

So, matrix $A - 3I_3$ is non-invertible matrix.

23. *(c)* Since, $\text{adj}(M') = (\text{adj } M)'$

$$= \text{adj}(M') - (\text{adj } M)'$$

$$[\because \text{adj}(A') = (\text{adj } A)']$$

$= 0$, a null matrix.

24. *(a)* Given,

$$A = \begin{bmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{bmatrix}$$

$\therefore \quad A^2 = A \cdot A$

$$= \begin{bmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25x + 5x^2 & 5x + 25x^2 + 5x \\ 0 & x^2 & 5x^2 + 25 \\ 0 & 0 & 25 \end{bmatrix}$$

$$= 25(25x^2 - 0)$$

$$= 25(25x^2)$$

Since, given that $|A|^2 = 25$

Now, $25(25x^2) = 25$

$\Rightarrow \quad 25x^2 = 1$

$\Rightarrow \quad x^2 = \frac{1}{25}$

$\Rightarrow \quad x = \pm \frac{1}{5}$

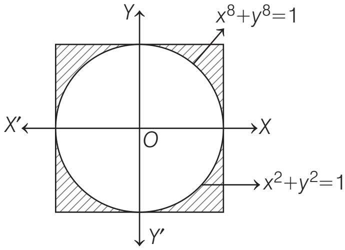
$\Rightarrow \quad x = \frac{1}{5}$

25. *(d)* Since, product of two non-null matrix can be a null matrix.

Therefore, may be

$$A \neq O_3, B \neq O_3.$$

26. *(a)*



From the graph, it is clear that there is common region which satisfies $x^2 + y^2 = 1$ and $x^8 + y^8 < 1$

$\because \quad P \cap T = P$

27. *(d)* We have,

$$f(x) = x^2 - \frac{x^2}{1 + x^2}$$

$$\therefore \quad f(-x) = (-x)^2 - \frac{(-x)^2}{1 + (-x)^2}$$

$$= x^2 - \frac{x^2}{1 + x^2} = f(x)$$

$\therefore f(-x) = f(x)$.

So, $f(x)$ is many one

Again, $f(x) = x^2 - \dfrac{x^2}{1 + x^2}$

$$= \frac{x^2 + x^4 - x^2}{1 + x^2} = \frac{x^4}{1 + x^2}$$

$\therefore$ Range of $f(x)$ will be $[0, \infty)$

But, co-domain of $f(x) = R$

$\therefore$ Range $\neq$ co-domain. So, $f(x)$ is onto.

28. *(d)* We observe the following properties:

Reflexivity : Let a be an arbitrary element of R, Then, $a \in R$

$\Rightarrow \quad 1 + a \cdot a = 1 + a^2 > 0 \qquad [\because a^2 > 0 \text{ for all } a \in R]$

$\Rightarrow \quad (a, a) \in R_1 \qquad$ [By def. of R_1]

Thus, $(a, a) \in R_1$ for all $a \in R$. So, R_1 is reflexive on R.

Symmetry : Let $(a, b) \in R$.

Then, $(a, b) \in R_1$

$\Rightarrow \quad 1 + ab > 0 \qquad [\because ab = b \text{ for all}]$

$\Rightarrow \quad 1 + ba > 0 \quad \Rightarrow \quad (b, a) \in R_1$

Thus, $(a, b) \in R_1 \neq (b, a) \in R$, for all $a, b \in R$

So, R_1 is symmetric on R.

Transitivity : we observe that $\left(1, \frac{1}{2}\right) \in R_1$ and $\left(\frac{1}{2}, -1\right) \in R_1$ but $(1, -1) \notin R_1$ because

$$1 + 1 \times (-1) = 0 \not> 0$$

So, R_1 is not transitive on R.

29. *(c)* Let $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

$$P(C) = \frac{1}{4}, P(D) = \frac{1}{5}$$

Now, $P(A \cup B \cup C \cup D)$

$$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$$

$$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})\, P(\bar{D})$$

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)$$

$$= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) = 1 - \frac{1}{5} = \frac{4}{5}$$

30. *(d)* Given that,

$$\sigma(X) = 26 = \sqrt{\text{var}(x)}$$

$$\sigma(1 - 4x) = \sqrt{\text{var}(1 - 4x)}$$

$$= \sqrt{16\,\text{var}(x)}$$

$$= 4 \times \sqrt{\text{var}(x)} = 4 \times 26 = 10.4$$

31. *(a)* Given that, $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = t$

$\therefore \quad t - t^{-1} - 4 = 0$

$\Rightarrow \quad t - \dfrac{1}{t} - 4 = 0$

$\Rightarrow \quad t^2 - 1 - 4t = 0$

$$t = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

Now, $e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 + \sqrt{5}$ and $e^{\sin x} = 2 - \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 + \sqrt{5} \qquad [e^x > 0]$

Also, $-1 \leq \sin x \leq 1$

$\therefore \quad \dfrac{1}{e} \leq e^{\sin x} \leq e$

$\therefore \quad e^{\sin x} \neq 2 + \sqrt{5}$

Hence, there is no value of x.

32. *(d)* Given, that angles of a triangles are $2x$, $3x$ and $7x$.

