IAT 2022

Biology

1. Match the features of a plant cell with their corresponding examples

Column I	Column II
i. Cell with cytoplasm and no nucleus	p. Pollen grain
ii. Cell lacking both cytoplasm and nucleus	q. Synergid
iii. Cell containing more than two nuclei	r. Tracheid
iv. Haploid cell produced by mitosis	s. Tapetum
	t. Mature sieve tube
	u. Phloem companion cell

Choose the CORRECT combination from the options below.

A. i and u; ii and r; iii and s; iv and p

B. i and t; ii and r; iii and s; iv and q

C. i and u; ii and r; iii and q; iv and p

D. i and r; ii and t; iii and s; iv and q

2. Shown below are some of the reactions that occur in the metabolic pathway leading to complete oxidation of glucose during aerobic respiration.

i. Pyruvate → Acetyl CoA

ii. Dihydroxy acetone phosphate → Glyceraldehyde-3-phosphate

iii. Oxaloacetate → Citrate

iv. Fumarate → Malate

Choose the CORRECT sequence of reactions during the complete oxidation of glucose.

A. i; ii; ii; ii; iv

B. i; iii; iv; ii

C. ii; i; iii; iv

D. ii; iii; i; iv

3. In a given species of flowering plant, the colour of the seeds is exclusively determined by the colour of its seed coat. Seed coat colour in this species is governed by a nuclear gene with two alleles. The $WHITE(W)$ allele is dominant over brown (w) allele. If a plant with brown seeds (ww) is crossed as a female with pollen from a white seed (WW) plant, what will be the seed colour and the genotype of the embryo in the resultant seeds obtained from this cross?

A. Brown seeds with Ww embryo

B. White seeds with Ww embryo

C. Brown seeds with ww embryo

D. White seeds with ww embryo

4. During the CoVID-19 pandemic, SARS-CoV2 virus mutated multiple times giving rise to many variants. What is the genetic material of the SARS-CoV2 virus?

A. Single stranded RNA

B. Single stranded DNA

C. Double stranded RNA

D. Double stranded DNA

5. Lymph is an important body fluid. Choose the INCORRECT statement about the lymph.

A. Fat digested in the intestine is absorbed through the lymph.

B. It is the interstitial fluid generated by the passage of liquid between the cells of the capillary.

C. It is colourless and has similar mineral composition as of plasma.

D. It remains as an interstitial fluid and is never put back into circulation.

6. Interneurons play an important role in the execution of spinal cord-mediated reflex action. Where are these interneurons located?

A. Dorsal root ganglion

B. White matter

C. Gray matter

D. Muscle spindle

7. Osmoreceptors are sensitive to changes in ionic concentrations and volumes of body fluids. Which among the following best describes the function of these osmoreceptors?

A. They stimulate renin production to induce vasoconstriction and increase blood pressure

B. They stimulate pituitary gland to facilitate the secretion of mineralocorticoids to adjust the changes in mineral composition of the body fluid.

C. They stimulate atrial wall to release atrial natriuretic factor to induce vasodilation and reduce blood pressure.

D. They stimulate hypothalamus to facilitate the release of the anti-diuretic hormone and increase water reabsorption.

8. A single point mutation in gene 'X' results in breathing difficulty, hypertension as well as partial sterility. Which among the following best explains the observed phenotypes?

A. Incomplete dominance

B. Pleiotropy

C. Linkage

D. Partial dominance

9. In a cross between individuals of the genotypes PpQQRrSS and ppqqRrSS, what will be the expected number of progenies with the genotype ppQQRrSS in a population of 400 individuals, assuming independent assortment?

A. 0 B. 25 C. 100 D. 200

10. Organisms in which of the following phyla are triploblastic, acoelomate and have bilateral symmetry?

A. Arthropoda B. Mollusca

C. Platyhelminthes D. Hemichordata

11. Consider the following biomes: Tropical Rainforest (P); Tundra (Q); Desert (R) and Coastal zone (S). The most probable order of Net Primary Productivity of these biomes is

A. P>Q>S>R B. P>S>Q>R C. Q>P>S>R D. R>Q>S>P

12. Nine percent of a population cannot taste a certain food item because of a recessive allele of the gene IAT. Assuming the population is in Hardy-Weinberg equilibrium, what will be the frequency of dominant and recessive alleles, respectively?

A. 0.7 and 0.3 B. 0.9 and 0.1

C. 0.3 and 0.7 D. 0.1 and 0.9

13. As opposed to DNA replication within the cell, discontinuous synthesis of DNA does NOT occur in a polymerase chain reaction (PCR). Why?

A. The replication fork is formed and DNA ligase activity of the Taq polymerase joins the discontinuous fragments.

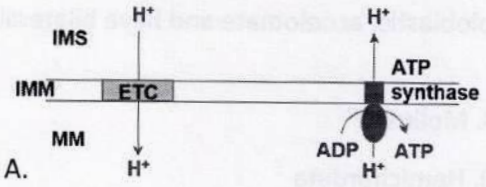
B. The replication fork is formed and Taq polymerase extends DNA in both $3'$ to $5'$ and $5'$ to $3'$ direction.

C. Denaturation step in PCR substitutes for the replication fork and Taq polymerase extends DNA only in the $3'$ to $5'$ direction.

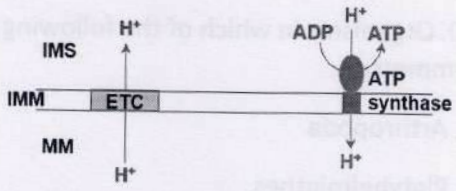
D. Denaturation step in PCR substitutes for the replication fork and Taq polymerase extends DNA only in the $5'$ to $3'$ direction.

14. Which of the following figures CORRECTLY represents the chemiosmotic hypothesis of ATP synthesis occurring in a mitochondrion in a cell? (Keys for the figure; IMS: Intermembrane space, IMM: Inner mitochondrial membrane, MM: Mitochondrial matrix, ETC: Electron transport chain)

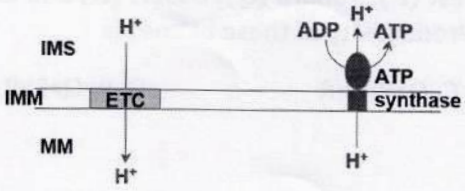
A.



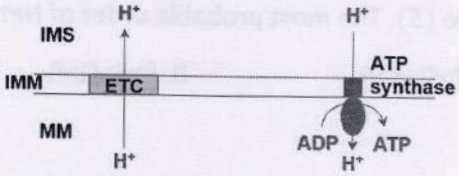
B.



C.



D.



15. Let 'X' be the perpendicular distance from the centromere of each chromosome to the equatorial plane of a human cell. Which of the following stages of the cell cycle is most likely to have the HIGHEST average value of X'?

A. Early anaphase

B. Early metaphase

C. Late anaphase

D. Metaphase

Chemistry

16. The products of the reaction between aqueous solutions of $K_4[Fe(CN)_6]$ and $\frac{1}{2}H_2O_2$ are

A. $K_3[Fe(CN)_6]$ and KOH

B. $K_3[Fe(CN)_6]$ and H_2O

C. $K_3[Fe(CN)_6]$, H_2O and H_2O

D. $K_4[Fe(CN)_5(OH)]$ and HCN

17. For the colourless complex $[M(H_2O)_6]^{n+}$, where M is a $3d$ transition metal, the CORRECT ground state d-electron configuration of M is

A. $(t_{2g})^3(e_g)^0$ B. $(t_{2g})^6(e_g)^4$ C. $(t_{2g})^6(e_g)^2$ D. $(t_{2g})^4(e_g)^2$

18. Choose the correct statement about the structure of C_{60} fullerene

A. 6-membered rings are fused with 5-membered rings ONLY.

B. 6-membered rings are fused with 6-membered rings ONLY.

C. 5-membered rings are fused with 6-membered rings ONLY.

D. 5-membered rings are fused with both 5-membered and 6-membered rings.

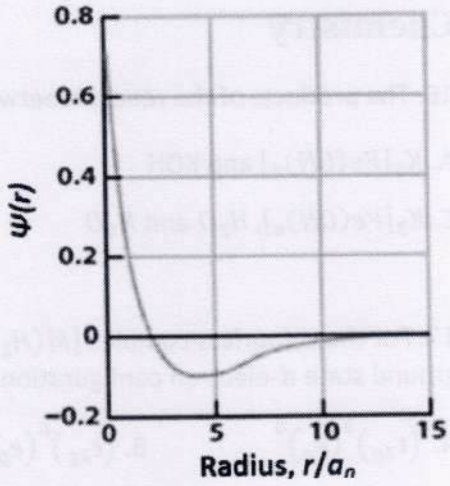
19. The first to fifth ionization energies (IE) of two p-block elements **X** and Y are given below.

	IE_1 (eV)	IE_2 (eV)	IE_3 (eV)	IE_4 (eV)	IE_5 (eV)
X	6.0	18.8	28.4	120.0	153.7
Y	8.2	16.3	33.5	45.1	166.7

The number of valence electrons in X and Y respectively are

A. 1,4 B. 3,4 C. 3,5 D. 4,5

20. Which of the following expressions represents the hydrogen atom wave function $\psi(r)$ shown in the figure below? (r is the distance of the electron from the nucleus and a_0 is a constant)



A. $\frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/2a_0}$

B. $\frac{1}{\sqrt{6\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(\frac{r}{a_0}\right) e^{-r/2a_0}$

C. $\frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right) e^{-r/2a_0}$

D. $\frac{1}{\sqrt{3\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(3-\frac{2r}{a_0}+\frac{2r^2}{9a_0^2}\right) e^{-r/3a_0}$

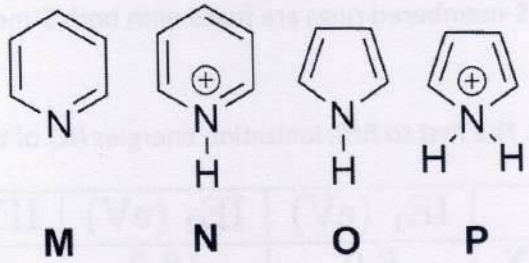
21. Which of the following molecules are aromatic?

A. M and O only

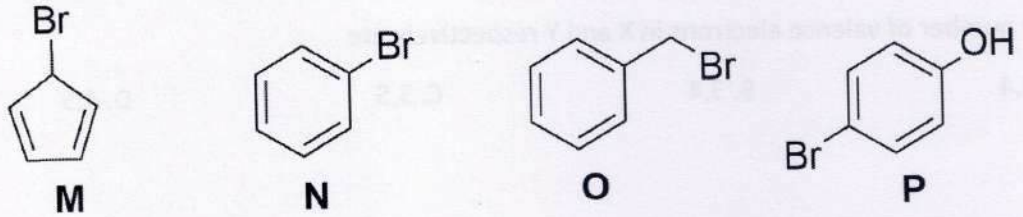
B. N and P only

C. M, N and O only

D. M, N, O, and P



22. Which of the following are aryl bromides?



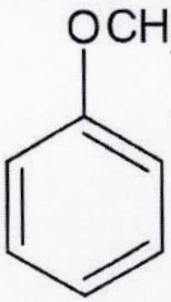
A. N and P only

B. N and O only

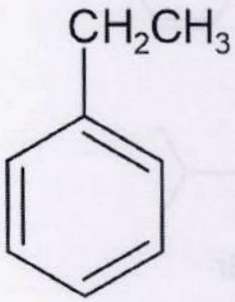
C. M, O and P only

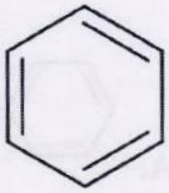
D. N, O and P only

23. Benzamide is treated with Br_2 and NaOH (aq) to form the product X, which is then reacted with $NaNO_2$ and HCl (aq) at $0 - 5°C$ to form Y. Y is immediately treated with ethanol to give **Z**. What is Z?

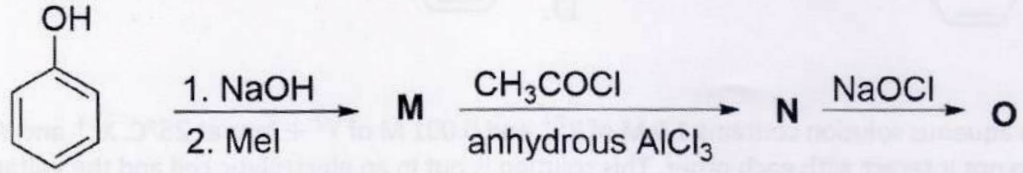
A. 

