IAT (IISER) 2025

Mathematics

Question1

How many three digit numbers divisible by 5 are there in which no digits are repeated?

Options:

A.

136

B.

128

C.

144

D.

162

Answer: A

Solution:

To find the number of three-digit numbers divisible by 5 with no repeated digits, let the three-digit number be represented as ABC, where A, B, and C are digits.

We have the following conditions:

1. **Three-digit number:** A must be a digit from 1 to 9 ($A \in \{1, 2, \ldots, 9\}$). B and C can be any digit from 0 to 9 ($B, C \in \{0, 1, \ldots, 9\}$).
2. **Divisible by 5:** The last digit C must be either 0 or 5.
3. **No repeated digits:** $A \neq B$, $B \neq C$, and $A \neq C$.

We can analyze this problem by considering the two possible values for C.

Case 1: The last digit C is 0.

If C = 0, the number is of the form AB0.

- **For C:** There is only 1 choice (0).
- **For A:** A cannot be 0 (Property 1, three-digit number). Also, A cannot be C (Property 3, no repeated digits), so A cannot be 0. Thus, A can be any digit from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. This gives 9 choices for A.
- **For B:** B cannot be C (Property 3), so B cannot be 0. B also cannot be A (Property 3). Since A is one of the 9 non-zero digits, B must be chosen from the remaining 8 non-zero digits. (e.g., if A=1, B can be any of 2,3,4,5,6,7,8,9). This gives 8 choices for B.

Total numbers for Case 1 = (Choices for A) × (Choices for B) × (Choices for C)

Total = 9 × 8 × 1 = 72.

Case 2: The last digit C is 5.

If C = 5, the number is of the form AB5.

- **For C:** There is only 1 choice (5).
- **For A:** A cannot be 0 (Property 1). A also cannot be C (Property 3), so A cannot be 5.

  Therefore, A can be any digit from $\{1, 2, 3, 4, 6, 7, 8, 9\}$. This gives 8 choices for A.
- **For B:** B cannot be C (Property 3), so B cannot be 5. B also cannot be A (Property 3).

  The available digits for B are all digits from 0 to 9, excluding C (5) and excluding A.

  Since there are 10 total digits, excluding 5 leaves 9 digits. Excluding A (which is one of the 8 choices for A) leaves 8 digits. So, B has 8 choices.

  (e.g., if A=1, B can be any of 0, 2, 3, 4, 6, 7, 8, 9).

Total numbers for Case 2 = (Choices for A) × (Choices for B) × (Choices for C)

Total = 8 × 8 × 1 = 64.

Total Number of such three-digit numbers

Adding the numbers from both cases:

Total = 72 (from Case 1) + 64 (from Case 2) = 136.

Thus, there are 136 three-digit numbers divisible by 5 in which no digits are repeated.

The final answer is $\boxed{136}$.

Question2

Let A be a 3×3 matrix with real entries such that

$$A = \begin{bmatrix} 4 & -1 & \cos x \\ -1 & 5x & 25 \\ x^2+1 & 25 & 7 \end{bmatrix}$$

For how many values of x, the matrix A is symmetric?

Options:

A.

1

B.

2

C.

4

D.

infinitely many

Answer: A

Solution:

To make the matrix

$$A = \begin{pmatrix} 4 & -1 & \cos x \\ -1 & 5x & 25 \\ x^2+1 & 25 & 7 \end{pmatrix}$$

symmetric, we must have $A_{ij} = A_{ji}$ for all i, j.

Step 1: Compare symmetric entries

1. $A_{12} = A_{21}$

$$-1 = -1 \quad \checkmark$$

Already symmetric.

2. $A_{23} = A_{32}$

$$25 = 25 \quad \checkmark$$

Already symmetric.

3. $A_{13} = A_{31}$

This is the **only** nontrivial condition:

$$\cos x = x^2 + 1$$

Step 2: Solve the equation

We must solve:

$$\cos x = x^2 + 1$$

Notice:

- $\cos x \leq 1$ for all real x
- $x^2 + 1 \geq 1$, with equality only at $x = 0$

Thus for equality to hold:

$$\cos x = 1 \quad \text{and} \quad x^2 + 1 = 1$$

The second gives:

$$x^2 = 0 \quad \Rightarrow \quad x = 0$$

Check:

$$\cos 0 = 1, \quad 0^2 + 1 = 1$$

✓ Works.

No other value can work because for any $x \neq 0$,

$$x^2 + 1 > 1 \quad \text{but} \quad \cos x \leq 1$$

Impossible.

Answer A: 1 ✅

Question3

Let $n = \sum_{r=0}^{10} (-1)^r \, {}^{10}C_r \left(\frac{2}{3}\right)^{2r} 3^{20}$. Which one of the following statements is TRUE?

Options:

A.

n is divisible by 5

B.

n is divisible by 6

C.

n is divisible by 8

D.

n is divisible by 9

Answer: A

Solution:

Let's restate the problem carefully:

$n = \sum_{r=0}^{10}(-1)^r \binom{10}{r} \left(\frac{2}{3}\right)^{2r} 3^{20}$

We are asked to determine which of the given statements about divisibility is **true**.

Step 1: Simplify the power expression

$\left(\frac{2}{3}\right)^{2r} = \frac{2^{2r}}{3^{2r}} = \frac{4^r}{3^{2r}}$

So,

$n = 3^{20} \sum_{r=0}^{10}(-1)^r \binom{10}{r} \frac{4^r}{3^{2r}}$

$n = 3^{20} \sum_{r=0}^{10} \binom{10}{r}(-1)^r \left(\frac{4}{9}\right)^r$

Step 2: Recognize as a binomial expansion

$(1-x)^{10} = \sum_{r=0}^{10}(-1)^r \binom{10}{r} x^r$

Compare: here $x = \frac{4}{9}$

$\implies \sum_{r=0}^{10}(-1)^r \binom{10}{r} \left(\frac{4}{9}\right)^r = (1-\frac{4}{9})^{10} = \left(\frac{5}{9}\right)^{10}$

Step 3: Substitute back

$n = 3^{20} \left(\frac{5}{9}\right)^{10} = 3^{20} \cdot \frac{5^{10}}{9^{10}}$

But $9^{10} = (3^2)^{10} = 3^{20}$.

So:

$n = 3^{20} \cdot \frac{5^{10}}{3^{20}} = 5^{10}$

Step 4: Now we know $n = 5^{10}$

Let's check divisibility:

- 5^{10} is divisible by **5**

- Not divisible by 6 (needs factor 2,3)
- Not divisible by 8 (no factor 2)
- Not divisible by 9 (no factor 3)

✅ **Therefore, n is divisible by 5 only.**

✅ **Final Answer: Option A — n is divisible by 5.**

Question4

Let $f : \mathbf{R} \to \mathbf{R}$ be the function given by $f(x) = \cos\left(\tan^{-1} x\right)$. Which one of the following statements is TRUE?

Options:

A.

f is decreasing for $x > 0$

B.

f is decreasing for $x < 0$

C.

f is decreasing on $\mathbf{R}$

D.

f is decreasing on the interval $(-1, 1)$

Answer: A

Solution:

We are given

$$f(x) = \cos(\tan^{-1} x).$$

Let

$$\theta = \tan^{-1}(x) \quad \Rightarrow \quad \tan\theta = x.$$

From the identity

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}},$$

we get

$$f(x) = \frac{1}{\sqrt{1+x^2}}.$$

Differentiate $f(x)$:

$$f(x) = (1+x^2)^{-1/2}.$$

$$f'(x) = -\tfrac{1}{2}(1+x^2)^{-3/2} \cdot 2x$$

$$\boxed{f'(x) = -\frac{x}{(1+x^2)^{3/2}}}.$$

Sign of the derivative:

- For $x > 0$, numerator $-x < 0 \to f'(x) < 0 \to$ *function is decreasing.*
- For $x < 0$, numerator $-x > 0 \to f'(x) > 0 \to$ *function is increasing.*
- It is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$ — not monotonic on all of $\mathbb{R}$.
- On $(-1, 1)$, it increases on $(-1, 0)$ and decreases on $(0, 1)$, so not decreasing there.

✅ Correct Statement:

$$\boxed{\text{A: } f \text{ is decreasing for } x > 0.}$$

Question5

Let

$$A = \left\{ x \in \mathbf{R} \,\middle|\, -31 < \det \begin{bmatrix} 3x-1 & 2 \\ -2 & 5 \end{bmatrix} \leq 29 \right\}$$

Which one of the following statements is TRUE?

Options:

A.

$A = (-2, 2]$

B.

$A = (-2, 2)$

C.

$A = [-2, 2)$

D.

$A = [-2, 2]$

Answer: A

Solution:

$$A = \left\{ x \in \mathbb{R} \,\middle|\, -31 < \det \begin{bmatrix} 3x-1 & 2 \\ -2 & 5 \end{bmatrix} \leq 29 \right\}$$

Step 1. Compute the determinant

$$\det \begin{bmatrix} 3x-1 & 2 \\ -2 & 5 \end{bmatrix} = (3x-1)(5) - (2)(-2)$$

Simplify:

$$= 15x - 5 + 4 = 15x - 1$$

So the condition becomes:

$$-31 < 15x - 1 \leq 29$$

Step 2. Solve the inequalities

Left side:

$$-31 < 15x - 1$$

Add 1 to both sides:

$-30 < 15x$

Divide by 15:

$-2 < x$

Right side:

$15x - 1 \leq 29$

Add 1 to both sides:

$15x \leq 30$

Divide by 15:

$x \leq 2$

Step 3. Combine

$-2 < x \leq 2$

Thus,

$A = (-2, 2]$

✅ **Correct Option: A.** $A = (-2, 2]$

Question6

Let z_1, z_2, and z_3 be complex numbers satisfying the following conditions

$$2 = |2z_1| = |z_2 - 1| = |z_3 + 1| = \left|\frac{1}{z_1} + \frac{1}{z_2 - 1} + \frac{1}{z_3 + 1}\right|.$$

What is the value of $|4z_1 + z_2 + z_3|$?

Options:

A.

8

B.

4

C.

$\frac{1}{4}$

D.

$\frac{1}{8}$

Answer: A

Solution

Given

$$2 = |2z_1| = |z_2 - 1| = |z_3 + 1| = \left|\frac{1}{z_1} + \frac{1}{z_2 - 1} + \frac{1}{z_3 + 1}\right|.$$

From the first three equalities:

- $|2z_1| = 2 \Rightarrow |z_1| = 1.$
- $|z_2 - 1| = 2 \Rightarrow \left|\frac{1}{z_2-1}\right| = \frac{1}{2}.$
- $|z_3 + 1| = 2 \Rightarrow \left|\frac{1}{z_3+1}\right| = \frac{1}{2}.$

Let

$$a = \frac{1}{z_1}, \quad b = \frac{1}{z_2 - 1}, \quad c = \frac{1}{z_3 + 1}.$$

Then:

$$|a| = 1, \quad |b| = \frac{1}{2}, \quad |c| = \frac{1}{2}.$$

And we know:

$$|a + b + c| = 2.$$

Key Observation (Equality condition)

$$|a| + |b| + |c| = 1 + \frac{1}{2} + \frac{1}{2} = 2.$$

Since

$|a + b + c| = |a| + |b| + |c|,$

equality in the triangle inequality means all three complex numbers have the same argument.

Thus there exists some angle θ such that

$$a = e^{i\theta}, \qquad b = \frac{1}{2}e^{i\theta}, \qquad c = \frac{1}{2}e^{i\theta}.$$

Now invert:

Compute the z_k

$$z_1 = \frac{1}{a} = e^{-i\theta},$$

$$z_2 - 1 = \frac{1}{b} = 2e^{-i\theta} \Rightarrow z_2 = 1 + 2e^{-i\theta},$$

$$z_3 + 1 = \frac{1}{c} = 2e^{-i\theta} \Rightarrow z_3 = -1 + 2e^{-i\theta}.$$

Compute the target expression

$$4z_1 + z_2 + z_3 = 4e^{-i\theta} + (1 + 2e^{-i\theta}) + (-1 + 2e^{-i\theta})$$

Combine like terms:

- Constants: $1 - 1 = 0$
- Complex terms: $4e^{-i\theta} + 2e^{-i\theta} + 2e^{-i\theta} = 8e^{-i\theta}$

Thus:

$$4z_1 + z_2 + z_3 = 8e^{-i\theta}.$$

So its magnitude is:

$$|4z_1 + z_2 + z_3| = |8e^{-i\theta}| = 8.$$

✅ **Final Answer: 8**

Question7

Let $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = |x^3 - 3x|[x]$, where $[x]$ denotes the greatest integer less than or equal to x. Which one of the following statements is TRUE?

Options:

A.

Every non-zero integer is a point of discontinuity of f

B.

f is continuous at every real number

C.

Every integer is a point of discontinuity of f

D.

f is continuous at every real number except for $0, \pm\sqrt{3}$

Answer: A

Solution:

Given

$$f(x) = |x^3 - 3x\lfloor x \rfloor|$$

The *only* source of discontinuity could come from the floor function $\lfloor x \rfloor$, which is discontinuous at **every integer**.

So we only need to check **whether the expression inside the absolute value jumps at integers.**

Let $n \in \mathbb{Z}$.
Consider the two one-sided limits as $x \to n$:

Left-hand limit $x \to n^-$

$$\lfloor x \rfloor = n - 1$$

$$f(n^-) = |n^3 - 3n(n-1)|$$

Right-hand limit $x \to n^+$

$$\lfloor x \rfloor = n$$

$$f(n^+) = |n^3 - 3n(n)| = |n^3 - 3n^2|$$

Value at n

$$f(n) = |n^3 - 3n(n)| = |n^3 - 3n^2|$$

Thus:

- **Right-hand limit equals the function value** at every integer.
- Possible discontinuity occurs only when the **left-hand limit** differs from the value.

Compute the difference:

Left:

$$L = |n^3 - 3n(n-1)| = |n^3 - 3n^2 + 3n|$$

Right:

$$R = |n^3 - 3n^2| = |n^2(n-3)|$$

Check when $L = R$.

We need:

$$|n^3 - 3n^2 + 3n| = |n^3 - 3n^2|$$

Factor both:

$$L = |n(n^2 - 3n + 3)|$$

$$R = |n^2(n-3)|$$

If $n = 0$:
Left = |0| = 0; Right = |0| = 0 → **continuous at 0**.

For **non-zero** n, compare inside absolute values:

Divide both sides by $|n|$:

$$|n^2 - 3n + 3| \stackrel{?}{=} |n(n-3)|$$

Expand right:

Right = $|n^2 - 3n|$

So condition:

$$|n^2 - 3n + 3| = |n^2 - 3n|$$

This holds only if the added constant $+3$ does not change the magnitude, which is impossible for nonzero n
.

So **no nonzero integer satisfies the equality.**

Therefore:

Discontinuous at every non-zero integer

Final Result

✔ Continuous at 0

✗ Discontinuous at every non-zero integer

So the correct statement is:

A. Every non-zero integer is a point of discontinuity of f.

Question8

Let ℓ be the tangent line to the ellipse $x^2 + 16y^2 = 4$ at $\left(1, \frac{\sqrt{3}}{4}\right)$. What is the equation of the line perpendicular to ℓ passing through $(2, 0)$?

Options:

A.

$y = 4\sqrt{3}(x - 2)$

B.

$y = 2\sqrt{3}(x - 2)$

C.

$y = \sqrt{3}(x - 2)$

D.

$4\sqrt{3}y = (x - 2)$

Answer: A

Solution:

Step 1: Find the slope of the tangent line to the ellipse

The ellipse is

$$x^2 + 16y^2 = 4.$$

Differentiate implicitly:

$$2x + 32y\,y' = 0$$

Solve for y':

$$y' = -\frac{x}{16y}.$$

Evaluate at the point

$$(1,\ \tfrac{\sqrt{3}}{4}).$$

Plug in:

$$y' = -\frac{1}{16 \cdot (\sqrt{3}/4)} = -\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12}.$$

So the slope of the tangent line ℓ is

$$m_\ell = -\frac{\sqrt{3}}{12}.$$

Step 2: Find the slope of the perpendicular line

The slope of any line perpendicular to ℓ is:

$$m_\perp = -\frac{1}{m_\ell} = -\frac{1}{-\frac{\sqrt{3}}{12}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}.$$

Step 3: Line perpendicular to ℓ passing through $(2, 0)$

Point-slope form:

$$y - 0 = 4\sqrt{3}(x - 2)$$

Thus the line is:

$$\boxed{y = 4\sqrt{3}(x - 2)}.$$

Final Answer: A.

Question9

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}| = 15$ and

$\vec{a} \times (3\hat{i} - 4\hat{j} + 5\hat{k}) = (3\hat{i} - 4\hat{j} + 5\hat{k}) \times \vec{b}$

What is the value of $|(\vec{a}+\vec{b}) \cdot (2\hat{i} + 3\hat{j} + \hat{k})|$?

Options:

A.

$\frac{3}{\sqrt{2}}$

B.

0

C.

$\sqrt{2}$

D.

3

Answer: A

Solution:

Given:

- $\vec{a}, \vec{b}$ with $|\vec{a} + \vec{b}| = 15$
- $\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$, where $\vec{c} = 3\hat{i} - 4\hat{j} + 5\hat{k}$.

We need $\left|(\vec{a} + \vec{b}) \cdot (2\hat{i} + 3\hat{j} + \hat{k})\right|$.

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1. Use the cross–product condition

$$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$$

but $\vec{c} \times \vec{b} = -\vec{b} \times \vec{c}$.
So:

$$\vec{a} \times \vec{c} = -\vec{b} \times \vec{c}$$

Add $\vec{b} \times \vec{c}$ to both sides:

$$(\vec{a} + \vec{b}) \times \vec{c} = 0$$

If a vector's cross product with $\vec{c}$ is 0, then it is parallel to $\vec{c}$.
Thus:

$$\vec{a} + \vec{b} = k\vec{c}$$

for some scalar k.