Since, $2x + 3x + 7x = 180$

$\Rightarrow \quad 12x = 180^\circ \Rightarrow x = 15^\circ$

So, angles are 30°, 45° and 105°

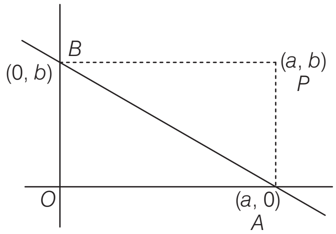
Now, $\dfrac{a}{\sin 30^\circ} = 2R$

$\Rightarrow \quad \dfrac{a}{\frac{1}{2}} = 2 \times 10 \qquad [\because R = 10 \text{ cm}]$

$\Rightarrow \quad 2a = 20 \Rightarrow a = 10$ cm

33. *(b)* Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$

Since, the line passing through a fixed point(x_1, y_1).



$\therefore \quad \frac{x_1}{a}+\frac{y_1}{b}=1$

Since, $OAPB$ is a rectangle, therefore the coordinate of P will be (a, b).

Hence, locus of P is

$$\frac{x_1}{x}+\frac{y_1}{y}=1$$

34. *(b)* Given line,

$$\sqrt{3}x+y=1$$

$\Rightarrow \quad y=-\sqrt{3}x+c$

We know that, $m=-\sqrt{3}$ $\quad [\because y=mx+c]$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan 60^\circ=\left|\frac{m_1-(-\sqrt{3})}{1+m_1(-\sqrt{3})}\right| \quad [\because \theta=60^\circ]$$

$$\sqrt{3}=\left|\frac{m_1+\sqrt{3}}{1-\sqrt{3}}m_1\right| \quad [\tan 60^\circ=\sqrt{3}]$$

$$\pm\sqrt{3}=\frac{m_1+\sqrt{3}}{1-\sqrt{3}m_1}$$

taking + sign	taking – sign
$+\sqrt{3}=\frac{m_1+\sqrt{3}}{1-\sqrt{3}m_1}$	$-\sqrt{3}=\frac{m_1+\sqrt{3}}{1-\sqrt{3}m_1}$
$\sqrt{3}(1-\sqrt{3}m_1)=m_1+\sqrt{3}$	$\Rightarrow -\sqrt{3}(1-\sqrt{3}m_1)$ $=m_1+\sqrt{3}$
$\Rightarrow \sqrt{3}-3m_1=m_1+\sqrt{3}$	$\Rightarrow -\sqrt{3}-3m_1=m_1+\sqrt{3}$
	$\Rightarrow \quad 3m_1-m_1=2\sqrt{3}$
$\Rightarrow \quad 4m_1=0$	$\Rightarrow \quad 2m_1=2\sqrt{3}$
$\Rightarrow \quad m_1=0$	$\Rightarrow \quad m_1=\sqrt{3}$

$\therefore$ given point $(3, -2)$

$\therefore \quad y=mx+c$

$$-2=\sqrt{3}(3)+c$$

$$-2=3\sqrt{3}+c$$

$$c=-2-3\sqrt{3}$$

Hence, required equation

$\therefore \quad y=mx+c$

$$y=\sqrt{3}x-2-3\sqrt{3}$$

$$y-\sqrt{3}x+2+3\sqrt{3}=0$$

$$y-x\sqrt{3}+2+3\sqrt{3}=0$$

35. *(a)* Let (α, β) be the given point, let $Q(x, y)$ be the foot of the perpendicular, and let O be the origin.

The line can have any direction

$$\angle PQO=90^\circ$$

Point Q lies on the circle having diameter OP.

The locus of point Q.

$$(x-0)(x-\alpha)+(y-0)(y-\beta)=0$$

$\Rightarrow \quad x^2-x\alpha+y^2-y\beta=0$

$\Rightarrow \quad x^2+y^2-\alpha x-\beta y=0$

36. *(b)* Let coordinate of the point be $(\alpha, -\alpha)$

Since, $(\alpha, -\alpha)$ lie on $2ax+4ay+c=0$

and $\quad 7bx+3by-d=0$

$\therefore \quad 2a\alpha-4a\alpha+c=0$

$\Rightarrow \quad -2a\alpha+c=0$

$\Rightarrow \quad \alpha=\frac{c}{2a} \quad \ldots(i)$

Also, $7b\alpha-3b\alpha-d=0$

$\Rightarrow \quad 4b\alpha-d=0$

$\Rightarrow \quad \alpha=\frac{d}{4b} \quad \ldots(ii)$

From Eqs. (i) and (ii),

$$\frac{c}{2a}=\frac{d}{4b}$$

$\Rightarrow \quad 2ad=4bc$

$\Rightarrow \quad \frac{ad}{bc}=\frac{4}{2}$

$\Rightarrow \quad \frac{ad}{bc}=\frac{2}{1}$

$\Rightarrow \quad ad:bc=2:1$

37. *(a)* In a circle, AB is a diameter where the coordinate of A is (p, q) and let the coordinate of $B(x_1, y_1)$.

Equation of circle in diameter form is

$$(x-p)(x-x_1)+(y-q)(y-y_1)=0$$

$\Rightarrow x^2 - (p + x_1)x + px_1 + y^2 - (y_1 + q)y + qy_1 = 0$

$\Rightarrow x^2 - (p + x_1)x + y^2 - (y_1 + q)y + px_1 + qy_1 = 0$

Since, this circle touches X-axis

$\therefore \quad y = 0$

$\Rightarrow \quad x^2 - (p + x_1)\,x + px_1 + qy_1 = 0$

Also, the discriminant of above equation will be equal to zero because circle touches X-axis.

