JEE(Main) 2025 | DATE: 29-01-2025 (SHIFT-2) | PAPER-1 | MEMORY BASED

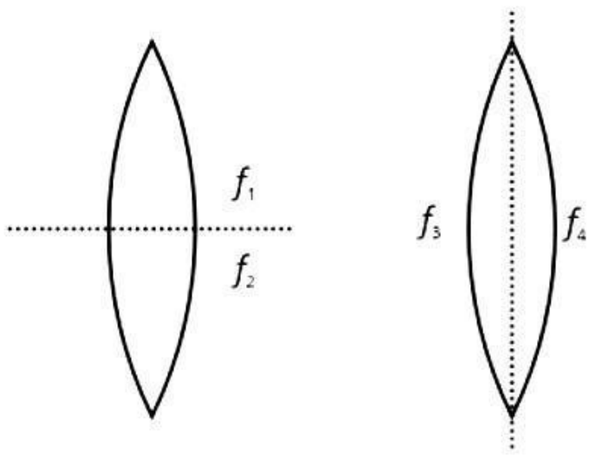
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. An equiconvex lens is cut in two ways as shown. If the focal lengths of the parts are as mentioned in the diagram. Find $\frac{f_1}{f_3}$



(1) 2 (2) 4

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

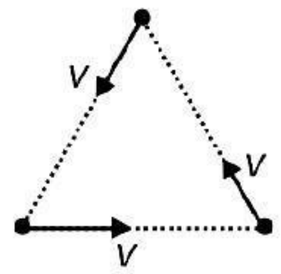
Answer (3)

Sol. $f_1 = f_2 = f$

$f_3 = f_4 = 2f$

$\frac{f_1}{f_3} = \frac{1}{2}$

2. Three identical particles, each of mass m move under the influence of mutual attractive forces. Initially the are on the vertices of an equilateral triangle of side 'a' and have equal speed v directed towards the adjacent particle as shown. The net angular momentum about the centre just before collision is



(1) $\frac{3mva}{2}$ (2) $\frac{2}{3}mva$

(3) $\frac{\sqrt{3}}{2}mva$ (4) $\frac{2}{\sqrt{3}}mva$

Answer (3)

Sol. Net angular momentum remains conserved as there is zero net torque about centroid.

$\Rightarrow \quad L_i = L_f$

$L_f = \frac{3amv}{2\sqrt{3}} = \frac{\sqrt{3}}{2}mva$

3. A solenoid of radius 10 cm carrying current 0.29 A and having total 200 turns. If magnetic field inside solenoid is 2.9×10^{-4} T. Find length of solenoid.

(1) 6π cm (2) 8π cm

(3) 4.5π cm (4) 16π cm

Answer (2)

Sol. $B = \mu_0 n_i$

$2.9 \times 10^{-4} = 4\pi \times 10^{-7} \times \frac{200}{L} \times 0.29$

$L = 8\pi$ cm

4. Match the physical quantities with their corresponding dimensions

	Column-I		**Column-II**
(A)	Young's modulus	(i)	$[AL^2]$
(B)	Magnetic moment	(ii)	$[ML^2T^{-2}A^{-1}]$
(C)	Magnetic flux	(iii)	$[AL^{-1}]$
(D)	Magnetic intensity	(iv)	$[ML^{-1}T^{-2}]$

(1) A → (iv), B → (i), C → (ii), D → (iii)

(2) A → (iv), B → (ii), C → (i), D → (iii)

(3) A → (iii), B → (i), C → (ii), D → (iv)

(4) A → (iii), B → (ii), C → (i), D → (iv)

Answer (1)

Sol. Young's modulus $\equiv \frac{F}{A} \equiv \left[ML^{-1}T^{-2}\right]$

Magnetic moment $\equiv IA \equiv \left[AL^{2}\right]$

Magnetic flux $\equiv Vt \equiv \left[ML^{2}T^{-2}A^{-1}\right]$

Magnetic intensity $\equiv H = nI \equiv \left[AL^{-1}\right]$

5. Two particles of same mass are performing SHM vertically with two different springs of spring constants K_1 and K_2. If amplitude of both is same. Find ratio of the maximum speed of two particles.

(1) $\sqrt{\frac{K_1}{K_2}}$ (2) $\sqrt{K_2K_1}$

(3) $\sqrt{\frac{K_2}{K_1}}$ (4) $\sqrt{\frac{K_1+K_2}{K_1-K_2}}$

Answer (1)

Sol. $V_{max} = A\omega$

$$\frac{V_1}{V_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{K_1}{K_2}}$$

6. A physical quantity Q is given as $Q = \frac{ab^4}{cd}$, if the percentage error is a, b, c and d are 2%, 1%, 2% and 1%, the % error in Q will be

(1) 5% (2) 15%

(3) 9% (4) 2%

Answer (3)

Sol. $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{4\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}$

= 2% + 4% + 2% + 1%

= 9%

7. **Assertion :** On increasing the pressure, the volume decrease is more in an isothermal process than in an adiabatic process.

Reason : Adiabatic process is given by PV^{γ}.

