Electrostatic Potential and Capacitance

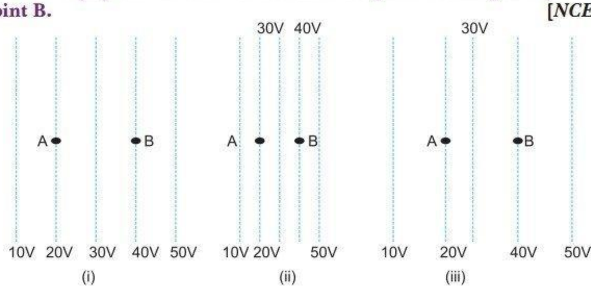
Multiple Choice Questions

Choose and write the correct option(s) in the following questions.

1. **Some charge is being given to a conductor. Then, its potential**
   (*a*) is maximum at surface.
   (*b*) is maximum at centre.
   (*c*) remains the same throughout the conductor.
   (*d*) is maximum somewhere between surface and centre.

2. **A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge** [*NCERT Exemplar*]
   (*a*) remains a constant because the electric field is uniform.
   (*b*) increases because the charge moves along the electric field.
   (*c*) decreases because the charge moves along the electric field.
   (*d*) decreases because the charge moves opposite to the electric field.

3. **Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.** [*NCERT Exemplar*]

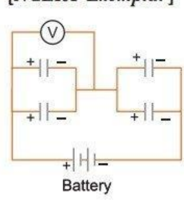


   (*a*) The work done in Fig. (*i*) is the greatest.
   (*b*) The work done in Fig. (*ii*) is least.
   (*c*) The work done is the same in Fig. (*i*), Fig. (*ii*) and Fig. (*iii*).
   (*d*) The work done in Fig. (*iii*) is greater than Fig. (*ii*) but equal to that in Fig. (*i*).

4. **The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard:** [*NCERT Exemplar*]
   S_1 : At any point inside the sphere, electric intensity is zero.
   S_2 : At any point inside the sphere, the electrostatic potential is 100 V.
   Which of the following is a correct statement?
   (*a*) S_1 is true but S_2 is false.
   (*b*) Both S_1 and S_2 are false.
   (*c*) S_1 is true, S_2 is also true and S_1 is the cause of S_2.
   (*d*) S_1 is true, S_2 is also true but the statements are independent.

5. **Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately** [*NCERT Exemplar*]
   (*a*) spheres (*b*) planes
   (*c*) paraboloids (*d*) ellipsoids

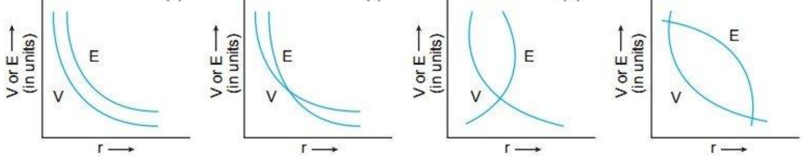
6. **Four capacitors, each 50 μF are connected as shown. The *DC* voltmeter *V* reads 100 V. The charge on each plate of each capacitor is**



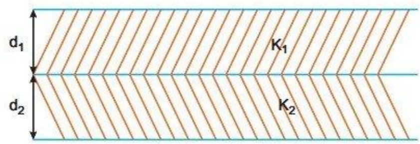
   (*a*) 2×10^{-3} C (*b*) 5×10^{-3} C
   (*c*) 0.2 C (*d*) 0.5 C

7. **The variation potential V with r and electric field E with r for a point charge is correctly shown in the graphs.**

(a) (b) (c) (d)



8. **A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant k_1 and the other has thickness d_2 and dielectric constant k_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d\ (= d_1 + d_2)$ and effective dielectric constant k. The k is** [*NCERT Exemplar*]



(a) $\dfrac{k_1 d_1 + k_2 d_2}{d_1 + d_2}$ (b) $\dfrac{k_1 d_1 + k_2 d_2}{k_1 + k_2}$ (c) $\dfrac{k_1 k_2 (d_1 + d_2)}{k_1 d_1 + k_2 d_2}$ (d) $\dfrac{2k_1 k_2}{k_1 + k_2}$

9. **Equipotential surfaces** [*NCERT Exemplar*]

(a) are closer in regions of large electric fields compared to regions of lower electric fields.

(b) will be more crowded near sharp edges of a conductor.

(c) will be more crowded near regions of large charge densities.

(d) will always be equally spaced.

10. **A 2 μF capacitor is charged to 200 volt and then the battery is disconnected. When it is connected in parallel to another uncharged capacitor, the potential difference between the plates of both is 40 volt. The capacitance of the other capacitor is**

(a) 2 μF (b) 4 μF (c) 8 μF (d) 16 μF

11. **n identical capacitors joined in parallel are charged to a common potential V. The battery is disconnected. Now, the capacitors are separated and joined in series. For the new combination:**

(a) energy and potential difference both will remain unchanged

(b) energy will remain same, potential difference will become nV

(c) energy and potential both will become n times

(d) energy will become n times, potential difference will remain V.

12. **The capacitance of a capacitor becomes $\frac{7}{6}$ times its original value if a dielectric slab of thickness $t = \frac{2}{3}d$ is introduced in between the plates, where d is the separation between the plates. The dielectric constant of the slab is**

(a) $\dfrac{14}{11}$ (b) $\dfrac{11}{14}$

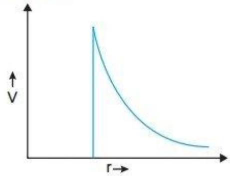
(c) $\dfrac{7}{11}$ (d) $\dfrac{11}{7}$

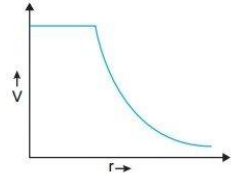
13. **The plates of a parallel plate capacitor are 4 cm apart, the first plate is at 300 V and the second plate at – 100 V. The voltage at 3 cm from the second plate is**

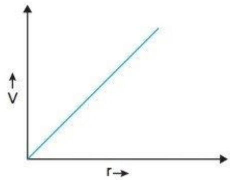
(a) 200 V (b) 400 V

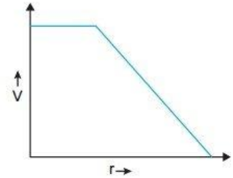
(c) 250 V (d) 500 V

14. **In the case of a charged metallic sphere, potential (*V*) changes with respect to distance (*r*) from the centre as**

(*a*)


(*b*)


(*c*)


(*d*)


15. **A conducting sphere of radius *R* is given a charge *Q*. The electric potential and the electric field at the centre of the sphere respectively are**

(*a*) zero and $\frac{Q}{4\pi\varepsilon_0 R^2}$ (*b*) $\frac{Q}{4\pi\varepsilon_0 R}$ and zero

(*c*) $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{Q}{4\pi\varepsilon_0 R^2}$ (*d*) both are zero

16. **Four point charges $-Q$, $-q$, $2q$ and $2Q$ are placed, one at each corner of the square. The relation between *Q* and *q* for which the potential at the centre of the square is zero is**

(*a*) $Q = \frac{1}{2}q$ (*b*) $Q = -q$

(*c*) $Q = -\frac{1}{2}q$ (*d*) $Q = q$

17. **An electric dipole consisting of charges $+q$ and $-q$ separated by a distance *L* is in stable equilibrium in a uniform electric field $\vec{E}$. The electrostatic potential energy of the dipole is** **[*CBSE 2020 (55/1/1)*]**

(*a*) qLE (*b*) zero

(*c*) $-qLE$ (*d*) $-2\,qEL$

18. **If a positive charge in displaced against the electric field in which it was situated, then** **[*CBSE 2020 (55/3/1)*]**

(*a*) work will be done by the electric field on the charge.

(*b*) the intensity of the electric field decreases.

(*c*) energy of the system will decrease.

(*d*) energy will be provided by external source displacing the charge.

19. **The capacitors of capacitances C_1 and C_2 are connected in parallel. If a charge *Q* is given to the combination, the ratio of the charge on the capacitor C_1 to the charge on C_2 will be** **[*CBSE 2020 (55/3/1)*]**

(*a*) $\frac{C_1}{C_2}$ (*b*) $\sqrt{\frac{C_1}{C_2}}$

(*c*) $\sqrt{\frac{C_2}{C_1}}$ (*d*) $\frac{C_2}{C_1}$

20. A charge Q is kept at the centre of a circle of radius r. A test charge q_0, is carried from a point X to the point Y on this circle such that arc XY subtends an angle of 60° at the centre of the circle. The amount of work done in this process will be [*CBSE 2020 (55/3/2)*]

(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{2r}$

(*b*) $\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}Qq_0}{2r}$

(*c*) zero

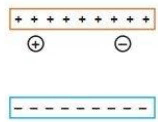
(*d*) $\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}Qq_0}{r}$

21. A charge Q is uniformly distributed over the surface of a spherical shell of radius R. The work done in bringing a test charge Q_0 from its centre to its surface is [*CBSE 2020 (55/3/3)*]

(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{QQ_0}{R}$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{QQ_0}{2R}$ (*c*) $\frac{QQ_0}{\varepsilon_0 R}$ (*d*) zero

22. A free electron and a free proton are placed between two oppositely charged parallel plates. Both are closer to the positive plate than the negative plate.

[*CBSE Sample Paper-2022, Term-1*]



Which of the following statements is true?

I. The force on the proton is greater than the force on the electron.

II. The potential energy of the proton is greater than that of the electron.

III. The potential energy of the proton and the electron is the same.

(*a*) I only (*b*) II only (*c*) III and I only (*d*) II and I only

23. Two charges 14 μC and – 4 μC are placed at (–12 cm, 0, 0) and (12 cm, 0, 0) in an external electric field $E = \left(\frac{B}{r^2}\right)$, (where $B = 1.2 \times 10^6$ N/Cm^2) and r is in metres. The electrostatic potential energy of the configuration is [*CBSE 2022 (55/2/4), Term-1*]

(*a*) 97.9 J (*b*) 102.1 J (*c*) 2.1 J (*d*) – 97.9 J

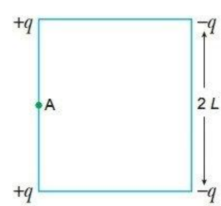
24. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from 2 μF to X μF, the decrease in energy of the capacitor is 2×10^{-2} J. The value of X is

[*CBSE 2022 (55/2/4), Term-1*]

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

25. Four charges $-q$, $-q$ and $+q$ are placed at the corners of a square of side 2 L is shown in figure. The electric potential at point A midway between the two charges + q and +q is

[*CBSE 2022 (55/2/4), Term-1*]



(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$

(*b*) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$

(*c*) $\frac{1}{4\pi\varepsilon_0}\frac{q}{2L}\left(1-\frac{1}{\sqrt{5}}\right)$

(*d*) zero

26. Three charges $2q$, $-q$ and $-q$ lie at the vertices of a triangle. The value of E and V at centroid of triangle will be [*CBSE Sample Paper-2022, Term-1*]

(*a*) $E \neq 0$ and $V \neq 0$

(*b*) $E = 0$ and $V = 0$

(*c*) $E \neq 0$ and $V = 0$

(*d*) $E = 0$ and $V \neq 0$

27. Two parallel plate capacitors X and Y, have the same area of plates and same separation between plates. X has air and Y with dielectric of constant 2 , between its plates. They are connected in series to a battery of 12 V. The ratio of electrostatic energy stored in X and Y is

[*CBSE Sample Paper-2022, Term-1*]

(*a*) 4:1 (*b*) 1:4 (*c*) 2:1 (*d*) 1:2

28. A capacitor plates are charged by a battery with 'V' volts. After charging, battery is disconnected and a dielectric slab with dielectric constant 'K' is inserted between its plates, the potential across the plates of a capacitor will become [*CBSE Sample Paper-2022, Term-1*]

(*a*) zero (*b*) $V/2$ (*c*) V/K (*d*) KV

29. Three capacitors 2μF, 3μF and 6μF are joined in series with each other. The equivalent capacitance is [*CBSE Sample Paper-2022, Term-1*]

(*a*) 1/2 μF (*b*) 1 μF (*c*) 2 μF (*d*) 11 μF

30. Which of the following is NOT the property of equipotential surface?

[*CBSE Sample Paper-2022, Term-1)*]

(*a*) They do not cross each other.
(*b*) The rate of change of potential with distance on them is zero.
(*c*) For a uniform electric field they are concentric spheres.
(*d*) They can be imaginary spheres.

31. A point P lies at a distance x from the mid point of an electric dipole on its axis. The electric potential at point P is proportional to [*CBSE 2023 (55/2/1)*]

(*a*) $\frac{1}{x^2}$ (*b*) $\frac{1}{x^3}$ (*c*) $\frac{1}{x^4}$ (*d*) $\frac{1}{x^{1/2}}$

Answers

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*b*)
8. (*c*)	**9.** (*a*), (*b*), (*c*)	**10.** (*c*)	**11.** (*b*)	**12.** (*a*)	**13.** (*a*)	**14.** (*b*)
15. (*b*)	**16.** (*b*)	**17.** (*c*)	**18.** (*d*)	**19.** (*a*)	**20.** (*c*)	**21.** (*d*)
22. (*b*)	**23.** (*a*)	**24.** (*a*)	**25.** (*a*)	**26.** (*c*)	**27.** (*c*)	**28.** (*c*)
29. (*b*)	**30.** (*c*)	**31.** (*a*)				

Assertion-Reason Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***
(b) ***Both A and R are true but R is not the correct explanation of A.***
(c) ***A is true but R is false.***
(d) ***A is false and R is also false.***

1. Assertion(*A*) : A capacitor can be given only a limited amount of charge.
Reason (*R*) : After a limited value of charge, the dielectric strength of dielectric between the capacitor plates breaks down.

2. Assertion(*A*) : An applied electric field polarises a polar dielectric.
Reason (*R*) : The molecules of a polar dielectric possess a permanent dipole moment, but in the absence of electric field, these dipoles are randomly oriented and when electric field is applied these dipoles align along the direction of electric field.

