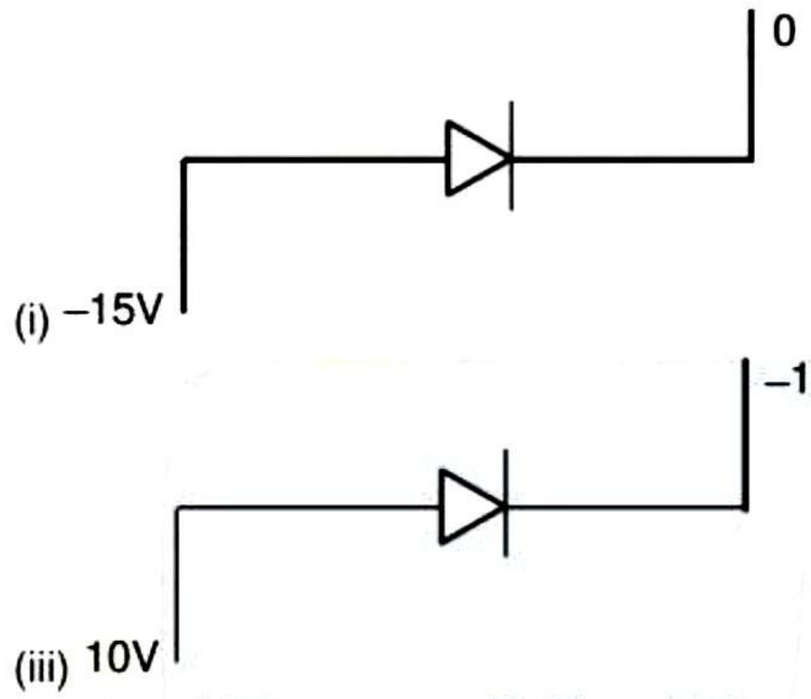
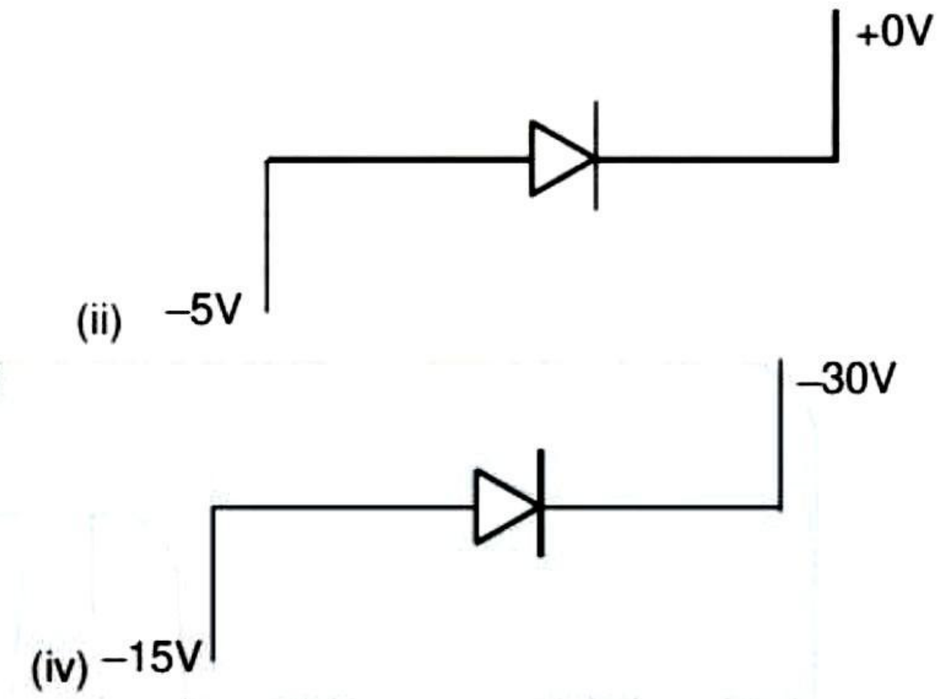
PART : PHYSICS

1. Forward bias will be :





(1) (i) and (ii) (2) (ii) and (iii) (3) (iii) and (iv) (4) (i) and (iv)

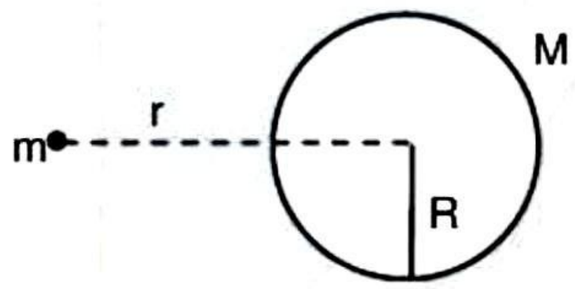
Ans. **(3)**

2. Solid sphere of mass M, radius R exerts force F on a point mass. Now a concentric spherical mass $\frac{M}{7}$ is removed from it. What is new force ?

(1) $\frac{F}{7}$ (2) $\frac{6}{7}F$ (3) $\frac{5}{7}F$ (4) $\frac{3}{7}F$

Ans. **(2)**

Sol.

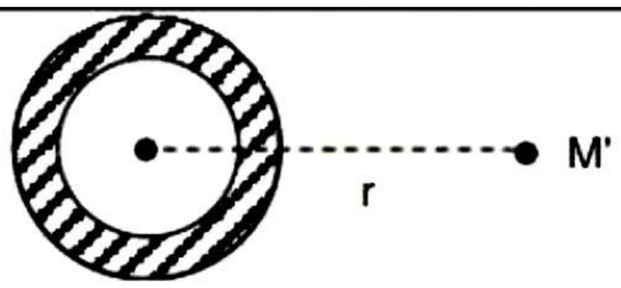


$$F = \frac{GMm}{r^2}$$

$\frac{M}{7}$ removed

$$\text{mass left} = M - \frac{M}{7}$$

$$M' = \frac{6M}{7}$$



$$F' = \frac{G\frac{6M}{7}xM}{r^2}$$

$$F' = \frac{6}{7}\frac{GMm}{r^2}$$

$$F' = \frac{6}{7}F$$

3. Find dimension of $\frac{B}{\mu_0}$.

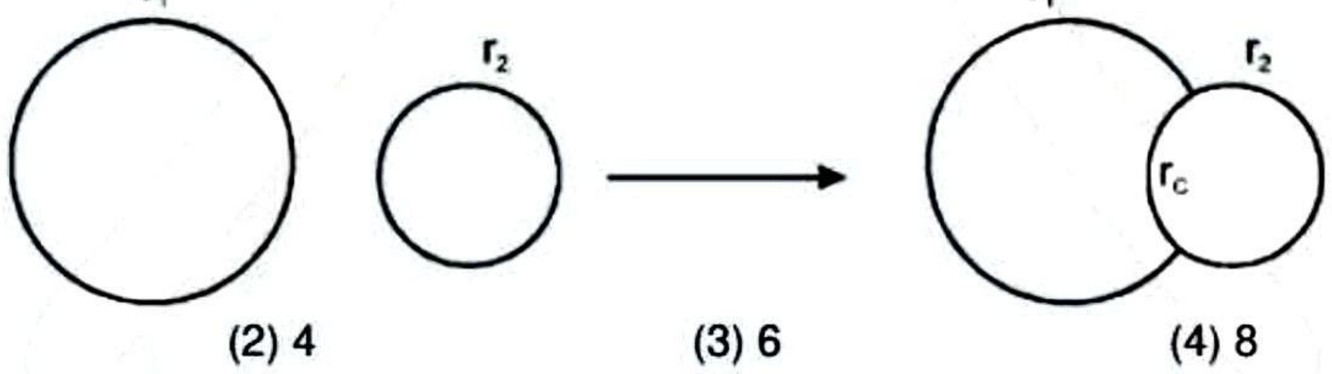
(1) AL (2) AL^{-1} (3) MAL (4) MAT^{-1}

Ans. **(2)**

Sol. $\therefore H = \frac{B}{\mu_0}$ or $B = \frac{\mu_0 i}{2r}$

$[AL^{-1}]$ $\therefore \frac{B}{\mu_0} = \frac{i}{2r} = [AL^{-1}]$

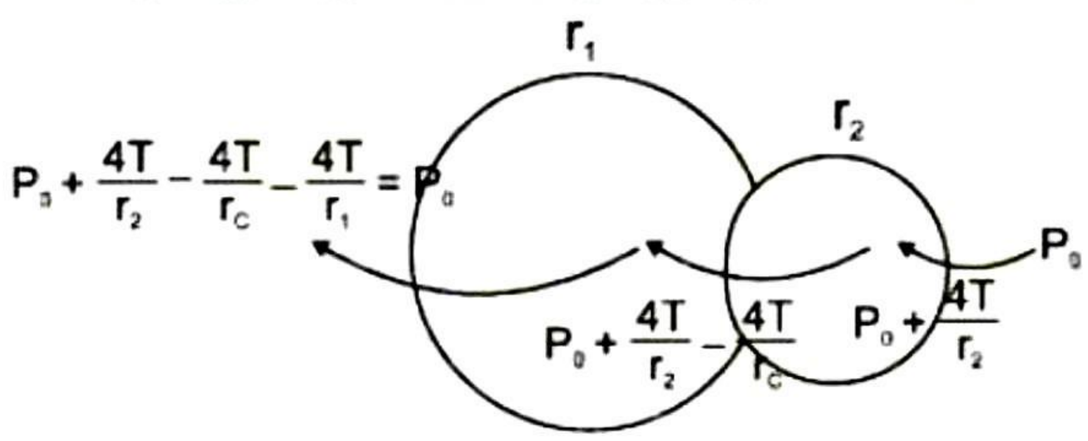
4. Two soap bubbles of radius r_1 and r_2 combine .Find radius of curvature of the common surface separating them.(r_1 = 4 cm, r_2 = 2cm)



(1) 2 (2) 4 (3) 6 (4) 8

Ans. **(2)**

Sol. $P_0 + \frac{4T}{r_2} - \frac{4T}{r_c} - \frac{4T}{r_1} = P_0, \quad \frac{1}{r_c} = \frac{1}{r_2} - \frac{1}{r_1}$



$$r_c = \frac{r_1\ r_2}{r_1 - r_2} = \frac{4 \times 2}{4 - 2} = 4$$

5. If light of wavelength 550 nm is incident on a metallic surface. If work function of Cs and Li are 1.9 eV and 2.5 eV respectively. Which can emit photo electron

