Probability

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to [CBSE 2023 (65/5/1)]

   (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$

2. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is [CBSE 2023 (65/5/1)]

   (a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$

3. If $P\left(\frac{A}{B}\right) = 0.3$, $P(A) = 0.4$ and $P(B) = 0.8$, then $P\left(\frac{B}{A}\right)$ is equal to [CBSE 2023 (65/1/1)]

   (a) 0·6 (b) 0·3 (c) 0.06 (d) 0.4

4. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is [CBSE 2023 (65/2/1)]

   (a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{2}$

5. From the set {1, 2, 3, 4, 5}, two numbers a and b ($a \neq b$) are chosen at random. The probability that $\frac{a}{b}$ is an integer is [CBSE 2020 (65/4/1)]

   (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

6. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$. Then $P(B' \cap A)$ equals [NCERT Exemplar]

   (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$

7. A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is [CBSE 2020 (65/4/1)]

(*a*) $\frac{1}{18}$ (*b*) $\frac{1}{36}$ (*c*) $\frac{1}{12}$ (*d*) $\frac{1}{24}$

8. You are given that A and B are two events such that $P(B) = \frac{3}{5}, P(A \mid B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A)$ equals

(*a*) $\frac{3}{10}$ (*b*) $\frac{1}{5}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{5}$

9. Three persons, A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(*a*) 0.024 (*b*) 0.188 (*c*) 0.336 (*d*) 0.452

10. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is [*NCERT Exemplar*]

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{4}{7}$

11. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{8}$ (*d*) $\frac{3}{4}$

12. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(*a*) $\frac{3}{28}$ (*b*) $\frac{2}{21}$ (*c*) $\frac{1}{28}$ (*d*) $\frac{167}{168}$

13. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is

(*a*) $\frac{33}{56}$ (*b*) $\frac{9}{64}$ (*c*) $\frac{1}{14}$ (*d*) $\frac{3}{28}$

14. Two dice are thrown. If it is known that the sum of number on the dice was less than 6, the probability of getting a sum 3, is

(*a*) $\frac{1}{18}$ (*b*) $\frac{5}{18}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{2}{5}$

15. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability, that both cards are queens is

(*a*) $\frac{1}{13} \times \frac{1}{13}$ (*b*) $\frac{1}{13} \times \frac{1}{12}$ (*c*) $\frac{1}{13} \times \frac{1}{17}$ (*d*) $\frac{1}{13} \times \frac{4}{51}$

16. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is

(*a*) $\frac{15}{196}$ (*b*) $\frac{131}{392}$ (*c*) $\frac{15}{56}$ (*d*) $\frac{15}{29}$

17. Three persons A, B and C, fire at a target in turn, standing with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(*a*) 0.025 (*b*) 0.188 (*c*) 0.339 (*d*) 0.475

18. The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

(*a*) 8 (*b*) 16 (*c*) 32 (*d*) 48

19. **Two dice are thrown together. Let A be the event 'getting 6 on the first die' and B be the event 'getting 2 on the second die', then $P(A \cap B)$ is**

(a) $\frac{1}{36}$ (b) $\frac{7}{4}$ (c) $\frac{9}{20}$ (d) None of these

20. **In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics if she has failed in Mathematics is**

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{9}{20}$ (d) $\frac{1}{3}$

21. **A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is, $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is** [*NCERT Exemplar*]

(a) $\frac{1}{12}$ (b) $\frac{1}{40}$ (c) $\frac{13}{120}$ (d) $\frac{10}{13}$

22. **A mapping is selected at random from set $A = \{1, 2, ..., 10\}$ into itself. The probability that mapping selected is an injective, is**

(a) $\frac{10}{10^9}$ (b) $\frac{9!}{10^9}$ (c) $\frac{9}{\underline{|10}}$ (d) none of these

23. **If two events are independent, then**

(a) they must be mutually exclusive.
(b) the sum of their probabilities must be equal to 1.
(c) Both (a) and (b) are correct.
(d) None of the above is correct.

24. **If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ is equals to**

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{16}$

Answers

1. (c)	2. (c)	3. (a)	4. (d)	5. (b)	6. (d)	7. (c)
8. (c)	9. (b)	10. (d)	11. (c)	12. (a)	13. (d)	14. (c)
15. (a)	16. (c)	17. (b)	18. (c)	19. (a)	20. (b)	21. (d)
22. (b)	23. (d)	24. (c)				

Solutions of Selected Multiple Choice Questions

1. $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\frac{7}{10}}{\frac{4}{5}} = \dfrac{35}{40} = \dfrac{7}{8}$

$\therefore$ Option (c) is correct.

2. $\because$ P(No head comes up) = P(All outcomes are tails)

$= P(TTTTT) = P(T)\,P(T)\,P(T)\,P(T)\,P(T)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^5} = \frac{1}{32}$

P(At least one head) = 1 – P(No head) = $1 - \frac{1}{32} = \frac{31}{32}$

$\therefore$ Option (c) is correct.

3. Given, $P\left(\frac{A}{B}\right) = 0.3, P(A) = 0.4$ and $P(B) = 0.8$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow 0.3 = \frac{P(A \cap B)}{0.8}$

$\Rightarrow \quad P(A \cap B) = 0.3 \times 0.8 = 0.24$

$\Rightarrow \quad P(A \cap B) = 0.24$

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} = \frac{0.24}{0.4} = 0.6$

$\Rightarrow \quad P\left(\frac{B}{A}\right) = 0.6$

$\therefore$ Option (*a*) is correct.

4. It is given that sum of numbers obtained on throwing a pair of dice is 9.

$\therefore$ Sample space = {(3, 6), (6, 3), (4, 5), (5, 4)} $\Rightarrow$ $n(S) = 4$

and, favourable out comes = {(4, 5), (5, 4)} $\Rightarrow$ $n(E) = 2$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$\therefore$ Option (*d*) is correct.

5. In $\frac{a}{b}$, b can be any of the five values and correspondingly a can assume four values.

Thus, the number of fractions possible is $5 \times 4 = 20$

Now, out of these, only five *i.e.*, $\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}$ and $\frac{4}{2}$ will be integers.

$\therefore \quad n(E) = 5$

$\therefore$ Requried probability $= \frac{n(E)}{n(S)} = \frac{5}{20} = \frac{1}{4}$

$\therefore$ Option (*b*) is correct.

6. Here, $P(A) = 0.4, P(B) = 0.3$ and $P(A \cup B) = 0.5$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$

$\therefore \quad P(B' \cap A) = P(A) - P(A \cap B)$

$= 0.4 - 0.2 = 0.2 = \frac{1}{5}$

$\therefore$ Option (*d*) is correct.

7. P (Both balls are white) $= \frac{{}^3C_2}{{}^9C_2}$

$$= \frac{\frac{3!}{2!\,1!}}{\frac{9!}{2!\,7!}} = \frac{1}{12}$$

$\therefore$ Option (*c*) is correct.

10. Here, S = {(B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G, B, B), (B, G, B), (B, B, G)}

E_1 = Event that a family has atleast one girl, then

E_1 = {(G, B, B), (B, G, B), (B, B, G), (G, G, B), (B, G, G), (G, B, G), (G, G, G)}

E_2 = Event that the eldest child is a girl, then

E_2 = {(G, B, B), (G, G, B), (G, B, G), (G, G, G)}

$\because \quad E_1 \cap E_2$ = {(G, B, B), (G, G, B), (G, B, G), (G, G, G)}

$\therefore \quad P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{4/8}{7/8} = \frac{4}{7}$

$\therefore$ Option (*d*) is correct.

21. Let E_1 = Event that both A and B solve the problem

$$\therefore \quad P(E_1) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Let E_2 = Event that both A and B got incorrect solution of the problem

$$\therefore \quad P(E_2) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

Let E = Event that they got same answer

Here, $\quad P(E/E_1) = 1, P(E/E_2) = \frac{1}{20}$

$$\therefore \quad P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)} = \frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1) + P(E_2)P(E/E_2)}$$

$$= \frac{\frac{1}{12} \times 1}{\frac{1}{12} \times 1 + \frac{1}{2} \times \frac{1}{20}} = \frac{1/12}{\frac{10+3}{120}} = \frac{120}{12 \times 13} = \frac{10}{13}$$

$\therefore$ Option (d) is correct.

22. $n(a) = 10$

$\therefore$ Number of mapping (function) from A to $A = 10^{10}$.

Out of 10^{10}, number of injective functions are $\lfloor 10$.

$$\therefore \quad P(A) = \frac{\lfloor 10}{10^{10}} = \frac{10 \times 9!}{10^{10}} = \frac{9!}{10^9}$$

$\therefore$ Option (b) is correct.

