Time : 1 hr. M.M. : 40

Answers & Solutions

for

GUJCET - ME- 2018

(Mathematics)

Important Instructions:

1. The Mathematics test consists of 40 questions. Each question carries 1 mark. For each correct response, teh candidate will get 1 mark. For each incorrect response, mark will be deducted. The maximum marks are 40.
2. This Test is of 1 hour duration
3. Use Black Ball Point Pen only for writing particulars on OMR Answer Sheet and marking answers by darketing the circle ‘’.
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room / Hall. The candidates are allowed to take away this Test Booklet with them.
6. The Set No. for this Booklet is . Mark sure that the Set No. printed on the Answer sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report teh matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
7. The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8. Do not write your Seat No. anywhere else, except in the specified space in the Test Booklet/Answer Sheet.
9. Use of White fluid for correction is not permissible on the Answer sheet.
10. Each candidate must show on demand his / her Admission Card to the Invigilator.
11. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.
12. Use of Manual Calculator is permissible.
13. The candidate Should not leave the Examination Hall handling over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak-01). Cases where a candidate has not signed the Attedance Sheet (Patrak-01) will be deemed not to have handed over the Answer Sheet and Will be dealt with as an unfair means case.
14. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
15. No Part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16. The candiates will write the Correct Test Booklet Set No. as given in the Test Booklet / Answer Sheet in the Attendance Sheet. (Patrak - 01).

PART–A : MATHEMATICS

1. Where does $f(x)=x+\sqrt{1-x}; 0<x<1$ decrease?

(A) $(0,1)$ (B) $\left(0,\frac{3}{4}\right)$

(C) $\left(\frac{3}{4},1\right)$ (D) $\left(\frac{3}{4},\infty\right)$

Answer (C)

Sol. $f(x)=x+\sqrt{1-x}$ $\quad 0<x<1$

$$f'(x)=1-\frac{1}{2\sqrt{1-x}}=0 \Rightarrow x=\frac{3}{4}$$

$\therefore$ Interval of decrease is $\left(\frac{3}{4},1\right)$

2. If $f'(x)=2-\frac{5}{x^4}$ and $f(1)=\frac{14}{3}$, then $f(-1)=$

(A) $\frac{11}{3}$ (B) $-\frac{8}{3}$

(C) $-\frac{14}{3}$ (D) 0

Answer (B)

Sol. $f'(x)=2-\frac{5}{x^4}$

$$\Rightarrow \quad f(x)=2x+\frac{5}{3x^3}+c$$

Now, $f(1)=\frac{14}{3}$

$\therefore c=1$

$$\Rightarrow \quad f(x)=2x+\frac{5}{3x^3}+1$$

$\therefore f(-1)=\frac{-8}{3}$

3. $\int\frac{\cos\alpha}{\sin x\cos(x-\alpha)}dx=..................+c$ where $0<x<\alpha<\frac{\pi}{2}$ and $\alpha-$ constant

(A) $\log|\cot x+\tan\alpha|$ (B) $\log|\tan x+\cot\alpha|$

(C) $-\log|\tan x+\cot\alpha|$ (D) $-\log|\cot x+\tan\alpha|$

Answer (D)

Sol. $\int\frac{\cos\alpha\, dx}{\sin x\cos(x-\alpha)}$

$$=\int(\cot x+\tan(x-\alpha))dx$$

$$=\ln\left|\frac{1}{\cot x\cos\alpha+\sin\alpha}\right|+c$$

$$=-\ln(\cot x\cos\alpha+\sin\alpha)+c$$

$$=-\ln(\cot x+\tan\alpha)+c$$

4. $\int\frac{e^{\cot^{-1}x}}{1+x^2}(x^2-x+1)dx=............+c$

(A) $x\cdot e^{\cot^{-1}x}$ (B) $e^{\cot^{-1}x}$

(C) $\frac{e^{\cot^{-1}x}}{1+x^2}$ (D) $-e^{\cot^{-1}x}$

Answer (A)

