Date: 24/08/2020

Test Booklet Set No.

12

Gujarat

Questions & Solutions

for

GUJCET 2020 (MATHEMATICS)

INSTRUCTIONS TO CANDIDATES

1. The **Mathematics** test consists of 40 questions. Each question carries 1 mark. For each correct response, the candidate will get **1 mark**. For each incorrect response **1/4 mark** will be deducted. The maximum marks are **40**.
2. This test is of 1hour duration.
3. Use **Black Ball Point Pen** only for writing particulars on OMR Answer Sheet and marking answers by darkening the circle (•).
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room/Hall. The candidates are allowed to take away this Test Booklet with them.
6. The Set No. for this Booklet is 01. Make sure that the Set No. Printed on the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
7. The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8. Do not write your Seat No. anywhere else, except in the specified space in the Test Booklet/Answer Sheet.
9. Use of White fluid for correction is not permissible on the Answer Sheet.
10. Each candidate must show on demand his/her Admission Card to the Invigilator.
11. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.
12. Use of simple (manual) Calculator is permissible.
13. The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak-01). Cases where a candidate has **not** signed the Attendance Sheet (Patrak-01) will be deemed not to have handed over the Answer Sheet and will be dealt with as an unfair means case.
14. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
15. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16. The candidates will write the Correct Test Booklet Set No. as given in the Test Booklet/Answer Sheet in the Attendance Sheet. (Patrak-01)

MATHEMATICS

1. If $|\vec{a}| = 3$ then value of

$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times k|^2 =$ ____________.

(A) 9 (B) 18

(C) 27 (D) 36

Answer (B)

Sol. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $|\vec{a}| = x^2 + y^2 + z^2$

Now, $|\vec{a} \times \hat{i}|^2 = y^2 + z^2$, $|\vec{a} \times \hat{j}|^2 = x^2 + z^2$

and $|\vec{a} \times \hat{k}|^2 = x^2 + y^2$

$\therefore \quad |\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2(x^2 + y^2 + z^2)$

$= 2|\vec{a}|^2 = 18$

2. The co-ordinates of the foot of perpendicular drawn from origin to the plane $2x - 3y + 4z - 6 = 0$ is ________

(A) $\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$ (B) $\left(\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

(C) $\left(\frac{12}{29}, \frac{18}{29}, \frac{24}{29}\right)$ (D) $\left(-\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

Answer (A)

Sol. Let (x_1, y_1, z_1) be foot of perpendicular from origin to plane $2x - 3y + 4z - 6 = 0$.

$$\therefore \quad \frac{x_1 - 0}{2} = \frac{y_1 - 0}{-3} = \frac{z_1 - 0}{4} = -\frac{(-6)}{2^2 + 3^2 + 4^2}$$

$$\therefore \quad \frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = \frac{6}{29}$$

$$\therefore \quad (x_1, y_1, z_1) = \left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$$

3. The angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$ is ________.

(A) $\cos^{-1}\frac{8}{21}$ (B) $\tan^{-1}\frac{8}{\sqrt{377}}$

(C) $\sin^{-1}\frac{8}{\sqrt{377}}$ (D) $\sin^{-1}\left(\frac{21}{8}\right)$

Answer (B)

Sol. Let angle between line and plane be θ.

Line is $\frac{x+1}{2} = \frac{y-0}{3} = \frac{z-3}{6}$

with direction ratios (2, 3, 6)

And plane is $10x + 2y - 11z - 3 = 0$ direction ratios of normal to plane is $(10, 2, -11)$

$$\therefore \quad \sin\theta = \left|\frac{20 + 6 - 66}{\sqrt{4 + 9 + 36}\sqrt{100 + 4 + 121}}\right|$$

$$\sin\theta = \left|\frac{-40}{7 \times 15}\right|$$

$$\sin\theta = \frac{8}{21} \text{ or } \tan\theta = \frac{8}{\sqrt{377}}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{8}{\sqrt{377}}\right)$$

4. If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12k) + 13 = 0$ then the values of p are ______________.

(A) $1, \frac{7}{3}$ (B) $1, \frac{4}{3}$

(C) $2, \frac{4}{3}$ (D) $\frac{7}{3}, 2$

Answer (A)

Sol. Equation of plane is $3x + 4y - 12z + 13 = 0$

Now as $(1, 1, p)$ and $(-3, 0, 1)$ are equidistant from the plane

$$\left|\frac{3 + 4 - 12P + 13}{\sqrt{9 + 16 + 144}}\right| = \left|\frac{-9 + 0 - 12 + 13}{\sqrt{9 + 16 + 144}}\right|$$

