JEE (Main)-2026 Session-1

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

22 January 2026 Shift – 2

Time: 3 hrs. **M.M: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A: Attempt all questions.

(5) Section - B: Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION-A

1. Let n be the number obtained on rolling a fair die. If the probability that the system

$x - ny + z = 6$

$x + (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

Has a unique solution is $\frac{k}{6}$, then the sum of k and all possible values of n is :

(1) 21 (2) 24

(3) 20 (4) 22

Ans. (4)

Sol. $x - ny + z = 6$

$x + (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & (n-2) & n+1 \\ 0 & n-1 & 1 \end{vmatrix} \neq 0$$

$\Rightarrow n^2 - 3n + 2 \neq 0$

$n \neq 1, 2$

for unique solution n = 3, 4, 5, 6

Now

P (probability when system of equations has unique solution) = $\frac{4}{6}$

So k = 4

Now required sum = 4 + (3 + 4 + 5 + 6) = 22

2. If the mean deviation about the median of the numbers k, 2k, 3k, ..., 1000k is 500, then k^2 is equal to :

(1) 16 (2) 4

(3) 1 (4) 9

Ans. (2)

Sol. $\because$ median $= \frac{1001k}{2} = X_M$

$\therefore$ mean deviation about median $= \frac{\sum |X_i - X_M|}{n}$

$$= \frac{2\left(\frac{k}{2} + \frac{3k}{2} + \frac{5k}{2} + \ldots 500 \text{ terms}\right)}{1000}$$

$$= \frac{2.\frac{k}{2}(500)^2}{1000} = \frac{500k}{2} = 500 \text{ (given)}$$

$\therefore k = 2$

$\therefore k^2 = 4$

3. The number of elements in the relation R = {(x,y): $4x^2 + y^2 < 52$, x, y $\in$ Z} is

(1) 77 (2) 89

(3) 67 (4) 86

Ans. (1)

Sol. $4x^2 + y^2 < 52$, $x, y \in Z$

$\downarrow \quad \downarrow$

$0 \quad 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7 \rightarrow 1 \times 15 = 15$

$\pm 1 \quad 0, \pm 1, \pm 2, \pm 3 \ldots\ldots\ldots \pm 6 \rightarrow 2 \times 13 = 26$

$\pm 2 \quad 0, \pm 1, \pm 2, \pm 3 \ldots\ldots\ldots \pm 5 \rightarrow 2 \times 11 = 22$

$\pm 3 \quad 0, \pm 1, \pm 2 \pm 3 \rightarrow 2 \times 7 = 14$

Number of elements = 77

4. Let $S = \left\{z \in \mathbb{C} : 4z^2 + \bar{z} = 0\right\}$. Then $\sum_{z \in S} |z|^2$ is equal to :

(1) $\frac{3}{16}$ (2) $\frac{7}{64}$

(3) $\frac{1}{16}$ (4) $\frac{5}{64}$

Ans. (1)

Sol. $4z^2 + \bar{z} = 0$

let $z = x + iy$

$4(x + iy)^2 + x - iy = 0$

$4x^2 - 4y^2 + 8xyi + x - iy = 0$

$4x^2 - 4y^2 + x = 0$ & $y(8x - 1) = 0$

$\Rightarrow y = 0$ or $x = \frac{1}{8}$

If $y = 0$, $4x^2 + x = 0$

$x = 0, \frac{-1}{4}$

$\therefore z_1 = 0 + 0.i \quad |z_1|^2 = 0$

$z_2 = 0 - \frac{1}{4}i \qquad |z_2|^2 = \frac{1}{16}$

If $x = \frac{1}{8}$,

$4 \times \frac{1}{64} - 4y^2 + \frac{1}{8} = 0$

$\Rightarrow 4y^2 = \frac{3}{16} \Rightarrow y = \pm\frac{\sqrt{3}}{8}$

$\therefore\ z_3 = \frac{1}{8} + \frac{\sqrt{3}}{8}i \qquad |z_3|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$

$z_4 = \frac{1}{8} - \frac{\sqrt{3}}{8}i \qquad |z_4|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$

$\therefore\ \sum_{i=1}^{n} |z_i|^2 = 0 + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$

5. If $\lim_{x\to 0} \frac{e^{(a-1)x} + 2\cos bx + (c-2)e^{-x}}{x\cos x - \log_e(1+x)} = 2$, then $a^2 + b^2 + c^2$ is equal to :

(1) 5 (2) 3

(3) 7 (4) 9

Ans. **(3)**

Sol. $\lim_{x\to 0} \frac{\left(1 + (a-1)x + \frac{(a-1)^2x^2}{2!}\right) + 2\left(1 - \frac{b^2x^2}{2!}\right) + (c-2)\left(1 - x + \frac{x^2}{2!}\right)}{x\left(1 - \frac{x^2}{2!}\right) - \left(x - \frac{x^2}{2}...\right)} = 2$

$\lim_{x\to 0} \frac{(1+2+c-2) + x(a-1-c+2) + x^2\left(\frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2}\right)\right)}{\frac{x^2}{2} - \frac{x^3}{2!} + ...} = 2$

For which

$\because\ c + 1 = 0 \Rightarrow c = -1$

$\because\ a - c = -1 \Rightarrow a = -2$

$\because\ \frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2}\right) = 1$

$\frac{9}{2} - b^2 - \frac{3}{2} = 1 \Rightarrow b^2 = 2$

$a^2 + b^2 + c^2 = 4 + 2 + 1 = 7$

6. If $y = y(x)$ satisfies the differential equation $16\left(\sqrt{x + 9\sqrt{x}}\right)\left(4 + \sqrt{9 + \sqrt{x}}\right)\cos y\, dy = (1 + 2\sin y)dx$, $x > 0$ and $y(256) = \frac{\pi}{2}$, $y(49) = \alpha$, then $2\sin\alpha$ is equal to :

(1) $2\sqrt{2} - 1$ (2) $2\left(\sqrt{2} - 1\right)$

(3) $3\left(\sqrt{2} - 1\right)$ (4) $\sqrt{2} - 1$

Ans. **(1)**

Sol. $\int \frac{\cos y}{1 + 2\sin y}\, dy = \int \frac{dx}{16\left(\sqrt{9\sqrt{x} + x}\right)\left(4 + \sqrt{9 + \sqrt{x}}\right)}$

$4 + \sqrt{9 + \sqrt{x}} = t$

$\frac{1}{2\sqrt{9 + \sqrt{x}}} \times \frac{dx}{2\sqrt{x}} = 1dx$

$\frac{1}{2}\ell n|1 + 2\sin y| = \int \frac{4dt}{16t} + C$

$\frac{1}{2}\ell n|1 + 2\sin y| = \frac{1}{4}\ell n|4 + \sqrt{9 + \sqrt{x}} + C$

$\frac{1}{2}\ell n(2\sin y + 1) = \frac{1}{4}\ell n|4 + \sqrt{9 + \sqrt{x}}| + C$

Substituting $\left(256, \frac{\pi}{2}\right)$

$\frac{1}{2}\ell n3 = \frac{1}{2}\ell n3 + C \qquad C = 0$

Substituting $(49, \alpha)$

$\frac{1}{2}\ell n(2\sin\alpha + 1) = \frac{1}{4}\ell n8$

$\ell n(2\sin\alpha + 1) = \frac{1}{2}\ell n8$

$\ell n(2\sin\alpha + 1) = \ell n2\sqrt{2}$

$2\sin\alpha + 1 = 2\sqrt{2}$

$2\sin\alpha = 2\sqrt{2} - 1$

7. Among the statements

(S1) : If A(5, –1) and B(–2, 3) are two vertices of a triangle, whose orthocentre is (0, 0), then its third vertex is (–4, –7) and

(S2) : If positive numbers 2a, b, c are three consecutive terms of an A.P., then the lines ax + by + c = 0 are concurrent at (2, –2),

(1) Only (S1) is correct

(2) Only (S2) is correct

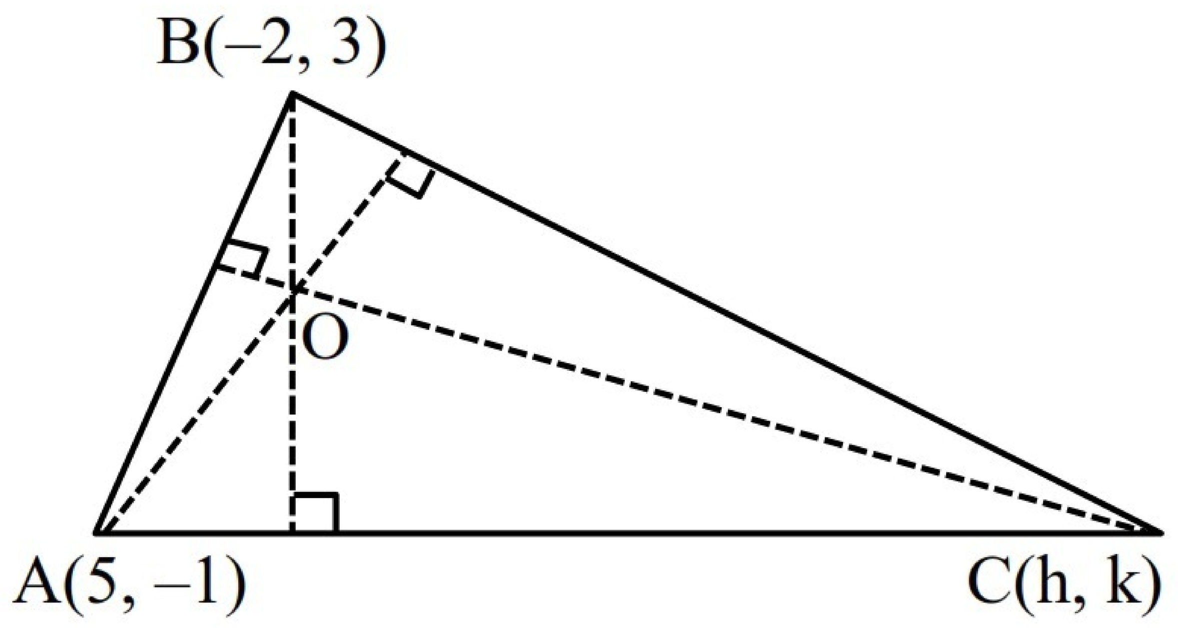
(3) Both are incorrect

(4) Both are correct

Ans. (4)

Sol. Solution of statement-1

$m_{AO}.m_{BC} = -1$



$\Rightarrow 5h - k + 13 = 0$ ....(1)

& $m_{AB}.m_{OC} = -1$

$\Rightarrow 4k = 7h$ ....(2)

$\Rightarrow$ third vertex is (–4, –7)

$\therefore$ Statement 1 is correct.

Solution of statement-2

2a, b, c $\rightarrow$ A.P.

$$b = \frac{2a+c}{2}$$

$\Rightarrow 2a - 2b + c = 0$

$\because$ lines ax + by + c = 0 are concurrent then

$$\frac{x}{2} = \frac{y}{-2} = \frac{1}{1}$$

x = 2 and y = –2

$\therefore$ Point of concurrency is (2, –2)

$\therefore$ Statement 2 is correct.

8. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}$, $\lambda \in Z$ be two vectors, Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz-plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $\left(\vec{c}\cdot\vec{d}\right)^2$ is equal to :

(1) 26 (2) 104
(3) 208 (4)52

Ans. (3)

Sol. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{b} = \lambda\hat{j} + 2\hat{k}$; $\lambda \in Z$

$\vec{c} = \vec{a} \times \vec{b} = (-2-\lambda)\hat{i} - 4\hat{j} + 2\lambda\hat{k}$

$|\vec{c}| = \sqrt{53}$

$\Rightarrow 5\lambda^2 + 4\lambda - 33 = 0$

$\lambda = 2.2$ or -3

$\Rightarrow \boxed{\lambda = -3}$

$\vec{c} = \hat{i} - 4\hat{j} - 6\hat{k}$

let $\vec{d} = y\hat{j} + z\hat{k}$

$|\vec{d}| = 2$

$\Rightarrow y^2 + z^2 = 4$

$(\vec{c}\,.\,\vec{d})^2 = (4y+6z)^2 \le \left(\sqrt{4^2+6^2} \times \sqrt{y^2+z^2}\right)^2 \le 208$

9. If $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a solution of the system of equations AX = B, where adj $A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, then |x + y + z| is equal to :

(1) 3 (2) $\frac{3}{2}$
(3) 1 (4) 2

Ans. (4)

Sol. $X = A^{-1}B = \left(\frac{\text{adj}A}{|A|}\right)B$

$$= \pm\frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$

$= \pm \frac{1}{10}\begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix} = \pm \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\therefore |x + y + z| = 2$

10. Let L be the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L, whose distance from the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c)\in S}(a+b+c)$ is equal to :

(1) 34 (2) 28

(3) 40 (4) 6

Ans. (1)

Sol. M is the point of intersection of L_1 & L_2

$\Rightarrow 2\lambda - 1 = 2\mu - 1,\ 3\lambda - 1 = 3\mu - 1,\ 6\lambda - 3 = 9$

$\Rightarrow \lambda = 2 = \mu$

$\Rightarrow$ M (3, 5, 9)

Now let point P be (2K – 1, 3K – 1, 6K –3) on L_2 such that PM = 7

$\Rightarrow \sqrt{(2K-4)^2 + (3K-6)^2 + (6K-12)^2} = 7$

$\Rightarrow 49K^2 + 196 - 196K = 49$

$\Rightarrow K^2 + 4 - 4K = 1$

$\Rightarrow K^2 - 4K + 3 = 0$

$\Rightarrow K = 1, 3$

So points P & Q are (1, 2, 3) & (5, 8, 15)

So sum of all co-ordinates of P & Q = 34

11. Let P $(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S′. If the length of its latus rectum is 8, then the square of the area of $\Delta PSS'$ is equal to :

(1) 4200 (2) 900

(3) 1462 (4) 2700

Ans. (4)

Sol. $P\left(10, 2\sqrt{15}\right)$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \boxed{\frac{100}{a^2} - \frac{60}{b^2} = 1}$ ....(1)

$\because$ length of latus rectum = 8

$\frac{2.b^2}{a} = 8 \Rightarrow \boxed{\frac{b^2}{a} = 4}$ ....(2)

From (1) & (2)

$\frac{100}{a^2} - \frac{60}{4a} = 1$

$400 - 60a = 4a^2$

$4a^2 + 60a - 400 = 0$

$a^2 + 15a - 100 = 0$

a = 5 & –20 (rejected)

$\Rightarrow b = \sqrt{20}$

$\therefore$ Hyperbola is $\boxed{\frac{x^2}{25} - \frac{y^2}{20} = 1}$

$\therefore$ Focal length $S_1S_2 = 2ae = 2.5.\left(\sqrt{1+\frac{4}{5}}\right) = 6\sqrt{5}$

$\therefore$ Area of $\Delta PS_1S_2 = \frac{1}{2}.6\sqrt{5}.2\sqrt{15} = 30\sqrt{3} = A$

$\therefore A^2 = 2700$

12. The area of the region

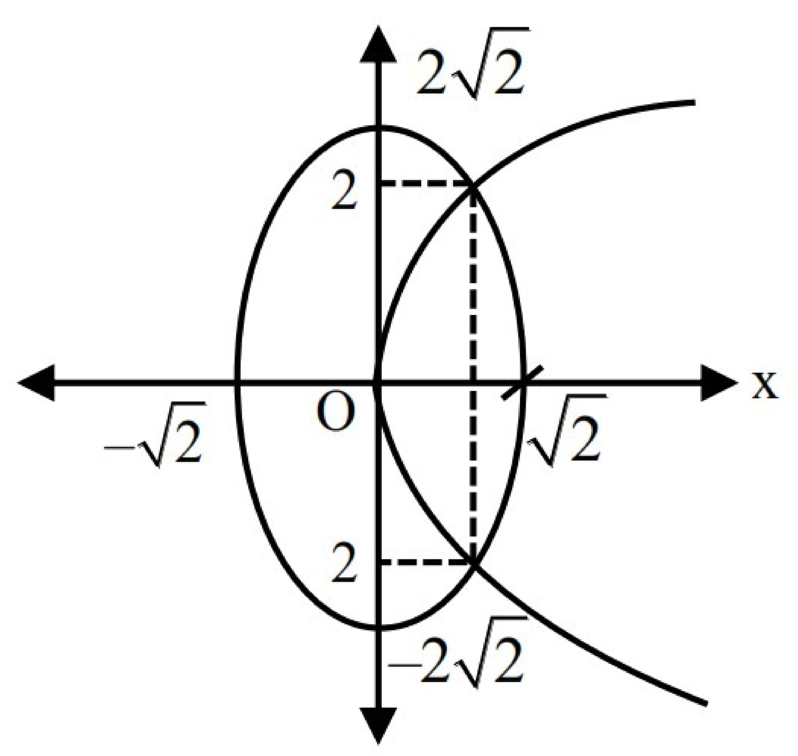
$A = \{(x, y) : 4x^2 + y^2 \le 8 \text{ and } y^2 \le 4x\}$ is :

(1) $\frac{\pi}{2} + 2$ (2) $\pi + \frac{2}{3}$

(3) $\pi + 4$ (4) $\frac{\pi}{2} + \frac{1}{3}$

Ans. (2)

Sol.



