IAT 2021

Biology

1. Which one of the following epithelial cell types is commonly found in the inner surface of the fallopian tubes?

A. Cuboidal B. Columnar C. Squamous D. Ciliated

2. Which of the following is not an asexual reproductive structure?

A. Conidia of Penicillium

B. Isogametes of Cladophora

C. Zoospores of Chlamydomonas

D. Gemmules in a sponge

3. Which one of the following is an example of palindromic DNA sequence?

A. 5' GACTTC 3'
3' CTGAAG 5'

B. 5' GAATTC 3'
3' CTTAAG 5'

C. 5' GAAGTC 3'
3' CTTCAG 5'

D. 5' GACCAG 3'
3' CTGGTC 5'

4. Some individuals start sneezing when the pollen content is high in the air. Primarily which Ig isotype will these individuals produce as an immune response?

A. IgM B. IgA C. IgE D. IgG

5. Which of these factors is least likely to cause deviation from the Hardy- Weinberg equilibrium?

A. Reduction in population size

B. Mutation

C. Gene flow

D. Genetic drift

6. Motivated by the classic experiment by Frederick Griffith and the work of Avery, Macleod and McCarty to identify the transforming principle, a scientist redesigned and performed the experiment with S-strain and R-strain of Streptococcus pneumoniae as summarized in the table below.

Treatment	Experimental Condition
T1	S-strain (heat-killed) injected into mice
T2	S-strain (heat-killed) + R-strain (live) together injected into mice
T3	Nucleic acids isolated from S-strain (heat-killed) + R-strain (live), incubated with RNase, and injected into mice
T4	Nucleic acids isolated from S-strain (heat-killed) + R-strain (live), incubated with DNase, and injected into mice
T5	S-strain (live) injected into mice
T6	R-strain (live) injected into mice

From the options below, identify the outcomes of the treatments (T1 to T6) on the viability of the mice.

A. T1: live, T2: die, T3: live, T4: die, T5: die, T6: live

B. T1: live, T2: live, T3: die, T4: live, T5: live, T6: live

C. T1: live, T2: die, T3: die, T4: die, T5: live, T6: die

D. T1: live, T2: die, T3: die, T4: live, T5: die, T6: live

7. Which of the following is/are produced by a plant during photosynthesis with far-red light?

A. ATP, NADPH and H^+

B. Only ATP

C. NADPH and H^+

D. Only NADPH

8. In plants, ammonium ions are produced by protonation of ammonia. Which enzyme uses these ammonium ions to convert an alpha-keto acid into an amino acid?

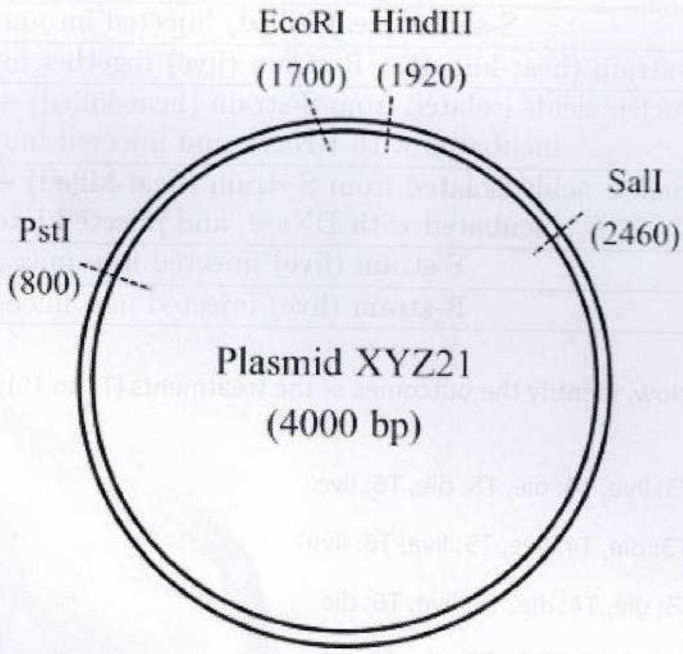
A. Transacetylase

B. Nitrogenase

C. Glutamate dehydrogenase

D. Lactate dehydrogenase

9. The map of a 4000 base pair (bp) plasmid DNA marking the locations of different restriction enzyme cut sites is shown in the figure below. The numbers in brackets indicate the base pair positions where the enzymes cut. This plasmid is completely digested first with the combination of restriction enzymes PstI and HindIII, and then with EcoRI and SalI. The final digested plasmid is analyzed by agarose gel electrophoresis.



Which one of the following options correctly represents the bands sizes (in bp) obtained on the gel?

A. 220, 540, 900 and 2340

B. 220, 540, 1200 and 2040

C. 540, 760, 1200 and 1500

D. 320, 540, 800 and 2340

10. Which one of the following physiological functions is common between the small intestine and the renal tubules?

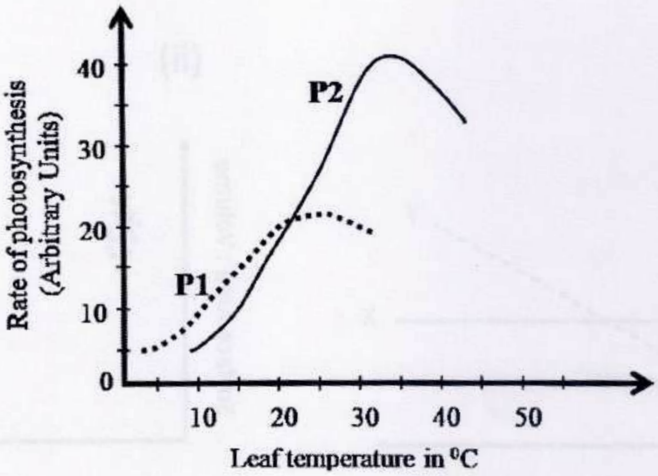
A. Absorption of glucose

B. Excretion of waste materials

C. Excretion of water

D. Absorption of proteins

11. The following plot represents the change in the rate of photosynthesis with leaf temperature for two plants, P1 and P2.



Which one of the following statements correctly describes the characteristics of P1 and P2 plants?

A. Plant P1 is a C_4 plant having tropical adaptation whereas Plant P2 is a C_3 plant with temperate adaptation

B. Plant P1 is a C_4 plant having temperate adaptation whereas Plant P2 is a C_3 plant with tropical adaptation

C. Plant P1 is a C_3 plant having temperate adaptation whereas Plant P2 is a C_4 plant with tropical adaptation

D. Plant P1 is a C_3 plant having tropical adaptation whereas Plant P2 is a C_4 plant with temperate adaptation

12.Which of the following floral formulae represents a zygomorphic and superior ovary?

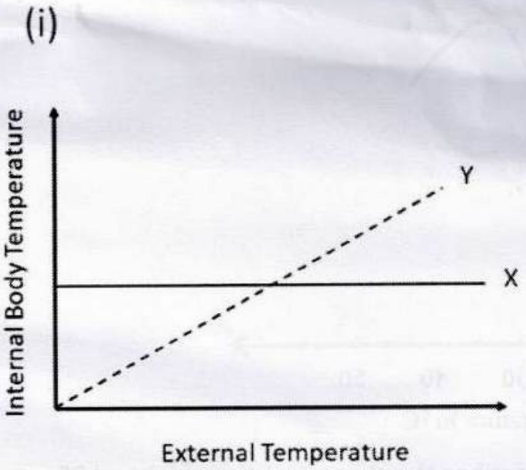
A. $\% ⚥ K_{(5)} C_{(5)} A_2 G_{\underline{1}}$

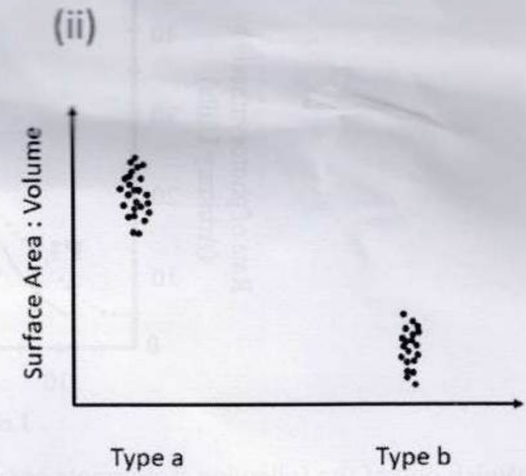
B. $\oplus ⚥ K_{(5)} C_{(5+5)} A_{(\infty)} \underline{G}_1$

C. $\% ⚥ K_{(5)} C_{(5+5)} A_{(\infty)} G_{\underline{1}}$

D. $\% ⚥ K_{(5)} C_{1+2+(2)} A_{(9)+1} \underline{G}_1$

13. The figure (i) represents two categories of animals (X and Y) with respect to their response to the external environment. The figure (ii) represents two broad categories of animals (Type a and Type b) with respect to their body surface area to volume ratio.





Which combination most suitably represents the following animals in the order: humming bird; crocodile; frog; polar bear?

A. Xa; Yb; Ya; Xb B. Xb; Ya; Yb; Xa C. Ya; Yb; Xa; Xb D. Yb; Xb; Xa; Ya

14. Match the following pairs of interacting species to the corresponding names of the interactions

	Name of interacting species		Name of interaction
i	Herbivores and Plants	a	Mutualism
ii	Cuckoo and Crow	b	Predation
iii	Sea anemone and Clown fish	c	Parasitism
iv	Fungus and Cyanobacteria (Lichens)	d	Commensalism

Pick the correct options from the below:

A. i and b; ii and d; iii and c; iv and a

B. i and c; ii and b; iii and d; iv and a

C. i and b; ii and c; iii and d; iv and a

D. i and b; ii and a; iii and c; iv and d

15. The following events are associated with meiosis

i. Appearance of recombinant modules

ii. Formation of meiotic spindle

iii. Formation of chiasmata

iv. Formation of synaptonemal complex

Which of the following is the correct sequence of these events during meiosis?

A. $i \rightarrow iii \rightarrow iv \rightarrow ii$

B. $iii \rightarrow iv \rightarrow ii \rightarrow i$

C. $iv \rightarrow iii \rightarrow i \rightarrow ii$

D. $iv \rightarrow i \rightarrow iii \rightarrow ii$

Chemistry

16. The F-P-Cl bond angles in the most stable structure of PF_3Cl_2 are close to

A. 90° only

B. 90° and 120°

C. 90° and 180°

D. 90°, 120°, and 180°

17. The correct statement about the bond angles and bond lengths in Al_2Cl_6 is (Cl_t= terminal Cl; Cl_b= bridging Cl)

A. $\angle Cl_t - Al - Cl_t > \angle Cl_b - Al - Cl_b$ *and* $Al - Cl_b > Al - Cl_t$.

B. $\angle Cl_t - Al - Cl_t > \angle Cl_b - Al - Cl_b$ *and* $Al - Cl_t > Al - Cl_b$.

C. $\angle Cl_t - Al - Cl_t = \angle Cl_b - Al - Cl_b$ *and* $Al - Cl_t > Al - Cl_b$.

D. $\angle Cl_b - Al - Cl_b > \angle Cl_t - \mathrm{Al} - \mathrm{Cl}_t$ *and* $Al - Cl_b > Al - Cl_t$

18. Among CH_3SiCl_3, $(CH_3)_2SiCl_2$ *and* $(CH_3)_3SiCl$, which one is used to synthesize straight chain (linear) and which one is used to prepare branched chain (cross-linked) silicone polymer, respectively?