3. **Assertion** (A) : Electric field is always normal to equipotential surfaces and along the direction of decreasing order of potential.

Reason (R) : Negative gradient of electric potential is electric field. [*CBSE Sample Paper 2021*]

4. **Assertion** (A) : The capacitance of a parallel plate capacitor increases with increase of distance between the plates.

Reason (R) : Capacitance of a parallel plate capacitor *i.e.*, $C \propto d$

5. **Assertion** (A) : The capacitance of a conductor does not depend on the charge given to it.

Reason (R) : The capacitance of a conductor depends only on geometry and size of conductor.

6. **Assertion** (A) : When a charged capacitor is filled completely with a metallic slab, its capacitance is increased by a large amount.

Reason (R) : The dielectric constant for metal is infinite.

7. **Assertion** (A) : The surface of a conductor is always an equipotential surface.

Reason (R) : A conductor contains free electrons which can move freely to equalise the potential.

8. **Assertion** (A) : When charged capacitors are connected in parallel, the algebraic sum of charges remains constant but there is a loss of energy.

Reason (R) : During sharing a charges, the energy conservation law does not hold.

9. **Assertion** (A) : In the absence of an externally applied electric field, the displacement per unit volume of a polar dielectric material is always zero.

Reason (R) : In polar dielectrics, each molecule has a permanent dipole moment but these are randomly oriented in the absence of an externally applied electric field.

[*AIIMS 2018*]

10. **Assertion** (A) : Work done in moving a charge around a closed path, in an electric field is always zero. [*CBSE 2023 (55/2/1)*]

Reason (R) : Electrostatic force is a conservative force.

Answers

1. (*a*) 2. (*a*) 3. (*b*) 4. (*d*) 5. (*a*) 6. (*a*) 7. (*a*)
8. (*c*) 9. (*a*) 10. (*a*)

Case-based/Passage-based Questions

Read the paragraph given below and answer the questions that follow:

1. **Faraday Cage:** A Faraday cage or Faraday shield is an enclosure made of a conducting material. The fields within a conductor cancel out with any external fields, so the electric field within the enclosure is zero. These Faraday cages act as big hollow conductors you can put things in to shield them from electrical fields. Any electrical shocks the cage receives, pass harmlessly around the outside of the cage.

[*CBSE Sample Paper 2021*]

(*i*) Which of the following material can be used to make a Faraday cage?

(*a*) Plastic (*b*) Glass (*c*) Copper (*d*) Wood

(*ii*) Example of a real-world Faraday cage is

(*a*) car (*b*) plastic box (*c*) lightning rod (*d*) metal rod

(iii) What is the electrical force inside a Faraday cage when it is struck by lightning?

(*a*) The same as the lightning

(*b*) Half that of the lightning

(*c*) Zero

(*d*) A quarter of the lightning

(iv) An isolated point charge $+q$ is placed inside the Faraday cage. Its surface must have charge equal to

(*a*) zero (*b*) $+q$

(*c*) $-q$ (*d*) $+2q$

OR

A point charge of 2 μC is placed at centre of Faraday cage in the shape of cube with surface of 9 cm edge. The number of electric field lines passing through the cube normally will be

(*a*) 1.9×10^5 Nm^2/C entering the surface

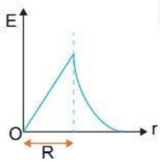
(*b*) 1.9×10^5 Nm^2/C leaving the surface

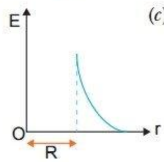
(*c*) 2.0×10^5 Nm^2/C leaving the surface

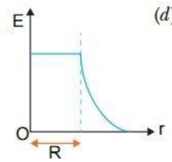
(*d*) 2.0×10^5 Nm^2/C entering the surface

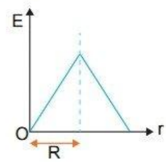
2. **Electrostatics:** Electrostatics deals with the study of forces, fields and potentials arising from static charges. Force and eletric field, due to a point charge is basically determined by Coulomb's law. For symmetric charge configurations, Gauss's law, which is also based on Coulomb's law, helps us to find the electric field. A charge a system of charges like a dipole experience a force/torque in an electric field. Work is required to be done to provide a specific orientation to a dipole with respect to an electric field. *[CBSE 2023 (55/4/1), Modified]*

(i) Consider a uniformly charged thin conducting shell of radius R. Which of the following graph showing the variation of $|\vec{E}|$ with distance r from the centre, for points $0 \leq r \leq 3r$.

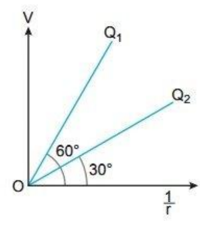
(*a*)


(*b*)


(*c*)


(*d*)


(ii) The figure shows the variation of potential V with $\frac{1}{r}$ for two point charges Q_1 and Q_2, where V is the potential at a distance r due to a point charge. The ratio, $\frac{Q_1}{Q_2}$ will be



(*a*) 1:3 (*b*) 3:1

(*c*) 2:1 (*d*) 1:2

(iii) An electric dipole of dipole moment of 6×10^{-7} C-m is kept in a uniform electric field of 10^4 N/C such that the dipole moment and the electric field are parallel. The potential energy of the dipole will be

(*a*) 2×10^3J (*b*) -2×10^3J (*c*) -6×10^{-3}J (*d*) 6×10^{-3}J

(iv) A dipole is placed parallel to electric field. If W is the work done in rotating the dipole from 0° to 60°, then work done in rotating it from 0° to 180° is

(*a*) 2 W (*b*) 3 W (*c*) 4 W (*d*) $\frac{W}{2}$

OR

The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volt. The electric field at the point (1 m, 0, 2 m) in volt/metre is

(*a*) 8 along negative *x*-axis

(*b*) 8 along positive *x*-axis

(*c*) 16 along negative *x*-axis

(*d*) 16 along positive *z*-axis

Explanations

1. (*i*) (*c*) Copper (Electric field inside a conductor is zero.)

(*ii*) (*a*) car (Body of the car is made up of conductor.)

(*iii*) (*c*) Zero (As electric field inside it is zero.)

(*iv*) (*c*) $-q$ (As from Gauss's law q_{in} must be zero for electric field inside it is zero.)

OR

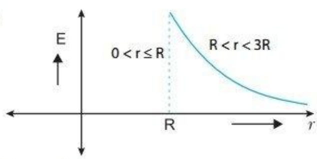
(*c*) $q = 2\ \mu C = 2 \times 10^{-6}$ C

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{Nm^2}{C^2}$$

Now, total number of electric field lines $= \frac{q_{in}}{\varepsilon_0} = \frac{2 \times 10^{-6}}{8.85 \times 10^{-12}}$

$$= 2.2 \times 10^5 \frac{Nm^2}{C} \text{ leaving the surface}$$

2. (*i*) (*b*)



(*ii*) (*b*) As we know,

$$V = \frac{KQ}{r} \quad \Rightarrow \quad Q = \frac{V}{k\left(\frac{1}{r}\right)} \qquad [k = \text{contant} = 9 \times 10^9\ Nm^2/C^2]$$

So, slope of V verses $\frac{1}{r}$ curve gives value of charge.

From the graph,

$$\text{Ratio, } \frac{Q_1}{Q_2} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{\tan 60°}{\tan 30°} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3.$$

(*iii*) (*c*) Here, $p = 6 \times 10^{-7}$ Cm, $E = 10^4$ N/C, $\theta = 0°$

Potential energy, $U = pE\cos\theta = -6 \times 10^{-7} \times 10^4 \times \cos 0° = -6 \times 10^{-3}$J

(*iv*) (*c*) $W = PE(\cos\theta_1 - \cos\theta_2)$

$$= PE(\cos 0° - \cos 60°)$$

$$= PE\left(1 - \frac{1}{2}\right) = \frac{1}{2}PE$$

And $W' = PE(\cos 0° - \cos 180°) = PE\{1 - (-1)\} = 2PE$

$$\text{Now, } \frac{W}{W'} = \frac{\frac{1}{2}PE}{2PE} = \frac{1}{4}$$

$\therefore \quad W' = 4W$

OR

(a) $E_x = -\frac{dV}{dx} = -\frac{d}{dx}(4x^2)$

$= -8x = -8 \times 1 = -8$ V/m

$\therefore \vec{E} = -8\hat{x}$ or 8 V/m along negative x-axis

CONCEPTUAL QUESTIONS

Q. 1. A charged particle ($+q$) moves in a uniform electric field ($\vec{E}$) in the direction opposite to $\vec{E}$. What will be the effect on its electrostatic potential energy during its motion?
[*CBSE 2020 (55/4/3)*]

Ans. As we know, $\Delta U = -W = -q\int E.dl$

When charge $+q$ moves in opposite direction to electric field then work done is negative.

$\Delta U = -W$

so, change in potential energy is positive, the potential energy increases.

Q. 2. Why is the electrostatic potential inside a charged conducting shell constant throughout the volume of the conductor? **[*CBSE 2019 (55/5/1)*]**

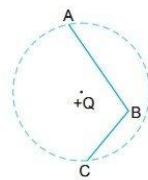
Ans. $E = 0$ inside the conductor & has no tangential component on the surface.

$\therefore$ No work is done in moving charge inside or on the surface of the conductor. & Potential is constant. **1**

[Even if a student writes "because $E = 0$ inside the conductor" - award full marks Or No work is done in moving a charge inside the conductor - award full marks.]

[*CBSE Marking Scheme 2019 (55/5/1)*]

Q. 3. In the given figure, charge $+Q$ is placed at the centre of a dotted circle. Work done in taking another charge $+q$ from A to B is W_1 and from B to C is W_2. Which one of the following is correct: $W_1 > W_2$, $W_1 = W_2$ and $W_1 < W_2$? **[*CBSE Sample Paper 2018*]**



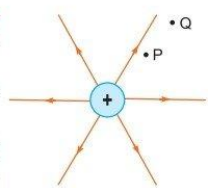
Ans. The points A and C are at same distance from the charge $+Q$ at the centre, so

$V_A = V_C$

Therefore, $V_A - V_B = V_C - V_B$

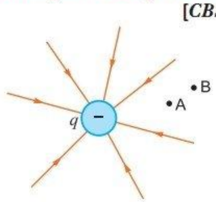
Hence, the magnitude of work done in taking charge $+q$ from A to B or from B to C will be the same *i.e.*, $W_1 = W_2$.

Q. 4. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason. **[*CBSE (F) 2014*]**



Ans. The work done by the field is negative. This is because the charge is moved against the force exerted by the field.

Q. 5. The field lines of a negative point charge are as shown in the figure. Does the kinetic energy of a small negative charge increase or decrease in going from B to A? **[*CBSE Patna 2015*]**



Ans. The kinetic energy of a negative charge decreases while going from point B to point A, against the movement of force of repulsion.

Q. 6. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero? **[*CBSE Delhi 2016*]**

Ans. The potential due to a point charge decreases with increase of distance. So, $V_A - V_B$ is positive.

Explanation: Let the distance of point A and B from charge Q be r_A and r_B respectively.

$$V_A = \frac{+Q}{4\pi\varepsilon_0 r_A} \text{ and } V_B = \frac{+Q}{4\pi\varepsilon_0 r_B}$$

$$V_A - V_B = \frac{+Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

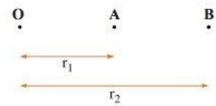
Also $r_A < r_B$

$$\Rightarrow \frac{1}{r_A} > \frac{1}{r_B} \quad \Rightarrow \frac{1}{r_A} - \frac{1}{r_B} > 0 \quad \Rightarrow \quad \frac{1}{r_A} - \frac{1}{r_B} \text{ has positive value}$$

Also Q is positive.

Hence $V_A - V_B$ is positive.

Q. 7. A point charge Q is placed at point 'O' as shown in figure. Is the potential at point A, *i.e.*, V_A, greater, smaller or equal to potential, V_B, at point B, when Q is (*i*) positive, and (*ii*) negative charge? **[*CBSE (F) 2017*]**



Ans. (*i*) If Q is positive,

$$V_A = \frac{KQ}{r_1} \quad \text{and} \quad V_B = \frac{KQ}{r_2}$$

Clearly, $V_A > V_B$

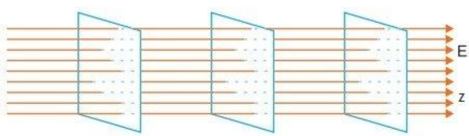
(*ii*) If Q is negative,

$$V_A = -\frac{KQ}{r_1} \quad \text{and} \quad V_B = -\frac{KQ}{r_2}$$

Clearly, $V_A < V_B$

Q. 8. Draw the equipotential surfaces corresponding to a uniform electric field in the *z*-direction. **[*CBSE 2019 (55/1/1)*]**

Ans. The equipotential surfaces are the equidistant planes normal to the *z*-axis, *i.e.*, planes parallel to the *X–Y* plane.



Q. 9. A point charge Q is placed at point O as shown in the figure. The potential difference $V_A - V_B$ is positive. Is the charge Q negative or positive? **[*CBSE (F) 2016*]**

Ans. We know that, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$$\Rightarrow \qquad V \propto \frac{1}{r}$$

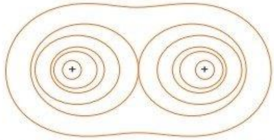
The potential due to a point charge decreases with increase of distance.

$$V_A - V_B > 0 \qquad \Rightarrow \qquad V_A > V_B$$

Hence, the charge Q is positive.

Q. 10. Depict the equipotential surfaces for a system of two identical positive point charges placed a distance '*d*' apart. *[CBSE Delhi 2010]*

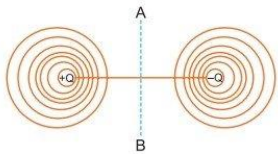
Ans. Equipotential surfaces due to two identical charges is shown in figure.



