

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

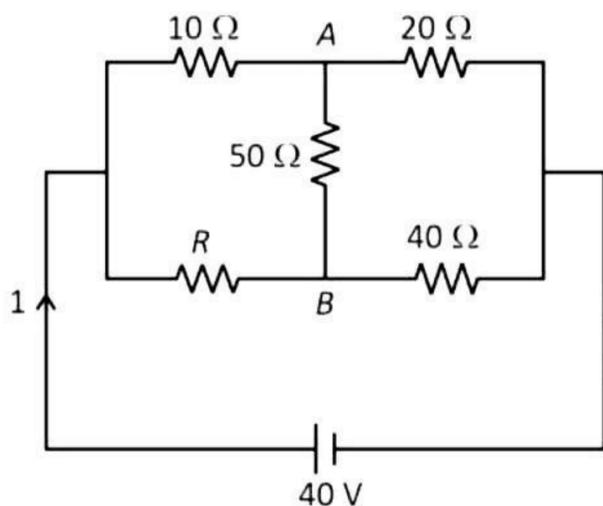
1. Bohr's model is applicable for single electron atom of atomic number z . Dependency of frequency of rotation of electron in n^{th} principal quantum number is proportional to

- | | |
|---------------------|-----------------------|
| (1) $\frac{z}{n^2}$ | (2) $\frac{z^2}{n^3}$ |
| (3) $\frac{n^3}{z}$ | (4) $\frac{z}{n}$ |

Answer (2)

Sol. $f = \frac{v}{2\pi r} \propto \frac{z}{n \left(\frac{n^2}{z} \right)} = \frac{z^2}{n^3}$

2. In the given circuit, find I if the potentials at A and B are equal



- (1) 1A
- (2) 2A
- (3) 3A
- (4) 4A

Answer (2)

Sol. Given potential at A and B are equal.

\Rightarrow This is a wheat-stone Bridge

i.e., $\frac{R}{10\Omega} = \frac{40\Omega}{20\Omega}$

or $R = 20\Omega$

Equivalent resistance = 20Ω

$I = \frac{40V}{20\Omega} = 2A$

3. In an electromagnetic wave, the magnetic field is given as

$\vec{B} = \left(\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j} \right) 30 \sin(\omega t - kz)$, the corresponding electric field is

- (1) $\left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j} \right) 9 \times 10^9 \sin(\omega t - kz)$
- (2) $\left(\frac{1}{2} \hat{i} - \frac{\sqrt{3}}{2} \hat{j} \right) 9 \times 10^9 \sin(\omega t - kz)$
- (3) $\left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j} \right) 9 \times 10^9 \cos(\omega t - kz)$
- (4) $\left(\frac{1}{2} \hat{i} - \frac{\sqrt{3}}{2} \hat{j} \right) 9 \times 10^9 \cos(\omega t - kz)$

Answer (2)

Sol. $E = BC$

$= 30 \times 3 \times 10^8 = 9 \times 10^9 \text{ N/C}$

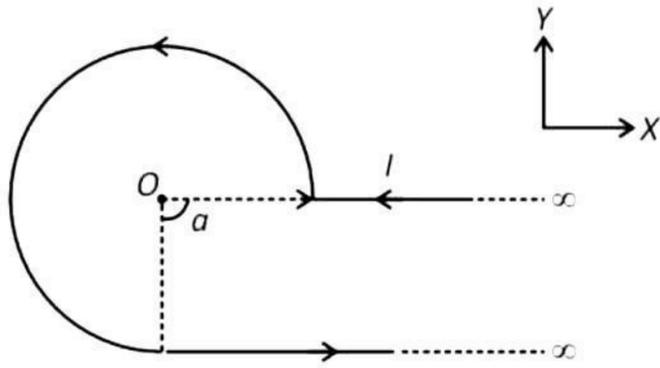
$\vec{E} = \hat{B} \times \hat{C}$

$= \left(\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j} \right) \times k$

$= \left(\frac{\sqrt{3}}{2} \hat{j} + \frac{1}{2} \hat{i} \right)$

$\vec{E} = \left(\frac{1}{2} \hat{i} - \frac{\sqrt{3}}{2} \hat{j} \right) 9 \times 10^9 \sin(\omega t - kz)$

4. The magnetic field \vec{B} at the centre O of the given arrangement is



- (1) $\frac{+\mu_0 I}{8\pi a}(3\pi + 2)\hat{k}$
- (2) $\frac{-\mu_0 I}{8\pi a}(3\pi + 2)\hat{k}$
- (3) $\frac{+\mu_0 I}{8\pi a}(3\pi - 2)\hat{k}$
- (4) $\frac{-\mu_0 I}{8\pi a}(3\pi - 2)\hat{k}$

Answer (1)

Sol. $\vec{B}_{\text{net}} = \vec{B}_{\text{circle}} + \vec{B}_{\text{wire 1}} + \vec{B}_{\text{wire 2}}$

$$= \frac{3}{4} \left(\frac{\mu_0 I}{2a} \right) (+\hat{k}) + \vec{0} + \frac{\mu_0 I}{4\pi a} (+\hat{k})$$

$$= \frac{+\mu_0 I}{8\pi a} (3\pi + 2)\hat{k}$$

5. A cube of side 10 cm having bulk modulus of 1.4×10^{11} Pa is placed in atmosphere. Now it is subjected to extra pressure of 7×10^6 Pa then magnitude of change in volume of cube is

- (1) 0.03 mL
- (2) 0.3 mL
- (3) 0.05 mL
- (4) 0.2 mL

Answer (3)

Sol. $B = \frac{\Delta P}{\Delta V/V}$

$$|\Delta V| = \frac{\Delta PV}{B}$$

$$|\Delta V| = \frac{7 \times 10^6 \times 10^{-3}}{1.4 \times 10^{11}}$$

$$|\Delta V|_{\text{mL}} = \frac{5 \times 10^3}{10^{11}} \times 10^6$$

$$= 0.05 \text{ mL}$$

6. Choose the correct option representing the energy density between the plates of a parallel plate capacitor with plate area A , plate separation d and potential difference V .

