

# IAT 2024

## Biology

1. What will be the sequence of RNA synthesized using the following DNA template strand?

5'-GTCTAGGCTTCTC-3'

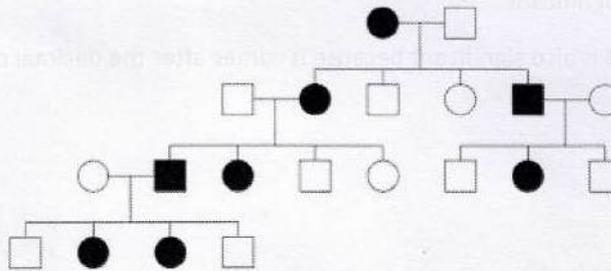
A. 5'-GAGAAGCCUAGAC-3'

B. 5'-GUCUAGGCUUCUC-3'

C. 5'-CAGAUCCGAAGAG-3'

D. 5'-CUCUUCGGAUCUG-3'

2. The following pedigree diagram shows the inheritance of a rare genetic disorder (filled shapes depict affected individuals).



Which of the following is the most likely pattern of inheritance of the disorder?

A. X-linked recessive

B. X-linked dominant

C. Autosomal recessive

D. Autosomal dominant

3. Which of the following proteins plays a direct role in muscle contraction?

A. Trypsin

B. Insulin

C. Myoglobin

D. Troponin

4. Which of the following is NOT derived from the epidermal cell layer in plants?

A. Subsidiary cells from rice leaf

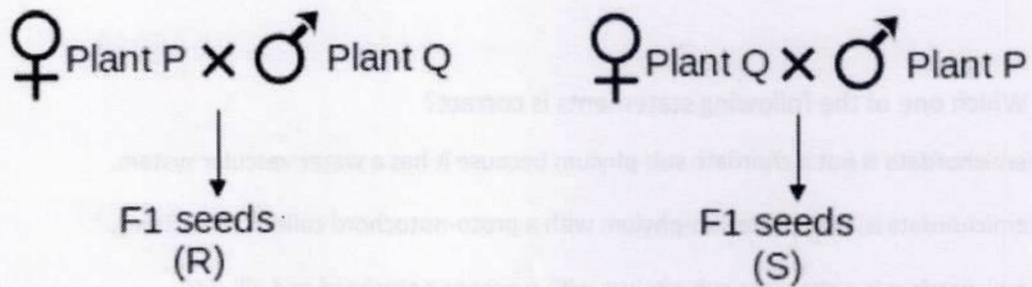
B. Casparian strip from rice root

C. Trichomes from maize leaf

D. Bulliform cells from grass

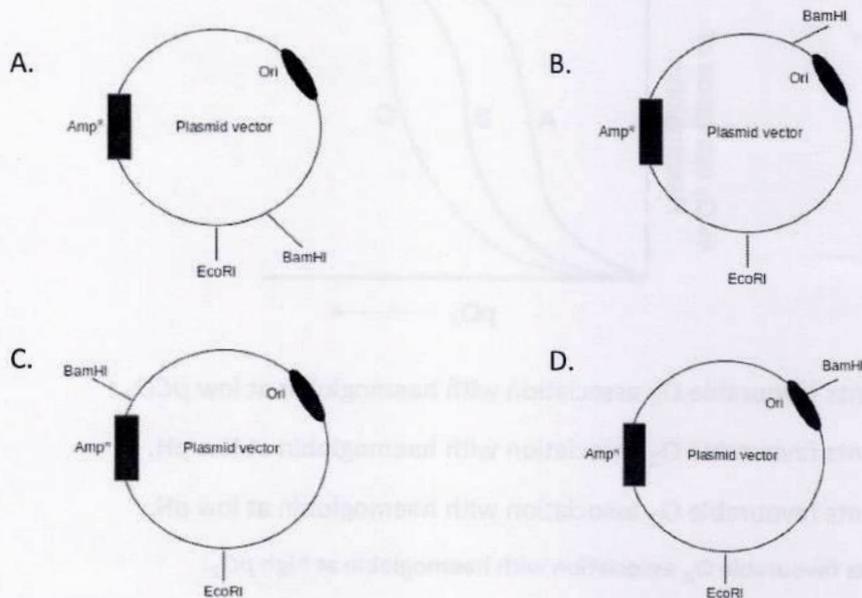


11. Two species of a flowering plant, P ( $2n=20$  chromosomes) and Q ( $2n=30$  chromosomes) are reciprocally crossed with each other as male or female as shown below to produce F1 seeds. Which of the following seed tissues from both the F1 seeds (R and S) will have the same chromosome numbers?



- A. Endosperm
- B. Embryo
- C. Embryo and seed coat
- D. Embryo and endosperm

12. Which of the following plasmid vectors can be used for cloning of a gene, with restriction enzymes BamHI and EcoRI, and ampicillin-containing nutrient agar for selection? [Ori - origin of replication; Amp<sup>R</sup> - gene for ampicillin resistance]



## Chemistry

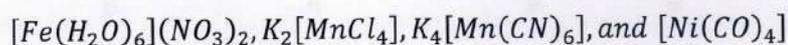
16. If an element with  $Z = 120$  is discovered, then which group of elements will it belong to?

- A. Noble gases
- B. Alkali metals
- C. Halogens
- D. Alkaline earth metals

17. Which one of the following statements is correct about  $N_2$ ,  $CO$ , and  $NO^+$ ?

- A. These are isoelectronic and have identical bond order.
- B. These are isoelectronic and have different bond orders.
- C. These are not isoelectronic but have identical bond order.
- D. These are neither isoelectronic nor have identical bond order.

18. Which of the following complexes exhibit(s) magnetic moment close to 2 BM?

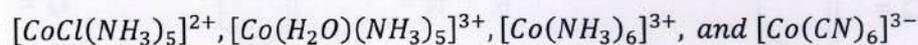


- A.  $K_4[Mn(CN)_6]$  and  $[Ni(CO)_4]$
- B.  $K_2[MnCl_4]$  and  $K_4[Mn(CN)_6]$
- C.  $[Fe(H_2O)_6](NO_3)_2$  and  $K_2[MnCl_4]$
- D. Only  $K_4[Mn(CN)_6]$

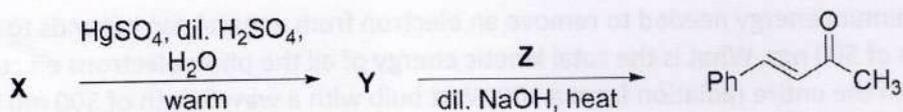
19. According to the VSEPR theory, what are the most stable shapes of  $XeF_4$  and  $SF_4$ , respectively?

- A. See-saw and square planar
- B. Both see-saw
- C. Square planar and see-saw
- D. Both square planar

20. The following complex ions absorb in the ultraviolet-visible region of light. Which one of these shows violet colour?

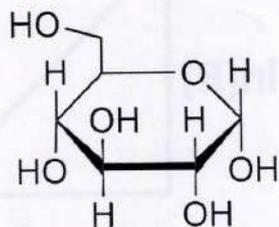


- A.  $[Co(H_2O)(NH_3)_5]^{3+}$
- B.  $[CoCl(NH_3)_5]^{2+}$
- C.  $[Co(NH_3)_6]^{3+}$
- D.  $[Co(CN)_6]^{3-}$

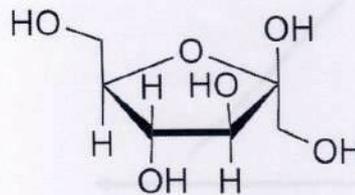


- A.  $\text{X} = \text{H}_3\text{C}-\text{C}\equiv\text{H}$        $\text{Z} = \text{PhCHO}$       B.  $\text{X} = \text{H}_3\text{C}-\text{C}\equiv\text{CH}_3$        $\text{Z} = \text{PhCHO}$
- C.  $\text{X} = \text{Ph}-\text{C}\equiv\text{H}$        $\text{Z} = \text{CH}_3\text{CHO}$       D.  $\text{X} = \text{Ph}-\text{C}\equiv\text{CH}_3$        $\text{Z} = \text{CH}_3\text{CHO}$

25. What are the correct structural descriptions for M and N?



**M**



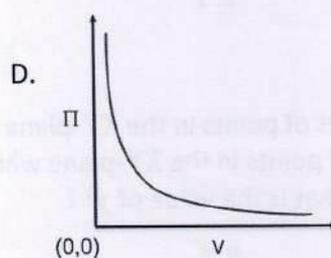
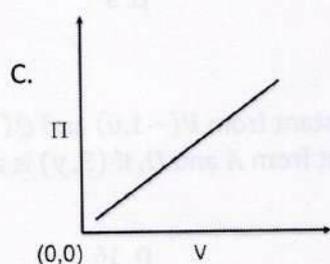
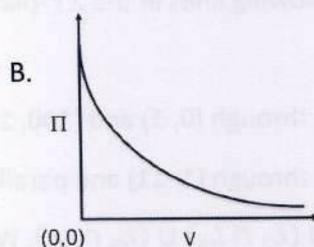
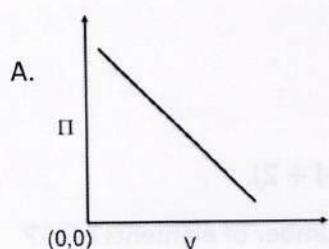
**N**

- A. M is  $\beta$ -D-(+)-glucopyranose and N is  $\beta$ -D-(-)-fructofuranose
- B. M is  $\alpha$ -D-(+)-glucopyranose and N is  $\beta$ -D-(-)-fructofuranose
- C. M is  $\alpha$ -D-(+)-glucopyranose and N is  $\alpha$ -D-(-)-fructofuranose
- D. M is  $\alpha$ -D-(+)-glucofuranose and N is  $\beta$ -D-(-)-fructopyranose

26. Consider an exothermic reaction:  $2\text{A}(\text{s}) \rightarrow \text{B}(\text{s}) + \text{C}(\text{g}) + \text{D}(\text{g})$ . The correct statement about the reaction is

- A. Spontaneous only at very low temperatures
- B. Spontaneous only at very high temperatures
- C. Non-spontaneous at all temperatures
- D. Spontaneous at all temperatures

29. Which one of these plots correctly describes the variation of osmotic pressure ( $\Pi$ ) of a fixed amount of a solute against the volume ( $V$ ) of the solution at a fixed temperature?



30. Consider the following data for KCl solution at a particular temperature. What is the value of the limiting molar conductivity?

Concentration ( $\text{mol L}^{-1}$ )	Molar Conductivity ( $\text{S cm}^2 \text{mol}^{-1}$ )
$1 \times 10^{-4}$	149.1
$9 \times 10^{-4}$	147.1

A.  $149.2 \text{ S cm}^2 \text{mol}^{-1}$

B.  $150.1 \text{ S cm}^2 \text{mol}^{-1}$

C.  $151.1 \text{ S cm}^2 \text{mol}^{-1}$

D.  $152.1 \text{ S cm}^2 \text{mol}^{-1}$

35. Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be a strictly decreasing function with  $|f(t)| < \pi/2$  for all  $t \in \mathbf{R}$ . Let  $g: [0, \pi] \rightarrow \mathbf{R}$  be a function defined by  $g(t) = \sin(f(t))$ . Which one of the following statements is Correct?

- A.  $g$  is increasing on  $[0, \pi]$ .
- B.  $g$  is decreasing on  $[0, \pi]$ .
- C.  $g$  is increasing on  $(0, \pi/2)$  and decreasing on  $(\pi/2, \pi)$ .
- D.  $g$  is decreasing on  $(0, \pi/2)$  and increasing on  $(\pi/2, \pi)$ .

36. Let  $f, g: \mathbf{R} \rightarrow \mathbf{R}$  be functions. If  $g$  is continuous, then which one of the following cases implies that  $f$  is continuous?

- A.  $g(x) = (f(x))^2$
- B.  $g(x) = |f(x)|$
- C.  $g(x) = (f(x))^3$
- D.  $g(x) = \sin(f(x))$

37. What is the largest area of a rectangle, whose sides are parallel to the coordinate axes, that can be inscribed under the graph of the curve  $y = 1 - x^2$  and above the  $x$ -axis?

- A.  $\frac{2}{3\sqrt{3}}$
- B.  $\frac{4}{3\sqrt{3}}$
- C.  $\frac{1}{3}$
- D.  $\frac{4}{3}$

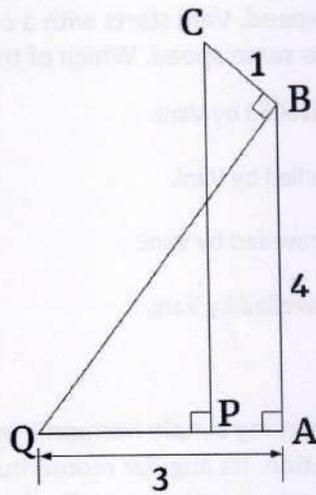
38. Let  $M$  be the set of all  $3 \times 3$  matrices with real entries. Consider the relation  $R$  on  $M$  given by  $R = \{(A, B) \in M \times M: \det(A - B) \text{ is an integer}\}$ . Which one of the following statements is Correct?

- A.  $R$  is reflexive and symmetric, but not transitive.
- B.  $R$  is reflexive, but neither symmetric nor transitive.
- C.  $R$  is an equivalence relation.
- D.  $R$  is symmetric and transitive, but not reflexive.

39. What is the value of  ${}^{23}C_0 + {}^{23}C_2 + {}^{23}C_4 + \dots + {}^{23}C_{22}$ ?

- A.  $2^{23}$
- B.  $2^{22} - 1$
- C.  $2^{23} + 1$
- D.  $2^{22}$

45. In the given figure, the angles  $\angle BAQ = \angle CPQ = \angle CBQ = \frac{\pi}{2}$ ; and the lengths  $QA = 3$  units,  $AB = 4$  units, and  $BC = 1$  unit. What is the length of  $PQ$ ?