$\therefore \quad (p + x_1)^2 = 4(px_1 + qy_1)$

$\Rightarrow \quad p^2 + x_1^2 + 2px_1 = 4px_1 + 4qy_1$

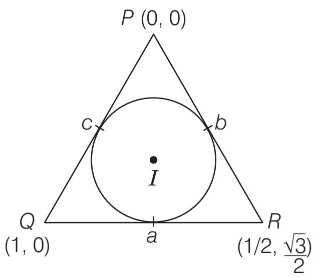
$\Rightarrow \quad x_1^2 - 2px_1 + p^2 = 4qy_1$

$\Rightarrow \quad (x_1 - p)^2 = 4qy_1$

Therefore, the locus of point B is

$$(x - p)^2 = 4qy$$

38. *(c)* $I = \left(\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}, \dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right)$



$$a = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1,\ b = 1,\ c = 1$$

Centre of the circle.

$$= \left(\frac{1 \times 0 + 1 \times 1 + 1 \times \frac{1}{2}}{1 + 1 + 1}, \frac{1 \times 0 + 1 \times 0 + 1 \times \sqrt{3}/2}{1 + 1 + 1}\right)$$

$$= \left(\frac{0 + 1 + \frac{1}{2}}{3}, \frac{0 + 0 + \frac{\sqrt{3}}{2}}{3}\right)$$

$$= \left(\frac{\frac{3}{2}}{3}, \frac{\frac{\sqrt{3}}{2}}{3}\right)$$

$$= \left(\frac{3}{2} \times \frac{1}{3}, \frac{\sqrt{3}}{2} \times \frac{1}{3}\right)$$

$$= \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$$

39. *(c)* Given, equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1,$$

Here, $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$

[i.e. comparing with standard equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$]

We know that, foci = $(\pm\, ae, 0)$,

where, $ae = \sqrt{a^2 + b^2} = \sqrt{\cos^2\alpha + \sin^2\alpha} = 1$

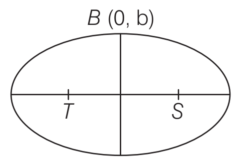
$\Rightarrow$ foci $(\pm\, 1, 0)$, where vertices are $(\pm\, \cos\alpha, 0)$

Eccentricity, $ae = 1$ or $e = \dfrac{1}{\cos\alpha}$.

Hence, foci remains constant with change in α.

40. *(c)* Equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Foci are $S(ae, 0)$, $T(-\,ae, 0)$.

$B(0, b)$ is the end of the minor axis.

STB is an equilateral triangle.

$SB = ST$

$\Rightarrow \quad SB^2 = ST^2$

$\Rightarrow \quad a^2e^2 + b^2 = 4a^2e^2$

$\Rightarrow \quad b^2 = 3a^2e^2$

$\Rightarrow \quad a^2(1 - e^2) = 3a^2e^2$

$\Rightarrow \quad 1 - e^2 = 3e^2$

$\Rightarrow \quad 4e^2 = 1$

$\Rightarrow \quad e^2 = \dfrac{1}{4}$

$\Rightarrow \quad e = \dfrac{1}{2}$

Eccentricity of the ellipse, $e = \dfrac{1}{2}$

41. *(a)* Given equation is

$$3x^2 - 3y^2 - 18x + 12y + 2 = 0$$

It can be written as

$$\frac{(x - 3)^2}{\left(\sqrt{\frac{13}{3}}\right)^2} - \frac{(y - 2)^2}{\left(\sqrt{\frac{13}{3}}\right)^2} = 1$$

here, $a = b = \sqrt{\frac{13}{3}}$

$\therefore \quad e = \sqrt{1 + \frac{b^2}{a^2}}$

$= \sqrt{1+1}$ $\quad [\because a = b]$

$= \sqrt{2}$

$\therefore$ Equation of the directrix are

$$x = 3 \pm \sqrt{\frac{13}{6}}$$

42. *(c)* Given equation of ellipse is

$$3x^2 + 4y^2 = 48$$

$\Rightarrow \quad \frac{3x^2}{48} + \frac{4y^2}{48} = 1 \quad \Rightarrow \quad \frac{x^2}{16} + \frac{y^2}{12} = 1$

Here, $\quad a = 4$ and $b = 2\sqrt{3}$

$\therefore \quad e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{12}{16}} = \sqrt{\frac{4}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore$ Coordinates of P are $= (2, 3)$

$\because (4\cos\theta, 2\sqrt{3}\sin\theta) = (2, 3) \quad \left[\because p = \left(ae, \frac{b^2}{a}\right)\right]$

By comparing, we get

$$4\cos\theta = 2 \text{ and } 2\sqrt{3}\sin\theta = 3$$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}$ and $\sin\theta = \sin\frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{3}$

$\therefore$ eccentric angle of P is $\frac{\pi}{3}$.

43. *(b)* Direction ratios of line joining the points $(1, 2, -3)$ and $(-1, -2, 1)$ is $(-1-1, -2-2, 1+3)$

i.e. $(-2, -4, 4)$

Let the direction ratios of the normal to the plane is (a, b, c), then

$$2a + 3b + 4c = 0 \quad \ldots\text{(i)}$$

and $\quad -2a - 4b + 4c = 0 \quad \ldots\text{(ii)}$

By Eqs. (i) and (ii), we get

$$\frac{a}{3(4) - (4)(-4)} = \frac{b}{(-2)(4) - (2)(4)}$$

$$= \frac{c}{(2)(-4) - (-2)(3)}$$

$\Rightarrow \quad \frac{a}{12+16} = \frac{b}{-8-8} = \frac{c}{-8+6}$

$\Rightarrow \quad \frac{a}{28} = \frac{b}{-16} = \frac{c}{-2}$

$\Rightarrow \quad \frac{a}{14} = \frac{b}{-8} = \frac{c}{-1}$

Hence, direction ratios are $(14, -8, -1)$.