- (1) $\frac{\epsilon_0 V^2}{2d^2}$
- (2) $\frac{\epsilon_0 V d^2}{2}$
- (3) $\frac{\epsilon_0 A V^2}{2d}$
- (4) $\frac{\epsilon_0 A V^2}{2d^2}$

Answer (1)

Sol. $E = \frac{V}{d}$

$$\text{Energy density} = \frac{1}{2} \epsilon_0 E^2$$

$$= \frac{1}{2} \frac{\epsilon_0 V^2}{d^2}$$

7. Which of the following phenomenon is not explained by wave theory of light

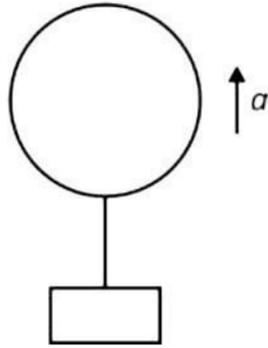
- (1) Reflection of light
- (2) Refraction of light
- (3) Diffraction
- (4) Compton effect

Answer (4)

Sol. Compton effect is based on particle nature of light.

8. A balloon system having mass m is moving up with acceleration a , find the mass to be removed from it to have acceleration $3a$.

(Neglect the volume of mass attached)



- (1) $\frac{2ma}{3a+g}$
 (2) $\frac{2ma}{2a+g}$
 (3) $\frac{ma}{3a+g}$
 (4) $\frac{ma}{g-3a}$

Answer (1)

Sol. $F_B - mg = ma$... (i)

$F_B - (m-x)g = 3(m-x)a$... (ii)

On solving

$$x = \frac{2ma}{3a+g}$$

9. Mass M and radius R of a planet is related with mass M_e and Radius R_e of earth as $M_e = 8M_p$ and $R_e = 2R_p$. If escape speed for earth is 11.2 km/sec, then escape speed for the planet is

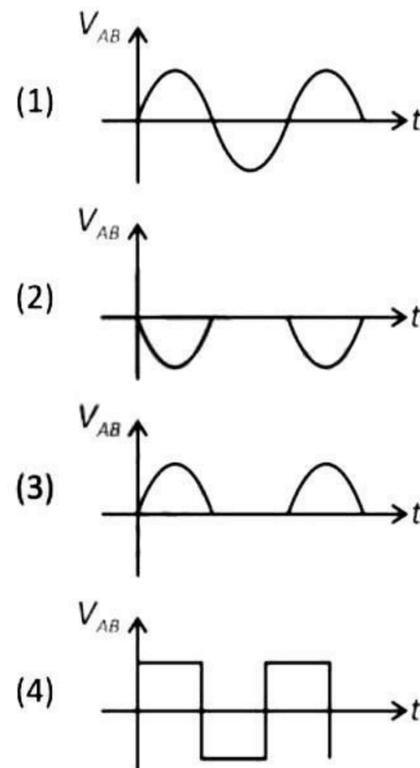
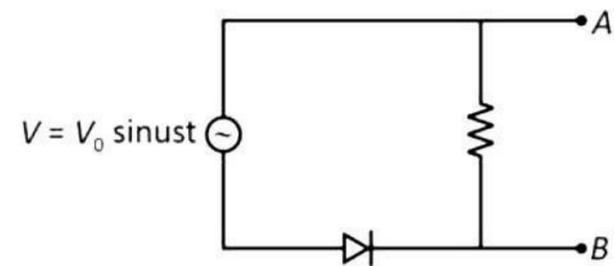
- (1) $11.2\sqrt{2}$ km/sec
 (2) 5.6 km/sec
 (3) $5.6\sqrt{2}$ km/sec
 (4) 11.2 km/sec

Answer (2)

Sol. $\frac{v'}{v_e} = \sqrt{\frac{GM}{RG(8M)}} = \frac{1}{2}$

$$v' = \frac{v_e}{2} = \frac{11.2}{2} = 5.6$$

10. The correct variation of voltage across AB is given by (consider that the threshold voltage of the diode is very small)



Answer (2)

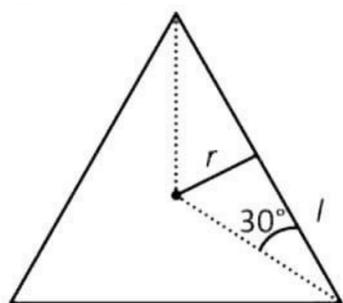
Sol. The diode will only conduct in negative half.

11. An equilateral triangle frame of side l is carrying current i , find magnetic field at its centroid

- (1) $\frac{3\mu_0 i}{4\pi l}$ (2) $\frac{3\mu_0 i}{\pi l}$
 (3) $\frac{9\mu_0 i}{2\pi l}$ (4) $\frac{\mu_0 i}{\pi l}$

Answer (3)

Sol.



$$B = 3 \left(\frac{\mu_0 i}{4\pi l \tan 30^\circ} \right) 2 \cos 30^\circ$$

$$= \frac{3\mu_0 i}{2\pi l} \sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{9\mu_0 i}{2\pi l}$$

12. Select the correct match for dimensions

Column-I

Column-II

(A) Angular Momentum (I) $[MLT^{-2}]$

(B) Force (II) $[ML^2T^{-1}]$

(C) Energy (III) $[ML^{-1}T^{-2}]$

(D) Pressure (IV) $[ML^2T^{-2}]$

(1) A-(II), B(III), C-(I), D-(IV)

(2) A-(I), B(II), C-(III), D-(IV)

(3) A-(II), B(I), C-(IV), D-(III)

(4) A-(II), B(I), C-(III), D-(IV)

Answer (3)

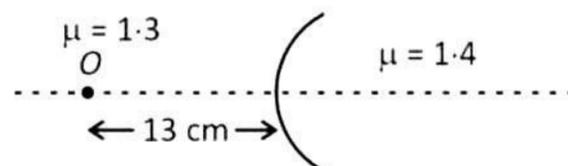
Sol. Angular momentum = $[ML^2T^{-1}]$

Force = $[MLT^{-2}]$

Energy = $[ML^2T^{-2}]$

Pressure = $[ML^{-1}T^{-2}]$

13. In the figure shown the object kept at a distance 13 cm from the interface forms a real image which is double in size. The radius of curvature of the interface is



(1) $\frac{3}{2}$ cm

(2) $\frac{2}{3}$ cm

(3) $\frac{3}{4}$ cm

(4) $\frac{4}{3}$ cm

Answer (2)