$A = \int_0^2 2\sqrt{x}dx + 2\int_1^{\sqrt{2}}\sqrt{8-4x^2}dx$

$= \frac{8}{3}\left(x^{\frac{3}{2}}\right)\Big|_0^1 + 4\int_1^{\sqrt{2}}\sqrt{2-x^2}dx$

$= \frac{8}{3} + 4 \times \frac{1}{2}\left[x\sqrt{2-x^2} + 2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]\Big|_1^{\sqrt{2}}$

$= \frac{8}{3} + 2\left[2 \times \frac{\pi}{2} - 1 - 2 \times \frac{\pi}{4}\right]$

$= \frac{8}{3} + 2\pi - 2 - \pi = \pi + \frac{2}{3}$ sq. units

13. Let α, β be the roots of the quadratic equation $12x^2 - 20x + 3\lambda = 0$, $\lambda \in \mathbb{Z}$. If $\frac{1}{2} \le |\beta - \alpha| \le \frac{3}{2}$, then the sum of all possible values of λ is :

(1) 6 (2) 1

(3) 3 (4) 4

Ans. (3)

Sol. $\frac{1}{2} \le |\alpha - \beta| \le \frac{3}{2}$

$\frac{1}{4} \le |\alpha - \beta|^2 \le \frac{9}{4}$

$\frac{1}{4} \le (\alpha + \beta)^2 - 4\alpha\beta \le \frac{9}{4}$

$\frac{1}{4} \le \frac{25}{9} - 4 \times \frac{\lambda}{4} \le \frac{9}{4}$

$-\frac{91}{36} \le -\lambda \le \frac{-19}{36}$

$\frac{19}{36} \le \lambda \le \frac{91}{36}$

$\lambda = 1, 2$

Sum = 3

14. Let the domain of the function

$$f(x) = \log_3 \log_5 \left(7 - \log_2 (x^2 - 10x + 85)\right) + \sin^{-1}\left(\left|\frac{3x-7}{17-x}\right|\right)$$

be (α, β]. Then α + β is equal to :

(1) 10 (2) 12

(3) 9 (4) 8

Ans. (3)

Sol. Let $x^2 - 10x + 85 = \lambda$

∴ Domain for first term

$\lambda > 0$ …(1)

& $7 - \log_2\lambda > 0 \Rightarrow \lambda < 2^7$ …(2)

& $\log_5(7 - \log_2\lambda) > 0 \Rightarrow \lambda < 2^6$ …(3)

∴ from (1), (2) & (3)

$0 < \lambda < 2^6$

$0 < x^2 - 10x + 85 < 64$

$\Rightarrow x \in (3, 7)$ …(A)

& domain for second term $-1 \le \frac{3x-7}{x-17} \le 1$

$\Rightarrow x \in [-5, 6]$ …(B)

From (A) & (B), domain of function will be (3, 6]

$\Rightarrow \alpha = 3, \beta = 6$

$\Rightarrow \alpha + \beta = 9$

15. Let [•] denote the greatest integer function, and let $f(x) = \min\left\{\sqrt{2}x, x^2\right\}$. Let S = {x ∈ (–2, 2) : the function $g(x) = |x|[x^2]$ is discontinuous at x}.

Then $\sum_{x \in S} f(x)$ equals :

(1) $2 - \sqrt{2}$ (2) $2\sqrt{6} - 3\sqrt{2}$

(3) $1 - \sqrt{2}$ (4) $\sqrt{6} - 2\sqrt{2}$

Ans. (3)

Sol. $g(x) = |x|[x^2]$

points of discontinuity of g(x) in (–2, 2) are

$\left(\pm 1, \pm\sqrt{2}, \pm\sqrt{3}\right)$

$\therefore S = \left\{-1, 1, -\sqrt{2}, \sqrt{2}, -\sqrt{3}, \sqrt{3}\right\}$

$\because f(x) = \min\left\{\sqrt{2}x, x^2\right\}$

$\therefore \sum_{x \in S} f(x) = -\sqrt{2} + 1 - 2 + 2 - \sqrt{6} + \sqrt{6}$

$= 1 - \sqrt{2}$

16. Let S and S' be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and P (α, β) be a point on the ellipse in the first quadrant. If $(SP)^2 + (S'P)^2 - SP \bullet S'P = 37$, then $\alpha^2 + \beta^2$ is equal to :

(1) 15 (2) 11

(3) 17 (4) 13

Ans. (4)

Sol. ∵ P lies on ellipse $\Rightarrow \frac{\alpha^2}{25} + \frac{\beta^2}{9} = 1$

$\because PS + PS' = 2a \Rightarrow PS + PS' = 10$

$\therefore (PS)^2 + (PS')^2 - PS.PS' = 37$

$(PS + PS')^2 - 3PS.PS' = 37$

$100 - 3PS.PS' = 37$

$3PS.PS' = 63 \Rightarrow PS.PS' = 21$

$\because$ PS & PS′ are $\left(5 \pm \frac{4}{5} \cdot \alpha\right)$

$\therefore PS.PS' = 25 - \frac{16}{25}\alpha^2 = 21$

$\frac{16}{25}\alpha^2 = 4$

$\alpha = \frac{5}{2} \Rightarrow \alpha^2 = \frac{25}{4}$

$\therefore \beta^2 = \frac{27}{4}$

$\therefore \alpha^2 + \beta^2 = \frac{52}{4} = 13$

17. Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S. Let P be any point on S. Then the locus of the point, which internally divides OP in the ratio 3 : 1, is :

(1) $3y^2 = 2x$ (2) $2y^2 = 3x$

(3) $3x^2 = 2y$ (4) $2x^2 = 3y$

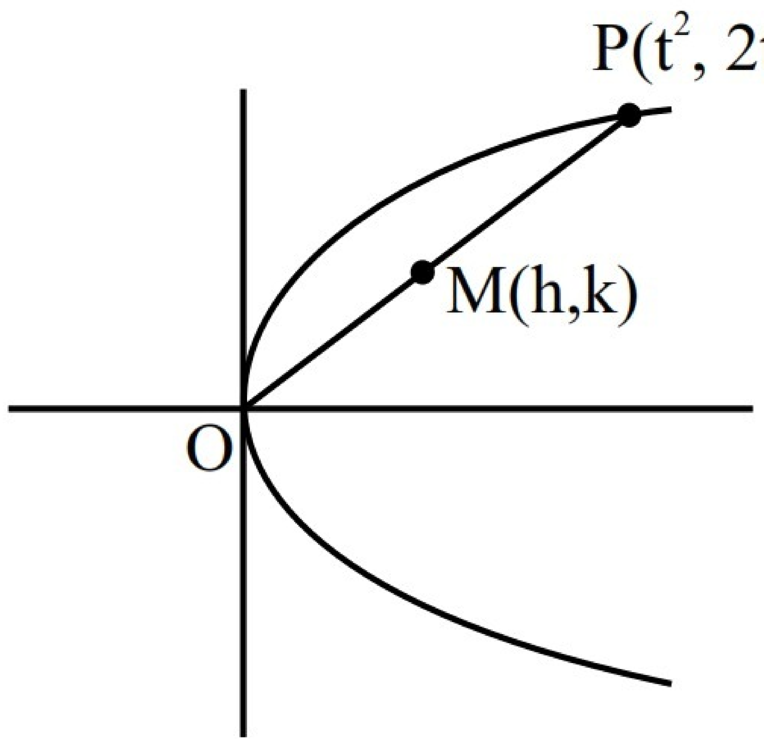
Ans. **(2)**

Sol. $y^2 = 4x$

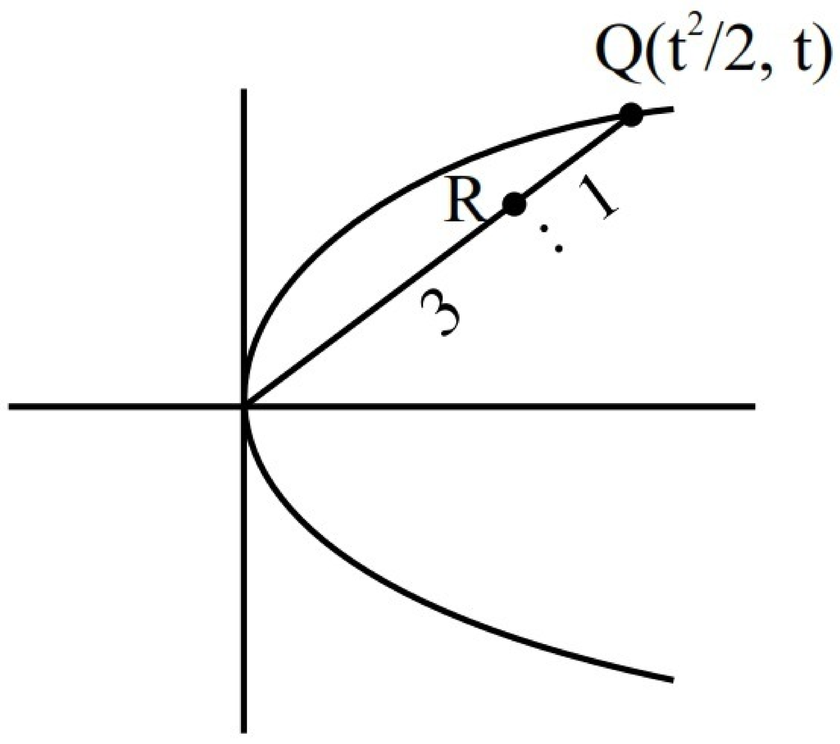
Locus of mid point of OP

$M(h, k) \Rightarrow h = \frac{t^2}{2}, k = t$

$\Rightarrow k^2 = 2h \Rightarrow y^2 = 2x$



$S : y^2 = 2x$



R(h, k)

$\Rightarrow h = \frac{\frac{3t^2}{2}}{4}, k = \frac{3t}{4}$

$t^2 = \frac{8h}{3}, t = \frac{4k}{3}$

$\Rightarrow \frac{16k^2}{9} = \frac{8h}{3} \Rightarrow 2k^2 = 3h$

Locus of R : $2y^2 = 3x$

18. Let $f(x) = [x]^2 - [x + 3] - 3, x \in \mathbb{R}$ where $[\bullet]$ is the greatest integer function. Then

(1) $f(x) > 0$ only for $x \in [4, \infty)$

(2) $f(x) < 0$ only for $x \in [-1, 3)$

(3) $\int_0^2 f(x)dx = -6$

(4) $f(x) = 0$ for finitely many values of x.

Ans. **(2)**

Sol. $f(x) = [x]^2 - [x] - 6 = ([x] + 2)([x] - 3)$

(1) $f(x) > 0 \Rightarrow [x] \in (-\infty, -2) \cup (3, \infty)$

$\Rightarrow x \in (-\infty, -2) \cup [4, \infty)$

(2) $f(x) < 0 \Rightarrow [x] \in (-2, 3)$

$\Rightarrow x \in [-1, 3)$

option (2) is correct

(3) $\int_0^2 f(x)dx = \int_0^1 (0 - 0 - 6)\,dx + \int_1^2 (1 - 1 - 6)\,dx$

$= -6 - 6$

$= -12$

(4) $f(x) = 0 \Rightarrow [x] = 3$ or $[x] = -2$

infinitely many solutions

19. Let f and g be functions satisfying $f(x+y) = f(x)f(y)$, $f(1) = 7$ and $g(x+y) = g(xy)$, $g(1) = 1$, for all $x, y \in \mathbb{N}$. $\sum_{x=1}^{n}\left(\frac{f(x)}{g(x)}\right) = 19607$, then n is equal to :

(1) 7 (2) 5

(3) 6 (4) 4

Ans. **(2)**

Sol. $f(x+y) = f(x).f(y) \Rightarrow f(x) = a^x$

$(\because f(1) = 7 \Rightarrow = a^1 = 7)$

So $f(x) = 7^x$

Now

$g(x+y) = g(xy)$ (put $y = 1$)

$\Rightarrow g(x+1) = g(x)$

so $g(1) = g(2) = g(3) = \ldots = g(n) = 1$

Given $\sum_{x=1}^{n}\frac{f(x)}{g(x)} = 19607$

$\sum_{x=1}^{n}\frac{7^x}{1} = 19607$

$\Rightarrow 7\left(\frac{7^n - 1}{7-1}\right) = 19607$

$7^n - 1 = \frac{6}{7} \times 19607$

$7^n = 16807 \Rightarrow n = 5$

20. Let C_r denote the coefficient of x^r in the binomial expansion of $(1 + x)^n$, $n \in \mathbb{N}$, $0 \le r \le n$.

If $P_n = C_0 - C_1 + \frac{2^2}{3}C_2 - \frac{2^3}{4}C_3 + \ldots.. + \frac{(-2)^n}{n+1}C_n$,

then the value of $\sum_{n=1}^{25}\frac{1}{P_{2n}}$ equals.

(1) 580 (2) 525

(3) 650 (4) 675

Ans. **(4)**

Sol. $P_n = \sum_{r=0}^{n}\frac{{}^nC_r(-2)^r}{r+1} = \sum_{r=0}^{n}\frac{1}{(n+1)}\,{}^{n+1}C_{r+1}(-2)^r$

$= \frac{-1}{2(n+1)}\sum_{r=0}^{n}{}^{n+1}C_{r+1}(-2)^{r+1}$

$= \frac{-1}{2(n+1)}\left[(1-2)^{n+1} - 1\right]$

$P_n = \frac{1}{2(n+1)}\left[1-(-1)^{n+1}\right]$

$P_{2n} = \frac{1}{2(2n+1)}\left[1-(-1)^{2n+1}\right]$

$P_{2n} = \frac{1}{2n+1}$

$\sum_{n=1}^{25}\frac{1}{P_{2n}} = \sum_{n=1}^{25}(2n+1)$

$= 3 + 5 + \ldots + 51$

$= \frac{25}{2}[51 + 3]$

$= 25 \times 27 = 675$

SECTION-B

21. Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}$, $\lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle θ, $\frac{\pi}{6} < \theta < \frac{\pi}{2}$, with the positive z-axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to ______.

