JEE (Main)-2026 Session-1

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

24 January 2026 Shift – 2

Time: 3 hrs. **M.M: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A: Attempt all questions.

(5) Section - B: Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION-A

1. Let $f(x) = \int \frac{7x^{10} + 9x^8}{(1 + x^2 + 2x^9)^2} dx$, $x > 0$, $\lim_{x \to 0} f(x) = 0$ and $f(1) = \frac{1}{4}$. If $A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$ and $B = adj(adj\ A)$ be such that $|B| = 81$, then α^2 is equal to

(1) 2 (2) 3

(3) 1 (4) 4

Ans. (4)

Sol. $f(x) = \frac{\int\left(\frac{7}{x^8} + \frac{9}{x^{10}}\right)}{\left(\frac{1}{x^9} + \frac{1}{x^7} + 2\right)^2} dx$

Put $t = \frac{1}{x^9} + \frac{1}{x^7} + 2 \Rightarrow \frac{dt}{dx} = \frac{-9}{x^{10}} - \frac{7}{x^8}$

$f(x) = \int \frac{-dt}{t^2} = \frac{1}{t} + C$

$f(x) = \frac{1}{\frac{1}{x^9} + \frac{1}{x^7} + 2} + C$

$= \frac{x^9}{1 + x^2 + 2x^9} + C$

Given $f(1) = \frac{1}{4} = \frac{1}{4} + C \Rightarrow C = 0$

$f(x) = \frac{x^9}{1 + x^2 + 2x^9}$

$f'(x) = \frac{(1 + x^2 + 2x^9) - 9x^8 - x^9(2x + 18x^8)}{(1 + x^2 + 2x^9)^2}$

$f'(x) = \frac{36 - 20}{16} = 1$

$A = \begin{pmatrix} 0 & 0 & 1 \\ 4 & 1 & 1 \\ \alpha^2 & \frac{1}{4} & 1 \end{pmatrix}$

$|A| = |1 - \alpha^2| = 3$

$1 - \alpha^2 = 3, -3 \Rightarrow \alpha^2 = -2, 4$

Value of $\alpha^2 = 4$

$B = adj(adjA)$

$|B| = 81 = |A|^4 \Rightarrow |A| = 3$

2. Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$, be 30. If its eccentricity is the maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$, then $(a^2 + b^2)$ is equal to -

(1) 516 (2) 256

(3) 496 (4) 276

Ans. (3)

Sol. $f(t) = \frac{-3}{4} + 2t - t^2$

$f(t)\Big|_{maximum} = \frac{1}{4} = e \Rightarrow e^2 = \frac{1}{16} \Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{16}$ ..(1)

$\because \frac{2b^2}{a} = 30 \Rightarrow b^2 = 15a$ ...(2)

By (1) & (2)

$16(a^2 - 15a) = a^2 \Rightarrow 15a^2 - 16 \times 15a = 0$

$a = 16$

$b^2 = 240$

$a^2 + b^2 = 256 + 240$

$= 496$

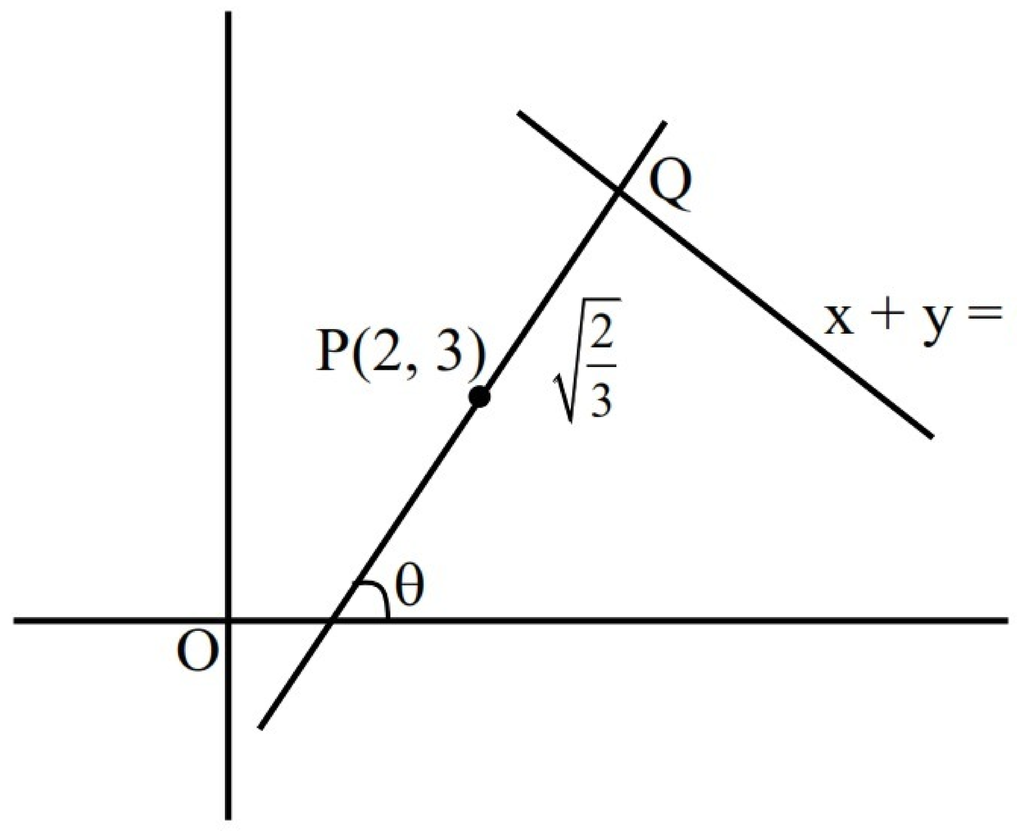
3. Let the angles made with the positive x-axis by two straight lines drawn from the point P(2, 3) and meeting the line x + y = 6 at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2. Then the value of $(\theta_1 + \theta_2)$ is :

(1) $\frac{\pi}{12}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. (3)

Sol.



Let Q is $\left(\sqrt{\frac{2}{3}}\cos\theta+2, \sqrt{\frac{2}{3}}\sin\theta+3\right)$

so, $x+y=6$

$\sqrt{\frac{2}{3}}(\cos\theta + \sin\theta) + 5 = 6$

$\sin\theta + \cos\theta = \sqrt{\frac{3}{2}}$

$1 + \sin2\theta = \frac{3}{2}$

$\sin2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{12}$ & $\frac{5\pi}{6}$

So $\theta_1 + \theta_2 = \frac{\pi}{2}$

4. Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to

(1) $\frac{\sqrt{21}}{2}$ (2) 13

(3) $\frac{\sqrt{35}}{2}$ (4) 7

Ans. (Bonus)

NTA (3)

Sol. $\vec{v} = x\vec{a} + y\vec{b} = x(2\hat{i}-\hat{j}-\hat{k}) + y(\hat{i}+3\hat{j}-\hat{k})$

$\vec{v} = (2x+y)\hat{i} + (3y-x)\hat{j} + (-x-y)\hat{k}$

$\left|\frac{\vec{v}.\vec{c}}{|\vec{c}|}\right| = \frac{1}{\sqrt{14}}$

$\vec{v}.\vec{c} = 2(2x+y) + 3y - x - 3x - 3y$

$= 2y$

$\left|\frac{2y}{\sqrt{14}}\right| = \frac{1}{\sqrt{14}} \Rightarrow |2y| = 1$

$|\vec{v}| = \sqrt{(2x+y)^2 + (3y-x)^2 + (x+y)^2}$

$= \sqrt{6x^2 + 11y^2 + 4xy - 6xy + 2xy}$

$= \sqrt{6x^2 + \frac{11}{4}} = \frac{\sqrt{24x^2+11}}{2}$

Now if we take $x^2 = 1$ then option $\frac{\sqrt{35}}{2}$ matches most probably NTA thought could been this.

5. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be an A.P. of four terms such that each term of the A. P. and its common difference l are integers. If $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 48$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4 + l^4 = 361$ then the largest term of the A.P. is equal to

(1) 27 (2) 24 (3) 21 (4) 23

Ans. (1)

Sol. a_1, a_2, a_3, a_4 as $a-3d, a-d, a+d, a+3d$

where $\boxed{d=\frac{\ell}{2}}$

$\because a_1+a_2+a_3+a_4=48 \Rightarrow 4a=48 \Rightarrow a=12$

& $a_1a_2a_3a_4+\ell^4=361 \Rightarrow (a^2-9d^2)(a^2-d^2)+16d^4$

$=361$

$\Rightarrow (144-9d^2)(144-d^2)+16d^4=361$

$\Rightarrow 25d^4-1440d^2+(144)^2=361$

$(5d^2-144)^2=19^2$

$\therefore 5d^2-144=19$ or -19

$d^2=\frac{163}{5}$ or $d^2=\frac{125}{5}=25$

$d=\sqrt{\frac{163}{5}}$ or $d=5$

$\therefore \ell=2\sqrt{\frac{163}{5}}$ or $\ell=10$

(rejected)

$\because$ common difference is an integer

$\therefore$ largest term $=12+15=27$

6. Let the image of parabola $x^2=4y$, in the line $x-y=1$ be $(y+\alpha)^2=b(x-c)$, $a, b, c \in \mathbb{N}$. Then $a+b+c$ is equal to

(1) 12 (2) 4

(3) 6 (4) 8

Ans. **(3)**

Sol. Parametric point P on $x^2=4y$ is $P(2t, t^2)$

$\therefore$ mirror image of P in $x-y=1$ is

$$Q\equiv\left(2t-\frac{2.1.(2t-t^2-1)}{2}, t^2+\frac{2.2(1).(2t-t^2-1)}{2}\right)$$

$Q\equiv(t^2+1, 2t-1)\equiv(h,k)$

$\therefore$ locus of Q is $x=\frac{(y+1)^2}{4}+1$ which is the required parabola.

$\therefore \boxed{(y+1)^2=4(x-1)}$

$\therefore a=1, b=4, c=1$

$\therefore \boxed{a+b+c=6}$

7. $\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\frac{1}{3^2}+\frac{1}{3}\times\frac{4}{7}+\frac{4^2}{7^2}\right)+\left(\frac{1}{3^3}+\frac{1}{3^2}\times\frac{4}{7}+\frac{1}{3}\times\frac{4^2}{7^2}+\frac{4^3}{7^3}\right)+ \ldots\ldots$ upto infinite terms is equal to -

(1) $\frac{5}{2}$ (2) $\frac{7}{4}$ (3) $\frac{4}{3}$ (4) $\frac{6}{5}$

Ans. **(1)**

Sol. Let $a=\frac{4}{7}$, $b=\frac{1}{3}$

Multiply N^r and D^r by $(a-b)=\frac{4}{7}-\frac{1}{3}=\frac{5}{21}$

$\frac{1}{a-b}[(a^2-b^2)+(a^3-b^3)+(a^4-b^4)+\ldots.\infty]$

$$\frac{1}{a-b}\left[\frac{a^2}{1-a}-\frac{b^2}{1-b}\right]=\frac{21}{5}\left[\frac{\frac{16}{49}}{1-\frac{4}{7}}-\frac{\frac{1}{9}}{1-\frac{1}{3}}\right]$$

$$=\frac{21}{5}\left[\frac{16}{21}-\frac{1}{6}\right]=\frac{21}{5}\left[\frac{96-21}{21.6}\right]$$

$$=\frac{75}{5.6}=\frac{15}{6}=\frac{5}{2}$$

8. Let $P=[p_{ij}]$ and $Q=[q_{ij}]$ be two square matrices of order 3 such that $q_{ij}=2^{(i+j-1)}p_{ij}$ and $\det(Q)=2^{10}$. Then the value of det(adj(adj P)) is :

(1) 32 (2) 16

(3) 81 (4) 124

Ans. **(2)**

Sol.
$$\begin{vmatrix}2p_{11} & 2^2p_{12} & 2^3p_{13}\\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23}\\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33}\end{vmatrix}=2^{10}$$

$$2^2\cdot2\cdot2^3\begin{vmatrix}p_{11} & p_{12} & p_{13}\\ 2p_{21} & 2p_{22} & 2p_{23}\\ 2^2p_{31} & 2^2p_{32} & 2^2p_{33}\end{vmatrix}=2^{10}$$

$$2^9\begin{vmatrix}p_{11} & p_{12} & p_{13}\\ p_{21} & p_{22} & p_{23}\\ p_{31} & p_{32} & p_{33}\end{vmatrix}=2^{10} \Rightarrow |P|=2$$

$|\text{adj}(\text{adj}(P))|=|P|^{(n-1)^2}=|P|^4=2^4=16$

9. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is :

(1) 1580 (2) 1578

(3) 1579 (4) 1581

Ans. (1)

Sol. ADIPRUU

$A \rightarrow \frac{6!}{2!} = 360$

$D \rightarrow \frac{6!}{2!} = 360$

$P \rightarrow \frac{6!}{2!} = 360$

$R \rightarrow \frac{6!}{2!} = 360$

UA $\rightarrow$ 5! = 120

UDAP $\rightarrow$ 3! = 6

UDAR $\rightarrow$ 3! = 6

UDAU $\rightarrow$ 3! = 6

UDAYPRU $\rightarrow$ 1

UDAYPUR $\rightarrow$ 1

Total = 1580

10. The largest value of n, for which 40^n divides 60!, is

(1) 13 (2) 11

(3) 12 (4) 14

Ans. (4)

Sol. $40^n = 2^{3n} \times 5^n$

$$E_2(60!) = \left[\frac{60}{2}\right] + \left[\frac{60}{2^2}\right] + \left[\frac{60}{2^3}\right] + \left[\frac{60}{2^4}\right] + \left[\frac{60}{2^5}\right]$$

= 30 + 15 + 7 + 3 + 1 = 56

$$E_5(60!) = \left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right]$$

= 12 + 2 = 14

$40^n = (2^3)^n \times 5^n = (2^3 \times 5)^n$

$60! = 2^{56} \times 5^{14} \ldots = 2^{14}.(2^3.5)^{14}$

$\therefore$ Maximum value of n is 14.

