COMEDK 2025 Evening Shift

Mathematics

Question1

A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases} \quad \text{is}$$

Options:

A. neither one-one nor onto

B. one-one but not onto

C. onto but not one-one

D. one-one and onto

Answer: D

Solution:

We can rewrite the rule by setting $n = 2k$ or $n = 2k + 1$ (with $k \geq 0$ in both cases):

• For $n = 2k + 1$ (odd):

$f(2k + 1) = \frac{(2k+1)-1}{2} = k \quad (\geq 0)$.

• For $n = 2k$ (even, $k \geq 1$):

$f(2k) = -\frac{2k}{2} = -k \quad (< 0)$.

Injective (one-one):

– If $n_1 \neq n_2$, then either their parities differ (one gives a nonnegative value, the other a negative one) or they share parity but different k's, so $f(n_1) \neq f(n_2)$.

Surjective (onto):

– Given any integer m,

• if $m \geq 0$, choose $n = 2m + 1$, then $f(n) = m$;

• if $m < 0$, choose $n = -2m$, then $f(n) = m$.

Hence f is both one-one and onto.

Answer: Option D.

Question2

The value of $\frac{\sin^2 20^\circ+\cos^4 20^\circ}{\sin^4 20^\circ+\cos^2 20^\circ}$ is :

Options:

A. 0

B. 2

C. 1

D. $\frac{1}{2}$

Answer: C

Solution:

Let's set

$s = \sin^2 20^\circ, \quad c = \cos^2 20^\circ$

so that $s + c = 1$.

Rewrite the expression:

$\frac{\sin^2 20^\circ+\cos^4 20^\circ}{\sin^4 20^\circ+\cos^2 20^\circ} = \frac{s+c^2}{s^2+c}$.

Substitute $c = 1 - s$:

$$\text{Numerator} = s + (1 - s)^2 = s + 1 - 2s + s^2 = 1 - s + s^2,$$
$$\text{Denominator} = s^2 + (1 - s) = s^2 + 1 - s = 1 - s + s^2.$$

Since numerator = denominator, the value is

$\frac{1-s+s^2}{1-s+s^2} = 1$.

Answer: 1 (Option C)

Question3

$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)}dx = a\log|1+\sin x| - b\log|2+\sin x| + c$ then the value of a and b is ----------

Options:

A. $a = -2, b = 4$

B. $a = 2, b = 4$

C. $a = -2, b = -4$

D. $a = 2, b = -4$

Answer: C

Solution:

Let $I = \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)}\,dx$.

Substitute $t = \sin x$, so $dt = \cos x\,dx$ and $\sin 2x = 2\sin x\cos x = 2t\cos x$.

Then

$I = \int \frac{2t\cos x}{(t+1)(t+2)}\,dx = \int \frac{2t}{(t+1)(t+2)}\,dt.$

Do partial-fractions:

$\frac{2t}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2},$

so $2t = A(t+2) + B(t+1) = (A+B)t + (2A+B)$.

Matching coefficients gives

$A + B = 2, \quad 2A + B = 0 \implies A = -2,\ B = 4.$

Thus

$I = \int\left(-\frac{2}{t+1} + \frac{4}{t+2}\right)dt = -2\ln|t+1| + 4\ln|t+2| + C.$

Re-substitute $t = \sin x$:

$I = -2\ln|1+\sin x| + 4\ln|2+\sin x| + C.$

Matching

$I = a\ln|1+\sin x| - b\ln|2+\sin x| + C$

we read off

$a = -2, \qquad -b = 4 \implies b = -4.$

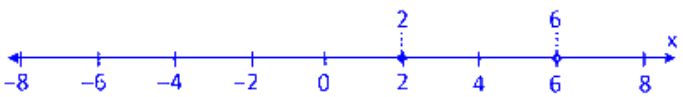
Answer: $a = -2,\ b = -4$ (Option C).

Question4

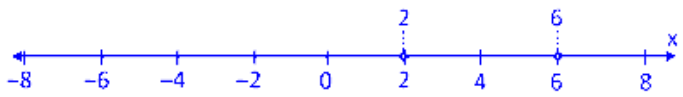
The solution for the following system of inequalities $3x - 7 < 5 + x$ and $11 - 5x \leq 1$ on a real number line is

Options:

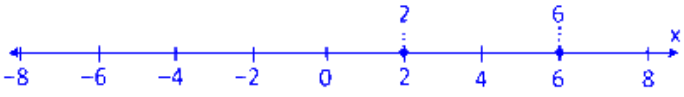
A.



B.



C.



D.

Answer: A

Solution:

Solve the system

1) $3x - 7 < 5 + x$

$$3x - 7 < 5 + x$$

$$3x - x < 5 + 7$$

$$2x < 12$$

$$x < 6$$

2) $11 - 5x \leq 1$

$$11 - 5x \leq 1$$

$$-5x \leq -10$$

Dividing by **−5** flips the inequality:

$$x \geq 2$$

Combine both:

$$2 \leq x < 6$$

Now check the graphs

- Starts at **2**, **closed point**
- Ends at **6**, **open point**
- Shade between 2 and 6.

Why answer A is correct

Option **A** shows:

- A **closed dot** at **2**
- An **open dot** at **6**
- The number line shaded **between** them.

That matches the solution perfectly.

✅ **Yes — Answer A is correct.**

Question5

Kiran purchased 3 pencils, 2 notebooks and one pen for ₹41. From the same shop Manasa purchased 2 pencils, one notebook and 2 pens for ₹ 29 , while Shreya purchased 3 pencils, 2 notebooks and 2 pens for ₹ 44. The above situation can be represented in matrix form as $AX = B$. Then $|\text{adj } A|$ is equal to

Options:

A. 9

B. -9

C. -1

D. 1

Answer: D

Solution:

First, form the coefficient matrix

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 2 & 2 \end{pmatrix}.$$

Compute its determinant:

$$\det A = 3\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 3(1\cdot 2 - 2\cdot 2) - 2(2\cdot 2 - 2\cdot 3) + 1(2\cdot 2 - 1\cdot 3) = -6 + 4 + 1 = -1.$$

For a 3×3matrix,

$\det\big(\text{adj } A\big) = (\det A)^{2} = (-1)^{2} = 1.$

Hence $|\text{adj } A| = 1.$

Answer: Option D.

Question6

The area of a triangle formed by the lines joining the vertex of the parabola $x^2 = \lambda y$ to the ends of its latus rectum is 18 sq units then the value of λ is

Options:

A. 24

B. 12

C. 6

D. 18

Answer: B

Solution:

Let $x^2 = \lambda y$. Writing it as $x^2 = 4ay$ gives $4a = \lambda$, so

$a = \frac{\lambda}{4}, \quad$ focus at $(0, a), \quad$ latus-rectum endpoints at $\left(\pm 2a, a\right)$.

• Endpoints: $\left(\pm 2a, a\right) = \left(\pm \frac{\lambda}{2}, \frac{\lambda}{4}\right)$

• Vertex: $(0, 0)$

• Base of the triangle = distance between endpoints = λ

• Height = vertical distance from $(0, 0)$ to $y = \frac{\lambda}{4} = \frac{\lambda}{4}$

Thus area

Area $= \frac{1}{2} \cdot (\text{base}) \cdot (\text{height}) = \frac{1}{2} \cdot \lambda \cdot \frac{\lambda}{4} = \frac{\lambda^2}{8}$

Setting $\dfrac{\lambda^2}{8} = 18$ gives $\lambda^2 = 144$ and hence $\lambda = 12$.

Answer: 12.

Question7

If $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $Y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then B equals :

Options:

A. $-X\cos\theta + Y\sin\theta$

B. $X\sin\theta + y\cos\theta$

C. $X\cos\theta - Y\sin\theta$

D. $X\cos\theta + Y\sin\theta$

Answer: D

Solution:

B can be written as the linear combination

$B = \cos\theta\, X + \sin\theta\, Y$

since

$\cos\theta\, X + \sin\theta\, Y = \cos\theta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \sin\theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$

That is Option D.

Question8

Two numbers are selected at random from integers 1 to 9 . If their sum is even, what is the probability that both the numbers are odd?

Options:

A. $\frac{4}{9}$

B. $\frac{5}{8}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$

Answer: B

Solution:

Count the odds and evens in {1,…,9}:

Odds: 1,3,5,7,9 → 5 numbers

Evens: 2,4,6,8 → 4 numbers

Total ways to pick 2 numbers (without replacement):

$\binom{9}{2} = 36$

Ways their sum is even:

– Both odd:

$\binom{5}{2} = 10$

– Both even:

$\binom{4}{2} = 6$

Total even-sum pairs = 10 + 6 = 16

Of those 16 even-sum pairs, the ones with both odd are 10.

So the conditional probability is

$\frac{\text{both odd}}{\text{sum even}} = \frac{10}{16} = \frac{5}{8}.$

Answer: Option B, $\frac{5}{8}$.

Question9

If the standard deviation of $0, 1, 2, 3 \cdots\cdots\cdots 9$ is ' k ' then the standard deviation of $10, 11, 12, 13, \cdots\cdots\cdots\cdots 19$ will be :

Options:

A. $k + 4$

B. $k + 10$

C. k

D. $k + 8$

Answer: C

Solution:

The two sets

• A = {0,1,2,…,9}

• B = {10,11,12,…,19}

differ only by adding 10 to each element of A. Since adding a constant shifts the mean but doesn't change how spread-out the data are, their standard deviations are equal.

So the standard deviation of B is still k.

Answer: Option C.

Question10

If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $(AI)^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ where I is the identity matrix then

Options:

A. $\alpha = a^2 + b^2, \beta = 2ab$

B. $\alpha = 2ab, \beta = a^2 + b^2$

C. $\alpha = a^2 + b^2, \beta = ab$

D. $\alpha = a^2 + b^2, \beta = a^2 - b^2$

Answer: A

Solution:

First note that $(A\,I)^2 = A^2$ since I is the identity. Now compute

$$A^2 = \begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & a\,b + b\,a \\ b\,a + a\,b & b^2 + a^2 \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{pmatrix}.$$

So $\alpha = a^2 + b^2$ and $\beta = 2ab$, which is Option A.

Question11

Let $f(x) = x\sqrt{4ax - x^2}, a > 0$ then $f'(x)$ at $x = 2a$ is :

Options:

A. Does not exist

B. Zero

C. Decreasing

D. Increasing

Answer: D

Solution:

Let $f(x) = x\sqrt{4ax - x^2}$, so that its domain is $0 \le x \le 4a$. We compute $f'(x)$ by the product–chain rule:

Write $g(x) = 4ax - x^2$, so $\sqrt{g(x)}$'s derivative is

$\frac{g'(x)}{2\sqrt{g(x)}} = \frac{4a-2x}{2\sqrt{4ax-x^2}}$.

Then

$f'(x) = \sqrt{4ax - x^2} + x \cdot \frac{4a-2x}{2\sqrt{4ax-x^2}}$.

At $x = 2a$:

$\sqrt{4a\,(2a) - (2a)^2} = \sqrt{8a^2 - 4a^2} = 2a,$

and

$x \cdot \frac{4a-2x}{2\sqrt{4ax-x^2}} = 2a \cdot \frac{4a-4a}{2 \cdot 2a} = 0.$

Hence

$f'(2a) = 2a > 0.$

Since $f'(2a)$ exists and is positive, f is increasing at $x = 2a$. Answer: Option D.

Question12

The angle between two lines is 45° and slope of one line is $\frac{1}{4}$ then which is the possible value of the slope of the other line.

Options:

A. $\frac{5}{4}$

B. $\frac{5}{3}$

C. $\frac{4}{5}$

D. $\frac{3}{5}$

Answer: B

Solution:

We use the fact that for two lines of slopes m_1 and m_2, the angle θ between them satisfies

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

Here $\theta = 45^\circ$ so $\tan 45^\circ = 1$, and $m_1 = \frac{1}{4}$. Hence

$1 = \left|\frac{m_2 - \frac{1}{4}}{1 + \frac{1}{4}\, m_2}\right|$.

This gives two cases:

$$\frac{m_2 - \frac{1}{4}}{1 + \frac{1}{4}m_2} = 1$$

Multiply by $1 + \frac{1}{4}m_2$:

$m_2 - \frac{1}{4} = 1 + \frac{1}{4}m_2$

$\implies 4m_2 - 1 = 4 + m_2 \implies 3m_2 = 5 \implies m_2 = \frac{5}{3}$.

$$\frac{m_2 - \frac{1}{4}}{1 + \frac{1}{4}m_2} = -1$$

$m_2 - \frac{1}{4} = -1 - \frac{1}{4}m_2$

$\implies 4m_2 - 1 = -4 - m_2 \implies 5m_2 = -3 \implies m_2 = -\frac{3}{5}$.

So the two possible slopes are $\frac{5}{3}$ or $-\frac{3}{5}$. Among the given choices only $\frac{5}{3}$ appears.

Answer: Option B, $\frac{5}{3}$.

Question13

Given $Z = 80x + 120y$, subject to constraints are $x + 3y \leq 30; 3x + 4y \leq 60; x \geq 0; y \geq 0$. P is one of the corner points of the feasible region for the given Linear Programming Problem. Then the coordinate of P is

Options:

A. $(0, 15)$

B. $(20, 0)$

C. $(6, 12)$

D. $(30, 0)$

Answer: B

Solution:

Let’s check which of the given points satisfies all the constraints

$x + 3y \le 30, \quad 3x + 4y \le 60, \quad x \ge 0, \quad y \ge 0.$

Option A: $(0, 15)$

– $x + 3y = 0 + 45 = 45 > 30$ ✗

Option B: $(20, 0)$

– $x + 3y = 20 + 0 = 20 \le 30$ ✓

– $3x + 4y = 60 + 0 = 60 \le 60$ ✓

Option C: $(6, 12)$

– $x + 3y = 6 + 36 = 42 > 30$ ✗

Option D: $(30, 0)$

– $3x + 4y = 90 > 60$ ✗

Only $(20, 0)$ satisfies all constraints.

Answer: Option B, $(20, 0)$.

Question14

If $\sin x + \sin^2 x = 1$ then $\cos^8 x + 2\cos^6 x + \cos^4 x$ is equal to :

Options:

A. 0

B. 1

C. -1

D. 2

Answer: B

Solution:

Let’s set $s = \sin x$. The equation

$s + s^2 = 1$

gives

$s^2 + s - 1 = 0 \implies s = \frac{-1 \pm \sqrt{5}}{2}$.

Only $s = \frac{\sqrt{5}-1}{2}$ lies in $[-1, 1]$.

Then

$\cos^2 x = 1 - s^2 = 1 - \left(\frac{\sqrt{5}-1}{2}\right)^2 = \frac{\sqrt{5}-1}{2} = s$.

So $\cos^2 x = s$, and

$\cos^8 x + 2\cos^6 x + \cos^4 x = \cos^4 x\big(\cos^4 x + 2\cos^2 x + 1\big) = \cos^4 x\,(1+\cos^2 x)^2.$

Replacing $\cos^2 x = s$ gives

$= s^2\,(1+s)^2.$

But from $s + s^2 = 1$ we have $s^2 = 1 - s$. Hence

$s^2\,(1+s)^2 = (1-s)(1+s)^2 = (1-s)(1+2s+s^2) = (1-s)\big(2+s\big) = 2 - 2s + s - s^2 = 2 - s - (1-s) = 1.$

Answer: 1. (Option B)

Question15

$\int_0^1 x(1-x)^{99}dx =$

Options:

A. $\frac{1}{10010}$

B. $\frac{1}{1010}$

C. $-\frac{1}{10100}$

D. $\frac{1}{10100}$

Answer: D

Solution:

We can use the Beta-function formula

$\int_0^1 x^a(1-x)^b\,dx = B(a+1, b+1) = \frac{a!\,b!}{(a+b+1)!}.$

Here $a = 1$, $b = 99$, so

$\int_0^1 x\,(1-x)^{99}\,dx = B(2, 100) = \frac{1!\,99!}{101!} = \frac{1}{100\cdot 101} = \frac{1}{10100}.$

That matches Option D.

Question16

Value of the determinant of a matrix A of order 3×3 is 7 . Then the value of the determinant formed by the cofactors of matrix A is

Options:

A. 7

B. 49

C. 14

D. 343

Answer: B

Solution:

The determinant of the cofactor-matrix of an $n \times n$ matrix A is the same as that of its adjugate $\text{adj}(A)$. Since

$\text{adj}(A) = \det(A)\, A^{-1}$, and

for an $n \times n$ matrix $\det(kM) = k^n \det(M)$,

we get

$\det\big(\text{adj}(A)\big) = \det\big(\det(A)\, A^{-1}\big) = \det(A)^n \;\det(A^{-1}) = \det(A)^{n-1}$.

Here $n = 3$ and $\det(A) = 7$, so

$\det(\text{cofactor-matrix of } A) = \det\big(\text{adj}(A)\big) = 7^{3-1} = 7^2 = 49$.

Answer: Option B (49).

Question17

The relationship between a and b for the continuous function

$$f(x) = \begin{cases} ax + 1, & \text{if } x \le 3 \\ bx + 3, & \text{if } x > 3 \end{cases} \textbf{ at } x = 3 \textbf{ is}$$

Options:

A. $3b = 3a + 2$

B. $3a = b + 2$

C. $3a = 3b + 2$

D. $a = 3b - 2$

Answer: C

Solution:

For continuity at $x = 3$, the left- and right-hand values must match:

Left side: $f(3^-) = a \cdot 3 + 1 = 3a + 1$

Right side: $f(3^+) = b \cdot 3 + 3 = 3b + 3$

Setting them equal gives

$3a + 1 = 3b + 3 \quad \Longrightarrow \quad 3a = 3b + 2$.

That’s Option C.

Question18

If the function $f(x) = \mu \sin x + \frac{1}{3}\sin 3x$ has its derivative equal to zero at $x = \frac{\pi}{3}$, then the value of ' μ ' is

Options:

A. 0

B. -1

C. 1

D. 2

Answer: D

Solution:

First, compute the derivative:

$f'(x) = \mu \cos x + \frac{1}{3} \cdot 3 \cos 3x = \mu \cos x + \cos 3x$.

We require

$f'\left(\frac{\pi}{3}\right) = 0 \implies \mu \cos \frac{\pi}{3} + \cos \pi = 0$.

Since $\cos \frac{\pi}{3} = \frac{1}{2}$ and $\cos \pi = -1$, this becomes

$\mu \cdot \frac{1}{2} - 1 = 0 \quad \implies \quad \mu = 2$.

Answer: 2 (Option D).

Question19

A man is moving away from a tower 41.6 m high at a rate of $2\ \mathrm{m/s}$. If the eyelevel of the man is 1.6 m above the ground, then the rate at which the angle of elevation of the top of the tower changes, when he is at a distance of 30 m from the foot of the tower is :

Options:

A. $-\frac{4}{125}\mathrm{rad/sec}$

B. $\frac{4}{625}\mathrm{rad/sec}$

C. $-\frac{2}{125}\mathrm{rad/sec}$

D. $\frac{1}{625}\mathrm{rad/sec}$

Answer: A

Solution:

Let x(t) be the horizontal distance of the man from the foot of the tower. His eye-level sees the top of the tower at height

$h = 41.6 - 1.6 = 40$ m,

so

$\theta(t) = \arctan \frac{40}{x(t)}$.

Since $\tan\theta = \frac{40}{x}$,

differentiate both sides w.r.t. t:

$\sec^2\theta \, \frac{d\theta}{dt} \; = \; -40\, x^{-2} \, \frac{dx}{dt}$.

Hence

$\frac{d\theta}{dt} = -\frac{40}{x^2 \sec^2\theta} \, \frac{dx}{dt} = -\frac{40}{x^2(1+\tan^2\theta)} \, \frac{dx}{dt} = -\frac{40}{x^2+40^2} \, \frac{dx}{dt}$.

At the instant x=30 m and dx/dt=2 m/s:

$\frac{d\theta}{dt} = -\frac{40 \cdot 2}{30^2+40^2} = -\frac{80}{900+1600} = -\frac{80}{2500} = -\frac{4}{125}$ rad/s.

Answer: Option A, $-\frac{4}{125}$ rad/s.

Question20

$\sin^{-1}(x-1) + \cos^{-1}(x-3) + \tan^{-1}\left(\frac{x}{2-x^2}\right) = \cos^{-1} k + \pi$, then the value of ' k ' is

Options:

A. 0

B. $-\frac{1}{\sqrt{2}}$

C. 1

D. $\frac{1}{\sqrt{2}}$

Answer: D

Solution:

Let's pin down the only possible x and then find k.

Domains

$\sin^{-1}(x-1)$ requires $-1 \le x-1 \le 1$, so $0 \le x \le 2$.

$\cos^{-1}(x-3)$ requires $-1 \le x-3 \le 1$, so $2 \le x \le 4$.

$\Rightarrow$ Intersection: $x = 2$.

Compute each term at $x = 2$

$\sin^{-1}(2-1) = \sin^{-1}(1) = \frac{\pi}{2}$

$\cos^{-1}(2-3) = \cos^{-1}(-1) = \pi$

$\tan^{-1}\left(\frac{2}{2-2^2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

Sum:

$\frac{\pi}{2} + \pi - \frac{\pi}{4} = \frac{2\pi}{4} + \frac{4\pi}{4} - \frac{\pi}{4} = \frac{5\pi}{4}.$

Equate to the right-hand side:

$\frac{5\pi}{4} = \cos^{-1}(k) + \pi \quad \Longrightarrow \quad \cos^{-1}(k) = \frac{\pi}{4} \quad \Longrightarrow \quad k = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$

Answer: $\dfrac{1}{\sqrt{2}}$ (Option D).

Question21

Given that z is a real number and $z = \frac{\lambda+4i}{1+\lambda i}$ where $\lambda \in R$, then the possible value of λ is :

Options:

A. -2

B. $2i$

C. 5

D. $\pm 2i$

Answer: A

Solution:

Let

$z = \frac{\lambda+4i}{1+\lambda i}$

with $\lambda \in \mathbb{R}$. Write $z = \frac{A+Bi}{C}$ by multiplying numerator and denominator by $1 - \lambda i$:

Numerator

$(\lambda + 4i)(1 - \lambda i) = \lambda - i\lambda^2 + 4i - 4i^2\lambda = (\lambda + 4\lambda) + i(4 - \lambda^2) = 5\lambda + i(4 - \lambda^2).$

Denominator

$(1 + \lambda i)(1 - \lambda i) = 1 + \lambda^2.$

So

$z = \frac{5\lambda}{1+\lambda^2} \; + \; i\,\frac{4-\lambda^2}{1+\lambda^2}.$

For z to be real, its imaginary part must vanish:

$\frac{4-\lambda^2}{1+\lambda^2} = 0 \quad \Longrightarrow \quad 4 - \lambda^2 = 0 \quad \Longrightarrow \quad \lambda^2 = 4 \quad \Longrightarrow \quad \lambda = \pm 2.$

Answer: $\boxed{\lambda = \pm 2}$.

(From the given choices the real one is -2, but the full solution set is ± 2.)

Question22

If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

Options:

A. $\mathrm{Re}(z) > 0, \mathrm{Im}(z) < 0$

B. $\mathrm{Im}(z) = 0$

C. $\mathrm{Re}(z) = 0$

D. $\mathrm{Re}(z) > 0, \mathrm{Im}(z) > 0$

Answer: B

Solution:

Let

$a = \frac{\sqrt{3}}{2} + \frac{i}{2} = e^{i\pi/6}, \qquad \bar{a} = e^{-i\pi/6}.$

Then

$z = a^5 + \bar{a}^5 = e^{i5\pi/6} + e^{-i5\pi/6} = 2\cos\left(\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}.$

Since z is a real number, its imaginary part vanishes:

$\Im(z) = 0.$

Answer: Option B.

Question23

The solution of the differential equation: $x\cos y dy = (xe^x \log x + e^x)dx$ is

Options:

A. $\sin y - e^x \log x = c$

B. $\sin y = e^x \log x + c$

C. $\sin y = e^x + \log x + c$

D. $\sin y = \frac{e^x}{x} + c$

Answer: B

Solution:

First separate variables:

$x\cos y\,dy \;=\; \left(xe^{x}\ln x + e^{x}\right)dx \;\Rightarrow\; \cos y\,dy \;=\; e^{x}\left(\ln x + \frac{1}{x}\right)dx.$

Notice

$\frac{d}{dx}\left(e^{x}\ln x\right) = e^{x}\ln x + e^{x}\cdot\frac{1}{x} = e^{x}\left(\ln x + \frac{1}{x}\right).$

Integrate both sides:

$\int \cos y\,dy = \int e^{x}(\ln x + \frac{1}{x})\,dx \;\Longrightarrow\; \sin y = e^{x}\ln x + C.$

Hence the general solution is

$\sin y = e^{x}\ln x + C,$

which is Option B.

Question24

Area of the region bounded by the curve $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \pi$ is ----------

Options:

A. 3 sq units

B. 4 sq units

C. 1 sq units

D. 2 sq units

Answer: A

Solution:

We want the **area of the region bounded by** $y = \cos x$ **between** $x = -\frac{\pi}{2}$ **and** $x = \pi$.

Step 1: Understand the curve

The function is $y = \cos x$.

- On $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos x \geq 0$.
- On $\left[\frac{\pi}{2}, \pi\right]$, $\cos x \leq 0$.

So when finding area, we use the absolute value:

Area $= \int_{-\pi/2}^{\pi} |\cos x|\,dx$.

Step 2: Break into two intervals

$\int_{-\pi/2}^{\pi} |\cos x|\,dx = \int_{-\pi/2}^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} (-\cos x)\,dx.$

Step 3: Compute each part

1. $\int_{-\pi/2}^{\pi/2} \cos x \, dx = [\sin x]_{-\pi/2}^{\pi/2} = \sin(\pi/2) - \sin(-\pi/2) = 1 - (-1) = 2.$
2. $\int_{\pi/2}^{\pi} (-\cos x) \, dx = -[\sin x]_{\pi/2}^{\pi} = -(\sin \pi - \sin \frac{\pi}{2}) = -(0 - 1) = 1.$

Step 4: Add them

Area $= 2 + 1 = 3.$

✅ Final Answer:

The area is **3 square units**.

Correct Option: A (3 sq units)

Question25

If a quadratic function in x has the value 19 when $x = 1$ and has a maximum value 20 when $x = 2$, then the function is

Options:

A. $f(x) = x^2 - 4x + 16$

B. $f(x) = -x^2 + 5x + 16$

C. $f(x) = x^2 + 4x + 16$

D. $f(x) = -x^2 + 4x + 16$

Answer: D

Solution:

Let's use the vertex form of a downward-opening parabola with vertex at $(2, 20)$:

Write

$f(x) = -a\,(x - 2)^2 + 20.$

Impose $f(1) = 19$:

$19 = -a\,(1 - 2)^2 + 20 \implies -a \cdot 1 + 20 = 19 \implies a = 1.$

So

$f(x) = -(x - 2)^2 + 20 = -(x^2 - 4x + 4) + 20 = -x^2 + 4x + 16.$

That matches Option D.

Question26

If A and B are two events such that $P(\bar{A}) = 0.3, P(B) = 0.4, P(A \cap \bar{B}) = 0.5$, then find the value of $P(B/A \cup \bar{B})$

Options:

A. 0.33

B. 0.7

C. 0.8

D. 0.25

Answer: D

Solution:

Let's compute step by step:

$P(A) = 1 - P(\bar{A}) = 1 - 0.3 = 0.7.$

$P(A \cap B) = P(A) - P(A \cap \bar{B}) = 0.7 - 0.5 = 0.2.$

$P(\bar{B}) = 1 - P(B) = 1 - 0.4 = 0.6.$

$P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B}) = 0.7 + 0.6 - 0.5 = 0.8.$

Finally,

$P\big(B \mid A \cup \bar{B}\big) = \frac{P(A \cap B)}{P(A \cup \bar{B})} = \frac{0.2}{0.8} = 0.25.$

Answer: 0.25 (Option D).

Question27

If $\lim\limits_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then the value of K is :

Options:

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. 8

D. $\frac{8}{3}$

Answer: D

Solution:

Left side :

$\lim_{x\to1}\frac{x^4-1}{x-1}=\lim_{x\to1}\frac{(x-1)(x^3+x^2+x+1)}{x-1}=x^3+x^2+x+1\big|_{x=1}=1+1+1+1=4.$

Right side :

$\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}=\lim_{x\to k}\frac{(x-k)(x^2+xk+k^2)}{(x-k)(x+k)}=\frac{x^2+xk+k^2}{x+k}\big|_{x=k}=\frac{3k^2}{2k}=\frac{3k}{2}.$

Setting them equal :

$4=\frac{3k}{2}\implies 3k=8\implies k=\frac{8}{3}.$

Answer : $\frac{8}{3}$ (Option D).

Question28

$\int\left(e^{x\log_e 6}\right)e^x dx=\phi(x)+c$ then $\phi(x)=$

Options:

A. $6^x e^x$

B. $\frac{e^x}{\log 6e}$

C. $\frac{6^x}{1+\log_e 6}$

D. $\frac{(6e)^x}{1+\log_e 6}$

Answer: D

Solution:

First note that

$e^{x\ln 6}\,e^x=6^x\,e^x=(6e)^x\,.$

Hence

$\int\left(e^{x\ln 6}e^x\right)dx=\int(6e)^x\,dx=\frac{(6e)^x}{\ln(6e)}+C\,.$

Since $\ln(6e)=\ln 6+1$, we get

$\phi(x)=\frac{(6e)^x}{1+\ln 6}\,.$

Answer: Option D.

Question29

The number of solutions of $\frac{dy}{dx}=\frac{y+1}{x-1}$, when $y(1)=2$ is :

Options:

A. two

B. one

C. zero

D. infinite

Answer: C

Solution:

We separate variables:

$\frac{dy}{y+1} = \frac{dx}{x-1}$

Integrating,

$\ln|y+1| = \ln|x-1| + C \quad \Longrightarrow \quad y+1 = A(x-1)\,.$

For any constant A, at $x = 1$ this gives

$y(1) + 1 = A \cdot 0 \quad \Longrightarrow \quad y(1) = -1\,,$

so it's impossible to have $y(1) = 2$. Hence there are no solutions satisfying the given initial condition.

Answer: zero.

Question30

Evaluate: $\lim_{x\to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$

Options:

A. 1

B. 0

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: C

Solution:

Use the binomial (Taylor) expansion for small x:

$(1 \pm x)^{1/3} = 1 \pm \frac{x}{3} + O(x^2).$

Subtract:

$\sqrt[3]{1+x} - \sqrt[3]{1-x} = \left(1 + \frac{x}{3} + O(x^2)\right) - \left(1 - \frac{x}{3} + O(x^2)\right) = \frac{2x}{3} + O(x^2).$

Divide by x and let $x \to 0$:

$\lim_{x\to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x} = \frac{2}{3}.$

Answer: Option C, $\frac{2}{3}$.

Question31

A geometric progression consists of an even number of terms. If the sum of all the terms is five times the sum of the terms occupying the odd places, then the common ratio of the geometric progression is

Options:

A. $r = 4$

B. $r = 3$

C. $r = 6$

D. $r = 2$

Answer: A

Solution:

Let the GP have $2n$ terms, first term a and common ratio $r \neq 1$.

Sum of all terms:

$S_{\text{total}} = a + ar + \cdots + ar^{2n-1} = a\,\frac{r^{2n}-1}{r-1}$.

Sum of terms in odd positions (there are n of them):

$S_{\text{odd}} = a + ar^2 + \cdots + ar^{2(n-1)} = a\,\frac{r^{2n}-1}{r^2-1}$.