$|20 - 12p| = -8 \Rightarrow 20 - 12p = 8 \Rightarrow p = 1$

and $20 - 12p = -8 \Rightarrow 12p = 28 \Rightarrow p = \frac{7}{3}$

5. The maximum value of $Z = 3x + 4y$ subject to constraints $x + y \le 4, x \ge 0, y \ge 0$ is ________.

(A) 16 (B) 12

(C) 0 (D) not possible

Answer (A)

Sol. $z = 3x + 4y$

At $(0, 0)$; $z = 0$

At $(4, 0)$; $z = 12$

At $(0, 4)$; $z = 16$

$\therefore$ Maximum value = 16

6. If A and B are independent events such that $P(A) = p$, $P(B) = 2p$ and P(Exactly one of A and B) = $\frac{5}{9}$, then p = ____________.

(A) $\frac{1}{3}, \frac{5}{12}$ (B) $\frac{1}{2}, \frac{3}{4}$

(C) $\frac{1}{12}, \frac{5}{3}$ (D) $\frac{2}{15}, \frac{5}{12}$

Answer (A)

Sol. As A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = 2p^2$$

and P(exactly one of A and B)

$$= P(A \cup B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B) = \frac{5}{9}$$

$\Rightarrow$ $p + 2p - 2 \cdot 2p^2 = \frac{5}{9}$

$\Rightarrow$ $36p^2 - 27p + 5 = 0$

$\Rightarrow$ $(12p - 5)(3p - 1) = 0$

$\therefore$ $p = \frac{5}{12}, \frac{1}{3}$

7. For the probability distribution

X	1	2	3	4
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2) =$ _________.

(A) 7 (B) 5

(C) 3 (D) 10

Answer (D)

Sol. $E(X^2) = 1 \times \frac{1}{10} + 4 \times \frac{1}{5} + 9 \times \frac{3}{10} + 16 \times \frac{2}{5}$

$$= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5}$$

$$= \frac{1 + 8 + 27 + 64}{10}$$

$$= \frac{100}{10} = 10$$

8. If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$ then ________.

(A) $P\left(\frac{B}{A}\right) = 1$ (B) $P\left(\frac{A}{B}\right) = 0$

(C) $P\left(\frac{B}{A}\right) = 0$ (D) $P\left(\frac{A}{B}\right) = 1$

Answer (D)

Sol. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

9. Let $f : R \to R$ be defined by $f(x) = 2x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2 + 1}$, then gof is ____________.

(A) $\frac{2x^2 - 5}{4x^4 + 20x^2 + 26}$ (B) $\frac{2x^2 - 5}{4x^4 - 20x^2 + 26}$

(C) $\frac{2x^2}{x^4 + 2x^2 - 4}$ (D) $\frac{2x^2}{4x^4 - 20x^2 + 26}$

Answer (B)

Sol. $g(f(x)) = \frac{f(x)}{(f(x))^2 + 1}$

$$= \frac{2x^2 - 5}{(2x^2 - 5)^2 + 1}$$

$$= \frac{2x^2 - 5}{4x^4 - 20x^2 + 26}$$

10. Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$. Then the range of f is _________.

(A) $[1, \infty)$ (B) $[4, \infty)$

(C) R (D) $[5, \infty)$

Answer (A)

Sol. $f(x) = (x-2)^2 + 1$

As $x \in [2, \infty)$

$0 \le x - 2 < \infty$

$1 \le (x-2)^2 + 1 < \infty$

$\therefore$ Range is $[1, \infty)$

11. On R, binary operation * is defined by $a * b = a + b + ab$ then identity and inverse of * are ………….. respectively.

(A) $0, \frac{a}{1-a}$ (B) $1, \frac{a}{1+a}$

(C) $0, -\frac{a}{1+a}$ (D) $1, \frac{a}{1-a}$

Answer (C)

Sol. If $a * e = e * a = a$ then e is identity

$a + e + ae = a$

$e(1+a) = 0$

$e = 0$ (Identity)

Now if $a * b = e$ then b is inverse of a

$a + b + ab = 0$

$a = -b(1+a)$

inverse $b = \frac{-a}{1+a}$

12. $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) =$ ____________.

