IAT 2020

Biology

1. Which of the following root modifications is observed in maize?

A. Fibrous roots B. Lateral root C. Prop roots D. Stilt

2. Which of the following chemical intermediates are products of light in photosynthesis?

A. ATP and NADPH
B. H_2O and NADPH
C. CO_2 and NADPH
D. CO_2 and ATP

3. Which of the following organisms produce Cry proteins?

A. Bacillus thuringiensis
B. Saccharomyces cerevisiae
C. Escherichia coli
D. Pseudomonas syringe

4. A hypothetical plant species shows two flower colour phenotypes- red and white. Red is dominant over white. In a population, this flower colour locus was found to be in Hardy-Weinberg equilibrium. If 490 out of a total population size of 1000 plants were found to be white-flowered, how many of the red-flowered plants will be heterozygous?

A. 700 B. 90 C. 580 D. 420

5. Troponin controls binding of actin to myosin motor, thereby regulating muscle contraction. Binding of which metal ions to troponin is crucial for this purpose?

A. Na B. K C. Mg D. Ca

6. The following word diagram depicts a food chain in a forest.

Trees (Aspen, willow) → Herbivore (Elk) → Predator (Wolf)

Which is the most likely outcome of depletion in predator population?

A. Increased browsing by the herbivores stimulates the plant growth leading to increased primary productivity

B. Overgrowth of herbivore population causing reduction in tree populations and therefore, if the process continues the entire ecosystem will breakdown

C. No change in the ecosystem as the herbivore and the tree populations will continue to maintain equilibrium

D. Overgrowth of herbivore population without any change in the remaining part of the ecosystem

7. Which of the following is the correct description of interferon?

A. A synthetic neurotransmitter used to treat neuronal diseases

B. A class of proteins secreted by B-lymphocytes that categorically attack and destroy virus-infected cells

C. Signalling molecules that interfere with its own synthesis inside a secretory cell

D. A class of proteins secreted by virus infected cells and are involved in protecting uninfected cells

8. Which of the following products are expected when lipase acts on diglycerides?

A. Fatty acids and glucose

B. Glucose and galactose

C. Fatty acids and glycerol

D. Monoglycerides and glycerol

9. Radial symmetry is NOT found in which of the following taxa?

A. Echinodermata

B. Cnidaria

C. Ctenophora

D. Cyclostomata

10. Reabsorption of which of these molecules in kidney occurs via passive transport?

A. Urea

B. Glucose

C. Sodium ions

D. Amino acids

11. Match the human cell types or (column I) to the number of chromatids (column II)

Column I	Column II
(a) G_2 phase cells	(i) 46
(b) G_1 phase cells	(ii) 92
(c) Spermatogonia (pre-replication)	(iii) 23
(d) Ovum	

A. a-ii, b-i, c-iii, d-iii

B. a-ii, b-i, c-i, d-iii

C. a-ii, b-iii, c-i, d-iii

D. a-i, b-ii, c-iii, d-iii

12. Which of the following processes is NOT involved in functionalisation of mRNA (from hnRNA) in eukaryotes?

A. Amino acylation

B. 5′-end capping

C. Polyadenylation

D. Splicing

13. Which of the following processes increases the uptake of water and minerals by roots in legumes?

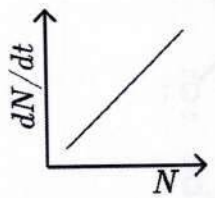
A. Active uptake by xylem

B. Pumping through plasmodesmata

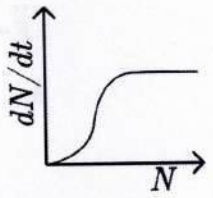
C. Pumping through cortical cells

D. Formation of mycorrhizae

14. Which of the following graphs correctly represents the relationship between population growth rate (dN/dt) and population size (N), if the population shows logistic growth?



A.



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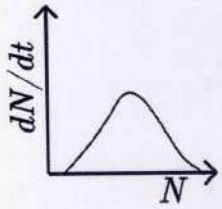

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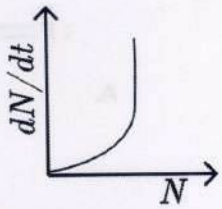


D.

15. Which of the following processes gives rise to new organisms from unfertilized ova in some species of plants and animals?

A. Autogamy and parthenogenesis

B. Parthenogenesis and apomixis

C. Syngamy and apomixis

D. Xenogamy and autogamy

Chemistry

16. How many atoms are present in a hexagonal close-packed unit cell?

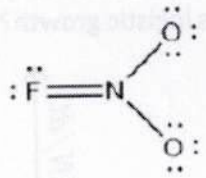
A. 4 B. 2 C. 8 D. 6

17. The molar conductivity of $0.00241\ M$ acetic acid is $32.87\ Scm^2mol^{-1}$. If the limiting molar conductivity at $298K$ is $390.5\ Scm^2mol^{-1}$, what is the dissociation constant ($mol\ L^{-1}$)?

A. 5.36×10^{-5} B. 1.86×10^{-5} C. 5.36×10^{-4} D. 1.86×10^{-4}

18. What is the most appropriate Lewis structure of NO_2F?

A.



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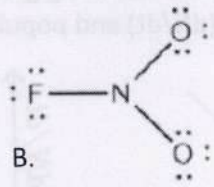

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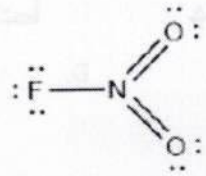
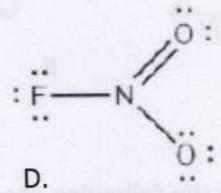


D.



19. Common disaccharide sucrose is held together by a glycosidic linkage between

A. C1 of β-D-glucose and C2 of α-D-fructose.

B. C2 of α-D-glucose and C1 of β-D-fructose.

C. C1 of α-D-glucose and C2 of β-D-fructose.

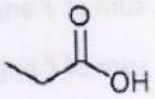
D. C1 of α-D-glucose and C2 of α-D-fructose.

20. Select the correct option based on the increasing order of boiling point of the molecules given below.

I

II

III



IV

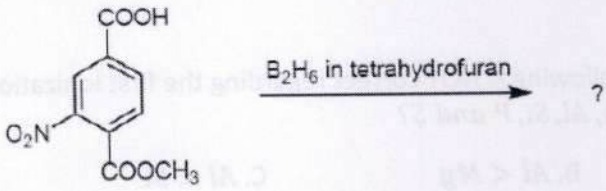
A. $I < III < IV < II$

B. $II < I < IV < III$

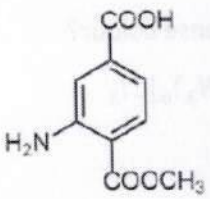
C. $I < III < II < IV$

D. $III < I < II < IV$

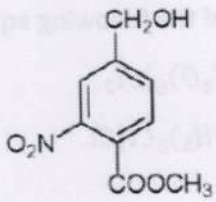
21. Identify the major product that forms easily and faster in the following reaction. (Tetrahydrofuran= solvent)



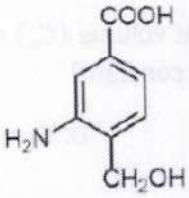
A.



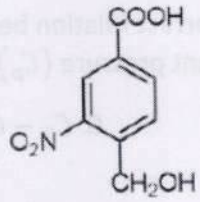
B.



C.



D.



22. A first-order reaction is found to have a half-life of one second (sec). What will be the time required for 99.9% completion of the reaction?

A. 0.693 sec B. 2 sec C. 10 sec D. 69.3 sec

23. The total number of nodes for a given principal quantum number n, and azimuthal quantum number ℓ, are

A. sum of ℓ angular nodes and $n + \ell - 1$ radial nodes

B. sum of ℓ angular nodes and $n - \ell - 1$ radial nodes

C. sum of $\ell + 1$ angular nodes and $n - \ell - 1$ radial nodes

D. sum of $\ell + 1$ angular nodes and $n + \ell - 1$ radial nodes

24. Which among the following is NOT correct?

A. H_3PO_4 is a tribasic acid

B. H_3PO_3 is a dibasic acid.

C. H_3PO_2 is a monobasic acid.

D. H_3PO_2 is a tribasic acid

25. Which among the following is NOT correct regarding the first ionization potential (IP_1) of period III elements Mg, Al, Si, P and S?

A. $P < S$

B. $Al < Mg$

C. $Al < Si$

D. $Si < P$

26. Which one of the following aqueous solutions gives the most intense colour?

A. 0.1 M $[Mn(H_2O)_6]Cl_2$

B. 0.1 M $[Mn(NH_3)_6]Cl_3$

C. 0.1 M $[Mn(NH_3)_5Cl]Cl$

D. 0.1 M $KMnO_4$

27. What is the correct relation between the heat capacity at constant volume (C_v) and heat capacity at constant pressure (C_p) for an ideal gas? (R=Universal gas constant)

A. $C_v + C_p = 2R$

B. $C_P - C_V = R$

C. $C_v - C_p = 2R$

D. $C_v + C_p = R$

28. The number of unpaired electron(s) in the square planar complex $[Ni(CN)]^{2-}$ is/are

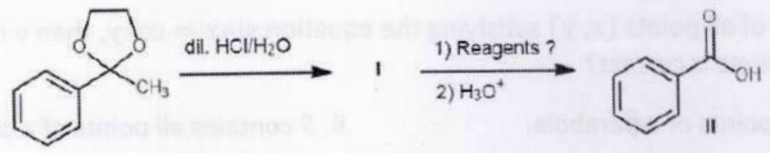
A. 2

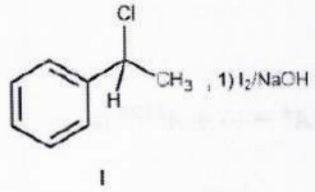
B. 1

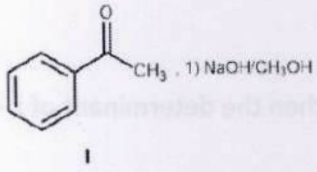
C. 4

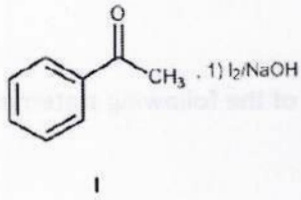
D. 0

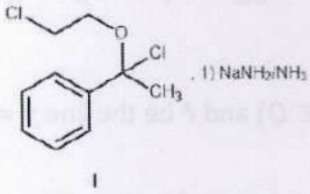
29. Select the most appropriate compound I and the reagents for the synthesis of compound II



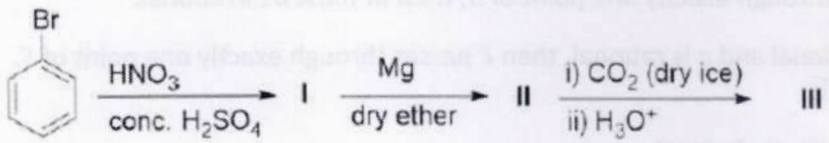
A. 