(1) Assertion is correct and Reason is false

(2) Assertion is correct and Reason is correct

(3) Assertion is false and Reason is correct

(4) Assertion is false and Reason is false

Answer (2)

Sol. Isothermal PV = Constant

$$\frac{dP}{DV} = -\frac{P}{V}$$

$$dP = -\frac{P}{V}(dV)_{isothermal} \quad \ldots(i)$$

Adiabatic $PV^{\gamma} = \text{Constant}$

$$\frac{dP}{dV} = -\gamma\frac{P}{V}$$

$$dP = -\frac{\gamma P}{V}(dV)_{adiabatic} \quad \ldots(ii)$$

If dP is same in both process

$$(dV)_{isothermal} = \gamma(dV)_{adiabatic}$$

8. Two planet A and B are revolving around a massive star such that $r_A = 2r_B$ and $m_A = 4\sqrt{3}\, m_B$. Find ratio of angular momentum of planet B to planet A.

(1) $8\sqrt{3}$ (2) $\frac{1}{8\sqrt{3}}$

(3) $\frac{1}{2\sqrt{3}}$ (4) $\frac{1}{3\sqrt{2}}$

Answer (2)

Sol. $L = mvr$

$$L = m\sqrt{\frac{GM}{r}}r$$

$$L = m\sqrt{GMr}$$

$$\frac{L_B}{L_A} = \frac{1}{4\sqrt{3}} \cdot \frac{1}{2} = \frac{1}{8\sqrt{3}}$$

9. A capacitor C_1 = 6 μF, initially charged with a cell of emf 5V is disconnected and connected to another capacitor C_2 = 12μF which is initially neutral. The charges on C_1 and C_2 after connection are

(1) 0 μC, 30 μC (2) 10 μC, 20 μC

(3) 20 μC, 10 μC (4) 30 μC, 0 μC

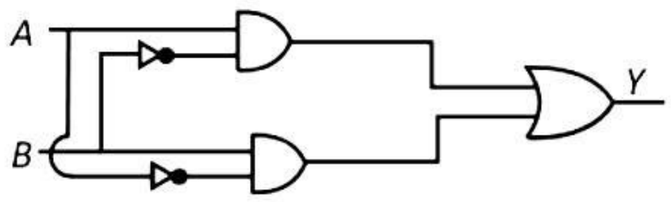
Answer (2)

Sol. Potential difference at equilibrium

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{(6\mu F)(5\ V)}{(6\mu F) + (12\mu F)} = \frac{5}{3}\ V$$

$$q_1 = C_1V = (6\mu F)\left(\frac{5}{3}\ V\right) = 10\ \mu C$$

10. The truth table for the logical circuit shown below is



(1)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(4)

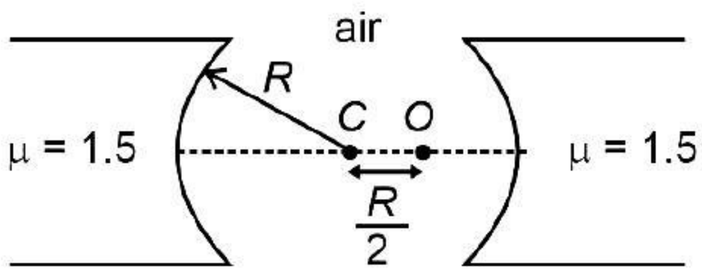
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Answer (2)

Sol. $Y = A\bar{B} + B\bar{A}$

This is a *XOR* gate

11. Figure shows two spherical surfaces of radius *R* having common centre. If the object is placed at *O*, find the distance between the first images formed by both the surfaces



(1) $\frac{4R}{35}$ (2) $\frac{4R}{27}$

(3) $\frac{4R}{70}$ (4) $\frac{2R}{35}$

Answer (1)

Sol. For right surface

$$\frac{1.5}{v_1} - \frac{1}{-R/2} = \frac{0.5}{-R}$$

$$\frac{1.5}{v_1} = \frac{-2}{R} - \frac{0.5}{R}$$

$$v_1 = \frac{-3R}{5}$$

For left surface

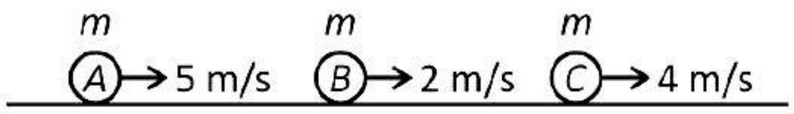
$$\frac{1.5}{v_2} - \frac{1}{-\frac{3R}{2}} = \frac{0.5}{-R}$$

$$v_2 = \frac{-9R}{7}$$

$$d = 2R - \left(\frac{3R}{5} + \frac{9R}{7}\right)$$

$$d = \frac{4R}{35}$$

12. Three particles of same mass are moving a shown. (all collisions are elastic)



S_1 : After all collisions velocities are 4 m/s, 2 m/s and 5 m/s.

S_2 : Velocities are get interchanged in elastic collision of same mass.

(1) S_1 : Correct, S_2 : Correct

(2) S_1 : Incorrect, S_2 : Correct

(3) S_1 : Incorrect, S_2 : Incorrect

(4) S_1 : Correct, S_2 : Incorrect

Answer (2)

Sol. Aster 1st collision

A → 2 m/s B → 5 m/s C → 4 m/s

After 2nd collision

A → 2 m/s B → 4 m/s C → 5 m/s

13. An electromagnetic wave propagates in +*X*-direction. Then, electric field and magnetic field are directed along

(1) *X*, *Y*

(2) *Y*, *Z*

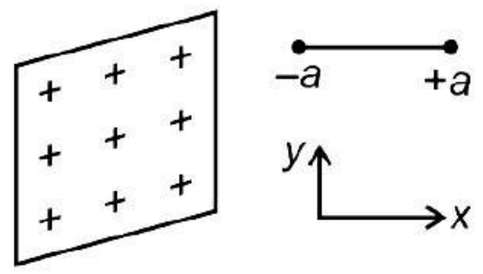
(3) *Z*, *Y*

(4) *Y*, *X*

Answer (2)