A. $(CH_3)_2SiCl_2$ *and* $(CH_3)_3SiCl$

B. $(CH_3)SiCl_3$ *and* $(CH_3)_2SiCl_2$

C. $(CH_3)_2SiCl_2$ and $(CH_3)SiCl_3$

D. $(CH_3)_3SiCl$ *and* $(CH_3)_2SiCl_2$

19. Which one of the following statements is INCORRECT about a complex of a divalent ion with atomic number 25? (BM= Bohr Magneton)

A. Complex with weak field ligands in tetrahedral geometry, magnetic moment of 1.73 BM.

B. Complex with weak field ligands in tetrahedral geometry, magnetic moment of 5.92 BM.

C. Complex with weak field ligands in octahedral geometry, magnetic moment of 5.92 BM.

D. Complex with strong field ligands in octahedral geometry, magnetic moment of 1.73 BM.

20. $Ni(CO)_4$ is diamagnetic because

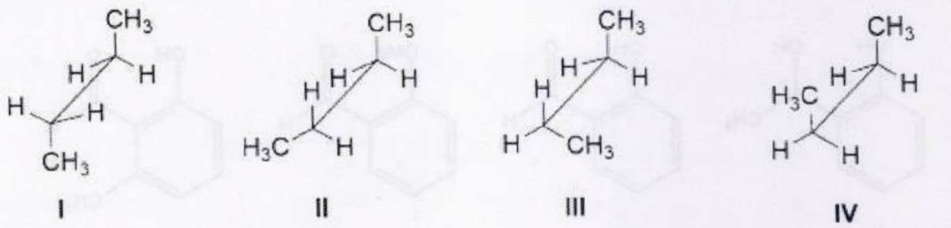
A. Ni has completely filled 3d orbitals.

B. it is a square planar complex.

C. CO is a strong field ligand.

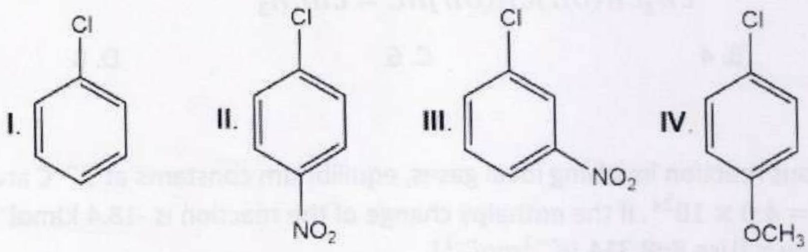
D. it has synergic bonding.

21. The correct order for the relative energies of the following sawhorse conformations is



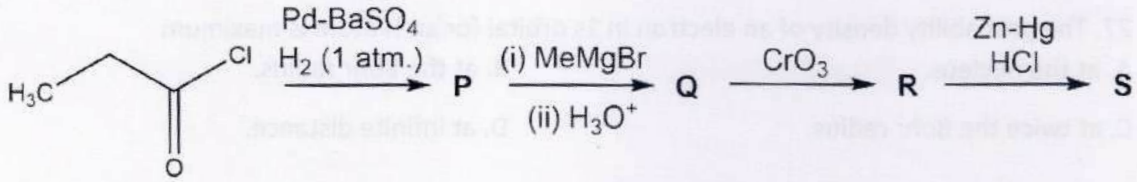
A. I > II = III > IV.   B. IV > III > II > I.   C. II ≈ III > IV > I.   D. IV > III = II > I.

22. Among the haloarenes ($I - IV$) shown below, the correct order of reactivity for the substitution reaction



A. I > IV > III > II.   B. II > III ≈ I > IV.   C. II > III > IV > I.   D. IV > I ≈ III > II.

23. Consider the following reaction sequence



(i) The products P and R can form addition product with HCN.

(ii) The product Q has no chiral center.

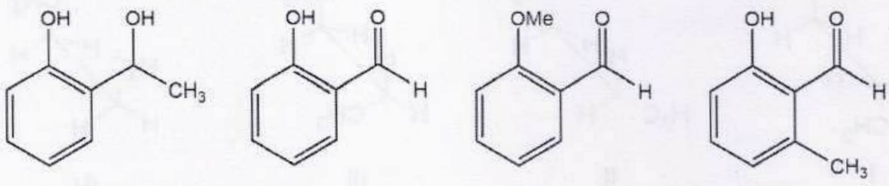
(iii) The product R can undergo Cannizzaro reaction.

(iv) The product S is a saturated hydrocarbon.

Select

A. (i) and (iv)   B. (i) and (ii)   C. (ii) and (iii)   D. (iii) and (iv)

24. The major product of the reaction of salicylaldehyde with one equivalent of MeMgBr, followed by acid neutralization is



A. I B. II C. III D. IV

25. The total number of stereoisomers possible for the following structure is

$$CH_3CH(OH)CH(OH)HC = CHCH_3$$

A. 2 B. 4 C. 6 D. 8

26. For a homogeneous reaction involving ideal gases, equilibrium constants at 27°C are $K_p = 9.98 \times 10^{27}$ and $K_c = 4.0 \times 10^{24}$. If the enthalpy change of the reaction is -18.4 kJmol⁻1, the internal energy change is [Use R=8.314 $JK^{-1}mol^{-1}$]

A. -13.41kJ mol^{-1}

B. - 15. 91 kJ mol^{-1}

C. - 20. 89 kJ mol^{-1}

D. - 23. 39 kJ mol^{-1}

27. The probability density of an electron in 1s orbital for an H atom is maximum

A. at the nucleus.

B. at the Bohr radius.

C. at twice the Bohr radius.

D. at infinite distance.

28. Which of the following statements is INCORRECT?

A. Half-life of a zero order reaction is proportional to the initial concentration of reactant

B. Half-life of a zero order reaction is proportional to the rate constant.

C. Half-life of a first order reaction is independent of the initial concentration of reactant.

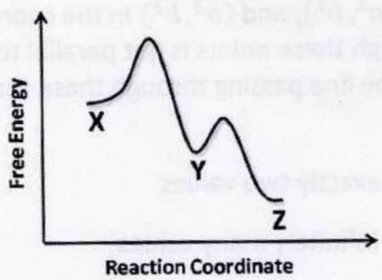
D. Half-life of a first order reaction is inversely proportional to the rate constant.

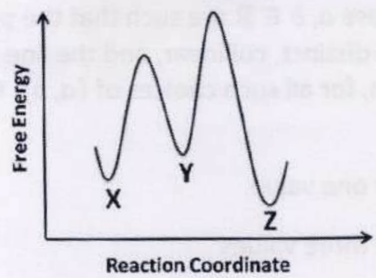
29. The time required for reducing 1 mole of MnO_4^-(aq.) to $Mn^2 +$ (aq.) when a current of 2.5 A is passed during electrolysis is

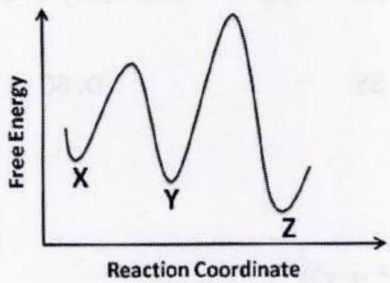
A. 64.8 μs B. 12.9 μs C. 10.7 hours D. 53.6 hours

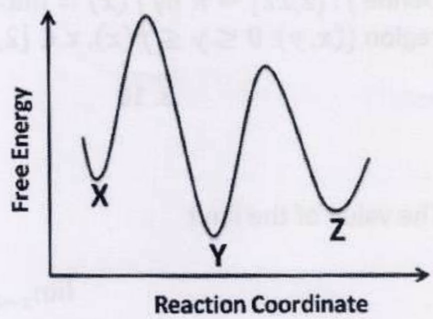
30. Based on the relative stabilities and barriers, which among the following schematic energy diagrams would best correspond to the reaction scheme shown below? [k and k' are similar in magnitude]

$$X \underset{2k}{\overset{4k}{\rightleftharpoons}} Y \underset{3k'}{\overset{8k'}{\rightleftharpoons}} Z$$

A.


B.


C.


D.


Mathematics

31. What is the maximum value of $\cos^4(x) + \sin^2(x) + \cos(x)$, when $x \geq 0$?

A. 1   B. 2   C. $\sqrt{2}$ D. $2\sqrt{2}$

32. Out of a pack of ten cards numbered 1 to 10, a boy draws a card at random and keeps it back. Then a girl draws a card at random from the same pack. If the boy's card reads m, and the girl's card reads n, then what is the probability that $m > n$, given that m is even?

A. 1/2   B. 1/3   C. 1/4   D. 1/5

33. Suppose $a, b \in \mathbf{R}$ are such that the points (a, b), (a^2, b^2), and (a^3, b^3) in the coordinate plane are distinct, collinear, and the line passing through these points is not parallel to the y-axis. Then, for all such choices of (a, b), the slope of the line passing through these three points can take

A. exactly one value

B. exactly two values

C. exactly three values

D. infinitely many values

34. Define $f: [2,22] \to R$ by $f(x) = \max\{n(1 - |x - (2n + 1)|): n = 1,2, \ldots, 10\}$. The area of the region $\{(x, y): 0 \leq y \leq f(x), x \in [2,22]\}$ is

A. 5   B. 10   C. 55   D. 60

35. The value of the limit

$$\lim_{x \to 0^+} (\sin x)^{\sqrt{x}} (e^x + x)^{\frac{1}{x}}$$

is:

A. 0   B. 1   C. e D. e^2

36. Let $f: \mathbf{R} \to \mathbf{R}$ be a continuous function satisfying $f(x) = e^{x^2/2} + \int_0^x t\, f(t)dt$ for all x.

A. $2 < f(\sqrt{2}) < 3$

B. $3 < f(\sqrt{2}) < 4$

C. $4 < f(\sqrt{2}) < 5$

D. $5 < f(\sqrt{2}) < 6$

37. For each $a \in \mathbf{R}$, define $p_a(z) = z^2 + 2e^{a-e^a}z + e^{a-e^a}$. Then which of the following is correct?

A. p_a has a real root for all $a \in \mathbf{R}$

B. p_a has only non-real complex roots for all $a \in \mathbf{R}$.

C. p_a has a real root if and only if $a \geq 1$.

D. p_a has a real root if and only if $a \leq -1$

38. The number of functions $f: \{1,2,3,4,5\} \to \{1,2,3,4,5\}$ such that $f(f(n)) = n$ for all $n \in \{1,2,3,4,5\}$ is

A. 25 B. 31 C. 41 D. 120

39. Let $P_1: x + y + z = 1, P_2: 2x + y + z = 3$ be two planes, and let L denote the line of intersection of P_1 and P_2. Let P be the plane passing through the point (1,2,1),and normal to L. Which of the following equations represents P?

A. $y - z = 1$ B. $x + z = 2$ C. $x + 2y + z = 6$ D. $x + y + 2z = 5$

40. Let $f(x) = \ln(1 + x)$ for $x \geq 0$.The value of the following is:

$$\int_0^{\frac{\pi}{2}} \frac{f(\sqrt[3]{\cos\theta})}{f(\sqrt[3]{\sin\theta}) + f(\sqrt[3]{\cos\theta})} d\theta$$

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$

41. The area enclosed by the curves $y = 1 + |\sin x|, y = -|\sin x|$ and the lines $x = 0, x = 2\pi$ is

A. $8 + 2\pi$ B. $8 + 4\pi$ C. $8 + 6\pi$ D. $8 + 8\pi$

42. Consider the function $f: \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^2|x|$. Then, which of the following statements is correct?

A. f is differentiable but f' is not differentiable

B. f is continuous but not differentiable.

C. f' is differentiable but f'' is not differentiable

D. f'' is differentiable.

43. Let A be a 2×2 matrix such that $A^2 + A + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Let I denote the 2×2 identity matrix. Which of the following statements is correct?