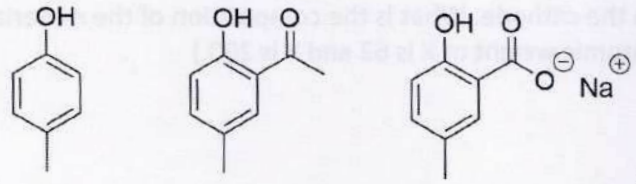
B. 

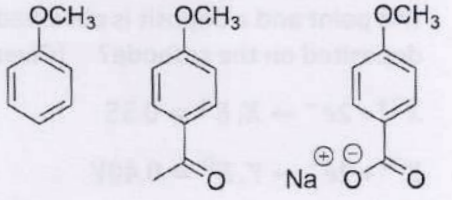
C. 

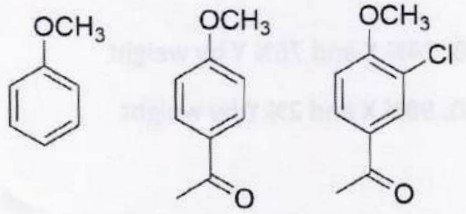
D. 

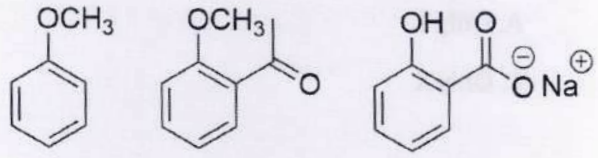
24. In the following reaction sequence, the major products **M**, **N**, and O respectively are



A. 

B. 

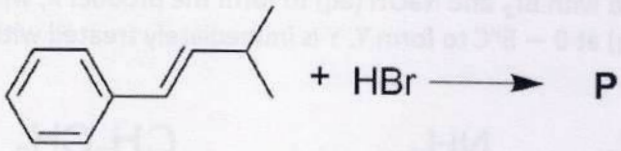
C. 

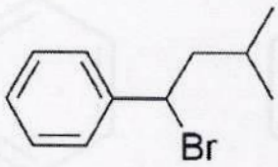
D. 

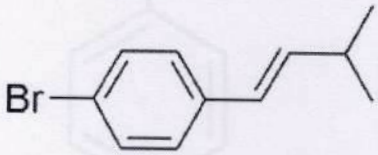
25. The van der Waals equation for a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$. What is the dimension of $\left(\frac{a}{b}\right)$?

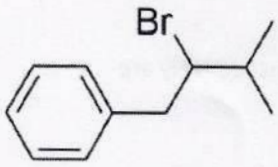
A. ML^2T^{-1} B. $ML^{-1}T^{-2}$ C. MLT^{-2} D. ML^2T^{-2}

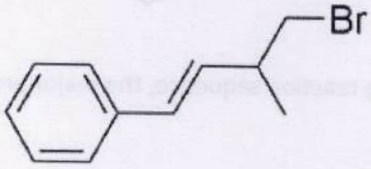
26. The major product P of the following reaction is



A. 

C. 

B. 

D. 

27. An aqueous solution contains 1.0 M of X^{2+} and 0.001 M of Y^2 + ions at 25°C. X^{2+} and Y^{2+} ions do not interact with each other. This solution is put in an electrolytic cell and the voltage is gradually increased till a current begins to flow through the cell. The voltage is maintained at this point and a deposit is observed on the cathode. What is the composition of the material deposited on the cathode? (Given: Atomic weight of X is 63 and Y is 200.)

$X^{2+} + 2e^- \rightarrow X, E^0 = 0.35$

$Y^{2+} + 2e^- \rightarrow Y, E^0 = 0.40V$

(E^0 is the standard reduction potential.)

A. Only Y

B. 24% X and 76% Y by weight

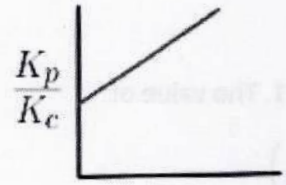
C. Only X

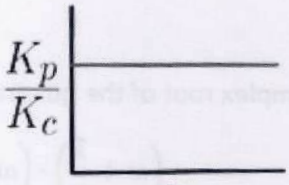
D. 98% X and 2% Y by weight

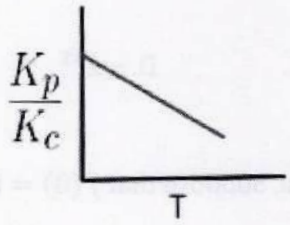
28. 2 g of naphthoic acid (molecular weight =172 g mol$^-$1) dissolved in 20 mL of benzene shows a freezing point depression of 2 K. For benzene, the freezing point depression constant, $K_f = 5$ K kg mol$^-$1and the density is 0.88 g mL^{-1}. What is the magnitude of the van't Hoff factor?

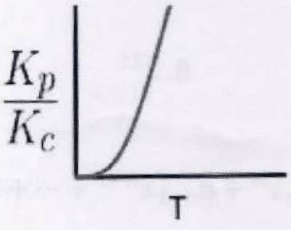
A. 0.605 B. 605.0 C. 0.688 D. 688.0

29. For the reaction involving ideal gases, $A(g) + 2B(g) = 2C(g) + 3D(g)$, which of the following plots is qualitatively correct? (K_p and K_c are the equilibrium constants in terms of pressure and concentration respectively. T is the absolute temperature.)

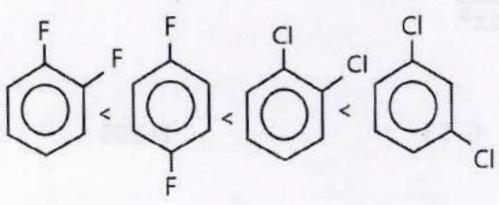
A. 

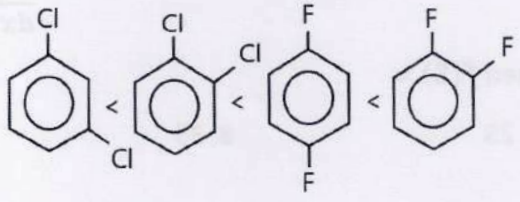
B. 

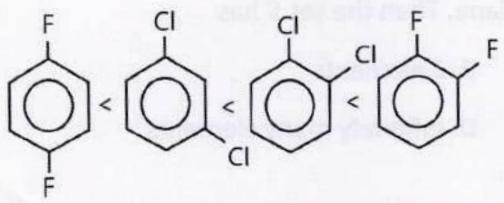
C. 

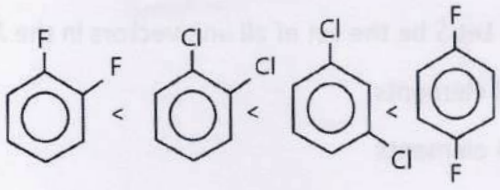
D. 

30. Identify the correct order of the molecules with respect to the magnitude of their dipole moment:

A. 

B. 

C. 

D. 

Mathematics

31. A randomly chosen card from a deck of 52 cards is given to be a black card (Spade or Club). What is the probability that it is either a face card (King, Queen or Jack) or a Spade?

A. 7/13 B. 8/13 C. 9/13 D. 19/26

32. Let ω be a complex root of the quadratic polynomial $x^2 + x + 1$. The value of

$$\left(\omega + \frac{1}{\omega}\right) \cdot \left(\omega^2 + \frac{1}{\omega^2}\right) \cdots \left(\omega^{100} + \frac{1}{\omega^{100}}\right)$$

is

A. 2^{33} B. 2^{31} C. -2^{33} D. -2^{31}

33. Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial. Suppose that $f(0) = 0$,

$$\frac{df}{dx}]_{x=0} = 1, \frac{d^2f}{dx^2}]_{x=0} = 4$$

and

$$\frac{d^3f}{dx^3} = \frac{d^5f}{dx^5}$$

Then $f(5) =$

A. 25 B. 35 C. 55 D.105

34. Let S be the set of all unit vectors in the XY-plane. Then the set S has

A. 8 elements B. 2 elements

C. 4 elements D. Infinitely many elements

35. A lab reports that the global average temperature in the year 2020 was 14.9°C and predicts that the global average temperature will increase at the rate of 1% per year. What will be the global average temperature in the year 2035?

A. 15.049°C B. 17.298°C C. 17.135°C D. 17.471°C

36. Let X be the set of all 2×2 matrices with real entries and $R \subset X \times X$ be the relation $R = \{(A, B): AB = BA\}$. Which of the following statements is true?

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalance relation

37. For a natural number n, let C_n be the curve in the XY-plane given by $y = x^n$, where $0 \leq x \leq 1$. Let A_n denote the area of the region bounded between C_n and $C_n + 1$. Then the largest value of A_n is

A. 1/2 B. 1/3 C. 1/6 D. 1/12

38. Let f be a continuous function on [0,1] and F be its antiderivative. If $F(0) = 1$ and $\int_0^1 f(x)dx = 1$, then $F(1)$ is

A. 0 B. 1/2 C. 1 D. 2

39. Let a be a nonzero real number and $f: \mathbf{R} \to \mathbf{R}$ be a continuous function such that $f'(x) > 0$ for all $x \in R$. Consider $g(x) = f(2a^2x - ax^2)$. Then g has

A. Local maxima at $x = a$ if $a > 0$ B. Local maxima at $x = a$ if $a < 0$

C. Local minima at $x = a$ if $a > 0$ D. A point of inflection at $x = a$

40. Let A be the matrix $\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 1 & 1 & 1 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. For any natural number k, the determinant of A^k is

A. 0 B. 1 C. -1 D. $(-1)^k$

41. Consider the vectors $\vec{a} = \hat{\imath} + x\hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + x\hat{k}$, $\vec{c} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$. The values of x for which there is at least one nonzero vector perpendicular to the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are

A. 0,2 B. - 2, 2 C. 7/2, 0 D. 4, -2

42. Consider the tangent lines to the circle $x^2 + y^2 = 1$ at points $P = (1,0)$ and $Q = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. If R is the point of intersection of these two tangent lines, then $\angle PRQ$ is:

$A. \frac{\pi}{4}$ $B. \frac{3\pi}{4}$ $C. \frac{5\pi}{6}$ $D. \frac{\pi}{6}$

43. The function given by $f(x) = 2x^3 - 15x^2 + 36x - 5$ is

A. Increasing on the interval (0,2)

B. Decreasing on the interval (-3,0)

C. Increasing on the interval (2,3)

D. Decreasing on the interval $(3, \infty)$

44. The value of the integral

$$\int_1^{100} \frac{[x]}{x}\, dx$$

where $[x]$ is the greatest integer less than or equal to x for any real number x, is

$A. log\left(\frac{100^{98}}{98!}\right)$ $B. log\left(\frac{100^{99}}{98!}\right)$

$C. log\left(\frac{100^{98}}{99!}\right)$ $D. log\left(\frac{100^{99}}{99!}\right)$

45. For arbitrary constants α, β, the differential equation representing the family of curves $y = (\alpha x + \beta)e^x$ is

A. $y'' - 2y' + y = 0$

B. $y'' - y' + y = 0$

C. $y'' - 2y' - y = 0$

D. $y'' - y' - y = 0$

Physics

46. A particle experiences an acceleration $\vec{a} = \alpha\vec{v}$, where $\vec{v}$ is the velocity of the particle and α is a constant. If the distances traveled by the particle in the time intervals $t_2 - t_1$ and $t_3 - t_1$ are S_{12} and S_{13}, respectively, which of the following relations is true?