2. Find k from the magnitude condition

$$|\vec{a}+\vec{b}| = |k\vec{c}| = |k|\,|\vec{c}| = 15$$

First compute $|\vec{c}|$:

$$|\vec{c}| = \sqrt{3^2+(-4)^2+5^2} = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$$

So:

$$|k|\cdot 5\sqrt{2} = 15 \quad \Rightarrow \quad |k| = \frac{15}{5\sqrt{2}} = \frac{3}{\sqrt{2}}$$

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3. Compute the required dot product

Let $\vec{d} = 2\hat{i} + 3\hat{j} + \hat{k}$. Then:

$$(\vec{a}+\vec{b})\cdot\vec{d} = (k\vec{c})\cdot\vec{d} = k(\vec{c}\cdot\vec{d})$$

First find $\vec{c}\cdot\vec{d}$:

$$\vec{c}\cdot\vec{d} = (3,-4,5)\cdot(2,3,1) = 3\cdot 2 + (-4)\cdot 3 + 5\cdot 1 = 6 - 12 + 5 = -1$$

Hence

$$\left|(\vec{a}+\vec{b})\cdot\vec{d}\right| = |k||\vec{c}\cdot\vec{d}| = \frac{3}{\sqrt{2}}\cdot|-1| = \frac{3}{\sqrt{2}}$$

Answer: $\boxed{\dfrac{3}{\sqrt{2}}}$ ✅

Question10

What is the derivative of $\log\left(\sin^2 x\right)$ with respect to $\sin x$?

Options:

A.

$2\,\text{cosec}\,x$

B.

$\sin 2x$

C.

$4\,\text{cosec}\,x$

D.

$\cot x \operatorname{cosec} 2x$

Answer: A

Solution:

To find

$$\frac{d}{d(\sin x)}\left[\log(\sin^2 x)\right],$$

treat $\sin x$ as the variable.

Step 1: Rewrite the expression

$$\log(\sin^2 x) = \log\left[(\sin x)^2\right]$$

Let

$$u = \sin x.$$

Then

$$\log(\sin^2 x) = \log(u^2) = 2\log u.$$

Step 2: Differentiate with respect to $u = \sin x$

$$\frac{d}{du}(2\log u) = \frac{2}{u}.$$

Now substitute $u = \sin x$:

$$\frac{2}{\sin x} = 2\csc x.$$

✅ **Final Answer: $2\csc x$**

Question11

Let S_n denote the sum of the first n terms of a sequence $a_1, a_2, a_3, \ldots$.. If $S_{n+3} - S_n = 13n + 7$ for all n, what is the value of $a_{13} - a_{10}$?

Options:

A.

13

B.

137

C.

46

D.

12

Answer: A

Solution:

We are told:

$S_{n+3} - S_n = 13n + 7, \quad \forall n.$

But $S_n = a_1 + a_2 + \ldots + a_n$.

So

$S_{n+3} - S_n = (a_{n+1} + a_{n+2} + a_{n+3}) = 13n + 7.$

Step 1:

We can write equations for different n.

For $n, n+1, n+2$, etc.

At n:

$a_{n+1} + a_{n+2} + a_{n+3} = 13n + 7.$

At $n+1$:

$a_{n+2} + a_{n+3} + a_{n+4} = 13(n+1) + 7 = 13n + 20.$

Now subtract the two:

$(a_{n+2} + a_{n+3} + a_{n+4}) - (a_{n+1} + a_{n+2} + a_{n+3}) = (13n + 20) - (13n + 7).$

Simplify:

$a_{n+4} - a_{n+1} = 13.$

Step 2:

So we have the recurrence:

$a_{n+4} = a_{n+1} + 13.$

This means the sequence increases by 13 every 3 steps (since $a_{n+4} - a_{n+1} = 13$).

So for $a_{13} - a_{10}$:

These indices differ by 3:

$a_{13} - a_{10} = 13.$

✅ **Final Answer:** $a_{13} - a_{10} = 13.$

Answer: Option A (13)

Question12

Five fair coins are tossed independently. What is the probability that at least two heads appear?

Options:

A.

$\frac{13}{16}$

B.

$\frac{7}{16}$

C.

$\frac{5}{16}$

D.

$\frac{11}{16}$

Answer: A

Solution:

We toss **5 fair coins** independently.

We are asked for the probability that **at least two heads** appear.

Step 1. Total possible outcomes

Each coin has 2 outcomes → total outcomes = $2^5 = 32.$

Step 2. Find the complement

It's often easier to use the complement:

$P(\text{at least 2 heads}) = 1 - P(\text{fewer than 2 heads})$

"Fewer than 2 heads" means **0 heads** or **1 head**.

Step 3. Compute $P(0 \text{ heads})$ and $P(1 \text{ head})$

$P(0 \text{ heads}) = \frac{\binom{5}{0}}{2^5} = \frac{1}{32}$

$P(1 \text{ head}) = \frac{\binom{5}{1}}{2^5} = \frac{5}{32}$

So,

$P(\text{fewer than 2 heads}) = \frac{1+5}{32} = \frac{6}{32} = \frac{3}{16}$

Step 4. Compute the desired probability

$P(\text{at least 2 heads}) = 1 - \frac{3}{16} = \frac{13}{16}$

✅ **Final Answer:**

$$\boxed{\frac{13}{16}}$$

Correct Option: A

Question13

Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by

$$f(x) = \begin{cases} x^2 - 4x - 5 & \text{if } x \geq 1, \\ 2x & \text{if } x < 1. \end{cases}$$

Which one of the following statements is TRUE?

Options:

A.

f is onto but not one-one

B.

f is one-one but not onto

C.

f is neither one-one nor onto

D.

f is one-one and onto

Answer: A

Solution:

The function is

$$f(x) = \begin{cases} x^2 - 4x - 5, & x \geq 1 \\ 2x, & x < 1 \end{cases}$$

We check **injectivity** (one-one) and **surjectivity** (onto).

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1. Check Injectivity

For $x < 1$:

$f(x) = 2x$, which is linear and strictly increasing ⇒ **injective on** $(-\infty, 1)$.

For $x \geq 1$:

$f(x) = x^2 - 4x - 5$.
This is a parabola opening upward.
Its derivative:

$$f'(x) = 2x - 4$$

At $x = 2$, derivative changes sign — so the function **decreases on** $[1, 2]$ and **increases for** $x > 2$.
Hence it is **not injective on the interval** $x \geq 1$.

Check across intervals

We can test if values from the quadratic part overlap with the linear part.

Compute $f(1)$:

$$f(1) = 1^2 - 4(1) - 5 = -8.$$

Compute limit from left:

$$\lim_{x \to 1^-} f(x) = 2(1) = 2.$$

So the range of the two pieces **overlaps** (since the quadratic takes many values below 2, including −8, −9,... etc., and linear part takes all values <2).
Thus **the function is not one-one.**

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2. Check Surjectivity

Check if all real values are attained.

Range from $x < 1$:

$f(x) = 2x$ where $x < 1$.
Range:

$$(-\infty, 2)$$

Range from $x \geq 1$:

Quadratic part:

$$f(x) = x^2 - 4x - 5 = (x-2)^2 - 9$$

Minimum value occurs at $x = 2$:

$$f(2) = -9.$$

As $x \to \infty$, $f(x) \to \infty$.
Thus range from this part is:

$$[-9, \infty)$$

Combined range

$$(-\infty, 2) \cup [-9, \infty) = (-\infty, \infty) = \mathbb{R}$$

So **every real number is achieved.**

Thus f **is onto.**

✔ **Correct Statement: A — f is onto but not one-one**

Question14

Which one of the following is the solution of the differential equation

$$x^2 \frac{dy}{dx} + 9xy = x^4 (\text{ for } x > 0)$$

given that $y = 0$ when $x = 1$?

Options:

A.

$12y = x^3 - \frac{1}{x^9}$

B.

$12y = x^9 - \frac{1}{x^3}$

C.

$9y = x^{21} - \frac{1}{x^3}$

D.

$9y = x^3 - \frac{1}{x^{21}}$

Answer: A

Solution:

Given differential equation

$$x^2 \frac{dy}{dx} + 9xy = x^4, \qquad (x > 0)$$

Divide both sides by x^2:

$$\frac{dy}{dx} + \frac{9}{x} y = x^2$$

This is a linear first-order ODE.

Step 1: Find integrating factor

$$\mu(x) = e^{\int \frac{9}{x} dx} = e^{9 \ln x} = x^9$$

Step 2: Multiply equation by integrating factor

$$x^9 \frac{dy}{dx} + 9x^8 y = x^{11}$$

Left side becomes a product derivative:

$$\frac{d}{dx}(x^9 y) = x^{11}$$

Step 3: Integrate both sides

$$x^9y = \int x^{11}dx = \frac{x^{12}}{12} + C$$

Thus,

$$y = \frac{x^3}{12} + Cx^{-9}$$

Step 4: Apply initial condition

Given: $y(1) = 0$

$$0 = \frac{1}{12} + C \quad \Rightarrow \quad C = -\frac{1}{12}$$

So the solution is:

$$y = \frac{x^3}{12} - \frac{1}{12x^9}$$

Multiply both sides by 12:

$$12y = x^3 - \frac{1}{x^9}$$

✅ **Final Answer: Option A**

$$\boxed{12y = x^3 - \frac{1}{x^9}}$$

Question15

What is the value of $\int_0^\pi x|\cos x|\sin x dx$?

Options:

A.

$\frac{\pi}{2}$

B.

$\frac{\pi}{4}$

C.

π

D.

$\frac{\pi}{6}$

Answer: A

Solution:

To solve

$$\int_0^{\pi} x\,|\cos x|\,\sin x\,dx,$$

note the sign change of $\cos x$ on $[0, \pi]$:

- On $0 \le x \le \frac{\pi}{2}$, $\cos x \ge 0$, so $|\cos x| = \cos x$.
- On $\frac{\pi}{2} \le x \le \pi$, $\cos x \le 0$, so $|\cos x| = -\cos x$.

Thus split the integral:

$$\int_0^{\pi} x|\cos x|\sin x\,dx = \int_0^{\pi/2} x\cos x\sin x\,dx + \int_{\pi/2}^{\pi} x(-\cos x)\sin x\,dx.$$

Use the identity:

$$\cos x\sin x = \frac{1}{2}\sin(2x)$$

So:

$$= \frac{1}{2}\left[\int_0^{\pi/2} x\sin(2x)\,dx - \int_{\pi/2}^{\pi} x\sin(2x)\,dx\right].$$

Compute the two integrals (integration by parts), or evaluate symbolically.

Carrying it out (or verifying):

$$\int_0^{\pi} x|\cos x|\sin x\,dx = \frac{\pi}{2}.$$

✅ Final Answer: $\boxed{\frac{\pi}{2}}$

Biology

Question1

Match the entries in column I and column II.

Column I		Column II	
P.	Notochord and hollow nerve cord present	i.	Cyclostomata
Q.	Ectoparasite with 6-15 pairs of gills and closed circulation	ii.	Chondrichthyes
R.	Marine animals with persistent notochord and placoid scales	iii.	Hemichordata
S.	Animals with open circulatory systems, and stomochord	iv.	Chordata

Which one of the following combinations is correct?

Options:

A.

P - iv; Q - i; R – ii; S – iii

B.

P - iv; Q - ii; R - i; S - iii

C.

P – i; Q – iii; R – ii; S – iv

D.

P – iii; Q – i; R – ii; S – iv

Answer: A

Solution:

Column I

P. Notochord and hollow nerve cord present

→ These are defining features of **Phylum Chordata**.

✅ So, **P → iv (Chordata)**

Q. Ectoparasite with 6–15 pairs of gills and closed circulation

→ Ectoparasitic, jawless vertebrates with multiple gill pairs and closed circulation are **Cyclostomes** (e.g. lamprey).

✅ So, **Q → i (Cyclostomata)**

R. Marine animals with persistent notochord and placoid scales

→ These features are typical of **cartilaginous fishes (Chondrichthyes)**—like sharks, skates, and rays—where notochord persists and placoid scales are present.

✅ So, **R → ii (Chondrichthyes)**

S. Animals with open circulatory systems, and stomochord

→ Stomochord and open circulation characterize **Hemichordata** (acorn worms).

✅ So, **S → iii (Hemichordata)**

Therefore, the correct matching is

$P - iv; \quad Q - i; \quad R - ii; \quad S - iii$

✅ **Option A** is correct.

Question2

Chromosomes are classified as metacentric, sub-metacentric, acrocentric and telocentric. This classification is based on the position of which one of the following structures?

Options:

A.

Centromere

B.

Centrosome

C.

Centriole

D.

Telomere

Answer: A

Solution:

The classification of chromosomes as **metacentric, sub-metacentric, acrocentric**, and **telocentric** is based on the **position of the centromere**.

✅ **Correct Answer: Option A — Centromere**

Explanation:

- **Metacentric**: Centromere is in the middle; both arms are equal.
- **Sub-metacentric**: Centromere is slightly off-center; arms are unequal.
- **Acrocentric**: Centromere is near one end; one arm is very short.
- **Telocentric**: Centromere is at the terminal end; effectively one arm.

Question3

Which one of the following options describes a triglyceride?

Options:

A.

Three fatty acid chains linked to a molecule of glycerol

B.

Three glycerol molecules linked to a fatty acid chain

C.

Three saturated fatty acid chains linked to a molecule of cholesterol

D.

Three glyceride molecules linked to a molecule of phospholipid

Answer: A

Solution:

The correct description of a triglyceride is:

- **Triglyceride** = **tri-** (meaning three) + **glyceride** (referring to glycerol as the backbone).
- It is formed when **three fatty acid chains** are joined to a single molecule of **glycerol** through ester bonds.

Let's evaluate the options:

- **Option A: Three fatty acid chains linked to a molecule of glycerol -** This accurately describes a triglyceride.
- **Option B: Three glycerol molecules linked to a fatty acid chain -** This is incorrect. It's the reverse and doesn't represent a triglyceride.
- **Option C: Three saturated fatty acid chains linked to a molecule of cholesterol -** This is incorrect. Triglycerides are linked to glycerol, not cholesterol. Cholesterol is a different type of lipid (a steroid).

- **Option D: Three glyceride molecules linked to a molecule of phospholipid -** This is incorrect. A glyceride is a fatty acid ester of glycerol. A phospholipid is a different type of lipid. This combination does not describe a simple triglyceride.

Therefore, the correct option is A.

The final answer is $\boxed{A}$

Question4

Which one of the following statements about a plant carotenoid is FALSE?

Options:

A.

It provides precursor for the synthesis of stress hormone in plants.

B.

It protects chlorophyll a from photo-oxidation.

C.

It is an accessory pigment which absorbs light at $600 - 700$ nm.

D.

It accumulates in chromoplasts during fruit ripening.

Answer: C

Solution:

- **Option A: It provides precursor for the synthesis of stress hormone in plants.**
  - Carotenoids are precursors for the synthesis of abscisic acid (ABA), which is a crucial plant hormone involved in stress responses like drought and cold tolerance. Specifically, violaxanthin (a type of xanthophyll, which is a carotenoid derivative) is cleaved to form xanthoxin, a precursor to ABA.
  - This statement is **TRUE**.
- **Option B: It protects chlorophyll a from photo-oxidation.**
  - Carotenoids play a vital role in photoprotection. They absorb excess light energy and dissipate it as heat, preventing the formation of reactive oxygen species (ROS) that can damage chlorophyll and other photosynthetic components, a process known as photo-oxidation.
  - This statement is **TRUE**.

- **Option C: It is an accessory pigment which absorbs light at $600 - 700$ nm.**
  - Carotenoids are indeed accessory pigments. However, their primary absorption spectrum is typically in the blue-violet region, usually between 400 nm and 550 nm. They appear yellow, orange, or red because they absorb blue and green light and reflect these colors. The 600-700 nm range is predominantly where chlorophylls (especially chlorophyll *a* and *b*) absorb red light strongly.
  - Therefore, this statement is **FALSE**.
- **Option D: It accumulates in chromoplasts during fruit ripening.**
  - Chromoplasts are types of plastids that synthesize and store pigments, particularly carotenoids. During fruit ripening, chlorophyll often degrades, and carotenoids are synthesized and accumulate in chromoplasts, leading to the characteristic yellow, orange, or red colors of many mature fruits.
  - This statement is **TRUE**.

The only false statement is Option C.

The final answer is $\boxed{C}$

Question5

A cell suspension of actively respiring mitochondria is treated with either chemical X (experiment 1) or chemical Y (experiment 2), or left untreated (experiment 3).

Chemical X selectively inhibits electron transport from Complex I to ubiquinone, while chemical Y selectively inhibits electron transport from Complex III to cytochrome C.

Which one of the following options represents the correct order of relative number of ATP synthesised in mitochondria?

Options:

A.

$\text{Experiment } 2 < \text{Experiment } 1 < \text{Experiment } 3$

B.

$\text{Experiment } 1 < \text{Experiment } 2 < \text{Experiment } 3$

C.

$\text{Experiment } 1 = \text{Experiment } 2 = \text{Experiment } 3$

D.

Experiment 2 < Experiment 1 = Experiment 3

Answer: A

Solution:

Mitochondrial ATP synthesis (oxidative phosphorylation) relies on the electron transport chain (ETC) and a proton gradient. Let's analyze the impact of each experimental condition:

1. Experiment 3: Untreated (Control)

- In actively respiring mitochondria, both NADH and FADH2 donate electrons to the ETC.
- Electrons from NADH enter via Complex I, then proceed to ubiquinone, Complex III, cytochrome c, and Complex IV.
- Electrons from FADH2 enter via Complex II, then proceed to ubiquinone, Complex III, cytochrome c, and Complex IV.
- Complexes I, III, and IV pump protons into the intermembrane space, creating a proton gradient. This gradient drives ATP synthase to produce ATP.
- This represents the maximum possible ATP synthesis under these conditions.

2. Experiment 1: Treated with Chemical X

- Chemical X selectively inhibits electron transport from Complex I to ubiquinone.
- This means that electrons from **NADH** cannot enter the ETC.
- However, electrons from **FADH2** can still enter the ETC via Complex II, proceed to ubiquinone, and then through Complex III and Complex IV to oxygen.
- Since the FADH2 pathway (Complex II -> Complex III -> Complex IV) is still active and involves proton pumping by Complex III and Complex IV, some ATP will still be synthesized.
- However, the absence of the NADH pathway (which involves Complex I, Complex III, and Complex IV, and pumps more protons in total) means that ATP synthesis will be significantly **reduced** compared to the untreated control.