Sol. $\int\frac{e^{\cot^{-1}x}}{1+x^2}(x^2-x+1)dx$

Let $\cot^{-1}x=t$

$$\Rightarrow \quad -\frac{1}{1+x^2}dx=dt$$

$$\int e^t(\cot t-\operatorname{cosec}^2 t)dt$$

$$e^t\cot t+c$$

$$x\,e^{\cot^{-1}x}+c$$

5. $\int_0^{\pi/2}(x-[\cos x])dx=.......$ where [t] = greatest integer less or equal to t

(A) $\frac{\pi^2}{8}-\frac{\pi}{8}$ (B) $\frac{\pi^2}{8}$

(C) $\frac{\pi^2}{8}-1$ (D) $\frac{\pi}{4}$

Answer (B)

Sol. $\int_0^{\frac{\pi}{2}}(x-[\cos x])dx$

$\left[\frac{x^2}{2}-0\right]_0^{\pi/2}$

$\frac{\pi^2}{8}$

6. If $\int_{\log 2}^{a}\frac{e^x}{\sqrt{e^x-1}}dx=2$, then a =

(A) 2log2 (B) log2

(C) log5 (D) 0

Answer (C)

Sol. $\int_{\log 2}^{a}\frac{e^x}{\sqrt{e^x-1}}dx=2$

$\Rightarrow \left[2\sqrt{e^x-1}\right]_{\log 2}^{a}=2$

$\Rightarrow 2\sqrt{e^a-1}-2=2$

$\Rightarrow 2\sqrt{e^a-1}=4$

$\Rightarrow \sqrt{e^a-1}=2$

$\Rightarrow e^a-1=4$

$\Rightarrow e^a=5$

$\Rightarrow a=\log_e 5$

7. $\int_0^{\sqrt{2}}\sqrt{2-x^2}dx=$

(A) π (B) $-\frac{\pi}{2}$

(C) 0 (D) $\frac{\pi}{2}$

Answer (D)

Sol. $\int_0^{\sqrt{2}}\sqrt{2-x^2}dx$

$=\left[\frac{x}{2}\sqrt{2-x^2}+\frac{2}{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^{\sqrt{2}}$

$=\frac{\pi}{2}$

8. Area of the region bounded by rays |x|+y=1 and X-axis is

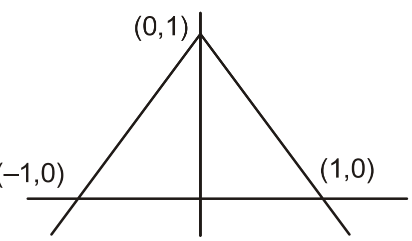
(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Answer (B)

Sol. Area bounded by rays

$|x|+y=1$ and X-axis



$2\left(\frac{1}{2}\times1\times1\right)=1$

9. If area bounded by the curves $x=ay^2$ and $y=ax^2$ is 1, then a= ______ (a>0)

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Answer (B)

Sol. Area bounded by $y^2=4ax$ and $x^2=4by$

given by $\left|\frac{16ab}{3}\right|=1$

$\therefore \left|\frac{16}{3}\times\frac{1}{4a}\times\frac{1}{4a}\right|=1$

$a^2=\frac{1}{3}$

$a=\frac{1}{\sqrt{3}}$ (a > 0)

10. The solution of the differential equation $2x\frac{dy}{dx}-y=0;y(1)=2$ represents ....

(A) Parabola (B) Straightline

(C) Circle (D) Ellipse

Answer (A)

Sol. $2x\frac{dy}{dx} - y = 0$

$\frac{dy}{dy} = \frac{dx}{2x} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{2x}$

$\ln y = \ln\sqrt{x} + \ln c$

$y = c\sqrt{x}$ as y(1) = 2

c = 2

$y = 2\sqrt{x}$

$x = \frac{y^2}{4}$

$\therefore$ Parabola

11. Particular solution of differential equation $e^{\frac{dy}{dx}} = x;\ y(1) = 3;\ x > 0$ is.....

(A) $y = \log x - x + 4$ (B) $y^2 = \log x + 4$

(C) $\log y = x^2 + 4$ (D) $2y = x^2 + 5$

Answer (A)*

Sol. $\frac{dy}{dx} = \log x$

$y = x\log x - x + c$

as y (1) = 3

3 = –1 + c

c = 4

$y = x\log x - x + 4$

***Note**: option (A) can be taken, but 'x' is missing*

12. The population of a city increases at the rate 3% per year. If at time t the population of city is p, then find equation of p in time t.