Given $S_{\text{total}} = 5\,S_{\text{odd}}$, we have

$\frac{r^{2n}-1}{r-1} = 5\,\frac{r^{2n}-1}{r^2-1} \quad\Longrightarrow\quad \frac{1}{r-1} = \frac{5}{r^2-1} \;\Longrightarrow\; r^2 - 1 = 5(r-1)$,

$r^2 - 5r + 4 = 0 \quad\Longrightarrow\quad (r-1)(r-4) = 0$.

Since $r \neq 1$, the common ratio is

$\boxed{r = 4.}$

Question32

Find the function ' f ' which satisfies the equation $\frac{df}{dx} = 2f$, given that $f(0) = e^3$

Options:

A. $2x + 3$

B. $\log(2x + 3)$

C. e^{2x+3}

D. $\frac{x^2}{2}$

Answer: C

Solution:

We solve the ODE by separating variables:

$\frac{df}{dx} = 2f \implies \frac{df}{f} = 2\,dx$

Integrate both sides:

$\int \frac{df}{f} = \int 2\,dx \implies \ln|f| = 2x + C$

Exponentiate:

$f(x) = e^{2x+C} = e^C\, e^{2x}$

Use the initial condition $f(0) = e^3$:

$e^C = e^3 \implies C = 3$

Therefore

$f(x) = e^{2x+3}$

This matches Option C.

Question33

In a game, a man wins ₹ 1000 if he gets an even number greater than or equal to 4 on a fair dice and loses ₹ 200 for getting any other number on the dice. If he decides to throw the dice until he wins or maximum of three times, then his expected gain/loss in (₹) is ----------

Options:

A. $\frac{2200}{9}$ loss

B. $\frac{3800}{9}$ gain

C. $\frac{2200}{9}$ gain

D. $\frac{3800}{9}$ loss

Answer: B

Solution:

Let

• p= P(win on a throw)=2/6=1/3, q=1–p=2/3

• Win gives +₹1000, a loss on each throw is –₹200

• He stops as soon as he wins or after 3 throws

Possible outcomes, their probs and gains:

Win on 1st:

– Prob = p

– Gain = +1000

Lose then win on 2nd:

– Prob = q·p

– Gain = –200 +1000 = +800

Lose twice, then win on 3rd:

– Prob = $q^2 \cdot p$

– Gain = –2·200 +1000 = +600

Lose all three:

– Prob = q^3

– Gain = –3·200 = –600

So the expected gain is

$E[G] = p \cdot 1000 \; + \; (q\,p) \cdot 800 \; + \; (q^2\,p) \cdot 600 \; + \; (q^3)\,(-600).$

Plug in $p = 1/3$, $q = 2/3$:

$E[G] = \frac{1}{3} \cdot 1000 + \frac{2}{3} \cdot \frac{1}{3} \cdot 800 + \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} \cdot 600 + \left(\frac{2}{3}\right)^3 (-600) = \frac{3800}{9} \approx 422.22.$

That's a net gain of $\frac{3800}{9}$.

Answer: Option B.

Question34

The number of words that can be formed with the letters of the word 'DEFINITE' if two vowels are together and the other two are also together but separated from the first two is

Options:

A. 720

B. 1680

C. 1440

D. 2880

Answer: C

Solution:

Identify the consonants and vowels in “DEFINITE”:

• Consonants: D, F, N, T (4 distinct letters)

• Vowels: E, E, I, I (two E’s and two I’s)

Arrange the 4 consonants in a row:

• 4! = 24 ways.

There are 5 “gaps” around these consonants (before the first, between each pair, after the last).

We need to choose 2 of those gaps in which to place our two vowel-blocks (each block is two adjacent vowels).

• Number of ways = $\binom{5}{2} = 10$.

Now distribute the 4 vowels into the two chosen gaps so that each gap gets exactly 2 vowels (order matters within a block), and overall we use E,E,I,I.

• This is the number of length-4 sequences with 2 E’s and 2 I’s, split into two ordered pairs.

• Total such sequences =

$\frac{4!}{2!\,2!} = 6$.

Multiply everything together:

Total = consonant-arrangements × gap-choices × vowel-assignments

$24 \times 10 \times 6 \;=\; 1440$.

Answer: 1440 (Option C)

Question35

The relation $R = \{(1,1),(2,2),(3,3)\}$ on the set $\{1,2,3\}$ is

Options:

A. symmetric only

B. an equivalence relation

C. transitive only

D. reflexive only

Answer: B

Solution:

The relation

$R = \{(1,1),(2,2),(3,3)\}$

on the set $S = \{1,2,3\}$ is actually the identity relation on S. Let’s check the three properties:

Reflexive

• For every $x \in S$, $(x,x) \in R$.

• Indeed, $(1,1),(2,2),(3,3)$ are all in R.

Symmetric

• Whenever $(x, y) \in R$, we need $(y, x) \in R$.

• Here every pair is of the form (x, x), so its “reverse” is itself, which is in R.

Transitive

• Whenever $(x, y) \in R$ and $(y, z) \in R$, we need $(x, z) \in R$.

• Again, the only pairs are loops (x, x), so composing them gives $(x, x) \in R$.

Since R is reflexive, symmetric and transitive, it is an equivalence relation.

Answer: Option B.

Question36

If the length of the diagonal of a square is increasing at the rate of 0.1 cm/sec. What is the rate of increase of its area when the side is $\frac{15}{\sqrt{2}}$ cm ?

Options:

A. $3 \text{ cm}^2/\text{sec}$

B. $0.15 \text{ cm}^2/\text{sec}$

C. $1.5 \text{ cm}^2/\text{sec}$

D. $3\sqrt{2} \text{ cm}^2/\text{sec}$

Answer: C

Solution:

Let D be the diagonal and A the area. Since s is the side,

• $D = s\sqrt{2}$

• $A = s^2 = \frac{D^2}{2}$

Differentiate $A = \frac{D^2}{2}$ with respect to t:

$\frac{dA}{dt} = \frac{1}{2} \cdot 2D \frac{dD}{dt} = D \ \frac{dD}{dt}$.

When $s = \frac{15}{\sqrt{2}}$, the diagonal is

$D = s\sqrt{2} = 15$.

Given $\frac{dD}{dt} = 0.1$, we get

$\frac{dA}{dt} = 15 \times 0.1 = 1.5 \text{ cm}^2/\text{s}$.

Answer: Option C, $1.5 \text{ cm}^2/\text{s}$.

Question37

Two finite sets have m and n elements. The total number of proper subsets of the first set is 119 more than the total number of subsets of the second set. Find the value of $m - n$

Options:

A. 4

B. 6

C. 8

D. 1

Answer: A

Solution:

Let the first set have m elements and the second n elements.

Number of proper subsets of the first set:

$2^m - 1$.

Number of all subsets of the second set:

2^n.

The condition says

$(2^m - 1) - 2^n = 119 \quad \Longrightarrow \quad 2^m - 2^n = 120.$

Factor out 2^n:

$2^n\left(2^{\,m-n} - 1\right) = 120.$

Set $k = m - n$. Then

$2^n\left(2^k - 1\right) = 120.$

Check the given options for k:

• $k = 4$: $2^4 - 1 = 15$, so $2^n = 120/15 = 8 \Rightarrow n = 3$.

• $k = 6$: $2^6 - 1 = 63$, so $2^n = 120/63$ (not an integer).

• $k = 8$ or $k = 1$ similarly fail to make 2^n an integer.

Only $k = 4$ works.

Answer: $m - n = 4$.

Question38

Given that n number of arithmetic means are inserted between two pairs of numbers $a, 2b$ and $2a, b$; where $a, b \in R$. If the m^{th} means in the two cases

are the same, then the ratio $a : b$ is equal to

Options:

A. $m : (n - m + 1)$

B. $n : (n - m + 1)$

C. $(n - m + 1) : m$

D. $(n - m + 1) : n$

Answer: A

Solution:

Let the n arithmetic means between two numbers x and y define an A.P. with common difference

$d = \frac{y-x}{n+1}$,

so that the mth inserted mean (which is term number $m + 1$) is

$x + m\,d = x + m\frac{y-x}{n+1}$.

Between a and $2b$:

common difference $d_1 = \frac{2b-a}{n+1}$,

mth mean $= a + m\,\frac{2b-a}{n+1}$.

Between $2a$ and b:

common difference $d_2 = \frac{b-2a}{n+1}$,

mth mean $= 2a + m\,\frac{b-2a}{n+1}$.

Setting them equal:

$a + m\,\frac{2b-a}{n+1} \;=\; 2a + m\,\frac{b-2a}{n+1}$.

Multiply through by $n + 1$:

$(n+1)a + m(2b - a) = 2a(n+1) + m(b - 2a)$.

Rearrange:

$2bm - mb = a\big(n + 1 - m\big) \implies bm = a\,(n + 1 - m) \implies \frac{a}{b} = \frac{m}{n+1-m}$.

Hence

$a : b \;=\; m : (n - m + 1)$,

which is Option A.

Question39

$$\int_{-2}^{2} \frac{|x-3|}{x-3}\,dx =$$

Options:

A. -4

B. -2

C. 0

D. 2

Answer: A

Solution:

We note that

$\frac{|x-3|}{x-3} = \begin{cases} 1, & x-3>0, \\ -1, & x-3<0. \end{cases}$

On the interval $[-2,2]$, we have $x-3<0$ everywhere, so the integrand is constantly -1. Hence

$\int_{-2}^{2} \frac{|x-3|}{x-3}\,dx = \int_{-2}^{2}(-1)\,dx = -1\cdot\big(2-(-2)\big) = -4.$

Answer: -4 (Option A).

Question40

If $x = a\left[\left\{\cos t + \frac{1}{2}\log\left(\tan^2\frac{t}{2}\right)\right\}\right]$ and $y = a\sin t$ then $\frac{dy}{dx} =$

Options:

A. $\frac{a^2\cos^3 t}{\sin t}$

B. $\tan t$

C. a $\tan t \sec t$

D. $\sec^2 t$

Answer: B

Solution:

Let's set

$x = a\Big(\cos t + \tfrac{1}{2}\ln\tan^2\tfrac{t}{2}\Big) = a\big(\cos t + \ln\tan\tfrac{t}{2}\big), \qquad y = a\sin t.$

Differentiate:

$\frac{dy}{dt} = a\cos t,$

$\frac{dx}{dt} = a\Big(-\sin t + \frac{d}{dt}\ln\tan\tfrac{t}{2}\Big).$

Now

$\frac{d}{dt}\ln\tan\frac{t}{2} = \frac{1}{\tan(t/2)}\cdot\sec^2\frac{t}{2}\cdot\frac{1}{2} = \frac{1}{2}\frac{\sec^2(t/2)}{\tan(t/2)} = \frac{1}{\sin t}.$

So

$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\sin t}\right) = a\,\frac{1-\sin^2 t}{\sin t} = a\,\frac{\cos^2 t}{\sin t}.$

Therefore

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{a\,\frac{\cos^2 t}{\sin t}} = \frac{\cos t\ \sin t}{\cos^2 t} = \tan t.$

Answer: Option B.

Question41

If the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$ is 4 sq units. Then the value of ' a ' is

Options:

A. $\frac{16}{\sqrt{\pi}}$

B. $\frac{4}{\pi}$

C. $\frac{2}{\sqrt{\pi}}$

D. $\frac{4}{\sqrt{\pi}}$

Answer: D

Solution:

The curve $y = \sqrt{a^2 - x^2}$ for $x \in [0, a]$ is a quarter-circle of radius a. Hence its area is

• Area = $\frac{1}{4}\pi a^2$.

Setting this equal to 4 gives

$\frac{\pi a^2}{4} = 4 \implies a^2 = \frac{16}{\pi} \implies a = \frac{4}{\sqrt{\pi}}.$

Answer: $\frac{4}{\sqrt{\pi}}$ (Option D).

Question42

The function $y = \frac{\log x}{x^3}$ is strictly increasing function for

Options:

A. $0 < x < e^{\frac{1}{3}}$

B. $x > e^{\frac{1}{3}}$

C. $x < 2$

D. $x < e^{\frac{1}{3}}$

Answer: A

Solution:

First, rewrite the function as

$y = \frac{\ln x}{x^3}, \quad x > 0.$

To find where it's increasing, compute its derivative:

Set

$u = \ln x, \quad v = x^3$

so

$u' = \frac{1}{x}, \quad v' = 3x^2.$

By the quotient rule,

$y' = \frac{u'v - uv'}{v^2} = \frac{\frac{1}{x} \cdot x^3 \, - \, \ln x \cdot 3x^2}{x^6} = \frac{x^2 - 3x^2 \ln x}{x^6} = \frac{1 - 3\ln x}{x^4}.$

Since $x^4 > 0$ for all $x > 0$, the sign of y' is that of the numerator:

$y' > 0 \iff 1 - 3\ln x > 0 \iff \ln x < \frac{1}{3} \iff x < e^{1/3}.$

Conclusion: y is strictly increasing exactly on

$0 < x < e^{1/3}.$

That's Option A.

Question43

If $P = \{5m : m \in N\}$ and $Q = \{5^m : m \in N\}$, where N is set of natural numbers, then

Options:

A. $P = Q$

B. $P \subset Q$

C. $Q \subset P$

D. $P \cup Q = N$

Answer: C

Solution:

We have

$P = \{5m : m \in \mathbb{N}\} = \{5, 10, 15, 20, 25, \ldots\}$,

$Q = \{5^m : m \in \mathbb{N}\} = \{5, 25, 125, 625, \ldots\}$.

• Every element of Q is of the form

$5^m = 5 \cdot 5^{\,m-1}$,

so it's a multiple of 5 and hence lies in P.

• But P contains numbers like 10 or 15 which are not powers of 5, so $P \neq Q$.

Thus

$Q \subset P$,

i.e. Option C.

Question44

The image of a point $P(3, 5, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(a, b, c)$. Then $a + b + c =$

Options:

A. -17

B. -7

C. 3

D. 7

Answer: D

Solution:

Let L be the line

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$,

so a point on L is $A = (0, 1, 2)$ and its direction vector is $\mathbf{v} = (1, 2, 3)$. To reflect $P = (3, 5, 3)$ across L:

Compute $\overrightarrow{AP} = P - A = (3, 4, 1)$.

Find the foot H of the perpendicular from P to L by projecting $\overrightarrow{AP}$ onto $\mathbf{v}$:

$t = \frac{\overrightarrow{AP} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} = \frac{3 \cdot 1 + 4 \cdot 2 + 1 \cdot 3}{1^2 + 2^2 + 3^2} = \frac{14}{14} = 1$.

Hence

$H = A + t\,\mathbf{v} = (0, 1, 2) + (1, 2, 3) = (1, 3, 5)$.

The reflection P' satisfies $P' = 2H - P$:

$P' = (2 \cdot 1 - 3,\ 2 \cdot 3 - 5,\ 2 \cdot 5 - 3) = (-1, 1, 7)$.

Finally,

$a + b + c = -1 + 1 + 7 = 7.$

Answer: 7.

Question45

If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$ then the angle between $\vec{a}$ and $\vec{b}$ is

Options:

A. $\frac{\pi}{4}$

B. π

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

Solution:

First write the two identities:

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\,|\sin\theta|$

Setting them equal gives

$|\vec{a}|\,|\vec{b}| \cos\theta \;=\; |\vec{a}|\,|\vec{b}|\,|\sin\theta| \;\;\Longrightarrow\;\; \cos\theta = |\sin\theta|.$

So

$\tan\theta = \pm 1 \;\Longrightarrow\; \theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$.

But $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}| \geq 0$ forces $\cos\theta \geq 0$, excluding $3\pi/4$.

Answer: $\theta = \dfrac{\pi}{4}$. (Option A)

Question46

How many natural numbers are there between 100 and 1000 such that at least one of their digits is 6?

Options:

A. 243

B. 251

C. 252

D. 258

Answer: C

Solution:

Let's count 3-digit numbers (100 to 999) with at least one digit 6 by using the complement.

Total 3-digit numbers

$999 - 100 + 1 = 900$.

Numbers with no digit equal to 6

Hundreds digit: can be 1–9 except 6 ⇒ 8 choices

Tens digit: can be 0–9 except 6 ⇒ 9 choices

Units digit: can be 0–9 except 6 ⇒ 9 choices

⇒ total without any 6's

$8 \times 9 \times 9 = 648$.

Numbers with at least one digit 6

$900 - 648 = 252$.

Answer: Option C (252).

Question47

For any vector $\vec{p}$, the value of $\left[2\left\{|\vec{p}\times\hat{i}|^2+|\vec{p}\times\hat{j}|^2+|\vec{p}\times\hat{k}|^2\right\}\right]$ is

Options:

A. $4|\vec{p}|^2$

B. $2|\vec{p}|^2$

C. $4|\vec{p}|$

D. $2|\vec{p}|$

Answer: A

Solution:

Let $\vec{p} = (p_x, p_y, p_z)$. Then

$|\vec{p}\times\hat{i}|^2 = p_y^2 + p_z^2$

$|\vec{p}\times\hat{j}|^2 = p_x^2 + p_z^2$

$|\vec{p} \times \hat{k}|^2 = p_x^2 + p_y^2$

So

$|\vec{p} \times \hat{\imath}|^2 + |\vec{p} \times \hat{\jmath}|^2 + |\vec{p} \times \hat{k}|^2 = 2(p_x^2 + p_y^2 + p_z^2) = 2\,|\vec{p}|^2.$

Multiplying by 2 gives

$2\{\cdots\} = 2 \cdot 2|\vec{p}|^2 = 4|\vec{p}|^2,$

which is Option A.

Question48

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx =$

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{2}$

Answer: C

Solution:

You can use the symmetry trick $x \mapsto \frac{\pi}{2} - x$:

Let

$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx.$

Under $x \to \frac{\pi}{2} - x$

the integrand becomes

$\frac{1}{1+\sqrt{\tan(\frac{\pi}{2}-x)}} = \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,.$

Adding the two expressions gives

$\frac{1}{1+\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} = 1,$

so

$2I = \int_{\pi/6}^{\pi/3} 1\,dx = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}.$

Hence

$I = \frac{\pi}{12}.$

Answer: Option C, $\frac{\pi}{12}$.

Question49

A pot contains 5 red and 2 green balls. A ball is drawn at random from this pot. If a drawn ball is green, then a red ball is added to the pot. If a drawn ball is red, then a green ball is added to the pot, while the original ball drawn is not replaced in the pot. Now a second ball is drawn at random from the pot, what is the probability that the second ball drawn is a red ball?

Options:

A. $\frac{12}{49}$

B. $\frac{32}{49}$

C. $\frac{3}{7}$

D. $\frac{27}{49}$

Answer: B

Solution:

Let G_1/R_1 be the event that the first ball is green/red, and R_2 the event that the second ball is red. By the law of total probability,

P(G_1)=2/7.

– If a green is drawn, it's removed and one red is added.

New composition: 6 R, 1 G ⇒

$P(R_2 \mid G_1) = \frac{6}{7}$.

P(R_1)=5/7.

– If a red is drawn, it's removed and one green is added.

New composition: 4 R, 3 G ⇒

$P(R_2 \mid R_1) = \frac{4}{7}$.

So

$P(R_2) = P(G_1)\,P(R_2 \mid G_1) \;+\; P(R_1)\,P(R_2 \mid R_1) = \frac{2}{7} \cdot \frac{6}{7} \;+\; \frac{5}{7} \cdot \frac{4}{7} = \frac{12+20}{49} = \frac{32}{49}$.

Answer: Option B.

Question50

If $A = \begin{bmatrix} 4 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ then A^{-1} exists if :

Options:

A. $\lambda = 2$

B. $\lambda = 0$

C. $\lambda \neq 2$

D. $\lambda \neq -2$

Answer: D

Solution:

To decide when A^{-1} exists, compute $\det(A)$:

$$\det(A) = 4\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} - \lambda\begin{vmatrix} 0 & 5 \\ 1 & 3 \end{vmatrix} - 3\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix}$$

Evaluate each minor:

- $\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1$
- $\begin{vmatrix} 0 & 5 \\ 1 & 3 \end{vmatrix} = 0 - 5 = -5$
- $\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = 0 - 2 = -2$

Combine:

$\det(A) = 4 \cdot 1 - \lambda \cdot (-5) - 3 \cdot (-2) = 4 + 5\lambda + 6 = 5(\lambda + 2).$

Since A^{-1} exists exactly when $\det(A) \neq 0$, we require

$\lambda + 2 \neq 0 \implies \lambda \neq -2.$

Answer: Option D.

Question51

The value of $\tan\left\{\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{2}\right\}$ is

Options:

A. -1

B. $\frac{1}{\sqrt{2}}$

C. 1

D. $-\frac{1}{\sqrt{2}}$

Answer: A

Solution:

Let θ = cos⁻¹(√2⁄2). Since cos θ = √2⁄2, we have θ = π⁄4. Thus

The angle inside the tangent is

$\theta - \frac{\pi}{2} = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$.

Therefore

$\tan\left(-\frac{\pi}{4}\right) = -1$.

Answer: –1 (Option A).

Question52

If the line $(3x + 14y + 7) + k(5x + 7y + 6) = 0$ is perpendicular to x-axis then the value of ' k ' is

Options:

A. -2

B. 2

C. $-\frac{3}{5}$

D. $\frac{1}{3}$

Answer: A

Solution:

First combine terms:

$(3x + 14y + 7) + k(5x + 7y + 6) = 0$

becomes

$(3 + 5k)x + (14 + 7k)y + (7 + 6k) = 0$.

A line perpendicular to the x-axis is vertical, so its coefficient of y must be zero:

$14 + 7k = 0 \quad \Longrightarrow \quad k = -2$.

Answer: -2 (Option A).

Question53

A straight line makes positive intercepts on the coordinate axes whose sum is 5 . If the line passes through the point $P(-3,4)$ then the equation of a line is

Options:

A. $2x - y + 10 = 0$

B. $2x + 3y = 6$

C. $3x + 2y = 6$

D. $x + 4y = 13$

Answer: B

Solution:

Let the intercepts be a on the x–axis and b on the y–axis, with

1) $a + b = 5$,

2) the line in intercept form:

$\frac{x}{a} + \frac{y}{b} = 1$,

3) it passes through $P(-3, 4)$, so

$\frac{-3}{a} + \frac{4}{b} = 1$.

Since $b = 5 - a$, substitute into the last equation:

$\frac{-3}{a} + \frac{4}{5-a} = 1 \implies -3(5-a) + 4a = a(5-a) \implies a^2 + 2a - 15 = 0 \implies a = 3$ (positive root),

hence $b = 5 - 3 = 2$. The line is

$\frac{x}{3} + \frac{y}{2} = 1 \implies 2x + 3y = 6.$

Answer: Option B) $2x + 3y = 6$.

Question54

If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the foci of the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is

Options:

A. 5

B. 9

C. 1

D. 7

Answer: D

Solution:

Hyperbola

$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

multiply both sides by 25:

$\frac{x^2}{144/25} - \frac{y^2}{81/25} = 1$

so

$a_h^2 = \frac{144}{25}, \quad b_h^2 = \frac{81}{25}, \quad c_h^2 = a_h^2 + b_h^2 = \frac{144+81}{25} = \frac{225}{25} = 9$

hence $c_h = 3$.

Ellipse

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

has

$a_e^2 = 16, \quad c_e^2 = a_e^2 - b^2 = 16 - b^2.$

Requiring the same focal distance, $c_e = c_h = 3$, gives

$16 - b^2 = 9 \quad \Longrightarrow \quad b^2 = 7.$

Answer: Option D (7).

Question55

The equation of a line passing through origin with direction angles $\frac{2\pi}{3}, \frac{\pi}{4}, \frac{\pi}{3}$ is

Options:

A. $x = \frac{y}{\sqrt{2}} = z$

B. $\frac{x}{-1} = \frac{y}{-\sqrt{2}} = z$

C. $x = \frac{y}{-\sqrt{2}} = z$

D. $x = \frac{y}{-\sqrt{2}} = -z$

Answer: D

Solution:

First find the direction cosines

• l = cos α = cos (2π/3) = –½

• m = cos β = cos (π/4) = √2/2

• n = cos γ = cos (π/3) = ½

So one choice of direction ratios is

$(l, m, n) \propto \left(-\frac{1}{2},\ \frac{\sqrt{2}}{2},\ \frac{1}{2}\right) \sim (-1,\ \sqrt{2},\ 1).$

Multiplying all three by –1 gives an equivalent set

$(1,\ -\sqrt{2},\ -1),$

and the symmetric equation through the origin is

$\frac{x}{1} = \frac{y}{-\sqrt{2}} = \frac{z}{-1},$

which we can rewrite as

$x = \frac{y}{-\sqrt{2}} = -z.$

That matches Option D.

Question56

Two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect at a point. Then the value of ' k ' is

Options:

A. $\frac{13}{2}$

B. $-\frac{13}{2}$

C. $\frac{7}{2}$

D. $\frac{9}{2}$

Answer: D

Solution:

Let the first line be given by the parameter t and the second by s:

• $x = 1 + 2t, \quad y = -1 + 3t, \quad z = 1 + 4t$

• $x = 3 + s, \ \ y = k + 2s, \ \ z = s$

For intersection we need

$1 + 2t = 3 + s$

$-1 + 3t = k + 2s$

$1 + 4t = s$

From (3):

$s = 1 + 4t.$

Plug into (1):

$1 + 2t = 3 + (1 + 4t) \implies 1 + 2t = 4 + 4t \implies -3 = 2t \implies t = -\frac{3}{2}.$

Then $s = 1 + 4(-\frac{3}{2}) = 1 - 6 = -5.$

Now use (2) to find k:

$-1+3\left(-\frac{3}{2}\right) \;=\; k+2(-5) \;\Longrightarrow\; -1-\frac{9}{2}=k-10 \;\Longrightarrow\; -\frac{11}{2}=k-10 \;\Longrightarrow\; k=10-\frac{11}{2}=\frac{20-11}{2}=\frac{9}{2}.$

Answer: $\frac{9}{2}$ (Option D).

Question57

Solve the following differential equation $\cos^2 x\,\frac{dy}{dx}+y=\tan x$, given that $y(0)=1$. Hence find $y\left(\frac{\pi}{4}\right)$

Options:

A. 2

B. $\frac{2}{e}$

C. e

D. 1

Answer: B

Solution:

Divide through by $\cos^2 x$:

$\frac{dy}{dx}+\sec^2 x\,y \;=\; \tan x\;\sec^2 x.$

The integrating factor is

$\mu(x)=\exp\!\left(\int \sec^2 x\,dx\right)=\exp(\tan x).$

Multiply the equation by $\mu(x)$:

$\frac{d}{dx}\left(y\,e^{\tan x}\right)=\tan x\sec^2 x\;e^{\tan x}.$

Notice $\tan x\sec^2 x\,dx=u\,du$ if $u=\tan x$. Hence

$\int \tan x\sec^2 x\;e^{\tan x}\,dx=\int u\,e^{u}\,du=(u-1)e^{u}+C=(\tan x-1)e^{\tan x}+C.$

So

$y\,e^{\tan x}=(\tan x-1)e^{\tan x}+C \quad\Longrightarrow\quad y=\tan x-1+C\,e^{-\tan x}.$

Apply $y(0)=1$. Since $\tan 0=0$,

$1=0-1+C\cdot e^{0} \quad\Longrightarrow\quad C=2.$

Thus

$y(x)=\tan x-1+2\,e^{-\tan x}.$

Finally, at $x=\pi/4$, $\tan(\pi/4)=1$:

$y\!\left(\tfrac{\pi}{4}\right)=1-1+2\,e^{-1}=\tfrac{2}{e}.$

Answer: Option B, $\frac{2}{e}$.

Question58

If $\tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}, \quad 0 < \alpha, \beta < \frac{\pi}{2}$ then 2β is equal to

Options:

A. $\frac{\pi}{8} - \alpha$

B. $\frac{\pi}{4} - \alpha$

C. $\frac{3\pi}{8} - \frac{\alpha}{2}$

D. $\frac{3\pi}{4} - \alpha$

Answer: B

Solution:

First, note that in the first quadrant

$\tan\alpha = \frac{1}{7} \implies \alpha = \arctan\frac{1}{7}, \quad 0 < \alpha < \frac{\pi}{2}$.

$\sin\beta = \frac{1}{\sqrt{10}} \implies \cos\beta = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}}, \quad \tan\beta = \frac{1/\sqrt{10}}{3/\sqrt{10}} = \frac{1}{3}, \quad \beta = \arctan\frac{1}{3}$.

Compute $\tan(2\beta) = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$.

On the other hand

$\tan\left(\frac{\pi}{4} - \alpha\right) = \frac{\tan\frac{\pi}{4} - \tan\alpha}{1+\tan\frac{\pi}{4}\tan\alpha} = \frac{1-\frac{1}{7}}{1+1\cdot\frac{1}{7}} = \frac{\frac{6}{7}}{\frac{8}{7}} = \frac{3}{4}$.

Since both 2β and $\frac{\pi}{4} - \alpha$ lie in $(0, \frac{\pi}{2})$ and have the same tangent, they must be equal.

Answer:

$2\beta = \frac{\pi}{4} - \alpha$,

so the correct choice is Option B.

Question59

An unbiased die is tossed twice. What is the probability of getting a 4,5 or 6 on the first toss and a $1, 2, 3$ or 4 on the second toss?

Options:

A. $\frac{5}{6}$

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: D

Solution:

Let A = “first toss is 4, 5 or 6” and B = “second toss is 1, 2, 3 or 4.” Since the tosses are independent,

• P(A) = 3/6 = 1/2

• P(B) = 4/6 = 2/3

So

$P(A \text{ and } B) = P(A) \times P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}.$

That matches Option D.

Question60

If $y = x + e^x$ then $\frac{d^2x}{dy^2} =$

Options:

A. e^x

B. $\frac{-e^x}{(1+e^x)^2}$

C. $\frac{-e^x}{(1+e^x)^3}$

D. $\frac{-1}{(1+e^x)^3}$

Answer: C

Solution:

First, note

$\frac{dy}{dx} = 1 + e^x \quad \Longrightarrow \quad \frac{dx}{dy} = (1 + e^x)^{-1}.$

Then by the chain rule,

$\frac{d^2x}{dy^2} = \frac{d}{dy}\left((1 + e^x)^{-1}\right) = \frac{d}{dx}\left((1 + e^x)^{-1}\right) \cdot \frac{dx}{dy}.$

Compute the derivative w.r.t. x:

$\frac{d}{dx}\left((1 + e^x)^{-1}\right) = -(1 + e^x)^{-2} \cdot e^x = -\frac{e^x}{(1+e^x)^2}.$

Multiplying by $\frac{dx}{dy} = (1 + e^x)^{-1}$ gives

$\frac{d^2x}{dy^2} = -\frac{e^x}{(1+e^x)^2} \cdot \frac{1}{1+e^x} = -\frac{e^x}{(1+e^x)^3}.$

So the answer is option C.

Chemistry

Question1

An aqueous solution of an electrolyte $\mathrm{A_3\ B}$ is prepared by dissolving 0.5625 g in 750 ml of water and is found to be 80% ionised. If $\mathrm{K_b}$ for water is $0.52\ \mathrm{K\ kg\ mol^{-1}}$, calculate the boiling point of the solution at 1.0 atm pressure.

Options:

A. 374.33 K

B. 371.68 K

C. 373.18 K

D. 377.2 K

Answer: A

Solution:

Determine the van't Hoff factor (i):

The electrolyte is $\mathrm{A_3\ B}$. When it dissolves in water, we can assume it dissociates into 3 ions of A and 1 ion of B, meaning a total of 4 ions per formula unit if it were completely dissociated. So, $n = 4$.

The solution is 80% ionized, which means the degree of ionization (α) is 0.80.