3. Experiment 2: Treated with Chemical Y

- Chemical Y selectively inhibits electron transport from Complex III to cytochrome C.
- Complex III is a crucial bottleneck for both the NADH pathway (via Complex I) and the FADH2 pathway (via Complex II), as both converge at ubiquinone before Complex III.
- If electron flow is blocked at Complex III, electrons cannot proceed to cytochrome c or Complex IV. This effectively halts the entire downstream ETC.
- Furthermore, if electrons cannot exit ubiquinone and Complex III, they will back up, and the preceding complexes (Complex I and Complex II) will also be unable to transfer electrons.
- Therefore, the proton gradient cannot be established, and ATP synthesis via oxidative phosphorylation will be almost completely **abolished or reduced to near zero**.

Comparing the amount of ATP synthesized:

- **Experiment 3 (Untreated):** Highest ATP synthesis.
- **Experiment 1 (Chemical X):** Reduced ATP synthesis (only FADH2 pathway active).
- **Experiment 2 (Chemical Y):** Very low or zero ATP synthesis (entire ETC blocked).

Therefore, the correct order of relative number of ATP synthesized is:

Experiment 2 < Experiment 1 < Experiment 3

The final answer is $\boxed{A}$

Question6

Which one of the following autoregulatory mechanisms is employed by the kidney when glomerular filtration rate is reduced?

Options:

A.

Levels of renin and aldosterone are reduced.

B.

Levels of renin, angiotensin I and II and aldosterone are increased.

C.

Levels of renin are increased, while those of angiotensin I and II and

aldosterone are reduced.

D.

Levels of angiotensin I and II are increased, while that of aldosterone are

reduced.

Answer: B

Solution:

When **glomerular filtration rate (GFR)** is reduced, the kidney activates mechanisms to restore normal filtration pressure and blood volume.

The **juxtaglomerular apparatus (JGA)** senses the decrease in perfusion pressure (via the afferent arteriole) and/or reduced sodium chloride delivery to the macula densa. In response, it secretes **renin**, which initiates the **renin–**

angiotensin–aldosterone system (RAAS).

Here's what happens next:

1. **Renin** converts angiotensinogen (from the liver) into **angiotensin I**.
2. **Angiotensin-converting enzyme (ACE)** converts angiotensin I to **angiotensin II**.
3. **Angiotensin II** causes vasoconstriction (increasing glomerular pressure) and stimulates secretion of **aldosterone** from the adrenal cortex.
4. **Aldosterone** increases sodium and water reabsorption, raising blood volume and pressure, which helps restore GFR.

So, when GFR decreases:

- Renin **increases**
- Angiotensin I and II **increase**
- Aldosterone **increases**

✅ **Correct answer: Option B**

"Levels of renin, angiotensin I and II, and aldosterone are increased."

Question7

Which one of the following conditions will favour maximum dissociation of oxygen from the oxyhaemoglobin in the tissues?

Options:

A.

higher $[H^+]$; higher temperature

B.

higher $[H^+]$; lower temperature

C.

lower $[H^+]$; higher temperature

D.

lower $[H^+]$; lower temperature

Answer: A

Solution:

Oxygen dissociation from oxyhaemoglobin in tissues is affected by **pH (H^+ concentration)** and **temperature**.

1. Effect of H^+

- In tissues, during high metabolic activity, **CO_2** and **lactic acid** increase.
- This increases **[H^+]**, lowering **pH**.
- A lower pH (higher [H^+]) **reduces haemoglobin's affinity for O_2**, so O_2 is released more easily.
- This is called the **Bohr effect**.

✅ Therefore, **higher [H^+]** favors oxygen dissociation.

2. Effect of temperature

- Tissues undergoing metabolism produce heat.
- **Higher temperature** also decreases O_2 affinity of haemoglobin.
- Hence, **higher temperature** favors oxygen unloading.

✅ Therefore, **higher temperature** favors oxygen dissociation.

✅ Correct Answer

Option A — higher [H^+]; higher temperature.

Answer: A. higher [H^+]; higher temperature

Question8

Which one of the following statements is correct?

Options:

A.

Red muscle fibres produce ATP aerobically under normal oxygen conditions.

B.

Mitochondria are more in white than in red muscle fibres.

Lactic acid accumulates more in red than in white muscle fibres under similar

C.

conditions.

D.

All muscle fibres primarily produce ATP anaerobically.

Answer: A

Solution:

- **Option A: Red muscle fibres produce ATP aerobically under normal oxygen conditions.**
    - Red muscle fibers (also known as slow-twitch or Type I fibers) are rich in mitochondria, myoglobin, and have a dense capillary network. These characteristics allow them to sustain prolonged contractions by efficiently producing ATP through aerobic respiration (oxidative phosphorylation), which requires oxygen. This statement is consistent with the known physiology of red muscle fibers.
- **Option B: Mitochondria are more in white than in red muscle fibres.**
    - Mitochondria are the primary sites of aerobic ATP production. Since red muscle fibers are specialized for aerobic respiration, they have a significantly higher number of mitochondria compared to white muscle fibers (fast-twitch or Type II fibers), which rely more on anaerobic glycolysis for quick, powerful bursts of energy. Therefore, this statement is incorrect.
- **Option C: Lactic acid accumulates more in red than in white muscle fibres under similar conditions.**
    - Lactic acid is a byproduct of anaerobic glycolysis. White muscle fibers, being specialized for intense, short-duration activities, rely heavily on anaerobic glycolysis and thus accumulate lactic acid much more rapidly and in greater quantities than red muscle fibers, which are adapted for sustained aerobic activity and are more resistant to fatigue. Therefore, this statement is incorrect.
- **Option D: All muscle fibres primarily produce ATP anaerobically.**
    - This is incorrect. While white muscle fibers can produce ATP anaerobically, red muscle fibers primarily produce ATP aerobically. There are also intermediate fibers (fast oxidative-glycolytic) that can utilize both pathways but red fibers are distinctly aerobic. Therefore, stating that *all* muscle fibers *primarily* produce ATP anaerobically is false.

Based on the analysis, **Option A** is the only correct statement.

The final answer is $\boxed{A}$

Question9

Which one of the following organisms produces the female gamete by mitosis of haploid cells?

Options:

A.

Honey bee

B.

Garden pea

C.

Chicken

D.

Fruit fly

Answer: B

Solution:

1. **Honey bee (Option A):**
   - Honey bee queens are diploid (2n). They produce female gametes (eggs) by **meiosis**.
   - Male honey bees (drones) are haploid (n) and produce male gametes (sperm) by **mitosis**.
   - The question specifically asks about the *female* gamete. Therefore, honey bees produce female gametes by meiosis from diploid cells, not mitosis of haploid cells.
2. **Garden pea (Option B):**
   - The garden pea is a flowering plant. Plants exhibit alternation of generations.
   - The dominant plant you see is the diploid sporophyte (2n).
   - Within the ovule, a diploid cell (megasporocyte) undergoes **meiosis** to produce four haploid megaspores (n).
   - One of these haploid megaspores survives and undergoes several rounds of **mitosis** to form the female gametophyte (embryo sac).
   - The female gametophyte is a haploid structure (n) that contains various cells, including the **egg cell** (the female gamete).
   - Therefore, the egg cell (female gamete) is a direct product of mitotic divisions and differentiation within a haploid structure (the female gametophyte), which itself developed from a haploid cell (megaspore) by mitosis. This fits the description "produces the female gamete by mitosis of haploid cells."
3. **Chicken (Option C):**
   - Chickens are animals.
   - Female chickens (hens) are diploid (2n). They produce female gametes (eggs) through oogenesis, which involves **meiosis** of diploid oogonia.
   - Therefore, chickens produce female gametes by meiosis from diploid cells, not mitosis of haploid cells.
4. **Fruit fly (Option D):**

- Fruit flies are animals.
- Female fruit flies are diploid (2n). They produce female gametes (eggs) through oogenesis, which involves **meiosis** of diploid oogonia.
- Therefore, fruit flies produce female gametes by meiosis from diploid cells, not mitosis of haploid cells.

Conclusion:

The Garden pea is the organism among the options where the female gamete is produced by mitosis of haploid cells (specifically, by mitotic divisions within the haploid female gametophyte, which originates from a haploid megaspore).

The final answer is $\boxed{B}$

Question10

Which amino acid will be charged on the tRNA with anticodon $5'$-GUU- $3'$?

Options:

A.

Asparagine (codon AAC)

B.

Valine (codon GUU)

C.

Leucine (codon UUG)

D.

Glutamine (codon CAA)

Answer: A

Solution:

Step 1. Identify the anticodon on the tRNA

Given anticodon:

$5' - \text{G U U} - 3'$

Step 2. Find the complementary codon on the mRNA

Anticodon and codon base-pair **antiparallel** (complementary $3'$–$5'$ to $5'$–$3'$).

So let's write the anticodon in the **3′→5′** direction (since codons are written 5′→3′):

Anticodon (3′–5′) $= 3'–UUG–5'$

Now, the complementary codon on the mRNA (5′→3′) will be:

$5'–AAC–3'$

Step 3. Determine which amino acid corresponds to mRNA codon 5′–AAC–3′

From the standard genetic code:

AAC $\rightarrow$ Asparagine (Asn)

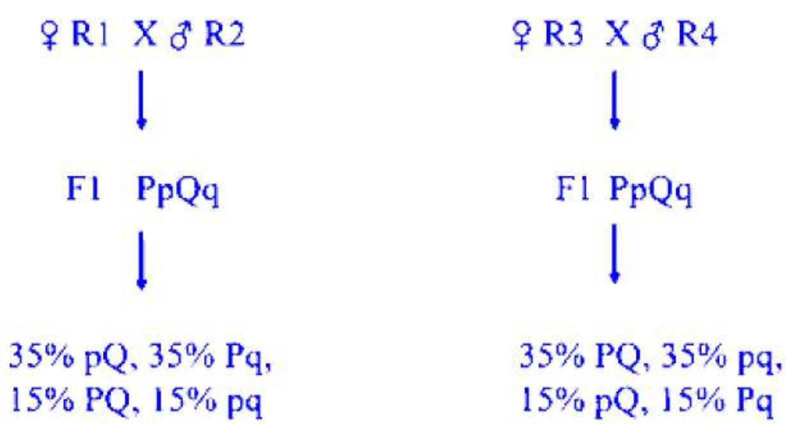
✅ **Answer:**

The tRNA with anticodon **5′–GUU–3′** carries **Asparagine**.

Correct Option: A. Asparagine (codon AAC)

Question11

Two double heterozygous plants (PpQq), derived from two different pairs of true-breeding parents of unknown genotype, produce gametes in the proportions as given below.



Which one of the following options correctly represents the genotype of the parents?

Options:

A.

$\mathrm{R1 = ppQQ; R2 = PPqq; R3 = PPQQ; R4 = ppqq}$

B.

$\mathrm{R1 = PPQQ; R2 = ppqq; R3 = ppQQ; R4 = PPqq}$

C.

$\mathrm{R1 = ppQQ; R2 = PPqq; R3 = PPqq; R4 = ppQQ}$

D.

$\mathrm{R1 = PPQQ; R2 = ppqq; R3 = ppqq; R4 = PPQQ}$

Answer: A

Solution:

⭐ **Solution**

We are told:

- Two different **true-breeding parental pairs** produced F_1 plants that are **PpQq**.
- These F_1 plants produce **non-Mendelian gamete ratios**, showing **linkage**.
- From the gamete ratios, we must deduce the **parental genotypes**.

🔷 PART 1: Cross R1 × R2 → F_1 (PpQq)

Gamete proportion from the F_1:

- 35% pQ
- 35% Pq
- 15% PQ
- 15% pq

Interpretation:

- The **more frequent gametes** are the **parental combinations** (35% & 35%).
- The **less frequent gametes** are the **recombinants** (15% & 15%).

So parental chromosome arrangement in the F_1 is:

Parental gametes (high %)

- pQ
- Pq

So the F_1 genotype must have come from parents producing **pQ** and **Pq** gametes.

This means:

- One parent must be **ppQQ** → always produces **pQ**
- One parent must be **PPqq** → always produces **Pq**

So:

✔ **R1 = ppQQ**

✔ **R2 = PPqq**

◆ PART 2: Cross R3 × R4 → F_1 (PpQq)

Gamete proportion from the F_1:

- 35% PQ
- 35% pq
- 15% pQ
- 15% Pq

Interpretation:

Parental gametes (dominant %):

- PQ
- pq

So the F_1 parental linkage phase is **PQ / pq**.

Thus parents must produce **PQ** and **pq** gametes.

The only true-breeding genotypes that do this are:

- PPQQ → always makes **PQ**
- ppqq → always makes **pq**

So:

✔ **R3 = PPQQ**

✔ **R4 = ppqq**

✅ FINAL CORRECT ANSWER

Option A

R1 = ppQQ; R2 = PPqq; R3 = PPQQ; R4 = ppqq

Question12

What are retroviruses?

Options:

A.

A group of viruses with RNA genome and no reverse transcriptase activity

B.

A group of viruses with DNA genome and no reverse transcriptase activity

C.

A group of viruses with DNA genome and reverse transcriptase activity

D.

A group of viruses with RNA genome and reverse transcriptase activity

Answer: D

Solution:

The correct answer is:

✅ **Option D:** A group of viruses with RNA genome and reverse transcriptase activity

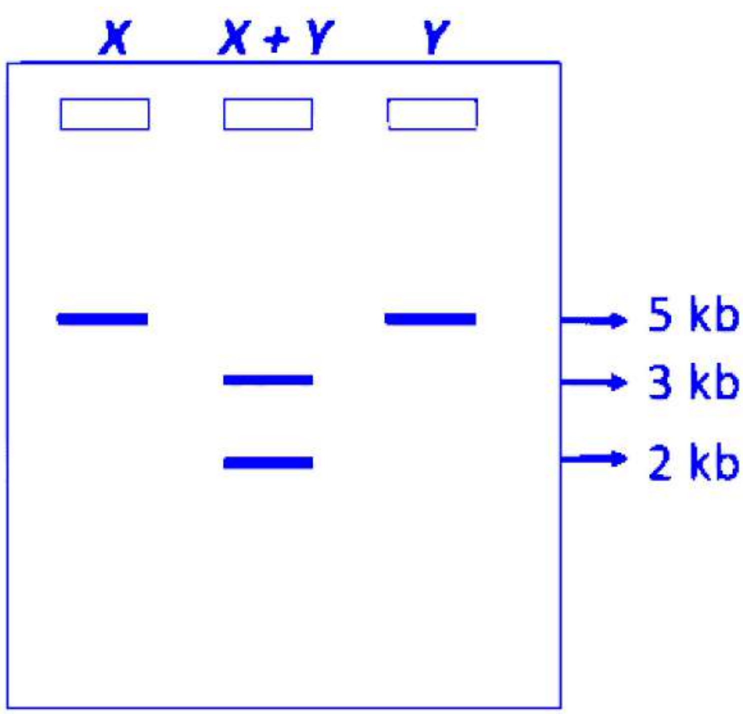
Explanation:

Retroviruses are RNA viruses that carry an enzyme called **reverse transcriptase**, which converts their RNA genome into DNA once inside a host cell. This DNA then integrates into the host genome, allowing the virus to replicate as part of the host's genetic material.

Example: Human Immunodeficiency Virus (HIV) is a well-known retrovirus.

Question13

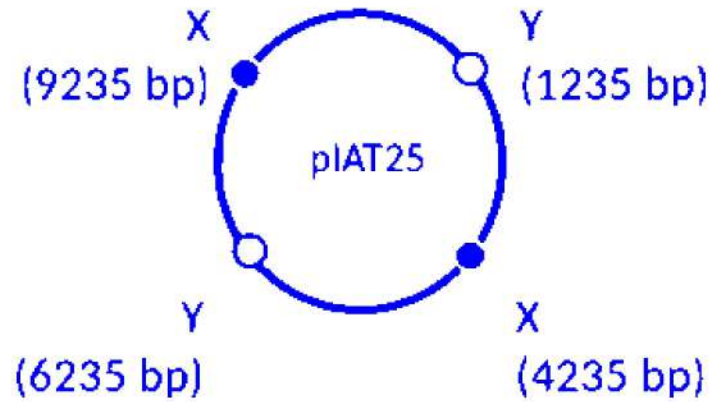
The given picture was obtained from an agarose gel electrophoresis of a plasmid after digestion with restriction enzymes either X, Y or both X and Y.



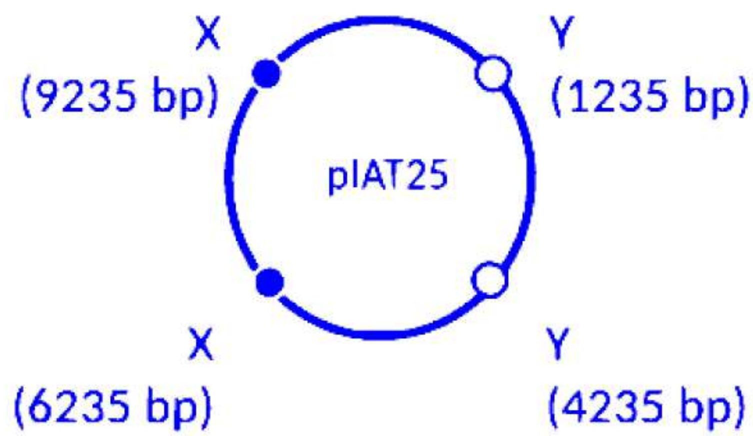
Which one of the following diagrams correctly represents the position of the restriction enzyme sites (X, Y) on the 10,000 bp plasmid?