(A) $p = 3e^{\frac{3t}{100}}$ (B) $p = e^{\frac{3t}{100}}$

(C) $p = ce^{\frac{3t}{100}}$ (D) $p = \frac{3}{100}e^{3t}$

Answer (C)

Sol. $\frac{dp}{dt} = \frac{3}{100}p$

$\Rightarrow \int\frac{dp}{p} = \int\frac{3}{100}\,dt$

$\Rightarrow \ln p = \frac{3}{100}t + \ln c$

$\Rightarrow p = ce^{\frac{3}{100}t}$

13. If $\bar{a}$ is unit vector, then $|\bar{a}\times\hat{i}|^2 + |\bar{a}\times\hat{j}|^2 + |\bar{a}\times\hat{k}|^2 =$

(A) 1 (B) 0

(C) 2 (D) 3

Answer (C)

Sol. Let $a = x\hat{i} + y\hat{j} + z\hat{k}$;

as $x^2 + y^2 + z^2 = 1$

$|a\times i|^2 + |a\times j|^2 + |a\times k|^2$

$= y^2 + z^2 + x^2 + z^2 + x^2 + y^2$

$= 2(x^2 + y^2 + z^2) = 2$

14. If for unit vectors $\bar{a}$ and $\bar{b}$, $\bar{a} + 2\bar{b}$ and $5\bar{a} - 4\bar{b}$ are perpendicular to each other, then $(\bar{a} \wedge \bar{b}) =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\cos^{-1}\frac{1}{3}$ (D) $\cos^{-1}\frac{2}{7}$

Answer (A)

Sol. $(\bar{a} + 2\bar{b})\cdot(5\bar{a} - 4\bar{b}) = 0$

$5 - 4(\bar{a}\cdot\bar{b}) + 10(\bar{a}\cdot\bar{b}) - 8 = 0$

$-3 + b(\bar{a}\cdot\bar{b}) = 0$

$\bar{a}\cdot\bar{b} = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

15. If a vector $\bar{x}$ makes angles with measure $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ with positive directions of X-axis and Y-axis respectively, then $\bar{x}$ made angle of measure ...... with positive direction of Z-axis

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{3}$

Answer (C)

Sol. $\cos\alpha = \frac{1}{2};\ \cos\beta = -\frac{1}{\sqrt{2}}$

Now, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$

$\cos^2\gamma = 0$

$\gamma = 90°$

16. If a plane has X-intercept l, Y-intercept m and Z-intercept n, and perpendicular distance of plane from origin is k, then

(A) $\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{k^2}$ (B) $l^2 + m^2 + n^2 = k^2$

(C) $l^2 + m^2 + n^2 = \frac{1}{k^2}$ (D) $\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2} = k^2$

Answer (A)

Sol. $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$

As distance from origin is k,

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2}}} = k$$

$$\Rightarrow \frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{k^2}$$

17. Lines $\bar{r} = (3+t)\hat{i} + (1-t)\hat{j} + (-2-2t)\hat{k},\ t \in R$ and $x = 4 + k,\ y = -k,\ z = -4 - 2k,\ k \in R$, then relation between lines is

(A) Coincident (B) Parallel

(C) Skew (D) Perpendicular

Answer (A)

Sol. $\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + t(\hat{i} - \hat{j} - 2\hat{k})$

and

$\frac{x-4}{1} = \frac{y-0}{-1} = \frac{z+4}{-2}$

As direction ratio is same and (3, 1, –2) point is satisfying other line $\therefore$ lines are coincident.

18. The equation of plane containing intersecting lines $\frac{x+3}{3} = \frac{y}{1} = \frac{z-2}{2}$ and $\frac{x-3}{4} = \frac{y-2}{2} = \frac{z-6}{3}$ is

(A) $2x - y + z + 9 = 0$ (B) $x + y - 2z + 7 = 0$

(C) $x + y + z + 5 = 0$ (D) $x + 2y - 2z + 9 = 0$

Answer (B)

Sol. Direction ratio normal to the plane is given by

$$\begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 4 & 2 & 3 \end{vmatrix}$$

$= -\hat{i} - \hat{j} + 2\hat{k}$

and plane is passing through (–3, 0, 2)

$\Rightarrow -1(x+3) - 1(y-0) + 2(z-2) = 0$

$\Rightarrow -x - 3 - y + 2z - 4 = 0$

$\Rightarrow x + y - 2z + 7 = 0$

19. The number of binary operations on the set {1,2,3} is

(A) 9^3 (B) 27

(C) 3^9 (D) 3!