Q. 11. Draw an equipotential surface for a system consisting of two charges Q, – Q separated by a distance *r* in air. Locate the points where the potential due to the dipole is zero.

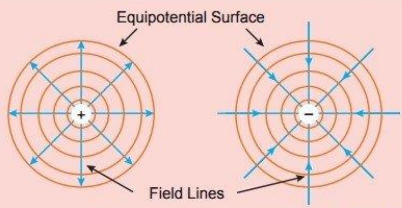
[CBSE Delhi 2017, (AI) 2008, 2013, 2019 (55/2/1), 2020 (55/4/2)]

Ans. The equipotential surface for the system is as shown. Electric potential is zero at all points in the plane passing through the dipole equator *AB*.



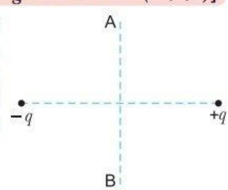
Q. 12. Draw the equipotential surfaces due to an isolated point charge. *[CBSE 2019 (55/1/2)]*

Ans. 1



[CBSE Marking Scheme 2019 (55/1/2)]

Q. 13. A charge '*q*' is moved from a point *A* above a dipole of dipole moment '*p*' to a point *B* below the dipole in equatorial plane without acceleration. Find the work done in the process. *[CBSE Central 2016]*



Ans. Work done in the process is zero. Because, equatorial plane of a dipole is equipotential surface and work done in moving charge on equipotential surface is zero.

$$W = qV_{AB} = q \times 0 = 0$$

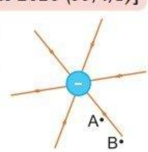
Q. 14. The work done in moving a charge particle between two points in an uniform electric field does not depend on the path followed by the particle. Why? *[CBSE 2020 (55/4/1)]*

Ans. Because the electrostatic force is conservative in nature 1

Alternatively:-

Electric field is conservative in nature / work done by or against the electric field does not depend upon the path followed. *[CBSE Marking Scheme 2020 (55/4/1)]*

Q. 15. Figure shows the field lines due to a negative point charge. Give the sign of the potential energy difference of a small negative charge between the points *A* and *B*. *[CBSE (F) 2014]*



Ans. $$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

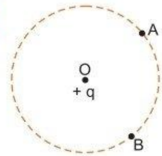
Since $r_A < r_B$

$$\therefore \quad \frac{kq_1q_2}{r_A} > \frac{kq_1q_2}{r_B}$$

$$\therefore \quad U_A > U_B$$

Therefore, $U_A - U_B$ is positive.

Q. 16. What is the amount of work done in moving a point charge Q around a circular arc of radius 'r' at the centre of which another point charge 'q' is located? **[*CBSE North 2016*]**



Ans. The potential of points A and B are same being equal to

$$V_A = V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

where R is the radius of the circle.

Work done $W = q\,(V_B - V_A) = q\,(V_A - V_A) = 0.$

Q. 17. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? **[*NCERT*]**

Ans. Electrostatic energy stored in capacitor, $U = \frac{1}{2}CV^2$

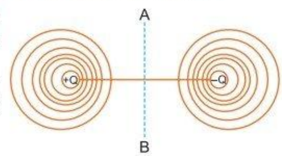
Here $C = 12\text{ pF} = 12 \times 10^{-12}\text{ F}, V = 50\text{ V}$

$$\therefore \quad U = \frac{1}{2}\times 12 \times 10^{-12} \times (50)^2 = \mathbf{1.5 \times 10^{-8}\ J}$$

Q. 18. Do free electrons travel to region of higher potential or lower potential? **[*NCERT Exemplar*]**

Ans. Free electrons would travel to regions of higher potentials as they are negatively charged.

Q. 19. Show that the equipotential surfaces are closed together in the regions of strong field and far apart in the regions of weak field. Draw equipotential surfaces for an electric dipole. **[*CBSE Sample Paper 2016*]**



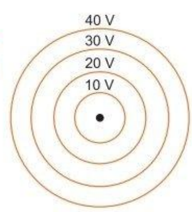
Ans. Equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field.

$$E = -\frac{dV}{dr}$$

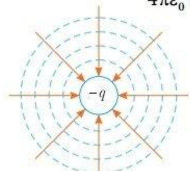
E = negative potential gradient

For same change in dV, $E \propto \frac{1}{dr}$ where 'dr' represents the distance between equipotential surfaces.

Q. 20. Concentric equipotential surfaces due to a charged body placed at the centre are shown. Identify the polarity of the charge and draw the electric field lines due to it. **[*HOTS*][*CBSE Sample Paper 2016*]**



Ans. For a single charge the potential is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$



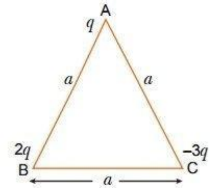
This shows that V is constant if r is constant. Greater the radius smaller will be the potential. In the given figure, potential is decreasing towards the centre of concentric equipotential surface. This shows that the polarity of charge is negative ($-q$). The direction of electric field will be radially inward. The field lines are directed from higher to lower potential.

Very Short Answer Questions

Each of the following questions are of 2 marks.

Q. 1. Obtain an expression for electrostatic potential energy of a system of three charges q, $2q$ and $-3q$ placed at the vertices of an equilateral triangle of side a. [*CBSE 2023 (55/4/1)*]

Ans. For the system of three charges at vertices of equilateral triangle,



P.E of each pair of charges, $U = \frac{k\, q_1 q_2}{r}$

then, $U_{AB} = \frac{k(q)(2q)}{a} = \frac{2kq^2}{a}$

And $U_{BC} = \frac{k(2q)(-3q)}{a} = \frac{-6kq^2}{a}$

$U_{CA} = \frac{k(q)(-3q)}{r} = \frac{-3kq^2}{a}$

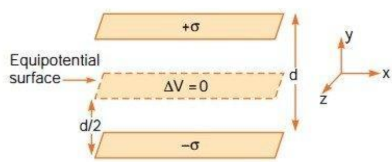
$\therefore$ Total Energy, $U = U_{AB} + U_{BC} + U_{CA}$

$$= \frac{kq^2}{a}[2-6-3] = -\frac{7kq^2}{a} \qquad \left[k = \frac{1}{4\pi\varepsilon_0}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\left(\frac{7q^2}{a}\right)$$

Q. 2. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the X-Z plane at a distance 'd' apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge '$-q$' remains stationary between the plates, what is the magnitude and direction of this field? [*CBSE Delhi 2011*]

Ans. The equipotential surface is at a distance $d/2$ from either plate in X-Z plane. For a particle of charge $(-q)$ at rest between the plates, then



(*i*) weight mg acts vertically downward

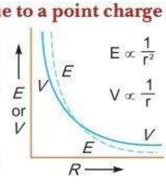
(*ii*) electric force qE acts vertically upward.

So, $mg = qE$

$E = \frac{mg}{q}$, vertically downward, *i.e.*, along $-Y$-axis.

Q. 3. Plot a graph comparing the variation of potential 'V' and electric field 'E' due to a point charge 'Q' as a function of distance 'R' from the point charge. [*CBSE Delhi 2012*]

Ans. The graph of variation of potential and electric field due to a point charge Q with distance R from the point charge is shown in figure.



Q. 4. What is electrostatic shielding? How is this property used in actual practice? Is the potential in the cavity of a charged conductor zero? [*CBSE South 2016*]

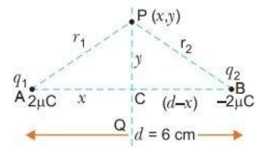
Ans. Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence. The field inside a conductor is zero. This is known as electrostatic shielding.

- Sensitive instruments are shielded from outside electrical influences by enclosing them in a hollow conductor.
- During lightning it is safest to sit inside a car, rather than near a tree. The metallic body of a car becomes an electrostatic shielding from lightening.

Potential inside the cavity is not zero. Potential is constant.

Q. 5. Two charges 2 μC and – 2 μC are placed at points *A* and *B* 6 cm apart.
(*a*) Identify an equipotential surface of the system.
(*b*) What is the direction of the electric field at every point on this surface? [*NCERT*]

Ans. (*a*) Let $P(x, y)$ be a point on zero potential surface. Let A (location of charge $q = 2\ \mu C$) be origin of coordinate system.



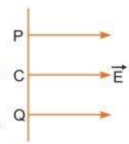
Distance $r_1 = \sqrt{x^2 + y^2}$, Distance $r_2 = \sqrt{(d - x)^2 + y^2}$

where $d = 6\text{ cm} = 6 \times 10^{-2}$ m.

Potential at P due to charges $q_1 = +2\ \mu C$ and $q_2 = -2\ \mu C$ is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = 0 \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-6}}{\sqrt{x^2+y^2}} + \frac{1}{4\pi\varepsilon_0}\frac{(-2\times10^{-6})}{\sqrt{(d-x)^2+y^2}} = 0$$

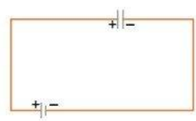
or $$\frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{(d-x)^2+y^2}} \Rightarrow x^2 + y^2 = (d-x)^2 + y^2 \Rightarrow x = \frac{d}{2} = \mathbf{3\ cm}$$



So, plane passing through mid point of line joining A and B has zero potential everywhere.

(*b*) The direction of electric field is normal to surface PCQ everywhere as shown in figure.

Q. 6. Why does current in a steady state not flow in a capacitor connected across a battery? However momentary current does flow during charging or discharging of the capacitor. Explain. [*CBSE (AI) 2017*]



Ans. (*i*) In the steady state no current flows through capacitor because, we have two sources (battery and fully charged capacitor) of equal potential connected in opposition.

(*ii*) During charging or discharging there is a momentary flow of current as the potentials of the two sources are not equal to each other.

Q. 7. Consider two identical point charges located at points (0, 0) and (*a*, 0).
1. Is there a point on the line joining them at which the electric field is zero?
2. Is there a point on the line joining them at which the electric potential is zero?
Justify your answers for each case. [*CBSE 2023 (55/2/1)*]

Ans. 1. Yes, electric field will zero at mid point.

Electric field being a vector quantity, its resultant is zero.

2. No, potential cannot be zero on line joining the charges.

Electric potential being a scalar quantity, the net potential due to two identical charges cannot be zero.

Q. 8. Two small conducting balls *A* and *B* of radius r_1 and r_2 have charges q_1 and q_2 respectively. They are connected by a wire. Obtain the expression for charges on *A* and *B*, in equilibrium.

Ans. According to law of conservation of charge, [*CBSE 2023 (55/4/1)*]

$$q_i = q_f$$

$$q_1 + q_2 = q'_1 + q'_2 = Q$$

When two balls are connected with wire,

$$V_1 = V_2$$

$$\Rightarrow \quad \frac{kq'_1}{r_1} = \frac{kq'_2}{r_2} \text{ or } \frac{q'_1}{r_1} = \frac{q'_2}{r_2}$$

$$\Rightarrow \quad q'_1 r_2 = q'_2 r_1$$

$$\Rightarrow \quad q'_1 r_2 = (Q - q'_1) r_1$$

$$\Rightarrow \quad q'_1 r_2 = Q r_1 - q'_1 r_1$$

$$\Rightarrow \quad q'_1 (r_1 + r_2) = Q r_1$$

$$\therefore \quad q'_1 = \frac{Q r_1}{r_1 + r_2} = \frac{(q_1 + q_2) r_1}{r_1 + r_2}$$

And, $q'_2 = Q - q'_1 = Q - \frac{Q r_1}{r_1 + r_2}$

$$= \frac{Q r_2}{r_1 + r_2} = \frac{(q_1 + q_2) r_2}{r_1 + r_2}$$

Q. 9. **(*a*) A parallel plate capacitor (C_1) having charge Q is connected, to an identical uncharged capacitor C_2 in series. What would be the charge accumulated on the capacitor C_2?**

(*b*) Three identical capacitors each of capacitance 3 μF are connected, in turn, in series and in parallel combination to the common source of V volt. Find out the ratio of the energies stored in two configurations. ***[CBSE South 2016]***

Ans. (*a*) Since the capacitor C_2 is uncharged so when connected to an identical capacitor C_1 charged to Q then charge Q is equally shared and charge acquired by capacitor C_2 is $\frac{Q}{2}$.

(*b*) We have $C_{series} = \frac{3\mu F}{3} = 1\mu F$

Also, $C_{parallel} = (3 + 3 + 3) = 9\ \mu F$

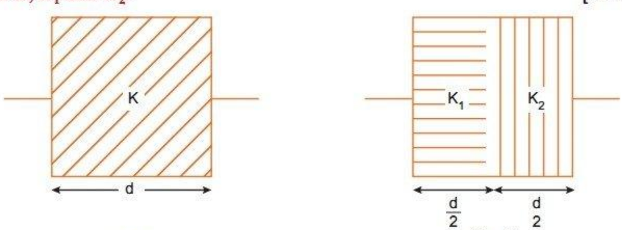
Energy stored $= \frac{1}{2} CV^2$

$\therefore$ Energy in series combination $= \frac{1}{2} \times 1 \times 10^{-6} \times V^2 \quad \Rightarrow \quad U_{Series} = \frac{10^{-6}}{2} V^2$

$\therefore$ Energy in parallel combination $= \frac{1}{2} \times 9 \times 10^{-6} \times V^2 \quad \Rightarrow \quad U_{parallel} = \frac{10^{-6} \times 9}{2} V^2$

$\therefore$ $U_{series} : U_{parallel} = \mathbf{1 : 9}$

Q. 10. **The space between the plates of a parallel plate capacitor is completely filled in two ways. In the first case, it is filled with a slab of dielectric constant K. In the second case, it is filled with two slabs of equal thickness and dielectric constants K_1 and K_2 respectively as shown in the figure. The capacitance of the capacitor is same in the two cases. Obtain the relationship between K, K_1 and K_2.** ***[CBSE 2020 (55/1/1)]***



(Case 1) (Case 2)

Ans. **Case I:** $C_1 = \frac{K \varepsilon_0 A}{d}$

Case II: It is filled with two slabs of equal thickness with dielectric constants K_1 and K_2 respectvely in given such way. So it behave as a capacitors are connected in series.