Sol. $\hat{C} = \hat{E} \times \hat{B}$

14. A dipole is placed such that its axis is perpendicular to the infinite charged sheet. Select the correct options



(a) $T_{net} = 0$, F_{net} is along –ve x-axis

(b) $T_{net} = 0$, $U = \min$

(c) $T_{net} = 0$, $F_{net} = 0$

(d) T_{net} and U both are maximum

(1) (a), (b), (c) and (d)

(2) (b) and (c)

(3) (a) and (c)

(4) (b) and (d)

Answer (2)

Sol. $T = \vec{P} \times \vec{E} = PE\sin\theta = 0$

$U = -\vec{P} \cdot \vec{E} = -PE$

$\therefore$ T = 0, U = min

$F_{net} = 0$

15. A cup of coffee take a time '*t*' to cool from 90°C to 80°C in a surrounding of 20°C. If a similar cup of coffee is cooled from 80°C to 60°C in the same surrounding, it takes a time

(1) $\frac{13t}{5}$

(2) $\frac{5t}{13}$

(3) $\frac{12t}{5}$

(4) $2t$

Answer (1)

Sol. From Newtons law of cooling

$$-\frac{\theta_2-\theta_1}{t}=C\left(\frac{\theta_2+\theta_1}{2}-\theta_s\right)$$

$$\Rightarrow \quad -\left(\frac{80^\circ C-90^\circ C}{t}\right)=C\left(\frac{90^\circ C+80^\circ C}{2}-20^\circ C\right)$$

$$\frac{10^\circ C}{t}=C(65^\circ C)$$

$$C=\frac{2}{13t}$$

Also,

$$-\left(\frac{60^\circ C-80^\circ C}{t'}\right)=C\left(\frac{60^\circ C+80^\circ C}{2}-20^\circ C\right)$$

$$\frac{20^\circ C}{t'}=C(50^\circ C)$$

$$C=\frac{2}{5t'}$$

$$\frac{2}{13t}=\frac{2}{5t'}$$

$$t'=\frac{13t}{5}$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A converting lens of focal length 24 cm, made of glass (μ_{glass} = 1.5) is immersed completely in water (μ_{water} = 1.33). It will now behave like a converging lens of focal length _____ cm.

Answer (96)

Sol. $$f_{air}\left(\mu_{glass-1}\right)=f_{water}\left(\frac{\mu_{glass}}{\mu_{water}}-1\right)$$

$$(+24\text{ cm})(1.5-1)=f_{water}\left(\frac{1.5}{1.33}-1\right)$$

$$24\times\frac{1}{2}=f\times\frac{1}{8}$$

$$f = 12 \times 8$$

$$f_{water} = 96 \text{ cm}$$

22. Find the number of spectral lines in H-atom when de-excite from n = 4 to ground state

Answer (6)

Sol. Number = 3 × 2

= 6

23. For a certain mechanical system the rate of accretion $\frac{dm}{dt}$ is proportional to $\sqrt{v}$, where m is mass, t is time and v is velocity, then the power is proportional to $v^{n/2}$ where n is _____.

Answer (5)

Sol. $$F=\left(\frac{dm}{dt}\right)v=\left(R\sqrt{v}\right)v=Rv^{3/2}$$

$$P=Fv=\left(Rv^{3/2}\right)v=Rv^{5/2}$$

24.

25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which one of the following forms most stable carbocation ?

   (1) $(Ph)_3C$-Br

   (2) $C_6H_5CH_2Br$

   (3) $C_6H_5CH(Br)CH_3$

   (4) $CH_3CH_2CH_2CH_2Br$

Answer (1)

Sol. $(Ph)_3C - Br$ Forms $Ph-\overset{\oplus}{\underset{\underset{Ph}{|}}{C}}-Ph$ as the most stable intermediate among the given compounds.

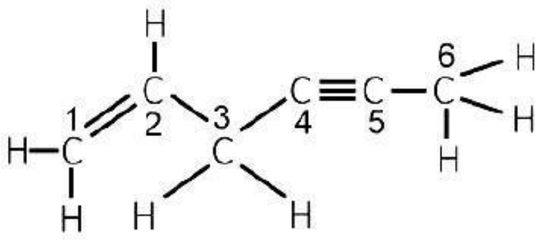
2. Number of σ and π bonds respectively in hex-1-en-4-yne are

   (1) 13, 3 (2) 14, 3

   (3) 3, 14 (4) 14, 13

Answer (1)

Sol.



Hex-1-en-4-yne

$\Rightarrow$ 13 σ and 3π bonds

3. Which element in group 15 has the lowest Ionisation Energy

   (1) Bi (2) P

   (3) As (4) Sb

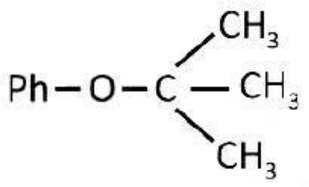
Answer (1)

Sol. $\underset{1402}{N} > \underset{1012}{P} > \underset{947}{As} > \underset{834}{Sb} > \underset{703\ kJ/mol}{Bi}$

4. Which of the following ether react with HBr to form phenol?

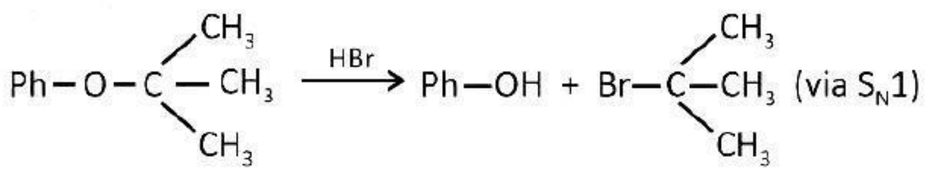
   (1) $Ph - CH_2 - O - CH_2 - CH_3$

   (2) $Ph - CH_2 - OCH_3$

   (3) 

   (4) $Ph - CH_2 - O - CH_2 - Ph$

Answer (3)

Sol. 