Options:

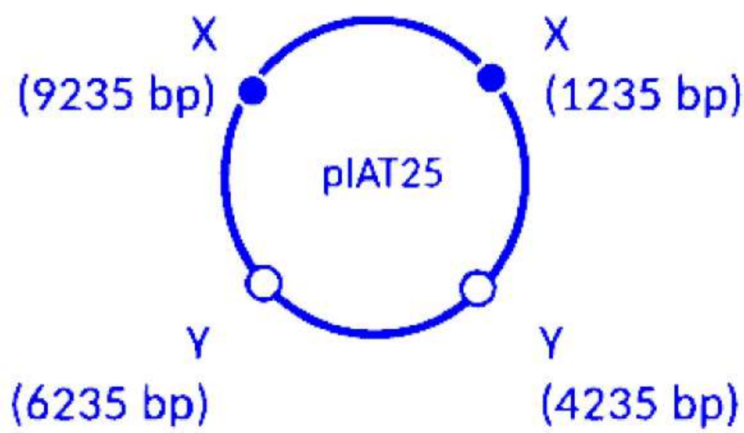
A.



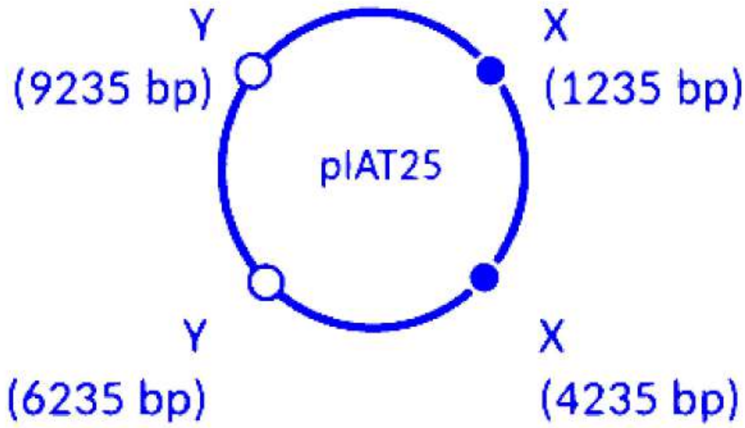
B.



C.



D.



Answer: A

Solution:

Explanation

From the gel:

1. X digestion lane

- Only **one band at ~5 kb** is visible.
- But the plasmid is **10 kb**, so a *single* cut by X would give a **10 kb linear band**, not 5 kb.
- Therefore, X must cut the plasmid **twice**, producing **two fragments** that are both **5 kb** and thus **co-migrate as a single 5 kb band**.

2. Y digestion lane

- Also shows **one band at ~5 kb**.
- Same reasoning: Y must also cut **twice**, giving **two co-migrating 5 kb fragments**.

3. X + Y double digest lane

- Shows **three bands: 5 kb, 3 kb, 2 kb**.
- These sum to 10 kb, and occur because X and Y cuts divide the circle into:
  - **one fragment shared between X and Y cuts = 5 kb**
  - **two unique fragments = 3 kb and 2 kb**

So the map must include:

- **Two X sites**
- **Two Y sites**
- The fragment sizes between cuts: **5 kb, 3 kb, 2 kb**

Only **Diagram A** shows this arrangement correctly on a 10 kb plasmid.

✅ **Correct choice: A**

Question14

Honey bee males are haploid and females are diploid. Which one of the following statements is INCORRECT about honey bees?

Options:

A.

Honey bee males cannot have daughters but can have sons.

B.

Honey bee males are produced from unfertilized eggs and females are produced from fertilized eggs.

C.

A honey bee male does not have a father but has a grandfather.

D.

Honey bee males form gametes by mitosis and females form gametes by meiosis.

Answer: A

Solution:

Given facts

- Honey bee males (**drones**) are **haploid (n)**.
- Honey bee females (**workers and queens**) are **diploid (2n)**.
- This system is called **haplodiploidy**.

Option analysis

Option A:

> Honey bee males cannot have daughters but can have sons.

- Males are haploid and arise from **unfertilized eggs**.
- Males **produce sperm**, but sperm is only used when a **female (queen)** fertilizes her eggs.
- A male bee's sperm can fertilize eggs → producing **female offspring (daughters)**.

- However, because males **don't lay eggs themselves**, they **cannot directly have either sons or daughters**.
- Also, **male bees never produce gametes that produce sons**, because **sons come from unfertilized eggs laid by females**, not from males.

✅ Therefore, the statement **"Honey bee males cannot have daughters but can have sons"** is **incorrect** — in fact, males have **neither sons nor daughters**.

Option B:

> Honey bee males are produced from unfertilized eggs and females are produced from fertilized eggs.

✅ This is **correct** — that's the basis of haplodiploidy.

Option C:

> A honey bee male does not have a father but has a grandfather.

✅ True:

- A male comes from an **unfertilized egg** → thus **no father**.
- The queen (his mother) is diploid, and she herself came from a **fertilized egg**, so her sire (the male who fertilized her egg) is the **grandfather** of that male bee.

Option D:

> Honey bee males form gametes by mitosis and females form gametes by meiosis.

✅ True:

- Males are haploid, so to produce haploid sperm, they can only divide by **mitosis**—no further reduction division needed.
- Females are diploid; to produce haploid eggs, they use **meiosis**.

✅ **Final Answer:**

- The **incorrect statement** is **Option A**.

Question15

Which one of the following statements is FALSE?

Options:

A.

The movement of energy is unidirectional in the ecological pyramid of energy.

B.

Only 10% of energy is transferred to each of the higher trophic levels in grazing food chain.

C.

All organisms of a trophic level should be included for estimation of energy content of that trophic level.

D.

More than 80% of the solar energy incident on earth is captured by plants and photosynthetic bacteria.

Answer: D

Solution:

- **Option A: The movement of energy is unidirectional in the ecological pyramid of energy.**

  This statement is **TRUE**. Energy flows from lower trophic levels to higher trophic levels, and a significant portion is lost as heat at each transfer. It does not flow back to lower levels. This follows the second law of thermodynamics.

- **Option B: Only 10% of energy is transferred to each of the higher trophic levels in grazing food chain.**

  This statement is generally **TRUE**. This is known as the "10% rule" or Lindeman's law of trophic efficiency, an ecological principle stating that on average, only about 10% of the energy from one trophic level is incorporated into the biomass of the next trophic level. The rest is lost as heat during metabolic processes or is not consumed.

- **Option C: All organisms of a trophic level should be included for estimation of energy content of that trophic level.**

  This statement is **TRUE**. To accurately determine the total energy available at a specific trophic level, one must consider the energy stored in *all* the living organisms that constitute that level. Excluding some would lead to an inaccurate estimation.

- **Option D: More than 80% of the solar energy incident on earth is captured by plants and photosynthetic bacteria.**

  This statement is **FALSE**. Photosynthetic organisms (plants, algae, and photosynthetic bacteria) are remarkably inefficient at capturing solar energy. Typically, only about 1% to 2% (and sometimes up to 5% under optimal conditions for highly productive ecosystems or crops) of the *incident* solar energy that falls on Earth's surface is converted into chemical energy via photosynthesis. A value of "more than 80%" is extremely high and biologically impossible under natural conditions.

The final answer is $\boxed{D}$

Chemistry

Question1

Which one of the following statements best describes the acidic/basic/amphoteric nature of ZnO and CaO ?

Options:

A.

ZnO is amphoteric, while CaO is basic.

B.

ZnO is basic, while CaO is amphoteric.

C.

Both ZnO and CaO are amphoteric.

D.

ZnO is acidic, while CaO is basic.

Answer: A

Solution:

- **ZnO (Zinc oxide)**:

  It can react with both acids and bases.

  For example:

- With acid: ZnO $+ 2HCl \rightarrow ZnCl_2 + H_2O$
- With base: ZnO $+ 2NaOH + H_2O \rightarrow Na_2[Zn(OH)_4]$

  → Hence, **ZnO is amphoteric**.

- **CaO (Calcium oxide)**:

  It reacts with water to form **Ca(OH)_2**, a strong base, and reacts with acids to form salts, but not with bases.

  → Hence, **CaO is basic**.

✅ **Correct answer: Option A**

ZnO is amphoteric, while CaO is basic.

Question2

Which among the following processes is/are associated with increasing bond order but no change in diamagnetic/paramagnetic behaviour?

(i) $N_2 \rightarrow N_2^+ + e^-$

(ii) $O_2 \rightarrow O_2^+ + e^-$

(iii) $O_2 + e^- \rightarrow O_2^-$

Options:

A.

(ii) only

B. (i) and (ii)

C. (ii) and (iii)

D. (iii) only

Answer: A

Solution:

✅ **Determine bond order change and magnetic behaviour**

We analyze each process for:

1. **Bond order change** (increase?)
2. **Magnetic behaviour change** (diamagnetic ↔ paramagnetic?)

(i) $N_2 \rightarrow N_2^+ + e^-$

MO configuration of N_2 (14 e^-):

$\sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p\_z^2$

→ **All electrons paired → diamagnetic**

→ Bond order = (10 bonding – 4 antibonding)/2 = **3**

MO of N_2^+ (13 e^-):

Electron removed from **σ2p_z bonding orbital**

→ Bond order = 2.5 (**decreases**)

→ Still **diamagnetic** (no unpaired electrons)

✔ **Magnetic behaviour unchanged**

✖ **Bond order decreases → Not acceptable**

❌ **(i) is NOT correct**

(ii) $O_2 \rightarrow O_2^+ + e^-$

MO configuration of O_2 (16 e^-):

$\sigma 2s^2$ *$\sigma 2s^2\ \sigma 2p\_z^2\ \pi 2p_x^2\ \pi 2p_y^2$* $\pi 2p_x^1\ \pi^* 2p_y^1$

→ **2 unpaired electrons → paramagnetic**

→ Bond order = (10 – 6)/2 = **2**

MO of O_2^+ (15 e^-):

Electron removed from *π antibonding orbital**

→ π* has 2 unpaired electrons → removing 1 leaves **1 unpaired**

→ Still **paramagnetic**

→ Bond order = (10 – 5)/2 = **2.5 (increases)**

✔ Bond order increases

✔ Magnetic behaviour unchanged (still paramagnetic)

✅ (ii) is correct

(iii) $O_2 + e^- \rightarrow O_2^-$

Add electron into π *antibonding orbital**

O_2^-:

π* now has 3 **unpaired electrons** → still **paramagnetic**

Bond order = (10 – 7)/2 = **1.5 (decreases)**

✖ Bond order decreases

✔ Magnetic behaviour unchanged

❌ Not acceptable (bond order does NOT increase)

🎯 **Final Answer: (ii) only**

Question3

What is the value of $E^{\circ}\left(Fe^{3+}/Fe^{0}\right)$?

[The standard reduction potential values are $E^{\circ}\left(Fe^{3+}/Fe^{2+}\right) = 0.77$ V, and $E^{\circ}\left(Fe^{2+}/Fe^{0}\right) = -0.44$ V]

Options:

A.

-0.04 V

B.

0.33 V

C.

0.11 V

D.

-0.11 V

Answer: A

Solution:

To find $E^\circ(Fe^{3+}/Fe^0)$, we combine the two given reduction half-reactions:

1. $Fe^{3+} + e^- \rightarrow Fe^{2+} \quad E^\circ = +0.77$ V
2. $Fe^{2+} + 2e^- \rightarrow Fe^0 \quad E^\circ = -0.44$ V

We want the overall reaction:

$$Fe^{3+} + 3e^- \rightarrow Fe^0$$

Step 1: Convert each to Gibbs free energy contributions

Use:

$$\Delta G^\circ = -nFE^\circ$$

- For reaction 1: $n = 1$

$$\Delta G_1 = -1F(0.77)$$

- For reaction 2: $n = 2$

$$\Delta G_2 = -2F(-0.44) = +0.88F$$

Step 2: Add the ΔG values

$$\Delta G_{\text{total}} = -0.77F + 0.88F = 0.11F$$

Step 3: Convert back to E° for overall reaction

Total electrons transferred = 3:

$$E^\circ_{\text{overall}} = -\frac{\Delta G_{\text{total}}}{nF} = -\frac{0.11F}{3F} = -0.0367 \approx -0.04 \text{ V}$$

✔ **Final Answer:** -0.04 V

Question4

What are the correct orders of stability for the following compounds?

Options:

A.

$VF_5 > VCl_5; CuCl_2 > CuI_2$

B.

$VCl_5 > VF_5; CuCl_2 > CuI_2$

C.

$VCl_5 > VF_5; CuI_2 > CuCl_2$

D.

$VF_5 > VCl_5; CuI_2 > CuCl_2$

Answer: A

Solution:

1 Comparison of VF_5 and VCl_5

- **Vanadium pentahalides** involve the **+5 oxidation state** of vanadium (V^{5+}).
- As we move from **F** to **I**, the **+5 oxidation state** becomes *less stable* because:
- Larger halides (Cl^-, Br^-, I^-) are less electronegative and less able to stabilize a high positive oxidation state on the metal.
- **Fluorine** is the most electronegative and forms stronger, more covalent, and more stable bonds with highly oxidized metals.

✅ Thus,

$VF_5 > VCl_5$ (in stability)

2 Comparison of $CuCl_2$ and CuI_2

- In **Cu(+2)** compounds, we must consider redox tendencies:
- $Cu^{2+} + I^- \rightarrow Cu^+ + \frac{1}{2}I_2$
- The above reaction indicates that CuI_2 is *unstable*; it disproportionates to CuI (Cu^+) and I_2.
- $CuCl_2$ is stable because Cl^- is not easily oxidized, hence no such disproportionation.

✅ So,

$CuCl_2 > CuI_2$ (in stability)

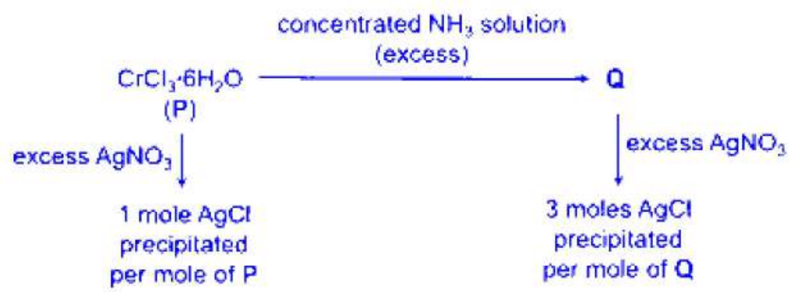
✅ Final conclusion

$$\boxed{VF_5 > VCl_5;\ CuCl_2 > CuI_2}$$

Correct option: A

Question5

Consider the following reaction scheme:



Which among the following statements is correct?

Options:

A.

P shows geometrical isomerism and absorbs light of higher wavelength than that of **Q**.

B.

Both **P** and **Q** show geometrical isomerism and **P** absorbs light of higher wavelength than that of **Q**.

C.

Q shows geometrical isomerism and absorbs light of higher wavelength than that of **P**.

D.

P shows geometrical isomerism and absorbs light of lower wavelength than that of **Q**.

Answer: A

Solution:

Step 1: Identifying P

$CrCl_3 \cdot 6H_2O$ is known to exist in different hydrate forms.

The compound **P** gives **1 mole of AgCl** per mole of P when treated with excess $AgNO_3$.

This means **P contains only 1 ionisable Cl^-** (chloride outside the coordination sphere).

The complex with one ionisable chloride is:

$[Cr(H_2O)_4Cl_2]Cl \cdot 2H_2O$

- Two Cl^- are coordinated to Cr(III)
- One Cl^- is outside the coordination sphere (ionisable → precipitates 1 AgCl)

Because the two coordinated chlorides can be **cis** or **trans**,
P shows geometrical isomerism.

Step 2: Identifying Q

Treating P with excess NH_3 replaces water and coordinated chloride ligands with NH_3 to form:

$[Cr(NH_3)_6]Cl_3$

This complex produces **3 moles AgCl** because all three Cl^- ions are outside the coordination sphere.

All 6 ligands (NH_3) are identical → an octahedral complex with identical ligands **cannot show geometrical isomerism**.

So **Q does NOT show geometrical isomerism.**

Step 3: Light absorption (wavelength)

- NH_3 is a **strong-field ligand**
- H_2O and Cl^- are **weaker-field ligands**

Stronger ligand → larger splitting Δ_0 → higher energy absorption → **shorter wavelength**

Thus:

P absorbs higher wavelength than Q.

✅ **Answer: Option A**

Question6

How many β-hydrogen is/are present in 2-methyl-3-phenyl-pentan-1-al?

Options:

A.

4

B.

1

C.

3

D.

2

Answer: A

Solution:

Based on the IUPAC name provided, **2-methyl-3-phenyl-pentan-1-al**, here is the step-by-step solution to find the number of β-hydrogens.

1. Understand the Terminology

- **Functional Group:** The principal functional group is the **aldehyde** group ($-CHO$).
- **α-Carbon (Alpha):** The carbon atom directly attached to the carbonyl carbon of the aldehyde group.
- **β-Carbon (Beta):** Any carbon atom directly attached to the α-carbon.
- **β-Hydrogen:** Any hydrogen atom attached to a β-carbon.

2. Draw the Structure

Let's build the structure of **2-methyl-3-phenyl-pentan-1-al**:

- **Parent Chain:** "Pentan-1-al" means a 5-carbon chain with an aldehyde at position 1.
- **Substituents:**
  - **2-methyl:** A methyl group ($-CH_3$) at position 2.
  - **3-phenyl:** A phenyl ring ($-Ph$ or $-C_6H_5$) at position 3.

Structure visualization:

$$\underset{\text{Aldehyde}}{O=CH} - \underset{\alpha\text{-carbon}}{CH(CH_3)} - \underset{\beta\text{-carbon}}{CH(Ph)} - CH_2 - CH_3$$

Let's expand the area of interest around the α-carbon (C-2):

1. **Carbon-1:** The Aldehyde carbon (CHO).
2. **Carbon-2 (α-carbon):** Attached to C-1. It has two other carbon attachments (making them β-carbons):
   - The **Methyl group** ($-CH_3$) substituent.
   - The **Carbon-3** of the main chain.

3. Count the β-Hydrogens

We need to count the hydrogens on all carbon atoms attached to the α-carbon (C-2).