44. *(c)* The given points are $A(1, 2, 3)$ and $B(3, 4, 5)$

The direction ratios of line segment AB is given by

$$(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$$

i.e. $\quad (3-1), (4-2), (5-3) = (2, 2, 2) = (1, 1, 1)$

Since, the plane bisects AB at right angles, AB is the normal to the plane. Which is n

Therefore, $\mathrm{n} = \hat{i} + \hat{j} + \hat{k}$

Let c be the mid-point of AB

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$= c\left(\frac{1+3}{2}, \frac{2+4}{2}, \frac{3+5}{2}\right)$$

$$= c(2, 3, 4)$$

Let this vector be $\mathrm{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Hence, the required equation of vector

$$\{\mathrm{r} - (2\hat{i} + 3\hat{j} + 4\hat{k})\}(\hat{i} + \hat{j} + \hat{k}) = 0$$

We know that, $\mathrm{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow \quad \{(x\hat{i} + y\hat{j} + z\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})\}(\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

$$= (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$\Rightarrow x + y + z = 2 + 3 + 4$

$\Rightarrow x + y + z = 9$

45. *(a)* It is clear that, the limit of the interior angle of a regular polygon of n sides as $n \to \infty$ is π or 180°.

46. *(a)* Given,

$$h\{f(x)\} = x$$

differentiating w.r.t.x, we get

$$h'\{f(x)\}.f'(x) = 1$$

$\Rightarrow \quad f'(x) = \frac{1}{h'\{f(x)\}}$

$\Rightarrow \quad f'(x) = \frac{1}{\frac{1}{1 + \log\{f(x)\}}}$

$$\left[\because h'(x) = \frac{1}{1 + \log x} \text{ (given)}\right]$$

$\Rightarrow \quad f'(x) = 1 + \log(f(x))$

47. *(c)* We have,

$$f(x) = \cos x^2$$

Let T be the period of $f(x)$. Then

$$f(x + T) = f(x)$$

$\Rightarrow \quad \cos(x + T)^2 = \cos x^2$

But there is no value of T for which

$$\cos(x + T)^2 = \cos x^2$$

$\therefore$ $f(x)$ is not periodic

48. *(c)* Let $\quad L = \lim_{x \to 0^+} (e^x + x)^{1/x}$

$$\Rightarrow \log L = \lim_{x \to 0^+} \frac{\log(e^x + x)}{x}$$

$$\Rightarrow \log L = \lim_{x \to 0^+} \frac{\frac{1}{(e^x + x)} \cdot e^x + 1}{1}$$

[using L' Hospital rule]

$$\Rightarrow \log L = \lim_{x \to 0^+} \frac{e^x + 1}{e^x + x}$$

$\Rightarrow \log L = 2$

$\Rightarrow \quad L = e^2$

49. *(a)* Let $g(x) = e^{-x} f(x)$

$\therefore \quad g'(x) = e^{-x} f'(x) - e^{-x} f(x)$

$\quad\quad = e^{-x}(f'(x) - f(x))$

Since, $f'(x) > f(x)$, so $f'(x) - f(x) > 0$

$\therefore$ $e^{-x}(f'(x) - f(x)) > 0$

$\Rightarrow \quad g'(x) > 0$

$\therefore$ $g(x)$ is an increasing function.

Now, $g(x) > g(0)\ \forall\ x > 0$

$\Rightarrow e^{-x} f(x) > e^0 f(0)$

$\Rightarrow \quad e^{-x} f(x) > 0 \quad\quad [\because f(0) = 0]$

$\Rightarrow \quad f(x) > 0, \forall x > 0$

50. *(b, c)* Given that $f : [1, 3] \to R$ be a continuous and differentiable in $(1, 3)$.

and $f'(x) = |f(x)|^2 + 4$

By applying *LMVT*, there exist at least one point $c \in (1, 3)$ such that

$$\frac{f(3) - f(1)}{3 - 1} = f'(c) \left[\because f'(c) = \frac{f(b) - f(a)}{b - a}\right]$$

$\Rightarrow \quad \dfrac{f(3) - f(1)}{2} = f'(c)$

$\Rightarrow \quad \dfrac{f(3) - f(1)}{2} = [f\,|\,c\,|^2 + 4]$

$(\because f'(c) = |f(c)|^2 + 4)$

$\Rightarrow \quad f(3) - f(1) = 2 \cdot |f(c)|^2 + 8$

$\Rightarrow \quad f(3) - f(1) \geq 8$

It is clear from the given options that (*B*) and (*C*) the correct.