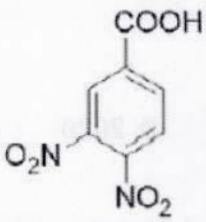
B. 

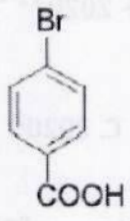
C. 

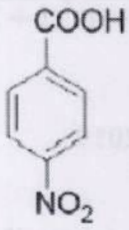
D. 

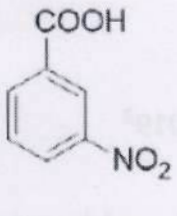
30. Identify the final major product III in the following reaction sequence:



A. 

B. 

C. 

D. 

Mathematics

31. If S is the set of all points (x, y) satisfying the equation $\sin x = \cos y$, then which of the following statements is correct?

A. S contains all points of a parabola.

B. S contains all points of a circle.

C. S contains all points of a hyperbola.

D. S contains all points of a straight line.

32. If $A = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$, then the determinant of $I - A + A^2 - A^3 + A^4 - \cdots + A^{2020}$ is

A. 2020 B. $a^{2020} - ba^{2019} + \cdots - b^{2019}a + b^{2020}$ C. 2020^3 D. 1

33. Let $S = \{(a, b): a, b \in Q\}$ and ℓ be the line $y = mx + c$. Which of the following statements is not correct?

A. If ℓ passes through two points of S, then m must be rational.

B. If m is rational and c is irrational, then ℓ passes through at least one point of S.

C. If ℓ passes through exactly one point of S, then m must be irrational.

D. If m is irrational and c is rational, then ℓ passes through exactly one point of S.

34. If $p(t) = \frac{t(t-1)\cdots(t-2019)}{2019!}$, then the value of

$$\int_0^1 \left(\frac{1}{t+1} + \frac{1}{t+2} + \cdots + \frac{1}{t+2020}\right) p(-t-1)dt$$

is:

A. 2019^2 B. 2019 C. 2020^2 D. 2020

35. Let a, b be nonzero real numbers. If $p(x) = x^n + c_{n-1}x^{n-1} + \cdots + c_0$, where $c_{n-1}, \ldots, c_0$ are integers and $n \geq 3$, then which of the following statements is correct?

A. If $a + ib$ is a root of $p(x)$, then n must be even.

B. If a is a root of $p(x)$, then n must be odd.

C. If $a + ib$ is a root of $p(x)$, then $(a + ib)^2$ can never be a root of $p(x)$.

D. If a is a root of $p(x)$, then a must be an integer.

36. The number of skew-symmetric matrices $A = [a_ij]_{3\times3}$, where $a_ij \in \{-3,-2,-1,0,1,2,3\}$ is:

A. 7^3 B. 3^7 C. 21^3 D. 7^6

37. Define binary operation $*$ on Q by $a * b = a + b - 3$. The inverse of 2 with respect to $*$ is

A. 4 B. 2 C. 3 D. 5

38. Let $[x]$ denote the greatest integer less than or equal to x. The number of positive integer solutions of the equation

$$\left[\frac{x}{19}\right] = \left[\frac{x}{20}\right]$$

are:

A. 190 B. 188 C. 189 D. 187

39. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be two functions such that

$$g \circ f(x) = \begin{cases} x^2, & \text{if } x \geq 0, \\ e^x - 1, & \text{if } x < 0. \end{cases}$$

Then which of the following statements is correct?

A. g is onto. B. f is onto. C. g is one-one. D. f is one-one.

40. $F(x) = \int_0^{e^x} (t^3 + 2t^2 - t - 2)\ dt$, then for how many real numbers x does $F'(x) = 0$?

A. 0 B. 3 C. 2 D. 1

41. Let C be a circle which passes through (-2, 1) and (0, 3) and whose centre lies on the line $y - x = 1$. Then which of the following points lies on C?

A. $(\sqrt{2}+1,1)$ B. $(1,\sqrt{2}+1)$ C. $(1,\sqrt{3}+1)$ D. $(1,\sqrt{3}-1)$

42. For what value of t does $f(x) = x^3 + tx^2 + (2-t)x - 3$ have only real roots?

A. $t = 0$. B. $t = 4$. C. $t = 2$. D. $t = -1$

43. If $f(x) = ax^2 + bx + c$ and $f\left(\frac{1}{n}\right) = \frac{n+1}{n^2}$ for all $n \in N$, then what is the value of $lim_{x\to 0} f'(x)$?

A. 2 B. 0 C. 1 D. -1

44. A bag contains 8 balls, of which 5 are green and 3 are red. The probability that two balls chosen at random are of the same colour is:

A. 12/28 B. 14/28 C. 15/28 D. 13/28

45. Consider the differential equation $f'' + pf' + qf = 0$ where p and q are constants, $p^2 = 4q$ and $q \neq 0$. If y is a nonzero solution of the equation, then which of the following statements is correct?

A. $(1 + x)y$ can never be a solution

B. y^2 can never be a solution

C. y' can never be a solution

D. xy can never be a solution

Physics

46. The potential energy of a point particle of mass m undergoing rectilinear motion along the x-axis is given by

$$V(x) = Ax + Bx^2$$

What is the maximum speed attained by the particle if it starts from rest at $x = \frac{A}{B}$?

$A. \frac{3A}{\sqrt{2Bm}}$

$B. \frac{A}{\sqrt{2Bm}}$

$C. \frac{2A}{\sqrt{Bm}}$

$D. A\sqrt{\frac{2}{Bm}}$

47. Two identical containers kept at the same temperature carry the same gas with different molecular mean free paths l_1 and l_2. The gas from the first container is completely transferred into the second at the same temperature. What is the molecular mean free path of the resultant mixture in the second container?

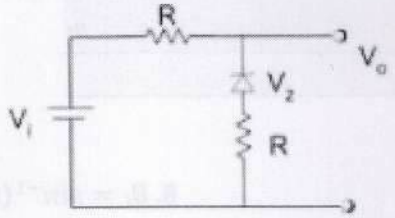
$A. \frac{l_1 l_2}{l_1 + l_2}$

$B. \sqrt{l_1 l_2}$

$C. l_1 + l_2$

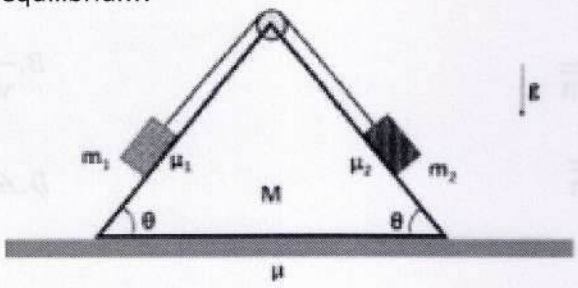
$D. \sqrt{l_1^2 + l_2^2}$

48. Consider the circuit with a Zener diode whose breakdown voltage $V_z = 4$ V. What is the voltage V_0 if $V_i = 20$ V?



A. 8V B. 12V C. 10V D. 16V

49. Consider a mass-pulley system as shown in the figure. There is a wedge of mass M and equal wedge angles θ lying on a rigid horizontal table. The coefficient of friction between the wedge and the table is μ. There are two blocks of mass m_1 and m_2 lying on the incline of the wedge. The coefficients of friction between the blocks and wedge are μ_1 and μ_2 as shown in the figure. Consider $m_1 > m_2$ and the coefficients of friction (μ, μ_1 and μ_2) to be less than $\tan\theta$. Gravity is acting downwards with acceleration due to gravity g. What should be the value of $\frac{m_1}{m_2}$ so that the system is in equilibrium?



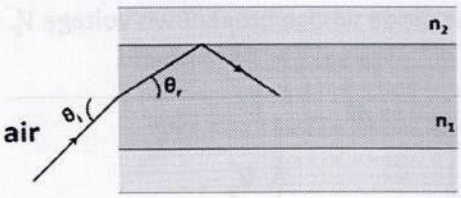
A. $\frac{\mu_1\mu_2 cos\theta + sin\theta}{sin\theta - \mu cos\theta}$

B. $\frac{\mu_1 cos\theta + sin\theta}{sin\theta - \mu_2 cos\theta}$

C. $\frac{\mu_2 cos\theta + sin\theta}{sin\theta - \mu_1 cos\theta}$

D. $\frac{\mu cos\theta + sin\theta}{sin\theta - \mu_1 cos\theta}$

50. What is the maximum angle of incidence θ_i in the figure for which the ray would satisfy the condition for total internal reflection at the boundary of media of refractive indices n_1 and n_2, where $n_1 > n_2$?



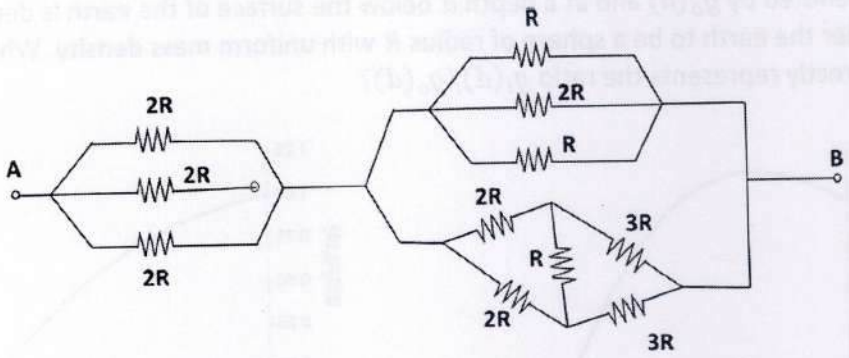
A. $\theta_i = sin^{-1}(n_1^2 - n_2^2)$

B. $\theta_i = sin^{-1}(n_1 - n_2)^2$

C. $\theta_i = sin^{-1}\left(\sqrt{\frac{n_2}{n_1}}\right)$

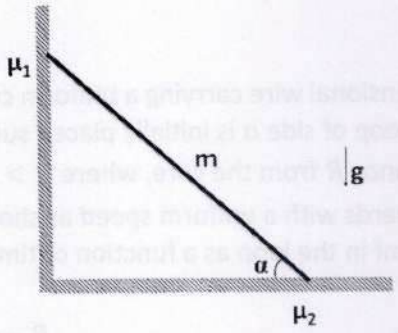
D. $\theta_i = sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$

51. What is the equivalent resistance between the points A and B in the figure?



A. $39R/10$ B. $39R/29$ C. $68R/87$ D. $87R/68$

52. Consider a solid rod of mass m and uniform density resting against a vertical wall and horizontal floor as shown in the figure. The coefficients of friction of the rod with the wall and with the floor are given to be μ_1 and μ_2 respectively. Gravity is acting downwards with acceleration due to gravity g. What should be the value of the inclination angle α so that the rod stays in equilibrium?