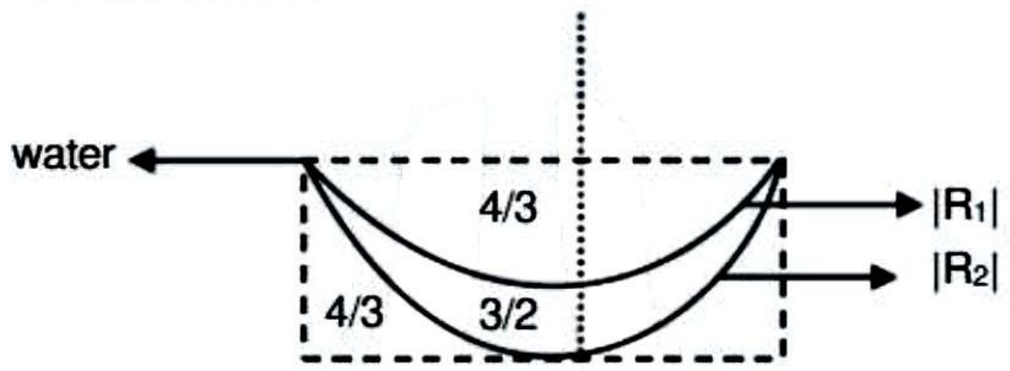
(1) Cs (2) Li (3) CsLi (4) None

Ans. **(1)**

Sol. Energy of light $E = \frac{hc}{\lambda} = \frac{1240}{550} = 2.25$ eV

Energy of light is only greater than work function of Cs(ϕ_0 = 1.5 eV), then only Cs will emit photo electrons

6. Find the power of combination of lens.



(1) $\frac{1}{6}\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$ (2) $\frac{1}{3}\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$ (3) $\frac{1}{5}\left[\frac{1}{R_2}+\frac{1}{R_1}\right]$ (4) $\frac{1}{8}\left[\frac{1}{R_2}+\frac{1}{R_1}\right]$

Ans. **(1)**

Sol. $\frac{1}{f_1}=\left[\frac{4}{3}-1\right]\left[\frac{1}{\infty}-\frac{1}{-R_1}\right]=\frac{1}{3R_1}$

$\frac{1}{f_2}=\left[\frac{3}{2}-1\right]\left[\frac{1}{-R_1}-\frac{1}{-R_2}\right]=\frac{1}{2}\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$

$\frac{1}{f_3}=\left[\frac{4}{3}-1\right]\left[\frac{1}{-R_2}-\frac{1}{\infty}\right]=\frac{1}{3}\left[-\frac{1}{R_2}\right]$

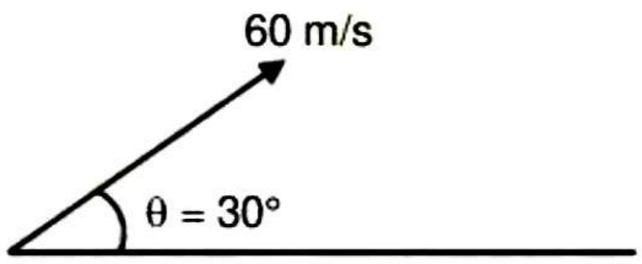
$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$

$P_{eq} = P_1 + P_2 + P_3$

$=\frac{1}{3R_1}+\frac{1}{2R_2}-\frac{1}{2R_1}-\frac{1}{3R_2}$

$P_{eq}=\frac{2R_2+3R_1-3R_2-2R_1}{6R_1R_2}=\frac{R_1-R_2}{6R_1R_2}=\frac{1}{6}\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$

7.

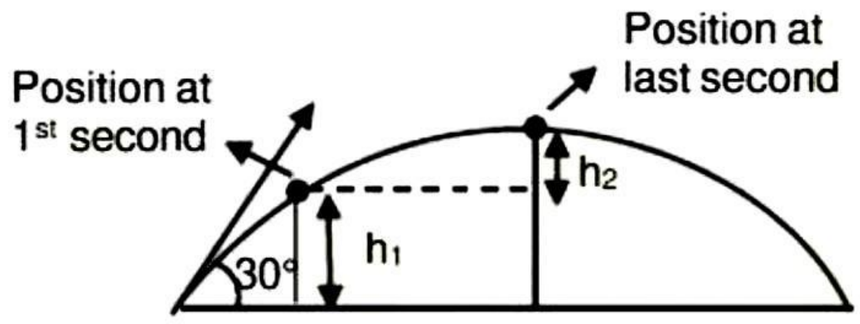


An object is thrown with speed 60 m/s making an angle 30° with the horizontal. Find the ratio of height covered in first second and last second of the upward journey.

(1) 3 : 2 (2) 5 : 1 (3) 5 : 6 (4) 6 : 1

Ans. **(2)**

Sol.



$u_y = 60 \sin 30° = 30$ m/s

$$y = u_y t - \frac{1}{2} gt^2$$

$$h_1 = 30 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 25 \text{ m}$$

$$\mathbf{h_2} = \frac{1}{2} gt^2 = \frac{1}{2} \times (10)(1)^2 = 5$$

$$\frac{h_1}{h_2} = \frac{25}{5} = \frac{5}{1}$$

8. Statement-1 Fringe come closer in denser medium in YDSE

Statement-2 Light travel slower in denser medium.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

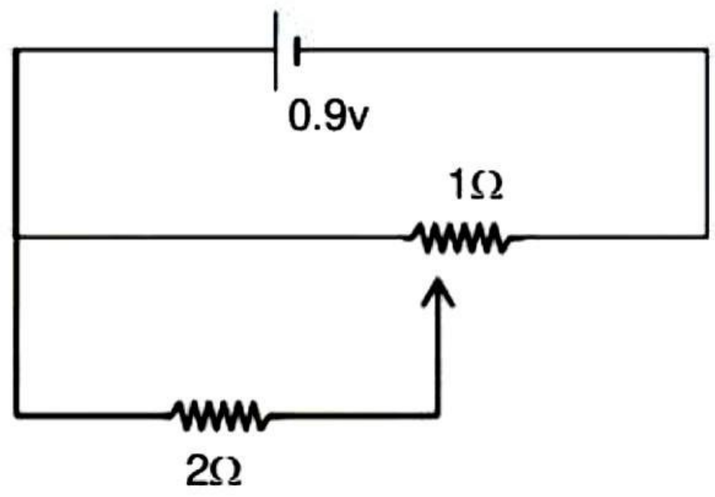
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

Ans. **(1)**

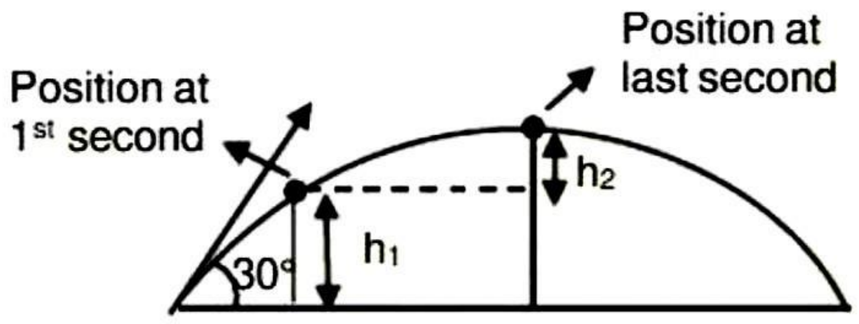
9. Find current in the circuit. Jockey is at middle point of 1Ω .



(1) 10 Amp. (2) 0.1 Amp (3) 1 Amp (4) 2 Amp

Ans. **(3)**

Sol.



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Ans. (1)

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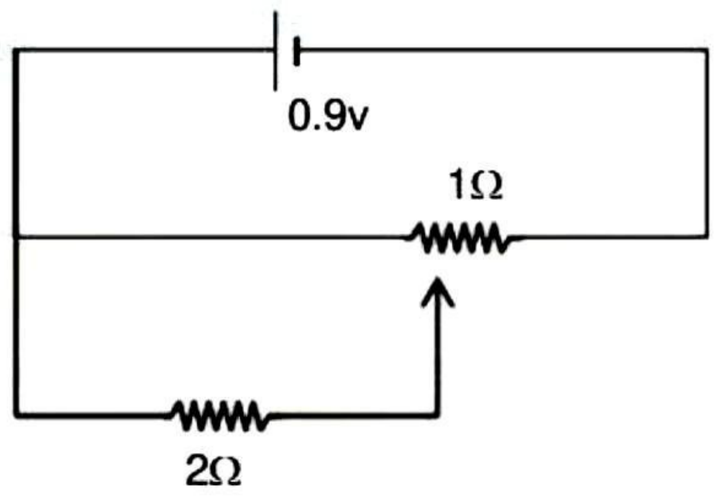

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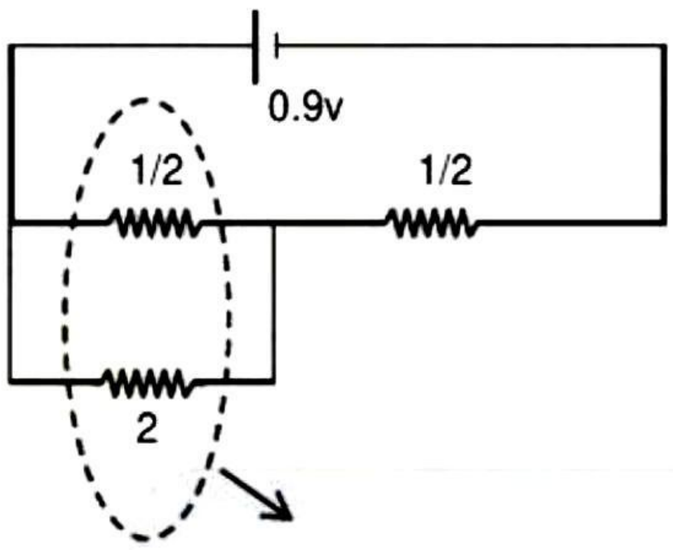
Ans. (3)

