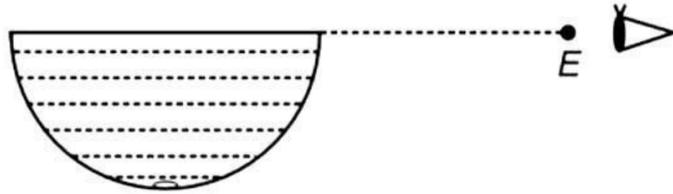


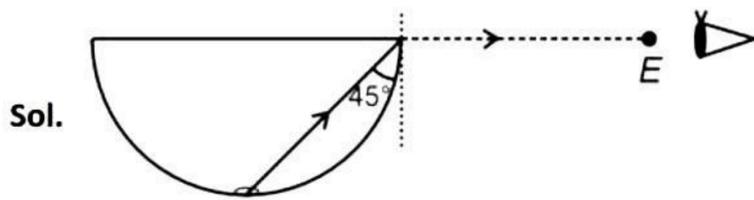


5. A coin is placed at the bottom of a hemispherical container filled with a liquid of refractive index  $\mu$ . Find the least refractive index if the coin is visible to an observer at  $E$ .



- (1)  $\sqrt{3}$                       (2)  $\sqrt{2}$   
 (3)  $\frac{\sqrt{3}}{2}$                       (4)  $3\sqrt{2}$

**Answer (2)**

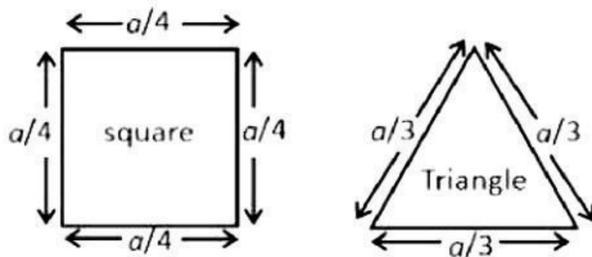


$$\theta_c = 45^\circ$$

$$\sin 45^\circ = \frac{1}{\mu}$$

$$\mu = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

6. In the given figure, the square and the triangle have same resistance per unit length. Find the ratio of their resistances about adjacent corners.



- (1)  $\frac{32}{27}$                       (2)  $\frac{27}{32}$   
 (3)  $\frac{8}{9}$                       (4)  $\frac{9}{8}$

**Answer (2)**

- Sol.** Let the resistance per unit length be  $\lambda$ , then

For square,

$$R_{\text{Square}} = \frac{\left(\frac{3\lambda a}{4}\right)\left(\frac{\lambda a}{4}\right)}{\frac{3\lambda a}{4} + \frac{\lambda a}{4}} = \frac{3\lambda a}{16}$$

For triangle,

$$R_{\text{Triangle}} = \frac{\left(\frac{2\lambda a}{3}\right)\left(\frac{\lambda a}{3}\right)}{\frac{2\lambda a}{3} + \frac{\lambda a}{3}} = \frac{2\lambda a}{9}$$

$$\frac{R_{\text{Square}}}{R_{\text{Triangle}}} = \frac{3\lambda a}{16} \times \frac{a}{2\lambda a} = \frac{27}{32}$$

7. **Assertion** : Work done by central force is independent of path.

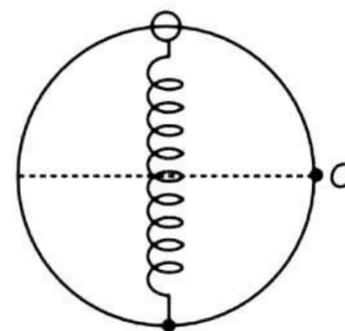
**Reason** : Potential energy is associated with every force.

- (1) Both Assertion and Reason are correct  
 (2) Assertion is correct, Reason is incorrect  
 (3) Assertion is incorrect, Reason is correct  
 (4) Both Assertion and Reason are incorrect

**Answer (4)**

- Sol.** Not all central force/s are conservative so work done by central force might depend on path.

8. There is smooth ring of radius  $R$  in vertical plane. A spring of natural length  $R$  & elastic constant  $K$  is vertical along a diameter. The free end is connected to bead of mass  $m$  & when slightly disturbed it reaches point  $C$  with speed  $v$  where  $v$  is



$$(1) \sqrt{\frac{KR^2(\sqrt{2}-1)+2mgR}{m}}$$

$$(2) \sqrt{\frac{2KR^2(\sqrt{2}-1)+2mgR}{m}}$$

$$(3) \sqrt{\frac{2KR^2(\sqrt{2}-1)+mgR}{m}}$$

$$(4) \sqrt{\frac{KR^2(\sqrt{2}-1)+mgR}{m}}$$

**Answer (2)**

**Sol.** Loss in PE = gain in KE

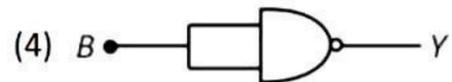
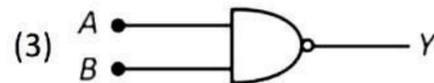
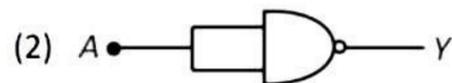
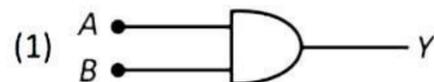
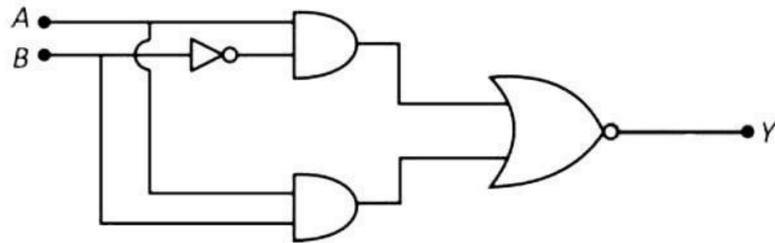
$$\frac{1}{2}K(R^2) - \frac{1}{2}K(\sqrt{2}-1)^2 R^2 + mgR = \frac{1}{2}mv^2$$

$$\frac{1}{2}KR^2 \{1 - 2 - 1 + 2\sqrt{2}\} + mgR = \frac{1}{2}mv^2$$

$$KR^2(\sqrt{2}-1) + mgR = \frac{1}{2}mv^2$$

$$\sqrt{\frac{2KR^2(\sqrt{2}-1)+2mgR}{m}} = v$$

9. The equivalent logic gate for the circuit shown below is



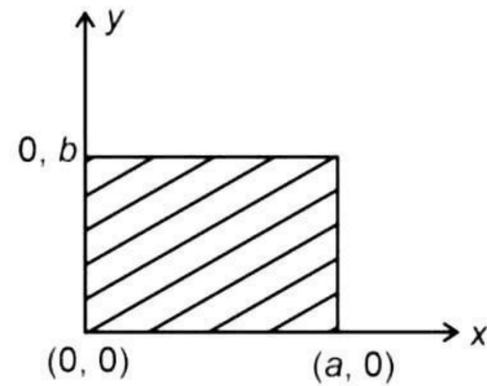
**Answer (2)**

**Sol.**  $Y = \overline{A\overline{B}} + AB$

$$Y = \overline{A(\overline{B} + B)}$$

$$Y = \overline{A}$$

10. Surface mass density varies as  $\sigma = \frac{\sigma_0 x}{ab}$  for the given plane sheet. Find the position of centre of mass for the distribution given



(1)  $\frac{2a}{3}, \frac{2b}{3}$

(2)  $\frac{2a}{3}, \frac{b}{2}$

(3)  $\frac{a}{3}, \frac{b}{2}$

(4)  $\frac{a}{2}, \frac{b}{2}$

**Answer (2)**

**Sol.** As there is no variation of mass density in y direction. So

$$y_{cm} = \frac{b}{2}$$

Now for x direction

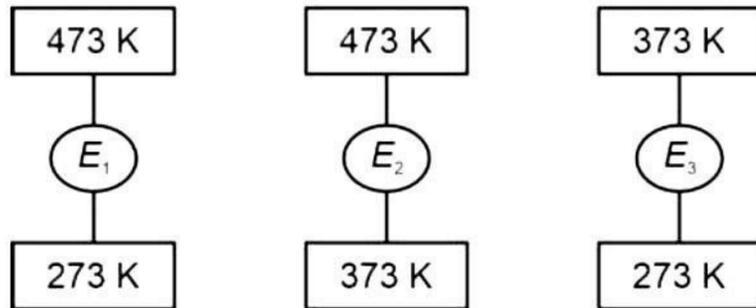
$$dm = \left(\frac{\sigma_0 x}{ab}\right) b \cdot dx$$

$$x_{cm} = \frac{\int_0^a x dm}{\int_0^a dm} = \frac{\frac{\sigma_0}{a} \int_0^a x^2 dx}{\frac{\sigma_0}{a} \int_0^a x dx}$$

$$\Rightarrow x_{cm} = \frac{a^3 \times 2}{3 \times a^2} = \frac{2}{3}a$$

$$\text{So } r_{cm} = \frac{2}{3}a, \frac{b}{2}$$

11.  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  are the efficiencies of the three Carnot engines  $E_1$ ,  $E_2$  and  $E_3$  operating between temperatures shown in the figure. Choose the correct option relating the efficiencies.