51. *(c)* Let $f(x) = x^2 + 2x + 3$

$$a = f(x)_{\min} = \frac{-D}{4a} = \frac{-(4 - 12)}{4} = \frac{8}{4} = 2$$

$$\text{and } b = \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2} = \lim_{\theta \to 0} \frac{1 - 1 + 2\sin^2\theta/2}{\theta^2}$$

$$= \lim_{\theta \to 0} \frac{2\sin^2\theta/2}{(\theta/2)^2 \cdot 4} = \lim_{\theta \to 0} \frac{1}{2} \cdot \left[\frac{\sin^2\theta/2}{(\theta/2)^2}\right]$$

$$= \frac{1}{2} \cdot \lim_{\theta \to 0} \frac{\sin^2\theta/2}{(\theta/2)^2}$$

$$= \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$b = \frac{1}{2}$$

Now, $\sum_{r=0}^{n} a^r \cdot b^{n-r}$

$$= \sum_{r=0}^{n} (2)^r \left(\frac{1}{2}\right)^{n-r}$$

$$= \sum_{r=0}^{n} 2^r \cdot 2^{(r-n)}$$

$$= \sum_{r=0}^{n} 2^{2r-n}$$

$$= 2^{-n} \sum_{r=0}^{n} 2^{2r}$$

$$= 2^{-n}[1 + 2^2 + 2^4 + 2^6 + \ldots + 2^{2n}]$$

$$= 2^{-n}\left[\frac{1 \cdot (2^2)^{n+1} - 1}{2^2 - 1}\right] \left\{\because S_n = \frac{a(r^n - 1)}{r - 1}\right\}$$

$$= 2^{-n}\left[\frac{4^{n+1} - 1}{3}\right]$$

$$= \frac{4^{n+1} - 1}{3 \cdot 2^n}$$

52. *(a)* Let $a = 4$, $b = 1$ and $n = 2$

$\therefore \quad I(n) = a^{1/n} - b^{1/n}$

$$= 4^{1/2} - 1^{1/2} = 2 - 1 = 1$$

and $J(n) = (a-b)^{1/n}$

$$= (4-1)^{1/2} = \sqrt{3}$$

$\therefore \quad J(n) > I(n)$

Similarly, we can prove for other values of a, b, n.

53. *(d)* Given,

$$|\hat{\alpha}| = |\hat{\beta}| = |\hat{\gamma}| = 1$$

and $\quad \hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$

Now, $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$

$\Rightarrow \quad (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma} = \frac{1}{2}\hat{\beta} + \frac{1}{2}\hat{\gamma}$

On comparing we get,

$$-(\hat{\alpha} \cdot \hat{\beta}) = \frac{1}{2}$$

$\Rightarrow \quad |\hat{\alpha}||\hat{\beta}|\cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \cos\theta = -\frac{1}{2} \qquad [\because |\hat{\alpha}| = |\hat{\beta}| = 1]$

$\Rightarrow \quad \theta = 120^\circ$ or $\frac{2\pi}{3}$

54. *(d)* The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} - \lambda\hat{k}$ respectively.

$$BA = (3\hat{i} - 2\hat{j} - \hat{k}) - (2\hat{i} - 3\hat{j} + 2\hat{k}) = \hat{i} + \hat{j} - 3\hat{k}$$

$$CA = (3\hat{i} - 2\hat{j} - \hat{k}) - (5\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} - \hat{j} - 3\hat{k}$$

$$DA = (3\hat{i} - 2\hat{j} - \hat{k}) - (4\hat{i} - \hat{j} + \lambda\hat{k})$$

$$= -\hat{i} - \hat{j} - (1+\lambda)\hat{k}$$

These are coplanar, so

$$\begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ -1 & -1 & -(1+\lambda) \end{vmatrix} = 0$$

$$= \begin{vmatrix} -1 & -1 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1+\lambda \end{vmatrix} = 0$$

$-1[1 + \lambda - 3] + 1[2 + 2\lambda - 3] + 3[2 - 1] = 0$

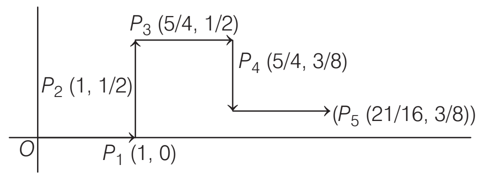
$\Rightarrow \quad -1[\lambda - 2] + 1[2\lambda - 1] + 3 = 0$

$\Rightarrow \quad -\lambda + 2 + 2\lambda - 1 + 3 = 0$

$\Rightarrow \quad \lambda + 4 = 0$

$\Rightarrow \quad \lambda = -4$

55. *(b)* According to the given information in question, we can draw the situation of particle at different stages as following



Here, $x_1 = 1, x_2 = 1, x_3 = \frac{5}{4}, x_4 = \frac{5}{4}$ and $x_5 = \frac{21}{16}$

$$y_1 = 0, y_2 = \frac{1}{2}, y_3 = \frac{1}{2}, y_4 = \frac{3}{8} \text{ and } y_5 = \frac{3}{8}$$

$\therefore \quad x_n = 1 + \frac{1}{4} + \frac{1}{4^2} + \ldots \infty = \frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

and $\ y_n = 0 + \frac{1}{2} - \frac{1}{8} + \ldots \infty$

$$= \frac{\frac{1}{2}}{1 - \left(-\frac{1}{4}\right)} = \frac{\frac{1}{2}}{\left(1 + \frac{1}{4}\right)} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{2}{5}$$

$\therefore \quad (\alpha, \beta) = \left(\frac{4}{3}, \frac{2}{5}\right)$

56. *(a)* $|z| + |z-1|$ a is complex number

$$|z| + |-(1-z)| \qquad |a| = |-a|$$

$$|z| + |1-z| \qquad \{\because |a| = |-a|\}$$

$$|z| + |1-z| \ge |z + 1 - z|$$

$$\{\because |z_1| + |z_2| \ge |z_1 + z_2|\}$$

$$|z| + |1-z| \ge |1|$$

$$|z| + |1-z| \ge 1$$

$$|z| + |-(z-1)| \ge 1$$

$$|z| + |z-1| \ge 1$$

Minimum value of $|z| + |z-1| = 1$

57. *(c)* Given, system of equations

$$\lambda x + y + 3z = 0$$

$$2x + \mu y - z = 0$$

$$5x + 7y + z = 0$$

System has infinitely many solutions in R, if

$$\begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda(\mu + 7) - 1(2 + 5) + 3(14 - 5\mu) = 0$