The van't Hoff factor (i) accounts for the effective number of particles in solution. For dissociation, the formula is:

$i = 1 + \alpha(n - 1)$

Plugging in the values:

$i = 1 + 0.80(4 - 1)$

$i = 1 + 0.80(3)$

$i = 1 + 2.40$

$i = 3.40$

Calculate the molality (m) of the solution:

Molality is defined as moles of solute per kilogram of solvent.

Based on our assumption, the moles of solute ($\mathrm{A_3\ B}$) = 0.5625 mol.

The volume of water is 750 ml. Since the density of water is approximately 1 g/mL, the mass of water is 750 g, which is 0.750 kg.

Now, let's calculate the molality:

$m = \frac{\text{moles of solute}}{\text{mass of solvent (in kg)}}$

$m = \frac{0.5625 \text{ mol}}{0.750 \text{ kg}}$

$m = 0.75 \text{ mol kg}^{-1}$

Calculate the boiling point elevation (ΔT_b):

The formula for boiling point elevation is:

$\Delta T_b = i \cdot K_b \cdot m$

We are given K_b for water as $0.52 \text{ K kg mol}^{-1}$.

$\Delta T_b = 3.40 \cdot 0.52 \text{ K kg mol}^{-1} \cdot 0.75 \text{ mol kg}^{-1}$

$\Delta T_b = 1.326 \text{ K}$

Calculate the boiling point of the solution:

The normal boiling point of pure water at 1.0 atm is 100°C, which is 373.15 K. Sometimes, for simplicity in problems, it's taken as 373 K. Let's see which one matches the options better.

$T_{\text{b, solution}} = T_{\text{b, pure water}} + \Delta T_b$

Using $T_{\text{b, pure water}} = 373 \text{ K}$:

$T_{\text{b, solution}} = 373 \text{ K} + 1.326 \text{ K}$

$T_{\text{b, solution}} = 374.326 \text{ K}$

This value is very close to Option A.

The final answer is $\boxed{374.33 \text{ K}}$.

Question2

Arrange the complex ions in the increasing order of their magnetic moments

A. $[Fe(H_2O)_6]^{2+}$
B. $[Fe(CN)_6]^{4-}$
C. $[Fe(CN)_6]^{3-}$
D. $[FeF_6]^{3-}$

Options:

A. $\text{D} < \text{A} < \text{B} < \text{C}$

B. $\text{B} < \text{C} < \text{A} < \text{D}$

C. $\text{D} < \text{C} < \text{B} < \text{A}$

D. $\text{A} < \text{D} < \text{C} < \text{B}$

Answer: B

Solution:

Let's count the unpaired electrons in each complex (spin-only magnetic moment $\mu = \sqrt{n(n+2)}$, where n=number of unpaired electrons):

$[Fe(CN)_6]^{4-}$

• Fe^{2+} is d^6, CN^- is strong-field → low-spin: $t_{2g}^6\, e_g^0$ ⇒ n=0

$[Fe(CN)_6]^{3-}$

• Fe^{3+} is d^5, CN^- is strong-field → low-spin: $t_{2g}^5\, e_g^0$ ⇒ n=1

$[Fe(H_2O)_6]^{2+}$

• Fe^{2+} is d^6, H_2O is weak-field → high-spin: $t_{2g}^4\, e_g^2$ ⇒ n=4

$[FeF_6]^{3-}$

• Fe^{3+} is d^5, F^- is weak-field → high-spin: $t_{2g}^3\, e_g^2$ ⇒ n=5

Ordering by increasing n:

$[Fe(CN)_6]^{4-}$ (0) < $[Fe(CN)_6]^{3-}$ (1) < $[Fe(H_2O)_6]^{2+}$ (4) < $[FeF_6]^{3-}$ (5)

That matches Option B.

Question3

When the temperature of a reaction $\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C}$ is increased from 300 K to 310 K the rate constant increases by 12%. What is the Activation energy of the reaction?

Options:

A. $163.9\ \mathrm{kJ/mol}$

B. $85.69\ \mathrm{kJ/mol}$

C. $192.17\ \mathrm{kJ/mol}$

D. $8.76\ \mathrm{kJ/mol}$

Answer: D

Solution:

We start with the Arrhenius relation in the form

$$\ln \frac{k_2}{k_1} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

Here

• $T_1 = 300$ K, $T_2 = 310$ K

• $\frac{k_2}{k_1} = 1.12$

• $R = 8.314\,\mathrm{J\cdot mol^{-1}\cdot K^{-1}}$

Compute the temperature-term:

$\frac{1}{T_1} - \frac{1}{T_2} = \frac{1}{300} - \frac{1}{310} = \frac{310-300}{300\cdot 310} = \frac{10}{93\,000} \approx 1.0753 \times 10^{-4}\,\mathrm{K^{-1}}$

Take the log-ratio:

$\ln(1.12) \approx 0.1133$

Solve for E_a:

$E_a = R\,\frac{\ln(1.12)}{\frac{1}{300}-\frac{1}{310}} = 8.314\ \frac{0.1133}{1.0753\times10^{-4}} \approx 8.76 \times 10^{3}\ \text{J/mol} = 8.76\ \text{kJ/mol}$

That matches Option D.

Question4

The first electron gain enthalpy of oxygen is $-141\ \mathrm{kJ\ mol^{-1}}$, its second electron gain enthalpy is :

Options:

A. A positive value

B. Less negative value than the first

C. Maximum negative value than the first

D. Same value as that of the first

Answer: A

Solution:

The second electron gain enthalpy corresponds to the process

$\mathrm{O^- + e^- \longrightarrow O^{2-}}$

Because you're trying to add an electron to a negatively charged ion, there's strong Coulombic repulsion and you must *input* energy. Hence ΔH is positive.

Answer: Option A (a positive value).

Question5

Two moles of an ideal gas at 1 bar pressure and 298 K is expanded into vacuum to double the volume. The work done is :

Options:

A. 2 J

B. 0 J

C. 1.5 J

D. 10 J

Answer: B

Solution:

When an ideal gas expands freely into a vacuum, the external pressure is zero, so:

• Work done by the gas

$w = \int P_{\text{ext}}\, dV = 0 \times \Delta V = 0$

Answer: Option B (0 J).

Question6

The order of a reaction $\mathrm{W} + \mathrm{X} - - - - \rightarrow \mathrm{Y} + \mathrm{Z}$ with respect to W is 3 and with respect to X is 1. If the concentrations of both W and X are tripled, the rate of reaction will increase by _____ times.

Options:

A. 81

B. 27

C. 51

D. 15

Answer: A

Solution:

The rate law is

rate $= k\,[W]^3\,[X]^1$.

If both [W] and [X] are tripled, the new rate becomes

$\text{rate}_{\text{new}} = k\,(3[W])^3\,(3[X]) = k\,3^3\,[W]^3 \;\times\; 3^1\,[X] = 27 \times 3\,k\,[W]^3[X] = 81\ \text{rate}_{\text{old}}$.

So the rate increases by 81-fold.

Answer: 81 (Option A).

Question7

Identify the correct statement.

Options:

A. Fructose is an example of Invert sugar

B. Leucine and Tryptophan are classified as Essential amino acids

C. Curdling of milk is not a denaturation process

D. Amylose is a branched chain polymer of $\alpha - \mathrm{D} - (+)$ - glucose units

Answer: B

Solution:

The correct statement is Option B.

• Option A is incorrect: “invert sugar” is the 1:1 mixture of glucose and fructose formed by sucrose hydrolysis, not pure fructose.

• Option B is correct: leucine and tryptophan are two of the nine essential amino acids (they must be supplied by the diet).

• Option C is incorrect: curdling (coagulation) of milk proteins involves denaturation and aggregation of casein.

• Option D is incorrect: amylose is a largely unbranched polymer of α-D-(+)-glucose; the branched starch component is amylopectin.

Question8

For the cell reaction $4\mathrm{Br}^- + \mathrm{O}_2 + 4\mathrm{H}^+ \rightarrow 2\mathrm{Br}_2 + 2\mathrm{H}_2\mathrm{O}$ at 298 K , the E^0 cell $= 0.16$ V. What would be the K_c (Equilibrium constant) value if the reverse reaction were to take place?

Options:

A. 2.012×10^{-10}

B. 8.47×10^{-9}

C. 1.422×10^{-11}

D. 7.031×10^{-10}

Answer: C

Solution:

First, write the two key relations at 298 K:

ΔG° = −n F E°

ΔG° = −RT ln K

Equating and solving for K (forward reaction):

$\ln K_\mathrm{f} = \frac{nFE^0}{RT} \quad \Longrightarrow \quad \log_{10} K_\mathrm{f} = \frac{nE^0}{0.05916}$

Here n = 4, E° = 0.16 V, so

• $\log_{10}$Kf = 4×0.16/0.05916 ≃ 10.82

• Kf ≃ 10^10.82 ≃ 6.6×10^10

The reverse reaction has Kc = 1/Kf ≃ 1/(6.6×10¹⁰) ≃ 1.5×10⁻¹¹, which matches option C (1.422×10⁻¹¹).

Question9

The bond angles in the following molecules decreases in the order. BF_3, NH_3, PF_3 and XeF_2

Options:

A. $NH_3 > PF_3 > XeF_2 > BF_3$

B. $XeF_2 > BF_3 > NH_3 > PF_3$

C. $BF_3 > NH_3 > XeF_2 > PF_3$

D. $PF_3 > BF_3 > NH_3 > XeF_2$

Answer: B

Solution:

We can rank these by their typical bond angles:

• XeF_2 (linear):

F–Xe–F = 180°

• BF_3 (trigonal planar):

F–B–F = 120°

• NH_3 (trigonal pyramidal):

H–N–H ≈ 107.5°

• PF_3 (trigonal pyramidal):

F–P–F ≈ 98°

Putting them in decreasing order gives:

$XeF_2\ (180^\circ) > BF_3\ (120^\circ) > NH_3\ (107.5^\circ) > PF_3\ (98^\circ)$

That matches Option B.

Question10

Two statements, one Assertion and the other Reason, are given. Choose the correct option.

Assertion : During the electrolysis of aqueous $NaCl$, Cl_2 is liberated at the anode in preference to O_2 and water gets reduced to H_2 at cathode.

Reason : The reaction at Anode with lower oxidation potential is not preferred due to over potential of Oxygen.

Options:

A. Both Assertion and Reason are incorrect.

B. Both Assertion and Reason are correct.

C. Assertion is correct but Reason is incorrect.

D. Assertion is incorrect but Reason is correct.

Answer: B

Solution:

Option B

Both the Assertion and the Reason are correct, and the Reason correctly explains the Assertion.

Explanation:

Cathode reactions

Reduction of Na^+:

$$\mathrm{Na^+ + e^- \rightarrow Na\ (s)}, \quad E^\circ = -2.71\ \mathrm{V}$$

Reduction of water:

$$\mathrm{2H_2O + 2e^- \rightarrow H_2 + 2OH^-}, \quad E^\circ = -0.83\ \mathrm{V}$$

Since -0.83 V is less negative than -2.71 V, water is reduced and H_2 is liberated.

Anode reactions

Oxidation of water:

$$\mathrm{2H_2O \rightarrow O_2 + 4H^+ + 4e^-}, \quad E^\circ = +1.23\ \mathrm{V}$$

Oxidation of chloride:

$$\mathrm{2Cl^- \rightarrow Cl_2 + 2e^-}, \quad E^\circ = +1.36\ \mathrm{V}$$

Thermodynamically, water oxidation (E° = +1.23 V) should occur before chloride oxidation (E° = +1.36 V).

However, the kinetic overpotential for O_2 evolution on typical electrodes is large, so chloride is oxidized preferentially to Cl_2.

Thus:

Assertion is correct (Cl_2 at anode; H_2 at cathode).

Reason is correct (water oxidation is suppressed by O_2 overpotential).

Question11

An aqueous solution of volume V ml contains a non-volatile solute of unknown mass W_B g and molar mass M_B g/mol. If the Osmotic pressure of

the solution is 1.013 bar, which one of the following is the mathematical expression to be used to calculate W_B ?

Options:

A. $W_B = \frac{\pi \mathrm{M_B} V}{760 * \mathrm{RT} * 1000}$

B. $W_B = \frac{\pi \mathrm{M_B} V 1000}{76 \mathrm{RT}}$

C. $W_B = \frac{\pi \mathrm{M_B} V}{\mathrm{RT}}$

D. $W_B = \frac{\pi \mathrm{M_B} V}{1000 \mathrm{RT}}$

Answer: D

Solution:

First, recall the van 't Hoff law for dilute solutions:

$\pi V_l = n R T$

where

• π is the osmotic pressure,

• V_l is the volume in litres,

• n = W_B/M_B is the number of moles of solute.

So

$n = \pi V_l/(R T)$

and

$W\_B = n M\_B = \pi M\_B V_l/(R T)$.

Since V_l = V (mL)/1000, this becomes

$W_B = \frac{\pi M_B (V/1000)}{R T} = \frac{\pi M_B V}{1000 R T}$

That matches Option D.

Question12

The ratio of $\mathrm{N_2}$ and $\mathrm{O_2}$ gases in the atmosphere is $4:1$. The ratio of the mole fractions of the dissolved gases $\mathrm{N_2}$: $\mathrm{O_2}$ in rain water will be approximately ..... (At 293 K, $\mathrm{K_H}$ for Nitrogen and Oxygen in kbar units are 76.48 and 34.86 respectively.)

Options:

A. 3 : 1

B. 2 : 1

C. 4 : 1

D. 1 : 4

Answer: B

Solution:

We apply Henry's law in the form

$x_i = \frac{P_i}{K_{H,i}}$,

so the ratio of mole fractions in water is

$\frac{x_{N_2}}{x_{O_2}} = \frac{P_{N_2}/K_{H,N_2}}{P_{O_2}/K_{H,O_2}} = \frac{P_{N_2}}{P_{O_2}} \times \frac{K_{H,O_2}}{K_{H,N_2}}$.

• In air, $P_{N_2} : P_{O_2} = 4 : 1$.

• Given $K_{H,N_2} = 76.48$ kbar, $K_{H,O_2} = 34.86$ kbar,

$\frac{x_{N_2}}{x_{O_2}} = 4 \times \frac{34.86}{76.48} \approx 4 \times 0.456 \approx 1.82 \approx 2 : 1.$

Answer: 2 : 1 (Option B).

Question13

Identify the correct coefficients (a), (b), (c) and (d) in the following equations

i) $\mathrm{xMnO_4^{--}}$ + **(a)** $\mathrm{SO_3^{2-}}$ + **(b)** $\mathrm{H^+} \cdots \longrightarrow \mathrm{xMn^{2+}}$ + **(a)** $\mathrm{SO_4^{2-}}$ + **(c)** $\mathrm{H_2O}$

ii) **(c)** $\mathrm{S_2O_3^{2-}}$ + **(d)** $\mathrm{MnO_4^-} + \mathrm{H_2O} \cdots \rightarrow$ **(d)** $\mathrm{MnO_2}$ + **(b)** $\mathrm{SO_4^{2-}} + \mathrm{xOH^-}$

Options:

A. $(a) = 3$ (b) $= 8$ $(c) = 6$ $(d) = 6$

B. $(a) = 5$ (b) $= 6$ $(c) = 3$ (d) $= 8$

C. (a) $= 2$ (b) $= 8$ $(c) = 4$ (d) $= 4$

D. $(a) = 2$ (b) $= 6$ $(c) = 6$ $(d) = 5$

Answer: B

Solution:

Let's balance each equation by the half-reaction method.

1) In acidic medium

Reduction:

$\mathrm{MnO_4^-} + 8\,H^+ + 5e^- \longrightarrow \mathrm{Mn^{2+}} + 4\,H_2O$

Oxidation:

$SO_3^{2-} + H_2O \longrightarrow SO_4^{2-} + 2H^+ + 2e^-$

– LCM of electrons = 10 → multiply first by 2, second by 5 and add:

$2MnO_4^- + 5SO_3^{2-} + 6H^+ \longrightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O$

→ x=2, a=5, b=6, c=3.

2) In basic medium

Oxidation (thiosulfate → sulfate):

$S_2O_3^{2-} + 5H_2O \longrightarrow 2SO_4^{2-} + 10H^+ + 8e^-$

Reduction (permanganate → MnO_2):

$MnO_4^- + 2H_2O + 3e^- \longrightarrow MnO_2 + 4OH^-$

– LCM of electrons = 24 → multiply oxidation by 3, reduction by 8, add, then neutralize H^+ with OH^-:

$3S_2O_3^{2-} + 8MnO_4^- + H_2O \longrightarrow 6SO_4^{2-} + 8MnO_2 + 2OH^-$

→ c=3, d=8 (and indeed b=6, x=2 again).

Answer:

a = 5, b = 6, c = 3, d = 8 (Option B)

Question14

The decomposition of PH_3 follows first order kinetics. The time required for $3/4^{th}$ of PH_3 to decompose is 75.76 s . Calculate the fraction of original amount of PH_3 which will remain after 2.0 minutes.

Options:

A. 0.354

B. 0.111

C. 0.954

D. 0.898

Answer: B

Solution:

Let N/N_0 be the fraction remaining and k the first-order rate constant.

At $t_1 = 75.76$ s, three-quarters has decomposed, so

$\frac{N}{N_0} = \frac{1}{4} = e^{-k\,t_1} \implies k = -\frac{\ln(1/4)}{75.76} = \frac{\ln 4}{75.76} \approx 0.01827\ \text{s}^{-1}.$

After $t_2 = 2.0$ min $= 120$ s,

$\frac{N}{N_0} = e^{-k\,t_2} = e^{-\,(0.01827)(120)} \approx e^{-2.193} \approx 0.111.$

So about 0.111 of the original PH_3 remains.

Answer: Option B (0.111).

Question15

An example of disproportionation reaction is :

Options:

A. $2H_2O_2 \rightarrow 2H_2O + O_2$

B. $2KMnO_4 \rightarrow\ K_2MnO_4 + MnO_2 + O_2$

C. $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

D. $2NaI + Cl_2 \rightarrow 2NaCl + I_2$

Answer: A

Solution:

Disproportionation is a redox reaction in which the same element is simultaneously oxidized and reduced. Examining the options:

• Option A

$2H_2O_2\ \rightarrow\ 2H_2O + O_2$

– In H_2O_2, oxygen is in the –1 state.

– In H_2O, it's –2 (reduction).

– In O_2, it's 0 (oxidation).

⇒ The same element (O) is both reduced and oxidized.

• Option B

$2KMnO_4\ \rightarrow\ K_2MnO_4 + MnO_2 + O_2$

– Mn goes from +7 to +6 and +4 (only reductions).

– O goes from –2 to 0 (only oxidation).

⇒ No single element undergoes both.

• Option C and D

– These are redox between different elements (Mn vs. I, Cl vs. I), not self-redox.

Answer: Option A.

Question16

Identify the Alkane (molecular formula C_8H_{18}) which yields only a single monochloride on Chlorination in the presence of sunlight.

Options:

A. 2,2,4-Trimethylpentane

B. 2,4-Dimethylhexane

C. 2,2,3,3 - Tetramethylbutane

D. 3,4-Dimethylhexane

Answer: C

Solution:

To see which alkane gives only one monochloride, note that the number of different chlorinated products equals the number of distinct H-types in the molecule. Only if all 18 H's are equivalent will you get a single monochloride.

• In 2,2,3,3-tetramethylbutane (C_8H_{18}) every H sits on one of six methyl groups that are all related by the molecule's symmetry.

• Hence there is only one kind of H-atom, so chlorination can give only one monochloride.

Answer: Option C, 2,2,3,3-Tetramethylbutane.

Question17

For a reaction,

$$A + B \rightleftharpoons 2C$$

1.0 mole of A, 1.5 mole of B and 0.5 mole of C were taken in a 1 L vessel. At equilibrium, the concentration of C was $1.0\ \text{mol L}^{-1}$. The equilibrium constant for the reaction is $x/15$. The value of ' x ' is:

Options:

A. 32

B. 22

C. 18

D. 16

Answer: D

Solution:

Write down initial and equilibrium concentrations (in a 1 L vessel):

• $[A]_0 = 1.0$ M, $[B]_0 = 1.5$ M, $[C]_0 = 0.5$ M

• $[C]_{eq} = 1.0$ M

Define the reaction extent, ξ, for

A + B ⇌ 2 C

so that

$[C]_{eq} = 0.5 + 2\,\xi = 1.0 \;\Rightarrow\; \xi = 0.25$

Find [A]eq and [B]eq:

$[A]_{eq} = 1.0 - \xi = 0.75\,, \quad [B]_{eq} = 1.5 - \xi = 1.25$

Write the equilibrium constant:

$K_c = \frac{[C]^2}{[A][B]} = \frac{1.0^2}{0.75 \times 1.25} = \frac{1}{0.9375} = 1.0667$

Since $K_c = \frac{x}{15}\,, \quad x = 15 \times 1.0667 \approx 16$

Answer: x = 16.

Question18

The correct order of reactivity of halogens with alkanes is :

Options:

A. $\mathrm{F_2} > \mathrm{Cl_2} > \mathrm{Br_2} > \mathrm{I_2}$

B. $\mathrm{F_2} > \mathrm{Br_2} > \mathrm{Cl_2} > \mathrm{I_2}$

C. $\mathrm{Cl_2} > \mathrm{F_2} > \mathrm{Br_2} > \mathrm{I_2}$

D. $\mathrm{I_2} > \mathrm{Br_2} > \mathrm{Cl_2} > \mathrm{F_2}$

Answer: A

Solution:

The correct order is

$\mathrm{F_2} > \mathrm{Cl_2} > \mathrm{Br_2} > \mathrm{I_2}$

(Option A).

Why?

Bond-dissociation energy (BDE):

• F–F is weakest → easiest to homolytically cleave → most reactive

• Cl–Cl stronger than F–F but weaker than Br–Br → intermediate reactivity

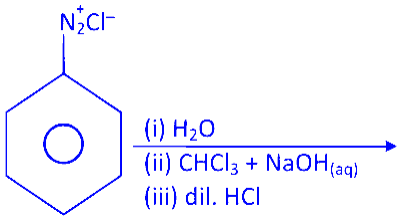
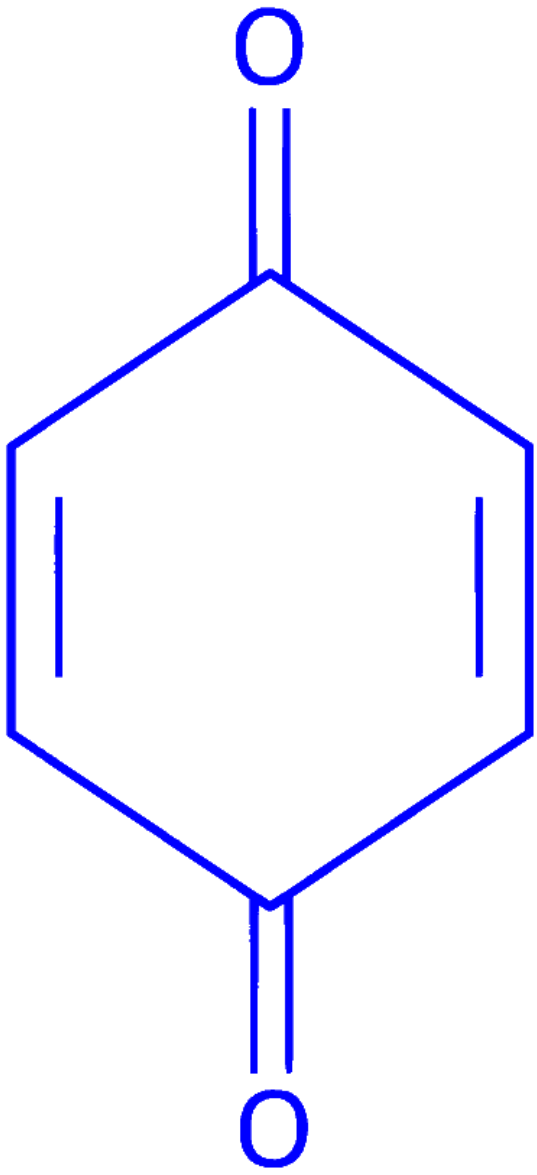
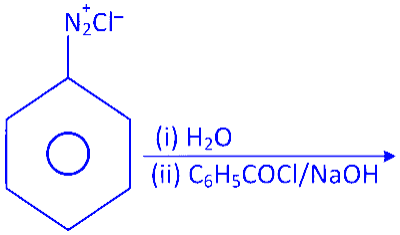
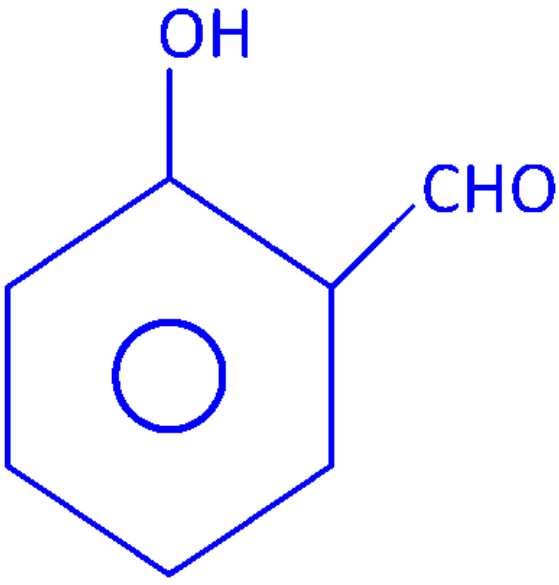
• Br–Br still stronger → lower reactivity

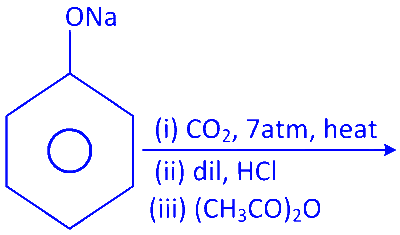
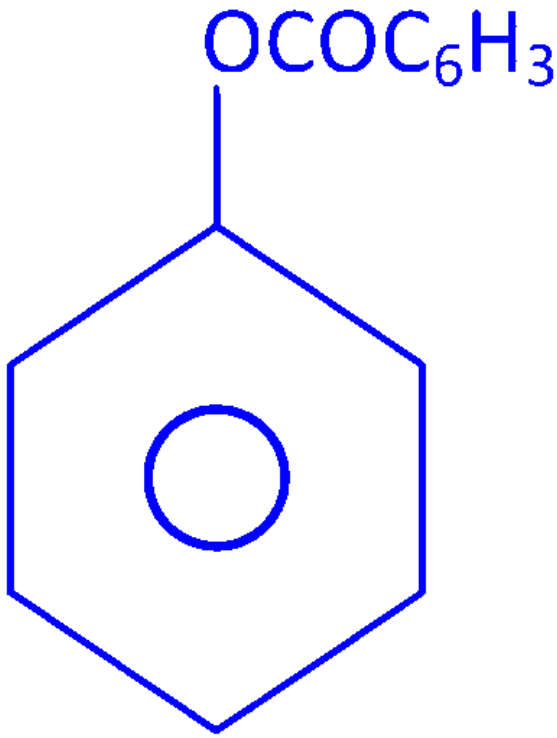
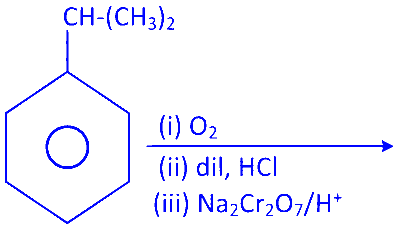
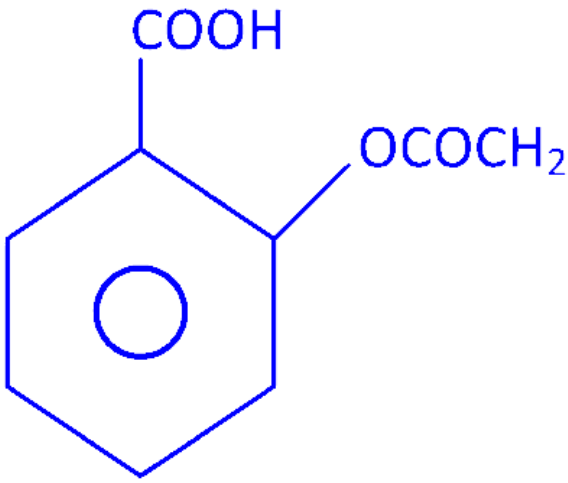
• I–I is strongest among these or gives endothermic substitution → effectively unreactive

Reaction enthalpy with a C–H bond becomes less exothermic down the group, so reactivity decreases in the same order.

Question19

Match the reactions given in Column I with the major product formed given in Column II.

S.No.	Reactions	S.No.	Major product formed
A.		P.	
B.		Q.	

S.No.	Reactions	S.No.	Major product formed
C.		R.	
D.		S.	

Options:

A. $A = Q \quad B = R \quad C = S \quad D = P$

B. $A = Q \quad B = P \quad C = S \quad D = R$

C. $A = R \quad B = S \quad C = Q \quad D = P$

D. $A = S \quad B = R \quad C = Q \quad D = P$

Answer: A

Question20

The number of grams of bromine that will completely react with 5 g of pentene is : [Atomic mass of $\mathrm{Br} = 80\mathrm{u}$]

Options:

A. 22.4 g

B. 8.4 g

C. 11.4 g

D. 54.2 g

Answer: C

Solution:

Step 1: Find the molar mass of pentene (C_5H_{10}).

Each carbon (C) has a mass of 12, so 5 × 12 = 60.

Each hydrogen (H) has a mass of 1, so 10 × 1 = 10.

Add both: 60 + 10 = 70 g/mol.

Step 2: Find the molar mass of bromine (Br_2).

Each bromine (Br) atom has a mass of 80, so 2 × 80 = 160 g/mol.

Step 3: Write the chemical reaction.

$C_5H_{10} + Br_2 \rightarrow C_5H_{10}Br_2$

Step 4: Use the reaction to make a proportion.

70 grams of pentene react with 160 grams of bromine.

Step 5: Set up the ratio for 5 grams of pentene.

For 5 g: $\frac{160}{70} \times 5 = \frac{800}{70} \approx 11.43$ g of Br_2 reacts.

Final Answer: 11.4 g (Option C).

Question21

Two statements, one Assertion and the other Reason are given. Choose the right option.

Assertion : Insulin is called a protein whereas Glycyl alanine is not called a protein

Reason : A polypeptide with amino acid residue less than 100 can also be called as a protein if it has a well-defined conformation of a protein.

Options:

A. Assertion is correct but Reason is incorrect

B. Both Assertion and Reason are incorrect

C. Both Assertion and Reason are correct

D. Assertion is incorrect but Reason is correct

Answer: C

Solution:

Option C

Both the Assertion and the Reason are correct, and the Reason correctly explains the Assertion.

• Insulin is a 51-residue polypeptide with a well-defined tertiary structure and biological function, so it's classified as a protein.

• Glycyl-alanine is only a dipeptide (2 residues), too short to fold into a stable, functional protein conformation.

• In general, any polypeptide—no matter how short—that adopts a specific three-dimensional structure and has a defined function is called a protein, even if it has fewer than 100 amino acids.

Question22

Choose the correct metal/ ion from the brackets which ----------

A. has chemical reactivity similar to that of the first few members of the Lanthanoids $(\mathrm{Zn}, \mathrm{Ca}, \mathrm{Fe}, \mathrm{Cu})$.

