

# **JEE (Main)-2026**

## **Session-1**

**24 January 2026 Shift-1**

### **Memory-Based Answers & Solutions (Physics, Chemistry, and Mathematics)**

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#### **IMPORTANT INSTRUCTIONS:**

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- (4) Section - A : Attempt all questions.
- (5) Section - B : Attempt all questions.
- (6) Section - A (01 – 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.
- (7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

# PHYSICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. A spring of stiffness  $k = 15 \text{ N/m}$  is cut into a ratio of 3 : 1. Find the spring constant of smaller length spring thus formed.

- (1) 15 N/m                      (2) 30 N/m  
 (3) 45 N/m                      (4) 60 N/m

**Answer (4)**

**Sol.**  $\frac{1}{k} = \frac{4}{k_1}$

$\Rightarrow k_1 = 4k$

$\Rightarrow k_1 = 60 \text{ N/m}$

2. EM waves and their source are given

**Column-I**

**Column-II**

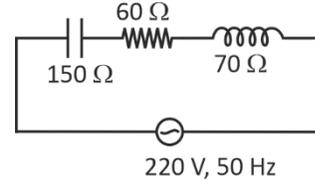
- |                   |  |
|-------------------|--|
| (a) X-rays        | (p) Hot bodies & Molecules                       |
| (b) Infrared rays | (q) Oscillating current in antennas              |
| (c) Microwaves    | (r) Magnetron                                    |
| (d) Radio waves   | (s) Fast moving electrons Striking a metal plate |

- (1) (a)-(p); (b)-(s); (c)-(r); (d)-(q)  
 (2) (a)-(s); (b)-(p); (c)- (r); (d)-(q)  
 (3) (a)-(s); (b)-(p); (c)- (s); (d)-(q)  
 (4) (a)-(s); (b)-(r); (c)- (p); (d)-(q)

**Answer (2)**

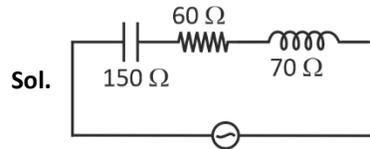
**Sol.** Theoretical

3. For the given AC circuit find the power factor.



- (1)  $\frac{4}{5}$                                       (2)  $\frac{3}{5}$   
 (3)  $\frac{3}{4}$                                       (4)  $\frac{4}{3}$

**Answer (2)**



$X = |X_L - X_C| = 80 \Omega$

$R = 60 \Omega$

$\Rightarrow \tan \theta = \frac{80}{60} = \frac{4}{3}$

$\Rightarrow \cos \theta = \text{Power factor} = \frac{3}{5}$

4. In H-like atom ratio of speed in two orbits is 3 : 2, then ratio of energy is

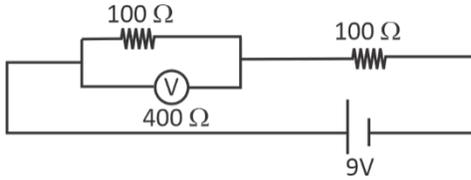
- (1) 2 : 3                                      (2) 9 : 4  
 (3) 2 : 1                                      (4) 5 : 3

**Answer (2)**

**Sol.**  $v = \frac{Z}{n}$

$\text{KE} = \left( \frac{Z}{n} \right)^2 = 9 : 4$

5. A voltmeter of  $400\ \Omega$  resistance is in parallel with  $100\ \Omega$  resistor. And the combination is connected with  $100\ \Omega$  resistor and a battery of 9 volt in series as shown. Find the reading of voltmeter.



- (1) 5 volts                      (2) 3 volts  
 (3) 4 volts                      (4) 6 volts

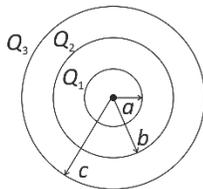
**Answer (3)**

**Sol.**  $R_{eq} = \frac{400 \times 100}{500} + 100 = 180$

$$I = \frac{9}{180}$$

So,  $V(\text{voltmeter}) = \frac{9}{180} \times 80 = 4$  volts.

6. Three concentric uniformly charged shells are kept as show. Find potential of the each shell.



(1)  $V_A = \frac{kQ_1}{a} + \frac{kQ_2}{b} + \frac{kQ_3}{c}$

$$V_B = \frac{k(Q_1 + Q_2 + Q_3)}{b}$$

$$V_C = \frac{k(Q_1 + Q_2 + Q_3)}{c}$$

(2)  $V_A = \frac{kQ_1}{a} + \frac{kQ_2}{b} + \frac{kQ_3}{c}$

$$V_B = \frac{k(Q_1 + Q_2)}{b} + \frac{kQ_3}{c}$$

$$V_C = \frac{k(Q_1 + Q_2 + Q_3)}{c}$$

(3)  $V_A = \frac{kQ_1}{a} + \frac{k(Q_2 + Q_3)}{c}$

$$V_B = \frac{k(Q_1 + Q_2)}{b} + \frac{kQ_3}{c}$$

$$V_C = \frac{k(Q_1 + Q_2 + Q_3)}{c}$$

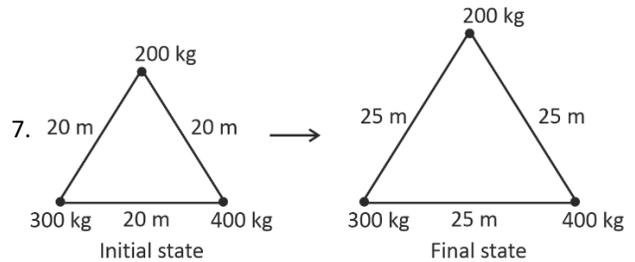
(4)  $V_A = \frac{kQ_1}{a} + \frac{kQ_2}{b} + \frac{kQ_3}{c}$

$$V_B = \frac{k(Q_1 + Q_2)}{a} + \frac{kQ_3}{b}$$

$$V_C = \frac{k(Q_1 + Q_2 + Q_3)}{c}$$

**Answer (2)**

**Sol.** Formula based



Find the work done.