11. The sum of all values of α, for which the shortest distance between the lines $\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha}$ and $\frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha}$ is $\sqrt{2}$, is

(1) 8 (2) –6

(3) 6 (4) –8

Ans. (2)

Sol. $$\sqrt{2} = \frac{\begin{vmatrix} -1 & 1 & 3 \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}{\begin{vmatrix} \hat{i} & j & k \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}$$

$$\sqrt{2} = \frac{-1(-2\alpha+2\alpha) - 1(2\alpha^2+\alpha^2) + 3(2\alpha+\alpha)}{\left|\hat{i}(-2\alpha+2\alpha) - \hat{j}(2\alpha^2+\alpha^2) + \hat{k}(2\alpha+\alpha)\right|}$$

$$\sqrt{2} = \frac{-3\alpha^2 + 9\alpha}{\sqrt{9\alpha^4 + 9\alpha^2}}$$

$$\sqrt{2} = \frac{-\alpha + 3}{\sqrt{\alpha^2 + 1}}$$

$\Rightarrow 2\alpha^2 + 2 = \alpha^2 + 9 - 6\alpha$

$\alpha^2 + 6\alpha - 7 = 0$

$(\alpha + 7)(\alpha - 1) = 0$

$\alpha = -7, 1$

sum = –7 + 1 = – 6

option (2)

12. Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by x = 0, x = 1, $y^2 = x$ and $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$. Then (f(0) + f(1)) is equal to

(1) 9 (2) 14

(3) 7 (4) 12

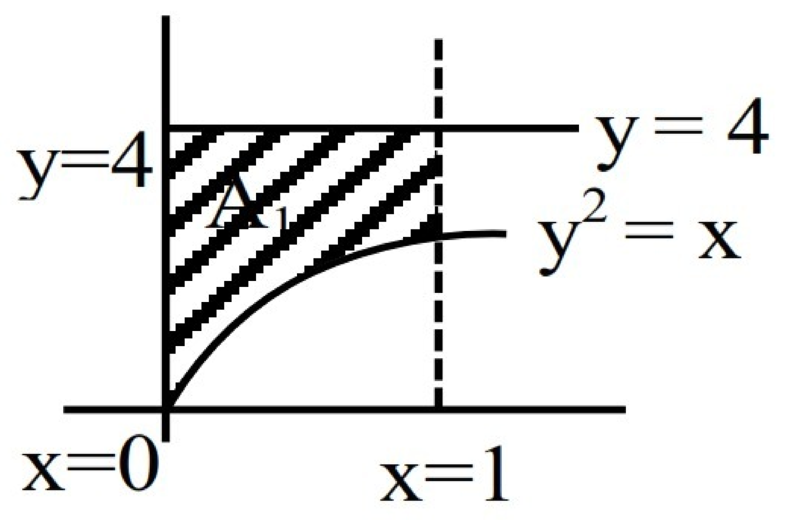
Ans. (3)

Sol. at $\alpha = 0 \Rightarrow f(0)$

$x = 0, x = 1, y^2 = x$

$y = |0 \cdot x - 5| - |1 - 0 \cdot x| + 0 \cdot x^2$

y = 4



$$A_1 = \int_0^1 (4-\sqrt{x})dx$$

$$= 4x - \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$$

$$= 4 - \frac{2}{3}(1) = \frac{10}{3}$$

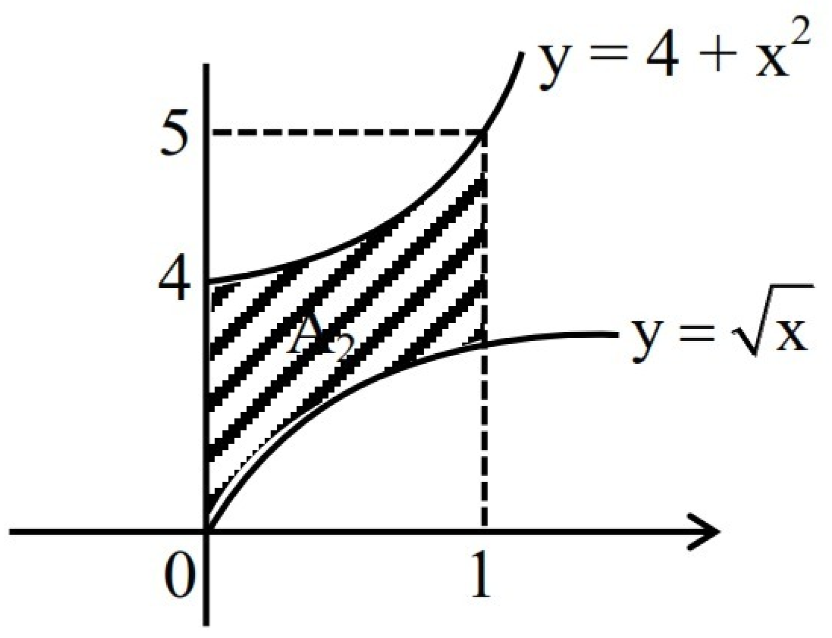
at $\alpha = 1 \Rightarrow f(1)$

$x = 0, x = 1, y^2 = x,$

$y = |x-5| - |1-x| + x^2$ in $x \in (0, 1)$

$y = 5 - x - (1-x) + x^2$

$y = 4 + x^2$



$$A_2 = \int_0^1 \left((4+x^2) - (\sqrt{x})\right)dx$$

$$= 4x + \frac{x^3}{3} - \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$$

$$= 4 + \frac{1}{3} - \frac{2}{3} = \frac{11}{3}$$

$$|f(0)+f(1)| = |A_1 + A_2| = \left|\frac{10}{3} + \frac{11}{3}\right| = \left|\frac{21}{3}\right| = 7$$

option (3)

13. If the domain of the function $f(x) = \sin^{-1}\left(\frac{1}{x^2-2x-2}\right)$, is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$, then $\alpha + \beta + \gamma + \delta$ is equal to

(1) 2 (2) 4

(3) 3 (4) 5

Ans. (2)

Sol. $-1 \le \frac{2}{x^2-2x-2} \le 1$

$$\frac{1+x^2-2x-2}{x^2-2x-2} \ge 0 \Rightarrow \frac{(x-1)^2-2}{(x-1)^2-3} \ge 0$$

$$\Rightarrow \frac{\left(x-1-\sqrt{2}\right)\left(x-1+\sqrt{2}\right)}{\left(x-1-\sqrt{3}\right)\left(x-1+\sqrt{3}\right)} \ge 0$$

$$x \in \left(-\infty, 1-\sqrt{3}\right) \cup \left[1-\sqrt{2}, 1+\sqrt{2}\right] \cup \left(1+\sqrt{3}, 0\right) \quad \ldots(1)$$

$$1 - \frac{1}{x^2-2x-2} \ge 0 \Rightarrow \frac{x^2-2x-3}{x^2-2x-2} \ge 0$$

$$\Rightarrow \frac{(x+1)(x-3)}{\left(x-1+\sqrt{3}\right)\left(x-1-\sqrt{3}\right)} \ge 0$$

$$x \in \left(-\infty, -1\right] \cup \left(1-\sqrt{3}, \sqrt{3}+1\right) \cup \left[3, \infty\right) \quad \ldots(2)$$

$(1) \cap (2)$

$$\Rightarrow x \in \left(-\infty, -1\right] \cup \left[1-\sqrt{2}, 1+\sqrt{2}\right] \cup \left[3, \infty\right)$$

$\therefore \alpha + \beta + \gamma + \delta = 4$

14. Let $X = \{x \in \mathbb{N} : 1 \le x \le 19\}$ and for some $a, b \in \mathbb{R}$, $Y = \{ax + b : x \in X\}$. If the mean and variance of the elements of Y are 30 and 750, respectively, then the sum of all possible values of b is

(1) 20 (2) 80

(3) 100 (4) 60

Ans. (4)

Sol. $\Sigma y_i = a\Sigma x_i + \Sigma b$

$= a \times (1 + 2 + \ldots.. + 19) + 19b$

$$\frac{\Sigma y_i}{19} = \frac{a \times 19 \times 20}{2 \times 19} + b$$

$30 = 10a + b \quad \ldots\ldots(1)$

$$\text{Variance of } X = \frac{\Sigma x_i^2}{19} - \left(\frac{\Sigma x_i}{19}\right)^2$$

$$= \frac{19 \times 20 \times 39}{19 \times 6} - (10)^2 = 30$$

Variance of Y = a^2 (variance of X)

$750 = a^2 \times 30$

$a^2 = 25 \Rightarrow a = \pm 5$

if $a = +5 \Rightarrow b = 30 - 50 = -20$ ...from (i)

if $a = -5 \Rightarrow b = 30 + 50 = 80$ ...from (i)

sum of values of b = 80 – 20 = 60

options (4)

15. Consider the following three statements for the function $f : (0, \infty) \to \mathbb{R}$ defined by

$f(x) = |\log_e x| - |x - 1|$:

(I) f is differentiable at all x > 0.

(II) f is increasing in (0, 1).

(III) f is decreasing in $(1, \infty)$.

Then.

(1) All (I), (II) and (III) are TRUE.

(2) Only (I) is TRUE.

(3) Only (II) and (III) are TRUE.

(4) Only (I) and (III) are TRUE.

Ans. **(4)**

Sol. $f(x) = |\ell nx| - |x - 1|$

$$= \begin{cases} \ell nx - (x-1) & x \geq 1 \\ -\ell nx + (x-1) & 0 < x < 1 \end{cases}$$

$$= \begin{cases} \ell nx - x + 1 & x \geq 1 \\ -\ell nx + x - 1 & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x} - 1 & x \geq 1 \\ -\frac{1}{x} + 1 & 0 < x < 1 \end{cases}$$

$f'(1^+) = f'(1^-) = 0 \Rightarrow f(x)$ is differentiable $\forall\ x > 0$

$f'(x) < 0 \quad \forall x > 1$

$f'(x) < 0 \quad \forall 0 < x < 1$

$\Rightarrow f(x)$ is decreasing $\forall\ x \in (0, \infty)$

Option (4)

16. Let y = y (x) be a differentiable function in the interval $(0, \infty)$ such that y(1) = 2.

and $\lim_{t \to x} \left(\frac{t^2 y(x) - x^2 y(t)}{x - t} \right) = 3$ for each x > 0.

Then 2y(2) is equal to

(1) 18

(2) 23

(3) 27

(4) 12

Ans. **(2)**

Sol. $\lim_{t \to x} \frac{2t\, f(x) - x^2 f'(t)}{-1} = 3$

$x^2 f'(x) - 2x\, f(x) = 3$

$$\frac{dy}{dx} - \frac{2y}{x} = \frac{3}{x^2}$$

$$\text{I.F.} = e^{-\int \frac{2}{x} dx} = e^{-2\log_e x} = 1/x^2$$

$$y \cdot \frac{1}{x^2} = \int \frac{3}{x^4} dx$$

$$\frac{y}{x^2} = -\frac{1}{x^3} + c \Rightarrow y = cx^2 - \frac{1}{x} = f(x)$$

$f(1) = 2 = c - 1 \Rightarrow c = 3$

$$f(x) = 3x^2 - \frac{1}{x}$$

$$f(2) = 12 - \frac{1}{2} \Rightarrow 2f(2) = 23$$

17. Let f be a function such that $3f(x) + 2f\left(\frac{m}{19x}\right) = 5x$, $x \neq 0$, where $m = \sum_{i=1}^{9} (i)^2$. Then f(5) – f(2) is equal to

(1) –9 (2) 36

(3) 18 (4) 9

Ans. **(3)**

Sol. $m = \frac{9 \times 10 \times 19}{6} = 15 \times 19$

$3f(x) + 2f\left(\frac{15}{x}\right) = 5x$

Replace x by $\frac{15}{x}$

$3f\left(\frac{15}{x}\right) + 2f(x) = \frac{75}{x}$

$9f(x) - 4f(x) = 15x - \frac{150}{x}$

$5f(x) = 15x - \frac{150}{x}$

$f(x) = 3x - \frac{30}{x}$

$f(5) = 15 - \frac{30}{5} = 9$

$f(2) = 6 - 15 = -9$

$f(5) - f(2) = 18$

18. The smallest positive integral value of a, for which all the roots of $x^4 - ax^2 + 9 = 0$ are real and distinct, is equal to

(1) 9 (2) 3

(3) 4 (4) 7

Ans. (4)

Sol. $x^4 - ax^2 + 9 = 0$ ...(1)

let $x^2 = t$

$t^2 - at + 9 =$ ...(2)

for roots of equation (1) to be real & distinct roots of equation (2) must be positive & distinct.