A. $\frac{S_{13}}{S_{12}} = \frac{\log[\alpha(t_3 - t_1)]}{\log[\alpha(t_2 - t_1)]}$

B. $\frac{S_{13}}{S_{12}} = \frac{\exp[\alpha(t_3 - t_1)]}{\exp[\alpha(t_2 - t_1)]}$

C. $\frac{S_{13}}{S_{12}} = \frac{\exp[\alpha(t_3 - t_1)] - 1}{\exp[\alpha(t_2 - t_1)] - 1}$

D. $\frac{S_{13}}{S_{12}} = \frac{\log[\alpha(t_3 - t_1)] - 1}{\log[\alpha(t_2 - t_1)] - 1}$

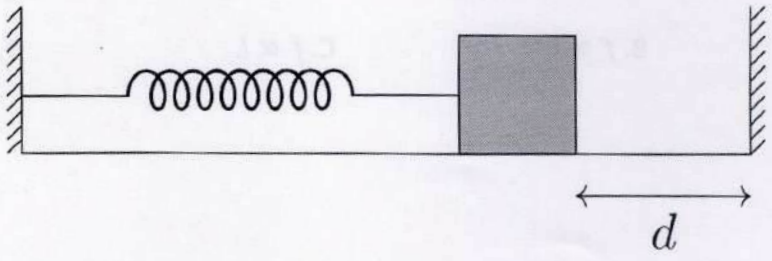
47. A point mass m attached to a massless string is undergoing circular motion in a vertical plane. The length of the string is R and the acceleration due to gravity is g. If the minimum value of the tension in the string is $2mg$, the maximum speed of this circular motion of the point mass is

A. $\sqrt{6gR}$ B. $\sqrt{7gR}$ C. $\sqrt{(7/2)gR}$ D. $4\sqrt{gR}$

48. An object is released from rest from the inner edge of a hemispherical bowl, and it falls under gravity. The coefficient of kinetic friction between the object and the bowl is μ. If the object covers an angular displacement θ with respect to the center of the hemisphere when it stops for the first time, which of the following expressions is correct?

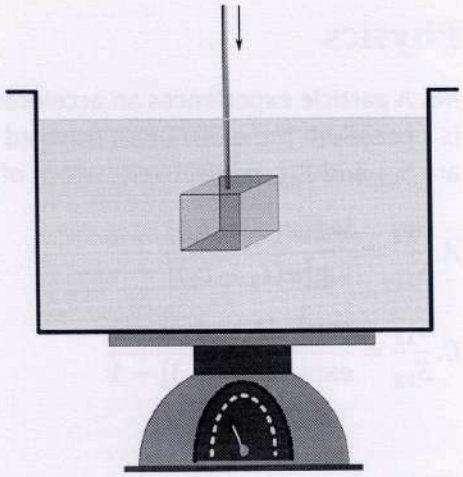
A. $\mu = \cot(\theta)$ B. $\mu = \cot\left(\frac{\theta}{2}\right)$ C. $\mu = \tan(\theta)$ D. $\mu = \tan(\theta/2)$

49. As shown in the figure a block is resting on a frictionless floor and is attached to the free end of a spring. The right edge of the block in equilibrium is at a distance d from the wall. When the spring is compressed by a distance $d/2$ and released, the time-period of the motion is T. Similarly, when compressed by a distance $2d$ and released, the time-period is T_0. Considering elastic collision between the block and the wall, what is the value of the quantity T_0/T?



A. 1/4 B.1 C. 3/4 D. 2/3

50. A liquid of density ρ in a container weighs W. A cubic block of side L and density $\rho_b < \rho$ is pushed by a stick to completely submerge the block in the liquid without touching the bottom. If g is the acceleration due to the gravity and liquid displaced by the stick is negligible, what is the new weight of the container as registered by the weighing machine below?



A. $W + \rho g L^3$

B. $W + \rho_b g L^3$

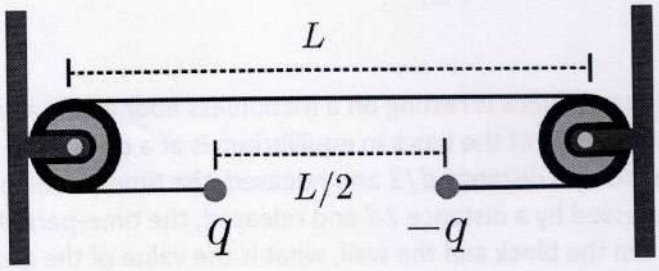
C. $W + (\rho - \rho_b) g L^3$

D. $W + (\rho_b - \rho) g L^3$

51. A monoatomic ideal gas at pressure P and volume V is first adiabatically compressed to volume $V/8$ and then is allowed to expand isothermally back to its original volume. What is the final pressure of the gas?

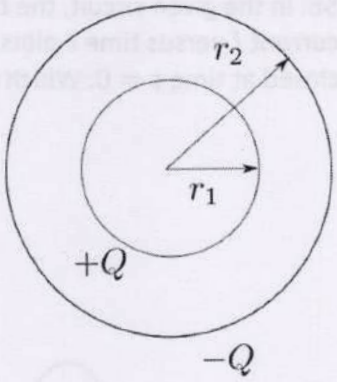
A. P B. $2P$ C. $4P$ D. $P/2$

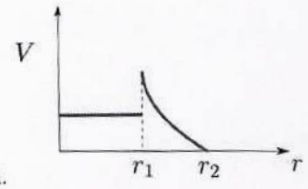
52. A uniform taut string with two point charges q and $-q$ attached to its ends passes over two massless pullies kept L distance apart as shown in the fgure. If f is the fundamental frequency of the part of the string over pullies, which of the following statements is correct?

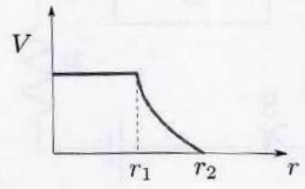


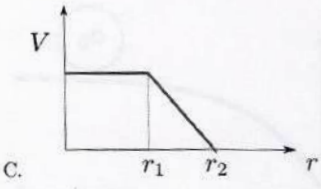
A. $f \propto L^{-2}$ B. $f \propto L^{-1}$ C. $f \propto L$ D. $f \propto L^2$

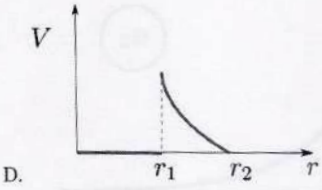
53. Two uniformly charged concentric thin spherical shells of radii r_1 and r_2 have charges $+Q$ and $-Q$, respectively. How does the electrostatic potential vary with distance from the center of the shells?



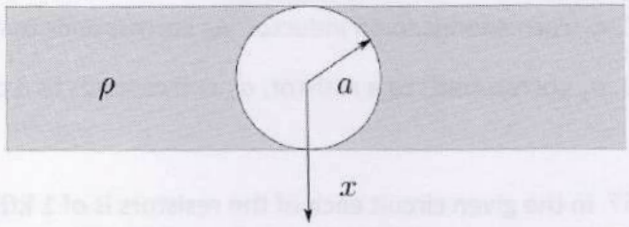
A.


B.


C.


D.


54. Consider a very long cylinder of radius a having a uniform positive charge density ρ. A sphere of radius a has been carved out (see figure), leaving no charge in that region. The distance radially outward to the cylinder, as measured from the center of the sphere is x. At what value of x will the electric field be maximum?

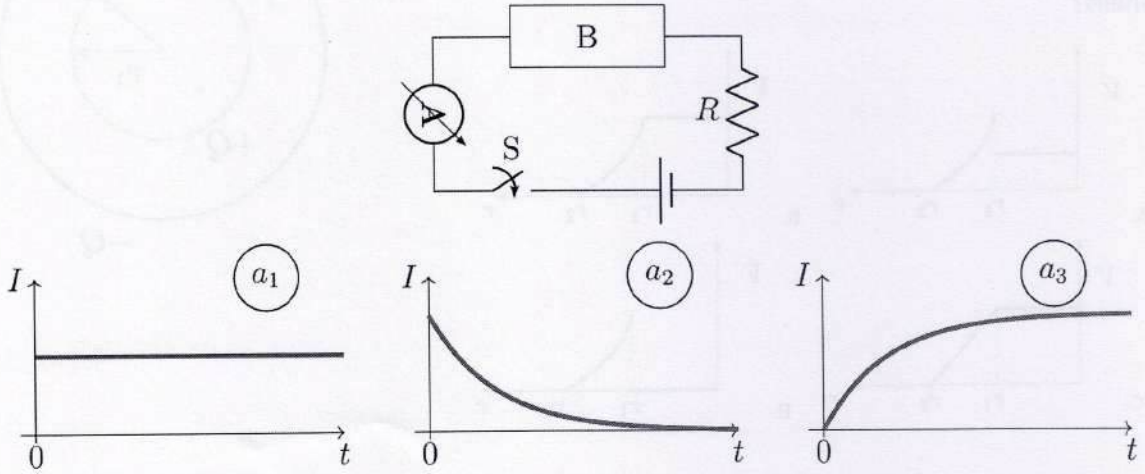


A. a B. $(2/3)\,a$ C. $(3/2)\,a$ D. $(4/3)\,a$

55. Consider that in a fission of a single Plutonium (Pu^{239}) atom 207 MeV energy is released. Assuming all atoms of Plutonium undergo fission, which of the following options is the closest estimate of the amount of Plutonium required for a 20,000 units of TNT explosion? (1 unit of TNT= 4.184×10^{9} J, 1 eV= 1.6×10^{-19} J)

A. 60.0 g B. 0.5 kg C. 1 kg D. 10 kg

56. In the given circuit, the box B either contains a capacitor, or an inductor or a resistor. The current I versus time t plots for three cases (a_1, a_2, a_3) are shown in figures. The switch S is closed at time $t = 0$. Which of the following is correct?



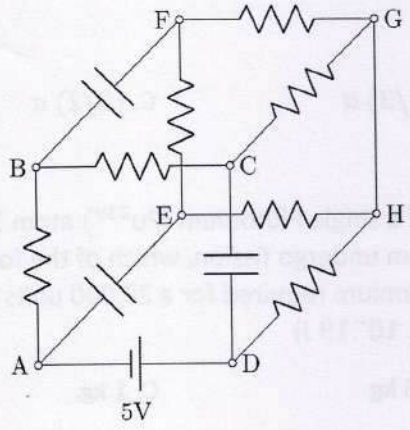
A. a_1 corresponds to an inductor, a_2 corresponds to a capacitor, a_3 correspond to a resistor.

B. a_1 corresponds to a resistor, a_2 corresponds to an inductor, a_3 corresponds to capacitor

C. a_1 corresponds to an inductor, a_2 corresponds to a resistor, a_3 corresponds to capacitor.

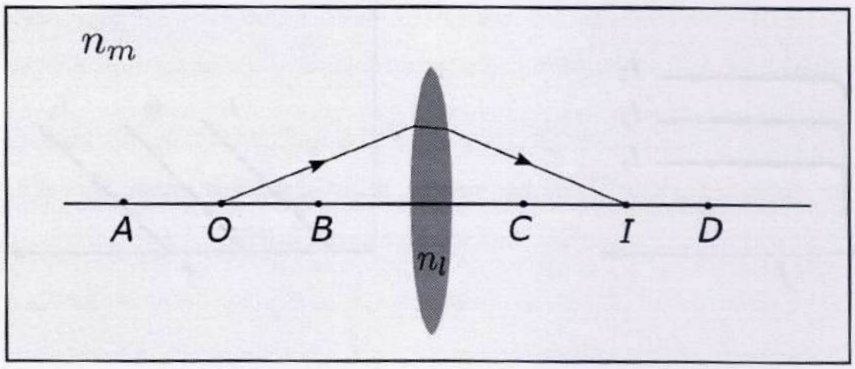
D. a_1 corresponds to a resistor, a_2 corresponds to a capacitor, a_3 corresponds to inductor.

57. In the given circuit each of the resistors is of 1 kΩ resistance and each of the capacitors has 4μF capacitance. What is the charge in the capacitor between the points B and F?



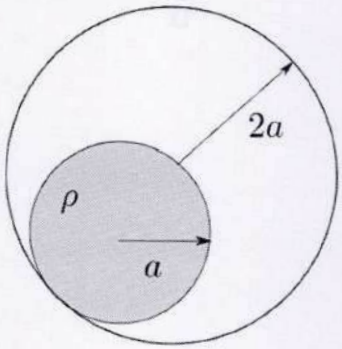
A. $10\mu C$ B. $15\mu C$ C. $(40/7)\mu C$ D. $(60/7)\mu C$

58. A thin lens made of material of refractive index n_l forms the image at the position I of a point object held at O on the central axis of the lens, as is shown in the ray diagram below. Consider that the refractive index of the medium is n_m and $n_l > n_m$ for the figure given. Where does the image form when $n_l < n_m$?