(A) $\cos^{-1}\left(\frac{84}{85}\right)$ (B) $\cos^{-1}\left(\frac{24}{85}\right)$

(C) $\sin^{-1}\left(\frac{24}{85}\right)$ (D) $\sin^{-1}\left(\frac{84}{85}\right)$

Answer (A)

Sol. $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right)$

$= \sin^{-1}\left(\frac{3}{5} \times \frac{15}{17} - \frac{4}{5} \times \frac{8}{17}\right)$

$= \sin^{-1}\left(\frac{13}{85}\right)$

$= \cos^{-1}\left(\frac{84}{85}\right)$

13. $\tan^2(\sec^{-1} 3) + \text{cosec}^2(\cot^{-1} 2) + \cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right) =$ ____________.

(A) 15 (B) 16

(C) 14 (D) 13

Answer (D)

Sol. $\tan^2(\sec^{-1} 3) + \text{cosec}^2(\cot^{-1} 2) + \cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right)$

$= \tan^2(\tan^{-1}(2\sqrt{2})) + \text{cosec}^2(\text{cosec}^{-1}\sqrt{5}) + \cos^2\left(\frac{\pi}{2}\right)$

= 8 + 5 = 13

14. If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$ then ____________.

(A) $1 - a^2 + bc = 0$ (B) $1 + a^2 + bc = 0$

(C) $1 + a^2 - bc = 0$ (D) $1 - a^2 - bc = 0$

Answer (D)

Sol. $A^2 = I$

$\begin{bmatrix} a & b \\ c & -a \end{bmatrix}\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab - ab \\ ac - ac & bc + a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + bc = 1$

$\Rightarrow 1 - a^2 - bc = 0$

15. If A is a square matrix such that $A^2 = I$ then $(A-I)^3 + (A+I)^3 - 7A$ is equal to ____________.

(A) $I + A$ (B) $I - A$

(C) A (D) $3A$

Answer (C)

Sol. $(A-I)^3 + (A+I)^3 - 7A$

$= A^3 - I - 3A^2 + 3A + A^3 + I + 3A^2 + 3A - 7A$

$= 2A^3 - A$

$= A(2A^2 - I)$

$= A(2I - I) = A$

16. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and inverse of A is $\frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ x & -3 & 1 \end{bmatrix}$ then x = ____________.

(A) 5 (B) 3

(C) 2 (D) 4

Answer (A)

Sol. We Know that $A\,A^{-1} = I$

Solving, we get $x = 5$

17. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\tan t & t & 2t \\ \tan t & t & t \end{vmatrix}$. Then $\lim\limits_{t\to 0}\frac{f(t)}{t^2}$ is equal to ____________.

(A) 3 (B) 1

(C) –1 (D) 0

Answer (D)

Sol. $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\tan & t & 2t \\ \tan t & t & t \end{vmatrix}$

$\Rightarrow f(t) = t\left[-t\cos t - (2t\tan t - 2t\tan t) + 1(\tan t)\right]$

$\Rightarrow f(t) = -t^2\cos t + t\tan t$

$\therefore \lim\limits_{t\to 0}\frac{f(t)}{t^2}\ \lim\limits_{t\to 0}\left(-\cos t + \frac{\tan t}{t}\right) = 0$

18. If $x, y \in R$ and $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$ $= 2y + 6$, then y = ______________.

(A) 0 (B) 3

(C) –3 (D) 6

Answer (C)

Sol. Applying $C_1 \to C_1 - C_2$, we get

$$\begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^y - b^{-x})^2 & 1 \\ 4 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 2y + 6$$

$\Rightarrow 2y + 6 = 0$

$\Rightarrow y = -3$

19. For ΔABC, the value of $\begin{vmatrix} 0 & \sin A & \tan B \\ -\sin(B+C) & 0 & \cos C \\ \tan(A+C) & -\cos C & 0 \end{vmatrix}$ = ____________.

(A) –1 (B) 0

(C) 1 (D) sin$A$cos$C$

Answer (B)

Sol. Given determinant is of skew symmetric matrix of odd order so value = 0

20. If function $f(\alpha) = \begin{cases} \frac{1-\cos 6\alpha}{36\alpha^2} & \text{if } \alpha \neq 0 \\ k & \text{if } \alpha = 0 \end{cases}$ is continuous at $\alpha = 0$ then k = __________.

(A) $-\frac{1}{2}$ (B) 1

(C) $\frac{1}{2}$ (D) 0

Answer (C)

Sol. $k = \lim\limits_{\alpha\to 0}\frac{1-\cos 6\alpha}{36\alpha^2} = \frac{1}{2}$

21. If $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ and $\frac{dy}{dx} = \frac{2^{x+1}\log 2}{f(x)}$ then $f(0)$ = ____________.