B. has stable $4\mathrm{f}^7$ electronic configuration, but acts as a strong reducing agent and converts to M^{3+} state. $\left(\mathrm{Eu}^{2+}, \mathrm{Ce}^{2+}, \mathrm{Pr}^{2+}, \mathrm{Dy}^{2+}\right)$

C. is a colorless ion $\left(\mathrm{Tm}^{3+}, \mathrm{Lu}^{3+}, \mathrm{Gd}^{3+}, \mathrm{Sm}^{3+}\right)$.

D. shows stable +2 oxidation state and is diamagnetic ($\mathrm{Ce}, \mathrm{Sm}, \mathrm{Ho}, \mathrm{Yb}$)

Options:

A. A: Cu B: Dy^{2+} C: Sm^{3+} D: Ho

B. A : Zn B: Ce^{2+}, C: Gd^{3+} D: Sm

C. A: Fe B: Pr^{2+} C: Tm^{3+} D: Ce

D. A: Ca B: Eu^{2+} C: Lu^{3+} D : Yb

Answer: D

Solution:

The only fully consistent choice is **Option D**:

A. Ca

– Lanthanoids are very electropositive, reactive metals; Ca behaves most like them.

B. Eu^{2+}

– Eu^{2+} has a half-filled $4f^7$ shell and readily oxidizes to Eu^{3+}.

C. Lu^{3+}

– With a full $4f^{14}$ configuration, Lu^{3+} has no $f \rightarrow f$ transitions in the visible and is colorless.

D. Yb^{2+}

– Yb^{2+} is $4f^{14}$ (fully paired electrons) and so is diamagnetic.

Question23

Which of the following alkene on reductive ozonolysis gives ketones only as the product

Options:

A. 1, 2-butadiene

B. 1, 4-cyclohexadiene

C. But-2-ene

D. 2,3-dimethylbut-2-ene

Answer: D

Solution:

Reductive ozonolysis converts each C=C into two C=O fragments. If a vinylic carbon has no H's (i.e. it's disubstituted by two C-groups), it gives a ketone; if it has one H, it gives an aldehyde.

Let's check each option:

1,2-Butadiene (CH_2=C=CH–CH_3)

– The terminal C (CH_2) has two H's → formaldehyde.

– The internal C's have one or zero H's → mixture of aldehydes/ketones.

1,4-Cyclohexadiene

– Each double bond carbon is CH– (one H, one ring C) → all aldehydes.

But-2-ene (CH_3–CH=CH–CH_3)

– Each vinylic C is CH (one H, one CH_3) → two acetaldehyde molecules.

2,3-Dimethylbut-2-ene

Structure:

CH_3–C*(CH_3)*=*C*(CH_3)–CH_3

– Each starred C has two alkyl substituents (no H) → ketones only.

– In fact:

$(CH_3)_2C=C(CH_3)_2 \xrightarrow{O_3/Zn} 2\,CH_3COCH_3$

(two molecules of acetone)

Answer: Option D.

Question24

The oxidation number of potassium in K_2O, K_2O_2 and KO_2 respectively are:

Options:

A. $+1, +2, +4$

B. $+1, +2, +1$

C. $+1, +1, +1$

D. $+2, +1, +1$

Answer: C

Solution:

Let's assign the usual oxidation state of oxygen in each compound and solve for K:

K_2O

• O is –2

• 2·(K) + (–2) = 0 ⇒ 2K = 2 ⇒ K = +1

K_2O_2 (peroxide)

• In a peroxide each O is –1

• 2·(K) + 2·(–1) = 0 ⇒ 2K = 2 ⇒ K = +1

KO_2 (superoxide)

• The O_2^- unit has overall charge –1, so total O oxidation = –1

• K + (–1) = 0 ⇒ K = +1

So the oxidation numbers of K are +1, +1 and +1 respectively. That's option C.

Question25

For a cell $2M_{(S)} + O_2(g) + 4H^+ \rightarrow 2M^{2+}(aq) + 2H_2O_{(l)}$ the E^0 cell $= +1.67\ V$. When $[M^{2+}]$ is $1.0 \times 10^{-3} M$ and $p(O_2)$ is 0.1 atm , the EMF of

the cell becomes +1.57 V . Calculate the pH of the electrochemical cell.

Options:

A. 3.49

B. 2.95

C. 5.24

D. 12.01

Answer: B

Solution:

First, note that in the overall reaction

2 M(s) + O_2(g) + 4 H^+ → 2 M^{2+}(aq) + 2 H_2O(l)

the number of electrons transferred is n = 4.

Write the reaction quotient Q (solids and pure liquids = 1):

$$Q = \frac{[\mathrm{M^{2+}}]^2}{p(\mathrm{O_2})\,[\mathrm{H^+}]^4}$$

Apply the Nernst equation at 25 °C:

$$E = E^0 - \frac{RT}{nF}\ln Q = E^0 - \frac{0.05916}{n}\log_{10} Q = E^0 - \frac{0.05916}{4}\log_{10}\frac{[\mathrm{M^{2+}}]^2}{p(\mathrm{O_2})\,[\mathrm{H^+}]^4}$$

Insert the given values (E = +1.57 V, E^0 = +1.67 V, $[M^{2+}] = 1.0\times10^{-3}$ M, $p(O_2)$ = 0.1 atm):

$$1.57 = 1.67 - \frac{0.05916}{4}\log_{10}\frac{(1.0\times10^{-3})^2}{0.1\,[\mathrm{H^+}]^4}$$

so

$$1.67 - 1.57 = 0.01479\ \log_{10}\frac{10^{-6}}{0.1\,[\mathrm{H^+}]^4}$$

$$0.10 = 0.01479\left(\log_{10}10^{-6} - \log_{10}0.1 - 4\log_{10}[\mathrm{H^+}]\right)$$

$$0.10 = 0.01479\left(-6 - (-1) - 4\log_{10}[\mathrm{H^+}]\right) = 0.01479\left(-5 + 4\,pH\right)$$

Solve for pH:

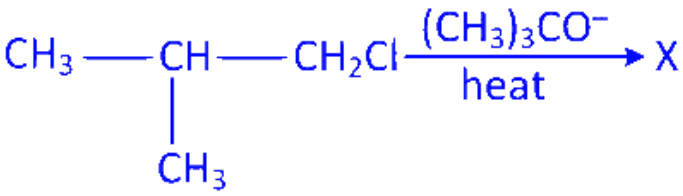
$$\frac{0.10}{0.01479} = -5 + 4\,pH \quad\Longrightarrow\quad 6.76 = -5 + 4\,pH \quad\Longrightarrow\quad pH = \frac{6.76+5}{4} = 2.94$$

Rounded to two decimal places, pH ≃ 2.95.

Answer: Option B (2.95).

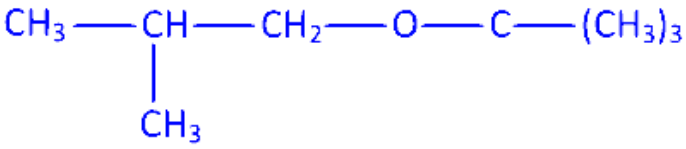
Question26

What would be the major product formed in the reaction?

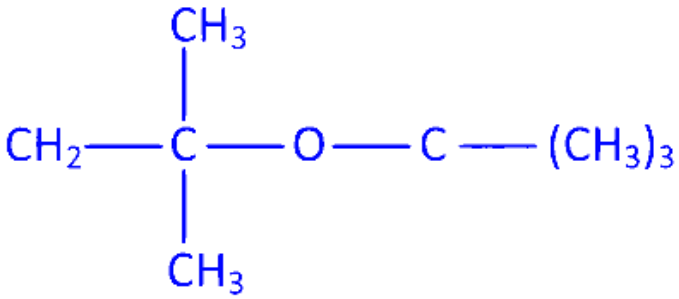


Options:

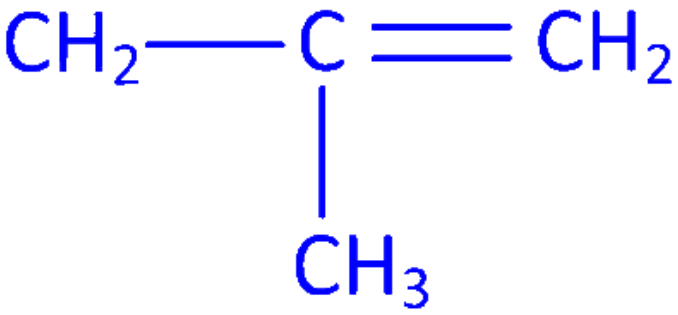
A.



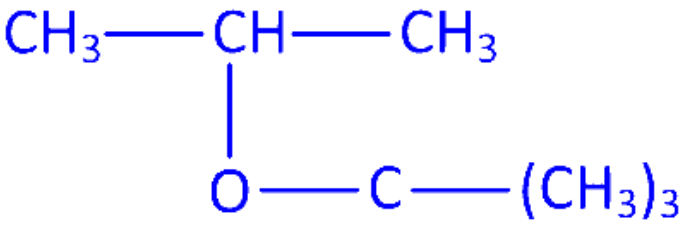
B.



C.



D.



Answer: C

Solution:

The substrate is **2-methyl-1-chloropropane** (isobutyl chloride) reacting with **potassium tert-butoxide** $(CH_3)_3CO^-$ and heat.

- $(CH_3)_3CO^-$ is a **strong, bulky base** → it favors **E2 elimination** over S_N2 substitution.
- In E2, the base abstracts a β-hydrogen while the leaving group (Cl^-) departs, forming a double bond.
- The carbon bearing Cl is primary (CH_2Cl); the only β-carbon is the adjacent **secondary** carbon (CH), which has a β-H.

Removing that β-H and Cl gives the alkene:

$$CH_3 - CH(CH_3) - CH_2Cl \xrightarrow[(CH_3)_3CO^-]{\Delta} (CH_3)_2C = CH_2$$

This is **2-methylpropene (isobutene)**, which corresponds to **option C**.

Question27

When $0.1\ \text{mol L}^{-1}$ of KCl was filled in a Conductivity cell the resistance was 80 ohms at 298 K . (Conductivity of 0.1 M KCl at 298 K is $1.29\ \text{S m}^{-1}$. The same cell when filled with an unknown electrolyte of concentration 0.025 M , had a resistance of 92 ohms. What is the Molar conductivity of the electrolyte at the given concentration?

Options:

A. $220.6\ \text{S cm}^2\ \text{mol}^{-1}$

B. $0.2206\ \text{S cm}^2\ \text{mol}^{-1}$

C. $448\ \text{S cm}^2\ \text{mol}^{-1}$

D. $0.449\ \text{S cm}^2\ \text{mol}^{-1}$

Answer: C

Solution:

Determine the cell constant K.

For 0.1 M KCl:

Resistance $R_1 = 80\ \Omega$

Conductivity $\sigma_1 = 1.29\ \text{S/m}$

Since

$R = \frac{K}{\sigma} \quad \Longrightarrow \quad K = R\sigma$

we get

$K = 80 \times 1.29 = 103.2\ \text{m}^{-1}$.

Find the conductivity of the unknown.

Measured resistance $R_2 = 92\ \Omega$

$\sigma_2 = \frac{K}{R_2} = \frac{103.2}{92} = 1.1217\ \mathrm{S/m}.$

Compute molar conductivity Λ_m.

Concentration $c = 0.025\ \mathrm{mol/L} = 25\ \mathrm{mol/m^3}$.

$\Lambda_m = \frac{\sigma_2}{c} = \frac{1.1217}{25} = 0.04487\ \mathrm{S\ m^2/mol}.$

To convert to $\mathrm{S\ cm^2/mol}$, multiply by 10^4:

$\Lambda_m = 0.04487 \times 10^4 = 448.7\ \mathrm{S\ cm^2/mol}.$

That rounds to about **448 S cm² mol⁻¹**, which is Option C.

Question28

An optically active compound $[\mathrm{X}]$ (Molecular formula $\mathrm{C_8H_{11}\ N}$) reacts with $\mathrm{CHCl_3}$ / Ethanolic KOH on heating to form an isocyanide. On reaction with $\mathrm{NaNO_2/HCl}$, it yields an alcohol with liberation of $\mathrm{N_2}$. Identify the compound $[\mathrm{X}]$.

Options:

A. 2-Ethylaniline.

B. 1-Phenylethanamine.

C. 3, 4-Dimethylaniline.

D. 2, 4-Dimethylaniline.

Answer: B

Solution:

Looking at the clues:

$C_8H_{11}N$ and optically active ⇒ one stereocentre

Gives positive carbylamine test ($CHCl_3$/KOH → R–NC) ⇒ primary amine

$R–NH_2 + HNO_2/HCl \rightarrow R–OH + N_2\uparrow$ ⇒ behaviour of a **primary aliphatic** amine

Only 1-phenylethanamine ($Ph–CH(NH_2)–CH_3$) fits all three:

It has the chiral centre at C–NH_2.

It's a primary aliphatic amine (so it gives an isocyanide with $CHCl_3$/KOH).

With $NaNO_2$/HCl it deaminates to $Ph–CH(OH)–CH_3 + N_2$.

Answer: 1-Phenylethanamine (Option B).

Question29

Choose the correct statement from the options given

Options:

A. The decreasing order of basic nature of the following amines is: Methylamine $>$ Dimethylamine $>$ Trimethylamine $>$ Aniline.

B. The intermolecular bonding in primary amines is stronger than in secondary amines.

C. Benzene diazonium chloride when reacted with Aniline in presence of dil. HCl at 273 K yields $C_6H_5 - N = N - NH - C_6H_5$

D. On heating an aliphatic primary amine with $CHCl_3$ in presence of Ethanolic KOH , a Nitrile is formed

Answer: B

Solution:

Option B

Primary amines (RNH_2) have two N–H bonds and can act as hydrogen-bond donors twice (and as acceptors), whereas secondary amines (R_2NH) have only one N–H. That extra H–bonding capability makes the intermolecular bonding in primary amines stronger than in secondary amines.

Question30

The number of electrons with azimuthal quantum numbers $l = 1$ and $l = 2$ for Cr in the ground state electronic configurations are respectively. [Given: $Cr(Z = 24)$]

Options:

A. 5 and 16

B. 12 and 5

C. 12 and 8

D. 12 and 16

Answer: B

Solution:

Chromium (Z = 24) has the ground-state configuration

Cr : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^5\,4s^1$.

• l = 0 (s-electrons): $1s^2+2s^2+3s^2+4s^1 \rightarrow$ 7 electrons

• l = 1 (p-electrons): $2p^6+3p^6 \rightarrow 6+6 = 12$ electrons

• l = 2 (d-electrons): $3d^5 \rightarrow$ 5 electrons

Hence, the numbers of electrons with

l = 1 and l = 2 are 12 and 5, respectively.

Answer: Option B.

Question31

The ratio of the difference between the radii of 3^{rd} and 4^{th} orbits of the He^{+} and those of Li^{2+} is:

Options:

A. $2 : 3$

B. $3 : 1$

C. $1 : 3$

D. $3 : 2$

Answer: D

Solution:

Let r_n be the radius of the n-th Bohr orbit in a hydrogen-like ion of charge Z. We have

$r_n = \frac{n^2 a_0}{Z}$,

where a_0 is the Bohr radius.

For He^{+} $(Z = 2)$

• $r_3 = \frac{9\,a_0}{2} = 4.5\,a_0$

• $r_4 = \frac{16\,a_0}{2} = 8\,a_0$

• $\Delta r_{He^{+}} = r_4 - r_3 = 8a_0 - 4.5a_0 = 3.5\,a_0$

For Li^{2+} $(Z = 3)$

• $r_3 = \frac{9\,a_0}{3} = 3\,a_0$

• $r_4 = \frac{16\,a_0}{3} = \frac{16}{3}\,a_0$

• $\Delta r_{Li^{2+}} = r_4 - r_3 = \frac{16}{3}a_0 - 3a_0 = \frac{7}{3}\,a_0$

Ratio

$\frac{\Delta r_{He^{+}}}{\Delta r_{Li^{2+}}} = \frac{3.5\,a_0}{\frac{7}{3}\,a_0} = \frac{\frac{7}{2}}{\frac{7}{3}} = \frac{7}{2} \times \frac{3}{7} = \frac{3}{2} = 3 : 2.$

Answer: Option D (3 : 2).

Question32

Consider the reaction

$$\mathrm{Fe_2O_3(\ s) + 3CO(\ g) \rightleftharpoons 2Fe(l) + 3CQ_2(\ g)}$$

In accordance with Le-Chatlier's principle, which of the following will not disturbs the equilibrium?

Options:

A. Removal of CO_2

B. Addition of CO

C. Removal of CO

D. Addition of Fe_2O_3

Answer: D

Solution:

The equilibrium constant (in terms of partial pressures) is

$K_p = \frac{(P_{CO_2})^3}{(P_{CO})^3}$

Notice that neither the solid Fe_2O_3 nor the liquid Fe appears in K_p—their activities are taken as unity. By Le Chatelier's principle:

Changing the pressures of the gases (CO or CO_2) will shift the equilibrium.

Changing the amount of a pure solid (or liquid) does **not** affect K_p or the position of the equilibrium.

Among the options, only "Addition of Fe_2O_3" won't disturb the equilibrium.

Answer: D

Question33

The electronic configuration of the element with atomic number 78 is :

Options:

A. $[Xe]4f^{14}5d^96s^1$

B. $[Kr]4f^{14}5d^86s^2$

C. $[\mathrm{Xe}]4f^{14}5d^{10}$

D. $[Kr]4f^{14}5d^{10}$

Answer: A

Solution:

Platinum (Z = 78) is one of the d-block exceptions that shifts an electron from 6s into 5d for extra stability. Its ground-state configuration is

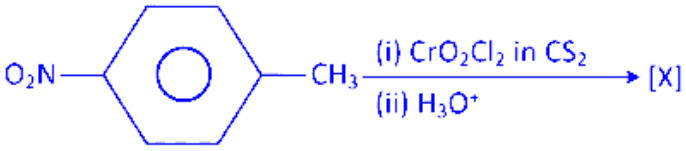
Option A

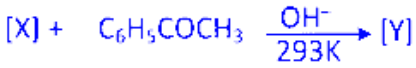
$[\mathrm{Xe}]\, 4f^{14}\, 5d^{9}\, 6s^{1}$

– the half-filled 6s and nearly filled 5d give a more stable arrangement than [Xe] $4f^{14}\ 5d^{8}\ 6s^{2}$.

Question34

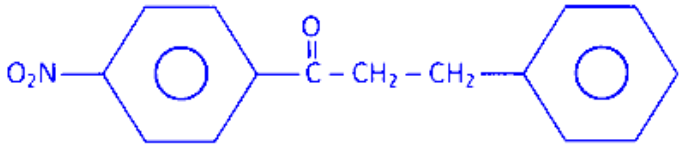
Identify the final product $[\mathrm{Y}]$ in the following reaction.



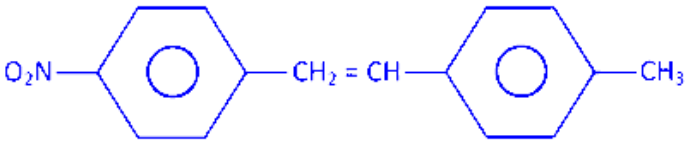


Options:

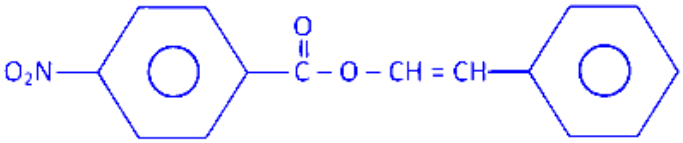
A.



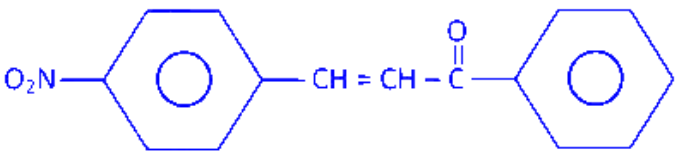
B.



C.



D.



Answer: D

Solution:

1. **First step: Oxidation of the side-chain methyl group**

The starting compound is **p-nitrotoluene**.

Reagents:

- **(i) CrO_2Cl_2 in CS_2**
- **(ii) H_3O^+**

These conditions oxidize an *aromatic side-chain methyl group (–CH_3)* **all the way to the carboxylic acid**.

$$\text{p-NO}_2\text{–C}_6\text{H}_4\text{–CH}_3 \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CrO_2Cl_2} \text{p-NO}_2\text{–C}_6\text{H}_4\text{–COOH}$$

So,

$$[X] = \text{p-nitrobenzoic acid.}$$

---

2. **Second step: Reaction with acetophenone under OH^-**

$$[X] + C_6H_5COCH_3 \xrightarrow[293\ K]{OH^-} [Y]$$

This is a **Claisen–Schmidt condensation** (a type of aldol condensation), between:

- The **acid derivative** (after forming carboxylate) and
- **Acetophenone** (which provides the enolate).

The reaction forms an **α,β-unsaturated carbonyl compound** attached to the aromatic ring of the carboxylic acid.

Question35

Choose the incorrect statement.

Options:

A. Cresols are less acidic than Phenol because electron releasing groups do not favour formation of Phenoxide ion.

B. Ethanol acts as Nucleophile when the $O-H$ bond is broken and acts as Electrophile on getting protonated.

C. In the acid catalysed hydration of Ethene, an Oxonium ion is formed when it reacts with $H_3O^+$and a Carbocation is formed when the Oxonium ion reacts with water.

D. In the reaction between Ethanol and Conc. H_2SO_4 at 413 K , an Oxonium ion is formed when Ethanol gets protonated.

Answer: C

Solution:

The incorrect statement is Option C.

Explanation:

In the acid-catalysed hydration of ethene the actual sequence is:

Protonation of ethene by H_3O^+ gives a carbocation:

$CH_2{=}CH_2 + H_3O^+ \longrightarrow CH_3{-}CH_2{}^+ + H_2O$

Nucleophilic attack of water on that carbocation produces an oxonium ion:

$CH_3{-}CH_2{}^+ + H_2O \longrightarrow CH_3{-}CH_2OH_2{}^+$

Deprotonation of the oxonium ion yields ethanol.

Option C reverses this order by claiming an oxonium ion forms first and then a carbocation forms when it reacts with water, which is not correct.

Question36

Find the correct matches.

A.	$Cr_2O_7^{2-}$	P.	is predominantly a basic compound.
B.	$FeCr_2O_4$	Q.	is predominantly a acidic compound
C.	CrO_3	R.	forms yellow chromate on reaction with Na_2CO_3 in air.
D.	CrO	S.	is interconvertible with MO_4^{2-} depending on the pH of the medium.

Options:

A. $A = R \quad B = S \quad C = P \quad D = Q$

B. $A = S \quad B = R \quad C = P \quad D = Q$

C. $A = R \quad B = Q \quad C = S \quad D = P$

D. $A = S \quad B = R \quad C = Q \quad D = P$

Answer: D

Solution:

The only consistent matching is

A → S

B → R

C → Q

D → P

so the correct choice is **D**.

Briefly:

• $Cr_2O_7^{2-} \rightleftharpoons 2\ CrO_4^{2-} + 2\ H^+$ (pH-dependent interconversion) → S

• $FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow Na_2CrO_4$ (yellow chromate) → R

• CrO_3 is a strongly acidic oxide → Q

• CrO is a basic oxide → P

Question37

Which of the following compounds has electrons symmetrically distributed in both $t_{2\ g}$ and e_g orbitals?

Options:

A. $[CoF_6]^{3-}$

B. $[Mn(CN)_6]^{4-}$

C. $[Cr(NH_3)_6]^{3+}$

D. $[FeCl_6]^{3-}$

Answer: D

Solution:

Let's work through the d-electron count and spin state for each:

$[CoF_6]^{3-}$

– Co^{3+} is d^6; F^- is a weak-field ligand ⇒ high-spin

– High-spin d^6:

$t_{2g}^4\, e_g^2$

– t_2g has one orbital doubly occupied ⇒ not symmetric

$[Mn(CN)_6]^{4-}$

– Mn^{2+} is d^5; CN^- is a strong-field ligand ⇒ low-spin

– Low-spin d^5:

$t_{2g}^5\, e_g^0$

– eg set empty (trivial symmetry) but t_2g is unevenly filled (2,2,1)

$[Cr(NH_3)_6]^{3+}$

– Cr^{3+} is d^3; regardless of field strength d^3 ⇒

$t_{2g}^3\, e_g^0$

– eg empty, only t_2g occupied

$[FeCl_6]^{3-}$

– Fe^{3+} is d^5; Cl^- is weak-field ⇒ high-spin

– High-spin d^5:

$t_{2g}^3\, e_g^2$

– t_2g^3 ⇒ one electron in each of the three t_2g orbitals

eg^2 ⇒ one electron in each of the two e_g orbitals

– both sets are equally (symmetrically) occupied

Answer: Option D, $[FeCl_6]^{3-}$.

Question38

Identify the two incorrect electrochemical reactions shown as taking place in the given cells at the respective electrodes.

Cell	Electrode	Reaction
Lead-Storage Cell	Cathode	$Pb + 2SO_4{}^{2-} + 4H^+ + 2e^- \rightarrow PbSO_4 + 2H_2O + SO_2$
Leclanche Cell	Cathode	$MnO_2 + NH_4^+ + 2e^- \rightarrow MnO + OH + NH_3$
Mercury Cell	Anode	$Zn/Hg + 2OH^- \rightarrow ZnO(s) + H_2O + 2e^-$
Fuel Cell	Cathode	$O_2(\ g) + 2H_2O + 4e^- \rightarrow 4OH^-$

Options:

A. C & D

B. A & C

C. B & D

D. A & B

Answer: D

Solution:

The two incorrect half-reactions are A and B.

Lead–acid (storage) cell, cathode (A)

Shown:

$Pb + 2\,SO_4^{2-} + 4\,H^+ + 2\,e^- \rightarrow PbSO_4 + 2\,H_2O + SO_2$

Correct cathode reaction on discharge is

$PbO_2 + 4\,H^+ + SO_4^{2-} + 2\,e^- \ \rightarrow\ PbSO_4 + 2\,H_2O$

Leclanché (dry) cell, cathode (B)

Shown:

$MnO_2 + NH_4^+ + 2\,e^- \rightarrow MnO + OH^- + NH_3$

In reality the reduction of MnO_2 in the presence of NH_4^+ is typically written as

$MnO_2 + NH_4^+ + e^- \rightarrow MnOOH + NH_3$

Options C (mercury cell anode) and D (alkaline fuel-cell cathode) are correctly written.

Answer: **A & B**.

Question39

Choose the correct statement.

Options:

A. The plot of $[A]$ versus time " t " for a zero order reaction is a horizontal line parallel to the x-axis which represents " t "

B. For a first order reaction the time taken for $3/4^{\text{th}}$ completion is 2 times the half-life period.

C. For a second order reaction the rate of reaction with respect to reactant becomes 27 times when its concentration is tripled.

D. Molecularity of a chemical reaction is always greater than order of the reaction.

Answer: B

Solution:

Option B is correct.

For a first-order reaction

The half-life is

$t_{1/2} = \frac{\ln 2}{k}$.

The time for ¾ completion (i.e. [A] drops to ¼ of its initial value) is

$t_{3/4} = \frac{\ln 4}{k} = \frac{2\ln 2}{k} = 2\,t_{1/2}$.

Question40

Consider the following reaction

$$\text{Propyne} + CH_3MgBr \xrightarrow{\text{dry ether}} \mathbf{X}$$

Identify ' X ' from the following.

Options:

A. But-2-yne

B. Propane

C. Methane

D. But-2-ene

Answer: C

Solution:

When a terminal alkyne meets a Grignard reagent, the strongly basic CH_3MgBr simply deprotonates it. In this case:

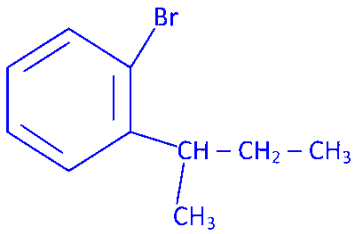
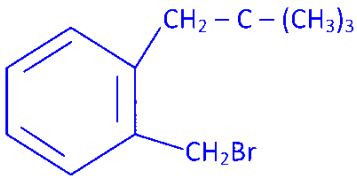
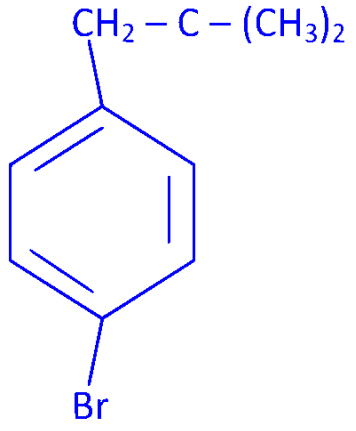
$CH_3–C \equiv CH \ + \ CH_3MgBr \ \longrightarrow \ CH_3–C \equiv C^-MgBr^+ \ + \ CH_4$

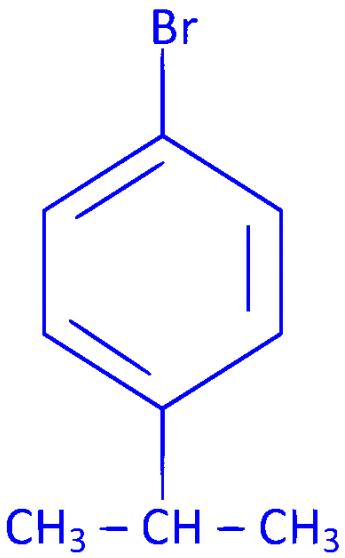
So the product X is CH_4, i.e. methane.

Answer: Option C (Methane).

Question41

Match the IUPAC names in Column II with the correct structures given in Column I.

	Structures of compounds		IUPAC name
A		P	1-Bromomethyl-3-(2,2-dimethylpropyl)benzene
B		Q	1-Bromo-4-(1-methylethyl)benzene
C		R	1-Bromo-2-(1-methylpropyl)benzene

	Structures of compounds		IUPAC name
D		S	1-Bromo-4-(2-methylpropyl)benzene

Options:

A. $\mathrm{A} = \mathrm{R} \quad \mathrm{B} = \mathrm{P} \quad \mathrm{C} = \mathrm{S} \quad \mathrm{D} = \mathrm{Q}$

B. $\mathrm{A} = \mathrm{S} \quad \mathrm{B} = \mathrm{R} \quad \mathrm{C} = \mathrm{Q} \quad \mathrm{D} = \mathrm{P}$

C. $\mathrm{A} = \mathrm{Q} \quad \mathrm{B} = \mathrm{P} \quad \mathrm{C} = \mathrm{S} \quad \mathrm{D} = \mathrm{R}$

D. $\mathrm{A} = \mathrm{S} \quad \mathrm{B} = \mathrm{R} \quad \mathrm{C} = \mathrm{P} \quad \mathrm{D} = \mathrm{Q}$

Answer: A

Question42

An organic compound $[\mathrm{X}]$ (Molecular formula $\mathrm{C_6H_{12}O_2}$) reacts with dil. $\mathrm{H_2SO_4}$ to form an alcohol [B] and a carboxylic acid [C]. Reaction of compound $[\mathrm{B}]$ with Jones reagent yielded compound $[\mathrm{C}]$. When compound $[\mathrm{C}]$ was heated with $\mathrm{P_2O_5}$ an Anhydride was formed. Compound $[\mathrm{X}]$ is ---------- --

Options:

A. $\mathrm{C_2H_5 - COO - (CH_2)_2 - CH_3}$

B. $\mathrm{CH_3 - COO - (CH_2)_3 - CH_3}$

C. $\mathrm{CH_3 - COO - CH_2 - CH - (CH_3)_2}$

D. $\mathrm{CH_3 - COO - C - (CH_3)_3}$

Answer: A

Solution:

The key steps are:

X is an ester, on acid-hydrolysis it gives

– a carboxylic acid C and

– an alcohol B.