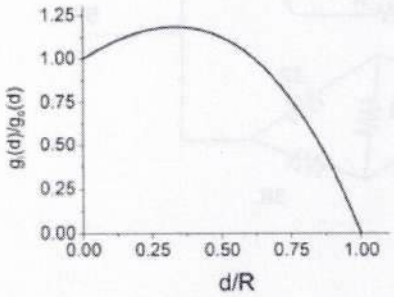
$A.\tan^{-1}\left(\frac{\mu_1}{\mu_2}\right)$ $B.\tan^{-1}\left(\frac{1-\mu_2}{2\mu_1\mu_2}\right)$ $C.\tan^{-1}\left(\frac{1-\mu_1\mu_2}{2\mu_2}\right)$ $D.\tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

53. The charge distribution inside a sphere of radius $R = 2a/b$ is given by the volume charge density $\rho = ar^2 - br^3$, where a and b are constants and r is the radial distance from the center of the sphere. At what distance from the center of the sphere, will the electric field go to zero?

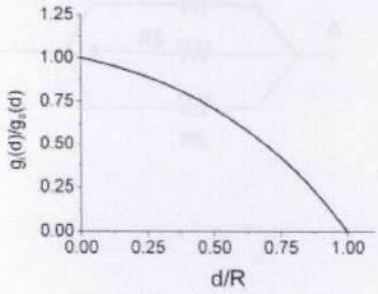
A. $4a/3b$ B. $3a/2b$ C. a/b D. $6a/5b$

54. The acceleration due to earth's gravity on a point particle at a height h above the surface of the earth is denoted by $g_o(h)$ and at a depth d below the surface of the earth is denoted by $g_i(d)$. Consider the earth to be a sphere of radius R with uniform mass density. Which of the following correctly represents the ratio $g_i(d)/g_o(d)$?

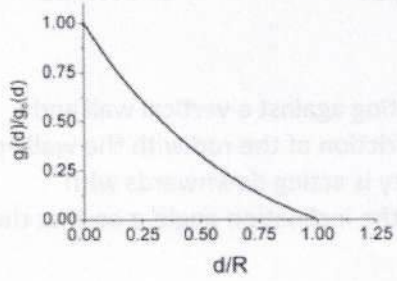
A.



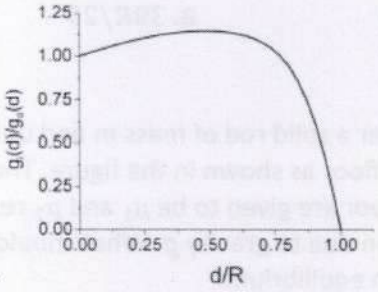
B.



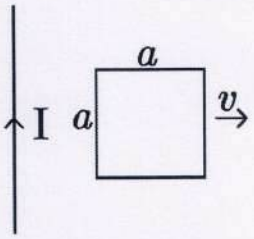
C.



D.



55. Consider an infinite one-dimensional wire carrying a uniform current I as shown in the figure. A square loop of side a is initially placed such that the center of the loop is at a distance R from the wire, where $R > \frac{a}{2}$. The square loop is then moved rightwards with a uniform speed as shown in the figure. What is the induced emf in the loop as a function of time t?



A. $\dfrac{\mu_0 I a^2 v}{2\pi\left[(R+vt)^2-\frac{a^2}{4}\right]}$

B. $\dfrac{3\mu_0 I a^2 v}{4\pi\left[(R+vt)^2-\frac{a^2}{4}\right]}$

C. $\dfrac{\mu_0 I a^2 v}{4\pi\left[(R+vt)^2-\frac{a^2}{4}\right]}$

D. $\dfrac{3\mu_0 I a^2 v}{2\pi\left[(R+vt)^2-\frac{a^2}{4}\right]}$

56. The electric field of an electromagnetic wave is given by $\vec{E}(x,t) = E_0\hat{z}\cos(ax+bt)$

If c is the speed of light, then the value of b is

A. ac B. $2\pi ac$ C. $\sqrt{2}ac$ D. $\frac{ac}{2\pi}$

57. An electron of mass m_e has a speed u_n in the n^{th} Bohr orbit of a hydrogen atom. The normalized speed V_n is given as $V_n = u_n/c = 1/685$ and the mass of the electron in the units of energy is given as $M_e = m_e c^2 = 0.51 \times 10^6$ eV, where c is the velocity of light in a vacuum. It is given that $\frac{e^2}{2\epsilon_0 hc} = 1/137$. The Planck's constant h is given as 4.13×10^{-15} eV-sec. The electron makes a transition from n^{th} orbit to the ground state of the atom. What is the frequency of the emitted photon?

A. 3.29×10^{15} Hz B. 2.47×10^{15} Hz C. 2.93×10^{15} Hz D. 3.16×10^{15}Hz

58. In the photoelectric effect experiment, the frequency of the incident light is changed from red to blue while keeping the energy per unit area per unit time of the incident radiation fixed. What will happen to the saturation current?

A. Saturation current doesn't depend on frequency and therefore will be same for red and blue light

B. Saturation current depends on intensity and therefore will be same for red and blue light

C. Saturation current for blue colour light is larger than red colour light

D. Saturation current for blue colour light is smaller than red colour light

59. An ideal rigid diatomic gas undergoes expansion at constant pressure when ΔQ heat is added to it. What is the ratio of the change in the internal energy ΔU to the heat supplied ΔQ?

$A.\frac{\Delta U}{\Delta Q} = \frac{3}{5}$ $B.\frac{\Delta U}{\Delta Q} = \frac{5}{7}$ $C.\frac{\Delta U}{\Delta Q} = \frac{7}{9}$ $D.\frac{\Delta U}{\Delta Q} = \frac{2}{3}$

60. The fundamental constants are given to be permittivity constant (ϵ_0), Planck's constant (h), velocity of light (c), Newton's constant (G) and Boltzmann's constant (k_B). Which of the following quantities constructed out of the above constant has the dimensions of an electric charge?

$A.\sqrt{\frac{hG}{\epsilon_0^3}}$ $B.\sqrt{hc}$ $C.k_B\sqrt{hc}$ $D.\sqrt{\epsilon_0 hc}$

ANSWER KEY 2020

Question Number	Answer	Question Number	Answer	Question Number	Answer	Question Number	Answer
1	(D)	16	(D)	31	(D)	46	(A)
2	(A)	17	(B)	32	(D)	47	(A)
3	(A)	18	(D)	33	(B)	48	(B)
4	(D)	19	(C)	34	(C)	49	(C)
5	(D)	20	(C)	35	(D)	50	(D)
6	(B)	21	(B)	36	(A)	51	(A)
7	(D)	22	(C)	37	(A)	52	(C)
8	(C)	23	(B)	38	(C)	53	(D)
9	(D)	24	(D)	39	(D)	54	(A)
10	(A)	25	(A)	40	(D)	55	(A)
11	(B)	26	(D)	41	(C)	56	(A)
12	(A)	27	(B)	42	(B)	57	(D)
13	(D)	28	(D)	43	(C)	58	(D)
14	(C)	29	(C)	44	(D)	59	(B)
15	(B)	30	(C)	45	(B)	60	(D)

SOLUTIONS

1. The stems of maize and sugarcane have supporting roots coming out of the lower nodes of the stem. These are called **stilt roots**.

2. The membrane system of the chloroplast is responsible for trapping the light energy and also for the synthesis of **ATP and NADPH**. Since these reactions are directly light driven, they are called light reactions (photochemical reactions).

3. Specific Bt toxin genes were isolated from **Bacillus thuringiensis** and incorporated into the several crop plants such as cotton. The toxin is coded by a gene crylAc named Cry.

4. Let the allelic frequency of the red phenotype be denoted by p and that of the white one by q. In a Hardy-Weinberg Equilibrium, sum of allelic frequencies is 1, i.e. p + q = 1.

$(p+q)^2 = p^2 + 2pq + q^2 = 1$

$q^2 = \frac{490}{1000} = 0.49$ $\qquad$ $q = \sqrt{0.49} = 0.7$

$p = 1 - q = 1 - 0.7 = 0.3$ $\qquad$ $p^2 = 0.09$

$2pq = 1 - (q^2 + p^2) = 1 - (0.49 + 0.09) = 0.42$

No. of heterozygous red-flowered plants = $0.42 \times 1000 = \mathbf{420}$.

5. The junction between a motor neuron and the sarcolemma of the muscle fibre is called the neuromuscular junction or motor-end plate. A neural signal reaching this junction releases Acetyl choline which generates an action potential in the sarcolemma. This spreads through the muscle fibre and causes the release of calcium ions into the sarcoplasm. Increase in $\mathbf{Ca^{2+}}$ level leads to the binding of calcium with a subunit of troponin on actin filaments and thereby remove the masking of active sites for myosin.

6. If the predator population (wolves) is depleted, the population of herbivores (elk) will increase unchecked because there are fewer predators to control their numbers. This increase in herbivores will lead to more intense grazing on trees (aspen and willow), which can result in a significant reduction in tree populations. Over time, this reduction in primary producers (trees) can destabilize the entire ecosystem, potentially leading to its breakdown if the process continues without intervention.

7. Interferons are proteins produced and released by virus-infected cells as part of the immune response. They help protect uninfected cells by signalling them to enhance their antiviral defences, thus interfering with the replication of viruses.

8. Diglycerides are lipids consisting of two fatty acid molecules esterified with a molecule of glycerol. Lipases are enzymes which break down these bonds, giving the constituent molecules.

9. **Cyclostomata** is a class under the phylum Chordata which being the highest phylum has bilateral symmetry.

10. **Urea** is reabsorbed by the tubular epithelial cells in different segments of nephrons passively along with other nitrogenous wastes to maintain osmolarity.

11. DNA replication occurs in the S phase, due to which the chromosome number doubles from 46 (2n) to **92** (4n) in the S and $\mathbf{G_2}$ phases. After mitosis, the cell returns to its 2n state, hence the $\mathbf{G_1}$ phase has **46** chromosomes. **Spermatogonia** before replication are diploid and have **46** chromosomes. They later divide to form haploid spermatocytes. **Ova** are haploid as they are gametes, and hence have **23** chromosomes.