(i) $D > 0 \Rightarrow a^2 - 36 > 0 \Rightarrow a \in (-\infty, -6) \cup (6, \infty)$

(ii) $\frac{-b}{2a} > 0 \Rightarrow \frac{a}{2} > 0 \Rightarrow a > 0$

(iii) $f(0) > 0 \Rightarrow 9 > 0 \Rightarrow a \in R$

By (i) $\cap$ (ii) $\cap$ (iii)

$\therefore a \in (6, \infty)$

$\therefore$ least integral value of a is 7

19. Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5k$ and $\vec{b} = \hat{i} - \hat{j} + 3k$. If $\vec{c}$ is a vector such that $2\left(\vec{a} \times \vec{c}\right) + 3\left(\vec{b} \times \vec{c}\right) = \vec{0}$ and $\left(\vec{a} - \vec{b}\right).\vec{c} = -97$, then $\left|\vec{c} \times k\right|^2$ is equal to

(1) 193 (2) 233

(3) 218 (4) 205

Ans. (3)

Sol. $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = 0$

$\Rightarrow (2\vec{a} + 3\vec{d}) \times \vec{c} = 0 \Rightarrow \vec{c} = \lambda(2\vec{a} + 3\vec{d})$

$\Rightarrow \vec{c} = \lambda\,(7i - 13j + 19k)$

Now $(\vec{a} - \vec{b}) \cdot \vec{c} - = \lambda\,(7 + 52 + 38) + 97\lambda = -97$

$\Rightarrow \lambda = -1$

Now $\vec{c} = -7i + 13j - 19k$

$\Rightarrow \vec{c} \times \vec{k} - = 7j + 13i \Rightarrow |\vec{c} \times \hat{k}|^2 = 7^2 + 13^2 = 218$

20. Let [t] denote the greatest integer less than or equal to t. If the function

$$f(x) = \begin{cases} b^2 \sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x + \sin x)\cos x\right]\right) & , x < 0 \\ \frac{\sin x - \frac{1}{2}\sin 2x}{x^3} & , x > 0 \\ a & , x = 0 \end{cases}$$

is continuous at x = 0, then $a^2 + b^2$ is equal to

(1) $\frac{5}{8}$ (2) $\frac{9}{16}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. (3)

Sol. $f(0) = a$

$$\text{RHL} = \lim_{x \to 0^+} \frac{\sin x(1 - \cos x)}{x^3} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \to 0^-} \left(b^2 \sin\frac{\pi}{2}\left[\frac{\pi}{2}(\sin x + \cos x)\cos x\right]\right) = b^2$$

$\therefore\ a = \frac{1}{2}$ & $b^2 = \frac{1}{2}$

so $(a^2 + b^2) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

SECTION-B

21. If f(x) satisfies the relation $f(x) = e^x + \int_0^1 \left(y + xe^x\right)$ f(y) dy, then e + f(0) is equal to ________.

Ans. (2)

Sol. $f(x) = e^x + \int_0^1 yf(y)dy + xe^x \int_0^1 f(y)dy$

$f(x) = e^x + A + Bxe^x$

$A = \int_0^1 yf(y)\,dy = \int_0^1 y(A + e^y + By\,e^y)\,dy$

$A = \frac{A}{2} + (0 - (-1)) + B(e-1)$

$\frac{A}{2} + B(1-e) = 1$

$B = \int_0^1 f(y)\,dy$

$B = \int_0^1 (e^y + A + By\,e^y)dy$

$B = (e - 1) + A + B(0 - (-1))$

$B = e - 1 + A + B \Rightarrow A = 1 - e$

$f(x) = e^x + A + Bxe^x$

$f(0) = 1 + A = 1 - e + 1 = 2 - e$

$e + f(0) = 2$

22. Let (h, k) lie on the circle C : $x^2 + y^2 = 4$ and the point (2h + 1, 3k + 2) lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to ____________.

Ans. (9)

Sol. Let P ≡ (2cos θ, 2sin θ)

$\therefore$ coordinates of Q = (4cosθ + 1, 6sinθ + 3)

$\therefore$ locus of Q is $\left(\frac{x-1}{4}\right)^2 + \left(\frac{y-3}{6}\right)^2 = 1$

$\therefore\ e^2 = 1 - \frac{16}{36} = \frac{5}{9}$

$\therefore\ \boxed{\frac{5}{e^2} = 9}$

23. Let z = (1 + i) (1 + 2i) (1 + 3i).... (1 + ni), where i $= \sqrt{-1}$. If $|z|^2 = 44200$, then n is equal to –

Ans. (5)

Sol. $|z|^2 = 2^3.5^2.13.17$

$\prod_{r=1}^{n}(1+r^2) = 2^3.5^2.13.17 = (2).(5).(2.5).(17).(2.13) = 2.5.10.17.26$

so n = 5

24. Let S be a set of 5 elements and P(S) denote the power set of S. Let E be an event of choosing an ordered pair (A, B) from the set P(S) × P(S) such that $A \cap B = \varnothing$. If the probability of the event E is $\frac{3^p}{2^q}$, where p, q ∈ ℕ, then p + q is equal to

Ans. (15)

Sol. S = {a, b, c, d, e}

P(S) contains 32 elements

both set A and set B are subsets of P(S)

Every element has 4 choices

A	B
✓	✓
✓	x
x	✓
x	x

Favourable cases = 3^5

Total cases = 4^5

$P = \frac{3^5}{4^5} = \frac{3^5}{2^{10}}$

m = 5, n = 10

m + n = 15

25. The number of elements in the set $\left\{x \in [0, 180°] : \tan(x + 100°) = \tan(x + 50°)\tan x \tan(x - 50°)\right\}$ is______.

Ans. (4)

Sol. $\frac{\tan(x+100°)}{\tan x} = \tan(x+50°)\tan(x-50°)$

$\frac{\sin(x+100°)\cos x}{\cos(x+100°)\sin x} = \frac{\sin(x+50°)\sin(x-50°)}{\cos(x+50°)\cos(x-50°)}$

Apply C & D

$\frac{\sin(2x+100°)}{\sin 100°} = \frac{\cos 100°}{-\cos 2x}$

2sin(2x + 100°) cos2x + sin200° = 0

sin(4x+100°) + sin 100° + sin200° = 0

sin(4x + 100°) = –2sin150° cos50°

sin(4x + 100°) = – cos50° = sin(–40°)

$\therefore$ 4x + 100° = nπ + $(-1)^n$ · (–40°)

$x = \frac{n\pi + (-1)^{n+1}(40°) - 100°}{4}$

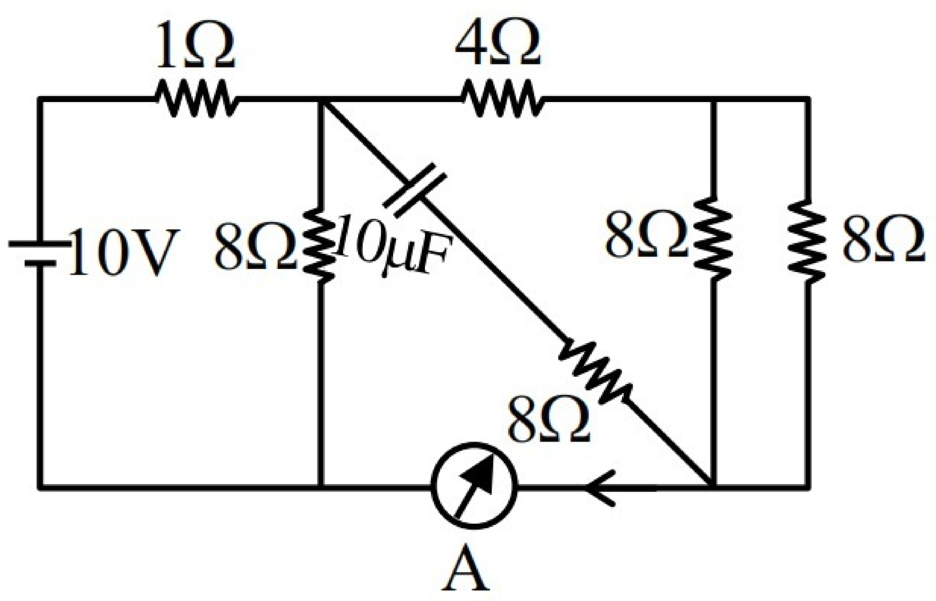
$\therefore$ x = 30°, 55°, 120°, 145° in (0, π)

$\therefore$ no. of solutions = 4

PHYSICS

SECTION-A

26. The reading of the ammeter (A) in steady state in the following circuit (assuming negligible internal resistance of the ammeter) is ________A.

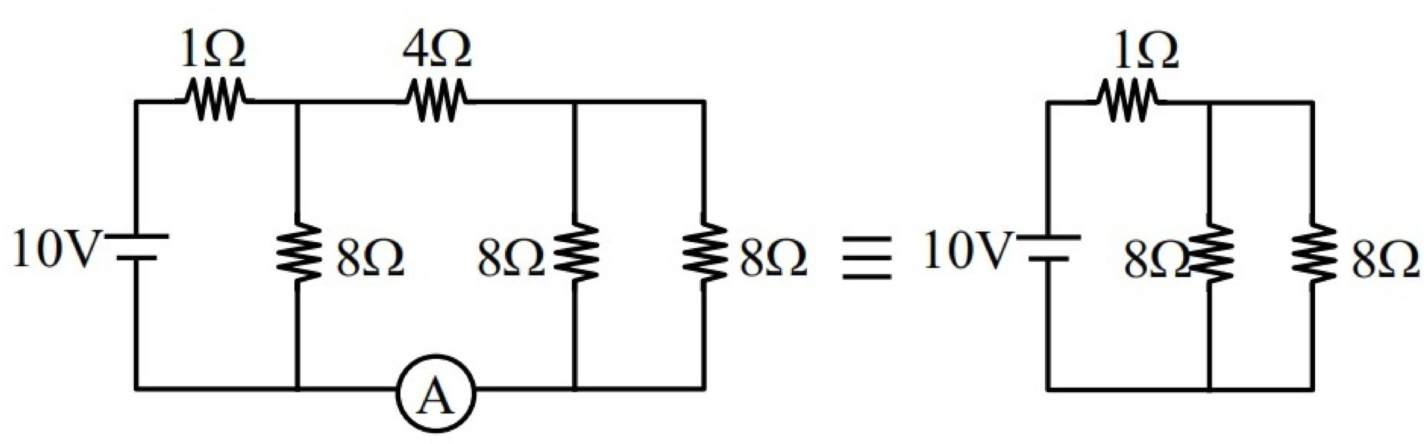


(1) 2 (2) 1

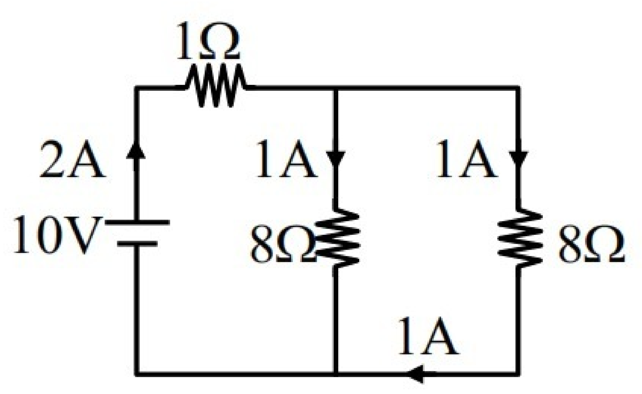
(3) 1/2 (4) 0

Ans. (2)

Sol. In steady state



I = 2A



Ammeter reading is 1A.

27. In a vernier callipers, 50 vernier scale divisions are equal to 48 main scale divisions. If one main scale division = 0.05 mm, then the least count of the vernier callipers is _____mm.

(1) 0.002 (2) 0.05

(3) 0.02 (4) 0.005

Ans. (1)

Sol. $LC = 1\text{ MSD} - 1\text{ MSD} = 1\text{ MSD} - \frac{48}{50}\text{MSD}$

$$= \frac{2}{50}\text{MSD} = \frac{2}{50} \times .05\text{mm} = 0.002\text{mm}$$

28. Five persons P_1, P_2, P_3, P_4 and P_5 recorded object distance (u) and image distance (v) using same convex lens having power +5D as (25,96), (30,62), (35,37), (45,35) and (50,32) respectively. Identify correct statement

(1) Readings recorded by all persons are correct

(2) Reading recorded by P_3 persons are incorrect

(3)Reading recorded by P_3 and P_2 persons are incorrect

(4) Reading recorded by P_4 and P_5 persons are incorrect

Ans. (2)

Sol. P = +5D

$$\frac{1}{f} = 5 \Rightarrow f = 20\text{ cm}$$

⇒ If object is between f & 2f image will be beyond 2f & magnified.

⇒ If object is beyond 2f, image will be between f & 2f & diminished.

Hence reading of P_3 are incorrect.

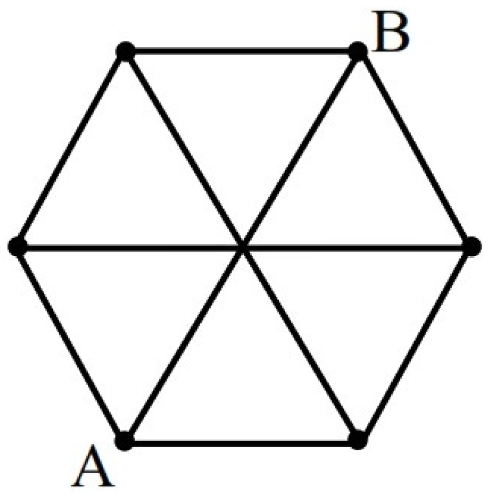
29. A regular hexagon is formed by six wires each of resistance r Ω and the corners are joined to centre by wires of same resistance. If the current enters at one corner and leaves at the opposite corner, the equivalent resistance of the hexagon between the two opposite corners will be

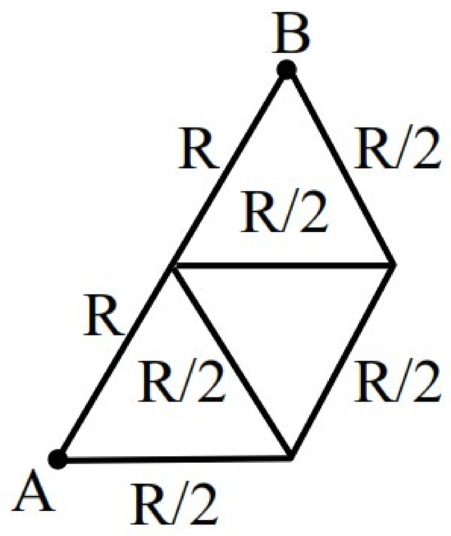
(1) $\frac{4}{5}r$ (2) $\frac{5}{8}r$

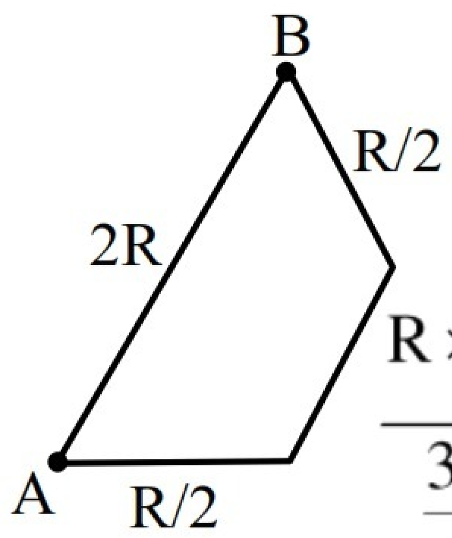
(3) $\frac{3}{4}r$ (4) $\frac{3}{5}r$

Ans. (1)

Sol.