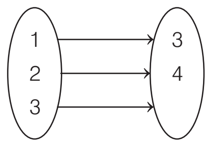
$\Rightarrow \quad \lambda(\mu + 7) - 7 + 3(14 - 5\mu) = 0$

$\Rightarrow \quad \lambda\mu + 7\lambda - 7 + 42 - 15\mu = 0$

$\Rightarrow \quad \lambda\mu + 7\lambda - 15\mu + 35 = 0$

By checking the options, we get option (c) $(\lambda = 1, \mu = 3)$ satisfies the given equation.

58. *(c)* Let $A = \{1, 2, 3\}$
and $B = \{2, 3, 4\}$



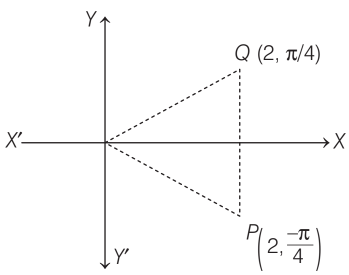
here, $f^{-1}(3) = 1$

$f^{-1}(4) = 2$ and $f^{-1}(2) = 3$

It is clear that option (C) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$ is correct.

59. *(b)* We have,
$gof : S \to U$ is an onto function.
Let Z be any arbitrary element such that $Z \in U$.
Now, $gof : S \to U$ is onto
$\Rightarrow gof(x) = Z$, for $x \in S$
$\Rightarrow g(f(x)) = Z$
$\Rightarrow g(y) = Z$, where $y = f(x) \in T$ for all $Z \in U$
$\therefore$ For all $Z \in U$, there exists $y = f(x) \in T$ such that $g(y) = Z$
$\therefore$ $g : T \to U$ is an onto function.

60. *(a)* Since, Q is the point for which PQ is bisected perpendicularly by the initial line (X-axis). Therefore, Q will be the image of P in X-axis
$\therefore$ Coordinates of Q is $(2, \pi/4)$



61. *(b)* Let the length of conjugate axis = b
and the length of transvers axis = a
Given that, $b > a$
Now, $b^2 > a^2$

$\therefore$ Eccentricity $(e) = \sqrt{1 + \frac{b^2}{a^2}}$

$$e^2 = \left(\sqrt{1 + \frac{b^2}{a^2}}\right)^2 > 2$$

$\therefore$ $e > \sqrt{2}$

62. *(a)* Given, $\lim_{x \to 0^+} \frac{x}{p}\left[\frac{q}{x}\right]$

$$= \lim_{x \to 0^+} \frac{x}{p}\left(\frac{q}{x} - \left(\frac{q}{x}\right)\right)$$

$$= \frac{[q]}{p} - \frac{x}{p}\left\{\frac{q}{x}\right\}$$

$$= \frac{[q]}{p} - 0 = \frac{[q]}{p}$$

63. *(b)* Given,

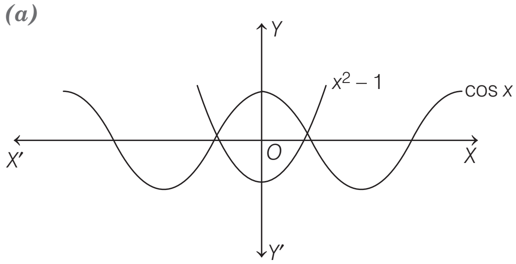
$$f(x) = x^4 - 4x^3 + 4x^2 + c$$

$\because$ $f(1) = 1 + c$

and $f(2) = 2^4 - 4(2)^3 + 4(2)^2 + c$
$= 16 - 32 + 16 + c$
$= c$

Now, $f(1) \cdot f(2) = c(1 + c)$
Let $f(1) \cdot f(2) < 0$
$\Rightarrow c(1 + c) < 0 \Rightarrow c \in (-1, 0)$
$\therefore$ For $c \in (-1, 0)$, $f(x)$ has exactly one zero is $(1, 2)$.

64. *(a)*



It is clear from the graph that it intersects at exactly two points.

65. *(c)* We have,

$$\frac{dx}{dt} = 1 \text{ and } y = \frac{10}{x}$$

Now, $y = \frac{10}{x}$

On differentiating both the sides w.r.t. t, we get

$$\Rightarrow \frac{dy}{dt} = \frac{-10}{x^2} \times \frac{dx}{dt}$$

$$= \frac{-10}{(5)^2} \times 1$$

$$= \frac{-10}{25} = \frac{-2}{5} \qquad \left[x = 10, \frac{dx}{dt} = 1\right]$$

$\therefore$ y decreases at the rate of $\frac{2}{5}$ unit per second.