(Given:  $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ )

- (1)  $1.7342 \times 10^{-7} \text{ J}$                       (2)  $1.6253 \times 10^{-7} \text{ J}$   
 (3)  $2.5232 \times 10^{-7} \text{ J}$                       (4)  $6.6325 \times 10^{-7} \text{ J}$

**Answer (1)**

**Sol.**  $U_i = -\frac{G \times 200 \times 300}{20} - \frac{G \times 200 \times 400}{20} - \frac{G \times 300 \times 400}{20}$

$$U_i = -\frac{26 \times 10^4 \times G}{20} = -8.671 \times 10^{-7} \text{ J}$$

$$U_f = -\frac{G \times 200 \times 300}{25} - \frac{G \times 200 \times 400}{25} - \frac{G \times 300 \times 400}{25}$$

$$U_f = -\frac{26}{25} \times 10^4 \times 6.67 \times 10^{-11}$$

$$U_f = -6.9368 \times 10^{-7} \text{ J}$$

$$W = \Delta U = 1.7342 \times 10^{-7} \text{ J}$$

8. Match the two lists given below .

	List-I		List-II
a.	Magnetic flux	1.	$M^1L^2T^{-2}A^{-2}$
b.	Magnetic permeability	2.	$M^1L^2T^{-2}A^{-1}$
c.	Magnetic induction	3.	$M^1L^1T^{-2}A^{-2}$
d.	Self-induction	4.	$M^1L^0T^{-2}A^{-1}$

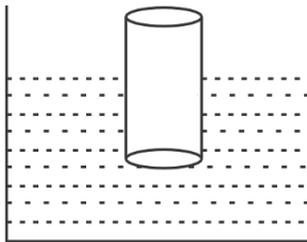
(1) a-2, b-3, c-4, d-1      (2) a-3, b-2, c-1, d-4

(3) a-4, b-3, c-1, d-2      (4) a-1, b-2, c-3, d-4

**Answer (1)**

**Sol.** Theoretical

9. A cylinder of mass  $m$ , length  $l$  and area of cross section  $A$  is in equilibrium in liquid of density  $\rho$ . Find time period of small vertical oscillations.



(1)  $2\pi\sqrt{\frac{mA}{\rho g}}$       (2)  $2\pi\sqrt{\frac{mg}{\rho A}}$

(3)  $2\pi\sqrt{\frac{m}{\rho A^2 g}}$       (4)  $2\pi\sqrt{\frac{m}{\rho Ag}}$

**Answer (4)**

**Sol.**  $F = (\rho Ag)x$

$$T = 2\pi\sqrt{\frac{m}{\rho Ag}}$$

10. A dipole is placed in uniform magnetic field  $B = 800$  gauss at an angle  $30^\circ$  then it experiences the torque of  $16 \times 10^{-3}$  N-m. Find the work done in slowly moving the dipole from stable equilibrium to unstable equilibrium.

(1)  $12.8 \times 10^{-3}$  J      (2)  $5 \times 10^{-3}$  J

(3)  $24.5 \times 10^{-3}$  J      (4)  $7.6 \times 10^{-3}$  J

**Answer (1)**

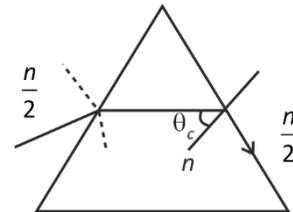
**Sol.**  $|\vec{\tau}| = |\vec{M} \times \vec{B}| = \frac{|\vec{M} \times \vec{B}|}{2}$

$$\Rightarrow |\vec{M}| = \frac{2 \times 16 \times 10^{-3}}{800 \times 10^{-4}} = \frac{4}{10}$$

$$\text{So } \Delta W = 2|\vec{M}||\vec{B}| = \frac{2 \times 4}{10} \times 800 \times 10^{-4}$$

$$\Rightarrow \Delta W = 64 \text{ mJ}$$

11. A light ray incident on the prism such that deviation is minimum and angle of incidence on 2<sup>nd</sup> surface is critical angle. Find prism angle.



(1)  $90^\circ$

(2)  $60^\circ$

(3)  $105^\circ$

(4)  $74^\circ$

**Answer (2)**

**Sol.**  $\sin\theta_c = \frac{1}{2}$

$$\theta_c = 30^\circ$$

$$r_1 = r_2 = 30^\circ$$

$$A = 30 + 30 = 60^\circ$$



15. Velocity of electron in  $n^{\text{th}}$  shell of a hydrogen like atom is  $3 \times 10^5$  m/s and velocity of electron in  $m^{\text{th}}$  shell of that atom is  $2.5 \times 10^5$  m/s. Find ratio of radius of  $m^{\text{th}}$  shell to  $n^{\text{th}}$  shell.

- (1)  $\frac{25}{40}$  (2)  $\frac{25}{36}$   
 (3)  $\frac{36}{25}$  (4)  $\frac{36}{35}$

**Answer (3)**

**Sol.**  $V \propto \frac{z}{n}$  and  $r \propto \frac{n^2}{z}$

$$\frac{V_n}{V_m} = \frac{m}{n} = \frac{6}{5}$$

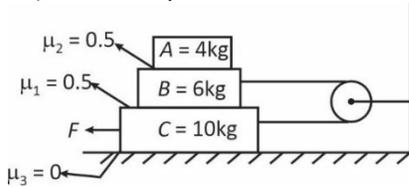
$$\frac{r_m}{r_n} = \frac{m^2}{n^2} = \frac{36}{25}$$

16.  
17.  
18.  
19.  
20.

### SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. For the given arrangement find the value of  $F$  (in Newton) so that body  $c$  moves with constant velocity.



**Answer (100)**

**Sol.**  $f_{r(\max)} = \frac{1}{2} \times 10 \times 10 = 50$  N

Since  $B$  also moves with uniform velocity

So, friction between  $A$  &  $B$  is zero.

Now,  $F - T - 50 = 0$

$\Rightarrow F = 50 + T$  ....(1)

Also,  $T - 50 = 0$   $T = 50$  N

So,  $F = 100$  N

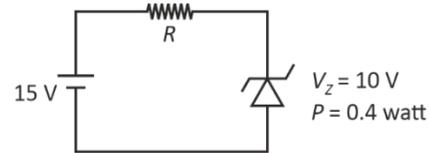
22. For a microscope focal length of objective is 2 cm and focal length of eyepiece is 4 cm. Tube length is  $L = 10$  cm. Magnification for normal adjustment is  $5^x$ . Find the value of  $x$ .