A. Both A and $A + I$ are invertible

B. A is invertible but $A + I$ may not be invertible.

C. $A + I$ is invertible but A may not be invertible.

D. Neither $A + I$ nor A may be invertible.

44. If three real numbers a, b, c are in arithmetic progression, the value of the determinant

$$\begin{vmatrix} x^2+3 & x^2+4 & x^2+5 \\ x^2+4 & x^2+5 & x^2+6 \\ x^2+a & x^2+b & x^2+c \end{vmatrix}$$

is

A. 0 B. $2a$ C. $a + c - b$ D. x^2+2b

45. Consider the two data sets

$$\begin{aligned} S_1 &= \{1,2,4,8,9,11,15,20,27,29,33\}, \\ S_2 &= \{51,52,54,58,59,61,65,70,77,79,83\}. \end{aligned}$$

Let m_1, m_2 and v_1, v_2 be the means and the variances of S_1 and S_2, respectively. Then, which of the following relations is correct?

A. $m_2 = m_1, v_2 = v_1$

B. $m_2 = m_1 + 50, v_2 = v_1 + 50$

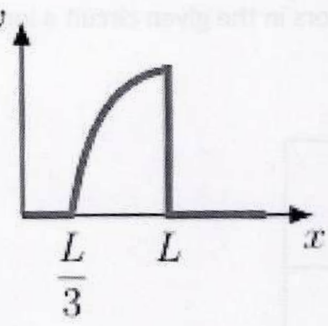
C. $m_2 = m_1 + 50, v_2 = v_1$

D. $m_2 = m_1 + 50, v_2 < v_1$

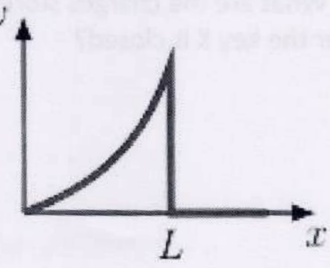
Physics

46. A door of mass M and width L is hinged at one end and rotates about a vertical axis without friction. A bullet of mass m ($m \ll M$) fired perpendicularly to the door at a speed v gets embedded in it at a distance x from its axis of rotation. Assuming the door was stationary initially, how does the resultant angular speed ω of the door vary as a function of x?

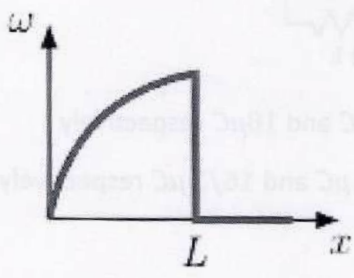
A.



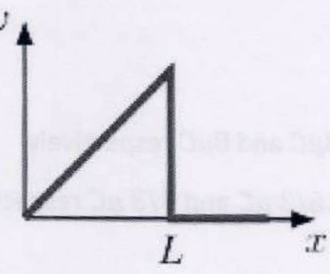
C.



B.



D.



47. A body of mass m executes simple harmonic motion along a line with time period T and energy E. What is the magnitude of the maximum acceleration of the body?

$A.\frac{\pi}{T}\sqrt{\frac{E}{m}}$ $\quad$ $B.\frac{2\pi}{T}\sqrt{\frac{E}{m}}$

$C.\frac{\sqrt{2}}{T}\sqrt{\frac{E}{m}}$ $\quad$ $D.\frac{2\sqrt{2}\pi}{T}\sqrt{\frac{E}{m}}$

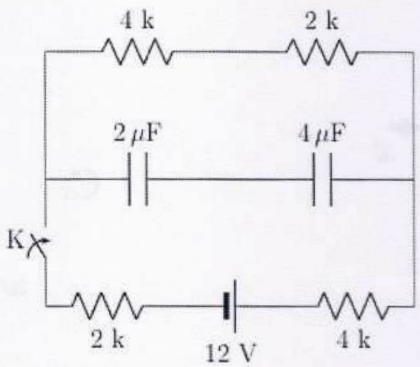
48. An athlete runs on a straight track. She starts from rest and runs with a constant acceleration for the first 2 seconds, reaching a speed of 9 ms^{-1}. She then continues at this constant speed for some time before slowing down to a halt at a constant deceleration. The total time taken, from start to finish, is 12 seconds. If the magnitude of her acceleration is twice the magnitude of deceleration, then what is the total distance covered by her?

A. 72 m $\quad$ B. 81 m $\quad$ C. 90 m $\quad$ D. 108 m

49. A capacitor of capacitance C consists of two large parallel metal plates. The coefficient of linear expansion of the metal is α. What is the change in capacitance if the temperature of the plates rises by ΔT, while the gap between the plates is kept fixed?

A. $\alpha \Delta T C$ B. $-\alpha \Delta T C$ C. $2\alpha \Delta T C$ D. $-2\alpha \Delta T C$

50. What are the charges stored in the 2 μF and 4 μF capacitors in the given circuit a long time after the key K is closed?



A. $8\mu C$ and $8\mu C$ respectively

B. $18\mu C$ and $18\mu C$ respectively

C. $16/3\ \mu C$ and $8/3\ \mu C$ respectively

D. $8/3\ \mu C$ and $16/3\ \mu C$ respectively

51. Consider a point charge $+q$ moving with a constant velocity $\vec{v} = v\hat{k}$ in vacuum in the presence of an electric field $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$ and a magnetic field $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$. Unit vectors $\hat{\imath}, \hat{\jmath}$, and $\hat{k}$ are in the directions of x, y,and z axes, respectively. Which of the following relations is correct?

A. $E_x = vB_x, E_y = vB_y$

B. $E_x = vB_y, E_y = -vB_x$

C. $E_x = -vB_y, E_y = vB_x$

D. $E_x = -vB_x, E_y = -vB_y$

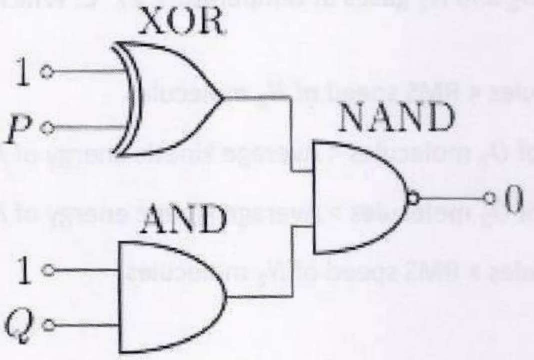
52. Four point charges $q, -2q, -3q$ and $4q$ are placed at the four vertices of a regular tetrahedron of side L, while a charge $5q$ is placed at its center. What is the total electrostatic energy of the system? (Vacuum permittivity is denoted by ε_0.)

A. $-\frac{15q^2}{4\pi\varepsilon_0 L}$ B. $\frac{15q^2}{4\pi\varepsilon_0 L}$ C. $-\frac{30q^2}{4\pi\varepsilon_0 L}$ D. $\frac{30q^2}{4\pi\varepsilon_0 L}$

53. The half life of a radioactive element is 2000 hours. Approximately how much time is required for the decay of 2/3 of its nuclei?

A. 1170 hours B. 2830 hours C. 3000 hours D. 3170 hours

54. A 2-input exclusive OR (XOR) gate with inputs X and Y produces the output $\bar{X}Y + X\bar{Y}$. In the Boolean circuit shown below, which values of the inputs P and Q will produce the output 0?



A. $P = 0, Q = 0$ B. $P = 1, Q = 0$ C. $P = 0, Q = 1$ D. $P = 1, Q = 1$

55. In an experiment on the photoelectric effect, the de Broglie wavelength of the emitted electron is λ_B. The energy of the photon incident on the metal is five times the work function. If h is Planck's constant and m_e is the electron mass, then what is the work function?

A. $\frac{h^2}{4m_e\lambda_B^2}$ B. $\frac{h^2}{8m_e\lambda_B^2}$

C. $\frac{h^2}{10m_e\lambda_B^2}$ D. $\frac{h^2}{12m_e\lambda_B^2}$

56. An ambulance traveling at a speed 20 m s^{-1} emits a sound of frequency 540 Hz from its siren. Sunanda is driving a car which approaches the ambulance from the opposite direction at a speed of 20 m s^{-1}. What will be the change in detected frequency by Sunanda, as she crosses the ambulance? (Given, the speed of sound in air is 340 m s^{-1}.)

A. 72.00 Hz B. 127.5 Hz C. 128. 8 Hz D. 135.5 Hz

57. Consider a Young's double slit experiment with monochromatic light of wavelength 600 nm. The intensity of the light is I_0 at a point on the screen where the path difference is 600 nm. What would be the intensity of light at a point on the screen where the path difference is 100 nm?

A. $(1/2)I_0$ B. $(1/4)I_0$ C. $(\sqrt{3}/2)I_0$ D. $(3/4)I_0$

58. Consider a mixture of O_2 and N_2 gases at temperature 27 °C. Which of the following relations is correct?

A. RMS speed of O_2 molecules < RMS speed of N_2 molecules

B. Average kinetic energy of O_2 molecules < Average kinetic energy of N_2 molecules

C. Average kinetic energy of O_2 molecules > Average kinetic energy of N_2 molecules

D. RMS speed of O_2 molecules > RMS speed of N_2 molecules

59. Two identical objects A and B are at initial temperatures T_A and $T_B (T_A > T_B)$, respectively. The specific heat capacity of the material of these objects increases with temperature. If these two objects are brought in contact then their final equilibrium temperature is T. Assuming that there is no heat exchange with the surroundings, then

A. $T < \frac{T_A + T_B}{2}$ B. $T > T_A$ C. $T = \frac{T_A + T_B}{2}$ D. $T > \frac{T_A + T_B}{2}$

60. Which one of the following expressions has the dimension of electrical resistance where e is the charge of an electron and h is Planck's constant?

A. e/h

B. e^2/h

C. h/e^2

D. h/e

ANSWER KEY 2021

Question Number	Answer	Question Number	Answer	Question Number	Answer	Question Number	Answer
1	(D)	16	(B)	31	(B)	46	(D)
2	(B)	17	(A)	32	(A)	47	(D)
3	(B)	18	(C)	33	(B)	48	(B)
4	(C)	19	(A)	34	(C)	49	(C)
5	(A)	20	(A)	35	(D)	50	(A)
6	(D)	21	(D)	36	(D)	51	(B)
7	(B)	22	(B)	37	(B)	52	(A)
8	(C)	23	(A)	38	(C)	53	(D)
9	(A)	24	(B)	39	(A)	54	(C)
10	(A)	25	(D)	40	(B)	55	(B)
11	(C)	26	(C)	41	(A)	56	(B)
12	(D)	27	(A)	42	(C)	57	(D)
13	(A)	28	(B)	43	(A)	58	(A)
14	(C)	29	(D)	44	(A)	59	(D)
15	(D)	30	(A)	45	(C)	60	(C)

SOLUTIONS

1. **Ciliated epithelial tissue** is found in the fallopian tube which helps in the movement of the ovum.

2. **Isogametes of Cladophora** are sexual structures (gametes) which undergo fusion to form zygote.

3. Palindromic sequences are the same on both strands when read in opposite directions, i.e. along the same polarity.

4. **IgE** is primarily involved in allergic reactions, such as sneezing in response to high pollen levels. It binds to allergens (like pollen) and triggers the release of histamines from mast cells and basophils, leading to allergy symptoms.

5. Five factors are known to affect Hardy-Weinberg equilibrium. These are gene migration or gene flow, genetic drift, mutation, genetic recombination and natural selection. **Reduction in population size** is not a directly influencing factor for Hardy-Weinberg Equilibria.