24. $P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3},$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \text{ (Given)}$$

$$\Rightarrow \quad P(A \cap B) = \frac{1}{4} \times P(B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{12}$$

$$P(A \cup B) = \frac{6+4-1}{12} = \frac{9}{12} = \frac{3}{4}$$

$$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

$\therefore$ Option (c) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion (A) and Reason (R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

[CBSE 2023 (65/5/1)]

2. **Assertion (A) :** The probability of obtaining an even prime number on each die, when a pair of dice is rolled is $\frac{1}{36}$.

Reason (R) : If $P(A/B) > P(A)$, then $P(B/A) > P(B)$.

3. **Assertion (A) :** If $P(A) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{3}$, Then the value of $P(B/A) = \frac{2}{3}$.

Reason (R) : $P(B/A) = \frac{P(A \cap B)}{P(A)}$.

4. **Assertion (A) :** A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7	8
$P(X)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

then $a = \frac{1}{81}$.

Reason (R) : The sum of probabilities of a probability distribution is always 1

i.e., $\Sigma P(X) = 1$.

5. **Assertion (A) :** If $P(A) = 0.2$, $P(B) = 0.3$, and A and B are independent events then $P(A \cap B) = 0.06$.

Reason (R) : When A and B are independent events than $P(A \cap B) = P(A) . P(B)$.

6. **Assertion (A) :** If $P(A) = \frac{3}{8}$ and $P(B) = \frac{5}{8}$, $P(A \cup B) = \frac{3}{4}$ then $P(A'/B') = \frac{2}{3}$.

Reason (R) : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

7. **Assertion (A) :** Two dice are thrown. Then the probability of getting an odd number on first and a multiple of 3 on the other die, is $\frac{1}{6}$.

Reason (R) : $P(A) + P(\overline{A}) = 1$.

8. **Assertion (A) :** The mean of the following distribution

X	0	1	2
$P(X)$	1/4	1/2	1/4

is 1.

Reason (R) : Mean of the distribution is $\Sigma p_i x_i$.

Answers

1. (*a*) 2. (*b*) 3. (*a*) 4. (*a*) 5. (*a*) 6. (*b*) 7. (*b*) 8. (*a*)

Solutions of Assertion-Reason Questions

1. A = Event of getting two heads

B = Event of getting at least one head

$A = \{HH\}, B = \{HT, TH, HH\}$

$A \cap B = \{HH\}$

$P(A \cap B) = \frac{1}{4}$, $P(B) = \frac{3}{4}$

$$\therefore\ P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Both (*A*) and (*R*) are true and (*R*) is the correct explanation of (*A*).

$\therefore$ Option (*a*) is correct.

2. Probability that on each die, an even prime number will be obtained $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Clearly, both Assertion (*A*) and Reason (*R*) are true, but Reason (*R*) is not the correct explanation of Assertion (*A*).

Hence, option (*b*) is correct.

3. We have, $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{3}}{\frac{1}{2}} \Rightarrow P(B/A) = \frac{2}{3}$.

Clearly, both Assertion (*A*) and Reason (*R*) are true and Reason (*R*) is the correct explanation of Assertion (*A*).

Hence, option (*a*) is correct.

4. We have, $\Sigma P(X) = 1$

$$\Rightarrow a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$\Rightarrow 81a = 1 \Rightarrow a = \frac{1}{81}$$

Clearly, both Assertion (*A*) and Reason (*R*) are true and Reason (*R*) is the correct explanation of Assertion (*A*).

Hence, option (*a*) is correct.

5. We have, $P(A) = 0.2,\quad P(B) = 0.3$

$$\therefore\quad P(A \cap B) = P(A) \times P(B) = 0.2 \times 0.3 = 0.06$$

(When *A* and *B* are independent events)

Clearly, both Assertion (*A*) and Reason (*R*) are true and Reason (*R*) is the correct explanation of Assertion (*A*).

Hence, option (*a*) is correct.

6. $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P[(A \cup B)']}{1 - P(B)}$

$$= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{\frac{1}{4}}{1 - \frac{5}{8}} = \frac{\frac{1}{4}}{\frac{3}{8}} \quad \Rightarrow \quad P(A'/B') = \frac{2}{3}$$

Clearly, both Assertion (*A*) and Reason (*R*) are true but Reason (*R*) is not the correct explanation of Assertion (*A*).

Hence, option (*b*) is correct.

7. Let *E* be the event of getting an odd number on first die and a multiple of 3 on the other die.

$E = \{(1, 3), (1, 6), (3, 3), (3, 6), (5, 3), (5, 6)\}$

$\therefore P(E) = \frac{6}{36} = \frac{1}{6}$

So statement *A* is true.

Also statement *R* is true but *R* does not gives correct explanation of statement *A*.

$\therefore$ Option (*b*) is correct.

8. We have

X	0	1	2
$P(X)$	1/4	1/2	1/4

$$\text{Mean } (\mu) = \Sigma X_i P(X_i) = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

So statement A is correct.

Also statement R is correct and gives correct explanation of the statement A.

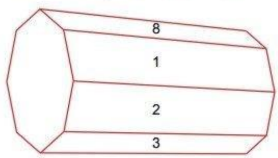
$\therefore$ Option (*a*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.** *[CBSE 2023 (65/3/2)]*

An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X	1	2	3	4	5	6	7	8
$P(X)$	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2 + p$

(*i*) Find the value of p.

(*ii*) Find $P(X > 6)$.

(*iii*) (*a*) Find $P(X = 3m)$, where m is a natural number.

OR

(*iii*) (*b*) Find the mean $E(X)$.

Sol. (*i*) We have

$$\sum_{i=1}^{8} p_i = 1$$

$$\Rightarrow \quad p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$$

$$\Rightarrow \quad 10p^2 + 9p - 1 = 0$$

$$\Rightarrow \quad 10p^2 + 10p - p - 1 = 0$$

$$\Rightarrow \quad 10p(p + 1) - 1(p + 1) = 0$$

$$\Rightarrow \quad (p + 1)(10p - 1) = 0$$

$\Rightarrow \quad 10p - 1 = 0 \qquad (\because p + 1 \neq 0 \;\Rightarrow\; p \neq -1)$

$\Rightarrow \quad p = \dfrac{1}{10}$

(*ii*) $P(X > 6) = P(7) + P(8) = 2p^2 + 7p^2 + p = 9p^2 + p$

$$= 9 \times \frac{1}{100} + \frac{1}{10} = \frac{9+10}{100} = \frac{19}{100}$$

(*iii*) (*a*) $P(X = 3m) = P(3) + P(6)$

$$= 2p + p^2 = p^2 + 2p = \left(\frac{1}{10}\right)^2 + 2 \times \frac{1}{10}$$

$$= \frac{1}{100} + \frac{2}{10} = \frac{1+20}{100} = \frac{21}{100}$$

OR

(*iii*) (*b*) $E(X) = 1 \times p + 2 \times 2p + 3 \times 2p + 4p + 5 \times 2p + 6 \times p^2 + 7 \times 2p^2 + 8 \times (7p^2 + p)$

$$= p + 4p + 6p + 4p + 10p + 6p^2 + 14p^2 + 56p^2 + 8p$$

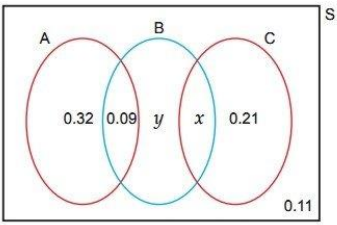
$$= 76p^2 + 33p = 76 \times \frac{1}{100} + \frac{33}{10} = \frac{76+330}{100} = \frac{406}{100}$$

$$= 4.06$$

2. Read the following passage and answer the following questions. *[CBSE 2023 (65/1/1)]*

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, *A*, *B* and *C* performed by the people of a society. Further, it is given that probability of a member performing type *C* Yoga is 0·44.



(*i*) Find the value of *x*.

(*ii*) Find the value of *y*.

(*iii*) (*a*) Find $P\left(\dfrac{C}{B}\right)$.

OR

(*iii*) (*b*) Find the probability that a randomly selected person of the society does Yoga of type *A* or *B* but not *C*.

Sol. (*i*) Given that probability of a member performing type C yoga is 0.44.