5. Consider the following thermochemical reactions and choose the correct option.

   C(diamond) $\rightarrow$ C(graphite) + x KJ

   C(diamond) + $O_2 \rightarrow CO_2$ + y KJ

   C(graphite) + $O_2 \rightarrow CO_2$ + z KJ

   (1) $x = y + z$ (2) $x = y - z$

   (3) $x + y = z$ (4) $x + y = -z$

Answer (2)

Sol. (1) C(diamond) $\rightarrow$ C(graphite) $\Delta H_1 = -xkJ$

(2) C(diamond) + $O_2(g) \rightarrow CO_2(g)$ $\Delta H_2 = -ykJ$

(3) C(graphite) + $O_2(g) \rightarrow CO_2(g)$ $\Delta H_3 = -zkJ$

From (1), (2) and (3), we get

$\Delta H_1 = \Delta H_2 - \Delta H_3$

$-x = -y + z$

$x = y - z$

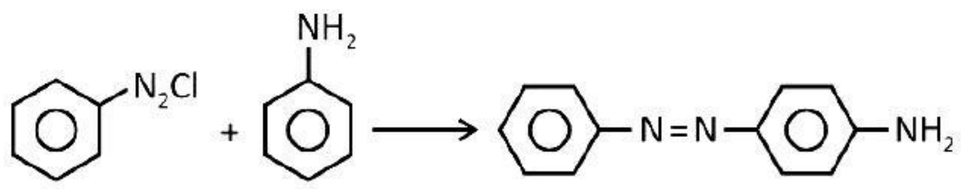
6. Which of the following will give azo dye test?

(1) Aniline (2) Anisole

(3) Benzene (4) Benzaldehyde

Answer (1)

Sol.



(Yellow coloured azo dye)

7. Which of the following is an essential amino acid?

(1) Alanine (2) Glycine

(3) Valine (4) Aspartic acid

Answer (3)

Sol. Tryptophan, Threonine, Histidine, Valine, Isoleucine, Phenylalanine, Methionine, Arginine, Leucine and Lysine are essential amino acids.

8. A drug becomes ineffective when it decomposes to 50 % its concentration. If 16 mg of said drug becomes 4 mg in 12 months, find the time in which drug becomes ineffective given that decomposition of drug follows first order kinetics.

(1) 6 months (2) 3 months

(3) 2 months (4) 12 months

Answer (1)

Sol. Drug $\xrightarrow{1^{st}\text{ order}}$ Products

Initial mass of drug = 16 mg

Mass of drug after 12 months = 4 mg

$t_{3/4}$ = 12 months

$2t_{1/2}$ = 12 months

$t_{1/2}$ = 6 months

$\therefore$ Drug becomes ineffective in 6 months.

9. Which of the following gives O_2 predominantly on electrolysis among the following?

A. Aq. $AgNO_3$ (Pt electrodes)

B. Aq. $AgNO_3$ (Ag electrodes)

C. Conc. H_2SO_4 (Pt electrodes)

D. Dilute H_2SO_4 (Pt electrodes)

(1) A, B only

(2) B, C only

(3) A, B, C only

(4) A, D only

Answer (4)

Sol. Aq. $AgNO_3$ (Pt electrodes)

Cathode : $Ag^+ + e^- \rightarrow Ag$

Anode : $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

Dilute H_2SO_4 (Pt electrodes)

Cathode : $2H_2O + 2e^- \rightarrow H_2 + OH^-$

Anode : $2H_2O \rightarrow O_2 + 4H^+ + e^-$

10. Determine the type of oxide formed by an element (A) which has the smallest size among following.

Li, Na, K, Be, B, Mg

(1) A_2O_3 (2) AO

(3) AO_2 (4) A_2O_2

Answer (1)

Sol. Among the given elements, boron (A) has the smallest size. The oxide of A is A_2O_3.

11. **Statement-I:** In partition chromatography a thin film of liquid acts as stationary phase.

Statement-II: Paper chromatography is not a type of partition chromatography.

(1) Statement-I is correct and statement-II is incorrect

(2) Statement-I is incorrect and statement-II is correct

(3) Both statement-I and statement-II are correct

(4) Both statement-I and statement-II are incorrect

Answer (1)

Sol. Paper chromatography is a type of partition chromatography, in which liquid acts as stationary phase.

12. 7.3g Benzalacetone is synthesized from 10.6 g of benzaldehyde using acetone as other reactant. Percentage yield of Benzalacetone is

(1) 50%

(2) 27%

(3) 90%

(4) 40%

Answer (1)

Sol. $C_6H_5CHO + CH_3COCH_3 \xrightarrow{\text{Base}} C_6H_5 - CH = CH - \overset{\overset{O}{\|}}{C} - CH_3$

1 mole 1 mole

(106 g) (146 g)

10.6 g should give 14.6 g for 100% yield

10.6 g give 7.3 of Benzalacetone in this question. So,

percentage yield = $\frac{7.3}{14.6} \times 100 = 50\%$

13. Some substances can effectively convert heat energy to electrical energy. For the conversion of thermal energy to electrical energy, the substance should have:

(1) Low thermal and low electrical conductivity

(2) High thermal and high electrical conductivity

(3) High thermal and low electrical conductively

(4) Low thermal and high electrical conductivity

Answer (4)

Sol. Substance should have low thermal and high electrical conductivity as it should readily conduct electricity while poorly transferring heat.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 0.41 g of $BaSO_4$ is obtained from 0.2 g of organic compound in Carius method. What is the percentage of sulphur present in organic compound?

