PART : PHYSICS

1. A drop of radius R is split into 27 drops of equal radius, the work done is 10 J. If the same big drop is split into 64 equal drops the work done is-

(1) 10 J (2) 15 J (3) 20 J (4) $\frac{75}{4}$ J

Ans. **(2)**

Sol. $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$r = \frac{R}{3}$

initial surface area = $4\pi R^2$

Final surface area = $4\pi r^2 \times 27$

Change in surface area = $4\pi\,(27r^2 - R^2)$

$$= 4\pi\left(27\left(\frac{R}{3}\right)^2 - R^2\right) = 8\pi R^2$$

Work done by External Agent = $T \times 8\pi R^2 = 10$ ...... (i)

$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow r = \frac{R}{4}$

change in surface area $= 4\pi(64r^2 - R^2)$

$$= 4\pi\left(64\left(\frac{R}{4}\right)^2 - R^2\right)$$

$= 12\pi R^2$

Work done by External Agent = $T \times 12\,\pi R^2 = 12 \times \frac{10}{8} = 15J$

2. A satellite is revolving in a stable circular orbit of radius R and time period is T. If orbital radius of an another satellite is 1.03 R, then the percentage charge in time period of the second satellite as compared to the first will be :

(1) 1.5% (2) 4.5% (3) 7.5% (4) 9%

Ans. **(2)**

Sol. $T^2 \propto n^3$

$T \propto n^{3/2}$

$\frac{dT}{T} = \frac{3}{2}\frac{dr}{r} = \frac{3}{2} \times 3\% = 4.5\%$

3. In a parallel plate capacitor, the length, width and separation between the plates are respectively ℓ = 5 cm, b = 3 cm and d = 1 μm. What will be the dimensions of an another capacitor, so that its capacitance becomes 10 times

(1) ℓ = 50 cm, b = 30 cm, d = 10 μm
(2) ℓ = 50 cm, b = 10 cm, d = 10 μm
(3) ℓ = 10 cm, b = 30 cm, d = 50 μm
(4) ℓ = 40 cm, b = 10 cm, d = 50 μm

Ans. **(1)**

Sol. $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0(\ell b)}{d}$

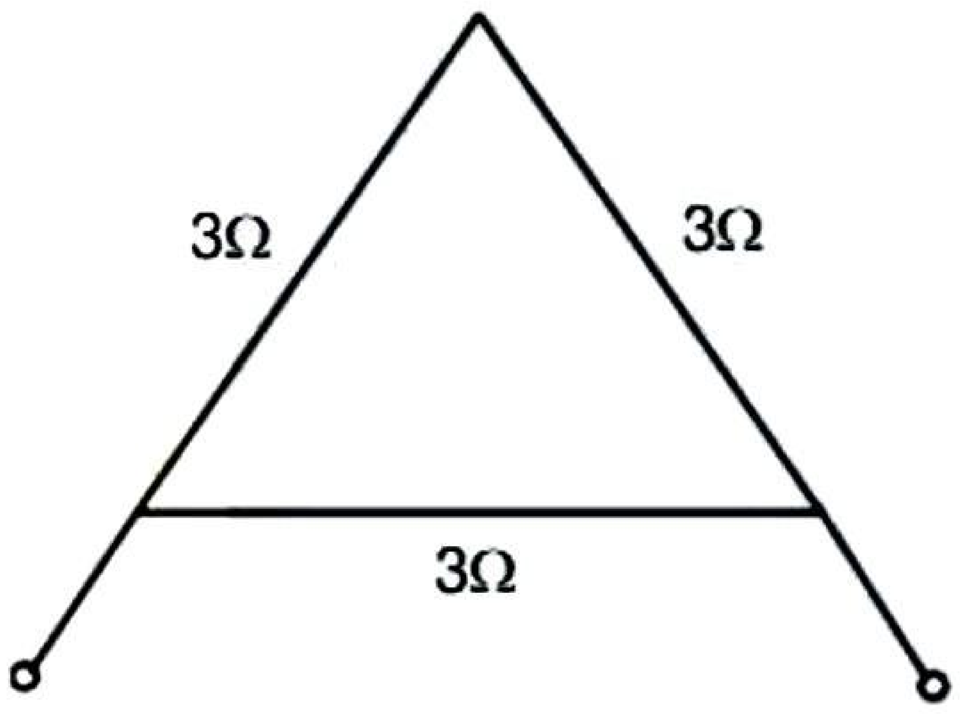
In option (A) $\ell \to 10$ times, b $\to$ 10 times, b $\to$ 10 so C will also be 10 times.

4. Resistance of uniform wire is 9Ω. If it is bent in the form of an equilateral triangle, then equivalent resistance between its two vertices will be :

(1) 1Ω (2) 2Ω (3) 3 Ω (4) 6Ω

Ans. **(2)**

Sol.



$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6}$$

$R_{eq} = 2\Omega$

5. A Solid cylinder of mass m and radius r is released from rest at the top of a rough inclined plane making an angle of 45° with the horizontal. Assuming the cylinder rolls without sipping find the acceleration of the axis of the cylinder

(1) $\frac{g}{2}$ (2) $\frac{g}{\sqrt{2}}$ (3) $\frac{2g}{3\sqrt{2}}$ (4) $\frac{g}{3\sqrt{2}}$

Ans. **(3)**

Sol. $a = \frac{g\sin\theta}{1 + K^2/R^2}$

$$\frac{K^2}{R^2} = \frac{1}{2}$$

$$\text{Sin}45° = \frac{1}{\sqrt{2}}$$

$$a = \frac{g \times \frac{1}{\sqrt{2}}}{3/2} = \frac{2g}{3\sqrt{2}}$$

6. If $I = I_A \text{ Sin } \omega t + I_B \text{ Cos } \omega t$, Then find rms value of current.

(1) $I_{rms} = I_A + I_B$ (2) $I_{rms} = \sqrt{I_A^2 + I_B^2}$ (3) $I_{rms} = \frac{1}{2}\sqrt{I_A^2 + I_B^2}$ (4) $I_{rms} = \sqrt{\frac{I_A^2 + I_B^2}{2}}$

Ans. **(4)**

Sol. $I_{max} = \sqrt{I_A^{\,2} + I_B^{\,2}}$

$I_{rms} = \sqrt{\dfrac{I_A^{\,2} + I_B^{\,2}}{2}}$

7. Work done by a force $F = (\alpha + \beta x^2)$ from $x = 0$ to $x = 1$ is 5J. If $\alpha = 1$, then find value of β.

(1) 4 (2) 8 (3) 12 (4) 16

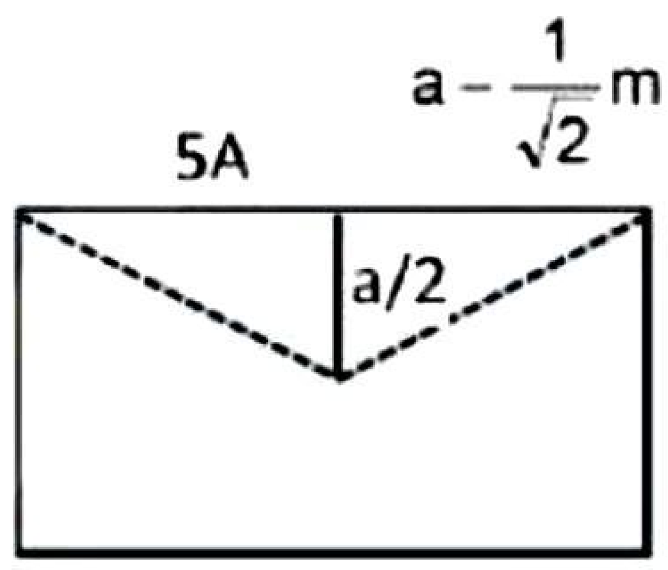
Ans. **(3)**

Sol. $W = \int F dx$

$5 = \alpha x + \beta \dfrac{x^3}{3}\Big|_0^1$

$5 = 1 \times 1 + \dfrac{\beta}{3} \times 1 \Rightarrow \beta = 12$

8. In a square loop of side length $\dfrac{1}{\sqrt{2}}$m, current of 5 Amp is flowing. Find magnetic field as its centre (in μT).