**Answer (2)**

**Sol.**  $M = \frac{1}{f_o} \cdot \frac{D}{f_e}$

$$M = \frac{10}{2} \cdot \frac{25}{5} = 25 = (5)^2$$

23. For the given circuit the breakdown voltage of Zener diode is  $V_Z = 10$  volts and it can with-stand the power dissipation of 0.4 watt. Find the value of resistance  $R$  ( $\Omega$ )



**Answer (125)**

**Sol.** Current in Zener diode is

$$I_{(Z)} = \frac{4}{10 \times 10} = \frac{4}{100} \text{ Ampere}$$

$$\text{So, } R = \frac{(15 - 10)}{4} \times 100 = 125 \Omega$$

24. If potential varies as distance  $r$  as  $v(r) = ar^3 + b$ . Total magnitude of charge  $Q$  inclosed within a sphere of unit radius is  $Q = \alpha(\pi a \epsilon_0)$ . Find the value of  $\alpha$ .

**Answer (12)**

**Sol.**  $v(r) = ar^3 + b$

$$\Rightarrow E(r) = \frac{-dv}{dr} = -3ar^2$$

$$\text{So } \int E \cdot ds = \frac{|q_{in}|}{\epsilon_0}$$

$$\Rightarrow 3ar^2 \cdot 4\pi r^2 = \frac{|q_{in}|}{\epsilon_0}$$

$$\Rightarrow |q_{in}| = 12\pi a \epsilon_0 (1)^4 = 12\pi a \epsilon_0$$

$$\text{So, } \alpha = 12$$

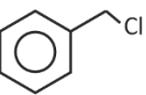
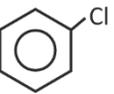
# CHEMISTRY

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

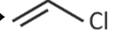
1. Match List-I with List-II.

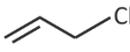
List-I	List-II
A. Vinyl halide	(I) 
B. Allyl halide	(II) 
C. Benzyl halide	(III) 
D. Aryl halide	(IV) 

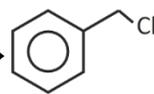
Select the correct option.

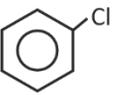
- (1) A(II), B(I), C(III), D(IV)    (2) A(I), B(II), C(III), D(IV)  
 (3) A(I), B(II), C(IV), D(III)    (4) A(II), B(I), C(IV), D(III)

**Answer (2)**

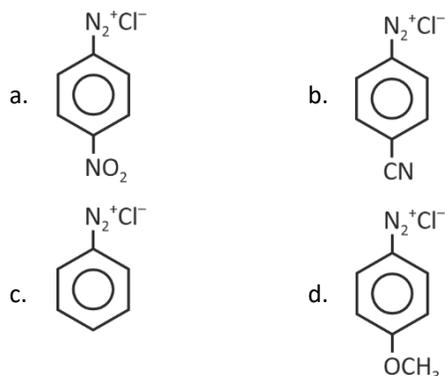
**Sol.** Vinyl halide → 

Allyl halide → 

Benzyl halide → 

Aryl halide → 

2. The correct order of stability of following diazonium ions is

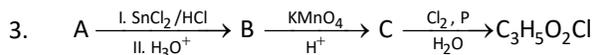


- (1)  $a < b < c < d$                       (2)  $a < b < d < c$   
 (3)  $c < d < b < a$                       (4)  $d < c < b < a$

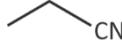
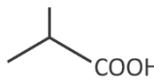
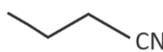
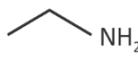
**Answer (1)**

**Sol.** Stronger the electron withdrawing group attached at para position of  $-N_2^+$  in diazonium ion, lesser is the stability and more electrophilicity.

Stability : (d) > (c) > (b) > (a)

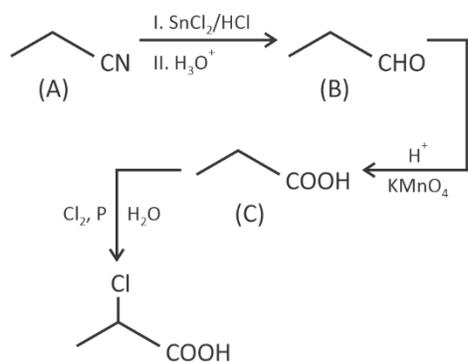


Final product has one chiral centre. Structure of A is

- (1) 
- (2) 
- (3) 
- (4) 

**Answer (1)**

Sol.



4. Which of following compound contains 3 unpaired electrons?

- (1)  $\text{V}_2\text{O}_5$
- (2)  $[\text{TiF}_6]^{3-}$
- (3)  $[\text{CoF}_6]^{4-}$
- (4)  $[\text{Fe}(\text{CN})_6]^{3-}$

Answer (3)

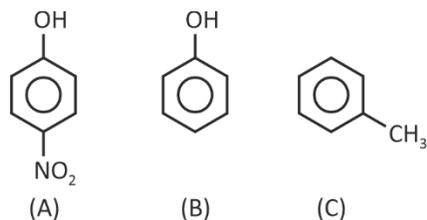
Sol.  $\text{V}_2\text{O}_5$  : 0 unpaired electrons

$[\text{TiF}_6]^{3-}$  :  $\text{Ti}^{3+}$  :  $[\text{Ar}] 4s^0 3d^1$  : 1 unpaired  $e^-$

$[\text{CoF}_6]^{4-}$  :  $\text{Co}^{2+}$  :  $[\text{Ar}] 4s^0 3d^7$  : 3 unpaired  $e^-$

$[\text{Fe}(\text{CN})_6]^{3-}$  :  $\text{Fe}^{3+}$  :  $[\text{Ar}] 4s^0 3d^5$  : 1 unpaired  $e^-$

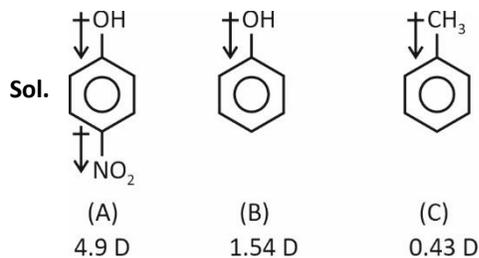
5. Consider the following molecules.