B is oxidized by Jones to the same acid C ⇒ B must be a primary alcohol whose oxidation product is C.

Let C have n carbons ⇒ B has n carbons ⇒ total carbons in X = 2n = 6 ⇒ n = 3.

So C = propanoic acid (CH_3CH_2COOH) and B = propan-1-ol ($CH_3CH_2CH_2OH$).

X must be propyl propanoate:

$CH_3CH_2COOCH_2CH_2CH_3$

That corresponds to Option A.

Question43

In the decomposition of limestone to lime, the values of ΔH^o and ΔS^o are $+179.1\ \text{kJ mol}^{-1}$ and $160.2\ \text{J K}^{-1}\ \text{mol}^{-1}$. Calculate the temperature above which conversion of limestone to lime will be spontaneous, if values of ΔH^o and ΔS^o remain unchanged with temperature. [Assuming pressure 1 bar]

Options:

A. 1018 K

B. 1200 K

C. 1028 K

D. 1118 K

Answer: D

Solution:

To find the temperature above which the decomposition becomes spontaneous, set ΔG° = 0 and solve for T:

ΔG° = ΔH° – T ΔS°

0 = ΔH° – T ΔS° ⇒ T = ΔH°/ΔS°

Plugging in (in SI units):

• ΔH° = +179.1 kJ mol⁻¹ = 179 100 J mol⁻¹

• ΔS° = +160.2 J K⁻¹ mol⁻¹

$T = \frac{179\,100}{160.2} \approx 1\,118\ \text{K}$

So above about 1 118 K the reaction becomes spontaneous.

Answer: Option D (1 118 K).

Question44

The mole fraction of an unknown solute in 1560 g of Benzene is 0.5 . What is the molality of the solution? (M. M of Benzene:78 amu)

Options:

A. 12.8

B. 10.3

C. 3.25

D. 16.9

Answer: A

Solution:

Let $n_{(benzene)}$ be the moles of benzene and $n_{(solute)}$ the moles of solute.

Calculate moles of benzene:

$n_{benzene} = \frac{1560\text{ g}}{78\text{ g/mol}} = 20\text{ mol}$

Use the mole-fraction definition:

$X_{solute} = \frac{n_{solute}}{n_{solute}+n_{benzene}} = 0.5$

So

$n_{solute} = 0.5\,(n_{solute} + 20) \longrightarrow n_{solute} = 20\text{ mol}$

Molality (m) = moles of solute per kilogram of solvent:

$m = \frac{n_{solute}}{\text{kg of benzene}} = \frac{20\text{ mol}}{1.560\text{ kg}} \approx 12.8\text{ mol/kg}$

Answer: 12.8 (Option A)

Question45

Which of the coordination compounds [(i) to (v)] are used for the following processes :

A: Electroplating

B: Removal of excess copper from human body

C: Estimation of hardness of water

(i) DMG (ii) Na_2EDTA (iii) Cis-platin (iv) D-penicillamine (v) $K\left[Au(CN)_2\right]$

Options:

A. $\mathrm{A} = \mathrm{iv} \quad \mathrm{B} = \mathrm{i} \quad \mathrm{C} = \mathrm{iii}$

B. $A = i \quad B = iii \quad C = iv$

C. $\mathrm{A} = \mathrm{v} \quad \mathrm{B} = \mathrm{iv} \quad \mathrm{C} = \mathrm{ii}$

D. $A = \mathrm{iii} \quad B = \mathrm{ii} \quad C = i$

Answer: C

Solution:

Here's how they match up:

Electroplating

• Compound (v): $K[Au(CN)_2]$

Removal of excess copper from the body

• Compound (iv): D-penicillamine

Estimation of water hardness

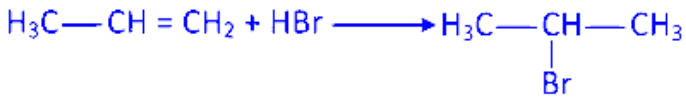
• Compound (ii): Na_2EDTA

So the correct choice is Option C:

A = v, B = iv, C = ii.

Question46

Identify the type of reaction:



Options:

A.

Electrophilic addition

B.

Free radical substitution

C.

Nucleophilic addition

D.

Free radical addition

Answer: A

Solution:

The reaction shown is:

$$CH_3-CH=CH_2+HBr \longrightarrow CH_3-CHBr-CH_3$$

This is the **addition of HBr to an alkene**.
Here's why the correct answer is **Electrophilic addition**:

Explanation

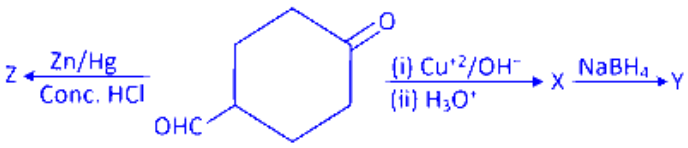
- Alkenes have a π-bond rich in electrons.
- **HBr** adds across the double bond.
- The **H^+ (electrophile)** attacks the electron-rich double bond first, forming a carbocation.
- Then **Br^- (nucleophile)** attacks the carbocation, giving the final bromoalkane.

This mechanism is the classic **electrophilic addition** mechanism of hydrogen halides to alkenes (Markovnikov addition).

✅ **Correct Answer: A. Electrophilic addition**

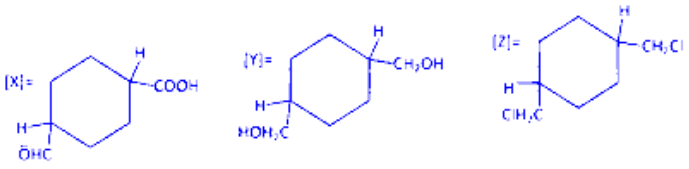
Question47

Identify X, Y and Z formed in the reaction:

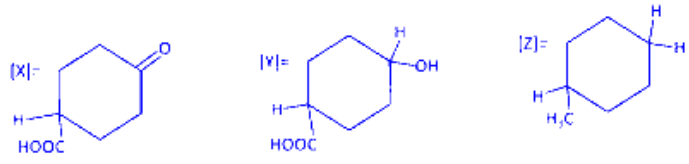


Options:

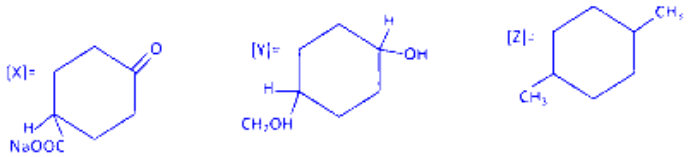
A.



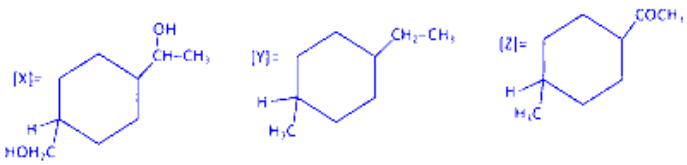
B.



C.



D.



Answer: B

Solution:

Solution (Explained Step-by-Step)

The given reaction sequence is:

1. **Starting compound:** A cyclohexane ring bearing **one aldehyde (–CHO)** and **one ketone (C=O)** group.
2. **Step (i): Cu^{2+} / OH^- oxidation**
   - Aldehydes are oxidized to **carboxylic acids**.
   - Ketones remain unchanged.
     → So **X** = cyclohexane ring with **carboxylic acid (–COOH)** and **ketone**.
3. **Step (ii): H_3O^+**
   - Just acid workup → no further structural change.
     → X stays the same.
4. **Next step: $NaBH_4$ reduction**
   - $NaBH_4$ reduces **ketones to alcohols**, but **does NOT reduce carboxylic acids**.
     → So **Y** = cyclohexane ring with **COOH** and **–OH** (from reduction of ketone).
5. **Final step: Zn/Hg, conc. HCl (Clemmensen reduction)**
   - Clemmensen reduces **carbonyl groups**, but **does NOT reduce COOH**.
   - Since Y has a **COOH** and an **alcohol**, the alcohol gets reduced to **–CH_2–**.
     → So **Z** = cyclohexane ring with **COOH** only.

Matching with answer choices:

✓ Option B matches X, Y, and Z exactly.

✅ **Correct Answer: B**

Question48

Which of the following is a water soluble vitamin?

Options:

A. Vitamin C

B. Vitamin E

C. Vitamin A

D. Vitamin D

Answer: A

Solution:

Option A – Vitamin C is water-soluble.

Explanation:

• Water-soluble vitamins dissolve in water and include the B-complex vitamins and vitamin C.

• Vitamins A, D, E (and K) are fat-soluble.

Question49

At 300 K the vapour pressure of an ideal solution containing 1.0 mole each of volatile liquids X and Y is 1000 mm . Keeping the temperature constant, when 2.0 moles of liquid X is added to the solution, its vapour pressure increases by 200 mm . Calculate the vapour pressure of X and Y in their pure state

Options:

A. $P_X^0 = 1400 \quad P_Y^0 = 600$

B. $P_X^0 = 1700 \quad P_Y^0 = 400$

C. $P_X^0 = 1200 \quad P_Y^0 = 480$

D. $P_X^0 = 1000 \quad P_Y^0 = 500$

Answer: A

Solution:

Let's apply Raoult's law in the two cases:

First, with 1 mol X and 1 mol Y

• Mole fractions:

$x_X = \frac{1}{1+1} = 0.5, \quad x_Y = 0.5$

• Total pressure:

$P_1 = x_X P_X^0 + x_Y P_Y^0 = 0.5 P_X^0 + 0.5 P_Y^0 = 1000$

⇒

$P_X^0 + P_Y^0 = 2000 \qquad (1)$

After adding 2 mol X (now 3 mol X, 1 mol Y)

• Mole fractions:

$x_X = \frac{3}{3+1} = 0.75, \quad x_Y = 0.25$

• Total pressure:

$P_2 = 0.75P_X^0 + 0.25P_Y^0 = 1000 + 200 = 1200$

⇒ multiply by 4:

$3P_X^0 + P_Y^0 = 4800 \qquad (2)$

Solve (1) and (2):

Subtract (1) from (2):

$3P_X^0 + P_Y^0 \; - \; (P_X^0 + P_Y^0) = 4800 - 2000$

$2P_X^0 = 2800 \implies P_X^0 = 1400 \text{ mm}$

$P_Y^0 = 2000 - 1400 = 600 \text{ mm}$

Answer:

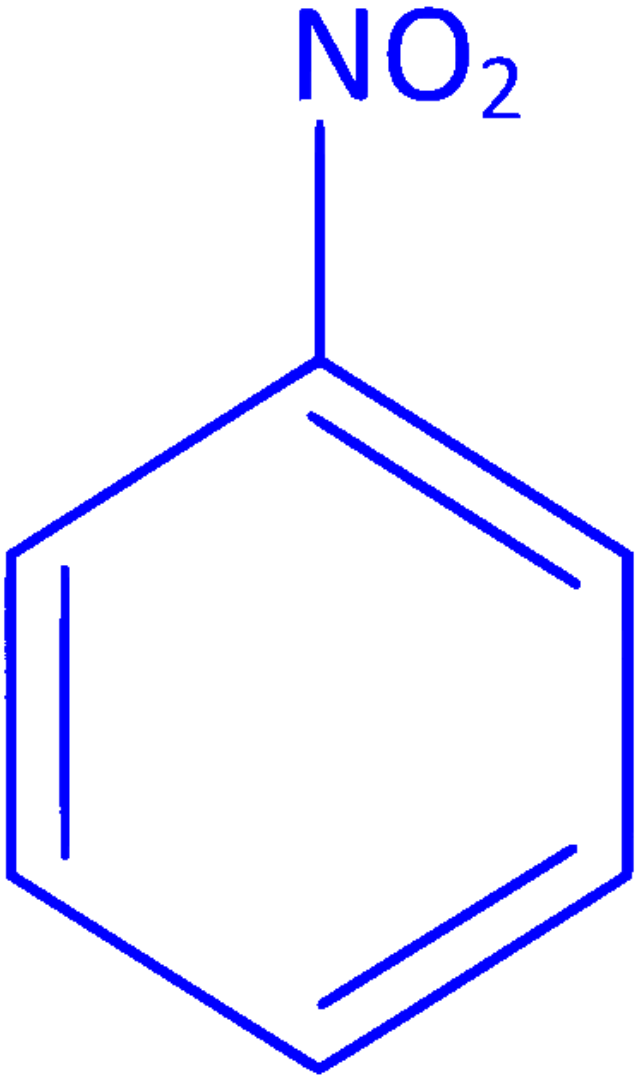
P_X^0=1400 mm, P_Y^0=600 mm (Option A)

Question50

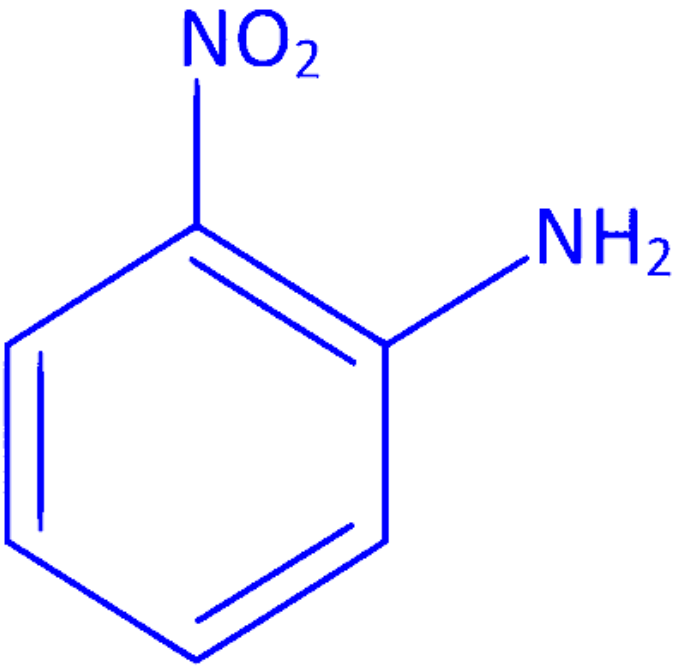
The maximum polarity and dipole moment among the following is :

Options:

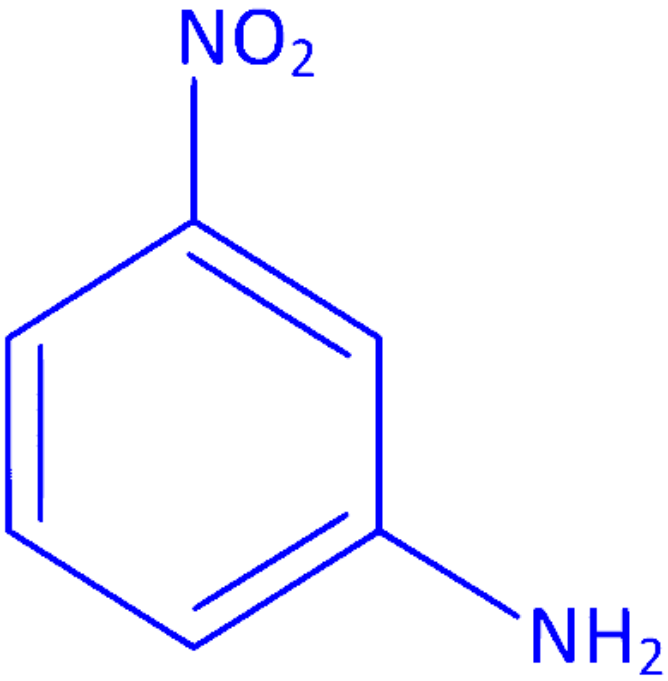
A.



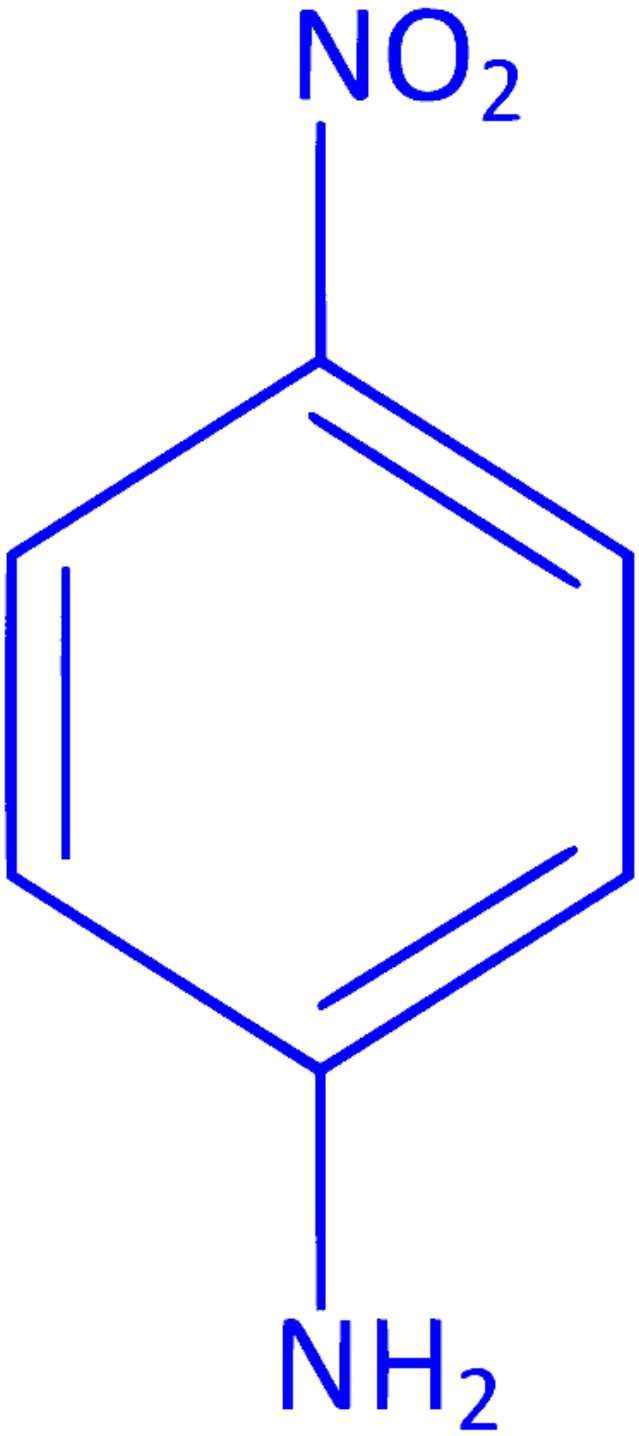
B.



C.



D.



Answer: D

Solution:

The question asks which of the given nitro–aniline substituted benzenes has the **maximum polarity and dipole moment**.

We compare dipole moments by looking at **vector addition** of the substituent dipoles:

- **NO_2** group → strong **–M and –I** withdrawing, dipole vector **toward NO_2**
- **NH_2** group → strong **+M donating**, **–I weakly withdrawing**, dipole vector generally **toward NH_2**
- Overall molecule → result of vector addition of these two dipoles

Case-by-case

A: Nitrobenzene

Only one substituent → moderate dipole moment (~4.0 D).
But others with two substituents may have higher dipole due to vector sum.

B: o-nitroaniline (ortho-5 position shown)

NO_2 and NH_2 are close, but their dipole vectors are not colinear.
The angle reduces the net dipole—**not maximum**.

C: m-nitroaniline (meta-position)

Meta position often leads to some cancellation because resonance donation does not apply strongly at meta.
Dipole moment moderate.

D: p-nitroaniline (para-position)

Here the **NO_2 and NH_2 groups lie directly opposite each other**.

- Both groups create dipoles **along the same axis**
- Their effects **add up strongly**, because
  - NO_2 pulls electron density strongly
  - NH_2 pushes electron density
  - This leads to a **large charge separation across the benzene ring**

Therefore, the **net dipole moment is maximum** in **para-nitroaniline** among the given options.

Final Answer: ✓ D (para-nitroaniline)

Question51

During a chemical reaction $\mathrm{X} \rightarrow \mathrm{Y}$, the rates of reaction starting with initial concentrations of X as $4.0 \times 10^{-3} \mathrm{M}$ and $2.0 \times 10^{-3} \mathrm{M}$ are $4.8 \times 10^{-4}\ \mathrm{mol\ L^{-1}/s}$ and $1.2 \times 10^{-4}\ \mathrm{mol\ L^{-1}/s}$ respectively. What is the order of reaction with respect to X ?

Options:

A. 2

B. 3

C. 1.5

D. 1

Answer: A

Solution:

Let the rate law be

rate $= k\,[X]^n$.

From the two experiments:

• Experiment 1: $[X]_1 = 4.0 \times 10^{-3}\,M,\ \text{rate}_1 = 4.8 \times 10^{-4}$

• Experiment 2: $[X]_2 = 2.0 \times 10^{-3}\,M,\ \text{rate}_2 = 1.2 \times 10^{-4}$

Taking the ratio of rates gives

$\frac{\text{rate}_1}{\text{rate}_2} = \frac{k\,[X]_1^n}{k\,[X]_2^n} = \left(\frac{[X]_1}{[X]_2}\right)^n.$

Plugging in the numbers:

$\frac{4.8\times10^{-4}}{1.2\times10^{-4}} = \left(\frac{4.0\times10^{-3}}{2.0\times10^{-3}}\right)^n \implies 4 = 2^n \implies n = 2.$

Answer: Option A (second order).

Question52

Identify the Carbonyl compound which will not be formed when hydration of Alkynes is carried out with dil. H_2SO_4/Hg^{2+} at 333 K .

Acetone, Butanal, Ethanal, Butanone.

Options:

A. Acetone

B. Butanone

C. Butanal

D. Ethanal

Answer: C

Solution:

Hydration of alkynes in the presence of dilute H_2SO_4/Hg^{2+} proceeds via Markovnikov addition to give an enol, which quickly tautomerizes to a ketone (or, in the special case of acetylene, to acetaldehyde). In particular:

Terminal alkyne general reaction

$R\text{–}C \equiv C\text{–}H + H_2O \xrightarrow{H_2SO_4,\ Hg^{2+}} R\text{–}CO\text{–}CH_3$

Examples

• Acetylene (R=H)

$H-C\equiv C-H \xrightarrow{H_2O} CH_3CHO$

(ethanal)

• 1-Propyne (R=CH3)

$CH_3-C\equiv C-H \xrightarrow{H_2O} CH_3COCH_3$

(acetone)

• 1-Butyne (R=CH2CH3)

$CH_3CH_2-C\equiv C-H \xrightarrow{H_2O} CH_3CH_2COCH_3$

(butanone)

Notice that no pathway leads to an aldehyde with its –CHO group at the end of a longer chain (butanal, $CH_3CH_2CH_2CHO$). All terminal alkynes give methyl ketones (or ethanal in the acetylene case), and internal alkynes give ketones only.

Answer: Option C) Butanal.

Question53

Choose the incorrect statement from the following.

Options:

A. The energy difference between the staggered and eclipsed conformation of ethane is $12.55\ \mathrm{kJ/mol}$

B. Staggered conformation of ethane is more stable than the eclipsed conformation.

C. The distance between non-bonded H -atoms of ethane in eclipsed conformation is $2.29\overset{o}{A}$

D. A 90° rotation converts staggered conformation of ethane into eclipsed conformation.

Answer: D

Solution:

The incorrect statement is Option D.

• In ethane a staggered → eclipsed interconversion requires a 60° rotation, not 90°.

• All the others are true:

– ΔE ≃12.6 kJ/mol between staggered and eclipsed.

– The staggered form is more stable.

– The eclipsed H···H separation is about 2.27 Å (≈2.29 Å).

Question54

Identify $[\mathrm{X}]$ used in the given reaction.

$$[\mathrm{X}] + \textbf{Copper}\ /573\ \mathrm{K} \rightarrow [\mathrm{Y}]$$

$$[\mathrm{Y}] \xrightarrow[\text{(ii)}\quad H_2O/H^+]{\text{(i)}\quad CH_3MgBr} \textbf{Tert.butyl alcohol. (major product)}$$

Options:

A. Propan-1-ol

B. Butan-1-ol

C. Propan-2-ol

D. Butan-2-ol

Answer: C

Solution:

Here's how to pin down [X]:

Copper at 573 K is a classic dehydrogenation catalyst for alcohols, turning secondary alcohols into ketones.

$$CH_3\text{-}CHOH\text{-}CH_3 \xrightarrow{Cu\ /573\ K} CH_3\text{-}CO\text{-}CH_3 + H_2$$

So [X] must be propan-2-ol and [Y] is acetone.

Acetone then reacts with methyl Grignard and water to give tert-butyl alcohol:

(i)

$$CH_3\text{-}CO\text{-}CH_3 + CH_3MgBr \longrightarrow (CH_3)_3C\text{-}OMgBr$$

(ii)

$$(CH_3)_3C\text{-}OMgBr + H_2O/H^+ \longrightarrow (CH_3)_3C\text{-}OH$$

Thus [X] = propan-2-ol (Option C).

Question55

Which one of the following is the product (Z) formed at the end of the given reaction:

$$\begin{aligned}&\text{Propan-1-ol} \ + \mathrm{SOCl_2} \rightarrow [\mathrm{X}]\\&[\mathrm{X}] + \mathrm{HC} \equiv \mathrm{C} - \mathrm{Na} \rightarrow [\mathrm{Y}]\\&[\mathrm{Y}] + \mathrm{H_2SO_4/HgSO_4}, 330\ \mathrm{K} \rightarrow [\mathrm{Z}]\end{aligned}$$

Options:

A. Pentan-2-ol.

B. Pent-2-ene.

C. Pentanal.

D. Pentan-2-one.

Answer: D

Solution:

Let’s follow the three steps:

$\mathrm{CH_3}CH_2CH_2OH + \mathrm{SOCl_2} \longrightarrow \mathrm{CH_3}CH_2CH_2Cl \quad (X)$

(1-Propanol → propyl chloride)

$\mathrm{CH_3}CH_2CH_2Cl + \mathrm{HC \equiv C{-}Na} \longrightarrow \mathrm{HC \equiv C{-}CH_2}CH_2CH_3 \quad (Y)$

(SN2 gives 1-pentyne)

Acid-catalyzed hydration (Markovnikov) with $HgSO_4$:

– Enol formation:

$\mathrm{HC \equiv C{-}(CH_2)_2}CH_3 + H_2O \longrightarrow CH_2 = C(OH)(CH_2)_2CH_3$

– Tautomerization to the ketone:

$CH_2 = C(OH)(CH_2)_2CH_3 \longrightarrow CH_3COCH_2CH_2CH_3$

That final product, $CH_3COCH_2CH_2CH_3$, is pentan-2-one.

Answer: Option D.

Question56

Two statements, one Assertion and the other Reason are given. Identify the correct option

Assertion : Primary and secondary amides on treatment with $\mathrm{Br_2}$ and alcoholic NaOH yield primary and secondary amines respectively and the reaction involves stepping down the series.

Reason : The reaction occurs due to the migration of alkyl group from Carbonyl carbon atom to the Nitrogen atom with elimination of carbonyl group as the carbonate salt.

Options:

A. Assertion is correct but Reason is incorrect.

B. Both Assertion and Reason are incorrect.

C. Both Assertion and Reason are correct.

D. Assertion is incorrect but Reason is correct.

Answer: D

Solution:

Assertion Analysis:

The assertion states that "Primary and secondary amides on treatment with $\mathrm{Br_2}$ and alcoholic NaOH yield primary and secondary amines respectively and the reaction involves stepping down the series."

Primary amides: It's true that primary amides (like $RCONH_2$) undergo the Hoffmann bromamide degradation (with Br_2 and alcoholic NaOH) to form primary amines (RNH_2). This reaction indeed involves "stepping down the series" because the product amine has one less carbon atom than the original amide. For example, propanamide (3 carbons) gives ethylamine (2 carbons).

Secondary amides: However, secondary amides (like RCONHR') **do not** undergo the Hoffmann bromamide degradation to yield secondary amines. This reaction is specific to primary amides.

Since the statement claims it works for *both* primary and secondary amides, the Assertion is incorrect.

Reason Analysis:

The reason states that "The reaction occurs due to the migration of alkyl group from Carbonyl carbon atom to the Nitrogen atom with elimination of carbonyl group as the carbonate salt."

This is a perfectly accurate description of the mechanism of the Hoffmann bromamide degradation.

Migration: The crucial step is indeed the migration of the alkyl (or aryl) group from the carbonyl carbon to the nitrogen atom.

Elimination of carbonyl group: The carbonyl carbon ($C=O$) is lost as a carbonate ion (CO_3^{2-}).

So, the Reason is correct.

Conclusion:

The Assertion is incorrect.

The Reason is correct.

Therefore, the correct option is:

Option D

Assertion is incorrect but Reason is correct.

Question57

The enthalpies of combustion of H_2, C (graphite) and $\mathrm{C}_2\mathrm{H}_6(\mathrm{~g})$ are $-286.0, -394.0$ and $-1560.0 \mathrm{~kJ~mol}^{-1}$ at $25^{\circ}\mathrm{C}$ and 1 atm pressure. The enthalpy of formation of ethane is :

Options:

A. $-97.0 \mathrm{~kJ~mol}^{-1}$

B. $-86.0 \mathrm{~kJ~mol}^{-1}$

C. $-92.0 \mathrm{~kJ~mol}^{-1}$

D. $-78.0 \mathrm{~kJ~mol}^{-1}$

Answer: B

Solution:

We seek the standard enthalpy of formation for

$$2\,\mathrm{C}(\mathit{graphite}) + 3\,\mathrm{H_2}(g) \;\longrightarrow\; \mathrm{C_2H_6}(g)\,, \quad \Delta H_f^\circ = ?$$

Given (all at 25 °C, 1 atm):

$\mathrm{H_2}(g) + \frac{1}{2}\,\mathrm{O_2}(g) \rightarrow \mathrm{H_2O}(l)\,,\ \Delta H = -286.0\ \mathrm{kJ/mol}$

$\mathrm{C}(graphite) + \mathrm{O_2}(g) \rightarrow \mathrm{CO_2}(g)\,,\ \Delta H = -394.0\ \mathrm{kJ/mol}$

$\mathrm{C_2H_6}(g) + \frac{7}{2}\,\mathrm{O_2}(g) \rightarrow 2\,\mathrm{CO_2}(g) + 3\,\mathrm{H_2O}(l)\,,\ \Delta H = -1560.0\ \mathrm{kJ/mol}$

By Hess's law, consider two pathways from elements to the same products (2 CO_2 + 3 H_2O):

A. Direct combustion of ethane:

$\mathrm{C_2H_6} + 3.5\,\mathrm{O_2} \rightarrow 2\,\mathrm{CO_2} + 3\,\mathrm{H_2O},\ \Delta H = -1560.0$

B. Form products from elements in two steps:

i) 2 C + 2 O_2 → 2 CO_2, ΔH = 2×(−394.0) = −788.0

ii) 3 H_2 + 1.5 O_2 → 3 H_2O, ΔH = 3×(−286.0) = −858.0

Total ΔH = −788.0 + (−858.0) = −1646.0

Now link A and B via the formation reaction (call its enthalpy x):

Step 1: 2 C + 3 H_2 → C_2H_6, ΔH = x

Step 2: C_2H_6 + 3.5 O_2 → 2 CO_2 + 3 H_2O, ΔH = −1560.0

Sum gives 2 C + 3 H_2 + 3.5 O_2 → 2 CO_2 + 3 H_2O with ΔH = x − 1560.0

But we also know from B that the same overall reaction has ΔH = −1646.0. Hence

x − 1560.0 = −1646.0

x = −1646.0 + 1560.0 = −86.0 kJ/mol

Answer: −86.0 kJ/mol (Option B).

Question58

Which of the following is the most stable free radical?