$\therefore \quad x + 0.21 = 0.44$

$\Rightarrow \quad x = 0.44 - 0.21 = 0.23$

$\Rightarrow \quad x = 0.23$

(*ii*) $0.32 + 0.09 + y + x + 0.21 = 1 - 0.11$

$\Rightarrow \quad 0.32 + 0.09 + y + 0.23 + 0.21 = 0.89$

$\Rightarrow \quad 0.85 + y = 0.89 \quad \Rightarrow y = 0.89 - 0.85 = 0.04$

$\Rightarrow y = 0.04$

(*iii*) (*a*) $P(C/B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{x}{0.09 + y + x} = \dfrac{0.23}{0.09 + 0.04 + 0.23}$

$= \dfrac{0.23}{0.36} = \dfrac{23}{36}$

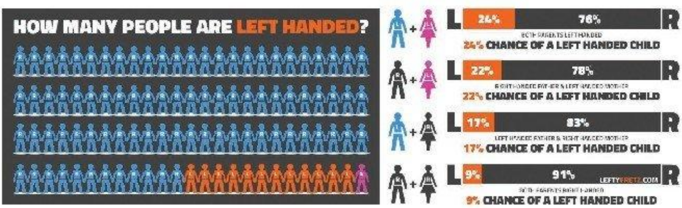
OR

(*iii*) (*b*) Required probability $= 0.32 + 0.09 + y = 0.41 + y$

$= 0.41 + 0.04 = 0.45$

3. Read the following passage and answer the following questions. [*CBSE 2023 (65/2/1)*]

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child areas follows:

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \dfrac{1}{4}$ and L denotes the event that child is left handed.

(*i*) Find $P(L/C)$.

(*ii*) Find $P\left(\dfrac{\bar{L}}{A}\right)$.

(*iii*) (*a*) Find $P\left(\dfrac{A}{L}\right)$.

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

Sol. (*i*) $P(L/C) = \dfrac{17}{100}$

(*ii*) $P(L/A) = 1 - \dfrac{24}{100} = \dfrac{76}{100}$

(*iii*) (*a*) $P(A/L) = \dfrac{P(A).\,P(L/A)}{P(A).P(L/A) + P(B).P(L/B) + P(C).P(L/C) + P(D).P(L/D)}$

$$= \frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}}$$

$$= \frac{24}{24 + 22 + 17 + 9} = \frac{24}{72} = \frac{1}{3}$$

OR

(*iv*) (*b*) We have,

$$P(L/B \text{ or } C) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$$

4. Read the following passage and answer the following questions. **[*CBSE 2022 (Term-2) (65/3/2)*]**

A shopkeeper sells three types of flower seed A_1, A_2, A_3. They are sold in the form of a mixture, where, the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

(*a*) Calculate the probability that a randomly chosen seed will germinate.

(*b*) Calculate the probability that the seed is of type A_2, given that a randomly chosen seed germinates.

Sol.

E be the event that seed germinates.

(a) $P(E) = P(A_1).P(E/A_1) + P(A_2).P(E/A_2) + P(A_3).P(E/A_3)$

$P(A_1) = 4/(4+4+2) = 4/10$ $\quad P(A_2) = 4/10$ $\quad P(A_3) = 2/10$.

$A_1 \to$ event of choosing flower seed A_1

$A_2 \to$ event of choosing flower seed A_2

$A_3 \to$ event of choosing flower seed A_3.

$E/A_1 \to$ event E occuring given A_1 has occured.

similarly E/A_2 and E/A_3.

$P(E) = \dfrac{4}{10} \times \dfrac{45}{100} + \dfrac{4}{10} \times \dfrac{60}{100} + \dfrac{2}{10} \times \dfrac{35}{100}$

$= \dfrac{180 + 240 + 70}{1000}$

$P(E) = \frac{490}{1000} = 0.49$

(b) $P(A_2/E) = \frac{P(A_2) \cdot P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$

[Baye's Theorem]

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$

$= \frac{24}{49}$

Answer: a ⇒ 0.49 or $\frac{49}{100}$

b ⇒ $\frac{24}{49}$

[*Topper's Answer* 2022]

CONCEPTUAL QUESTIONS

1. An unbiased coin is tossed 4 times. Find the probability of getting atleast one head.

[*CBSE 2020 (65/3/1)*]

Sol. When an unbiased coin is tossed once, then

$P(H) = P(T) = \frac{1}{2}$, where H and T denote head and tail respectively.

∴ Probability of getting atleast one head

$= 1 - P$ (No head)

$= 1 - P$ (all tails)

$= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= 1 - \frac{1}{16} = \frac{15}{16}$

2. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

[*CBSE (AI) 2017*]

Sol. Here, A = Event that "number obtained is even".

B = Event that "number obtained is red".

$P(A) = \frac{3}{6} = \frac{1}{2}; P(B) = \frac{3}{6} = \frac{1}{2};\ P(A \cap B) = \frac{1}{6}; P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

i.e., $P(A \cap B) \neq P(A).P(B)$

Hence, A and B are not independent events.

3. **Write the probability of an even prime number on each die, when a pair of dice is rolled.**

Sol. The probability of getting even number on each die = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

(As there is only one even prime number on each die *i.e.*, 2).

4. **Two Independent events A and B are given such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A$ and not $B)$.**

Sol. We have,

$$P(A \text{ and not } B) = P(A \cap B') = P(A) - P(A \cap B)$$
$$= 0.3 - 0.18 \qquad [\because P(A \cap B) = P(A) \times P(B)]$$
$$= 0.12$$

5. **The probability distribution of X is:**

X	0	1	2	3
$P(X)$	0.2	k	k	$2k$

Write the value of k.

Sol. We have,

$$\Sigma P(X) = 1 \quad \Rightarrow \quad 0.2 + 4k = 1$$
$$\Rightarrow \quad 4k = 0.8 \quad \Rightarrow \quad k = 0.2$$

6. **The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\overline{A}) + P(\overline{B})$.** [*NCERT Exemplar*]

Sol. We know that, $A \cup B$ denotes the occurrence of atleast one of A and B and $A \cap B$ denotes the occurrence of both A and B, simultaneously.

Thus, $\quad P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$

Also, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \Rightarrow \quad 0.6 = P(A) + P(B) - 0.3$

$$\Rightarrow \quad P(A) + P(B) = 0.9$$
$$\Rightarrow \quad [1 - P(\overline{A})] + [1 - P(\overline{B})] = 0.9 \qquad [\because P(A) = 1 - P(\overline{A}) \text{ and } P(B) = 1 - P(\overline{B})]$$
$$\Rightarrow \quad P(\overline{A}) + P(\overline{B}) = 2 - 0.9 = 1.1$$

7. **Let A and B be two events. If $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cup B) = 0.6$ then find $P\left(\frac{A}{B}\right)$.**

[*NCERT Exemplar*]

Sol. Since, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.6 = 0.2 + 0.4 - P(A \cap B)$$
$$\Rightarrow \quad P(A \cap B) = 0.6 - 0.6 = 0$$
$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.4} = 0$$

Very Short Answer Questions

1. **Prove that if E and F are independent events, then the events E and F' are also independent.**

[*CBSE Delhi 2017*]

Sol. Since, E and F are independent events.

$$\Rightarrow \quad P(E \cap F) = P(E).\ P(F)$$

Now, $\quad P(E \cap F') = P(E) - P(E \cap F)$

$$= P(E) - P(E).\,P(F) = P(E)(1 - P(F))$$

$$\Rightarrow \quad P(E \cap F') = P(E).\,P(F')$$

Hence, E and F' are independent events.

2. From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24. **[*CBSE Chennai 2015*]**

Sol. Numbers divisible by 6 from 1 to 100 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96

Numbers divisible by 8 from 1 to 100 = 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96

∴ Numbers divisible by 6 or 8 but not by 24 from 1 to 100 = 6, 8, 12, 16, 18, 30, 32, 36, 40, 42, 54, 56, 60, 64, 66, 78, 80, 84, 88, 90.

∴ Required probability $= \frac{20}{100} = \frac{1}{5}$.

3. The random varibale X has a probability distribution $P(X)$ of the following from, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'. **[*CBSE 2019 (65/1/1)*]**

Sol.

$\sum_{i=0}^{2} P(X_i) = 1$ $\left[\because \sum_{i=0}^{2} P(x_i) = 1\right]$ [∵ Sum of all probabilities = 1] ~~disjoint~~ exhaustive & exclusive

$\Rightarrow \quad k + 2k + 3k = 1$

$\Rightarrow \quad 6k = 1$

$k = \frac{1}{6}$

[*Topper's Answer 2019*]

4. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not. **[*CBSE 2019 (65/1/1)*]**

Sol.

~~P(A~~ $P(A) = \frac{3}{6} = \frac{1}{2}$ $\left[\because P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}\right]$

$P(B) = \frac{3}{6} = \frac{1}{2}$ [⇒ Sample space S = {1r, 2r, 3r, 4g, 5g, 6g} A = {2r, 4g, 6g} B = {1r, 2r, 3r} A∩B = {2r}]

$P(A \cap B) = P(\text{no. which is even and red}) = \frac{1}{6}$ [∵ No. which is even and red = {2r}]

$P(A).\,P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(A \cap B) \neq P(A).\,P(B)$

∴ Events ~~are not~~ NOT INDEPENDENT.