Sol.



$$\frac{\frac{1}{2}\times 2}{\frac{1}{2}+2} \Rightarrow \frac{2}{5}$$

$$\text{Req.} \quad = \frac{2}{5} + \frac{1}{2} \Rightarrow \frac{4+5}{10} \Rightarrow \frac{9}{10} \Rightarrow 0.9\ \Omega$$

$$i = \frac{v}{\text{Req.}} = \frac{0.9}{0.9} \Rightarrow 1 \text{ Ans .}$$

10. For H, radius of first and second excited states are 5.3×10^{-11} m and 8.48×10^{-10} m ratio of debroglie wave lengths is :

(1) $\lambda_1 : \lambda_2 :: 16 : 15$ (2) $\lambda_1 : \lambda_2 :: 14 : 15$

(3) $\lambda_1 : \lambda_2 :: 15 : 16$ (4) $\lambda_1 : \lambda_2 :: 17 : 15$

Ans. **(3)**

Sol. $2\pi r = n \cdot \lambda$

$2\pi (5.3 \times 10^{-11}) = 2(\lambda_1)$

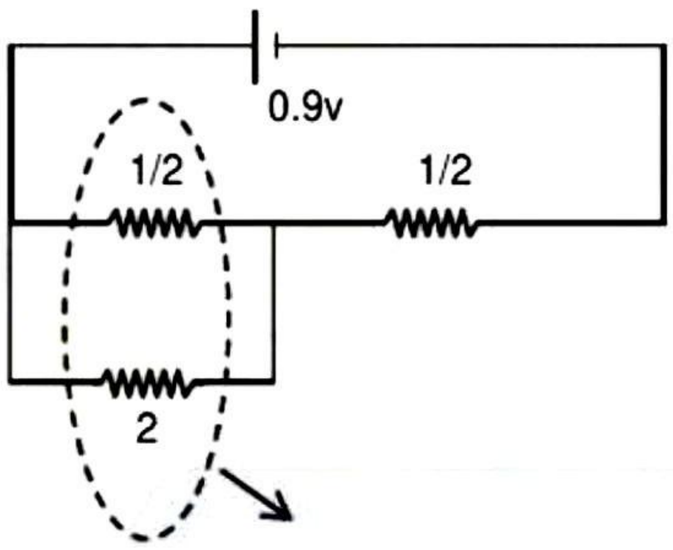
$2\pi (8.48 \times 10^{-10}) = 3(\lambda_2)$

$$\frac{5.3}{8.48} \times 10^{-1} \times \frac{3}{2} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{0.53}{8.48} \times \frac{3}{2} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{15}{16}$$

Sol.



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Ans. **(3)**

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$2\pi (8.48 \times 10^{-10}) = 3(\lambda_2)$

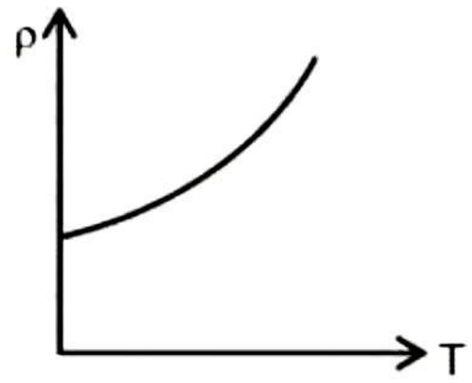
$$\frac{5.3}{8.48}\times 10^{-1}\times\frac{3}{2}=\frac{\lambda_1}{\lambda_2}$$

$$\frac{0.53}{8.48}\times\frac{3}{2}=\frac{\lambda_1}{\lambda_2}$$

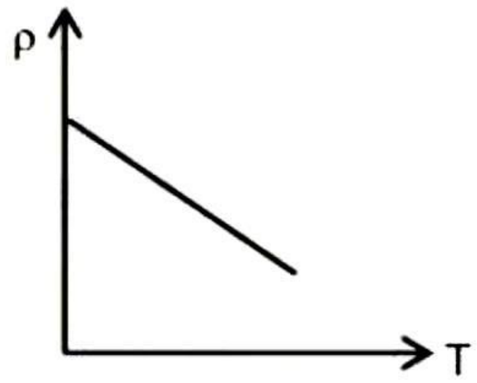
$$\frac{\lambda_1}{\lambda_2}=\frac{15}{16}$$

11. Which of the following represent correct relation between resistivity of conductor (F) and temperature (T)

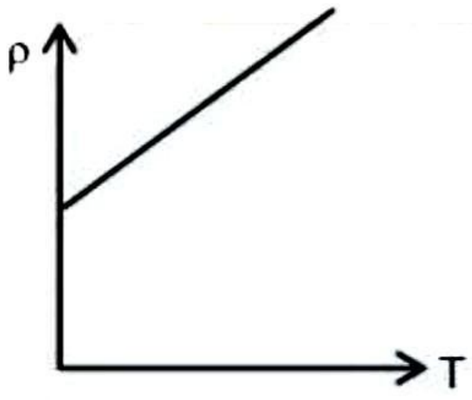
(1)



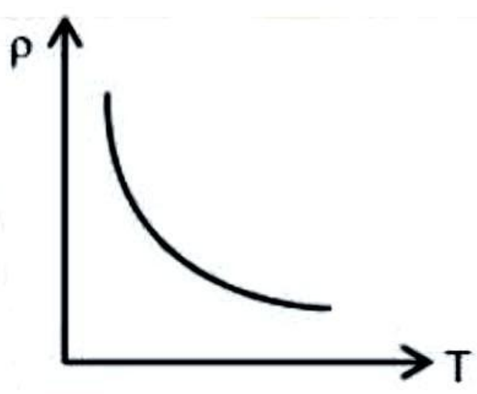
(2)



(3)



(4)



Ans. **(1)**

12. Two organ pipe, one is open and other is closed and the densities of gas filled in the ratio of 1 : 16. 9 th harmonic frequency of closed organ pipe is equal to 4th harmonics of open organ pipe. Find the length of the open organ pipe if length of closed pipe is 10 cm & Bulk Modulus is same for both.

(1) 32.5 cm (2) 35.5 cm (3) 25.5 cm (4) 45.5 cm

Ans. **(2)**

Sol.

$f_1 = \frac{v_1}{2\ell_1}$ $f_2 = \frac{v_2}{4\ell_2}$

Given $\frac{\rho_1}{\rho_2} = \frac{1}{16}$

Acceleration to equation

$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

$9f_2 = 4f_1$ $= 4$

$\Rightarrow 9\frac{v_2}{4\ell_2} = 4\frac{v_1}{2\ell_1}$

$\Rightarrow \ell_1 = \frac{16\ell_2 v_1}{18 v_2}$

$\Rightarrow \ell_1 = \frac{16}{18} \times 10 \times 4$

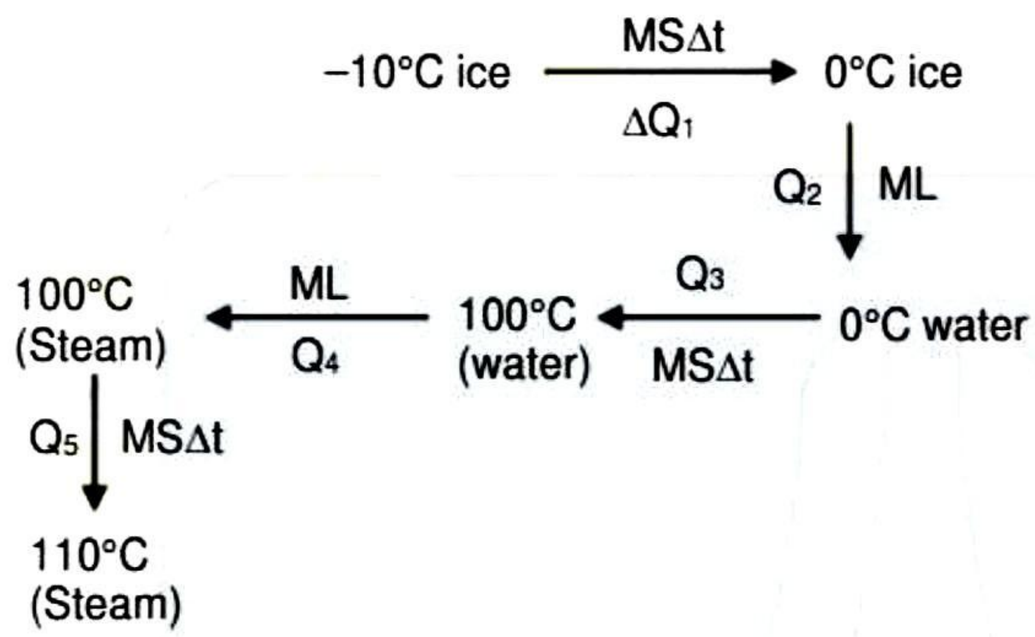
$\Rightarrow \ell_1 = \frac{8}{9} \times 10 \times 4 = 35.5$ cm

13. Ice at –10°C is to be converted into steam at 110°C. Mass of ice is 10^{-3} kg. What amount of heat is required ?

(1) $\Delta Q = 730$ cal (2) $\Delta Q = 900$ cal (3) $\Delta Q = 1210$ cal (4) $\Delta Q = 870$ cal

Ans. **(1)**

Sol.