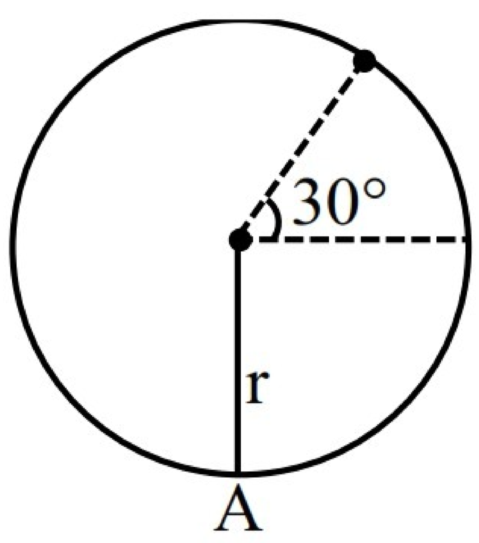


$$\frac{R\times\frac{R}{2}}{\frac{3R}{2}}=\frac{R}{3}$$

$$R_{eq}=\frac{2R\times\frac{4R}{3}}{2R+\frac{4R}{3}}=\frac{8R^2}{10R}=\frac{4}{5}R$$

30. In case of vertical circular motion of a particle by a thread of length r if the tension in the thread is zero at an angle 30° shown in figure, the velocity at the bottom point (A) of the circular path is

(g = gravitational acceleration)

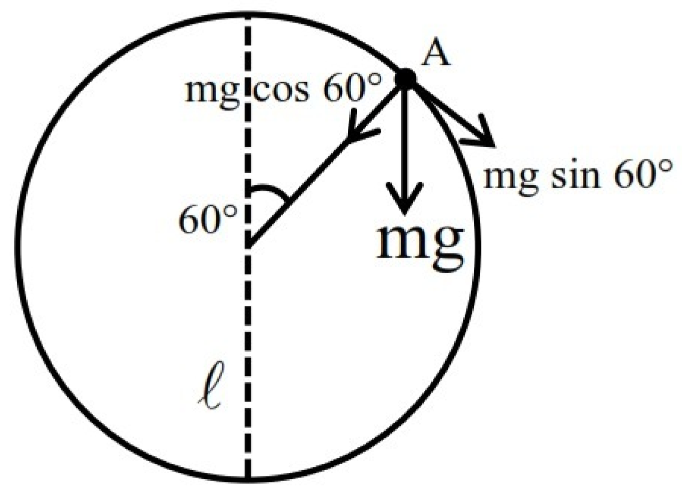


(1) $\sqrt{5gr}$ (2) $\sqrt{\frac{7}{2}gr}$

(3) $\sqrt{4gr}$ (4) $\sqrt{\frac{5}{2}gr}$

Ans. (2)

Sol.



$$T+mg\cos 60°=\frac{mV^2}{\ell}$$

$T=0$

$V^2=\frac{g\ell}{2}$ here V is the speed at point A

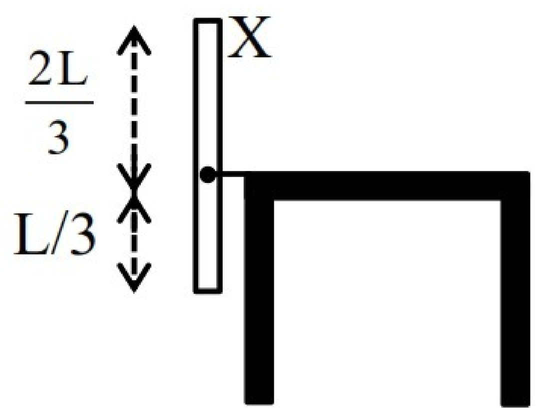
M.E.C.

$$\frac{1}{2}mu^2=mg(\ell+\ell\cos 60°)+\frac{1}{2}mV^2$$

$$u^2=3g\ell+\frac{g\ell}{2}$$

$$u=\sqrt{\frac{7g\ell}{2}}$$

31. A thin uniform rod (X) of mass M and length L is pivoted at a height $\left(\frac{L}{3}\right)$ as shown in the figure. The rod is allowed to fall from a vertical position and lie horizontally on the table. The angular velocity of this rod when it hits the table top, is_______. (g = gravitational acceleration)



(1) $\sqrt{\frac{3}{2}\frac{g}{L}}$ (2) $\frac{3}{\sqrt{2}}\sqrt{\frac{g}{L}}$

(3) $\frac{1}{\sqrt{2}}\sqrt{\frac{g}{L}}$ (4) $\sqrt{\frac{3g}{L}}$

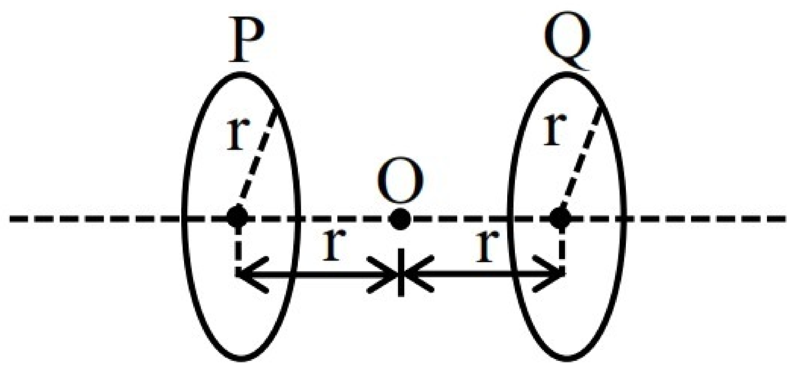
Ans. (4)

Sol. $mg\frac{\ell}{6}=\frac{1}{2}I\omega^2$

Here $I=\frac{m\ell^2}{12}+\frac{m\ell^2}{36}=\frac{m\ell^2}{9}$

$$mg\frac{\ell}{6}=\frac{m\ell^2}{18}\omega^2\Rightarrow\omega^2=\frac{3g}{\ell}$$

$$\omega=\sqrt{\frac{3g}{\ell}}$$

32. 

Two identical circular loops P and Q each of radius r are lying in parallel planes such that they have common axis. The current through P and Q are I and 4I respectively in clockwise direction as seen from O. The net magnetic field at O is:

(1) $\frac{3\mu_o I}{4\sqrt{2}r}$ toward P

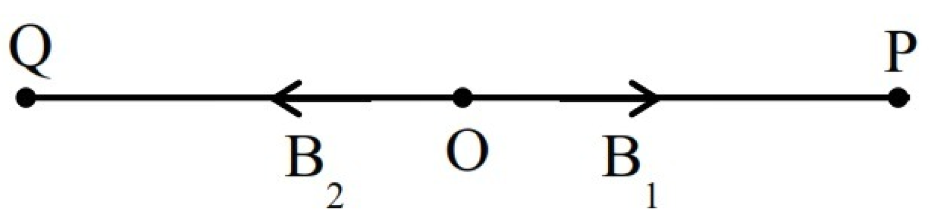
(2) $\frac{\mu_o I}{4\sqrt{2}r}$ toward P

(3) $\frac{\mu_o I}{4\sqrt{2}r}$ towards Q

(4) $\frac{3\mu_o I}{4\sqrt{2}r}$ towards Q

Ans. (4)

Sol. $B_{net} = B_1 - B_2$



$$= \frac{4\mu_0 iR^2}{2(R^2+R^2)^{3/2}} - \frac{\mu_0 iR^2}{2(R^2+R^2)^{3/2}}$$

$$= \frac{3\mu_0 i}{4\sqrt{2}R}$$

33. When a light of a given wavelength falls on a metallic surface the stopping potential for photoelectrons is 3.2 V. If a second light having wavelength twice of first light is used, the stopping potential drops to 0.7 V. The wavelength of first light is_________m.

($h = 6.63 \times 10^{-34}$ J.s, $e = 1.6 \times 10^{-19}$ C, $c = 3 \times 10^8$ m/s)

(1) 2.9×10^{-8} (2) 2.2×10^{-8}

(3) 3.1×10^{-7} (4) 2.5×10^{-7}

Ans. (4)

Sol. $q.(3.2) = \frac{hc}{\lambda} - \phi$ ...(1)

$q(0.7) = \frac{hc}{2\lambda} - \phi$ ...(2)

Eq. (1) – Eq. (2)

$q.(2.5) = \frac{hc}{2\lambda}$

$2.5 = \left(\frac{hc}{e}\right)\left(\frac{1}{2\lambda}\right)$

$2.5 = \frac{12400}{2(\lambda)}$

$\lambda = \frac{12400}{5}$ Å

$\lambda = 2480$ Å

$\lambda = 2.48 \times 10^{-7}$ m

34. The fifth harmonic of a closed organ pipe is found to be in unison with the first harmonic of an open pipe. The ratio of lengths of closed pipe to that of the open pipe is 5/x. The value of x is______.

(1) 4 (2) 2 (3) 1 (4) 3

Ans. (2)

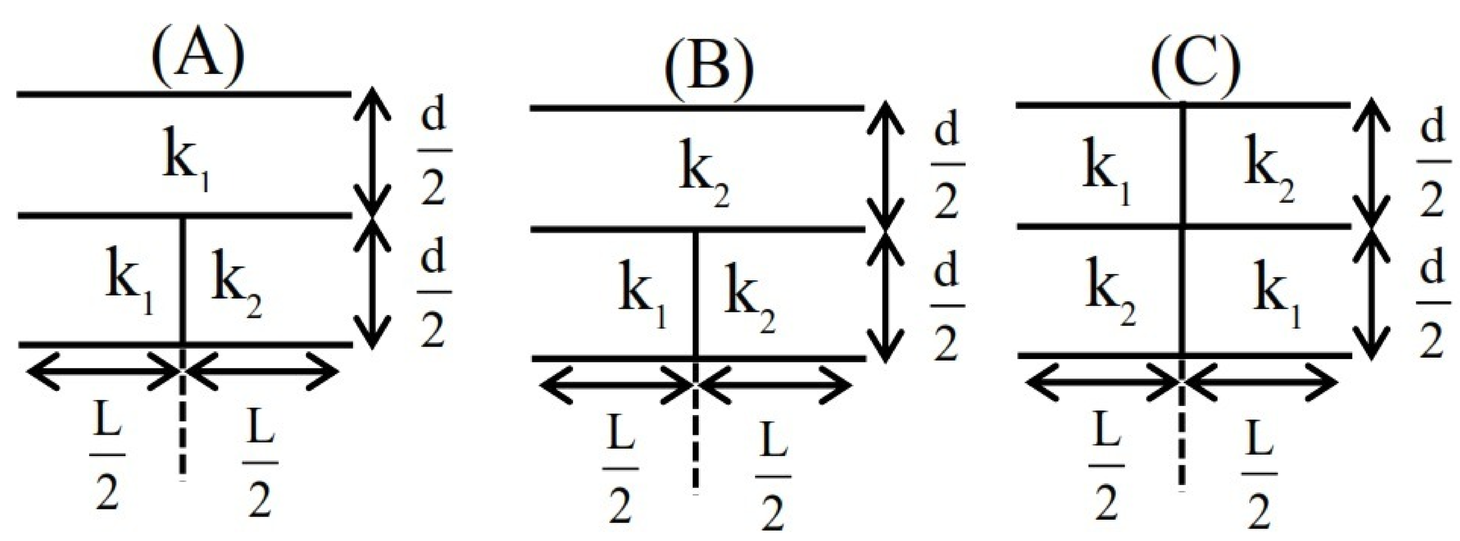
Sol. $f_{5\,closed} = f_{1\,open}$

$$\frac{5v}{4L_{closed}} = \frac{v}{2L_{open}}$$

$$\frac{L_{closed}}{L_{open}} = \frac{5}{2}$$

x = 2

35. Three parallel plate capacitors each with area A and separation d are filled with two dielectric (k_1 and k_2) in the following fashion. Which of the following is true? ($k_1 > k_2$)

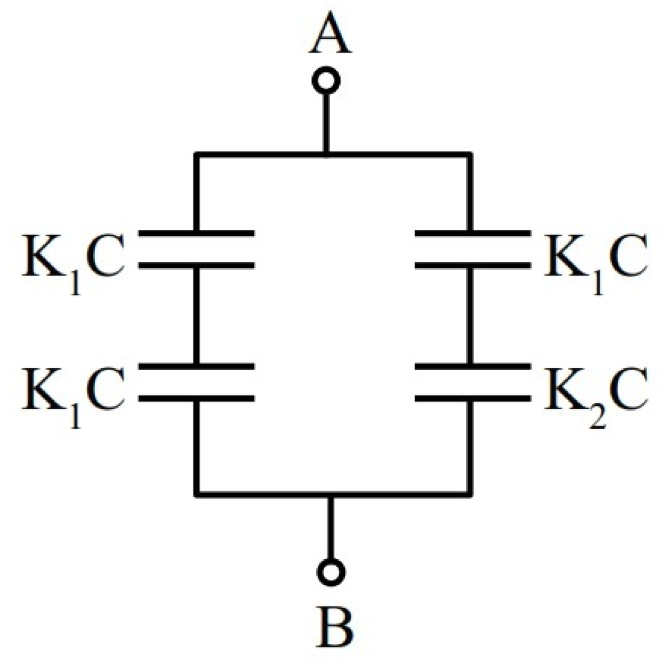


(1) $C_B > C_C > C_A$ (2) $C_C > C_B > C_A$

(3) $C_C > C_A > C_B$ (4) $C_A > C_C > C_B$

Ans. (4)

Sol. For C_A :