66. *(b,d)* We have,

$$I_n = \int_0^1 x^n \tan^{-1} x \, dx$$

$$= \left[\tan^{-1} x \cdot \frac{x^{n+1}}{n+1}\right]_0^1 - \int_0^1 \frac{x^{n+1}}{n+1} \cdot \frac{1}{1+x^2} dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1} \int_0^1 \frac{x^2 \cdot x^{n-1}}{1+x^2} dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1} \int_0^1 \frac{(1+x^2)\, x^{n-1} - x^{n-1}}{1+x^2} dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1} \int_0^1 x^{n-1} \, dx + \frac{1}{n+1} \int_0^1 \frac{x^{n-1}}{1+x^2} dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1} \left[\frac{x^n}{n}\right]_0^1 + \frac{1}{n+1}$$

$$\left[[x^{n-1} \cdot \tan^{-1} x]_0^1 - \int_0^1 \tan^{-1} x \cdot (n-1)\, x^{n-2} \, dx\right.$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n(n+1)} + \frac{\pi}{4(n+1)} - \frac{n-1}{n+1} \int_0^1 x^{n-2} \tan^{-1} x \, dx$$

$$\Rightarrow \quad I_n = \frac{\pi}{2(n+1)} - \frac{1}{n(n+1)} - \frac{n-1}{n+2} I_{n-2}$$

$$\Rightarrow \quad I_n + \frac{n-1}{n+2} I_{n-2} = \frac{\pi}{2(n+1)} - \frac{1}{n(n+1)}$$

Put $n = n + 2$, we get

$$I_{n+2} + \frac{n+1}{n+3} I_n = \frac{\pi}{2(n+3)} - \frac{1}{(n+2)\,(n+3)}$$

$$\Rightarrow \quad (n+3)\, I_{n+2} + (n+1)\, I_n = \frac{\pi}{2} - \frac{1}{n+2}$$

$$\therefore\ a_n = n+3,\ b_n = n+1,\ c_n = \frac{\pi}{2} - \frac{1}{n+2}$$

$\therefore$ a_n and b_n are in AP.

67. *(c)* Let B travels x units, $v = u + at$

According to problem, $ht = f(t + m)$

$$\Rightarrow \quad ht = ft + fm$$

$$\Rightarrow \quad ht - ft = fm$$

$$\Rightarrow \quad t(h - f) = fm$$

$$\Rightarrow \quad \frac{h-f}{f} = \frac{m}{t}$$

$$\Rightarrow \quad t = m\left(\frac{f}{h-f}\right)$$

$$\Rightarrow \quad t^2 = m^2 \left(\frac{f}{h-f}\right)^2 \quad \ldots\text{(i)}$$

Again, $n + x = \frac{1}{2} f(t+m)^2$

$$\Rightarrow \quad h + \frac{1}{2} ht^2 = \frac{1}{2} f(t+m)^2 \quad \ldots\text{(ii)}$$

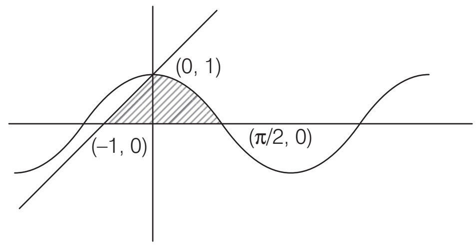
From Eqs. (i) and (ii)

$$\frac{m}{n} = \frac{2(h-f)}{mfh}$$

$$\Rightarrow \quad (h-f)n = \frac{m^2 fh}{2}$$

68. *(b)* We have,

$$y = x + 1 \text{ and } y = \cos x$$



$$\therefore \text{ Require area} = \int_{-1}^0 (x+1)\, dx + \int_0^{\pi/2} \cos x \, dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^0 + [\sin x]_0^{\pi/2}$$

$$= \left[(0) - \left(\frac{1}{2} - 1\right)\right] + [1 - 0]$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ sq unit.}$$

69. *(b)* We have,

x_1, x_2 be the roots of equation $x^2 - 3x + a = 0$

$\therefore \quad x_1 + x_2 = 3$ and $x_1 x_2 = a$

Also, x_3, x_4 be the roots of equation $x^2 - 12x + b = 0$

$\therefore \quad x_3 + x_4 = 12$ and $x_3 x_4 = b$

Again, x_1, x_2, x_3, x_4 are in GP

$\therefore \quad x_1 = A,\ x_2 = AR,\ x_3 = AR^2,\ x_4 = AR^3$

Now, $x_1 + x_2 = 3$

$$\Rightarrow \quad A(1 + R) = 3 \quad \ldots\text{(i)}$$

and $\quad x_3 + x_4 = 12$

$$\Rightarrow \quad AR^2(1 + R) = 12 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$R^2 = 4 \Rightarrow R = \pm 2$$

When, $R = 2$, $A = 1$ and when, $R = -2$, $A = -3$

$\therefore$ Numbers are either 1, 2, 4, 8 or − 3, 6, − 12, 24

But $x_1 < x_2 < x_3 < x_4$.