$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

$Q_1 = Ms\,(0 - (-10))$ $\quad S_{ice} = \frac{1}{2}$ KC/kg°C

$Q_1 = 10^{-3} \times \frac{1}{2} \times 10^3 \times 10$ $\quad L_{ice} = 80$ Kcal/kg

$Q_1 = 5$ cal

$Q_2 = 10^{-3} \times 80 \times 10^{-3} = 80$

$Q_3 = 10^{-3} \times 1 \times (100 - 0) \times 10^3$

$Q_3 = 100$

$Q_4 = 10^{-3} \times 540$ cal $\quad L_v = 540$ Kcal/kg

$Q_4 = 540$ cal

$Q_5 = 10^{-3} \times \frac{1}{2} \times 10^3\,(110 - 100)$

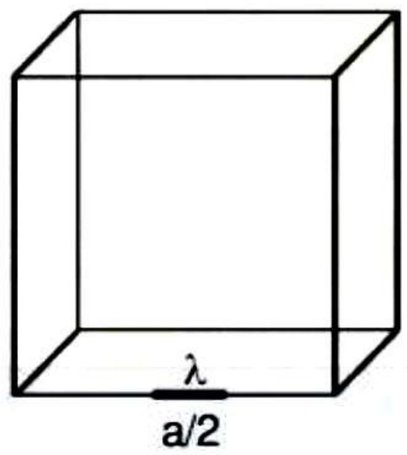
$Q_5 = \frac{1}{2} \times 10 = 5$

$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

$\Delta Q = 5 + 80 + 100 + 540 + 5$

$\Delta Q = 730$ cal

14. Wire of length $\frac{a}{2}$ of linear charge density λ is placed on edge of cube then find the flux passing through cube.



(1) $\frac{\lambda a}{2\varepsilon_0}$ (2) $\frac{\lambda a}{4\varepsilon_0}$ (3) $\frac{\lambda a}{8\varepsilon_0}$ (4) $\frac{\lambda a}{\varepsilon_0}$

Ans. **(3)**

Sol. $\phi = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda \frac{a}{2}}{4\varepsilon_0} = \frac{\lambda a}{8\varepsilon_0}$

15. The radius of ground state of H atom is a_0. The radius of first excited state of He^+ is :

(1) a_0 (2) $2a_0$ (3) $3a_0$ (4) $4a_0$

Ans. **(2)**

Sol. $r_0 = a_0$ $r_n = a_0 \times \frac{n^2}{Z}$

$r_{He}^{+} = a_0 \times \frac{Z^2}{2}$

$n = 2 = 2a_0$

16. Which of the following is not true ?

(1) decay constant does not depend on temperatures

(2) decay constant increases with temperature

(3) $t_{1/2} = \frac{\ln(2)}{\lambda}$

(4) None

Ans. **(2)**

17. **Statement -1** Vernier scale division always has small division than main scale division

Stetement- 2 Vernier constant is number of division of vernier scale multiply by main scale division

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is false

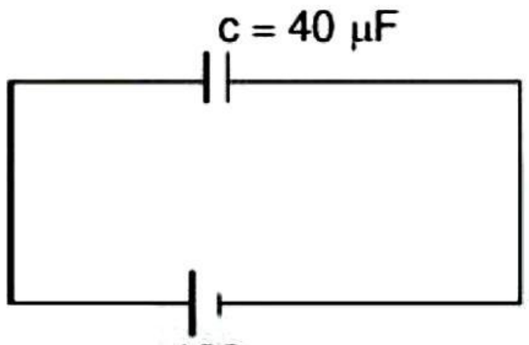
Ans. **(3)**

18. Capacitor having capacity of 40 μF is connected with 100 v battery. If dielectric constant (k = 2) is inserted between plates of capacitor, then change in charge of capacitor plate and change in energy stored in capacitor will be :

(1) 4 mc, 0.2 J (2) 6 mc, 0.2 J (3) 8 mc, 2 J (4) 2 mc, 4 J

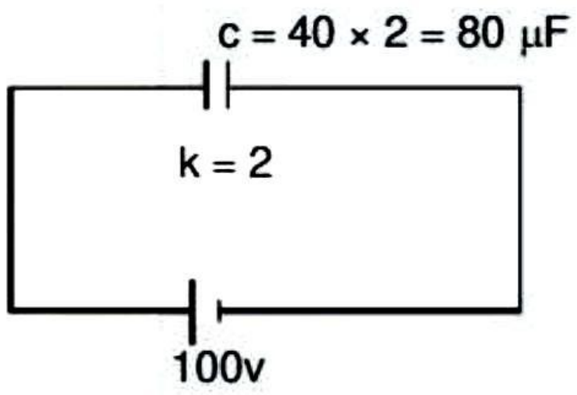
Ans. **(1)**

Sol.



$Q_i = 100 \times 40\ \mu C$

$Q_i = 4 \times 10^3\ \mu C$



$Q_F = 80 \times 100 = 8 \times 10^3 \mu C$

$\Delta Q = q_F - q_F = (8 \times 10^3 - 4 \times 10^3)\ \mu c$

$= 4 \times 10^3\ \mu c$

$= 4 \times 10^3 \times 10^{-6} \times 10^3$

$= 4$ mc

$E_1 = \frac{1}{2} C_1 V^2 \rightarrow$ Initial energy

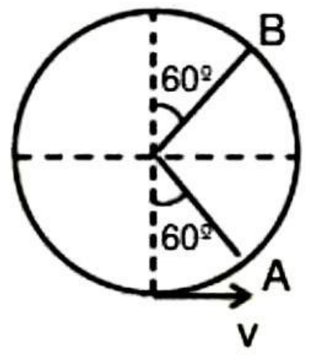
$E_2 = \frac{1}{2} C_2 V^2 \rightarrow$ Final energy

$E_2 - E_1 = \frac{1}{2} V^2 (C_2 - C_1)$

$= \frac{1}{2} \times (100)^2 (80 - 40)$

$= 20 \times 10^{-2} = 0.2$ J

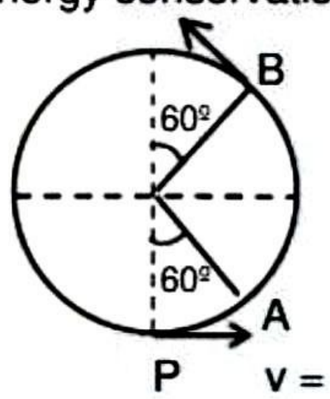
19. The particle shown in the fig is just able to complete the vertical circular motion find the ratio of kinetic energy at A to the kinetic energy at B.



(1) 3 : 2 (2) 4 : 1 (3) 2 : 1 (4) 1 : 2

Ans. **(3)**

Sol. Energy conservatism at (P) and (A)



$$0 + \frac{1}{2} m \times 5gl = mgl(1-\cos\theta) + \frac{1}{2} m v_A^2$$

$$\frac{1}{2} m v_A^2 = mgl\left(\frac{5}{2} - \frac{1}{2}\right) = 2mgl$$

energy conservator at (P) and (B)

$$O + \frac{1}{2} \times m \times 5gl = \frac{1}{2} m v_B^2 + mgl(1+\cos\theta)$$

$$\frac{1}{2} m v_B^2 = mgl\left(\frac{5}{2} - \frac{3}{2}\right) = mgl$$

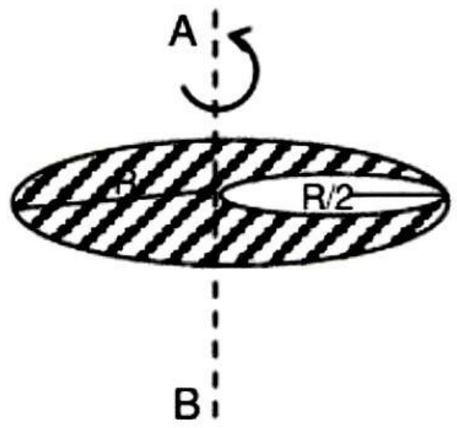
$$\frac{\frac{1}{2} m v_A^2}{\frac{1}{2} m v_B^2} = \frac{2mgl}{mgl} = \frac{2}{1}$$

20. A mass of a disc is M and radius R. A cavity of radius $\frac{R}{2}$ is created. Find the moment of inertia about an axis passing through the centre of disc.

(1) $\frac{17MR^2}{32}$ (2) $\frac{13MR^2}{32}$ (3) $\frac{7MR^2}{32}$ (4) $\frac{19MR^2}{32}$

Ans. **(2)**

Sol.



$$I = I_{disc} - I_{cavity}$$

$$= \frac{MR^2}{2} - \left[\frac{M}{4}\frac{(R/2)^2}{2} + \frac{M}{4}(R/2)^2\right]$$

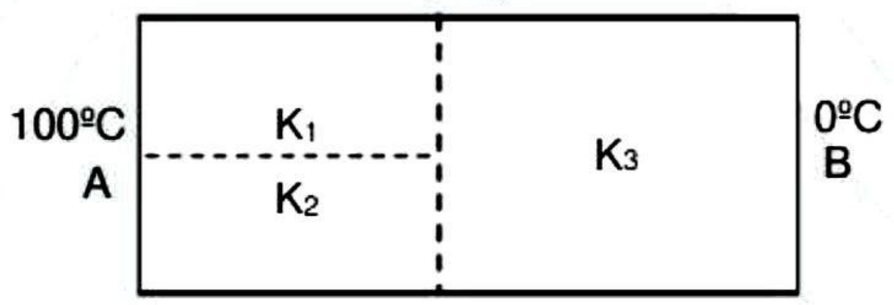
$$= \frac{MR^2}{2} - \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right]$$

$$= \frac{MR^2}{2} - \left(\frac{MR^2 + 2MR^2}{32}\right)$$

$$= \frac{16MR^2 - MR^2 - 2MR^2}{32}$$

$$\frac{13MR^2}{32}$$

21. Unit of all quantities is in S.I. System.



Given $K_1 = 60$, $K_2 = 120$ one K_3 135 ºC find final junction T = ?