- (1)  $\eta_2 + \eta_3 > \eta_1$       (2)  $\eta_2 + \eta_3 = \eta_1$   
 (3)  $\eta_2 + \eta_3 < \eta_1$       (4)  $\eta_1 + \eta_2 = \eta_3$

**Answer (1)**

**Sol.**  $\eta_1 = 1 - \frac{273}{473} = \frac{200}{473}$

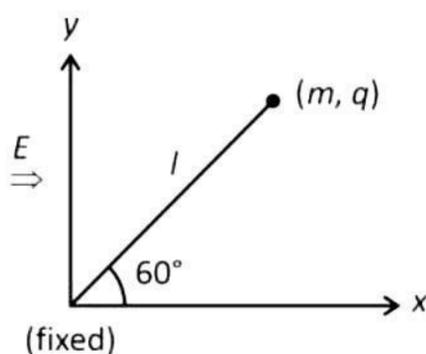
$$\eta_2 = 1 - \frac{373}{473} = \frac{100}{473}$$

$$\eta_3 = 1 - \frac{273}{373} = \frac{100}{373}$$

$$\eta_1 - \eta_2 = \frac{100}{473} < \eta_3$$

*i.e.*,  $\eta_2 + \eta_3 > \eta_1$

12. A simple pendulum of length  $l$  and bob of mass  $m$  is placed on smooth horizontal surface as shown. When electric field of strength  $E$  is switched on, the bob passes the  $x$ -axis with speed  $v$  then



(1)  $v = \sqrt{\frac{2qEl}{m}}$

(2)  $v = \sqrt{\frac{qEl}{m}}$

(3)  $v = \sqrt{\frac{qEl}{2m}}$

(4)  $v = 2\sqrt{\frac{qEl}{m}}$

**Answer (2)**

**Sol.**  $W = \Delta K$

$$qE(l - l \cos 60^\circ) = \frac{1}{2}mv^2$$

$$\frac{qEl}{m} = v^2$$

13. **Statement-I** : Velocity of sound in solids is more compared to that in gases.

**Statement-II** : Bulk modulus of gas is more than that of solids.

- (1) Statement-I is correct statement-II is correct  
 (2) Statement-I is correct statement-II is incorrect  
 (3) Statement-I is incorrect statement-II is correct  
 (4) Statement-I is incorrect statement-II is incorrect

**Answer (2)**

**Sol.** Speed of sound in medium depends on elastic and inertia property of medium.

for gas  $v = \sqrt{\frac{B}{\rho}}$

for solids  $v = \sqrt{\frac{Y}{\rho}}$

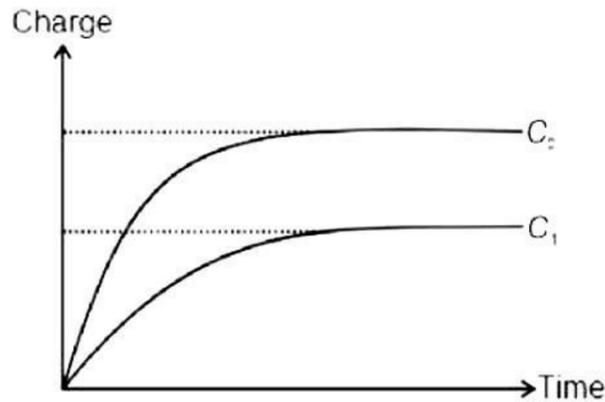
The elastic property of solids happens to be many fold greater than that of elastic property of gases.

Bulk modulus of gas depends on the process,

$$B = \frac{-\Delta P}{\Delta V/V}, \text{ which varies between } 0 \text{ to } \infty \text{ therefore in}$$

general statement 2 is incorrect.

14. Two capacitors  $C_1$  and  $C_2$  are connected across same battery and store energies  $U_1$  and  $U_2$  respectively at steady state. Choose the correct option by observing the graph of charge vs time shown below.



- (1)  $C_1 > C_2$   
 $U_1 > U_2$
- (2)  $C_1 < C_2$   
 $U_1 < U_2$
- (3)  $C_1 > C_2$   
 $U_1 < U_2$
- (4)  $C_1 < C_2$   
 $U_1 > U_2$

**Answer (2)**

**Sol.** Steady state

$$\frac{C_1}{C_2} = \frac{Q_1}{Q_2} < 1 \quad (\text{Connected across same potential difference})$$

$$\frac{C_1}{C_2} = \frac{U_1}{U_2} < 1$$

15. Energy of photon of wavelength  $\lambda$  is  $E_0$  which is equal to kinetic energy of proton of mass  $m_p$ . The ratio of de Broglie wavelengths of proton and photon is

- (1)  $\frac{1}{c} \sqrt{\frac{2E_0}{m_p}}$
- (2)  $\frac{1}{c} \sqrt{\frac{E_0}{2m_p}}$
- (3)  $\frac{2}{c} \sqrt{\frac{E_0}{m_p}}$
- (4)  $\frac{1}{2c} \sqrt{\frac{E_0}{m_p}}$

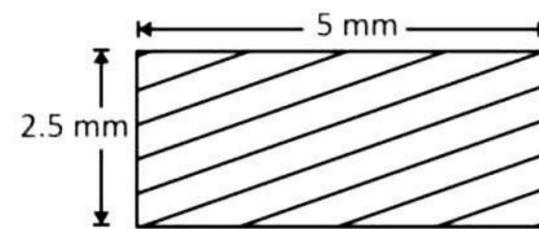
**Answer (2)**

**Sol.**  $\frac{1}{2} m_p v^2 = E_0 \Rightarrow p = \sqrt{2m_p E_0} \quad \lambda_p = \frac{h}{\sqrt{2m_p E_0}}$

for photon  $\frac{hc}{\lambda} = E_0 \Rightarrow \lambda_{ph} = \frac{hc}{E_0}$

$$\Rightarrow \frac{\lambda_p}{\lambda_{ph}} = \frac{h}{\sqrt{2m_p E_0}} \times \frac{E_0}{hc} = \frac{1}{c} \sqrt{\frac{E_0}{2m_p}}$$

16. The lengths of a rectangular sheet is measured from a screw gauge of pitch 0.75 mm and number of division on circular scale = 15. Find maximum possible error in measurement of area.