(1) 3 (2) 9 (3) 11 (4) 8

Ans. **(4)**

Sol. $B_C = \dfrac{\mu_0 i}{\pi l} 2\sqrt{2}$

$= \dfrac{\mu_0 \times 5}{\pi \dfrac{1}{\sqrt{2}}} \times 2\sqrt{2}$

$= \dfrac{\mu_0}{\pi} \times 20 = \dfrac{4\pi \times 10^{-7} \times 10}{\pi}$

$80 \times 10^{\ 7}$

$8 \times 10^{\ 7} = N \times 10^{\ 7}$

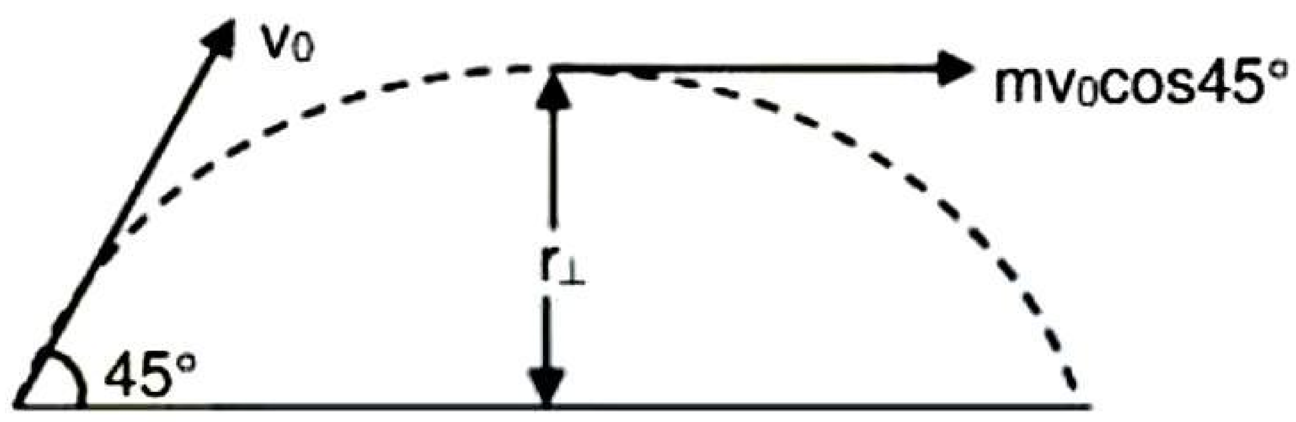
$N = 8$

9. A body projected with initial velocity V_0 at 45° angle in X-Y plane. Angular momentum of the particle at highest point about point of projection is :

(1) $\frac{mV_0^3}{4g}$ (2) $\frac{mV_0^3}{4\sqrt{2}g}$ (3) $\frac{mV_0^2}{4\sqrt{2}g}$ (4) $\frac{mV_0}{2\sqrt{2}g}$

Ans. **(2)**

Sol.



$\vec{L} = \vec{r} \times \vec{p}$

$\dot{L} = mv_0\cos45°$

$L = mvH$

$= m \times v_0\cos45° \times \frac{u^2 \sin^2 45°}{2g}$

$= mv_0 \times \frac{1}{\sqrt{2}} \times \frac{v_0^2}{4g}$

$L = \frac{mv_0^3}{4\sqrt{2}g} \Rightarrow \dot{L} = \frac{mv_0^3}{4\sqrt{2}g}(-\hat{k})$

10. An electron jumps from principle quantum state A to C by releasing photon of wavelength 2000 Å and from state B to C by releasing of photon of wavelength 6000 Å, then find wavelength of photon for transition from A to B.

(1) 2000 Å (2) 3000 Å (3) 4000 Å (4) 8000 Å

Ans. **(2)**

Sol. $E_A - E_C = \frac{hc}{2000}$

$E_B - E_C = \frac{hc}{6000}$

$E_A - E_B = \frac{hc}{\lambda}$

$E_A - E_B = \frac{hc}{2000} - \frac{hc}{6000} = \frac{hc}{\lambda}$

$\Rightarrow \frac{3-1}{6000} = \frac{1}{\lambda}$

$\lambda = \frac{6000}{2} = 3000\text{Å}$

11. Initial velocity of an electron is $v_0\hat{i}$ and its initial De-Broglie wavelength is λ_0. A uniform electric field of $\vec{E} = -E_0\hat{k}$ is applied. The De-Broglie wavelength as a function of time will be :-

(1) $\lambda(t) = \lambda_0$

(2) $\lambda(t) = \dfrac{h}{\sqrt{\left(\dfrac{1}{\lambda_0}\right)^2 + \left(\dfrac{eE_0t}{h}\right)^2}}$

(3) $\lambda(t) = \dfrac{1}{\sqrt{\left(\dfrac{1}{\lambda_0}\right)^2 + \left(\dfrac{eE_0t}{h}\right)^2}}$

(4) $\lambda(t) = \dfrac{1}{\sqrt{\left(\dfrac{1}{\lambda_0}\right)^2 + \left(\dfrac{eE_0t}{mh}\right)^2}}$

Ans. **(3)**

Sol. $\lambda_0 = \dfrac{h}{mV_0} \Rightarrow mV_0 = \dfrac{h}{\lambda_0}$

$$\vec{V} = \vec{u} + \vec{a}t \Rightarrow \vec{V}(t) = v_0\hat{i} + \frac{(-e)(-E_0\hat{k})}{m}t$$

$$\vec{V} = v_0\hat{i} + \frac{eE_0}{m}t\hat{k} \Rightarrow |V| = \sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}$$

$$\lambda_{ab}(t) = \frac{h}{m|v|} = \frac{h}{m\sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}} = \frac{h}{\sqrt{\left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{eE_0}{m}t\right)^2}}$$

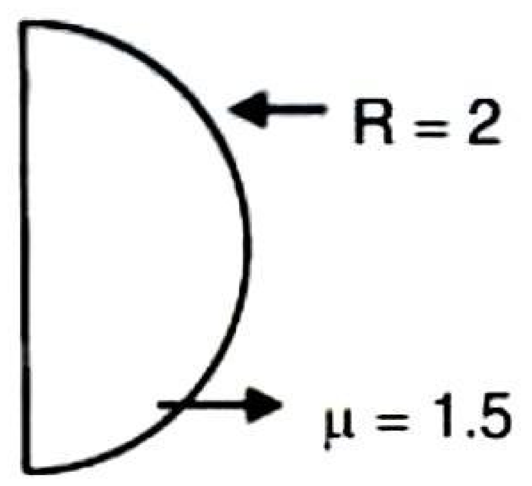
$$\lambda_{ab}(t) = \frac{1}{\sqrt{\left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{eE_0t}{h}\right)^2}}$$

12. Radius of curvature of a plano convex lens is 2 cm and refractive index is 1.5 has focal length f_1 in air and f_2 in a medium of refractive index 1.2, calculate f_1/f_2

(1) 2 : 1 (2) 1 : 2 (3) 3 : 2 (4) 1 : 4

Ans. **(2)**

Sol.



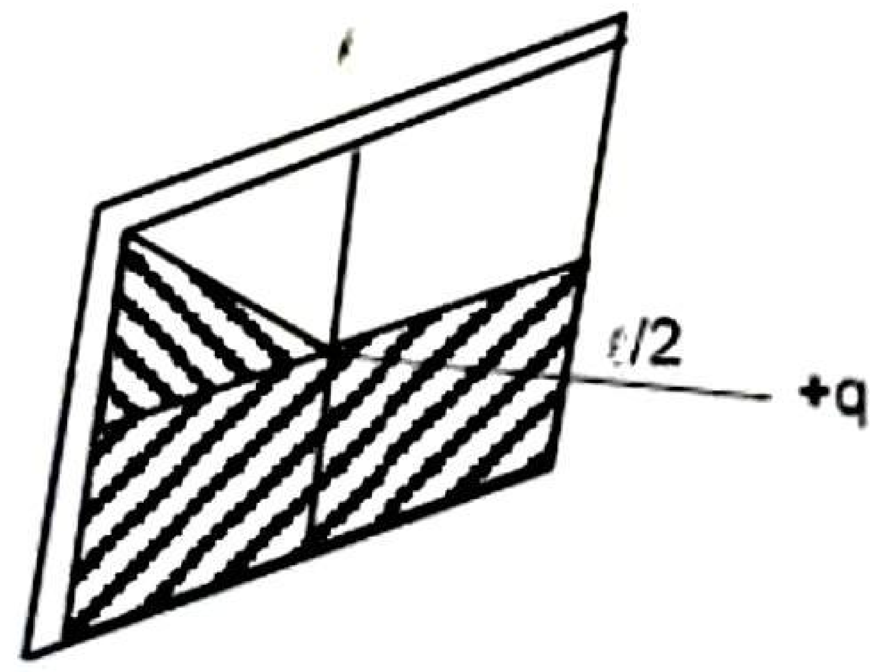
$$\frac{1}{f_1} = (1.5 - 1) \times \frac{1}{2} = \frac{1}{4} \Rightarrow f_1 = 4$$

$$\frac{1}{f_2} = \left(\frac{1.5}{1.2} - 1\right) \times \frac{1}{2} = \frac{0.3}{2 \times 1.2} = \frac{1}{8}$$

$f_2 = 8$

$$\frac{f_1}{f_2} = \frac{4}{8} = \frac{1}{2}$$

13. A point charge +1C is placed at a distance $\frac{\ell}{2}$ from center of a square surface of side length ℓ. If the flux passing through the shaded region is then $\phi = \frac{5}{x\varepsilon_0}$, then write the value of x



Ans. 48.00

Sol. flux passing through the complete square surface = $\frac{q}{6\varepsilon_0}$

8 triangular surface .... $\phi = \frac{5}{6\varepsilon_0}$

1 triangular surface .... $\frac{q}{48\varepsilon_0}$

5 triangular surface .... $\phi = \frac{q}{48\varepsilon_0} \times 5$

$$\phi = \frac{5q}{48\varepsilon_0} \text{ where } q = 1C = \frac{5}{48\varepsilon_0} = \frac{5}{x\varepsilon_0}$$

x = 48

14. Find minimum order of maxima of wavelength λ_1 on screen in YDSE where maxima of $\lambda_1 = 480$ nm coincide with maxima of $\lambda_2 = 600$ nm.