The correct order of dipole moment is

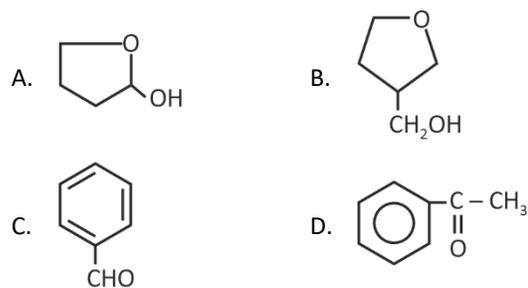
- (1)  $A > B > C$
- (2)  $A > C > B$
- (3)  $B > A > C$
- (4)  $C > A > B$

Answer (1)



Dipole moment  $A > B > C$

6. Which of the following compounds with give positive Tollen's reagent test?



- (1) A, B and C only
- (2) A and C only
- (3) A, C and D only
- (4) B, C and D only

Answer (2)

Sol. Aldehydes and compounds with hemiacetal linkage gives positive Tollen's test. A and C give +ve T.R. test.

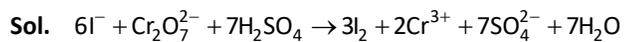
7.  $\text{K}_2\text{Cr}_2\text{O}_7 + \text{I}^- + \text{H}^+ \rightarrow \text{I}_2$  ( $x$  = number of moles of  $e^-$  exchanged per mol  $\text{I}_2$ )

$\text{K}_2\text{Cr}_2\text{O}_7 + \text{S}^{2-} \rightarrow \text{S}$  ( $y$  = number of moles of  $e^-$  exchanged for mole of S)

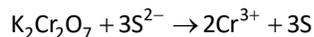
$x + y$  is

- (1) 12
- (2) 9
- (3) 4
- (4) 6

Answer (3)



$x = 2$



$y = 2$

8. Match the column

	Column-I		Column-II
(A)	IF <sub>3</sub>	(I)	$sp^3d^3$ , Pentagonal bipyramidal
(B)	IF <sub>5</sub>	(II)	$sp^3d$ , T-shaped
(C)	IF <sub>7</sub>	(III)	$sp^3$ , Tetrahedral
(D)	ClO <sub>4</sub> <sup>-</sup>	(IV)	$sp^3d^2$ , Square pyramidal

(1) (A)-(I); (B)-(II); (C)-(III); (D)-(IV)

(2) (A)-(II); (B)-(I); (C)-(IV); (D)-(III)

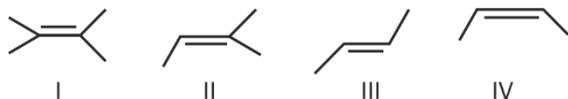
(3) (A)-(III); (B)-(IV); (C)-(I); (D)-(III)

(4) (A)-(II); (B)-(III); (C)-(IV); (D)-(I)

**Answer (3)**

Sol. (A)-(II); (B)-(IV); (C)-(I); (D)-(III)

9. Consider the following alkene



The correct stability order of alkenes is

(1) II > I > III > IV

(2) I > II > IV > III

(3) I > II > III > IV

(4) III > I > II > IV

**Answer (3)**

Sol. Alkene stability  $\propto$  no. of  $\alpha$ -hydrogen

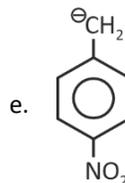
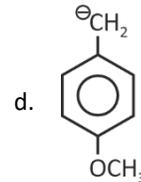
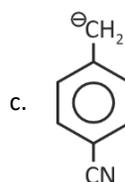
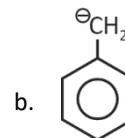
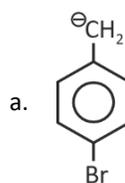
I  $\rightarrow$  12  $\alpha$ -H

II  $\rightarrow$  9  $\alpha$ H

III & IV  $\rightarrow$  6  $\alpha$ H

$\therefore$  Trans alkene is more stable than cis.

10. The correct order of stability of following species is



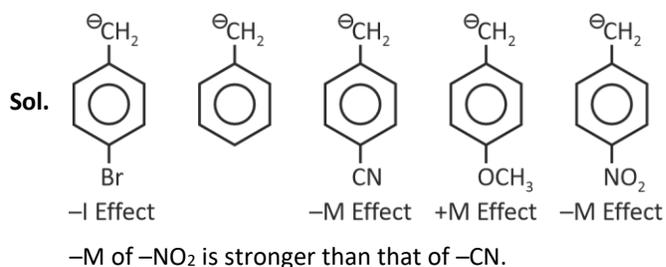
(1)  $e > c > a > b > d$

(2)  $d > c > b > a > e$

(3)  $e > a > c > b > d$

(4)  $e > a > b > c > d$

**Answer (1)**





15. Non-volatile solute A of mass 0.3 g (Molecular mass = 60 g/mol), and non-volatile solute B of mass 0.9 g (Molecular mass = 180 g/mol) are dissolved in 100 mL H<sub>2</sub>O at 27°C. (Take  $i = 1$ ;  $d_{\text{H}_2\text{O}} = 1 \text{ g/mL}$ )

If  $K_b = 0.52 \text{ K}\cdot\text{kg}\cdot\text{mol}^{-1}$ , then elevation of boiling point is

- (1) 0.52 K
- (2) 0.052 K
- (3) 0.026 K
- (4) 0.083 K

**Answer (2)**

**Sol.** mol of A =  $\frac{0.3}{60} = \frac{1}{200}$ , mol of B =  $\frac{0.9}{180} = \frac{1}{200}$

mass of solvent = 100 mL  $\times$  (1 g/mL) = 100 g

$$\Delta T_f = K_f \times m = 0.52 \times \left( \frac{\frac{1}{200} + \frac{1}{200}}{0.1} \right) = 0.052 \text{ K}$$

16. A solution contains two group-IV cations, X<sup>2+</sup> and Y<sup>2+</sup>, each at an initial concentration of 0.1 M. H<sub>2</sub>S gas is passed through the solution to form a saturated solution. Given

$K_{sp}$  of YS =  $2 \times 10^{-27} \text{ M}^2$

$K_{sp}$  of XS =  $1 \times 10^{-27} \text{ M}^2$

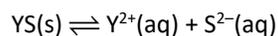
What is the minimum concentration of sulphide in [S<sup>2-</sup>] required to begin precipitation of YS?