- (1) 0.225 mm<sup>2</sup>
- (2) 0.375 mm<sup>2</sup>
- (3) 0.75 mm<sup>2</sup>
- (4) 0.30 mm<sup>2</sup>

**Answer (2)**

**Sol.** Least count of screw gauge

$$= \frac{0.75}{15} = 0.05 \text{ mm}$$

Now,  $S = lb$

$$\Rightarrow \frac{\Delta S}{S} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\Rightarrow \frac{\Delta S}{S} = \left( \frac{0.05}{5} \right) + \frac{0.05}{2.5} = \frac{1}{100} + \frac{2}{100} = \frac{3}{100}$$

$$\Delta S = \left( \frac{3}{100} \right) \times 5 \times 2.5 = 0.375 \text{ mm}^2$$



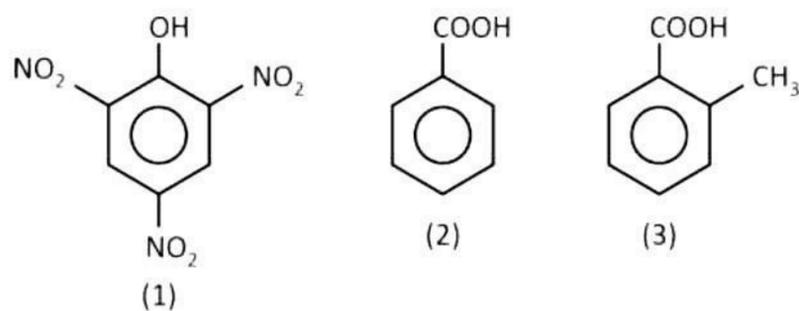
# CHEMISTRY

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. What is the rate of reaction for releasing  $\text{CO}_2(\text{g})$  with aq.  $\text{NaHCO}_3$  among following?



- (1) (1) > (2) > (3)                      (2) (3) > (2) > (1)  
 (3) (1) > (3) > (2)                      (4) (2) > (3) > (1)

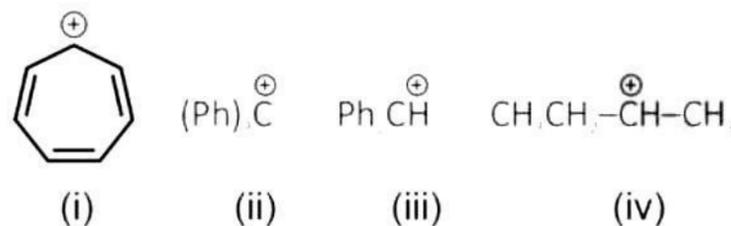
**Answer (3)**

**Sol.**  $\text{pK}_a$  (Benzoic acid) : 4.27

$\text{pK}_a$  (o-Toluic acid) : 3.91  $\rightarrow$  due to ortho effect.

$\text{pK}_a$  (Picric acid) : 0.3

2. Consider the following carbocations



The correct increasing order of stability of these carbocations is:

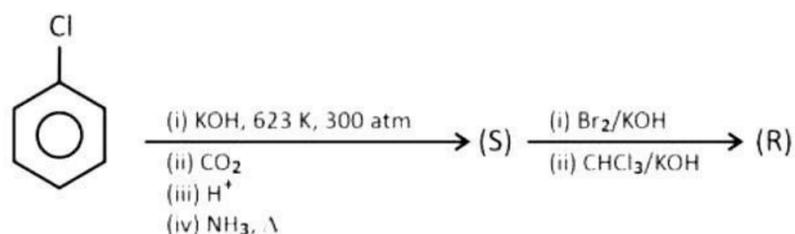
- (1) (i) < (ii) < (iii) < (iv)                      (2) (iv) < (iii) < (ii) < (i)  
 (3) (ii) < (iii) < (iv) < (i)                      (4) (iv) < (iii) < (i) < (ii)

**Answer (2)**

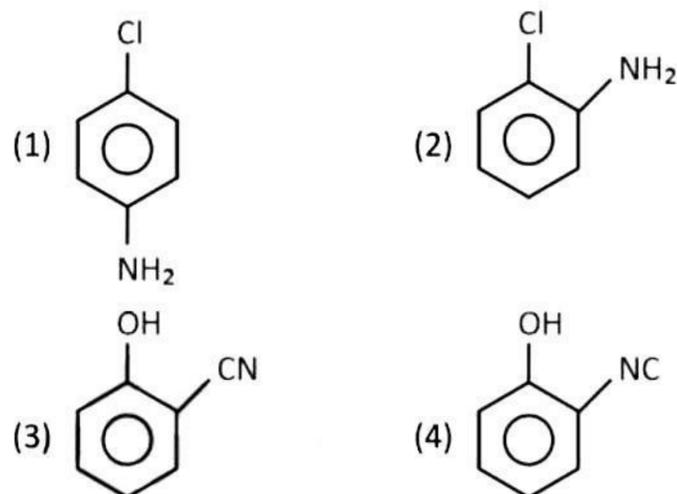
**Sol.** Tropylium carbocation is most stable due to presence of aromaticity.

$\therefore$  (i) > (ii) > (iii) > (iv) is the correct order of stability of carbocation.

3. In the given reaction sequence:

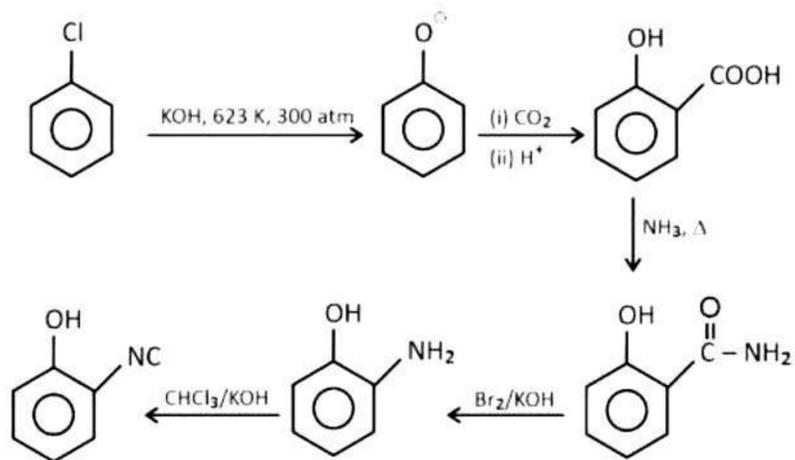


The compound R is



**Answer (4)**

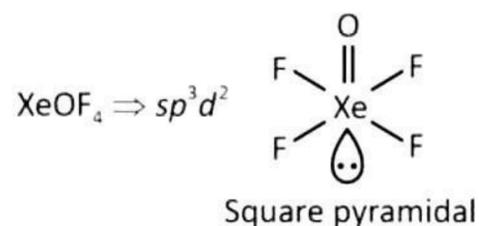
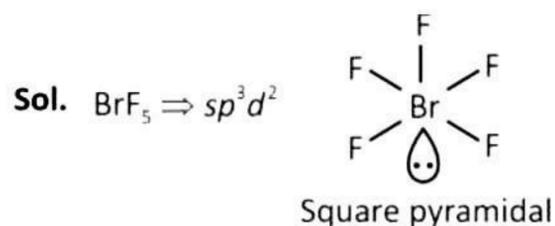
**Sol.**



4. Which of the following pair have square pyramidal shape?

- (1)  $\text{BrF}_5, \text{XeOF}_4$                       (2)  $\text{SbF}_5, \text{BrF}_5$   
 (3)  $\text{PCl}_5, \text{XeOF}_4$                       (4)  $\text{PCl}_5, \text{SbF}_5$

**Answer (1)**



5. Which of the following set of quantum numbers have same energy?

- (a)  $n = 2, l = 2, m = +1$   
 (b)  $n = 2, l = 1, m = -1$   
 (c)  $n = 3, l = 2, m = 0$   
 (d)  $n = 3, l = 2, m = 1$

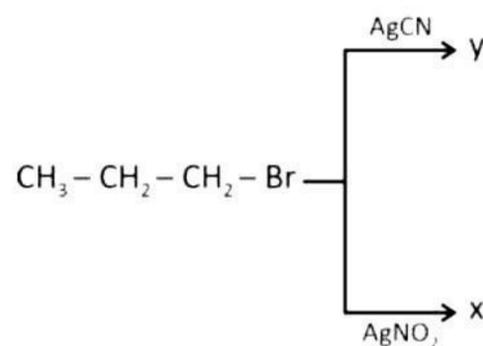
- (1) a, b  
 (2) b, c  
 (3) c, d  
 (4) a, c

**Answer (3)**

- Sol.: (a)  $\Rightarrow 2d =$  does not exist  
 (b)  $\Rightarrow 2p$   
 (c)  $\Rightarrow 3d$  same energy  
 (d)  $\Rightarrow 3d$  same energy

The value of  $n + l$  is same for (c) and (d) both represents 3d orbital

6. Consider the following reaction



The major product x and y respectively are

- (1)  $\text{CH}_3\text{CH}_2\text{CH}_2\text{ONO}$  &  $\text{CH}_3\text{CH}_2\text{CH}_2\text{CN}$   
 (2)  $\text{CH}_3\text{CH}_2\text{CH}_2\text{NO}_2$  &  $\text{CH}_3\text{CH}_2\text{CH}_2\text{CN}$   
 (3)  $\text{CH}_3\text{CH}_2\text{CH}_2\text{NO}_2$  &  $\text{CH}_3\text{CH}_2\text{CH}_2\text{NC}$   
 (4)  $\text{CH}_3\text{CH}_2\text{CH}_2\text{ONO}$  &  $\text{CH}_3\text{CH}_2\text{CH}_2\text{CN}$

**Answer (3)**

Sol.  $\text{CN}^-$  and  $\text{NO}_2^-$  are ambidentate nucleophile but since  $\text{AgCN}$  and  $\text{AgNO}_2$  are covalent compound, so only the nitrogen can donate electrons.