(1) 5 (2) 4 (3) 3 (4) 1

Ans. (1)

Sol. $\lambda_1 = 480$ nm

$\lambda_2 = 600$ nm

$n_1 = ?$

$y = n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{600}{480} = \frac{5}{4}$

$n_1 = 5$

15. In a process pressure of the gas is directly proportional to temperature then choose correct option

(A) Process is isochoric

(B) Work done in process is zero

(C) Internal energy increases with increase in temperature

(1) A and B are correct (2) A and C are correct

(3) A, B and C are correct (4) B and C are correct

Ans. **(3)**

Sol. $P \propto T$ (volume is constant)

process is isochoric

work = $P\Delta V = 0$

$\Delta U = nC_v\Delta T$

ΔU increase if temperature increase

16. A particle executes SHM with its time period 2 second and has amplitude of 1 cm. What is the ratio of total distance and displacement in 12.5 second :

(1) 25 : 1 (2) 5 : 1 (3) 4 : 5 (4) 3 : 2

Ans. **(1)**

Sol.

x = −1cm x = 0 x = 1 cm

12 second + 0.5 second

$$\text{Distance} = \frac{4}{2} \times 12 + \frac{4}{2} \times 0.5$$

$$= 24 + 1 = 25 \text{ cm}$$

Displacement = 1 cm

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{25}{1}$$

17. Find the maximum possible velocity for the given angle of banking θ on a curved road of radius of curvature r having coefficient of friction μ.

(1) $v_{max} = \sqrt{\frac{gr(\mu + \tan\theta)}{(1-\mu\tan\theta)}}$ (2) $v_{max} = \sqrt{\frac{gr(\mu - \tan\theta)}{(1-\mu\tan\theta)}}$

(3) $v_{max} = \sqrt{\frac{gr(1 + \tan\theta)}{(1-\mu\tan\theta)}}$ (4) $v_{max} = \sqrt{\frac{gr(\mu - \tan\theta)}{(1+\mu\tan\theta)}}$

Ans. **(1)**

18. What is the fractional decrease in focal length of a lens when optical power is increased from 2.5 D to 2.6 D.

(1) 0.05 (2) 0.04 (3) 0.10 (4) 0.25

Ans. **(2)**

Sol. $f = \frac{1}{P}$

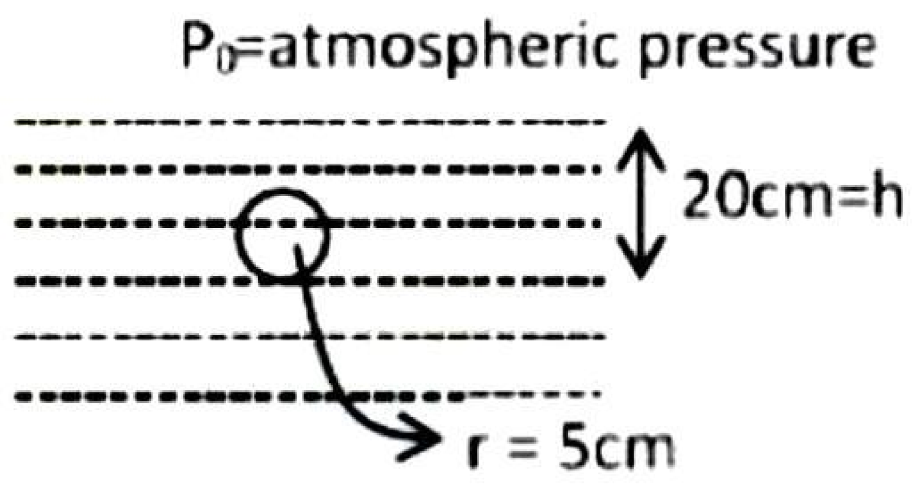
$$\frac{\Delta f}{f} = \frac{\Delta P}{P} = \frac{2.6 - 2.5}{2.5} = \frac{1}{25} = 0.04$$

19. A water bubble is at a depth of 20 cm and radius of bubble is 1 cm. If the inner pressure of the bubble is greater than the atmospheric pressure by 2100 N/m^2 then find the surface tension?

(1) 0.6 (2) 0.5 (3) 0.8 (4) 0.4

Ans. **(2)**

Sol.



$P_{in} - P_0 = 2100$ .... (i)

$P_{in} - P_0 = \frac{2T}{R} + h\rho g$ ...(ii)

From (i) and (ii)

$2100 = \rho gh + \frac{2T}{R}$

$\frac{2T}{R} = 2100 - \rho gh$

$T = 0.5$

20. If the distance between two parallel plates of a capacitor is d, A is the area of each plate, and E is the electric field between both the plates. Find the energy stored in capacitor.

(1) $\frac{1}{2}E^2A\epsilon_0 d$ (2) $\frac{1}{4}E^2A\epsilon_0 d$ (3) $\frac{3}{4}E^2A\epsilon_0 d$ (4) $E^2A\epsilon_0 d$

Ans. **(1)**

Sol. Energy density (u) = $\frac{1}{2}\epsilon_0 E^2$

Energy stored = energy density × volume

$= \frac{1}{2}\epsilon_0 E^2 \times (A \times d)$

$= \frac{1}{2}E^2 A \epsilon_0 d$

PART : CHEMISTRY

1. Difference of B.P and M.P in oxygen and sulpher can be explained by
(1) Atomicity (2) Atomic mass (3) Electronegativity (4) Electron gain enthalphy

Ans. **(1)**

Sol. The large difference between the melting and boiling points of oxygen and sulpher may be explained on the basis of their atomicity. O_2 exist as diatomic molcule where sulpher exist as polyatomic molecule S_8

2. Which of the following strong oxidising agent?
(1) Eu^{+2} (2) Ce^{2+} (3) Ce^{4+} (4) Eu^{4+}

Ans. **(4)**

Sol. M^{4+} will reduce itself to stable (+3) so, it will be good Oxidizing agent.

3. If 280 kg CO and 2320 kg of fe_3O_4 are made to react according to

$$fe_3O_4 + 4\,CO \rightarrow 4CO_2 + 3Fe$$

what is the weight of Fe produce (in kg)
Given : Mol. Wt. of CO and Fe_3O_4 are 28 and 232 u

Ans. **(420)**

Sol.

	fe_3O_4	+	4 CO	→	3Fe + $4CO_2$
n_i	10000		10000 (LR)		
n_f					$10000 \times \frac{3}{4} = 7500$

$\therefore$ $W_{Fe} = 7500 \times 56\text{ g} = 420\text{ kg}$

4. A reaction is non spontaneous at freezing point and spontaneous at boiling point select the correct option
(1) Both ΔH and ΔS are positive (2) $\Delta H > 0$, $\Delta S < 0$
(3) $\Delta H < 0$, $\Delta S > 0$ (4) Both ΔH and ΔS are negative

Ans. **(1)**

Sol.

Case I	**Case II**
At freezing point	At boiling point
$\Delta G > 0$	$\Delta G < 0$
$\Delta G - T\Delta S > 0$	$\Delta H - T\Delta S < 0$
$\lvert\Delta G\rvert > \lvert T\Delta S\rvert$	$\lvert\Delta H\rvert < \lvert T\Delta S\rvert$

5. Standard potential
(i) $Fe^{3+} + 3e^- \rightarrow Fe$; $E_1^0 = Z$ volt
(ii) $Ag^+ + 1e^- \rightarrow Ag$; $E_2^0 = Y$ volt
(iii) $Fe^{2+} + 2e^- \rightarrow Fe$; $E_3^0 = X$ volt
Find Eº of reaction $Fe^{2+} + Ag^+ \rightarrow Fe^{3+} + Ag\,(s)$
(1) (Z – 2X + 3Y) volt (2) (X – 2Y + 3Z) volt (3) (Y – 2Z + 3X) volt (4) (Y – 3Z + 2X) volt

Ans. **(4)**

Sol. $Fe^{3+} + e^- \rightarrow Fe^{2+}$
(iv) = (i) – (iii)

$$\Delta G_4^0 = \Delta G_1^0 - \Delta G_3^0$$

$$-1.f.E_4^0 = -3.f.E_1^0 + 2.f.E_3^0$$

$$E_4^0 = 3E_1^0 - 2E_3^0 = 3Z - 2x$$

$$E^0 = E^0_{\frac{Ag^+}{Ag}} - E^0_{\frac{Fe^{2+}}{Fe^{3+}}} = Y - E^0_{\frac{Fe^{2+}}{Fe^{3+}}} = Y - 3Z + 2x$$

6. In a process pressure of gas is directly proportional to temperature then choose correct option
(A) Process is isochoric
(B) Work done in process is zero
(C) Internal energy increases with increase in temperature
(1) A and B are correct (2) A and C are correct
(3) A, B and C are correct (4) B and C are correct

Ans. **(3)**

Sol. $P \propto T$ (V, n = constant)

7. 1 mole of a complex with molecular formula CO $(NH_3)_5$ Cl_3 produces 3 mole ions upon complete ionisation. Upon adding excess $AgNO_3$, 2 mole AgCl are precipitated. Complex is:
(1) $[Co(NH_3)_5Cl]Cl_2$ (2) $[Co(NH_3)_4Cl_2]Cl.NH_3$
(3) $[Co(NH_3)_3Cl_3].2NH_3$ (4) $[Co(NH_3)_4Cl]Cl_2.NH_3$

Ans. **(1)**

Sol. $2Cl^-$ should be outside square bracket. Also, Co^{3+} shows CN = 6
$\therefore$ Complex : $[Co(NH_3)_5Cl]\ Cl_2$

8. Consider H_2O, NH_3, CH_4 and select correct statements:
(A) All are sp^3 hybridised
(B) Bond angle H–O–H, H–N–H, H–C–H are 104.5°, 107°, 109.5°
(C) Dipole moment: $CH_4 < NH_3 < H_2O$
(D) H_2O and NH_3 are lewis base and lewis acid
(E) NH_3 in H_2O is basic in nature
(1) A, B, C and E (2) A, B and D (3) A, C, D (4) B, D, E

Ans. **(1)**

Sol. Both NH_3 and H_2O behave as lewis base.