C_2 = Series combination of two capacitors (let)

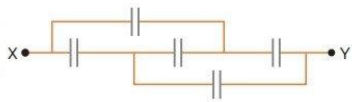
$$C' = \frac{K_1 \varepsilon_0 A}{d/2},\quad C'' = \frac{K_2 \varepsilon_0 A}{d/2}$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{C'} + \frac{1}{C''} = \frac{d}{2K_1\varepsilon_0 A} + \frac{d}{2K_2\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

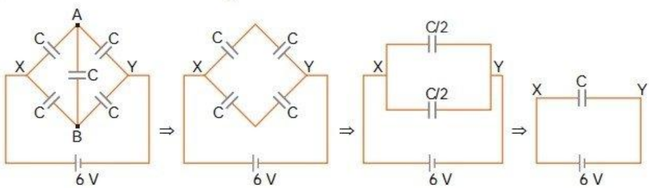
$$\therefore \quad C_2 = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

$$\because \quad C_1 = C_2 \text{ (given)} \quad \Rightarrow \quad \frac{K\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right) \quad \Rightarrow \quad \mathbf{K = \frac{2K_1 K_2}{K_1 + K_2}}$$

Q. 11. Find the equivalent capacitance of the network shown in the figure, when each capacitor is of 1 μF. When the ends *X* and *Y* are connected to a 6 V battery, find out (*i*) the charge and (*ii*) the energy stored in the network. [*CBSE Patna 2015*]



Ans. The given circuit can be rearranged as



It is known as wheatstone bridge of the capacitor.

Since $V_A = V_B$, so the bridge capacitor between points A and B can be removed.

(*i*) The equivalent capacitor of the network

$$C_{eq} = \frac{C \times C}{C + C} + \frac{C \times C}{C + C}$$

$$= \frac{C}{2} + \frac{C}{2} = C = 1\mu\text{F}$$

Charge in the network, $Q = C_{eq} V = C \times V$

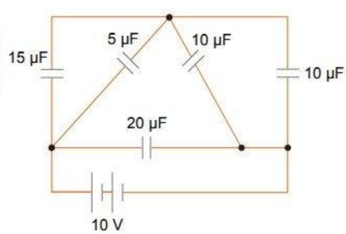
$$= 1\,\mu\text{F} \times 6\text{ V} = \mathbf{6\,\mu C}$$

(*ii*) Energy stored in the capacitor,

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 1\,\mu\text{F} \times (6)^2$$

$$= \mathbf{18\,\mu J}$$

Q. 12. The figure shows a network of five capacitors connected to a 10 V battery. Calculate the charge acquired by the 5 μF capacitor. [*CBSE 2019 (55/3/3)*]



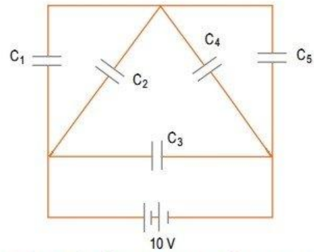
Ans. Net capacitance of parallel C_1 & $C_2 = C_1 + C_2$

$C_{12} = 15 + 5 = 20\,\mu\text{F}$

Net capacitance of parallel C_4 & $C_5 = C_4 + C_5$

$C_{45} = 10 + 10 = 20\,\mu\text{F}$

C_{12}, C_{45} in series, $C_{1245} = \dfrac{C_{12}C_{45}}{C_{12}+C_{45}} = \dfrac{20\times20}{20+20} = 10\,\mu F$



C_3 in parallel with $C_{1245} = C_{1245} + C_3 = 10 + 20 = 30\,\mu F$

P.D. across $C_{1245} = 10$ V

P.D. across $C_{12} = C_{45} = 5$ V

Charge on $5\,\mu F$, $Q = CV$

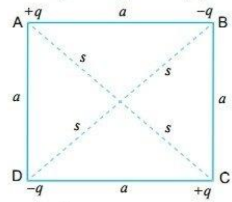
$= 5 \times 10^{-6} \times 5\text{ C} = \mathbf{25 \times 10^{-6}\text{ C}}$

Q. 13. Four charges $+q, -q, +q$ and $-q$ are to be arranged respectively at the four corners of a square *ABCD* of side '*a*'.

(*a*) Find the work required to put together this arrangement.

(*b*) A charge q_0 is brought to the centre of the square, the four charges being held fixed. How much extra work is needed to do this? **[*HOTS*][*CBSE (F) 2015*]**

Ans. (*a*) Work done in bringing charge $+q$ at point A



$$W_A = 0$$

Work done in bringing charge $-q$ to the point B

$$W_B = W_{AB} = -q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$$

Work done in bring the charge $+q$ to the point C

$$W_C = W_{AC} + W_{BC}$$

$$= q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a\sqrt{2}} + q \times \left(-\frac{1}{4\pi\varepsilon_0}\frac{q}{a}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a\sqrt{2}} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$$

Work done in bringing a charge $-q$ to the point D

$$W_D = W_{AD} + W_{BD} + W_{CD}$$

$$= -q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + (-q)\left(\frac{1}{4\pi\varepsilon_0}\frac{-q}{a\sqrt{2}}\right) + (-q) \times \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$$

Total work done $W = W_A + W_B + W_C + W_D$

$$= 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a\sqrt{2}} - 4 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}(\sqrt{2}-4)$$

(*b*) Work done in bringing a charge from infinity to a point is given by

$W = q_0 V_p$ (V_p = Electric potential at the point)

Electric potential at the centre of the square is, where $s = \dfrac{a}{\sqrt{2}}$

$$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{s}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{s}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{s}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{s}\right) = 0$$

and electric potential at infinity is always zero.

Hence, work done $W = 0$.

Q. 14. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

[*HOTS*][*NCERT Exemplar*]

Ans. Since two spheres are at the same potential, therefore

$$V_1 = V_2$$

$$\frac{Q_1}{4\pi\varepsilon_0 R_1} = \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2} \qquad \ldots(i)$$

Given, $R_1 > R_2$, $\therefore$ $Q_1 > Q_2$

$\Rightarrow$ Larger sphere has more charge

Now, $\sigma_1 = \frac{Q_1}{4\pi R_1^2}$ and $\sigma_2 = \frac{Q_2}{4\pi R_2^2}$

$$\Rightarrow \quad \frac{\sigma_2}{\sigma_1} = \frac{Q_2}{Q_1} \cdot \frac{R_1^2}{R_2^2}$$

$$\Rightarrow \quad \frac{\sigma_2}{\sigma_1} = \frac{R_2}{R_1} \cdot \frac{R_1^2}{R_2^2} = \frac{R_1}{R_2} \qquad \text{[From equation (i)]}$$

Since $R_1 > R_2$, therefore $\sigma_2 > \sigma_1$.

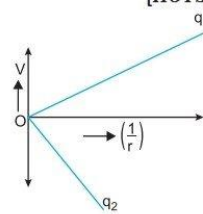
Charge density of smaller sphere is more than that of larger one.

Q. 15. The two graphs are drawn below, show the variations of electrostatic potential (V) with $\frac{1}{r}$ (r being the distance of field point from the point charge) for two point charges q_1 and q_2.

(*i*) What are the signs of the two charges?

(*ii*) Which of the two charges has the larger magnitude and why? **[*HOTS*]**

Ans. (*i*) The potential due to positive charge is positive and due to negative charge, it is negative, so, q_1 is **positive** and q_2 is **negative**.



(*ii*) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

The graph between V and $\frac{1}{r}$ is a straight line passing through the origin with slope $\frac{q}{4\pi\varepsilon_0}$.

As the magnitude of slope of the line due to charge q_2 is greater than that due to q_1, q_2 has larger magnitude.

Q. 16. Two identical capacitors of 12 pF each are connected in series across a 50 V battery. Calculate the electrostatic energy stored in the combination. If these were connected in parallel across the same battery, find out the value of the energy stored in this combination. [*CBSE 2019 (55/5/1)*]

Ans. Net capacitance in series combination is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \quad \Rightarrow \quad \frac{1}{C_s} = \frac{1}{12} + \frac{1}{12}$$

$$\Rightarrow \quad C_s = 6 \text{ pF}$$

$$E_s = \frac{1}{2}C_s V^2 = \frac{1}{2} \times 6 \times 10^{-12} \times 50 \times 50$$

$$= 7500 \times 10^{-12} \text{ J}$$

$$= 7.5 \times 10^{-9} \text{ J}$$

Net capacitance in parallel combination is given by

$$C_p = 12 \text{ pF} + 12 \text{ pF} = 24 \text{ pF}$$

$$E_p = \frac{1}{2}C_p V^2 = \frac{1}{2} \times 24 \times 10^{-12} \times 50 \times 50$$

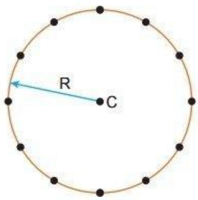
$$= \mathbf{3 \times 10^{-8}\ J}$$

Short Answer Questions

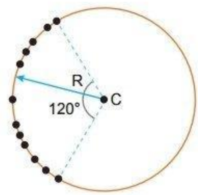
Each of the following questions are of 3 marks.

Q. 1. **(*a*) Twelve negative charges of same magnitude are equally spaced and fixed on the circumference of a circle of radius *R* as shown in Fig. (*i*). Relative to potential being zero at infinity, find the electric potential and electric field at the center *C* of the circle.**

(*b*) If the charges are unequally spaced and fixed on an arc of 120° of radius *R* as shown in Fig. (*ii*), find electric potential at the centre *C*. [*CBSE 2023 (55/1/1)*]



(i)

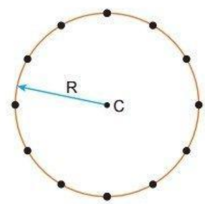


Ans. (*a*) Potential due to single charge at *C*, $V = \dfrac{kq}{r_1}$

Here, $q_1 = q_2 = ... = q_{12} = -q$ and $r_1 = r_2 = ... r_{12} = R$

So, net electric potential at *C*,

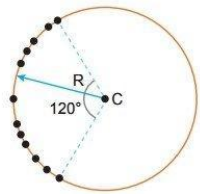
$$V_C = V_1 + V_2 + ... + V_{12}$$
$$= -kq\left[\frac{1}{R}\times 12\right]$$
$$= -\frac{\mathbf{12}kq}{R}.$$



Due to symmetry of distribution of charges.

All electric fields are cancel to each other.

So, $E = \mathbf{0}$



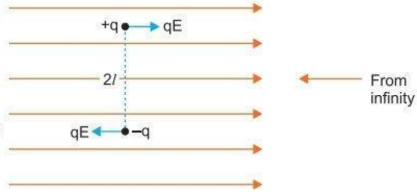
(*b*) Potential is a scalar and orientation is irrelevant.

Hence, Net potential at C,

$$V_C = -\frac{\mathbf{12}kq}{R}.$$

Q. 2. **Show that the potential energy of a dipole making angle θ with the direction of the field is given by $U(\theta) = -\vec{P}\cdot\vec{E}$. Hence find out the amount of work done in rotating it from the position of unstable equilibrium to the stable equilibrium.** [*CBSE East 2016*]

Ans. The potential energy of an electric dipole in an electric field is defined as the work done in bringing the dipole from infinity to its present position in the electric field.



Suppose the dipole is brought from infinity and placed at orientation θ with the direction of electric field. The work done in this process may be supposed to be done in two parts.

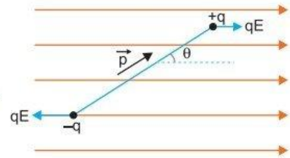
(*i*) The work done (W_1) in bringing the dipole perpendicular to electric field from infinity.

(*ii*) Work done (W_2) in rotating the dipole such that it finally makes an angle θ from the direction of electric field.

Let us suppose that the electric dipole is brought from infinity in the region of a uniform electric field such that its dipole moment $\vec{P}$ always remains perpendicular to electric field. The electric forces on charges $+q$ and $-q$ are qE and $-qE$, along the field direction and opposite to field direction respectively.

As charges $+q$ and $-q$ traverse equal distance under equal and opposite forces; therefore, net work done in bringing the dipole in the region of electric field perpendicular to field-direction will be zero, *i.e.*, $W_1 = 0$.

Now the dipole is rotated and brought to orientation making an angle θ with the field direction (*i.e.*, $\theta_0 = 90°$ and $\theta_1 = \theta$), therefore, work done



$$W_2 = pE\,(\cos\theta - \cos\theta_1)$$

$$= pE\,(\cos 90° - \cos\theta) = -pE\cos\theta$$

∴ Total work done in bringing the electric dipole from infinity, *i.e.*,

Electric potential energy of electric dipole

$$U = W_1 + W_2 = 0 - pE\cos\theta = -pE\cos\theta$$

In vector form $U = -\vec{p}\cdot\vec{E}$

For rotating dipole from position of unstable equilibrium ($\theta_0 = 180°$) to the stable equilibrium ($\theta = 0°$)

∴ $$W_{req} = pE(\cos 180° - \cos 0°)$$

$$= pE(-1-1) = -2pE$$

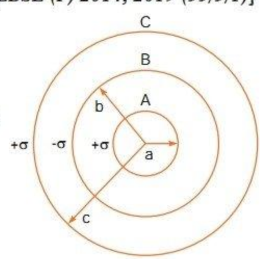
Q. 3. Three concentric metallic shells *A*, *B* and *C* of radii *a*, *b* and *c* ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively as shown in the figure.
If shells *A* and *C* are at the same potential, then obtain the relation between the radii *a*, *b* and *c*.