7. Match the following column and choose the correct option.

	Column-I		Column-II
(A)	$\text{H}_2\text{O}_2 \rightarrow \text{H}_2\text{O} + \text{O}_2$	(P)	Combustion reaction
(B)	$\text{NaH} \rightarrow \text{Na} + \text{H}_2$	(Q)	Disproportionation reaction
(C)	$\text{CH}_4 + \text{O}_2 \rightarrow \text{CO}_2 + \text{H}_2\text{O}$	(R)	Decomposition reaction
(D)	$\text{Fe} + \text{CuSO}_4 \rightarrow \text{FeSO}_4 + \text{Cu}$	(S)	Displacement reaction

- (1) A-(Q), B-(P), C-(R), D-(S)  
 (2) A-(R), B-(Q), C-(S), D-(P)  
 (3) A-(Q), B-(R), C-(P), D-(S)  
 (4) A-(R), B-(Q), C-(P), D-(S)

**Answer (3)**

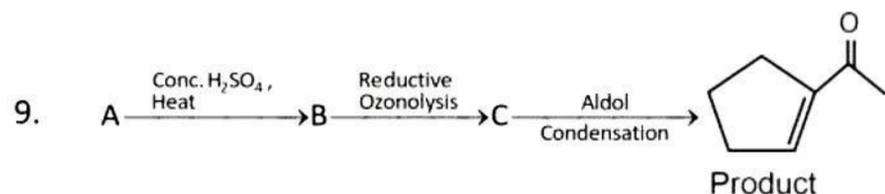
- Sol.  $\text{H}_2\text{O}_2 \rightarrow \text{H}_2\text{O} + \text{O}_2$  : Disproportionation reaction  
 $\text{NaH} \rightarrow \text{Na} + \text{H}_2$  : Decomposition reaction  
 $\text{CH}_4 + \text{O}_2 \rightarrow \text{CO}_2 + \text{H}_2\text{O}$  : Combustion reaction  
 $\text{Fe} + \text{CuSO}_4 \rightarrow \text{FeSO}_4 + \text{Cu}$  : Displacement reaction

8. Among the following the incorrect order of atomic radius is

- (1)  $B > Al > Mg > F$       (2)  $Al > B > N > F$   
 (3)  $Mg > Al > Be > O$       (4)  $Mg > Be > N > F$

**Answer (1)**

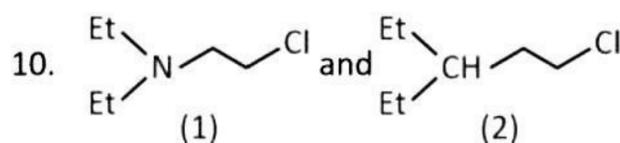
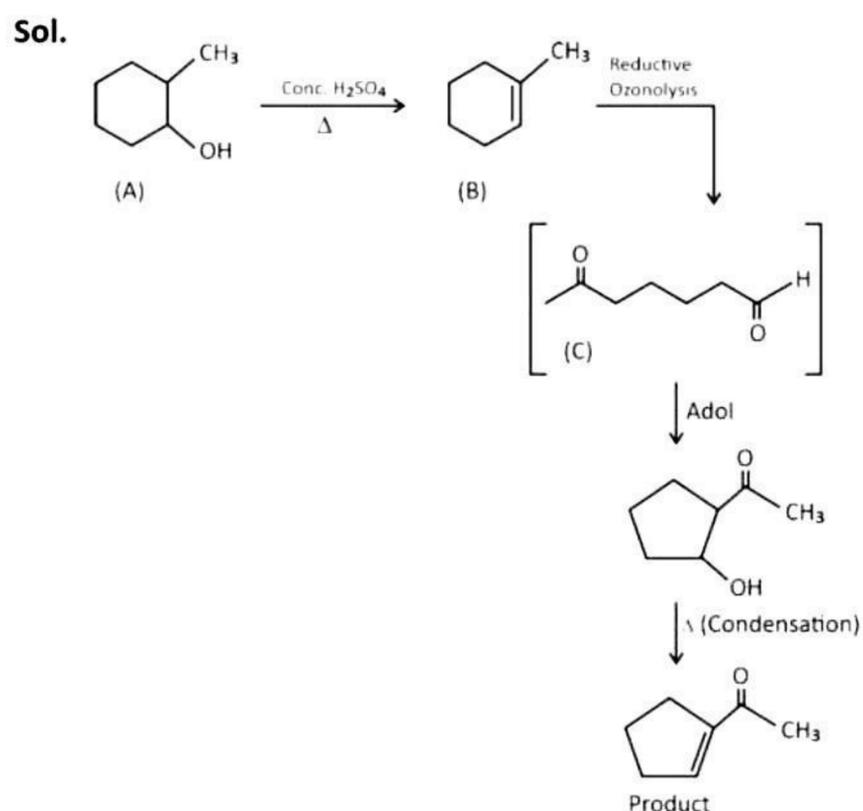
**Sol.** As we move down the group size increases whereas on moving left to right in a period size decreases



Identify the Compound A

- (1) (2)   
 (3) (4)

**Answer (1)**

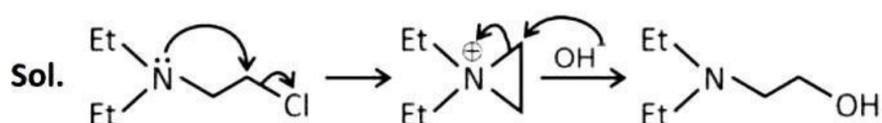


**Statement-I:** Compound (2) shows faster alkaline hydrolysis compared to (1).

**Statement-II:** Compound (1) shows substitution via neighbouring group participation.

- (1) Statement-I is correct and statement-II is incorrect  
 (2) Statement-I is incorrect and statement-II is correct  
 (3) Statement-I and statement-II both are correct  
 (4) Statement-I and statement-II both are incorrect

**Answer (2)**



11. Which of the following has same energy in absence of electric and magnetic field for hydrogen atom?

- (1) 2s, 3p      (2) 3s, 2p  
 (3) 2s, 2p      (4) 3s, 4f

**Answer (3)**

**Sol.** For hydrogen atom in absence of electric and magnetic field, energy only depends on principal quantum number (n). For same the value of n, energy will be same. Hence 2s and 2p have same energy.