(1) 20°C (2) 35°C (3) 40°C (4) 45°C

Ans. **(3)**

Sol.

$$R_1 = \frac{L}{K,A} \text{ , } R_2 = \frac{L}{K_2 A}$$

$$R_{ev} = \frac{R_1R_2}{R_1+R_2} = \frac{L^2/K_1K_2A^2}{\frac{L}{A}\left(\frac{1}{K_1}+\frac{1}{K_2}\right)}$$

$$\text{So } \frac{Leq}{K_{eq}} = \frac{L/K_1K_2A}{\frac{K_2K_1}{K_1K_2}} = \frac{L}{(K_1+K_2)A}$$

$$\Rightarrow \frac{1}{2k_{ev}} = \frac{1}{K_1+K_2} \Rightarrow K_{eq} = \frac{K_2+K_1}{2} = \frac{60+120}{2} = 90$$

$$\frac{100-T}{\frac{L}{90(2A)}} = \frac{T-0}{\frac{L}{135(2A)}}$$

$\Rightarrow 90 \times 100 = T$

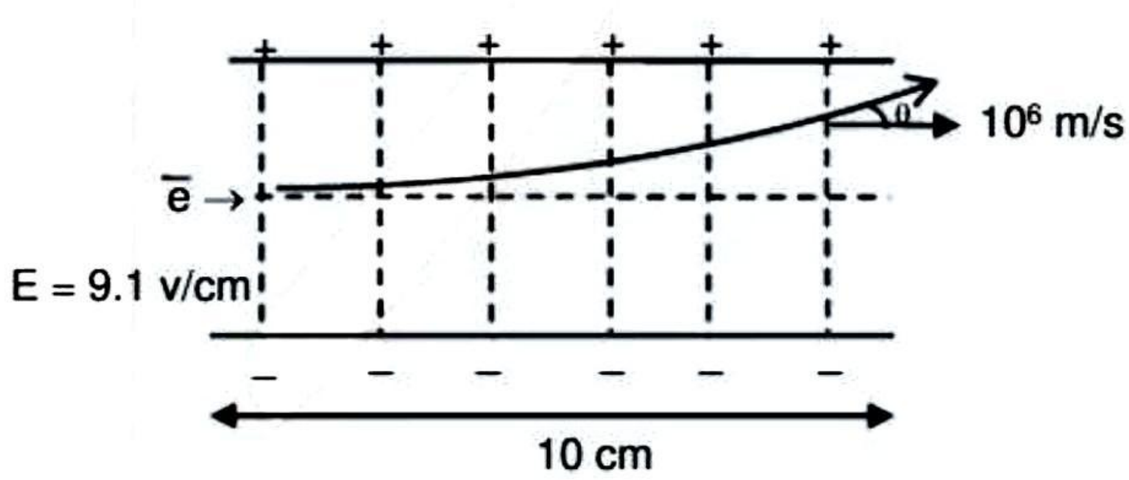
$\Rightarrow T = 40^{\circ}C$

22. An electron projected horizontally between two horizontal charged plates , emerges with horizontal speed 10^6 m/s if length of plates is 10cm and electric field between plate is 9.1 volt/cm then vertical component of velocity of electron when it emerges wile be (e & m are given)

(1) 16×10^4 (2) 16×10^6 (3) 1.6×10^5 (4) 32×10^7

Ans. **(3)**

Sol.



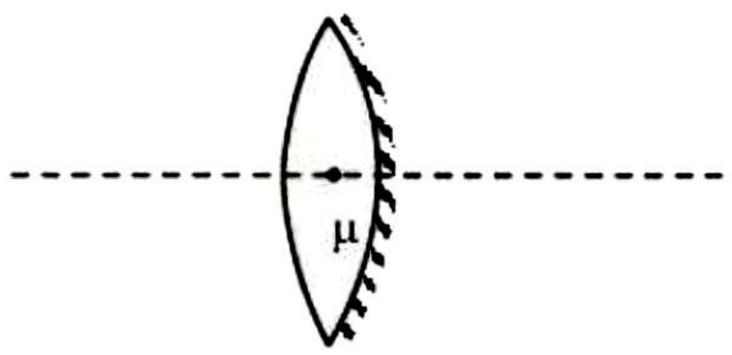
To coure the 10 cm horizontally time taken by electron will be

→ there is no force on horizontal direction so velocity will remain constant. 10^6m/s

$$t = \frac{10cm}{10^6 m/s} = \frac{10cm}{10^6 \times 100} = 10^{-7} \text{ sec.}$$

$$V_y = \frac{eE}{mt} = 1.6 \times 10^7$$

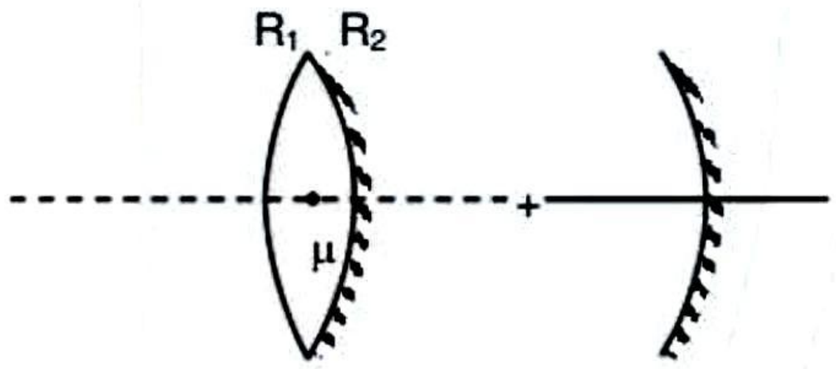
23. The object is placed in front of convex lens then one surface of lens get silvered. the R is radius of curvature of lens of μ is refractive index. Whose should object be placed so that image formed on object. (options are in R and μ):



(1) $\frac{-R}{(\mu-1)}$ (2) $\frac{-R}{(2\mu+1)}$ (3) $\frac{-R}{(2\mu-1)}$ (4) $\frac{R}{(\mu-1)}$

Ans. **(3)**

Sol. For lens



$$\frac{1}{f_\ell} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$R_2 = -R_0$

$R_1 = R$

$$\frac{1}{f_\ell} = (\mu-1)\left(\frac{2}{R}\right) \qquad .....(1)$$

focal length of mirror :

$$f_m = -\frac{R}{2}$$

$$\frac{1}{f_m} = -\frac{2}{R} \qquad .....(2)$$

equaling focal length of system :

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_\ell} = \frac{-2}{R} - \frac{2(2)(\mu-1)}{R}$$

$$\frac{1}{f_{eq}} = \frac{-2}{R}(1+2\mu-2)$$

$$\frac{1}{f_{eq}} = \frac{-2}{R}(2\mu-1)$$

$$f_{eq} = \frac{-R}{2(2\mu-1)}$$

Image will form when object is placed at centre of curvature :

So $R_{eq} = 2f$

$$R_{eq} = 2\left(\frac{-R}{2(2\mu-1)}\right)$$

$$R_{eq} = \frac{-R}{(2\mu-1)}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. For complex ion $[NiCl_4]^{2-}$ what is the charge on metal and shape of complex respectively?

   (1) +2, Tetrahedral (2) +2, Square planar

   (3) +4, Tetrahedral (4) +4, Square Planar

Answer (1)

Sol. $[NiCl_4]^{2-} \Rightarrow Ni^{2+} \rightarrow 3d^8$

Cl^- ligand is weak field ligand and hybridisation is sp^3. Shape of complex is tetrahedral.

2. Compare boiling point of given solutions

   (i) 10^{-4} M NaCl (ii) 10^{-3} M NaCl

   (iii) 10^{-2} M NaCl (iv) 10^{-4} M urea

   (1) I > II > III > IV (2) III > II > I > IV

   (3) II > I > III > IV (4) III > I > II > IV

Answer (2)

Sol. Higher the elevation in boiling point, higher will be the boiling point

$\Delta Tb \propto i \times m$

For urea i = 1

For NaCl i = 2

Boiling point order III > II > I > IV

3. The correct decreasing order of electronegativity is

   (1) F > Cl > I > Br (2) Cl > F > Br > I

   (3) F > Cl > Br > I (4) Br > F > I > Cl

Answer (3)

Sol. The correct order is

F > Cl > Br > I

4. Which of the following has maximum size out of Al^{3+}, Mg^{2+}, F^-, Na^+?

   (1) Al^{3+} (2) Mg^{2+}

   (3) F^- (4) Na^+

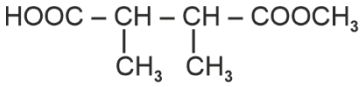
Answer (3)

Sol. For isoelectronic species, more the negative charge more will be the size, also more the positive charge smaller will be the size.

The correct order of ionic size is :

$Al^{3+} < Mg^{2+} < Na^+ < F^-$

5. The IUPAC name of given specie is



   (1) 2, 3-dimethyl methyl carboxy butanoic acid

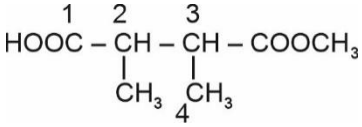
   (2) 4-methoxy carbonyl-2, 3-dimethyl propanoic acid

   (3) 3-methoxycarbonyl-2-methyl butanoic acid

   (4) 1-carboxy-2, 3-dimethyl methyl butanoate

Answer (3)

Sol.



3-methoxycarbonyl-2-methyl butanoic acid

6. Compare crystal field splitting energy(Δ) for given complexes

   (i) $K_4[Fe(CN)_6]$ (ii) $[Cu(NH_3)_4]^{+2}$s

   (iii) $K_4[Fe(SCN)_6]$ (iv) $[Fe(en)_3]Cl_3$

   (1) I > II > III > IV (2) II > I > IV > III

   (3) IV > I > III > II (4) IV > III > I > II

Answer (2)

Sol. $K_4[Fe(CN)_6] \Rightarrow d^6 \Rightarrow$ SFL,

$K_2[Cu(NH_3)_4] \Rightarrow d^9 \Rightarrow dsp^2$

$K_4[Fe(SCN)_6] \Rightarrow d^6 \Rightarrow$ WFL

$[Fe(en)_3]Cl_3 \Rightarrow d^5 \Rightarrow$ SFL

Splitting energy $\propto$ Strength of ligand $\propto$ Charge of CA.

$\Delta_{sp} > \Delta_o$

II > I > IV > III

7. Consider the given equilibrium reaction

$$CO_2(g) + C(s) \rightleftharpoons 2CO(g)$$

If initial pressure of CO_2 is 0.6 atm and after equilibrium is established, total pressure is 0.8 atm. Then, find K_p.