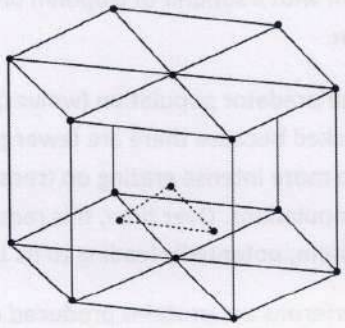
12. Amino acids are activated in the presence of ATP and linked to their cognate tRNA to enable peptide bond formation between two amino acids for translation of mRNA into proteins. This process is commonly called charging of tRNA or **amino acylation** of tRNA. All the other 3 processes are involved in functionalisation of mRNA in eukaryotes.

13. The mycobiont of the mycorrhizae produces filaments which increase the surface area available to the leguminous plant for water and mineral uptake.

14. Logistic growth is characterized by a population growth rate that first increases, then reaches a maximum, and finally decreases as the population size approaches the carrying capacity of the environment.

15. **Parthenogenesis** refers to the development of embryo from an egg directly, without fertilization. **Apomixis** refers to the development of seed without fertilization.

16. In a hexagonal close-packed (hcp) unit cell, there are a total of 6 atoms present:



- 2 atoms fully inside the unit cell (one at the top and one at the bottom layer)
- 12 atoms at the vertices, each shared with six other unit cells, contributing 1/6 per atom
- 2 atoms in the middle layer, each shared with two other unit cells, contributing 1/2 per atom

17.

$$\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} \qquad (2.26)$$

But we know that for a weak electrolyte like acetic acid (Class XI, Unit 7),

$$K_a = \frac{c\alpha^2}{(1-\alpha)} = \frac{c\Lambda_m^2}{\Lambda_m^{\circ 2}\left(1-\frac{\Lambda_m}{\Lambda_m^\circ}\right)} = \frac{c\Lambda_m^2}{\Lambda_m^\circ(\Lambda_m^\circ - \Lambda_m)} \qquad (2.27)$$

Here,

$$c = 2.41 \times 10^{-3}\ M$$

$$\Lambda_m = 32.87\ Scm^2mol^{-1}$$

$$\Lambda_m^\circ = 390.5\ Scm^2mol^{-1}$$

Putting in values (use calculator),

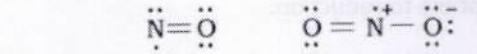
$$K_a = \frac{2.41 \times 10^{-3}(32.87)^2}{390.5(390.5 - 32.87)}$$

$$\boxed{K_a = 1.86 \times 10^{-5}}$$

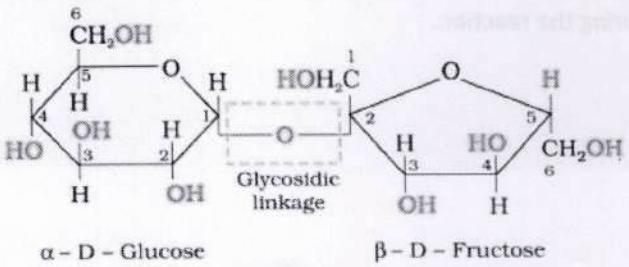
18. With NO_2F you'll need to form a double bond between one of the Oxygen atoms and a Nitrogen atom to fill the octets and still use only the 24 valence electrons available for the molecule.

Odd-electron molecules

In molecules with an odd number of electrons like nitric oxide, NO and nitrogen dioxide, NO_2, the octet rule is not satisfied for all the atoms



19. Sucrose is a disaccharide composed of glucose and fructose. The glycosidic linkage in sucrose involves the C1 carbon of glucose and the C2 carbon of fructose.



Sucrose

20. The larger the force of attraction between the molecules more would be its boiling point. Or the larger the electronegativity difference between the molecule, the more would be its polarity or boiling point. The force of attraction generally increases in the following order: electrostatic force of attraction > hydrogen bonding > dipole-dipole attraction > van der Waals force of attraction.

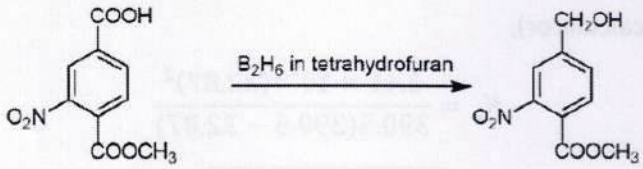
By the logic of which,

Alkanes < Alkenes < Alkynes < Haloalkanes < Ether < Ester < Amine < Aldehyde < Ketone < Alcohol < Carboxylic Acid < Amide.

I- Alkane II- Alcohol III- Aldehyde IV- Carboxylic Acid

$$I < III < II < IV$$

21.



Explanation:

Nitro Group (NO_2): The nitro group is generally resistant to reduction by diborane under typical reaction conditions. Diborane is not a strong enough reducing agent to convert the nitro group to an amine NH_2. Therefore, the nitro group will remain unchanged in this reaction.

Carboxylic Acid Group $(COOH)$: Diborane is highly effective at reducing carboxylic acids to primary alcohols. The carboxylic acid group $COOH$ on the benzene ring will be reduced to a hydroxymethyl group CH_2OH. This occurs because the carboxylic acid has a free hydroxyl group that can coordinate with diborane, making it highly susceptible to reduction.

Ester Group $(COOCH_3)$: The ester group is less reactive toward diborane compared to the carboxylic acid. This lower reactivity is due to the resonance stabilization of the ester, which makes the carbonyl carbon less electrophilic and less prone to nucleophilic attack by diborane. Moreover, the ester lacks a free hydroxyl group, which further limits its interaction with diborane. As a result, the ester group is likely to remain intact during the reaction.

22.

Show that in a first order reaction, time required for completion of 99.9% is 10 times of half-life ($t_{1/2}$) of the reaction.

When reaction is completed 99.9%, $[R]_n = [R]_0 - 0.999[R]_0$

$$k = \frac{2.303}{t}\log\frac{[R]_0}{[R]}$$

$$= \frac{2.303}{t}\log\frac{[R]_0}{[R]_0 - 0.999[R]_0} = \frac{2.303}{t}\log 10^3$$

$$t = 6.909/k$$

For half-life of the reaction

$$t_{1/2} = 0.693/k$$

$$\frac{t}{t_{1/2}} = \frac{6.909}{k} \times \frac{k}{0.693} = 10$$

The time required for 99.9% completion of the reaction is approximately 10 seconds, which corresponds to option **C.** 10 sec.

23. The correct answer is **B**. The total number of nodes for a given principal quantum number (n) and azimuthal quantum number (ℓ) is the sum of (ℓ) angular nodes and ($n - \ell - 1$) radial nodes.

So, the total number of nodes = (ℓ) (angular nodes) + ($n - \ell - 1$) (radial nodes).

24. The correct answer is:

D. H_3PO_2 is a tribasic acid.

This statement is incorrect because H_3PO_2 (hypophosphorous acid) is a monobasic acid, not tribasic. It can only donate one proton (H^+) per molecule in an aqueous solution. The correct classifications are:

- H_3PO_4 (phosphoric acid) is a tribasic acid.
- H_3PO_3 (phosphorous acid) is a dibasic acid.
- H_3PO_2 (hypophosphorous acid) is a monobasic acid.

25. The first ionization potential (IP_1) generally increases across a period from left to right due to increasing nuclear charge, which holds the electrons more tightly, making it harder to remove an electron. However, there are some exceptions due to electronic configurations.

For the period III elements Mg, Al, Si, P, and S, the first ionization potentials would be arranged as follows:

Al < S < Si < P < Mg

Reasoning:

Mg: The ionization potential of magnesium is relatively high because it has a stable, fully filled $3s^2$ orbital.

Al: The ionization potential of aluminum is lower than magnesium because the electron being removed comes from the 3p orbital, which is higher in energy and easier to remove than the 3s electrons of magnesium.

Si: The ionization potential increases again after aluminum because silicon has a more positive effective nuclear charge, making it harder to remove an electron from the 3p orbital.

P: Phosphorus has a half-filled $3p^3$ configuration, which is relatively stable, so its ionization potential is higher than that of silicon.

S: The ionization potential of sulfur is slightly lower than that of phosphorus because adding an additional electron to the 3p orbital causes electron-electron repulsion, making it slightly easier to remove an electron

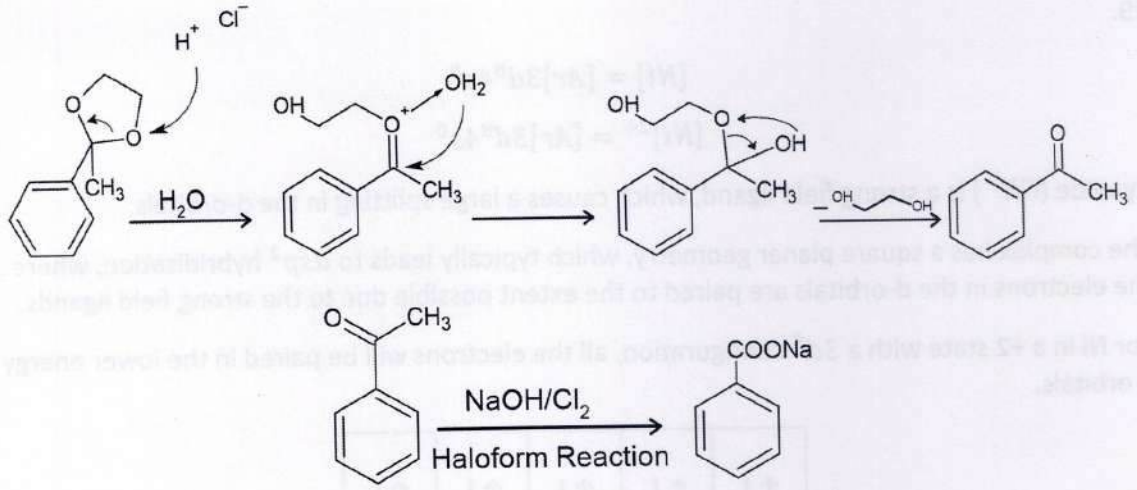
26. The intensity of color in a solution is generally associated with the presence of transition metal ions and the extent of d-d transitions or charge transfer transitions they undergo. In the case of manganese compounds, the Mn ion can exhibit different oxidation states, each with distinct colors:

1. $[Mn(H_2O)_6]Cl_2$: This complex contains Mn^{2+} ions. Mn^{2+} typically has a pale pink color due to weak d-d transitions.
2. $[Mn(NH_3)_6]Cl_3$: This complex contains Mn^{3+} ions. Mn^{3+} is not stable in aqueous solution and typically oxidizes water to produce MnO_2, which is a brown precipitate. The solution would not be intensely colored.
3. $[Mn(NH_3)_5Cl]Cl$: This complex likely contains Mn^{2+} ions, similar to the first complex, but with different ligands, leading to slightly different colors.
4. $KMnO_4$: Potassium permanganate ($KMnO_4$) contains Mn in the +7 oxidation state, and it is well known for its intense purple color due to strong charge transfer transitions.