12. Which of the following reaction(s)/test(s) can be used to distinguish acetaldehyde and acetone?

- (A) Iodoform Test      (B) Cannizzaro  
 (C) Aldol Condensation      (D) Fehling's Test  
 (E) Tollen's Test      (F) Clemmensen's Reduction

- (1) (D), (E) Only  
 (2) (A), (B), (C), (F) only  
 (3) (B), (C), (F) only  
 (4) (B), (C), (D), (E) only

**Answer (1)**

**Sol.**  $\text{CH}_3\text{CHO}$  will react with Fehling's solution and Tollen's reagent while  $\text{CH}_3\text{COCH}_3$  will not. Iodoform, Aldol and Clemmensen's reduction will be shown by both

13. Which of the following give violet colour in Borax bead test?

- (1)  $\text{Cr}^{3+}$
- (2)  $\text{Mn}^{2+}$
- (3)  $\text{Co}^{3+}$
- (4)  $\text{Fe}^{2+}$

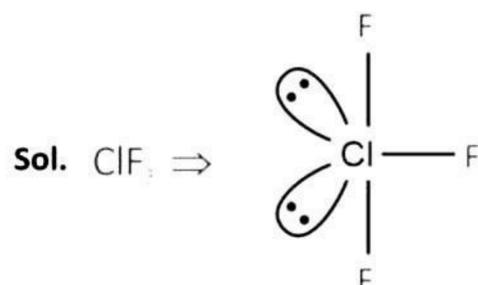
**Answer (2)**

**Sol.** Colour of Borax bead in oxidising flame is violet for  $\text{Mn}^{2+}$ .

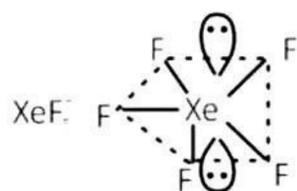
14. Which of the following compounds have the same number of lone pair on central atom as  $\text{ClF}_3$ .

- (1)  $\text{XeF}_5^-$
- (2)  $\text{XeF}_2$
- (3)  $\text{BrF}_5$
- (4)  $\text{I}_3^-$

**Answer (1)**



No. of lone pairs on central atom = 2



No. of lone pairs on central atom = 2

$\text{XeF}_2 \Rightarrow$  no. of lone pairs on central atom = 3

$\text{I}_3^- \Rightarrow$  no. of lone pairs on central atom = 3

$\text{BrF}_5 \Rightarrow$  no. of lone pair on central atom = 1

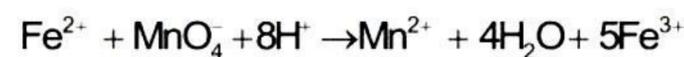
15. **Statement 1:** For titration of oxalic acid using  $\text{KMnO}_4$ , warming of acid solution is required whereas in case of Ferrous Ammonium Sulphate, it is done at room temperature.

**Statement 2:**  $\text{Fe}^{2+}$  converts to  $\text{Fe}^{3+}$  during titration.

- (1) Both Statement 1 and statement 2 are correct
- (2) Statement 1 is correct and statement 2 is incorrect
- (3) Statement 1 is incorrect and statement 2 is correct
- (4) Both Statement 1 and statement 2 both are incorrect

**Answer (1)**

**Sol.** In case of oxalic acid warming of solution ( $50^\circ\text{C}$ - $60^\circ\text{C}$ ) is done as rate of reaction is very slow at room temperature.



- 16.
- 17.
- 18.
- 19.
- 20.

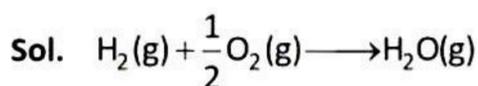
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### SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21.  $\Delta_f H$  of  $\text{H}(\text{g})$  is 222 kJ/mol,  $\Delta_f H$  of  $\text{O}(\text{g})$  is 250 kJ/mol,  $\Delta_f H$  of  $\text{H}_2\text{O}(\text{g})$  is -248 kJ/mol. What is the value of Bond Energy of O - H bond in  $\text{H}_2\text{O}$  in kJ/mol?

**Answer (471)**



$$\Delta H_f = \sum (\text{B.E. of reactant}) - \sum (\text{B.E. of product})$$

$$-248 = 444 + 250 - (2 \times \text{B.E.}_{\text{O-H}})$$

$$2\text{B.E.}_{\text{O-H}} = 942$$

$$2\text{B.E.}_{\text{O-H}} = 471 \text{ kJ/mole}$$

22. 70% by mass solution of  $\text{HNO}_3$  is taken having density 1.41 g/mL. Calculate the molarity of solution

**Answer (16)**

**Sol.**  $M = \frac{10 \times d}{M_0}$

$$= \frac{10 \times 70 \times 1.41}{63}$$

$$= 15.66 \text{ M}$$

23. 1g of a non-electrolyte solute (MW = 256 g/mol) is dissolved in 50g of solvent, freezing point of solution is lowered by 0.40 K. Calculate the Molal depression constant of solvent.

**Answer (5)**

**Sol.**  $\Delta T_f = i k_f \times m$

$$\Delta T_f = i k_f \times \frac{W_{\text{solute}}}{MW_{\text{solute}} \times W_{\text{solvent}}} \times 1000$$

$$0.4 = k_f \times \frac{1 \times 1000}{256 \times 50}$$

$$k_f = \frac{0.4 \times 256 \times 50}{1000}$$

$$k_f = 5.12 \text{ K kg mol}^{-1}$$

24. A compound contains 14.4% carbon, 1.2% hydrogen and 84.4% chlorine, calculate empirical formula mass of compound.

(Molar mass of C = 12, H = 1, Cl = 35.5)

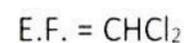
**Answer (84)**

**Sol.** Let mass of compound = 100g

	Mole	Molar ratio
C = 14.4g $\Rightarrow \frac{14.4}{12} = 1.2$	1.2	1

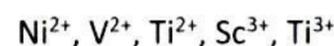
$$\text{H} = 1.2\text{g} \Rightarrow \frac{1.2}{1} = 1.2 \quad 1$$

$$\text{Cl} = 84.4\text{g} \Rightarrow \frac{84.4}{35.5} = 2.4 \quad 2$$



E.F. mass = 84

25. How many of the following ions have same value of spin only magnetic moment?



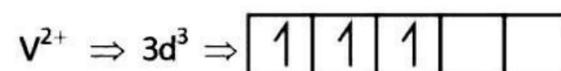
**Answer (2)**

- Sol.** The ions having same number of unpaired electrons having same value of spin only magnetic moment.

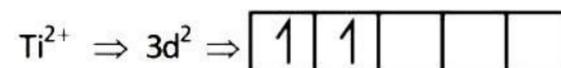
$$\mu = \sqrt{n(n+2)} \text{ BM} \quad (n = \text{Number of unpaired electron})$$



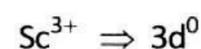
$n = 2$



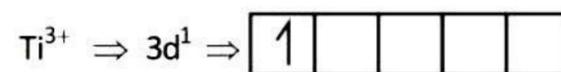
$n = 3$



$n = 2$



$n = 0$



$n = 1$

$\text{Ni}^{2+}$  and  $\text{Ti}^{2+}$  have same value of spin only magnetic moment.



3. If the image of the point  $P(4, 4, 3)$  in the line  $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{1}$  is  $Q(\alpha, \beta, \gamma)$ . Then  $(\alpha + \beta + \gamma)$  is equal to

- (1) 7  
 (2)  $\frac{31}{3}$   
 (3)  $\frac{11}{3}$   
 (4) 8

**Answer (2)**

**Sol.**  $P(4, 4, 3)$

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{1} = \lambda$$

Any point of line  $R(2\lambda + 1, \lambda + 2, \lambda + 1)$

$$\overline{PR} : (2\lambda - 3)\hat{i} + (\lambda - 2)\hat{j} + (\lambda - 2)\hat{k}$$

$$\overline{PR} \cdot \langle 2, 1, 1 \rangle = 0$$

$$2(2\lambda - 3) + (\lambda - 2) + 2(\lambda - 2) = 0$$

$$6\lambda = 10$$

$$\lambda = \frac{5}{3}$$

$$\therefore R\left(\frac{13}{3}, \frac{11}{3}, \frac{8}{3}\right)$$

Now,  $Q(\alpha, \beta, \gamma)$

$$\frac{\alpha+4}{2} = \frac{13}{3}, \frac{\beta+4}{2} = \frac{11}{3}, \frac{\gamma+3}{2} = \frac{8}{3}$$

$$\alpha = \frac{14}{3}, \beta = \frac{10}{3}, \gamma = \frac{7}{3}$$

$$\alpha + \beta + \gamma = \frac{14+10+7}{3} = \frac{31}{3}$$

4. If  $\int_0^x tf(t)dt = x^2 f(x)$  and  $f(2) = 3$ , then  $f(6)$  equals to

- (1) 1  
 (2) 6  
 (3) 3  
 (4) 2

**Answer (1)**

**Sol.**  $\int_0^x tf(t)dt = x^2 f(x)$

Differentiating both sides w.r.t 'x'

$$xf(x) = x^2 f'(x) + 2xf(x)$$

$$\frac{x^2 dy}{dx} + xy = 0$$

$$\frac{dy}{y} = \frac{-dx}{x}$$

$$\ln y + \ln x = \ln c$$

$$yx = c$$

$$\text{As } f(2) = 3$$

$$6 = c$$

$$\therefore yx = 6$$

$$\therefore \text{Put } x = 6$$

$$y(6) = 6$$

$$y = 1$$

Option (1) is correct

5. Let  $R$  be a relation such that  $R = \{(x, y) : x, y \in Z \text{ and } (x + y) \text{ is even}\}$ , then the relation  $R$  is