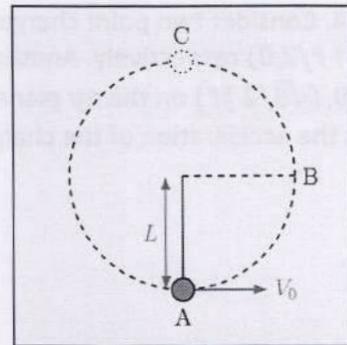
A. 2 unit

B. 2.2 unit

C.  $\sqrt{2}$  unit

D.  $3 - \sqrt{2}$  unit

50. A solid bob of a material having density twice that of water is suspended with a massless and inextensible string of length  $L$ . The whole set-up is placed inside a water-filled tank. The bob is imparted a horizontal velocity  $V_0$  at the lowest point A, while the other end of the string is fixed, such that the bob completes a semi-circular trajectory in the vertical plane. The string becomes slack only when the bob reaches the topmost point C. Assume that the effects of viscosity and water currents are negligible. The acceleration due to gravity is  $g$ . What is the expression for  $V_0$ ?



- A.  $\sqrt{5gL}$       B.  $\sqrt{(5/2)gL}$       C.  $\sqrt{2gL}$       D.  $\sqrt{(3/2)gL}$

51. Consider a solid sphere of radius  $R$  floating in a pond with half of the sphere submerged. The sphere is pushed vertically downwards at the topmost point and released, such that it executes a simple harmonic motion. Acceleration due to gravity is  $g$ . What is the time period of oscillation?

- A.  $2\pi\sqrt{\frac{2R}{g}}$       B.  $2\pi\sqrt{\frac{R}{g}}$       C.  $2\pi\sqrt{\frac{3R}{2g}}$       D.  $2\pi\sqrt{\frac{2R}{3g}}$

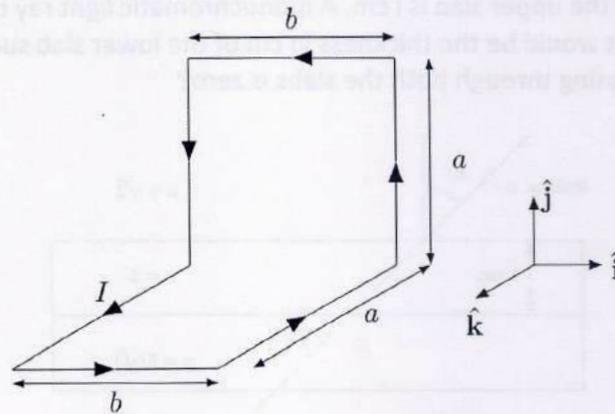
52. One mole of an ideal gas of volume  $V$  and temperature  $T$  is allowed to expand adiabatically to volume  $2V$  while doing no external work. The universal gas constant is  $R$ . What is the pressure of the gas after expansion?

- A.  $\frac{RT}{V}$       B.  $\frac{RT}{2V}$       C.  $\frac{2RT}{V}$       D.  $\frac{RT}{4V}$

53. Two identical boxes contain the same ideal gas. Let  $(n_1, \lambda_1, T_1)$  and  $(n_2, \lambda_2, T_2)$  be the number density, mean free path and temperature of the gas in the first and the second box, respectively. One of the boxes is empty into the other one. What will be the mean free path  $\lambda$  and temperature  $T$  of the gas now?

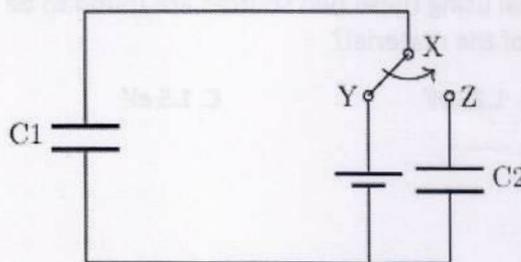
- A.  $\lambda = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}, T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$       B.  $\lambda = \frac{n_1\lambda_1 + n_2\lambda_2}{n_1 + n_2}, T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$   
 C.  $\lambda = \frac{n_1\lambda_1 + n_2\lambda_2}{n_1 + n_2}, T = \sqrt{T_1T_2}$       D.  $\lambda = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}, T = \sqrt{T_1T_2}$

56. A conducting wire carrying a steady current  $I$  is shaped as shown in the figure below. All connected straight segments meet at right angles. What is the magnetic moment of the current loop?



- A.  $Iab(\hat{j} - \hat{k})$       B.  $Iab(\hat{j} + \hat{k})$       C.  $\sqrt{2}Iab(\hat{j} + \hat{k})$       D.  $Iab(\hat{k} - \hat{j})$

57. Consider the shown circuit. The capacitors  $C_1$  and  $C_2$  have capacitances  $2 \mu\text{F}$  and  $8 \mu\text{F}$ , respectively. The switch can connect point X to either Y or Z. Initially XY is connected until the capacitor is fully charged by the battery. Then the switch connects X and Z, and the final charges on  $C_1$  and  $C_2$  are  $Q_1$  and  $Q_2$ , respectively. What is the value of the ratio  $\frac{Q_2}{Q_1 + Q_2}$ ?



- A.  $1/2$       B.  $1/4$       C.  $1/5$       D.  $4/5$

58. Atomic masses of two oxygen isotopes  $^{16}_8\text{O}$  and  $^{18}_8\text{O}$  are  $15.99491 \text{ u}$  and  $17.99916 \text{ u}$ , respectively, where  $\text{u}$  is the atomic mass unit. Masses of proton and neutron are given by  $1.00727 \text{ u}$  and  $1.00866 \text{ u}$ , respectively. The speed of light is  $c$ . What is the difference between the binding energies of  $^{18}_8\text{O}$  and  $^{16}_8\text{O}$  nuclei in units of  $uc^2$ ?

- A.  $0.01307$       B.  $2.00425$       C.  $0.99559$       D.  $3.01291$

## ANSWER KEY 2024

Question Number	Answer						
1	(A)	16	(D)	31	(D)	46	(C)
2	(B)	17	(A)	32	(C)	47	(C)
3	(D)	18	(D)	33	(A)	48	(D)
4	(B)	19	(C)	34	(B)	49	(A)
5	(C)	20	(B)	35	(B)	50	(B)
6	(A)	21	(A)	36	(C)	51	(D)
7	(D)	22	(D)	37	(B)	52	(B)
8	(C)	23	(C)	38	(A)	53	(A)
9	(D)	24	(A)	39	(D)	54	(C)
10	(A)	25	(B)	40	(D)	55	(B)
11	(B)	26	(D)	41	(C)	56	(B)
12	(A)	27	(C)	42	(B)	57	(D)
13	(C)	28	(A)	43	(A)	58	(A)
14	(D)	29	(D)	44	(A)	59	(C)
15	(B)	30	(B)	45	(B)	60	(A)

9. At lower light intensities, there exists a linear relationship between incident light and photosynthetic ( $\text{CO}_2$  fixation) rate. At higher light intensities, the rate becomes stable as other factors become limiting.

10. Succinate dehydrogenase - Inner mitochondrial membrane

Pyruvate dehydrogenase - Mitochondrial matrix

Lactate dehydrogenase - Cytoplasm

ATP synthase - Thylakoid membrane

11. **Embryo** is formed after halving the number of chromosomes from each parent via meiosis (here,  $\frac{20+30}{2} = 25$ ), while seed coat and endosperm's chromosome count depend upon female species.

12. The restriction sites for BamHI and EcoRI should not be too close to the ori site or gene for ampicillin resistance to ensure proper and safe cutting of the plasmid.

13.  $2^{30} = 1$  billion;  $2^{20} = 1$  million.

14. Hemichordata was earlier considered as a sub-phylum under phylum Chordata. But now it is placed as a separate phylum under non-chordata. Hemichordates have a rudimentary structure in the collar region called stomochord, a structure similar to notochord.

15. Curve C does not favour  $\text{O}_2$  association.  $\text{O}_2$  affinity increases with decrease in  $[\text{H}^+]$ .

16. Seeing the periodic table, an element with  $Z=120$  would be placed in Group 2, below radium (Ra) in the periodic table.

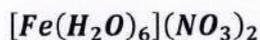
17. **Isoelectronic Series:** An isoelectronic series is a group of molecules or ions that have the same number of electrons. These species have identical electronic configurations and typically exhibit similar chemical and physical properties.

The image shows a standard periodic table with several annotations. A box labeled "Alkali earth metals" points to Group 2. A box labeled "119 120" is placed below the last two elements of Group 2, indicating the predicted positions of elements with atomic numbers 119 and 120. A box labeled "f-block transition elements" points to the lanthanide and actinide series at the bottom of the table.

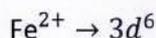
**Bond Order:** The bond order in a molecule can be determined using Molecular Orbital Theory. The bond order is calculated as:

$$\text{Bond Order} = \frac{1}{2} (\text{Number of bonding electrons} - \text{Number of antibonding electrons})$$

Applying to  $\text{N}_2$ ,  $\text{CO}$ , and  $\text{NO}^+$ :



Iron in this complex is in the +2 oxidation state (since the complex is neutral overall and each  $\text{NO}_3$  ion is -1). The electron configuration of  $\text{Fe}^{2+}$  is  $[\text{Ar}]3d^6$ . Water ( $\text{H}_2\text{O}$ ) is a weak field ligand, so it doesn't cause pairing of electrons.



The 3d orbitals will have 4 unpaired electrons in a weak field.

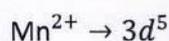
$$\mu = \sqrt{4(4 + 2)} = \sqrt{24} \approx 4.90 \text{ BM}$$



Manganese in this complex is in the +2 oxidation state (since K is +1 and the complex is neutral overall).

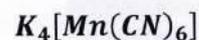
The electron configuration of  $\text{Mn}^{2+}$  is  $[\text{Ar}]3d^5$ .

Chloride ( $\text{Cl}^-$ ) is a weak field ligand, so it doesn't cause pairing of electrons.



The 3d orbitals will have 5 unpaired electrons.

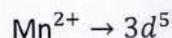
$$\mu = \sqrt{5(5 + 2)} = \sqrt{35} \approx 5.92 \text{ BM}$$



Manganese in this complex is in the +2 oxidation state (since K is +1 and the complex is neutral overall).

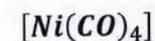
The electron configuration of  $\text{Mn}^{2+}$  is  $[\text{Ar}]3d^5$ .

Cyanide ( $\text{CN}^-$ ) is a strong field ligand, causing pairing of electrons.



In a strong field, the 3d orbitals will have 1 unpaired electron.

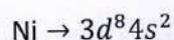
$$\mu = \sqrt{1(1 + 2)} = \sqrt{3} \approx 1.73 \text{ BM}$$



Nickel in this complex is in the 0 oxidation state.

The electron configuration of Ni is  $[\text{Ar}]3d^84s^2$ .

Carbon monoxide ( $\text{CO}$ ) is a strong field ligand, causing pairing of electrons.



In a strong field, all electrons are paired.

arrangement of atoms. Enantiomers typically have identical physical properties except for their interaction with plane-polarized light and reactions in chiral environments.

**4. Positional Isomers:** Positional isomers are a **type of structural isomer where the basic carbon skeleton remains unchanged**, but the position of a functional group, substituent, or multiple bonds differ. This variation can lead to different physical and chemical properties.

22. To determine the most acidic compound among phenol, benzoic acid, picric acid, and o-nitrophenol, we must consider the electron-withdrawing effects of the substituents and their ability to stabilize the conjugate base.

**1. Phenol:** Phenol has a hydroxyl group (-OH) attached to a benzene ring. It is relatively acidic due to the resonance stabilization of the phenoxide ion, but it lacks strong electron-withdrawing groups to further stabilize the conjugate base.

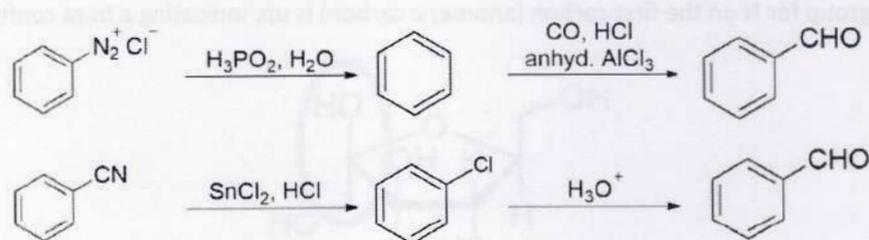
**2. Benzoic Acid:** Benzoic acid has a carboxyl group (-COOH) attached to a benzene ring. The carboxyl group is a strong electron-withdrawing group, significantly stabilizing the conjugate base (benzoate ion) through resonance and inductive effects, making benzoic acid more acidic than phenol.

**3. Picric Acid (2,4,6-trinitrophenol):** Picric acid has three nitro groups at the 2, 4, and 6 positions. Nitro groups are very strong electron-withdrawing groups that greatly stabilize the conjugate base through resonance and inductive effects. The extensive stabilization provided by the three nitro groups makes picric acid extremely acidic.

**4. o-Nitrophenol:** o-Nitrophenol has a nitro group at the para position relative to the hydroxyl group. The nitro group is a strong electron-withdrawing group and significantly stabilizes the phenoxide ion through resonance and inductive effects, making o-nitrophenol more acidic than phenol but less acidic than compounds with additional or stronger electron-withdrawing groups.

Picric acid is the most acidic compound among the given options. The three nitro groups provide strong electron-withdrawing effects, which greatly stabilize the conjugate base, making picric acid significantly more acidic than the others.

23.



26. To determine the spontaneity, we need to consider the Gibbs free energy change ( $\Delta G$ ) for the reaction. The Gibbs free energy change is given by the equation:

$$\Delta G = \Delta H - T\Delta S$$

[Where:  $-\Delta H$  is the change in enthalpy,  $-T$  is the temperature in Kelvin,  $-\Delta S$  is the change in entropy.]

For an exothermic reaction,  $\Delta H < 0$ .

Change in Entropy ( $\Delta S$ ): The reaction produces two gases ( $C(g)$  and  $D(g)$ ) from a solid ( $A(s)$ ). Generally, the production of gases from solids increases the entropy ( $\Delta S > 0$ ) because gases have higher entropy than solids due to their greater freedom of movement and higher number of microstates.