9. Which of the following is not true combination from given statement.
(a) Elements in periodic table are linearly arranged with atomic weight
(b) Elements in periodic table are linearly arranged with atomic number
(c) Element having similar electronic configuration are arranged in same group
(d) Using periodic table we can identify in which subshell last electron enters
(e) Isotopes of an element are placed in periodic table.
(1) a, e only (2) b, e only (3) b, c, d only (4) a, c, d, e only

Ans. **(1)**

10. If the k_{sp} of $Cr(OH)_3$ is $1.6 \times 10^{-30}\ M^4$ the molar solubility of salt in water is
(1) $\left(\frac{1.6\times10^{-30}}{27}\right)^{1/4}$ (2) $\left(\frac{16}{27}\times10^{-30}\right)^{1/4}$ (3) $\left(\frac{160}{27}\times10^{-30}\right)^{1/4}$ (4) None of these

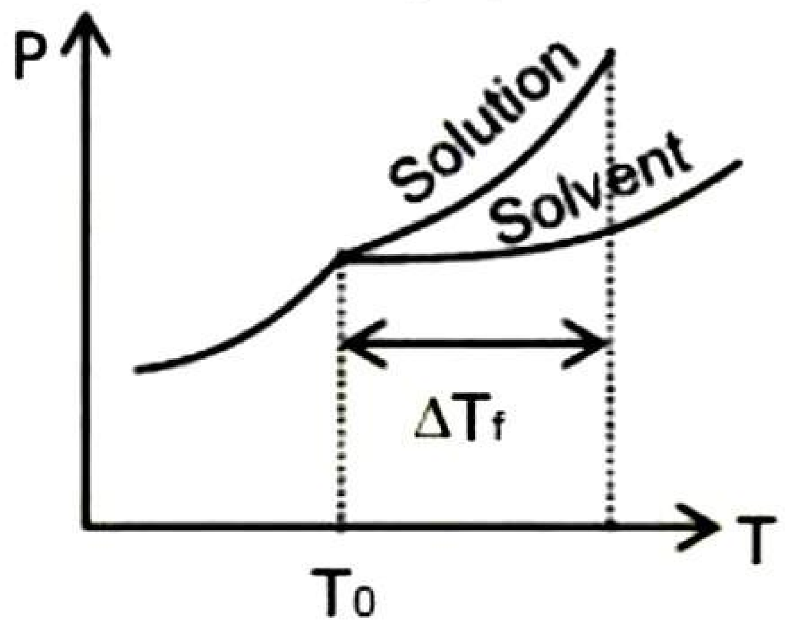
Ans. **(1)**

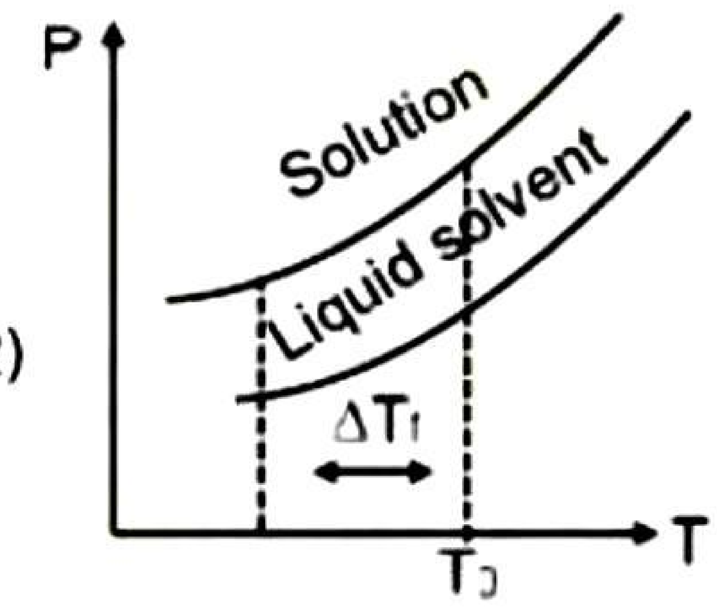
Sol. $Cr(OH)_3(s) \rightleftharpoons \underset{s}{Cr^{3+}} + \underset{3s}{3OH^-}$

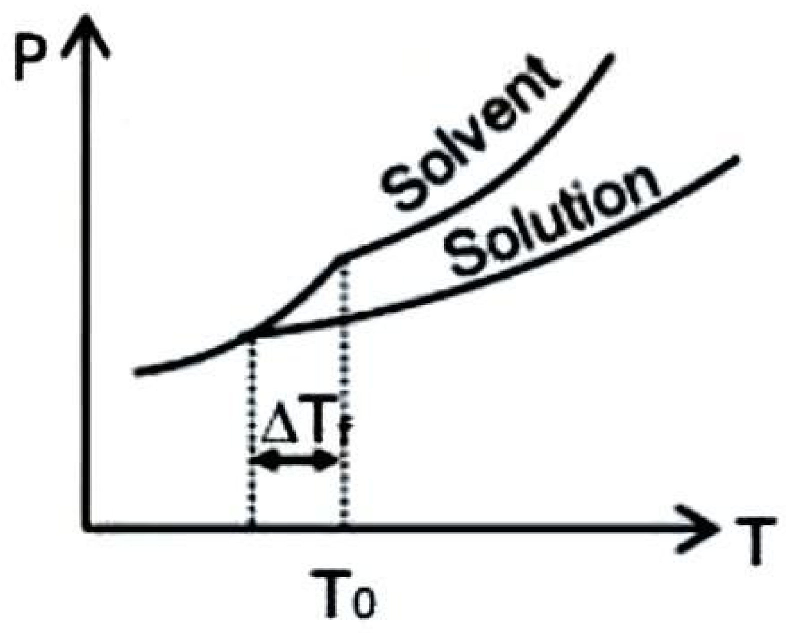
$k_{sp} = [Cr^{3+}]\,[OH^-]^3 = s(3s)^3$
$= 27\ S^4$

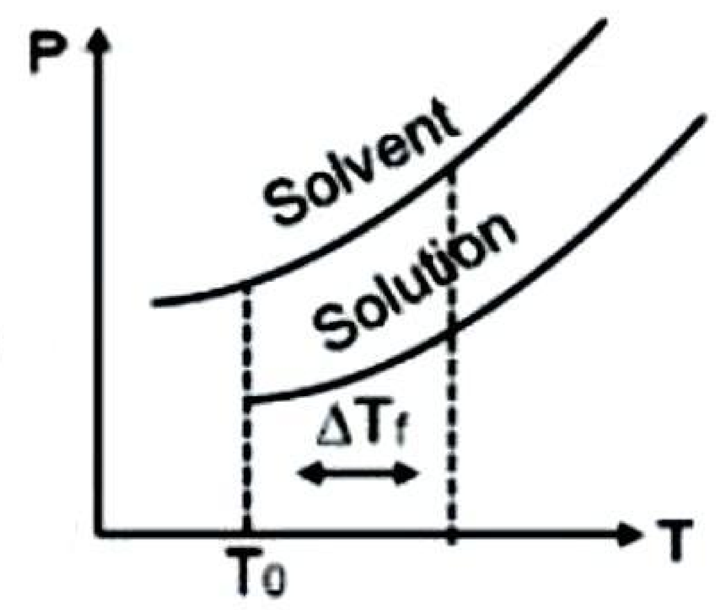
$$S = \left(\frac{1.6\times10^{-30}}{27}\right)^{1/4}$$

11. Select the correct graph.

(1) 

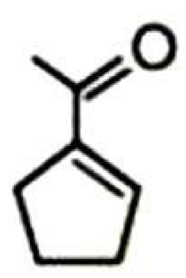
(2) 

(3) 

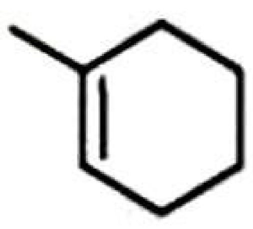
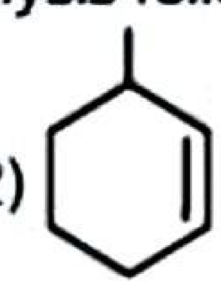
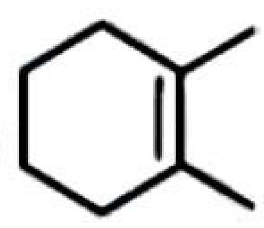
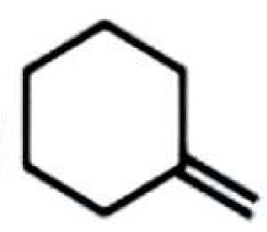
(4) 

Ans. **(3)**

Sol. T at which $VP_{solid} = VP_{liquid}$ is freezing point of solution is less than that of solvent.