- (1)  $2 \times 10^{-26}$
- (2)  $10^{-26}$
- (3)  $3.2 \times 10^{-14}$
- (4) 0.1

**Answer (1)**

**Sol.** For precipitation

$$Q_{ip} > K_{sp}$$



$$[Y^{2+}] [S^{2-}] = K_{sp}(YS)$$

$$[Y^{2+}] = 0.1 \text{ M}$$

$$[S^{2-}] = \frac{K_{sp}(YS)}{0.1}$$

$$= \frac{2 \times 10^{-27}}{0.1}$$

$$= 2 \times 10^{-26} \text{ M}$$

17. What is the hybridisation and spin only magnetic moment of complex [Co(CO)<sub>6</sub>]Cl<sub>3</sub>?

- (1)  $d^2sp^3$ , 0 BM
- (2)  $sp^3d^2$ , 4.90 BM
- (3)  $d^2sp^3$ , 4.90 BM
- (4)  $sp^3d^2$ , 0 BM

**Answer (1)**

**Sol.** CO is SFL with Co<sup>3+</sup>

$$3d^6 \Rightarrow t_{2g}^6 e_g^0$$

hybridisation =  $d^2sp^3$

$$\mu (\text{spin only}) = \sqrt{n(n+2)} \text{ BM}$$

$$n = 0$$

$$\mu \text{ spin only} = 0 \text{ BM}$$

18.

19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Two solutes A and B of 0.3 g and 0.9 g respectively (molar mass of A and B are 30 g/mol and 90 g/mol respectively) are dissolved in 100 mL water. (Take solutes to be non-electrolyte). Calculate osmotic pressure at 300 K (in atm)

**Answer (5)**

**Sol.**  $n_A = \frac{0.3}{30} = 10^{-2}$  mol

$$n_B = \frac{0.9}{90} = 10^{-2}$$
 mol

$$[A] = \frac{10^{-2}}{100} \times 1000 = 0.1 \text{ M}$$

$$[B] = \frac{10^{-2}}{100} \times 1000 = 0.1 \text{ M}$$

$$\pi = i CRT$$

$$\pi = 1 \times 0.2 \times 0.0821 \times 300 = 4.926 \text{ atm} \approx 5$$

22. Minimum energy transition of Balmer series (energy line having minimum energy) of H-atom has energy of L eV. If the value of minimum energy of Lyman series (energy line having minimum energy) of H-atom in terms of L is y, then the value of 10y is \_\_\_\_\_.

**Answer (54)**

**Sol.**  $(\Delta E_{\min})_{\text{Balmer}} = 13.6 \left( \frac{1}{4} - \frac{1}{9} \right) \text{ eV}$

$$= 13.6 \frac{5}{36} = L \text{ eV}$$

$$(\Delta E_{\min})_{\text{Lyman}} = 13.6 \left( \frac{1}{1} - \frac{1}{4} \right) = 13.6 \frac{3}{4} \text{ eV}$$

$$= 13.6 \times \frac{5}{36} \times \frac{36}{5} \times \frac{3}{4}$$

$$5.4 L = y$$

$$10y = 54$$

23. Find % of 'N' in 0.5 g organic compound which gives 34 mL N<sub>2</sub> (g) at 715 mm Hg pressure and 300 K.  
(Aq. tension = 15 mm Hg)

(Report to nearest integer)  $R = 0.0821 \frac{\text{Lit-atm}}{\text{K-mol}}$

**Answer (7)**

$$\frac{715 - 15}{760} \times 34 \times 10^{-3}$$

**Sol.**  $\% N = \frac{0.082 \times 300}{0.5} \times 28 \times 100 = 7.12\%$

24. Find the value of  $\log \left( \frac{k_{\text{catalysed}}}{k_{\text{uncatalysed}}} \right)$  at 300K if the change in activation energy ( $\Delta E_a$ ) is 10 kJ/mol. ( $R = 8 \text{ JK}^{-1} \text{ mol}^{-1}$ )  
( $\ln x = 2.3 \log x$ )

**Answer (2)**

**Sol.**  $k = Ae^{-E_a/RT}$

$$E_{a1} \text{ (catalysed)}$$

$$E_{a2} \text{ (uncatalysed)}$$

$$\frac{k_{\text{cat}}}{k_{\text{uncat}}} = e^{\frac{-E_{a1} + E_{a2}}{RT}}$$

$$\log \frac{k_{\text{cat}}}{k_{\text{uncatalysed}}} = \frac{-E_{a1} + E_{a2}}{2.303RT}$$

$$(E_{a2} - E_{a1}) = 10000 \text{ J/mol}$$

$$\log \frac{K_{\text{cat}}}{K_{\text{uncatalysed}}} = \frac{10000}{300 \times 8 \times 2.3}$$

$$= \frac{4.167}{2.3}$$

$$= 1.81$$

$$\approx 2$$

25.