Answer (C)

Sol. Number of binary operations = n^{n^2}

$\Rightarrow 3^{3^2}$

$\Rightarrow 3^9$

20. Function $f : N \to Z;\ f(n) = \begin{cases} \frac{n}{2}, & n-\text{even} \\ -\left(\frac{n-1}{2}\right), & n-\text{odd} \end{cases}$

(A) One-one but not onto

(B) One-one and onto

(C) Not one-one but onto

(D) Not one-one and not onto

Answer (B)

Sol. $f(n) = \begin{cases} \frac{n}{2}; & n-\text{even} \\ -\left(\frac{n-1}{2}\right); & n-\text{odd} \end{cases}$

when n = 2, 4, 6, 8.....

$f(n) = \{1, 2, 3....\}$

when n = 1,3,5...

$f(n) = \{0, -1, -2, -3...\}$

Function is one-one and onto

21. The relation $S=\{(3,3),(4,4)\}$ on the set $A=\{3,4,5\}$ is

(A) Reflexive only

(B) Symmetric only

(C) Not reflexive but symmetric and transitive

(D) An equivalence relation

Answer (C)

Sol. $S=\{(3,3)(4,4)\}$

As (5,5) is not present, therefor it is not reflexive

But it is symmetric and transitive

22. $\cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=$

(A) $\cot^{-1}x$ (B) $\frac{\pi}{2}-\frac{1}{2}\tan^{-1}x$

(C) $-\frac{1}{2}\tan^{-1}x$ (D) $\frac{\pi}{2}-\frac{1}{2}\cot^{-1}x$

Answer (B)

Sol. $\cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

put $(x=\tan\theta)$

$\cot^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$=\cot^{-1}\left(\tan\frac{\theta}{2}\right)$

$=\cot^{-1}\left(\cot\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right)$

$=\frac{\pi}{2}-\frac{\theta}{2}$

$=\frac{\pi}{2}-\frac{1}{2}\tan^{-1}x$

23. If $\cos\left(2\tan^{-1}x\right)=\frac{1}{2}$, then value of x is

(A) $\pm\sqrt{3}$ (B) $\pm\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}-1$ (D) $1-\frac{1}{\sqrt{3}}$

Answer (B)

Sol. $\cos\left(2\tan^{-1}x\right)=\frac{1}{2}$

$2\tan^{-1}x=\pm\frac{\pi}{3}$

$\Rightarrow \tan^{-1}x=\pm\frac{\pi}{6}$

$\Rightarrow x=\pm\frac{1}{\sqrt{3}}$

24. $\sin^{-1}\left(\cos\left(\sin^{-1}x\right)\right)+\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right)=$

(A) $\frac{\pi}{4}$ (B) 0

(C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

Answer (C)

Sol. $\sin^{-1}\left(\cos\left(\sin^{-1}x\right)\right)+\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right)$

$=\sin^{-1}\left(\cos\left(\frac{\pi}{2}-\cos^{-1}x\right)\right)+\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right)$

$=\sin^{-1}\left(\sin\left(\cos^{-1}x\right)\right)+\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right)$

$=\frac{\pi}{2}$ $\left(\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right)$

25. If $x^4+y^4+z^4=0$ then $\begin{vmatrix}1 & xy & yz\\ zx & 1 & xy\\ yz & zx & 1\end{vmatrix}=$ ....

(where $x,y,z\in R$)

(A) x+y+z+3 (B) xyz+2

(C) 1 (D) 0

Answer (C)

Sol. As $x^4 + y^4 + z^4 = 0$

$x = y = z = 0$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

26. $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} =$ ....