So, $x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 8$

$\therefore \quad ab = x_1 x_2 x_3 x_4$

$= 1 \times 2 \times 4 \times 8 = 64$

70. *(a)* Let $Z = \dfrac{1 - i\cos\theta}{1 + 2i\cos\theta}$

$\therefore \quad \bar{z} = \dfrac{1 + i\cos\theta}{1 - 2i\cos\theta}$

Since, z is a real number, then $z - \bar{z} = 0$

$\Rightarrow \dfrac{1 - i\cos\theta}{1 + 2i\cos\theta} = \dfrac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$\Rightarrow (1 - i\cos\theta)(1 - 2i\cos\theta) = (1 + i\cos\theta)(1 + 2i\cos\theta)$

$\Rightarrow 1 - 2i\cos\theta - i\cos\theta - 2\cos^2\theta$

$= 1 + 2i\cos\theta + i\cos\theta - 2\cos^2\theta$

$\Rightarrow \quad 6\,i\cos\theta = 0$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = (2n + 1)\dfrac{\pi}{2}, n \in I$

71. *(b)* We have,

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

Again, we have

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)[(3-\lambda)^2 - 0] - 0 + 3[0 - 3(3 - \lambda)] = 0$

$\Rightarrow (3-\lambda)^3 - 9(3-\lambda) = 0$

$\Rightarrow (3-\lambda)[(3-\lambda)^2 - 9] = 0$

$\Rightarrow (3-\lambda)(9 + \lambda^2 - 6\lambda - 9) = 0$

$\Rightarrow (3-\lambda)(\lambda^2 - 6\lambda) = 0$

$\Rightarrow (3-\lambda)\lambda(\lambda - 6) = 0$

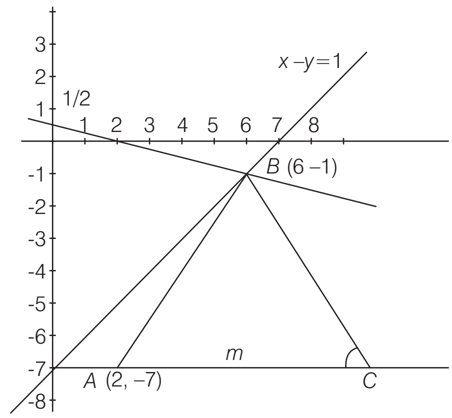
$\Rightarrow \lambda = 3, 0, 6$

72. *(a, b)* Given equations of lines are

$x - y = 7$ ...(i)

and $x + 4y = 2$ ...(ii)

By solving Eqs. (i) and (ii), we get the point $B(6, -1)$



Let the slope of AC be m

then, $AB = AC$

$$\therefore \quad \left|\frac{m + \frac{1}{4}}{1 - \frac{m}{4}}\right| = \left|\frac{-\frac{1}{4} - 1}{1 - \frac{1}{4}}\right|$$

$\Rightarrow \quad m = \dfrac{-23}{7}, 1$

When $m = -\dfrac{23}{7}$, then equation of line

$y + 7 = \dfrac{-23}{7}(x - 2)$

$\Rightarrow \quad 7y + 49 = -23x + 46$

$\Rightarrow \quad 23x + 7y + 3 = 0$

When $m = 1$, then equation of line

$y + 7 = (x - 2)$

$\Rightarrow \quad x - y - 9 = 0$

73. *(a, c)* Given equation of hyperbola can be write as

$$\frac{x^2}{1} - \frac{y^2}{(\sqrt{5})^2} = 1$$

here, $a = 1$ and $b = \sqrt{5}$

Since, $y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = mx \pm \sqrt{m^2 - 5}$

$\Rightarrow \quad (8 - 2m)^2 = (\sqrt{m^2 - 5})^2$

$\Rightarrow \quad (8 - 2m)^2 = m^2 - 5$

$\Rightarrow 64 + 4m^2 - 32m = m^2 - 5$

$\Rightarrow 3m^2 - 32m + 69 = 0$

$\Rightarrow \quad m = \dfrac{22}{3}, 3$ [by solving equation]

When $m = \frac{23}{3}$, then equation of tangent

$$(y - 8) = \frac{23}{3}(x - 2)$$

$\Rightarrow \quad 3y - 24 = 23x - 46$

$\Rightarrow \quad 23x - 3y - 46 + 24 = 0$

$\Rightarrow \quad 23x - 3y - 22 = 0$

When $m = 3$, then equation of tangent

$$(y - 8) = 3(x - 2)$$

$\Rightarrow \quad y - 8 = 3x - 6$

$\Rightarrow \quad 3x - y - 6 + 8 = 0$

$\Rightarrow \quad 3x - y + 2 = 0$

Hence, option (a) and (c) are correct.

74. *(a, c)* From option (a)

We have,

$$f(a) = f(b) = 0$$

$\Rightarrow \quad f'(a) \cdot f'(b) < 0$

Again, let $h(x) = f'(x) + f(x)\, g'(x)$

$\Rightarrow \quad h(a) = f'(a) + f(a)\, g'(a) = f'(a)$

and $h(b) = f'(b) + f(b)\, g'(b) = f'(b)$

$\therefore \quad h(a) \cdot h(b) = f'(a) \cdot f'(b) < 0$

$\therefore \quad h(x) = 0$ has root between (a, b)

From option (c)

We have, $g(a) = g(b) = 0$

$\Rightarrow \quad g'(a) \cdot g'(b) < 0$

Again, let $m(x) = g'(x) + kg(x)$

$\Rightarrow \quad m(a) = g'(a) + kg(a) = g'(a)$

$\Rightarrow \quad m(b) = g'(b) + kg(b) = g'(b)$

$\therefore \quad m(a) \cdot m(b) = g'(a) \cdot g'(b) < 0$

$\therefore \quad m(x) = 0$ has root between (a, b).

So, option (a) and (c) are correct.

75. *(d)* Given, $f(x) = \frac{x^3}{4} - \sin \pi x + 3$

differentiating w.r.t. x, we get

$$f'(x) = \frac{3x^2}{4} - \pi \cos(\pi x)$$

at $\quad x = -2,\ f(-2) = 1$

and at $\quad x = 2,\ f(2) = 5$