Options:

A. $(\mathrm{C_6H_5})_2\dot{\mathrm{C}}\mathrm{H}$

B. $\dot{\mathrm{C}}\mathrm{H_3}$

C. $(\mathrm{CH_3})_3\dot{\mathrm{C}}$

D. $\mathrm{CH_2} = \dot{\mathrm{C}}\mathrm{H}$

Answer: A

Solution:

The key to radical stability is how well the unpaired electron can be delocalized. Here's a quick comparison:

Methyl radical, $\dot{\mathrm{C}}\mathrm{H_3}$ (B)

– No resonance, only 3 hyperconjugative C–H bonds.

Vinyl radical, $CH_2 = \dot{C}H$ (D)

– Radical on an sp² carbon, very little stabilization.

Tertiary radical, $(CH_3)_3\dot{C}$ (C)

– Hyperconjugation from 9 C–H bonds, moderately stabilized.

Diphenylmethyl (benzhydryl) radical, $(C_6H_5)_2\dot{C}H$ (A)

– Unpaired electron is delocalized over two aromatic rings (resonance), giving the greatest stabilization.

Because resonance stabilization (especially over two phenyl rings) outweighs hyperconjugation, the most stable radical here is option A.

Question59

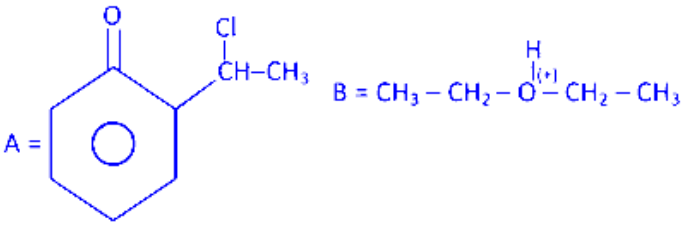
What are the intermediates formed during the reactions A and B ?

A. Reimer-Tiemann reaction.

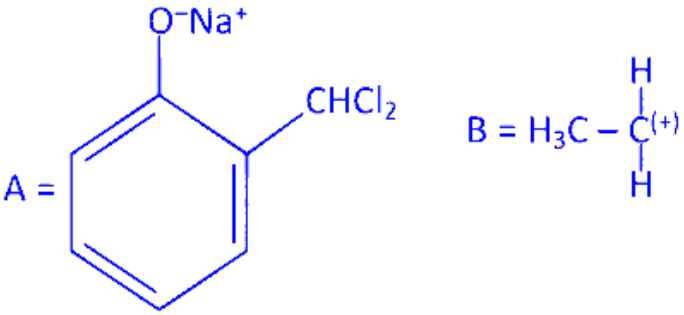
B. Dehydration of alcohols in the slow rate determining step.

Options:

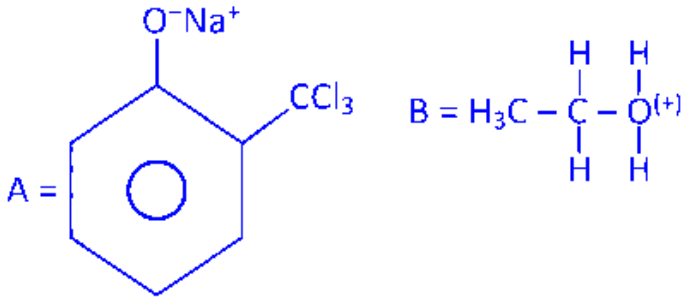
A.



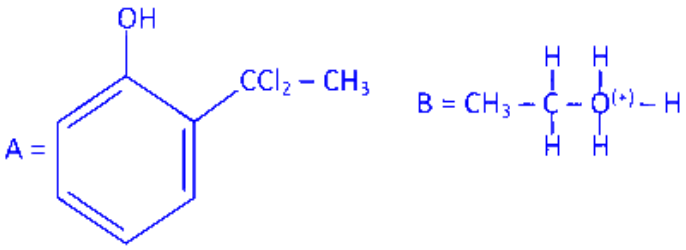
B.



C.



D.



Answer: B

Solution:

Reaction A – Reimer–Tiemann Reaction

In the Reimer–Tiemann reaction, **phenol** reacts with **chloroform ($CHCl_3$)** in the presence of **NaOH**.
The key **intermediate** formed is the **dichlorocarbene ($:CCl_2$)**, which then attacks the phenoxide ion to form:

Intermediate A:

Ortho-hydroxy benzylidene dichloride (O-$CHCl_2$ attached to phenoxide)
This corresponds to the structure shown in **Option B** (A = O^-Na^+ on phenyl, $CHCl_2$ substituent).

Reaction B – Dehydration of Alcohols (Slow Step)

During the acid-catalyzed dehydration of an alcohol, the **rate-determining step** is the formation of a **carbocation** after loss of water.

Intermediate B:

For a secondary alcohol, the slow step gives a **secondary carbocation (R–C^+H–R)**.
This also corresponds to **Option B**, where B is shown as a carbocation.

✔ **Correct Answer: Option B**

Question60

Arrange the following in the increasing order of their covalent character.

$CaF_2; CaCl_2; CaBr_2; CaI_2$

Options:

A. $CaF_2 < CaI_2 < CaBr_2 < CaCl_2$

B. $CaF_2 < CaCl_2 < CaI_2 < CaBr_2$

C. $CaCl_2 < CaF_2 < CaBr_2 < CaI_2$

D. $CaF_2 < CaCl_2 < CaBr_2 < CaI_2$

Answer: D

Solution:

According to Fajans' rules, the covalent character of an ionic compound increases when the anion is more easily polarizable. Keeping Ca^{2+} fixed, the halide polarizabilities increase down the group:

F^- (smallest, least polarizable)

Cl^-

Br^-

I^- (largest, most polarizable)

Hence the covalent character order (least → most) is:

$CaF_2 \ < \ CaCl_2 \ < \ CaBr_2 \ < \ CaI_2$

This corresponds to option D.

Physics

Question1

The interference pattern is obtained with two coherent light sources of intensity ratio $9:1$. The ratio of $\frac{I_{MAX}+I_{MIN}}{I_{MAX}-I_{MIN}}$ is $\frac{\alpha}{\beta}$. The values of α and β are:

Options:

A. 5 and 3

B. 3 and 1

C. 1 and 9

D. 9 and 1

Answer: A

Solution:

Let the two intensities be

$I_1 = 9I_0, \quad I_2 = 1I_0$

so the amplitudes are

$a_1 = \sqrt{9I_0} = 3\sqrt{I_0}, \quad a_2 = \sqrt{I_0}.$

Maximum intensity (in-phase):

$I_{\max} = (a_1 + a_2)^2 = (3\sqrt{I_0} + \sqrt{I_0})^2 = 16\, I_0.$

Minimum intensity (out-of-phase):

$I_{\min} = (a_1 - a_2)^2 = (3\sqrt{I_0} - \sqrt{I_0})^2 = 4\, I_0.$

The desired ratio is

$\frac{I_{\max}+I_{\min}}{I_{\max}-I_{\min}} = \frac{16I_0+4I_0}{16I_0-4I_0} = \frac{20}{12} = \frac{5}{3}.$

So $\alpha = 5$ and $\beta = 3$. (Option A)

Question2

If the mass numbers of two nuclei are in the ratio $5:2$ and their diameters are in ratio $2:6$. Then their nuclear densities will be in the ratio

Options:

A. $1:1$

B. $2:5$

C. $10:12$

D. $6:5$

Answer: A

Solution:

Let's denote the two nuclei by 1 and 2. Recall that

– Mass ∝ A (mass number)

– Volume ∝ R³ ∝ (Diameter)³

So

$\rho_1/\rho_2 \;=\; \frac{A_1/V_1}{A_2/V_2} \;=\; \frac{A_1/A_2}{(D_1/D_2)^3} \;=\; \frac{\frac{5}{2}}{(\frac{2}{6})^3} \;=\; \frac{5/2}{(1/3)^3} \;=\; (5/2)\times 27 \;=\; \frac{135}{2} \;\approx\; 67.5 : 1$

None of the given choices match 135 : 2, but in the liquid-drop model of the nucleus one shows that

$R \propto A^{1/3} \quad\Longrightarrow\quad \rho \;=\; \frac{A}{R^3} \;\approx\; \text{constant}$

for all nuclei. Hence to very good approximation

$\rho_1 : \rho_2 \;=\; 1:1$

Answer: Option A $(1:1)$.

Question3

Which of the following is correct in the case of the Bohr model of atoms?

A. Predicts continuous emission spectra for all atoms

B. Assumes that the angular momentum of electrons is quantised

C. Predicts same emission spectrum for singly ionised neon atom and hydrogen atom

D. Predicts same emission spectrum for singly ionised neon atom and singly ionised helium atom

Options:

A. C

B. B

C. A

D. D

Answer: B

Solution:

The only correct statement is B.

• A is false – Bohr's model predicts discrete (line) spectra, not continuous ones.

• B is true – it postulates

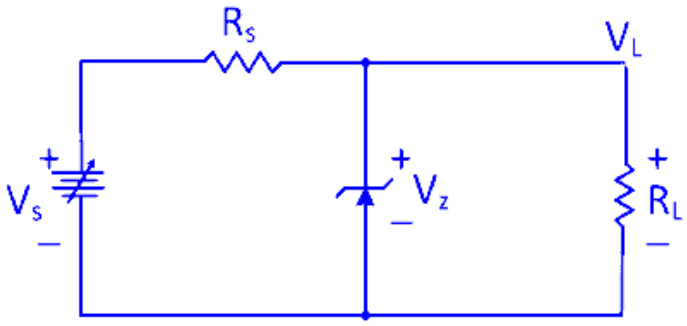
$mvr = n\hbar, \quad n = 1, 2, 3, \ldots$

i.e. the electron's angular momentum is quantised.

• C and D are false – the Bohr model only applies to hydrogen-like (one-electron) atoms, so it doesn't predict spectra for Ne^+ (which has 9 electrons) or compare Ne^+ with He^+.

Question4

The zener voltage in the circuit shown is $V_Z = 20$ V. The load resistance $R_L = 5k\Omega$ and the resistance $R_S = 10k\Omega$. If the input voltage is $Vs = 100$ V, then the current through the zener diode in milliampere is:



Options:

A.

8

B.

6

C.

4

D.

5

Answer: C

Solution:

✅ Given

- Zener voltage: **Vz = 20 V**
- Load resistor: **RL = 5 kΩ**
- Series resistor: **RS = 10 kΩ**
- Supply voltage: **VS = 100 V**

✅ Step 1: Current through series resistor (IS)

When the Zener regulates, the voltage across **RS** is:

$$V_{RS} = V_S - V_Z = 100 - 20 = 80V$$

So the current flowing through **RS** is:

$$I_S = \frac{80}{10\,000} = 0.008\ \text{A} = 8\ \text{mA}$$

✅ Step 2: Load current (IL)

Load voltage = Zener voltage = **20 V**

$$I_L = \frac{20}{5\,000} = 0.004\ \text{A} = 4\ \text{mA}$$

✅ Step 3: Zener current (IZ)

The series current splits into load current + Zener current:

$$I_Z = I_S - I_L = 8\ \text{mA} - 4\ \text{mA} = 4\ \text{mA}$$

🎉 Final Answer: 4 mA

Question5

The unit of universal gravitational constant is :

Options:

A. $\mathrm{Nm}^2\ \mathrm{kg}^2$

B. $\mathrm{Nm}^{-2}\ \mathrm{kg}^{-2}$

C. $\mathrm{Nm}^{-2}\ \mathrm{kg}^2$

D. $\mathrm{Nm}^2\ \mathrm{kg}^{-2}$

Answer: D

Solution:

Starting from Newton's law of gravitation:

$F = G\frac{m_1 m_2}{r^2}$

Solving for G:

$G = F\frac{r^2}{m_1 m_2}$

Now, in terms of units:

$[F] = \mathrm{N} = \frac{\mathrm{kg\ m}}{\mathrm{s}^2}$

$[r] = \mathrm{m}$

$[m_1] = [m_2] = \mathrm{kg}$

So

$[G] = \frac{(\frac{\mathrm{kg\ m}}{\mathrm{s}^2})\,\mathrm{m}^2}{\mathrm{kg}^2} = \mathrm{N\,m^2\,kg^{-2}}$

That matches Option D.

Question6

In the normal adjustment of an astronomical telescope, the objective and eyepiece are 36 cm apart. If the magnifying power of the telescope is 8 , find the focal lengths of the objective and eyepiece.

Options:

A. $\mathrm{F_O} = 28\ \mathrm{cm}, \mathrm{Fe^e} = 7\ \mathrm{cm}$

B. $\mathrm{F}_0 = 28\ \mathrm{cm},\ \mathrm{F_e} = 4\ \mathrm{cm}$

C. $\mathrm{F_o} = 32\ \mathrm{cm}, \mathrm{Fe^e} = 4\ \mathrm{cm}$

D. $\mathrm{F}_0 = 4\ \mathrm{cm},\ \mathrm{F}_e = 32\ \mathrm{cm}$

Answer: C

Solution:

We have in normal adjustment

• Tube length L = $f_o + f_e = 36$ cm

• Angular magnification M = $f_o / f_e = 8$

So

From $M = f_o/f_e = 8 \Rightarrow f_o = 8\, f_e$

Plug into $f_o + f_e = 36 \Rightarrow 8\, f_e + f_e = 36 \Rightarrow 9\, f_e = 36 \Rightarrow f_e = 4$ cm

Then $f_o = 8 \cdot 4 = 32$ cm

Answer: $f_o = 32$ cm, $f_e = 4$ cm (Option C).

Question7

A rectangular coil of 250 turns has an average area of $20\ \mathrm{cm} \times 15\ \mathrm{cm}$. The coil rotates with a speed of 60 cycles per second in a uniform magnetic field of $2 \times 10^{-2}\ \mathrm{T}$ about an axis perpendicular to the field. The peak value of the induced emf is:

Options:

A. 36π volt

B. 30π volt

C. 18π volt

D. 12π volt

Answer: C

Solution:

Let’s collect the data and apply the standard formula for a rotating coil:

Given

• Number of turns: $N = 250$

• Coil dimensions: $20\ \mathrm{cm} \times 15\ \mathrm{cm} \implies A = 0.20 \times 0.15 = 0.03\ \mathrm{m}^2$

• Rotation rate: $f = 60$ Hz, so $\omega = 2\pi f = 120\pi\ \mathrm{rad/s}$

• Magnetic field: $B = 2 \times 10^{-2}\ \mathrm{T}$

The induced emf varies as

$\mathcal{E}(t) = N\,B\,A\,\omega\ \sin(\omega t)$

so the peak (maximum) emf is

$\boxed{\mathcal{E}_{\max} = N B A \omega.}$

Substituting in:

$\mathcal{E}_{\max} = 250 \times (2 \times 10^{-2}) \times 0.03 \times (120\pi) = 18\pi \text{ V}.$

Answer: 18π V, i.e. option C.

Question8

A beam of incident parallel light falls on a diverging lens of focal length 20 cm in magnitude. If a converging lens of focal length 15 cm in magnitude is placed at a distance of 10 cm to the right of the diverging lens on the other side, then, the final image formed is:

Options:

A. Virtual and is at a distance of 30 cm to the right of the converging lens.

B. Virtual and is at a distance of 40 cm to the right of the diverging lens.

C. Real and is at a distance of 30 cm to the right of the converging lens.

D. Real and is at a distance of 40 cm to the right of the converging lens.

Answer: C

Solution:

Let's label:

• Diverging lens (L_1) at x=0, f_1=–20 cm

• Converging lens (L_2) at x=+10 cm, f_2=+15 cm

• Incoming rays are parallel ⇒ object for L_1 at infinity.

Image by L_1

Lens formula:

$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f_1}, \quad u_1 = +\infty,\ f_1 = -20$

$\Rightarrow 1/v_1 = 1/(-20) \ \Rightarrow v_1 = -20\,\text{cm}.$

So L_1 forms a virtual image 20 cm to its left.

That virtual image acts as an object for L_2.

Distance from that point (x=–20) to L_2 (x=+10) is

20+10 = 30 cm on the incoming side ⇒ $u_2 = +30\,\text{cm}.$

Image by L_2

$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f_2} \implies \frac{1}{v_2} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30} \implies v_2 = +30 \text{ cm}.$

$v_2 > 0$ means a real image 30 cm to the right of the converging lens.

Answer: Option C. The final image is real and 30 cm to the right of the converging lens.

Question9

The total number of degrees of freedom associated with $2\ \mathrm{cm}^3$ of Nitrogen gas at normal temperature and pressure is:

[Given Avogadro number as ' N ']

Options:

A. $\frac{N}{44800}$

B. $\frac{N}{4480}$

C. $\frac{N}{2240}$

D. $\frac{N}{22400}$

Answer: C

Solution:

First, find how many N_2 molecules are in 2 cm³ at NTP:

Volume per mole at NTP: 22.4 L = 22400 cm³

Number of moles in 2 cm³:

$n = \frac{2\ \mathrm{cm}^3}{22400\ \mathrm{cm}^3/\mathrm{mol}} = \frac{1}{11200}\ \mathrm{mol}$

Number of molecules:

$N_{\mathrm{mol}} = N \times n = \frac{N}{11200}$

At “normal” (room) temperature the vibrational mode of N_2 is essentially frozen, so each molecule has

• 3 translational + 2 rotational = 5 degrees of freedom.

Total degrees of freedom:

$\mathrm{dof}_{\mathrm{total}} = 5 \times \frac{N}{11200} = \frac{5N}{11200} = \frac{N}{2240}$.

Answer: Option C, $\frac{N}{2240}$.

Question10

A bob of a simple pendulum has a mass of 4 g and a charge of $20\mu\mathrm{C}$. If it is at rest in a uniform horizontal electric field of intensity $1000\mathrm{Vm}^{-1}$, the angle that the pendulum makes with the vertical at equilibrium is:

Options:

A. $\tan^{-1}\frac{1}{2}$

B. 60°

C. 30°

D. $\tan^{-1} 2$

Answer: A

Solution:

Let the pendulum bob of mass $m = 4\,\text{g} = 0.004\,\text{kg}$ carry a charge $q = 20 \times 10^{-6}\,\text{C}$ in a horizontal field $E = 1000\,\text{V/m}$. At equilibrium the horizontal electric force balances the component of tension:

Electric force:

$F_e = qE = (20 \times 10^{-6}\,\text{C})(1000\,\text{V/m}) = 0.02\,\text{N}.$

Weight:

$W = mg = (0.004\,\text{kg})(9.8\,\text{m/s}^2) = 0.0392\,\text{N}.$

At angle θ, equilibrium requires

$\tan\theta = \frac{F_e}{W} = \frac{0.02}{0.0392} \approx 0.51 \approx \frac{1}{2}.$

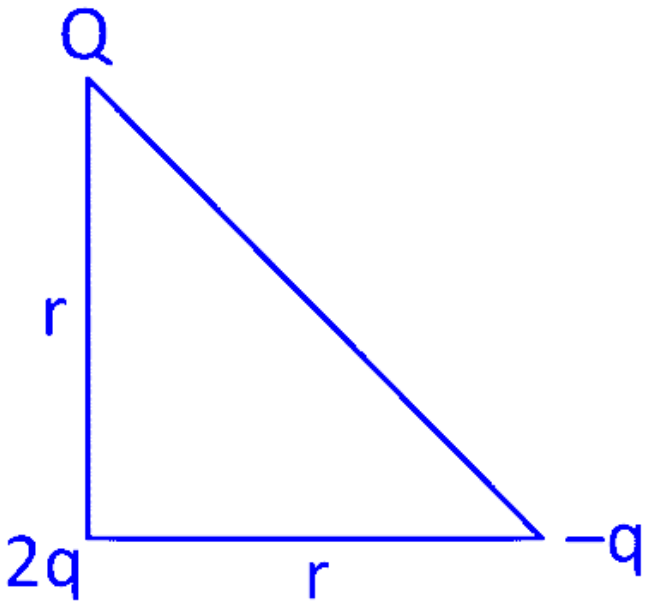
Hence

$\theta = \tan^{-1}\frac{1}{2},$

which is Option A.

Question11

Three charges, Q, $-q$ and $2q$ are placed at the vertices of a right-angled isosceles triangle. What is the value of q for the net electrostatic energy of the configuration to be zero?



Options:

A.

$Q\left[\frac{1}{2\sqrt{2}}-1\right]$

B.

$Q\left[1-\frac{1}{2\sqrt{2}}\right]$

C.

$Q\left[1+\frac{1}{2\sqrt{2}}\right]$

D.

$Q\left[\frac{2\sqrt{2}+1}{2}\right]$

Answer: B

Solution:

Take the sides of the right-angled isosceles triangle as shown:

- Distance between Q and $2q$: r
- Distance between $2q$ and $-q$: r
- Distance between Q and $-q$: $r\sqrt{2}$

Electrostatic potential energy of a system of point charges is the sum over all pairs:

$$U = \sum_{i<j} \frac{kq_i q_j}{r_{ij}}$$

So here:

1. Between Q and $2q$:

$$U_{Q,2q} = \frac{kQ \cdot 2q}{r} = \frac{2kQq}{r}$$

2. Between $2q$ and $-q$:

$$U_{2q,-q} = \frac{k(2q)(-q)}{r} = -\frac{2kq^2}{r}$$

3. Between Q and $-q$ (distance $r\sqrt{2}$):

$$U_{Q,-q} = \frac{k(Q)(-q)}{r\sqrt{2}} = -\frac{kQq}{r\sqrt{2}}$$

Total energy:

$$U = \frac{k}{r}\left(2Qq - 2q^2 - \frac{Qq}{\sqrt{2}}\right)$$

For the net electrostatic energy to be zero:

$$2Qq - 2q^2 - \frac{Qq}{\sqrt{2}} = 0$$

Factor out $q \neq 0$:

$$q\left(2Q - 2q - \frac{Q}{\sqrt{2}}\right) = 0$$

$$2Q - 2q - \frac{Q}{\sqrt{2}} = 0$$

$$-2q = -2Q + \frac{Q}{\sqrt{2}} = Q\left(-2 + \frac{1}{\sqrt{2}}\right)$$

$$q = -\frac{Q}{2}\left(-2 + \frac{1}{\sqrt{2}}\right) = \frac{Q}{2}\left(2 - \frac{1}{\sqrt{2}}\right) = Q\left(1 - \frac{1}{2\sqrt{2}}\right)$$

So,

$$\boxed{q = Q\left(1 - \frac{1}{2\sqrt{2}}\right)}$$

which corresponds to option **B**.

Question12

The minimum energy required by a hydrogen atom in ground state to emit radiation in Paschen series is nearly:

Options:

A. 13.6 eV

B. 12.75 eV

C. 10.75 eV

D. 1.5 eV

Answer: B

Solution:

To emit a Paschen-series photon (transition ending at $n = 3$), the atom must first be excited above $n = 3$. The lowest such level is $n = 4$.

Energy levels of hydrogen:

$E_n = -\frac{13.6\text{ eV}}{n^2}$

Ground-state energy $E_1 = -13.6$ eV; fourth-level energy

$E_4 = -\frac{13.6}{4^2} = -0.85\text{ eV}.$

Minimum excitation energy from $n = 1$ to $n = 4$:

$\Delta E = E_4 - E_1 = (-0.85) - (-13.6) = 12.75\text{ eV}.$

Answer: Option B, 12.75 eV.

Question13

An object of height h is placed midway between f and $2f$ in front of a biconvex lens. A real inverted image is captured on a screen placed a little beyond 2 f on the other side of the lens. If the whole arrangement is immersed in water without disturbing the object, lens and the screen positions, then the new image formed will be

Options:

A. Magnified virtual erect image on the same side of the lens where the object is placed

B. Magnified real inverted image on the same side of the lens where the object is placed

C. Diminished real inverted image on the other side of the lens between f and $2f$

D. Magnified real inverted image on the other side of the lens beyond 2f

Answer: A

Solution:

Option A

Reasoning:

In air, the thin lens equation is

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

with the object at $u = 1.5f$. That gives

$\frac{1}{v} = \frac{1}{f} - \frac{1}{1.5f} = \frac{1}{3}\frac{1}{f} \quad \Rightarrow \; v = 3f,$

a real, inverted image beyond $2f$.

Immersed in water ($n_w > 1$), the lens's focal length increases because its refractive-power drops. In fact

$\frac{1}{f_{\text{water}}} = \left(\frac{n_{\text{lens}}}{n_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

is smaller than $\frac{1}{f}$ in air, so $f_{\text{water}} > f$.

The object–lens distance remains $u = 1.5f$, but now $u

– virtual (on the same side as the object),

– erect,

– magnified.

That exactly matches Option A:

Magnified virtual erect image on the same side of the lens where the object is placed.

Question14

A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 2000 turns. The output power is delivered at 220 V by the transformer. If the current in the primary of the transformer is 2 A and its efficiency is 80%, the output current would be:

Options:

A. 32 A

B. 8 A

C. 16 A

D. 4 A

Answer: C

Solution:

Compute the input power

$P_{\text{in}} = V_p\, I_p = 2200\ \text{V} \times 2\ \text{A} = 4400\ \text{W}$

Account for the transformer's efficiency

$P_{\text{out}} = \eta P_{\text{in}} = 0.80 \times 4400 \text{ W} = 3520 \text{ W}$

Find the output current

$I_{\text{out}} = \frac{P_{\text{out}}}{V_s} = \frac{3520 \text{ W}}{220 \text{ V}} = 16 \text{ A}$

Answer: 16 A (Option C)

Question15

Two physical quantities having the same dimensional formula $\left[\mathrm{M}^{1}\ \mathrm{L}^{-1}\ \mathrm{T}^{-2}\right]$ are

Options:

A. Thrust and Strain

B. Moment of force and Thrust

C. Stress and Pressure

D. Work and thrust

Answer: C

Solution:

Stress and pressure both have the dimensions of force per unit area.

• A force F has dimensions

$[F] = [M\,L\,T^{-2}]$.

• Dividing by area (dimensions L^2) gives

$\frac{F}{L^2} = [M\,L\,T^{-2}]\,[L^{-2}] = [M\,L^{-1}\,T^{-2}]$.

Hence stress and pressure share the dimensional formula $[M^1\,L^{-1}\,T^{-2}]$, so the correct answer is Option C.

Question16

The stopping potential when a metal surface is illuminated by light of wavelength λ is 15 V . The stopping potential when the same surface is illuminated by light of wavelength 4λ is 3 V . The ratio of threshold wavelength to the initial incident wavelength λ is:

Options:

A. 1 : 16

B. 1 : 15

C. 16 : 1

D. 15 : 1

Answer: C

Solution:

Let $A = \dfrac{hc}{e\,\lambda}$ (in eV) and let ϕ_e be the work-function in eV. The photoelectric equations for stopping potentials $V_1 = 15$ V at λ and $V_2 = 3$ V at 4λ are

$V_1 = A - \phi_e \quad \Longrightarrow \quad 15 = A - \phi_e,$

$V_2 = \dfrac{A}{4} - \phi_e \quad \Longrightarrow \quad 3 = \frac{A}{4} - \phi_e.$

Subtracting (2) from (1):

$15 - 3 = \left(A - \frac{A}{4}\right) \quad \Longrightarrow \quad 12 = \frac{3A}{4} \quad \Longrightarrow \quad A = 16 \text{ eV}.$

Then from (1), $\phi_e = A - 15 = 1$ eV.

The threshold wavelength λ_0 satisfies

$\phi_e = \frac{hc}{e\,\lambda_0} \quad \Longrightarrow \quad \lambda_0 = \frac{hc}{e\,\phi_e} = \frac{A\,\lambda}{\phi_e} = 16\,\lambda.$

So the ratio $\lambda_0 : \lambda$ is $\boxed{16 : 1}$ (Option C).

Question17

A wire of negligible mass having uniform area of cross section ' A ' and young modulus ' Y ' is used to suspend a point mass ' m '. The point mass executes simple harmonic motion in a vertical plane with a period ' T ', then the length of the wire is :

Options:

A. $L = \frac{TY^2A}{4\pi^2m}$

B. $L = \frac{T^2YA}{4\pi m^2}$

C. $L = \frac{TY^2A}{4\pi m^2}$

D. $L = \frac{T^2YA}{4\pi^2m}$

Answer: D

Solution:

We model the wire as a spring with spring-constant

$k = \frac{YA}{L}$.

The period of small vertical oscillations is

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m\,L}{Y\,A}}$.

Squaring and solving for L gives

$T^2 = \frac{4\pi^2\,m\,L}{Y\,A} \quad \Longrightarrow \quad L = \frac{T^2\,Y\,A}{4\pi^2\,m}$.

That's Option D.

Question18

Applying a constant torque the speed of a flywheel is increased from 1800 rpm to 2400 rpm in 10 seconds. The number of revolutions made by the flywheel during this time is:

Options:

A. 350

B. 900

C. 700

D. 2100

Answer: A

Solution:

Let's work in revolutions per second (rps) and use the fact that with constant torque (hence constant angular acceleration) the average speed times time gives total revolutions.

Convert speeds:

$n_i = \frac{1800\ \text{rpm}}{60} = 30\,\text{rps}, \quad \mathrm{n_f} = \frac{2400\ \text{rpm}}{60} = 40\,\text{rps}.$

Average speed:

$\bar{n} = \frac{n_i + n_f}{2} = \frac{30+40}{2} = 35\,\text{rps}.$

Total revolutions in $t = 10$ s:

$N = \bar{n} \times t = 35 \times 10 = 350.$

Answer: 350 (Option A).

Question19

A short bar magnet placed with its axis at 45° with an external field of $400 \times 10^{-4}\ \mathrm{T}$ experiences a torque of 0.024 Nm . If a solenoid of cross-sectional area $10^{-4}\ \mathrm{m}^2$ and 500 turns replaces the short bar magnet such that they have the same magnetic moment, then the current flowing through the solenoid is:

Options:

A. $2\sqrt{2}$ A

B. $5\sqrt{2}$ A

C. $12\sqrt{2}$ A

D. $4\sqrt{2}$ A

Answer: C

Solution:

Let the magnetic moment of the bar-magnet be μ. From the torque

$\tau = \mu B \sin\theta$

we have

$\mu = \frac{\tau}{B\sin\theta} = \frac{0.024}{(400\times10^{-4})\,(\sin 45^\circ)} = \frac{0.024}{0.04\,\frac{\sqrt{2}}{2}} = 0.8485\ \text{A}\cdot\text{m}^2\,.$

When replaced by a solenoid of $N = 500$ turns and area $A = 10^{-4}\,\text{m}^2$, its magnetic moment is

$\mu = NIA \implies I = \frac{\mu}{NA} = \frac{0.8485}{500\times10^{-4}} = 16.97\ \text{A} = 12\sqrt{2}\ \text{A}\,.$

Answer: Option C, $12\sqrt{2}$ A.

Question20

If the distance between the Sun and Earth is doubled, then the duration of the year on earth will be :

[Given actual duration of the year = T]

Options:

A. $2\sqrt{2}T$

B. $\frac{T}{2}$

C. $\sqrt{2}T$

D. $\frac{T}{\sqrt{2}}$

Answer: A

Solution:

According to Kepler's third law, the square of the orbital period is proportional to the cube of the semi-major axis:

$T^2 \propto a^3$.

If the distance (i.e. a) is doubled, $a' = 2a$, then

Compute the new period T':

$(T')^2 \propto (2a)^3 = 2^3\,a^3 = 8\,a^3.$

Compare to the original:

$\frac{(T')^2}{T^2} = \frac{8\,a^3}{a^3} = 8 \quad \Longrightarrow \quad T' = T\sqrt{8} = 2\sqrt{2}\,T.$

So the new year would last

$\boxed{2\sqrt{2}\,T}$

(Option A).