12. 

is formed by ozonolysis followed by aldol condensation of Alkene. Alkene can be:

(1)  (2)  (3)  (4) 

Ans. **(1)**

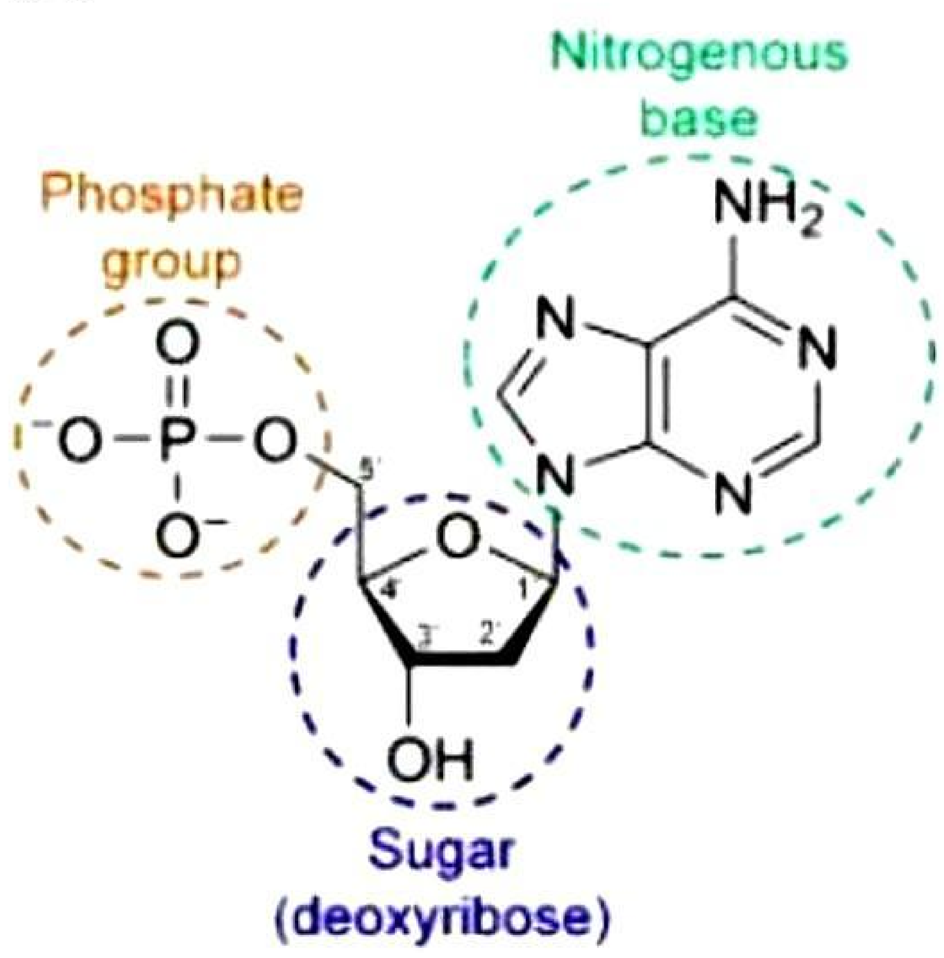
13. Ribose present in DNA

(A) It is a pentose sugar
(B) It is a present in pyranose form
(C) It is present in D-configuration
(D) It is reducing sugar in free form
(E) α anomeric form is present.

Correct options are :

(1) A, C, D (2) A, B, D (3) A, B, C, D, E (4) A, C, E

Ans. **(1)**

Sol. 

14. Arrange following for reaction rate with nucleophilic attack

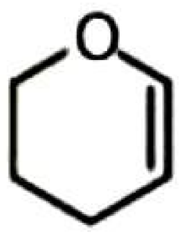
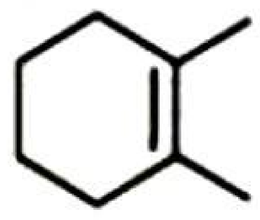
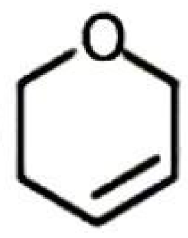
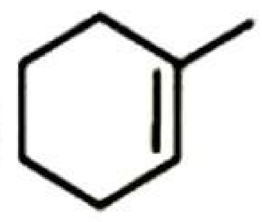
(a) Acetophenone (b) p-tolylaldehyde

(c) Benzaldehyde (d) p-Nitrobenzaldehyde

(1) d >c >b >a (2) a >c >b >d (3) d >b >c >a (4) d >a >b >c

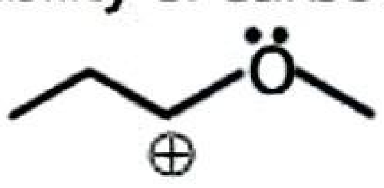
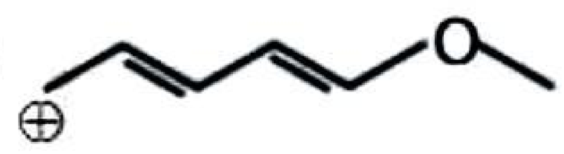
Ans. **(1)**

15. Which compound react fastest with HBr?

(1)  (2)  (3)  (4) 

Ans. **(1)**

16. Stability of carbocation is maximum is?

(1)  (2) 

(3)  (4) 

Ans. **(2)**

17. **Statement-I :** Dumas method is used for estimation of Nitrogen.

Statement-II : In Dumas method Nitrogen present in compound is converted to $(NH_4)_2SO_4$

(1) Both Statement I and statement II are true

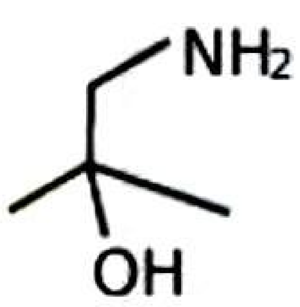
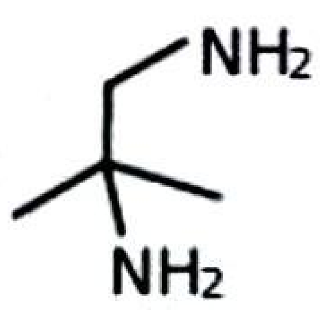
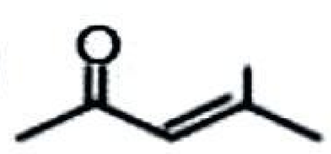
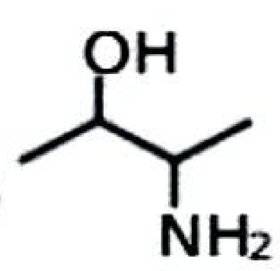
(2) Both statement I and statement II are false

(3) Statement I is true but statement II is false

(4) Statement I is false but statement II is true

Ans. **(3)**

18. $CH_3–C \equiv CH \xrightarrow[\text{2.HCN/HO}^{\ominus} \text{ 3.H}_2\text{/Ni}]{\text{1.HgSO}_4}$

(1)  (2)  (3)  (4) 

Ans. **(1)**

19. **Statement-I :** $CH_3–CH_2–CH_2–CH_2–Cl + OH^{\ominus} \longrightarrow$ Reaction is favoured in less polar solvent.

Statement-II : $CH_3–CH_2–CH_2–CH_2–Cl + R_3N \longrightarrow$ Reaction is favoured in more polar solvent.