- (1) Reflexive and symmetric but not transitive  
 (2) Reflexive and transitive but not symmetric  
 (3) Transitive only  
 (4) Equivalence relation

**Answer (4)**

**Sol. for reflexive**

$$\text{If } (x, x) \in Z$$

$$R : x + x = 2x \Rightarrow R \text{ is reflexive}$$

**For symmetric**

$$\text{If } (x, y) \in R \Rightarrow x + y = \text{even}$$

$$\Rightarrow y + x = \text{even } \forall (y, x) \in R$$

$$\Rightarrow R \text{ is symmetric}$$

$$\text{If } (x, y) \in R \Rightarrow x + y = \text{even}$$

$$(y, z) \in R \Rightarrow y + z = \text{even}$$

$$\Rightarrow x + 2y + Z \in \text{even} \Rightarrow x + Z = \text{even} - 2y \text{ even}$$

$$\Rightarrow \underline{x + z \in \text{even}}$$

$$\Rightarrow (x, z) \in R$$

$$\Rightarrow R \text{ is equivalence relation.}$$

6. Evaluate

$$\cos\left(\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{33}{65}\right)\right)$$

(1) 0

(2) 1

(3)  $\cos \frac{5}{13}$

(4) 2

**Answer (1)**

**Sol.**  $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$

$$= \cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\left[\frac{5}{13}\sqrt{\frac{1-33^2}{65^2}} + \frac{33}{65}\sqrt{1-\frac{5^2}{13^2}}\right]\right)$$

$$= \cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$$

$$= \cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5}\right)$$

$$= \cos\left(\frac{\pi}{2}\right) = 0$$

7. The sum of squares of real roots of the equation:  $x^2 + |2x - 3| - 4 = 0$ , is

(1)  $6(2 - \sqrt{2})$                       (2)  $3(2 - \sqrt{2})$

(3)  $3(2 + \sqrt{2})$                       (4)  $6(2 + \sqrt{2})$

**Answer (1)**

**Sol.**  $x^2 + |2x - 3| - 4 = 0$

(i)  $2x - 3 \geq 0 \Rightarrow x \geq \frac{3}{2}$

$$\Rightarrow x^2 + 2x - 3 - 4 = 0$$

$$x^2 + 2x - 7 = 0 \Rightarrow (x + 1)^2 = 8$$

$$\Rightarrow x = \pm 2\sqrt{2} - 1$$

$$\Rightarrow x = (2\sqrt{2} - 1)$$

as  $-2\sqrt{2} - 1 < \frac{3}{2}$

(ii)  $x \leq \frac{3}{2} \Rightarrow x^2 - (2x - 3) - 4 = 0$

$$\Rightarrow x^2 - 2x - 1 - 0 = 0 \Rightarrow (x - 1)^2 = 2$$

$$\Rightarrow x = \pm 2\sqrt{2} + 1 \Rightarrow x = -2\sqrt{2} + 1$$

as  $\sqrt{2} + 1 > \frac{3}{2}$

$\Rightarrow$  two roots are  $x = -\sqrt{2} + 1, 2\sqrt{2} - 1$

$\Rightarrow$  Sum of squares =  $12 - 6\sqrt{2} = 6(2 - \sqrt{2})$

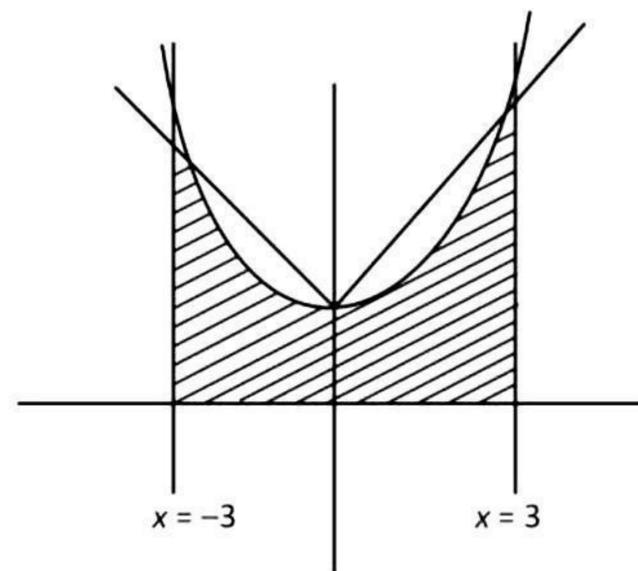
8. Area enclosed by

$$\{(x, y) : 0 \leq y \leq 2|x| + 1, 0 \leq y \leq x^2 + 1, |x| \leq 3\}$$
 is equals to

- (1)  $\frac{17}{3}$                                       (2)  $\frac{32}{3}$
- (3)  $\frac{64}{3}$                                       (4)  $\frac{80}{3}$

**Answer (3)**

**Sol.**



$$\text{Area} = 2 \left[ \int_0^3 (x^2 + 1) dx + \frac{1}{2} [5 + 7] \times 1 \right]$$

$$= \frac{64}{3}$$

9. There are 2 bad oranges mixed with 7 good oranges and 2 oranges are drawn at random. Let  $X$  be the number of bad oranges. The variance of  $X$  is

- (1)  $\frac{51}{268}$  (2)  $\frac{49}{162}$   
 (3)  $\frac{63}{108}$  (4)  $\frac{91}{206}$

**Answer (2)**

$X$	0	1	2
$P(X)$	$\frac{{}^7C_2}{{}^9C_2}$	$\frac{{}^7C_1 \cdot {}^2C_1}{{}^9C_2}$	$\frac{{}^2C_2}{{}^9C_2}$

$$\begin{aligned} \text{Variance} &= 0^2 \cdot \frac{{}^7C_2}{{}^9C_2} + 1^2 \cdot \frac{{}^7C_1 \cdot {}^2C_1}{{}^9C_2} + 2^2 \cdot \frac{{}^2C_2}{{}^9C_2} \\ &\quad - \left( \frac{0 \cdot {}^7C_2}{{}^9C_2} + \frac{1 \cdot {}^7C_1 \cdot {}^2C_1}{{}^9C_2} + \frac{2 \cdot {}^2C_2}{{}^9C_2} \right)^2 \\ &= \frac{7}{18} + \frac{4}{36} - \left( \frac{7}{18} + \frac{2}{36} \right)^2 \\ &= \frac{49}{162} \end{aligned}$$

10. Let  $f(x) = \frac{2^x}{2^x + \sqrt{2}}$ , then  $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$  is equal to