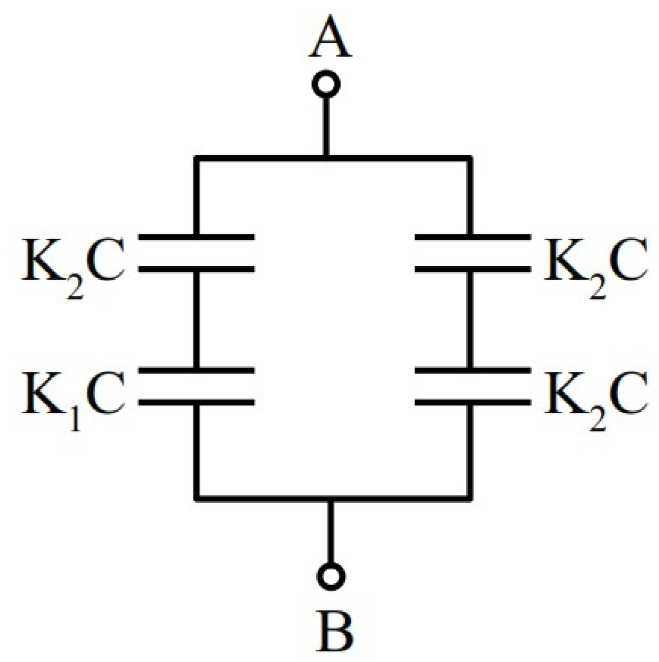


Let $\frac{\varepsilon_0 A}{d} = C$

$$\therefore \; C_A = \frac{K_1C}{2} + \frac{K_1K_2C}{K_1 + K_2}$$

$$= K_1C\left[\frac{K_1 + 2K_2}{2(K_1 + K_2)}\right]$$

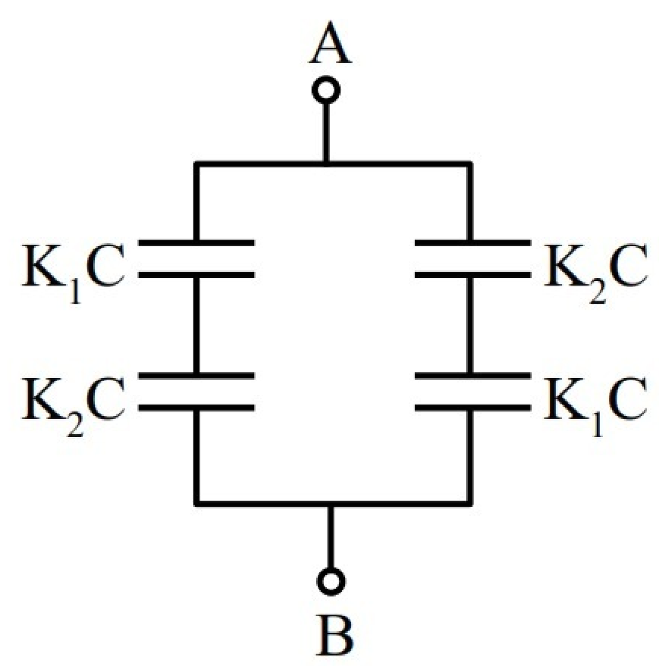
For C_B :



$$C_B = \frac{K_2C}{2} + \frac{K_1K_2C}{K_1 + K_2}$$

$$= K_2C\left[\frac{K_1 + 2K_2}{2(K_1 + K_2)}\right]$$

For C_C :



$$C_C = \frac{2K_1K_2C}{(K_1 + K_2)}$$

$C_A > C_C > C_B$

36. The binding energy for the following nuclear reactions are expressed in MeV.

${}_2He^3 + {}_0n^1 \rightarrow {}_2He^4 + 20$ MeV

${}_2He^4 + {}_0n^1 \rightarrow {}_2He^5 - 0.9$ MeV

If X_3, X_4, X_5 denote the stability of ${}_2He^3$, ${}_2He^4$ and ${}_2He^5$, respectively, then the correct order is :

(1) $X_4 > X_5 > X_3$

(2) $X_4 = X_5 = X_3$

(3) $X_4 > X_5 < X_3$

(4) $X_4 < X_5 < X_3$

Ans. (1)

Sol. $BE_{He^4} - BE_{He^3} = 20\text{MeV}$ ...(1)

$BE_{He^5} - BE_{He^4} = -0.9\text{MeV}$ ...(2)

From eq (1) & (2)

$BE_{He^4} > BE_{He^5} > BE_{He^3}$

$X_4 > X_5 > X_3$

37. A cubical block of density $\rho_b = 600$ kg/m^3 floats in a liquid of density $\rho_e = 900$ kg/m^3. If the height of block is $H = 8.0$ cm then height of the submerged part is _____ cm.

(1) 7.3 (2) 4.3

(3) 6.3 (4) 5.3

Ans. (4)

Sol. $Mg = F_b$

$dAHg = \rho Ahg$

600×8 cm $= 900 \times h$

$h = \frac{16}{3}$ cm

$h = 5.3$ cm

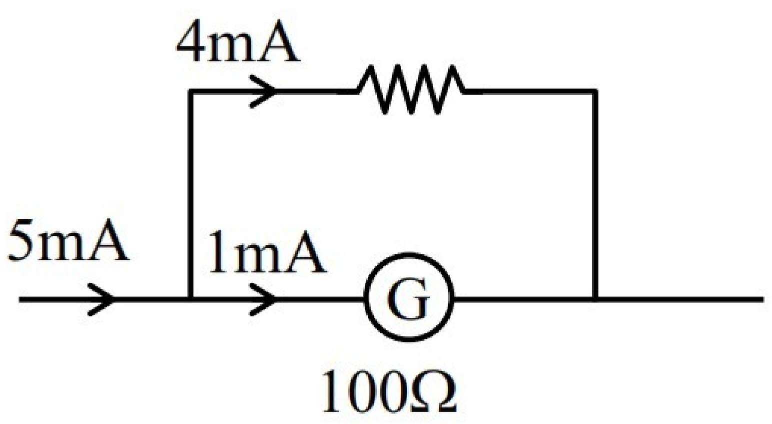
38. A moving coil galvanometer of resistance 100 Ω shows a full scale deflection for a current of 1 mA. The value of resistance required to convert this galvanometer into an ammeter, showing full scale deflection for a current of 5 mA, is _____ Ω

(1) 25 (2) 10

(3) 0.5 (4) 2.5

Ans. (1)

Sol.



$G = 100\ \Omega$

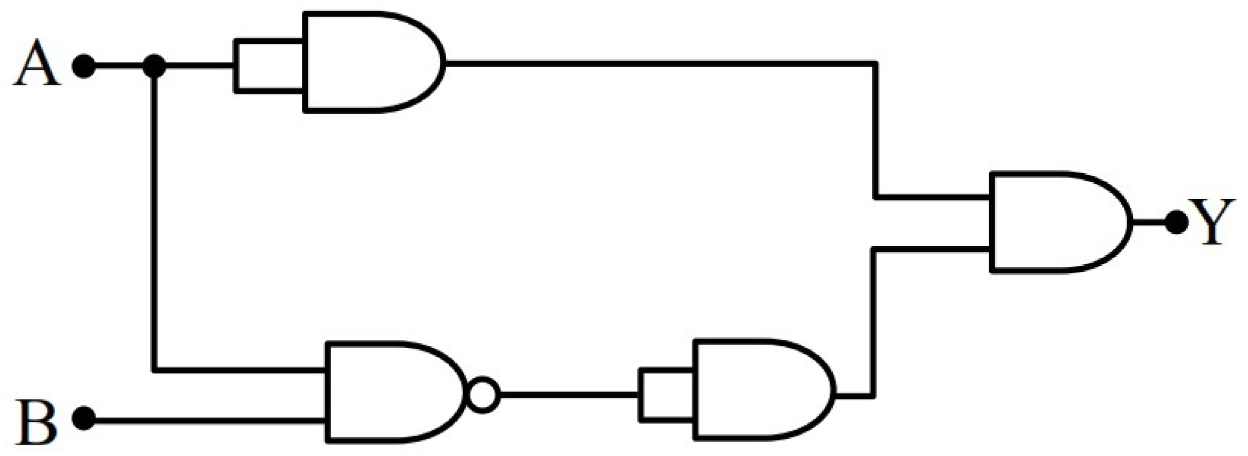
$i_g = 1mA$

$i = 5mA$

$$r_s = \frac{G}{\left(\frac{i}{i_g} - 1\right)}$$

$$= \frac{100}{\left(\frac{5}{1} - 1\right)} = 25\Omega$$

39. Identify the correct truth table of the given logical circuit.



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(3)

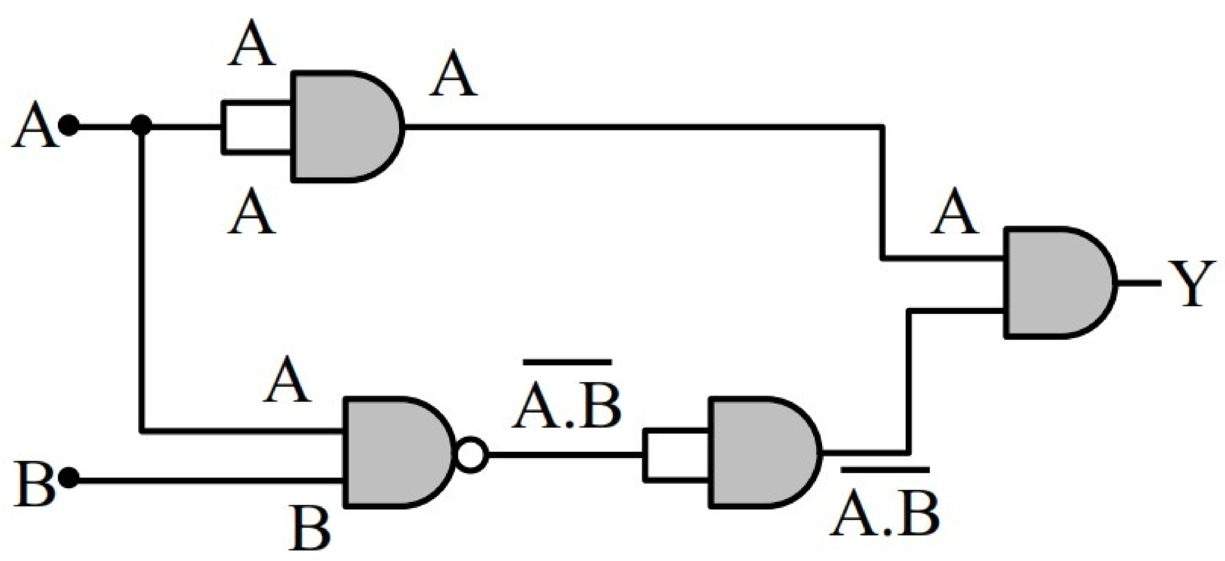
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

Ans. **(4)**

Sol.



$y = A.\ \overline{A.B}$

$= A.\ \left(\overline{A} + \overline{B}\right) = 0 + A\overline{B}$

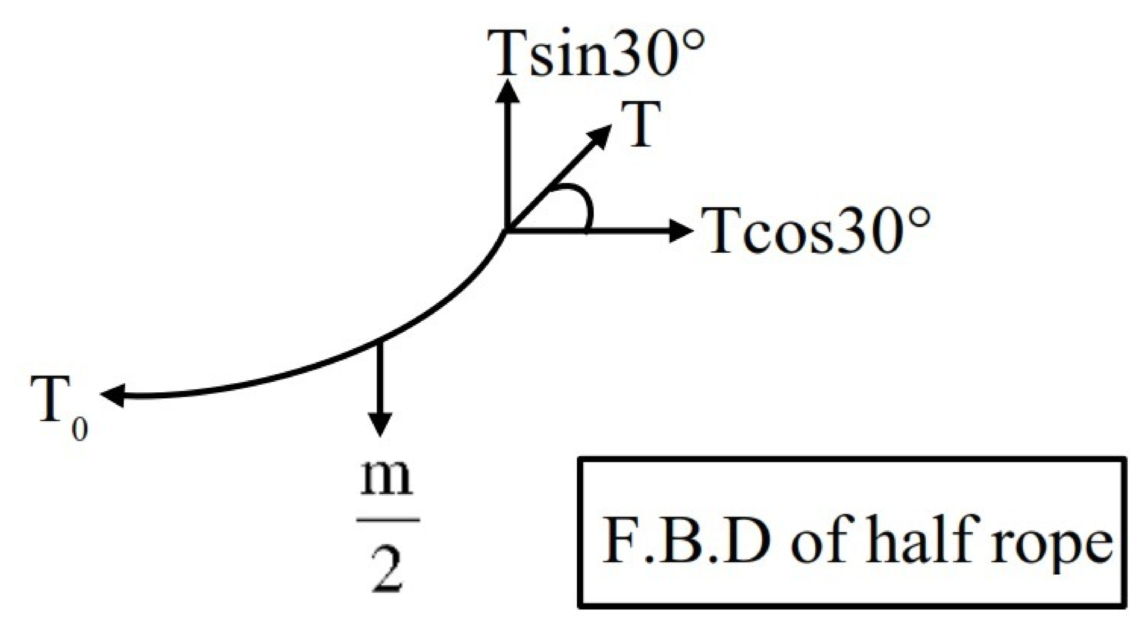
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

40. A flexible chain of mass *m* hangs between two fixed points at the same level. The inclination of the chain with the horizontal at the two points of support is 30°. Considering the equilibrium of each half of the chain, the tension of the chain at the lowest point is ______.

(1) $\frac{\sqrt{3}}{2}mg$ (2) $\frac{1}{2}mg$ (3) mg (4) $\sqrt{3}\,mg$

Ans. **(1)**

Sol.



$$T\sin30° = \frac{m}{2}g$$

$$T\cos30° = T_0$$

$$\tan30° = \frac{mg}{2T_0}$$

$$T_0 = \frac{\sqrt{3}}{2}mg$$

41. A point source is kept at the center of a spherically enclosed detector. If the volume of the detector increased by 8 times, the intensity will

(1) increase by 8 times (2) increase by 64 times

(3) decrease by 8 times (4) decrease by 4 times

Ans. **(4)**

Sol. $V \rightarrow 8V \Rightarrow R \rightarrow 2R$

$\Rightarrow A \rightarrow 4A$

$$\Rightarrow I \rightarrow \frac{I_0}{4}$$

42. In the Young's double slit experiment the intensity produced by each one of the individual slits is I_0. The distance between two slits is 2 mm. The distance of screen from slits is 10 m. The wavelength of light is 6000 Å. The intensity of light on the screen in front of one of the slits is ____.