A. At point A B. At point B C. At point C D. At point D

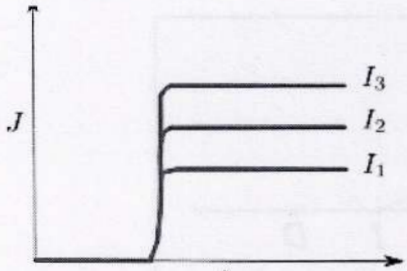
59. A conducting sphere of radius a initially contains a uniform volume charge density ρ. What is the electric flux Φ through a spherical surface of radius $2a$ centered at a point on the surface of the charged sphere (see figure) at a later time t? (ϵ_0 is the permittivity of free space.)



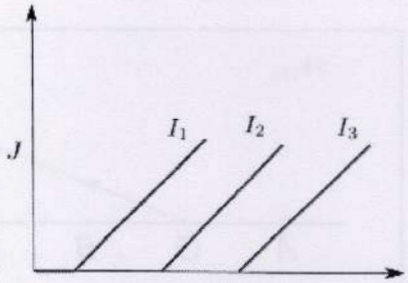
$A.\ \Phi = \frac{16}{\epsilon_0}\pi a^2\rho$ $C.\ \Phi = \frac{4}{3\epsilon_0}\pi a^3\rho$ $B.\ \Phi = \frac{32}{3\epsilon_0}\pi a^3\rho$ $D.\ \Phi = \frac{4}{\epsilon_0}\pi a^2\rho$

60. The figures in the options show the photocurrent J of a photoelectric material versus the frequency f of an incident light beam. The light beam can have three different intensities I_1, I_2, I_3, with $I_3 > I_2 > I_1$. Which of the options is correct?

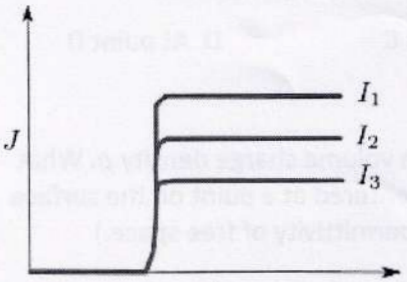
A.



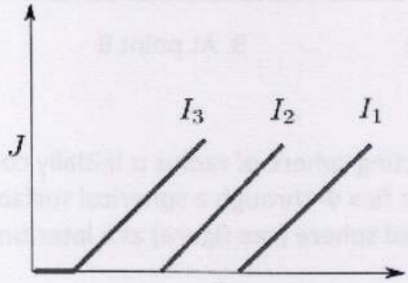
B.



C.



D.



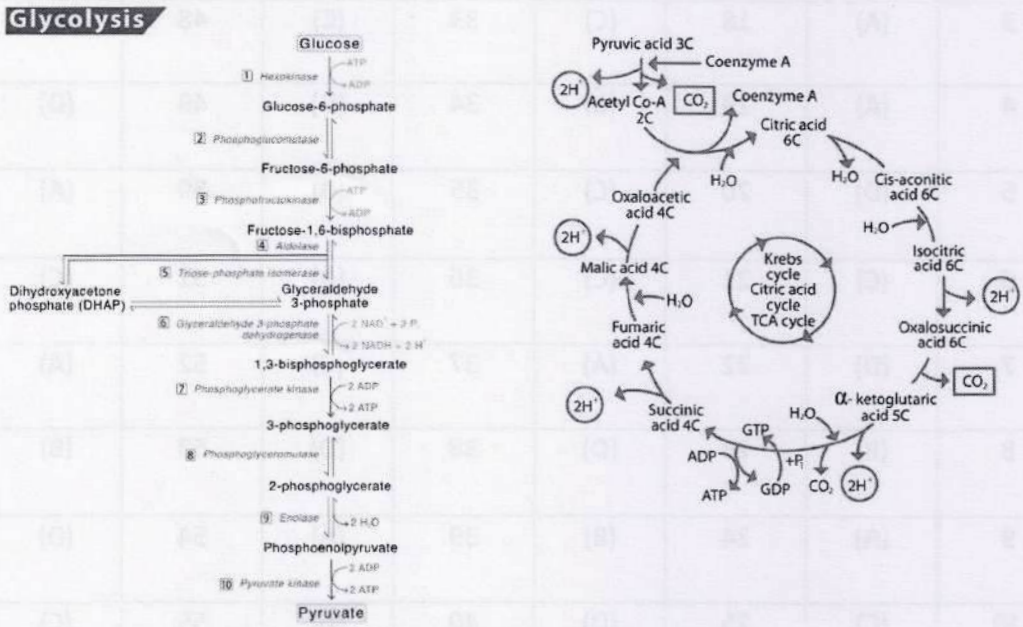
ANSWER KEY 2022

Question Number	Answer	Question Number	Answer	Question Number	Answer	Question Number	Answer
1	(B)	16	(A)	31	(B)	46	(C)
2	(C)	17	(B)	32	(C)	47	(B)
3	(A)	18	(C)	33	(C)	48	(B)
4	(A)	19	(B)	34	(D)	49	(D)
5	(D)	20	(C)	35	(B)	50	(A)
6	(C)	21	(C)	36	(A)	51	(C)
7	(D)	22	(A)	37	(C)	52	(A)
8	(B)	23	(D)	38	(D)	53	(B)
9	(A)	24	(B)	39	(A)	54	(D)
10	(C)	25	(D)	40	(B)	55	(C)
11	(B)	26	(A)	41	(A)	56	(D)
12	(A)	27	(C)	42	(B)	57	(A)
13	(D)	28	(A)	43	(A)	58	(B)
14	(D)	29	(D)	44	(D)	59	(C)
15	(C)	30	(C)	45	(A)	60	(A)

SOLUTIONS

1. **Sieve tube elements** lose their nucleus but retain cytoplasm for transporting nutrients.
Tracheid, a type of xylem cell, loses its cytoplasm and nucleus to facilitate water transport.
Tapetum is a nutritive tissue in anthers which can be multinucleated.
Synergids are haploid cells in the embryo sac of angiosperms formed by mitotic division.

2.



Glycolysis is followed by the Krebs cycle in aerobic respiration of glucose.

3. From the Punnett square, it is clear that all embryos will have the **Ww** genotype. However, as the seed coat colour is determined solely by the maternal genes, the seed coat colour will be **brown**.

♀ /♂	W	W
w	Ww	Ww
w	Ww	Ww

4. The SARS-CoV2 virus has single stranded RNA (ssRNA) as its genetic material, which allows it to mutate rapidly.

5. The lymphatic system of vessels drains lymph back to the major veins of the body.

6. Interneurons are located in the **grey matter** of the central nervous system. They are responsible for processing information locally and connecting sensory and motor neurons, playing a crucial role in reflex actions.

7. Osmoreceptors in the body are activated by changes in blood volume, body fluid volume and ionic concentration. An excessive loss of fluid from the body can activate these receptors which stimulate the hypothalamus to release antidiuretic hormone (ADH) or vasopressin from the neurohypophysis. ADH facilitates water reabsorption from latter parts of the tubule, thereby preventing diuresis.

8. **Pleiotropy** occurs when a single gene influences multiple, seemingly unrelated phenotypic traits.

9. As one parent is homozygous dominant QQ while the other is homozygous recessive qq, progeny will all be heterozygous Qq. Hence no amount of progeny has QQ alleles.

10. Organisms in the phylum **Platyhelminthes** (flatworms) are triploblastic, meaning they have three germ layers (ectoderm, mesoderm, and endoderm), acoelomate, meaning they lack a true body cavity (coelom), and have bilateral symmetry, meaning their body can be divided into two equal halves along one plane.

11. **Tropical Rainforest (P):** Highest NPP due to favourable conditions like high temperature, moisture, and sunlight.
Coastal Zone (S): Moderate NPP, coastal ecosystems like mangroves and estuaries have high productivity, though not as much as tropical rainforests.
Tundra (Q): Low NPP due to cold temperatures and short growing seasons.
Desert (R): Lowest NPP because of extreme arid conditions and minimal vegetation.

12. Let the allelic frequency of the normal gene be denoted by p and that of the IAT gene by q. In a Hardy-Weinberg Equilibrium, sum of allelic frequencies is 1, i.e.

$$p + q = (p + q)^2 = p^2 + 2pq + q^2 = 1$$

As the gene is recessive, the affected individuals will all be homozygous recessive.

$q^2 = \frac{9}{100}$

$q = \sqrt{\frac{9}{100}} = 0.3.$ Hence, frequency of recessive allele = **0.3**.

$p = 1 - q = 1 - 0.3 = 0.7.$ Hence, frequency of dominant allele = **0.7**.

13. In PCR, the **denaturation** step separates the double-stranded DNA into single strands. The Taq polymerase then extends the DNA in the **5' to 3' direction**. PCR does not involve the formation of Okazaki fragments or discontinuous synthesis because it operates on single-stranded templates, and the extension is continuous on each strand.

14. The complexes of the ETC produce H^+ ions by oxidation/dehydrogenation of various compounds. These protons move to the intermembrane space, creating a proton gradient where $[H^+]$ is low inside the matrix and high in the intermembrane space. The breakdown of this gradient through the F_0 channel of complex V leads to conformational change in F_1, producing ATP from ADP + P_i.

15. During **late anaphase**, the chromosomes are moving towards the poles of the cell. The centromeres are further from the equatorial plane compared to metaphase or early anaphase, as the chromatids are being pulled apart towards the poles.

16. The reaction between potassium ferrocyanide ($K_4[Fe(CN)_6]$) and hydrogen peroxide (H_2O_2) typically involves the oxidation of the ferrocyanide ion ($[Fe(CN)_6]^{4-}$) to ferricyanide ion ($[Fe(CN)_6]^{3-}$).

The balanced chemical equation for this reaction is:

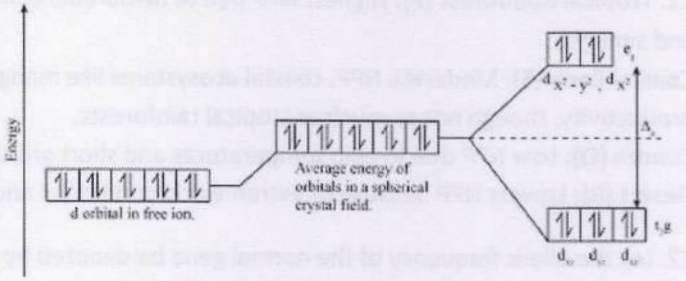
$$K_4[Fe(CN)_6](aq) + \frac{1}{2}H_2O_2(aq) \rightarrow K_3[Fe(CN)_6](aq) + KOH(aq)$$

17. A colorless complex typically indicates that there are no d-d transitions in the visible region, meaning the t_{2_g} and e_g orbitals are either fully filled or completely empty. i.e., The t_{2_g} and e_g orbitals need to either have 0 or 10 electrons in total.

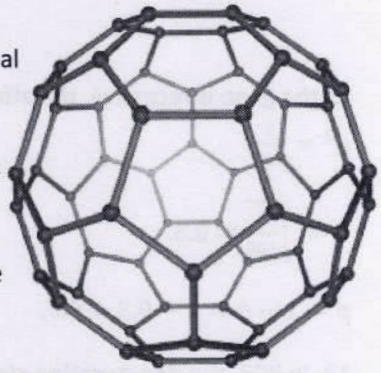
On evaluating the options,

$$(t_{2g})^6(e_g)^4$$

This configuration has 10 electrons in the d-orbitals, indicating that both the t_{2_g} and e_g sets are fully filled.



18.Fullerenes are spherical carbon molecules with a cage-like structure, composed entirely of carbon atoms arranged in hexagonal and pentagonal rings.



19. The significant jump occurs between IE_3 and IE_4 for X and IE_4 and IE_5, indicating that X has 3 valence electrons and Y has 4. After removing these electrons, the next ionization energy is significantly higher, suggesting removal of an electron from a more stable inner shell.