Among these, $KMnO_4$ (potassium permanganate) gives the most intense color due to the presence of Mn in the +7 oxidation state, which leads to strong charge transfer transitions that result in a deep purple color. Therefore, the answer is:

D. 0.1 M $KMnO_4$

27. When Acetophenone ethylene ketal reacts with dilute hydrochloric acid (HCl) in water (H_2O), it undergoes acid hydrolysis. The reaction breaks the ketal back into the original carbonyl compound **acetophenone** and ethylene glycol.



28. This is a standard result which should be remembered but can also be derived as the following:

The relationship between C_p and C_v for an ideal Gas

From the equation $q = nC\Delta T$, we can say:

At constant pressure P, we have

$$q_P = nCp\Delta T$$

This value is equal to the change in enthalpy, that is,

$$q_P = nC_P\Delta T = \Delta H$$

Similarly, at constant volume V, we have

$$qv = nCv\Delta T$$

This value is equal to the change in internal energy, that is,

$$qv = nCv\Delta T = \Delta U$$

We know that for one mole (n=1) of an ideal gas,

$$\Delta \mathrm{H} = \Delta \mathrm{U} + \Delta(\mathrm{pV}) = \Delta \mathrm{U} + \Delta(\mathrm{RT}) = \Delta \mathrm{U} + \mathrm{R}\,\Delta \mathrm{T}$$

Therefore, ΔH= ΔU+ RΔT

Substituting the values of ΔH and ΔU from above in the former equation,

$$C_P\Delta T = C_V\Delta T + R\Delta T$$

$$C_P = C_V + R$$

$$C_P - C_V = R$$

29.

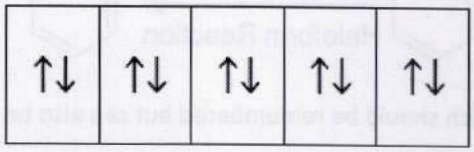
$$[Ni] = [Ar]3d^8 4s^2$$

$$[Ni]^{2+} = [Ar]3d^8 4s^0$$

Cyanide (CN^-) is a strong field ligand, which causes a large splitting in the d-orbitals.

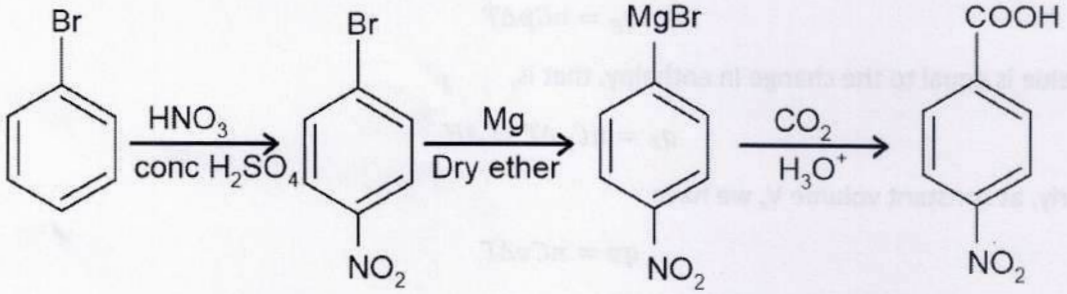
The complex has a square planar geometry, which typically leads to dsp^2 hybridization, where the electrons in the d-orbitals are paired to the extent possible due to the strong field ligands.

For Ni in a +2 state with a $3d^8$ configuration, all the electrons will be paired in the lower energy d orbitals.



Thus, there are no unpaired electrons in the complex $[Ni(CN)_4]^{2-}$.

30.



31. The equation $\sin(x) = \cos(y)$ represents a set of points (x, y) in the plane. We can rewrite this equation using the trigonometric identity $\cos(y) = \sin\left(\frac{\pi}{2} - y\right)$, which gives us:

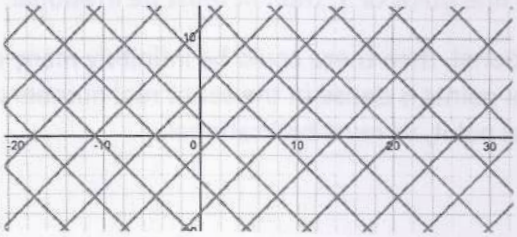
$$\sin(x) = \sin\left(\frac{\pi}{2} - y\right)$$

The solutions to this equation are given by the general formula for sine function equality

$$x = \frac{\pi}{2} - y + 2k\pi \quad \text{or} \quad x = \frac{\pi}{2} + y + (2k+1)\pi$$

Where k is any integer. These equations describe a set of parallel lines in the xy-plane. The first equation represents lines of the form $y = \frac{\pi}{2} - x + 2k\pi$, and the second equation represents lines of the form $y = x - \frac{3\pi}{2} - 2k\pi$.

So, the equation $\sin(x) = \cos(y)$ represents a series of parallel lines with specific separations in the plane.



32.

$$|A^n| = |A|^n$$

Applying This to the Given Matrix A. Where, $|A| = 1$:

$$|A^2| = |A|^2 = 1^2 = 1$$

$$|A^3| = |A|^3 = 1^3 = 1$$

and so on,

$$S = (I - A + A^2 - A^3 + \cdots + A^{2020})$$

Using the linearity property of determinants, we can write,

$$\det(S) = \det(I) - \det(A) + \det(A^2) - \det(A^3) + \cdots + \det(A^{2020})$$

$$= 1 - 1 + 1 - 1 + 1 - 1 + \cdots + 1$$

$$\boxed{\det(S) = 1}$$

33. Let's analyze each statement to determine which one is not correct.

Statement A

"If l passes through two points of S, then m must be rational."

S consists of points (a, b) where both a and b are rational numbers. If l passes through two such points (a_1, b_1) and (a_2, b_2), the slope m of the line l would be:

$$m = \frac{b_2 - b_1}{a_2 - a_1}$$

Since a_1, a_2, b_1, and b_2 are rational, m must also be rational. So, statement A is correct.

Statement C

"If l passes through exactly one point of S, then m must be irrational."

If m were rational and c rational, then there could be infinitely many rational points on the line l because the set of rational points is dense in the set of real numbers. Therefore, if l passes through exactly one rational point, m must be irrational.

So, statement C is correct.

Statement D

"If m is irrational and c is rational, then l passes through exactly one point of S."

For (a, b) on the line with $a, b \in \mathbb{Q}$, $b = ma + c$. If m is irrational and c is rational, for a specific rational a, ma is irrational, and adding rational c to irrational ma gives irrational b. Hence, there can only be at most one rational point on l, where both the numerator and denominator accidentally "cancel" out the irrationality (which is rare, but possible).

So, statement D is correct.

Statement B

"If m is rational and c is irrational, then l passes through at least one point of S."

The line equation is $y = mx + c$. For (a, b) to be a point on the line where $a, b \in \mathbb{Q}$, we need $b = ma + c$. If m is rational, ma is rational for any rational a. However, if c is irrational, $ma + c$ becomes irrational, which means b would be irrational, contradicting the condition that $b \in \mathbb{Q}$. Therefore, l cannot pass through any point in S.

So, statement B is **not correct**.

The incorrect statement is: **B.** If m is rational and c is irrational, then l passes through at least one point of S.

34. Given,

$$\int_0^1 \left(\frac{1}{(t+1)} + \frac{1}{(t+2)} + \cdots + \frac{1}{(t+2020)}\right) p(-t-1)dt$$

$$p(t) = \frac{t(t-1)\ldots.(t-2019)}{2019!}$$

The expression for $p(-t-1)$:

$$\Rightarrow p(-t-1) = \frac{(-t-1)(-t-2)\ldots.(-t-2020)}{2019!}$$

$$p(-t-1) = \frac{(t+1)(t+2)\ldots.(t+2020)}{2019!}$$

Taking log on both sides,

$$ln\big(p(-t-1)\big) = ln(t+1) + ln(t+2) + \ldots + ln(t+2020) - ln(2019!)$$

Let $p(-t-1) = y$ and then differentiating,

$$\frac{1}{y}\frac{dy}{dt} = \frac{1}{t+1} + \frac{1}{t+2} + \ldots \frac{1}{t+2020}$$

$$\boldsymbol{\frac{dy}{dt} = \left[\frac{1}{t+1} + \frac{1}{t+2} + \ldots \frac{1}{t+2020}\right] p(-t-1)}$$

On integrating both sides and multiplying with dt, we get

$$\int_0^1 \left(\frac{1}{t+1} + \frac{1}{t+2} + \cdots + \frac{1}{t+2020}\right) p(-t-1)dt = \int_{y(0)}^{y(1)} d\,y = [y]_{y(0)}^{y(1)}$$

Using

$$y(t) = p(-t-1) = \frac{(t+1)(t+2)\ldots.(t+2020)}{2019!}$$

We get,

$$y(1) = \frac{2 \cdot 3 \cdot 4 \cdots 2021}{2019!} = \frac{2021!}{2019!} = 2021 \times 2020$$

You can similarly find $y(0) = 2020$

$$= y(1) - y(0) = 2021 \times 2020 - 2020 = 2020(2021 - 1)$$

$$\boxed{= 2020^2}$$

35. To determine which statement is correct, let's analyse each option given and also carefully **observe that $p(x) = (1+x)^n$ where $n \geq 3$** and a, b are nonzero real numbers.

Option A: "If $a + ib$ is a root of $p(x)$, then n must be even."

The roots of $p(x) = (1+x)^n$ are determined by setting $(1+x)^n = 0$. Since $n \geq 3$ and $(1+x)^n = 0$ when $1 + x = 0$, the only root is $x = -1$. For a complex root like $a + ib$ to be a root, $(1 + a + ib)^n$ must be 0, which can only happen if $1 + a + ib = 0$, leading to $a = -1$ and $b = 0$. However, since b is nonzero, this condition cannot be satisfied. Thus, $a + ib$ cannot be a root.

Therefore, **Option A is incorrect**.

Option B: "If a is a root of $p(x)$, then n must be odd."

If a is a real root, then $(1 + a)^n = 0$ which leads to $1 + a = 0$ or $a = -1$. This is true for any $n \geq 3$, not necessarily odd. Therefore, **Option B is incorrect**.

Option C: "If $a + ib$ is a root of $p(x)$, then $(a + ib)^2$ can never be a root of $p(x)$."