6. **T1:** Heat killed bacteria cannot cause infections.
T2: Leads to transformation of R-strain bacteria to S-strain, causing infection.
T3: Transformation is induced by DNA, so RNase does not affect it.
T4: DNA is degraded by DNase which prevents transformation.
T5: S-strain bacteria is virulent and causes pneumonia, which leads to the death of the mice.
T6: R-strain bacteria are non-virulent and do not cause pneumonia, hence the mice live.

7. Far red light has wavelengths of 700 nm and above which is only sufficient for cyclic phosphorylation, which only produces **ATP**.

8. **Glutamate dehydrogenase** is an enzyme that combines ammonium ions with α-ketoglutaric acid to convert it into glutamic acid through reductive amination.

9. Cutting the plasmid by all 4 enzymes as mentioned, we get: 220 bp between EcoRI and HindIII, 540 bp between HindIII and SalI, 900 bp between PstI and EcoRI, and 2340 bp between SalI and PstI.

10. The small intestine absorbs glucose from digested food material, while the renal tubules selectively reabsorb glucose from the renal filtrate.

11. P1 is a C_3 plant adapted to temperate areas having lower temperatures as it shows highest photosynthetic rate around 20-25 °C while P2 is a C_4 plant adapted to tropical areas which usually have high temperatures and humidity, hence the increased photosynthetic rate around 30-35 °C.

12. Symbol for: Zygomorphic flower - **%** Superior ovary - **$\underline{G}$**

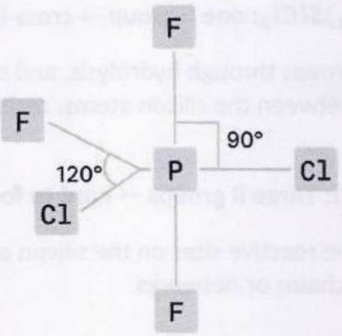
13. **X:** Homeotherms **Y:** Poikilotherms **a:** Small body size **b:** Large body size

Humming bird (Aves) and polar bear (Mammal) are warm-blooded (X), while crocodile (Reptile) and frog (Amphibian) are cold-blooded (Y). Humming bird and frog have a smaller body size (a) compared to crocodile and polar bear (b).

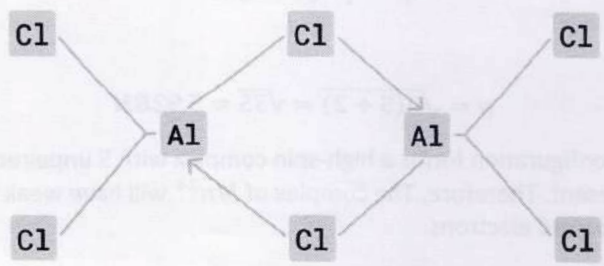
14. Herbivores consume plants, hence it is **predation**. The cuckoo lays its eggs in the crow's nest, relying on the crow to raise its young, making it a **parasite**. The clownfish gets protection from predators by living among the sea anemone's stinging tentacles, while the sea anemone is not affected, hence making this interaction **commensalism.** The cyanobacteria provide food through photosynthesis, and the fungus offers protection, which is a mutually beneficial relationship. Hence it is an example of **mutualism**.

15. **Synaptonemal complex** forms in the Zygotene stage, followed by appearance of **recombinant nodules** and crossing over in the Pachytene stage, terminalisation of synaptonemal complex and formation of **chiasmata** in the Diplotene stage, and finally formation of **meiotic spindle** in the Diakinesis stage.

16. According to Bent's rule, the most electronegative elements prefer to occupy the orbitals that has lesser s character. In the trigonal bipyramidal geometry, the axial bonds are pd hybridized and the equatorial bonds are sp^2 hybridized.



17. This compound is a dimer, consisting of two $AlCl_3$ units joined together through chlorine. $AlCl_3$ can form dimer through halogen bridging and exists as Al_2Cl_6 because it has vacant d-orbitals which can accommodate electron from chlorine atom. Each $AlCl_3$ unit forms a trigonal planar structure where the aluminum atom is at the center bonded to three chlorine atoms. Each Al-Cl bond consists of a pair of shared electrons.



The bond angles around the aluminum atoms in the dimer Al_2Cl_6 are slightly less than the ideal 120° due to the influence of the bridging chlorine atoms.

Bond Lengths:

Terminal Al-Cl Bonds: These bonds are shorter and stronger due to direct single bonds between aluminum and chlorine.

Bridging Al-Cl Bonds: These bonds are longer and weaker because the chlorine atoms are shared between two aluminum atoms.

Terminal Al-Cl bonds are shorter.

Bridging Al-Cl bonds are longer.

18. When one R group is present then, three Cl groups are eliminated by $3H_2O$ molecules leaving OH in place of Cl. Then it polymerizes to form the cross-linked silicone polymer. When two R groups are present, it polymerizes to form a straight chain.

$(CH_3)_2SiCl_2$: Two R group → straight chain

Each Si atom can link to two other Si atoms through the Cl atoms

$(CH_3)SiCl_3$: one R group → cross-linked

Three Cl atoms are replaced by OH groups through hydrolysis, and subsequent condensation reactions lead to the formation of cross-links between the silicon atoms, resulting in a branched-chain (cross-linked) silicone polymer

$(CH_3)_3SiCl$: Three R groups → hard to form polymer

Since polymerization requires multiple reactive sites on the silicon atom to link with other silicon atoms, $(CH_3)_3SiCl$ cannot easily form long chains or networks

Thus, the correct answer is:

$$\boxed{A.\,(CH_3)_2SiCl_2 \text{ and } (CH_3)SiCl_3}$$

19. Manganese (Mn) has an atomic number of 25. The divalent ion Mn^{2+} loses two electrons, resulting in the electronic configuration $[Ar]3d^5$.

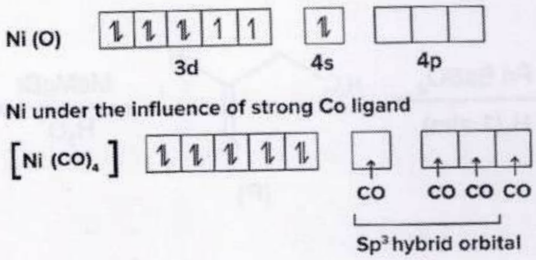
$$\mu = \sqrt{n(n+2)}BM$$

For $Mn^{2+}, n = 5$

$$\mu = \sqrt{5(5+2)} = \sqrt{35} \approx 5.92BM$$

Since Mn^{2+} with a $3d^5$ configuration forms a high-spin complex with 5 unpaired electrons, this suggests weak field ligands are present. Therefore, The complex of Mn^{2+} will have weak field ligands and it will be high-spin with five unpaired electrons.

20.



Nickel (Ni) has an atomic number of 28.

$$[Ar]3d^8 4s^2 \rightarrow [Ar]3d^{10} 4s^0$$

As CO is a SFL, this results in all $(3d)$ electrons being paired, making $Ni(CO)_4$ diamagnetic. Hence the option is correct.

21. Stability $\propto \frac{1}{\text{energy}}$

Eclipsed(IV): This is the highest-energy configuration due to the torsional strain caused by the alignment of substituents in close proximity, leading to increased steric hindrance and repulsion between electron clouds.

Gauche(II=III): This configuration is less energetic than the eclipsed form but still has some steric repulsion because the groups are close in a staggered arrangement, causing moderate torsional strain.

Antiperiplanar (I): This is the lowest-energy configuration. The groups are positioned in a staggered arrangement opposite each other, minimizing steric hindrance and torsional strain, making it the most stable of the three.

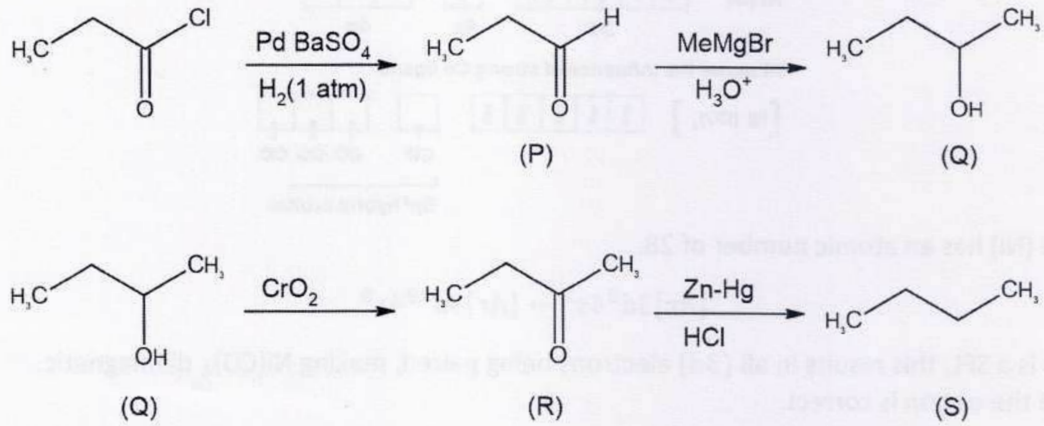
22.

O,P-directing groups					M-directing groups	
Very Strongly activating	Strongly activating	Moderately activating	Weakly activating	Deactivating	Very strongly Deactivating	Strongly Deactivating
+I & +M	-I << +M	-I < +M	+I < +M	+M< -I	-I & -M	-M(less)
-O-	$-NH_2$	-NHCOR	$-CH_3$	-Cl	$-NO_2$	-COOH
	-OH	-OCOR	$-CR_3$	-F	-CN	-COOR
	-OR	-OCOPh	$-CR_2$	-I	$-CO_3H$	-COOPh
	-NHR	-Ph	$-CH_2R$		-CHO	$-CF_3$
	$-NR_2$	$-CH=CH_2$			-COR	$-CCl_3$

So, if we consider the options:

at para position show -M and-I and at meta show -I OCH_3 at para position show +M and-I. so, it acts as an e-donating group.so, the option should be $II > III \approx I > IV$

23.

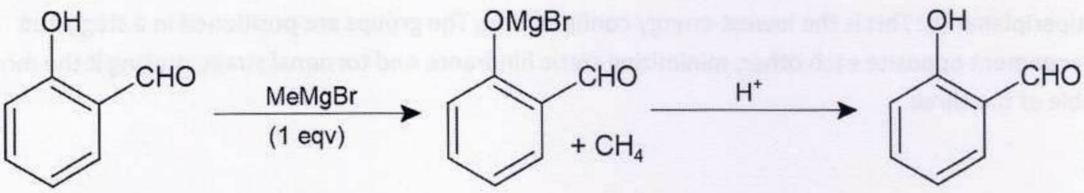


Since the P and R are aldehyde and ketone, they can form addition product. Q has one chiral C

R is a ketone can't undergo Cannizzaro's reaction

Since S is Butane, it is a saturated hydrocarbon

24.

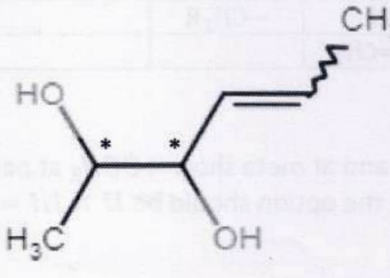


When salicylaldehyde reacts with MeMgBr, the Grignard reagent attacks the carbonyl carbon in the -CHO group, resulting in the formation of an alkoxide intermediate. This leads to:

In the second step, when we add H+, it protonates the alkoxide (O-MgBr) to form a hydroxyl group (-OH).