(1) $2I_0$ (2) I_0 (3) $\frac{I_0}{2}$ (4) $4I_0$

Ans. (2)

Sol. d = 2mm

D = 10 m

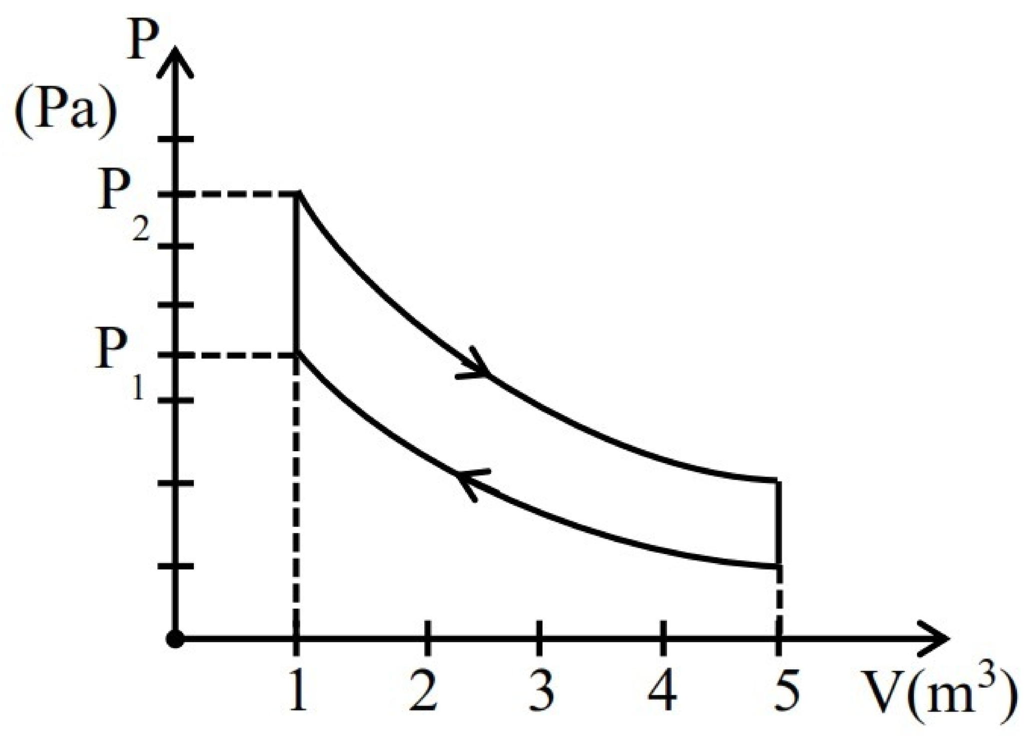
λ = 6000 Å

$y = \frac{d}{2}$ (in front of one slit)

$$I = 4I_0 \cos^2\left(\frac{2\pi}{\lambda}.\frac{y}{D}d\right)$$

$\Rightarrow I = I_0$

43. 10 mole of an ideal gas is undergoing the process shown in the figure. The heat involved in the process from P_1 to P_2 is α Joule (P_1 = 21.7 Pa and P_2 = 30 Pa, C_v = 21 J/K.mol, R = 8.3 J/mol.K). The value of α is ____.



(1) 24 (2) 15 (3) 21 (4) 28

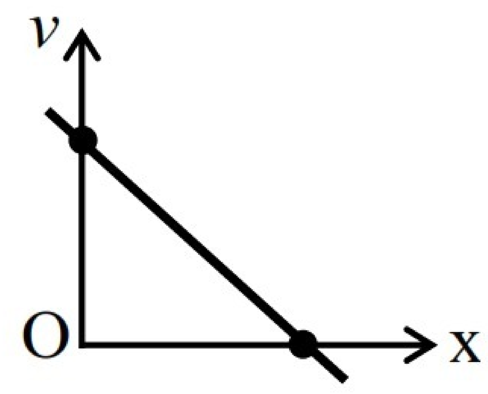
Ans. (3)

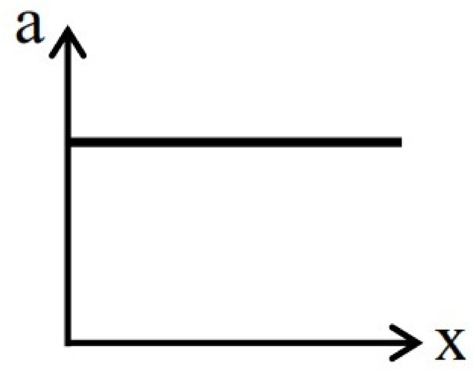
Sol. $\Delta Q = nC_v\Delta T$ (isochoric)

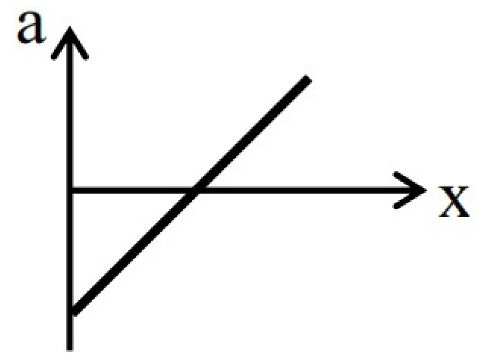
$$= \frac{C_v}{R}.nR\Delta T = \frac{C_v}{R}(P_2 - P_1)V$$

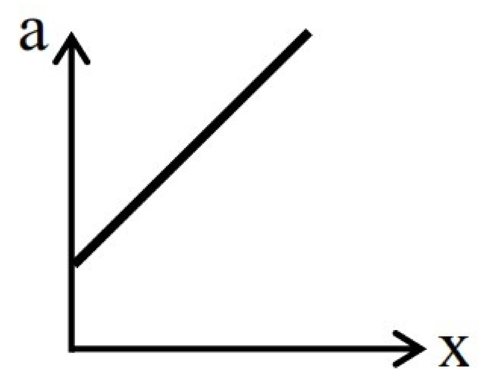
$$= \frac{21}{8.3}\times(30-21.7)\times 1 = 21J$$

44. The velocity (v) – Distance (x) graph is shown in figure. Which graph represents acceleration (a) versus distance (x) variation of this system?



(1)


(2)


(3)


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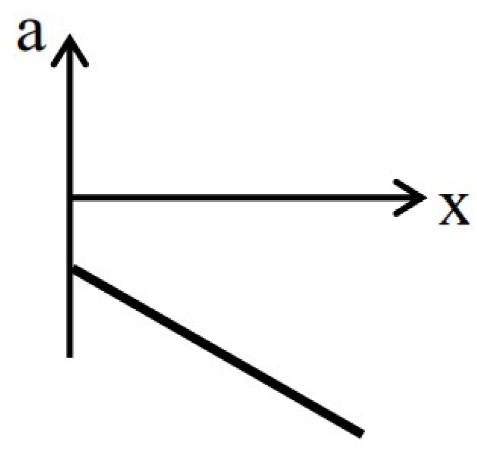
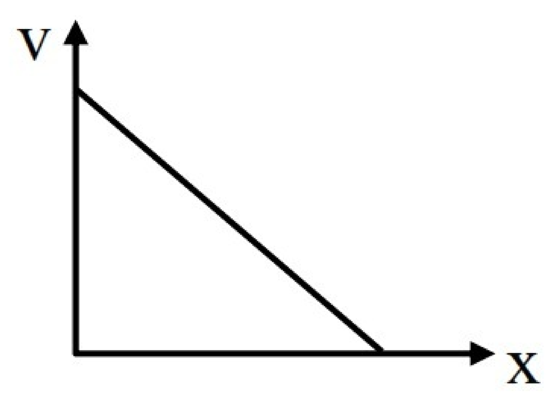


Ans. (2)

Sol.


Eq. of V vs x from graph

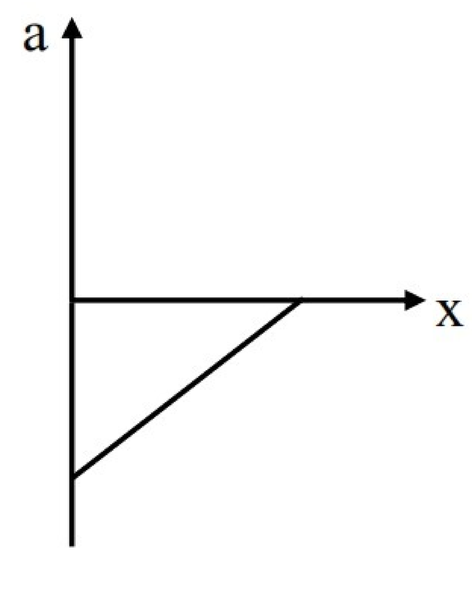
$V = C_1 - C_2x$

$$a = V\frac{dV}{dx}$$

$= (C_1 - C_2x) \times -C_2$

$a = C_2^2x - C_1C_2$

$\therefore$ graph is straight line +ve solve –ve intercept



45. Distance between an object and three times magnified real image is 40 cm. The focal length of the mirror used is _____ cm.

(1) – 15/2 (2) – 10

(3) – 20 (4) – 15

Ans. **(4)**

Sol. $m = -3 = \frac{v}{u}$

$v = -3u$

$|v| - |u| = 40$

$u = 20$ cm

$v = 60$ cm

$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{-60} + \frac{1}{-20} = \frac{1}{f}$

$f = -15$ cm

SECTION-B

46. When 300 J of heat given to an ideal gas with $C_p = \frac{7}{2}R$ its temperature raises from 20 °C to 50 °C keeping its volume constant. The mass of the gas is (approximately) ___ g. (R = 8.314 J/mol.K).

Ans. (481)

Sol. $C_V = C_P - R = \frac{5}{2}R$

$\Delta Q = nC_V \Delta T$

$300 = n \times \frac{5}{2} \times 8.314 \times 30$

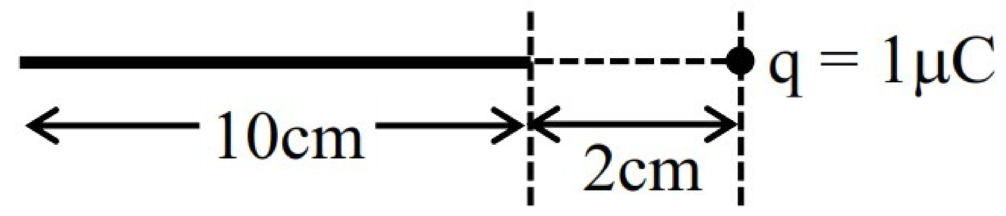
$n = 0.48$

$\frac{m}{M} = 0.48$

We cannot find mass (m) because molar mass (M) not given.

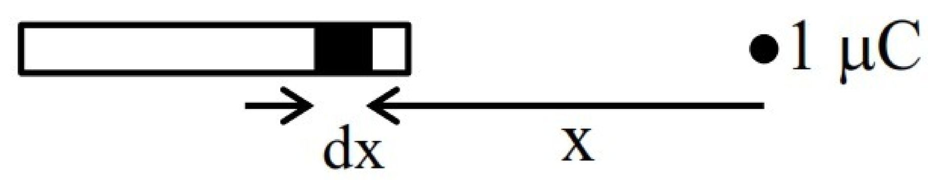
47. A point charge q = 1 μC is located at a distance 2 cm from one end of a thin insulating wire of length 10 cm having a charge *Q* = 24 μC, distributed uniformly along its length, as shown in figure. Force between q and wire is ____ N.

(Use $\frac{1}{4\pi\epsilon_0} = 9\times10^9 N.m^2/C^2$)



Ans. **(90)**

Sol.



$$F = \int dF = \int_{2cm}^{12cm} \frac{kq\lambda dx}{x^2} = kq\lambda\left(\frac{1}{2\times10^{-2}} - \frac{1}{12\times10^{-2}}\right)$$

$$F = (9\times10^9)(10^{-6})\left(\frac{24\times10^{-6}}{10^{-1}}\right)\left(\frac{5}{12}\right)\times10^2$$

$$= 9\times24\times\frac{5}{12} = 90N$$

48. In a meter bridge experiment to determine the value of unknown resistance, first the resistances 2 Ω and 3 Ω are connected in the left and right gaps of the bridge and the null point is obtained at a distance *l* cm from the left. Now when an unknown resistance x Ω is connected in parallel to 3 Ω resistance, the null point is shifted by 10 cm to the right of wire. The value of unknown resistance x is ___ Ω.

Ans. **(6)**

Sol. In case I

$\frac{2}{3} = \frac{\ell}{(100-\ell)}$ ......(1)

$\ell = 40$ cm

In case II

$\frac{2}{R} = \frac{\ell+10}{100-(\ell+10)}$

Put $\ell = 40$ cm & solve

$R = 2\Omega$

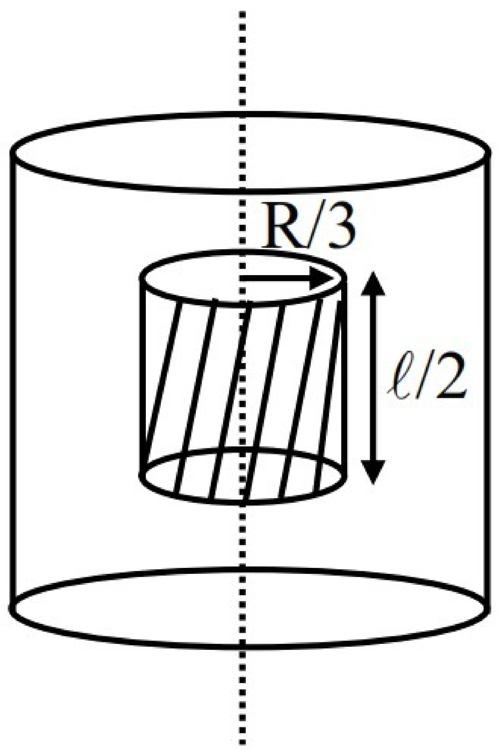
$\therefore \frac{3x}{3+x} = 2$

$x = 6\Omega$

49. A uniform solid cylinder of length L and radius R has moment of inertia about its axis equal to I_1. A small co-centric cylinder of length $L / 2$ and radius R/3 carved from this cylinder has moment of inertia about its axis equals to I_2. The ratio I_1 / I_2 is _____.