Sol.  $\sum x_i = 100$

$$\frac{\sum x_i^2}{10} - (10)^2 = 2$$

$$\Rightarrow \sum x_i^2 = 1020$$

$$\mu' = \frac{\sum(x_i) - \alpha + \beta}{10} \Rightarrow 100 - \alpha + \beta = 101$$

$$\Rightarrow \beta - \alpha = 1$$

$$\sigma' = \left( \frac{\sum x_i^2 - \alpha^2 + \beta^2}{10} \right) - \left( \frac{101}{10} \right)^2 = \frac{199}{100}$$

$$\Rightarrow \frac{1020 - \alpha^2 + \beta^2}{10} = \frac{199}{100} + \left( \frac{101}{100} \right)^2$$

$$= \frac{10400}{100} = 104$$

$$\Rightarrow 1020 - \alpha^2 + \beta^2 = 1040$$

$$\Rightarrow \beta^2 - \alpha^2 = 20$$

$$\beta - \alpha = 1$$

$$\Rightarrow (\beta + \alpha)(\beta - \alpha) = 20$$

$$\Rightarrow \alpha + \beta = 20$$

4. If  $F(t) = \int \frac{1 - \sin(\ln t)}{1 - \cos(\ln t)} dt$  and  $F(e^{\pi/2}) = -e^{\pi/2}$  then

$F(e^{\pi/4})$  is

(1)  $(-1 - \sqrt{2})e^{\pi/4}$

(2)  $(1 - \sqrt{2})e^{\pi/4}$

(3)  $(1 + \sqrt{2})e^{\pi/4}$

(4)  $(-2 - \sqrt{2})e^{\pi/4}$

**Answer (1)**

Sol.  $\int \frac{1 - \sin(\ln t)}{1 - \cos(\ln t)} dt$

Let  $\ln t = x$

$$t = e^x$$

$$dt = e^x dx$$

$$\int e^x \frac{(1 - \sin x)}{1 - \cos x} dt$$

$$\Rightarrow \int e^x \left( \frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} \right) dx$$

$$\Rightarrow \int e^x \left( \frac{1}{2} - \operatorname{cosec}^2 \frac{x}{2} - \cot \frac{x}{2} \right) dx$$

$$\Rightarrow \int e^x \left[ \underbrace{-\cot \frac{x}{2}}_{f(x)} + \underbrace{\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2}}_{f'(x)} \right]$$

$$\Rightarrow -e^x \cot \frac{x}{2} + c$$

$$f(t) = -t \cot \left( \frac{\ln t}{2} \right) + c$$

$$f(e^{\pi/2}) = -e^{\pi/2} + c = -e^{\pi/2}$$

$$\Rightarrow c = 0$$

$$f(e^{\pi/4}) = -e^{\pi/4} \cot \left( \frac{\pi}{8} \right)$$

$$= -e^{\pi/4} [\sqrt{2} + 1]$$

5. Consider a sequence 729, 81, 9, 1, .....

Let  $P_n$  = product of first  $n$  terms of the given sequence

$$\text{and } \sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = \frac{3^\alpha - 1}{2 \times 3^\beta}$$

Then the value of  $\alpha + \beta$  is

(1) 73

(2) 75

(3) 76

(4) 81

**Answer (1)**

**Sol.**  $3^6, 3^4, 3^2, 3^0, \dots$

$$P_n = 3^{6+4+2+\dots+n \text{ terms}}$$

$$= 3^{2 \left[ \frac{n}{2} \times 6 + (n-1)(-2) \right]} = 3^{n(6-n+1)} = 3^{n(7-n)}$$

$$\Rightarrow \sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = \sum_{n=1}^{40} 3^{7-n} = 3^7 \times \frac{1}{3} \left( \frac{1 - \frac{1}{3^{40}}}{1 - \frac{1}{3}} \right)$$

$$= 3^7 \left( \frac{3^{40} - 1}{2 \times 3^{40}} \right) = \frac{3^{40} - 1}{2 \cdot 3^{33}}$$

$$\Rightarrow \alpha + \beta = 73$$

6. If  $a_1, a_2, a_3, \dots, a_n$  are in A.P. and given that  $a_2 - a_1 = -\frac{3}{4}$  and  $a_1 + a_2 + \dots + a_n = \frac{525}{2}$  and  $a_n = \frac{1}{4} a_1$ . Then

$\sum_{i=1}^{17} a_i$  is equal to

(1) 276 (2) 238

(3) 189 (4) 258

**Answer (2)**

**Sol.**  $\therefore a_1, a_2, a_3, \dots, a_n$  are in A.P.

Given that  $a_2 - a_1 = -\frac{3}{4}$  = common difference ( $d$ ).

$$\therefore d = -\frac{3}{4}$$

and also given that  $a_n = \frac{1}{4} a_1$

$$\therefore S_n = \frac{n}{2} (a_1 + a_n) = \frac{525}{2}$$

$$\therefore n \left( a_1 + \frac{1}{4} a_1 \right) = 525$$

$$\therefore a_1 n = 420 \quad \dots(i)$$

$$\text{Now } \frac{n}{2} \left\{ 2a_1 + (n-1) \left( -\frac{3}{4} \right) \right\} = \frac{525}{2}$$

$$8 \times 420 - 3n^2 + 3n = 2100$$

$$\therefore 3n^2 - 3n - 1260 = 0$$

$$\therefore n^2 - n - 420 = 0$$

$$(n-21)(n+20) = 0$$

$$\therefore n = 21$$

$$\therefore a_1 = 20$$

$$\sum_{i=1}^{17} a_i = \frac{17}{2} \left\{ 40 + 16 \times -\frac{3}{4} \right\} = 238$$

7. Number of matrices A of order  $3 \times 2$  such that all of its elements are from the set  $\{-2, -1, 0, 1, 2\}$  such that trace of  $AA^T$  is 5, is equal to

(1) 120 (2) 312

(3) 192 (4) 126

**Answer (2)**

$$\text{Sol. } AA^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + d^2 & - & - \\ - & b^2 + e^2 & - \\ - & - & c^2 + f^2 \end{bmatrix}$$

$\Rightarrow$  sum of diagonal (trace) = 5

$$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 5$$

where  $a, b, c, d, e, f \in \{-2, -1, 0, 1, 2\}$

**Case A** 5 of them square is 1

$$\Rightarrow {}^6C_5 \times (2^5) = 6 \times 32 = 192$$

**Case B** one of them square 4 and another one is square is 1

$\Rightarrow \{4, 1, 0, 0, 0, 0\}$  are possible as square

$$\Rightarrow {}^6C_4 \times (2!) \cdot (2 \cdot 2) = 15 \times 8 = 12$$

$\Rightarrow$  number of such matrices

$$= 192 + 120 = 312$$

8. Out of 100 bulbs, 10 are defective and 90 are non-defective. If the probability of finding 7 defective bulbs out of 8 draws, with replacement, is  $\frac{K}{10^8}$ , then the value of  $K$  is