As established earlier, $(1 + x)^n = 0$ only gives real roots. Thus, if $a + ib$ were a root, $1 + a + ib$ would need to equal zero, and $a = -1, b = 0$. This makes the question of whether $(a + ib)^2$ can be a root irrelevant because no complex roots exist for $p(x)$.

Therefore, **Option C is irrelevant.**

Option D: "If a is a root of $p(x)$, then a must be an integer."

If a is a root, $(1 + a)^n = 0$ leads to $a = -1$. Since -1 is an integer, this statement holds true.

Option D is correct.

36. A skew-symmetric matrix A is defined as a square matrix that satisfies the condition $A^T = -A$. For a 3×3 skew-symmetric matrix, this implies:

$$A = \begin{bmatrix} 0 & a_{12} & a_{13} \\ -a_{12} & 0 & a_{23} \\ -a_{13} & -a_{23} & 0 \end{bmatrix}$$

All a_{11}, a_{22}, and a_{33} are all zero.

The elements a_{12}, a_{13}, and a_{23} can be any value from the set $\{-3, -2, -1, 0, 1, 2, 3\}$. The matrix is completely determined by these off-diagonal elements and the remaining elements get adjusted accordingly.

There are 3 off-diagonal elements (a_{12}, a_{13}, a_{23}), and each of these elements can take 7 possible values. Therefore, the total number of such matrices is:

$$\boxed{7 \times 7 \times 7 = 7^3}$$

37. To find the inverse of 2 with respect to the binary operation $*$ defined by $a * b = a + b - 3$, we need to find x such that $2 * x = e$, where e is the identity element for the operation.

First, let's find the identity element e:

$$a * e = a$$

$$a + e - 3 = a$$

$$e = 3$$

So, the identity element e is 3.

Now, we need to find x such that $2 * x = 3$:

$$2 * x = 2 + x - 3$$

$$2 + x - 3 = 3$$

$$x = 4$$

Therefore, the inverse of 2 with respect to the binary operation $*$ is $\boxed{4}$.

38.

$$\left\lfloor \frac{x}{19} \right\rfloor = n \quad \text{and} \quad \left\lfloor \frac{x}{20} \right\rfloor = n$$

where n is an integer. This implies:

$$n \leq \frac{x}{19} < n+1 \quad \text{and} \quad n \leq \frac{x}{20} < n+1$$

$$19n \leq x < 19(n+1)$$

$$20n \leq x < 20(n+1)$$

Finding the Intersection:

$$\max(19n, 20n) \leq x < \min\big(19(n+1), 20(n+1)\big)$$

$$20n \leq x < 19(n+1)$$

The number of integers in this range is:

$$19(n+1) - 20n = 19n + 19 - 20n = 19 - n$$

We also need to determine the range of n for which there are valid solutions. We need:

$$20n \leq 19(n+1)$$

$$20n \leq 19n + 19$$

$$n \leq 19$$

Thus, n ranges from 0 to 18 (since $n = 19$ would result in $x \geq 380$, which falls outside our desired range as $\frac{x}{20} = 19$).

For each integer n from 0 to 18, the number of valid x values is $19 - n$. We sum these values:

$$\sum_{n=0}^{18} (19 - n) = 19 + 18 + 17 \ldots + 1$$

Which is the sum of AP with a=1,n=19,d=1:

$$\frac{19}{2}(2 + 18) = 190$$

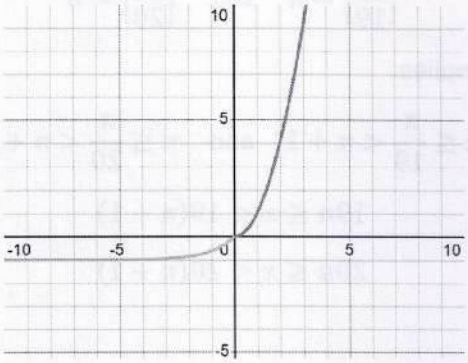
Since $x = 0$ is not a valid positive integer, we need to subtract 1 from the total count:

$$190 - 1 = 189$$

Therefore, the number of positive integer solutions is:

$$\boxed{189}$$

39.



The above is the plot for:

$$g \circ f(x) = \begin{cases} x^2, & \text{if } x \geq 0, \\ e^x - 1, & \text{if } x < 0. \end{cases}$$

It can be verified in general that $g \circ f$ is one-one implies that f is one-one. Similarly, $g \circ f$ is onto implies that g is onto.(NCERT Old Edition Class 12 page 14)

Note: There is an argument to be made about this question as it is based on a "general" case, but for now we will work with the NCERT statement.

So, since $g \circ f(x)$ is one-one, this implies that $f(x)$ will also be one-one in nature.

40. Given, $F(x) = \int_0^{e^x} (t^3 + 2t^2 - t - 2)\, dt$,

$$F(x) = \left[\frac{t^4}{4} + \frac{2t^3}{3} - \frac{t^2}{2} - 2t\right]_0^{e^x}$$

$$F(x) = \frac{e^{4x}}{4} + \frac{2e^{3x}}{3} - \frac{e^{2x}}{2} - 2e^x$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$F'(x) = e^{4x} + 2e^{3x} - e^{2x} - 2e^x$$

$$F'(x) = e^x(e^{3x} + 2e^{2x} - e^x - 2) = e^x\left(e^{2x}(e^x + 2) - 1(e^x + 2)\right)$$

$$F'(x) = e^x(e^x - 1)(e^x + 2)$$

For a root of $F'(x)$, one of $e^x, (e^x - 1), (e^x + 2)$ has to be equal to zero as e^x will never be equal to zero(although it will tend to zero at $-\infty$) and also never be equal to -2.

The one and only root will be $e^x = 1$, i.e. $x = 0$

only one root

41. The general equation of a circle is given as:

$$(x-h)^2+(y-k)^2=r^2$$

Where, (h,k) is the circle's center, r is the circle's radius and (x,y) is any point on a circle.

It is given that the center lies on the line $y=x+1$, hence $k=h+1$,

We can substitute the values and equate two equations which will be given as:

$$(-2-h)^2+\left(1-(h+1)\right)^2=r^2$$

$$(0-h)^2+\left(3-(h+1)\right)^2=r^2$$

$$\Rightarrow(-2-h)^2+\left(1-(h+1)\right)^2=(0-h)^2+\left(3-(h+1)\right)^2$$

$$(2+h)^2+h^2=h^2+(2-h^2)$$

$$(2+h)^2=(2-h)^2$$

$$\boxed{h=0,k=1}$$

You can now check the distance between the point $(0,1)$ and check if $r=2$ (same as the distance between $(0,1)$ and $(0,3)$ as the radius will be the same)

As $(\sqrt{2}+1,1)$ will always give an irrational distance you can eliminate it and with the same logic $(1,\sqrt{3}-1)$ can be eliminated as $(\sqrt{3}-2)^2$ will always be irrational.

Now, let us check the remaining two options:

$$(1,\sqrt{2}+1):\Rightarrow 1^2+\left(\sqrt{2}+1-1\right)^2=1+\left(\sqrt{2}\right)^2=1+2=3\neq 4$$

$$(1,\sqrt{3}+1)\Rightarrow 1^2+\left(\sqrt{3}+1-1\right)^2=1+\left(\sqrt{3}\right)^2=1+3=4$$

$$\boxed{(1,\sqrt{3}+1)\text{ lies on }c}$$

42.

$$f(x)=x^3+tx^2+(2-t)x-3$$

Testing $f(1)$:

$$f(1)=1+t+(2-t)-3=0$$

So, $x=1$ is a root.

Factoring $x-1$ out of the polynomial:

$$\frac{x^3+tx^2+(2-t)x-3}{x-1}=x^2+x(t+1)+3$$

For the polynomial to have all real roots, the quadratic $x^2 + x(t-1) + 3$ must have real roots. The discriminant of this quadratic must be non-negative:

$$D = (t+1)^2 - 4 \cdot 1 \cdot 3 \geq 0$$

$$(t+1)^2 - 12 \geq 0$$

$$(t+1)^2 \geq 12$$

Solving for t:

$$t + 1 \geq \sqrt{12} \quad \text{or} \quad t + 1 \leq -\sqrt{12}$$

But we can ignore the negative solution as no relevant option is given

$$t \geq \sqrt{12} - 1 = 2.464$$

From the given options (0, 4, 2, -1), the value of t that satisfies this condition is:

$$\boxed{4}$$

43. Given,

$$f(x) = ax^2 + bx + c$$

$$f\left(\frac{1}{n}\right) = \frac{1+n}{n^2}$$

Then,

$$f'(x) = 2ax + b$$

$$f'(0) = b$$

To find the value of b, we need to arrange the function in a way to present it in the form $ax^2 + bx + c$ to find the coefficient of x, which will be our final answer.

We can re-arrange as the following:

$$f\left(\frac{1}{n}\right) = \frac{1+n}{n^2} = \frac{1}{n^2} + \frac{1}{n}$$

Let $\frac{1}{n} = t$,

$$f(t) = t^2 + t + 0$$

Hence, $b = 1$ which will be our final answer

44. Total number of ways to choose 2 balls out of 8

$$={}^{8}C_2 = \frac{8!}{2!\,(8-2)!} = \frac{8\times 7}{2\times 1} = 28$$

Number of ways to choose 2 green balls out of 5

$$={}^{5}C_2 = \frac{5!}{2!\,(5-2)!} = \frac{5\times 4}{2\times 1} = 10$$

Number of ways to choose 2 red balls out of 3

$$={}^{3}C_2 = \frac{3!}{2!\,(3-2)!} = \frac{3\times 2}{2\times 1} = 3$$

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{10+3}{28}$$

Thus, the probability that two balls chosen at random are of the same color is:

$$\boxed{\frac{13}{28}}$$

45. Procedure for solving: Firstly put $f =$ option, then find f' and f''. Now put these values in the equation $f'' + pf' + qf = 0$ and find y'', y'in terms of y. Assume $y'' + py' + qy = 0$. If you can prove LHS=RHS in the equation, the answer is correct.