[*Topper's Answer 2019*]

5. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamonds. **[*CBSE 2019 (65/5/3)*]**

Sol. X be the random variable of drawing at least three cards are of diamonds.

P(At least three cards are of diamonds)

$$= P(X = 3) + P(X = 4)$$

$$= \frac{{}^{13}C_3 \times {}^{39}C_1}{{}^{52}C_4} + \frac{{}^{13}C_4}{{}^{52}C_4} = \frac{1}{{}^{52}C_4}\left[{}^{13}C_3 \times 39 + {}^{13}C_4\right]$$

$$= \frac{1}{\frac{52!}{4!\,48!}}\left[\frac{13!}{3!\;10!}\times 39+\frac{13!}{4!\;9!}\right]$$

$$= \frac{1}{\frac{49\times 50\times 51\times 52}{4\times 3\times 2\times 1}}\left[\frac{11\times 12\times 13\times 39}{3\times 2\times 1}+\frac{10\times 11\times 12\times 13}{4\times 3\times 2\times 1}\right]$$

$$= \frac{4\times 3\times 2\times 1}{49\times 50\times 51\times 52}\left[\frac{11\times 12\times 13\times 39\times 4+10\times 11\times 12\times 13}{4\times 3\times 2\times 1}\right]$$

$$= \frac{1}{49\times 50\times 51\times 52}\times 11\times 12\times 13\,\{156+10\}$$

$$= \frac{11\times 12\times 13\times 166}{49\times 50\times 51\times 52} = 0.04$$

6. The probability that *A* hits the target is $\frac{1}{3}$ and the probability that *B* hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

[CBSE 2022 (65/3/2) (Term-2)]

Sol.

P (A hits) = $\frac{1}{3}$ (P(A))

P (B hits) = $\frac{2}{5}$ (P(B))

P (A doesn't hit) = $1 - \frac{1}{3}$ [P(A) + P(A') = 1]

P(A') = $\frac{2}{3}$

P (B doesn't hit) = $1 - \frac{2}{5}$

P(B') = $\frac{3}{5}$

As these are independent events = P (not hitting)
= P(A') . P(B')
[P(A∩B) = P(A) . P(B)]

P (target is hit) = 1 − P (target not hit)

= 1 − P(A') × P(B')

= $1 - \frac{2}{3} \times \frac{3}{5}$

= $1 - \frac{2}{5}$

= $\frac{3}{5}$

Answer: Probability of target being hit is $\frac{3}{5}$

[Topper's Answer 2022]

7. **The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?** [*CBSE 2020 (65/1/1)*]

Sol. let A be the event of finding a green signal on 2 consecutive days

let B_i, $i=1,2,3$ be the probability of finding a green signal on i^{th} day.

$$P(B_1)=P(B_2)=P(B_3)=\frac{30}{100}$$

$$P(A) = P(B_1)P(B_2)P(\overline{B_3}) + P(\overline{B_1})P(B_2)P(B_3)$$

$$= \frac{30}{100}\times\frac{30}{100}\times\frac{70}{100} + \frac{70}{100}\times\frac{30}{100}\times\frac{30}{100}$$

$$= \frac{126000}{1000000} = 12.6\% \text{ or } \frac{126}{1000}$$

[*Topper's Answer 2020*]

8. **A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.** [*CBSE 2018*]

Sol. Let E_1 = "Event of obtaining the sum 8"

E_1 = {(2,6)(6,2)(3,5)(5,3)(4,4)}

and F = "Event that red die result in a number less than 4."

F = {(1,1) (2,1) (3,1) (4,1) (5,1) (6,1)
(1,2) (2,2) (3,2) (4,2) (5,2) (6,2)
(1,3) (2,3) (3,3) (4,3) (5,3) (6,3)}

$$P(E/F) = \frac{P(E\cap F)}{P(F)} \text{ or } \frac{n(E\cap F)}{n(F)}$$

$E \cap F$ = {(6,2),(5,3)}

$$n(E\cap F) = \frac{2}{36}, \quad n(F) = \frac{18}{36}$$

$$P(E_1/F) = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \boxed{\frac{1}{9}} \text{ Ans}$$

[*Topper's Answer 2018*]

9. **If $P(A) = 0.6$, $P(B) = 0.5$ and $P(B/A) = 0.4$, find $P(A \cup B)$ and $P(A/B)$.** [*CBSE 2019 (65/5/1)*]

Sol. We have $P(A) = 0.6$, $P(B) = 0.5$ and $P(B/A) = 0.4$

$\because$ $$P(B/A) = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B) = P(A)\,P(B/A)$$

$\Rightarrow$ $$P(A\cap B) = 0.6 \times 0.4 = 0.24$$

Now $$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= 0.6 + 0.5 - 0.24$$

$$= 1.1 - 0.24 = 0.86$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.5} = 0.48$$

10. The probability of two students A and B coming to school in time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time. **[*CBSE 2019 (65/5/3)*]**

Sol. $P(A) = \frac{2}{7}$ *i.e.;* A is coming on time, $P(B) = \frac{4}{7}$ *i.e.;* B is coming on time,

$P(A') = 1 - \frac{2}{7} = \frac{5}{7}$, $P(B') = 1 - \frac{4}{7} = \frac{3}{7}$

$\therefore$ Probability of only one of them coming to school on time $= P(A)P(B') + P(A')P(B)$

$$= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$$

11. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?

Sol. Let X denote the number of milk chocolate drawn.

X	$P(X)$
0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$
1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$
2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$

Most likely outcome is getting one chocolate of each type.

12. Given that E and F are events such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P(\overline{E} \mid \overline{F})$.

Sol. $P(\overline{E} \mid \overline{F}) = \frac{P(\overline{E} \cap \overline{F})}{P(\overline{F})} = \frac{P(\overline{E \cup F})}{P(\overline{F})} = \frac{1 - P(E \cup F)}{1 - P(F)}$ ...(*i*)

Now, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.8 + 0.7 - 0.6 = 0.9$

Substituting value of $P(E \cup F)$ in (*i*)

$$P(\overline{E} \mid \overline{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}$$

Short Answer Questions

1. There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin. **[*CBSE 2023 (65/3/2)*]**

Sol. Let B be the event of biased coin and C be the event of fair coin.

$\therefore \quad P(B) = P(C) = \frac{1}{2}$

Now, $P\left(\frac{H}{B}\right)=\frac{1}{4}, P\left(\frac{H}{C}\right)=\frac{1}{2}$

Using Bayes' Theorem

$$\therefore \quad P\left(\frac{B}{H}\right)=\frac{P(B).P\left(\frac{H}{B}\right)}{P(B).P\left(\frac{H}{B}\right)+P(C).P\left(\frac{H}{C}\right)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{4}}{\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{1+2}{4}}$$

$$=\frac{1}{3}$$

2. *A* and *B* throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first. **[*CBSE 2023 (65/2/1)*]**

Sol. Probability of getting a six when a die is thrown $=\frac{1}{6}$

Let *A* be the event of winning the game by *A* and *B* be the event of winning the game by *B*,

When *A* starts the game first.

Winning probability of *A* is given by

A or $A'B'A$ or $A'B'A'B'A$ or .... to ∞

$$=\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\ldots \text{ to } \infty$$

$$=\frac{1}{6}\left(1+\frac{25}{36}+\left(\frac{25}{36}\right)^2+\ldots \text{ to } \infty\right)$$

G.P.

$$=\frac{1}{6}\times\frac{1}{1-\frac{25}{36}}=\frac{1}{6}\times\frac{36}{11}=\frac{6}{11} \qquad \left(S_\infty=\frac{a}{1-r} \text{ for a G.P.}\right)$$

Winning probability of *B* is given by

$A'B$ or $A'B'A'B$ or $A'B'A'B'A'B$ or .... to ∞

$$=\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\ldots \text{ to } \infty$$

$$=\frac{5}{36}\left(1+\frac{25}{36}+\left(\frac{25}{36}\right)^2+\ldots. \text{ to } \infty\right)$$

$$=\frac{5}{36}\times\frac{1}{1-\frac{25}{36}}=\frac{5}{36}\times\frac{36}{11}=\frac{5}{11}$$

3. *A* and *B* are independent events such that $P(A\cap\overline{B})=\frac{1}{4}$ and $P(\overline{A}\cap B)=\frac{1}{6}$.