(A) $2(10!\cdot 11!\cdot 12!)$ (B) $2(10!\cdot 13!)$

(C) $-2(10!\cdot 11!\cdot 12!)$ (D) $2(10!\cdot 12!\cdot 13!)$

Answer (A)

Sol. $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

$$10!\,11!\,12!\begin{vmatrix} 1 & 1 & 1 \\ 11 & 12 & 13 \\ 12\times 11 & 13\times 12 & 13\times 14 \end{vmatrix}$$

$c_2 \to c_2 - c_1$ and $c_3 \to c_3 - c_1$

$$10!\,11!\,12!\begin{vmatrix} 1 & 0 & 0 \\ 11 & 1 & 2 \\ 12\times 11 & 24 & 50 \end{vmatrix}$$

$2(10!\,11!\,12!)$

27. If $s = p + q + r$, then value of

$\begin{vmatrix} s+r & p & q \\ r & s+p & q \\ r & p & s+q \end{vmatrix}$ is ....

(A) $2s^3$ (B) $2s^2$

(C) s^3 (D) $3s^3$

Answer (A)

Sol. $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 2s & p & q \\ 2s & s+p & q \\ 2s & p & s+q \end{vmatrix}$$

$R_3 \to R_3 - R_1$

$R_2 \to R_2 - R_1$

$$\begin{vmatrix} 2s & p & q \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} = 2s^3$$

28. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and B = ______________ then AB = BA, where $B \neq I$

(A) $\begin{bmatrix} x & y \\ 0 & x \end{bmatrix}$ (B) $\begin{bmatrix} x & x \\ y & 0 \end{bmatrix}$

(C) $\begin{bmatrix} x & y \\ 0 & y \end{bmatrix}$ (D) $\begin{bmatrix} x & 0 \\ y & y \end{bmatrix}$

Answer (A)

Sol. As AB = BA, and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\therefore$ By option $B = \begin{bmatrix} x & y \\ 0 & x \end{bmatrix}$

29. If $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, then A^3 =

(A) 81 A (B) 27 A

(C) 243 A (D) 729 A

Answer (A)

Sol. $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 243 & 243 & 243 \\ 243 & 243 & 243 \\ 243 & 243 & 243 \end{bmatrix}$$

$A^3 = 81A$

30. $\frac{d}{dx}\log_{|x|} e =$ .....

(A) $\frac{1}{(\log x)^2}$ (B) $\frac{1}{|x|}$

(C) $-\frac{1}{x(\log|x|)^2}$ (D) e^x

Answer (C)

Sol. $\frac{d}{dx}\log_{|x|} e$

$= \frac{d}{dx}(\log_e |x|)^{-1}$

$= -\frac{1}{x(\log|x|)^2}$

31. $\frac{d}{dx}\tan^{-1}\left(\frac{1-x}{1+x}\right) =$

(A) $\frac{-1}{1+x^2}$ (B) $\frac{1}{1+x^2}$

(C) $\frac{2}{1+x^2}$ (D) $\frac{-2}{1+x^2}$

Answer (A)

Sol. $\frac{d}{dx}\left(\tan^{-1}\left(\frac{1-x}{1+x}\right)\right)$

$= -\frac{1}{1+x^2}$

32. If $x = at^2, y = 2at$, then $\frac{d^2x}{dy^2} =$ ....

(A) $-2at^3$ (B) $\frac{-1}{t^2}$

(C) $\frac{-1}{2at^3}$ (D) $\frac{1}{2a}$

Answer (D)

Sol. $x = at^2, \; y = 2at$

$\frac{dx}{dy} = t$

$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dt}(t)\times\frac{dt}{dy}$

$= 1\times\frac{dt}{dy} = \frac{1}{2a}$

33. $\int x\, e^{x^2\log 2}\cdot e^{x^2}\, dx =$ __________ $+ c$

(A) $\frac{2^{x^2}\cdot e^{x^2}}{1+\log 2}$ (B) $\frac{e^{x^2\log 2}\cdot e^{x^2}}{\log 2}$

(C) $\frac{2^{x^2}\cdot e^{x^2}}{2(1+\log 2)}$ (D) $\frac{(2e)^{x^2}}{\log(2e)}$

Answer (C)