Ans. **(5)**

Sol. $\frac{\vec{a}.\hat{k}}{|\vec{a}|} = \cos\theta \Rightarrow \frac{\lambda}{\sqrt{3+\lambda^2}} = \cos\theta$

$\Rightarrow 0 < \frac{\lambda}{\sqrt{3+\lambda^2}} < \frac{\sqrt{3}}{2}$

$\Rightarrow \lambda > 0 \;\&\; 4\lambda^2 < 9 + 3\lambda^2 \Rightarrow \lambda^2 < 9$

$\Rightarrow \lambda \in (0, 3)$ ....(1)

$\Rightarrow \vec{a}\cdot\vec{b} < 0 \Rightarrow -\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda < 0$

$\Rightarrow \lambda^2 - 4\lambda + 4 > 0 \Rightarrow (\lambda - 2)^2 > 0$

$\Rightarrow \lambda \ne 2$ ....(2)

from (1) & (2)

$\lambda \in (0, 3) - \{2\}$

$\therefore \alpha = 0, \beta = 3, \gamma = 2$

$\Rightarrow \alpha + \beta + \gamma = 5$

22. Let [•] be the greatest integer function. If $\alpha = \int_0^{64}\left(x^{1/3} - \left[x^{1/3}\right]\right)dx$, then

$\frac{1}{\pi}\int_0^{\alpha\pi}\left(\frac{\sin^2\theta}{\sin^6\theta + \cos^6\theta}\right)d\theta$ is equal to ____.

Ans. (36)

Sol. $\because \int_0^{64} x^{\frac{1}{3}}dx = \frac{3}{4}.\left[x^{\frac{4}{3}}\right]_0^{64} = 192$ &

$$\int_0^{64}\left[x^{1/3}\right]dx = \int_0^{1}\left[x^{1/3}\right]dx + \int_1^{8}\left[x^{1/3}\right]dx + \int_8^{27}\left[x^{1/3}\right]dx + \int_{27}^{64}\left[x^{1/3}\right]dx = 156$$

So $\alpha = 192 - 156 = 36$

Now $E = \frac{1}{\pi}\int_0^{36\pi}\frac{\sin^2\theta}{\sin^6\theta + \cos^6\theta}d\theta$

$$= \frac{36}{\pi}\int_0^{\pi}\frac{\sin^2\theta}{\sin^6\theta + \cos^6\theta}d\theta$$

$$\Rightarrow E = \frac{36 \cdot 2}{\pi}\int_0^{\pi/2}\frac{\sin^2\theta\, d\theta}{\sin^6\theta + \cos^6\theta}$$

Let $J = \int_0^{\pi/2}\frac{\sin^2\theta}{\sin^6\theta + \cos^6\theta}d\theta$ ....(1)

Applying King

$J = \int_0^{\pi/2}\frac{\cos^2\theta}{\sin^6\theta + \cos^6\theta}d\theta$ ....(2)

Now $2J = \int_0^{\pi/2}\frac{1}{\sin^6\theta + \cos^6\theta}d\theta$ (add (1) & (2))

$$= \int_0^{\pi/2}\frac{\sec^6\theta}{\tan^6\theta + 1}d\theta$$

$$= \int_0^{\infty}\frac{(1+\lambda^2)}{\lambda^4 - \lambda^2 + 1}d\lambda$$

$$= \int_0^{\infty}\frac{1 + \frac{1}{\lambda^2}}{\lambda^2 - 1 + \frac{1}{\lambda^2}}d\lambda$$

$= \pi$

$\Rightarrow J = \frac{\pi}{2}$

$\Rightarrow E = \frac{36 \cdot 2}{\pi} \times J = 36$

23. Let $\cos(\alpha + \beta) = -\frac{1}{10}$ and $\sin(\alpha - \beta) = \frac{3}{8}$, where $0 < \alpha < \frac{\pi}{3}$ and $0 < \beta < \frac{\pi}{4}$.

If $\tan 2\alpha = \frac{3(1 - r\sqrt{5})}{\sqrt{11}(s + \sqrt{5})}$, r, s $\in \mathbb{N}$, then r + s is equal to______ .

Ans. (20)

Sol. $\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$\tan 2\alpha = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta).\tan(\alpha-\beta)}$$

$$\tan 2\alpha = \frac{\left(-\sqrt{99} + \frac{3}{\sqrt{55}}\right)}{1 - \left(\sqrt{99}\right)\left(\frac{3}{\sqrt{55}}\right)}$$

$$\tan 2\alpha = \frac{-3\sqrt{11} + \frac{3}{\sqrt{5}\times\sqrt{11}}}{1 + \frac{9\sqrt{11}}{\sqrt{5}\times\sqrt{11}}}$$

$$\tan 2\alpha = \frac{3\left(1 - 11\sqrt{5}\right)}{\sqrt{11}\left(9 + \sqrt{5}\right)}$$

$r = 11, s = 9$

$r + s = 20$

24. Suppose a, b, c are in A.P. and a^2, $2b^2$, c^2 are in G.P. If $a < b < c$ and $a + b + c = 1$, then $9(a^2 + b^2 + c^2)$ is equal to______ .

Ans. (9)

Sol. $a = b - d,\ c = b + d, \Rightarrow\ b = \frac{1}{3}$

$\Rightarrow 4b^4 = a^2c^2$

$\Rightarrow 4b^4 = [(b - d)(b + d)]^2$

$\Rightarrow \frac{4}{81} = \left(\frac{1}{9} - d^2\right)^2$

$\Rightarrow \left(\frac{1}{9} - d^2\right) = \pm\frac{2}{9}$

$d^2 = 1/3 \Rightarrow\ d = +\frac{1}{\sqrt{3}}$ (as $a < b < c$)

$\therefore\ 9(a^2 + b^2 + c^2)$

$= 9\left[\left(\frac{1}{3} - \frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} + \frac{1}{\sqrt{3}}\right)^2\right]$

$= 9\left[\frac{1}{3} + \frac{2}{3}\right] = 3 + 6 = 9$

25. Let S be the set of the first 11 natural numbers. Then the number of elements in $A = \{B \subseteq S : n(B) \geq 2$ and the product of all elements of B is even$\}$ is ______ .

Ans. (1979)

Sol. $A\ \{1,2,3.....11\}$

$\therefore\ n(B) \geq 2$ & product of all elements in B is even

Case (i) $n(B) = 2 \Rightarrow {}^{11}C_2 - {}^{6}C_2$

$n(B) = 3 \Rightarrow {}^{11}C_3 - {}^{6}C_3$

$n(B) = 4 \Rightarrow {}^{11}C_4 - {}^{6}C_4$

$n(B) = 5 \Rightarrow {}^{11}C_5 - {}^{6}C_5$

$n(B) = 6 \Rightarrow {}^{11}C_6 - {}^{6}C_6$

$n(B) = 7 \Rightarrow {}^{11}C_7$

:

:

$n(B) = 11 \Rightarrow {}^{11}C_{11}$

$\therefore$ number of set $B \Rightarrow \sum_{r=2}^{11} {}^{11}C_r - \sum_{r=2}^{6} {}^{6}C_r$

$= 2^{11} - (12) - (2^6 - 7)$

$= 2048 - 64 - 5$

$= 1979$

Alternate Solution :

Total subsets $= 2^{11}$

No. of subsets having odd terms only $= 2^6$

No. of subsets having one term only & also having even terms = 5

Req. ways $= 2^{11} - 2^6 - 5 = 1979$

PHYSICS

SECTION-A

26. If $\in$, E and t represent the free space permittivity, electric field and time respectively, then the unit of $\frac{\in E}{t}$ will be :

(1) Am (2) Am^2

(3) A/m^2 (4) A/m

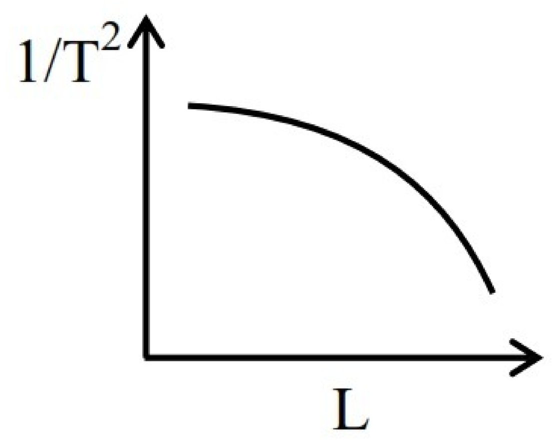
Ans. (3)

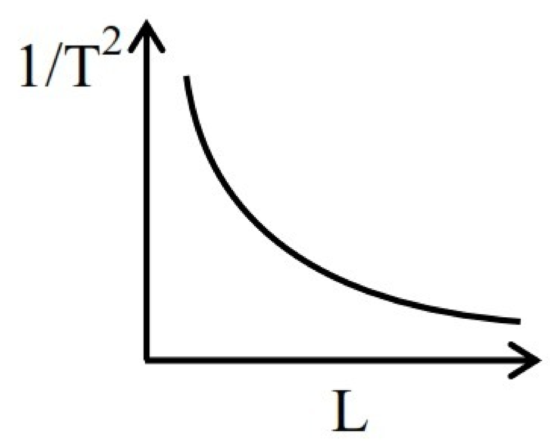
Sol. $\frac{\varepsilon E}{t} = \frac{\varepsilon}{t}\frac{1}{4\pi\varepsilon}\frac{q}{r^2}$

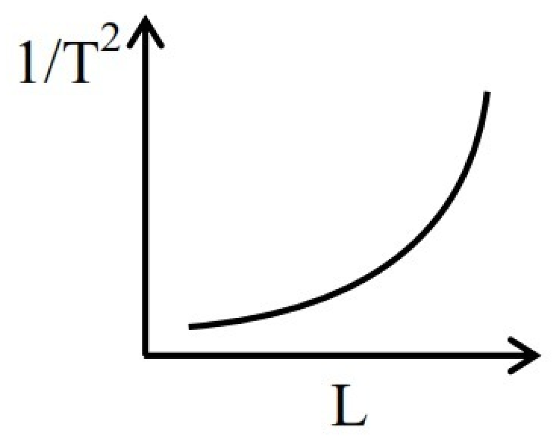
$\Rightarrow \frac{AT}{TL^2} = \left(AL^{-2}\right)$

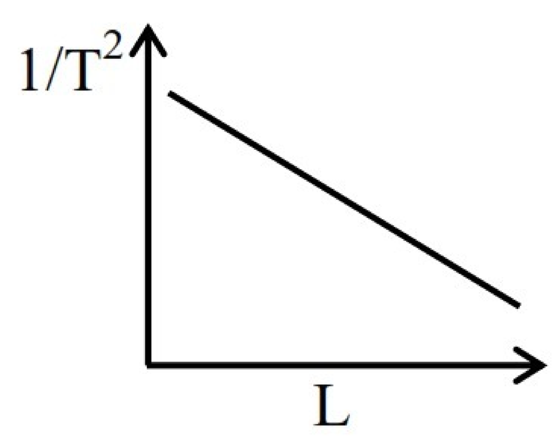
$\Rightarrow A/m^2$

27. Using a simple pendulum experiment g is determined by measuring its time period T. Which of the following plots represent the correct relation between the pendulum length L and time period T ?

(1)


(2)


(3)


(4)


Ans. (2)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$ $T^2 = \frac{4\pi^2\ell}{g}$

$\frac{1}{T^2} = \frac{g}{4\pi^2\ell}$

28. Consider two boxes containing ideal gases A and B such that their temperatures, pressures and number densities are same. The molecular size of A is half of that of B and mass of molecule A is four times that of B. If the collision frequency in gas B is 32×10^{18}/s then collision frequency in gas A is _____/s.

(1) 32×10^8 (2) 4×10^8

(3) 2×10^8 (4) 8×10^8

Ans. (2)

Sol. Collision frequency (z) $= \sqrt{2}\pi d^2 N\sqrt{\frac{8RT}{\pi M}}$

Temp, N are same

$Z \propto \frac{d^2}{\sqrt{M}}$

$d_A = \frac{d_B}{2}$

$M_A = 4M_B$

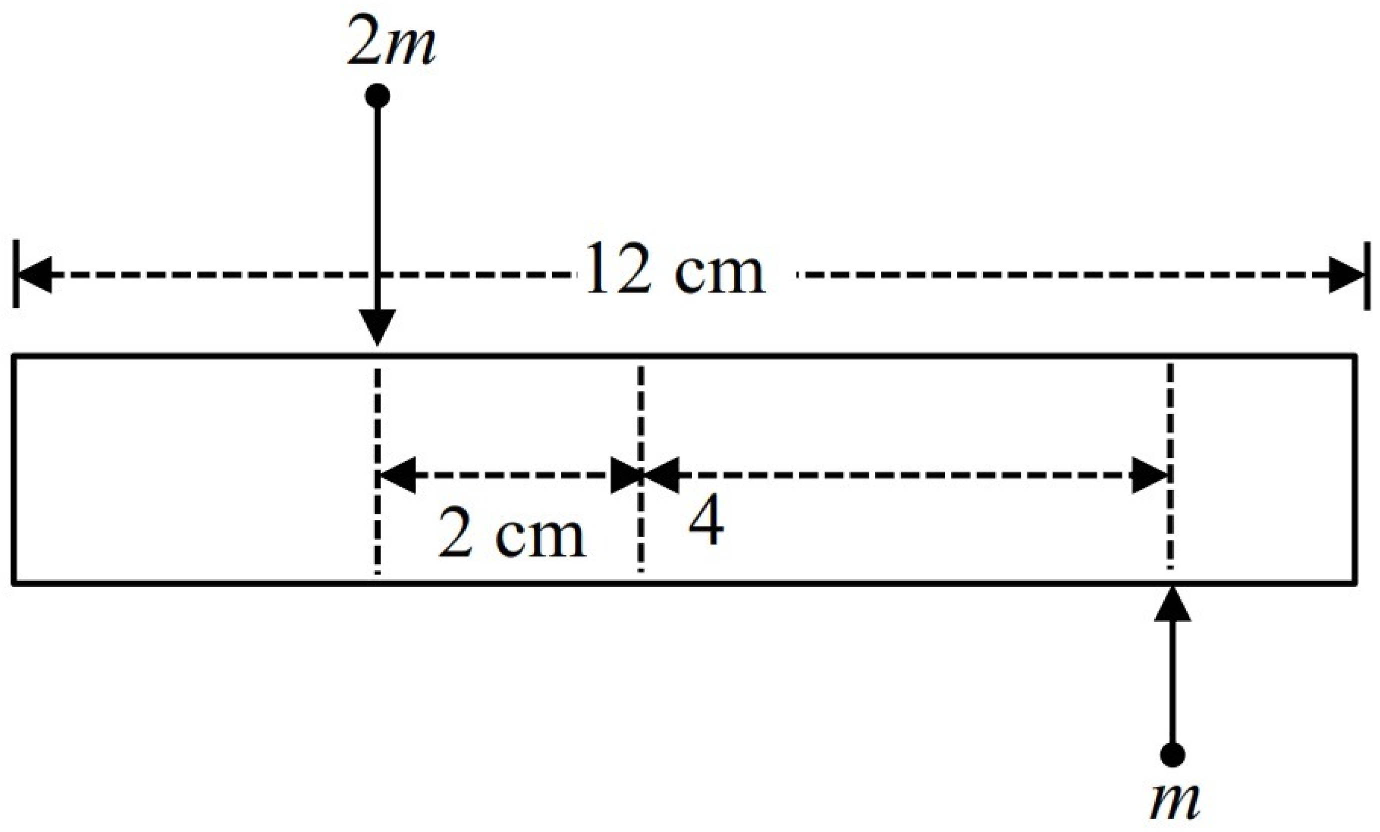
$\frac{Z_A}{Z_B} = \frac{d_A^2}{\sqrt{M_A}} \times \frac{\sqrt{M_B}}{d_B^2} = \left(\sqrt{\frac{M_B}{M_A}}\right)\left(\frac{d_A}{d_B}\right)^2$

$= \left(\sqrt{\frac{1}{4}}\right)\left(\frac{1}{2}\right)^2$

$\frac{Z_A}{Z_B} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\Rightarrow Z_A = \frac{32\times10^8}{8} = 4\times10^8\ /\ s$

29. A uniform bar of length 12 cm and mass 20 *m* lies on a smooth horizontal table. Two point masses m and 2 *m* are moving in opposite directions with same speed of v and in the same plane as the bar, as shown in figure. These masses strike the bar simultaneously and get stuck to it. After collision the entire system is rotating with angular frequency ω. The ratio of v and ω is :

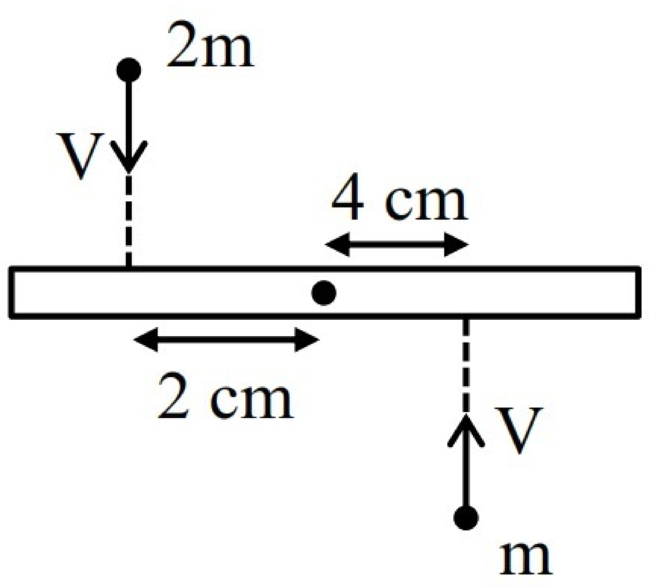


(1) 33 (2) $2\sqrt{88}$

(3) 66 (4) 32

Ans. **(1)**

Sol.