- (1)  $\frac{81}{2}$  (2) 41  
 (3)  $41\sqrt{2}$  (4) 81

**Answer (1)**

**Sol.**  $f(x) = \frac{2^x}{2^x + 2^{1/2}} = \frac{2^x}{2^x + \sqrt{2}}$

$$\begin{aligned} f(1-x) &= \frac{2^{1-x}}{2^{1-x} + 2^{1/2}} = \frac{\frac{2}{2^x}}{\frac{2}{2^x} + 2^{1/2}} = \frac{2}{2 + \sqrt{2} \cdot 2^x} \\ &= \frac{\sqrt{2}}{2^x + \sqrt{2}} \\ \Rightarrow f(x) + f(1-x) &= \frac{\sqrt{2} + 2^x}{\sqrt{2} + 2^x} = 1 \end{aligned}$$

$$\begin{aligned} &\Rightarrow \sum_{k=1}^{81} f\left(\frac{k}{82}\right) + f\left(\frac{2}{82}\right) + \left( f\left(\frac{3}{82}\right) \right) + \dots \\ &\dots + f\left(\frac{40}{82}\right) + f\left(\frac{41}{82}\right) + f\left(\frac{42}{82}\right) \\ &+ \dots + f\left(\frac{79}{82}\right) + f\left(\frac{80}{82}\right) + f\left(\frac{81}{82}\right) \\ &= \left[ f\left(\frac{1}{82}\right) + f\left(\frac{81}{82}\right) \right] + \left[ f\left(\frac{2}{82}\right) + f\left(\frac{80}{82}\right) \right] + \dots \\ &+ \left[ f\left(\frac{40}{82}\right) + f\left(\frac{42}{82}\right) + f\left(\frac{41}{82}\right) \right] \\ &= \left[ \frac{1+1+\dots+1}{40 \text{ times}} \right] + f\left(\frac{1}{2}\right) \\ &= 40 + \frac{\sqrt{2}}{\sqrt{2} + \sqrt{2}} = 40 + \frac{1}{2} = \frac{81}{2} \end{aligned}$$

11. If  $2a_{n+2} = 5a_{n+1} - 3a_n$ , where  $n = 0, 1, 2, \dots$ . If  $a_0 = 3$  and  $a_1 = 4$ , then the value of  $\sum_{k=1}^{100} a_k$  is equal to

- (1)  $3a_{100} - 91$  (2)  $3a_{99} - 91$   
 (3)  $3a_{100} + 91$  (4)  $3a_{99} + 91$

**Answer (1)**

**Sol.**  $2a_{n+2} = 5a_{n+1} - 3a_n = 0$

$$\begin{aligned} \Rightarrow 2t^2 - 5t + 3 &= 0 \\ \Rightarrow t &= 1, \frac{3}{2} \end{aligned}$$

$$\therefore a_n = A \cdot (1)^n + B \cdot \left(\frac{3}{2}\right)^n$$

$$a_0 = 3, a_1 = 4$$

$$\therefore A = 1, B = 2$$

$$a_n = 1 + 2 \cdot \left(\frac{3}{2}\right)^n$$

$$S_{100} = 100 - 6 \left( 1 - \left(\frac{3}{2}\right)^{100} \right)$$

$$= 3a_{100} - 99$$

$$\text{Using } \Rightarrow \left(\frac{3}{2}\right)^{100} = \left(\frac{a_{100}-1}{2}\right)$$

$$a_{100} = 2 \cdot \left(\frac{3}{2}\right)^{100} + 1$$

12. Let  $k_1$  and  $k_2$  be two randomly selected natural numbers.

The probability that  $(i)^{k_1} + (i)^{k_2}$  is non-zero is (where  $i = \sqrt{-1}$ )

$$(1) \frac{1}{2} \qquad (2) \frac{3}{4}$$

$$(3) \frac{1}{4} \qquad (4) \frac{1}{6}$$

**Answer (2)**

**Sol.**  $(i)^{k_1} + (i)^{k_2}$  is non zero

$$\Rightarrow k_1 : 4\lambda_1 + r_1, \quad r_1 \in \{0, 1, 2, 3\}$$

$$k_2 : 4\lambda_2 + r_2, \quad r_2 \in \{0, 1, 2, 3\}$$

The pairs to get zero will be

$$(1, -1), (i, -i)$$

$$\Rightarrow (i) \quad (1, -1) \text{ pair}$$

$$\Rightarrow (r_1, r_2) \in \{(2, 0), (0, 2)\}$$

$$(ii) \quad (i, -i) \text{ pair}$$

$$\Rightarrow (r_1, r_2) \in \{(1, 3), (3, 1)\}$$

$$\Rightarrow \text{probability } (i^{k_1} + i^{k_2} \neq 0)$$

$$= 1 - \text{probability } (i^{k_1} + i^{k_2} = 0)$$

$$= 1 - \frac{4}{16} = \frac{12}{16} = \frac{3}{4}$$

13. In  $\Delta ABC$ ,  $A(4\sin\theta, 4\cos\theta)$ ,  $B(-2\cos\theta, 0)$  and  $C(2, 2\sin\theta)$ . If locus of centroid is  $(3x-2)^2 + (3y)^2 = \alpha$ , then  $\alpha$  is

$$(1) 20$$

$$(2) 4$$

$$(3) 16$$

$$(4) 12$$

**Answer (1)**

$$\text{Sol. } h = \frac{4\sin\theta - 2\cos\theta + 2}{3} \Rightarrow 3h - 2 = 4\sin\theta - 2\cos\theta \dots(1)$$

$$k = \frac{4\cos\theta - 2\sin\theta}{3} \Rightarrow 3k = 4\cos\theta + 2\sin\theta \dots(2)$$

$$(1)^2 + (2)^2$$

$$(3h-2)^2 + (3k)^2 = (4\sin\theta - 2\cos\theta)^2 + (4\cos\theta + 2\sin\theta)^2$$

$$= 16\sin^2\theta + 4\cos^2\theta - 8\sin\theta\cos\theta +$$

$$16\cos^2\theta + 4\sin^2\theta - 8\sin\theta\cos\theta$$

$$(3h-2)^2 + (3k)^2 = 20$$

$$(3x-2)^2 + (3y)^2 = 20$$

14. Let  $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$  be an ellipse and a series of ellipse

are drawn that  $E_{i+1}$  has same centre, eccentricity as  $E_i$  and  $E_{i+1}$ 's major axis is minor axis of  $E_i$ . If  $S_i$  be the area

of  $E_i$ , then  $\left(\frac{5}{\pi} \sum_{i=1}^x S_i\right)$  is equal to

$$(1) 63$$

$$(2) 54$$

$$(3) 45$$

$$(4) 72$$

**Answer (2)**

**Sol.** Let  $b_i$  be minor axis of  $E_i$

$a_i$  be major axis of  $E_i$

$$\Rightarrow e_i = 1 - \frac{b_i^2}{a_i^2}$$

Now,  $b_{i+1}$  minor axis of  $E_{i+1}$

$a_{i+1}$  major axis of  $E_{i+1}$

$$\Rightarrow e_{i+1} = 1 - \frac{b_{i+1}^2}{a_{i+1}^2}, \text{ also } a_{i+1} = b_i \text{ and } e_i = e_{i+1}$$

$$= \frac{b_i^2}{a_i^2} = \frac{b_{i+1}^2}{(b_i)^2} \Rightarrow b_{i+1} = \frac{b_i^2}{a_i}$$

$$\Rightarrow \text{Area of } E_i = S_i = \pi a_i b_i$$

$$\Rightarrow S_{i+1} = \pi a_{i+1} b_{i+1}$$

$$= \pi(b_i) \left(\frac{b_i^2}{a_i}\right)$$

$$= \pi(b, a_i) \left( \frac{b_i}{a_i} \right)^2$$

$$S_{i-1} = S_i(1 - e_i^2)$$

$$\Rightarrow S_{i+1} = S_i \left( 1 - \left( 1 - \frac{4}{9} \right) \right) = S_i \cdot \frac{4}{9}$$

$$\Rightarrow S_i = 6\pi, S_2 = 6\pi \cdot \frac{4}{9}, S_3 = 6\pi \cdot \left( \frac{4}{9} \right)^2$$

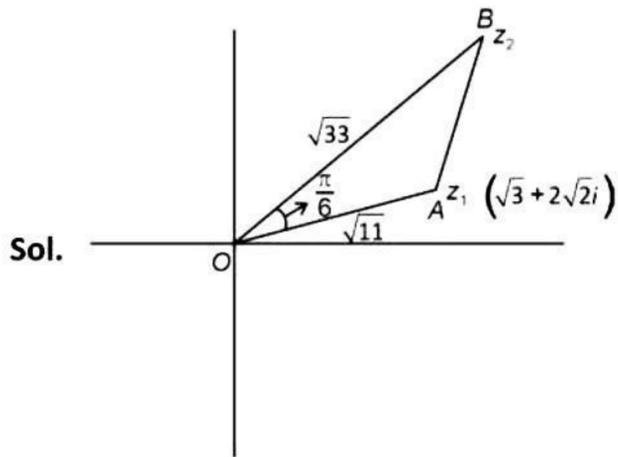
$$\Rightarrow \sum_{k=1}^{\infty} S_k = \left( \frac{6\pi}{1 - \frac{4}{9}} \right) = \frac{54\pi}{5}$$

$$\Rightarrow \frac{5}{\pi} \sum_{k=1}^{\infty} S_k = \frac{5}{\pi} \cdot \frac{54\pi}{5} = 54$$