Sol. Magnification = $m = \frac{v}{u} \frac{\mu_2}{\mu_1}$

$$\Rightarrow m = -2 = \frac{v}{-13} \frac{1.4}{1.3}$$

$$\Rightarrow v = +28 \text{ cm}$$

Using formula for refraction at a curved surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

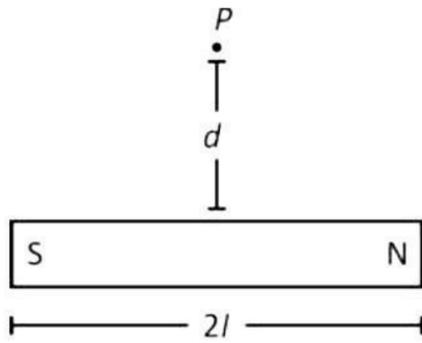
$$\frac{1.4}{+28 \text{ cm}} - \frac{1.3}{-13 \text{ cm}} = \frac{1.4 - 1.3}{R}$$

$$\frac{1}{2} + 1 = \frac{1}{R}$$

$$R = \frac{2}{3} \text{ cm}$$

14. Due to the bar magnet shown, if the % uncertainty in d is 1%, find uncertainty in the magnetic field at P .

[d : 10 units, l = 10 units]



- (1) 2%
 (2) 3%
 (3) 1.5%
 (4) 0.5%

Answer (3)

Sol. $B = \frac{2\mu_0 m}{4\pi r^2} \cos\theta$

$$r = \sqrt{(10)^2 + d^2}$$

$$\cos\theta = \frac{10}{\sqrt{(10)^2 + d^2}}$$

$$B = 2 \left(\frac{\mu_0}{4\pi} \right) \frac{10m}{(10^2 + d^2)^{3/2}}$$

$$\frac{dB}{dd} = \frac{3B}{2} \frac{2d}{(10^2 + d^2)}$$

$$\frac{dB}{B} = \left(\frac{3d^2}{10^2 + d^2} \right) \left(\frac{dd}{d} \right)$$

$$= 1.5\%$$

15. A capacitor of capacitance $1 \mu\text{F}$ is charged to potential of 20 V. Distance between plates is $10 \mu\text{m}$, then charge density on plates is

- (1) 17.7 nC/m^2 (2) $17.7 \mu\text{C/m}^2$
 (3) 8.85 nC/m^2 (4) $4.42 \mu\text{C/m}^2$

Answer (2)

Sol. $Q = 20 \times 10^{-6} \text{ C}$

$$\frac{\epsilon_0 A}{d} = 10^{-6}$$

$$A = \frac{10^{-6} \times 10 \times 10^{-6}}{8.85 \times 10^{-12}} = \frac{10}{8.85}$$

$$\sigma = \frac{20 \times 10^{-6}}{10} \times 8.85$$

$$= 17.7 \times 10^{-6}$$

$$= 17.7 \mu\text{C/m}^2$$

16. A ring of radius 3 cm has a soap film which is getting evaporated. Light of wavelength $\lambda = 580 \text{ nm}$ gives minimum transmission every 12 s. Find the rate of evaporation. (refractive index = 1.45)

- (1) $1.5 \pi \times 10^{-13} \text{ m}^3/\text{s}$
 (2) $15 \pi \times 10^{-13} \text{ m}^3/\text{s}$
 (3) $3 \pi \times 10^{-13} \text{ m}^3/\text{s}$
 (4) $3 \pi \times 10^{-12} \text{ m}^3/\text{s}$

Answer (2)

Sol. $2\mu l = n\lambda$

$$2\mu \Delta l = \Delta n \lambda$$

$$\left(\frac{\Delta l}{\Delta t} \right) = \frac{\Delta n}{\Delta t} \frac{\lambda}{2\mu}$$

$$= \frac{1}{12} \frac{580 \text{ nm}}{2 \times 1.45} = \frac{5}{3} \text{ nm/s}$$

$$\text{Rate of evaporation} = \pi R^2 \frac{\Delta l}{\Delta t}$$

$$= \pi (3 \times 10^{-4})^2 \times \left(\frac{5}{3} \times 10^{-9} \right) \text{ m}^3/\text{s}$$

$$= 15\pi \times 10^{-13} \text{ m}^3/\text{s}$$

17.
 18.
 19.
 20.

SECTION - B

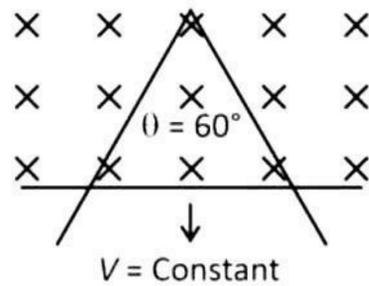
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. An electric dipole of moment $6 \times 10^{-6} \text{ cm}$ is placed parallelly in electric field of strength 10^6 N/C . Work done required to rotate the dipole by 180° is X joules, then X is

Answer (12)

Sol. $\omega = \Delta U = -pE \cos 180^\circ - (-pE \cos 0)$
 $= 2pE = 2 \times 6 \times 10^{-6} \times 10^6$

22. The figure shows a conducting rod sliding on two conducting rails having angle $(\theta = 60^\circ)$ in a uniform magnetic field with a constant velocity V . Find n if the motional emf E varies with time as $E = ct^n$.

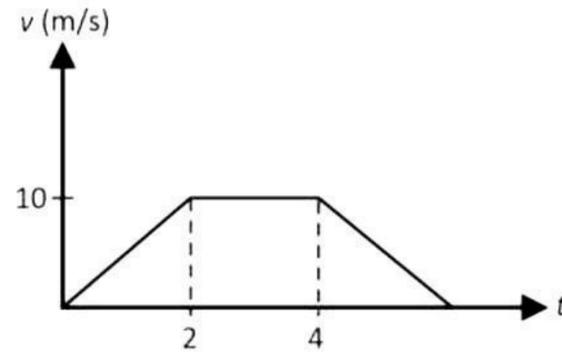


Answer (1)

Sol. Slide $= \frac{2}{\sqrt{3}}x$
 $= \frac{2}{\sqrt{3}}vt$
 $Emf = B \left(\frac{2}{\sqrt{3}}vt \right) v$
 $= \frac{2Bv^2}{\sqrt{3}}t$