Ans. (162)

Sol.



Original mass (M)

The removed mass (m)

$$m = \rho \times \pi \left(\frac{R}{3}\right)^2 \times \frac{L}{2}$$

$$= \frac{\rho.\pi R^2 L}{18} = \frac{M}{18}$$

$$I' = \frac{1}{2}.\frac{M}{18}.\frac{R^2}{9} = \frac{1}{324} MR^2$$

$$\frac{I}{I'} = \frac{\frac{1}{2}MR^2}{\frac{1}{324}MR^2} = 162$$

50. A soap bubble of surface tension 0.04 N/m is blown to a diameter of 7 cm. If (15000 – x) μJ of work is done in blowing it further to make its diameter 14 cm, then the value of x is ____.

($\pi = 22/7$)

Ans. (11304)

Sol. $W = \Delta u$

$$= S \times \left(8\pi r_2^2 - 8\pi r_1^2\right)$$

$$= 0.04 \times 2 \times \frac{22}{7}(147) \times 10^{-4}$$

$$W = 3696 \times 10^{-6}\ J$$

$$3696 = 15000 - x$$

$$x = 11304\ \mu J$$

CHEMISTRY

SECTION-A

51. Choose the **INCORRECT** statement

(1) Among the isotopes of carbon, ^{13}C is a radioactive isotope.

(2) Carbon exhibits negative oxidation states along with +4 and +2.

(3) Carbon cannot exceed its covalency more than four.

(4) CO_2 is the most acidic oxide among the dioxides of group of 14 elements.

Ans. **(1)**

Sol. **C^{13}** is not radioactive

C^{14} is radioactive

52. The unsaturated ether on acidic hydrolysis produces carbonyl compounds as shown below:-

$$CH_3-CH=CH-O-CH=CH_2$$

$$\downarrow H_3O^{(+)}$$

$$CH_3-CH_2-CH=O \text{ and } O=CH-CH_3$$

Based on this, predict the solution / reagent that will help to distinguish "P" and "Q" obtained in the following reaction.

$$CH_3-CH=CH-O-\underset{}{\overset{CH_3}{\overset{|}{C}}}=CH_2 \xrightarrow[\Delta]{H_3O^{(+)}} P+Q$$

(1) Lucas reagent

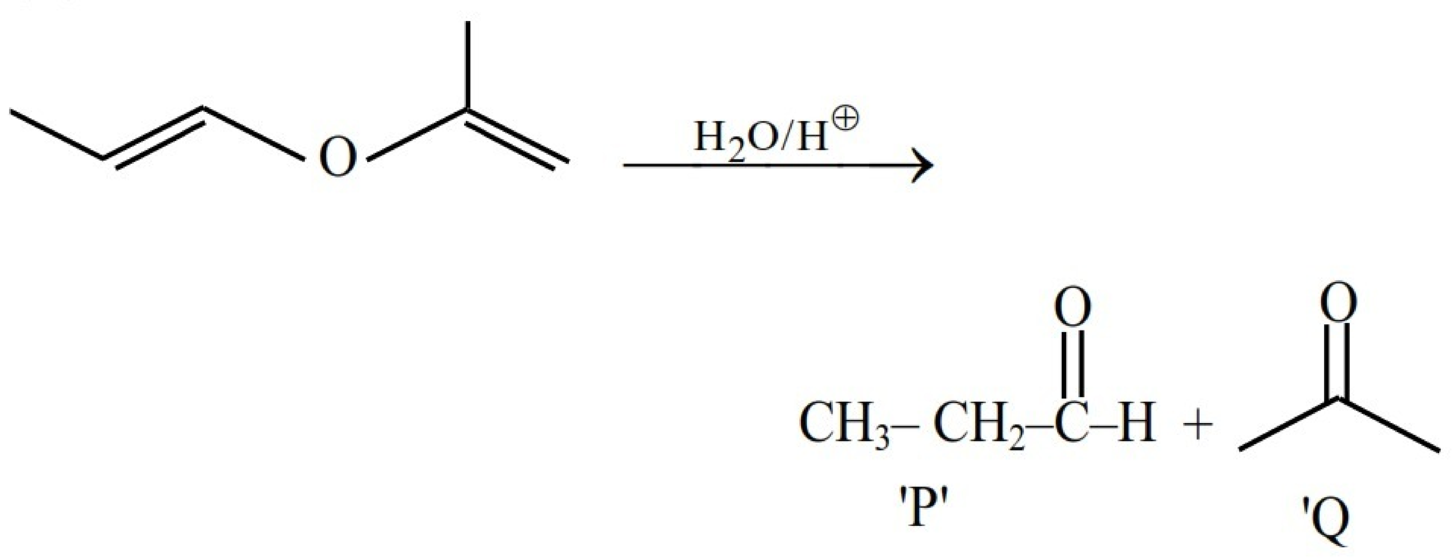
(2) 2,4-DNP reagent

(3) Saturated $NaHSO_3$ solution

(4) Fehling solution

Ans. **(4)**

Sol.



'P' and 'Q' can be differentiated by Fehling's test.

P gives positive Fehling test

Q gives negative Fehling test

53. The number of possible tripeptides formed involving alanine (ala), glycine (gly) and valine (val), where no amino acid has been used more than once is :

(1) 6 (2) 3

(3) 4 (4) 8

Ans. **(1)**

Sol. Gly ala val

Gly val ala

Val gly ala

Val ala gly

Ala val gly

Ala gly val

Total tri peptides = 6

54. Two liquids A and B form an ideal solution at temperature T K. At T K , the vapour pressures of pure A and B are 55 and 15 kNm^{-2} respectively. What is the mole fraction of A in solution of A and B in equilibrium with a vapour in which the mole fraction of A is 0.8 ?

(1) 0.5217 (2) 0.480

(3) 0.663 (4) 0.340

Ans. **(1)**

Sol. $$\frac{Y_A}{Y_B}=\frac{P_A^o}{P_B^0}\cdot\frac{X_A}{X_B}$$

$$\frac{0.8}{0.2}=\frac{55}{15}\times\frac{X_A}{X_B}$$

$$\frac{X_A}{X_B}=\frac{60}{55}=\frac{12}{11}$$

$$X_A=\frac{12}{23}=0.5217$$

55. Consider the following gaseous equilibrium in a closed container of volume "V" at T(K).

$P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$

2 moles each of $P_2(g)$, $Q_2(g)$ and PQ (g) are present at equilibrium. Now one mole each of 'P_2' and 'Q_2' are added to the equilibrium keeping the temperature at T(K). The number of moles of P_2, Q_2 and PQ at the new equilibrium, respectively , are -

(1) 2.67, 2.67, 2.67 (2) 1.21, 2.24, 1.56

(3) 1.66, 1.66, 1.66 (4) 2.56, 1.62, 2.24

Ans. (1)

Sol. $P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$

$t=t_{eq}$ 2 mole 2 mole 2 mole

$$K_{eq} = \frac{2^2}{2.2} = 1$$

Now 1 mole of each P_2 and Q_2 is added

So reaction will move in forward direction

$P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$

$t = t'_{eq.}$ $3 - x$ $3 - x$ $2 + 2x$

$$K_C = 1 = \frac{(2+2x)^2}{(3-x)(3-x)}$$

$$\frac{2+2x}{3-x} = 1$$

$2 + 2x = 3 - x$

$$x = \frac{1}{3}$$

At new equilibrium :

Moles of $P_2 = \frac{8}{3} = 2.67$

Moles of $Q_2 = \frac{8}{3} = 2.67$

Moles of PQ $= \frac{8}{3} = 2.67$

56. Pair of species among the following having same bond order as well as paramagnetic character will be-

(1) O_2^+, N_2^{2-} (2) O_2^-, N_2^+

(3) O_2^+, N_2^- (4) O_2^-, N_2^-

Ans. (3)

Sol.

Species	Bond order	Magnetic Nature
O_2^+	2.5	Paramagnetic
O_2^-	1.5	Paramagnetic
O_2^+	2.5	Paramagnetic
N_2^-	2.5	Paramagnetic
N_2^{2-}	2	Paramagnetic

57. The wavelength of light absorbed for the following complexes are in the order.

$\underset{(I)}{[Co(NH_3)_6]^{3+}}$; $\underset{(II)}{[Co(H_2O)_6]^{3+}}$; $\underset{(III)}{[Co(CN)_6]^{3-}}$;

$\underset{(IV)}{[Co(NH_3)_5(H_2O)]^{3+}}$; $\underset{(V)}{[CoF_6]^{3-}}$

(1) III < I < II < IV < V

(2) III < I < IV < V < II

(3) III < IV < I < II < V

(4) III < I < IV < II < V

Ans. (4)

Sol. Wavelength of light absorbed increases as C.F.S.E of complex decreases.

$[Co(CN)_6]^{3-}$ has maximum CFSE

$[CoF_6]^{3-}$ has least CFSE

Ligand field strength ↑ ; C.F.S.E ↑

Correct wavelength order.

V > II > IV > I > III

58. One mole of Cl_2 (g) was passed into 2 L of cold 2M KOH solution. After the reaction, the concentrations of Cl^-, ClO^- and OH^- are respectively (assume volume remains constant)

(1) 0.75 M , 0.75 M , 1 M

(2) 0.5 M , 0.5 M , 0.5 M

(3) 0.5 M , 0.5 M , 1 M

(4) 1 M , 1M , 1 M

Ans. (3)

Sol. $Cl_2 + 2KOH \rightarrow KCl + KClO + H_2O$

$t = 0$ 1 mole 4 mole

t_f 0 2 mole 1 mole 1 mole

$[OH^-] = 1$ M

$[Cl^-] = \frac{1}{2}$ M

$[ClO^-] = \frac{1}{2}$ M

59. A student has planned to prepare acetanilide from aniline using acetic anhydride. The student has started from 9.3g of aniline. However, the student has managed to obtain 11 g of dry acetanilide.

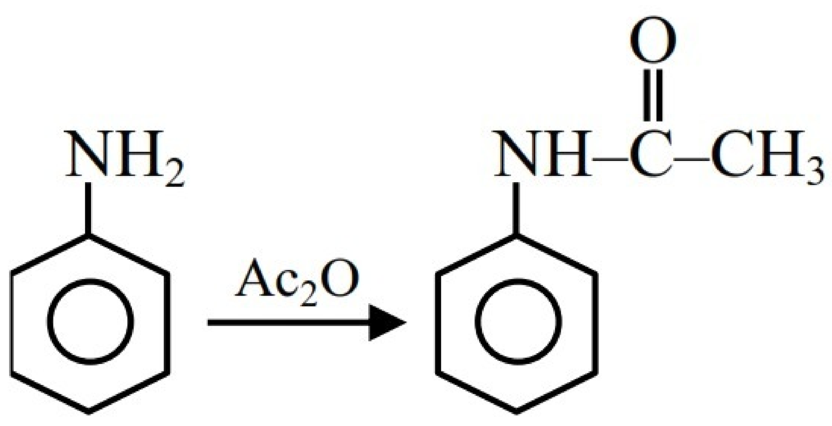
The % yield of this reaction is :-

(1) 81.5% (2) 97.5%

(3) 59.5% (4) 72.5%

Ans. **(1)**

Sol.



9.3 gm 11 gm

MW = 93 MW = 135

$n = \frac{9.3}{93} = 0.1$ $n = \frac{11}{135} = 0.08148$

$\% \text{ yield} = \frac{0.08148}{0.1} \times 100 = 81.5\%$

60. Find out the statements which are **not** true.

A. Resonating structure with more number of covalent bonds and lesser charge separation are more stable.

B. In electromeric effect, an unsaturated system shows + E effect with nucleophile and –E effect with electrophile.

C. Inductive effect is responsible for high melting point, boiling point and dipole moment of polar compounds.

D. The greater the number of alkyl groups attached to the doubly bonded carbon atoms, higher is the heat of hydrogenation.

E. Stability of carbanion increases with the increase in s-character of the carbon carrying the negative charge.

Choose the ***correct*** answer from the options given below.

(1) A, D & E only (2) B, D & E only

(3) A, C & D only (4) B & D only

Ans. **(4)**

Sol. Statement B & D are not true

61. The correct order of C, N, O and F in terms of second ionisation potential is

(1) F < N < C < O (2) C < O < N < F

(3) C < N < F < O (4) C < F < N < O

Ans. **(2)**

Sol. To compare second ionization potential configuration of mono-cation is observed

C^+	N^+	O^+	F^+
[He] $2s^2sp^1$	[He] $2s^22p^2$	[He] $2s^22p^3$ Half-filled stable.	[He] $2s^22p^4$

2^{nd} IE order

O > F > N > C

62. In the Group analysis of cations , Ba^{2+} & Ca^{2+} are precipitated respectively as

(1) sulphide & sulphide

(2) hydroxide & carbonate

(3) carbonate & carbonate

(4) chromate & sulphide

Ans. **(3)**

Sol. To identify Ba^{2+} & Ca^{2+}

Reagent $(NH_4)_2CO_3$ + NH_4Cl is used $BaCO_3$ & $CaCO_3$ are obtained as precipitates

63. The wavelength of spectral line obtained in the spectrum of LI^{2+} ion, when the transition takes place between two levels whose sum is 4 and difference is 2, is

(1) 2.28×10^{-7} cm

(2) 2.28×10^{-6} cm

(3) 1.14×10^{-7} cm

(4) 1.14×10^{-6} cm

Ans. **(4)**

Sol. $n_1 \rightarrow$ lower energy level

$n_2 \rightarrow$ higher energy level

$n_1 + n_2 = 4, \quad n_2 = 3$

$n_2 - n_1 = 2, \quad n_1 = 1$

Rydberg's formula :

$$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = R_H (3)^2 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda} = 8R_H$$

$$\lambda = \frac{1}{8R_H}$$

$$\lambda = \frac{1}{8 \times 1.1 \times 10^5}$$

$$\lambda = \frac{1000}{8.8} \times 10^{-8} \text{ cm}$$

$$\lambda = 113.63 \times 10^{-8} \text{ cm}$$

$$\lambda \simeq 1.14 \times 10^{-6} \text{ cm}$$

64. Given below are two statements :

Statement I : Cross aldol condensation between two different aldehydes will always produce four different products.