Now following these steps,

A)

$$f = (1+x)y$$
$$f' = y + (1+x)y'$$
$$f'' = (1+x)y'' + 2y'$$

Now putting in the equation $f'' + pf' + qf = 0$,

$$(1+x)y'' + 2y' + p[y + (1+x)y'] + qy(1+x)$$

Taking $(1+x)$ common,

$$(1+x)[\mathbf{y'' + py' + qy}] + 2y' + py = 0$$

$$\Rightarrow y' = -\frac{1}{2}py$$

On differentiating both sides:

$$y'' = -\frac{1}{2}py' \Longrightarrow y'' = \frac{1}{4}p^2y$$

Now on putting the values of y'', y' in $y'' + py' + qy$:

$$\frac{1}{4}p^2y - \frac{1}{2}p^2y + qy = 0$$

$$y\left(-\frac{1}{4}p^2 + q\right) = 0$$

as $p^2 = 4q$:

LHS=RHS $\therefore (1 + x)y$ **can** be a solution

B)

$$f = y^2 \qquad f' = 2y\frac{dy}{dx} = 2yy' \qquad f'' = 2(y')^2 + 2y(y'')$$

Similar to previous option:

$$2(y')^2 + 2y(y'') + p[2yy'] + qy^2 = 0$$

Taking y common:

$$2(y')^2 + y(2y'' + 2py' + qy) = 0$$

$$2(y')^2 + y(2y'' + 2py' + 2qy - qy) = 0$$

$$2(y')^2 + y(2[\mathbf{y'' + py' + qy}] - qy) = 0$$

$$y' = \frac{1}{2}qy^2$$

Differentiating on both sides:

$$y'' = \frac{1}{2}q(2yy') = q(yy') = qy\frac{1}{2}qy^2 = \frac{1}{2}(q^2y^3)$$

Putting in $y'' + py' + qy$:

$$\frac{1}{2}q^2y^3 + \frac{1}{2}qy^2 + qy^2 = 0$$

For this to be true:

$$q = 0 \text{ or } y = 0$$

Which is not possible according to the question.

Therefore, y^2 can **never** be a solution

Do try the remaining.

46. Expression for potential energy is given as:

$$V(x) = Ax + Bx^2$$

The maximum speed of the particle is when potential energy will be minimum and hence kinetic energy maximum

$$\frac{d(V(x))}{dx} = 0 \Rightarrow A + 2Bx = 0 \Rightarrow x = -\frac{A}{2B}$$

$$\frac{d^2(V(x))}{dx^2} = 2 \Rightarrow \text{Minima}$$

Using the work-energy theorem:

$$PE_i + KE_i = PE_f + KE_f$$

$$KE_f = PE_i - PE_f = \left(A\left(\frac{A}{B}\right) + B\left(\frac{A}{B}\right)^2\right) - \left(A\left(-\frac{A}{2B}\right) + B\left(\frac{-A}{2B}\right)^2\right)$$

$$= \frac{A^2}{B} + \frac{A^2}{B} - \left(-\frac{A^2}{2B} + \frac{A^2}{4B}\right) = \frac{A^2}{B}\left(2 + \frac{1}{4}\right)$$

$$\Rightarrow KE_f = \frac{9A^2}{4B}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{9A^2}{4B}$$

$$\boxed{v = \sqrt{\frac{9A^2}{2Bm}} = \frac{3A}{\sqrt{2Bm}}}$$

47. The formula for the mean free path (λ) of a gas molecule is:

$$\lambda = \frac{k_B T}{\sqrt{2}\pi d^2 P}$$

Where:

- k_B is the Boltzmann constant
- T is the absolute temperature of the gas
- d is the diameter of the gas molecules
- P is the pressure of the gas

Here,

$$T_1 = T_2 = T$$

And since it is the same gas,

$$d_1 = d_2 = d$$

For convenience let,

$$\frac{k_B T}{\sqrt{2}\pi d^2} = k$$

Then,

$$l_1 = \frac{k}{P_1},\ l_2 = \frac{k}{P_2} \Rightarrow P_1 = \frac{k}{l_1},\ P_2 = \frac{k}{l_2}$$

Then,

$$l_{net} = \frac{k}{P_{net}}$$

(Here, $P_{net} = P_1 + P_2$)

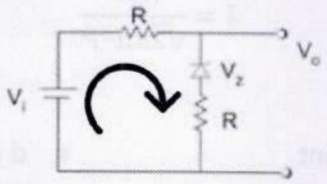
$$l_{net} = \frac{k}{\frac{k}{l_1} + \frac{k}{l_2}} = \frac{1}{\frac{1}{l_1} + \frac{1}{l_2}}$$

$$\boxed{l_{net} = \frac{l_1 l_2}{l_1 + l_2}}$$

48. When the input voltage V_i exceeds the Zener breakdown voltage V_Z, the Zener diode enters breakdown mode and maintains a constant voltage across itself, which is V_Z.This characteristic allows the Zener diode to regulate the output voltage.

In this circuit, the breakdown voltage V_Z of the Zener diode is given as 4 V. Therefore, regardless of the value ofV_i (as long as V_i is greater thanV_Z), the voltage across the Zener diode remains at 4 V.

Using Kirchhoff's Voltage Law (KVL) around the loop, we have:



$$V_i - iR - V_Z - iR = 0$$

$$20 - iR - 4 - iR = 0$$

$$16 = 2iR$$

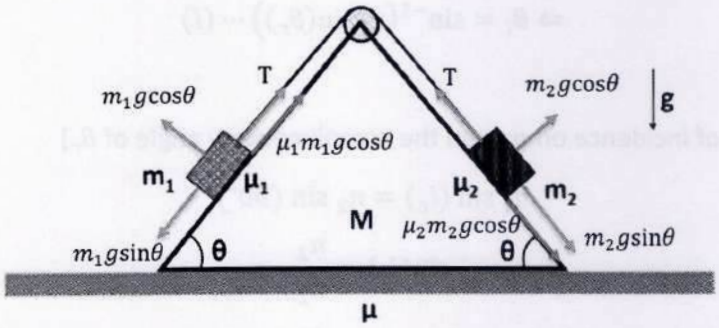
$$iR = 8$$

The potential at V_o will be calculated as:

$$V_i - iR = 20 - 8$$

$$\boxed{V_o = 12V}$$

49.



The forces acting on the blocks can be described as follows:

For m_1:

Normal Force: $N_1 = m_1 g\cos\theta$

Friction Force: $f_1 = \mu_1 N_1 = \mu_1 m_1 g\cos\theta$

Component of weight down the incline: $m_1 g\sin\theta$

Since m_1 is in equilibrium:

$$m_1 g\sin\theta = T + f_1$$

$$T = m_1 g(\sin\theta - \mu_1\cos\theta)$$

For m_2:

Normal Force: $N_2 = m_2 g\cos\theta$

Friction Force: $f_2 = \mu_2 N_2 = \mu_2 m_2 g\cos\theta$

Component of weight down the incline: $m_2 g\sin\theta$

Since m_2 is in equilibrium:

$$m_2 g\sin\theta = T - f_2$$

$$T = m_2 g(\sin\theta + \mu_2\cos\theta)$$

Equating the two tensions:

$$m_1 g(\sin\theta - \mu_1\cos\theta) = m_2 g(\sin\theta + \mu_2\cos\theta)$$

Cancelling out g:

$$m_1(\sin\theta - \mu_1\cos\theta) = m_2(\sin\theta + \mu_2\cos\theta)$$

The value of $\frac{m_1}{m_2}$ for the system to be in equilibrium is:

$$\boxed{\frac{m_1}{m_2} = \frac{\mu_2\cos\theta + \sin\theta}{\sin\theta - \mu_1\cos\theta}}$$

50. Using Snell's law which states,

$$\mu_1 \sin i = \mu_2 \sin r$$

For $air \rightarrow n_1$,

$$1 \times \sin(\theta_i) = n_1 \sin(\theta_r)$$

$$\Rightarrow \theta_i = \sin^{-1}\big(n_1 \sin(\theta_r)\big) \cdots (i)$$

For $n_1 \rightarrow n_2$,

(let i_c be the angle of incidence on n_2 and the complimentary angle of θ_r)

$$n_1 \sin(i_c) = n_2 \sin(90°)$$

$$\sin(i_c) = \frac{n_2}{n_1}$$

$$\cos(\theta_r) = \frac{n_2}{n_1}$$

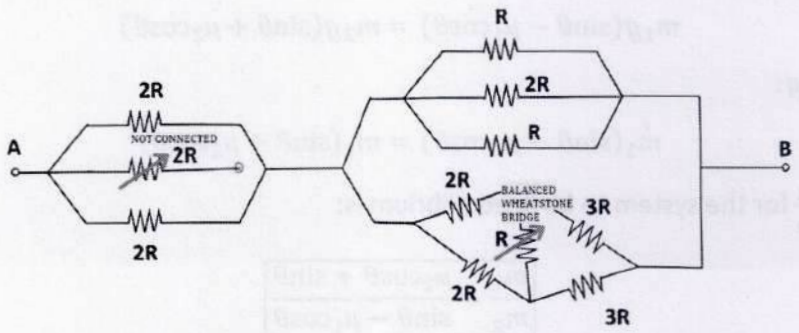
$$\sin(\theta_r) = \sqrt{1 - cos^2(\theta_r)} = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

Substituting into (i),

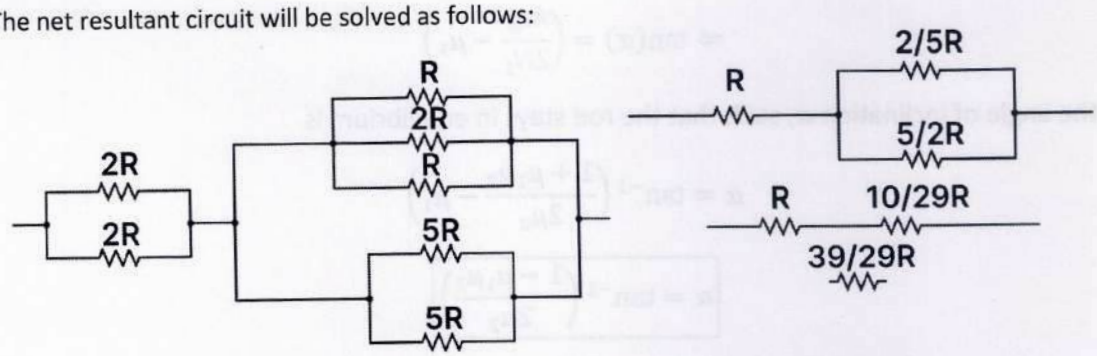
$$\theta_i = \sin^{-1}\big(n_1 \sin(\theta_r)\big) = \sin^{-1}\left(n_1 \times \sqrt{1 - \frac{n_2^2}{n_1^2}}\right)$$

$$\boxed{\theta_i = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)}$$

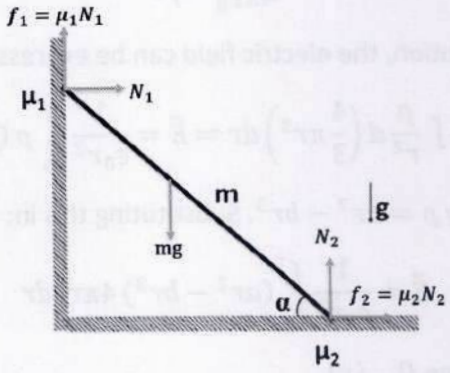
51.