20. On observing the graph, you can observe that $\psi(r)$ *has* one root.

These following equations have only exponential functions of distance from radius and will never be zero as e is a positive number.

$$\frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/2a_0} \; and \; \frac{1}{\sqrt{6\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(\frac{r}{a_0}\right)e^{-r/2a_0}$$

For the remaining equations, we only need to consider

$$2-\frac{r}{a_0} \text{ and } 3-\frac{2r}{a_0}+\frac{2r^2}{9a_0^2}$$

As the rest of the equations are either constants or an exponential function of *r*.

Let $\frac{r}{a_0}=x$,

$$3-\frac{2r}{a_0}+\frac{2r^2}{9a_0^2}=3-2x+\frac{2x^2}{9}$$

Here, $d=b^2-4ac=16-4(2/9)3=+ve$,

This means it will have two roots whereas the shown graph clearly has only one. Hence,

$$\psi(r)=\frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$$

is the best answer.

21. To determine which molecules are aromatic using Huckel's rule, we check if they follow the (4n + 2) π-electron rule, where n is a non-negative integer (0, 1, 2, ...).

Molecule M: It has 6 π-electrons (from three double bonds). Huckel's rule: 4n + 2 = 6, which gives n = 1. It is aromatic.

Molecule N: It has 6 π-electrons. Huckel's rule: 4n + 2 = 6, which gives n = 1. It is aromatic.

Molecule O: It has 6 π-electrons (4 from double bonds and 2 from a lone pair on nitrogen). Huckel's rule: 4n + 2 = 6, which gives n = 1. It is aromatic.

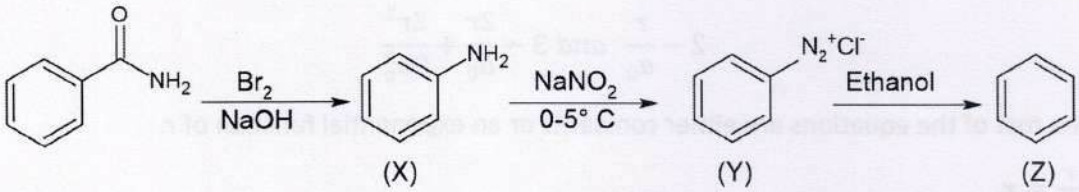
Molecule P: It has 4 π-electrons (from two double bonds). Huckel's rule: 4n + 2 = 4, which gives n = 0.5. It is not aromatic

Therefore, the aromatic molecules are M, N, and O.

22. Aryl bromides are organic compounds containing a bromine atom directly bonded to an aromatic ring. The general structure of an aryl bromide is represented as Ar-Br, where "Ar" denotes an aryl group, which is a functional group derived from an aromatic ring (such as benzene or naphthalene) by the removal of one hydrogen atom.

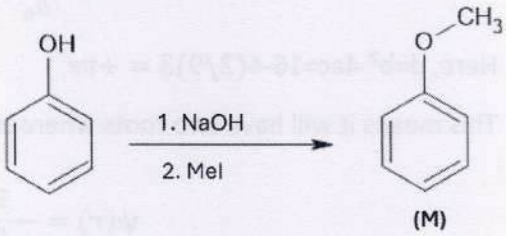
M-is not aryl, O does not have Br directly connected to benzene.

23.

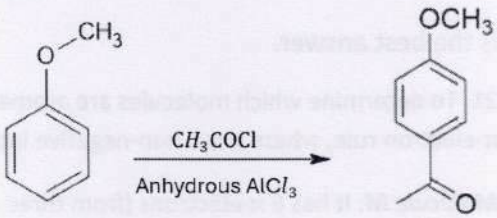


The first reaction is Hoffmann Bromamide degradation and the second reaction is diazotization.

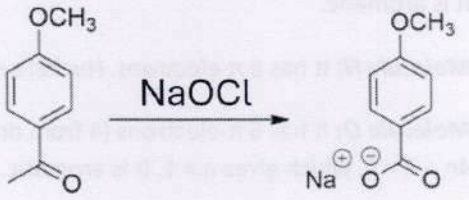
24.The phenoxide ion then undergoes methylation with methyl iodide (MeI) to form anisole (methoxybenzene, $C_6H_5OCH_3$).



Anisole undergoes Friedel-Crafts acylation with acetyl chloride in the presence of $AlCl_3$. The methoxy group (–OCH_3) is an ortho/para-directing group. However, the major product is usually the para product due to steric hindrance at the ortho position.



N undergoes the haloform reaction with NaOCl, where the methyl ketone group ($COCH_3$) is converted to a carboxylic acid group (COOH) and the methyl group is cleaved off as chloroform ($CHCH_3$).



25. Principle of Homogeneity states that dimensions of each of the terms of a dimensional equation on both sides should be the same. For the terms to be added they must be the same dimensions.

$$[P] = M^1L^{-1}T^{-2}, \qquad [V] = M^0L^3T^0$$

$$\left[\frac{a}{V^2}\right] = [P]$$

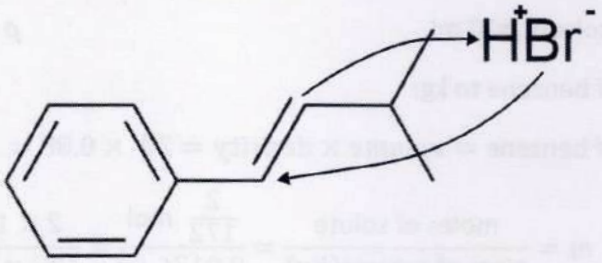
$$\Rightarrow [a] = [PV^2]$$

$$\Rightarrow [b] = [V]$$

$$\left[\frac{a}{b}\right] = \left[\frac{PV^2}{V}\right] = [PV] = [ML^2T^{-2}]$$

26. The alkene double bond acts as a nucleophile and attacks the proton (H^+) from HBr, resulting in the formation of a carbocation. The location of the carbocation formation is influenced by the stability of the possible carbocation intermediates.

Given the structure, the more stable carbocation would form on the carbon that can be stabilized by the phenyl group through resonance. The proton (H^+) is most likely to add to the carbon that is not adjacent to the phenyl ring, leading to a secondary benzylic carbocation which is stabilized by resonance with the benzene ring.



27. Given Reduction Potentials:

$$E^0_{X^{2+}/X} = 0.35\text{V} \qquad E^0_{Y^{2+}/Y} = 0.40\text{V}$$

The reduction potential for Y^{2+} is higher than for X^{2+}.This means that, under standard conditions, Y^{2+} would be reduced first. However, the significant difference in concentration affects the actual reduction process.

Nernst Equation:

The Nernst equation helps us account for the non-standard conditions

$$E = E^0 - \frac{0.0591}{n}\log Q$$

For X^{2+}:

$$E_{X^{2+}/X} = 0.35 - \frac{0.0591}{2}\log\frac{1}{1.0} = 0.35V$$

For Y^{2+}:

$$E_{Y^{2+}/Y} = 0.40 - \frac{0.0591}{2}\log\frac{1}{0.001} = 0.40 - 0.0886 = 0.3114V$$

Comparing Effective Potentials:

Effective potential for X^{2+} is 0.35 V.

Effective potential for Y^{2+} is 0.3114 V.

The material deposited on the cathode will be only X due to its higher effective potential despite the higher standard reduction potential of Y^{2+}

28. Given:

mass(solute)= 2 g, molecular weight(solute)= 172 g mol^{-1}

$$\text{Moles of naphthoic acid} = \frac{\text{given mass}}{\text{molar mass}} = \frac{2\text{ g}}{172\text{ g/mol}}$$

Given:

V (solvent)=20 mL $\rho = 0.88g\ mol^{-1}$

Convert 20 mL of benzene to kg:

$$\text{Mass of benzene} = volume \times density = 20\ \times 0.88 = 17.6\text{ g} = 0.0176\text{ kg}$$

$$m = \frac{\text{moles of solute}}{\text{mass of solvent(Kg)}} = \frac{\frac{2}{172}\text{ mol}}{0.0176\text{ kg}} = \frac{2 \times 10^4}{172 \times 176} \approx 0.66$$

Given:

$\Delta T_f = 2$ K $K_f = 5$ K kg mol^{-1} $m = 0.66$ mol/kg

$$\Delta T_f = K_f \times \text{m} \times \text{i} \Longrightarrow \text{i} = \frac{\Delta T_f}{K_f \times \text{m}}$$

$$i = \frac{2}{5 \times 0.66}$$

$$i \approx 0.605$$

29. K_p and K_c are related as per the following formula:

$$K_p = K_c(RT)^{\Delta n_{(g)}}$$

For the given equation:

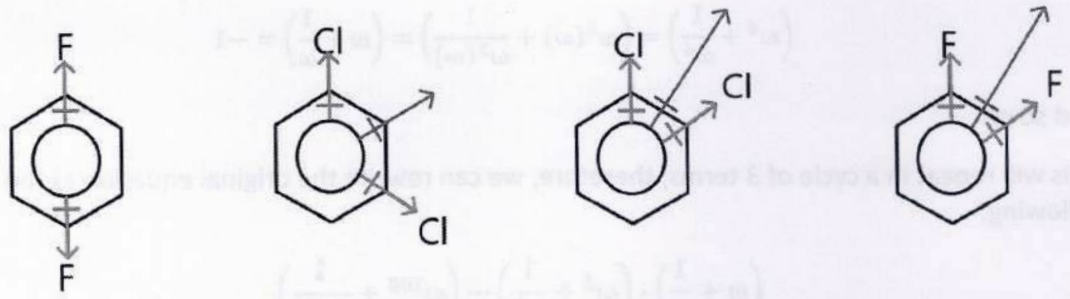
$$\Delta n_{(g)} = [2\ due\ to\ C + 3\ due\ to\ D] - [1\ due\ to\ A + 2\ due\ to\ B] = 5 - 3 = 2$$

Putting $\Delta n_{(g)} = 2$,

$$K_p = K_c(RT)^2$$

$$\frac{K_p}{K_c} = (RT)^2$$

$$\Rightarrow \frac{K_p}{K_c} \propto T^2$$



30.

i) Dipole moment=0 , (cancelled out)

ii) $\sqrt{P_{Cl}^2 + P_{Cl}^2 + 2P_{Cl}^2\cos(120)} = P_{Cl}$

iii) $\sqrt{P_{Cl}^2 + P_{Cl}^2 + 2P_{Cl}^2\cos(60)} = \sqrt{3}P_{Cl}$

iv) $\sqrt{P_F^2 + P_F^2 + 2P_F^2\cos(60)} = \sqrt{3}P_F$

$|P_F| > |P_{Cl}|$ as Fluorine is more electronegative

31.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Number of cards= (Number of black face cards) + (Number of Spades) − (Number of Spade face cards)

$$\text{Number of cards} = 6 + 13 - 3 = 16$$

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of black cards}} = \frac{16}{26}$$

$$\text{Probability} = \frac{8}{13}$$

32. Revision Note on ω:

ω is a primitive cube root of unity with the following key properties:

1. $\omega^3 = 1$
2. $\omega + \omega^2 = -1$

$$\left(\omega + \frac{1}{\omega}\right) \cdot \left(\omega^2 + \frac{1}{\omega^2}\right) \cdots \left(\omega^{100} + \frac{1}{\omega^{100}}\right)$$

Let us simplify a few terms by replacing 1 with ω^3:

$$\left(\omega + \frac{1}{\omega}\right) = \left(\omega + \frac{\omega^3}{\omega}\right) = (\omega + \omega^2) = -1$$

$$\left(\omega^2 + \frac{1}{\omega^2}\right) = \left(\omega^2 + \frac{\omega^3}{\omega^2}\right) = (\omega + \omega^2) = -1$$

$$\left(\omega^3 + \frac{1}{\omega^3}\right) = 1 + 1 = 2$$

$$\left(\omega^4+\frac{1}{\omega^4}\right)=\left(\omega^3(\omega)+\frac{1}{\omega^3(\omega)}\right)=\left(\omega+\frac{1}{\omega}\right)=-1$$

and so on

This will repeat in a cycle of 3 terms; therefore, we can rewrite the original equation as the following:

$$\left(\omega+\frac{1}{\omega}\right)\cdot\left(\omega^2+\frac{1}{\omega^2}\right)\cdots\left(\omega^{100}+\frac{1}{\omega^{100}}\right)$$

$$=\left[\left(\omega+\frac{1}{\omega}\right)\cdot\left(\omega^2+\frac{1}{\omega^2}\right)\cdot\left(\omega^3+\frac{1}{\omega^3}\right)\right]^{33}\left(\omega^{99}\omega+\frac{1}{\omega^{99}\omega}\right)$$

$$=[-1\times-1\times2]^{33}\times-1$$

$$=\boxed{-2^{33}}$$

33. For this question, we have a polynomial function whose degree in n and it is given that,

$$\frac{d^3f}{dx^3}=\frac{d^5f}{dx^5}$$

For this to be true, the degree of the function has to be 2.