Combining these observations: - Since  $\Delta H$  is negative (favorable for spontaneity) and  $\Delta S$  is positive (favorable for spontaneity),  $\Delta G = \Delta H - T\Delta S$  will be negative at all temperatures.

Therefore, the reaction will be spontaneous at all temperatures.

27. The energy of a photon with a wavelength of 300 nm:

The energy of a photon is given by:

$$E = \frac{hc}{\lambda}$$

Given:

$$h = 6.6 \times 10^{-34} \text{ J s}, c = 3 \times 10^8 \text{ m/s}, \lambda = 300 \text{ nm} = 300 \times 10^{-9} \text{ m}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}}$$

$$E = 6.6 \times 10^{-18} \text{ J}$$

Calculating the energy needed to remove an electron from the metal (work function):

$$\phi = \frac{hc}{\lambda}$$

Given:

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$$

$$\phi = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}}$$

$$\phi = 3.96 \times 10^{-18} \text{ J} \approx 4 \times 10^{-18}$$

Calculating the kinetic energy of a single photoelectron:

29.

$$\Pi = \frac{nRT}{V}$$

where: -  $\Pi$  is the osmotic pressure, -  $n$  is the number of moles of solute, -  $R$  is the universal gas constant, -  $T$  is the temperature in Kelvin, -  $V$  is the volume of the solution.

Given that the amount of solute ( $n$ ) and temperature ( $T$ ) are fixed, the relationship simplifies to:

$$\Pi \propto \frac{1}{V}$$

$$V \rightarrow 0$$

$$\Pi \rightarrow \infty$$

30. Given data:

$$C = 1 \times 10^{-4} \text{ mol/L} \rightarrow \Lambda_m = 149.1 \text{ Scm}^2/\text{mol}$$

$$C = 9 \times 10^{-4} \text{ mol/L} \rightarrow \Lambda_m = 147.1 \text{ Scm}^2/\text{mol}$$

Using the formula for extrapolation:

$$\Lambda_m = \Lambda_m^0 - K\sqrt{C}$$

(where  $\Lambda_m^0$  is the limiting molar conductivity,  $K$  is a constant, and  $C$  is the concentration.)

We have two equations:

$$149.1 = \Lambda_m^0 - K\sqrt{1 \times 10^{-4}}$$

$$147.1 = \Lambda_m^0 - K\sqrt{9 \times 10^{-4}}$$

Subtracting:

$$149.1 - 147.1 = (\Lambda_m^0 - 0.01K) - (\Lambda_m^0 - 0.03K)$$

$$2 = 0.02K$$

$$K = 100$$

Now, substitute  $K = 100$  back into one of the original equations to find  $\Lambda_m^0$

$$149.1 = \Lambda_m^0 - 0.01 \times 100$$

$$149.1 = \Lambda_m^0 - 1$$

$$\Lambda_m^0 = 149.1 + 1 = 150.1 \text{ S cm}^2/\text{mol}$$

So, the value of the limiting molar conductivity  $\Lambda_m^0$  for KCl is  $150.1 \text{ S cm}^2/\text{mol}$ .

on squaring both sides, we get:

$$(x + 1)^2 + y^2 = (x - 1)^2 + y^2$$

$$x^2 + 2x + 1 + y^2 = x^2 - 2x + 1 + y^2$$

$$4x = 0$$

$$x = 0$$

→  $A$  is the line  $x = 0$  or  $y$  - axis

$B$  is the sets of points which are equidistant from the  $y$ -axis and  $Q(1,0)$ , this represents a parabola which can be written as follows:

Since the distance from the  $y$  - axis is the  $x$ -coordinate:

so we can simply equate the  $x$  coordinate from the point  $Q(1,0)$

$$x = \sqrt{(x - 1)^2 + y^2}$$

on squaring both sides,

$$x^2 = x^2 - 2x + 1 + y^2$$

$$y^2 = 2x - 1$$

For the point  $(5, y)$ ,  $x = 5$

$$\Rightarrow y^2 = 2(5) - 1 = \boxed{9}$$

33. The given lines are:

$$L_1: x = 2 + \lambda, y = 3 + 2\lambda, z = 4 + 3\lambda$$

$$L_2: x = 4 + \lambda, y = 4, z = 4 + \lambda$$

The point  $(2,3,4)$  lies on  $L_1$ , and it is the point closest to  $L_2$ . To find which point of  $L_2$  is closest to  $L_1$ , we can use the fact that the vector connecting the closest points on the lines is perpendicular to both the direction vectors of the lines.

Direction vector of  $L_1$ , denoted as  $\mathbf{d}_1$ , is:

$$\mathbf{d}_1 = \langle 1, 2, 3 \rangle$$

Direction vector of  $L_2$ , denoted as  $\mathbf{d}_2$ , is:

$$\mathbf{d}_2 = \langle 1, 0, 1 \rangle$$

Let  $(x_1, y_1, z_1)$  be a point on  $L_1$  and  $(x_2, y_2, z_2)$  be a point on  $L_2$ . The parametric equations give:

A point on  $L_1$  is  $P_1(\lambda) = (2 + \lambda, 3 + 2\lambda, 4 + 3\lambda)$  A point on  $L_2$  is  $P_2(\mu) = (4 + \mu, 4, 4 + \mu)$

The vector connecting these points is:

$$s_n = \frac{n}{2}(2a + (n-1)d), a_n = a_1 + (n-1)d$$

on substituting,

$$2\left(\frac{n}{2}(2a_1 + (n-1)d)\right) = n(c + a_1 + (n-1)d)$$

$$2a_1 + (n-1)d = c + a_1 + (n-1)d$$

$$2a_1 = a_1 + c$$

$$\boxed{c = a_1}$$

Since  $c = a_1$ , our assumption is confirmed,

$$\boxed{a_1, a_2, a_3 \dots \text{ are in AP}}$$

35.  $f: \mathbf{R} \rightarrow \mathbf{R}$  is a strictly decreasing function.

$$|f(t)| < \pi/2 \text{ for all } t \in \mathbf{R}$$

Since  $f$  is strictly decreasing,

$$g(t) = \sin(f(t))$$

$$g'(t) = \cos(f(t)) \cdot f'(t)$$

Since  $f(t)$  is strictly decreasing,  $f'(t)$  will always be negative

$\cos(f(t))$ , is strictly positive on  $f(t) \in (-\pi/2, \pi/2)$ .

$$g'(t) = \cos(f(t)) \cdot f'(t) = (+)(-) < 0$$

Therefore, the composition  $g(t) = \sin(f(t))$  will be strictly decreasing on  $[0, \pi]$ .

36.

$$g(x) = (f(x))^3$$

For  $g(x)$  to be continuous,  $f(x)$  must also be continuous because the composition of continuous functions is continuous.

$$g(x) = |f(x)|$$

Suppose a function:

$$f(x) = \begin{cases} 2 & x \geq 0 \\ -2 & x < 0 \end{cases}$$

In this case  $|f(x)|$  will be continuous and equal to 2 for any value of  $x$  but  $f(x)$  is not continuous

$$g(x) = (f(x))^2$$

### 38. 1. Reflexivity:

For a relation  $R$  to be reflexive, every element must be related to itself. That is,  $(A, A) \in R$  for all  $A \in M$ .

- Consider any matrix  $A \in M$ .
- We need to check if  $(A, A) \in R$ .
- $\det(A - A) = \det(0) = 0$ .
- Since 0 is an integer,  $(A, A) \in R$ .

Therefore, the relation  $R$  is reflexive.

### 2. Symmetry:

For a relation  $R$  to be symmetric, if  $(A, B) \in R$ , then  $(B, A)$  must also be in  $R$ .

- Suppose  $(A, B) \in R$ .
- This means  $\det(A - B)$  is an integer.
- We need to check if  $\det(B - A)$  is also an integer.
- Since the determinant of a matrix and its negative are equal up to a sign,  
$$\det(B - A) = (-1)^3 \det(A - B) = -\det(A - B).$$
- If  $\det(A - B)$  is an integer, then  $-\det(A - B)$  is also an integer.

Therefore, the relation  $R$  is symmetric.

### 3. Transitivity

A relation  $R$  on a set  $M$  is transitive if for any  $A, B, C$  in  $M$ :  $(A, B) \in R$  and  $(B, C) \in R \Rightarrow (A, C) \in R$ . In our case,  $(A, B) \in R$  means that  $\det(A - B)$  is an integer.

Let,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\det(A - B) = 1$$

$$\det(B - C) = 0$$

$$\det(A - C) = 0.5$$

The relation  $R$  is not transitive on  $M$  because  $(A, B) \in R$  and  $(B, C) \in R$  do not imply that  $(A, C) \in R$ .

42. Given,

$$X\text{-coordinate} = |z - iz|$$

$$Y\text{-coordinate} = |z|^2$$

Let,  $z = x + iy$

$$\text{Then, } X\text{-coordinate} = |(x + iy) - i(x + iy)|$$

$$= |x + iy - ix - i^2y| = |x + iy - ix + y|$$

$$= |x + y + i(y - x)|$$

$$= \sqrt{(x + y)^2 + (y - x)^2}$$

$$= \sqrt{x^2 + y^2 + 2xy + x^2 + y^2 - 2xy}$$

$$X\text{-coordinate} = \sqrt{2(x^2 + y^2)}$$

$$Y\text{-coordinate} = |x + iy|^2 = x^2 + y^2$$

We can establish the relationship between X and Y as following:

$$X^2 = 2(x^2 + y^2)$$

$$\frac{X^2}{2} = \frac{2(x^2 + y^2)}{2}$$

$$\boxed{\frac{X^2}{2} = Y}$$

This equation represents a parabola

43. Given:

$$P(A_{\text{break}}) = \frac{1}{4}$$

$$P(B_{\text{break}}) = \frac{1}{4}$$

$$P(C_{\text{break}}) = \frac{1}{2}$$

Then,

$$P(A_{\text{work}}) = 1 - P(A_{\text{break}}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(B_{\text{work}}) = 1 - P(B_{\text{break}}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(C_{\text{work}}) = 1 - P(C_{\text{break}}) = 1 - \frac{1}{2} = \frac{1}{2}$$

The cases where the ship cannot complete the voyage are:

$$e^{\int \frac{1}{x} dx} = e^{\log(x)} = x$$

$$tx = \int x^2 dx$$

$$\sin(y)x = \frac{x^3}{3} + c$$

using the point  $y = \frac{\pi}{2}$  at  $x = \sqrt{3}$

$$(1)\sqrt{3} = \frac{3\sqrt{3}}{3} + c$$

$$\sqrt{3} = \sqrt{3} + c$$

$$\Rightarrow c = 0$$

Final equation,

$$x\sin(y) = \frac{x^3}{3}$$

$$y = \sin^{-1}\left(\frac{x^2}{3}\right)$$

For  $x = \sqrt{\frac{3}{2}}$ ,

$$y = \sin^{-1}\left(\frac{3/2}{3}\right) = \sin^{-1}\frac{1}{2}$$

$$y = \frac{\pi}{6}$$

46. Let: -  $v_1$  = Abhijit's constant speed. -  $v_2$  = Vani's speed when they meet. -  $a$  = Vani's constant acceleration. -  $t$  = time taken for them to meet.

Since Abhijit moves with a constant speed, the distance  $d_1$  he travels is:

$$d_1 = v_1 \cdot t$$

Vani starts from rest with a constant acceleration. Therefore, her speed when they meet is given by:

$$v_2 = a \cdot t$$

The distance  $d_2$  Vani travels can be found using  $s = ut + \frac{1}{2}at^2$

$$d_2 = \frac{1}{2}at^2$$

Since they meet on the track with the same speed:

$$v_1 = v_2 = a \cdot t$$

Therefore:

$$a = \frac{v_1}{t}$$

Substitute this value of  $a$  into the equation for  $d_2$ :

$$d_2 = \frac{1}{2} \left( \frac{v_1}{t} \right) t^2$$

$$d_2 = \frac{1}{2} v_1 t$$

So, we have:

$$d_1 = v_1 \cdot t$$

$$d_2 = \frac{1}{2} v_1 \cdot t$$

Therefore:

$$\boxed{d_1 = 2 \cdot d_2}$$

This means Abhijit travelled double the distance travelled by Vani.

47. Given,

The amplitude of SHM is  $\theta_0$

Let us form a general equation and the angle of the pendulum with the vertical.

48. Let  $T_1$  and  $T_2$  be the tensions in the cord on the sides of the masses  $M$  and  $M/2$ , respectively. The accelerations of the two masses will be  $a$ .

$$Mg - T_1 = Ma \Rightarrow T_1 = M(g - a) \quad (\text{Equation 1})$$

For the block with mass  $M/2$ :

$$T_2 - \frac{M}{2}g = \frac{M}{2}a \Rightarrow T_2 = \frac{M}{2}(g + a) \quad (\text{Equation 2})$$

Since the cord is inextensible and there is no slipping, the linear acceleration  $a_1$  of the blocks is related to the angular acceleration  $\alpha$  of the disc by:

$$a = R\alpha$$

The torque on the disc due to the tensions  $T_1$  and  $T_2$  is:

$$\text{Torque} = T_1R - T_2R = I\alpha$$

where  $I$  is the moment of inertia of the disc about its axis,  $I = \frac{1}{2}MR^2$ .

$$\Rightarrow (T_1 - T_2)R = \frac{1}{2}MR^2\alpha \quad (\text{Equation 3})$$

Substitute the tensions from Equations 1 and 2 into Equation 3:

$$M(g - R\alpha) - \frac{M}{2}(R\alpha + g) = \frac{1}{2}MR\alpha$$

Cancelling out  $M$  and multiply by 2:

$$2(g - R\alpha) - (R\alpha + g) = R\alpha$$

$$2g - 2R\alpha - R\alpha - g = R\alpha$$

$$g = 4R\alpha$$

$$\alpha = \frac{g}{4R}$$

49. Given,

$$x(t) = \sin^2(\omega t)\cos^3(\omega t)$$

$\sin(\omega t)$  has a period  $T_1 = \frac{2\pi}{\omega}$

$\cos(\omega t)$  also has a period  $T_1 = \frac{2\pi}{\omega}$

To find the period of  $x(t)$ , we should determine the smallest positive  $T$  such that  $x(t + T) = x(t)$ .