Sol. $\int x\cdot e^{x^2\log 2}\cdot e^{x^2}dx$

$\int x\cdot 2^{x^2}\cdot e^{x^2}dx$

$\int x\cdot(2e)^{x^2}\, dx$

$x^2 = t$

$xdx = \frac{dt}{2}$

$\frac{1}{2}\int(2e)^t\, dt$

$\frac{1}{2}\frac{(2e)^t}{\log 2e} + c$

$\frac{2^{x^2}\cdot e^{x^2}}{2(\log 2+1)} + c$

34. $\int\left(\frac{1}{x-3} - \frac{1}{x^2-3x}\right)dx =$ .... $+ c; x > 3$

(A) $\frac{1}{3}\log\left(\sqrt{x}(x-3)\right)$ (B) $\frac{2}{3}\log\left(\sqrt{x}(x-3)\right)$

(C) $\frac{2}{3}\log\left(x(x-3)\right)$ (D) $\frac{1}{3}\log\left(x(x-3)\right)$

Answer (B)

Sol. $\int\left(\frac{1}{x-3} - \frac{1}{x^2 3x}\right)dx$

$\frac{2}{3}\int\frac{1}{x-3}dx + \frac{1}{3}\int\frac{1}{x}dx$

$\frac{2}{3}\ln(x-3) + \frac{1}{3}\ln x + c$

$\frac{2}{3}\ln\left(\sqrt{x}(x-3)\right) + c$

35. What is the mean of f (x) = 3x + 2 where x is a random variable with probability distribution

X=x	1	2	3	4
P(X=x)	$1/6$	$1/3$	$1/3$	$1/6$

(A) $\frac{15}{2}$ (B) $\frac{5}{3}$

(C) $\frac{5}{2}$ (D) $\frac{19}{2}$

Answer (D)

Sol. Mean $\left(\bar{x}\right)=\sum x_i\,p\left(x_i\right)$

$$=\frac{1}{6}+\frac{2}{3}+1+\frac{2}{3} \qquad =\frac{5}{2}$$

$\therefore$ Mean of $f\left(x\right)=3\left(\frac{5}{2}\right)+2$

$$=\frac{19}{2}$$

36. The probability that an event A occurs in a single trial of an experiment is 0.3. Six independent trials of the experiment are performed. What is the variance of probability distribution of occurrence of event A ?

(A) 0.18 (B) 1.26

(C) 12.6 (D) 1.8

Answer (B)

Sol. Variance = npq = $6\times0.3\times0.7=1.26$

37. The probability that A speaks truth is $\frac{4}{5}$, while this probability fo B $\frac{3}{5}$. The probability of at least one of them is true when asked to speak on an event is ....

(A) $\frac{2}{25}$ (B) $\frac{23}{25}$

(C) $\frac{3}{25}$ (D) $\frac{4}{25}$

Answer (B)

Sol. $P\left(A\right)=\frac{4}{5}$; $P\left(\bar{A}\right)=\frac{1}{5}$

$P\left(B\right)=\frac{3}{5}$; $P\left(\bar{B}\right)=\frac{2}{5}$

required probability = $\frac{4}{5}\times\frac{2}{5}+\frac{3}{5}\times\frac{1}{5}+\frac{4}{5}\times\frac{3}{5}$

$$=\frac{23}{25}$$

38. The corner points of the feasible region determined by the system of linear constraints are (0,10), (5,5), (15,15), (0,20). Let Z = px + qy where p, q > 0. Condition on p and q so that the maximum of z occurs at both the points (15,15) and (0,20) is

(A) p = 2q (B) p = q

(C) q = 2p (D) q = 3p

Answer (D)

Sol. $15p+15q=20q$

3p = q

39. What is the approximate value of $\sqrt[5]{242.999}$?

(A) $\frac{1115}{405}$ (B) $\frac{121499}{40500}$

(C) $\frac{1214999}{405000}$ (D) $\frac{1214999}{4050}$

Answer (C)

Sol. $\Delta x=-\frac{1}{5x^{4/5}}\times.001$

Here x = 243

$\therefore \Delta x=-\frac{1}{405000}$

Approximate value = $\frac{1214999}{405000}$

40. The length of subtangent at any point of the curve log y = 25x is

(A) Proportional to y

(B) Proportional to x

(C) Zero

(D) Constant

Answer (D)

Sol. Length of subtangent $=y_1\cot\theta$

$$= y\times\frac{1}{25y}=\frac{1}{25}$$

$\therefore$ constant

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