Answer (28)

Sol. Moles of $BaSO_4 = \frac{0.41}{233}$ mol

Mass of sulphur $= \frac{0.41}{233} \times 32$ g

$= 0.056$ g

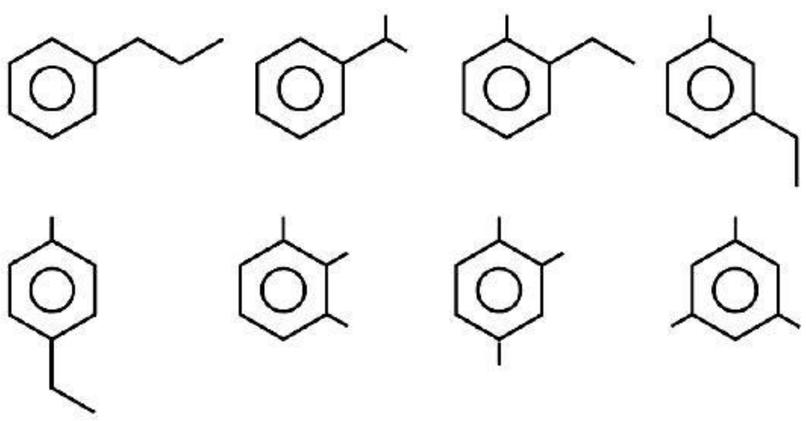
% of sulphur in organic compound $= \frac{0.056}{0.2} \times 100$

$= 28\%$

22. The number of benzenoid structural isomers having molecular formula C_9H_{12} which do not give Baeyer's reagent test is ?

Answer (8)

Sol. D.U. = $\frac{18+2-12}{2} = 4$



Baeyer's Reagent (cold dil. $KMnO_4$) reacts with alkene and alkynes and not with benzene.

23. How many maximum spectral lines are observed when a sample of hydrogen atoms de-excited from n = 4 to n = 1?

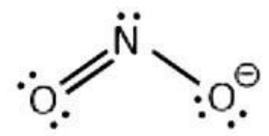
Answer (6)

Sol. Maximum number of spectral lines $= \frac{n(n-1)}{2}$

$= \frac{4(4-1)}{2} = \frac{12}{2} = 6$

24. Find number of non-bonding electron in NO_2^- ion is _____.

Answer (12)

Sol. 

Number of non-bonding electrons will be

$= 4 + 2 + 6$

$= 12$

25. Find spin only magnetic moment of yellow coloured complex compound

$K_3[Co(NO_2)_6]$, $Cu_2[Fe(CN)_6]$, $Zn_2[Fe(CN)_6]$, $Cu_3[Fe(CN)_6]_2$

Answer (0)

Sol. $Cu_2[Fe(CN)_6]$ = Chocolate brown ppt

$Zn_2[Fe(CN)_6]$ = White ppt

$Cu_3[Fe(CN)_6]_2$ = Green ppt

$K_3[Co(NO_2)_6]$ = Yellow ppt

In $K_3[Co(NO_2)_6]$, Co^{3+} with SFL(NO_2^-) has electronic configuration $t_{2g}^6 eg^0$

Number of unpaired e^- = 0

So, $\mu = 0$

MATHMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If the letters of the word "KANPUR" are arranged in dictionary, then the 440^{th} word is

   (1) PRKAUN

   (2) PRKUAN

   (3) PRKNAU

   (4) PRKUNA

Answer (1)

Sol. A K N P R U

A ---------5!

K ---------5!

N ---------5!

PA ---------4!

PK ---------4!

PN ---------4!

PRA ---------3!

438

$PRKANU \rightarrow 439^{th}$

$PRKAUN \rightarrow 440^{th}$

2. Let $f(x)=\int_0^x t\left(t^2-3t+20\right)dt, x\in(1,3)$ and range of $f(x)$ is (α, β), then $\alpha+\beta$ is equal to

   (1) $\frac{185}{4}$ (2) $\frac{185}{2}$

   (3) $\frac{185}{3}$ (4) $\frac{37}{4}$

Answer (2)

Sol. $f(x)=\frac{x^4}{4}-x^3+10x^2$

$f'(x)=x^3-3x^2+20x$

$=x(x^2-3x+20)$

in (1, 3) $f'(x)$ is positive

$\therefore$ $f(x)$ is increasing in (1, 3)

$\therefore$ $\alpha=f(1)=\frac{37}{4}$

$\beta=f(3)=83.25=\frac{333}{4}$

$\therefore$ $\alpha+\beta=92.5=\frac{185}{2}$

3. The value of the limit

$\lim\limits_{x\to0}(\text{cosec}\,x)\left(\sqrt{2\cos^2x+3\cos x}-\sqrt{\cos^2x+\sin x+4}\right)$ is

(1) 0 (2) 1

(3) $\frac{1}{2\sqrt{5}}$ (4) $-\frac{1}{2\sqrt{5}}$

Answer (4)