The weight of the ball ( $W$ ) is:

$$W = V\rho_{\text{ball}}g = V(2\rho)g$$

The net force ( $F_{\text{net}}$ ) acting on the ball when submerged in water is the difference between the weight of the ball and the buoyant force:

$$F_{\text{net}} = W - F_b = V(2\rho)g - V\rho g = V\rho g$$

The effective acceleration ( $g_{\text{eff}}$ ) is the net force per unit mass:

$$g_{\text{eff}} = \frac{F_{\text{net}}}{m} = \frac{V\rho g}{V(2\rho)} = \frac{g}{2}$$

We can use this expression to calculate the velocity ( $V_0$ ) on the basis of the standard results of vertical circular motion which can be also be derived by using TME, but here we will only use the standard result:

$$v_{AC} = \sqrt{5rg_{\text{eff}}}$$

Hence,

$$V_0 = \sqrt{\frac{5}{2}Lg}$$

51. Given: Radius of the sphere:  $R$  Half of the sphere is submerged in the pond.

Solution:

First, identify equilibrium conditions by equating the buoyant force and the sphere's weight. Next, consider the forces when the sphere is displaced: the additional buoyant force acts as the restoring force. Use this to set up the equation of motion for simple harmonic motion (SHM). Compare the resulting equation with the standard SHM form to find the angular frequency. Finally, derive the time period using the relationship between angular frequency and the time period of SHM.

#### Buoyant Force and Equilibrium:

The buoyant force when the sphere is at equilibrium (half-submerged) is equal to the weight of the sphere.

$$V_{\text{sub}} = \frac{1}{2} \times \frac{4}{3} \pi R^3 = \frac{2}{3} \pi R^3$$

#### Weight of the sphere:

$$W = mg = (\rho \times V)g = \rho \times \frac{4}{3} \pi R^3 \times g$$

$$\omega = \sqrt{\frac{3g}{2R}}$$

The time period  $T$  is given by  $T = \frac{2\pi}{\omega}$ .

Substituting  $\omega$ ,

$$T = 2\pi \sqrt{\frac{2R}{3g}}$$

52. Given,

Initial volume =  $V$

Final volume =  $2V$

Initial temperature =  $T$

Number of moles = 1

The process is adiabatic, meaning no heat is exchanged with the surroundings.

However, since it is stated that no external work is done, this implies that the process may involve a free expansion. In free expansion, the gas expands into a vacuum and does no work on the surroundings, and since it is adiabatic, the internal energy and hence the temperature of the gas remains constant.

Therefore, the temperature after expansion is still  $T$ .

Using the ideal gas law:

$$PV = nRT$$

For the initial state:

$$P_i V = RT$$

For the final state, the volume is  $2V$ , and the temperature remains  $T$ :

$$P_f \cdot 2V = RT$$

Solving for the final pressure  $P_f$ :

$$P_f = \frac{RT}{2V}$$

53. You may be familiar with the formula:

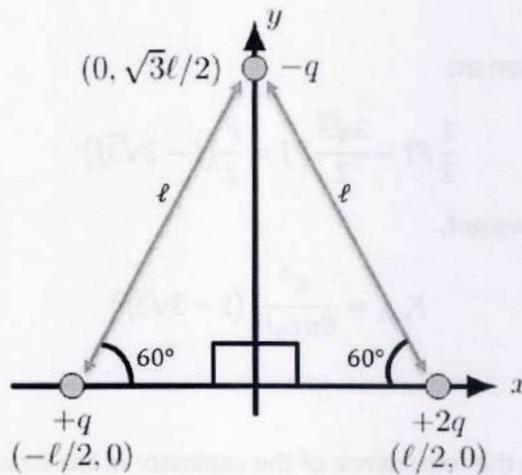
$$\lambda = \frac{k_B T}{\sqrt{2\pi d^2 P}}$$

Where:

$$T_f = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

54. Using the Pythagorean theorem, The distance between  $-q$  and the positive charges

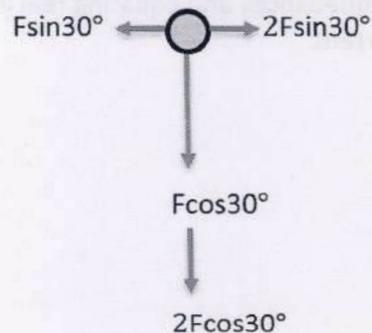
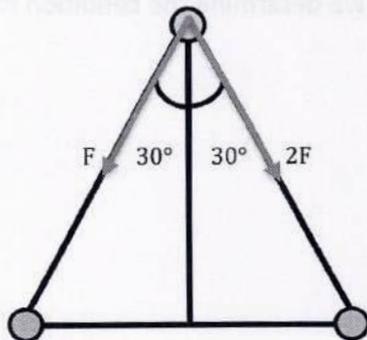
$$\sqrt{\frac{l^2}{4} + \frac{3l^2}{4}} = l$$



Let,

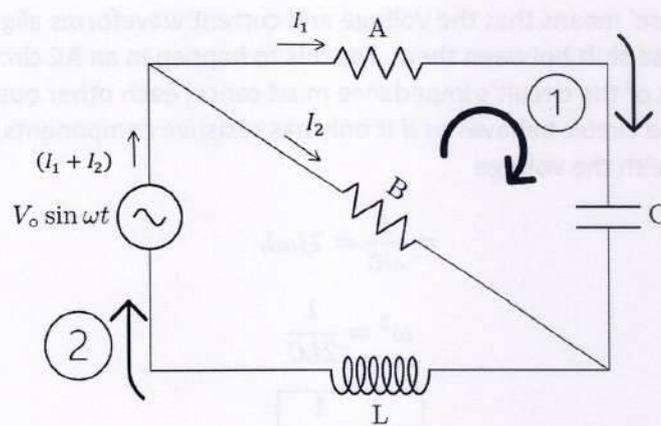
$$|F| = \left| \frac{q^2}{4\pi\epsilon_0 l^2} \right|$$

Then, the forces can be redrawn as the following:



Simplifying:





KCL in loop 1:

$$i_2 R - \frac{i_2 j}{\omega C} - i_1 R = 0 \quad (\text{equation 1})$$

$$i_1 R = i_2 \left( R - \frac{j}{\omega C} \right)$$

$$i_1 = i_2 \left( 1 - \frac{j}{\omega RC} \right)$$

KCL in loop 2:

$$i_2 R + i_2 \left( \frac{-j}{\omega C} \right) + (i_1 + i_2) j \omega L = V_{\text{avg}} = 0$$

$$i_2 \left( R - \frac{j}{\omega C} \right) = (i_1 + i_2) j \omega L$$

On substituting,

$$i_2 \left( R - \frac{j}{\omega C} \right) = \left( i_2 \left( 1 - \frac{j}{\omega RC} \right) + i_2 \right) j \omega L$$

Cancelling out  $i_2$ ,

$$R - \frac{j}{\omega C} = \left( 2 - \frac{j}{\omega RC} \right) j \omega L$$

Since  $j = \sqrt{-1}$ ,  $j^2 = -1$

$$R - \frac{j}{\omega C} = 2j\omega L + \frac{L}{RC}$$

57. When the capacitor  $C_1$  with capacitance  $C_1 = 2\mu\text{F}$  is connected to a battery of (let) potential  $V$ , the charge  $Q$  on  $C_1$  can be calculated using the formula:

$$Q = C_1 \times V$$

After disconnecting  $C_1$  from the battery, it is then connected to another capacitor  $C_2$  with capacitance  $C_2 = 8\mu\text{F}$ . Since the capacitors are connected in parallel, they will share the charge. The total charge  $Q$  is conserved and distributed between the two capacitors.

The total initial charge is:

$$Q = C_1 \times V$$

After the connection, let  $Q_1$  and  $Q_2$  be the charges on  $C_1$  and  $C_2$ , respectively. The final voltage across both capacitors will be the same (let's denote it as  $V_f$ ):

$$Q_1 = C_1 \times V_f$$

$$Q_2 = C_2 \times V_f$$

The total charge conservation gives us:

$$Q_1 + Q_2 = Q$$

Substituting the expressions for  $Q_1$  and  $Q_2$ :

$$C_1 \times V_f + C_2 \times V_f = C_1 \times V$$

$$V_f(C_1 + C_2) = C_1 \times V$$

Solving for  $V_f$ :

$$V_f = \frac{C_1}{C_1 + C_2} (V)$$

Finding the Ratio  $\frac{Q_2}{Q_1 + Q_2}$ :

Now we calculate  $Q_2$  using  $V_f$ :

$$Q_2 = C_2 \times V_f = C_2 \times \frac{C_1 \times V}{C_1 + C_2}$$

The total charge  $Q_1 + Q_2 = Q$ :

$$Q = C_1 \times V$$

The ratio  $\frac{Q_2}{Q_1 + Q_2}$  is:

$$\frac{Q_2}{Q_1 + Q_2} = \frac{C_2 \times V_f}{C_1 \times V} = \frac{C_2 \times \frac{C_1 \times V}{C_1 + C_2}}{C_1 \times V}$$

Calculate the binding energies in units of  $u \cdot c^2$ :

- Binding energy for  $^{16}_8\text{O}$ :

$$E_{b,16} = \Delta m_{16} \cdot c^2 = 0.13253 u \cdot c^2$$

- Binding energy for  $^{18}_8\text{O}$ :

$$E_{b,18} = \Delta m_{18} \cdot c^2 = 0.1456 u \cdot c^2$$

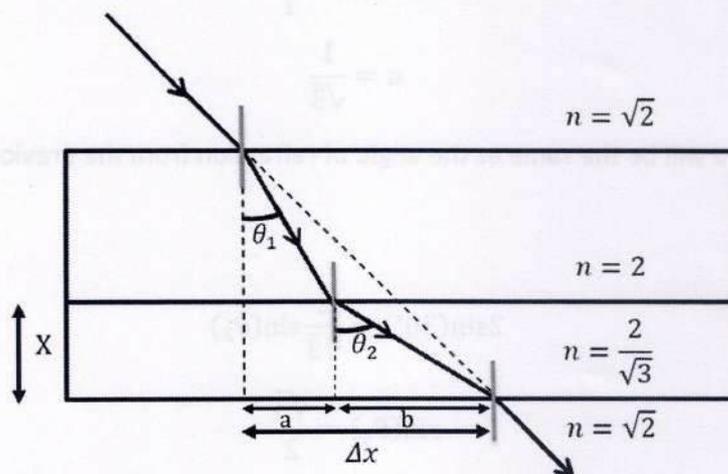
Calculate the difference in binding energies:

$$\Delta E_b = E_{b,18} - E_{b,16} = 0.1456 u \cdot c^2 - 0.13253 u \cdot c^2 = 0.01307 u \cdot c^2$$

Therefore, the difference between the binding energies of the  $^{18}_8\text{O}$  and  $^{16}_8\text{O}$  nuclei is

$$\boxed{0.01307 u \cdot c^2}$$

59.



As we can see from the diagram,

$$\Delta x = a + b$$

and

$$\tan(45^\circ) = \frac{\Delta x}{1 + X}$$

$$\Delta x = 1 + X$$

IAT PYQ

$$1 - \frac{1}{\sqrt{3}} = X\sqrt{3} - X$$

$$\frac{1}{\sqrt{3}}(\sqrt{3} - 1) = X(\sqrt{3} - 1)$$

$$X = \frac{1}{\sqrt{3}}$$

60. Given:

1. Stopping Potential for  $\lambda$ :  $V_1 = 1 \text{ V}$
2. Stopping Potential for  $\lambda/2$ :  $V_2 = 3 \text{ V}$

Photoelectric Equation:

$$eV = \frac{hc}{\lambda} - \phi$$

For Wavelength  $\lambda$ :

$$e \cdot 1 = \frac{hc}{\lambda} - \phi$$

$$\frac{hc}{\lambda} = e + \phi \quad (\text{Equation 1})$$

For Wavelength  $\lambda/2$ :

$$e \cdot 3 = \frac{hc}{\lambda/2} - \phi$$

$$\frac{hc}{\lambda} = \frac{3e + \phi}{2} \quad (\text{Equation 2})$$

Now, equating Equations 1 and 2:

$$e + \phi = \frac{3e + \phi}{2}$$

$$2e + 2\phi = 3e + \phi$$

$$\phi = e = 1 \text{ eV}$$

Thus, the work function  $\phi$  of the material is 1 eV

$$1 + X = a + b$$

We can calculate  $a$  and  $b$  by simple trigonometry, once we calculate  $\theta_1$  and  $\theta_2$

Using Snell's law:

$$n_1 \sin(i) = n_2 \sin(r)$$

For calculating  $\theta_1$ ,

$$\sqrt{2} \sin(45^\circ) = 2 \sin(\theta_1)$$

$$\sin(\theta_1) = \frac{1}{2}$$

$$\theta_1 = 30^\circ$$

$$\tan(\theta_1) = \frac{a}{1}$$

$$a = \frac{1}{\sqrt{3}}$$

The angle of incidence will be the same as the angle of refraction from the previous bending, i.e.  $i_2 = \theta_1 = 30^\circ$

For calculating  $\theta_2$ ,

$$2 \sin(30^\circ) = \frac{2}{\sqrt{3}} \sin(\theta_2)$$

$$\sin(\theta_2) = \frac{\sqrt{3}}{2}$$

$$\theta_2 = 60^\circ$$

$$\tan(\theta_2) = \frac{b}{X}$$

$$b = X\sqrt{3}$$

Now, using:

$$1 + X = a + b$$

and substituting  $a$  and  $b$ :

$$1 + X = \frac{1}{\sqrt{3}} + X\sqrt{3}$$

Simplifying:

$$\frac{Q_2}{Q_1 + Q_2} = \frac{C_2}{C_1 + C_2}$$

Substituting the values:

$$\frac{Q_2}{Q_1 + Q_2} = \frac{8 \mu\text{F}}{2 \mu\text{F} + 8 \mu\text{F}} = \frac{8}{10}$$

$$\boxed{\frac{Q_2}{Q_1 + Q_2} = \frac{4}{5}}$$

58. To find the difference in binding energies between the  ${}^8_{16}\text{O}$  and  ${}^8_{18}\text{O}$  nuclei in units of  $u \cdot c^2$ , we will use the mass defect formula. The binding energy is the energy equivalent of the mass defect, which is the difference between the mass of the constituent protons and neutrons and the actual mass of the nucleus.