[*CBSE (F) 2014, 2019 (55/5/1)*]

Ans. Charge on shell *A*, $q_A = 4\pi a^2\sigma$

Charge on shell *B*, $q_B = -4\pi b^2\sigma$

Charge of shell *C*, $q_C = 4\pi c^2\sigma$



Potential of shell *A*: Any point on the shell *A* lies inside the shells *B* and *C*.

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

Any point on *B* lies outside the shell *A* and inside the shell *C*. Potential of shell *B*,

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

Any point on shell *C* lies outside the shells *A* and *B*. Therefore, potential of shell *C*.

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{c} + \frac{q_B}{c} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2\sigma}{c} - \frac{4\pi b^2\sigma}{c} + \frac{4\pi c^2\sigma}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right]$$

Now, we have

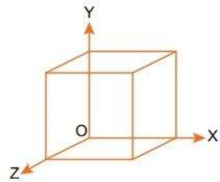
$$V_A = V_C$$

$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

$$a - b = \frac{(a-b)(a+b)}{c}$$

or $\boldsymbol{a + b = c}$

Q. 4. A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m. **[*CBSE 2020 (55/2/1)*]**



Find the

(*i*) electric field, and

(*ii*) total electric flux through the cube.

Ans. (*i*) We have, $E = -\frac{dV}{dx} = -\frac{d}{dx}(10x + 5)$ 1

$\therefore \quad \boldsymbol{\vec{E} = -10\,\hat{i}}$ **N/C** ½

(*ii*) Electric flux through the cube, ϕ = sum of electric flux through 6 faces.

Electric flux through perpendicular Y and Z axis = 0 ½

$\because E$ is along x axis

Electric flux through faces perpendicular to x – axis, ½

$$\phi = \phi_1 + \phi_2$$ ½

$= 10 \times (0.2)^2 - 10 \times (0.2)^2 = 0$ [*CBSE Marking Scheme 2020 (55/2/1)*]

Q. 5. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to the another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process? **[*NCERT*]**

Ans. Given, $C_1 = 600$ pF $= 600 \times 10^{-12}$ F, $V_1 = 200$ V

Initial energy stored, $U_{initial} = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2 = 12 \times 10^{-6}$ J

When another uncharged capacitor C_2=600 pF is connected across capacitor C_1 then common potential difference

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + 0}{C_1 + C_2} = \frac{C_1V_1}{C_1 + C_2}$$

$$= \frac{600 \times 10^{-12} \times 200}{(600 + 600) \times 10^{-12}} = 100 \text{ V}$$

$\therefore$ Final electrostatic energy, $U_{\text{final}} = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(600 + 600) \times 10^{-12} \times (100)^2 = 6 \times 10^{-6}$ J

$\therefore$ Energy lost, $\Delta U = U_{\text{initial}} - U_{\text{final}} = 12 \times 10^{-6} - 6 \times 10^{-6} = \boldsymbol{6 \times 10^{-6}}$ **J**

Q. 6. A parallel plate capacitor (*A*) of capacitance *C* is charged by a battery to voltage *V*. The battery is disconnected and an uncharged capacitor (*B*) of capacitance 2*C* is connected across *A*. Find the ratio of **[*CBSE 2023 (55/2/1)*]**

(*i*) final charges on *A* and *B*.

(*ii*) total electrostatic energy stored in *A* and *B* finally and that stored in *A* initially.

Ans. (*i*) Total charge on A, $q_A = CV$

After nn with capacitor B,

$$V_{\text{common}} = \frac{q_A + q_B}{C_A + C_B} = \frac{CV}{2C + C} = \frac{V}{3}$$

Now, final charge , $q'_A = C\left(\frac{V}{3}\right) = \frac{CV}{3}$

and $q'_B = 2C\left(\frac{V}{3}\right) = \frac{2CV}{3}$

So, Ratio, $\frac{q'_A}{q'_B} = \frac{\frac{CV}{3}}{\frac{2CV}{3}} = \mathbf{\frac{1}{2}}$

(*ii*) Initial energy stored on A,

$$U_A = \frac{1}{2}CV^2$$

Now, finally energy stored on A and B both,

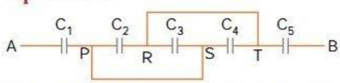
$$E_T = E_A + E_B$$

$$= \frac{1}{2}C\left(\frac{V}{3}\right)^2 + \frac{1}{2}2C\left(\frac{V}{3}\right)^2$$

$$= \frac{1}{2}3C\left(\frac{V^2}{q}\right) = \frac{CV^2}{6} = \frac{U_A}{3}$$

Now, Ratio, $\frac{U_T}{U_A} = \frac{\frac{U_A}{3}}{U_A} = \mathbf{\frac{1}{3}}$

Q. 7. (*i*) Find equivalent capacitance between A and B in the combination given below. Each capacitor is of 2 μF capacitance.



(*ii*) If a dc source of 7 V is connected across *AB*, how much charge is drawn from the source and what is the energy stored in the network? **[*CBSE Delhi 2017*]**

Ans. (*i*) Capacitors C_2, C_3 and C_4 are in parallel

$$C_{234} = C_2 + C_3 + C_4 = 2\ \mu F + 2\ \mu F + 2\ \mu F$$

$\therefore$ $C_{234} = 6\ \mu F$

Capacitors C_1, C_{234} and C_5 are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{234}} + \frac{1}{C_5} = \frac{1}{2} + \frac{1}{6} + \frac{1}{2}$$

$$= \frac{7}{6}$$

$$\therefore C_{eq} = \mathbf{\frac{6}{7}\ \mu F}$$

(*ii*) Charge drawn from the source

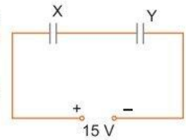
$$Q = C_{eq}V$$

$$= \frac{6}{7} \times 7\ \mu C = \mathbf{6\ \mu C}$$

Energy stored in the network, $U = \frac{Q^2}{2C} = \frac{6\times6\times10^{-12}\times7}{2\times6\times10^{-6}}$ J

$$= 21\times10^{-6}\ \text{J} = \mathbf{21\ \mu J}$$

Q. 8. Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium $\varepsilon_r = 4$.



(i) Calculate the capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Calculate the potential difference between the plates of X and Y.

(iii) Estimate the ratio of electrostatic energy stored in X and Y. ***[CBSE Delhi 2016]***

Ans. (i) Capacitance of X, $C_X = \frac{\varepsilon_0 A}{d}$

Capacitance of Y, $C_Y = \frac{\varepsilon_r \varepsilon_0 A}{d} = 4\frac{\varepsilon_0 A}{d}$

$\therefore \quad \frac{C_Y}{C_X} = 4 \Rightarrow C_Y = 4C_X$ ...(i)

As X and Y are in series, so

$$C_{eq} = \frac{C_X C_Y}{C_X + C_Y} \Rightarrow 4\ \mu\text{F} = \frac{C_X . 4C_X}{C_X + 4C_X}$$

$\Rightarrow \quad C_X = \mathbf{5\ \mu F}$ and $C_Y = 4C_X = \mathbf{20\ \mu F}$

(ii) In series charge on each capacitor is same, so

P.d. $V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$

$\therefore \quad \frac{V_X}{V_Y} = \frac{C_Y}{C_X} = 4 \Rightarrow V_X = 4V_Y$ ...(ii)

Also $\quad V_X + V_Y = 15$ ...(iii)

From (ii) and (iii),

$$4V_Y + V_Y = 15 \Rightarrow V_Y = 3\ \text{V}$$

$$V_X = 15 - 3 = 12\ \text{V}$$

Thus potential difference across X, V_X = **12 V,** P.d. across Y, V_Y = **3 V**

(iii) $\frac{\text{Energy stored in } X}{\text{Energy stored in } Y} = \frac{Q^2/2C_X}{Q^2/2C_Y} = \frac{C_Y}{C_X} = \frac{4}{1} \Rightarrow \frac{U_X}{U_Y} = \frac{\mathbf{4}}{\mathbf{1}}$

Q. 9. A capacitor of 4 μF is charged by a battery of 12 V. The battery is disconnected and a dielectric slab of dielectric constant 8 is inserted in between the plates of the capacitor of fill the space completely. Find the change in the (*a*) charge stored in the capacitor, (*b*) potential difference between the places of the capacitor, and (*c*) energy stored in the capacitor. [*CBSE 2020 (55/2/3)*]

Ans. (*a*) Charge, $q = CV = 48 \times 10^{-6}$ C ½

Charge remains same or no change. ½

(*b*) Initial potential difference, $V_0 = 12$ V

After battery is removed,

$V = \frac{V_0}{K} = \frac{12}{8} = 1.5$ volt ½

Decrease in potential difference = 12 – 1.5 = 10.5 V ½

(*c*) Energy stored initially,

$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\times4\times10^{-6}\times12\times12 = 288\times10^{-6}$ J ½

Final energy stored, $U = \frac{1}{2}CV^2 = \frac{1}{2}C_0 \frac{V_0^2}{K} = \frac{1}{2} \times \frac{4 \times 10^{-6} \times 12 \times 12}{8} = 36 \times 10^{-6}$J

$\therefore$ Energy reduces, $\Delta U = U_0 - U = (288 - 36) \times 10^{-6}$ J $= 252 \times 10^{-6}$ J ½

Note : (*b*) Even if a student writes, voltage becomes $\frac{1}{8}$ award full one mark.

(*c*) Even if a student, without evaluating U_o and U, writes U becomes $\frac{1}{k}$, award $\frac{1}{2}$ mark; and award $\frac{1}{2}$ mark for the correct formulae for U and U_o

[CBSE Marking Scheme 2020 (55/2/3)]

Q. 10. A 200 μF parallel plate capacitor having plate separation of 5 mm is charged by a 100 V *dc* source. It remains connected to the source. Using an insulated handle, the distance between the plates is doubled and a dielectric slab of thickness 5 mm and dielectric constant 10 is introduced between the plates. Explain with reason, how the (*i*) capacitance, (*ii*) electric field between the plates, (*iii*) energy density of the capacitor will change? *[CBSE 2019 (55/2/1)]*

Ans. Dielectric slab of thickness 5 mm is equivalent to an air capacitor of thickness $= \frac{5}{10}$ mm.

Effective separation between the plates with air in between is = (5 + 0.50) mm = 5.5 mm

(*i*) Effective new capacitance,

$$C' = 200\,\mu\text{F} \times \frac{5\text{ mm}}{5.5\text{ mm}} = \frac{2000}{11}\,\mu\text{F}$$

$$\approx \mathbf{182\,\mu F}$$

(*ii*) Effective new electric field,

$$E' = \frac{100\,V}{5.5 \times 10^{-3}\text{m}} = \frac{200000}{11}\text{ V/m, where } E = \frac{V}{d} = \frac{100}{5 \times 10^{-3}} = 20000\text{ V/m}$$

$$\approx \mathbf{18182\ V/m}$$

(*iii*) $$\frac{\text{New energy density}}{\text{Original energy density}} = \frac{\frac{1}{2}\varepsilon_0 E'^2}{\frac{1}{2}\varepsilon_0 E^2} = \left(\frac{E'}{E}\right)^2 = \left(\frac{10}{11}\right)^2$$

New Energy density will be $\left(\frac{10}{11}\right)^2$ of the original energy density = $\mathbf{\frac{100}{121}}$ the original energy density.

Q. 11. In a parallel plate capacitor with air between the plates, each plate has an area of 6 × 10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

[NCERT] [HOTS]

Ans. Capacitance of parallel plate air capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

Given $A = 6 \times 10^{-3}$ m^2, d = 3 mm = 3×10^{-3} m, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.

$$\therefore \quad C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = \mathbf{17.7 \times 10^{-12}\ F}$$

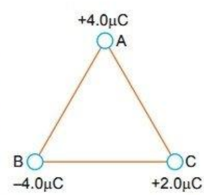
Charge on each plate of capacitor,

$$Q = CV = 17.7 \times 10^{-12} \times 100 = 1.77 \times 10^{-9}\text{ coulomb} = \mathbf{1.77\ nC}$$

Q. 12. State the significance of negative value of electrostatic potential energy of a system of charges.

Three charges are placed at the corners of an equilateral triangle ABC of side 2.0 m as shown in figure. Calculate the electric potential energy of the system of three charges.

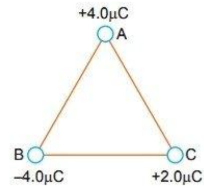
[CBSE, 2023] [55/2/1]



Ans. Negative value of electrostatic potential energy of a system signifies that the system has attractive forces.

Potential energy due to between two charges,

$$U = \frac{1}{4\pi \in_0} \times \frac{q_1 q_2}{r}$$



P.E due to system of three charges,

$$U = \frac{1}{4\pi \in_0}\left[\frac{q_A q_B}{r} + \frac{q_B q_C}{r} + \frac{q_C q_A}{r}\right]$$

$$= \frac{9\times 10^9}{2}[-16-8+8]\times 10^{-12}$$

$$= -7.2 \times 10^{-2}\ \text{J}$$

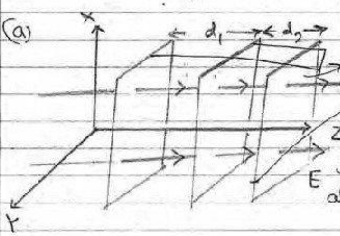
Q. 13. **(*a*) Draw equipotential surfaces corresponding to the electric field that uniformly increases in magnitude along with the *z*-directions.**

(*b*) Two charges –*q* and +*q* are located at point (0, 0, –*a*) and (0, 0, *a*). What is the electrostatic potential at the points (0, 0, ± *z*) and (*x*, *y*, 0)? **[*CBSE 2019 (55/1/2)*]**

Ans.

towards right

(a)



(Parallel to ~~Along~~ x-y plane)

Plane surface (equipotential surfaces)

Also, $d_2 < d_1$ as electric field is uniformly increasing along the z-direction (axis)

(b) Electrostatic potential at point (x, y, 0) is zero because it is lying on the equitorial plane of the dipole and is equidistant from both +q and –q.