(1) Both Statement I and statement II are true (2) Both statement I and statement II are false

(3) Statement I is true but statement II is false (4) Statement I is false but statement II is true

Ans. **(1)**

PART : MATHEMATICS

1. Evaluate :

$$\lim_{x\to 0} \operatorname{cosec} x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$$

(1) 0 (2) 1 (3) $\frac{1}{2\sqrt{5}}$ (4) $-\frac{1}{2\sqrt{5}}$

Ans. **(4)**

Sol. $= \lim_{x\to 0} \frac{1}{\sin x}\frac{(2\cos^2 x+3\cos x-\cos^2 x-\sin x-4)}{\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}}$

$= \frac{1}{2\sqrt{5}} \lim_{x\to 0}\frac{(\cos^2 x+3\cos x-\sin x-4)}{\sin x} \quad \left(\frac{0}{0}\right)$

By L' Hospital Rule

$= \frac{1}{2\sqrt{5}} \lim_{x\to 0}\frac{2\cos x.(-\sin x)-3\sin x-\cos x}{\cos x}$

$= -\frac{1}{2\sqrt{5}}$

2. Two students A & B throw two dices one by one. A wins the game when sum of numbers is 5 and B wins when the sum of numbers is 8. If A start the game then find probability that A wins the game

(1) $\frac{11}{19}$ (2) $\frac{12}{19}$ (3) $\frac{9}{19}$ (4) $\frac{10}{19}$

Ans. **(3)**

Sol. P (sum 5) = $\frac{4}{36}$

P (sum 8) = $\frac{5}{36}$

$P(\text{A wins the game}) = P(A) + P(\overline{A}\overline{B}A) + P(\overline{A}\overline{B}\overline{A}\overline{B}A) + \ldots\ldots\ldots\ldots\infty$

$= \frac{4}{36} + \left(\frac{32}{36}\right)\left(\frac{31}{36}\right)\left(\frac{4}{36}\right) + \ldots\ldots\ldots\ldots\ldots\ldots\infty$

$= \frac{\frac{4}{36}}{1-\left(\frac{32}{36}\right)\left(\frac{31}{36}\right)}$

$= \frac{4}{36}\times\frac{36^2}{(1296-992)}$

$= \frac{144}{304} = \frac{36}{76} = \frac{9}{19}$

3. If the 5^{th}, 6^{th} and 7^{th} term of the binomial expansion of $(1+x^2)^{n+4}$ are in A.P. Then greatest binomial coefficient in the expansion of $(1+x^2)^{n+4}$ is:

(1) ${}^{16}C_8$ (2) ${}^{14}C_7$ (3) ${}^{12}C_6$ (4) ${}^{13}C_7$

Ans. **(2)**

Sol. ${}^{n+4}C_4$, ${}^{n+4}C_5$, ${}^{n+4}C_6$ are in A.P.

$2\ {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6$

$$\frac{2\,\lfloor n+4}{\lfloor n-1\,\lfloor 5} = \frac{\lfloor n+4}{\lfloor n\,\lfloor 4} + \frac{\lfloor n+4}{\lfloor n-2\,\lfloor 6}$$

$2(6)n = (6)(5) + n(n-1)$

$12n = 30 + n^2 - n$

$n^2 - 13n + 30 = 0$

$(n-3)(n-10) = 0$

$n = 3, 10$

But $n = 3$ is not possible.

Now in expansion of $(1+x^2)^{14}$, greatest bionomial coefficient $= {}^{14}C_7$

4. If A is 3 × 3 order matrix, such that $|A| = 2$ then value of $|adj(adj(adj(adj\ A)))|$ is equal to

(1) 2^{32} (2) 2^{16} (3) 2^{20} (4) 2^8

Ans. **(2)**

Sol. $|A|=2$

$|adj(adj(adj(adj\ A)))|$

$|adj(adj(adj\ A))|^2$

$|adj(adj\ A)|^4$

$|adj\ A|^8$

$|A|^{16}$

2^{16}

5. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots\ldots\ldots\ldots$, If $\sqrt{2026 \times S_{2025}}$ = sum of 6 terms of an A.P. whose first term is $-p$ and common difference is p then find difference of 20^{th} term and 15^{th} term of that A.P.

Ans. **(25)**

Sol. T_n of $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots\ldots\ldots\ldots\ldots\ldots$ is $\frac{1}{n^2+n} = \frac{1}{n(n+1)}$

$$S_{2025} = \sum_{n=1}^{2025}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots\ldots\ldots + \left(\frac{1}{2025} - \frac{1}{2026}\right)$$

$$= 1 - \frac{1}{2026} = \frac{2025}{2026}$$

Now,

$$\sqrt{2026 \times \frac{2025}{2026}} = \frac{6}{2}\left[-2p + (6-1)p\right]$$

$45 = 3(3p)$ $p = 5$

Now,

$A_{20} - A_{15} = [-5 + (19)(5)] - [-5 + (14)(5)]$

$= 95 - 70 = 25$

6. If $(x+2)^2 f(x) + 3(x+2)^2 = \int_0^x (t+2)f(t)dt$ then $f(2) = ?$

(1) $\frac{9}{2}$ (2) $-\frac{9}{2}$ (3) $\frac{7}{2}$ (4) $-\frac{7}{2}$

Ans. **(2)**

Sol. $(x+2)^2 f(x) + 3(x+2)^2 = \int_0^x ((t+2)f(t)dt$

$2(x+2) f(x) + (x+2)^2 f'(x) + 6(x+2) = (x+2) f(x)$

$2 f(x) + (x+2) f'(x) + 6 = f(x)$

$(x+2) f'(x) = -f(x) - 6$

$(x+2)\frac{dy}{dx} = -y - 6$

$\frac{dy}{y+6} = -\int \frac{dx}{x+2}$

$\ell n(y+6) = -\ell n(x+2) + c$ ____(1)

$(x+2)^2 f(x) + 3(x+2)^2 = \int_0^x (t+2)f(t)dt$

put x = 0

$4f(0) + 12 = 0$

$f(0) = -3$ _____(2)

put x = 0 is equation (1)

$\ell n3 = -\ell n2 + c$

$c = \ell n6$

$\ell n(y+6) = -\ell n(x+2) + \ell n6$

$(y+6) = \frac{6}{(x+2)}$

then x = 2

$y + 6 = \frac{6}{4}$

$y = \frac{3}{2} - 6$

$y = -\frac{9}{2}$

so $f(2) = -\frac{9}{2}$

7. If α and β are real numbers such that $\sec^2(\tan^{-1}(\alpha)) + \text{cosec}^2(\cot^{-1}(\beta)) = 36$ and $\alpha + \beta = 8$, $(\alpha > \beta)$ then $\alpha^3 + \beta^3$ is:

(1) 150 (2) 152 (3) 148 (4) 146

Ans. **(2)**

Sol. $1 + \tan^2(\tan^{-1}\alpha) + 1 + \cot^2(\cot^{-1}\beta) = 36$

$2 + \alpha^2 + \beta^2 = 36$

$\alpha^2 + \beta^2 = 34$

$(\alpha + \beta)^2 - 2\alpha\beta = 34$

$8^2 - 2\alpha\beta = 34 \quad \Rightarrow \quad 2\alpha\beta = 30 \quad \Rightarrow \quad \alpha\beta = 15$

So, $\alpha + \beta = 8$ and $\alpha\beta = 15$

$\alpha = 5$ and $\beta = 3$

$\alpha^3 + \beta^3 = 125 + 27 = 152$

8. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$, $\vec{c}$ is a vector perpendicular to $\vec{b}$, also $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If $\vec{c}.\vec{a} = 5$ then $|\vec{c}|$ is equal to

(1) $\sqrt{\frac{13}{6}}$ (2) $\sqrt{\frac{11}{6}}$ (3) $\sqrt{\frac{5}{6}}$ (4) $\sqrt{\frac{7}{6}}$

Ans. **(2)**

Sol. Let $\vec{c} = \lambda\vec{b} \times (\vec{a} \times \vec{b})$

$= \lambda[((\vec{b}.\vec{b})\vec{a} - (\vec{b}.\vec{a})\vec{b}]$

$= \lambda[11\vec{a} - 2\vec{b}]$

$= \lambda[11\hat{i} + 22\hat{j} + 33\hat{k} - 6\hat{i} - 2\hat{j} + 2\hat{k}]$

$= \lambda[5\hat{i} + 20\hat{j} + 35\hat{k}]$

$\vec{c} = 5\lambda[\hat{i} + 4\hat{j} + 7\hat{k}]$

Now, $\vec{c}.\vec{a} = 5$

$5\lambda[1 + 8 + 21] = 5$

$\lambda = \frac{1}{30}$

$\vec{c} = \frac{1}{6}(\hat{i} + 4\hat{j} + 7\hat{k})$

$|\vec{c}| = \frac{1}{6}\sqrt{1 + 16 + 49}$

$|\vec{c}| = \frac{\sqrt{66}}{6} = \sqrt{\frac{11}{6}}$

9. If system of equations

$2x - y + z = 4$,

$4x - \lambda y + 3z = 12$ and

$100x - 41y + \mu z = 212$ has infinite solutions then $\mu - 2\lambda$ is

(1) 59 (2) 55 (3) 56 (4) 57

Ans. **(2)**

Sol. Let equations are P_1, P_2 and P_3

Then

$KP_1 + P_2 : x(2K + 4) + y(-K-\lambda) + z(K+3) = 4K + 12$

$P_3 : 100x - 41y + \mu z = 212$

Now comparing coefficient

$$\frac{2K+4}{100} = \frac{-K-\lambda}{-41} = \frac{K+3}{\mu} = \frac{4K+12}{212}$$

Solving $\dfrac{2K+4}{100} = \dfrac{4K+12}{212}$

$$\frac{K+2}{50} = \frac{K+3}{53}$$

$53K + 106 = 50K + 150$

$3K = 44$

$K = \dfrac{44}{3}$

Now

$$\frac{\frac{88}{3}+4}{100} = \frac{\frac{-44}{3}-\lambda}{-41} = \frac{\frac{44}{3}+3}{\mu} = \frac{\frac{176}{3}+12}{212}$$

$$\frac{1}{3} = \frac{44+3\lambda}{123} \text{ and } \frac{53}{3\mu} = \frac{212}{3\times 212} = \mu = 53$$

$41 = 44 + 3\lambda$

$\lambda = -1$

Now $\mu - 2\lambda = 53 - 2(-1) = 55$

10. Find the product of all rational roots of the equation $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 3$.

Ans. **(14)**

Sol. $(x^2 - 9x + 11)^2 - (x^2 - 9x + 23) = 0$

Let $x^2 - 9x + 11 = t$.