Calculate the mass defect for each isotope:

For  ${}^8_{16}\text{O}$ :

- Number of protons ( $Z$ ): 8
- Number of neutrons ( $N$ ):  $16 - 8 = 8$
- Mass of 8 protons:  $8 \times 1.00727 u = 8.05816 u$
- Mass of 8 neutrons:  $8 \times 1.00866 u = 8.06928 u$
- Total mass of nucleons:  $8.05816 u + 8.06928 u = 16.12744 u$
- Mass defect ( $\Delta m$ ) for  ${}^8_{16}\text{O}$ :

$$\Delta m_{16} = 16.12744 u - 15.99491 u = 0.13253 u$$

For  ${}^8_{18}\text{O}$ :

- Number of protons ( $Z$ ): 8
- Number of neutrons ( $N$ ):  $18 - 8 = 10$
- Mass of 8 protons:  $8 \times 1.00727 u = 8.05816 u$
- Mass of 10 neutrons:  $10 \times 1.00866 u = 10.0866 u$
- Total mass of nucleons:  $8.05816 u + 10.0866 u = 18.14476 u$
- Mass defect ( $\Delta m$ ) for  ${}^8_{18}\text{O}$ :

$$\Delta m_{18} = 18.14476 u - 17.99916 u = 0.1456 u$$

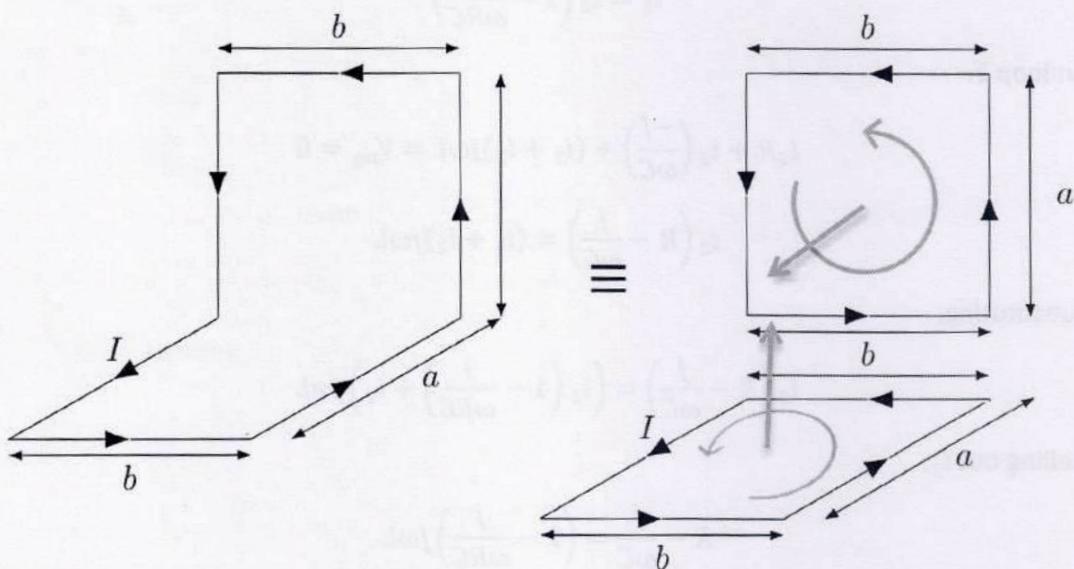
In this context, 'in phase' means that the voltage and current waveforms align perfectly over time, without any phase shift between them. For this to happen in an AC circuit, the imaginary (reactive) components of the circuit's impedance must cancel each other out. When this cancellation occurs, the circuit behaves as if it only has resistive components, leading to the current being in sync with the voltage

$$\Rightarrow \frac{j}{\omega C} = 2j\omega L$$

$$\omega^2 = \frac{1}{2LC}$$

$$\omega = \frac{1}{\sqrt{2LC}}$$

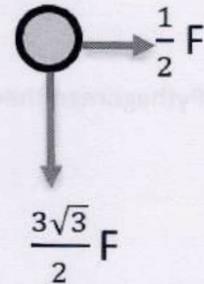
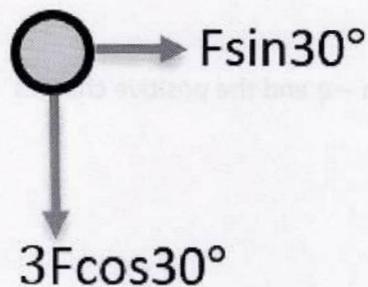
56. The magnetic moment of a current-carrying loop can be found by breaking the loop into sub-loops because of the superposition principle, where the total magnetic moment is the vector sum of the moments of the sub-loops. This approach simplifies calculations, especially for loops with complex shapes, by allowing the use of simpler geometric sub-loops. The total magnetic moment is given by  $\mathbf{m}_{\text{total}} = I \cdot \mathbf{A}_{\text{total}}$ , where the area vector of the entire loop is the sum of the area vectors of the sub-loops, since the same current flows through all parts.



$$\mathbf{m}_{\text{total}} = I \cdot (\vec{A}_{\text{total}}) = I \cdot (\vec{A}_1 + \vec{A}_2)$$

$$\mathbf{m}_{\text{total}} = I \cdot (ab(\hat{k}) + ab(\hat{j}))$$

$$\mathbf{m}_{\text{total}} = Iab(\hat{k} + \hat{j})$$



Vectorially this can be written as:

$$\frac{1}{2} F \hat{i} - \frac{3\sqrt{3}}{2} F \hat{j} = \frac{F}{2} (\hat{i} - 3\sqrt{3} \hat{j})$$

On substituting value of  $F$  we get,

$$F_{net} = \frac{q^2}{8\pi\epsilon_0 l^2} (\hat{i} - 3\sqrt{3} \hat{j})$$

Using  $F = ma$ ,  $a = \frac{F_{net}}{m}$

55. The term  $\frac{j}{\omega C}$  represents the impedance of the capacitor in the circuit. In AC circuits, capacitive impedance is given by  $Z_C = \frac{j}{\omega C}$  where  $j = \sqrt{-1}$  and  $\omega$  is the angular frequency of the source.

Similarly,

$$Z_L = j\omega L$$

We'll apply Kirchhoff's Current Law (KCL) to both loops of the circuit. By analyzing the phasor (complex) impedances and equating real and imaginary parts, we determine the condition for in-phase current.

- $k_B$  is the Boltzmann constant
- $T$  is the absolute temperature of gas

- $d$  is the diameter of the gas molecules
- $P$  is the pressure of the gas

Using:

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$\lambda = \frac{k_B V}{\sqrt{2} \pi d^2 n R}$$

Here,

$$V_1 = V_2$$

(as both the gasses are the same)

$$d_1 = d_2$$

So for convenience,

$$\frac{k_B V}{\sqrt{2} \pi d^2 R} = K$$

So,

$$\lambda = \frac{K}{n}$$

$$\lambda_1 = \frac{K}{n_1}, \lambda_2 = \frac{K}{n_2} \Rightarrow n_1 = \frac{K}{\lambda_1}, n_2 = \frac{K}{\lambda_2}$$

$$n_{net} = n_1 + n_2 = \frac{K}{\lambda_{net}} \Rightarrow \frac{K}{\lambda_1} + \frac{K}{\lambda_2} = \frac{K}{\lambda_{net}}$$

$$\boxed{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_{net}}}$$

The initial and final total internal energy will be equal, for an ideal gas the net internal energy is given as:

$$U = \frac{3}{2} nRT$$

$$U_1 = \frac{3}{2} n_1 R T_1$$

$$U_2 = \frac{3}{2} n_2 R T_2$$

$$U_f = \frac{3}{2} n_f R T_f$$

$$U_1 + U_2 = U_f$$

Cancelling out  $\frac{3}{2}R$ :

$$n_1 T_1 + n_2 T_2 = (n_1 + n_2) T_f$$

At equilibrium, the buoyant force  $F_b$  is equal to the weight of the displaced water.

$$F_b = \rho_w \times V_{\text{sub}} \times g = \rho_w \times \frac{2}{3}\pi R^3 \times g$$

Since the sphere is floating with half of it submerged,

$$F_b = W$$

$$\rho_w \times \frac{2}{3}\pi R^3 \times g = \rho \times \frac{4}{3}\pi R^3 \times g$$

$$\rho_w \times 2 = \rho \times 4$$

$$\rho_w = 2\rho$$

Additional buoyant force:

$\Delta F_b = \rho_w \times A \times x \times g$ , where  $A = \pi R^2$  (cross-sectional area of the sphere).

Net force acting on the sphere when displaced by  $x$  is the restoring force,

$$F = -\Delta F_b = -\rho_w \times \pi R^2 \times x \times g$$

Using  $\rho_w = 2\rho$ ,

$$F = -2\rho \times \pi R^2 \times x \times g$$

Since  $F = ma$  and here,  $a = y$

$$my = -2\rho \times \pi R^2 \times x \times g$$

Using  $m = \rho \times \frac{4}{3}\pi R^3$ ,

$$\rho \times \frac{4}{3}\pi R^3 \times y = -2\rho \times \pi R^2 \times x \times g$$

Simplifying

$$\frac{4}{3}Ry = -2gx$$

$$y = -\frac{3}{2R}gx$$

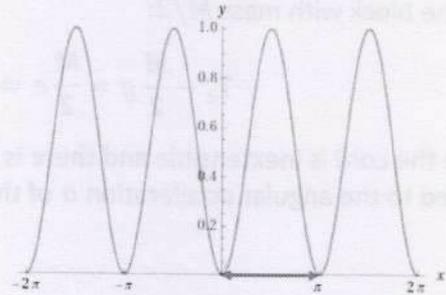
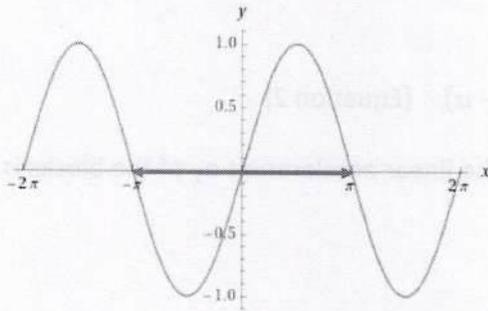
Comparison with SHM Equation:

The standard form of SHM equation is  $y = -\omega^2 x$ , where  $\omega$  is the angular frequency.

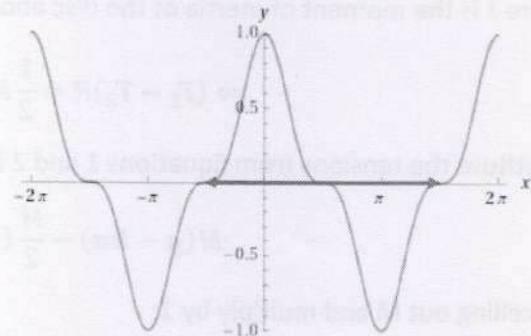
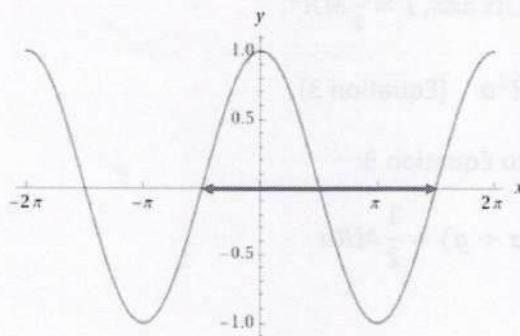
$$\omega^2 = \frac{3g}{2R}$$

Since both  $\sin(\omega t)$  and  $\cos(\omega t)$  have a period of  $\frac{2\pi}{\omega}$ , we need to see how this affects  $x(t)$ .

and since,  $\sin(\omega t)$  has a period of  $\frac{2\pi}{\omega}$ , squaring it reduces the period by half to  $\frac{\pi}{\omega}$ .



$\cos^3(\omega t)$  retains the period  $\frac{2\pi}{\omega}$  since the cubic power does not change the period.



The smallest common period of both  $\sin^2(\omega t)$  and  $\cos^3(\omega t)$  is the least common multiple (LCM) of  $\frac{\pi}{\omega}$  and  $\frac{2\pi}{\omega}$ .

Since  $\frac{2\pi}{\omega}$  is exactly twice  $\frac{\pi}{\omega}$ , the LCM is  $\frac{2\pi}{\omega}$ .