25. There are two carbon atoms attached to OH groups:

$$CH_3CH(OH)CH(OH)HC = CHCH_3$$



For a molecule with n chiral centers and m double bonds that can exhibit geometrical isomerism (cis/trans or E/Z), the total number of stereoisomers is given by: Total number of stereoisomers=$2^n \times 2^m = 2^{n+m}$

Each of these carbons is attached to four different groups, making them chiral centers. The double bond can exhibit cis/trans (E/Z) isomerism.

Each chiral center can have two configurations (R or S). With two chiral centers, the number of possible configurations is (2^2)=4.

Total number of stereoisomers=2(cis/trans)×4 (chiral centers)=8

26.

$$\Delta H = \Delta U + \Delta n_g RT$$

Where symbols gold their usual meanings

Given: - $\Delta H = -18.4$ kJ/mol $= -18400$ J/mol, $R = 8.314$ J/K · mol , $T = 27°\text{C} = 300$ K

- Equilibrium constants are given but are not needed for this calculation.

$$K_p = K_c(RT)^{\Delta n_g}$$

where: - $K_p = 9.98 \times 10^{27}$ - $K_c = 4.0 \times 10^{24}$

Using the relationship:

$$9.98 \times 10^{27} = 4.0 \times 10^{24}(8.314 \times 300)^{\Delta n_g}$$

$$9.98 \times 10^{27} = 4.0 \times 10^{24}(2494.2)^{\Delta n_g}$$

$$2494.2^{\Delta n_g} = 2495$$

So, $\Delta n_g \approx 1$.

Now we can calculate the internal energy change:

$$\Delta H = \Delta U + \Delta n_g RT$$

$$-18400 = \Delta U + (1 \times 8.314 \times 300)$$

$$-18400 = \Delta U + 2494.2$$

$$\Delta U = -18400 - 2494.2$$

$$\Delta U = -20894.2 \text{ J/mol}$$

$$\Delta U \approx -20.89 \text{ kJ/mol}$$

Therefore, the correct answer is: $\boxed{-20.89 \text{ kJ/mol}}$

27. S orbitals are spherically symmetric orbitals. Thus, the probability of finding the electron at a given distance is equal in all directions.

1s

The probability of finding an electron is maximum near the nucleus.

And, total number of nodes =n−l. For 1s-orbital, the number of nodes = 0. Thus, the probability density is maximum at the nucleus and decreases sharply with the distance from the nucleus.

28. The half-life of a zero-order reaction:

$$t_{1/2} = \frac{[A]_0}{2k}$$

-Direct Proportionality to Initial Concentration:

-Inverse Proportionality to Rate Constant

For first-order reactions, the half-life $(t_{1/2})$ is:

$$t_{1/2} = \frac{0.693}{k}$$

-Inverse Proportionality to Rate Constant

-Independence from Initial Concentration

29. Construct the half-reduction reaction for MnO_4^- to Mn^{2+} is:

$$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$$

This shows that 5 moles of electrons ($5e^-$) are required to reduce 1 mole of MnO_4^- to Mn^{2+}.

$$Q = n \times F$$

where n is the number of moles of electrons.

$$Q = 5 \times 96500 = 482500C$$

$$Q = I \times t \Rightarrow t = \frac{Q}{I}$$

Substituting the values of Q and I:

$$t = \frac{482500}{2.5} = 193000 \text{ s} \qquad t = \frac{193000}{3600} = 53.6 \text{ hours}$$

⇒ The time required for reducing 1 mole of MnO_4^- to Mn^{2+} when a current of 2.5 A is passed during electrolysis is 53.6 hours.

30. Analyze the relative stabilities and barriers to determine which energy diagram best corresponds to the reaction scheme.

$X \rightleftharpoons Y \rightleftharpoons Z$ with rate constants:

Given,

$$X \xrightarrow{4k} Y \text{ and } Y \xrightarrow{8k'} Z$$

$$X \xleftarrow{2k} Y \text{ and } Y \xleftarrow{3k'} Z$$

Analysis:

⋆ Higher rate constants indicate lower energy barriers.

Also, since both reactions will be exothermic in nature, the first peak must be higher than the second one and an overall downward graph should be seen.

Option (A) fits best

31. This question cannot be solved by simply differentiating, you will have to eliminate the options

We can easily eliminate 1, $\sqrt{2}$ by simply putting $x = 0$, which gives us the value of 2

Now to eliminate the rest we can use the following inequality:

$$\cos^4(x) \leq \cos(x)$$

We can write,

$$\cos^4(x) + \sin^2(x) + \cos(x) \leq \cos(x) + \sin^2(x) + \cos(x)$$

Now we can easily find the maximum value of the function $\cos(x) + \sin^2(x) + \cos(x)$ by differentiating and equating to zero:

$$-\sin(x) + 2\sin(x)\cos(x) - \sin(x) = 0$$

$$2\sin(x)\cos(x) = 2\sin(x)$$

$$\cos(x) = 1$$

The maximum value of $\cos(x) + \sin^2(x) + \cos(x)$ is 2

Then using the inequality, we can say that

$$\cos^4(x) + \sin^2(x) + \cos(x) \leq 2$$

So we can now also eliminate $2\sqrt{2}$

The final answer would be $\boxed{2}$

32. Given, Cards are numbered 1-10, to find the probability we need to consider case by case outcome(s) of each possible value:

- If $m = 2$, the possible values for n are $\{1\}$. (1 value)
- If $m = 4$, the possible values for n are $\{1,2,3\}$. (3 values)
- If $m = 6$, the possible values for n are $\{1,2,3,4,5\}$. (5 values)
- If $m = 8$, the possible values for n are $\{1,2,3,4,5,6,7\}$. (7 values)
- If $m = 10$, the possible values for n are $\{1,2,3,4,5,6,7,8,9\}$. (9 values)

Each number {2,4,6,8,10} is equally as likely(1/5) to occur, for each outcome we use theorem of total probability,

$$P(A) = P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2) \ldots P(E_5) \times P(A|E_5)$$

Here,

$$P(E_1) = P(E_2) = P(E_3) = P(E_4) = P(E_5) = \frac{1}{5}$$

$$P(A) = \frac{1}{5}[P(A|E_1) + P(A|E_2) + P(A|E_3) + P(A|E_4) + P(A|E_5)]$$

$$P(A|E_1) = \frac{1}{10} \qquad P(A|E_2) = \frac{3}{10} \ldots \qquad P(A|E_5) = \frac{9}{10}$$

$$P(A) = \frac{1}{5}\left[\frac{1+3+5+7+9}{10}\right] = \frac{25}{50} = \frac{1}{2}$$

33. Given, The points $(a,b), (a^2,b^2), (a^3,b^3)$ are all colliner, and not parallel to y-axis, i.e. $a \neq 1,0$ (**only** $\boldsymbol{a}$), because then all x(=1/0) coordinates will be the same and line will be parallel to y-axis)

⇒ the value of slope for any set of two points will be equal

using the formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{b^2 - b}{a^2 - a} = \frac{b^3 - b^2}{a^3 - a^2}$$

$$\frac{b^2 - b}{a^2 - a} = \frac{b(b^2 - b)}{a(a^2 - a)}$$

$$\frac{b}{a} = 1 \Rightarrow a = b$$

Also remember that b can be equal 0,1 and the remaining points can be parallel to x-axis. But in both cases the slope will be same i.e. $m = 0$

Then the value of $m = 0$ or for $b = 1 \; or \; 0, a = 0$ and $m = 1$ for $b = a = cons$ will be our **two solutions**

34. This is an exciting question; we have to establish a certain pattern that is not visible at first:

for $x \in [2,22]$, let us observe a few functions:

$$f(2) = \max\{n(1 - |2 - 2n - 1|)\} = \max\{n(1 - |1 - 2n|)\}$$

The maximum value here is 0 on $n = 1$, as on any higher value $(1 - |1 - 2n|)$ will be negative

Similarly,

$$f(3) = \max\{n(1 - |3 - 2n - 1|)\} = \max\{n(1 - |2 - 2n|)\}$$

Here, the maximum value will be at $n = 1 \to |2 - 2n| = 0$ and be equal to 1

$$f(4) = \max\{n(1 - |3 - 2n|)\} \to n = 2 \to \max = 0$$

$$f(5) = \max\{n(1 - |4 - 2n|)\} \to n = 2 \to \max = 2$$

...

$$f(21) = \max\{n(1 - |20 - 2n|)\} \to n = 10 \to \max = 0$$

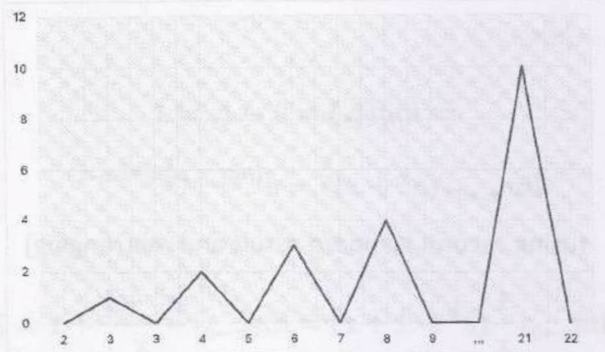
$$f(22) = \max\{n(1 - |21 - 2n|)\} \to n = 10 \to \max = 10$$

As you can see there is a pattern of:

$$\max\{f(even)\} = 0$$

$$\max\{f(odd)\} = \frac{(x-1)}{2}$$

We can now make the graph as follows:



Now we can quite literally see the pattern, the area enclosed would be $\frac{1}{2} \times b \times h$

The base for all triangles is 2 and now adding all areas:

$$\frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times 2 \times 2 \cdots \frac{1}{2} \times 2 \times 10 = 1 + 2 + 3 \cdots 10$$

$$= \frac{10}{2}(2 + 9) = \boxed{55}$$

35. Given limit,

$$lim_{x\to 0^+}\ (sinx)^{\sqrt{x}}(e^x+x)^{\frac{1}{x}}$$

$$lim_{x\to c}\big(f(x)\times \mathrm{g}(x)\big) = lim_{x\to c}\big(f(x)\big)\times lim_{x\to c}(\mathrm{g}(x))$$

Standard results:

$$i)\ lim_{x\to 0}\frac{e^x-1}{x}=1$$

$ii)$ For an indeterminate form of

$$lim_{x\to a}f(x)^{g(x)}=1^{\infty}$$

We can write:

$$lim_{x\to a}f(x)^{g(x)}=e^{lim_{x\to a}g(x)\times(f(x)-1)}$$

for the above limit let's evaluate as two limits:

1) $(sinx)^{\sqrt{x}}=t_1$

$$lim_{x\to 0^+}log(t_1)=lim_{x\to 0^+}\frac{ln(sinx)}{1/\sqrt{x}}$$

using L-Hospital's rule,

$$lim_{x\to 0^+}\frac{ln(sinx)}{1/\sqrt{x}}=lim_{x\to 0^+}\frac{(1/sinx)\times cosx}{-1/2(x^{-\frac{3}{2}})}=-\frac{2x}{sinx}\times\sqrt{x}\times cosx$$

Since, $lim_{x\to 0}\frac{x}{sinx}=1$

$$\Rightarrow log(t_1)=0\to t_1=1$$

$$lim_{x\to 0^+}(e^x+x)^{\frac{1}{x}}=e^{lim_{x\to 0^+}\frac{1}{x}(e^x+x-1)}$$

(using second standard result and rearranging)

$$lim_{x\to 0^+}\frac{1}{x}(e^x-1+x)=lim_{x\to 0^+}\frac{e^x-1}{x}+1=2$$

(using first standard result)

$$\Rightarrow lim_{x\to 0^+}(e^x+x)^{\frac{1}{x}}=e^2$$

On multiplying both results we get,

$$\boxed{lim_{x\to 0^+}\ (sinx)^{\sqrt{x}}(e^x+x)^{\frac{1}{x}}=e^2}$$

36. Given, $f(x) = e^{x^2/2} + \int_0^x t\, f(t)dt$

On differentiating both sides we get,

$$f'(x) = xe^{x^2/2} + \frac{d}{dx}\left(\int_0^x t\, f(t)dt\right)$$

Using Leibnitz theorem which states,

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f\,(t)dt = f\big(b(x)\big)b'(x) - f\big(a(x)\big)a'(x)$$