$t^2 - (t + 12) = 0 \quad \Rightarrow \quad (t-4)(t+3) = 0$

$t = 4$

$x^2 - 9x + 11 = 4$

$x^2 - 9x + 7 = 0$

$D = 81 - 28 = 53$

D is not a perfect square $\Rightarrow$ Irrational value of x.

$t = -3$

$x^2 - 9x + 11 = -3$

$x^2 - 9x + 14 = 0$

$(x-7)(x-2) = 0$

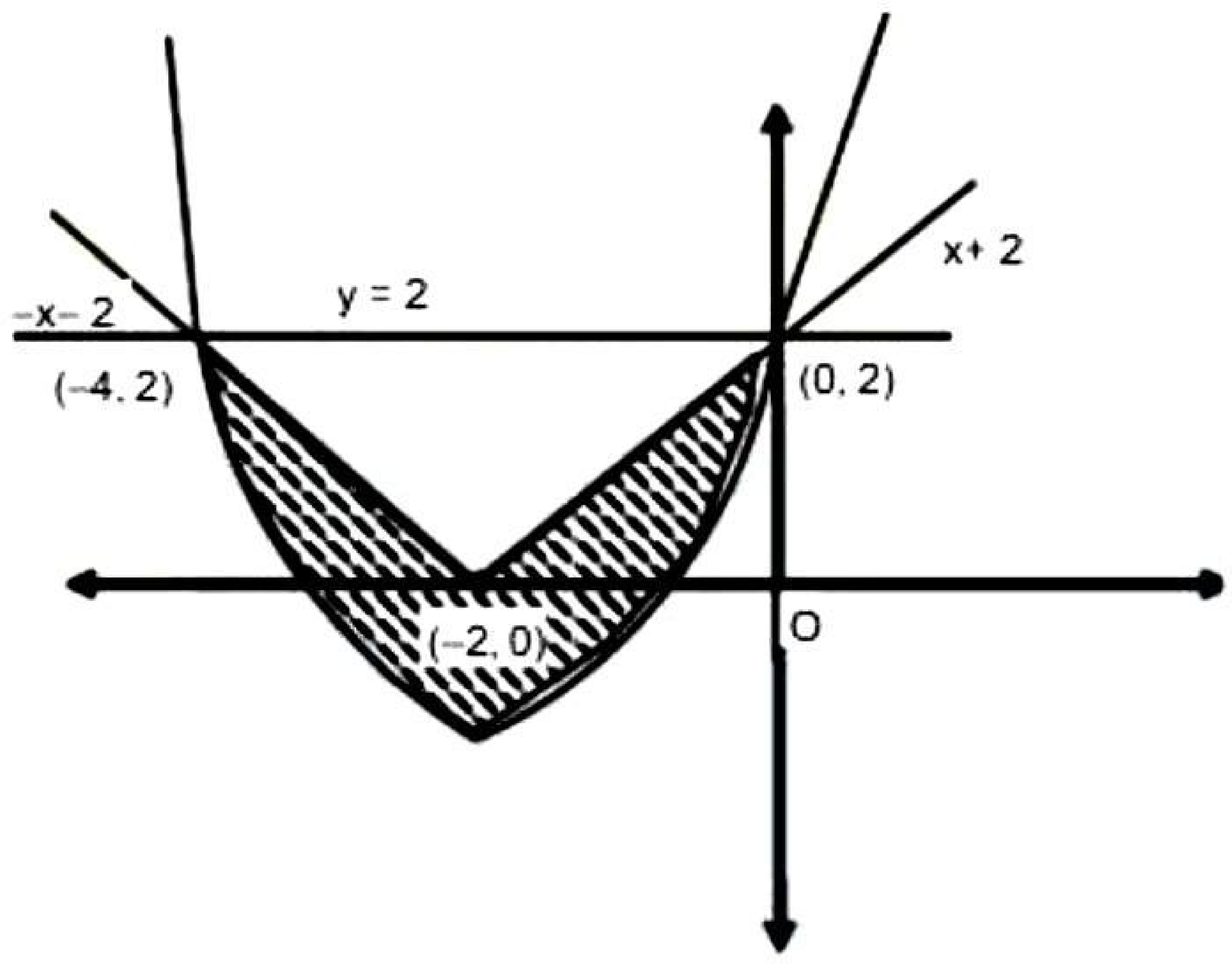
Product of roots $= 7 \times 2 = 14$.

11. The area of the region bounded by S(x, y) such that $S = \{(x, y) : x^2 + 4x + 2 \le y \le |x + 2|\}$ is (in sq. units)

(1) $\frac{24}{5}$ (2) 5 (3) $\frac{20}{3}$ (4) 7

Ans. **(3)**

Sol.



Area of shaded region = $\int_{-4}^{0} 2-\left(x^2+4x+2\right)dx - \frac{1}{2}\times 4\times 2$

$= \int_{-4}^{0}\left(-x^2-4x\right)dx - 4$

$= -\int_{-4}^{0}\left(x^2+4x\right)dx - 4$

$= -\left[\frac{x^3}{3}+2x^2\right]_{-4}^{0} - 4$

$= -\left[0-\left(\frac{-64}{3}\right)-32\right]-4$

$= 32-\frac{64}{3}-4$

$= 28-\frac{64}{3}$

$= \frac{84-64}{3}$

$= \frac{20}{3}$ sq. units

12. Find number of three digits numbers which are divisible by 2 & 3 both but not by 4 and 9.

Ans. **(125)**

Sol. Required numbers are which are divisible by 6

102, 114, 120, 132, 138, 150, 156, 168, …………,996 (A.P.) (Total numbers are 150)

Now

Numbers which are divisible by 36

108, 120, 138, …………….., 972 (A.P.) (Total numbers are 25)

Therefore, the required three digit numbers which are divisible by 6 and not by 36 are 150 – 25 = 125.

13. If $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, then I(9,14) + I(10,13) is equal to

(1) I (7, 11) (2) I (9, 13) (3) I (11, 15) (4) I (8, 11)

Ans. (2)

Sol. Let $x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\, d\theta$

$$\therefore I(m, n) = \int_0^{\frac{\pi}{2}} \sin^{2m-2}\theta\cos^{2n-2}\theta\ \ 2\sin\theta\cos\theta\, d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta\cos^{2n-1}\theta\ d\theta$$

$$\therefore I(9,14) + I(10,13) = 2\int_0^{\frac{\pi}{2}} \sin^{17}\theta\cos^{27}\theta\, d\theta + 2\int_0^{\frac{\pi}{2}} \sin^{19}\theta\cos^{25}\theta\, d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} \sin^{17}\theta\cos^{25}\theta(\cos^2\theta + \sin^2\theta)\, d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} \sin^{17}\theta\cos^{25}\theta\, d\theta$$

= I(9,13)

14. Let $f(x) = \dfrac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$ then $f\left(\dfrac{1}{15}\right) + f\left(\dfrac{2}{15}\right) + \ldots\ldots.. + f\left(\dfrac{59}{15}\right)$ is equal to

(1) 28.25 (2) 29.25 (3) 29 (4) 30

Ans. **(2)**

Sol. $f(x) = \dfrac{4(2^x+4)}{2(2^{2x}) + 16(2^x) + 32}$

$$= \frac{2(2^x+4)}{(2^x+4)^2} = \frac{2}{2^x+4}$$

Now $f(x) + f(4-x) = \dfrac{2}{2^x+4} + \dfrac{2}{2^{4-x}+4} = \dfrac{2}{2^x+4} + \dfrac{2\times 2^x}{16+4\times 2^x} = \dfrac{2}{2^x+4} + \dfrac{2^x}{2(4+2^x)} = \dfrac{1}{2}$

So $f\left(\dfrac{1}{15}\right) + f\left(\dfrac{59}{15}\right) = \dfrac{1}{2}$

$f\left(\dfrac{2}{15}\right) + f\left(\dfrac{58}{15}\right) = \dfrac{1}{2}$

and $f\left(\dfrac{30}{15}\right) = \dfrac{2}{4+4} = \dfrac{1}{4}$ hence

$f\left(\dfrac{1}{15}\right) + f\left(\dfrac{12}{15}\right) + \ldots\ldots. + f\left(\dfrac{59}{15}\right) = 29 + \dfrac{1}{4} = 29.25$

15. Let y(x) be the solution of differential equation $\frac{dy}{dx}+\left(\frac{x}{1+x^2}\right)y=\frac{\sqrt{x}}{\sqrt{1+x^2}}$

If y(0) = 0 then y(1) is equal to

(1) $\frac{2}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{2}}{3}$ (4) $\sqrt{\frac{2}{3}}$

Ans. **(3)**

Sol. I.F. = $\int\frac{xdx}{1+x^2}$ = $e^{\frac{1}{2}\ln(1+x^2)}$ = $\sqrt{1+x^2}$