Using angular momentum conservation about COM of rod :

$L_i = L_f$

$$m\times V\times 4 + 2m\times V\times 2 = \left(\frac{20m(12)^2}{12} + m\times 4^2 + 2m\times 2^2\right)\omega$$

$8mV = (240m + 24\ m)\omega$

$8V = 264\ \omega$

$\frac{V}{\omega} = 33$

30. Three small identical bubbles of water having same charge on each coalesce to form a bigger bubble. Then the ratio of the potentials on one initial bubble and that on the resultant bigger bubble is :

(1) $1 : 3^{1/3}$ (2) $1 : 2^{2/3}$

(3) $3^{2/3} : 1$ (4) $1 : 3^{2/3}$

Ans. **(4)**

Sol. Using volume conservation

$$3\left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3}\pi R^3\right)$$

$R = 3^{1/3}\ r$

$$\frac{V_i}{V_f} = \frac{\frac{kq}{r}}{\frac{k3q}{R}} = \frac{R}{3r} = \frac{3^{1/3}r}{3r} = \frac{1}{3^{2/3}}$$

31. In parallax method for the determination of focal length of a concave mirror, the object should always be placed :

(1) between the focus (F) and the centre of curvature (C) of the mirror ONLY

(2) at any point beyond the focus (F) of the mirror

(3) beyond the centre of the curvature (C) of the mirror ONLY

(4) between the pole (P) and the focus (F) of the concave mirror ONLY

Ans. **(2)**

Sol. Image should be real. So object should be placed beyond focus.

32. The smallest wavelength of Lyman series is 91 nm. The difference between the largest wavelengths of *Paschen* and *Balmer* series is nearly ____ nm.

(1) 1875 (2) 1550

(3) 1217 (4) 1784

Ans. **(3)**

Sol.

n = 4 ————————

n = 3 ———————— paschen

n = 2 ———————— Balmer

n = 1 ———————— Lyman

Smallest wavelength of lyman

$$\frac{1}{\lambda} = R\left(\frac{1}{12} - \frac{1}{\infty^2}\right)$$

$$R = \frac{1}{\lambda} = \frac{1}{91}\text{nm}^{-1}$$

λ_{max} for balmer series

$n_1 = 2 \rightarrow n_2 = 3$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda_B} = \frac{1}{91}\left(\frac{5}{36}\right)$$

$$\lambda_B = \left(\frac{91 \times 36}{5}\right) = 655.2\text{nm}$$

λ_{max} paschen

$n_1 = 3 \quad \rightarrow \quad n_2 = 4$

$$\frac{1}{\lambda_p} = \frac{1}{91}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{1}{91} \times \frac{7}{144}$$

$$\lambda_p = \left(\frac{91 \times 144}{7}\right) = 1872\text{nm}$$

$$\Delta\lambda = \lambda_P - \lambda_B = 1872 - 655.2$$

$$\Delta\lambda = 1216.8$$

$$\Delta\lambda \approx 1217$$

33. In an open organ pipe ν_3 and ν_6 are 3rd and 6th harmonic frequencies, respectively.

If $\nu_6 - \nu_3$ = 2200 Hz then length of the pipe is ______ mm.

(Take velocity of sound in air is 330 m/s.)

(1) 275 (2) 225

(3) 200 (4) 250

Ans. (2)

Sol. $f = n\left(\frac{V_0}{2L}\right)$

$$\frac{6V_0}{2L} - \frac{3V_0}{2L} = 2200$$

$$\frac{3V_0}{2L} = 2200$$

$$\frac{3 \times 330}{2 \times L} = 2200$$

$$L = \frac{3 \times 330}{2 \times 2200}$$

$$L = 0.225 \text{ m}$$

$$L = 225 \text{ mm}$$

34. When a part of a straight capillary tube is placed vertically in a liquid, the liquid raises upto certain height h. If the inner radius of the capillary tube, density of the liquid and surface tension of the liquid decrease by 1 % each, then the height of the liquid in the tube will change by _____%.

(1) –1 (2) +3

(3) –3 (4) +1

Ans. (4)

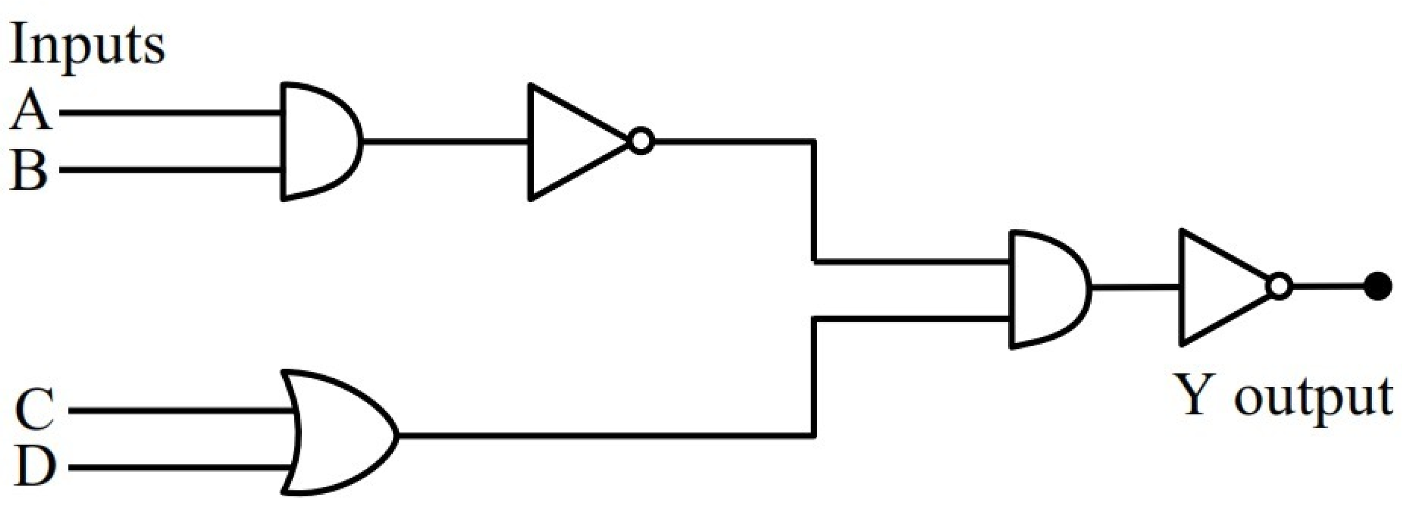
Sol. $h = \frac{2T\cos\theta}{\rho g r}$

$$\frac{\Delta h}{h}\% = \frac{\Delta T}{T}\% - \frac{\Delta\rho}{\rho}\% - \frac{\Delta r}{r}\%$$

$$\frac{\Delta h}{h}\% = 1 + 1 + 1$$

$$\frac{\Delta h}{h} = +1\%$$

35. The correct truth table for the given input data of the following logic gate is :



(1)

Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	1
1	1	1	1	0

(2)

Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	0
1	1	1	1	1

(3)

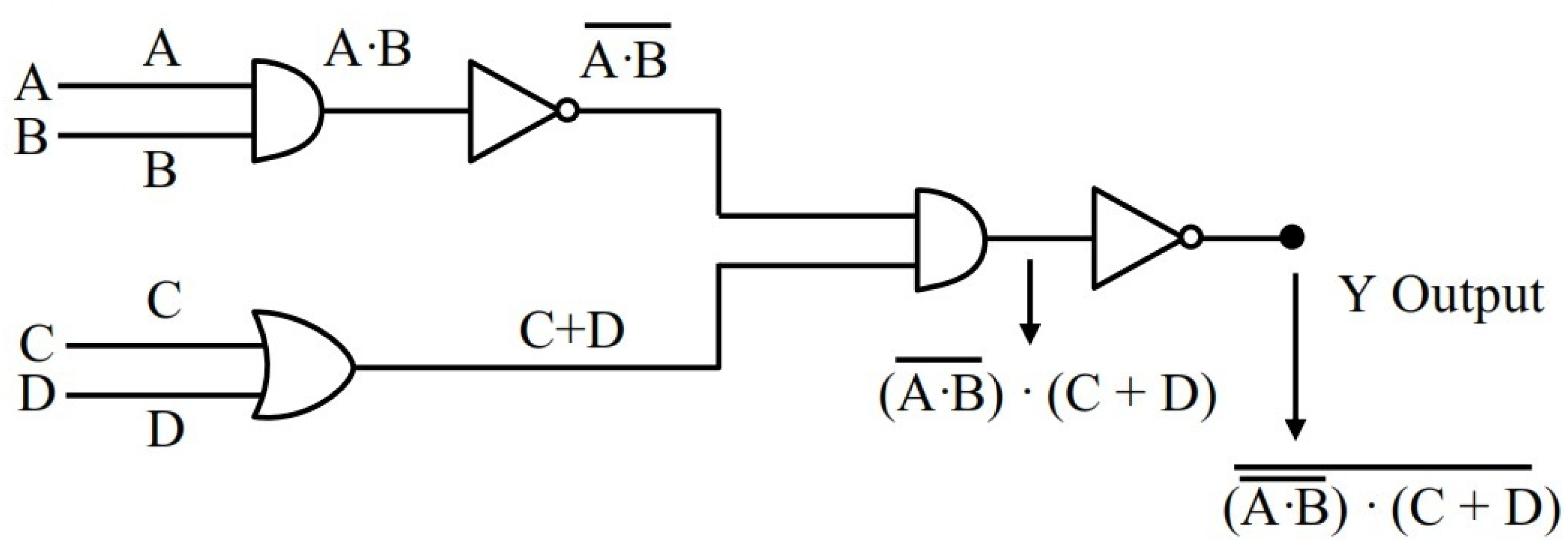
Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	0
1	0	1	0	1
1	1	1	1	1

(4)

Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	1
1	0	1	0	1
1	1	1	1	1

Ans. **(2)**

Sol.



$$Y = \overline{(\overline{A.B}).(C+D)} = \overline{\overline{A \cdot B}} + \left(\overline{C+D}\right)$$

$$Y = (A.B) + (\overline{C+D})$$

36. An electric power line having total resistance of 2 Ω, delivers 1 kW of power of 250 V. The percentage efficiency of transmission line is ____ .

(A) 96.9 (B) 86.5

(C) 100 (D) 92.5

Ans. **(1)**

Sol. $P_{out} = 1000$ W

$P = VI$

$1000 = 250 \times I$

$I = 4A$

$P_{loss} = I^2R = (4)^2 \times 2 = 3200$

$P_{net} = 1000 + 32 = 103200$

$$\eta = \left(\frac{P_{out}}{P_{net}}\right) \times 100 = \frac{1000}{1032} \times 100 = 96.9\%$$

37. The wavelength of light, while it is passing through water is 540 nm. The refractive index of water is $\frac{4}{3}$. The wavelength of the same light when it is passing through a transparent medium having refractive index of $\frac{3}{2}$ is __________nm.

(1) 380 (2) 840

(3) 480 (4) 540

Ans. **(3)**

Sol. $\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} \Big|\; v = f\lambda$

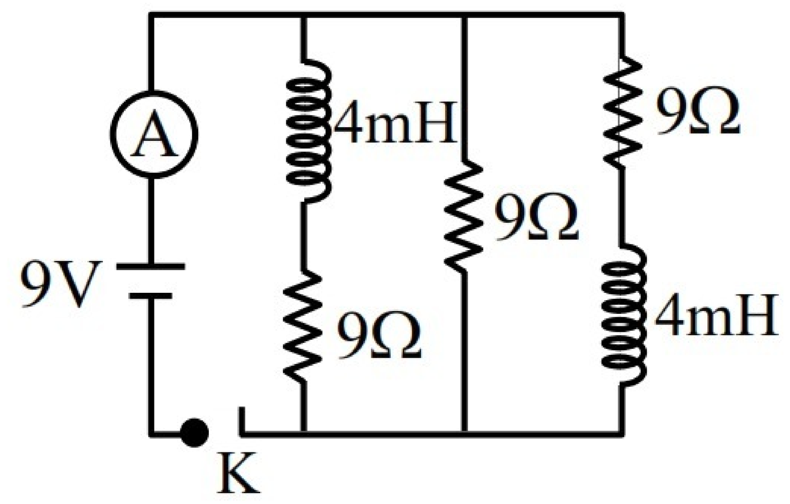
$$\frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1}$$

$$\frac{\mu_B}{3/2} = \frac{\lambda}{540}$$

$$\lambda = \left(\frac{4 \times 2}{3 \times 3} \times 540\right)$$

$\lambda = 480$ nm

38. Figure shows the circuit that contains three resistances (9 Ω each) and two inductors (4 mH each). The reading of ammeter at the moment switch K is turned ON, is _________A.



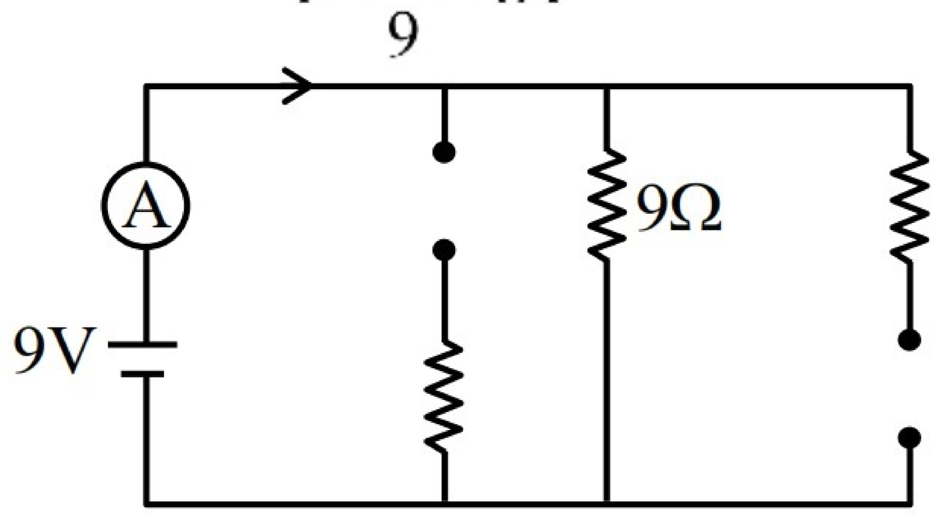
(1) 1 (2) zero

(3) 3 (4) 2

Ans. **(1)**

Sol. Just after closing the switch, inductor will behave as open circuit,

$$I = \frac{9}{9} = 1A$$



39. Given below are two statements :

Statement I : A satellite is moving around earth in the orbit very close to the earth surface. The time period of revolution of satellite depends upon the density of earth.