Sol. After rationalization,

$$\lim_{x\to0}\frac{1}{\sin x}\left(\frac{\left(2\cos^2x+3\cos x\right)-\left(\cos^2x+\sin x+4\right)}{\sqrt{2\cos^2x+3\cos x}+\sqrt{\cos^2x+\sin x+4}}\right)$$

$$\lim_{x\to0}\frac{\cos^2x+3\cos x-\sin x-4}{(\sin x)\left(\sqrt{5}+\sqrt{5}\right)}$$

$$=\lim_{x\to0}\frac{1}{2\sqrt{5}}\frac{\cos^2x+3\cos x-\sin x-4}{\sin x}$$

L'Hopital,

$$\Rightarrow \lim_{x\to0}\left(\frac{1}{2\sqrt{5}}\right)\left(\frac{2\cos x(-\sin x)-3\sin x-\cos x}{\cos x}\right)$$

$$=\frac{1}{2\sqrt{5}}\left[\frac{-1}{1}\right]=-\frac{1}{2\sqrt{5}}$$

4. Let the line L be $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{5}$ and foot of perpendicular from $(1, -2, -1)$ to L is (α, β, γ), then $\alpha+\beta+\gamma$ is

(1) $-\frac{69}{35}$ (2) $\frac{102}{35}$

(3) $\frac{69}{35}$ (4) $-\frac{102}{35}$

Answer (4)

Sol. $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{5}=\lambda$

Point on line $A(\lambda+1, 3\lambda+4, 5\lambda+7)$

$B(1, -2, -1)$

$\overrightarrow{AB}\cdot\langle 1, 3, 5\rangle=0$

$\lambda\cdot 1+3(3\lambda+6)+5(5\lambda+8)=0$

$35\lambda+18+40=0$

$\lambda=-\frac{58}{35}$

$(\alpha, \beta, \gamma)\equiv\left(-\frac{23}{35}, -\frac{34}{35}, -\frac{9}{7}\right)$

$\alpha+\beta+\gamma=-\frac{102}{35}$

5. If the exhaustive values of a for which the equation $2x^2+(a-5)x+15=3a$ has no real roots is (α, β), then $|4(\alpha+\beta)|$ is equal to

(1) 56 (2) 52

(3) 54 (4) 18

Answer (1)

Sol. No real roots $\Rightarrow$ discriminant is negative

$\Rightarrow (a-5)^2-4(2)(15-3a)<0$

$\Rightarrow a^2-10a+25-120+24a<0$

$a^2+14a-95<0$

$(a+19)(a-5)<0$

$\Rightarrow a\in(-19, 5)$

$\alpha=-19$

$\alpha=5$

$|4(\alpha+\beta)|$

$=|4(-19)|=56$

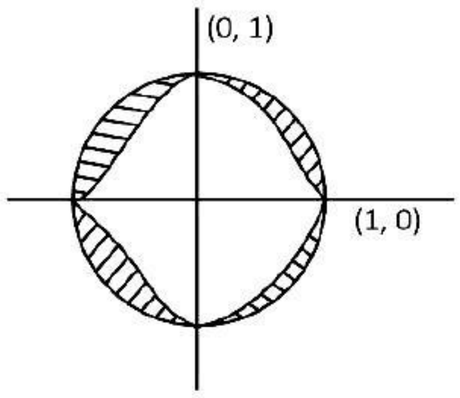
6. Area enclosed between the curves $|y|=1-x^2$ and $x^2+y^2=1$ is $(\pi-\alpha)$ sq. units, then 9α is

(1) 8 (2) 16

(3) 32 (4) 24

Answer (2)

Sol.



$\text{Area}=\pi-4\int_0^1(1-x^2)dx$

$=\pi-4\left[x-\frac{x^3}{3}\right]_0^1$

$=\pi-4\times\frac{2}{3}=\left(\pi-\frac{8}{3}\right)\text{sq unit}$

$=\pi-\alpha$

$\Rightarrow \pi=\frac{8}{3}$

$9\alpha=16$

7. If $\log y=x\log\frac{2}{5}$, $x\in\mathbb{N}\cup\{0\}$. Then sum of all values of y equals to

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{5}{4}$ (4) $\frac{8}{3}$

Answer (1)

Sol. $\log y=x\log\frac{2}{5}$

$\log y=\log\left(\frac{2}{5}\right)^x$

$$y=\left(\frac{2}{5}\right)^x$$

$x \in \mathbb{N} \cup \{0\}$

$\Rightarrow$ $y = 1, \frac{2}{5}, \left(\frac{2}{5}\right)^2$ ... which is in G.P.

Sum of all values of $\sum y = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$

8. There is an arithmetic progression $a_1, a_2, a_3, \ldots a_{2024}$ and $a_1 + (a_5 + a_{10} + a_{15} \ldots a_{2020}) + a_{2024} = 2233$. Find the value of $a_1 + a_2 + a_3 + \ldots a_{2024}$.

(1) 11034 (2) 11132

(3) 10432 (4) 20462

Answer (2)

Sol. $\because$ $a_1, a_2, a_3, \ldots, a_{2024}$ are in A.P.