(1) 0.4 (2) 0.2

(3) 0.6 (4) 0.8

Answer (1)

Sol. $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$

t = 0 0.6

$t = t_{eq}$ 0.6 – p 2p

P_t at equilibrium = 0.8 = 0.6 + p

0.2 = p

$$K_p = \frac{(p_{CO})^2}{(p_{CO_2})} = \frac{(2p)^2}{0.6-p} = \frac{4\times0.04}{0.6-0.2} = \frac{4\times0.04}{0.4} = 0.4$$

8. Statement-I: $CH_3 - O - CH_2 - Cl$ will show nucleophilic substitution by S_N1 mechanism in protic medium.

Statement-II: $CH_3 - \underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}} - CH_2 - Cl$ will not undergo nucleophilic substitution via S_N2 mechanism easily.

(1) Statement-I and statement-II both are correct

(2) Statement-I and statement-II both are incorrect

(3) Statement-I is correct but statement-II is incorrect

(4) Statement-I is incorrect but statement-II is correct

Answer (1)

Sol. $CH_3 - O - \overset{\oplus}{C}H_2$ stabilised by resonance.

9. Which of the following acids is also known as vitamin C?

(1) Adipic acid (2) Ascorbic acid

(3) Saccharic acid (4) Aspartic acid

Answer (2)

Sol. Ascorbic acid is also known as vitamin C.

10. An electron of He^+ is present in 3rd excited state. Find its de-Broglie wavelength.

(1) 6.64 Å (2) 1.66 Å

(3) 3.32 Å (4) 13.28 Å

Answer (1)

Sol. $n\lambda = 2\pi r$

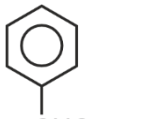
For 3rd excited state, n = 4

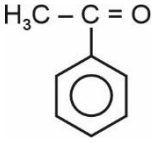
$$4\lambda = 2 \times \pi \times a_\circ \frac{n^2}{z}$$

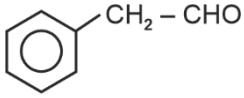
$$4\lambda = 2 \times \pi \times 0{\cdot}529 \frac{16}{2} \text{ Å}$$

$\lambda = 2 \times 3{\cdot}14 \times 0{\cdot}529 \times 2$ Å = 6·64 Å

11. Which of the following will show positive Fehling test?

(1) 

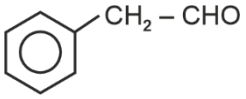
(2) 

(3) 

(4) $C_2H_5 - \underset{\displaystyle OH}{\overset{|}{CH}} - \underset{\displaystyle OH}{\overset{|}{CH}} - CH_3$

Answer (3)

Sol. Fehling test is given by Aldehydes except benzaldehyde



will give +ve Fehling test

12. $4f^7$ configuration is possible for

(a) Eu^{3+}, (b) Eu^{2+}, (c) Gd^{3+}, (d) Tb^{3+}, (e) Sm^{2+}

(1) (a) and (c)

(2) (b) and (c)

(3) (d) and (e)

(4) Only (c)

Answer (2)

Sol. Electronic configuration of:

$Eu^{3+} \Rightarrow 4f^6$ $\quad$ $Tb^{3+} \Rightarrow 4f^8$

$Eu^{2+} \Rightarrow 4f^7$ $\quad$ $Sn^{2+} \Rightarrow 4f^5$

$Gd^{3+} \Rightarrow 4f^7$

13. Given : $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

If the partial pressure of CO_2 gas at equilibrium is 0.4 atm and the total pressure is 1 atm, then the value of K_p at the same temperature is

(1) 0.027 atm^3

(2) 0.064 atm^3

(3) 0.144 atm^3

(4) 0.216 atm^3

Answer (3)

Sol. $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

Total pressure at equilibrium = 1.0 atm

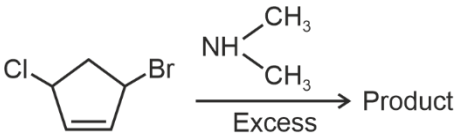
Partial pressure of CO_2 at equilibrium = 0.4 atm

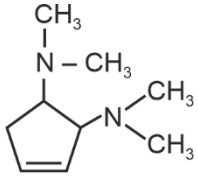
$\therefore$ Partial pressure of NH_3 at equilibrium = 0.6 atm

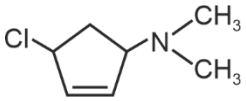
$K_p = (p_{NH_3})^2 (p_{CO_2})$

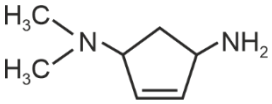
$= (0.6)^2(0.4)$

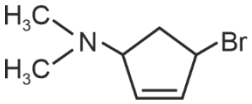
$= 0.144\ atm^3$

14. 

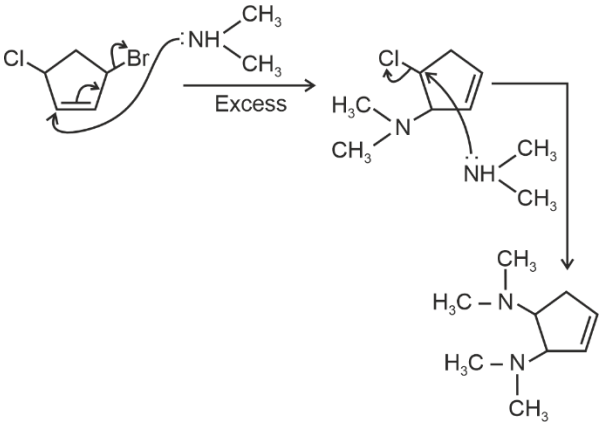
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

15. CO_2 gas is taken at 1 atm, 273K. Now it is allowed to pass through 0.1 M $Ca(OH)_2$ aq. solution. Excess amount of $Ca(OH)_2$ is neutralised with 40 mL of 0.1 M HCl. Then find volume of $Ca(OH)_2$ initially taken if 50% $Ca(OH)_2$ is react with CO_2

(1) 40 mL

(2) 20 mL

(3) 80 mL

(4) 50 mL

Answer (1)

Sol. g meq of $Ca(OH)_2$ = 2 × gm eq of HCl

$$0.1\times\frac{V_{mL}}{1000}\times 2 = 2\times 0.1\times\frac{40}{1000}\times 1$$

V_{mL} = 40 mL

16. In a closed insulated container, a liquid is stirred with a paddle to increase the temperature, which of the following is true?

(1) $w = 0, \Delta E = q \neq 0$ (2) $\Delta E = w \neq 0, q = 0$

(3) $\Delta E = w = 0, q \neq 0$ (4) $\Delta E = 0, w = q \neq 0$

Answer (2)

Sol. In closed insulated container a liquid stirred with a paddle to increase the temperature, it behaves as an adiabatic container, q = 0

From FLOT

$\Delta U = q + w; q = 0$

$\Delta E = w$ (but not zero)

17. Match the column and choose the correct option

	Column-I (Properties)		**Column-II (Order)**
(A)	Electronegativity	(1)	B < C < N < O
(B)	Cationic size	(2)	Li > Mg > Be
(C)	Metallic Character	(3)	K > Mg > Al
(D)	Electron affinity	(4)	Cl > F > Br > I

(1) A–1, B–2, C–3, D–4

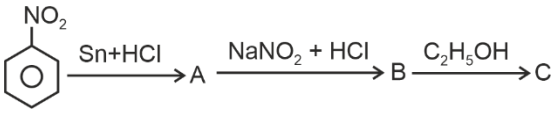
(2) A–4, B–3, C–2, D–1

(3) A–2, B–3, C–4, D–1

(4) A–3, B–2, C–4, D–1

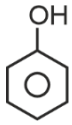
Answer (1)

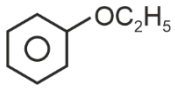
Sol. $\underset{76\,pm}{Li^+} > \underset{72\,pm}{Mg^{2+}} > \underset{31\,pm}{Be^{2+}}$

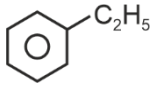
18. 

Identify C.

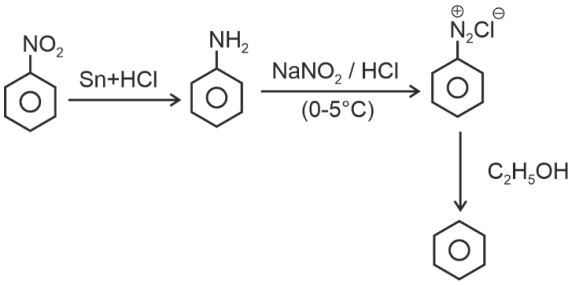
(1) 

(2) 

(3) 

(4) 

Answer (1)

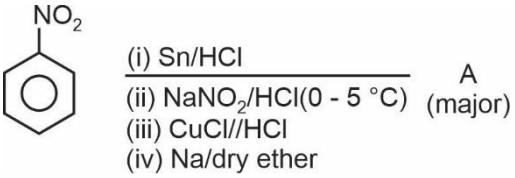
Sol. 

19.

20.

SECTION - B

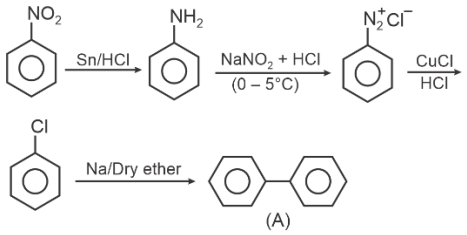
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 

Find molecular weight of (A) in g mol^{-1}

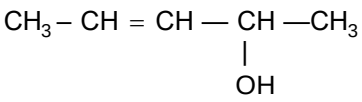
Answer (154)

Sol.



Molecular weight of (A) = 154 g mol^{-1}

22. Calculate Number of stereoisomers of



Answer (4)

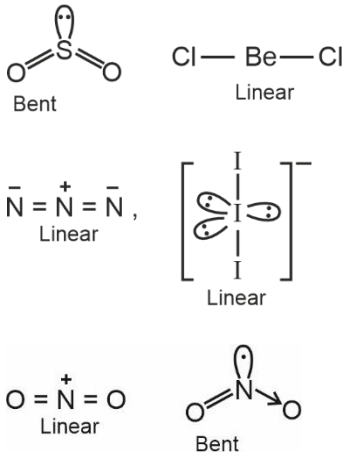
Sol. Number of centres which can show stereoisomerism in molecule = 2

Number of isomers = $2^2 = 4$

23. How many compounds have linear shape SO_2, $BeCl_2$, $N_3^-, I_3^-, NO_2^+, NO_2$?

Answer (4)

Sol.