$E \propto t$
 or $n = 1$

23. The velocity vs time graph of a particle moving along X-axis is plotted as shown. The distance travelled (in metre) by the particle in the interval $t = 0 \text{ s}$ to $t = 4 \text{ s}$ is



Answer (30)

Sol. Distance = displacement as direction of velocity does not change in the given interval.
 $\Rightarrow \text{Distance} = \frac{1}{2} (2\text{s} + 4\text{s}) \times 10 \text{ m/s}$
 [Area of trapezium with base 4s]
 $= 30\text{m}$

24. Distance between real object and its three times magnified image formed by concave mirror is 20 cm then radius of curvature of the mirror is X cm, then X is

Answer (15)

Sol. $\left| \frac{v}{u} \right| = 3$
 $|v| = 3|u|$
 $|u| = X$
 $|v| = 3X$
 $3X - X = 20$
 $X = 10 \text{ cm}$
 $\frac{1}{-30} - \frac{1}{10} = \frac{1}{f}$
 $-\frac{4}{30} = \frac{1}{f}$
 $R = \frac{2 \times 30}{4} = 15 \text{ cm}$

25.

6. 30 gm HNO_3 is added to a solution to prepare 75% w/w solution having density 1.25 g/mL. Volume of solution is
- (1) 32 mL
 - (2) 48 mL
 - (3) 36 mL
 - (4) 28 mL

Answer (1)

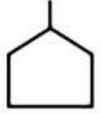
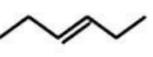
Sol. $M = \frac{10 \times \%w / w \times d}{M_0}$

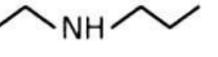
$$M = \frac{10 \times 75 \times 1.25}{63}$$

$$M = \frac{n}{V_{\text{mL}}} \times 1000$$

$$\frac{10 \times 75 \times 1.25}{63} = \frac{30}{63 \times V_{\text{mL}}} \times 1000$$

$$V_{\text{mL}} = 32 \text{ mL}$$

7. Statement-I  and  are ring chain isomers

Statement-II  NH_2 and  NH are functional isomers

- (1) Both Statement -I and Statement -II are correct statements
- (2) Statement -I is correct and Statement -II is not correct
- (3) Statement -I is wrong statement and Statement -II is correct statement
- (4) Both Statement -I and Statement -II are correct

Answer (1)

Sol. 1° amine and 2° amine are functional isomers

8. For an elementary reaction



When volume becomes $\frac{1}{3}$ rd, rate of reaction becomes

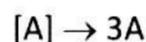
- (1) 8 times
- (2) 9 times
- (3) 6 times
- (4) 2 times

Answer (2)

Sol. For an elementary reaction

$$r = k[A]^1 [B]^1$$

When volume becomes $\frac{1}{3}$ rd



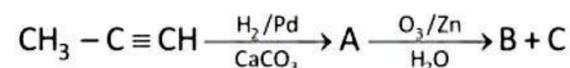
$$r' = k[3A]^1 [3B]^1$$

$$r' = k \cdot 3 \times 3 [A] [B]^1$$

$$r' = 9 \times r$$

rate of reaction becomes 9 times

9. Consider the following sequence of reaction

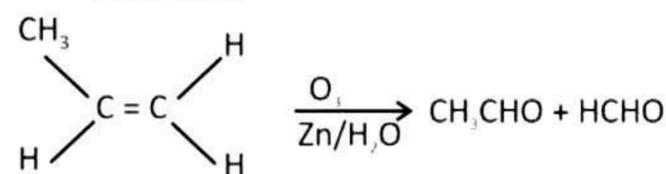


- (1) B = CH_3CHO (2) B = CH_3CHO
C = HCHO C = HCOOH

- (3) B = $\text{CH}_3 - \overset{\text{O}}{\parallel}{\text{C}} - \text{CH}_3$ (4) B = HCHO
C = HCHO C = CH_3COOH

Answer (1)

Sol. $\text{CH}_3 - \text{C} \equiv \text{CH} \xrightarrow[\text{hydrogenation}]{\text{Partial}}$



10. Match the following List-I with List-II.

| | List-I | | List-II |
|-----|-----------------------------------|-------|-----------|
| (A) | $[\text{CoF}_6]^{3-}$ | (i) | sp^3d^2 |
| (B) | $[\text{Co}(\text{NH}_3)_6]^{3+}$ | (ii) | d^2sp^3 |
| (C) | $[\text{NiCl}_4]^{2-}$ | (iii) | sp^3 |
| (D) | $[\text{Ni}(\text{CN})_4]^{2-}$ | (iv) | dsp^2 |

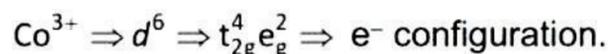
Choose the correct answer from the options given below:

- (1) (A)-(i), (B)-(ii), (C)-(iii), (D)-(iv)
- (2) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)
- (3) (A)-(i), (B)-(ii), (C)-(iv), (D)-(iii)
- (4) (A)-(ii), (B)-(i), (C)-(iii), (D)-(iv)

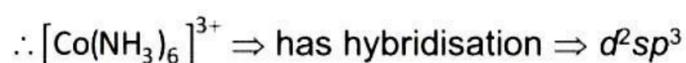
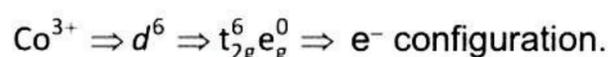
Answer (1)

Sol.

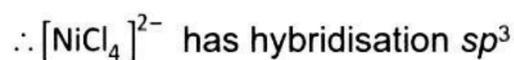
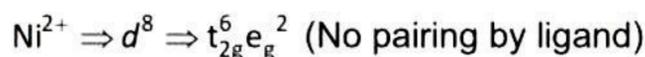
(A) $[\text{CoF}_6]^{3-} \Rightarrow$ Cobalt in +3 O.S. with Flourine ligand. Here, F^- act as weak field ligand



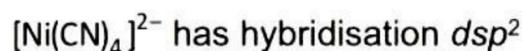
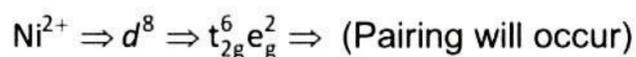
(B) $[\text{Co}(\text{NH}_3)_6]^{3+} \Rightarrow \text{Co}^{3+}$, NH_3 ligand act as SFL.