Then $a_1 + a_{2024} = a_2 + a_{2023} = \ldots = a_r + a_{2024} - r + 1 = l$

$\therefore$ $a_1 + (a_5 + a_{10} + \ldots + a_{2020}) + a_{2024} = 2023$

or, $(202l) + l = 2023$

or, $203l = 2233$

$\therefore$ $a_1 + a_2 + \ldots + a_{2024} = 1012 \times l$

$= 1012 \times \frac{2233}{203}$

$= 1012 \times 11$

$= 11132$

9. Two points (4, 2) and (0, 2) lie on the circle whose centre lies on $3x + 2y + 2 = 0$, then length of chord whose mid point is (1, 2), is

(1) $\sqrt{3}$ (2) $\sqrt{5}$

(3) $2\sqrt{3}$ (4) $2\sqrt{5}$

Answer (3)

Sol. Let the centre be $(-2\alpha, 3\alpha - 1)$

$$\sqrt{(-2\alpha-4)^2 + (3\alpha-3)^2} = \sqrt{(-2\alpha-0)^2 + (3\alpha-3)^2}$$

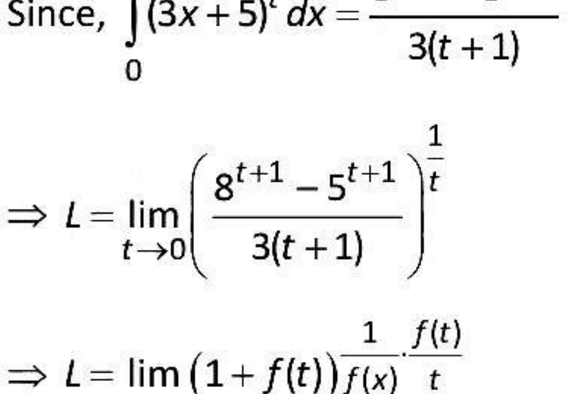
$\Rightarrow$ $(-2\alpha - 4)^2 = (-2\alpha)^2$

$\Rightarrow$ $-2\alpha - 4 = -2\alpha$

$\Rightarrow$ $\boxed{\alpha = -1}$

Centre will be (2, –4), radius $\sqrt{4+36} = \sqrt{40}$

$(x - 2)^2 + (y + 4)^2 = 40$



$\Rightarrow$ Length of chord $= 2\sqrt{3}$

10. If $\lim\limits_{t\to 0}\left(\int\limits_0^1 (3x+5)^t\,dx\right)^{\frac{1}{t}} = \frac{\alpha\left(\frac{8}{5}\right)^{\frac{p}{q}}}{4e}$, then α is

(1) 32 (2) 16

(3) 8 (4) 64

Answer (1)

Sol. Since, $\int\limits_0^1 (3x+5)^t\,dx = \frac{8^{t+1} - 5^{t+1}}{3(t+1)}$

$$\Rightarrow L = \lim_{t\to 0}\left(\frac{8^{t+1} - 5^{t+1}}{3(t+1)}\right)^{\frac{1}{t}}$$

$$\Rightarrow L = \lim_{t\to 0}\left(1 + f(t)\right)^{\frac{1}{f(x)}\cdot\frac{f(t)}{t}}$$

Where $f(t) = \frac{8^{t+1} - 5^{t+1}}{3(t+1)} - 1 = \frac{8^{t+1} - 5^{t+1} - 3t - 3}{3(t+1)}$

$\Rightarrow$ Since, $\lim\limits_{t\to 0} f(t) = 0$

$$L = \lim_{t\to 0} e^{\frac{f(t)}{t}} = e^{\lim\limits_{t\to 0}\frac{f(t)}{t}} = e^{\lim\limits_{x\to 0} f'(t)}$$

$$f'(t) = \frac{8.8^t \ln 8 - 5.5^t \ln 5 - 3}{3(t+1)} - \frac{8^{t-1} - 5^{t+1} - 3t - 3}{3(t+1)^2}$$

$$\lim_{t\to 0} f'(t) = \frac{8\ln 8 - 5\ln 5 - 3}{3} = \frac{1}{3}\ln\left(\frac{8^8}{5^5}\right) - 1$$

$$L = e^{\ln\left(\frac{8^8}{5^5}\right)^{\frac{1}{3}}-1} = \left(\frac{8^8}{5^5}\right)^{\frac{1}{3}}.e^{-1} = \frac{\left(\frac{8^8}{5^5}\right)^{\frac{1}{3}}}{e}$$

$$= \frac{1}{e}\left(\frac{8}{5}\right)^{\frac{5}{3}}.8^{\frac{1}{3}} = \frac{32\left(\frac{8}{5}\right)^{\frac{5}{3}}}{4e}$$

$\Rightarrow \alpha = 32$

11. The value of $\int_0^{\frac{\pi}{4}}\left(\sin\left|\left(4x-\frac{\pi}{2}\right)\right|+\sin[2x]\right)dx$ is

(where [·] denotes the greatest integer function)

(1) $\frac{1}{2}+\left(\frac{\pi-2}{4}\right)\sin 1$ (2) $\frac{1}{4}+\left(\frac{\pi-2}{2}\right)\sin 1$

(3) $\frac{1}{2}-\left(\frac{\pi-2}{4}\right)\sin 1$ (4) $\frac{1}{4}-\left(\frac{\pi-2}{2}\right)\sin 1$

Answer (1)