(A) 0 (B) –2

(C) 2 (D) 2 log 2

Answer (C)

Sol. Now $y = \sin^{-1}\left(\frac{2\cdot 2^x}{(2^x)^2 + 1}\right)$

Let $2^x = \tan\theta$

$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow y = 2\tan^{-1}(2^x)$

$\therefore \frac{dy}{dx} = \frac{2.2^x\ \log 2}{1+4^x} = \frac{2^{x+1}\log 2}{4^x+1}$

$\therefore f(x) = 4^x + 1$

$\Rightarrow f(0) = 2$

22. For function $f(x) = x + \frac{1}{x}, x \in [1, 2]$, the value of C for mean value theorem is ____________.

(A) 2 (B) $\sqrt{2}$

(C) 1 (D) $\sqrt{3}$

Answer (B)

Sol. We know

$$f'(C) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 1 - \frac{1}{C^2} = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow 1 - \frac{1}{C^2} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{C^2} = \frac{1}{2} = C = \pm\sqrt{2}$$

Now as $C \in (1, 2)$

$$\Rightarrow C = \sqrt{2}$$

23. The interval in which $y = x^2 e^{-x}$ is increasing is ___________.

(A) $(0, 2)$

(B) $(2, \infty)$

(C) $(-\infty, \infty)$

(D) $(-2, 0)$

Answer (A)

Sol. The interval in which $y = x^2 \cdot e^{-x}$ is increasing

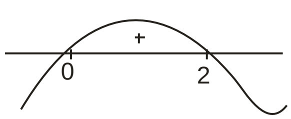
$$y = x^2 \cdot e^{-x}$$

$$\Rightarrow \frac{dy}{dx} = 2xe^{-x} + x^2(-e^{-x})$$

$$= e^{-x}[2x - x^2]$$

$$= -xe^{-x}[x - 2]$$

For increasing function $\frac{dy}{dx} > 0$



$\therefore$ Increasing in interval (0, 2).

24. The rate of change of volume of sphere with respect to its radius r at $r = 2$ is _____________.

(A) 24π

(B) 32π

(C) 16π

(D) 8π

Answer (C)

Sol. $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2$$

$$= 4\pi r^2$$

Now at $(r = 2)$ = 16π

25. The tangent to the curve given by $x = e^\theta \cdot \cos\theta, y = e^\theta \cdot \sin\theta$ at $\theta = \frac{\pi}{4}$ makes an angle with X-axis is ___________.

(A) $\frac{\pi}{2}$ (B) 0

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Answer (A)

Sol. $x = e^\theta \cos\theta$

$$\Rightarrow \frac{dx}{d\theta} = e^\theta \cos\theta - e^\theta \sin\theta$$

$$= e^\theta(\cos\theta - \sin\theta)$$

$$y = e^\theta \sin\theta$$

$$\Rightarrow \frac{dy}{d\theta} = e^\theta \sin\theta + e^\theta \cos\theta$$

$$= e^\theta(\sin\theta + \cos\theta)$$

$$= \frac{dy}{dx} = \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta}$$

Not defined at $\theta = \frac{\pi}{4}$

$\therefore$ $\theta = \frac{\pi}{2}$ (Angle formed by tangent with X-axis)

26. The minimum value of $f(x) = x \log x$ is ________.

(A) 0 (B) $-\frac{1}{e}$

(C) $\frac{1}{e}$ (D) e

Answer (B)

Sol. $y = x \log x$

$$\Rightarrow \frac{dy}{dx} = \log x + \left(x \times \frac{1}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \log x$$

$\therefore$ Minima at $x = \frac{1}{e}$

$$\therefore \quad f\left(\frac{1}{e}\right) = \frac{-1}{e}$$

27. If $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx = \frac{x^3}{3} + f(x) + C$, then $f(1)$ = ___________.

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{1}{2}$

Answer (B)

Sol. $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx$

$$\int \left(x^2 + \frac{1}{x^2 + 1}\right) dx$$

$$\frac{x^3}{3} + \tan^{-1}(x) + c$$

$$\therefore \quad f(x) = \tan^{-1}(x)$$

$$f(1) = \tan^{-1}(1)$$

$$\frac{\pi}{4}$$

28. $\int \frac{x+100}{(x+101)^2} e^x dx =$ _______ $+ C$.