24. In Carius method 180 mg of organic compound gives 143.5 mg of AgCl. Find the percentage of Cl in the organic compound. (Nearest integer)

Answer (20)

Sol. Mass of organic compound = 180 mg

Mass of AgCl = 143.5 mg

$$\text{Mass of Cl} = \frac{143.5}{143.5} \times 35.5 \text{ mg}$$

$$= 35.5 \text{ mg}$$

Percentage of Cl in the organic compound

$$= \frac{35.5 \times 100}{180}$$

$$= 19.72\% \simeq 20\%$$

25. Two ampere current is allowed to pass through molten $AlCl_3$ for 30 min. Find the mass (in mg) of aluminium deposited at cathode. (Nearest integer)

Answer (336)

Sol. Total charge passed = 2 × 30 × 60 C

$$\text{Number of Faradays passed} = \frac{2 \times 30 \times 60}{96500} \text{ F}$$

$$\text{Equivalents of Al deposited} = \frac{36}{965}$$

$$\text{Mass of Al deposited} = \frac{36 \times 9}{965} \text{ g}$$

$$= \frac{36 \times 9 \times 1000}{965} \text{ mg}$$

$$= 335.75 \text{ mg}$$

$$\simeq 336 \text{ mg}$$

PART : MATHEMATICS

1. In how many ways, a 5 letter word can be made using any distinct 5 alphabets such that the middle alphabet is 'M' and letter should be in increasing order.

(1) 2198 (2) 4031 (3) 9014 (4) 5148

Ans. **(4)**

Sol. There are 12 alphabets before M and 13 alphabets after M.

So, total number of ways = $^{12}C_2 \times {}^{13}C_2 = 66 \times 78 = 5148$

2. The value of $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2}$ is :

(1) $\frac{512}{3}$ (2) $\frac{2047}{12}$ (3) $\frac{1023}{12}$ (4) $\frac{2049}{12}$

Ans. **(2)**

Sol. $\frac{1}{12}\sum_{r=0}^{5}\frac{12}{2r+2}{}^{11}C_{2r+1} = \frac{1}{12}\sum_{r=0}^{5}{}^{12}C_{2r+2} = \frac{1}{12}\left[{}^{12}C_2 + {}^{12}C_4 + {}^{12}C_6 + {}^{12}C_8 + {}^{12}C_{10} + {}^{12}C_{12}\right]$

$= \frac{1}{12}\left[2^{12-1} - 1\right] = \frac{2047}{12}$

3. If $\sum_{r=0}^{n} T_1 = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$ then find $\lim_{n\to\infty}\sum_{r=0}^{n}\frac{1}{T_r}$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{2}$

Ans. **(1)**

Sol. $Tn = S_n - S_{n-1}$

$= \frac{(2n-1)(2n+1)(2n+3)(2n+5) - (2n-3)(2n-1)(2n+1)(2n+3)}{64}$

$T_n = \frac{(2n-1)(2n+1)(2n+3)}{8}$

$\frac{1}{T_n} = \frac{8}{(2n-1)(2n+1)(2n+3)}$

$\frac{1}{T_n} = 2\left(\underbrace{\frac{1}{(2n-1)(2n+1)}}_{T_{r-1}} - \underbrace{\frac{1}{(2n-1)(2n+3)}}_{T_r}\right)$

$S_n = 2\left(\frac{1}{1\times 3} - \frac{1}{(2n-1)(2n+3)}\right)$

$\lim_{n\to\infty} S_n = \frac{2}{3}$

4. If $e^{5(\ell nx)^2+3} = x^8$, then product of all real values of x

(1) $e^{2/5}$ (2) $e^{3/5}$ (3) $e^{8/5}$ (4) $e^{1/5}$

Ans. **(3)**

Sol. $e^{5(\ell nx)^2+3} = x^8$

$5(\ell nx)^2 + 3 = 8\ell nx$

$\Rightarrow \quad \ell nx = t$

$\Rightarrow \quad 5t^2 - 8t + 3 = 0$

$\Rightarrow \quad (t-1)(5t-3) = 0$

$t = \frac{3}{5}, 1$

$\ell nx = \frac{3}{5} \quad ; \quad \ell nx = 1$

$x = e^{3/5} \quad ; \quad x = e^1$

Product $= e^{3/5}.e^1 = e^{8/5}$ **Ans.**

5. In a bag there are 6 white and 4 black balls two balls are drawn randomly one by one without replacement then probability that the both balls are white is:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{9}{16}$

Ans. **(2)**

Sol. Probability $= \frac{{}^6C_2}{{}^{10}C_2} = \frac{6\times5}{10\times9} = \frac{2}{6} = \frac{1}{3}$

6. A be a 3×3 square matrix such that $|A| = -2$ if $\text{Det}(3\text{adj}(-6\text{adj}(3A))) = 2^n \times 3^m$, where $m \geq n$, then $4m + 2n$ is equal to –

Ans. **104**

Sol. $|3\text{adj}(-6(\text{adj}(3A)))|$

$= \quad 3^3 |\text{adj}(-6(\text{adj}(3A)))|$

$= \quad 3^3 |-6\ \text{adj}(3A)|^2$

$= \quad 3^3 \times ((-6)^3)^2 |\text{adj}(3A)|^2$

$= \quad 3^9 \times 2^6 |3A|^4$

$= \quad 3^9 \times 2^6 \times 3^{12} |A|^4$

$= \quad 3^{21} \times 2^6 \times 2^4$

$= \quad 3^{21} \times 2^{10}$

$m = 21$ and $n = 10$

So, $4m + 2n = 84 + 20 = 104.$

7. $a_1, a_2, a_3, a_4, \ldots$ are positive & increasing terms of G.P. If $a_1 . a_5 = 28$ and $a_2 + a_4 = 29$ then a_6 is equal to

(1) $\sqrt{28}$ (2) $28\sqrt{28}$ (3*) 784 (4) 28

Ans. **(3)**

Sol. Let $a_1 . a_5 = 28$

$a_1 . a_1 r^4 = 28$

$a_1^2 r^4 = 28$ ____(1)

also $a_2 + a_4 = 29$

$a_1 r + a_1 r^3 = 29$

$a_1^2 (r + r^3)^2 = 29^2$

$\frac{28}{r^4}(r + r^3)^2 = 29^2$

$28(1 + r^2)^2 = 841 r^2$

$28 + 28r^4 + 56r^2 = 841 r^2$

$28r^4 - 785r^2 + 28 = 0$

$r^2 - \frac{784}{28}, \frac{1}{28}$

$r^2 = 28$ or $\frac{1}{28}$

Now from (1)

$a_1^2 = \frac{28}{28^2} = \frac{1}{28}$

Now $a_6 = a_1 r^5$

$\frac{1}{\sqrt{28}} \times 28^2 . \sqrt{28} = 28^2 = 784$

8. Let f(x) be a real differentiable function such that f(0) = 1 and f(x + y) = f(x)f'(y) + f(y) f '(x) for all x, y $\in$ R, then $\sum_{n=1}^{100} \log_e f(n)$ is equal to –

Ans. **2525**

Sol. Put x =y =0

$$f(0) = 2f'(0) \text{ as } f(0) = 1$$

$$f'(0) = \frac{1}{2}$$

Now, put y =0 in given equation

$$f(x) = f(x).f'(0) + f(0).f'(x)$$

$$f(x) = \frac{1}{2} f(x) + f'(x)$$

$$f'(x) = \frac{1}{2} f(x)$$

$$\frac{dy}{dx} = \frac{y}{2}$$

$$\int \frac{dy}{y} = \int \frac{dx}{2}$$

$$\ell ny = \frac{x}{2} + C$$

$$f(0) = 1$$

$$0 = 0 + C$$

$$C = 0$$

$$y = e^{x/2}$$

$$\ell ny = \frac{x}{2}$$

Now, $\sum_{n-1}^{100} \ell n\, f(n) = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + ... + \frac{100}{2} = \frac{1}{2}\left[\frac{100 \times 101}{2}\right] = 2525$

9. Let the triangle PQR be the image of the triangle with vertices (1, 3) , (3, 1) and (2, 4) in the line $x+2y=2$. If the centroid of triangle PQR is the point (α, β) then value of $15\ (\alpha - \beta)$ is:

Ans. **(22)**

Sol. Centroid of the triangle whose vertices are (1, 3) , (3, 1) and (2, 4) is, $\left(2, \frac{8}{3}\right)$.

Image of centroid $\left(2, \frac{8}{3}\right)$ in the line, $x+2y=2$ is,

$$\frac{\alpha-2}{1} = \frac{\beta-\frac{8}{3}}{2} = -2\left(\frac{2+\frac{16}{3}-2}{1+4}\right)$$

$$\alpha - 2 = \frac{\beta-\frac{8}{3}}{2} = -\frac{2}{5}\left(\frac{16}{3}\right)$$

$$\alpha = -\frac{2}{15}\ ,\ \beta = -\frac{24}{15} \qquad \Rightarrow \qquad \alpha = -\frac{2}{15}\ ,\ \beta = -\frac{24}{15}$$

$$\Rightarrow \qquad 15\ (\alpha - \beta) = 15\left(-\frac{2}{15}+\frac{24}{15}\right) = 22$$

10. (1, 14) and (1, –12) are focii of hyperbola passing through (1, 6), then length of Latus rectum is equal to

(1) $\frac{144}{5}$ (2) $\frac{288}{5}$ (3) $\frac{144}{7}$ (4) $\frac{288}{15}$

Ans. **(2)**

Sol. Let P (1, 6)

By $|PS - PS'| = 2a$

$$\left|\sqrt{0+64}-\sqrt{0+324}\right| = 2a$$

$$|8-18| = 2a$$

$$a = 5$$

and $SS' = 2ae$

$$\sqrt{0+26^2} = 2ae$$

$$ae = \frac{26}{2} = 13$$

$$e = \frac{13}{5}$$

Now, $b^2 = a^2(e^2 - 1) = 25\left(\frac{169}{25}-1\right)$

$$b = 12$$

$$\text{Length of L.R.} = \frac{2b^2}{a} = \frac{2\times144}{5} = \frac{288}{5}$$