Find *P*(*A*) and *P*(*B*). **[*CBSE 2023 (65/1/1)*]**

Sol. Given *A* and *B* are independent events such that

$$P(A\cap B')=\frac{1}{4} \Rightarrow P(A).P(B')=\frac{1}{4}$$

$$\Rightarrow P(A)\{1-P(B)\}=\frac{1}{4}$$

Let $P(A) = x$

$$\Rightarrow 1 - P(B) = \frac{1}{4x} \quad \Rightarrow P(B) = 1 - \frac{1}{4x}$$

Also,

$$P(A' \cap B) = \frac{1}{6} \Rightarrow P(A').P(B) = \frac{1}{6}$$

$$\Rightarrow \{1 - P(A)\}\, P(B) = \frac{1}{6}$$

$$\Rightarrow (1 - x)\left(1 - \frac{1}{4x}\right) = \frac{1}{6}$$

$$\Rightarrow \quad \frac{(1-x)(4x-1)}{4x} = \frac{1}{6} \qquad \Rightarrow 4x - 1 - 4x^2 + x = \frac{2x}{3}$$

$$\Rightarrow \quad (5x - 4x^2 - 1) \times 3 = 2x \qquad \Rightarrow 15x - 12x^2 - 3 = 2x$$

$$\Rightarrow \quad 12x^2 - 13x + 3 = 0 \qquad \Rightarrow 12x^2 - 9x - 4x + 3 = 0$$

$$\Rightarrow \quad 3x(4x - 3) - 1(4x - 3) = 0 \qquad \Rightarrow (4x - 3)(3x - 1) = 0$$

$$\Rightarrow \quad x = \frac{3}{4} \text{ or } \frac{1}{3}$$

$$\Rightarrow \quad P(A) = \frac{3}{4} \text{ or } \frac{1}{3}$$

When $P(A) = \frac{3}{4}$ $\quad \Rightarrow P(B) = 1 - \frac{1}{4x} = 1 - \frac{1}{4 \times \frac{3}{4}} = 1 - \frac{1}{3} = \frac{2}{3}$

$$\Rightarrow P(B) = \frac{2}{3}$$

When $P(A) = \frac{1}{3}$ $\quad \Rightarrow P(B) = 1 - \frac{1}{4x} = 1 - \frac{1}{4 \times \frac{1}{3}} = 1 - \frac{3}{4} = \frac{1}{4}$

$$\Rightarrow P(B) = \frac{1}{4}$$

4. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that **[*CBSE Delhi 2014*]**

(*i*) the youngest is a girl? (*ii*) atleast one is a girl?

Sol. A family has 2 children,

then sample space = $S = \{BB, BG, GB, GG\}$, where B stands for boy and G for girl.

(*i*) Let A and B be two event such that

A = Both are girls = $\{GG\}$ and B = The youngest is a girl = $\{BG, GG\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2} \qquad [\because A \cap B = \{GG\}]$$

(*ii*) Let C be event such that

C = at least one is a girl = $\{BG, GB, GG\}$

$$\text{Now } P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \qquad [\because A \cap C = \{GG\}]$$

5. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee. **[*CBSE (East) 2016*]**

Sol. Let A and B be two events such that

A = selection of committee having exactly 2 boys.

B = selection of committee having at least one girl.

The required probability is $P\left(\frac{A}{B}\right)$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{{}^{11}C_4}$$

$$= \frac{\frac{4!}{1!3!} \times \frac{7!}{3! \times 4!} + \frac{4!}{2!2!} \times \frac{7!}{2!5!} + \frac{4!}{3!1!} \times \frac{7!}{1!6!} + \frac{4!}{4!0!}}{\frac{11!}{4!7!}}$$

$$= \frac{4 \times \frac{7 \times 6 \times 5}{3 \times 2} + \frac{4 \times 3}{2 \times 1} \times \frac{7 \times 6}{2 \times 1} + 4 \times 7 + 1}{\frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}} = \frac{140 + 126 + 28 + 1}{330} = \frac{295}{330} = \frac{59}{66}$$

$$P(A \cap B) = \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_4} = \frac{\frac{4!}{2!2!} \times \frac{7!}{2!5!}}{\frac{11!}{4!7!}} = \frac{\frac{4 \times 3}{2 \times 1} \times \frac{7 \times 6}{2 \times 1}}{330} = \frac{126}{330} = \frac{21}{55}$$

$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{\frac{21}{55}}{\frac{59}{66}} = \frac{21}{55} \times \frac{66}{59} = \frac{126}{295}$$

6. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black. **[*CBSE Delhi 2015*]**

Sol. Let E, F and A be three events such that

E = selection of bag A

and F = selection of bag B

A = getting one red and one black ball out of two

Here, $P(E) = P(\text{getting 1 or 2 in a throw of die}) = \frac{2}{6} = \frac{1}{3}$

$\therefore\ P(F) = 1 - \frac{1}{3} = \frac{2}{3}$

Also, $P(A/E) = P$ (getting one red and one black if bag A is selected) $= \frac{{}^6C_1 \times {}^4C_1}{{}^{10}C_2} = \frac{24}{45}$

and $P(A/F) = P$ (getting one red and one black if bag B is selected) $= \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2} = \frac{21}{45}$

Now, by theorem of total probability,

$$P(A) = P(E).\ P(A/E) + P(F).\ P(A/F)$$

$$= \frac{1}{3} \times \frac{24}{45} + \frac{2}{3} \times \frac{21}{45} = \frac{8 + 14}{45} = \frac{22}{45}$$

7. **Bag *I* contains 3 red and 4 black balls while another bag *II* contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag *II*.** **[*CBSE Delhi 2011*]**

Sol. Let E_1 be the event of choosing the bag I, E_2 the event of choosing the bag II and A be the event of drawing a red ball.

Then $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(A/E_1) = P$ (drawing a red ball from bag I) $= \frac{3}{7}$

and $P(A/E_2) = P$ (drawing a red ball from bag II) $= \frac{5}{11}$

Now, the probability of drawing a ball from bag II, being given that it is red, is $P(E_2/A)$. By using Bayes' theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = \frac{\frac{1}{2}\times\frac{5}{11}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}} = \frac{35}{68}$$

8. **Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.** **[*CBSE Delhi 2011*]**

Sol. Let E_1 , E_2 and A be event such that

E_1 = Selecting male person

E_2 = Selecting women (female person)

A = Selecting grey haired person.

Then $P(E_1) = \frac{1}{2}$, $\quad P(E_2) = \frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{5}{100}$, $\quad P\left(\frac{A}{E_2}\right) = \frac{0.25}{100}$

Here, required probability is $P\left(\frac{E_1}{A}\right)$.

$$\therefore \quad P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$\therefore \quad P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2}\times\frac{5}{100}}{\frac{1}{2}\times\frac{5}{100}+\frac{1}{2}\times\frac{0.25}{100}} = \frac{5}{5+0.25} = \frac{500}{525} = \frac{20}{21}$$

9. **Three persons *A*, *B* and *C* apply for a job of manager in a private company. Chances of their selection (*A*, *B* and *C*) are in the ratio 1 : 2 : 4. The probabilities that *A*, *B* and *C* can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of *C*.** **[*CBSE Delhi 2016*]**

Sol. Let E_1, E_2, E_3 and A be events such that

E_1 = person selected is A; $\quad E_2$ = person selected is B; $\quad E_3$ = person selected is C

A = changes to improve profit does not take place.

Now $\quad P(E_1)=\frac{1}{7}, P(E_2)=\frac{2}{7}, P(E_3)=\frac{4}{7}$

$$P\left(\frac{A}{E_1}\right)=1-\frac{8}{10}=\frac{2}{10}; \quad P\left(\frac{A}{E_2}\right)=1-\frac{5}{10}=\frac{5}{10}; \quad P\left(\frac{A}{E_3}\right)=1-\frac{3}{10}=\frac{7}{10}$$

We require $P\left(\frac{E_3}{A}\right)$

$$P\left(\frac{E_3}{A}\right)=\frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}=\frac{\frac{4}{7}\times\frac{7}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$$

$$=\frac{28}{70}\times\frac{70}{2+10+28}=\frac{28}{40}=\frac{7}{10}$$

10. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white? *[CBSE (Central) 2016]*

Sol. There may be three situations as events.

E_1 = bag contains 2 white balls, $\quad E_2$ = bag contains 3 white balls,

E_3 = bag contains all 4 white balls, $\quad A$ = getting two white balls.