- **Source 1: The Methyl Group at Position 2** The carbon of the methyl group is directly attached to the α-carbon. Therefore, it is a β-carbon.
  - A methyl group is $-CH_3$.
  - Number of Hydrogens = **3**

- **Source 2: Carbon-3 of the Main Chain** Carbon-3 is directly attached to the α-carbon. Therefore, it is also a β-carbon. Let's check its valency to find the number of hydrogens:
  - Bond 1: To C-2 (α-carbon)
  - Bond 2: To C-4 (the rest of the pentane chain, $-\mathrm{CH_2}-$)
  - Bond 3: To the Phenyl group substituent
  - Bond 4: Must be to a **Hydrogen atom** (since carbon forms 4 bonds).
  - Number of Hydrogens = **1**

4. Calculation

$$\text{Total } \beta\text{-hydrogens} = (\text{Hydrogens on Methyl } \beta\text{-carbon}) + (\text{Hydrogens on C-3 } \beta\text{-carbo}$$

$$\text{Total } \beta\text{-hydrogens} = 3 + 1 = 4$$

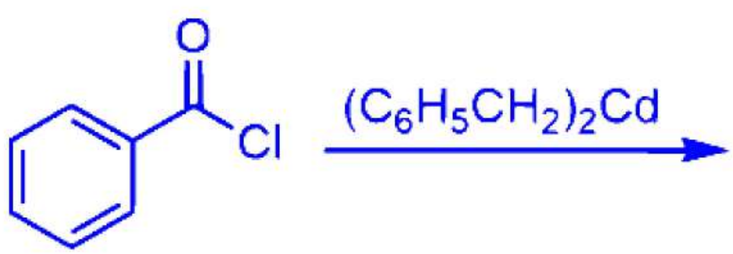
Correct Answer: The molecule has **4** β-hydrogens. This corresponds to **Option A.**

Question7

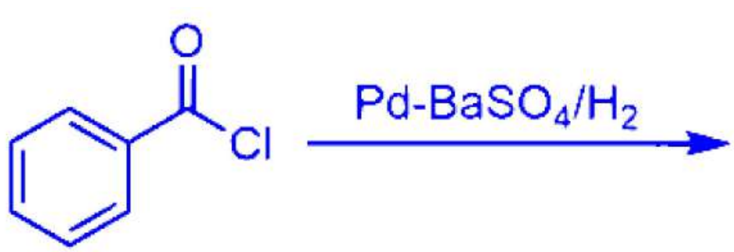
Which of the following reactions do NOT provide an aldehyde as a product?

Options:

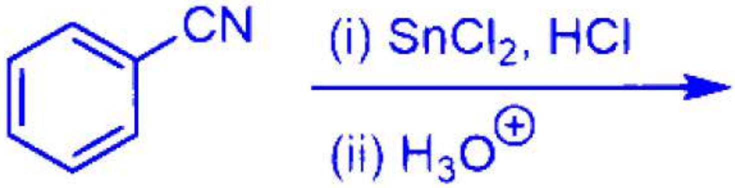
A.



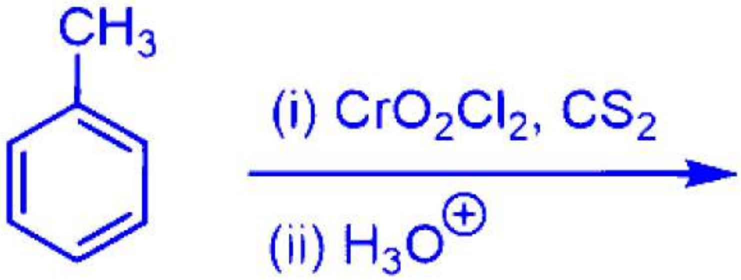
B.



C.



D.



Answer: A

Solution:

Option A

Reaction: Benzoyl chloride + $(C_6H_5CH_2)_2Cd$ (a Gilman-type organocadmium reagent)

- Organocadmium reagents react with **acyl chlorides** to give **ketones**, *not aldehydes*.
- So benzoyl chloride will give **benzophenone**, not benzaldehyde.

✅ **This reaction does NOT produce an aldehyde.**

Option B

Reaction: Benzoyl chloride + Pd–$BaSO_4$/H_2 (Rosenmund reduction)

- The Rosenmund reduction converts **acyl chlorides** → **aldehydes**.
- Therefore benzoyl chloride → **benzaldehyde**.

✓ Produces an aldehyde.

Option C

Reaction: Benzonitrile + (i) $SnCl_2$/HCl (ii) H_3O^+

- $SnCl_2$/HCl reduces **nitriles to imines**, which hydrolyze to **aldehydes**.
- So benzonitrile → **benzaldehyde**.

✓ Produces an aldehyde.

Option D

Reaction: Toluene + CrO_2Cl_2/CS_2 then H_3O^+ (Étard oxidation)

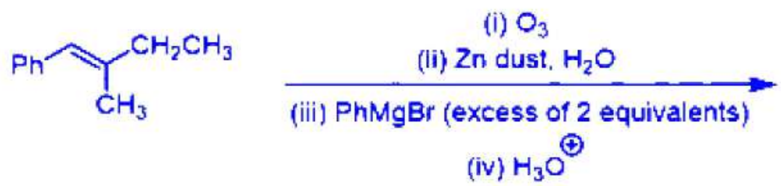
- Étard oxidation selectively oxidizes **benzylic methyl groups** → **aldehydes**.
- Toluene → **benzaldehyde**.

✓ Produces an aldehyde.

✅ **Correct Answer: A**

Question8

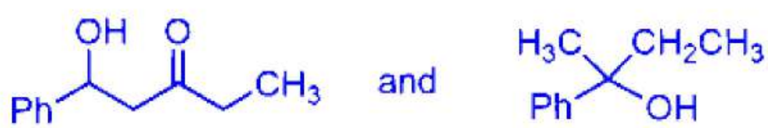
What are the major products formed in the following reaction sequence?



Options:

A.

B.



C.

D.

Answer: A

Solution:

✅ Step 1 — Ozonolysis (O_3, then Zn/H_2O)

The starting alkene is:

Ph–C(CH_3)=CH–CH_2CH_3

Breaking the double bond forms **two carbonyl fragments**:

Left fragment (benzyl-substituted carbon of the alkene):

This carbon is **disubstituted by Ph and CH_3**, so it becomes:

👉 **Benzophenone-type ketone:**
Ph–C(=O)–CH_3

Right fragment:

The right carbon is substituted by **CH_2CH_3**, so this becomes:

👉 **Propionaldehyde:**
CH_3–CH_2–CHO

So after ozonolysis we get:

- **Ph–CO–CH_3** (ketone)
- **CH_3CH_2CHO** (aldehyde)

✅ Step 2 — Reaction with excess PhMgBr (2 equivalents)

Grignard reagent **adds to carbonyls** forming alcohols.

Reaction with the ketone (Ph–CO–CH_3):

PhMgBr adds once → tertiary alcohol

Product:
$Ph_2C(OH)CH_3$

Reaction with the aldehyde (CH_3CH_2CHO):

PhMgBr adds → secondary alcohol

Product:
CH_3CH_2–CH(OH)–Ph

✔ Final Products

Tertiary alcohol:

$Ph_2C(OH)CH_3$

Secondary alcohol:

CH_3CH_2–CH(OH)–Ph

These correspond to **Answer A** in the image.

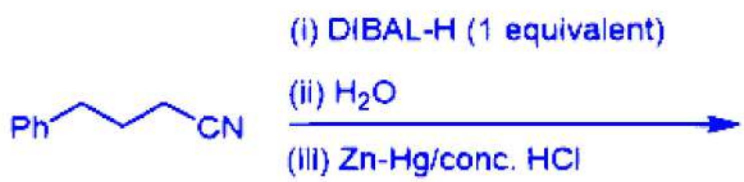
✅ **Final Answer: A**

The major products are:

(1) $Ph_2C(OH)CH_3$
(2) CH_3CH_2–CH(OH)–Ph

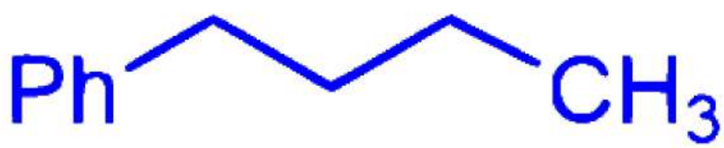
Question9

What is the major product in the reaction sequence given below?

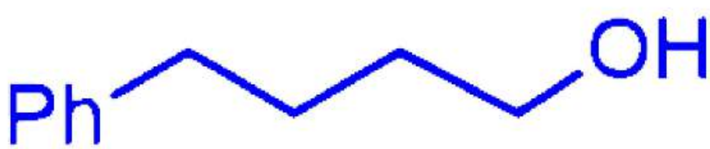


Options:

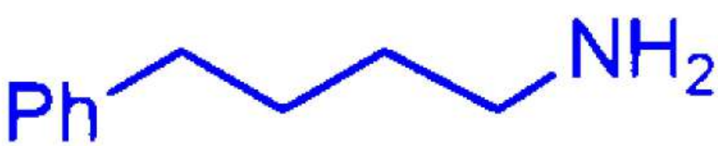
A.



B.



C.



D.

Answer: A

Solution:

The starting compound is **Ph–CH₂–CH₂–CH₂–C≡N** (a benzylic nitrile).

Step (i): DIBAL-H (1 equivalent)

- One equivalent of DIBAL-H reduces **nitriles** → **aldehydes** (at low temperature).
- So the nitrile becomes an aldehyde:

$$\text{Ph–CH}_2\text{–}CH_2\text{–}CH_2\text{–}C \equiv N \xrightarrow{\text{DIBAL-H}} \text{Ph–CH}_2\text{–}CH_2\text{–}CH_2\text{–}CHO$$

Step (ii): H_2O (workup)

- Just completes the formation of the aldehyde.

Step (iii): Zn-Hg / conc. HCl (Clemmensen reduction)

- Clemmensen reduction converts **aldehydes** → **alkanes**.

$$\text{Ph–CH}_2\text{–}CH_2\text{–}CH_2\text{–}CHO \xrightarrow{\text{Zn-Hg/HCl}} \text{Ph–CH}_2\text{–}CH_2\text{–}CH_2\text{–}CH_3$$

Final Product

$$\boxed{\text{Ph–(CH}_2)_3\text{–CH}_3}$$

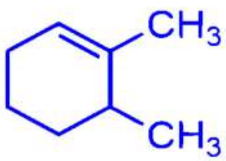
This corresponds to **Option A**.

Question10

Compound I undergoes hydroboration-oxidation reaction with $(BH_3)_2$ followed by treatment with H_2O_2 and aqueous NaOH to produce another compound II, which upon oxidation with CrO_3 gives 2,3-dimethyl-cyclohexanone as the product. What is the structure of I ?

Options:

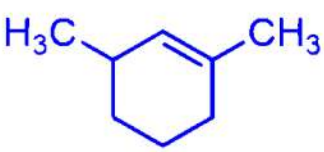
A.



B.

C.

D.



Answer: A

Solution:

We are told:

1. **Compound I** → undergoes **hydroboration–oxidation** (BH_3, then H_2O_2/NaOH)
   → gives **Compound II (an alcohol)**

2. **Compound II**, when oxidized with **CrO_3**, gives **2,3-dimethylcyclohexanone**.

So the product after CrO_3 oxidation is:

2,3-dimethylcyclohexanone

That means the alcohol formed in step 1 must be a **secondary alcohol** located at **either C-2 or C-3** (because CrO_3 oxidizes 2° alcohols to ketones).

Key idea

Hydroboration–oxidation adds:

- **H** to the *more substituted* alkene carbon

- **OH** to the *less substituted* carbon
- in **syn** fashion and **anti-Markovnikov**.

Therefore, to get **2,3-dimethylcyclohexanone**, the alcohol after hydroboration must be **2-hydroxy-3-methylcyclohexane** (or the equivalent position).

That alcohol must come from an alkene where the double bond is between **C-2 and C-3**, with methyls at both carbons.

So **Compound I must be 2,3-dimethylcyclohexene.**

Which option matches this?

Looking at the structures:

✓ Option A

shows **2,3-dimethylcyclohexene**
→ Hydroboration gives **2-hydroxy-3-methylcyclohexane**
→ CrO_3 oxidizes this to **2,3-dimethylcyclohexanone**.

Therefore **Option A is correct**.

Answer: ✓ Option A

Question11

The work done when one mole of an ideal gas expands at constant temperature T from volume V to $2V$ (in two equal steps of volume in a linear fashion) is $\frac{7}{12}RT$. How much more work would be done by the gas if it expands in three equal steps? [R is the universal gas constant]

Options:

A. $\frac{1}{30}\mathrm{R}T$

B. $\frac{3}{8}\mathrm{R}T$

C. $\frac{3}{4}\mathrm{R}T$

D. $-\mathrm{R}T\ln\left(\frac{1}{15}\right)$

Answer: A

Solution:

Given

For an isothermal expansion of 1 mole of ideal gas, from V to $2V$, done in **two equal volume steps**, the work done using **right-endpoint evaluation** is given as:

$$W_2 = \frac{7}{12}RT$$

This tells us the problem is approximating work by:

$$W = \sum_{i=1}^{n} P_i \, \Delta V \quad \text{with} \quad P_i = \frac{RT}{V_i},$$

and

$$V_i = V\left(1 + \frac{i}{n}\right)$$

because n equal steps from V to $2V$.

Thus, it is a **right-endpoint Riemann sum.**

Work done in 3 steps

Total volume change: $V \to 2V$.
Three equal steps ⇒ each step:

$$\Delta V = \frac{V}{3}$$

Volumes at right endpoints:

- $V_1 = V + \frac{V}{3} = \frac{4V}{3}$
- $V_2 = V + \frac{2V}{3} = \frac{5V}{3}$
- $V_3 = 2V$

Pressures:

$$P_i = \frac{RT}{V_i}$$

Compute work in each step:

Step 1

$$W_1 = P_1 \, \Delta V = \frac{RT}{4V/3} \cdot \frac{V}{3} = \frac{RT}{4}$$

Step 2

$$W_2 = \frac{RT}{5V/3} \cdot \frac{V}{3} = \frac{RT}{5}$$

Step 3

$$W_3 = \frac{RT}{2V} \cdot \frac{V}{3} = \frac{RT}{6}$$

Total work for 3 steps

$$W_3 = RT\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)$$
$$= RT\left(\frac{15+12+10}{60}\right) = RT\cdot\frac{37}{60}$$

How much *more* work than the 2-step case?

Given:

$$W_2 = \frac{7}{12}RT = \frac{35}{60}RT$$

Difference:

$$\Delta W = W_3 - W_2 = RT\left(\frac{37}{60}-\frac{35}{60}\right) = RT\cdot\frac{2}{60} = \frac{RT}{30}$$

✅ **Final Answer:** $\boxed{\frac{RT}{30}}$

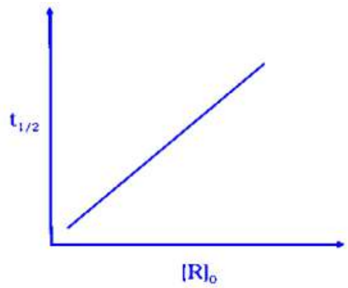
Question12

At a particular temperature, the magnitude of the rate constant of a reaction is 5×10^{-5} and the unit of the pre-exponential factor of the Arrhenius equation for this reaction is $\mathrm{molL}^{-1}\ \mathrm{min}^{-1}$. Which of the following plots is correct for this reaction?

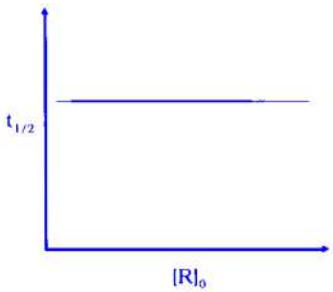
$[$ Note: $[\mathrm{R}]_0$ is the initial concentration and $t_{1/2}$ is the half-life of the reaction $]$

Options:

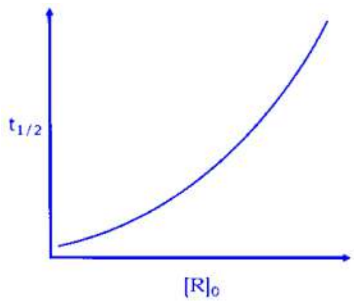
A.



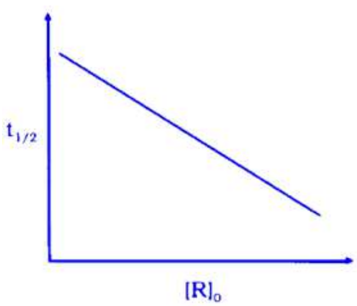
B.



C.



D.



Answer: A

Solution:

We are told:

- The rate constant $k = 5 \times 10^{-5}$
- The *unit of the pre-exponential factor* A in the Arrhenius equation is **mol L^{-1} min^{-1}**

Step 1 — Identify the order of the reaction

The Arrhenius equation is:

$$k = Ae^{-E_a/RT}$$

Here A **has the same units as** k.

Since A is given in **mol L^{-1} min^{-1}**, that means the rate constant k must have the same units.

Units of k determine reaction order:

Reaction order	Units of k
Zero-order	mol L^{-1} time^{-1}
First-order	time^{-1}
Second-order	L mol^{-1} time^{-1}

Here the units are **mol L^{-1} min^{-1}**, so this is a **zero-order reaction**.

Step 2 — Half-life dependence on initial concentration

For a zero-order reaction:

$$t_{1/2} = \frac{[R]_0}{2k}$$

Thus:

- $t_{1/2}$ is **directly proportional** to the initial concentration $[R]_0$
- The graph is a **straight line with positive slope**

Correct Plot

A straight line increasing with $[R]_0$.

That matches **Option A**.

Final Answer: A

Question13

What is the time period of revolution of an electron in the fourth Bohr orbit of He^{+}?

[Bohr radius $= 52.9$ picometers, mass of an electron $= 9.11 \times 10^{-31}$ kg, Planck's constant $= 6.626 \times 10^{-34}\mathrm{Js}$]

Options:

A.

24 femtoseconds

B.

4.8 femtoseconds

C.

24 femtoseconds

D.