Solution of differential equation

$y.\sqrt{1+x^2} = \int\sqrt{1+x^2}.\frac{\sqrt{x}}{\sqrt{1+x^2}}dx+C$

$y\sqrt{1+x^2} = \frac{2}{3}x^{3/2}+C$

when x = 0, y = 0 then C = 0

so $y = \frac{2x^{3/2}}{3\sqrt{1+x^2}}$ now at x = 1

$y = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$

16. Let $f(x)-6f\left(\frac{1}{x}\right) = \frac{35}{3x}-\frac{5}{2}$, if $\lim_{x\to 0}\left(\frac{1}{\alpha x}+f(x)\right) = \beta$

Then value of $\alpha + 2\beta$ is

(1) 0 (2) 1 (3) 3 (4) 4

Ans. **(4)**

Sol. $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}$ ....(1)

Replace $x \to \frac{1}{x}$

$f\left(\frac{1}{x}\right) - 6f(x) = \frac{35x}{3} - \frac{5}{2}$ ....(2)

from (1) and (2)

$f(x) = -2x - \frac{1}{3x} + \frac{1}{2}$

Now

$\beta = \lim_{x\to 0}\left(\frac{1}{\alpha x} - 2x - \frac{1}{3x} + \frac{1}{2}\right)$

$\Rightarrow \alpha = 3, \beta = \frac{1}{2}$

So, $\alpha + 2\beta = 3+1 = 4$

17. The line 2x+3y = 12 meets the x-axis at A and y-axis at B. The line through (5, 5), perpendicular to AB meets the x-axis, y-axis and AB at C, D and E respectively. If O is origin of coordinate axes then area of OCEB is

(1) $\frac{16}{3}$ (2) $\frac{14}{3}$ (3) $\frac{23}{3}$ (4) $\frac{17}{2}$

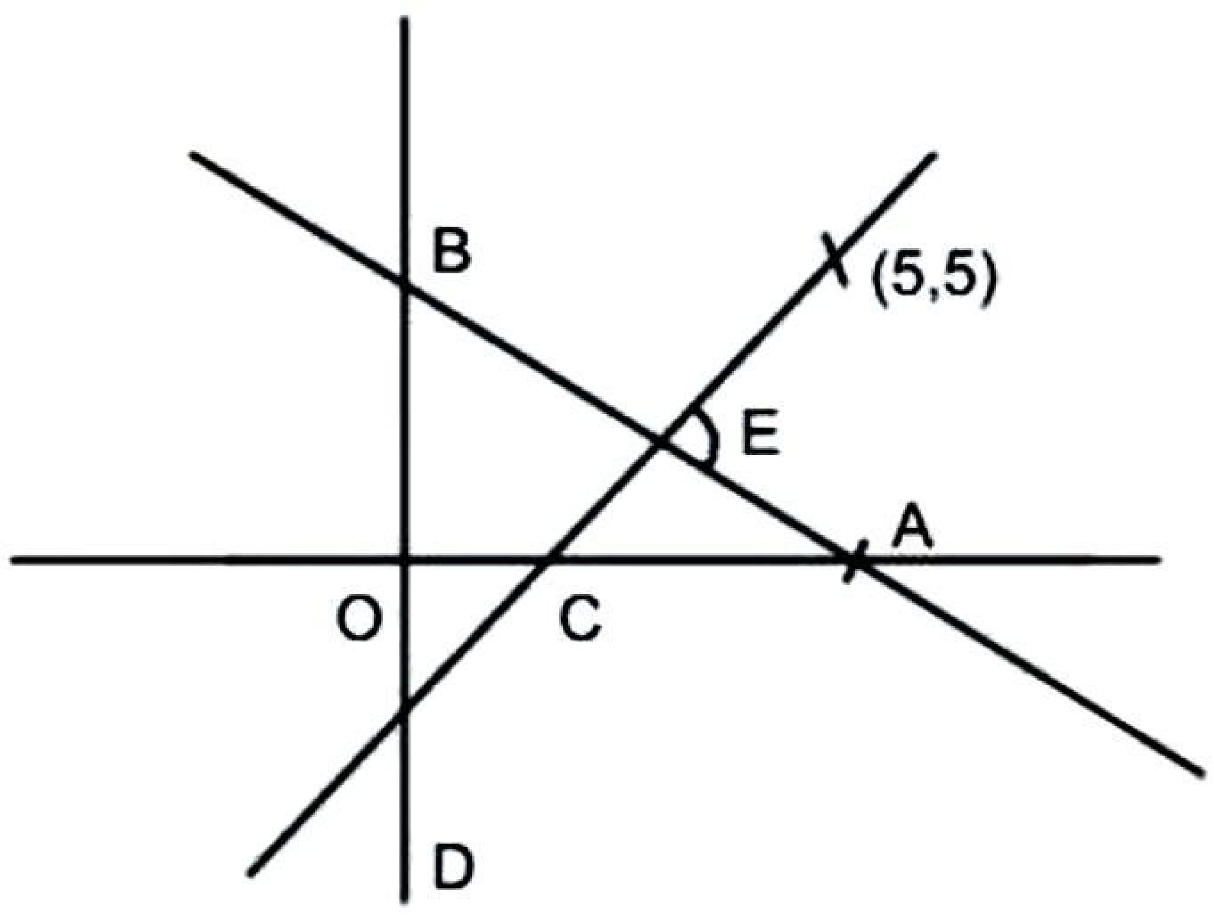
Ans. **(3)**

Sol. A(6,0), B(0,4)

AB is 2x+3y = 12

ED is 3x – 2y = 5

Solving E (3,2), C($\frac{5}{3}$,0) O(0,0)



$$\text{Area} = \frac{1}{2}\begin{pmatrix} 0 & \frac{5}{3} & 3 & 0 & 0 \\ 0 & \frac{3}{0} & 2 & 4 & 0 \end{pmatrix} = \frac{1}{2}\left(\frac{10}{3} + 12\right) = \frac{23}{3}$$

18. Let $P\left(\sqrt{3}, \frac{1}{2}\right)$ lies on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, also product of focal distance of point P is $\frac{7}{4}$ there are two ellipse satisfying given conditions then product of eccentricities is:

(1) $\left(\frac{1}{\sqrt{2}}\right)$ (2) $\left(\frac{1}{2}\right)$ (3) $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$ (4) $\left(\frac{\sqrt{3}}{4}\right)$

Ans. **(1)**

Sol. $PS_1 + PS_2 = 2a$ ____ (i) $P\left(\sqrt{3}, \frac{1}{2}\right)$

$PS_1 PS_2 = \frac{7}{4}$

Squaring (i) $PS_1^2 + PS_2^2 + 2PS_1.PS_2 = 4a^2$

$(ae - \sqrt{3})^2 + \frac{1}{4} + (ae + \sqrt{3})^2 + \frac{1}{4} + 2.\frac{7}{4} = 4a^2$

$2a^2e^2 + 10 = 4a^2$

$a^2e^2 + 5 = 2a^2$ _____(ii)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ satisfying $\left(\sqrt{3}, \frac{1}{2}\right)$ in the equation

$$\frac{3}{a^2}+\frac{1}{4b^2}=1 \qquad \{a^2 - a^2e^2 = b^2\}$$

$$\frac{3}{a^2}+\frac{1}{4a^2(1-e^2)}=1$$

$$3+\frac{1}{4(1-e^2)}=a^2 \qquad \left\{\text{from (ii) } a^2=\frac{5}{2-e^2}\right\}$$

$$3+\frac{1}{4(1-e^2)}=\frac{5}{(2-e^2)}$$

$$12(1-e^2)(2-e^2)+(2-e^2)=20(1-e^2)$$

Let $1 - e^2 = t$

$$12t(1+t)+(1+t)=20t$$

$$12t^2+12t+t+1=20t$$

$$12t^2-7t+1=0$$

$$(4t-1)(3t-1)=0$$

$$t=\frac{1}{4} \quad \text{or} \quad t=\frac{1}{3}$$

$$1-e^2=\frac{1}{4} \quad \text{or} \quad 1-e^2=\frac{1}{3}$$

$$e^2=\frac{3}{4} \quad \text{or} \quad e^2=\frac{2}{3}$$

$$\text{so } e_1^2\times e_2^2=\frac{1}{2}$$

$$e_1\times e_2=\frac{1}{\sqrt{2}}$$

19. For ten observations $x_1, x_2, x_3, \ldots, x_{10}$, mean is 5.5, $\sum_{i=1}^{10} x_1^2 = 377$, in which observations 4 and 5 are wrongly included. Replacing these wrong observations 4 and 5 by correct observations 6 and 8, then correct variance is:

(1) 6.7 (2) 8.7 (3) 7.5 (4) 7.6

Ans. **(4)**

Sol. $x_1 + x_2 + \ldots\ldots\ldots + x_8 + 4+5 = 55$ ....(1)

$x_1^2 + x_2^2 + \ldots\ldots\ldots + x_8^2 + 4^2 + 5^2 = 377$ ....(2)

For corrected data mean is

$$\frac{x_1+x_2+\ldots\ldots x_8+6+8}{10}=\frac{55-9+6+8}{10}=6$$

$$\sigma^2+\bar{x}^2=\frac{1}{n}\sum x_i^2$$

$$\sigma^2+6^2=\frac{1}{10}(377-4^2-5^2+36+64)$$

$$=\frac{1}{10}(477-41)=\frac{436}{10}=\sigma^2=7.6$$