Statement II : When semicarbazide reacts with a mixture of benzaldehyde and acetophenone under optimum pH, it forms a condensation product with acetophenone only.

In the light of the above statements, choose the ***correct*** answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Ans. (1)

Sol. **Statement I :** False

Cross aldol can give 2 or 4 products

Statement II : False

Benzaldehyde & Acetone both react with semi carbazide.

65. Given below are two statements :

Statement I : The dipole moment of R-CN is greater than R-NC and R-NC can undergo hydrolysis under acidic medium to produce $R\text{–}\overset{\displaystyle O}{\overset{\|}{C}}\text{–}OH$.

Statement II : R-CN hydrolyses under acidic medium to produce a compound which on treatment with $SOCl_2$, followed by the addition of NH_3 gives another compound(x). This compound (x) on treatment with NaOCl/NaOH gives a product, that on treatment with $CHCl_3$/KOH/Δ produces R-NC

In the light of the above statements, choose the ***correct*** answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. (4)

Sol. **Statement I :** False

$$R-NC \xrightarrow{H_3O^+} R-NH_2 + HCOOH$$

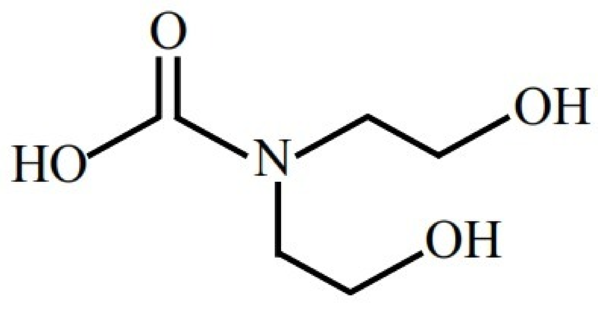
Statement II : True

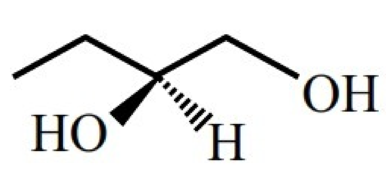
$$R\text{–}CN \xrightarrow{H_3O^+} RCOOH \xrightarrow{SOCl_2} RCOCl$$

$$\xrightarrow{NH_3} RCONH_2 \xrightarrow{NaOCl/OH^-} RNH_2$$

$$\xrightarrow{CHCl_3+KOH} R\text{–}NC$$

66. From the following, how many compounds contain at least one secondary alcohol ?


(I)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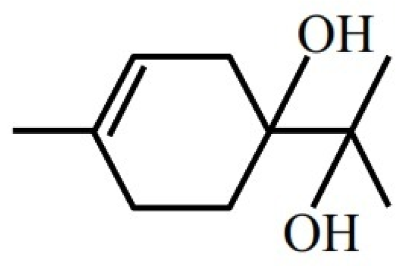
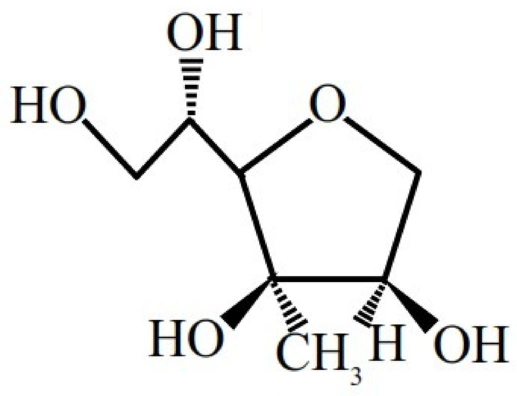
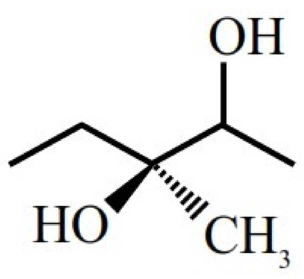
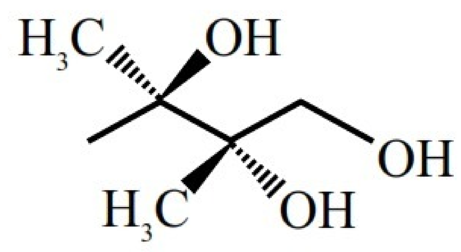

(III)


(IV)


(V)


(VI)

Choose the ***correct*** answer from the options given below :

(1) Five (2) Three

(3) Four (4) two

Ans. **(2)**

Sol. II, IV & V are secondary alcohol.

67. The heat of atomisation of methane and ethane are 'x' kJ mol^{-1} and 'y' kJ mol^{-1} respectively. The longest wavelength (λ) of light capable of breaking the C-C bond can be expressed in SI unit as :

(1) $\frac{hc}{1000}\left(\frac{y-6x}{4}\right)^{-1}$ (2) $\frac{N_A hc}{250(4y-6x)}$

(3) $\frac{N_A hc}{250(y-6x)}$ (4) $N_A hc\left(y-\frac{6x}{4}\right)^{-1}$

Ans. **(2)**

Sol. $CH_4(g) \rightarrow C(g) + 4H(g); \quad \Delta_r H = x$ kJ/mole

$C_2H_6(g) \rightarrow 2C(g) + 6H(g); \ \Delta_r H = y$ kJ/mole

$1000x = 4 \times \varepsilon_{C-H}$

$1000y = 1 \times \varepsilon_{C-C} + 6 \times \varepsilon_{C-H}$

$$\varepsilon_{C-C} = \left[y - \frac{3x}{2}\right] \times 1000 = \frac{hc}{\lambda}.N_A$$

$$('\lambda')\ \text{wavelength of photon} = \frac{hc\ N_A}{[4y-6x]\times 250}$$

68. At 298 K, the mole percentage of $N_2(g)$ in air is 80%. Water is in equilibrium with air at a pressure of 10 atm. What is the mole fraction of $N_2(g)$ in water at 298 K ? (K_H for N_2 is 6.5×10^7 mm Hg)

(1) 1.23×10^{-7} (2) 1.17×10^{-4}

(3) 9.35×10^{5} (4) 9.35×10^{-5}

Ans. **(4)**

Sol. $P_{N_2} = K_H.X_{N_2}$

$P_{N_2} = 0.8 \times 10 = 8$ atm

$8 \times 760 = 6.5 \times 10^7 \times X_{N_2}$

$$X_{N_2} = \frac{8\times 760}{6.5\times 10^7}$$

$X_{N_2} = 9.35 \times 10^{-5}$.

69. "X" is an oxoanion of the lightest element of group 7 (in the periodic table). The metal is in +6 oxidation state in "X". The color of the potassium salt of X is

(1) green (2) purple (3) yellow (4) orange

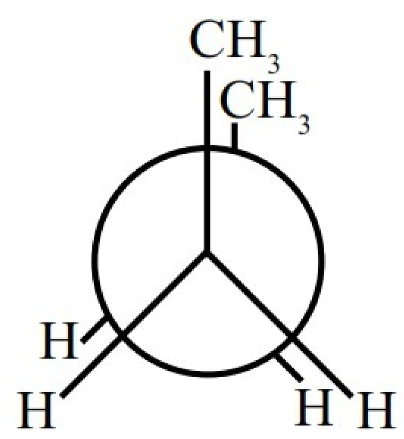
Ans. **(1)**

Sol. Lightest element of Group 7 $\Rightarrow$ Mn

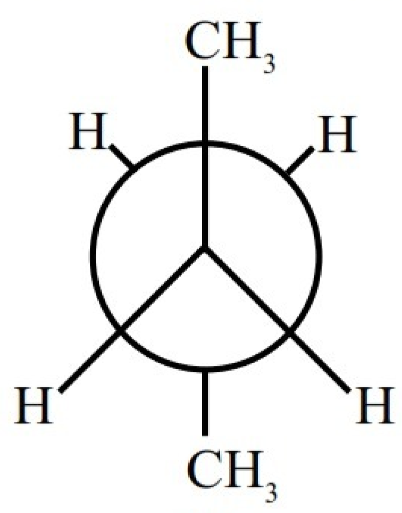
$K_2MnO_4 \Rightarrow$ Green

70. Given below are two statements :

Statement I : There are several conformers for n-butane. Out of those conformers,


(X)

is the least stable and most stable conformer is


(Y)

Statement II : As the dihedral angle increases, torsional strain decreases from (X) to (Y).

In the light of the above statements, choose the ***correct*** answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. **(4)**

Sol. Both Statements are correct.

SECTION-B

71. 0.25 g of an organic compound "A" containing carbon, hydrogen and oxygen was analysed using the combustion method. There was an increase in mass of $CaCl_2$ tube and potash tube at the end of the experiment. The amount was found to be 0.15 g and 0.1837 g, respectively. The percentage of oxygen in compound A is ________%. (Nearest integer)

(Given : molar mass in g mol^{-1} H : 1, C : 12, O : 16)

Ans. (73)

Sol. $C_xH_yO_z + O_2 \rightarrow CO_2 + H_2O$

t = 0   0.25 gm

t_{eq} –   0.18 gm   0.15 gm

Mass of 'C' = $\frac{0.18}{44} \times 12 = 0.049 \simeq 0.05$ gm

Mass of 'H' = $\frac{0.15}{18} \times 2 = 0.016 \simeq 0.017$ gm.

Mass of 'O' = 0.25 – 0.05 – 0.017 = 0.1833 gm

Mass % of 'O' = $\frac{0.1833}{0.25} \times 100 = 73.32\%$.

72. The half-life of ^{65}Zn is 245 days. After x days, 75% of original activity remained. The value of x in days is _______. (Nearest integer)

(Given : log 3 = 0.4771 and log 2 = 0.3010)

Ans. (102)

Sol. $t_{1/2} = \frac{\ell n 2}{K}$

$K = \frac{\ell n 2}{245}$

$t = \frac{1}{K} \ell n \frac{a_0}{a_t}$

$t_{25\%} = \frac{1}{K} \ell n \frac{4}{3}$

$t_{25\%} = \frac{1}{\frac{\ell n 2}{245}} \ell n \frac{4}{3}$

$t_{25\%} = 245 \frac{\ell n \frac{4}{3}}{\ell n 2} = 245 \left[\frac{2\log 2 - \log 3}{\log 2}\right]$

$= 245 \left[\frac{2 \times 0.3010 - 0.4771}{0.3010}\right] = 101.66$ day.

73. A chromium complex with a formula $CrCl_3.6H_2O$ has a spin only magnetic moment value of 3.87 BM and its solution conductivity corresponds to 1 : 2 electrolyte. 2.75 g of the complex solution was initially passed through a cation exchanger. The solution obtained after the process was reacted with excess of $AgNO_3$. The amount of AgCl formed in the above process is _______ g. (Nearest integer)

[Given : Molar mass in g mol^{-1} Cr : 52; Cl : 35.5, Ag : 108, O : 16, H : 1]

Ans. (3)

Sol. $[Cr(H_2O)_5Cl]Cl_2.H_2O + AgNO_3 \rightarrow 2AgCl$

2.75/266.5   –

= 0.0103 moles   0.02063 mole

Mass of AgCl = 0.02063 × 143.5 = 2.96 gm

74. Molar conductivity of a weak acid HQ of concentration 0.18 M was found to be 1/30 of the molar conductivity of another weak acid HZ with concentration of 0.02 of M. If $\lambda^0_{Q^-}$ happened to be equal with $\lambda^0_{Z^-}$, then the difference of the pK_a values of the two weak acids ($pK_a(HQ) - pK_a(HZ)$) is _______ (Nearest integer).

[Given : degree of dissociation (α) << 1 for both weak acids, λ° : limiting molar conductivity of ions]

Ans. (2)

Sol. $K_a(HQ) = C_1\alpha_1^2$ $\alpha_1 = \frac{\lambda_m(HQ)}{\lambda_m^\infty(HQ)}$

$K_a(HZ) = C_2\alpha_2^2$ $\alpha_2 = \frac{\lambda_m(HZ)}{\lambda_m^\infty(HZ)}$

$\frac{K_a(HQ)}{K_a(HZ)} = \frac{C_1}{C_2}.\left(\frac{\alpha_1}{\alpha_2}\right)^2 = \frac{0.18}{0.02}.\left[\frac{\lambda_m(HQ)}{\lambda_m(HZ)}\right]^2$

$\frac{K_a(HQ)}{K_a(HQ)} = 9 \times \left(\frac{1}{30}\right)^2 = \frac{1}{100}$

$pK_a(HQ) - pK_a(HZ) = 2$

75. Grignard reagent RMgBr(P) reacts with water and forms a gas (Q). One gram of Q occupies 1.4 dm^3 at STP. (P) on reaction with dry ice in dry ether followed by H_3O^+ forms a compound (Z). 0.1 mole of (Z) will weigh ________ g. (Nearest integer)

Ans. (6)

Sol. 1.4 dm^3 (or 1.4 mL) occupied by 1 gm

∴ Molecular weight of Q = $\frac{22.4}{1.4} = 16$

∴ Q is CH_4 gas and Grignard reagent is CH_3MgBr

$CH_3MgBr \xrightarrow[(ii)\ H_3O^+]{(i) CO_2 (dry\ ice)} CH_3COOH$

(Molecular weight 60)

∴ Weight of 0.1 mole of CH_3COOH = 6