The net resultant circuit will be solved as follows:



52. For equilibrium, the sum of vertical and horizontal forces must be zero, and the sum of torques about any point must also be zero.



From the diagram, we can see that:

$N_2 + f_1 = mg$ $\qquad$ $N_1 = f_2 = \mu_2 N_2$

$$f_1 = \mu_1 N_1 = \mu_1 \mu_2 N_2$$

$$N_2(1 + \mu_1 \mu_2) = mg$$

$$\Rightarrow N_2 = \frac{mg}{(1 + \mu_1 \mu_2)}$$

Torque Balance:

Balancing the torque, we get,

$$f_1 l\cos(\alpha) + N_1 l\sin(\alpha) = mg\frac{l}{2}\cos(\alpha)$$

where l is the length of the rod.

$$f_1 + N_1\tan(\alpha) = \frac{mg}{2}$$

$$N_1\tan(\alpha) = \frac{mg}{2} - f_1 = \frac{mg}{2} - \mu_1 N_1$$

$$\Rightarrow \tan(\alpha) = \left(\frac{mg}{2N_1} - \mu_1\right)$$

So, the angle of inclination α, such that the rod stays in equilibrium is

$$\alpha = \tan^{-1}\left(\frac{1+\mu_1\mu_2}{2\mu_2} - \mu_1\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{1-\mu_1\mu_2}{2\mu_2}\right)}$$

53. The electric field $\vec{E}$ inside a sphere with a given charge density ρ is given by:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r^2} dV$$

For a spherical charge distribution, the electric field can be expressed as:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r^2} d\left(\frac{4}{3}\pi r^3\right) dr = \vec{E} = \frac{1}{\epsilon_0 r^2}\int_0^r \rho\,(r) 4\pi r^2 dr$$

The charge density is given by $\rho = ar^2 - br^3$. Substituting this in:

$$\vec{E} = \frac{1}{\epsilon_0 r^2}\int_0^r (ar^2 - br^3)\, 4\pi r^2 dr$$

Calculating the enclosed charge $Q_{\text{enc}}(r)$:

$$Q_{\text{enc}}(r) = \int_0^r (ar^2 - br^3)\, 4\pi r^2 dr$$

$$Q_{\text{enc}}(r) = \frac{4\pi a r^5}{5} - \frac{4\pi b r^6}{6}$$

The electric field goes to zero when $Q_{\text{enc}}(r) = 0$:

$$\frac{4\pi a r^5}{5} - \frac{4\pi b r^6}{6} = 0$$

$$\frac{ar^5}{5} = \frac{br^6}{6}$$

$$\boxed{r = \frac{6a}{5b}}$$

54.

$$g_i(d) = \frac{g}{R}(R-d)$$

$$g_o(h) = \frac{GMR^2}{(R+d)^2}$$

$$\Rightarrow \frac{g_i(d)}{g_o(d)} = \frac{(R-d)}{R^3}(R+d)^2 = \frac{(R^2-d^2)(R+d)}{R^3} = \frac{R^3+R^2d-Rd^2-d^3}{R^3}$$

$$\frac{g_i(d)}{g_o(d)} = 1 + \frac{d}{R} - \frac{d^2}{R^2} - \frac{d^3}{R^3}$$

let $\frac{d}{R} = t$,

$$\Rightarrow \frac{g_i(d)}{g_o(d)} = 1 + t - t^2 - t^3$$

$$\Rightarrow \frac{d\left(\frac{g_i(d)}{g_o(d)}\right)}{dt} = 1 - 2t - 3t^2 = 0$$

$$3t^2 + 3t - t - 1 = 3t(1+t) - 1(1+t) = (3t-1)(t+1)$$

(for critical points)

Critical points:

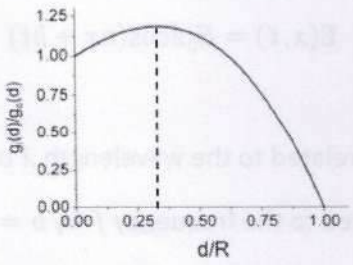
$$t = \frac{1}{3}, t = -1$$

Distance from the centre of the earth cannot be negative so we can ignore $t = -1$

So, the maximum value of the ratio $\frac{g_i(a)}{g_o(d)}$ occurs when (You can double differentiate):

$$t = \frac{d}{R} = \frac{1}{3}$$

The graph that corresponds best is:

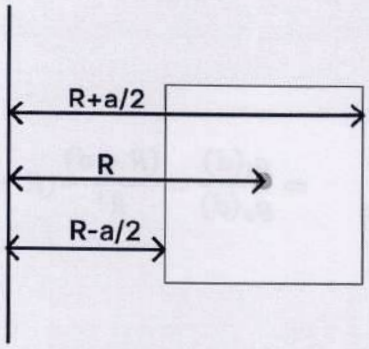


55. The magnetic field at a distance r from an infinitely long wire carrying a current I is given by Ampere's Law:

$$B = \frac{\mu_0 I}{2\pi r}$$

Where μ_0 is the permeability of free space, and r is the distance from the wire.

After some time t, the loop will have moved to some distance $R' = R + vt$.



The magnetic flux varies as:

$$\Phi_B = \int BdA = \int BadR'.$$

We take a/2 and $-a/2$ in the integration limits because of how the magnetic field varies across the width of the square loop as can be seen in the diagram.

$$\Phi_B = \frac{\mu_o Ia}{2\pi}\int_{R'-\frac{a}{2}}^{R'+\frac{a}{2}} \frac{dR'}{R'} \Rightarrow \frac{\mu_o Ia}{2\pi}\left(\ln\left(\frac{R'+\frac{a}{2}}{R'-\frac{a}{2}}\right)\right)$$

And, so the induced emf, as a function of time t is given by

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\frac{\mu_o Ia}{2\pi}\frac{d}{dt}\left(\ln\left(R+vt+\frac{a}{2}\right)+\ln\left(R+vt-\frac{a}{2}\right)\right) = \frac{\mu_o Ia}{2\pi}\left(\frac{v}{(R+vt)+\left(\frac{a}{2}\right)}\frac{v}{(R+vt)-\left(\frac{a}{2}\right)}\right)$$

$$\boxed{\frac{\mu_o Ia^2 v}{2\pi\left((R+vt)^2-\frac{a^2}{4}\right)}}$$

56. The electric field of the electromagnetic wave is given by:

$$\mathbf{E}(x,t) = E_0\hat{z}\cos(ax+bt)$$

In this equation:

- a is the wave number, which is related to the wavelength λ by $a = \frac{2\pi}{\lambda}$.

- b is the angular frequency, related to the frequency f by $b = 2\pi f$.

For an electromagnetic wave traveling in a vacuum, the relationship between the wavelength λ and frequency f is given by:

$$c = \lambda f$$

We can express this in terms of a and b as:

$$c = \frac{2\pi}{a} \cdot \frac{b}{2\pi}$$

Solving for b, we get:

$$c = \frac{b}{a}$$

$$\boxed{b = ac}$$

57. Dividing the following equations,

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \text{(Coulomb's law)}$$

$$mvr = \frac{nh}{2\pi}$$

We can derive,

$$v = \frac{e^2}{2h\epsilon_0(n)}$$

And we are given,

$$\frac{e^2}{2h\epsilon_0 c} = \frac{1}{137} \Rightarrow \frac{e^2}{2h\epsilon_0} = \frac{c}{137}$$

Therefore,

$$\boxed{u_n = \frac{c}{137n}}$$

It is given,

$$V_n = \frac{u_n}{c} = \frac{1}{137 \times n} = \frac{1}{685}$$

$$\Rightarrow n = 5$$

$$\Delta E = h\Delta v = 13.6\left(Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\right)$$

For $n = 5$ ($h = 4.13 \times 10^{15} eV \cdot s$)

$$v = \frac{13.6 \times 1}{4.13 \times 10^{-15}} \times \frac{24}{25}$$

(use calculator)

$$\boxed{v = 3.16 \times 10^{15} Hz}$$

58. In the photoelectric effect, the saturation current is determined by the number of photoelectrons emitted per unit time, which is directly proportional to the intensity (energy per unit area per unit time) of the incident light.

When the frequency of the incident light is changed from red to blue, while keeping the energy per unit area per unit time (intensity) of the incident radiation fixed, the following happens:

1. Energy of Photons: Blue light has a higher frequency than red light, so the energy of each photon of blue light ($E = h\nu$, where ν is the frequency) is greater than that of red light photons.

2. Number of Photons: Since the intensity (total energy per unit area per unit time) is kept constant, if the energy of each photon increases (as in the case of blue light), the number of photons hitting the surface per unit time must decrease to keep the total energy per unit area per unit time constant.

3. Photoelectrons Emission: The number of photoelectrons emitted per unit time (and hence the saturation current) depends on the number of incident photons. With a higher energy per photon (blue light), fewer photons are needed to maintain the same intensity, resulting in fewer photoelectrons emitted.

Thus, the correct answer is: D. Saturation current for blue color light is smaller than red color light

59. For diatomic gases,

$$C_V = \frac{5}{2}R \qquad C_P = C_V + R = \frac{7}{2}R.$$

So, the ratio,

$$\frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC_P\Delta T} = \frac{\frac{5}{2}R}{\frac{7}{2}R} = \frac{5}{7}$$

60. For deriving the dimensions of ϵ_0, h, G and k_B, use the following formulae:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \qquad E = h\nu \qquad F = G\frac{m_1 m_2}{r^2} \qquad E = k_B T$$

Where all symbols hold their usual meanings

The final results are:

$$[\epsilon_0] = [M^{-1}L^{-3}T^4I^2] \qquad [h] = [ML^2T^{-1}]$$
$$[G] = [M^{-1}L^3T^{-2}] \qquad [k_B] = [ML^2T^{-2}\theta^{-1}]$$

You can easily eliminate $\sqrt{hc}$, $k_B\sqrt{hc}$ as neither has any term of I, i.e. ϵ_0

$$\sqrt{\frac{hG}{\epsilon_0^3}} = \sqrt{\frac{[L^5T^{-3}]}{[M^{-3}L^{-9}T^{12}I^6]}} = \left[M^{3/2}L^7T^{-15/2}I^{-3}\right]$$

$$\sqrt{\epsilon_0 hc} = \sqrt{[M^{-1}L^{-3}T^4I^2]\cdot[ML^3T^{-2}]} = \sqrt{[T^2I^2]} = [IT]$$

$$\boxed{\left[\sqrt{\epsilon_0 hc}\right] = [IT]}$$