Proof:

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$$

The third derivative of $f(x)$ is: $\frac{d^3f}{dx^3}=n(n-1)(n-2)a_nx^{n-3}+\cdots\rightarrow$Degree=$n-3$

$\frac{d^5f}{dx^5}=n(n-1)(n-2)(n-3)(n-4)a_nx^{n-5}+\cdots\rightarrow$Degree=$n-5$

Given,

$$\frac{d^3f}{dx^3}=\frac{d^5f}{dx^5}$$

For this equation to hold, the degrees of both derivatives must be the same.

If $n\geq3$, then the degrees must match. Thus: $n-3=n-5$ This equation implies:

$$n-3=n-5\Longrightarrow-3=-5$$

This is a contradiction unless $n-3$ and $n-5$ are both not the degree of $\frac{d^3f}{dx^3}$ and $\frac{d^5f}{dx^5}$ $(n-2=0)$. Therefore, the only feasible solution for this equation is when $f(x)$ has a degree 2.

$$f(x)=a_2x^2+a_1x+a_0$$

$$f(0)=0\Longrightarrow a_0=0$$

$$f'(x)=2a_2x+a_1$$

$$f'(0) = 1 \Rightarrow a_1 = 1$$

$$f''(x) = 2a_2$$

$$f''(0) = 4 \Rightarrow a_2 = 2$$

Hence, $f(x) = 2x^2 + x\,f(x) \Rightarrow f(5) = 2(5)^2 + 5 = \boxed{55}$

34. A unit vector in the XY-plane is a vector of length 1. Any unit vector in the XY-plane can be represented in the form $((\cos\theta, \sin\theta))$, where (θ) is the angle the vector makes with the positive x-axis.

Given that (θ) can take any value from (0) to (2π), there are infinitely many angles (θ). Therefore, there are infinitely many unit vectors in the XY-plane, each corresponding to a different angle (θ).

Hence, the number of elements in the set (S) of all unit vectors in the XY-plane is infinite.

35.

$$T_{2020+n} = T_{2020} \times (1 + r)^n$$

$$T_{2035} = 14.9 \times (1 + 0.01)^{15}$$

The global average temperature in the year 2035 is predicted to be approximately $17.298°C$. (You should use a calculator for this)

36.

$$R = \{(A, B) \in X \times X \mid AB = BA\}$$

Reflexivity

A relation R on a set X is reflexive if every element is related to itself. In other words, $\forall A \in X$,

$$(A, A) \in R.$$

In this context, for R to be reflexive, we need $AA = AA$ for all $A \in X$.This is always true. Hence, $(A, A) \in R$ for all $A \in X$.

The relation R is reflexive.

Symmetry

A relation R on a set X is symmetric if for all $A, B \in X$,

$$(A, B) \in R \Rightarrow (B, A) \in R.$$

In this context, if $AB = BA$,then $BA = AB$.This is inherently true because equality is symmetric. Therefore, if $(A, B) \in R$,then $(B, A) \in R$.

Conclusion: The relation R is symmetric.

Transitivity

A relation R on a set X is transitive if for all $A, B, C \in X$,

$(A, B) \in R$ and $(B, C) \in R \Rightarrow (A, C) \in R$.

In this context, if $AB = BA$ and $BC = CB$, we need to show that $AC = CA$. This is not generally true for arbitrary 2×2 matrices. As a counterexample, consider the following matrices:

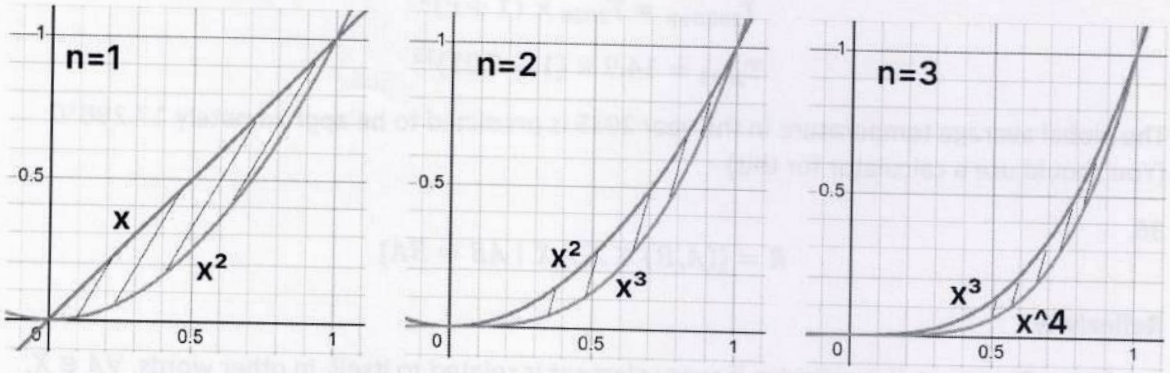
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I, \qquad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad C = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

For these matrices, $AB = BA$ and $BC = CB$,but $AC \neq CA$

(You can calculate any one element of both matrices to disprove).

Conclusion: The relation R is not transitive.

37. To understand the area it is asking us to maximize, observe the following diagram to visualise better:



Given,

-n is a **natural** number

The equation of the area will be given by:

$$A(x) = \int_0^1 (x^n - x^{n-1})\, dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \frac{1}{n+1} - \frac{1}{n+2}$$

$$= \frac{(n+2) - (n+1)}{(n+1)(n+2)}$$

$$A(x) = \frac{1}{(n+1)(n+2)}$$

For the largest value of $A(x)$, we need the smallest denominator (the smallest NATURAL number), i.e., $n = 1$

$$A_{max} = \frac{1}{2 \times 3} = \boxed{\frac{1}{6}}$$

38. Given, $F(0) = 1$

$$\int f(x) = F(X)$$

$$\int_0^1 f(x) = F(1) - F(0) = 1$$

$$F(1) = 1 + F(0) = 2$$

39.

$$g(x) = f(2a^2x - ax^2).$$

Using chain rule:

$$g'(x) = f'(2a^2x - ax^2) \cdot (2a^2 - 2ax) = f'(2a^2x - ax^2) \cdot 2a(a - x)$$

For maxima/minima,

$$g'(x) = 0$$

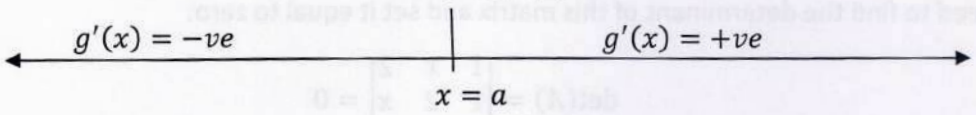
Since, it is given that

$$f'(x) > 0 \; \forall \; x \in R$$

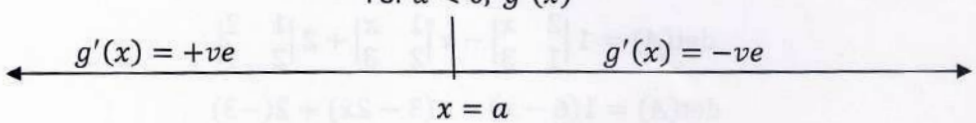
The only root will be $x = a$

You can also solve for the nature by finding $g''(x)$ but here we can also find it by evaluvating the nature of $g'(x)$

For $a > 0$, $g'(x)$



For $a < 0$, $g'(x)$



If the nature of the slope goes

$$+ve \rightarrow -ve \Rightarrow Maxima$$

$$-ve \rightarrow +ve \Rightarrow Minima$$

Hence, the option "Local maxima at $x = a$ if $a > 0$" is the most appropriate as we do not know the nature of $g''(x)$ which may not necessarily be 0, due to which we cannot comment if there is a point of inflection or not.

40.

$$A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 1 & 1 & 1 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\det(A) = \cos\theta \cdot (1 \cdot \cos\theta - 1 \cdot 0) - 0 \cdot (1 \cdot \cos\theta - 1 \cdot \sin\theta) + (-\sin\theta) \cdot (1 \cdot 0 - 1 \cdot \sin\theta)$$

$$\det(A) = \cos\theta \cdot \cos\theta - \sin\theta \cdot (-\sin\theta)$$

$$\det(A) = \cos^2\theta + \sin^2\theta$$

$$\det(A) = 1$$

$$\det(A^k) = \big(\det(A)\big)^k$$

Therefore, the determinant of (A^k) is $\boxed{1}$

41. To solve for the values of x such that there is at least one nonzero vector perpendicular to $\vec{a}$, $\vec{b}$, and $\vec{c}$, we need to find x for which the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are linearly dependent.

Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are linearly dependent if the determinant of the matrix formed by these vectors is zero. Let's form the matrix A

$$A = \begin{bmatrix} 1 & x & 2 \\ 1 & 2 & x \\ 2 & 1 & 3 \end{bmatrix}$$

We need to find the determinant of this matrix and set it equal to zero:

$$\det(A) = \begin{vmatrix} 1 & x & 2 \\ 1 & 2 & x \\ 2 & 1 & 3 \end{vmatrix} = 0$$

Expanding the determinant, we get:

$$\det(A) = 1\begin{vmatrix} 2 & x \\ 1 & 3 \end{vmatrix} - x\begin{vmatrix} 1 & x \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

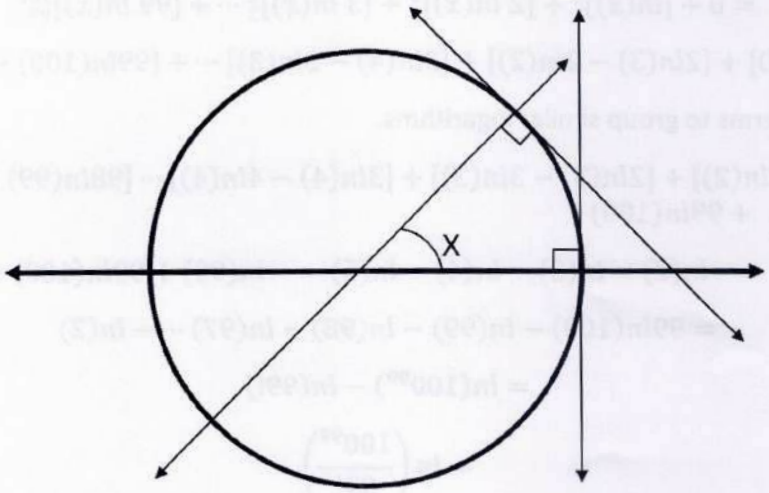
$$\det(A) = 1(6 - x) - x(3 - 2x) + 2(-3)$$

$$det(A) = 6 - x - 3x + 2x^2 - 6 = 2x^2 - 4x = 0$$

$$\Rightarrow 2x(x-2) = 0$$

The possible values of x are $\boxed{0,2}$

42.



$$\tan(x) = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$$

$$\Rightarrow x = 45^\circ$$

Now, you can simply use the sum of angles in quadrilaterals.

$$x + \theta + 90 + 90 = 360 \Rightarrow \boxed{\theta = 135^\circ}$$

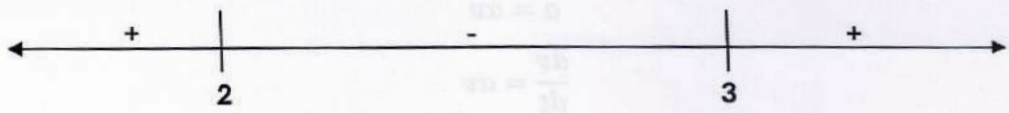
43. The first derivative: $f'(x) = 6x^2 - 30x + 36$

$$6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$$

The critical points are $x = 2$ and $x = 3$.

⋆ Using the wavy curve method:

For x<2 (choose $x = 1$):$f'(1) = 6(1-2)(1-3) = 6(-1)(-2) = 12 > 0$



Therefore, the function $f(x) = 2x^3 - 15x^2 + 36x - 5$ is increasing on the interval (0,2).