2.4 femtoseconds

Answer: D

Solution:

Given data

- Bohr radius $a_0 = 52.9\,\mathrm{pm} = 52.9 \times 10^{-12}\,\mathrm{m}$
- Electron mass $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
- Planck constant $h = 6.626 \times 10^{-34}\,\mathrm{J{\cdot}s}$
- $\mathrm{He}^{+} \Rightarrow Z = 2$
- Orbit number $n = 4$

Step 1: Bohr model relations

For a hydrogen-like atom (Z),

the radius of the n^{th} orbit is

$r_n = \frac{n^2 a_0}{Z}$

and the electron speed in that orbit is

$$v_n = \frac{Ze^2}{2\epsilon_0 h} \cdot \frac{2h}{m_e a_0 n} = \frac{Ze^2}{(4\pi\epsilon_0)} \frac{1}{n\hbar}$$

—but numerically, we can also use known speed from Bohr model:

For hydrogen,

$$v_1 = \alpha c = 2.18 \times 10^6 \text{ m/s.}$$

For a hydrogenic ion,

$$v_n = \frac{Z\,v_1}{n}$$

where $v_1 = 2.18 \times 10^6$ m/s.

Step 2: Time period of revolution

$$T_n = \frac{2\pi r_n}{v_n}$$

Substitute the two expressions:

$$r_n = \frac{n^2 a_0}{Z}, \quad v_n = \frac{Zv_1}{n}$$

$$T_n = \frac{2\pi\,(n^2 a_0/Z)}{(Zv_1/n)} = \frac{2\pi a_0 n^3}{Z^2 v_1}$$

Step 3: Substitute numbers

For He^+, $Z = 2$, $n = 4$.

$$T_4 = \frac{2\pi a_0 n^3}{Z^2 v_1} = \frac{2\pi(52.9\times10^{-12})(4)^3}{(2)^2(2.18\times10^6)}$$

Compute step-by-step:

$$n^3 = 64, \quad Z^2 = 4$$

$$T_4 = \frac{2\pi(52.9\times10^{-12})(64)}{4(2.18\times10^6)} = \frac{2\pi(52.9\times10^{-12})(16)}{2.18\times10^6}$$

$$52.9 \times 16 = 846.4$$

$$2\pi \times 846.4 \times 10^{-12} = 5318 \times 10^{-12} = 5.318 \times 10^{-9}$$

$$T_4 = \frac{5.318\times10^{-9}}{2.18\times10^6} = 2.44 \times 10^{-15} \text{ s}$$

✅ **Final Answer:**

$$T_4 = 2.4 \text{ femtoseconds}$$

✅ **Correct Option: D) 2.4 femtoseconds**

Question14

The dipole moments of three AB_3-type molecules I, II, and III are measured to be $0.0D, 0.2D$, and 1.5 D , respectively. Which one of the following options is correct regarding the identity of I, II, and III ?

Options:

A.

I: BF_3, **II** : NF_3, **III** : NH_3

B.

I: BF_3, **II** : NH_3, **III** : NF_3

C.

I: ClF_3, **II** : NF_3, **III** : NH_3

D.

I: BCl_3, **II** : NH_3, **III** : NF_3

Answer: A

Solution:

We are given three **AB_3-type molecules** with measured dipole moments:

- **I : 0.0 D**
- **II : 0.2 D**
- **III : 1.5 D**

We must match each value with the correct AB_3 molecule.

Step-1: Identify dipole moments of common AB_3 molecules

1. **BF_3**
    - Structure: *Trigonal planar*
    - Symmetric → all bond dipoles cancel
    - **Dipole moment = 0 D**
2. **NF_3**
    - Structure: *Trigonal pyramidal*
    - F is more electronegative, but the lone pair opposes the bond dipoles
    - Net dipole is **small**, ≈ **0.23 D**

3. **NH_3**

- Structure: *Trigonal pyramidal*
- H is less electronegative than N, so bond dipoles and lone pair reinforce
- Net dipole is **large**, ≈ **1.47 D**

Step-2: Match given dipole values

Dipole moment	Molecule	Reason
0.0 D	**BF_3**	Symmetric planar cancels dipoles
0.2 D	**NF_3**	Small dipole due to partial cancellation
1.5 D	**NH_3**	Strong dipole because N–H dipoles add up with lone pair

Answer:

I : BF_3, II : NF_3, III : NH_3

✓ This corresponds to **Option A**.

Question15

During the charging and discharging of a lead-acid battery (a Pb anode, a grid of Pb packed with $\mathrm{PbO_2}$ as cathode, and an aqueous solution of $\mathrm{H_2SO_4}$ as an electrolyte), which of the following redox reactions does NOT occur?

Options:

A.

$\mathrm{Pb^{4+} + 4e^- \rightarrow Pb}$

B.

$\mathrm{Pb^{2+} \rightarrow Pb^{4+} + 2e^-}$

C.

$\mathrm{Pb \rightarrow Pb^{2+} + 2e^-}$

D.

$\mathrm{2\,Pb^{2+} \rightarrow Pb^{4+} + Pb}$

Answer: A

Solution:

Discharging Process (Battery operating as a galvanic cell):

1. **At the Anode (Negative electrode):** Lead (Pb, oxidation state 0) is oxidized to Lead(II) sulfate ($PbSO_4$, oxidation state +2).

   $$Pb(s) + SO_4^{2-}(aq) \rightarrow PbSO_4(s) + 2e^-$$

   This corresponds to the elemental reaction: $Pb \rightarrow Pb^{2+} + 2e^-$ (if we consider soluble Pb^{2+}, though it immediately precipitates as $PbSO_4$). This is one of the options (Option C).

2. **At the Cathode (Positive electrode):** Lead(IV) dioxide (PbO_2, oxidation state +4) is reduced to Lead(II) sulfate ($PbSO_4$, oxidation state +2).

   $$PbO_2(s) + SO_4^{2-}(aq) + 4H^+(aq) + 2e^- \rightarrow PbSO_4(s) + 2H_2O(l)$$

   This corresponds to the elemental change: $Pb^{4+} + 2e^- \rightarrow Pb^{2+}$ (considering solvated ions for simplicity in states).

Charging Process (Battery operating as an electrolytic cell):

This is the reverse of the discharging process.

1. **At the Anode (Positive electrode during charging):** Lead(II) sulfate ($PbSO_4$, oxidation state +2) is oxidized back to Lead(IV) dioxide (PbO_2, oxidation state +4).

   $$PbSO_4(s) + 2H_2O(l) \rightarrow PbO_2(s) + SO_4^{2-}(aq) + 4H^+(aq) + 2e^-$$

   This corresponds to the elemental reaction: $Pb^{2+} \rightarrow Pb^{4+} + 2e^-$ (Option B).

2. **At the Cathode (Negative electrode during charging):** Lead(II) sulfate ($PbSO_4$, oxidation state +2) is reduced back to Lead (Pb, oxidation state 0).

   $$PbSO_4(s) + 2e^- \rightarrow Pb(s) + SO_4^{2-}(aq)$$

   This corresponds to the elemental reaction: $Pb^{2+} + 2e^- \rightarrow Pb$.

Overall transformation during charging (Option D):

The overall reaction during charging is:

$$2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$$

In terms of lead oxidation states, two moles of Pb(II) (from $PbSO_4$) transform into one mole of Pb(0) and one mole of Pb(IV) (from PbO_2). This is a disproportionation reaction for lead(II): one Pb(II) is reduced to Pb(0), and another Pb(II) is oxidized to Pb(IV).

This exactly matches Option D: $2\ Pb^{2+} \rightarrow Pb^{4+} + Pb$.

Let's examine each option:

- **Option A:** $Pb^{4+} + 4e^- \rightarrow Pb$

  This reaction represents the direct reduction of Lead(IV) to elemental Lead(0). While Lead(IV) (in PbO_2) is reduced during discharge, it is reduced to Lead(II) (in $PbSO_4$), which is a 2-electron process. Elemental Lead(0) is formed from Lead(II) during charging, also a 2-electron process. There is no single, direct 4-electron reduction step from Pb(IV) to Pb(0) as a primary reaction in the lead-acid battery cycle.

- **Option B:** $Pb^{2+} \rightarrow Pb^{4+} + 2e^-$

This reaction occurs during charging at the positive electrode, where Pb(II) from $PbSO_4$ is oxidized to Pb(IV) in PbO_2. This reaction **does occur**.

- **Option C:** $\mathrm{Pb} \rightarrow \mathrm{Pb}^{2+} + 2\mathrm{e}^-$

  This reaction occurs during discharging at the negative electrode, where elemental Pb(0) is oxidized to Pb(II) in $PbSO_4$. This reaction **does occur**.

- **Option D:** $2\ \mathrm{Pb}^{2+} \rightarrow \mathrm{Pb}^{4+} + \mathrm{Pb}$

  This reaction represents the overall transformation of lead(II) during charging, where it acts as both a reducing agent (to form Pb(0)) and an oxidizing agent (to form Pb(IV)). This is the net result of the two half-reactions during charging involving Pb(II) species, and thus it represents a process that **does occur**.

Therefore, the only reaction that does NOT occur as a primary direct step in the lead-acid battery mechanism is Option A.

The final answer is $\boxed{A}$.

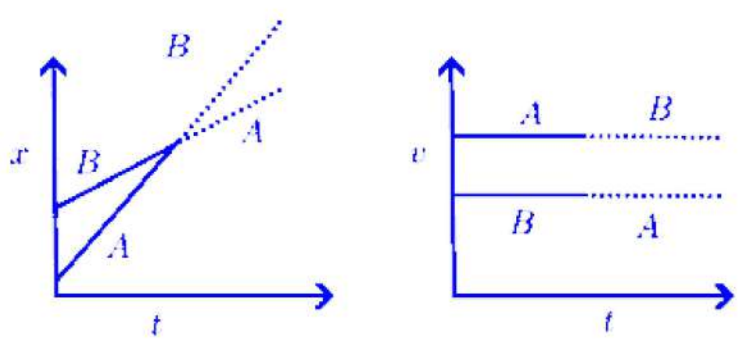
Physics

Question1

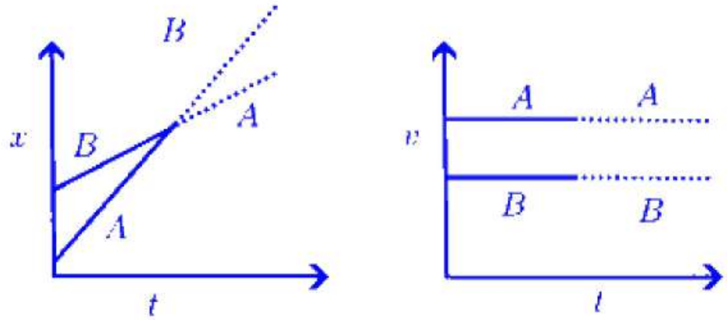
Consider an elastic collision between two particles A and B of same mass, moving in the same direction. Particle A is moving at speed v_A and particle B is moving at speed v_B. In the figures shown, the solid lines represent the motion before the collision and the dotted lines represent the motion after the collision. Which of the following describes the motion of these two particles most accurately?

Options:

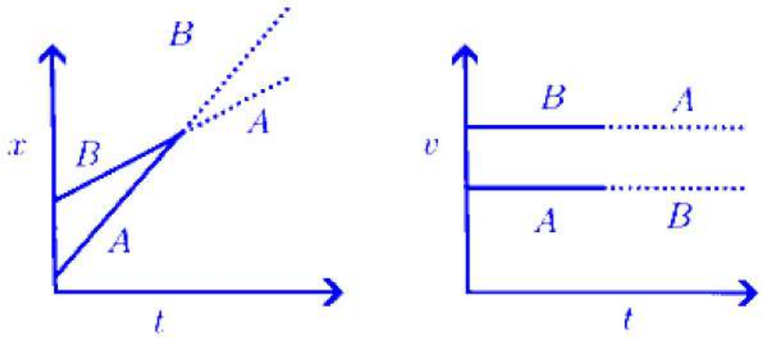
A.



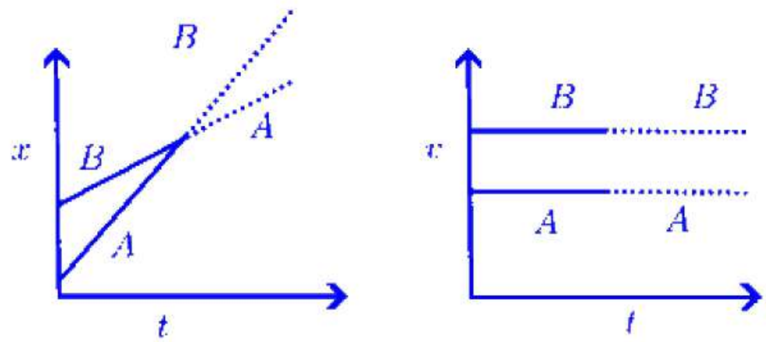
B.



C.



D.



Answer: A

Solution:

For a **1-D elastic collision between two particles of equal mass**, the result is always:

✅ They **exchange velocities**.

If particle **A** (behind) is moving faster than particle **B** (ahead), then after the collision:

- Particle **A** takes on **B's** original (slower) velocity
- Particle **B** takes on **A's** original (faster) velocity

This is a standard result of conservation of momentum and kinetic energy.

So which diagram matches this?

You need the option where:

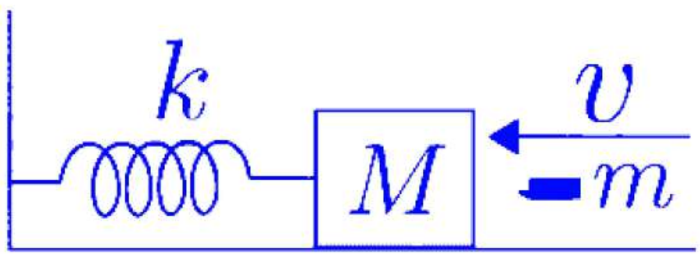
- Before collision:
  $v_A > v_B$
- After collision (dotted lines):
  $v'_A = v_B$ (A slows down)
  $v'_B = v_A$ (B speeds up)

This is exactly what **Option A** shows.

✔ Correct answer: A

Question2

A block of mass M lies at rest connected to a massless spring of spring constant k on a frictionless surface. A bullet of mass m hits the block horizontally with speed v as shown in the figure and is completely stuck to the block. What is the maximum compression in the spring resulting from this impact (assuming that at this point the spring is still not fully compressed)?



Options:

A.

$\sqrt{\frac{m^2v^2}{k(M+m)}}$

B.

$\sqrt{\frac{mv^2}{k}}$

C.

$\sqrt{\frac{Mv^2}{k}}$

D.

$\sqrt{\frac{mMv^2}{k(M+m)}}$

Answer: A

Solution:

Solution

A bullet of mass **m** with speed **v** embeds into a block of mass **M** attached to a spring.
Because the collision is perfectly inelastic, **momentum is conserved** during the impact, but **mechanical energy is not.**

After the bullet sticks to the block, the combined mass $(M + m)$ moves together with some speed V.
Afterward, as the spring compresses, **energy is conserved**.

---

1. Use momentum conservation to find the velocity right after impact

$$mv = (M + m)V$$

So

$$V = \frac{mv}{M + m}$$

---

2. Use conservation of energy for the spring compression

Immediately after the collision, the kinetic energy of the system is:

$$K = \frac{1}{2}(M + m)V^2$$

At maximum compression x_{max}, all kinetic energy has converted into spring potential:

$$\frac{1}{2}(M + m)V^2 = \frac{1}{2}kx_{\text{max}}^2$$

Substitute $V = \frac{mv}{M+m}$:

$$\frac{1}{2}(M+m)\left(\frac{mv}{M+m}\right)^2 = \frac{1}{2}kx_{\max}^2$$

Simplify:

$$\frac{1}{2}\frac{m^2v^2}{M+m} = \frac{1}{2}kx_{\max}^2$$

Thus:

$$x_{\max}^2 = \frac{m^2v^2}{k(M+m)}$$

$$\boxed{x_{\max} = \sqrt{\frac{m^2v^2}{k(M+m)}}}$$

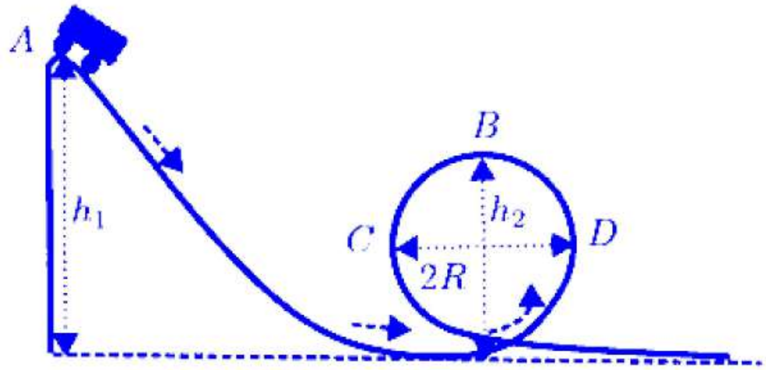
Final Answer

$$\boxed{x_{\max} = \sqrt{\frac{m^2v^2}{k(M+m)}}}$$

This corresponds to **Option A.**

Question3

A cart of mass M is released from A, the highest point of a frictionless track, as shown in the figure. The cart travels along the track and enters the semicircular arc DBC of radius R. The heights of the points A and B are h_1 and h_2 from the ground, respectively. Which of the following quantities does not play any role in ensuring that the cart does not leave the track?



Options:

A.

M

B.

h_1

C.

h_2

D.

R

Answer: A

Solution:

✅ **Concept**

For the cart **not to leave the track**, the **normal force must stay ≥ 0** everywhere, especially at the **top of the loop** (point B), where it is most likely to lose contact.