15. Let  $z_1 = \sqrt{3} + 2\sqrt{2}i$  and  $\sqrt{3}|z_1| = |z_2|$  and  $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$ , then the area of triangle with vertices  $z_1, z_2$  and origin is (in sq. units)

- (1)  $\frac{11\sqrt{3}}{4}$  (2)  $\frac{11\sqrt{2}}{3}$   
 (3)  $\frac{11}{4}$  (4)  $\frac{2\sqrt{2}}{3}$

Answer (1)



$$\text{Area of } \triangle OAB = \frac{1}{2} \times \sqrt{11} \times \sqrt{33} \sin \frac{\pi}{6}$$

$$= \frac{11\sqrt{3}}{4} \text{ square units}$$

16. Let  $f(x) = \begin{cases} 2x, & x < 0 \\ \min(1+x+[x], 1+2[x]), & 0 \leq x < 2 \\ 5, & x \geq 2 \end{cases}$

If  $\alpha$  is the number of points of discontinuity and  $\beta$  is the number of points of non-differentiability, then  $(\alpha + \beta)$  is equal to (where  $[.]$  denote greatest integer function)

- (1) 6 (2) 5  
 (3) 4 (4) 8

Answer (1)

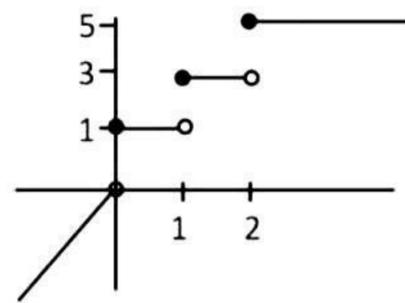
Sol.  $f(x) = \begin{cases} 2x, & x < 0 \\ \min(1+x+[x], 1+2[x]), & 0 \leq x < 2 \\ 5, & x \geq 2 \end{cases}$

$$1+x+[x] = 1+\{x\}+2[x]$$

Since  $\{x\} \geq 0 \forall x \in R$

$$\Rightarrow 1+x+[x] \geq 1+2[x]$$

$$\Rightarrow f(x) = \begin{cases} 2x, & x < 0 \\ 1+2[x], & 0 \leq x < 2 \\ 5, & x \geq 2 \end{cases}$$



Number of discontinuity = 3  $\Rightarrow \alpha = 3$

Number of point of non-differentiability

$$= 3 \Rightarrow \beta = 3$$

17. If  $\alpha = 1 + \sum_{n=1}^6 (-3)^{n-1} \cdot {}^{12}C_{2n-1}$ , then distance of point

$(12, \sqrt{3})$  from the line  $\alpha x - \sqrt{3}y + 100 = 0$  is,

- (1)  $\frac{109}{2}$   
 (2) 55  
 (3) 54  
 (4) 109

Answer (1)

$$\begin{aligned} \text{Sol. } \alpha &= 1 + \sum_{r=1}^6 (-3)^{r-1} \cdot {}^{12}C_{2r-1} \\ &= 1 + \sum_{r=1}^6 \left[ (\sqrt{3}i)^2 \right]^{r-1} \cdot {}^{12}C_{2r-1} \\ &= 1 + \frac{1}{\sqrt{3}i} \sum_{r=1}^6 (\sqrt{3}i)^{2r-1} \cdot {}^{12}C_{2r-1} \end{aligned}$$

$$\text{Let } \sqrt{3}i = x$$

$$\Rightarrow \alpha = 1 + \frac{1}{\sqrt{3}i} \sum_{r=1}^6 {}^{12}C_{2r-1} \cdot x^{2r-1}$$

$$\alpha = 1 + \frac{1}{\sqrt{3}i} \left[ {}^{12}C_1 \cdot x^1 + {}^{12}C_3 \cdot x^3 + {}^{12}C_5 \cdot x^5 + \dots + {}^{12}C_{11} \cdot x^{11} \right]$$

$$\text{Let } (1+x)^{12} = {}^{12}C_0 \cdot x^0 + {}^{12}C_1 \cdot x^1 + {}^{12}C_2 \cdot x^2 + \dots + {}^{12}C_{12} \cdot x^{12}$$

$$(1-x)^{12} = {}^{12}C_0 \cdot x^0 - {}^{12}C_1 \cdot x^1 + {}^{12}C_2 \cdot x^2 - {}^{12}C_3 \cdot x^3 + \dots + {}^{12}C_{12} \cdot x^{12}$$

$$(1+x)^{12} - (1-x)^{12} = 2 \left( {}^{12}C_1 \cdot x^1 + {}^{12}C_3 \cdot x^3 + \dots + {}^{12}C_{11} \cdot x^{11} \right)$$

$$\Rightarrow \alpha = 1 + \frac{1}{\sqrt{3}i} \left[ \frac{(1+x)^{12} - (1-x)^{12}}{2} \right]$$

$$\alpha = 1 + \frac{1}{\sqrt{3}i} \left[ \frac{(1+\sqrt{3}i)^{12} - (1-\sqrt{3}i)^{12}}{2} \right]$$

$$\text{Since, } \omega = \frac{-1}{2} + \frac{\sqrt{3}i}{2} \Rightarrow -2\omega = 1 - \sqrt{3}i$$

$$\omega^2 = \frac{-1}{2} - \frac{\sqrt{3}i}{2} \Rightarrow -2\omega^2 = 1 + \sqrt{3}i$$

$$\begin{aligned} \Rightarrow \alpha &= 1 + \frac{1}{\sqrt{3}i} \left( \frac{(-2\omega^2)^{12} - (2\omega)^{12}}{2} \right) \\ &= 1 + \frac{2^{11}}{\sqrt{3}i} (\omega^{24} - \omega^{12}) \end{aligned}$$

$$= 1 + \frac{2^{11}}{\sqrt{3}i} (1-1) = 1$$

$\alpha = 1$ ,  $\Rightarrow$  perpendicular distance from  $(12, \sqrt{3})$  is

$$\frac{|12 - \sqrt{3}(\sqrt{3}) + 100|}{\sqrt{1^2 + (\sqrt{3})^2}} = \frac{109}{2}$$

18.

19.

20.

### SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In an AP,  $T_m = \frac{1}{25}$ ,  $T_{25} = \frac{1}{20}$  and  $20 \sum_{r=1}^{25} T_r = 13$ , then

$$5m \sum_{r=m}^{2m} T_r \text{ equals}$$

**Answer (126)**

$$\text{Sol. } T_m = a + (m-1)d \cdot \frac{1}{25} \quad \dots(1)$$

$$T_{25} = a + 24d = \frac{1}{20}$$

$$20 \times \frac{2r}{2} \left[ a + \frac{1}{20} \right] = 13 \Rightarrow a = \frac{1}{20 \times 25}$$

$$20 \sum_{r=1}^{25} T_r = 20 \times \frac{25}{2} [2a + 24d] = 13$$

$$d = \frac{1}{20 \times 25}$$

Substitute a and d in (i)

$$\Rightarrow m = 20$$

$$\text{Now } 5m \sum_{r=m}^{2m} T_r = 5 \times 20 \left[ \sum_{r=20}^{40} T_r \right]$$

$$= 100 \left[ \frac{40}{2} [2a + 39d] \right] - \frac{19}{2} [2a + 18d]$$

$$= 100 \left[ \frac{40}{2} \times 41d - \frac{19}{2} \cdot 20d \right]$$

$$= 100 \left[ \frac{40}{2} \times \frac{41}{20 \times 25} - \frac{19 \cdot 20}{2 \times 20 \times 25} \right]$$

$$= 126$$

22.

23.

24.

25.