44. The integral is split into segments where $[x]$ is constant. For $n \leq x < n+1$, $[x] = n$.

$$\int_0^{100} \frac{[x]}{x} = \int_0^1 \frac{0}{x} + \int_1^2 \frac{1}{x} + \int_2^3 \frac{3}{x} \cdots + \int_{99}^{100} \frac{99}{x}$$

$\int \frac{1}{x} dx = ln \mid x \mid$, Here $ln|x| = ln(x)$ as all values being used are positive

$$= 0 + [ln(x)]_1^2 + [2\ ln(x)]_2^3 + [3\ ln(x)]_3^4 \cdots + [99\ ln(x)]_{99}^{100}$$

$$= [ln(2) - 0] + [2ln(3) - 2ln(2)] + [3ln(4) - 3ln(3)] \cdots + [99ln(100) - 99ln(99)]$$

Rearrange the terms to group similar logarithms.

$$= [ln(2) - 2ln(2)] + [2ln(3) - 3ln(3)] + [3ln(4) - 4ln(4)] \cdots [98ln(99) - 99ln(99)] + 99ln(100)$$

$$= -ln(2) - ln(3) - ln(4) - ln(5) \cdots - ln(99) + 99ln(100)$$

$$= 99ln(100) - ln(99) - ln(98) - ln(97) \cdots - ln(2)$$

$$= ln(100^{99}) - ln(99!)$$

$$= ln\left(\frac{100^{99}}{99!}\right)$$

45. Given,

$$y = (\alpha x + \beta)e^x$$

$$y' = \alpha e^x + (\boldsymbol{\alpha x + \beta})\boldsymbol{e^x}$$

$$y' = y + \alpha e^x \cdots (I)$$

On differentiating both sides,

$$y'' = y' + \alpha e^x \cdots (II)$$

$I - II$:

$$y' - y'' = y - y'$$

$$\boxed{y'' - 2y' + y = 0}$$

46. Given, $\vec{a} = \alpha \vec{v}$

$$a = \alpha v$$

$$\frac{dv}{dt} = \alpha v$$

On Integrating both sides:

$$\int \frac{dv}{v} = \int \alpha dt \qquad v = e^{\alpha t + c} = e^{\alpha t} \cdot e^{c}$$

$$log_e(v) = \alpha t + c \qquad \frac{dS}{dt} = e^{\alpha t} \cdot e^{c}$$

Integrate to find displacement:

$$\int dS = e^{c} \int e^{\alpha t} \cdot dt \qquad [S]_x^y = \left[\frac{e^c}{\alpha} \cdot e^{\alpha t}\right]_x^y$$

So,

$$S_{12} = [S]_{t_1}^{t_2} = \left[\frac{e^c}{\alpha} \cdot e^{\alpha t}\right]_{t_1}^{t_2}$$

$$S_{12} = \frac{e^c}{\alpha} \cdot (e^{\alpha t_2} - e^{\alpha t_1}) \qquad S_{13} = \frac{e^c}{\alpha} \cdot (e^{\alpha t_3} - e^{\alpha t_1})$$

$$\frac{S_{13}}{S_{12}} = \frac{\frac{e^c}{\alpha} \cdot (e^{\alpha t_3} - e^{\alpha t_1})}{\frac{e^c}{\alpha} \cdot (e^{\alpha t_2} - e^{\alpha t_1})}$$

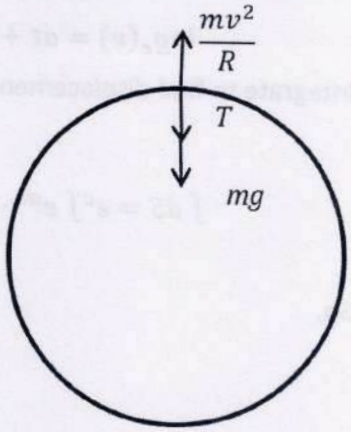
$$\frac{S_{13}}{S_{12}} = \frac{(e^{\alpha t_3} - e^{\alpha t_1})}{(e^{\alpha t_2} - e^{\alpha t_1})}$$

Taking $e^{\alpha t_1}$ common,

$$\frac{S_{13}}{S_{12}} = \frac{e^{t_1}(e^{\alpha t_3 - t_1} - 1)}{e^{t_1}(e^{\alpha t_2 - t_1} - 1)}$$

$$\boxed{\frac{S_{13}}{S_{12}} = \frac{(e^{\alpha t_3 - t_1} - 1)}{(e^{\alpha t_2 - t_1} - 1)} = \frac{\exp(\alpha t_3 - t_1) - 1}{\exp(\alpha t_3 - t_1) - 1}}$$

47. The point in a vertical circular motion with the least tension will be the top point. Meanwhile, the point with maximum speed will be the bottom point. The FBD can be made as shown.



Using the above FBD:

$$\frac{mv^2}{R} = T + mg$$

$$\frac{mv^2}{R} = 2mg + mg = 3mg$$

$$v_{top} = \sqrt{3Rg}$$

Now, using TME on the top and bottom points:

$$\frac{1}{2}mv_{top}^2 + mgh = \frac{1}{2}mv_{max}^2$$

Here, h will be the difference in the height between the two points $(2R)$ and you can cancel out m from all the terms

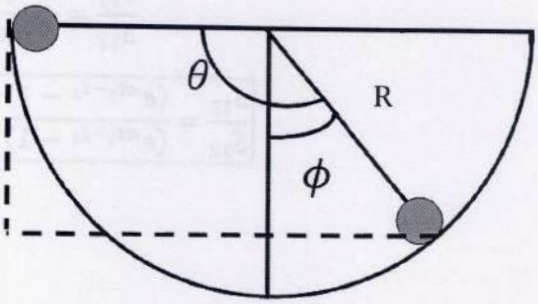
$$\frac{1}{2}(3Rg) + 2Rg = \frac{1}{2}(v_{max}^2)$$

$$v_{max}^2 = 3Rg + 4Rg = 7Rg$$

$$\boxed{v_{max} = \sqrt{7Rg}}$$

48. Let us take : $\phi = \frac{\pi}{2} + \theta$

Using work energy conservation, we know that



$$-W_{friction} = \Delta KE + \Delta PE$$

Since both initial and final kinetic energy are the same(0)

$$\Delta KE = 0$$

$$-W_{friction} = \Delta PE$$

Let the radius of the hemisphere =R

Then,

$$\Delta PE = mgh = mgR\cos(\phi)$$

For any curved surface, the magnitude of frictional force will not be constant so we have to integrate the function of friction

Let us consider a curved surface with dl being small strip where $f = \mu mg\cos\theta$ and dy =horizontal displacment and dx =vertical distance

Then,

$$dW = -\mu mg\cos\theta dl = -\mu mg(\cos\theta dl) = -\mu mgdx$$

$$\int dW = \int -\mu mgdx$$

$$\boxed{W = -\mu mgx}$$

On substitution,

$$\mu mgx = mgR\cos(\phi)$$

$$\mu x = R\cos(\phi)$$

Here,

$$x = R\big(1 + \sin(\phi)\big)$$

$$\mu R\big(1 + \sin(\phi)\big) = R\cos(\phi)$$

$$\mu = \frac{\cos(\phi)}{1 + sin(\phi)}$$

On substituting $\phi = \theta - \frac{\pi}{2} = -\left(\frac{\pi}{2} - \theta\right)$

$$\cos(\phi) = \cos -\left(\frac{\pi}{2} - \theta\right) = sin(\theta) \qquad \sin(\phi) = \sin -\left(\frac{\pi}{2} - \theta\right) = -cos(\theta)$$

$$\mu = \frac{\sin(\theta)}{1 - cos(\theta)}$$

$$1 - \cos(\theta) = 2\sin^2\frac{\theta}{2}$$

$$\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

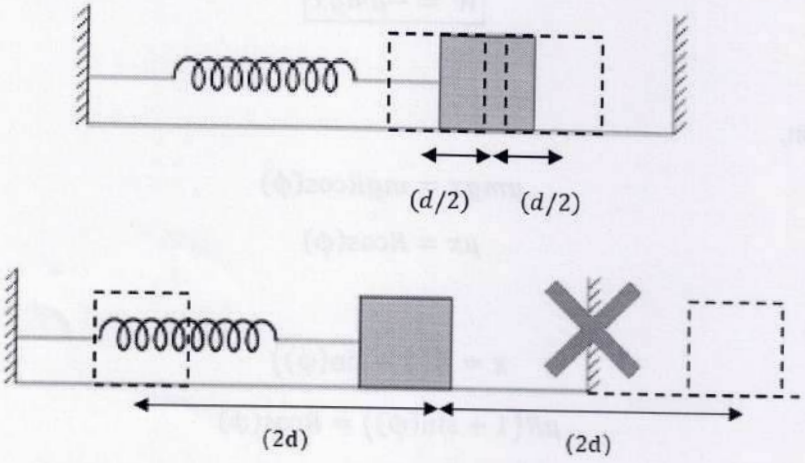
$$\mu - \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\sin^2\frac{\theta}{2}}$$

$$\boxed{\mu = \cot\left(\frac{\theta}{2}\right)}$$

49. The time period of oscillation for a harmonic oscillator is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The time period is not related to the Amplitude of the oscillation but since there is a wall present in the second oscillation with which the block collides elastically, the time period will be decreased by the time it takes for the block with no wall present to go from $\boldsymbol{d}$ to $\boldsymbol{2d}$ and back from $\boldsymbol{d}$.



(Approximate visual representation)

The time period can be calculated by the following:

$$T_0 = T - T_{d\to 2d\to d}$$

$$T_{d\to 2d\to d} \Rightarrow T_{1/2} - 2 \times T_{0\to D}$$

$$T_{0\to D}:$$

Using, $y = A\sin(\omega t)$

$$d = 2d\sin(\omega t)$$

$$\sin(\omega t) = \frac{1}{2}$$

$$\omega t = \frac{\pi}{6}$$

$$\frac{2\pi}{T}t = \frac{\pi}{6}$$

$$t = \frac{T}{12}$$

Substituting,

$$T_{d\to 2d\to d} = \frac{T}{2} - 2\frac{T}{12} = \frac{3T - T}{6} = \frac{T}{3}$$

Substituting again,

$$T_0 = T - \frac{T}{3} = \frac{2T}{3}$$

$$\boxed{\frac{T_0}{T} = \frac{2}{3}}$$

50. The original weight= W

Weight of cube:

$$mg = density \times volume \times g = \rho L^3 g$$

Why not ρ_b, one may wonder?

The principle of buoyancy states force applied is equal to the weight of the **fluid** displaced.

$$\boxed{\text{Net weight} = W + \rho g L^3}$$

51. **Adiabatic process**

For an adiabatic process involving an ideal gas, we use the relation:

$$PV^\gamma = \text{constant}$$

Where symbols hold their normal meanings. For a monoatomic ideal gas, $\gamma = \frac{5}{3}$.

Given the initial state (P, V) and the final volume after adiabatic compression $V_f = \frac{V}{8}$, the final pressure P_f can be found using:

$$PV^\gamma = P_f \left(\frac{V}{8}\right)^\gamma$$

Solving for P_f:

$$P_f = P\left(\frac{V}{\frac{V}{8}}\right)^\gamma = P(8)^\gamma = P \cdot 8^{5/3}$$

Isothermal process

In an isothermal process, the temperature remains constant, so the ideal gas law $PV = nRT$ applies and the product of pressure and volume remains constant.

Let the pressure after isothermal expansion back to the original volume V be $P_f{}'$. The initial state before isothermal expansion is $\left(P_f, \frac{V}{8}\right)$ and the final state is $\left(P_f{}', V\right)$.