(A) $\frac{1}{x+101} e^x$ (B) $\frac{x}{x+101} e^x$

(C) $\frac{1}{x+100} e^x$ (D) $(x+101)e^x$

Answer (A)

Sol. $\int \frac{(x+101)-1}{(x+101)^2} e^x \, dx$

Now we know $\int e^x \left(f(x) + f'(x)\right) dx$

$$= e^x f(x) + c$$

$$= \frac{e^x}{x+101} + c$$

29. $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx =$ _____ $+ C$.

(A) $-2\sqrt{\cot x}$ (B) $-2\sqrt{\tan x}$

(C) $2\sqrt{\cot x}$ (D) $\frac{1}{\sqrt{\cot x}}$

Answer (A)

Sol. $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx$

$$= \int \frac{\sqrt{\cos x}}{\sqrt{\sin x} \sin x \cos x} dx$$

$$= \int (\cos x)^{\frac{-1}{2}} (\sin x)^{\frac{-3}{2}} dx$$

As $m + n$ negative even integer put

$\therefore \quad \tan x = t$

$$= \int (\tan x)^{\frac{-3}{2}} (\cos x)^{-2} dx$$

$$= \int (t)^{\frac{-3}{2}} dt$$

$$= \frac{t^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} + c$$

$$= \frac{-2}{\sqrt{\tan x}} + c$$

$$= -2\sqrt{\cot x} + c$$

30. $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right)dx =$ ______________.

(A) π (B) 0

(C) $\frac{\pi}{2}$ (D) 1

Answer (B)

Sol. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right)dx$

as we know $\int_{-a}^{a} f(x) = 0$

if $f(x)+f(-x)=0$ (odd function)

$\therefore$ 0

31. $\int_4^9 \frac{\sqrt{x}}{\left(30-x^{\frac{3}{2}}\right)^2}dx =$ ______________.

(A) $\frac{19}{66}$ (B) $\frac{19}{33}$

(C) $\frac{38}{99}$ (D) $\frac{19}{99}$

Answer (D)

Sol. $\int_4^9 \frac{\sqrt{x}}{\left(30-x^{\frac{3}{2}}\right)^2}dx$ …(1)

Let $30-x^{\frac{3}{2}}=t$

$\Rightarrow \frac{-3}{2}(x)^{\frac{1}{2}}dx = dt$

$\Rightarrow \sqrt{x}dx = \frac{-2}{3}dt$

Now using (1),

$= \frac{-2}{3}\int_{22}^{3}\frac{1}{t^2}dt$

$= \frac{2}{3}\left[\frac{1}{t}\right]_{22}^{3}$

$= \frac{2}{3}\left[\frac{1}{3}-\frac{1}{22}\right]$

$= \frac{19}{99}$

32. If $f(a+b-x)=f(x)$ then $\int_a^b x\cdot f(x)dx$ is equal to ___________.

(A) $\frac{a+b}{2}\int_a^b f(x)dx$ (B) $\frac{a+b}{2}\int_a^b f(b+x)dx$

(C) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (D) $\frac{b-a}{2}\int_a^b f(x)dx$

Answer (A)

Sol. $I = \int_a^b x\cdot f(x)dx$

$= \int (a+b-x) f(a+b-x) = I$

add both

$\Rightarrow 2I = \int_a^b (a+b)f(x)dx$

$= \left(\frac{a+b}{2}\right)\int_a^b f(x)dx$

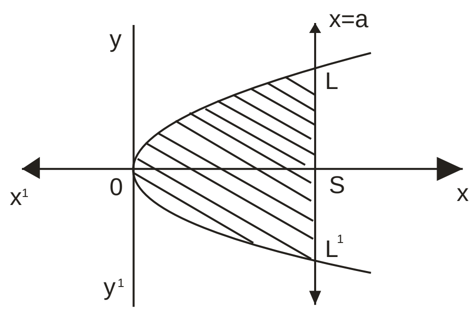
33. The area of the parabola $y^2 = 4ax$ bounded by its latus rectum is __________.