Statement II : The time period of revolution of the satellite is $T = 2\pi\sqrt{\frac{R_e}{g}}$ (for satellite very close to the earth surface), where R_e radius of earth and g acceleration due to gravity.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are false

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. **(2)**

Sol. $T = 2\pi\sqrt{\frac{R^3}{GM}}$

$\because M = \rho \cdot \frac{4}{3}\pi R^3$

$\therefore T = 2gp\sqrt{\frac{1}{G\rho\frac{4}{3}\pi}}$

Statement I is correct.

And $\therefore \frac{GM}{R^2} = g$

$\therefore T = 2\pi\sqrt{\frac{R}{g}}$

Statement II is correct

Ans. (2)

40. Which of the following are true for a single slit diffraction?

(A) Width of central maxima increases with increase in wavelength keeping slit width constant.

(B) Width of central maxima increases with decrease in wavelength keeping slit width constant.

(C) Width of central maxima increases with decrease in slit width at constant wavelength.

(D) Width of central maxima increases with increase in slit width at constant wavelength.

(E) Brightness of central maxima increases for decrease in wavelength at constant slit width.

Options :

(1) A, D, E only (2) A, D only

(3) B, D only (4) B, C only

Ans. (1)

Sol. $\beta_{cm} = \frac{2\lambda D}{a}$

(A) Correct $\beta \propto \lambda$

(B) Incorrect

(C) Correct $\beta \propto \frac{1}{d}$

(D) Incorrect

(E) Correct

Statement A, C & E are correct.

No option matching

41. Given below are two statements :

Statement I : An object moves from position r_1 to position r_2 under a conservative force field $\vec{F}$. The work done by the force is $W = -\int_{r_1}^{r_2} \vec{F} \cdot \overrightarrow{dr}$.

Statement II : Any object moving from one location to another location can follow infinite number of paths. Therefore, the amount of work done by the object changes with the path it follows for a conservative force.

In the light of the above statements, choose the **correct answer** from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are false

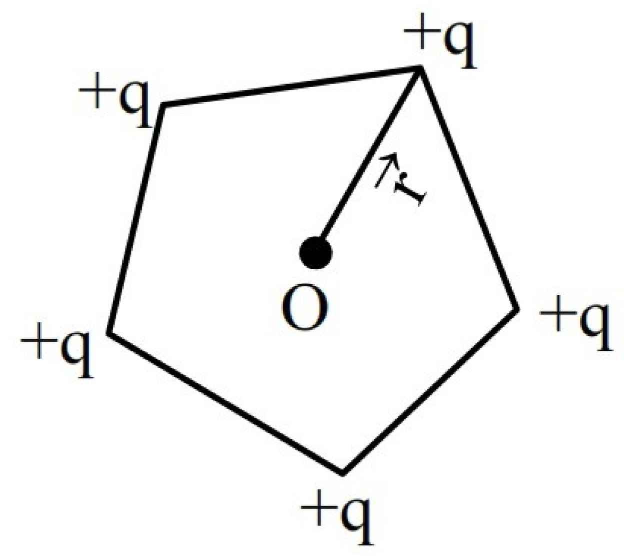
Ans. (3)

Sol. Statement-I : Incorrect

Correct equation is $W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$

Statement-II : Incorrect

42. Five positive charges each having charge q are placed at the vertices of a pentagon as shown in the figure. The electric potential (V) and the electric field $(\vec{E})$ at the center O of the pentagon due to these five positive charges are :



(1) $V = \frac{5q}{4\pi\varepsilon_0 r}$ and $\vec{E} = 0$

(2) $V = \frac{5q}{4\pi\varepsilon_0 r}$ and $\vec{E} = \frac{5\sqrt{3}q}{8\pi\varepsilon_0 r^2}\hat{r}$

(3) $V = \frac{5q}{4\pi\varepsilon_0 r}$ and $\vec{E} = \frac{5q}{4\pi\varepsilon_0 r^2}\hat{r}$

(4) $V = 0$ and $\vec{E} = 0$

Ans. (1)

Sol. Electric potential $\rightarrow V = \frac{5kq}{R}$

As regular polygon $\rightarrow \vec{E} = 0$

43. A laser beam has intensity of 4.0×10^{14} W/m^2. The amplitude of magnetic field associated with beam is __________ T. (Take $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 and c = 3×10^8 m/s)

(1) 2.0 (2) 18.3

(3) 5.5 (4) 1.83

Ans. (4)

Sol. $I = \frac{1}{2}\epsilon_0 E_0^2 \cdot C$

$\therefore E_0 = \sqrt{\frac{2I}{\epsilon_0 C}}$

& $\frac{E_0}{B_0} = C$

$\therefore B_0 = \frac{E_0}{C} = \frac{1}{C}\sqrt{\frac{2I}{\epsilon_0 C}}$

$\therefore B_0 = \frac{1}{3\times10^8}\sqrt{\frac{2\times4\times10^{14}}{8.85\times10^{-12}\times3\times10^8}}$

$B_0 = \frac{10}{3}\sqrt{\frac{8}{8.85\times3}}$

$B_0 = 1.83$ T

44. Light is incident on a metallic plate having work function 110×10^{-20} J. If the produced photoelectrons have zero kinetic energy then the angular frequency of the incident light is _________ rad/s. ($h = 6.63 \times 10^{-34}$ J.s)

(1) 1.04×10^{16} (2) 1.04×10^{13}

(3) 1.66×10^{16} (4) 1.66×10^{15}

Ans. (1)

Sol. $\phi = h\nu$

$$\nu = \frac{\phi}{h}$$

$$\omega = 2\pi\nu = \frac{2\pi\phi}{h} = \frac{2\times3.14\times110\times10^{-2}}{6.63\times10^{-34}}$$

$\omega = 1.04 \times 10^{16}$ rad/sec

45. Given below are two statements :

Statement I : For a mechanical system of many particles total kinetic energy is the sum of kinetic energies of all the particles.

Statement II : The total kinetic energy can be the sum of kinetic energy of the center of mass w.r.t. to the origin and the kinetic energy of all the particles w.r.t. the center of mass as the reference.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is true but **Statement II** is false

(3) **Statement I** is false but **Statement II** is true

(4) Both **Statement I** and **Statement II** are false

Ans. (1)

Sol. $KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$KE = \frac{1}{2}(m_1+m_2)v_{cm}^2 + \frac{1}{2}\frac{m_1m_2}{m_1+m_2}|\vec{v}_1 - \vec{v}_2|^2$$

So Ans. is (1)

SECTION-B

46. A conducting circular loop is rotated about its diameter at a constant angular speed of 100 rad/s in a magnetic field of 0.5T perpendicular to the axis of rotation. When the loop is rotated by 30° from the horizontal position, the induced EMF is 15.4 mV. The radius of the loop is ______ mm.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

Ans. (14)

Sol. $E = B\omega A\sin\omega t$

$$15.4\times10^{-3} = \frac{1}{2}\times100\times\frac{22}{7}r^2\times\frac{1}{2}$$

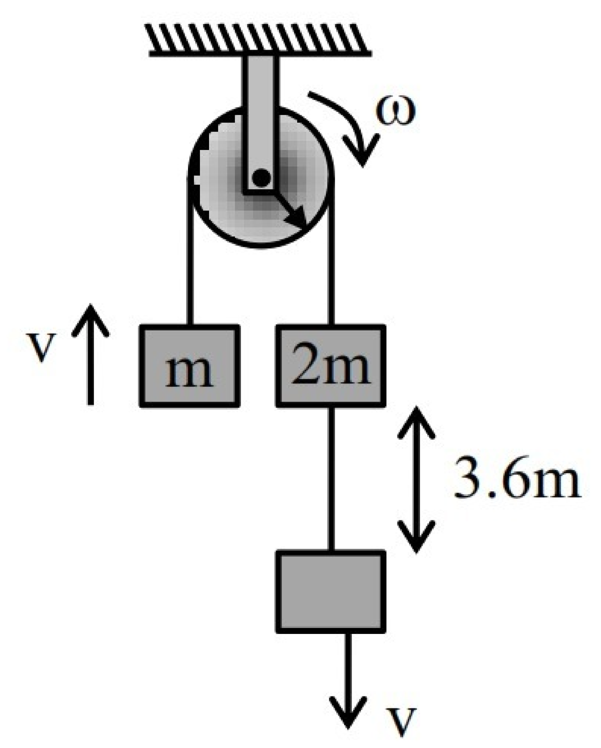
$$r = \sqrt{\frac{15.4\times28\times10^{-5}}{22}}$$

R = 14 mm

47. Two masses m and 2m are connected by a light string going over a pulley (disc) of mass 30m with radius r = 0.1 m. The pulley is mounted in a vertical plane and it is free to rotate about its axis. The 2m mass is released from rest and its speed when it has descended through a height of 3.6 m is _______ m/s. (Assume string does not slip and $g = 10$ m/s^2)

Ans. (2)

Sol.



Using energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + \frac{1}{2}\frac{30mR^2}{2}\times\frac{v^2}{R^2} = mgh$$

$9\,mv^2 = mgh$

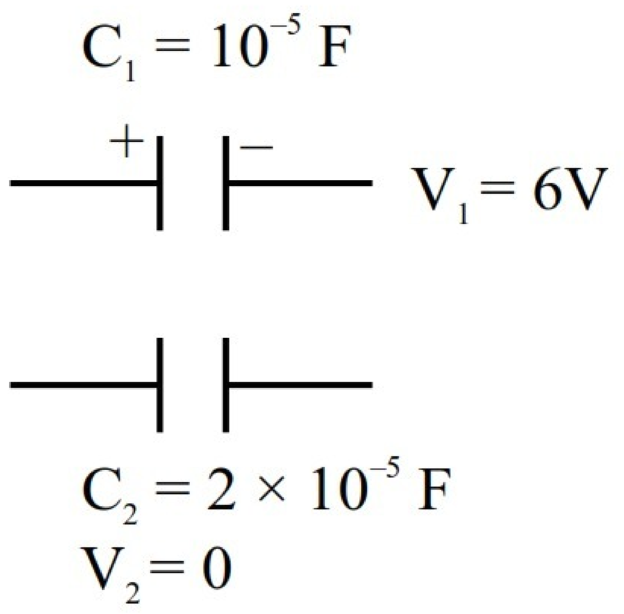
$$v = \sqrt{\frac{gh}{9}} = \sqrt{\frac{10\times3.6}{9}}$$

$v = \sqrt{4} = 2m/s$

48. A capacitor *P* with capacitance 10×10^{-6} F is fully charged with a potential difference of 6.0 V and disconnected from the battery. The charged capacitor *P* is connected across another capacitor *Q* with capacitance 20×10^{-6} F. The charge on capacitor *Q* when equilibrium is established will be $\alpha \times 10^{-5}$ C (assume capacitor *Q* does not have any charge initially), the value of α is _______ .

Ans. **(4)**

Sol.



$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{10^{-5} \times 6 + 0}{3 \times 10^{-5}}$$

V = 2 volt

$Q_2 = C_2V = 2 \times 10^{-5} \times 2 = 4 \times 10^{-5}$ C

49. A cylindrical conductor of length 2m and area of cross-section 0.2 mm^2 carries an electric current of 1.6 A when its ends are connected to a 2V battery. Mobility of electrons in the conductor is $\alpha \times 10^{-3}$ m^2/V.s. The value of α is :

(electron concentration = $5 \times 10^{28}/m^3$ and electron charge = 1.6×10^{-19} C)

Ans. **(1)**

Sol. $V_\ell = \mu E = \mu \times \frac{V}{\ell}$

$I = neAV_d$

$$V_d = \frac{I}{neA}$$

$$\mu = \frac{I\ell}{NneA}$$

$$\mu = \frac{1.6 \times 3}{2 \times 5 \times 10^{26} \times 1.6 \times 10^{-19} \times 2 \times 10^{-7}}$$

$\mu = 1 \times 10^{-3}$ m^2/v·s

$\alpha = 1$

50. An insulated cylinder of volume 60 cm^3 is filled with a gas at 27°C and 2 atmospheric pressure. Then the gas is compressed making the final volume as 20 cm^3 while allowing the temperature to rise to 77°C. The final pressure is _______ atmospheric pressure.

Ans. **(7)**

Sol. PV = nRT

$$\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$$

$$\frac{2 \times 10^5 \times 60}{300} = \frac{P_2 \times 20}{350}$$

$$P_2 = \frac{2 \times 10^5 \times 7 \times 3}{6}$$

$P_2 = 7 \times 10^5 = 7$ atm

CHEMISTRY

SECTION-A

51. At T(K), 100 g of 98% H_2SO_4 (w/w) aqueous solution is mixed with 100 g of 49% H_2SO_4 (w/w) aqueous solution. What is the mole fraction of H_2SO_4 in the resultant solution ?

(Given : Atomic mass H = 1 u ; S = 32 u; O = 16 u)

(Assume that temperature after mixing remains constant)

(1) 0.9 (2) 0.1

(3) 0.337 (4) 0.663

Ans. **(4)**

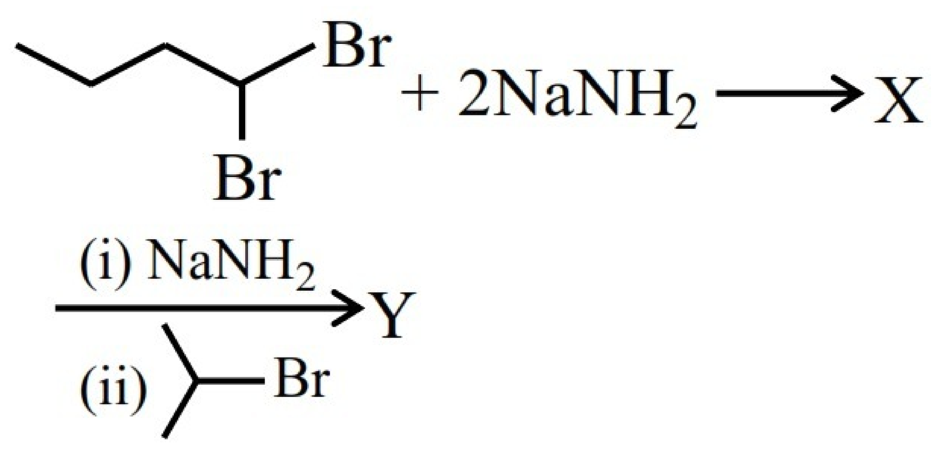
Sol. Total weight of H_2SO_4

$$= \left(100\times\frac{98}{100}\right) + \left(100\times\frac{49}{100}\right) = 147 \text{ gm}$$

Total weight of $H_2O = 200 - 147 = 53$ gm

$$\text{Mole fraction of } H_2SO_4 = \frac{\frac{147}{98}}{\left(\frac{147}{98}+\frac{53}{18}\right)} = 0.337$$

52. Consider the following reaction :



The product Y formed is :

(1) 2-methylhex-2-yne

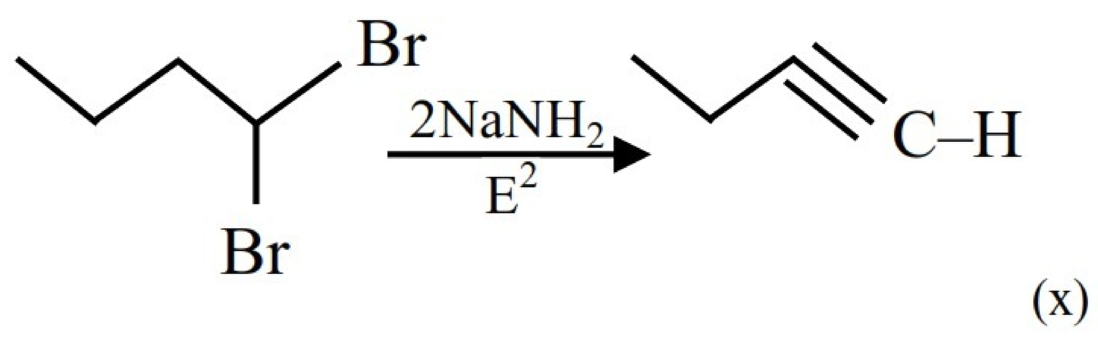
(2) 5-methylhex-2-yne

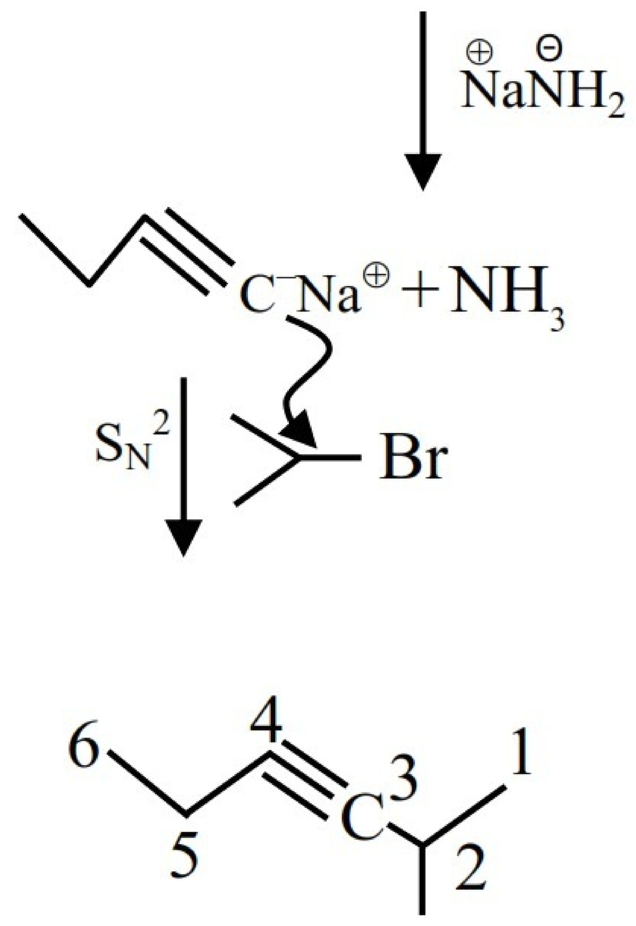
(3) 2-methylhex-3-yne

(4) Isopropylbut-1-yne

Ans. **(3)**

Sol.