- (1) 69                                      (2) 72  
 (3) 75                                      (4) 96

**Answer 2)**

**Sol.** Probability a bulb is defective on any draw

$$p = \frac{10}{100} = \frac{1}{10}$$

$$\Rightarrow q = \frac{9}{10}, n = 8$$

$$\Rightarrow P(X=7) = {}^8C_7 \left(\frac{1}{10}\right)^7 \left(\frac{9}{10}\right)^1 = \frac{72}{10^8}$$

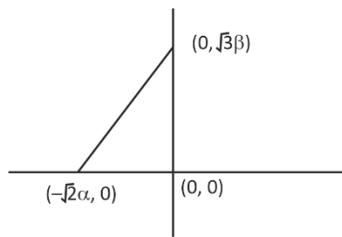
$$\Rightarrow K = 72$$

9. Let a circle passes through points  $A(-\sqrt{2}\alpha, 0)$ ,  $B(0, \sqrt{3}\beta)$  and  $O(0, 0)$  such that its radius is 4. Then the radius of locus of centroid of triangle  $OAB$  is

- (1)  $\frac{2}{3}$                                       (2)  $\frac{8}{3}$   
 (3)  $\frac{4}{3}$                                       (4)  $\frac{11}{3}$

**Answer (2)**

**Sol.**



$$\Rightarrow \text{radius} = \frac{\sqrt{(\sqrt{3}\beta)^2 + (-\sqrt{2}\alpha)^2}}{2} = 4$$

$$\Rightarrow 3\beta^2 + 2\alpha^2 = 64$$

Let the centroid of the triangle is

$$(h, k) \Rightarrow h = \frac{-\sqrt{2}\alpha}{3} \text{ and } k = \frac{-\sqrt{3}\beta}{3}$$

$$\Rightarrow \alpha = \frac{-3h}{\sqrt{2}}, \beta = \frac{3k}{\sqrt{3}}$$

$$\Rightarrow 2\left(\frac{9h^2}{2}\right) + 3\left(\frac{9k^2}{3}\right) = 64$$

$$\Rightarrow 9h^2 + 9k^2 = 64$$

$$\Rightarrow \text{Locus is } x^2 + y^2 = \frac{64}{9}$$

10. Let  $\cot\theta = \frac{5}{12}$  and  $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ .

Then the value of  $\cos 7\theta \left(\sin \frac{13\theta}{2} + \cos \frac{13\theta}{2}\right) +$

$\sin 7\theta \left(\sin \frac{13\theta}{2} - \cos \frac{13\theta}{2}\right)$  is

- (1)  $-\frac{1}{\sqrt{13}}$                                       (2)  $\frac{1}{\sqrt{13}}$   
 (3)  $-\frac{5}{\sqrt{13}}$                                       (4)  $\frac{5}{\sqrt{13}}$

**Answer (3)**

**Sol.**  $\because \cot\theta = \frac{5}{12}, \theta \in \left(\pi, \frac{3\pi}{2}\right)$

$$\Rightarrow \sin\theta = -\frac{12}{13}, \cos\theta = -\frac{5}{13}$$

$$\text{Let } P(\theta) = \cos 7\theta \left(\sin \frac{13\theta}{2} + \cos \frac{13\theta}{2}\right)$$

$$+ \sin 7\theta \left(\sin \frac{13\theta}{2} - \cos \frac{13\theta}{2}\right)$$



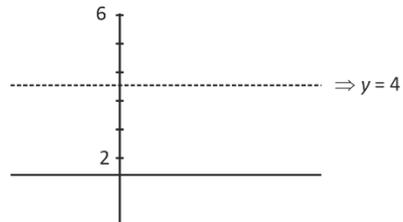
12. Let  $\left| \frac{z-6i}{z-2i} \right| = 1$ ,  $\left| \frac{z-8+2i}{z+2i} \right| = \frac{3}{5}$  then if  $\omega$  satisfy both

equation then find  $\sum |\omega|^2$ .

- (1) 398 (2) 385  
 (3) 413 (4) 433

**Answer (2)**

**Sol.**  $|z-6i| = |z-2i|$



$$z = x + iy$$

$$5|(x-8) + (y+2)i| = 3|(x+0)^2 + (y+2)^2|$$

$$\Rightarrow 25(x-8)^2 + 25(y+2)^2$$

$$= 9(x^2) + 9(y+2)^2$$

$$\Rightarrow 25x^2 - 16 \times 25x + 25 \times 64 + 25y^2 + 100y + 100$$

$$= 9x^2 + 9y^2 + 36y + 36$$

$$\Rightarrow 16x^2 + 16y^2 - 400x + 64y + 166y = 0$$

$$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$$

This circle intersects lines  $y = 4$

$$\text{at } x^2 + 16 - 25x + 16 + 104 = 0$$

$$x^2 - 25x + 136 = 0 \Rightarrow x = 8, 17$$

$\Rightarrow z$  can be  $(17, 4)$  and  $(8, 4)$

$$\Rightarrow \sum |z|^2 = \left( \sqrt{8^2 + 4^2} \right)^2 + \left( \sqrt{4^2 + 17^2} \right)^2$$

$$= 64 + 16 + 16 + 289 = 385$$

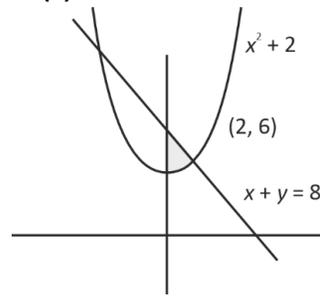
13. Let  $A_1$  be the area enclosed by  $y = x^2 + 2$ ,  $y$ -axis and  $x + y = 8$  and

Let  $A_2$  be the area enclosed by  $y = x^2 + 2$ ,  $y^2 = x$ ,  $x = 2$  and  $y$ -axis, then the value of  $A_1 - A_2$  is