(C) $[\text{NiCl}_4]^{2-} \Rightarrow \text{Ni}^{2+} \Rightarrow \text{Cl}^-$ ligand act as WFL.



(D) $[\text{Ni}(\text{CN})_4]^{2-} \Rightarrow \text{Ni}^{2+} \Rightarrow \text{CN}^-$ act as weak field ligand.

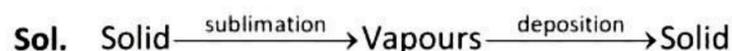


11. The correct name of I & II in the following process is :



- (1) I \rightarrow Sublimation
II \rightarrow Vaporisation
- (2) I \rightarrow Sublimation
II \rightarrow Decomposition
- (3) I \rightarrow Sublimation
II \rightarrow Deposition
- (4) I \rightarrow Deposition
II \rightarrow Sublimation

Answer (3)



12. Which of the following biomolecules doesn't contain $\text{C}_1 - \text{C}_4$ glycosidic linkage

- (1) Amylopectin
- (2) Maltose
- (3) Lactose
- (4) Sucrose

Answer (4)

Sol. Amylopectin \rightarrow branched chain polymer. The chain is formed by $\text{C}_1 - \text{C}_4$ glycosidic linkage and $\text{C}_1 - \text{C}_6$ glycosidic linkage

Maltose $\rightarrow \text{C}_1 - \text{C}_4$ glycosidic linkage

Lactose $\rightarrow \text{C}_1 - \text{C}_4$ glycosidic linkage

Sucrose $\rightarrow \text{C}_1 - \text{C}_2$ glycosidic linkage

13. Consider the following statements:

Statement I: In law of octaves, elements were arranged in increasing order of their atomic numbers.

Statement II: Lothar Meyer, plotted the physical properties against atomic weight.

Choose the correct answer from the options given below:

- (1) Both statement I and statement II are correct
- (2) Both statement I and statement II are incorrect
- (3) Statement I is correct but statement II is incorrect
- (4) Statement I is incorrect but statement II is correct

Answer (4)

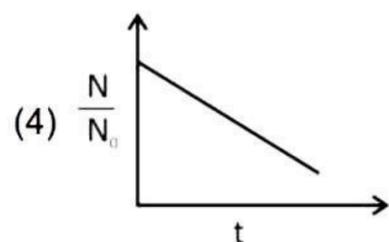
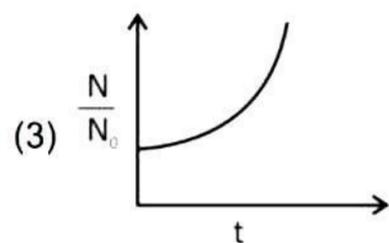
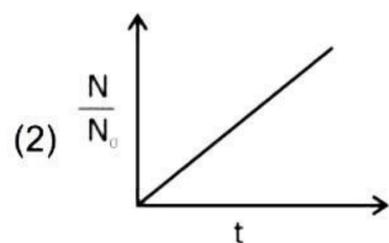
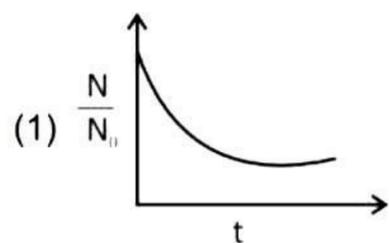
Sol. In law of octaves, elements were arranged in increasing order of their atomic weights.

\therefore Statement I is incorrect and statement II is correct statement.

14. The bacterial life grows as per 1st order kinetics.

Which of the following graph is correct between $\frac{N}{N_0}$

and t



Answer (3)

Sol. $\frac{dN}{dt} = kN$

$$\frac{dN}{N} = k dt$$

On integrating using proper limits

$$\int_{N_0}^N \frac{dN}{N} = k \int_0^t dt$$

$$[\ln N]_{N_0}^N = k[t]_0^t$$

$$\ln N - \ln N_0 = kt$$

$$\ln \frac{N}{N_0} = kt$$

$$\frac{N}{N_0} = e^{kt}$$

Value of $\frac{N}{N_0}$ increases exponentially.

15. Bohr model is applicable for single electron system having atomic number Z. Frequency of rotation of electron is directly proportional to

- (1) $\frac{Z}{n^2}$ (2) $\frac{Z^2}{n^3}$
 (3) $\frac{n^2}{Z}$ (4) $\frac{n^3}{Z^2}$

Answer (2)

Sol. $f = \frac{v}{2\pi r} \propto \frac{\frac{Z}{n^2}}{\frac{n}{Z}} \propto \frac{Z^2}{n^3}$

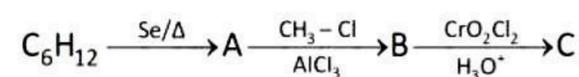
16. In which of the following detection of nitrogen is not possible by Lassaigne's extract method?

- (1) $\text{NH}_2\text{-NH}_2$
 (2) $\text{NH}_2\text{-}\overset{\text{O}}{\parallel}\text{C}\text{-NH}_2$
 (3) Aniline
 (4) Phenyl hydrazine

Answer (1)

Sol. Since, in hydrazine carbon is not present
 \therefore It cannot be detected by Lassaigne's test

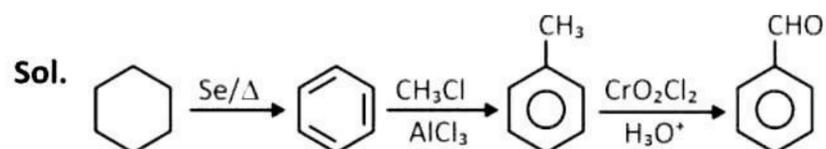
17. Consider the following sequence of reaction



Choose the correct option about major product

- (1) 'C' gives Fehling's solution test
 (2) 'C' can be prepared by reaction of PhMgBr with CO_2
 (3) 'C' can give Tollen's test
 (4) 'C' can give effervescence with NaHCO_3

Answer (3)



Benzaldehyde can give Tollen's test

18.