2–methyl hex–3–yne

53. $A + 2B \longrightarrow AB_2$

36.0 g of 'A' (Molar mass : 60 g mol^{-1}) and 56.0 g of 'B' (Molar mass : 80 g mol^{-1}) are allowed to react. Which of the following statements are **correct** ?

(A) 'A' is the limting reagent

(B) 77.0 g of AB_2 is formed

(C) Molar mass of AB_2 is 140 g mol^{-1}

(D) 15.0 g of A is left unreacted after the completion of reaction.

Choose the **correct** answer from the options given below :

(1) C and D only (2) A and C only

(3) B and D only (4) A and B only

Ans. **(3)**

Sol. $A \quad + \quad 2B \longrightarrow \quad AB_2$

Mole $\frac{36}{60}$ $\frac{56}{80}$

0.6 mole 0.7 mole

0.25 mole – 0.35 mole

(A) Molecular wt. of AB_2 is

$60 + 2 \times 80 = 220$ g/mol

(B) LR is AB.

(C) Wt. of A remaining = 0.25 × 60 = 15 g

(D) wt. of AB_2 formed = 0.35 × 220 = 77 gm.

54. Given below are two statements :

Statement-I L: The first ionization enthalpy of Cr is lower than that of Mn.

Statement-II : The second and third ionization enthalpies of Cr are higher than those of Mn.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement-I and Statement-II are false.

(2) Statement-I is true but Statement-II is false.

(3) Both Statement-I and Statement-II are true.

(4) Statement-I is false but Statement-II is true.

Ans. (2)

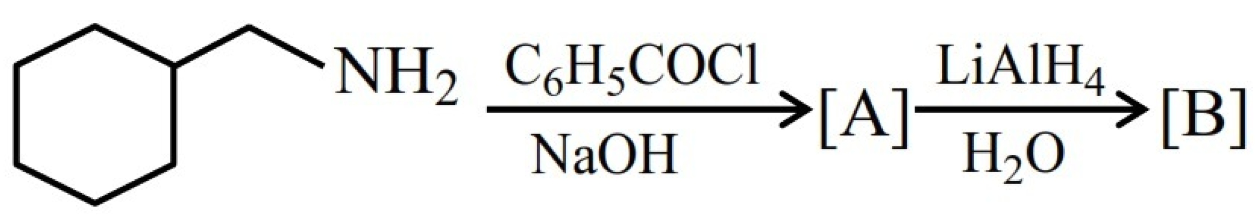
Sol. $Cr = (Ar)\ 3d^5 4s^1$

$Mn = (Ar)3d^5 4s^2$

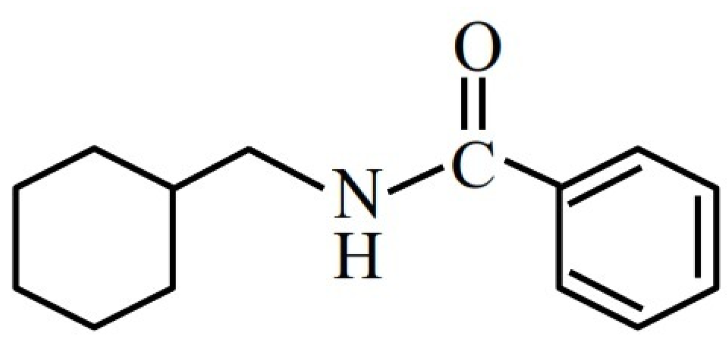
$IE_1\ (Cr) < IE_1\ (Mn)$

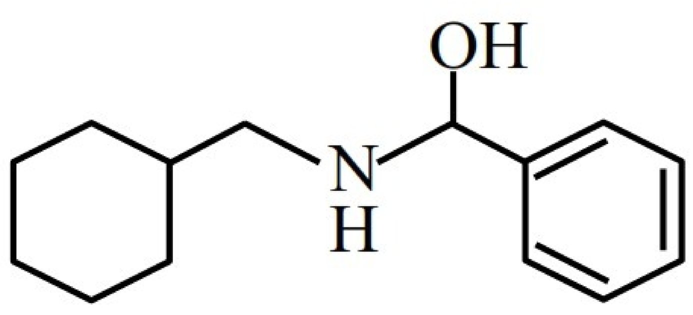
$IE_2\ (Cr) > IE_2\ (Mn)$

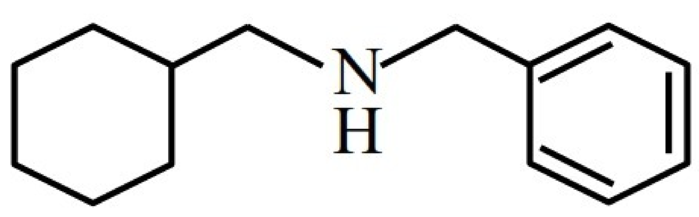
$IE_3\ (Cr) < IE_3\ (Mn)$

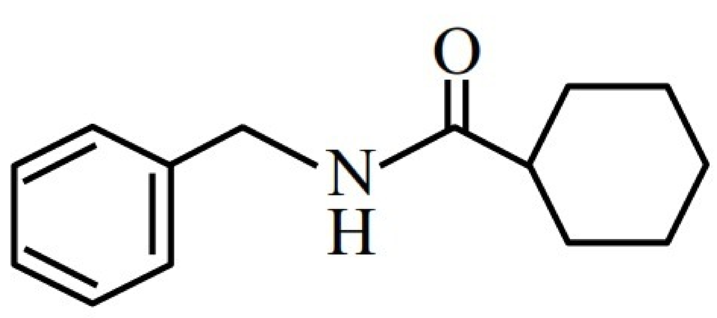
55. 

The final product [B] is :

(1) 

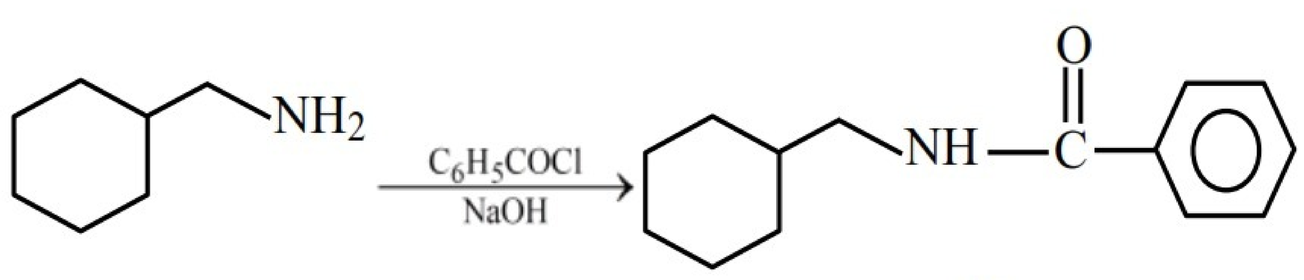
(2) 

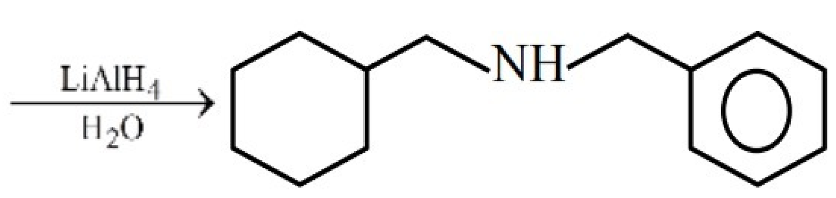
(3) 

(4) 

Ans. (3)

Sol.





56. When 1 g of compound (X) is subjected to Kjeldahl's method for estimation of nitrogen, 15 mL, 1M H_2SO_4 was neutralized by ammonia evolved. The percentage of nitrogen in compound (X) is :

(1) 21

(2) 0.42

(3) 42

(4) 0.21

Ans. (3)

Sol. eq. of H_2SO_4 = eq. of Ammonia

$\Rightarrow \dfrac{15\times1\times2}{1000}$ = moles of ammonia × 1

$\Rightarrow$ Moles of ammonia = moles of 'N'

$\Rightarrow$ Weight of nitrogen = $\dfrac{15\times1\times2}{1000}\times14 = 0.42$

% weight of 'N' = $\dfrac{0.42}{1}\times100 = 42\%$

57. Correct statements regarding Arrhenius equation among the following are :

(A) Factor $e^{-Ea/RT}$ corresponds to fraction of molecules having kinetic energy less than Ea.

(B) At a given temperature, lower the Ea, faster is the reaction.

(C) Increase in temperature by about 10°C doubles the rate of reaction.

(D) Plot of log k vs $\dfrac{1}{T}$ gives a straight line with slope $= -\dfrac{Ea}{R}$.

Choose the **correct** answer from the options given below :

(1) B and D only

(2) A and B only

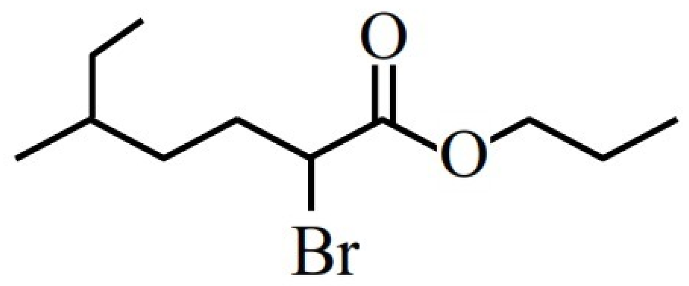
(3) A and C only

(4) B and C only

Ans. (4)

Sol. Fact based.

58. The IUPAC name of the following comopound is :



(1) n-propyl-2-bromo-5-methylheptanoate

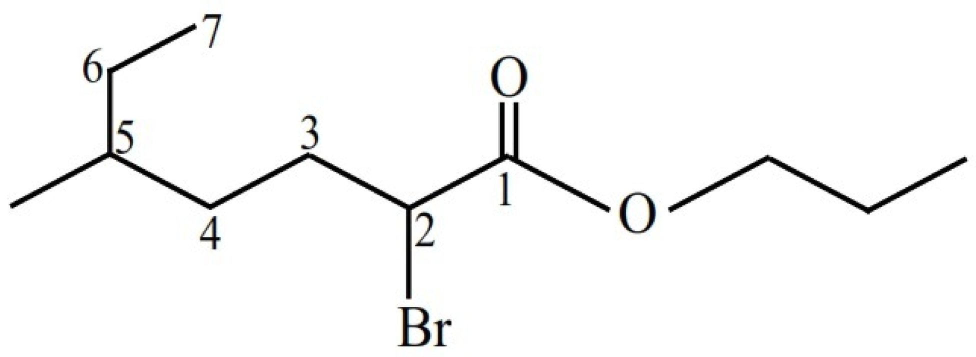
(2) 2-bromo-5-methylhexylpropanoate

(3) 2-bromo-5-methylpropanoate

(4) n-propyl-1-bromo-4-methylhexanoate

Ans. (1)

Sol.



Propyl-2-Bromo-5-Methylheptanoate

59. Given below are two statements :

Statement-I : Element 'X' and 'Y' are the most and least electronegative elements, respectively among N, As, Sb and P. The nature of the oxides X_2O_3 and Y_2O_3 is acidic and amphoteric, respectively.

Statement-II : BCl_3 is covalent in nature and gets hydrolysed in water. It produces $[B(OH)_4]^-$ and $[B(H_2O)_6]^{3+}$ in aqueous medium.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement-I and Statement-II are true.

(2) Statement-I is true but Statement-II is false.

(3) Both Statement-I and Statement-II are false.

(4) Statement-I is false but Statement-II is true.