We get,

$$f'(x) = xe^{x^2/2} + xf(x)$$

$$\text{let } f(x) = y$$

$$y' = xe^{x^2/2} + xy$$

$$\frac{dy}{dx} - xy = xe^{x^2/2}$$

This is a first order linear differential eqn,

$$\frac{dy}{dx} + Py = Q$$

$$I.F. = e^{\int P dx} = e^{\int (-x)dx} = e^{\frac{-x^2}{2}}$$

$$y\left(e^{\frac{-x^2}{2}}\right) = \int \left(e^{\frac{-x^2}{2}}\right) \times \left(e^{\frac{x^2}{2}}\right) xdx$$

$$y\left(e^{\frac{-x^2}{2}}\right) = \int xdx$$

$$y\left(e^{\frac{-x^2}{2}}\right) = \frac{x^2}{2} + c$$

Using, $f(x) = e^{x^2/2} + \int_0^x t\, f(t)dt$

At $x = 0$

$$f(0) = e^0 + \int_0^0 t\, f(t)dt$$

$f(0) = 1$, putting $y(0) = 1$

$$1(e^0) = 0 + c \Rightarrow c = 1$$

So,

$$f(\sqrt{2}) = e^{\frac{\sqrt{2}^2}{2}}\left(\frac{\sqrt{2}^2}{2} + 1\right)$$

$$f(\sqrt{2}) = e(2)$$

$$e \approx 2.7 \rightarrow 2e \approx 5.4$$

Hence, $f(\sqrt{2}) \approx 5.4 \rightarrow 5 < f(\sqrt{2}) < 6$

37. let, $z = x\, e^{a-e^a} = t,$

the given equation becomes, $x^2 - 2tx + t$

For real roots, $D \geq 0$

$$(-2t)^2 - 4(1)(t) \geq 0$$

$$4t^2 - 4t \geq 0$$

$$4(t-1)t \geq 0$$

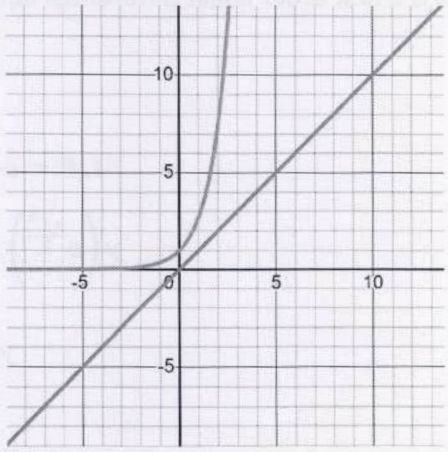
Using wavy curve method we get,

$$t \leq 0 \; or \; t \geq 1$$

For $t \leq 0$:

$e^{a-e^a} \leq 0$, this is not possible for any value of a

For $t \geq 1$:



$$e^{a-e^a} \geq 1$$

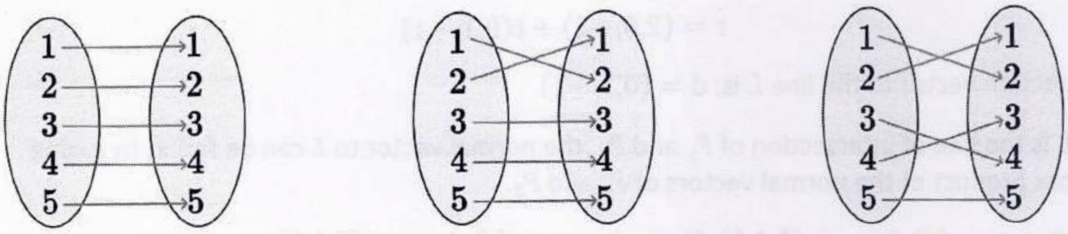
$$a - e^a \geq 0$$

$$a \geq e^a$$

This is also not possible for any value of a

38. $Each$ element n in the set $\{1,2,3,4,5\}$ must either: - Be a fixed point (i.e., $f(n) = n$). - Form a 2-cycle with another element m (i.e., $f(n) = m$ and $f(m) = n$).

For example, $f(1) = 2$ and $f(2) = 1$, while $f(3) = 3, f(4) = 4, f(5) = 5$



Case 1: All elements are fixed points

There is only one function in this case, which is the identity function: $f(n) = n$ for all n

Case 2: One 2-cycle, and 3 fixed points

Choose 2 out of 5 elements to form the 2-cycle. The number of ways to choose 2 elements out of 5 is: ${}^5C_2 = 10$, For each choice, there is exactly one way to define the 2-cycle.

Case 3: Two 2-cycles, and 1 fixed point

You can view this as a modified version of the second case in which instead of having three remaining points fixed, we have to make one 2 cycle out of 3 elements and do it for all possible cases in the second case.

$${}^3C_2 \times {}^5C_2 = 3 \times 10 = 30$$

Total number of involutions: $1 + 10 + 30 = 41$

39. $P_1: x + y + z = 1$ $P_2: 2x + y + z = 3$

Subtract the equation of P_1 from the equation of P_2:

$$(2x + y + z) - (x + y + z) = 3 - 1$$

$$\rightarrow x = 2$$

For $x = 2$, let's solve for y and z in terms of a parameter t.

From P_1:

$$2 + y + z = 1$$

$$y + z = -1$$

Let $y = t$, then $z = -1 - t$.

Thus, the parametric equations for the line L are:

$$x = 2, \quad y = t, \quad z = -1 - t$$

or

$$\mathrm{r} = (2,0,-1) + t(0,1,-1)$$

The direction vector of the line L is: $\mathrm{d} = (0,1,-1)$

Since L is the line of intersection of P_1 and P_2, the normal vector to L can be found by taking the cross product of the normal vectors of P_1 and P_2.

Normal vector of P_1 is $\mathrm{n}_1 = (1,1,1)$. Normal vector of P_2 is $\mathrm{n}_2 = (2,1,1)$.

Cross product:

$$\mathrm{n}_1 \times \mathrm{n}_2 = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \mathrm{i}(1\cdot 1 - 1\cdot 1) - \mathrm{j}(1\cdot 1 - 1\cdot 2) + \mathrm{k}(1\cdot 1 - 1\cdot 2)$$

$$= \mathrm{i}(0) - \mathrm{j}(-1) + \mathrm{k}(-1)$$

$$= \mathrm{j} - \mathrm{k} = (0,1,-1)$$

So, the normal vector to L is $\mathrm{n}_L = (0,1,-1)$, which aligns with the direction vector d.

The plane P has a normal vector $\mathrm{n}_L = (0,1,-1)$ and passes through the point $(1,2,1)$.

The equation of the plane can be written as:

$$0(x-1) + 1(y-2) - 1(z-1) = 0$$

$$y - 2 - (z-1) = 0 \; y - z - 1 = 0$$

Therefore, the equation of the plane P is $\boxed{y - z = 1}$.

40. I=Given, $f(x) = ln(1+x) for\, x \geq 0$

$$I = \int_0^{\frac{\pi}{2}} \frac{f(\sqrt[3]{cos\theta})}{f(\sqrt[3]{sin\theta}) + f(\sqrt[3]{cos\theta})} d\theta$$

but we do not need $f(x)$

using P_4(King's rule), which states: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

applying which we get,

$$\int_0^{\frac{\pi}{2}} \frac{f\left(\sqrt[3]{\cos\left(\frac{\pi}{2}-\theta\right)}\right)}{f\left(\sqrt[3]{\sin\left(\frac{\pi}{2}-\theta\right)}\right)+f\left(\sqrt[3]{\cos\left(\frac{\pi}{2}-\theta\right)}\right)}\, d\theta$$

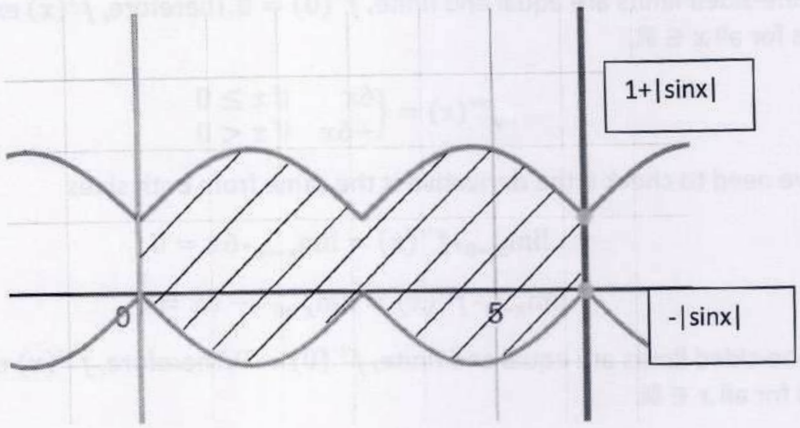
$$I = \int_0^{\frac{\pi}{2}} \frac{f(\sqrt[3]{sin\theta})}{f(\sqrt[3]{sin\theta})+f(\sqrt[3]{cos\theta})}\, d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{f(\sqrt[3]{cos\theta})+f(\sqrt[3]{sin\theta})}{f(\sqrt[3]{sin\theta})+f(\sqrt[3]{cos\theta})}\, d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\,\theta = \frac{\pi}{2}$$

$$\boxed{I = \frac{\pi}{4}}$$

41.



Since $|\sin x|$ is periodic with period π, we can simplify the calculation by finding the area for one period and then multiplying by the number of periods in the given range $[0,2\pi]$.

The total area A enclosed by these curves over the interval $[0,\pi]$ is given by the integral of the difference between the upper curve and the lower curve:

$$A_{\text{one period}} = \int_0^{\pi} [(1+\sin x)-(-\sin x)]\, dx = \int_0^{\pi} (1+2\sin x)\, dx$$

$$= \int_0^{\pi} 1\, dx + \int_0^{\pi} 2\sin x\, dx$$

$$= [x]_0^{\pi} + 2[-\cos x]_0^{\pi} = \pi + 2(-\cos(\pi)+\cos(0)) =$$

$$= \pi + 2(-(-1) + 1) = \pi + 4$$

Since the interval $[0,2\pi]$ includes two periods, the total area A_{total} is:

$$A_{\text{total}} = 2 \times (\pi + 4) = 2\pi + 8$$

Therefore, the area enclosed by the curves $y = 1 + |\sin x|$, $y = -|\sin x|$, and the lines $x = 0$ and $x = 2\pi$ is $\boxed{2\pi + 8}$ square units.