(0, 0, –z) –q (0, 0, –a) q (0, 0, a) (0, 0, +z)

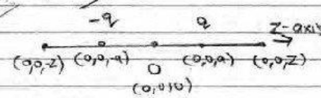
Now, potential at (0, 0, +z)

$$V_{+z} = V_q + V_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-a)} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(z+a)}$$

$$V_{+z} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z-a} - \frac{1}{z+a}\right]$$

$$V_{+z} = \frac{q}{4\pi\varepsilon_0}\left[\frac{z+a-z+a}{z^2-a^2}\right]$$

$$V_{+z} = \frac{2qa}{4\pi\varepsilon_0(z^2-a^2)}$$



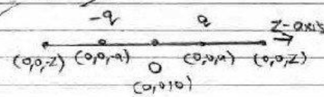
and $V_{-z} = V_{-q} + V_q$

$$= \frac{-q}{4\pi\varepsilon_0(z-a)} + \frac{q}{4\pi\varepsilon_0}\frac{1}{(z+a)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z+a} - \frac{1}{z-a}\right]$$

$$V_{+z} = \frac{q}{4\pi\varepsilon_0}\left[\frac{z+a-z+a}{z^2-a^2}\right]$$

$$V_{+z} = \frac{2qa}{4\pi\varepsilon_0(z^2-a^2)}$$



and $V_{-z} = V_{-q} + V_q$

$$= \frac{-q}{4\pi\varepsilon_0(z-a)} + \frac{q}{4\pi\varepsilon_0}\frac{1}{(z+a)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z+a} - \frac{1}{z-a}\right]$$

$$= \frac{q\,(-2a)}{4\pi\varepsilon_0(z^2-a^2)} = \frac{-2qa}{4\pi\varepsilon_0(z^2-a^2)}$$

Hence, electrostatic potential at point $(0,0,z)$ is $\frac{1}{4\pi\varepsilon_0}\frac{2qa}{z^2-a^2}$

and at $(0,0,-z)$ is $\frac{-1}{4\pi\varepsilon_0}\frac{2qa}{z^2-a^2}$ [*Toppers Answer 2019*]

Q. 14. Two capacitors of capacitance 10 μF and 20 μF are connected in series with a 6 V battery. After the capacitors are fully charged, a slab of dielectric constant (*K*) is inserted between the plates of the two capacitors. How will the following be affected after the slab is introduced:

(*a*) the electric field energy stored in the capacitors?

(*b*) the charges on the two capacitors?

(*c*) the potential difference between the plates of the capacitors?

Justify your answer. [*CBSE Bhubaneshwer 2015*]

Ans. Let Q be the charge on each capacitor. So, $Q = \frac{C_1C_2}{C_1+C_2}V$.

Initial electric field energy in each capacitor becomes

$$U_1 = \frac{1}{2}\frac{Q^2}{C_1} \text{ and } U_2 = \frac{1}{2}\frac{Q^2}{C_2}$$

Initial charge on each capacitor

$$Q = C_1V_1,\ \ Q = C_2V_2 \text{ and } Q = \frac{C_1C_2}{C_1+C_2}.V$$

where V_1 and V_2 are *p.d* across the capacitors

On inserting the dielectric slab the capacitance of each capacitor becomes

$$C'_1 = KC_1 \text{ and } C'_2 = KC_2$$

and equivalent capacitance becomes

$$C'_{eq} = \frac{KC_1 \times KC_2}{KC_1 + KC_2} = K\frac{C_1C_2}{C_1+C_2}$$

New charge on the capacitor becomes

$$Q' = C'_{eq}V' = K\left(\frac{C_1C_2}{C_1+C_2}\right)\times V$$

$$Q' = \frac{C_1C_2}{C_1+C_2}.V \times K$$

$$Q' = Q \times K$$

$$Q' = KQ$$

(*a*) New electric field energy becomes

$$U'_1 = \frac{Q'^2}{2KC_1} = \frac{KQ^2}{2C_1}$$

$$U'_2 = \frac{1}{2}\frac{Q'^2}{KC_2} = \frac{KQ^2}{2C_2}$$

i.e., electric field energy increases in each capacitor.

(*b*) $Q' = KQ$ (as stated above) *i.e.*, charges are increases on each capacitor.

(*c*) $$V'_1 = \frac{Q'}{C_1'} = \frac{KQ}{KC_1} = \frac{Q}{C_1}$$

and $$V'_2 = \frac{Q'}{C_2'} = \frac{KQ}{KC_2} = \frac{Q}{C_2}$$

i.e., *p.d* across each capacitor remains same.

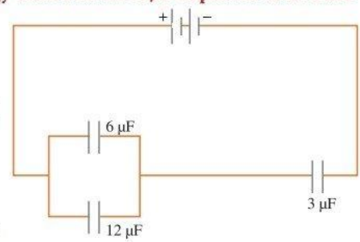
Q. 15. In the following arrangement of capacitors, the energy stored in the 6 μF capacitor is E. Find the value of the following:

(*i*) Energy stored in 12 μF capacitor.

(*ii*) Energy stored in 3 μF capacitor.

(*iii*) Total energy drawn from the battery.

[CBSE (F) 2016]



Ans. Given that energy stored in 6 μF is E.

(*i*) Let V be the voltage across 6 μF capacitor

Also, 6 μF and 12 μF capacitors are in parallel.

Therefore, voltage across 12 μF = Voltage across 6 μF capacitor

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\times 6\times V^2 \quad \Rightarrow \quad V = \sqrt{\frac{E}{3}}$$

Energy stored in $12\mu F = \frac{1}{2}\times 12\times\left(\sqrt{\frac{E}{3}}\right)^2 = \mathbf{2E}$

(*ii*) Since charge remains constant in series. Sum of charge on 6 μF capacitor and 12 μF capacitor is equal to charge on 3 μF capacitor.

Using $Q = CV$,

Charge on 3 μF capacitor = $(6 + 12) \times V = 18V$

Energy stored in 3 F capacitor $= \frac{Q^2}{2C} = \frac{(18V)^2}{2\times 3} = \frac{18\times 18}{6}\left(\frac{\sqrt{E}}{3}\right)^2 = \mathbf{18E}$

(*iii*) Total energy drawn from battery = $E + 2E + 18E = \mathbf{21E}$

Q. 16. In the figure given below, find the

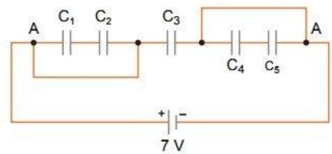
(*a*) equivalent capacitance of the network between points *A* and *B*.

Given: $C_1 = C_5 = 8$ μF, $C_2 = C_3 = C_4 = 4$ μF.

(*b*) maximum charge supplied by the battery, and

(*c*) total energy stored in the network.

[CBSE 2020 (55/2/1)]

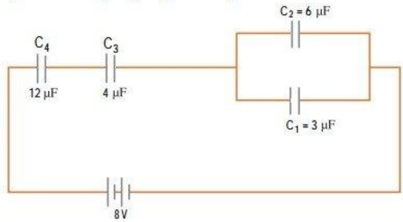


Ans. (*a*) Equivalent Capacitance, $C = C_4 = \mathbf{4\ \mu F}$ (as C_1, C_2, C_4, C_5 are short circuited)

(*b*) Charge, $Q = CV = 4 \times 7\ \mu C = \mathbf{28\ \mu C}$

(*c*) Energy stored, $U = \frac{1}{2}CV^2 = \frac{1}{2}\times 4\times 10^{-6}\times 7\times 7 = \mathbf{98\times 10^{-6} J}$

Q. 17. In a network, four capacitors C_1, C_2, C_3 and C_4 are connected as shown in the figure.



(a) Calculate the net capacitance in the circuit.

(b) If the charge on the capacitor C_1 is 6 μC, (i) calculate the charge on the capacitors C_3 and C_4, and (ii) net energy stored in the capacitors C_3 and C_4 connected in series.

[*CBSE 2019 (55/2/3)*]

Ans. (a) Capacitance across C_3 & C_4

$$C_{34} = \frac{12\times 4}{16} = 3\,\mu\text{F}$$

Capacitance across C_2 & C_1

$$C_{12} = 6+3 = 9\,\mu\text{F}$$

Equivalent capacitance **1**

$$C_{eq} = \frac{9\times 3}{12} = \frac{9\,\mu\text{F}}{4}$$

(b) (i) $Q_1 = 6\,\mu C,\ V_1 = \frac{Q_1}{C_1}$

$$= \frac{6\times 10^{-6}}{3\times 10^{-6}} = 2\text{ V}$$

$\therefore Q_2 = C_2 V_1 = 6\times 10^{-6}\times 2 = 12\,\mu C$ **1**

As C_3 & C_4 are in series they carry a charge of $18\,\mu C$ each

(ii) $Q = 18\,\mu\text{C}$

$C_{43} = 3\,\mu\text{F}$

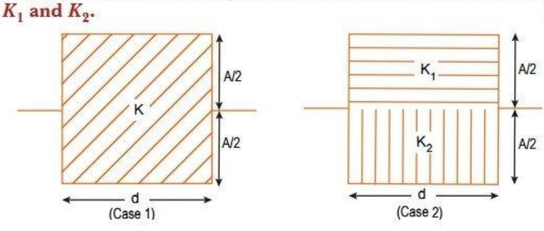
$$E_{34} = \frac{1}{2}\frac{Q^2}{C_{34}}$$ **½**

$$= \frac{1}{2}\times\frac{(18\times 10^{-6})^2}{3\times 10^{-6}}$$ **½**

$\therefore E_{34} = 54\times 10^{-6}\text{joule}$

[*CBSE Marking Scheme 2019 (55/2/3)*]

Q. 18. The space between the plates of a parallel plate capacitor is completely filled in two ways. In the first case, it is filled with a slab of dielectric constant K. In the second case, it is filled with two slabs of equal dimensions but dielectric constants K_1 and K_2, respectively as shown in the figure. The capacitance of the capacitor is same in the two cases. Obtain the relationship between K, K_1 and K_2. **[*CBSE 2020 (55/1/3)*]**



Ans. **Case I:** Let area of each plate = A

and initially capacitance of parallel plate capacitor, $C_1 = \dfrac{\varepsilon_0 A}{d}$

After inserting respective dielectric slabs, we have $C_1 = \dfrac{K\varepsilon_0 A}{d}$

Case II: It is filled with two slabs of equal dimension with dielectric constants K_1 and K_2 respectively in given such a way. So, it behave as capacitors are connected in parallel.

C_2 = parallel combination of two capacitors.

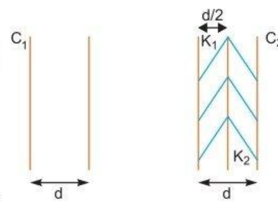
$$= \frac{K_1\varepsilon_0(A/2)}{d} + \frac{K_2\varepsilon_0(A/2)}{d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$\because \quad C_1 = C_2 \text{ (given)} \Rightarrow K = \frac{\mathbf{K_1 + K_2}}{\mathbf{2}}$$

Q. 19. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in C_2. Find the capacitances of the capacitor C_2 if area of the plates is A and distance between the plates is d.

[*HOTS*] [*CBSE (F) 2011*]

Ans. Capacitance, $C_1 = \dfrac{\varepsilon_0 A}{d}$

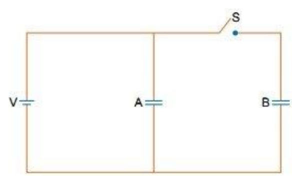


It is filled with two slabs of with dielectric constants K_1 and K_2 respectively as given such a way. Hence, it behave as capacitors are connected in series. So,

$$\frac{1}{C_2} = \frac{1}{K_1\dfrac{\varepsilon_0 A}{d/2}} + \frac{1}{K_2\dfrac{\varepsilon_0 A}{d/2}} = \frac{d}{2.K_1\varepsilon_0 A} + \frac{d}{2.K_2\varepsilon_0 A}$$

$$\Rightarrow \frac{1}{C_2} = \frac{d}{2\varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2}\right] \Rightarrow C_2 = \frac{2.\varepsilon_0 A}{d}\left[\frac{K_1K_2}{K_1 + K_2}\right] = \mathbf{C_1}\left[\frac{\mathbf{2K_1K_2}}{\mathbf{K_1 + K_2}}\right]$$

Q. 20. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **[*CBSE (AI) 2017*]**



Ans.