Ans. (2)

Sol. Electronegativity order : N > P > As > Sb

↑ N: Most electronegative; ↑ Sb: Least electronegative

X = N $X_2O_3 = N_2O_3$ (Acidic)

Y = Sb $Y_2O_3 = Sb_2O_3$ (Amphoteric)

Statement-I is true

$BCl_3 + 3H_2O \rightarrow B(OH)_3 + 3HCl$

Statement-II is false

60. Match List-I with List-II.

List-I Reaction of glucose with	**List-II** Product formed
A. Hydroxylamine	I. Glouconic acid
B. Br_2 water	II. Glucose pentacetate
C. Excess acetic anhydride	III. Saccharic acid
D. Concentrated HNO_3	IV. Glucoxime

Choose the **correct** answer from the options given below :

(1) A-I, B-III, C-IV, D-II

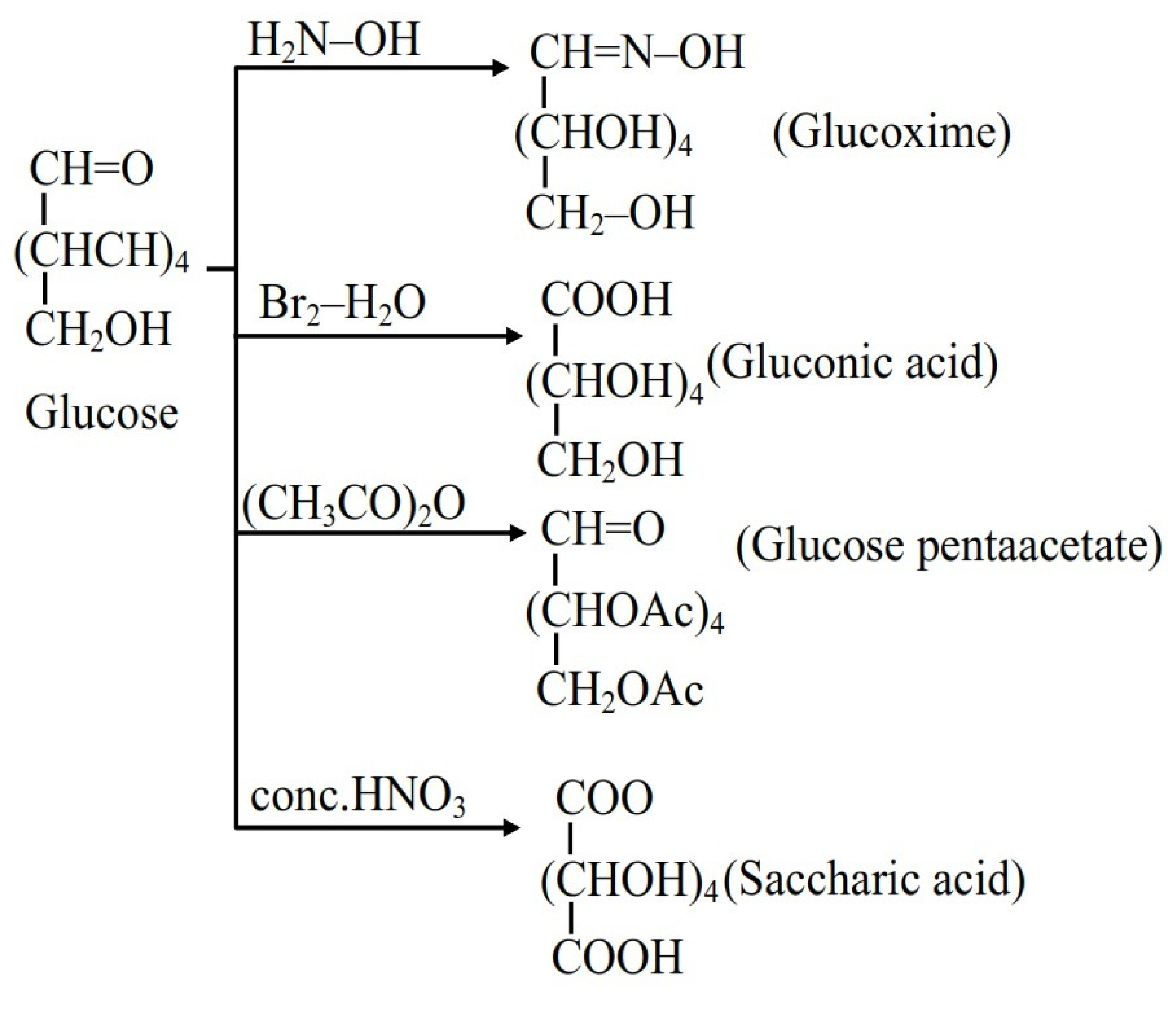
(2) A-IV, B-I, C-II, D-III

(3) A-III, B-I, C-IV, D-II

(4) A-IV, B-III, C-II, D-I

Ans. (2)

Sol.



61. Among H_2S, H_2O, NF_3, NH_3 and $CHCl_3$, identify the molecule (X) with lowest dipole moment value. The number of lone pairs of electrons present on the central atom of the molecule (X) is :

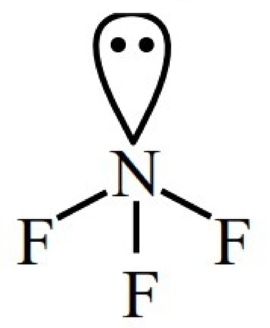
(1) 2 (2) 0

(3) 1 (4) 3

Ans. (3)

Sol.

Molecule	Dipole moment
H_2S	0.95
H_2O	1.85
NF_3	0.23 (minimum)
NH_3	1.47
$CHCl_3$	1.04



Number of lone pair on central atom = 1

62. Given below are two statements :

Statement-I : C < O < N < F is the correct order in terms of first ionization enthalpy values.

Statement-II : S > Se > Te > Po > O is the correct order in terms of the magnitude of electron gain enthalpy values.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Statement-I is false but Statement-II is true

(2) Both Statement-I and Statement-II are true.

(3) Both Statement-I and Statement-II are false.

(4) Statement-I is true but Statement-II is false.

Ans. (2)

Sol. C < O < N < F

$2p^2$ $2p^4$ $2p^3$ $2p^5$ $\longrightarrow$

$Z_{eff}\uparrow$ I. E. $\uparrow$

Statement-I is correct

	S >	Se >	Te >	Po >	O
$\lvert\Delta_{eg}H\rvert$	200	195	190	174	141 (in kJ/mol)

Statement-II is correct.

63. Which of the following mixture gives a buffer solution with pH = 9.25 ?

Given : pK_b $(NH_4OH) = 4.75$

(1) 0.2M NH_4OH (0.4 L) + 0.1M HCl (1L)

(2) 0.2M NH_4OH (0.5 L) + 0.1M HCl (0.5 L)

(3) 0.5M NH_4OH (0.2 L) + 0.2M HCl (0.5 L)

(4) 0.4M NH_4OH (1 L) + 0.1M HCl (1L)

Ans. (2)

Sol. $pOH = pK_b + \log \frac{Salt}{Base}$

$4.75 = 4.75 + \log \frac{Salt}{Base}$

Milimoles of [Salt] = milimoles of [Base]

Option (D) :

$NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

0.2M, 0.5L 0.1M, 0.5L

100 mmole 50 mmole

50 mmole – 50 mmole

Milimoles of NH_4OH = milimoles of NH_4Cl

64. The energy of first (lowest) Balmer line of H atom is x J. The energy (in J) of second Balmer line of H atom is :

(1) x^2 (2) $\frac{x}{1.35}$

(3) 2x (4) 1.35 x

Ans. (4)

Sol. Transition of first Balmer line

$n_1 = 2; n_2 = 3$

$\Delta E = x = 13.6\,(1)^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$ ..... (i)

Transition of 2^{nd} Balmer line

$n_1 = 2; n_2 = 4$

$\Delta E = 13.6\,(1)^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$ ..... (ii)

Divide Eq. (ii) by eq. (i)

$$\frac{\Delta E}{x}=\frac{\frac{1}{4}-\frac{1}{16}}{\frac{1}{4}-\frac{1}{9}}$$

$$\frac{\Delta E}{x}=\frac{\frac{3}{16}}{\frac{5}{36}}$$

$$\frac{\Delta E}{x}=\frac{27}{20}$$

$\Delta E = 1.35x$.

65. Identify the **correct** statements :

A. Hydrated salts can be used as primary standard.

B. Primary standard should not undergo any reaction with air.

C. Reactions of primary standard with another substance should be instantaneous and stoichiometric.

D. Primary standard should not be soluble in water.

E. Primary standard should have low relative molar mass.

Choose the **correct** answer from the options given below :

(1) A, B, C and E only

(2) A, B, and C only

(3) A, B and E only

(4) D and E only

Ans. (2)

Sol. Primary standard must be soluble for standard solution formation.

66. $[Ni(PPh_3)_2Cl_2]$ is a paramagnetic complex. Identify the **INCORRECT** statements about this complex.

A. The complex exhibits geometrical isomerism.

B. The complex is white in colour.

C. The calculated spin-only magnetic moment of the complex is 2.84 BM.

D. The calculated CFSE (Crystal Field Stabilization Energy) of Ni in this complex is $-0.8\Delta_0$.

E. The geometrical arrangement of ligands in this complex is similar to that in $Ni(CO)_4$.

Choose the **correct** answer from the options given below :

(1) A and B only

(2) A, B and D only

(3) C and D only

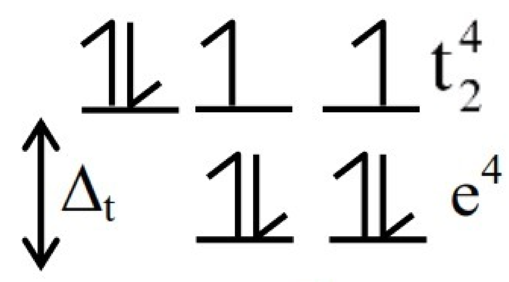
(4) C, D and E only

Ans. (2)

Sol. $[Ni(PPh_3)_2Cl_2]$

Given : Paramagnetic complex hence it must be tetrahedral so

Crystal field splitting :



$Ni^{+2} = (Ar)\ 3d^8$

(A) Tetrahedral complex not show geometrical isomerism.

(B) Complex is Blue colour

(C) Calculated spin only magnetic moment of the complex is 2.84 B.M.

(D) C.F.S.E $= -0.6\ \Delta_t(4) + 0.4\ \Delta_t(4)$

$= -0.8\ \Delta_t$ (not $-0.8\ \Delta_0$)

(E) $Ni(CO)_4$ also tetrahedral

Hence only A, B, D correct.

67. Consider the following reduction processes :

$Al^{3+} + 3e^- \rightarrow Al(s),\ E^o = -1.66$ V

$Fe^{3+} + e^- \rightarrow Fe^{2+},\ E^o = +0.77$ V

$Co^{3+} + e^- \rightarrow Co^{2+},\ E^o = +1.81$ V

$Cr^{3+} + 3e^- \rightarrow Cr(s),\ E^o = -0.74$ V

The tendency to act as reducing agent decreases in the order :

(1) $Al > Cr > Fe^{2+} > Co^{2+}$

(2) $Al > Fe^{2+} > Cr > Co^{2+}$

(3) $Al > Cr > Co^{2+} > Fe^{2+}$

(4) $Cr > Fe^{2+} > Al > Co^{2+}$

Ans. (1)

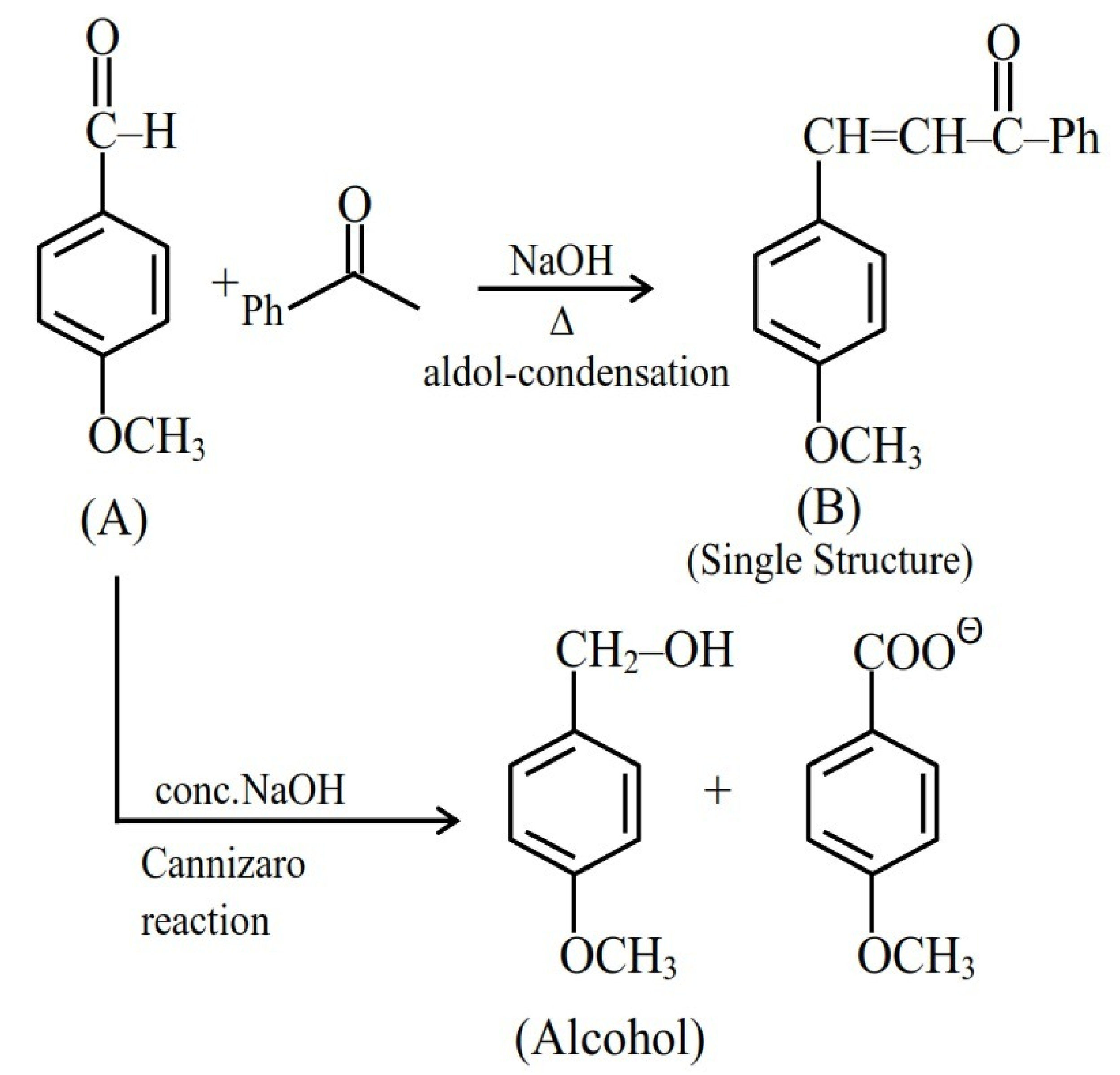
Sol. Reducing power $\propto \dfrac{1}{\text{Reduction potential}}$

68. The compound A, $C_8H_8O_2$ reacts with acetophenone to form a single product via cross-Aldol condensation. The compound A on reaction with conc. NaOH forms a substituted benzyl alcohol as

(1) 2-hydroxy acetophenone
(2) 4-methyoxy benzaldehyde
(3) 4-hydroxy benzylaldehyde
(4) 4-methyl benzoic acid

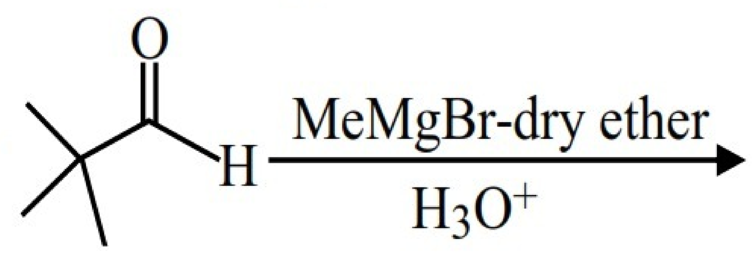
Ans. (2)

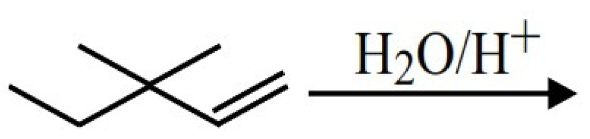
Sol.

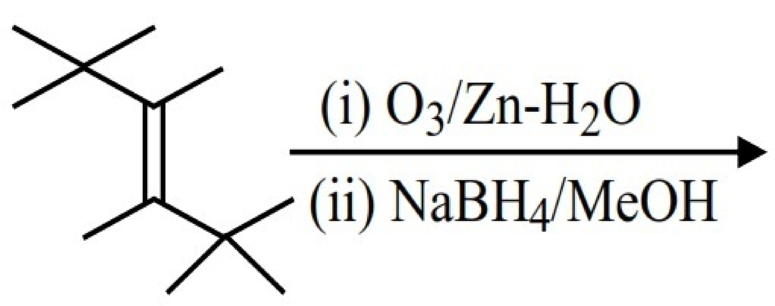


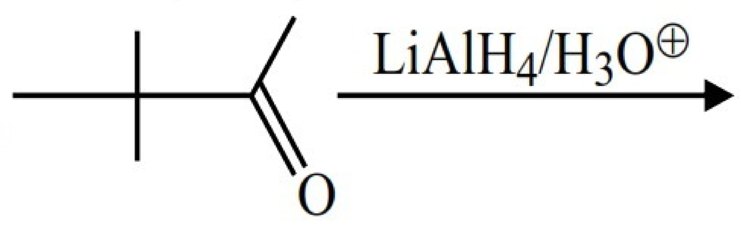
(On cross aldol reaction, a single structure is obtained but it can show geometrical isomerism).