From the ideal gas law:

$$P_f \cdot \frac{V}{8} = P_f{}' \cdot V$$

Solving for $P_f{}'$:

$$P_f{}' = \frac{P_f \cdot \frac{V}{8}}{V} = \frac{P_f}{8}$$

Substituting $P_f = P \cdot 8^{5/3}$ into the equation:

$$P_f{}' = \frac{P \cdot 8^{5/3}}{8} = P \cdot 8^{5/3-1} = P \cdot 8^{2/3}$$

So, the final pressure $P_f{}'$ is: $P_f{}' = P \cdot 8^{2/3}$

Thus, the final pressure of the gas after the entire process is: $P_f{}' = P \cdot 8^{2/3}$

$$\boxed{P_f{}' = 4P}$$

52. The tension present in the string will be the electrostatic force:

$$F_e = \frac{kq_1q_2}{r^2}$$

Where $k = \frac{1}{4\pi\epsilon_0}$

$$F_q = -\frac{Kq^2}{(L/2)^2} \qquad |F_q| = \left|\frac{4Kq^2}{L^2}\right|$$

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where: - f is the fundamental frequency, - L is the length of the string, - T is the tension in the string, - μ is the linear mass density of the string.

The tension T in the string is $T = \frac{4kq^2}{L^2}$, we can substitute this into the equation for f:

$$f = \frac{1}{2L}\sqrt{\frac{\frac{4kq^2}{L^2}}{\mu}}$$

$$f = \frac{1}{2L}\sqrt{\frac{4kq^2}{\mu L^2}}$$

$$\boxed{f = \frac{1}{2L^2}\sqrt{\frac{4kq^2}{\mu}}}$$

Since $\sqrt{\frac{4kq^2}{\mu}}$ is a constant, we can simplify this to:

$$f \propto \frac{1}{L^2}$$

53. **Inside the Inner Shell $(r < r_1)$**

Inside a uniformly charged shell, the electrostatic potential V is constant. Since the inner shell has a charge $+Q$ and the outer shell has a charge $-Q$, and given the spherical symmetry, the potential inside the inner shell$(r < r_1)$is:

$$V(r < r_1) = \frac{kQ}{r_1} - \frac{kQ}{r_2} = \text{constant}$$

Between the Two Shells $(r_1 \leq r < r_2)$

In this region, the potential is influenced by the charge of the inner shell $(+Q)$ and the charge of the outer shell $-Q$.The potential at a distance r from the center due to a spherical shell of charge Q and radius r_i for $r \geq r_i$ is given by:

$$V(r_1 \leq r < r_2) = \frac{kQ}{r} - \frac{kQ}{r_2}$$

Here, $\frac{kQ}{r_2}$ will be constant but $\frac{kQ}{r}$ will decrease propotional to $\frac{1}{r}$ until at $r = r_2$ when $\frac{kQ}{r} = \frac{kQ}{r_2}$

$$V(r = r_2) = 0$$

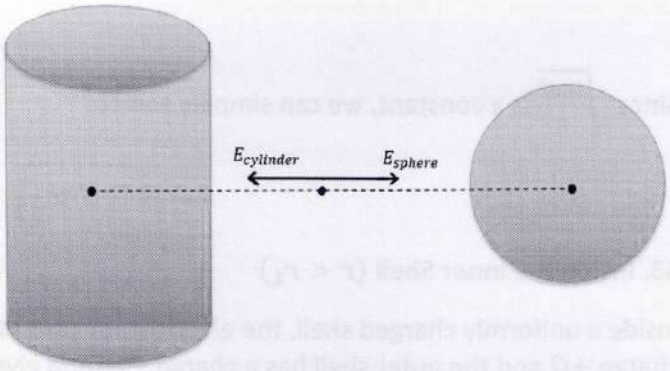
Outside the Outer Shell $(r \geq r_2)$

For $r \geq r_2$, the potentials from both shells must be considered as if they were point charges located at the center. Since the shells have charges $+Q$ and $-Q$, the net charge is zero. Therefore, the potential outside the outer shell will be zero:

$$V(r \geq r_2) = \frac{kQ}{r} - \frac{kQ}{r} = 0$$

54.

The infinite cylinder with a spherical cavity can be considered the superposition of an infinite cylinder ρ and sphere of charge density $-\rho$. From the principle of superposition, we have the following:



$$E_{net} = E_{cylinder} - E_{sphere}$$

$$E_{cylinder} = \frac{2k\lambda}{x}$$

Where: $k = \frac{1}{4\pi\epsilon_0}$

λ = linear charge density

$$\lambda = \rho \times Area = \rho \times \pi a^2$$

$$E_{cylinder} = \frac{2k\rho\pi a^2}{x} \qquad E_{sphere} = \frac{kq}{x^2}$$

$$\rho = \frac{q}{4/3\pi a^3} \Rightarrow q = \frac{4}{3}\rho\pi a^3$$

$$E_{sphere} = \frac{4k\rho\pi a^3}{3x^2}$$

Thus,

$$E_{net} = \frac{2k\rho a^2}{x} - \frac{4k\rho\pi a^3}{3x^2} = 2k\rho\pi a^2\left(\frac{1}{x} - \frac{2a}{3x^2}\right)$$

For maxima/minima:

$$\frac{dE}{dx} = 0$$

$$\frac{dE}{dx} = \frac{-1}{x^2} - \frac{-2a \times 2}{3x^3} = 0$$

$$\frac{1}{x^2} = \frac{4a}{3x^3}$$

$$\boxed{x = \frac{4a}{3}}$$

The other answer would be $x = 0$, but we know that the electric field would be 0 at the center

55. **Step 1:** Calculate the total energy in joules equivalent to 20,000 units of TNT.

$$1 \text{ unit of TNT} = 4.184 \times 10^9 \text{ J}$$

So, 20,000 units of TNT is:

$$20{,}000 \times 4.184 \times 10^9 = 8.368 \times 10^{13} \text{ J}$$

Step 2: Convert the Energy Released per Pu-239 Fission to Joules

$$\text{Energy released per Pu-239 fission} = 207 \text{ MeV}$$

$$207 \text{ MeV} = 207 \times 10^6 \text{ eV}$$

$$207 \times 10^6 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV} = 3.312 \times 10^{-11} \text{ J}$$

Step 3: Determine the Number of Fissions Required

The number of fissions required to produce 8.368×10^{13} J is:

$$\frac{8.368 \times 10^{13}}{3.312 \times 10^{-11}} \approx 2.526 \times 10^{24} \text{ fissions}$$

Step 4: Calculate the Amount of Pu-239 Required

First, find the number of Pu-239 atoms needed for this number of fissions. Given Avogadro's number,(6.022×10^{23}) and the atomic mass of Pu-239 (239 g/mole):

Number of moles of Pu-239 needed:

$$\frac{2.526 \times 10^{24}}{6.022 \times 10^{23}} \approx 4.195 \text{ moles}$$

Convert moles to grams:

$$4.195 \text{ moles} \times 239 \text{ g/mole} \approx 1002.605 \text{ g}$$

Thus, the closest estimate of the amount of Plutonium required is approximately 1 kg.

So, the closest estimate of the amount of Plutonium required for a 20,000 units of TNT explosion is around 1 kg.

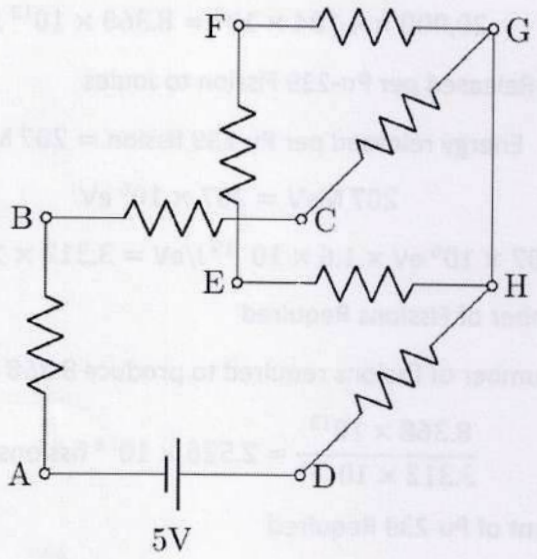
56. **There is no change in current over time for a resistor.** The best corresponding graph is a_1.

After a long time (steady state), the capacitor acts like an open circuit, meaning no current flows through it. The best corresponding graph is a_2.

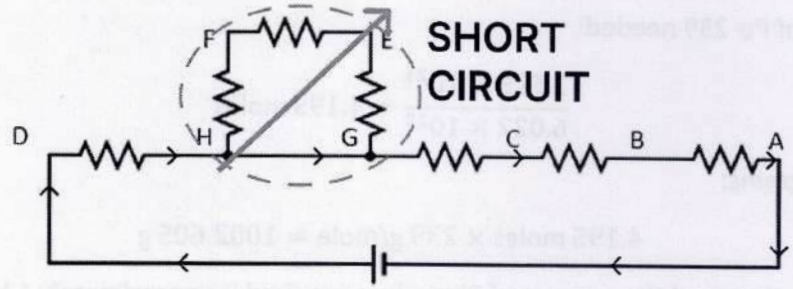
After a long time, an inductor in a DC circuit behaves like a short circuit, meaning it allows the current to pass through with almost no resistance. The best corresponding graph is a_3.

57. Once a capacitor is fully charged, it behaves like an open circuit for DC current. So, we can remove all the capacitors to simplify the circuit and then obtain potential difference across BF.

On simplifying we get the following circuit which can be remade as the following:



GH will short-circuit the three resistors and the potential at F will be equal to G.



The current can be calculated using Ohm's law:

$$R_{net} = 4 \times 1k\Omega$$

$$V = IR$$

$$I = \frac{5}{4} \times 10^{-3} A$$

The potential difference between F and B is the same as the potential difference between H and B, where the net resistance between the two points is $2k\Omega$. Hence, the potential difference is:

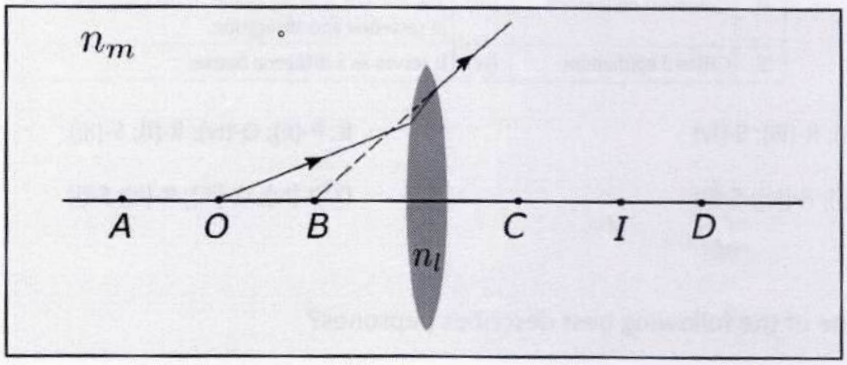
$$V_{HB} = I(R_{HB}) \Rightarrow V_{HB} = \frac{5}{4} \times 10^{-3} \times 2 \times 10^{3} = \frac{10}{4} V$$

Using the formula $Q = CV$:

$$Q = 4 \times 10^{-6} \cdot \frac{10}{4}$$

$$\boxed{Q = 10\mu C}$$

58. When a convex lens with a refractive index n_l is placed in a medium with a higher refractive index n_m (where $n_l < n_m$), it no longer acts as a converging lens. Instead, it behaves as a diverging lens. This happens because light rays bend away from the normal when entering the lens and towards the normal when exiting, causing the rays to spread out rather than converge. Consequently, the lens's focal length becomes negative, indicating a diverging effect.



59. We can clearly see the larger sphere completely contains the smaller sphere. All we need to do is find the charge due to the smaller sphere and then use the formula:

$$\phi = \frac{q_{enc}}{\epsilon_0}$$

Here, ρ is the charge density which will be given by the following:

$$\rho = \frac{charge}{volume} \qquad \rho = \frac{q_{enc}}{4/3\pi a^3}$$

$$q_{enc} = \frac{4}{3}\rho\pi a^3$$

Using the previously mentioned formula:

$$\boxed{\phi = \frac{4}{3\epsilon_0}\rho\pi a^3}$$

60. The number of ejected electrons depends on the number of incident photons, which in turn depends on the intensity of the light. The intensity of light (I) is directly proportional to the number of photons hitting the surface per unit time. Therefore, a higher intensity means more photons are hitting the surface per unit time.

If the light intensity remains constant over time, the rate at which electrons are emitted will also remain constant, leading to a steady (constant) photocurrent.

Since $I_3 > I_2 > I_1$, the number of emitted electrons (and hence the photocurrent) will be highest for I_3, followed by I_2, and lowest for I_1.