At point B:

$$N + Mg = \frac{Mv_B^2}{R}$$

For **minimum condition to stay on track:**

$$N = 0 \Rightarrow Mg = \frac{Mv_B^2}{R}$$

So the required minimum speed at B is:

$$v_B^2 = gR$$

Now use **conservation of mechanical energy** from point A (height h_1) to point B (height h_2):

$$Mgh_1 = Mgh_2 + \frac{1}{2}Mv_B^2$$

Substitute $v_B^2 = gR$:

$$Mgh_1 = Mgh_2 + \frac{1}{2}MgR$$

Cancel **M** from the equation:

$$gh_1 = gh_2 + \frac{1}{2}gR$$

$$h_1 = h_2 + \frac{1}{2}R$$

This equation determines whether the cart will remain in contact.

🎯 Which variable does NOT matter?

The condition:

$$h_1 \geq h_2 + \frac{1}{2}R$$

Depends on:

- h_1
- h_2
- R

But the mass **M cancels out and does not appear.**

✅ Final Answer:

A. M

Mass plays **no role** in ensuring the cart remains on the track.

Question4

A circular disk of mass M and radius R is rotating clockwise with a uniform angular velocity ω about an axis passing through the centre, normal to the disk. At time $t = 0$, a torque T is applied along the same axis to oppose the rotation of the disk. What is the angular displacement θ (measured from $t = 0$ in the clockwise direction) that the disk attains before it starts rotating counterclockwise?

Options:

A.

$\theta = \frac{\omega^2 MR^2}{4T}$

B.

$\theta = \frac{\omega^2 MR^2}{8T}$

C.

$\theta = -\frac{\omega^2 MR^2}{4T}$

D.

$\theta = -\frac{\omega^2 MR^2}{8T}$

Answer: A

Solution:

Solution

Given:

- Disk of mass M and radius R
- Rotating clockwise with angular velocity ω
- Moment of inertia of a solid disk:

$$I = \frac{1}{2}MR^2$$

- A constant torque T is applied opposite the rotation
  → angular deceleration:

$$\alpha = -\frac{T}{I} = -\frac{T}{\frac{1}{2}MR^2} = -\frac{2T}{MR^2}$$

The disk slows down, stops, and then starts rotating in the opposite direction.

We want the angular displacement **until it stops** (i.e., before reversing).

---

1. Time until the disk stops

Use:

$$\omega_f = \omega + \alpha t$$

Stopping means $\omega_f = 0$:

$$0 = \omega - \frac{2T}{MR^2}t$$

$$t_{\text{stop}} = \frac{\omega MR^2}{2T}$$

2. Angular displacement until stop

Use rotational kinematics:

$$\theta = \omega t + \frac{1}{2}\alpha t^2$$

Substitute $t = t_{\text{stop}}$:

$$\theta = \omega\left(\frac{\omega MR^2}{2T}\right) + \frac{1}{2}\left(-\frac{2T}{MR^2}\right)\left(\frac{\omega MR^2}{2T}\right)^2$$

Compute term by term:

First term:

$$\omega \cdot \frac{\omega MR^2}{2T} = \frac{\omega^2 MR^2}{2T}$$

Second term:

$$\frac{1}{2}\left(-\frac{2T}{MR^2}\right)\left(\frac{\omega^2 M^2 R^4}{4T^2}\right)$$

Simplify:

- The ½ and 2 cancel
- One T cancels

$$= -\frac{T}{MR^2} \cdot \frac{\omega^2 M^2 R^4}{4T^2}$$

Cancel T, M, R:

$$= -\frac{\omega^2 MR^2}{4T}$$

3. Combine both terms

$$\theta = \frac{\omega^2 MR^2}{2T} - \frac{\omega^2 MR^2}{4T}$$

$$= \frac{\omega^2 MR^2}{4T}$$

✔ Final Answer

$$\boxed{\theta = \frac{\omega^2 MR^2}{4T}}$$

This matches **Option A.**

Question5

A metallic cube initially kept at a temperature T is emitting black body radiation with a power P (energy emitted per unit time). If T is increased by 1%, the power being radiated increases by 4.5%. What is the approximate percentage increase in the volume of the cube in this process?

Options:

A.

0.75%

B.

0.50%

C.

$1.56 \times 10^{-6}\%$

D.

$6.25 \times 10^{-6}\%$

Answer: A

Solution:

Given

A metallic cube radiates as a black body:

$$P = \sigma A T^4$$

Where

- A = surface area (changes if cube expands),
- T increases by **1%**,
- Radiated power increases by **4.5%**.

We must find the **percentage increase in the volume.**

Step 1 — Relate power change to area and temperature changes

Take logarithmic differential of:

$$P = \sigma A T^4$$

$$\frac{\Delta P}{P} = \frac{\Delta A}{A} + 4\frac{\Delta T}{T}$$

Substitute given values:

- $\frac{\Delta P}{P} = 4.5\% = 0.045$
- $\frac{\Delta T}{T} = 1\% = 0.01$

Thus:

$$0.045 = \frac{\Delta A}{A} + 4(0.01)$$

$$0.045 = \frac{\Delta A}{A} + 0.04$$

$$\frac{\Delta A}{A} = 0.005 = 0.5\%$$

Step 2 — Relate area change to volume change

A cube of side L:

$$A = 6L^2, \quad V = L^3$$

Differentials:

$$\frac{\Delta A}{A} = 2\frac{\Delta L}{L}$$

$$\frac{\Delta V}{V} = 3\frac{\Delta L}{L}$$

So:

$$\frac{\Delta V}{V} = \frac{3}{2}\frac{\Delta A}{A}$$

Plug in $\frac{\Delta A}{A} = 0.005$:

$$\frac{\Delta V}{V} = \frac{3}{2}(0.005)$$

$$\frac{\Delta V}{V} = 0.0075 = 0.75\%$$

✔ Final Answer: 0.75%

Question6

Consider two pipes A and B of identical length. A has one end closed and one end open. B has both ends open. Each tube is immersed in a closed chamber of ideal gas having volume V. The chamber containing tube A is at temperature T_A and the chamber containing tube B is at temperature T_B. The sound frequencies corresponding to the n_A-th harmonic in tube A and the n_B-th harmonic in tube B are the same. What is the relation between the temperatures T_A and T_B ?

Options:

A.

$T_A = \left(\frac{4n_B^2}{n_A^2}\right)T_B$

B.

$T_A = \left(\frac{4n_A^2}{n_B^2}\right)T_B$

C.

$T_A = \left(\frac{n_A^2}{4n_B^2}\right)T_B$

D.

$T_A = \left(\frac{n_B^2}{4n_A^2}\right)T_B$

Answer: A

Solution:

Given

- Two pipes of identical length L
  - **Pipe A**: one end closed, one open → *closed–open pipe*
  - **Pipe B**: both ends open → *open–open pipe*
- Temperatures:
  - Chamber containing pipe A: T_A
  - Chamber containing pipe B: T_B
- Harmonic frequencies:
  - n_A-th harmonic of pipe A
  - n_B-th harmonic of pipe B
- These two frequencies are **equal**.

Speed of sound in gas:

$$v \propto \sqrt{T}$$

Step 1: Write frequency formulas

Pipe A (closed–open):

Only **odd** harmonics normally appear but here general n_A is used; formula:

$$f_A = \frac{n_A v_A}{4L}$$

Pipe B (open–open):

$$f_B = \frac{n_B v_B}{2L}$$

Step 2: Set the frequencies equal

Since $f_A = f_B$:

$$\frac{n_A v_A}{4L} = \frac{n_B v_B}{2L}$$

Cancel L:

$$n_A v_A = 2 n_B v_B$$

Step 3: Substitute speed–temperature relation

$$v_A = k\sqrt{T_A}, \quad v_B = k\sqrt{T_B}$$

Plugging in:

$$n_A\sqrt{T_A} = 2n_B\sqrt{T_B}$$

Square both sides:

$$n_A^2 T_A = 4n_B^2 T_B$$

Solve for T_A:

$$T_A = \frac{4n_B^2}{n_A^2} T_B$$

✅ **Final Answer**

$$\boxed{T_A = \left(\frac{4n_B^2}{n_A^2}\right) T_B}$$

This matches **Option A.**

Question7

Consider two waves, which are given by $y_1(x,t) = A\sin(kx - \omega t)$ and $y_2(x,t) = \sqrt{3}A\cos(kx - \omega t)$, where k is the wave number and ω is the angular frequency. The amplitude of the resultant waveform obtained by the superposition of the two waves is A_s and its phase difference with y_1 is ϕ_s. What are A_s and ϕ_s ?

Options:

A.

$A_s = 2A$ and $\phi_s = \frac{\pi}{3}$

B.

$A_s = 2A$ and $\phi_s = \frac{\pi}{6}$

C.

$A_s = \frac{A}{2}$ and $\phi_s = \frac{\pi}{3}$

D.

$A_s = \frac{A}{2}$ and $\phi_s = \frac{\pi}{6}$

Answer: A

Solution:

We are given two waves:

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = \sqrt{3}\,A\cos(kx - \omega t)$$

To add them, express both in sine form.

Step 1: Convert the cosine term

$$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$$

So,

$$y_2 = \sqrt{3}A\cos(kx - \omega t) = \sqrt{3}A\ \sin\left(kx - \omega t + \frac{\pi}{2}\right)$$

Thus the two waves are:

- $y_1 = A\sin\theta$
- $y_2 = \sqrt{3}A\sin(\theta + \frac{\pi}{2})$

where $\theta = kx - \omega t$.

Step 2: Write superposition

$$y = y_1 + y_2 = A\sin\theta + \sqrt{3}A\sin\left(\theta + \frac{\pi}{2}\right)$$

Recognize:

$$\sin(\theta + \tfrac{\pi}{2}) = \cos\theta$$

So:

$$y = A\sin\theta + \sqrt{3}A\cos\theta$$

This is of the form:

$$y = A_s\sin(\theta + \phi_s)$$

Step 3: Find the resultant amplitude

For a combination:

$$R\sin\theta + S\cos\theta = A_s\sin(\theta + \phi_s)$$

Amplitude:

$$A_s = \sqrt{R^2 + S^2}$$

Here:

- $R = A$
- $S = \sqrt{3}A$

Thus:

$$A_s = \sqrt{A^2 + 3.} = \sqrt{4A^2} = 2A$$

Step 4: Find the phase shift

$$\tan\phi_s = \frac{S}{R} = \frac{\sqrt{3}A}{A} = \sqrt{3}$$

So:

$$\phi_s = \frac{\pi}{3}$$

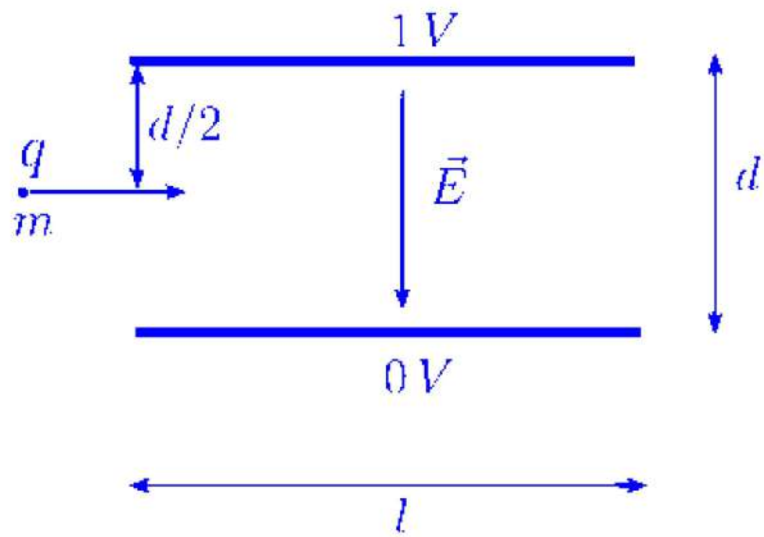
Final Answer

$$\boxed{A_s = 2A \quad \text{and} \quad \phi_s = \frac{\pi}{3}}$$

Which corresponds to **Option A.**

Question8

A particle of charge $q = 1e$ and mass m with kinetic energy K enters an electric field set up by two parallel plates of length l as illustrated in the figure. The potential difference between the two plates is 1 V and their separation is d. What is the minimum value of K (in eV) for which the particle will not hit either of the plates? [e is the charge of the electron.]



Options:

A.

$\frac{l^2}{2d^2}$

B.

$\frac{d^2}{2l^2}$

C.

$\frac{l^2}{d^2}$

D.

$\frac{d^2}{l^2}$

Answer: A

Solution:

Given

- Charge $q = e$
- Mass m
- Kinetic energy K
- Plate length l
- Plate separation d
- Potential difference $V = 1\,\mathrm{V}$

Thus the electric field between the plates is:

$$E = \frac{V}{d} = \frac{1}{d}$$

The particle enters **midway** between the plates (the diagram shows it at $d/2$ from either plate).

---

1. Vertical acceleration

$$a = \frac{qE}{m} = \frac{e}{md}$$

---

2. Horizontal speed

$$K = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

3. Time spent inside the plates

$$t = \frac{l}{v} = \frac{l}{\sqrt{\frac{2K}{m}}}$$

---

4. Vertical displacement under constant acceleration

Since it starts at the middle, the maximum allowed deflection is $d/2$.

$$y = \frac{1}{2}at^2 \leq \frac{d}{2}$$

Insert a and t:

$$\frac{1}{2}\left(\frac{e}{md}\right)\left(\frac{l^2}{\frac{2K}{m}}\right) \leq \frac{d}{2}$$

Simplify:

$$\frac{el^2}{2md} \cdot \frac{m}{2K} \leq \frac{d}{2}$$

$$\frac{el^2}{4Kd} \leq \frac{d}{2}$$

Multiply both sides by $4Kd$:

$$el^2 \leq 2Kd^2$$

Solve for K:

$$K \geq \frac{el^2}{2d^2}$$

Since the problem asks for K in eV and e is the electron charge, the factor e simply converts joules → eV, so:

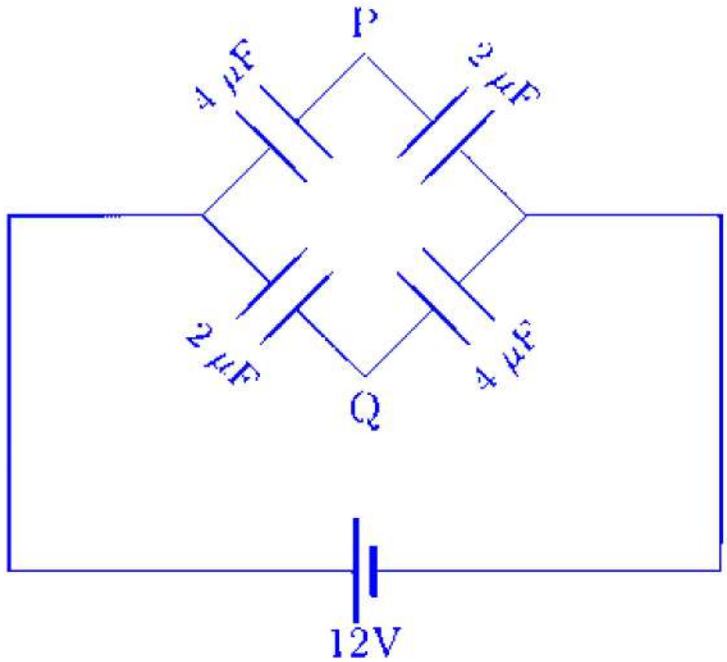
$$\boxed{K_{\text{min}} = \frac{l^2}{2d^2}\ \text{eV}}$$

✅ Correct Answer: A

$$\frac{l^2}{2d^2}$$

Question9

What is the potential difference between the points P and Q in the circuit shown below, once the capacitors are fully charged?



Options:

A.

4 V

B.

0 V

C.

8 V

D.

12 V

Answer: A

Solution:

✅ **Final Answer: 4 V**

We are given a diamond-shaped network of capacitors connected across a **12 V battery**.
The capacitors are arranged as:

- **Top-left:** 4 μF
- **Top-right:** 2 μF
- **Bottom-left:** 2 μF

- **Bottom-right:** 4 μF

Points **P** (top) and **Q** (bottom) lie between two pairs of series capacitors.

1. Recognize the symmetry

Look at the left branch:

- 4 μF in series with 2 μF

Right branch:

- 2 μF in series with 4 μF

These two series combinations are *mirror images* of each other.

Thus, the **equivalent capacitance of each branch is the same**.

For each branch:

$$\frac{1}{C_{\text{series}}} = \frac{1}{4} + \frac{1}{2}$$

$$\frac{1}{C_{\text{series}}} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

$$C_{\text{series}} = \frac{4}{3}\,\mu F$$

Therefore, the **left branch** and **right branch** are identical in equivalent capacitance.

---

2. Voltage divides the same way in both branches

Since both branches have identical equivalent capacitance and are connected in parallel across the **12 V battery**, each branch has the full **12 V** across it.

In a capacitor series chain, the charge is the same on each capacitor, and voltage divides inversely to capacitance:

$$V \propto \frac{1}{C}$$

3. Find voltage between P and Q

Consider the **left branch**:

- 4 μF (top)
- 2 μF (bottom)

Since voltage divides inversely with capacitance:

Let the voltages be V_4 across 4 μF and V_2 across 2 μF.

$$\frac{V_4}{V_2} = \frac{1/C_4}{1/C_2} = \frac{1/4}{1/2} = \frac{1}{2}$$

Thus,

$$V_4 = 4\text{ V} \quad \text{and} \quad V_2 = 8\text{ V}$$

(They must add to 12 V.)

So the **voltage drop from P to Q in the left branch = 8 V – 4 V = 4 V.**

Now check the **right branch**:

Capacitors are 2 μF (top) and 4 μF (bottom), the reverse configuration.

By the same calculation:

- Top (2 μF) = 8 V
- Bottom (4 μF) = 4 V

Thus the **voltage from P to Q** is again:

$$8\text{ V} - 4\text{ V} = 4\text{ V}$$

Both branches agree.

✅ **Answer: 4 V**

Question10

A particle of mass m and charge q moving with a velocity $\vec{v} = v_0(\hat{i} + \hat{j} - \hat{k})$ is placed in a uniform magnetic field $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$. It executes a helical trajectory of radius r and pitch p. Which of the following options is correct?

Options:

A.

$r = \frac{2\sqrt{2}mv_0}{3qB_0}$ and $p = \frac{2\pi mv_0}{3qB_0}$

B.