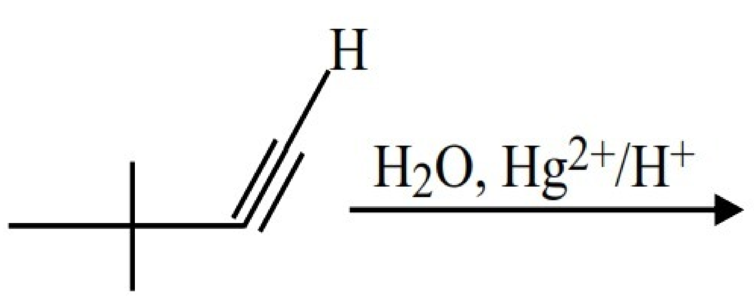
69. 3, 3-Dimenthyl-2-butanol **cannot** be prepared by :

A. 

B. 

C. 

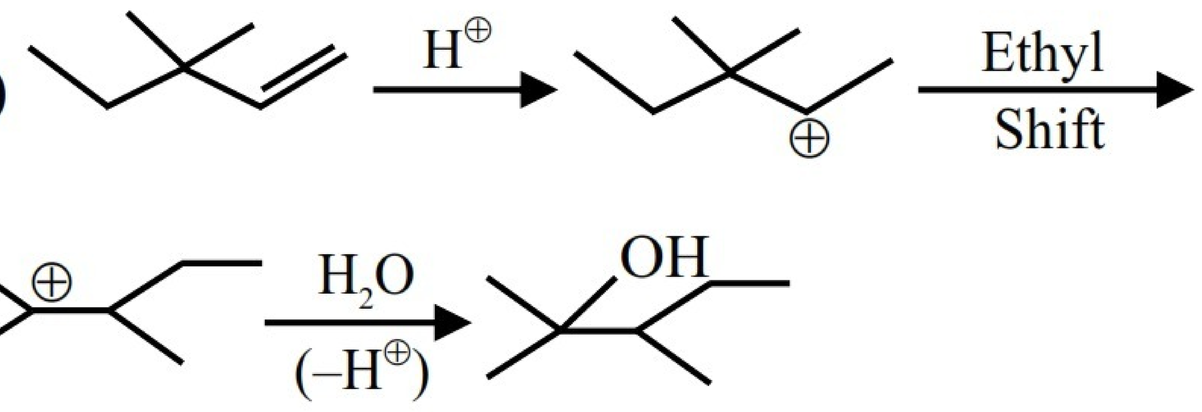
D. 

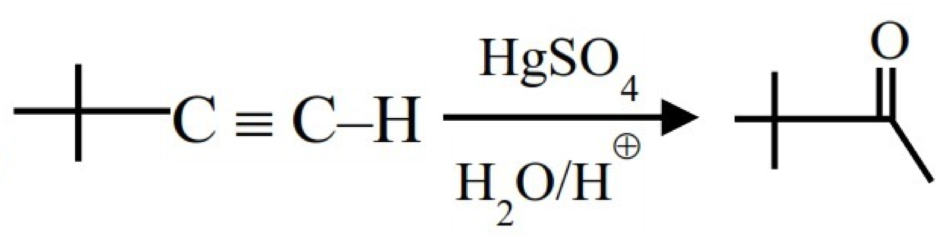
E. 

Choose the **correct** answer from the options given below :

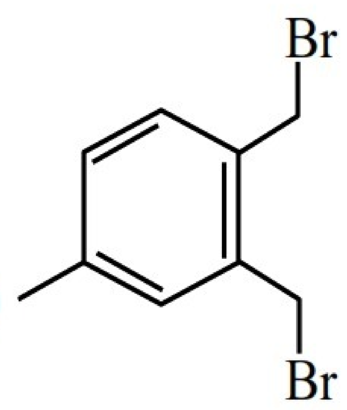
(1) B only (2) B and E only
(3) B and C only (4) B, C and E only

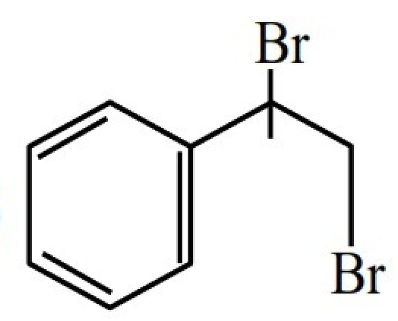
Ans. (2)

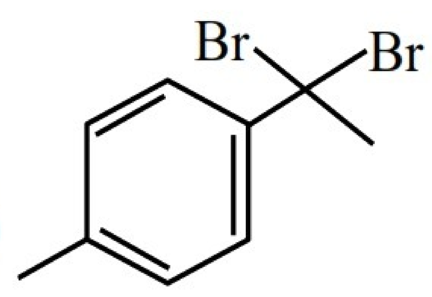
Sol. (B) 

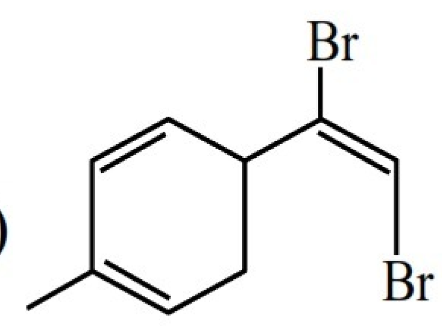
(E) 

70. The dibromo compound [P] (molecular formula : $C_9H_{10}Br_2$) when heated with excess sodamide followed by treatment with dilute HCl gives [Q]. On warming [Q] with mercuric sulphate and dilute sulphuric acid yield [R] which gives positive Iodoform test but negative Tollen's test. The compound [P] is :

(1) 

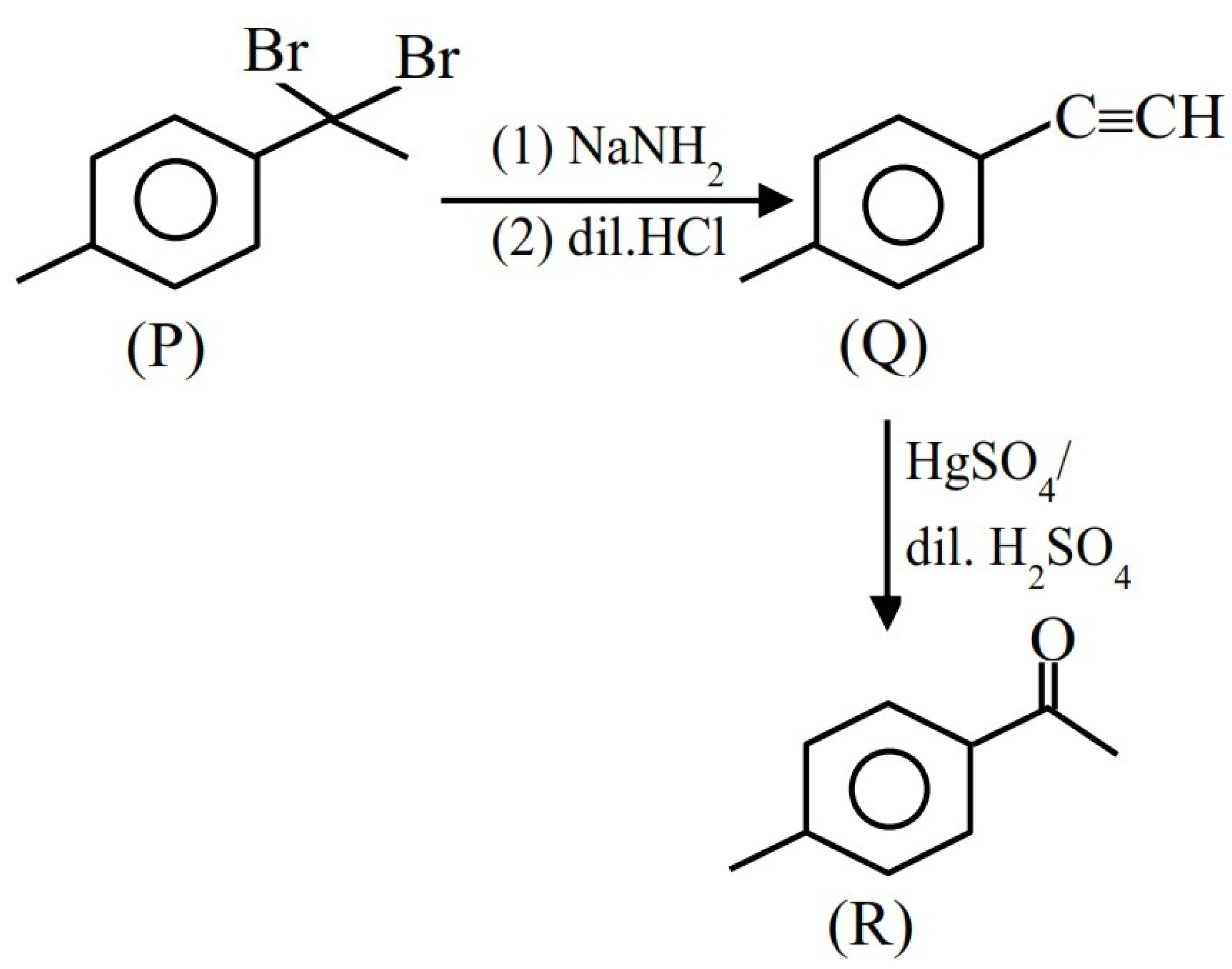
(2) 

(3) 

(4) 

Ans. (3)

Sol.



SECTION-B

71. Consider the following electrochemical cell :

$Pt \mid O_2(g)\ (1bar) \mid HCl\ (aq) \parallel M^{2+}(aq,\ 1.0\ M) \mid M(s)$

The pH above which, oxygen gas would start to evolve at anode is _______ (nearest integer).

$$\left[\text{Given}: \left.\begin{array}{l} E^0_{M^{2+}/M} = 0.994V \\ E^0_{O_2/H_2O} = 1.23V \end{array}\right\}\text{standard reduction potential} \quad \text{and } \frac{RT}{F}(2.303) = 0.059\text{ V at the given condition}\right]$$

Ans. **(4)**

Sol. For spontaneity $E_{cell} > 0$

At limiting condition :

E_{Oxi} (anode) = $-E_{Red}$ (cathode)

$$H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$$

$$E = E° - \frac{0.059}{2}\log\left[\frac{[H^+]^2 \times P_{O_2}^{1/2}}{1}\right]$$

$-0.997 = -1.23 + 0.059 \times pH$

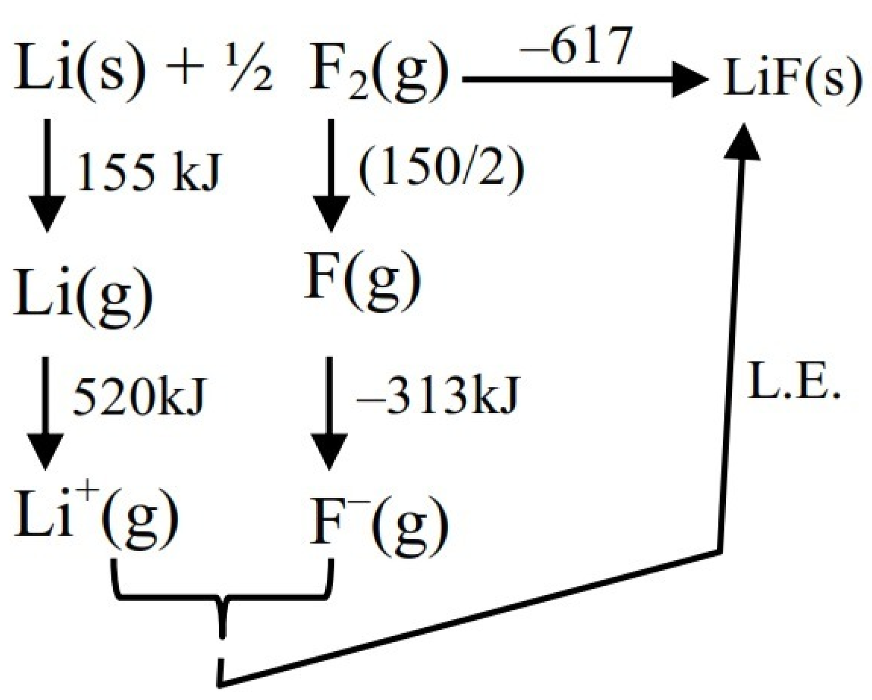
$pH = 3.94$

$pH \simeq 4$

72. If the enthalpy of sublimation of Li is 155 kJ mol^{-1}, enthalpy of dissociation of F_2 is 150 kJ mol^{-1}, ionization enthalpy of Li is 520 kJ mol^{-1}, electron gain enthalpy of F is –313 kJ mol^{-1}, standard enthalpy of formation of LiF is –594 kJ mol^{-1}. The magnitude of lattice enthalpy of LiF is _________ kJ mol^{-1} (Nearest integer).

Ans. **(1031)**

Sol.



$$-594 = 155 + 520 + \frac{150}{2} - 313 + (L.E.)$$

L.E. = – 1031 kJ/mol

73. Among the following oxides of 3d elements, the number of mixed oxides are_________.

Ti_2O_3, V_2O_4, Cr_2O_3, Mn_3O_4, Fe_3O_4, Fe_2O_3, Co_3O_4

Ans. **(3)**

Sol.

$Mn_3O_4 = MnO{\cdot}Mn_2O_3$

$Fe_3O_4 = FeO{\cdot}Fe_2O_3$

$Co_3O_4 = CoO{\cdot}Co_2O_3$

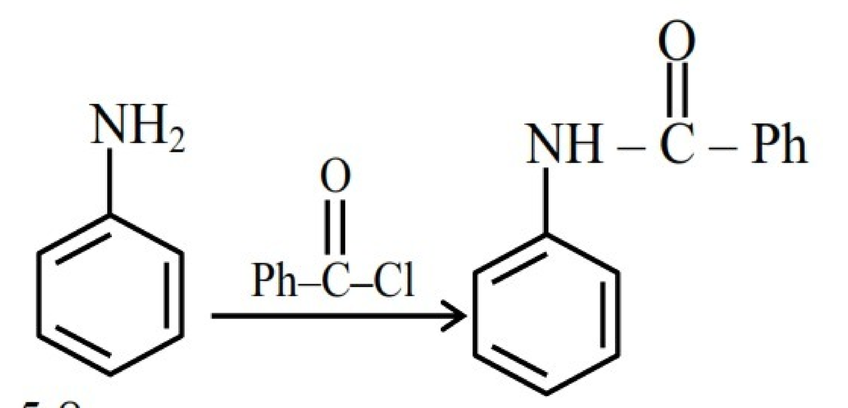
} Only three mixed oxides

74. The mass of benzanilide obtained from the benzoylation reaction of 5.8 g of aniline, if yield of product is 82%, is_________g (nearest integer).

(Given molar mass in g mol^{-1} H:1, C:12, N:14, O:16)

Ans. **(10)**

Sol.



5.8 gm

$n = \frac{5.8}{93}$

$n = 0.0623$

$n = 0.0623 \times \frac{82}{100}$

mass = 0.051 × 197

mass = 10.047

75. Consider $A \xrightarrow{k_1} B$ and $C \xrightarrow{k_2} D$ are two reactions. If the rate constant (k_1) of the $A \rightarrow B$ reaction can be expressed by the following equation $\log_{10} k = 14.34 - \frac{1.5 \times 10^4}{T/K}$ and activation energy of $C \rightarrow D$ reaction (Ea_2) is $\frac{1}{5}$th of the $A \rightarrow B$ reaction (Ea_1), then the value of (Ea_2) is ______kJ mol^{-1}. (Nearest Integer)

Ans. **(57)**

Sol. $\frac{E_{a_1}}{2.303R} = 1.5 \times 10^4$

$E_{a_1} = 1.5 \times 10^4 \times 2.303 \times 8.314$

$E_{a_1} = 28.7207 \times 10^4$ J

$E_{a_1} = 287.207$ kJ

$$E_{a_2} = \frac{E_{a_1}}{5} = \frac{287.207}{5} = 57.44 \text{ kJ}$$