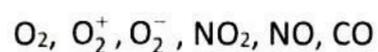
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Number of paramagnetic species among the following is:



Answer (5)

Sol. O_2 , Number of $e^- = 16$, paramagnetic

O_2^+ , Number of $e^- = 15$, paramagnetic

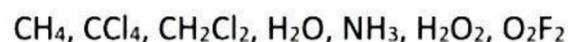
O_2^- , Number of $e^- = 17$, paramagnetic

NO_2 , odd e^- specie, paramagnetic

NO , Number of $e^- = 15$, paramagnetic

CO , Number of $e^- = 14$, diamagnetic

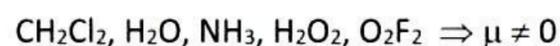
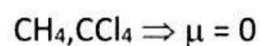
22. How many of the following molecules are polar?



Answer (5)

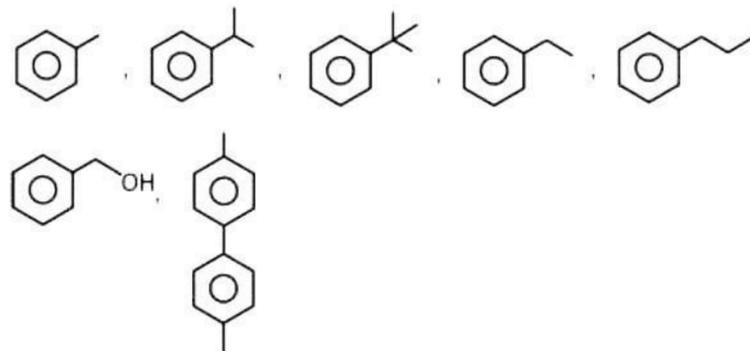
Sol. Compounds having permanent dipole moment

($\mu \neq 0$) are polar

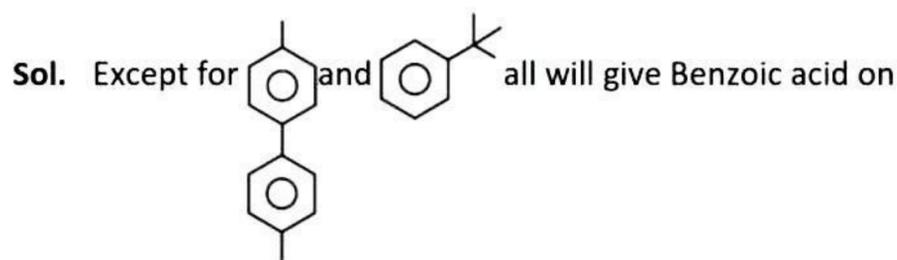


Number of polar molecules = 5

23. How many of the following will give Benzoic acid on reaction with hot alkaline $KMnO_4$?



Answer (5)

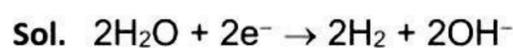


reaction with hot alkaline $KMnO_4$.

Note : Benzylic hydrogen must be present to give Benzoic acid.

24. By passing current in 600 mL of NaCl solution pH increases to 12. Find current (i) if electrolysis occur for 10 min (assume 100% efficiency)

Answer (1)



$pH = 12 \quad pOH = 2 \quad [OH^-] = 10^{-2}M$

g eq. of OH^- formed = no. of faraday of charge passed

$$10^{-2} \times \frac{600}{1000} \times 1 = \frac{i \times 10 \times 60}{96500}$$

$0.965A = i$

25.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f(x) = \int \frac{dx}{x^{1/4}(x^{1/4} + 1)}$. If $f(0) = -6$, then $f(2)$ is

- (1) $4 \left[\frac{1}{\sqrt{2}} - 2^{1/4} + \ln|1 + 2^{1/4}| \right] - 6$
 (2) $4 \left[\frac{1}{\sqrt{2}} - 2^{1/4} + \ln|1 + 2^{1/4}| \right] + 6$
 (3) $4 \left[\frac{1}{\sqrt{2}} + 2^{1/3} + \ln|2^{1/4}| \right] - 6$
 (4) $4 \left[3 + 2^{1/3} - \ln 2^{1/4} \right] + 6$

Answer (1)

Sol. $\int \frac{dx}{x^{1/4}(x^{1/4} + 1)}$
 $x^{1/4} = t \Rightarrow dx = 4t^3 dt$
 $\int \frac{4t^3 dt}{t(t+1)} = 4 \int \frac{t^2 dt}{t+1} = 4 \left[\int \left(\frac{t^2 - 1}{t+1} + \frac{1}{t+1} \right) dt \right]$
 $= 4 \left[\int (t-1) dt + \ln|t+1| \right] + c$
 $f(x) = 4 \left[\frac{x^{1/2}}{2} - x^{1/4} + \ln|x^{1/4} + 1| \right] + c$
 $f(0) = -6$
 $-6 = 4(0) + c$
 $\Rightarrow c = -6$
 $\Rightarrow f(x) = 4 \left[\frac{\sqrt{x}}{2} - x^{1/4} + \ln|1 + x^{1/4}| \right] - 6$
 $f(2) = 4 \left[\frac{1}{\sqrt{2}} - (2)^{1/4} + \ln|1 + 2^{1/4}| \right] - 6$

2. Evaluate $\sum_{r=1}^{13} \frac{1}{\sin \left[\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right] \sin \left[\frac{\pi}{4} + \frac{r\pi}{6} \right]}$

- (1) $2\sqrt{3} + 2$ (2) $2\sqrt{3} - 2$
 (3) $3\sqrt{2} + 2$ (4) $3\sqrt{2} - 4$

Answer (2)

Sol. $\sum_{r=1}^{13} \frac{1}{\sin \left[\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right] \cdot \sin \left[\frac{\pi}{4} + r \cdot \frac{\pi}{6} \right]}$
 $= \sum_{r=1}^{13} \frac{\sin \left\{ \left(\frac{\pi}{4} + r \cdot \frac{\pi}{6} \right) - \left(\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right) \right\}}{\sin \frac{\pi}{6} \cdot \sin \left[\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right] \cdot \sin \left[\frac{\pi}{4} + r \cdot \frac{\pi}{6} \right]}$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