Therefore, the time period  $T$  of the motion described by  $x(t) = \sin^2(\omega t)\cos^3(\omega t)$  is:

$$T = \frac{2\pi}{\omega}$$

50. Given:

Density of water ( $\rho_{\text{water}}$ ) =  $\rho$

Density of the ball ( $\rho_{\text{ball}}$ ) =  $2\rho$

The buoyant force ( $F_b$ ) is given by Archimedes' principle and is equal to the weight of the displaced water:

$$F_b = V\rho_{\text{water}}g$$

$$\theta(t) = \theta_0 \sin(\omega t)$$

where  $\theta_0$  is the maximum angular displacement (amplitude) and  $\omega$  is the angular frequency.  
The angular frequency

Then, angular velocity can be calculated by:

$$\frac{d}{dt}(\theta(t)) = \theta_0 \omega \cos(\omega t)$$

$\omega$  is related to the period  $T$  by:

$$\omega = \frac{2\pi}{T}$$

Angular Momentum  $L(t)$ :

The angular momentum  $L$  of the pendulum bob about the point of suspension is given by:

$$L(t) = I \cdot \dot{\theta}(t)$$

Where  $I$  is the moment of inertia of the pendulum bob. For a simple pendulum of mass  $m$  and length  $l$ , the moment of inertia is:

$$I = ml^2$$

$$L(t) = ml^2(-\theta_0 \omega \cos(\omega t))$$

The amplitude  $A$  of the angular momentum is the maximum value of  $L(t)$ , which occurs when  $\sin(\omega t) = \pm 1$ :

$$A = ml^2 \theta_0 \omega$$

Also remember that we are not given a fixed length  $l$ , so remember to relate it to  $T$ :

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$l = \frac{T^2}{4\pi^2} g$$

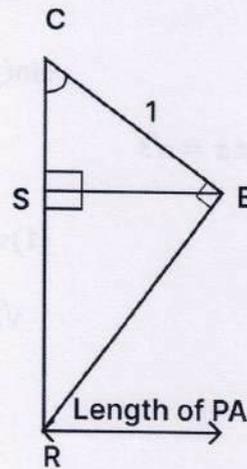
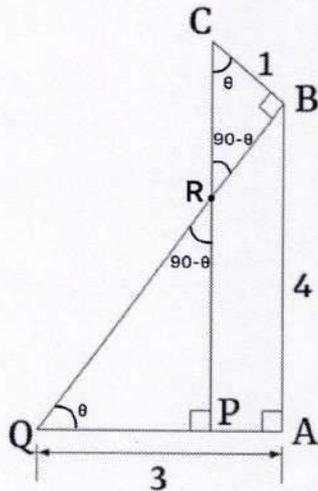
$$l^2 = \frac{T^4}{16\pi^4} g^2$$

substituting,

$$A = m \frac{T^4}{16\pi^4} g^2 \theta_0 \frac{2\pi}{T}$$

$$\boxed{A \propto T^3}$$

45.



Let  $\angle PQR = \theta$ ,

By angle sum property of triangles  $\angle PRQ = 90 - \theta$

Since opposite angles in intersecting lines are equal  $\angle BRC = 90 - \theta$

Again, by using angle sum property of triangles,  $\angle BCR = \theta$

Now let us construct a perpendicular to  $CR$  which intersect with point  $B$  which will be equal in length to  $PA$

Let us focus on  $\triangle BSC$

Where  $BC = 1$  unit

Here,

$$\sin \theta = \frac{BS}{CB} = \frac{4}{5} \text{ (from } \triangle AQB \text{)}$$

as in triangle  $AQB$ ,  $BQ = 5$  cm (use Pythagorean theorem)

So,

$$BS = \frac{4(1)}{5} = 0.8$$

Since  $PA = BS = 0.8$  cm

$$QP = QA - PA = 3 - 0.8 = \boxed{2.2 \text{ unit}}$$

1. All three engines fail.
2. Exactly two engines fail.

Case 1: All three engines fail

$$\begin{aligned}
 P(A_{\text{break}} \cap B_{\text{break}} \cap C_{\text{break}}) &= P(A_{\text{break}}) \times P(B_{\text{break}}) \times P(C_{\text{break}}) \\
 &= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{32}
 \end{aligned}$$

Case 2: Exactly two engines fail

A and B fail, and C works (calculated similarly as above):

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{32}$$

A and C fail, and B works:

$$= \frac{1}{4} \times \frac{3}{4} \times \frac{1}{2} = \frac{3}{32}$$

B and C fail, and A works:

$$= \frac{3}{4} \times \frac{1}{4} \times \frac{1}{2} = \frac{3}{32}$$

Total Probability of the Ship Failing to Complete the Voyage:

$$= \frac{1}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32} = \frac{8}{32} = \frac{1}{4}$$

Probability that the Ship Can Complete the Voyage  $P(\text{complete}) = 1 - P(\text{fail})$

$$= 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

44. Given,

$$\cos(y) \frac{dy}{dx} + \frac{1}{x} \sin(y) = x$$

Let,  $\sin(y) = t$

$$\frac{dt}{dx} = \cos(y) \cdot \frac{dy}{dx}$$

On substitution,

$$\frac{dt}{dx} + \frac{t}{x} = x$$

I.F.: (see NCERT pg 409, if not familiar)

39.

$$(1+x)^n = {}^nC_0 + {}^nC_1x + \dots + {}^nC_nx^n$$

$$\text{Let } x = 1$$

$$\Rightarrow C_0 + C_1 \cdot 1 + C_2 \cdot 1^2 + \dots + C_n \cdot 1^n = (1+1)^n = 2^n$$

$$\Rightarrow C_0 + C_1 + C_2 \cdot 1 + \dots + C_n = (1+1)^n = 2^n$$

$$\text{So, } {}^{23}C_0 + {}^{23}C_2 + {}^{23}C_4 + \dots + {}^{23}C_{22} = \boxed{2^{22}}$$

40. Given,

$$f(x+y) = f(x) + f(y)$$

$$f(1+1) = f(1) + f(1) \rightarrow f(2) = 20$$

$$f(1+1) = f(1) + f(1+1) = 3f(1) \rightarrow f(3) = 30$$

...

Clearly,  $f(x) = 10x$  which is one-and onto

41.

$$\int_{e^{-\pi/2}}^{e^{\pi/2}} (\sin^2(\log(x)) + \sin(2\log(x))) dx$$

Let's use the substitution  $t = \log(x)$ . Also note,  $x = e^t$

$$\text{Then, } dt = \frac{1}{x} dx \text{ or } dx = x dt = e^t dt$$

$$\int_{-\pi/2}^{\pi/2} (\sin^2(t) + \sin(2t)) e^t dt.$$

$$\text{Note, } \frac{d}{dx} (\sin^2(x)) = 2\sin(x)\cos(x)$$

Now, using  $\sin(2t) = 2\sin(t)\cos(t)$ :

$$\int_{-\pi/2}^{\pi/2} (\sin^2(t) + 2\sin(t)\cos(t)) e^t dt.$$

This is present in the form:

$$\int e^x [f(x) + f'(x)] dx \text{ which is equal to } \int e^x f(x) dx \text{ (NCERT XII page 327)}$$

So,

$$\int_{-\pi/2}^{\pi/2} (\sin^2(t) + 2\sin(t)\cos(t)) e^t dt = [\sin^2(t)e^t]_{-\pi/2}^{\pi/2} = \boxed{e^{\pi/2} - e^{-\pi/2}}$$

For the same function as before, i.e.  $f(x) = \begin{cases} 2 & x \geq 0 \\ -2 & x < 0 \end{cases}$

$(f(x))^2$  will be continuous and equal to 2 for any value of  $x$  but  $f(x)$  is not continuous

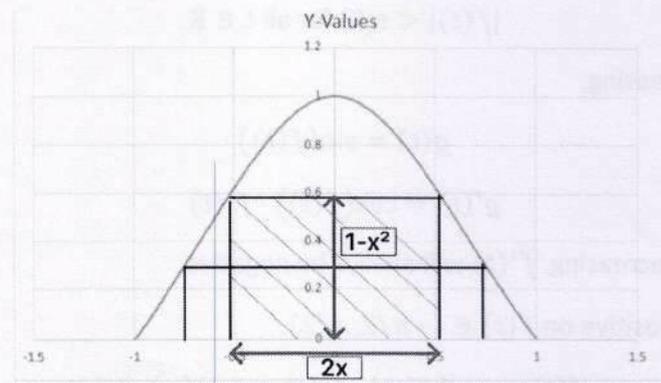
$$g(x) = \sin(f(x))$$

Suppose a function:

$$f(x) = \begin{cases} \frac{\pi}{2} & x \geq 0 \\ \frac{5\pi}{4} & x < 0 \end{cases}$$

In this case  $\sin(f(x))$  will be continuous and equal to 1 for any value of  $x$  but  $f(x)$  is not continuous

37.



As we can see from the figure, the area of any shape would be given by the equation:

$$A(x) = 2xy = 2x(1 - x^2) = 2[x - x^3]$$

$$A'(x) = 2[1 - 3x^2]$$

For maxima/minima of  $A(x)$ ,  $A'(x) = 0$

$$3x^2 = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{3}}$$

$$A''(x) = -12x$$

For maxima,  $A''(x)$  has to be negative so the correct answer is  $x = \frac{1}{\sqrt{3}}$

$$A(x) = 2x(1 - x^2) = 2 \times \frac{1}{\sqrt{3}} \left(1 - \frac{1}{3}\right) = \boxed{\frac{4}{3\sqrt{3}}}$$

$$\mathbf{P}_1 - \mathbf{P}_2 = \langle (2 + \lambda) - (4 + \mu), (3 + 2\lambda) - 4, (4 + 3\lambda) - (4 + \mu) \rangle$$

$$\mathbf{P}_1 - \mathbf{P}_2 = \langle -2 + \lambda - \mu, -1 + 2\lambda, 3\lambda - \mu \rangle$$

This means the dot product of  $\mathbf{P}_1 - \mathbf{P}_2$  with both direction vectors  $\mathbf{d}_1$  and  $\mathbf{d}_2$  should be zero.

Condition 1:  $\mathbf{P}_1 - \mathbf{P}_2$  is perpendicular to  $\mathbf{d}_1$ :

$$(-2 + \lambda - \mu) \cdot 1 + (-1 + 2\lambda) \cdot 2 + (3\lambda - \mu) \cdot 3 = 0$$

$$-2 + \lambda - \mu - 2 + 4\lambda + 9\lambda - 3\mu = 0$$

$$14\lambda - 4\mu - 4 = 0$$

$$7\lambda - 2\mu = 2 \quad (\text{Equation 1})$$

Condition 2:  $\mathbf{P}_1 - \mathbf{P}_2$  is perpendicular to  $\mathbf{d}_2$ :

$$(-2 + \lambda - \mu) \cdot 1 + (-1 + 2\lambda) \cdot 0 + (3\lambda - \mu) \cdot 1 = 0$$

$$-2 + \lambda - \mu + 3\lambda - \mu = 0$$

$$4\lambda - 2\mu = 2 \quad (\text{Equation 2})$$

From Equation 2:

$$2\lambda - \mu = 1 \Rightarrow \mu = 2\lambda - 1$$

$$7\lambda - 2(2\lambda - 1) = 2$$

$$7\lambda - 4\lambda + 2 = 2$$

$$3\lambda = 0 \Rightarrow \lambda = 0$$

Substitute  $\lambda = 0$  into  $\mu = 2\lambda - 1$ :

$$\mu = -1$$

Substitute  $\mu = -1$  into the parametric equation of  $L_2$ :

$$P_2(-1) = (4 + (-1), 4, 4 + (-1)) = (3, 4, 3)$$

Thus, the point on  $L_2$  that is closest to  $L_1$  is  $\boxed{(3, 4, 3)}$

34. Since,  $s_n = a_1 + a_2 + a_3 + \dots + a_n = \frac{n}{2}(c + a_n)$

Which closely resembles the formula of  $s_n = \frac{n}{2}(a_1 + a_n)$ , which is only applicable for the sum of an AP. Let us assume for once that  $a_1, a_2, a_3 \dots$  are in AP and if we get  $c = a_1$ , then our answer is confirmed.

With the assumption:

31. Given,  $L_1: 5x - 2y = 1$ ,  $L_2$ : the line passing through  $(0,1)$  and  $(100,101)$ ,  $L_3$ : the line passing through  $(1,11)$  and parallel to the vector  $-\hat{i} + 2\hat{j}$ .

Let us solve find the equations of each line in the form  $y = mx + c$ ,

$$L_1: 5x - 2y = 1 \Rightarrow y = \frac{1}{2}(5x - 1)$$

$$L_2: m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{101 - 1}{100 - 0} = 1$$

Using the point  $(0,1)$  and putting in the equation  $y = mx + c$ :

$$(1) = 1(0) + c \Rightarrow c = 1$$

$$L_2: y = x + 1$$

For  $L_3$  we are given that it is parallel to the vector  $-\hat{i} + 2\hat{j}$ . which means it has the same slope as the line connecting the points  $(-1,2)$  and origin  $(0,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{0 - (-1)} = -2$$

Using the point  $(1,11)$  and putting in the equation  $y = mx + c$ :

$$11 = -2(1) + c \Rightarrow c = 13$$

$$y = -2x + 13$$

So, we have lines as the following:

$$L_1: y = \frac{1}{2}(5x - 1)$$

$$L_2: y = x + 1$$

$$L_3: y = -2x + 13$$

You can either find all intersections of the three lines (which would be  $(1,2), (4,5), (3,7)$ ) or simply see that none of the lines are parallel and hence will all intersect at different points. Then,

$$(L_1 \cap L_2) \cup (L_2 \cap L_3) \cup (L_3 \cap L_1) = \{(1,2)(4,5)(3,7)\}$$

$\Rightarrow$  There are 3 elements in A

32. We need to determine the locus of points that satisfy the given condition of equidistant points,

Let us assume a point  $(x, y)$  and equate its distance from the points  $P(1,0)$ ,  $Q(-1,0)$

$$\sqrt{(x+1)^2 + y^2} = \sqrt{(x-1)^2 + y^2}$$

The kinetic energy  $KE$  of an ejected electron is given by:

$$KE = E - \phi$$

$$KE = 6.6 \times 10^{-18} - 4 \times 10^{-18} = 2.6 \times 10^{-18}$$

Calculate the number of photons emitted per second by the 100 Watt bulb:

Power  $P$  is given by energy per unit time:

$$P = \frac{E_{\text{total}}}{t}$$

The total energy emitted by the bulb per second (100 J/s) is divided by the energy of a single photon to find the number of photons emitted per second

$P = 100 \text{ W} = 100 \text{ J/s}$  Number of photons per second  $N$ :

$$N = \frac{100}{6.6 \times 10^{-18}}$$

$$N \approx 15 \times 10^{18}$$

Calculate the total kinetic energy of all photoelectrons ejected per second: The total kinetic energy is the kinetic energy of a single electron multiplied by the number of photoelectrons (which equals the number of incident photons)

$$\text{Total KE} = N \times KE$$

$$\text{Total KE} = 15 \times 10^{18} \times 2.6 \times 10^{-18}$$

$$\text{Total KE} = 39 \text{ J}$$

$$\text{Total KE} \approx 40 \text{ J}$$

28. On the basis of the unit of the reaction we can see that the reaction is a zero order reaction.

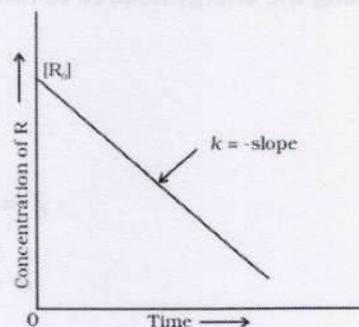
$$k = \frac{\text{Rate}}{[A]^x [B]^y}$$

$$= \frac{\text{concentration}}{\text{time}} \times \frac{1}{(\text{concentration})^n} \quad (\text{where } [A]=[B])$$