11. Let z be complex number such that $|z| = 1$ and z_1, z_2, z_3 are three points satisfying $|z| = 1$ such that $\arg(z_1) = -\frac{\pi}{4}$, $\arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$ also $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1| = \alpha + \beta\sqrt{2}$, then $3\beta + 2\alpha =$

(1) 4 (2) 8 (3) 2 (4) 6

Ans. **(1)**

Sol. $z_1 = \cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right) = \frac{1-i}{\sqrt{2}}$

$z_2 = \cos 0 + i\sin 0 = 1$

$z_3 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1+i}{\sqrt{2}}$

Now,

$|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2$

$= \left|\frac{1-i}{\sqrt{2}} + \frac{1-i}{\sqrt{2}} + \left(\frac{1+i}{\sqrt{2}}\right)^2\right|^2$

$= \left|\sqrt{2} - \sqrt{2}i + \frac{1}{2}(2i)\right|^2$

$= \left|\sqrt{2} + i(1-\sqrt{2})\right|^2$

$= 2 + (1-\sqrt{2})^2$

$= 2 + 1 + 2 - 2\sqrt{2}$

$= 5 - 2\sqrt{2} = \alpha + \beta\sqrt{2}$

Now $\alpha = 5$ and $\beta = -2$

So $3\beta + 2\alpha$

$= -6 + 10 = 4$

12. Let A = {1,2,3}, then the number of non-empty equivalence relations on set A is :

(1) 4 (2) 6 (3) 8 (4) 5

Ans. **(4)**

Sol. For equivalence relation, relation should be Reflexive, symmetric and transitive:

$R_1 = \{(1, 1) (2, 2) (3, 3)\}$

$R_2 = \{(1, 1) (2, 2) (3, 3) (1, 2) (2, 1)\}$

$R_3 = \{(1, 1) (2, 2) (3, 3) (2, 3) (3, 2)\}$

$R_4 = \{(1, 1) (2, 2) (3, 3) (1, 3) (3, 1)\}$

$R_5 = \{(1, 1) (2, 2) (3, 3) (1, 2) (2, 1), (2, 3), (3, 2), (1, 3) (3, 1)\}$

13. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. If $I_1 = \int_0^{\pi/4} f(x)dx$ and $I_2 = \int_0^{\pi/4} xf(x)dx$, then value of $7I_1 + 12I_2$ is:

Ans. **(1)**

Sol. $f(x) = (7\tan^6 x - 3\tan^2 x).\sec^2 x$

$$\therefore \quad I_1 = \int_0^{\frac{\pi}{4}} f(x)\,dx = \int_0^1 \left(7t^6 - 3t^2\right)\,dt = (t^7 - t^3)_0^1 = 0$$

Now $$I_2 = \int_0^{\frac{\pi}{4}} xf(x)dx = \int_0^1 \frac{\left(7t^6 - 3t^2\right)}{II} \frac{\tan^{-1}t}{I}\,dt$$

$$= \left(\tan^{-1}t\left(t^7 - t^3\right)\right)_0^1 - \int_0^1 \left(t^7 - t^3\right)\frac{1}{1+t^2}dt$$

$$= \int_0^1 \frac{t^3\left(1-t^4\right)}{1+t^2}\,dt = \int_0^1 t^3\left(1-t^2\right)\,dt$$

$$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

Now, $7I_1 + 12I_2 = 1$

14. Let x(y) is the solution of differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$ if $x(1) = 1$, then $x(2)$ is equal to:

(1) $\frac{3}{2} + \frac{3}{\sqrt{e}}$ (2) $\frac{3}{2} - \frac{3}{\sqrt{e}}$ (3) $-\frac{3}{2} - \frac{3}{\sqrt{e}}$ (4) $-\frac{3}{2} + \frac{3}{\sqrt{e}}$

Ans. **(2)**

Sol. $$\frac{dx}{dy} = \frac{-x}{y^2} + \frac{1}{y^3}$$

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

$$\text{I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

Now solution of differential equation

$$x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}}.\frac{1}{y^3}dy + C$$

Put $-\frac{1}{y}=t$

$\frac{1}{y^2}dy=dt$

$xe^{-\frac{1}{y}}=-\int e^t.tdt+C$

$xe^{-\frac{1}{y}}=-\left[te^t-e^t\right]+C$

$xe^{-\frac{1}{y}}=e^t(1-t)+C$

$xe^{-\frac{1}{y}}=e^{-\frac{1}{y}}\left(1+\frac{1}{y}\right)+C$

Now when $y=1 \quad \Rightarrow \quad x=1$

We get, $1.e^{-1}=e^{-1}(1+1)+C$

$C=-\frac{1}{e}$

Now $xe^{-\frac{1}{y}}=e^{-\frac{1}{y}}\left(1+\frac{1}{y}\right)-\frac{1}{e}$

Put $y=2$

$\frac{x}{\sqrt{e}}=\frac{1}{\sqrt{e}}\left(\frac{3}{2}\right)-\frac{1}{e}$

$x=\frac{3}{2}-\frac{3}{\sqrt{e}}$

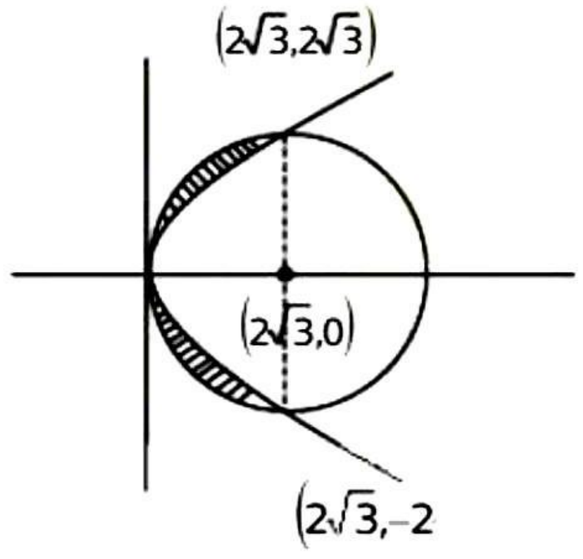
15. The area bounded by inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is

(1) $4(3\pi-8)$ (2) $3(2\pi-5)$ (3) $2(3\pi-8)$ (4) $2(3\pi-5)$

Ans. **(3)**

Sol.

Required area =



$$=2\left[\frac{1}{4}(12\pi)-\int_0^{2\sqrt{3}}\sqrt{2\sqrt{3}x}\right]=2\left[3\pi-\sqrt{2\sqrt{3}}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{2\sqrt{3}}=2\left[3\pi-\sqrt{2\sqrt{3}}\times\frac{2}{3}\left(2\sqrt{3}\right)^{3/2}\right]$$

$$=2\left[3\pi-\frac{2}{3}\times12\right]\quad=2[3\pi-8]$$

16. Let, $A=\{1,2,3,.....,10\}$ and $B=\left\{\frac{m}{n};m<n\ \&\ gcd(m,n)=1\ \&\ m,n\in A\right\}$. Then number of all elements in set B is equal to _____.

Ans. **(31)**

Sol. Number of elements in set B, corresponding to,

$m=1$	are		$=9$
$m=2$	are	$n=3,5,7,9$	$=4$
$m=3$	are	$n=4,5,7,8,10$	$=5$
$m=4$	are	$n=5,7,9$	$=3$
$m=5$	are	$n=6,7,8,9$	$=4$
$m=6$	are	$n=7$	$=1$
$m=7$	are	$n=8,9,10$	$=3$
$m=8$	are	$n=9$	$=1$
$m=9$	are	$n=10$	$=1$

$\therefore$ Total number $=9+4+5+3+4+1+3+1+1=31$

17. $f(x)=16\left(\sec^{-1}x\right)^2+\left(\operatorname{cosec}^{-1}x\right)^2$ then difference between the maximum and the minimum value of, f(x) is equal to _____.

(1) $\frac{1089}{68}\pi^2$ (2) $\frac{1089}{136}\pi^2$ (3) $\frac{1089}{17}\pi^2$ (4) $\frac{1089}{34}\pi^2$

Ans. **(1)**

Sol. $\Rightarrow\quad 16\left(\sec^{-1}x\right)^2+\left(\frac{\pi}{2}-\sec^{-1}x\right)^2$

$\Rightarrow\quad 17\left(\sec^{-1}x\right)^2-\pi\sec^{-1}x+\frac{\pi^2}{4}$

$$f(x)=17\left(\left(\sec^{-1}x-\frac{\pi}{34}\right)^2\right)+\frac{4\pi^2}{17}$$

$f(x)_{max}$ will be at, $\sec^{-1}x=\pi$

i.e. $17\left(\frac{33\pi}{34}\right)^2+\frac{4\pi^2}{17}=\frac{1105}{68}\pi^2$

$f(x)_{min}$ will be at, $\sec^{-1}x=\frac{\pi}{34}$

i.e. $f(x)_{min}=\frac{4\pi^2}{17}$

Now difference of maximum and minimum values of, f (x) is,

$\frac{1105}{68}\pi^2-\frac{4\pi^2}{17}=\frac{1089}{68}\pi^2$

18. Let the parabola $y = x^2 + px - 3$ cuts the coordinate axes at P, Q and R. A circle with centre $(-1, -1)$ passes through P, Q and R, then area of ΔPQR is

(1) 3 (2) 6 (3) 5 (4) 9

Ans. **(2)**

Sol. Parabola cuts $x-$ axis at $y=0$

$x^2 + px - 3 = 0$

and $y-$axis at $x=0$

$y=-3$

Now radius of circle $= \sqrt{(-1-0)^2+(-1+3)^2} = \sqrt{5}$

equation of circle

$(x+1)^2 + (y+1)^2 = 5$

point of $(x, 0)$ satisfying circle

$(x+1)^2 + 1 = 5$

$x + 1 = \pm 2$

$x = -3, 1$

Now sum of roots of (1) $= -p = -2$

$p = 2$

Solving $x^2 + 2x - 3 = 0$

$x = -3, 1$

So points are $(0, -3), (-3, 0)$ & $(1, 0)$

are $= \frac{1}{2} \times 4 \times 3 = 6$