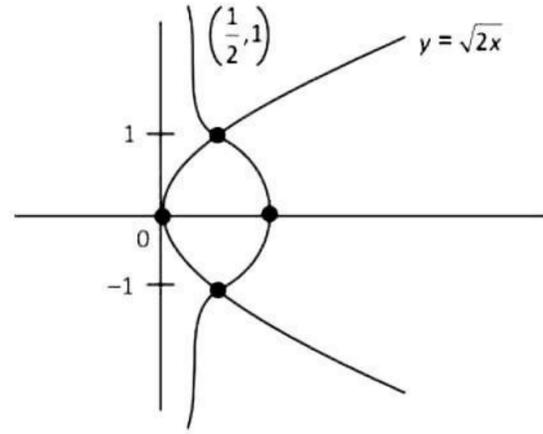
$= 2 \cdot \sum_{r=1}^{13} \cot \left(\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right) - \cot \left(\frac{\pi}{4} + r \cdot \frac{\pi}{6} \right)$
 $= 2 \cdot \cot \left(\frac{\pi}{4} + 0 \cdot \frac{\pi}{6} \right) - \cot \left(\frac{\pi}{4} + \frac{13\pi}{6} \right)$
 $= 2 \cdot \left\{ \cot \frac{\pi}{4} - \cot \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \right\}$
 $= 2 [1 - 2 + \sqrt{3}]$
 $= 2 [\sqrt{3} - 1]$

3. Area bounded between the curves $C_1 : x(1 + y^2) - 1 = 0$ and $C_2 : y^2 - 2x = 0$ is (in sq. unit)

- (1) $\frac{\pi}{2} - \frac{1}{3}$ (2) $\frac{\pi}{4} - \frac{1}{6}$
 (3) $2 \left(\frac{\pi}{2} - \frac{1}{6} \right)$ (4) $\frac{\pi}{6} + \frac{1}{2}$

Answer (1)

Sol.



$$\int_{-1}^1 (x_2 - x_1) dy$$

$$\Rightarrow \int_{-1}^1 \left(\frac{1}{1+y^2} - \frac{y_2}{2} \right) dy$$

$$= \tan^{-1} y - \frac{y_3}{6} \Big|_{-1}^1$$

$$= \left(\tan^{-1} \frac{1}{6} \right) - \left(\tan^{-1}(-1) - \left(\frac{-1}{6} \right) \right)$$

$$= 2 \tan^{-1} 1 - \frac{1}{3} = \frac{\pi}{2} - \frac{1}{3}$$

4. There are three bags such that bag 1 has 4 white, 6 blue, bag 2 has 6 white and 4 blue and bag 3 has 5 white and 5 blue balls. A bag is randomly selected and a ball is randomly picked out of it, it comes out to be white then probability that selected bag was bag 2.

- (1) $\frac{2}{5}$ (2) $\frac{2}{15}$
 (3) $\frac{1}{15}$ (4) $\frac{7}{15}$

Answer (1)

| | | |
|---------|---------|---------|
| 4W + 6B | 6W + 4B | 5W + 5B |
| BAG 1 | BAG 2 | BAG 3 |

$$P\left(\frac{B_2}{W}\right) = \frac{P(B_2) \cdot P\left(\frac{W}{B_2}\right)}{P(W)}$$

$$P(W) = \sum_{i=1}^3 P(B_i) P\left(\frac{W}{B_i}\right)$$

$$P(B_1) = \frac{1}{3} = P(B_2) = P(B_3)$$

$$P\left(\frac{W}{B_1}\right) = \frac{4}{10}, P\left(\frac{W}{B_2}\right) = \frac{6}{10}, P\left(\frac{W}{B_3}\right) = \frac{5}{10}$$

$$\Rightarrow P\left(\frac{B_2}{W}\right) = \frac{\frac{1}{3} \times \frac{6}{10}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{6}{10} + \frac{1}{3} \times \frac{5}{10}}$$

$$= \frac{\frac{6}{10}}{\frac{4}{10} + \frac{6}{10} + \frac{5}{10}} = \left(\frac{6}{15} \right) = \frac{2}{5}$$

5. If S is a set of words formed by all the letters of word "GARDEN", then find the probability that vowels are not in alphabetical order.

- (1) $\frac{1}{2}$ (2) $\frac{1}{3}$
 (3) $\frac{1}{4}$ (4) $\frac{1}{5}$

Answer (1)

Sol. AE GRDN

Only 2 vowels are there.

∴ Only half of the cases will have A before E and vice-versa.

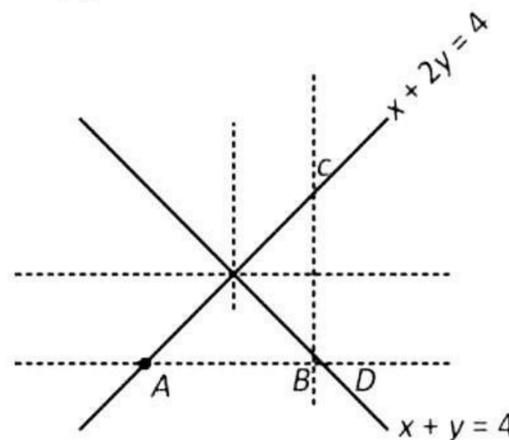
$$\text{Required probability} = \frac{1}{2}$$

6. In isosceles triangle two sides are $x + 2y = 4$, $x + y = 4$ than the sum of all possible value of slope of third side of triangle is

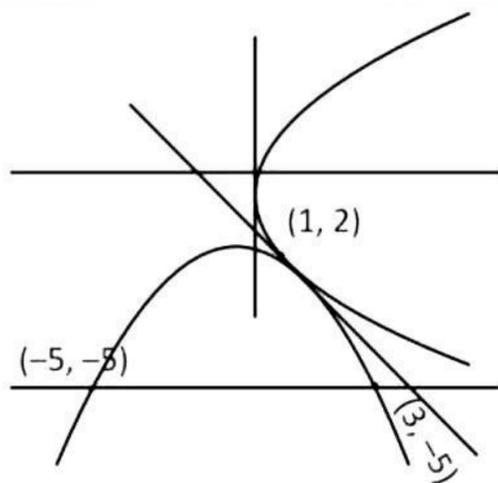
- (1) $\frac{3}{2}$ (2) $\frac{2}{3}$
 (3) $\frac{-3}{2}$ (4) $\frac{-2}{3}$

Answer (2)

Sol.



Sol.