We have required $P\left(\frac{E_3}{A}\right)=?$

Now, $\quad P(E_1)=\frac{1}{3}; \quad P(E_2)=\frac{1}{3}; \quad P(E_3)=\frac{1}{3}$

$$P\left(\frac{A}{E_1}\right)=\frac{^2C_2}{^4C_2}=\frac{1}{6}; \quad P\left(\frac{A}{E_2}\right)=\frac{^3C_2}{^4C_2}=\frac{3}{6}=\frac{1}{2}; \quad P\left(\frac{A}{E_3}\right)=\frac{^4C_2}{^4C_2}=1$$

Now, $\quad P\left(\frac{E_3}{A}\right)=\frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1}=\frac{\frac{1}{3}}{\frac{1}{18}+\frac{1}{6}+\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{10}{18}}=\frac{1}{3}\times\frac{18}{10}=\frac{3}{5}$$

11. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. *[CBSE Delhi 2017; (F) 2013]*

Sol. Let E_1, E_2 and E be three events such that

E_1 = six occurs; $\quad E_2$ = six does not occur and

E = man reports that six occurs in the throwing of the dice.

Now, $\quad P(E_1)=\frac{1}{6}, P(E_2)=\frac{5}{6}; \quad P\left(\frac{E}{E_1}\right)=\frac{4}{5}, P\left(\frac{E}{E_2}\right)=1-\frac{4}{5}=\frac{1}{5}$

We have to find $P\left(\frac{E_1}{E}\right)$

$$P\left(\frac{E_1}{E}\right)=\frac{P(E_1).P\left(\frac{E}{E_1}\right)}{P(E_1).P\left(\frac{E}{E_1}\right)+P(E_2).P\left(\frac{E}{E_2}\right)}=\frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}=\frac{4}{30}\times\frac{30}{4+5}=\frac{4}{9}$$

12. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine:

(i) k *(ii)* $P(X < 3)$ *(iii)* $P(X > 6)$ *(iv)* $P(0 < X < 3)$ [CBSE (AI) 2011]

Sol. $\because \sum_{i=1}^{n} p_i = 1$

$\therefore \quad 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0$

$\Rightarrow \quad 10k^2 + 10k - k - 1 = 0 \quad \Rightarrow \quad 10k(k+1) - 1(k+1) = 0$

$\Rightarrow \quad (k+1)(10k-1) = 0 \quad \Rightarrow \quad k = -1$ and $k = \frac{1}{10}$

(*i*) $\because$ k can never be negative as probability is never negative then $k = \frac{1}{10}$

(*ii*) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k = \frac{3}{10}$

(*iii*) $P(X > 6) = P(X = 7) = 7k^2 + k = 7 \times \frac{1}{100} + \frac{1}{10} = \frac{17}{100}$

(*iv*) $P(0 < X < 3) = P(X = 1) + P(X = 2) = k + 2k = 3k = \frac{3}{10}$

13. The probability distribution of a random variable X is given below:

X	1	2	3
$P(X)$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(*i*) Find the value of k.

(*ii*) Find $P(1 \le X < 3)$.

(*iii*) Find $E(X)$, the mean of X. [CBSE 2023 (65/1/1)]

Sol. Given probability distribution be

X	1	2	3
$P(X)$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(*i*) $\Sigma P(X_i) = 1 \Rightarrow \frac{k}{2} + \frac{k}{3} + \frac{k}{6} = 1$

$\Rightarrow \quad \frac{3k + 2k + k}{6} = 1 \quad \Rightarrow k = 1$

$\therefore \quad k = 1$

(*ii*) $P(1 \le X < 3) = P(1) + P(2) = \frac{k}{2} + \frac{k}{3} = \frac{5k}{6} = \frac{5 \times 1}{6} = \frac{5}{6}$

(*iii*) $E(X) = \Sigma p_i . x_i = 1 \times \frac{k}{2} + 2 \times \frac{k}{3} + 3 \times \frac{k}{6}$

$= \frac{k}{2} + \frac{2k}{3} + \frac{k}{2}$

$= \frac{3k + 4k + 3k}{6} = \frac{10k}{6} = \frac{5k}{3} = \frac{5 \times 1}{3} = \frac{5}{3} \quad [\because k = 1]$

14. Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. *[CBSE 2023 (65/2/1)]*

Sol. Let number of red balls = number of green balls = x

$\therefore$ Total number of balls = $x + x = 2x$

$\therefore \quad P(\text{getting a red ball}) = \dfrac{x}{2x} = \dfrac{1}{2}$

Let X = Number of red balls

When X = 0 *i.e.*, there is no red ball drawn.

$P(X = 0) = P(\text{IG}) \times P(\text{IIG})$ Where IG = first drawn ball is green

$= \dfrac{x}{2x} \times \dfrac{x}{2x} = \dfrac{1}{4}$ IIG = second drawn ball is green

When X = 1 *i.e.*, there is only one ball is red.

$P(X = 1) = P(\text{IR}) \times P(\text{IIG}) + P(\text{IG}) \times P(\text{IIR})$

$= \dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{2}{4} = \dfrac{1}{2}$

When X = 2 *i.e.*; both drawn ball is red.

$P(X = 2) = P(\text{IR}) \times P(\text{IIR}) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

Thus, probability distribution is

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\therefore$ Mean $(\overline{X}) = \Sigma P_i . X_i = \dfrac{1}{4} \times 0 + \dfrac{1}{2} \times 1 + \dfrac{1}{4} \times 2 = \dfrac{1}{2} + \dfrac{1}{2} = 1$

15. A pair of dice is thrown simultaneously. If *X* denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X. *[CBSE 2023 (65/3/2)]*

Sol. We have

$X = 0, 1, 2, 3, 4, 5$

When X = 0, favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$P(0) = \dfrac{6}{36}$

When X = 1, favourable outcomes = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 5), (5, 4), (4, 3), (3, 2), (2, 1)}

$P(X) = \dfrac{10}{36}$

When X = 2, favourable outcomes = {(1, 3), (2, 4), (3, 5), (4, 6), (6, 4), (5, 3), (4, 2), (3, 1)}

$P(X) = \dfrac{8}{36}$

When X = 3, favourable outcomes = {(1, 4), (2, 5), (3, 6), (6, 3), (5, 2), (4, 1)}

$\therefore P(X = 3) = \dfrac{6}{36}$

When X = 4, favourable outcomes = {(1, 5), (2, 6), (6, 2), (5, 1)}

$\therefore P(X = 3) = \dfrac{4}{36}$

When X = 5, favourable outcomes = {(1, 6), (6, 1)}

$\therefore P(X = 5) = \dfrac{2}{36}$

$\therefore$ Probability distribution is

X	0	1	2	3	4	5
P(X)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

16. Three numbers are selected at random (without replacement) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also, find the mean of the distribution. [*CBSE (South) 2016*]

Sol. First six positive integers are 1, 2, 3, 4, 5 and 6.

If three numbers are selected at random from above six numbers then the number of elements in sample space S is given by

i.e., $n(S) = {}^6C_3 = \dfrac{6!}{3!3!} = \dfrac{6 \times 5 \times 4}{3 \times 2} = 20$

Here X, smallest of the three numbers obtained, is random variable X may have value 1, 2, 3 and 4. Therefore, required probability distribution is given as

$P(X = 1)$ = Probability of event getting 1 as smallest number

$= \dfrac{{}^5C_2}{20} = \dfrac{5!}{2!3! \times 20} = \dfrac{5 \times 4}{2 \times 20} = \dfrac{10}{20} = \dfrac{1}{2}$ [${}^5C_2 \equiv$ selection of two numbers out of 2, 3, 4, 5, 6]

$P(X = 2)$ = Probability of events getting 2 as smallest number.

$= \dfrac{{}^4C_2}{20} = \dfrac{4!}{2!2! \times 20} = \dfrac{6}{20} = \dfrac{3}{10}$ [${}^4C_2 \equiv$ selection of two numbers out of 3, 4, 5, 6]

$P(X = 3)$ = Probability of events getting 3 as smallest number

$= \dfrac{{}^3C_2}{20} = \dfrac{3!}{2!1! \times 20} = \dfrac{3}{20}$ [${}^3C_2 \equiv$ selection of two numbers out 4, 5, 6]

$P(X = 4)$ = Probability of events getting 4 as smallest number.

$= \dfrac{{}^2C_2}{20} = \dfrac{1}{20}$ [${}^2C_2 \equiv$ selection of two numbers out of 5, 6]

Required probability distribution table is

X or x_i	1	2	3	4
$P(X)$ or p_i	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$

Mean $= E(X) = \Sigma p_i x_i$

$$= 1 \times \frac{1}{2} + 2 \times \frac{3}{10} + 3 \times \frac{3}{20} + 4 \times \frac{1}{20}$$

$$= \frac{1}{2} + \frac{6}{10} + \frac{9}{20} + \frac{4}{20} = \frac{10 + 12 + 9 + 4}{20} = \frac{35}{20} = \frac{7}{4}$$

17. In a game, a man wins ₹5 for getting a number greater than 4 and loses ₹1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses. [*CBSE (Central) 2016*]

Sol. Let X be random variable, which is possible value of winning or loosing of rupee occur with probability of getting a number greater than 4 in 1st, 2nd, 3rd or in any throw respectively.