Question21

The rms velocity of the gas molecule at $327^\circ\mathrm{C}$ is same as the rms velocity of the oxygen molecules at $27^\circ\mathrm{C}$. If the molecular weight of oxygen is 32 then the molecular weight of the given gas molecule is:

Options:

A. 32

B. 64

C. 96

D. 128

Answer: B

Solution:

First, convert temperatures to kelvin:

T_1 = 327 °C + 273 = 600 K

T_2 = 27 °C + 273 = 300 K

For any gas,

$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$,

where M is the molar mass.

Since the rms speeds are equal,

$\sqrt{\frac{3R\,T_1}{M_{\text{gas}}}} = \sqrt{\frac{3R\,T_2}{M_{O_2}}}$

⇒

$\frac{T_1}{M_{\text{gas}}} = \frac{T_2}{M_{O_2}}$

⇒

$M_{\text{gas}} = M_{O_2}\,\frac{T_1}{T_2} = 32\,\frac{600}{300} = 64.$

Answer: 64.

Question22

Select the correct statement from the following: The position of the centre of mass of a system :

Options:

A. Remains the same in any translatory motion

B. Depend on the choice of coordinates

C. Does not depend on the shape and size of the system

D. Depends on the distribution of its mass.

Answer: D

Solution:

The correct choice is Option D.

Explanation:

By definition the centre of mass of a system of point masses m_i at positions $\mathbf{r}_i$ is

$\mathbf{R}_{\text{cm}} = \frac{1}{M}\sum_i m_i \mathbf{r}_i\,, \quad M = \sum_i m_i.$

This formula shows that $\mathbf{R}_{\text{cm}}$ depends directly on how the mass is distributed (the values of each m_i and their positions $\mathbf{r}_i$).

Shape and size only matter insofar as they affect that mass distribution.

Although the numerical coordinates of $\mathbf{R}_{\text{cm}}$ change if you pick a different origin (Option B), the physical location of the centre of mass for a given distribution is determined solely by the masses and their arrangement.

Hence, “Depends on the distribution of its mass” is the fundamental property.

Question23

A particle of mass 3 g and charge $60\mu\text{C}$ is released from rest in a uniform electric field of intensity $10^5 NC^{-1}$. If the value of kinetic energy attained by the particle after moving through a distance of 2 cm is $m \times 10^{-2} J$, then the value of m is:

Options:

A. 12

B. 6

C. 5

D. 4

Answer: A

Solution:

The kinetic energy gained is equal to the work done by the field:

$K = qEd$

Here

• $q = 60 \times 10^{-6}$ C

• $E = 10^5$ N/C

• $d = 0.02$ m

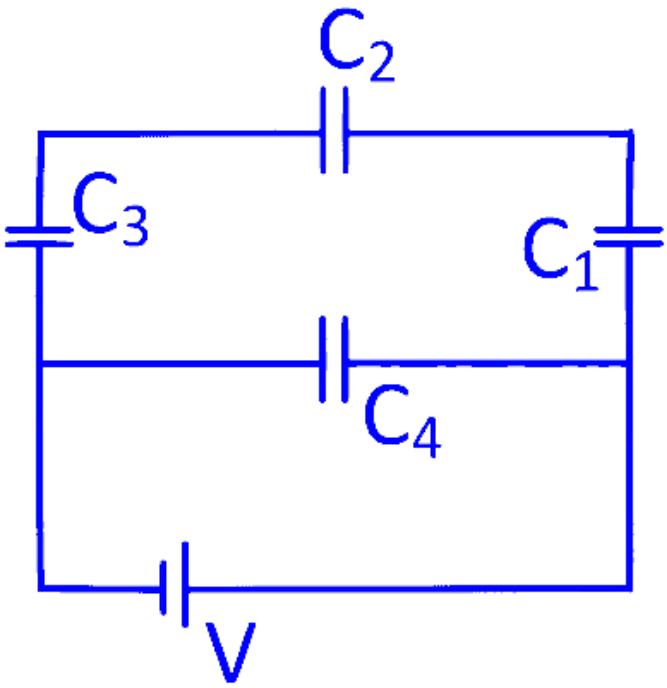
So

$K = (60 \times 10^{-6})(10^5)(0.02) = 0.12 \text{ J} = 12 \times 10^{-2} \text{ J}$

Hence $m = 12$. (Option A)

Question24

A network of capacitors is as shown below. If the voltage supply is 100 V , find the energy stored in the 6μ F capacitor.



$C_1 = 3\mu F, C_2 = 6\mu F, C_3 = 3\mu F$ **and** $C_4 = 4\mu F$

Options:

A.

1.2 mJ

B.

12 mJ

C.

2.2 mJ

D.

4.2 mJ

Answer: A

Solution:

Solution

We are asked to find the **energy stored in the 6 μF capacitor (C_2)** when the supply voltage is **100 V**.

To do that, we must first understand the circuit configuration.

1. Understanding the Circuit

From the diagram:

- C_2 (6 μF) is in **series** with the parallel combination of C_1 (3 μF) and C_3 (3 μF).
- That whole branch is in **parallel** with C_4 (4 μF).

Thus the arrangement is:

Top branch:

$$C_3 \quad \text{in series with} \quad C_2 \quad \text{in series with} \quad C_1$$

Middle branch:

$$C_4$$

Both branches connect across the 100 V supply.

---

2. Simplify the Top Branch

Step 1: Combine the parallel capacitors C_1 and C_3

But careful: In the diagram, C_1 and C_3 are NOT in parallel directly.

Actually the structure is:

Left plate of C_3 → C_2 → C_1 right plate

Middle node → C_4 → right plate

Left plate of C_3 and middle node are the same (connected by wire)

So:

C_2 is in parallel with (C_1 + C_3)

Wrong — let's analyze correctly:

Nodes:

- Node A: left of C_3 and bottom supply.
- Node B: between C_3, C_2, C_4.
- Node C: right of C_2 and C_1 and bottom supply.

Connections:

C_3: A ↔ B

C_2: B ↔ C

C_1: C ↔ B

C_4: B ↔ C (same as C_1)

Thus C_2 is between **B and C**, and C_1 and C_4 are also between **B and C**.

So:

C_1, C_2, and C_4 are all in parallel.

C_3 is connected between A and B.

Thus we really have:

$$C_3 \text{ in series with } (C_1 + C_2 + C_4)$$

Now compute.

3. Combine Parallel Capacitors

$$C_{\text{parallel}} = C_1 + C_2 + C_4 = 3 + 6 + 4 = 13\,\mu\text{F}$$

---

4. Total Series Combination

Series combination of:

$$C_3 = 3\,\mu\text{F}$$

$$C_{\text{parallel}} = 13\,\mu\text{F}$$

For capacitors in series:

$$C_{\text{eq}} = \frac{C_3 \cdot 13}{C_3 + 13} = \frac{3 \times 13}{16} = \frac{39}{16} = 2.4375\,\mu\text{F}$$

---

5. Charge on Each Series Capacitor

In series: Q is same across each branch.

$$Q = C_{\text{eq}} V = 2.4375 \times 10^{-6} \times 100 = 2.4375 \times 10^{-4}\,\text{C}$$

This charge is also the charge entering the parallel group.
Inside the parallel group, the voltage across each capacitor is:

$$V_{\text{parallel}} = \frac{Q}{C_{\text{parallel}}}$$

$$V_{\text{parallel}} = \frac{2.4375 \times 10^{-4}}{13 \times 10^{-6}} = 18.75\,\text{V}$$

Thus C_2 (6 µF) has voltage:

$$V_{C_2} = 18.75\,\text{V}$$

6. Energy Stored in the 6 µF Capacitor

$$E = \frac{1}{2} C V^2$$

$$E = \frac{1}{2}(6 \times 10^{-6})(18.75)^2$$

$$E = 3 \times 10^{-6} \times 351.56$$

$$E = 1.0547 \times 10^{-3}\,\text{J}$$

$$E \approx \mathbf{1.06\ mJ}$$

Closest answer choice:

✅ 1.2 mJ (Option A)

Final Answer: ❗ 1.2 mJ

Question25

A body of mass 10 kg is moving up on an inclined plane of 30° with an acceleration $2\ \text{ms}^{-2}$. Find the force required, if the coefficient of friction

between the object and the plane surface is $\frac{\sqrt{3}}{6}$, [given $g = 10\ \mathrm{ms}^{-2}$].

Options:

A. 1.95 N

B. 9.5 N

C. 19 N

D. 95 N

Answer: D

Solution:

Forces along the plane

– Down-slope component of weight:

$mg\sin\theta = 10 \cdot 10 \cdot \sin 30^\circ = 100 \cdot \frac{1}{2} = 50$ N

– Normal reaction:

$N = mg\cos\theta = 10 \cdot 10 \cdot \cos 30^\circ = 100 \cdot \frac{\sqrt{3}}{2} = 50\sqrt{3}$ N

– Friction (opposes upward motion):

$f = \mu N = \frac{\sqrt{3}}{6} \times 50\sqrt{3} = \frac{50 \cdot 3}{6} = 25$ N

Newton's 2nd law up the plane

Let P be the applied force up the plane. Then

$P - \left(mg\sin\theta + f\right) \;=\; ma$

$P = ma + mg\sin\theta + f$

Substitute numbers

$P = 10 \cdot 2 \;+\; 50 \;+\; 25 = 20 + 50 + 25 = 95$ N

Answer: 95 N (Option D).

Question26

A galvanometer having a resistance of 50Ω is shunted by a wire of resistance 10Ω. If the total current is 2 A , the current passing through the shunt is:

Options:

A. $\frac{3}{5}$ A

B. $\frac{2}{5}$ A

C. $\frac{5}{6}$ A

D. $\frac{5}{3}$ A

Answer: D

Solution:

Understand the setup: When a galvanometer is "shunted by a wire," it means the galvanometer and the shunt wire are connected in parallel. In a parallel circuit, the voltage across each branch is the same.

Identify the given values:

Resistance of the galvanometer (R_g) = 50Ω

Resistance of the shunt wire (R_s) = 10Ω

Total current (I_{total}) = 2 A

Apply the voltage equality: Let the current passing through the galvanometer be I_g and the current passing through the shunt be I_s. Since the voltage across parallel components is the same, we have:

$V_g = V_s$

$I_g R_g = I_s R_s$

Apply Kirchhoff's Current Law: The total current entering the parallel combination splits into the current through the galvanometer and the current through the shunt:

$I_{total} = I_g + I_s$

From this, we can express I_g as:

$I_g = I_{total} - I_s$

Substitute and solve for I_s: Now, let's substitute the expression for I_g into our voltage equality equation:

$(I_{total} - I_s)R_g = I_s R_s$

Expand the equation:

$I_{total}R_g - I_s R_g = I_s R_s$

Move all terms with I_s to one side:

$I_{total}R_g = I_s R_s + I_s R_g$

Factor out I_s:

$I_{total}R_g = I_s(R_s + R_g)$

Finally, solve for I_s:

$I_s = \frac{I_{total}R_g}{R_s+R_g}$

Plug in the numbers:

$I_s = \frac{(2\,\text{A})\times(50\Omega)}{10\Omega+50\Omega}$

$I_s = \frac{100\,\text{A}\ \Omega}{60\Omega}$

$I_s = \frac{10}{6}$ A

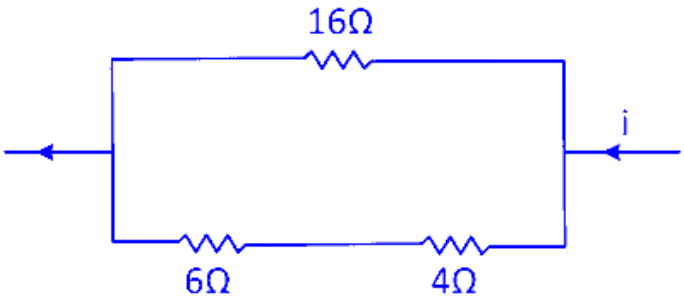
$I_s = \frac{5}{3}$ A

So, the current passing through the shunt is $\frac{5}{3}$ A.

Comparing this with the given options, it matches Option D.

Question27

The power dissipated across the 16Ω resistor in the circuit is 2 watts. The power dissipated in watt units across the 4Ω resistor is:



Options:

A.

2.38 W

B.

0.64 W

C.

1.28 W

D.

4.28 W

Answer: C

Solution:

Step 1: Use the given power on the 16 Ω resistor

Power on 16 Ω resistor is:

$$P = 2\,\text{W}$$

Voltage across the 16 Ω resistor:

$$V = \sqrt{PR} = \sqrt{2 \times 16} = \sqrt{32} \approx 5.657\,\text{V}$$

Since the 16 Ω resistor is in parallel with the 6 Ω–4 Ω series branch, this same voltage appears across the series branch.

Step 2: Current through the 6 Ω + 4 Ω series branch

Total resistance in the bottom branch:

$$R_{\text{series}} = 6 + 4 = 10\,\Omega$$

Current through the branch:

$$I = \frac{V}{R} = \frac{5.657}{10} = 0.5657\,\text{A}$$

Step 3: Power dissipated in the 4 Ω resistor

$$P_{4\Omega} = I^2 R = (0.5657)^2 \times 4$$

$$P_{4\Omega} \approx 0.32 \times 4 = 1.28\,\text{W}$$

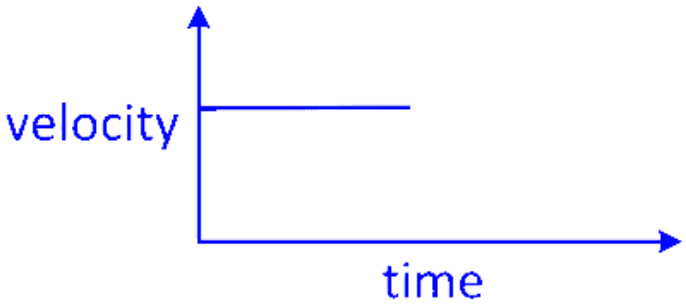
✅ **Final Answer: 1.28 W**

Question28

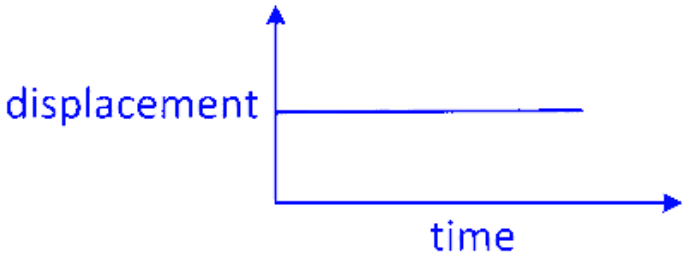
Select the graph which represents the motion of a particle along a straight line with uniform acceleration.

Options:

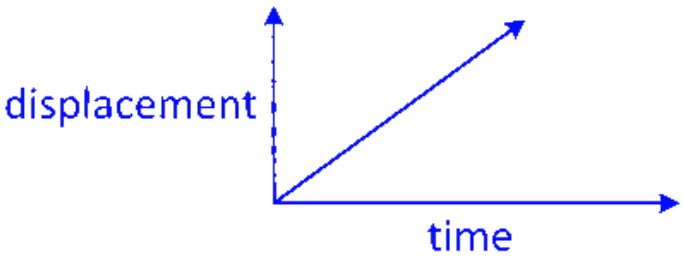
A.



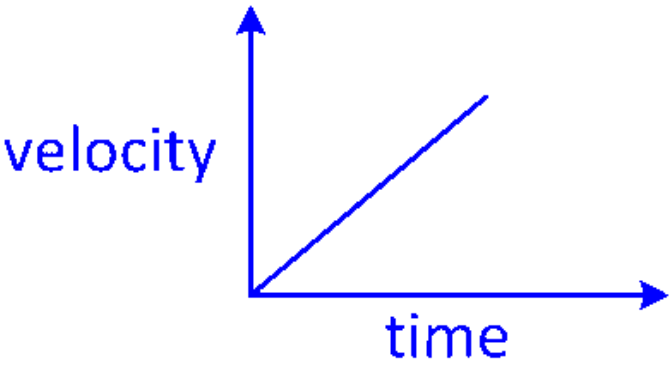
B.



C.



D.



Answer: D

Solution:

Uniform (constant) acceleration means:

- **Velocity–time graph**: a **straight line with non-zero slope** (velocity increases or decreases at a constant rate).
- **Displacement–time graph**: a **curve** (parabolic), **not** a straight line.

Check each option:

- **A:** Velocity is constant → **acceleration = 0** (not uniform non-zero acceleration).
- **B:** Displacement is constant → particle is at rest (no acceleration).
- **C:** Displacement increases linearly → constant velocity, **zero acceleration**.
- **D:** Velocity increases linearly with time → **constant, non-zero acceleration** ✔

✅ Correct Answer: **D**

Question29

A metal rod of susceptibility 799 is subjected to a magnetising field of $2000\mathrm{Am}^{-1}$. The permeability of the material of the rod is: (Given

$\mu_0 = 4\pi \times 10^{-7} \mathrm{TmA}^{-1}$)

Options:

A. $4.2\pi \times 10^{-7} TmA^{-1}$

B. $3.2\pi \times 10^{-4} \mathrm{TmA}^{-1}$

C. $2.4\pi \times 10^{-5} \mathrm{TmA}^{-1}$

D. $80\pi \times 10^{-7} TmA^{-1}$

Answer: B

Solution:

Magnetic susceptibility χ relates to relative permeability μr by

$\mu_r = 1 + \chi = 1 + 799 = 800.$

The absolute permeability μ is

$\mu = \mu_0 \, \mu_r = (4\pi \times 10^{-7}) \times 800 = 3200\pi \times 10^{-7} = 3.2\pi \times 10^{-4} \ \mathrm{T\,m\,A^{-1}}.$

That matches Option B.

Question30

A given volume of gas at NTP is allowed to expand 6 times of its original volume, first under isothermal condition and then under adiabatic condition. Which of the given statement is correct? [Given $\frac{c_p}{c_v} = \gamma = 1.4$]

Options:

A. The final pressure after the adiabatic expansion is 1.4 times greater than the final pressure after the isothermal expansion.

B. The final temperature after the adiabatic expansion is 1.4 times less than the final temperature after the isothermal expansion.

C. Pressure remains same in both adiabatic and isothermal expansion

D. The final pressure after the adiabatic expansion is less than the final pressure after the isothermal expansion.

Answer: D

Solution:

Let’s compare the two final pressures, starting from the same initial state (p_1, V_1, T_1) and expanding to $V_2 = 6V_1$.

Isothermal expansion

• Temperature stays at T_1.

• By Boyle's law:

$p_{2,\text{iso}} = \frac{p_1 V_1}{V_2} = \frac{p_1}{6}$.

Adiabatic expansion

• No heat exchange, so

$p_1 V_1^{\gamma} = p_{2,\text{ad}} V_2^{\gamma} \quad \Longrightarrow \quad p_{2,\text{ad}} = p_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = p_1\, 6^{-\gamma}$.

• With $\gamma = 1.4$:

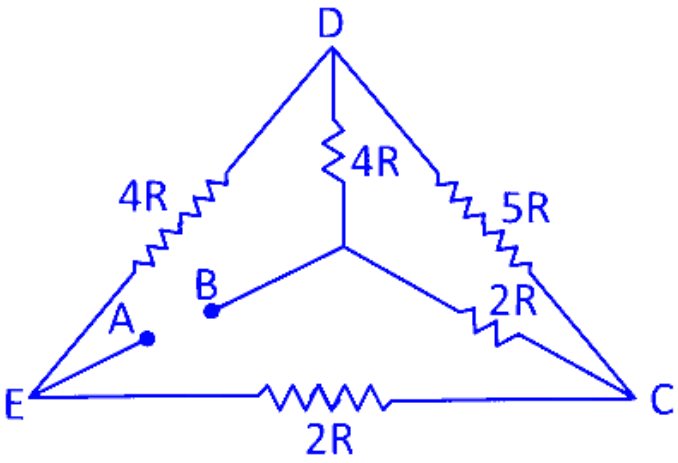
$6^{-1.4} \approx \frac{1}{6^{1.4}} \approx \frac{1}{12.3} \approx 0.08 \quad \Longrightarrow \quad p_{2,\text{ad}} \approx 0.08\, p_1$.

Since $p_{2,\text{ad}} \approx 0.08\, p_1$ is smaller than $p_{2,\text{iso}} = 0.167\, p_1$, the final pressure after adiabatic expansion is lower than after isothermal expansion.

Answer: Option D.

Question31

The net resistance between the points A and B in the circuit given below is:



Options:

A.

$\frac{8}{3}$R

B.

$\frac{8}{5}R$

C.

R

D.

$\frac{5}{3}$R

Answer: A

Solution:

Between **E and B**, there is a **direct wire (0 Ω)**.
So the current returns through the *other two paths* around the triangle:

Path 1:

$$E \to D \to B: \ 4R + 4R = 8R$$

Path 2:

$$E \to C \to B: \ 2R + 2R = 4R$$

These two are in **parallel**:

$$R_{eq} = \left(\frac{1}{8R} + \frac{1}{4R}\right)^{-1} = \frac{8}{3}R$$

Final Answer:

$$\boxed{\frac{8}{3}R}$$

Question32

The wavelength of a monochromatic light which is used in single slit diffraction is 800 nm . The width of the single slit for which the first minimum appears at $\theta = 45^\circ$ on the screen will be:

Options:

A. 1.13μ m

B. 1.23μ m

C. 2.13μ m

D. 1.3μ m

Answer: A

Solution:

For a single slit, the first minimum occurs when

$a \sin\theta = \lambda$

where

• a is the slit width,

• $\lambda = 800$ nm $= 8.00 \times 10^{-7}$ m,

• $\theta = 45^\circ$ so $\sin 45^\circ = \frac{\sqrt{2}}{2} \approx 0.7071$.

Plug in the numbers:

$a = \frac{\lambda}{\sin\theta} = \frac{8.00\times10^{-7}}{0.7071} \approx 1.13 \times 10^{-6}\ \text{m}$

Converting to micrometers:

$1.13 \times 10^{-6}\ \text{m} = 1.13\ \mu\text{m}$

That matches Option A.

Question33

A plane electromagnetic wave with frequency 40 MHz travels in free space. At a particular point in space and time, the magnetic field is $2 \times 10^{-8}T$. What will be the electric field at this point?

Options:

A. 16Vm^{-1}

B. 6Vm^{-1}

C. 8Vm^{-1}

D. 18Vm^{-1}

Answer: B

Solution:

The electric and magnetic fields in a free-space plane wave are related by

$E = c\,B$

where $c = 3 \times 10^{8}\ \text{m/s}$. Plugging in $B = 2 \times 10^{-8}\ \text{T}$,

$E = (3 \times 10^{8})(2 \times 10^{-8}) = 6\ \text{V/m}\,.$

So the correct choice is Option B: 6 V/m.

Question34

A series LCR circuit having $\text{R} = 44\Omega,\ \text{L} = 2\text{H}$ and $\text{C} = 25\mu\ \text{F}$ is connected to a variable frequency of 220 V . What is the average power transferred to the circuit in one complete cycle if the frequency of supply equals the natural frequency of the circuit?

Options:

A. 2.1 kW

B. 4.2 kW

C. 1.1 kW

D. 1.2 kW

Answer: C

Solution:

At resonance $X_L = X_C$, so the net impedance is purely resistive:

$Z = R = 44\ \Omega$.

The rms current is

$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220}{44} = 5\ \text{A}$.

The average power (over one cycle) is

$P = V_{\text{rms}}\, I_{\text{rms}} \cos\phi = V_{\text{rms}}\, I_{\text{rms}} = \frac{V_{\text{rms}}^2}{R} = \frac{220^2}{44} = 1100\ \text{W} = 1.1\ \text{kW}$.

Answer: Option C (1.1 kW).

Question35

Momentum of a body is increased to three times its original value. By what percentage will its kinetic energy change?

Options:

A. 300%

B. 900%

C. 200%

D. 800%

Answer: D

Solution:

Let the original momentum be p and the mass m. Then the kinetic energy is

$K_i = \frac{p^2}{2m}$.

When the momentum triples, $p' = 3p$, the new kinetic energy is

$K_f = \frac{(3p)^2}{2m} = 9\,\frac{p^2}{2m} = 9\,K_i$.

The change in kinetic energy is

$\Delta K = K_f - K_i = 9K_i - K_i = 8K_i$,

so the percent increase is

$\frac{\Delta K}{K_i} \times 100\% = 8 \times 100\% = 800\%$.

Answer: Option D, 800 %.

Question36

What is the dc component of the output voltage if a sinusoidal signal of 33 V peak voltage is the input of a half wave diode rectifier circuit?

Options:

A. 3.5 V

B. 2.9 V

C. 10.5 V

D. 12.5 V

Answer: C

Solution:

The average (DC) output of an ideal half-wave rectifier with peak input voltage V_p is

- $V_{DC} = \frac{1}{2\pi}\int_0^{2\pi} V_p \max(\sin\theta, 0)\, d\theta = \frac{V_p}{\pi}$

So here

$V_{DC} = \frac{33\ \text{V}}{\pi} \approx 10.5\ \text{V}$

Answer: Option C (10.5 V).

Question37

Radium having mass number 200 and binding energy per nucleon 5.6 MeV , splits into two fragments Cadmium of mass number 112 and Hassium of mass number 108. If the binding energy per nucleon for Cadmium and Hassium is approximately 8.0 MeV , then the energy Q released per fission will be:

Options:

A. 598 MeV

B. 176 MeV

C. 640 MeV

D. 475 MeV

Answer: C

Solution:

Total binding energy of the parent (Ra-200):

$B_E(\text{Ra}) = 200 \times 5.6 = 1120$ MeV

Total binding energies of the two fragments:

– Cadmium-112:

$B_E(\text{Cd}) = 112 \times 8.0 = 896$ MeV

– Hassium-108:

$B_E(\text{Hs}) = 108 \times 8.0 = 864$ MeV

Sum of products' binding energies:

$896 + 864 = 1760$ MeV

Energy released (Q-value) is the gain in binding energy:

$Q = B_E(\text{products}) - B_E(\text{parent}) = 1760 - 1120 = 640$ MeV

So the fission releases 640 MeV. That's option C.

Question38

In a particular case X , a certain length of insulated copper wire is bent to form double loops of equal radii. In another case Y the same copper wire is bent to form three loops of equal radii. If the same steady current is passed through the copper wire in both the cases, the ratio of magnetic field at the centre in the case X to that in case Y is:

Options:

A. $\frac{1}{4}$

B. $\frac{1}{9}$

C. $\frac{9}{4}$

D. $\frac{4}{9}$

Answer: D

Solution:

Let the total length of the wire be L, and the steady current be I.

Radius in each case

• Case X (2 loops):

$$L = 2 \times (2\pi r_X) \implies r_X = \frac{L}{4\pi}.$$

• Case Y (3 loops):

$$L = 3 \times (2\pi r_Y) \implies r_Y = \frac{L}{6\pi}.$$

Magnetic field at the centre of N coplanar loops of radius r:

$B = \frac{\mu_0 N I}{2 r}$.

Compute B_X and B_Y:

• For X ($N = 2$, $r = r_X$):

$B_X = \frac{\mu_0 \cdot 2 I}{2 r_X} = \frac{\mu_0 I}{r_X} = \frac{\mu_0 I}{L/(4\pi)} = \frac{4\pi \mu_0 I}{L}$.

• For Y ($N = 3$, $r = r_Y$):

$B_Y = \frac{\mu_0 \cdot 3 I}{2 r_Y} = \frac{3\mu_0 I}{2 (L/(6\pi))} = \frac{3\mu_0 I (6\pi)}{2L} = \frac{9\pi \mu_0 I}{L}$.

Ratio

$\frac{B_X}{B_Y} = \frac{\frac{4\pi \mu_0 I}{L}}{\frac{9\pi \mu_0 I}{L}} = \frac{4\pi}{9\pi} = \frac{4}{9}$.

Answer: Option D, $\frac{4}{9}$.

Question39

The fundamental frequency of sound produced in an open pipe of length L_1 is same as the frequency of the 3^{rd} harmonic of the sound produced in the closed pipe of length L_2 Then the ratio of $\frac{L_1}{L_2}$ is :

Options:

A. $\frac{L_1}{L_2} = \frac{3}{1}$

B. $\frac{L_1}{L_2} = \frac{1}{3}$

C. $\frac{L_1}{L_2} = \frac{2}{3}$

D. $\frac{L_1}{L_2} = \frac{3}{2}$

Answer: C

Solution:

Let v be the speed of sound.

Open pipe (both ends open), fundamental frequency:

$f_{\text{open}} = \frac{v}{2L_1}$.

Closed pipe (one end closed), only odd harmonics. The 3rd harmonic is $n = 3$:

$f_{\text{closed},3} = \frac{3v}{4L_2}$.

Setting them equal,

$\frac{v}{2L_1} = \frac{3v}{4L_2} \implies \frac{1}{2L_1} = \frac{3}{4L_2} \implies \frac{L_1}{L_2} = \frac{2}{3}$.

Answer: $\frac{L_1}{L_2} = \frac{2}{3}$.

Question40

Select the correct statement from the following:

Options:

A. Nuclear force is a long range force

B. Nuclear force is the weakest force in nature

C. Nuclear force is a non central force

D. Nuclear force depends on the charge of the nucleons

Answer: C

Solution:

The correct choice is Option C: “Nuclear force is a non-central force.”

Key points:

• The strong (nuclear) force is very short-range (~1–2 fm), not long-range.

• It’s the strongest of the four fundamental forces at those distances, not the weakest.

• Aside from a central component V(r), it has a tensor part (spin-dependent), so it cannot be described solely by V(r)—hence it’s non-central.

• It’s essentially charge-independent (pp, nn, np interactions are nearly the same), so it doesn’t depend on the nucleon’s electric charge.

Question41

A straight wire of mass 250 g and length 2.5 m carries a current of 4 A . It can be suspended in mid air by a uniform horizontal magnetic field of magnitude:

Options:

A. 0.145 T

B. 0.245 T

C. 0.625 T

D. 2.245 T

Answer: B

Solution:

First, equate the upward magnetic force to the weight of the wire:

• Magnetic force: $F_{\text{mag}} = I\,L\,B$

• Weight: $F_g = m\,g$

Setting $F_{\text{mag}} = F_g$ gives

$I\,L\,B = m\,g \quad \Longrightarrow \quad B = \frac{m\,g}{I\,L}$

Plug in the numbers:

• $m = 0.250$ kg

• $g = 9.8$ m/s²

• $I = 4$ A

• $L = 2.5$ m

$B = \frac{0.250 \times 9.8}{4 \times 2.5} = \frac{2.45}{10} = 0.245$ T

Answer: Option B, 0.245 T.

Question42

Select the correct statement from the following:

Options:

A. Shear modulus of an ideal liquid is infinite

B. Rubber is more elastic than steel

C. The bulk modulus of a perfect rigid body is infinite

D. The bulk modulus of an ideal liquid is zero

Answer: C

Solution:

The only correct statement is Option C.

Why? Recall the bulk modulus

$K = -V\,\frac{dP}{dV}$.

• For a perfect rigid body, the volume never changes under any pressure change (dV=0), so

$K \to \infty$.

By contrast:

An ideal (incompressible) liquid also has infinite bulk modulus, not zero.

Liquids have zero shear modulus (they can’t sustain shear stress).

Rubber’s Young’s modulus (~1 MPa) is far lower than steel’s (~200 GPa), so steel is “more elastic” (stiffer) in the usual sense.