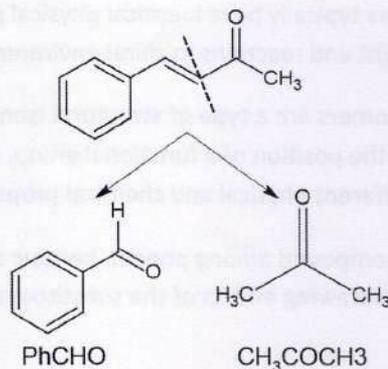
Taking SI units of concentration,  $\text{mol L}^{-1}$  and time, s, the units of  $k$  for different reaction order are listed in Table 3.3

**Table 3.3: Units of rate constant**

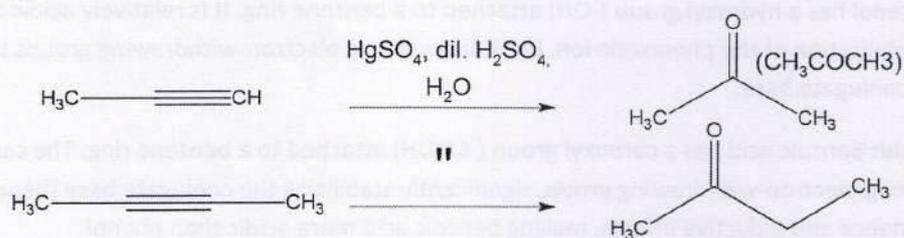
Reaction	Order	Units of rate constant
Zero order reaction	0	$\frac{\text{mol L}^{-1}}{\text{s}} \times \frac{1}{(\text{mol L}^{-1})^0} = \text{mol L}^{-1} \text{s}^{-1}$
First order reaction	1	$\frac{\text{mol L}^{-1}}{\text{s}} \times \frac{1}{(\text{mol L}^{-1})^1} = \text{s}^{-1}$
Second order reaction	2	$\frac{\text{mol L}^{-1}}{\text{s}} \times \frac{1}{(\text{mol L}^{-1})^2} = \text{mol}^{-1} \text{L s}^{-1}$



**Fig. 3.3:** Variation in the concentration vs time plot for a zero order reaction



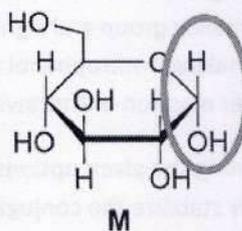
24.



25. Pyranose: Structure has a six-membered ring

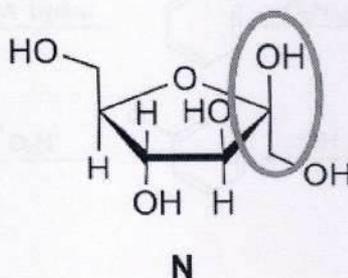
Furanose: Structure has a five-membered ring

The hydroxyl group on the first carbon (anomeric carbon) is down, indicating an alpha configuration.



M is  $\alpha$ -D-(+)-glucopyranose

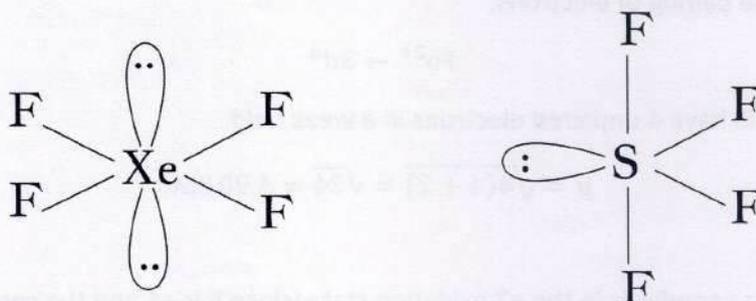
The hydroxyl group for N on the first carbon (anomeric carbon) is up, indicating a beta configuration.



N is  $\beta$ -D-(-)-fructofuranose

$$\mu = \sqrt{0(0 + 2)} = 0 \text{ BM}$$

19.



**XeF<sub>4</sub>:** With 4 bonding pairs and 2 lone pairs around xenon, the most stable shape is square planar.

**SF<sub>4</sub>:** With 4 bonding pairs and 1 lone pair around sulfur, the most stable shape is see-saw.

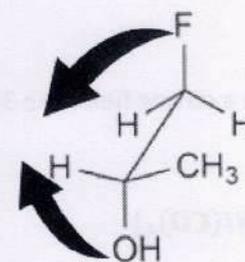
20. The data required to solve is experimental and has to be memorized from NCERT. The compound is the first compound in the table.

Table 5.3: Relationship between the Wavelength of Light absorbed and the Colour observed in some Coordination Entities

Coordination entity	Wavelength of light absorbed (nm)	Colour of light absorbed	Colour of coordination entity
[CoCl(NH <sub>3</sub> ) <sub>5</sub> ] <sup>2+</sup>	535	Yellow	Violet
[Co(NH <sub>3</sub> ) <sub>5</sub> (H <sub>2</sub> O)] <sup>3+</sup>	500	Blue Green	Red
[Co(NH <sub>3</sub> ) <sub>6</sub> ] <sup>3+</sup>	475	Blue	Yellow Orange
[Co(CN) <sub>6</sub> ] <sup>3-</sup>	310	Ultraviolet Not in visible region	Pale Yellow
[Cu(H <sub>2</sub> O) <sub>6</sub> ] <sup>2+</sup>	600	Red	Blue
[Ti(H <sub>2</sub> O) <sub>6</sub> ] <sup>3+</sup>	498	Blue Green	Violet

### 21. 1. Conformational Isomers (Conformers):

Conformational isomers are briefly separate variants of the same molecule that **can be converted into one another by rotation around single bonds**. They are usually interconvertible at room temperature and are not considered different compounds.

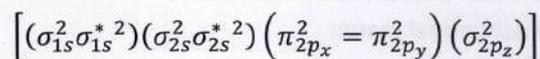


**2. Structural Isomers:** Structural isomers are **compounds that have the same molecular formula but different connectivity of their atoms**. This means the atoms are linked together in different ways, resulting in different structures and potentially different chemical properties.

**3. Enantiomers:** Enantiomers are **pairs of molecules that are non-superimposable mirror images of each other**. They have the same molecular formula and connectivity of atoms but differ in the spatial

Molecular Orbital Configurations:

**1. Nitrogen (N<sub>2</sub>):**



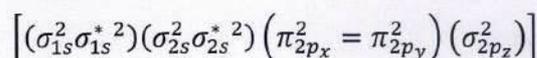
Bonding electrons: 10 (2 from  $\sigma_{2s}$ , 6 from  $\pi_{2p}$ , and 2 from  $\sigma_{2p_z}$ )

Antibonding electrons: 4 (2 from  $\sigma_{1s}^*$  and 2 from  $\sigma_{2s}^*$ )

Bond order calculation:  $\left[ \text{Bond Order} = \frac{1}{2} (10 - 4) = 3 \right]$

**2. Carbon Monoxide (CO):**

6 (electrons from C) + 8 (electrons from O) = 14 electrons



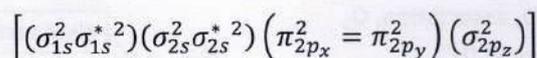
Bonding electrons: 10 (2 from  $\sigma_{2s}$ , 6 from  $\pi_{2p}$ , and 2 from  $\sigma_{2p_z}$ )

Antibonding electrons: 4 (2 from  $\sigma_{1s}^*$  and 2 from  $\sigma_{2s}^*$ )

Bond order calculation:  $\left[ \text{Bond Order} = \frac{1}{2} (10 - 4) = 3 \right]$

**3. Nitrosyl ion (NO<sup>+</sup>):**

7 (electrons from N) + 8 (electrons from O) - 1 (due to positive charge) = 14 electrons



Bonding electrons: 10 (2 from  $\sigma_{2s}$ , 6 from  $\pi_{2p}$ , and 2 from  $\sigma_{2p_z}$ )

Antibonding electrons: 4 (2 from  $\sigma_{1s}^*$  and 2 from  $\sigma_{2s}^*$ )

Bond order calculation:  $\left[ \text{Bond Order} = \frac{1}{2} (10 - 4) = 3 \right]$

**For N<sub>2</sub>, CO, and NO<sup>+</sup>:**

Bonding electrons: 10 (2 from  $\sigma_{2s}$ , 6 from  $\pi_{2p}$ , and 2 from  $\sigma_{2p_z}$ ) Antibonding electrons: 4 (2 from  $\sigma_{1s}^*$  and 2 from  $\sigma_{2s}^*$ ). The bond order for all N<sub>2</sub>, CO, and NO<sup>+</sup> is 3.

Isoelectronic Series: N<sub>2</sub>, CO, and NO<sup>+</sup> are isoelectronic with each other because they all have the same number of electrons, which is 14.

18.

$$\mu = \sqrt{n(n+2)} \text{ BM}$$

## SOLUTIONS

1. The RNA formed is complementary to the template strand but with opposite polarity and Uracil (U) instead of Thymine (T)
2. The disease is **autosomal** as there is no father-to-son, i.e. male to male transfer. It is **dominant** as it is continuous across generations, not skipping any.
3. **Troponin** is a complex protein distributed at regular intervals on tropomyosin fibres in thin filaments. Subunits of troponin mask active binding sites for myosin while at rest.
4. **Casparian strips** are suberized cells found in the tangential and radial walls of the endodermis, the innermost layer of the root cortex.
5. The exaggerated response of the immune system to certain antigens present in the environment is called **allergy**. The substances to which such an immune response is produced are called allergens.

Malfunctioning of kidneys can lead to accumulation of urea in blood, a condition called **uremia**, which is highly harmful and may lead to kidney failure.

**Myasthenia gravis:** Auto immune disorder affecting neuromuscular junction leading to fatigue, weakening and paralysis of skeletal muscle.

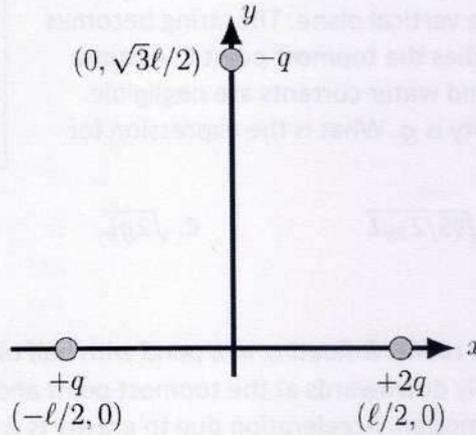
Over-secretion of GH stimulates abnormal growth of the body leading to gigantism and low secretion of GH results in stunted growth resulting in pituitary dwarfism. Excess secretion of growth hormone in adults especially in middle age can result in severe disfigurement (especially of the face) called **Acromegaly**, which may lead to serious complications, and premature death if unchecked.

6. The end products of meiosis II in sexually reproducing plants are called spores.
7. Typhoid fever could be confirmed by **Widal test**.

8. From  $\frac{dN}{dt} = rN \frac{(K-N)}{K}$ ,  $\frac{dN}{dt} = 0.01 \times 400 \times \frac{(500-400)}{500} = 0.8$ .



54. Consider two point charges  $+q$  and  $+2q$  fixed on the  $x - y$  plane at  $(-\ell/2, 0)$  and  $(+\ell/2, 0)$  respectively. Another point charge  $-q$  having mass  $m$  is released from rest at  $(0, (\sqrt{3}/2)\ell)$  on the  $xy$  plane, as shown in the figure. The permittivity of free space is  $\epsilon_0$ . What is the acceleration of the charge  $-q$  at the time of release?



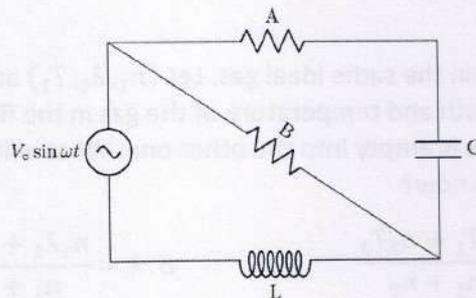
A.  $\frac{q^2}{8\pi\epsilon_0 ml^2} (3\hat{i} - \sqrt{3}\hat{j})$

B.  $\frac{q^2}{8\pi\epsilon_0 ml^2} (\hat{i} - \sqrt{3}\hat{j})$

C.  $\frac{q^2}{8\pi\epsilon_0 ml^2} (\hat{i} - 3\sqrt{3}\hat{j})$

D.  $\frac{q^2}{8\pi\epsilon_0 ml^2} (3\sqrt{3}\hat{i} - \hat{j})$

55. Consider the circuit diagram as shown in the figure. The source has a voltage  $V = V_0 \sin \omega t$ . Both the resistors A and B have the same resistance. The capacitor and the inductor have capacitance  $C$  and inductance  $L$ , respectively. For some frequency  $\omega$ , and certain initial charge in the capacitor, the current through the resistor A is in phase with the source. What is the value of  $\omega$ ?



A.  $\frac{1}{\sqrt{LC}}$

B.  $\frac{1}{\sqrt{2LC}}$

C.  $\frac{1}{2\sqrt{LC}}$

D.  $\frac{1}{\sqrt{3LC}}$

## Physics

46. On a circular track, two cyclists, Abhijit and Vani, start moving in opposite directions from a point. Abhijit moves with a constant speed. Vani starts with a constant acceleration from rest. They meet again on the track with the same speed. Which of the following is correct?