42. Given, $f(x) = x^2|x|$ which can be written as

$$f(x) = \begin{cases} x^3 & \text{if } x \geq 0 \\ -x^3 & \text{if } x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2 & \text{if } x \geq 0 \\ -3x^2 & \text{if } x < 0 \end{cases}$$

At $x = 0$, we need to check if the derivative is the same from both sides

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (3x^2) = 0$$

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} (-3x^2) = 0$$

Since the one-sided limits are equal and finite, $f'(0) = 0$.Therefore, $f'(x)$ exists and is continuous for all $x \in \mathbb{R}$.

$$f''(x) = \begin{cases} 6x & \text{if } x \geq 0 \\ -6x & \text{if } x < 0 \end{cases}$$

At $x = 0$,we need to check if the derivative is the same from both sides

$$\lim_{x \to 0^+} f''(x) = \lim_{x \to 0^+} 6x = 0$$

$$\lim_{x \to 0^-} f''(x) = \lim_{x \to 0^-} - 6x = 0$$

Since the one-sided limits are equal and finite, $f''(0) = 0$.Therefore, $f''(x)$ exists and is continuous for all $x \in \mathbb{R}$.

$$f'''(x) = \begin{cases} 6 & \text{if } x \geq 0 \\ -6 & \text{if } x < 0 \end{cases}$$

At $x = 0$,we need to check if the derivative is the same from both sides

$$\lim_{x \to 0^+} f'''(x) = \lim_{x \to 0^+} 6 = 6$$

$$\lim_{x \to 0^-} f'''(x) = \lim_{x \to 0^-} - 6 = -6$$

Since the one-sided limits are not equal and finite, $f'''(x)$ exists and is NOT continuous for all $x \in \mathbb{R}$.

43.

$$A^2 + A + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

This simplifies to:

$$A^2 + A = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A(A+1) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Converting to determinant

$$|A||A+1| = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = -1$$

$$\Rightarrow |A| \neq 0, |A+1| \neq 0$$

If A is not invertible, it means det(A)=0. ∴ Both A and A+I are invertible

44.

$$\begin{vmatrix} x^2+3 & x^2+4 & x^2+5 \\ x^2+4 & x^2+5 & x^2+6 \\ x^2+a & x^2+b & x^2+c \end{vmatrix}$$

Substituting $x = 0$ into the matrix to simplify to

$$\begin{vmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ a & b & c \end{vmatrix}$$

opening the determinant along first row, we get:

$$3(5c - 6b) - 4(4c - 6a) + 5(4b - 5a)$$

$$= 15c - 18b - 16c + 24a + 20b - 25a = -a - c + 2b$$

since a, b, c are in A.P., $2b = a + c$

$$\begin{vmatrix} x^2+3 & x^2+4 & x^2+5 \\ x^2+4 & x^2+5 & x^2+6 \\ x^2+a & x^2+b & x^2+c \end{vmatrix} = \boxed{0}$$

45. We can use logical reasoning about the properties of mean and variance to solve this problem without heavy calculations.

1) Relationship between the means

Consider the two data sets: $S_1 = \{1,2,4,8,9,11,15,20,27,29,33\}$

$$S_2 = \{51,52,54,58,59,61,65,70,77,79,83\}$$

Notice that each element in (S_2) is exactly 50 more than the corresponding element in (S_1). For example, $(51 = 1 + 50)$, $(52 = 2 + 50)$, and so on.

The mean of a data set is the sum of its elements divided by the number of elements. If we add a constant (in this case, 50) to each element in a data set, the mean of the new data set will be the mean of the original data set plus that constant.

Therefore: $m_2 = m_1 + 50$

2) Relationship between the variances

Variance measures how spread out the values in a data set are around the mean. It is calculated as the average of the squared differences from the mean.

If we add a constant to each element of a data set, the differences between each element and the mean do not change because both the elements and the mean have been increased by the same constant. Thus, the squared differences remain the same, and so does the variance.

Therefore: $v_2 = v_1$

Conclusion

Combining these two observations, we get: $m_2 = m_1 + 50$ $v_2 = v_1$

Thus, the correct answer is:

$$\boxed{m_2 = m_1 + 50, v_2 = v_1.}$$

46. The moment of inertia of the door about the hinge will be given by:

$$\frac{1}{3}ML^2$$

(You can easily derive this using the parallel axis theorem)

The angular momentum imparted on the door by the bullet will be given by:

$$mvr = mvx$$

This will be converted to the angular momentum given by the door:

$$mvx = I\omega = \frac{1}{3}ML^2\omega$$

$$\Rightarrow \omega = \frac{3mvx}{ML^2}$$

$$\boxed{\omega \propto x}$$

47. Eqn of SHM:

$$y = A\sin(\omega t) \qquad v = \frac{dy}{dx} = A\omega\cos(\omega t) \qquad a = \frac{dv}{dx} = -A\omega^2\sin(\omega t)$$

We can clearly see that, $a_{max} = A\omega^2$

Now let's arrange the terms wrt E, T, m

$$E = \frac{1}{2}kA^2 \qquad \star A = \sqrt{\frac{2E}{k}}$$

putting in expression,

$$a_{max} = \sqrt{\frac{2E}{k}}\omega^2$$

Now to eliminate k

$$\omega = \sqrt{\frac{k}{m}} \qquad \star \sqrt{k} = \omega \times \sqrt{m}$$

Again, putting in expression,

$$a_{max} = \sqrt{2E} \times \frac{\omega^2}{\omega} \qquad a_{max} = \sqrt{2E} \times \omega$$

Finally using,

$$\star\, \omega = \frac{2\pi}{T}$$

Putting in expression we get,

$$\boxed{a_{max} = \frac{2\pi\sqrt{2}}{T} \times \frac{E}{m}}$$

48. Let us break the motion down in 3 parts and analyze them individually

Phase-1) Acceleration Phase:

$u = 0$ m/s, $v = 9$ m/s, $t_1 = 2$ s

Using the equation $v = u + at$:

$$9 = 0 + a \cdot 2$$

$$\boxed{a = \frac{9}{2} = 4.5 \text{ m/s}^2}$$

Distance covered during acceleration, s_1, using $s = ut + \frac{1}{2}at^2$: $s_1 = 0 \cdot 2 + \frac{1}{2} \cdot 4.5 \cdot 2^2$

$$s_1 = \frac{1}{2} \cdot \frac{9}{2} \cdot 4$$

$$s_1 = 9 \text{ m}$$

Phase-2) Constant Speed Phase:

$v = 9$ m/s, t_2 (to be determined)

Phase-3) Deceleration Phase:

$u = 9$ m/s $v = 0$ m/s $a_d =$ deacceleration

Given that the magnitude of acceleration is twice the magnitude of deceleration:

$$a_d = \frac{9}{4} = 2.25 \text{ m/s}^2$$

Using the equation $v = u - a_d t_3$: $0 = 9 - \frac{9}{4}t_3$ $t_3 = \frac{9}{9/4} = 4\text{s}$

Now, the total time T is the sum of the times for each phase:

$$T = t_1 + t_2 + t_3$$

$$12 = 2 + t_2 + 4$$

$$t_2 = 12 - 6 = 6 \text{ s}$$

Distance covered during constant speed, s_2: $s_2 = v \cdot t_2 = 9 \cdot 6 = 54$ m

Distance covered during deceleration, s_3, using $s = ut - \frac{1}{2}a_d t^2$:

$$s_3 = 9 \cdot 4 - \frac{1}{2} \cdot \frac{9}{4} \cdot 4^2$$

$$s_3 = 36 - 9 \cdot 2$$

$$s_3 = 36 - 18 = 18 \text{ m}$$

Total distance covered: $s_{\text{total}} = s_1 + s_2 + s_3$

$$s_{\text{total}} = 9 + 54 + 18 = 81 \text{ m}$$

Thus, the total distance covered by the athlete is 81 meters.

49. The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

where symbols hold their usual meanings.

The linear expansion of a material is given by:

$$\Delta L = \alpha L \Delta T$$

For a two-dimensional object like a plate, both dimensions expand.

$$\Delta A = \beta A \Delta T$$

Where, $\beta = 2\alpha$

Which can also be shown by:

$$A' = [l(1 + \alpha\Delta T)]^2 = A(1 + \alpha\Delta T)^2$$

For small values of $\alpha\Delta T$, we can use the binomial approximation:

$$(1 + \alpha\Delta T)^2 \approx 1 + 2\alpha\Delta T$$

Thus, the new area A' becomes:

$$A' \approx A(1 + 2\alpha\Delta T)$$

The capacitance after the temperature change will be:

$$C' = \frac{\varepsilon_0 A'}{d} = \frac{\varepsilon_0 A(1 + 2\alpha\Delta T)}{d} = C(1 + 2\alpha\Delta T)$$

Therefore, the change in capacitance ΔC is:

$$\Delta C = C' - C\ \Delta C = C(1 + 2\alpha\Delta T) - C$$

$$\Delta C = C(2\alpha\Delta T)$$

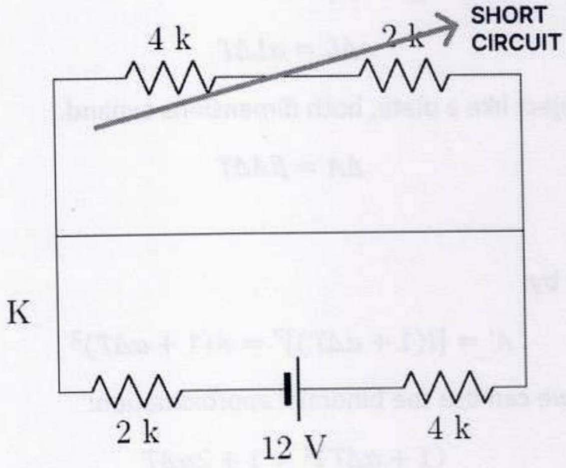
So, the change in capacitance when the temperature of the plates rises by ΔT is:

$$\boxed{\Delta C = 2C\alpha\Delta T}$$

50. It is given to us that the given circuit will operate for a long time,

This means that the capacitors will act as wires and short-circuit the above to resistances ($4k\Omega, 2k\Omega$) as they will have relative infinite resistance as compared to the capacitors which will have zero

The circuit can now be redrawn as following:



Now, the current in the circuit can be calculated as following:

$$R_{net} = 2k\Omega + 4k\Omega = 6k\Omega$$

$$I = \frac{V}{R_{net}} = \frac{12}{6 \times 10^3} = 2 \times 10^{-3}$$

Using Kirchoff's laws, the potential difference across the capacitors will be:

$$V' = V - ir = 12 - (2 \times 10^{-3})(6 \times 10^3) = 6V$$

$$C_{net} = \left(\frac{1}{2} + \frac{1}{4}\right)^{-1} \mu F = \frac{4}{3}\mu F$$

Then using $q = CV$,

$$q = 6 \times \frac{4}{3} \times 10^{-6} = 8\mu C$$

Since, the capacitors are in series the charge will be the same as the same amount of current flow exists throughout a series circuit as all the capacitors are being supplied with the same number or quantity of electrons.

The charge for both the capacitors after a long time will be the same and equal to

$$\boxed{8\mu C \text{ and } 8\mu C}$$

51. The Lorentz force law is given by:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Since the charge is moving with a constant velocity($\frac{dv}{dt} = 0, a = 0$), the net force acting on it must be zero. Hence,

$$\vec{E} + \vec{v} \times \vec{B} = 0$$

Given: $\vec{v} = v\hat{k}$

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

Let's calculate the cross product $\vec{v} \times \vec{B}$:

$$\vec{v} \times \vec{B} = (v\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath})$$

$(\hat{k} \times \hat{\imath} = \hat{\jmath}, \hat{k} \times \hat{\jmath} = -\hat{\imath})$

So,

$$\vec{v} \times \vec{B} = v(\hat{k} \times B_x\hat{\imath} + \hat{k} \times B_y\hat{\jmath})$$

$$\vec{v} \times \vec{B} = v(B_x\hat{\jmath} - B_y\hat{\imath})$$

$$\vec{v} \times \vec{B} = vB_x\hat{\jmath} - vB_y\hat{\imath}$$

$$\vec{E} + \vec{v} \times \vec{B} = 0$$

$$(E_x\hat{\imath} + E_y\hat{\jmath}) + (-vB_y\hat{\imath} + vB_x\hat{\jmath}) = 0$$

Equating the components:

For the $\hat{\imath}$ component:

$$E_x - vB_y = 0$$

$$E_x = vB_y$$

For the $\hat{\jmath}$ component: $E_y + vB_x = 0$ $E_y = -vB_x$

Therefore, the correct relations are:

$$E_x = vB_y, E_y = -vB_x$$

52. We need to calculate the potential energy due to all pairs of charges.

Let's denote the vertices of the tetrahedron as A, B, C, and D with charges $q_A = q, q_B = -2q, q_C = -3q, q_D = 4q$, respectively. The centre charge is $q_0 = 5q$.