Question43

A boy is running along a straight horizontal road with a constant speed $5\ \mathrm{ms}^{-1}$. While running he throws a stone with a velocity $30\ \mathrm{ms}^{-1}$ at an angle 60° with the horizontal. Then the time of flight of the stone is: (Given $g = 10\ \mathrm{ms}^{-2}$)

Options:

A. $3\sqrt{2}$

B. $\frac{\sqrt{3}}{2}$

C. $4\sqrt{3}$

D. $3\sqrt{3}$

Answer: D

Solution:

The time of flight depends only on the vertical component of the initial velocity. Here’s how to see it step by step:

Initial speed: $u = 30\ \mathrm{m/s}$

Launch angle: $\theta = 60^{\circ}$

Vertical component:

$u_y = u \sin\theta = 30 \cdot \frac{\sqrt{3}}{2} = 15\sqrt{3}\ \mathrm{m/s}$

Time of flight (up and down):

$T = \frac{2\,u_y}{g} = \frac{2 \cdot 15\sqrt{3}}{10} = 3\sqrt{3}\ \mathrm{s}$

So the correct answer is Option D: $3\sqrt{3}$.

Question44

In a pure inductive circuit, a sinusoidal voltage $V(t) = 200 \sin 250t$ is applied to a pure inductance of $\mathrm{L} = 0.02\mathrm{H}$. The current through the coil is:

Options:

A. $40 \sin\left[250t - \frac{\pi}{2}\right]$

B. $40 \cos\left[250t - \frac{\pi}{2}\right]$

C. $40 \sin \left[250t + \frac{\pi}{2}\right]$

D. $40 \cos \left[250t + \frac{\pi}{2}\right]$

Answer: A

Solution:

In a pure inductance L, the voltage and current satisfy

$v(t) = L \frac{di}{dt}$.

Here

- $v(t) = 200 \sin(250t)\,, \quad L = 0.02\,\mathrm{H}$
- $\frac{di}{dt} = \frac{200}{0.02}\sin(250t) = 10^4 \sin(250t)$

Integrate:

$i(t) = \int 10^4 \sin(250t)\, dt = -\frac{10^4}{250}\cos(250t) = -40\cos(250t)\,.$

Since

$-\cos(250t) = \sin\left(250t - \frac{\pi}{2}\right),$

we get

$$\boxed{i(t) = 40 \sin\left(250t - \frac{\pi}{2}\right)}$$

which is Option A.

Question45

In to a vessel containing pure water a clean glass tube of radius 3.6×10^{-4} m is held vertically with 12 cm of the tube above the water level. Now the capillary tube is moved down in to the water so that only 2 cm of its length is above the water surface. Angle of contact Θ at this position is (given surface tension of water $= 7.2 \times 10^{-2}\mathrm{Nm}^{-1}$ and $g = 10\ \mathrm{ms}^{-2}$)

Options:

A. $\Theta = 45^{\circ}$

B. $\Theta = 30^{\circ}$

C. $\Theta = 15^{0}$

D. $\Theta = 60^{\circ}$

Answer: D

Solution:

We use the capillary-rise formula

$h = \frac{2\sigma\cos\theta}{\rho\, g\, r}$.

For clean glass in pure water, θ≈0° so the maximum rise is

$h_{\max} = \frac{2\sigma}{\rho g r} = \frac{2\times 7.2\times 10^{-2}}{1000\times 10\times 3.6\times 10^{-4}} = 0.04\ \text{m} = 4\ \text{cm}.$

Since the tube top is 12 cm above the bath, the meniscus lies well below the rim and θ≈0°.

When the tube is pushed down so only 2 cm of it is above the outside level, the meniscus pins at the rim and the equilibrium rise must be h=2 cm. Solving for θ:

$\cos\theta = \frac{\rho g r\, h}{2\sigma} = \frac{1000\cdot 10\cdot 3.6\times 10^{-4}\cdot 0.02}{2\cdot 7.2\times 10^{-2}} = 0.5 \quad\Longrightarrow\quad \theta = 60^\circ.$

Answer: 60° (Option D).

Question46

A point marked on a ring of radius 2 cm is in contact with a horizontal plane. Now the ring is rolled forward half a revolution along the positive X - direction. Then the angle made by the displacement vector of the point with the X - axis is:

Options:

A. $\theta = \mathbf{tan^{-1}}\left(\frac{\mathbf{2}}{\mathbf{3\pi}}\right)$

B. $\boldsymbol{\theta} = \mathbf{tan^{-1}}\left(\frac{\mathbf{2}}{\boldsymbol{\pi}}\right)$

C. $\theta = \tan^{-1}\left(\frac{2\pi}{3}\right)$

D. $\theta = \cot^{-1}\left(\frac{2}{\pi}\right)$

Answer: B

Solution:

Let the plane be the x–axis and the vertical upward be the y–axis.

Initial position of the marked point P₀ = (0,0).

As the ring (radius R=2 cm) rolls half a turn (π rad) without slipping, its center moves horizontally by

$s = R\cdot\pi = 2\pi\ \text{cm}.$

So the center goes from (0,2) to (2π,2).

After a half-turn the point is at the top of the ring, i.e. 2 cm above the center. Hence its final position is

$P_f = \left(2\pi,\ 2+2\right) = (2\pi,\ 4).$

The displacement vector is

$\Delta\mathbf{r} = (2\pi,\ 4)$

so its angle θ with the x–axis satisfies

$\tan\theta = \frac{4}{2\pi} = \frac{2}{\pi} \quad \Longrightarrow \quad \theta = \tan^{-1}\left(\frac{2}{\pi}\right).$

Answer: Option B.

Question47

The instantaneous values of alternating current and voltages in a circuit are $\mathrm{I} = \frac{3}{\sqrt{2}}\sin(200\pi t)$ ampere and $\mathrm{V} = \frac{3}{\sqrt{2}}\sin\left(200\pi t + \frac{\pi}{6}\right)$ volt. What is the average power consumed in the circuit in watt ?

Options:

A. $\frac{\sqrt{3}}{8}$

B. $\frac{6\sqrt{3}}{8}$

C. $\frac{8\sqrt{2}}{9}$

D. $\frac{9\sqrt{3}}{8}$

Answer: D

Solution:

The average (real) power is given by

$P_{\text{avg}} = V_{\text{rms}}\, I_{\text{rms}} \cos\varphi$

where

$I_m = \frac{3}{\sqrt{2}}$ and $V_m = \frac{3}{\sqrt{2}}$ are the amplitudes,

$\varphi =$ phase difference $= +\frac{\pi}{6}$,

$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{3}{2}$,

$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} = \frac{3}{2}$.

So

$P_{\text{avg}} = \left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\cos\left(\frac{\pi}{6}\right) = \frac{9}{4}\,\frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{8}$ W.

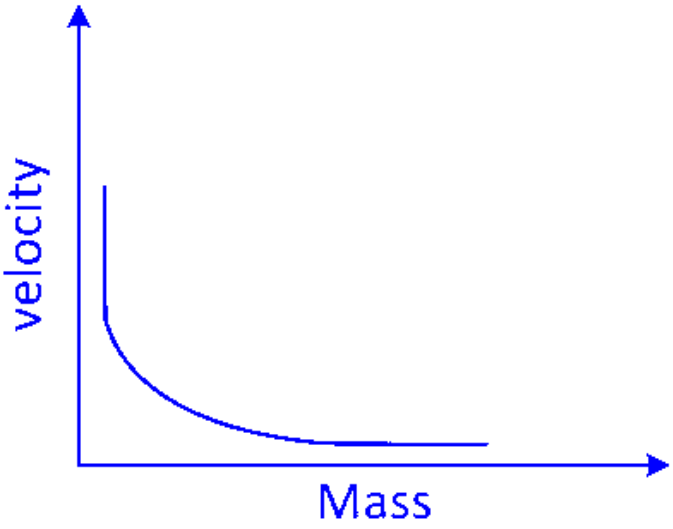
Answer: $\frac{9\sqrt{3}}{8}$ **W** (Option D).

Question48

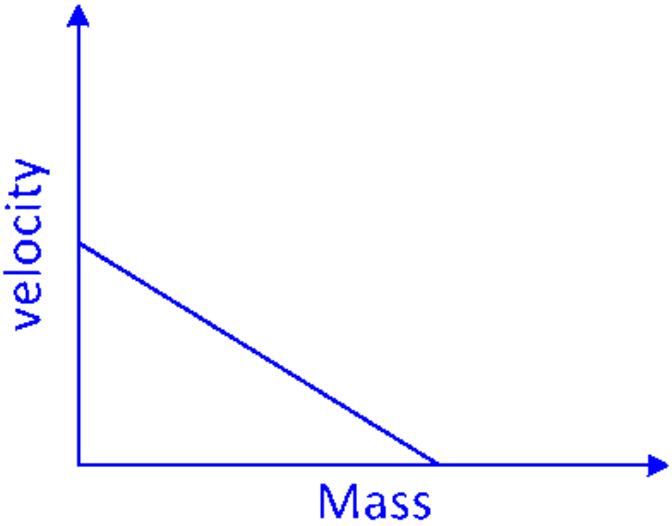
The velocity - mass graph of body with constant linear momentum is represented by the graph:

Options:

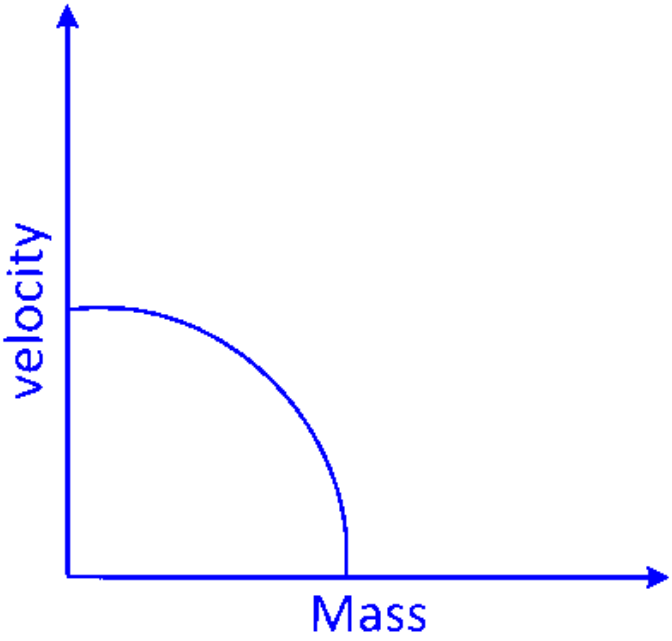
A.



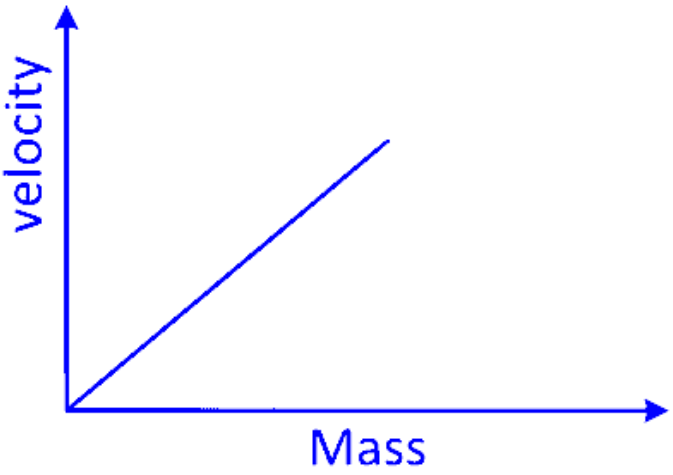
B.



C.



D.



Answer: A

Solution:

1. Recall the definition of linear momentum

Linear momentum p of a body is:

$$p = mv$$

where

- m = mass of the body
- v = velocity of the body

---

2. Use the condition "constant linear momentum"

The question says the body has **constant linear momentum**.
That means:

$$p = \text{constant}$$

So we can write:

$$mv = \text{constant}$$

---

3. Express velocity in terms of mass

We want a graph of **velocity vs mass**, so solve for v:

$$v = \frac{\text{constant}}{m}$$

Let the constant be k:

$$v = \frac{k}{m}$$

---

4. Understand the mathematical relation

The equation $v = \frac{k}{m}$ shows:

- v is **inversely proportional** to m.
- As m increases, v decreases.
- The curve is a **rectangular hyperbola** (a decreasing curve that gets closer and closer to the axes but never touches them).

---

5. Match this with the options

Now look at the shapes:

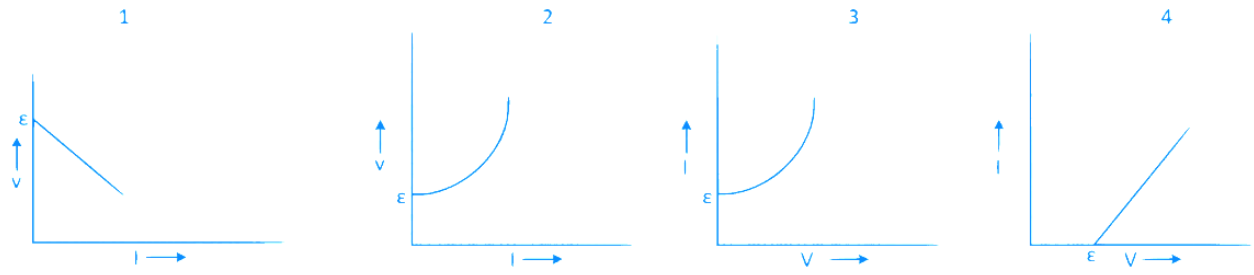
- **A:** Decreasing curve that looks like $v \propto \frac{1}{m}$ (rectangular hyperbola). ✅
- **B:** Straight line sloping down → linear decrease ($v = a - bm$), not inverse. ❌
- **C:** Quarter of a circle, not the shape of k/m. ❌
- **D:** Straight line increasing → $v \propto m$. ❌

Only **Option A** shows an inverse (hyperbolic) decrease of velocity with increasing mass.

Final answer: Option A ✔

Question49

A student measures the terminal potential difference V of a cell of emf ε and internal resistance r as a function of the current I flowing through it. Which of the following graphs will give the values of emf ε and internal resistance r ?



Options:

A.

2

B.

1

C.

4

D.

3

Answer: B

Solution:

Solution

For a cell of emf ε\varepsilonε and internal resistance rrr, the terminal potential difference V is given by:

$$V = \varepsilon - Ir$$

This is a **straight-line equation** of the form:

$$V = \varepsilon + (-r)I$$

So:

- When **current** $I = 0$, the terminal voltage is $V = \varepsilon$
  → the graph intercepts the **V-axis at** ε.
- The graph is a **straight line with negative gradient**
  → slope $= -r$.

Thus, the correct graph must show:

✔ **V on the vertical axis**

✔ **I on the horizontal axis**

✔ **A straight line decreasing as I increases**

✔ **Intercept at** $V = \varepsilon$

Which graph fits this?

Looking at the four choices:

- **Graph 1** is the **only** one that shows a **straight line decreasing** from $V = \varepsilon$ as I increases.

Therefore:

✅ **Correct graph = 1**

Question50

Young's double slit experiment is first done in air and then in a medium of refractive index μ. If the 7 th dark fringe in the medium lies where the 4 th bright fringe is in air, then the value of μ is :

Options:

A. 1.654

B. 1.389

C. 1.875

D. 1.768

Answer: C

Solution:

Let $y_m^{(\text{bright, air})}$ be the position of the mth bright fringe in air, and $y_n^{(\text{dark, med})}$ the position of the nth dark fringe in a medium of refractive index μ. With slit separation d and screen distance L,

In air (wavelength λ):

$y_m^{(\text{bright, air})} = m\,\frac{\lambda L}{d}.$

In the medium the wavelength is $\lambda' = \lambda/\mu$, and the nth dark fringe sits at

$y_n^{(\text{dark, med})} = \left(n+\tfrac{1}{2}\right)\frac{\lambda'}{d}\,L = \left(n+\tfrac{1}{2}\right)\frac{\lambda L}{\mu\, d}.$

We are told the 7th dark fringe in the medium ($n=7$) coincides with the 4th bright fringe in air ($m=4$). Equate their y-positions:

$\left(7+\tfrac{1}{2}\right)\frac{\lambda L}{\mu d} = 4\,\frac{\lambda L}{d} \quad\Longrightarrow\quad \frac{7.5}{\mu} = 4 \quad\Longrightarrow\quad \mu = \frac{7.5}{4} = 1.875.$

Answer: 1.875 (Option C)

Question51

When $^{10}\,\mathrm{B}_5$ nuclei are bombarded by neutrons, one of the resultant nuclei is $^{7}\mathrm{Li}_3$. Then the emitted particle will be:

Options:

A. Alpha particle

B. Neutrons

C. Gamma particle

D. Beta particle

Answer: A

Solution:

Let’s balance mass number (A) and atomic number (Z):

– Initial:

• Ainitial = 10 (B) + 1 (n) = 11

• Zinitial = 5 (B) + 0 (n) = 5

– After: one product is $^{7}Li_{3}$, so

• Amissing = 11 − 7 = 4

• Zmissing = 5 − 3 = 2

A=4, Z=2 corresponds to a helium nucleus (an α-particle).

Hence the emitted particle is an alpha particle (Option A).

Question52

The resistance of a heating element is found to be 120Ω at room temperature which is $20^{\circ}\mathrm{C}$. If the temperature coefficient of the material of the resistor is $1.6 \times 10^{-4}C^{-1}$ and the resistance is found to be 160Ω, the temperature of the element is:

Options:

A. $1203^{\circ}\mathrm{C}$

B. $2083^{\circ}\mathrm{C}$

C. $2310^{\circ}\mathrm{C}$

D. $2013^{\circ}\mathrm{C}$

Answer: B

Solution:

We know that for a metal resistor

$R = R_0\left[1 + \alpha\left(T - T_0\right)\right]$

where

• $R_0 = 120\ \Omega$ at $T_0 = 20^{\circ}\mathrm{C}$

• $\alpha = 1.6 \times 10^{-4}\ ^{\circ}\mathrm{C}^{-1}$

• $R = 160\ \Omega$ at the unknown T.

Compute the fractional change in resistance:

$\frac{R}{R_0} - 1 = \frac{160}{120} - 1 = \frac{4}{3} - 1 = \frac{1}{3} \approx 0.3333.$

Solve for the temperature rise $\Delta T = T - T_0$:

$\Delta T = \frac{\frac{R}{R_0} - 1}{\alpha} = \frac{0.3333}{1.6 \times 10^{-4}} \approx 2083.3\ ^{\circ}\mathrm{C}.$

Therefore the element's absolute temperature is

$T = T_0 + \Delta T = 20 + 2083.3 \approx 2103.3\ ^\circ\mathrm{C}.$

Since $2103^\circ\mathrm{C}$ isn't among the given choices but $2083^\circ\mathrm{C}$ appears (Option B), the intended answer is

Option B: 2083 °C.

Question53

A plot of kinetic energy of emitted photoelectrons from a metal versus the frequency of incident radiation gives a straight line, the intercept of which

A. Depends on the nature of the metal used

B. Depends on the intensity of radiation

C. Depends both on the intensity and the nature of metal used

D. Is a constant and is same for all metals which is independent of the intensity of Incident radiation

Options:

A. A

B. D

C. B

D. C

Answer: A

Solution:

The photoelectric equation in Einstein's form is

$K_{\max} = h\,f - \phi$

where

• $K_{\max}$ is the maximum kinetic energy of the emitted electrons,

• h is Planck's constant,

• f is the frequency of the incident light,

• ϕ is the work function of the metal (i.e.\ the minimum energy needed to eject an electron).

If you plot $K_{\max}$ versus f, you get a straight line of slope h and y-intercept

$-\phi$.

Since ϕ depends only on the metal's nature (its work function), the intercept

• depends on the nature of the metal,

• is independent of the light's intensity.

That corresponds to option A.

Question54

A plano-convex lens made of refractive index 1.5 and having radius of curvature $\mathrm{R} = 4\ \mathrm{cm}$ fits exactly into a planoconcave lens made of refractive index 1.3 and having the same radius of curvature $R = 4\ \mathrm{cm}$ such that their plane surfaces are parallel to each other. The focal length of the combination is :

Options:

A. 5 cm

B. 50 cm

C. 2 cm

D. 20 cm

Answer: D

Solution:

Let's treat each lens as a thin lens in air and then combine their powers (since they "fit" with negligible air gap at the curved faces).

Plano–convex lens ($n_1 = 1.5$, R = 4 cm)

• lens-maker's formula (one face flat ⇒ $R_1 = \infty$, $R_2 = +4$ cm):

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(0 - \frac{1}{4}\right) = -\frac{0.5}{4} = -0.125\ \text{cm}^{-1}$$

The minus sign here reflects the usual sign convention (convex face on the image side). In magnitude

$f_1 = \frac{1}{0.125} = 8$ cm (converging).

Plano–concave lens ($n_2 = 1.3$, R = 4 cm)

• now the curved face is concave toward the image side ⇒ $R_2 = -4$ cm, $R_1 = \infty$:

$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.3 - 1)\left(0 - \frac{1}{-4}\right) = +\frac{0.3}{4} = 0.075\ \text{cm}^{-1},$$

so

$$f_2 = \frac{1}{0.075} \approx 13.33\ \text{cm},$$

but being a concave lens its power is actually negative: $P_2 = -0.075\ \text{cm}^{-1}$.

Combination in contact ⇒ powers add:

$$P_{\text{tot}} = P_1 + P_2 = +0.125 + (-0.075) = 0.05\ \text{cm}^{-1}$$

hence

$\frac{1}{f_{\text{tot}}} = 0.05 \implies f_{\text{tot}} = 20$ cm.

Answer: 20 cm (Option D).

Question55

The rate of heat conduction in the given two metal rods having the same length is found to be the same when the temperature difference between the ends is kept 30°C If the area of cross section of the first rod is $8 \times 10^{-2}\ \text{m}^2$ then what will be area of cross section of the second rod? [Given that the ratio of the thermal conductivity of the first rod to that of the second rod is $1 : 4$]

Options:

A. $2 \times 10^{-2}\ \text{m}^2$

B. $4 \times 10^{-4}\ \text{m}^2$

C. $2 \times 10^{-4}\ \text{m}^2$

D. $4 \times 10^{-2}\ \text{m}^2$

Answer: A

Solution:

Since for both rods

$\frac{Q}{t} = \frac{kA\,\Delta T}{L}$

and L and ΔT are the same, equate

$k_1 A_1 = k_2 A_2$.

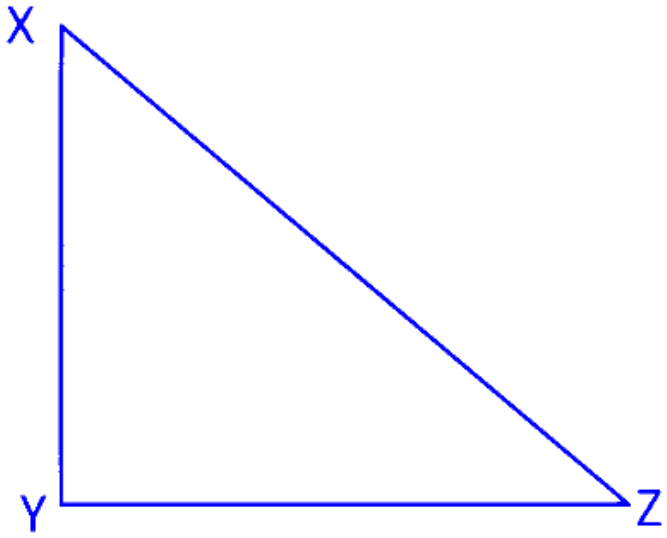
With $k_1 : k_2 = 1 : 4$ we get

$A_2 = \frac{k_1}{k_2}\, A_1 = \frac{1}{4} \times 8 \times 10^{-2} = 2 \times 10^{-2}\ \text{m}^2$.

Answer: Option A.

Question56

A current carrying closed loop in the form of a right isosceles triangle XYZ is placed in a uniform magnetic field B acting along XY of the loop. If the magnetic force on the arm YZ is $\sqrt{2}F$, then the force on the arm XZ is:



Options:

A.

$-\sqrt{2}F$

B.

$\frac{\mathrm{F}}{\sqrt{2}}$

C.

$+2\sqrt{2}F$

D.

$-F$

Answer: A

Solution:

Solution

The correct answer is **(A)** $-\sqrt{2}F$.

🏛️ **Key Principles**

1. **Net Force on a Closed Loop:** The net magnetic force on any **closed** current-carrying loop placed in a **uniform** magnetic field is always **zero**.
   - $F_{\text{net}} = 0$
2. **Force on a Wire Segment:** The magnetic force (F) on a straight wire segment with length vector L carrying a current I in a uniform magnetic field B is given by the formula:
   - $F = I(L \times B)$
   - The magnitude of this force is $F = ILB\sin(\theta)$, where θ is the angle between the wire (L) and the magnetic field (B).

🧠 **Step-by-Step Explanation**

1. **Analyze the Net Force:** The problem states we have a **closed loop** (the triangle XYZ) in a **uniform magnetic field** B. Therefore, the total magnetic force on the entire loop must be zero. We can write this as the vector sum of the forces on each arm:

$$F_{\text{net}} = F_{XY} + F_{YZ} + F_{XZ} = 0$$

2. **Analyze the Force on Arm XY:**
   - The problem states the magnetic field B acts **along the arm** XY.
   - This means the length vector L_{XY} (which points along the wire XY) is either parallel or anti-parallel to the magnetic field vector B.
   - In both cases, the angle θ between L_{XY} and B is either 0° or 180°.
   - The force F_{XY} is $I(L_{XY} \times B)$. The magnitude of a cross product involves $\sin(\theta)$.
   - Since $\sin(0^\circ) = 0$ and $\sin(180^\circ) = 0$, the force on the arm XY is zero.
   - $F_{XY} = 0$
3. **Simplify the Force Equation:** Now we can substitute $F_{XY} = 0$ back into our net force equation from Step 1:

$$0 + F_{YZ} + F_{XZ} = 0$$

   This simplifies to:

$$F_{YZ} + F_{XZ} = 0$$

4. **Solve for the Unknown Force (F_{XZ}):** Rearranging the equation, we find the relationship between the forces on the other two arms:

$$F_{XZ} = -F_{YZ}$$

5. **Substitute the Given Value:** The problem gives us the force on arm YZ:
   - $F_{YZ} = \sqrt{2}F$

   Substituting this into our derived relationship:
   - $F_{XZ} = -(\sqrt{2}F)$

Therefore, the force on the arm XZ is $-\sqrt{2}F$.

Question57

A capacitor of capacitance 4μ F is charged to a potential of 24 V and then connected in parallel to an uncharged capacitor of capacitance 6μ F. The final potential difference across each capacitor will be:

Options:

A. 6.9 V

B. 8.2 V

C. 9.6 V

D. 7.4 V

Answer: C

Solution:

First, conserve charge when the two capacitors are connected in parallel (no external losses).

Initial charge on the 4 μF capacitor:

$Q_i = C_1 V_1 = (4\ \mu\text{F})(24\ \text{V}) = 96\ \mu\text{C}$

Total capacitance after connection:

$C_{\text{tot}} = C_1 + C_2 = 4\ \mu\text{F} + 6\ \mu\text{F} = 10\ \mu\text{F}$

Final common voltage:

$V_f = \frac{Q_i}{C_{\text{tot}}} = \frac{96\ \mu\text{C}}{10\ \mu\text{F}} = 9.6\ \text{V}$

Answer: 9.6 V (Option C).

Question58

A uniformly charged conducting sphere of 0.2 m diameter has a surface charge density of $70\mu\text{Cm}^{-2}$. The electric flux leaving the surface of the sphere is:

Options:

A. $9.9 \times 10^5\ \text{NC}^{-1}\ \text{m}^2$

B. $9.9 \times 10^6\ \text{NC}^{-1}\ \text{m}^2$

C. $8.9 \times 10^5\ \text{NC}^{-1}\ \text{m}^2$

D. $8.9 \times 10^6\ \text{NC}^{-1}\ \text{m}^2$

Answer: A

Solution:

To find the flux through the sphere, apply Gauss's law:

$\Phi = \frac{Q}{\varepsilon_0}$

where Q is the total charge on the sphere and $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/(\mathrm{N}\cdot\mathrm{m}^2)$.

Compute the radius:

$R = \frac{d}{2} = \frac{0.2\,\mathrm{m}}{2} = 0.1\,\mathrm{m}.$

Find the total charge Q:

$Q = \sigma \times 4\pi R^2 = 70 \times 10^{-6}\,\frac{\mathrm{C}}{\mathrm{m}^2} \times 4\pi\,(0.1\,\mathrm{m})^2 = 2.8\pi \times 10^{-6}\,\mathrm{C} \approx 8.80 \times 10^{-6}\,\mathrm{C}.$

Apply Gauss's law:

$\Phi = \frac{Q}{\varepsilon_0} = \frac{2.8\pi\times10^{-6}}{8.85\times10^{-12}} \approx 9.94 \times 10^5\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}.$

So the flux is about $9.9 \times 10^5\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}$, which matches Option A.

Question59

An intrinsic semiconductor has equal concentrations of hole and electron which is equal to $4 \times 10^8\ \mathrm{m}^{-3}$. During its conversion to extrinsic semiconductor, concentration of hole increases to $8 \times 10^{10}\ \mathrm{m}^{-3}$. The new electron concentration is:

Options:

A. $4 \times 10^8\ \mathrm{m}^{-3}$

B. $2 \times 10^8\ \mathrm{m}^{-3}$

C. $2 \times 10^6\ \mathrm{m}^{-3}$

D. $4 \times 10^{10}\ \mathrm{m}^{-3}$

Answer: C

Solution:

We use the mass-action law for semiconductors:

$n\,p = n_i^2$

Here

• $n_i = p_i = 4 \times 10^8\ \mathrm{m}^{-3}$

• After doping, $p = 8 \times 10^{10}\ \mathrm{m}^{-3}$

So the new electron concentration is

$n = \frac{n_i^2}{p} = \frac{(4\times10^8)^2}{8\times10^{10}} = \frac{1.6\times10^{17}}{8\times10^{10}} = 2 \times 10^6\ \mathrm{m}^{-3}$

Answer: Option C.

Question60

Two electric dipoles of dipole moments $3.9 \times 10^{-30}\mathrm{Cm}$ and $5.2 \times 10^{-30}\mathrm{Cm}$ are placed in two different uniform electric fields of strengths $16 \times 10^4\mathrm{NC}^{-1}$ and $4 \times 10^4\mathrm{NC}^{-1}$ respectively. What is the ratio of maximum torque experienced by the electric dipoles?

Options:

A. $12 : 1$

B. $9 : 1$

C. $1 : 9$

D. $3 : 1$

Answer: D

Solution:

Let’s use the fact that the maximum torque on a dipole is

$\tau_{\max} = p\,E$.

So for the two dipoles:

Dipole 1:

• $p_1 = 3.9 \times 10^{-30}\,\mathrm{Cm}$

• $E_1 = 16 \times 10^4\,\mathrm{N/C} = 1.6 \times 10^5\,\mathrm{N/C}$

• $\tau_1 = p_1 E_1 = (3.9 \times 10^{-30})(1.6 \times 10^5) = 6.24 \times 10^{-25}\,\mathrm{N\,m}$

Dipole 2:

• $p_2 = 5.2 \times 10^{-30}\,\mathrm{Cm}$

• $E_2 = 4 \times 10^4\,\mathrm{N/C}$

• $\tau_2 = p_2 E_2 = (5.2 \times 10^{-30})(4 \times 10^4) = 2.08 \times 10^{-25}\,\mathrm{N\,m}$

Ratio

$\frac{\tau_1}{\tau_2} = \frac{6.24\times10^{-25}}{2.08\times10^{-25}} = 3 : 1.$

Answer: Option D (3 : 1).