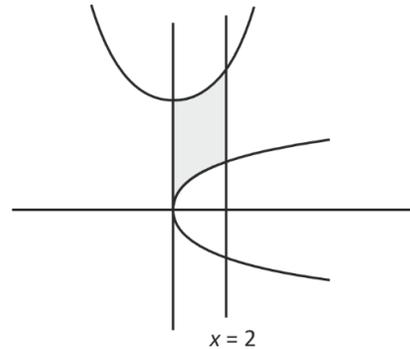
- (1)  $\frac{4+8\sqrt{2}}{3}$  (2)  $\frac{2+4\sqrt{2}}{3}$   
 (3)  $\frac{8+2\sqrt{2}}{3}$  (4)  $\frac{8-2\sqrt{2}}{3}$

**Answer (2)**

**Sol.**



$$A_1 = \int_0^2 ((8-x) - (x^2+2)) dx = \frac{22}{3}$$



$$A_2 = \int_0^2 ((x^2+2) - (\sqrt{x})) dx$$

$$= \left[ \frac{x^3}{3} + 2x - \frac{2x^{3/2}}{3} \right]_0^2$$

$$= \frac{8}{3} + 4 - \frac{2(2)^{3/2}}{3}$$

$$= \frac{20}{3} - \frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{22}{3} - \frac{20}{3} + \frac{4\sqrt{2}}{3} = \frac{2+4\sqrt{2}}{3}$$

14. If  $f(x) = \frac{e^x(\tan x - x) + \ln(\sec x + \tan x) - x}{\tan x - x}$ ,  $x \neq 0$ . If  $f(x)$

is continuous at  $x = 0$ , then  $f(0)$  is equal to

- (1)  $\frac{3}{2}$  (2) 1  
 (3)  $\frac{1}{2}$  (4) 2

**Answer (1)**

**Sol.**  $f(x) = e^x + \ln \frac{(\sec x + \tan x) - x}{\tan x - x}$

since  $\lim_{x \rightarrow 0} \frac{\ln(\sec x + \tan x) - x}{\tan x - x} = \lim_{x \rightarrow 0} \frac{\sec x - 1}{\sec^2 x - 1} = \frac{1}{2}$

Using L'Hospital

$\Rightarrow \lim_{x \rightarrow 0} f(x) = f(0) = 1 + \frac{1}{2} = \frac{3}{2}$

15.  $E_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$E_2: \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

Let eccentricity of both  $E_1$  and  $E_2$  be  $\frac{4}{5}$ ,  $2l_1^2 = 9l_2^2$

where  $l_1$  and  $l_2$  are the length of latus rectum of  $E_1$  and  $E_2$  respectively. Distance between the foci of  $E_1$  be 8.

Then distance between foci of ellipse  $E_2$  is

(1)  $\frac{32}{5}$

(2)  $\frac{16}{5}$

(3)  $\frac{8}{5}$

(4)  $\frac{4}{5}$

**Answer (1)**

**Sol.**  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

$l_1^2 = 1 - \frac{b^2}{a^2}$

$l_2^2 = 1 - \frac{B^2}{A^2}$

$\frac{16}{25} = 1 - \frac{b^2}{a^2}$

$\frac{16}{25} = 1 - \frac{B^2}{A^2}$

$\frac{b^2}{a^2} = \frac{9}{25}$  ... (1)

$\frac{B^2}{A^2} = \frac{9}{25}$  ... (2)

Now  $2l_1^2 = 9l_2^2$

$2\left(\frac{2b^2}{a}\right)^2 = 9\left(\frac{2B^2}{A}\right)$

$8\frac{b^4}{a^2} = 18\frac{B^2}{A}$

$\frac{b^4}{a^2} = \frac{9B^2}{4A}$

Also given:  $2ae = 8$

$2 \times \frac{4}{5}a = 8$

$a = 5$

$\Rightarrow b = 3$

Now  $\frac{81}{25} = \frac{9B^2}{4A}$

Now  $2Ae$

$\frac{36}{25}A = B^2$

$= 2 \times 4 \times \frac{4}{5}$

Sub in (2)

$= \frac{32}{5}$

$\frac{36A}{25A^2} = \frac{9}{25}$

$A = 4$

16. Find the number of numbers greater than 5000 and less than 9000, formed by using numbers 0, 1, 2, 5, 9 with repetition allowed and divisible by 3.

(1) 31

(2) 42

(3) 48

(4) 52

**Answer (2)**

**Sol.** As number is more than 5000 and less than 9000 then thousand place must be 5.

5	a	b	c
---	---	---	---

For  $(a, b, c) = (0, 0, 1) \rightarrow 3$  ways

$(0, 1, 9) \rightarrow 6$  ways

$(0, 2, 5) \rightarrow 6$  ways

$(0, 2, 2) \rightarrow 3$  ways

$(0, 5, 5) \rightarrow 3$  ways

$(1, 1, 2) \rightarrow 3$  ways

$(1, 1, 5) \rightarrow 3$  ways

$$(1, 9, 9) \rightarrow 3 \text{ ways}$$

$$(2, 2, 9) \rightarrow 3 \text{ ways}$$

$$(2, 5, 9) \rightarrow 6 \text{ ways}$$

$$(5, 5, 9) \rightarrow 3 \text{ ways}$$

$$\underline{42}$$

$\therefore$  Total 42 numbers are possible.

17.

18.

19.

20.

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### SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If  $S = \frac{1}{25!} + \frac{1}{23!3!} + \frac{1}{21!5!} + \dots$  upto 13 terms. Then

$$13S = \frac{2^\alpha}{\beta!}, \text{ then } \alpha + \beta \text{ is}$$

**Answer (49)**

**Sol.**  $\frac{1}{25!} + \frac{1}{23!3!} + \frac{1}{21!5!} + \dots$  till 13 term = S

$$26!S = \frac{26!}{25!1!} + \frac{26!}{23!3!} + \frac{26!}{21!5!} + \dots$$

$$= {}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \dots + {}^{26}C_{25}$$

$$26!S = 2^{25}$$

$$S = \frac{2^{25}}{26!}$$

$$13S = 13 \times \frac{2^{25}}{26 \times 25!}$$

$$= \frac{2^{24}}{25!} \Rightarrow \alpha = 24 \quad \beta = 25$$

$$\alpha + \beta = 49$$

22.

23.

24.

25.