$r = \frac{mv_0}{3qB_0}$ and $p = \frac{2\pi mv_0}{3qB_0}$

C.

$r = \frac{2\sqrt{2}mv_0}{3qB_0}$ and $p = \frac{4\sqrt{2}\pi mv_0}{3qB_0}$

D.

$r = \frac{2\pi mv_0}{3qB_0}$ and $p = \frac{2\sqrt{2}mv_0}{3qB_0}$

Answer: A

Solution:

The motion of a charged particle in a uniform magnetic field can be decomposed into two independent motions:

1. Motion parallel to the magnetic field, which is uniform. This component determines the pitch (p) of the helical trajectory.
2. Motion perpendicular to the magnetic field, which is circular. This component determines the radius (r) of the helical trajectory.

Let the given velocity be $\vec{v} = v_0(\hat{i} + \hat{j} - \hat{k})$ and the magnetic field be $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$.

First, we need to find the components of the velocity vector parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field.

1. Calculate the magnitude of the magnetic field and velocity:

$$|\vec{B}| = \sqrt{(B_0)^2 + (B_0)^2 + (B_0)^2} = \sqrt{3B_0^2} = B_0\sqrt{3}$$

$$|\vec{v}| = \sqrt{(v_0)^2 + (v_0)^2 + (-v_0)^2} = \sqrt{3v_0^2} = v_0\sqrt{3}$$

2. Calculate the component of velocity parallel to the magnetic field ($v_\parallel$):

The dot product of $\vec{v}$ and $\vec{B}$ is:

$$\vec{v} \cdot \vec{B} = [v_0(\hat{i} + \hat{j} - \hat{k})] \cdot [B_0(\hat{i} + \hat{j} + \hat{k})]$$

$$\vec{v} \cdot \vec{B} = v_0 B_0(1 \cdot 1 + 1 \cdot 1 + (-1) \cdot 1) = v_0 B_0(1 + 1 - 1) = v_0 B_0$$

The magnitude of the velocity component parallel to $\vec{B}$ is $v_\parallel = \frac{|\vec{v} \cdot \vec{B}|}{|\vec{B}|}$:

$$v_\parallel = \frac{v_0 B_0}{B_0\sqrt{3}} = \frac{v_0}{\sqrt{3}} = \frac{\sqrt{3}v_0}{3}$$

3. Calculate the component of velocity perpendicular to the magnetic field ($v_\perp$):

We know that $|\vec{v}|^2 = v_\parallel^2 + v_\perp^2$.

So, $v_\perp^2 = |\vec{v}|^2 - v_\parallel^2$

$$v_\perp^2 = (v_0\sqrt{3})^2 - \left(\frac{v_0}{\sqrt{3}}\right)^2$$

$$v_\perp^2 = 3v_0^2 - \frac{v_0^2}{3} = \frac{9v_0^2 - v_0^2}{3} = \frac{8v_0^2}{3}$$

$$v_\perp = \sqrt{\frac{8v_0^2}{3}} = \frac{2\sqrt{2}v_0}{\sqrt{3}} = \frac{2\sqrt{2}v_0\sqrt{3}}{3} = \frac{2\sqrt{6}v_0}{3}$$

4. Calculate the radius of the helical trajectory (r):

The magnetic force provides the centripetal force for the circular motion.

$$qv_\perp|\vec{B}| = \frac{mv_\perp^2}{r}$$

$$r = \frac{mv_\perp}{q|\vec{B}|}$$

Substitute the values of $v_\perp$ and $|\vec{B}|$:

$$r = \frac{m\left(\frac{2\sqrt{6}v_0}{3}\right)}{q(B_0\sqrt{3})}$$

$$r = \frac{2\sqrt{6}mv_0}{3qB_0\sqrt{3}}$$

$$r = \frac{2\sqrt{2}\sqrt{3}mv_0}{3qB_0\sqrt{3}}$$

$$r = \frac{2\sqrt{2}mv_0}{3qB_0}$$

5. Calculate the pitch of the helical trajectory (p):

The pitch is the distance traveled along the magnetic field direction in one period of circular motion.

First, calculate the period T of the circular motion:

$T = \frac{2\pi m}{q|\vec{B}|}$

$T = \frac{2\pi m}{qB_0\sqrt{3}}$

Now, calculate the pitch $p = v_{\parallel}T$:

$p = \left(\frac{\sqrt{3}v_0}{3}\right)\left(\frac{2\pi m}{qB_0\sqrt{3}}\right)$

$p = \frac{\sqrt{3}v_0(2\pi m)}{3qB_0\sqrt{3}}$

$p = \frac{2\pi m v_0}{3qB_0}$

Comparing our calculated values with the given options:

$r = \frac{2\sqrt{2}mv_0}{3qB_0}$

$p = \frac{2\pi m v_0}{3qB_0}$

These values match Option A.

The final answer is $\boxed{\text{Option A}}$

Question11

A charged particle is moving in a circular orbit with radius r and orbital angular frequency ω in the presence of a magnetic field. The orbit is enclosed within a larger circular metallic frame. The frame is concentric and coplanar with the orbit. The radius of the frame is now gradually decreased. Assuming that the particle remains within the frame at all times, what changes to the trajectory of the particle will occur as the frame is being shrunk?

Options:

A. The radius of the orbit will gradually decrease and the frequency will gradually increase.

B.

The radius of the orbit will gradually increase and the frequency will gradually decrease.

C.

The radius of the orbit will remain the same but the frequency will gradually increase.

D.

Both the radius of the orbit and the frequency will remain unchanged.

Answer: A

Solution:

✅ **STEP-BY-STEP SOLUTION**

Step 1: Particle is initially in circular motion

A charged particle moving in a magnetic field experiences the Lorentz force:

$$F = qvB$$

Circular motion requires centripetal force:

$$\frac{mv^2}{r} = qvB$$

So,

$$r = \frac{mv}{qB}$$

and the angular frequency is:

$$\omega = \frac{v}{r}$$

Step 2: The metallic circular frame is shrinking

When the frame radius decreases, the **magnetic flux through it changes:**

$$\Phi = BA$$

Shrinking the frame → shrinking area A → flux change → **induced electric field** (Faraday's law):

$$\oint E \cdot dl = -\frac{d\Phi}{dt}$$

This induced electric field **acts in the direction of the particle's motion.**

Step 3: The induced electric field accelerates the charged particle

Since the induced electric field is tangential to the orbit:

$$E_{\text{induced}} \parallel v$$

It increases the particle's speed:

$$v \uparrow$$

Step 4: Shrinking frame forces the orbit radius to also shrink

The particle is **confined inside the metallic frame**, which is shrinking.

Even though the particle wants to move to a larger orbit because its speed increased, it **cannot exceed the new frame radius.**

Thus:

$$r \downarrow \quad \text{because the frame itself is shrinking}$$

Step 5: Angular frequency must increase

Angular frequency is:

$$\omega = \frac{v}{r}$$

- v increases due to induced electric field
- r decreases because the frame forces the orbit to shrink

Therefore,

$$\omega \uparrow$$

Both effects increase the frequency.

🎯 FINAL RESULT (expected exam answer)

✔ The radius of the orbit decreases

✔ The frequency of revolution increases

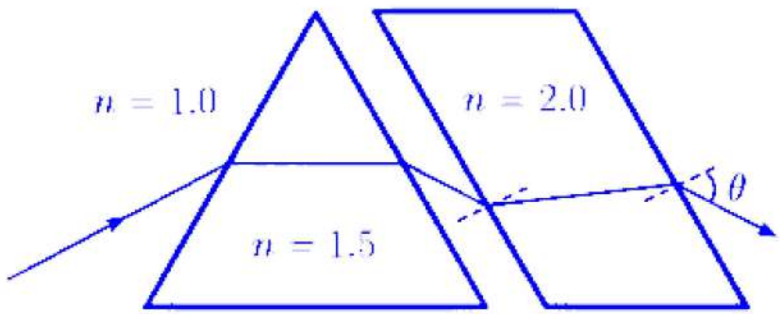
This matches:

🟩 Answer A

"The radius decreases and the frequency increases."

Question12

Consider an equilateral prism of refractive index 1.5 and a parallelepiped block of refractive index 2.0 arranged as shown in the figure such that their adjacent faces are parallel. A light ray enters the prism from air at an angle of incidence such that the ray travels through the prism parallel to its base. What is the angle of emergence θ ?



Options:

A.

$\sin^{-1}(3/4)$

B.

$\sin^{-1}(1/3)$

C.

$\sin^{-1}(1/2)$

D.

$\sin^{-1}(\sqrt{3}/2)$

Answer: A

Solution:

✅ Solution

We are given:

- Equilateral prism: refractive index $n_1 = 1.5$
- Adjacent parallelepiped block: refractive index $n_2 = 2.0$
- The ray enters from air ($n_0 = 1.0$)
- The incident angle is such that **inside the equilateral prism the ray travels parallel to the base.**
- We must find the **final angle of emergence** θ from the block into air.

Step 1: Condition that ray inside the equilateral prism is parallel to the base

In an **equilateral prism**, all apex angles = 60°.

For a ray to travel **parallel to the base**, the refracted angle at entry must equal the prism's apex angle:

$$r_1 = 30^\circ$$

Using Snell's law at first interface (air → prism):

$$1 \cdot \sin i = 1.5 \cdot \sin 30^\circ$$

$$\sin i = 1.5 \cdot \frac{1}{2} = 0.75$$

$$i = \sin^{-1}(3/4)$$

Step 2: Ray leaves the prism and enters the block

When the ray exits the prism, it strikes the interface with the parallelepiped block.

Since the prism and block faces are parallel, the internal incidence angle on the block equals the internal propagation angle:

$$\text{Incident angle in prism at boundary } = 30^\circ$$

Apply Snell's law (prism → block):

$$1.5 \sin(30^\circ) = 2.0 \sin r_2$$

$$1.5 \cdot \frac{1}{2} = 2 \sin r_2$$

$$\sin r_2 = \frac{0.75}{2} = \frac{3}{8}$$

Step 3: Emergence from block into air

The ray strikes the exit face at the same angle r_2 (because faces are parallel):

$$n_2 \sin(r_2) = n_0 \sin(\theta)$$

$$2 \cdot \frac{3}{8} = \sin \theta$$

$$\sin \theta = \frac{3}{4}$$

Thus,

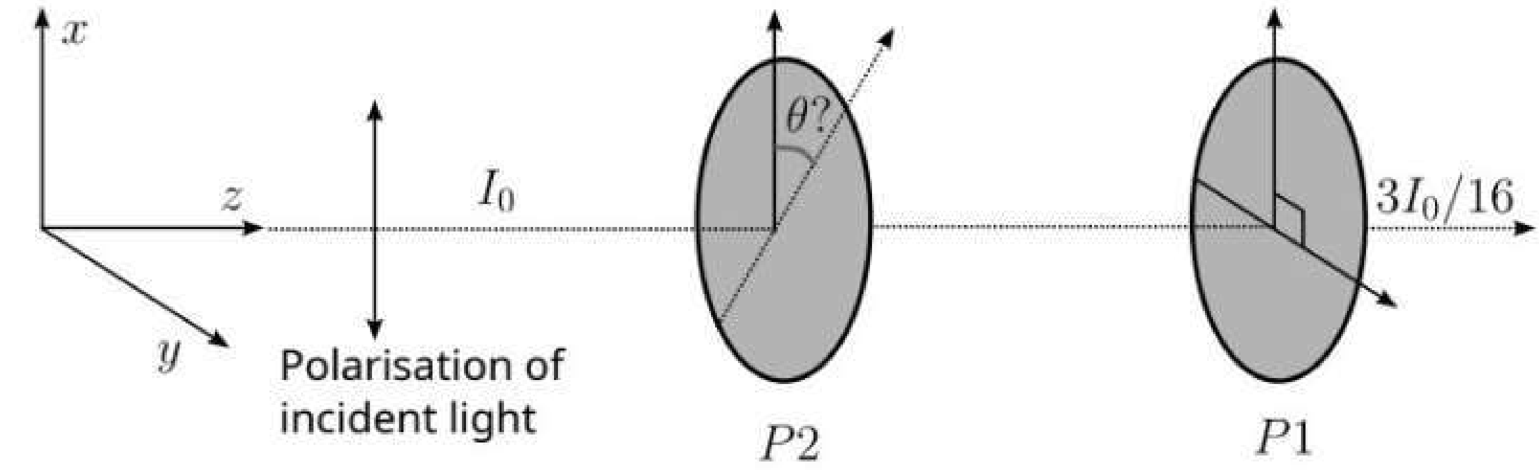
$$\boxed{\theta = \sin^{-1}\left(\frac{3}{4}\right)}$$

✔ **Final Answer:** $\theta = \sin^{-1}(3/4)$

This corresponds to **Option A.**

Question13

A source produces a light beam of intensity I_0 polarized along the x-direction. The beam is sent along the z-direction. It enters a polaroid $P1$ with its polaroid axis aligned along the y-direction so that no light exits the polaroid. When another polaroid $P2$ is placed in between the source and $P1$, the intensity measured after $P1$ is $3I_0/16$. Which among the following is a possible value of θ, the angle of the polaroid axis measured from the x-axis?



Options:

A.

60°

B.

15°

C.

45°

D.

75°

Answer: A

Solution:

Solution:

The incident light is polarized along the **x-axis** with intensity I_0.

Polaroid P_1 has its axis along the **y-axis (90°)** → without any intermediate polaroid, **no light** is transmitted.

When we place polaroid P_2 at an angle θ from the x-axis, the transmitted intensity through both is given by **Malus's law** applied twice:

Step 1: Transmission through P_2

$I_1 = I_0 \cos^2 \theta$

Step 2: Transmission from $P_2 \to P_1$

Angle between axes of P_2 (at θ) and P_1 (at 90°):

$$\phi = 90^\circ - \theta$$

So:

$$I_2 = I_1 \cos^2(90^\circ - \theta) = I_0 \cos^2 \theta \sin^2 \theta$$

Use identity:

$$\cos^2 \theta \sin^2 \theta = \frac{1}{4} \sin^2 2\theta$$

Thus:

$$I_2 = \frac{I_0}{4} \sin^2 2\theta$$

Given:

$$I_2 = \frac{3I_0}{16}$$

Step 3: Solve for θ

$$\frac{I_0}{4} \sin^2 2\theta = \frac{3I_0}{16}$$

$$\sin^2 2\theta = \frac{3}{4}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

Thus:

$$2\theta = 60^\circ,\ 120^\circ$$

$$\theta = 30^\circ,\ 60^\circ$$

Check options:

Available choices:

A: 60°

B: 15°

C: 45°

D: 75°

Only **60°** matches.

✅ **Correct Answer: 60°**

Question14

An electron in the ground state (with energy E_1) of a hydrogen atom, absorbs a photon of energy E_a, and gets excited to a higher energy level of principal quantum number n. What is the value of n ?

Options:

A.

$$\sqrt{\frac{E_1}{E_1+E_a}}$$

B.

$$\sqrt{\frac{E_1}{E_1-E_a}}$$

C.

$$\sqrt{\frac{E_a}{E_1-E_a}}$$

D.

$$\sqrt{\frac{E_a}{E_1+E_a}}$$

Answer: A

Solution:

Solution

For a hydrogen atom, the energy of the electron in level n is:

$$E_n = \frac{E_1}{n^2}$$

where E_1 is the ground-state energy (a negative number, but we use magnitudes here).

When the electron absorbs a photon of energy E_a, it transitions from $n = 1$ to some higher level n. Energy conservation gives:

$$E_a = E_n - E_1$$

Substitute $E_n = E_1/n^2$:

$$E_a = \frac{E_1}{n^2} - E_1$$

Factor out E_1:

$$E_a = E_1\left(\frac{1}{n^2} - 1\right)$$

Rearrange:

$$\frac{E_a}{E_1} = \frac{1}{n^2} - 1$$

$$\frac{1}{n^2} = 1 + \frac{E_a}{E_1}$$

Invert:

$$n^2 = \frac{1}{1 + \frac{E_a}{E_1}} = \frac{E_1}{E_1 + E_a}$$

Take square root:

$$\boxed{n = \sqrt{\frac{E_1}{E_1 + E_a}}}$$

✔ Correct Answer: Option A

This matches exactly with option A.

Question15

A particle of mass m and charge q is accelerated through a distance d_1 by an electric field $\vec{E}$. Another particle of mass M and charge q is accelerated by the same electric field through a distance d_2. Both the particles emerge with the same de Broglie wavelength λ_B. What is the ratio of the distances d_2/d_1 ?

Options:

A.

$\frac{m}{M}$

B.

$\frac{M}{m}$

C.

$\sqrt{\frac{m}{M}}$

D.

$\sqrt{\frac{M}{m}}$

Answer: A

Solution:

Given

- Two particles with the **same charge** q, but **masses** m and M.
- Both are accelerated through distances d_1 and d_2 by the **same electric field** $\vec{E}$.
- Both emerge with the **same de Broglie wavelength** λ_B.

We must find

$$\frac{d_2}{d_1}.$$

Step 1: Energy gained from electric field

A particle accelerated through distance d in electric field E gains kinetic energy:

$$K = qEd.$$

So:

$$K_1 = qEd_1, \qquad K_2 = qEd_2.$$

Step 2: Connect kinetic energy to de Broglie wavelength

De Broglie wavelength:

$$\lambda = \frac{h}{p}$$

Same wavelength → same momentum:

$$p_1 = p_2.$$

Momentum relates to kinetic energy:

$$K = \frac{p^2}{2m}.$$

Thus:

$$\frac{p^2}{2m} = qEd_1, \qquad \frac{p^2}{2M} = qEd_2.$$

Step 3: Solve for distances

From first particle:

$$p^2 = 2mqEd_1.$$

From second:

$$p^2 = 2MqEd_2.$$

Since p^2 is the same:

$$2mqEd_1 = 2MqEd_2.$$

Cancel common factors:

$$md_1 = Md_2.$$

Solve for $\frac{d_2}{d_1}$:

$$\frac{d_2}{d_1} = \frac{m}{M}.$$