Obviously X may have value ₹5, ₹4, ₹3 and – ₹3 respectively.

Now, $P(X = 5) = P$ (getting number greater than 4 in first throw)

$$= \frac{2}{6} = \frac{1}{3}$$

$P(X = 4) = P$ (getting number greater than 4 in 2nd throw)

$$= \frac{4}{6} \times \frac{2}{6} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

$P(X = 3) = P$ (getting number greater than 4 in 3rd throw)

$$= \frac{4}{6} \times \frac{4}{6} \times \frac{2}{6} = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$$

$P(X = -3) = P$ (getting number greater than 4 in no throw)

$$= \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

Therefore, probability distribution is as

X or x_i	5	4	3	–3
$P(X)$ or p_i	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{27}$

$\therefore$ Expected value of the amount he wins/loses = $E(X)$

$$E(X) = \Sigma x_i p_i = 5 \times \frac{1}{3} + 4 \times \frac{2}{9} + 3 \times \frac{4}{27} + (-3) \times \frac{8}{27}$$

$$= \frac{5}{3} + \frac{8}{9} + \frac{12}{27} - \frac{24}{27} = \frac{45 + 24 + 12 - 24}{27} = \frac{57}{27} = ₹\frac{19}{9} = ₹\,2\frac{1}{9}$$

18. There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X. *[CBSE (F) 2017]*

Sol. If two cards, from four cards having numbers 1, 2, 3, 4 each are drawn at random then sample space S is given by

$$S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (4, 1), (4, 2), (4, 3), (3, 1), (3, 2), (3, 4)\}$$

Let X, sum of the numbers, be random variable. X may have values 3, 4, 5, 6, 7.

Now $P(X = 3)$ = Probability of event getting (1, 2), (2, 1) $= \frac{2}{12} = \frac{1}{6}$

$P(X = 4)$ = Probability of event getting (1, 3), (3, 1) $= \frac{2}{12} = \frac{1}{6}$

$P(X = 5)$ = Probability of event getting (1, 4), (4, 1), (2, 3), (3, 2) $= \frac{4}{12} = \frac{1}{3}$

$P(X = 6)$ = Probability of event getting (4, 2), (2, 4) $= \frac{2}{12} = \frac{1}{6}$

$P(X = 7)$ = Probability of event getting (4, 3), (3, 4) $= \frac{2}{12} = \frac{1}{6}$

Thus, probability distribution is represented in tabular form as

X	3	4	5	6	7
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\therefore \text{ Mean} = \Sigma X.P(X) = \frac{3}{6} + \frac{4}{6} + \frac{5}{3} + \frac{6}{6} + \frac{7}{6} = \frac{3 + 4 + 10 + 6 + 7}{6} = \frac{30}{6} = 5$$

19. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3) = a$ such that $\sum p_i x_i^2 = 2\sum p_i x_i$, find the value of p. [CBSE Delhi 2017]

Sol. Given X is a random variable with values 0, 1, 2, 3. Given probability distributions are as

$X(x_i)$	0	1	2	3
$P(x)(p_i)$	p	p	a	a
$x_i p_i$	0	p	$2a$	$3a$
$x_i^2 p_i$	0	p	$4a$	$9a$

$\therefore$ $\sum x_i p_i = 0 + p + 2a + 3a = p + 5a$

$\sum x_i^2 p_i = 0 + p + 4a + 9a = p + 13a$

According to question

$\sum p_i x_i^2 = 2\sum p_i x_i$

$p + 13a = 2p + 10a \Rightarrow p = 3a$

Also $p + p + a + a = 1 \Rightarrow 2p + 2a = 1$

$2a = 1 - 2p \Rightarrow a = \frac{1-2p}{2}$

$\therefore$ $p = 3 \times \frac{(1-2p)}{2} \Rightarrow 2p = 3 - 6p$

$\Rightarrow$ $8p = 3 \Rightarrow p = \frac{3}{8}$

20. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean of the distribution. [CBSE Sample Paper 2018]

Sol. Let X be the number of bad oranges in two draws of orange from the lot. Here, X is random variable and may have value 0, 1, 2.

Now, $P(X = 0) = \frac{^{16}C_2}{^{20}C_2} = \frac{16!}{2! \times 14!} \times \frac{2! \times 18!}{20!} = \frac{18 \times 17 \times 16 \times 15}{20 \times 19 \times 18 \times 17} = \frac{60}{95}$

$P(X = 1) = \frac{^{4}C_1 \times {}^{16}C_1}{^{20}C_2} = \frac{4 \times 16 \times 2}{20 \times 19} = \frac{32}{95}$;

$P(X = 2) = \frac{^{4}C_2}{^{20}C_2} = \frac{4 \times 3}{20 \times 19} = \frac{3}{95}$

Now, required probability distribution is

X	0	1	2
$P(X)$	$\frac{60}{95}$	$\frac{32}{95}$	$\frac{3}{95}$

Now, mean $= \sum X_i P(X_i) = 0 \times \frac{60}{95} + 1 \times \frac{32}{95} + 2 \times \frac{3}{95} = \frac{32}{95} + \frac{6}{95} = \frac{38}{95} = \frac{2}{5}$

21. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails. [CBSE 2020 (65/4/1)]

Sol. A coin is biased such that

$$P(H) = \frac{3}{4} \text{ and } P(T) = \frac{1}{4}$$

Let X = Number of tails when coin is tossed twice.

$\therefore$ When $X = 0$ (*i.e.* no tail)

$$\Rightarrow \quad P(X=0) = P(H).P(H) = \frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$$

when $X = 1$ (*i.e.*, only one tail)

$$\Rightarrow \quad P(X=1) = P(T).P(H) + P(H).P(T)$$

$$\Rightarrow \quad P(X=1) = \frac{1}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{1}{4} = \frac{6}{16}$$

when $X = 2$ (*i.e.*, both tails).

$$P(X=2) = P(T).P(T) = \frac{1}{4}\times\frac{1}{4} = \frac{1}{16}$$

Probability distribution is given by

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

We have, mean $= \Sigma P(X_i).X_i$

$$\therefore \quad \text{Mean} = 0\times\frac{9}{16} + 1\times\frac{6}{16} + 2\times\frac{1}{16} = \frac{6}{16} + \frac{2}{16} = \frac{8}{16} = \frac{1}{2}$$

Long Answer Questions

1. **There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.** **[*CBSE 2019 (65/3/1)*]**

Sol. We have events

E_1: bag I is selected; E_2: bag II is selected; A : getting a red ball

So, $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P(A/E_1) = \frac{3}{9} = \frac{1}{3}$ and $P(A/E_1) = \frac{5}{5+n}$

Now, $P(E_2/A) = \dfrac{P(E_2).P(A/E_2)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$

$$\Rightarrow \quad \frac{3}{5} = \frac{\frac{1}{2}\times\frac{5}{5+n}}{\frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{5}{5+n}} = \frac{\frac{5}{5+n}}{\frac{1}{3} + \frac{5}{5+n}}$$

$$\Rightarrow \quad \frac{3}{5} = \frac{5}{5+n}\times\frac{3(5+n)}{(5+n)+15} = \frac{15}{20+n}$$

$$\Rightarrow \quad 60 + 3n = 75$$

$$\Rightarrow \quad 3n = 15 \quad \Rightarrow \quad n = 5$$

2. **A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.** **[CBSE Delhi 2014]**

Sol. Let E_1, E_2, E_3, E_4 and A be event defined as

$E_1 =$ the lost card is a spade card, $E_2 =$ the lost card is a non spade card

and $A =$ drawing three spade cards from the remaining cards.

Now, $P(E_1) = \frac{13}{52} = \frac{1}{4}, P(E_2) = \frac{39}{52} = \frac{3}{4}$

$$P\left(\frac{A}{E_1}\right) = \frac{^{12}C_3}{^{51}C_3} = \frac{220}{20825}; \qquad P\left(\frac{A}{E_2}\right) = \frac{^{13}C_3}{^{51}C_3} = \frac{286}{20825}$$

Here, required probability $= P\left(\frac{E_1}{A}\right)$

$$\therefore \quad = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{4}\times\frac{220}{20825}}{\frac{1}{4}\times\frac{220}{20825} + \frac{3}{4}\times\frac{286}{20825}}$$

$$= \frac{220}{220 + 3\times 286} = \frac{220}{1078} = \frac{10}{49}$$

3. **There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.** **[CBSE 2020 (65/1/1)]**

Sol.