(A) $\frac{16}{3}a^2$ (B) $\frac{4}{3}a^2$

(C) $\frac{8}{3}a^2$ (D) $4a^2$

Answer (C)

Sol. For parabola $y^2 = 4ax$



Area required = area OLSL'

= 2x Area OSL

$= 2\times\int_0^a y dx$

Now parabola equation is

$y^2 = 4ax$

$\Rightarrow y = \pm\sqrt{4ax}$

Since OSL is in 1st quadrant

$y=\sqrt{4ax}$

Area required = $2\times\int_0^a \sqrt{4ax}dx$

$= 2\sqrt{4a}\int_0^a \sqrt{x}dx$

$= 4\sqrt{a}\int_0^a \sqrt{x}dx$

$= \frac{8}{3}a^2$

34. The area enclosed by the curve $x=4\cos\theta, y=3\sin\theta$ is ____________.

(A) 4π (B) 6π

(C) 8π (D) 12π

Answer (D)

Sol. $\left(\frac{x}{4}\right)^2+\left(\frac{y}{3}\right)^2=1$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$

And area of ellipse = πab

$= \pi\times4\times3$

$= 12\pi$

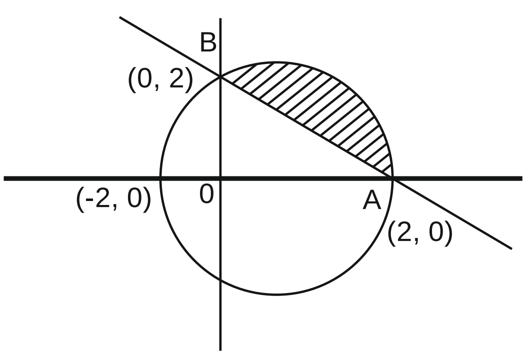
35. The smallest area enclosed by circle $x^2+y^2=4$ and line $x + y = 2$ is ____________.

(A) $\pi+2$ (B) $\pi-2$

(C) π (D) 2π

Answer (B)

Sol. The Smallest area enclosed by circle $x^2+y^2=4$ and line $x+y=2$



Required area

$= \frac{1}{4}$(Area of circle) – area of triangle ΔOAB

= Area = $\frac{\pi}{4}\times(2)^2-\frac{1}{2}\times2\times2 = \pi-2$

36. The order and degree of differential equation $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}=\frac{d^2y}{dx^2}$ are p and q respectively then $p + q$ = _________.

(A) 6 (B) 4

(C) 2 (D) 5

Answer (B)

Sol. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx}\right)^2$

$\therefore$ Order = $2=p$

Degree = 2 = q

$\therefore$ $p+q=4$

37. Integrating factor of differential equation $\left(\tan^{-1}y-x\right)dy=\left(1+y^2\right)dx$ is _________.

(A) e^{1+y^2} (B) e^y

(C) $e^{\tan^{-1}x}$ (D) $e^{\tan^{-1}y}$

Answer (D)

Sol. $\frac{dy}{dx}=\frac{\left(1+y^2\right)}{\left(\tan^{-1}y-x\right)}$

$\Rightarrow \frac{dx}{dy}=\frac{\tan^{-1}y}{1+y^2}\frac{-x}{1+y^2}$

$\Rightarrow \frac{dx}{dy}+\frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$

Now integrating factor = $e^{\int\frac{dy}{1+y^2}}$

$e^{\tan^{-1}y}$

38. The differential equation $y\frac{dy}{dx}+x=k$ represents ____________.

(A) circles (B) hyperbolas

(C) parabolas (D) ellipses

Answer (A)

Sol. $\frac{dy}{dx} = \frac{k-x}{y}$

$y\,dy = dx(k-x) \quad \frac{y^2}{2} = kx - \frac{x^2}{2} + c$

$x^2 + y^2 = 2kx + 2c$

$\therefore$ Circle

39. If $\vec{a} = 2\hat{i} - \hat{j} + k, \vec{b} = \hat{i} + \hat{j} - 2k, \vec{c} = \hat{i} + 3\hat{j} - k$, if $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$, then the value of λ is ____________.

(A) 0 (B) 2

(C) –2 (D) 3

Answer (C)

Sol. As $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

$\Rightarrow \vec{a}.(\lambda\vec{b} + \vec{c}) = 0$

$\Rightarrow (2\hat{i} - \hat{j} + k).(\lambda[\hat{i} + \hat{j} - 2k] + [\hat{i} + 3\hat{j} - k]) = 0$

$\Rightarrow 2(\lambda + 1) - (\lambda + 3) + 1(-2\lambda - 1) = 0$

$\Rightarrow 2\lambda + 2 - \lambda - 3 - 2\lambda - 1 = 0$

$\Rightarrow \lambda = -2$

40. For three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfies $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 2$ then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ = __________.

(A) 29

(B) $\frac{29}{2}$

(C) $-\frac{9}{2}$

(D) $-\frac{29}{2}$

Answer (D)

Sol. $(\vec{a} + \vec{b} + \vec{c})^2 =$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow (9 + 16 + 4) + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = \frac{-29}{2}$

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