19. Initially:

$C_A^0 = \frac{\varepsilon_0 A}{d} = C \quad V_0$

$C_B^0 = \frac{\varepsilon_0 A}{d} = C \quad V_0$

Later

$C_A = K C_A^0 = KC$

$C_B = KC$

Total electrostatic energy

stored before dielectric

is $\frac{1}{2}C_AV_A^2 + \frac{1}{2}C_BV_B^2$

$\frac{1}{2}\times CV^2 + \frac{1}{2}CV^2$

$\times 2 \times \frac{1}{2}CV^2 = CV^2$

After across A

$\frac{1}{2}\times C_A \cdot V^2$

$\frac{1}{2}\times KC\times V^2$

$= K\times\frac{1}{2}CV^2$

across B $\frac{1}{2}C_B\times V_{new}^2$

$\frac{1}{2}C_B\times\frac{V_0^2}{K^2} = \frac{1}{2}\times KC\times\frac{V_0^2}{K^2}$

$\frac{1}{2}\frac{CV_0^2}{K} = \frac{1}{2}\frac{CV^2}{K}$

Total energy after insertion $= \frac{KCV^2}{2} + \frac{CV^2}{2K}$

$\frac{CV^2}{2}\left[K+\frac{1}{K}\right]$

$\frac{CV^2}{2}\left[\frac{K^2+1}{K}\right]$

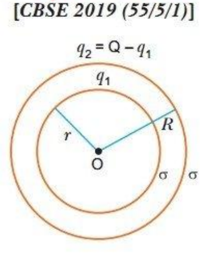
Ratio is $\frac{CV^2\times 2K}{CV^2(K^2+1)}$

$= \frac{2K}{K^2+1}$

[Toppers Answer 2017]

Q. 21. A charge Q is distributed over the surfaces of two concentric hollow spheres of radii r and R ($R >> r$), such that their surface charge densities are equal. Derive the expression for the potential at the common centre. **[CBSE 2019 (55/5/1)]**

Ans. If charge q_1 is distributed over the smaller sphere and q_2 over the larger sphere, then



$$Q = q_1 + q_2 \qquad ...(i)$$

If σ is the surface charge density of the two spheres, then

$$\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

or $$q_1 = 4\pi r^2\sigma \quad \text{and} \quad q_2 = 4\pi R^2\sigma$$

From (*i*), we have

$$Q = 4\pi r^2\sigma + 4\pi R^2\sigma$$

$$= 4\pi\sigma(r^2 + R^2)$$

or $$\sigma = \frac{Q}{4\pi(r^2 + R^2)}$$

The potential at a point inside the charged sphere is equal to the potential at its surface. So, the potential due to the smaller sphere at the common centre,

$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r}$$

Also, the potential due to the larger sphere at the common centre,

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{R}$$

∴ Potential at common centre,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

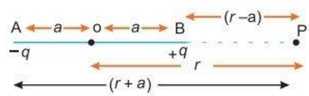
$$= \frac{1}{4\pi\varepsilon_0}\times\left[\frac{4\pi r^2\sigma}{r} + \frac{4\pi R^2\sigma}{R}\right]$$

$$= \frac{(r+R)\sigma}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q(r+R)}{r^2 + R^2}\right] \text{ (By putting the value of } \sigma)$$

Q. 22. (*a*) Derive an expression for the electric potential at any point along the axial line of an electric dipole.

(*b*) Find the electrostatic potential at a point on equatorial line of an electric dipole.

Ans. (*a*) Potential at point P



$$V_P = V_{-q} + V_{+q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)} + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)} - \frac{1}{(r+a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r+a-r+a}{(r-a)(r+a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{2a}{(r^2-a^2)} = \frac{q\times 2a}{4\pi\varepsilon_0(r^2-a^2)}$$

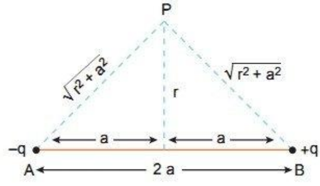
$$= \frac{1}{4\pi\varepsilon_0}\times\frac{p}{(r^2-a^2)} \text{ (where } p \text{ is the dipole moment)}$$

For a short dipole, $a^2 << r^2$, so $V = V = \frac{1}{4\pi\varepsilon_0}\times\frac{p}{r^2}$

(*b*) Let P be a point on the equatorial line of an electric dipole due to charges $-q$ and $+q$ with separation $2a$

The distance of point P from centre of dipole $= r$

$$AP = BP = \sqrt{r^2 + a^2}$$



Electrostatic potential at P, $V_P = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{BP}-\frac{q}{AP}\right)$

$$\Rightarrow \qquad V_P = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2+a^2}}-\frac{q}{\sqrt{r^2+a^2}}\right] = \mathbf{0}$$

That is electrostatic potential at each equatorial point of an electric dipole is **zero**.

Q. 23. If N drops of same size each having the same charge, coalesce to form a bigger drop. How will the following vary with respect to single small drop?

(*i*) Total charge on bigger drop

(*ii*) Potential on the bigger drop

(*iii*) Capacitance [*CBSE Sample Paper 2017*]

Ans. Let r, q and v be the radius, charge and potential of the small drop.

The total charge on bigger drop is sum of all charge on small drops.

(*i*) $\therefore \quad Q = \boldsymbol{Nq}$ (where Q is charge on bigger drop)

(*ii*) The volume of N small drops $= N\frac{4}{3}\pi r^3$

Volume of the bigger drop $= \frac{4}{3}\pi R^3$

Hence, $\quad N\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \Rightarrow \quad R = N^{1/3}r$

Potential on bigger drop, $V = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{R}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Nq}{N^{1/3}r} = \frac{1}{4\pi\varepsilon_0}\frac{N^{2/3}.q}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.N^{2/3} = \boldsymbol{N^{2/3}v} \quad \left[\therefore v = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right]$$

(*iii*) Capacitance $= 4\pi\varepsilon_0 R$

$= 4\pi\varepsilon_0 N^{1/3}r$

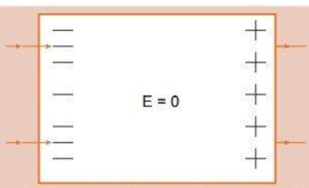
$= N^{1/3}\,(4\pi\varepsilon_0 r)$

$= \mathbf{N^{1/3}C}$ [where C is capacitance of the small drop]

Q. 24. (*a*) Explain briefly, using a proper diagram, the difference in behaviour of a conductor and a dielectric in the presence of external electric field.

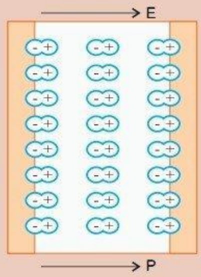
(*b*) Define the term polarization of a dielectric and write the expression for a linear isotropic dielectric in terms of electric field. [*CBSE 2019 (55/3/1)*]

Ans. ½



(*a*) **For conductor:** Due to induction the free electrons collect on the left face of slab creating equal positive charge on the right face. Internal electric field is equal and opposite to external field; hence net electric field (inside the conductor) is zero. ½

½



For dielectric: Due to alignment of atomic dipoles along $\vec{E}$, the net electric field within the dielectric decreases. ½

(*b*) The net dipole moment developed per unit volume in the presence of external electric field is called polarization vector $\vec{P}$. ½

Expression: $\vec{P} = \chi_e \vec{E}$ ½

[*CBSE Marking Scheme 2029 (55/3/1)*]

Long Answer Questions

Each of the following questions are of 5 marks.

Q. 1. Derive an expression for the electric potential at a point due to an electric dipole. Mention the contrasting features of electric potential of a dipole at a point as compared to that due to a single charge. [*CBSE Delhi 2008, 2017*]

Ans. Potential at a point due to a dipole.

Suppose, the negative charge $-q$ is placed at a point A and the positive charge q is placed at a point B (fig.), the separation $AB = 2a$. The middle point of AB is O. The potential is to be evaluated at a point P where $OP = r$ and $\angle POB = \theta$. Also, let $r >> a$.

Let AA' be the perpendicular from A to PO and BB' be be the perpendicular from B to PO. Since a is very small compared to r,

$$AP = A'P = OP + OA'$$
$$= OP + AO \cos\theta$$
$$= r + a\cos\theta$$

Similarly, $BP = B'P = OP - OB'$

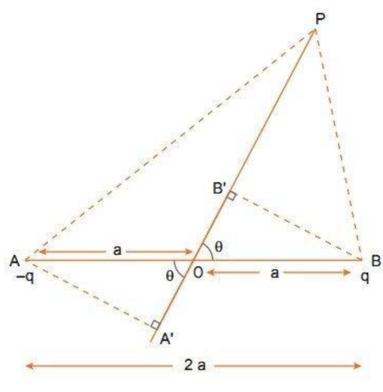
$$= r - a\cos\theta$$

The potential at P due to the charge $-q$ is

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r + a\cos\theta}$$

The potential at P due to the charge q is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r - a\cos\theta}$$



The net potential at P due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r - a\cos\theta} - \frac{q}{r + a\cos\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q\,2a\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Special Cases:

(*i*) When point P lies on the axis of dipole, then $\theta = 0°$

$\therefore \quad \cos\theta = \cos 0^0 = 1$

$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

(*ii*) When point P lies on the equatorial plane of the dipole, then

$\therefore \cos\theta = \cos 90° = 0$

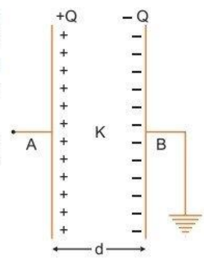
$\therefore \quad V = 0$

It may be noted that the electric potential at any point on the equitorial line of a dipole is zero.

Q. 2. Briefly explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor, whose plates are separated by a dielectric medium.

Ans. Principle of a Capacitor: A capacitor works on the principle that the capacitance of a conductor increases appreciably when an earthed conductor is brought near it.

Parallel Plate Capacitor: Consider a parallel plate capacitor having two plane metallic plates A and B, placed parallel to each other (see fig.). The plates carry equal and opposite charges $+Q$ and $-Q$ respectively.



In general, the electric field between the plates due to charges $+Q$ and $-Q$ remains uniform, but at the edges, the electric field lines deviate outward. If the separation between the plates is much smaller than the size of plates, the electric field strength between the plates may be assumed uniform.

Let A be the area of each plate, 'd' the separation between the plates, K the dielectric constant of medium between the plates. If σ is the magnitude of charge density of plates, then

$$\sigma = \frac{Q}{A}$$

The electric field strength between the plates

$$E = \frac{\sigma}{K\epsilon_0} \text{ where } \varepsilon_0 = \text{permittivity of free space.} \qquad ...(i)$$

The potential difference between the plates, $V_{AB} = Ed = \frac{\sigma d}{K\varepsilon_0}$ ...(*ii*)

Putting the value of σ, we get

$$V_{AB} = \frac{(Q/A)d}{K\varepsilon_0} = \frac{Qd}{K\varepsilon_0 A}$$

$\therefore$ Capacitance of capacitor,

$$C = \frac{Q}{V_{AB}} = \frac{Q}{(Qd/K\varepsilon_0 A)} \text{ or } C = \frac{K\varepsilon_0 A}{d} \qquad ...(iii)$$

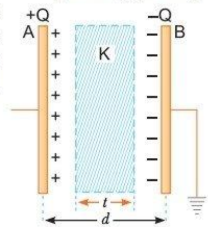
This is a general expression for capacitance of parallel plate capacitor. Obviously, the capacitance is directly proportional to the dielectric constant of medium between the plates.

For air capacitor ($K=1$); capacitance $C = \frac{\varepsilon_0 A}{d}$. This is expression for the capacitance of a parallel plate air capacitor. It can be seen that the capacitance of parallel plate (air) capacitor is

(*a*) directly proportional to the area of each plate.
(*b*) inversely proportional to the distance between the plates.
(*c*) independent of the material of the plates.

Q. 3. Derive an expression for the capacitance of a parallel plate capacitor when a dielectric slab of dielectric constant K and thickness $t = \frac{d}{2}$ but of same area as that of the plates is inserted between the capacitor plates. (d = separation between the plates). **[CBSE (F) 2010]**

Ans. Consider a parallel plate capacitor, area of each plate being A, the separation between the plates being d. Let a dielectric slab of dielectric constant K and thickness $t < d$ be placed between the plates. The thickness of air between the plates is $(d - t)$. If charges on plates are $+Q$ and $-Q$, then surface charge density $\sigma = \frac{Q}{A}$



The electric field between the plates in air, $E_1 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

The electric field between the plates in slab, $E_2 = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{K\varepsilon_0 A}$

$\therefore$ The potential difference between the plates,

V_{AB} = work done in carrying unit positive charge from one plate to another

$= \Sigma Ex$ (as field between the plates is not constant).

$$= E_1(d-t) + E_2 t = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{K\varepsilon_0 A}t$$

$$\therefore \quad V_{AB} = \frac{Q}{\varepsilon_0 A}\left[d - t + \frac{t}{K}\right]$$

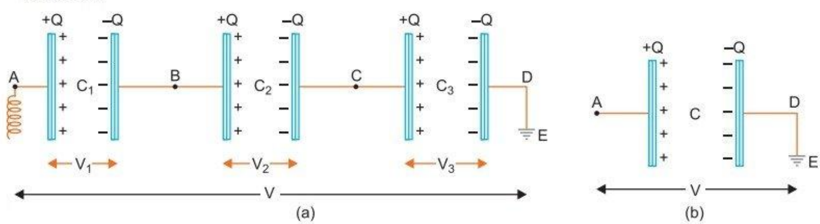
$$\therefore \quad \text{Capacitance of capacitor, } C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)}$$

or,
$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Here, $t = \frac{d}{2}$ $\quad\therefore\quad$ $C = \frac{\varepsilon_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{K}\right)} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$

Q. 4. Derive an expression for equivalent capacitance of three capacitors when connected (*i*) in series and (*ii*) in parallel.

Ans. (*i*) In fig. (*a*) three capacitors of capacitances C_1, C_2, C_3 are connected in series between points A and D.



In series first plate of each capacitor has charge $+Q$ and second plate of each capacitor has charge $-Q$ *i.e.*, charge on each capacitor is Q.

Let the potential differences across the capacitors C_1, C_2, C_3 be V_1, V_2, V_3 respectively. As the second plate of first capacitor C_1 and first plate of second capacitor C_2 are connected together, their potentials are equal. Let this common potential be V_B. Similarly the common potential of second plate of C_2 and first plate of C_3 is V_C. The second plate of capacitor C_3 is connected to earth, therefore its potential $V_D=0$. As charge flows from higher potential to lower potential, therefore $V_A > V_B > V_C > V_D$.