Sol. $\int_0^{\frac{\pi}{4}}\left(\sin\left|4x-\frac{\pi}{2}\right|+\sin[2x]\right)dx$

$$= \int_0^{\frac{\pi}{4}}\sin\left|4x-\frac{\pi}{2}\right|dx+\int_0^{\frac{\pi}{4}}\sin[2x]dx$$

$$= \int_0^{\frac{\pi}{8}}\sin\left|\frac{\pi}{2}-4x\right|dx+\int_{\frac{\pi}{8}}^{\frac{\pi}{4}}\sin\left[4x-\frac{\pi}{2}\right]dx+\int_0^{\frac{1}{2}}0dx$$

$$+\int_{\frac{1}{2}}^{\frac{\pi}{4}}\sin(1)dx$$

$$= \int_0^{\frac{\pi}{8}}\cos 4x dx+\int_{\frac{\pi}{8}}^{\frac{\pi}{4}}\cos 4x dx+\sin 1\cdot\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

$$= \left[\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{8}}-\left[\frac{\sin 4x}{4}\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}}+\frac{(x-2)\sin 1}{4}$$

$$= \frac{1}{4}+\frac{1}{4}+\frac{(\pi-2)\sin 1}{4}$$

$$= \frac{(\pi-2)\sin(1)+2}{4} = \frac{1}{2}+\left(\frac{\pi-2}{4}\right)\sin 1$$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $a_{ij} = \left(\sqrt{2}\right)^{i+j}$, $A = \left[a_{ij}\right]_{3\times 1}$. If sum of third row of A^2 is $\alpha+\beta\sqrt{2}$, then $\alpha+\beta$ is

Answer (224)

Sol. $\begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}\begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix} = \begin{bmatrix} 28 & 28\sqrt{2} & 56 \\ 28\sqrt{2} & 56 & 56\sqrt{2} \\ 56 & 56\sqrt{2} & 112 \end{bmatrix}$

$56+112+56\sqrt{2}$

$168+56\sqrt{2}$

$\alpha+\beta\sqrt{2}$

$\alpha+\beta = 224$

22. If 3^{107} is divided by 23, then remainder is

Answer (06)

Sol. Notice that, $3^4 \equiv (12) \pmod{23}$

$\Rightarrow 3^8 \equiv 144 \equiv 6 \pmod{23}$

$3^{11} \equiv 1 \pmod{23}$

$(3^{11})^9 \equiv 1 \pmod{23}$

$3^{99} \equiv 1 \pmod{23}$

$3^8 \cdot 3^{99} \equiv 1 \pmod{23}$

$\Rightarrow 3^{107} \equiv 6 \pmod{23}$

23. If α, β are the values of m where

$x + y + 2z = 1$

$x + 2y + 4z = m$

$x + 4y + 8 = m^2$ have infinitely many solutions.

Then $\sum_{n=1}^{10}\left(n^{\alpha}+n^{\beta}\right)$ is equal to

Answer (440)

Sol. For infinite solution

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 4 & 8 \end{vmatrix} = 0$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 2 \\ m & 2 & 4 \\ m^2 & 4 & 8 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & m & 4 \\ 1 & m^2 & 8 \end{vmatrix} = 0 \Rightarrow m^2 + 3m - 2$$

$\Rightarrow \; m^2 - 3m + 2 = 0$

$m = 2, 1$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & m \\ 1 & 4 & m^2 \end{vmatrix} = 0 \Rightarrow \; m^2 - 3m + 2 = 0$$

$\Rightarrow \; m = 2, 1 \; \Rightarrow \; \alpha = 1, \beta = 2$

$$\sum_{n=1}^{10}(n)^1 + (n)^2 = \frac{10\times 11}{2} + \frac{10\times 11\times 21}{6}$$

$= 440$

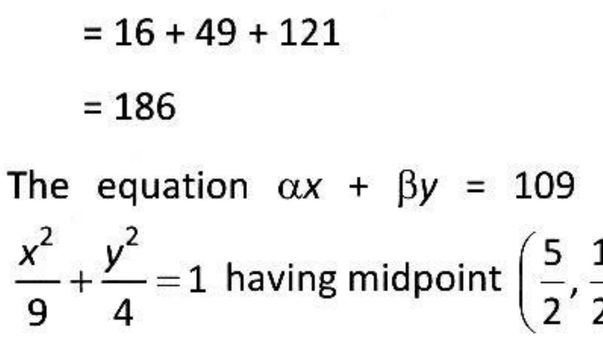
24. If the domain of $\log_{x-1}\left(\frac{2x^2-9x+4}{x^2-4x+5}\right)$ is (α, ∞) and $\log_5(18x - x^2 - 77)$ is (β, γ), then the value of $\alpha^2 + \beta^2 + \gamma^2$ is

Answer (186)

Sol. $\frac{2x^2-9x+4}{x^2-4x+5} > 0$ ...(i)

$x - 1 > 0, x - 1 \neq 1$

$\Rightarrow \; (2x - 1)(x - 4) > 0$



$\therefore \quad x \in (4, \infty)$

$\therefore \quad \alpha = 4$

$\log_5(18x - x^2 - 77)$

$\Rightarrow \; 18x - x^2 - 77 > 0$

$\Rightarrow \; x^2 - 18x + 77 < 0$

$\Rightarrow \; (x - 7)(x - 11) < 0$

$x \in (7, 11)$

$\therefore \quad \beta = 7, \gamma = 11$

$\therefore \quad \alpha^2 + \beta^2 + \gamma^2$

$= 16 + 49 + 121$

$= 186$

25. The equation $\alpha x + \beta y = 109$ is chord of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ having midpoint $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is

Answer (58)

Sol. Chord with given middle point

$T = S_1$

$$\frac{5}{18}x + \frac{y}{8} = \frac{25}{36} + \frac{1}{16} = \frac{109}{144}$$

$40x + 18y = 109$

$\equiv \quad \alpha x + \beta y = 109$

$\Rightarrow \quad \alpha = 40 \qquad \beta = 18$

$\alpha + \beta = 58$

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