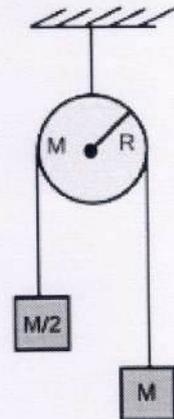
- A. Abhijit travelled the same distance travelled by Vani.
- B. Abhijit travelled half the distance travelled by Vani.
- C. Abhijit travelled double the distance travelled by Vani.
- D. Abhijit travelled  $4/3$  of the distance travelled by Vani.

47. Consider a simple pendulum undergoing simple harmonic motion with a time period  $T$ , and a fixed amplitude  $\theta_0$  of angular oscillation. Its angular momentum about the point of suspension exhibits an oscillatory behaviour with an amplitude  $A$ . Which of the following relations between  $A$  and  $T$  is correct?

- A.  $A \propto T^1$
- B.  $A \propto T^2$
- C.  $A \propto T^3$
- D.  $A \propto T^4$

48. An inextensible cord of negligible mass passes over the rim of a solid disc of mass  $M$  and radius  $R$ . The disc is free to rotate about an axis passing through the centre perpendicular to the plane of the screen, as shown in the figure. Two blocks of masses  $M$  and  $M/2$  are attached to the two free ends of the cord. Assume that there is no slipping of the cord on the disc. The acceleration due to gravity is  $g$ . What is the value of the angular acceleration of the disc?

- A.  $g/R$
- B.  $g/2R$
- C.  $g/3R$
- D.  $g/4R$



49. Consider the motion of a particle along the  $x$ -axis. The position of the particle varies with time as  $x(t) = \sin^2(\omega t)\cos^3(\omega t)$ , where  $\omega$  is a constant. What is the time period of the motion?

- A.  $\frac{2\pi}{\omega}$
- B.  $\frac{2\pi}{3\omega}$
- C.  $\frac{2\pi}{5\omega}$
- D.  $\frac{2\pi}{15\omega}$

40. Let  $f: \mathbf{Q} \rightarrow \mathbf{Q}$  be a function such that  $f(x + y) = f(x) + f(y)$  for all  $x, y \in \mathbf{Q}$ , and  $f(1) = 10$ . Which one of the following statements is Correct?

A.  $f$  is neither injective nor surjective

B.  $f$  is injective but not surjective

C.  $f$  is surjective but not injective

D.  $f$  is bijective

41. Let  $I = \int_{e^{-\pi/2}}^{e^{\pi/2}} (\sin^2(\log(x)) + \sin(\log(x^2))) dx$ . What is the value of  $I$  ?

A. 0

B.  $\frac{\pi e^2}{2}$

C.  $e^{\pi/2} - e^{-\pi/2}$

D.  $e^{\pi} - 1$

42. Consider the following subset of the  $XY$ -plane.

$$S = \{(|z - iz|, |z|^2) : z \text{ is a complex number}\}$$

Which one of the following statements is correct?

A.  $S$  is a circle

B.  $S$  is a parabola.

C.  $S$  is an ellipse but not a circle

D.  $S$  is a hyperbola.

43. A ship sets off on a voyage with three engines, labelled A, B, and C, which work independently. The ship can complete the voyage only if at least two of these engines keep working. The probability that engine A breaks down is  $1/4$ , that engine B breaks down is  $1/4$ , and that engine C breaks down is  $1/2$ . What is the probability that the ship can complete the voyage?

A.  $3/4$

B.  $1/2$

C.  $1/32$

D.  $1/4$

44. Consider the differential equation  $\cos(y) \frac{dy}{dx} + \frac{1}{x} \sin(y) = x$ , ( $x > 0$ ); given that,  $y = \frac{\pi}{2}$  at  $x = \sqrt{3}$ . Which one of the following is the value of  $y$  at  $x = \sqrt{\frac{3}{2}}$ ?

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{2}$

D.  $\frac{\pi}{4}$

## Mathematics

31. Consider the following lines in the  $XY$ -plane:

$$L_1: 5x - 2y = 1,$$

$L_2$ : The line passing through  $(0, 1)$  and  $(100, 101)$ ,

$L_3$ : The line passing through  $(1, 11)$  and parallel to the vector  $-\hat{i} + 2\hat{j}$ .

Let  $A = (L_1 \cap L_2) \cup (L_2 \cap L_3) \cup (L_3 \cap L_1)$ . What is the total number of elements of  $A$  ?

- A. 0                                      B. 1                                      C. 2                                      D. 3

32. Let  $A$  be the set of points in the  $XY$ -plane which are equidistant from  $P(-1,0)$  and  $Q(1,0)$ . Let  $B$  be the set of points in the  $XY$ -plane which are equidistant from  $A$  and  $Q$ . If  $(5, y)$  is a point in  $B$ , then what is the value of  $y^2$ ?

- A. 1                                      B. 4                                      C. 9                                      D. 16

33. Consider the lines  $L_1$  and  $L_2$  given below:

$$L_1: x = 2 + \lambda, y = 3 + 2\lambda, z = 4 + 3\lambda$$

$$L_2: x = 4 + \lambda, y = 4, z = 4 + \lambda$$

If  $(2, 3, 4)$  is the point of  $L_1$  that is closest to  $L_2$ , then which point of  $L_2$  is closest to  $L_1$ ?

- A.  $(3, 4, 3)$                               B.  $(3, 4, 4)$                               C.  $(5, 4, 5)$                               D.  $(4, 4, 4)$

34. Let  $a_1, a_2, a_3, \dots$  be a sequence of real numbers. Let  $s_n = a_1 + a_2 + \dots + a_n$ . If  $2s_n = n(c + a_n)$  for some real number  $c$  and for all  $n = 1, 2, 3, \dots$ , then which one of the following statements is Correct?

- A.  $a_1, 2a_2, 3a_3, \dots$  is an Arithmetic Progression.  
B.  $a_1, a_2, a_3, \dots$  is an Arithmetic Progression.  
C.  $a_1, 2a_2, 3a_3, \dots$  is a Geometric Progression.  
D.  $a_1, a_2, a_3, \dots$  is a Geometric Progression.

27. The minimum energy needed to remove an electron from a metal corresponds to a wavelength of 500 nm. What is the total kinetic energy of all the photoelectrons ejected per second when the entire radiation from a 100 Watt bulb with a wavelength of 300 nm falls on the surface of the metal? [Planck's constant =  $6.6 \times 10^{34}$  Js; speed of light =  $3 \times 10^8$  ms<sup>-1</sup>]

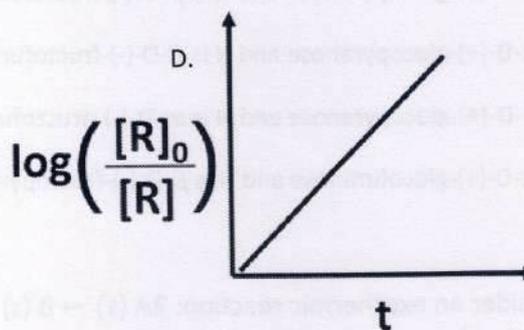
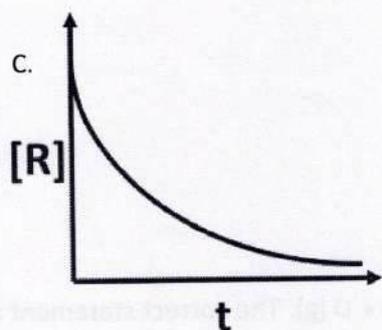
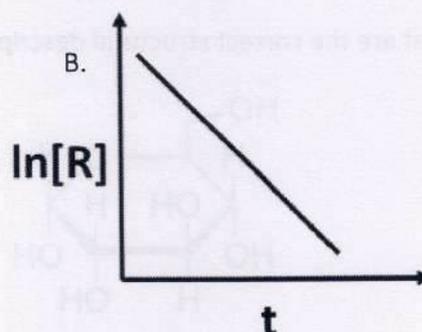
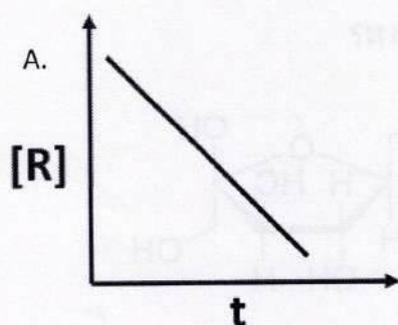
A.  $1.6 \times 10^{-19}$  J

B.  $2.6 \times 10^{-19}$  J

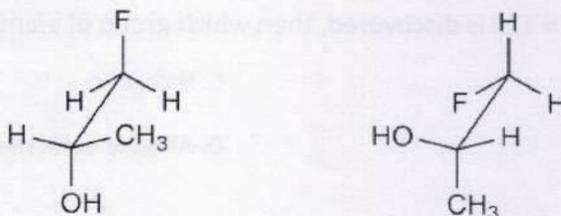
C. 40 J

D. 80 J

28. For a reaction  $R \rightarrow P$  with a rate constant of  $3 \times 10^{-3}$  mol L<sup>-1</sup> s<sup>-1</sup>, which one of the following plots is correct? (Given  $[R]_0$  is the initial concentration of R and  $[R]$  is the concentration of R at time t)



21. What is the relationship between the structures depicted below?



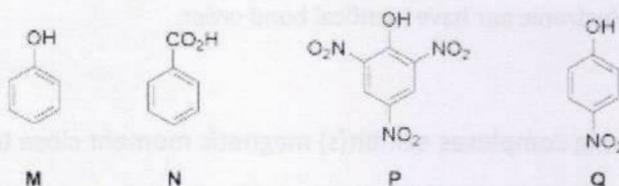
A. Conformational isomers

B. Structural isomers

C. Enantiomers

D. Positional isomers

22. What is the correct order of acidity for the following compounds?



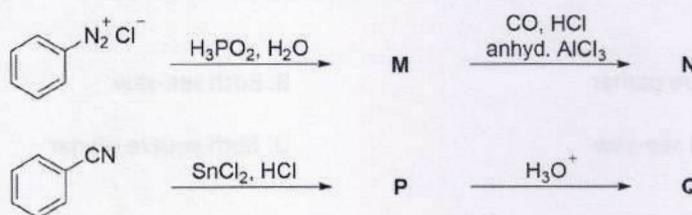
A.  $N > P > Q > M$

B.  $P > Q > N > M$

C.  $N > P > M > Q$

D.  $P > N > Q > M$

23. What are the products N and Q in the following reaction sequences?



A.  $N = Q =$

B.  $N =$   $Q =$

C.  $N = Q =$

D.  $N =$   $Q =$

24. What are X and Z in the following sequence of reactions?

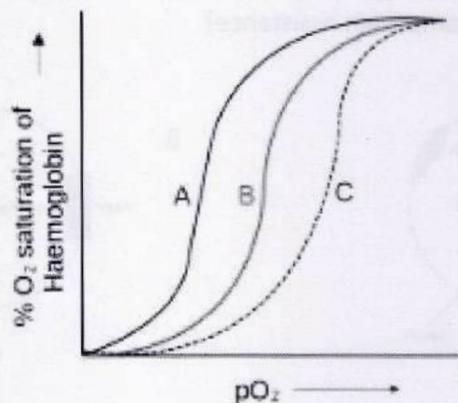
13. Polymerase chain reaction (PCR) is used to amplify a gene of interest (GOI). If, after 30 cycles of PCR, 1 billion copies of GOI are produced, approximately how many copies of GOI were present at the end of the 20<sup>th</sup> cycle?

- A. 10 million                      B. 0.66 billion                      C. 1 million                      D. 0.1 billion

14. Which one of the following statements is correct?

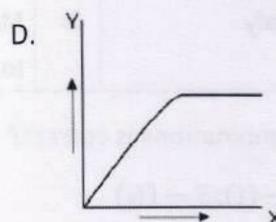
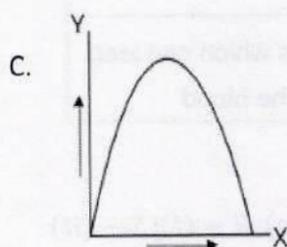
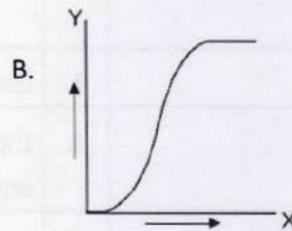
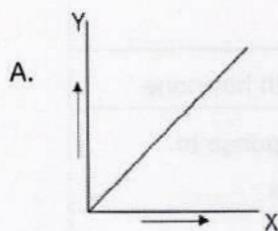
- A. Hemichordata is not a chordate sub-phylum because it has a water vascular system.  
B. Hemichordata is a chordate sub-phylum with a proto-notochord called stomochord.  
C. Hemichordata is a chordate sub-phylum with a proper notochord and gill slits.  
D. Hemichordata is not a chordate sub-phylum, with a proto-notochord called stomochord.

15. Which of the following statements is correct about the oxygen (O<sub>2</sub>) dissociation curves A and C relative to curve B?



- A. Curve C represents favourable O<sub>2</sub> association with haemoglobin at low  $pCO_2$ .  
B. Curve A represents favourable O<sub>2</sub> association with haemoglobin at low pH.  
C. Curve A represents favourable O<sub>2</sub> association with haemoglobin at low pH.  
D. Curve C represents favourable O<sub>2</sub> association with haemoglobin at high  $pO_2$ .

9. Which of the following graphs represents the correct relationship between light intensity (X-axis) and the rate of photosynthesis (Y-axis)?



10. Match the enzymes in Column I with the cellular compartments in Column II.

Column I		Column II	
P	Succinate dehydrogenase	i	Cytoplasm
Q	Pyruvate dehydrogenase	ii	Inner mitochondrial membrane
R	Lactate dehydrogenase	iii	Mitochondrial matrix
S	ATP synthase	iv	Thylakoid membrane
		v	Inner chloroplast membrane

Which of the following combinations is correct?

A. P – (ii); Q – (iii); R – (i); S – (iv)

B. P – (iii); Q – (ii); R – (i); S – (iv)

C. P – (iv); Q – (i); R – (iii); S – (ii)

D. P – (iii); Q – (i); R – (iv); S – (ii)