To find image of $P(t^2, 2t)$

$$\frac{x - t^2}{1} = \frac{y - 2t}{1} = \frac{-2(t^2 + 2t + 1)}{1^2 + 1^2} = -(t + 1)^2$$

$$\Rightarrow x = t^2 - (t + 1)^2 = -2t - 1$$

$$y = 2t - (t + 1)^2 = -t^2 - 1$$

$$t = \left(\frac{-1 - x}{2} \right)$$

$$\Rightarrow (y + 1) = - \left(\frac{-1 - x}{2} \right)^2$$

$$(y + 1) = - \frac{(x + 1)^2}{4}$$

$$\Rightarrow (x + 1)^2 = -4(y + 1)$$

$$\Rightarrow X^2 = -4y, \text{ the vertex is}$$

$$X = 0, y = 0$$

$$\Rightarrow \text{Focus} \Rightarrow (-1, -2)$$

Other method would have been to find image of $(1, 0)$ about $x + y + 1 = 0$

$$\Rightarrow y = -5 \text{ intersect}$$

$$(-4) (-4) = (x + 1)^2$$

$$\Rightarrow (x + 1) = \pm 4$$

$$\Rightarrow x = 3, -5$$

$$\Rightarrow a = 8$$

$$d = \frac{1}{2} \times a \times \text{height} = \frac{1}{2} \times 8 \times (-2 - (-5))$$

$$= 4 \cdot 3 = 12 \text{ Sq. unit}$$

$$\Rightarrow \boxed{a + d = 20}$$

11. For positive integer n , $4a_n = n^2 + 5n + 6$ and

$$S_n = \sum_{k=1}^n \frac{1}{a_k}. \text{ Then the value of } 507(S_{2025}) \text{ is}$$

- (1) 675 (2) 540
(3) 1350 (4) 725

Answer (1)

$$\text{Sol. } S_n = \sum_{k=1}^n \frac{4}{k^2 + 5k + 6}$$

$$= \sum_{k=1}^n \frac{4}{(k+2)(k+3)} = 4 \sum_{k=1}^n \left(\frac{1}{k+2} - \frac{1}{k+3} \right)$$

$$= 4 \left[\frac{1}{3} - \frac{1}{4} \right]$$

$$= 4 \left[\frac{1}{4} - \frac{1}{5} \right]$$

$$= 4 \left[\frac{1}{n+2} - \frac{1}{n+3} \right]$$

$$S_n = 4 \left[\frac{1}{3} - \frac{1}{n+3} \right]$$

$$S_{2025} = 4 \left[\frac{1}{3} - \frac{1}{2028} \right]$$

$$S_{2025} = 4 \left[\frac{675}{2028} \right]$$

$$507 S_{2025} = 675$$

12. The number of natural numbers between 212 to 999 such that sum of their digit is 15 is equal to

- (1) 63 (2) 61
(3) 62 (4) 65

Answer (2)

$$\text{Sol. } 717 \rightarrow 3!$$

$$726 \rightarrow 3!$$

$$735 \rightarrow 3!$$

$$744 \rightarrow 3$$

$$636 \rightarrow 3$$

$$645 \rightarrow 3!$$

$$\hline 555 \rightarrow 1$$

$$915 \rightarrow 3!$$

$$924 \rightarrow 3!$$

$$933 \rightarrow 3$$

$$\hline 816 \rightarrow 3!$$

$$825 \rightarrow 3!$$

$$834 \rightarrow 3!$$

$$\text{Total : 61}$$

13.
14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $B = QPQ^T$ and matrix A is defined as $A = Q^T B^{10} Q$ (where $P = \begin{bmatrix} \sqrt{2} & -2 \\ 0 & 1 \end{bmatrix}$), then trace of matrix A is

Answer (33)

- Sol.** $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
 $B = QPQ^T$
 $A = Q^T B^{10} Q$
 $Q^T Q = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
 $= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
 $\Rightarrow Q^T Q = Q Q^T$
 $A = Q^T (QPQ^T)^{10} Q$
 $= IPQ^T B^9 Q = PQ^T B^9 Q$
 $\Rightarrow A = PQ^T (QPQ^T)^8 Q$
 $= P^2 Q^T B^8 Q = P^9 Q^T B^1 Q$
 $= P^9 Q^T (QPQ^T) Q = P^{10}$
Trace $(P^{10}) = (d_1^{10} + d_2^{10})$ where d_1 and d_2 are diagonal elements
 $\Rightarrow \text{Trace}(A) = (\sqrt{2})^{10} + 1^{10}$
 $= 2^{10/2} + 1 = 2^5 + 1 = 33$

22. $f: [0, 3] \rightarrow b$, $f(x) = 2x^3 - 15x^2 + 36x + 7$ is an onto function
 $g: [0, \infty) \rightarrow d$, $g(x) = \frac{x^{2025}}{x^{2025} + 1}$ is also an onto function.
Find the number of elements in the set $S = \{x : x \in Z, x \in b \text{ or } x \in d\}$

Answer (30)

- Sol.** $f(x) = 2x^3 - 15x^2 + 36x + 7$
 $f'(x) = 6x^2 - 30x + 36 = 0$
 $\Rightarrow x^2 - 5x + 6 = 0$
 $\therefore x = 1, 5$
 $f(0) = 7, f(2) = 35, f(3) = 34$
 $\therefore b = [7, 35]$

$$g(x) = \frac{x^{2025}}{1 + x^{2025}}$$

- $d = [0, 1)$
 $\therefore S = [0, 7, 8, 9, \dots, 35]$
Number of elements = 30

23. The maximum interior angle of a polygon is 171° with n sides such that its angles are in Arithmetic progression with common difference of 6° . Then n is equal to

Answer (10)

- Sol.** Sum of interior angle
 $\Rightarrow \frac{n}{2}(2a + (n-1)d) = 180^\circ(n-2)$
 $\Rightarrow 171 = a + (n-1)d$
 $\Rightarrow \frac{n}{2}(171 + a) = 180(n-2)$
 $a = 171 - 6(n-1) = 177 - 6n$
 $\Rightarrow \frac{n}{2}(171 + 177 - 6n) = 180(n-2)$
 $\Rightarrow n(174 - 3n) = 180n - 360$
 $\Rightarrow 3n^2 + 6n - 360 = 0$
 $n^2 + 2n - 120 = 0$
 $(n+12)(n-10) = 0$
 $\Rightarrow n = 10$

24.
25.