Pairwise Interaction Energies

The electrostatic potential energy U_{ij} between two point charges q_i and q_j separated by a distance r_{ij} is given by: $U_{ij} = \frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}$

Energies between the vertices (pairs of charges at vertices):

Each pair of vertices is separated by a distance L. There are ${}^4C_2 = 6$ pairs.

$$U_{vertices} = U_{AB} + U_{AC} + U_{AD} + U_{BC} + U_{BD} + U_{CD}$$

$$= \frac{1}{4\pi\epsilon_0 L}(q_A \cdot q_B + q_A \cdot q_C + q_A \cdot q_D + q_B \cdot q_C + q_B \cdot q_D + q_C \cdot q_D)$$

$$U_{\text{vertices}} = \frac{1}{4\pi\epsilon_0 L}(-2q^2 - 3q^2 + 4q^2 + 6q^2 - 8q^2 - 12q^2)$$

$$U_{\text{vertices}} = -\frac{15q^2}{4\pi\epsilon_0 L}$$

Total energy between center and vertices:

Let the distance of the centroid from the edge=R The following expression will give the total electrostatic energy:

$$U_{net} = U_{A0} + U_{B0} + U_{C0} + U_{D0}$$

$$= \frac{q_A \times q_1}{4\pi\epsilon_0 R} + \frac{q_A \times q_2}{4\pi\epsilon_0 R} + \frac{q_A \times q_3}{4\pi\epsilon_0 R} + \frac{q_A \times q_4}{4\pi\epsilon_0 R}$$

$$= \frac{q_A}{4\pi\epsilon_0 R}(q_1 + q_2 + q_3 + q_4)$$

$$= \frac{5}{4\pi\epsilon_0 R}(1 - 2 - 3 + 4)$$

$$= 0$$

Thus, the total electrostatic energy of the system is: $\boxed{-\frac{15q^2}{4\pi\epsilon_0 L}}$

53. The decay process can be described by the exponential decay formula:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

where: - *N(t)* is the remaining quantity of the substance after time t. - N_0 is the initial quantity of the substance. - $T_{1/2}$ is the half-life of the substance. - t is the time elapsed.

Given: - Half-life $T_{1/2} = 2000$ hours. - We want to find the time t for the decay of 2/3 of the nuclei, meaning $\frac{1}{3}$ of the nuclei remain.

Thus, we set $N(t) = \frac{N_0}{3}$ and solve for t:

$$\frac{N_0}{3} = N_0 \left(\frac{1}{2}\right)^{\frac{t}{2000}}$$

$$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t}{2000}}$$

$$\ln\left(\frac{1}{3}\right) = \frac{t}{2000} \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{1}{2}\right)} \times 2000$$

$$t = \frac{-\ln(3)}{-\ln(2)} \times 2000 = \frac{\ln(3)}{\ln(2)} \times 2000$$

Using approximate values for the natural logarithms:

$\ln(3) \approx 1.09$ or $2.303 \times .477$ $\ln(2) \approx 0.69$ or $2.303 \times .301$

Thus:

$$t \approx \frac{1.0986}{0.6931} \times 2000 \approx 1.585 \times 2000 \approx 3170 \text{ hours}$$

54.

P	Q	1 XOR P	1 AND Q	(1 XOR P) AND (1 AND Q)	NAND (Result)
0	0	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	1	0	1	0	1

55.

$$E_{\text{photon}} = \phi + K.E.$$

where E_{photon} is the energy of the incident photon, ϕ is the work function of the metal, and $K.E.$ is the kinetic energy of the emitted electron.

Given Condition:

$$E_{\text{photon}} = 5\phi$$

Using the photoelectric effect equation:

$$5\phi = \phi + K.E.$$

$$K.E. = 5\phi - \phi = 4\phi$$

The de Broglie wavelength λ_B of the emitted electron is related to its momentum p by:

$$\lambda_B = \frac{h}{p}$$

The kinetic energy $K.E.$ of the electron is also given by:

$$K.E. = \frac{1}{2}\frac{(m_e v)^2}{m_e} = \frac{p^2}{2m_e}$$

Solving for p:

$$p = \sqrt{2m_e K.E.}$$

Substituting p in the de Broglie wavelength equation:

$$\lambda_B = \frac{h}{\sqrt{2m_e K.E.}}$$

Substituting $K.E. = 4\phi$:

$$\lambda_B = \frac{h}{\sqrt{8m_e\phi}}$$

Solve for ϕ: Rearrange the equation to solve for the work function ϕ:

$$\lambda_B\sqrt{8m_e\phi} = h$$

$$\lambda_B^2 \cdot 8m_e\phi = h^2$$

$$\phi = \frac{h^2}{8m_e\lambda_B^2}$$

56. To determine the change in detected frequency by Sunanda as she crosses the ambulance, we need to use the Doppler effect formulas for sound.

Determine the frequency detected by Sunanda as she approaches the ambulance:

When Sunanda is moving towards the ambulance and the ambulance is moving towards her, the observed frequency f' can be calculated using the formula:

$$f' = f\left(\frac{v + v_o}{v - v_s}\right)$$

where:

f is the emitted frequency (540 Hz)

v is the speed of sound in air (340 m/s)

v_o is the speed of the observer (Sunanda's car) towards the source (20 m/s)

v_s is the speed of the source (ambulance) towards the observer (20 m/s)

Putting in the values:

$$f' = 540\left(\frac{340 + 20}{340 - 20}\right) = 540\left(\frac{360}{320}\right) = 540\left(\frac{9}{8}\right) = 607.5 \text{ Hz}$$

Determining the frequency detected by Sunanda as she moves away from the ambulance:

When Sunanda moves away from the ambulance after crossing it, the observed frequency f'' can be calculated using the formula:

$$f'' = f\left(\frac{v - v_o}{v + v_s}\right)$$

where the terms are as previously defined, but now v_o is msoving away from the source.

Putting in the values:

$$f'' = 540\left(\frac{340 - 20}{340 + 20}\right) = 540\left(\frac{320}{360}\right) = 540\left(\frac{8}{9}\right) = 480 \text{ Hz}$$

Calculating the change in detected frequency:

The change in detected frequency f is the difference between the frequency detected as she approaches and the frequency detected as she moves away:

$$\Delta f = f' - f'' = 607.5 \text{ Hz} - 480 \text{ Hz} = 127.5 \text{ Hz}$$

Therefore, the change in detected frequency by Sunanda as she crosses the ambulance is 127.5 Hz.

57. Formula for the intensity I at a point where the path difference is δ is given by:

$$I = I_0 \cos^2\left(\frac{\pi\delta}{\lambda}\right)$$

Given: Wavelength, $\lambda = 600$ nm, Path difference for the maximum intensity, $\delta_0 = 600$ nm, Path difference for the point 2, $\delta = 100$ nm.

First, let's calculate the phase difference ϕ corresponding to the given path difference $\delta = 100$ nm:

$$\phi = \frac{2\pi\delta}{\lambda}$$

$$\phi = \frac{2\pi \cdot 100\,10^{-9}}{600\,10^{-9}} = \frac{\pi}{3}$$

Now, using the intensity formula:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Since $\phi = \frac{\pi}{3}$:

$$I = I_0 \cos^2\left(\frac{\pi}{3}/2\right) = I_0 \cos^2\left(\frac{\pi}{6}\right)$$

Therefore, the intensity I at the point where the path difference is 100 nm is:

$$I = I_0 \cdot \frac{3}{4}$$

So, the intensity at the point where the path difference is 100 nm is $\frac{3}{4} I_0$.

58. We can use the following formulas from kinetic theory of gases:

RMS Speed:

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

$$v_{rms} \propto \frac{1}{m}$$

Since N_2 has a lower molar mass.

$$\boxed{v_{rms}(N_2) > v_{rms}(O_2)}$$

Average Kinetic Energy:

$$\langle KE \rangle = \frac{3}{2} k_B T$$

where k_b is the Boltzmann constant

The average kinetic energy of both O_2 and N_2 molecules is the same because it depends only on the temperature, not the type of gas.

59. To compare the final equilibrium temperature T_{eq} to the arithmetic mean of the initial temperatures, $\frac{T_A+T_B}{2}$, let's consider how the specific heat capacity $c(T)$ affects the heat exchange process.

Note:

If the specific heat capacity c were constant (independent of temperature), the final equilibrium temperature would be exactly the arithmetic mean:

$$T_{\text{eq}} = \frac{T_A + T_B}{2}$$

This result follows directly from the fact that the heat lost by object A would exactly equal the heat gained by object B, with no variation in specific heat.

But here we are given increasing Specific Heat Capacity with Temperature

When $c(T)$ increases with temperature, more heat is required to change the temperature of the hotter object A by a certain amount than is required to change the temperature of the cooler object B by the same amount.

Due to this, we can make the following observations:

Heat Loss from A:

Since $c(T)$ increases with temperature, the hotter object A loses heat less efficiently as its temperature decreases. Thus, the integral $\int_{T_A}^{T_{\text{eq}}} c\,(T)\,dT$ is larger than it would be if $c(T)$ were constant.

Heat Gain by B:

Conversely, the cooler object B gains heat more efficiently as its temperature increases. Thus, the integral $\int_{T_B}^{T_{\text{eq}}} c\,(T)\,dT$ is smaller than it would be if $c(T)$ were constant.

Comparison to Arithmetic Mean:

Given that the specific heat capacity increases with temperature, T_{eq} will be **closer to T_A than** T_B. As a result:

$$\boxed{T_{\text{eq}} > \frac{T_A + T_B}{2}}$$

This inequality indicates that the equilibrium temperature T_{eq} is greater than the simple average of the initial temperatures $\frac{T_A+T_B}{2}$. This is due to the fact that the hotter object A "resists" cooling down more strongly than the cooler object B "resists" heating up, which skews the equilibrium temperature towards the higher initial temperature T_A.

60. **Planck's Constant(h)**

Planck's constant (h) is given by the equation: $E = h\nu$ (where (E) is energy and ν is frequency)

$$[E] = \frac{1}{2}[mv^2] = [M(LT^{-1})^2] = [ML^2T^{-2}]$$

Frequency ν has dimensions: $[\nu] = [T]^{-1}$ Dimensions of Planck's Constant (h) are:

$$[h] = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

Charge of an Electron (e)

Just remember that charge is not a fundamental unit but current is:

$$Q = I \cdot T$$

The dimensions of (e) (or Q) are:

$$[e] = [Q] = [IT]$$

Electrical Resistance (R)

Electrical resistance (R) is defined by Ohm's law:

$$V = I \cdot R$$

Dimensions of Electric Potential (V): The dimensions of electric potential are:

$$[V] = \frac{[E]}{[Q]}$$

Using the dimensions of energy [E] and charge [Q]

$$[V] = \frac{[ML^2T^{-2}]}{[IT]} = [ML^2T^{-3}I^{-1}]$$

$$\boxed{[R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}I^{-1}]}{[I]} = [ML^2T^{-3}I^{-2}]}$$

We can clearly see that for I^{-2} to be present we must have $\frac{1}{e^2}$

$$\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[IT]^2} = [ML^2T^{-3}I^{-2}]$$