For the first capacitor, $V_1 = V_A - V_B = \frac{Q}{C_1}$ ...(*i*)

For the second capacitor, $V_2 = V_B - V_C = \frac{Q}{C_2}$ ...(*ii*)

For the third capacitor, $V_3 = V_C - V_D = \frac{Q}{C_3}$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get

$$V_1 + V_2 + V_3 = V_A - V_D = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ...(iv)$$

If V be the potential difference between A and D, then

$$V_A - V_D = V$$

$\therefore$ From (*iv*), we get

$$V = (V_1 + V_2 + V_3) = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ...(v)$$

If in place of all the three capacitors, only one capacitor is placed between A and D such that on giving it charge Q, the potential difference between its plates become V, then it will be called **equivalent capacitor**. If its capacitance is C, then

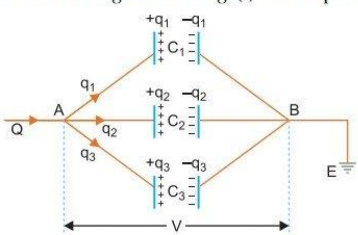
$$V = \frac{Q}{C} \quad ...(vi)$$

Comparing (*v*) and (*vi*), we get

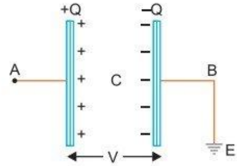
$$\frac{Q}{C} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus in series arrangement, "*The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitors.*"

(*ii*) Parallel Arrangement: In fig. (*c*) three capacitors of capacitance C_1, C_2, C_3 are connected in parallel.



(c)



(d)

In parallel the potential difference across each capacitor is same V (say). Clearly the potential difference between plates of each capacitor

$$V_A - V_B = V \text{ (say)}$$

The charge Q given to capacitors is divided on capacitors C_1, C_2, C_3.

Let q_1, q_2, q_3 be the charges on capacitors C_1, C_2, C_3 respectively.

Then $\quad Q = q_1 + q_2 + q_3 \qquad ...(i)$

and $\quad q_1 = C_1V, q_2 = C_2V, q_3 = C_3V$

Substituting these values in (*i*), we get

$$Q = C_1V + C_2V + C_3V \quad or \quad Q = (C_1 + C_2 + C_3)V \qquad ...(ii)$$

If, in place of all the three capacitors, only one capacitor of capacitance C be connected between A and B; such that on giving it charge Q, the potential difference between its plates be V, then it will be called **equivalent capacitor.** If C be the capacitance of equivalent capacitor, then

$$Q = CV \qquad ...(iii)$$

Comparing equations (*ii*) and (*iii*), we get

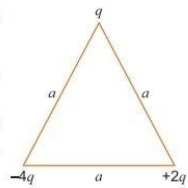
$CV=(C_1+C_2+ C_3)V$ or $\qquad \mathbf{C=(C_1+C_2+ C_3)}$

Important Note: It may be noted carefully that the formula for the total capacitance in series and parallel combination of capacitors is the reverse of corresponding formula for combination of resistors in current electricity.

Q. 5. (*a*) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point.

Draw a sketch of equipotential surfaces due to a single charge (– *q*), depicting the electric field lines due to the charge.

(*b*) Obtain an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side '*a*' as shown below. [*CBSE North 2016*]



Ans. (*a*) The work done in moving a charge from one point to another on an equipotential surface is zero. If the field is not normal to an equipotential surface, it would have a non zero component along the surface. This would imply that work would have to be done to move a charge on the surface which is contradictory to the definition of equipotential surface.

Mathematically

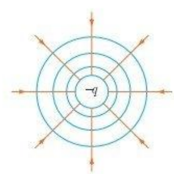
Work done to move a charge dq on a surface can be expressed as

$$dW = dq(\vec{E}.\vec{dr})$$

But $dW = 0$ on an equipotential surface

$\therefore \quad \vec{E} \perp \vec{dr}$

Equipotential surfaces for a charge $-q$ is shown alongside.



(*b*) Work done to dissociate the system = – Potential energy of the system

$$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{(-4q)(q)}{a} + \frac{(2q)(q)}{a} + \frac{(-4q)(2q)}{a}\right]$$

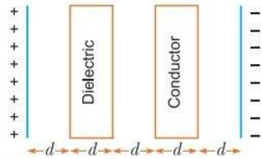
$$= -\frac{1}{4\pi\varepsilon_0}\left[-4q^2 + 2q^2 - 8q^2\right] = +\left[\frac{\mathbf{10q^2}}{\mathbf{4\pi\varepsilon_0 a}}\right]$$

Q. 6. (*i*) Compare the individual dipole moment and the specimen dipole moment for H_2O molecule and O_2 molecule when placed in

(*a*) Absence of external electric field

(*b*) Presence of external eclectic field. Justify your answer.

(*ii*) Given two parallel conducting plates of area A and charge densities $+\sigma$ and $-\sigma$. A dielectric slab of constant K and a conducting slab of thickness d each are inserted in between them as shown.



(*a*) **Find the potential difference between the plates.**

(*b*) **Plot E versus x graph, taking $x = 0$ at positive plate and $x = 5d$ at negative plate.**

[*CBSE Sample Paper 2016*]

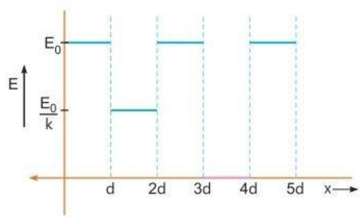
Ans. (*i*)

	Non-Polar (O_2)	Polar (H_2O)
(a) Absence of electric field		
Individual	No dipole moment exists	Dipole moment exists
Specimen	No dipole moment exists	Dipole are randomly oriented. Net P = 0
(b) Presence of electric field		
Individual	Dipole moment exists (molecules become polarised)	Torque acts on the molecules to align them parallel to $\vec{E}$
Specimen	Dipole moment exists	Net dipole moment exists parallel to $\vec{E}$

(*ii*) (*a*) The potential difference between the plates is given by

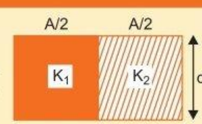
$$V = E_0 d + \frac{E_0}{K} d + E_0 d + 0 + E_0 d \Rightarrow V = 3E_0 d + \frac{E_0}{K} d$$

(*b*) E versus x graph



Questions for Practice

1. **Choose and write the correct option in the following questions.**



(*i*) A parallel plate condenser is filled with two dielectrics as shown. Area of each plate is A metre^2 and the separation is d metre. The dielectric constants are K_1 and K_2 respectively. Its capacitance in farad will be

(*a*) $\frac{\varepsilon_0 A}{d}(K_1 + K_2)$ (*b*) $\frac{\varepsilon_0 A}{d} \cdot \frac{K_1 + K_2}{2}$ (*c*) $\frac{\varepsilon_0 A}{d} 2(K_1 - K_2)$ (*d*) $\frac{\varepsilon_0 A}{d}\left(\frac{K_1 - K_2}{2}\right)$

(*ii*) A parallel plate capacitor is charged to V volt by a battery. The battery is disconnected and the separation between the plates is halved. The new potential difference across the capacitor will be **[*CBSE 2020 (55/3/2)*]**

(*a*) $\frac{V}{2}$ (*b*) V (*c*) $2V$ (*d*) $\frac{V}{4}$

(iii) The electric potential V at any point (x, y, z) is given by $V = 3x^2$ where x is in metres and V in volts. The electric field at the point (1 m, 0, 2 m) is [*CBSE 2022 (55/2/4), Term-1*]

(a) 6 V/m along – x-axis (b) 6 V/m along + x-axis

(c) 1.5 V/m along – x-axis (d) 1.5 V/m along + x-axis

(iv) An electric dipole of moment p is placed parallel to the uniform electric field. The amount of work done in rotating the dipole by 90° is [*CBSE Sample Paper-2022, Term-1)*]

(a) $2pE$ (b) pE (c) $pE/2$ (d) zero

(v) Equipotential surface associated with an electric field, which is increasing in magnitude along the X-direction, are

(a) planes parallel to YZ-plane.

(b) planes parallel to XZ-plane.

(c) planes parallel to XY-plane.

(d) coaxial cylinder of increasing radii around the X-axis.

(vi) Two identical metal plates, separated by a distance d form a parallel-plate capacitor. A metal sheet of thickness $d/2$ is inserted between the plates. The ratio of the capacitance after the insertion of the sheet to that before insertion is

(a) $\sqrt{2}:1$ (b) 2 : 1 (c) 1 : 1 (d) 1 : 2

(vii) A particle A has charge $+q$ and particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through same electrical potential difference, the ratio of their speeds $v_A : v_B$ will become

(a) 2 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

2. ***In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.***

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false and R is also false.***

Assertion (*A*) **:** The capacitance of a parallel plate capacitor increases when a dielectric constant of medium between the plates.

Reason (*R*) **:** Capacitance of a parallel plate capacitor is directly proportional to dielectric constant of medium between the plates.

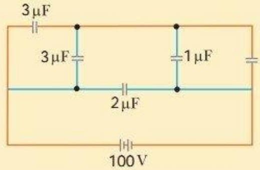
3. What is the electrostatic potential due to an electric dipole at an equatorial point?

4. A hollow metal sphere of radius 10 cm is charged such that the potential on its surface is 5V. What is the potential at the centre of the sphere?

5. Why is the electrostatic potential inside a charged conducting shell constant throughout the volume of the conductor?

6. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same. [*CBSE Central 2016*]

7. The figure shows a network of five capacitors connected to a 100 V supply. Calculate the total energy stored in the network.



8. Two capacitors are connected in series. Derive an expression of the equivalent capacitance of the combination. [*CBSE 2023 (55/3/1)*]

9. Two point charges $+q$ and $-q$ are located at points $(3a, 0)$ and $(0, 4a)$ respectively in x-y plane. A third charge Q is kept at the origin. Find the value of Q, in terms of q and a, so that the electrostatic potential energy of the system is zero. [*CBSE 2023 (55/3/1)*]

10. Find the energy lost when the charged capacitor is disconnected from the source and connected in parallel with the uncharged capacitor. Where does this loss of energy appear? [*CBSE Sample Paper 2017*]

11. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. [*NCERT*]

12. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon. [*NCERT*]

13. Three capacitors each of capacitance 9 pF are connected in series:
   (*a*) What is the total capacitance of the combination?
   (*b*) What is the potential difference across each capacitor if the combination is connected to 120 V supply? [*NCERT*]

14. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
   (*a*) What is the total capacitance of the combination?
   (*b*) Determine the charge on each capacitor if the combination is connected to a 100 V supply. [*NCERT*]

15. Why do the equipotential surfaces due to a uniform electric field not intersect each other? [*CBSE (F) 2012*]

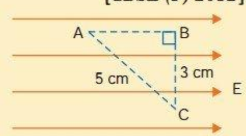
16. "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify. [*CBSE Delhi 2014*]

17. Why is the potential inside a hollow spherical charged conductor constant and has the same value as on its surface? [*CBSE (F) 2012*]

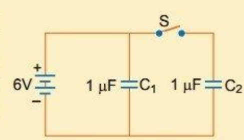
18. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere? [*CBSE (AI) 2011*]

19. Why is there no work done in moving a charge from one point to another on an equipotential surface? [*CBSE (F) 2012*]

20. Three points A, B and C lie in a uniform electric field (E) of 5×10^3 NC^{-1} as shown in the figure. Find the potential difference between A and C. [*CBSE (F) 2009*]



21. Figure shows two identical capacitors, C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially switch '*S*' is closed. After sometimes '*S*' is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (*i*) charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted? [*CBSE Delhi 2011*]



22. A parallel plate capacitor is charged by a battery, which is then disconnected. A dielectric slab is then inserted in the space between the plates. Explain what changes, if any, occur in the values of
   (*i*) capacitance
   (*ii*) potential difference between the plates
   (*iii*) electric field between the plates, and
   (*iv*) the energy stored in the capacitor.

[*CBSE Delhi 2010, (AI) 2009, 2012*]

23. A parallel plate is charged by a battery. When the battery remains connected, a dielectric slab is inserted in the space between the plates. Explain what changes if any, occur in the values of

(*i*) potential difference between the plates (*ii*) electric field strength between the plates

(*iii*) capacitance (*iv*) charge on the plates

(*v*) energy stored in the capacitor. [*CBSE Delhi 2010*]

24. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. [*CBSE (AI) 2014*]

25. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination. [*CBSE Delhi 2015*]

26. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. [*CBSE Delhi 2017*]

27. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination? If these were connected in parallel across the same battery, how much energy will be stored in the combination now?

Also find the charge drawn from the battery in each case. [*CBSE Delhi 2017*]

28. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

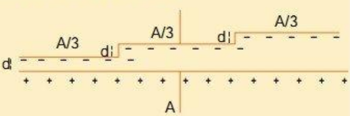
[*HOTS*] [*CBSE (AI) 2013*]

29. (*a*) Deduce the expression for the potential energy of a system of two charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$ respectively in an external electric field.

(*b*) Three point charges, $+Q$, $+2Q$ and $-3Q$ are placed at the vertices of an equilateral triangle ABC of side l. If these charges are displaced to the mid-points A_1, B_1 and C_1 respectively, find the amount of the work done in shifting the charges to the new locations.

30. A capacitor is made of a flat plate of area A and second plate having a stair like structure as shown in figure below. If width of each stair is $A/3$ and height is d. Find the capacitance of the arrangement.

[*CBSE Sample Paper 2017*]



Answers

1. (*i*) (*b*) (*ii*) (*a*) (*iii*) (*a*) (*iv*) (*b*) (*v*) (*a*) (*vi*) (*b*) (*vii*) (*b*)

2. (*a*) **6.** $3:\sqrt{2}$ **7.** 0.02J **9.** $\frac{12q}{5}$C **11.** 10 cm **12.** 2.7×10^{6} V

13. (*a*) 3 PF (*b*) 40V **14.** (*a*) 9pF (*b*) 2×10^{-10}C, 3×10^{-10}C, 4×10^{-10}C

18. 10V **20.** 200V **21.** (*i*) 18μC (*ii*) 2V **24.** 1:2

25. 38.2 mF, 11.8 mF, 38.2×10^{-4} C, 11.8×10^{-4} C

26. 2×10^{-4} C, 2×10^{-4} C and $\frac{50}{3}$ V, $\frac{100}{3}$ V

27. 75×10^{-10}J, 3×10^{-8}J, 3×10^{10}C, 1.2×10^{-9} C.

29. (*b*) $\frac{-7KQ^2}{l}$J **30.** $\frac{11A\varepsilon_0}{18d}$