I	II
3R	4R
5B	3B

let B_1 be the event that a black ball is transferred to Bag 2.

let R_1 be the event that a red ball is transferred to Bag 2.

let B_2 be the event that a black ball is picked from Bag 2.

$P(B_1) = \frac{5}{8}$ $\quad P(R_1) = \frac{3}{8}$ $\quad P(B_2/B_1) = \frac{4}{8}$ $\cdot$ $P(B_2/R_1) = \frac{3}{8}$

According to Bayes theorem

$$P(B_1/B_2) = \frac{P(B_1)P(B_2/B_1)}{P(B_1)P(B_2/B_1) + P(R_1)P(B_2/R_1)}$$

$$= \frac{\frac{5}{8}\times\frac{4}{8}}{\frac{5}{8}\times\frac{4}{8} + \frac{3}{8}\times\frac{3}{8}} = \frac{20}{20+9} = \frac{20}{29}$$

[Topper's Answer 2020]

4. **In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly?** **[CBSE 2023 (65/5/1)]**

Sol. Let E_1, E_2, E are the events such that

E_1 : Student knows the answer

E_2 : Student guess the answer

E : The answer is correct

Given, $P(E_1) = \frac{3}{5}, P(E_2) = \frac{2}{5}$

$P(E | E_2) = \frac{1}{3}, P(E | E_1) = 1$

$P(E_1 | E) = ?$

By Bayes' Theorem

$$P(E_1 | E) = \frac{P(E_1)\, P(E \mid E_1)}{P(E_1)\, P(E \mid E_1) + P(E_2)\, P(E \mid E_2)}$$

$$= \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}} = \frac{\frac{3}{5}}{\frac{3}{5} + \frac{2}{15}}$$

$$\Rightarrow P(E_1 | E) = \frac{\frac{3}{5}}{\frac{9+2}{15}} = \frac{\frac{3}{5}}{\frac{11}{15}} = \frac{3}{5} \times \frac{15}{11} = \frac{9}{11}$$

5. **There are three coins. One is two headed coin (having head on both faces), another is a biased coin that comes up head 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?** *[CBSE Delhi 2009, (F) 2011]*

Sol. Let E_1, E_2, E_3 and A be event defined as:

E_1 = selection of a two headed coin; E_2 = selection of a biased coin

E_3 = selection of an unbiased coin; A = coin shows head after tossing

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{75}{100} = \frac{3}{4}, \quad P\left(\frac{A}{E_3}\right) = \frac{1}{2}$$

Here, required probability = $P\left(\frac{E_1}{A}\right)$

By using Bayes' theorem,

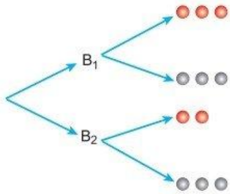
$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)}$$

$$= \frac{1}{\frac{4+3+2}{4}} = \frac{4}{9}$$

6. **A bag contains 5 red and 3 black balls and another bag contains 2 red and 6 black balls. Two balls are drawn at random (without replacement) from one of the bags and both are found to be red. Find the probability that balls are drawn from first bag.** **[*CBSE 2019 (65/5/3)*]**

Sol.



Let E be the event of drawing two red balls.

Let B_1 be the event that first bag is chosen and B_2 be the event that second bag is chosen.

$$P(B_1) = P(B_2) = \frac{1}{2}, \quad P(E/B_1) = \frac{5}{8}\times\frac{4}{7} = \frac{20}{56}$$

$$P(E/B_2) = \frac{2}{8}\times\frac{1}{7} = \frac{2}{56}$$

$$\therefore \; P(B_1/E) = \frac{P(B_1)\times P(E/B_1)}{P(B_1)\times P(E/B_1) + P(B_2)P(E/B_2)}$$

$$= \frac{\frac{1}{2}\times\frac{20}{56}}{\frac{1}{2}\times\frac{20}{56}+\frac{1}{2}\times\frac{2}{56}}$$

$$= \frac{20}{20+2} = \frac{20}{22} = \frac{10}{11}$$

7. **In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.** **[*CBSE (F) 2012*]**

Sol. Let E_1, E_2, A be events such that

E_1 = student selected is girl

E_2 = student selected is boy

A = student selected is taller than 1.75 metres.

Here $P\left(\frac{E_1}{A}\right)$ is required.

Now, $P(E_1) = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = \frac{40}{100} = \frac{2}{5}$, $P\left(\frac{A}{E_1}\right) = \frac{1}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{4}{100}$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{1}{100}}{\frac{3}{5}\times\frac{1}{100}+\frac{2}{5}\times\frac{4}{100}} = \frac{\frac{3}{500}}{\frac{3}{500}+\frac{8}{500}} = \frac{3}{500}\times\frac{500}{11} = \frac{3}{11}$$

8. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B? **[*CBSE (F) 2011*]**

Sol. Let E_1, E_2 and A be event such that

E_1 = Production of items by machine A

E_2 = Production of items by machine B

A = Selection of defective items.

$$P(E_1) = \frac{60}{100} = \frac{3}{5}, P(E_2) = \frac{40}{100} = \frac{2}{5},$$

$$P\left(\frac{A}{E_1}\right) = \frac{2}{100} = \frac{1}{50}, P\left(\frac{A}{E_2}\right) = \frac{1}{100}$$

$P\left(\frac{E_2}{A}\right)$ is required.

By Bayes' theorem

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{2}{5}\times\frac{1}{100}}{\frac{3}{5}\times\frac{1}{50}+\frac{2}{5}\times\frac{1}{100}} = \frac{\frac{2}{500}}{\frac{3}{250}+\frac{2}{500}} = \frac{2}{500}\times\frac{500}{6+2} = \frac{1}{4}$$

9. A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from the bag I to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red, then find the probability that one red and one white ball are transferred from the bag I to the bag II. **[*CBSE Sample Paper 2016*]**

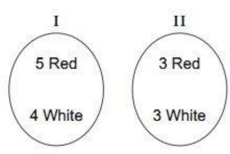
Sol. Let E_1, E_2, E_3 and A be event such that

E_1 = Both transferred balls from bag I to bag II are red.

E_2 = Both transferred balls from bag I to bag II are white.

E_3 = Out of two transferred balls one is red and other is white.

A = Drawing a red ball from bag II



$$P(E_1) = \frac{{}^5C_2}{{}^9C_2} = \frac{5\times4}{9\times8} = \frac{20}{72} = \frac{5}{18}$$

$$P(E_2) = \frac{{}^4C_2}{{}^9C_2} = \frac{4\times3}{9\times8} = \frac{12}{72} = \frac{3}{18}$$

$$P(E_3) = \frac{{}^5C_1\times{}^4C_1}{{}^9C_2} = \frac{5\times4\times2}{9\times8} = \frac{40}{72} = \frac{10}{18}$$

$$P\left(\frac{A}{E_1}\right) = \frac{5}{8};\quad P\left(\frac{A}{E_2}\right) = \frac{3}{8};\quad P\left(\frac{A}{E_3}\right) = \frac{4}{8}$$

We require $P\left(\frac{E_3}{A}\right)$.

Now, by Bayes' theorem

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{10}{18} \times \frac{4}{8}}{\frac{5}{18} \times \frac{5}{8} + \frac{3}{18} \times \frac{3}{8} + \frac{10}{18} \times \frac{4}{8}}$$

$$= \frac{\frac{40}{144}}{\frac{25}{144} + \frac{9}{144} + \frac{40}{144}} = \frac{40}{144} \times \frac{144}{74} = \frac{20}{37}$$

10. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtaind exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die? *[CBSE 2018]*

Sol. Let E_1 = "Event that the girl threw 3, 4, 5 or 6"

E_2 = "Event that the girl threw 1, 2."

and A = "Event that the girl gets exactly one tail"

$P(E_1) = \frac{4}{6} = \frac{2}{3}$, $P(E_2) = \frac{2}{6} = \frac{1}{3}$

$P(A/E_1) = \frac{1}{2}$, $P(A/E_2) = 3/8$

Now, $$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$P(E_1/A) = \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{8}}$$

$$P(E_1/A) = \frac{\frac{2}{6}}{\frac{2}{6} + \frac{1}{8}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}} = \frac{\frac{1}{3}}{\frac{8+3}{24}}$$

$$P(E_1/A) = \frac{1}{3} \times \frac{24}{11} = \boxed{\frac{8}{11}}$$ Ans

[Topper's Answer 2018]

11. An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of three balls. Find the mean of X. *[CBSE (F) 2010]*

Sol. Let X be the number of red balls in a random draw of three balls.

As there are 3 red balls, possible values of X are 0, 1, 2, 3.

$$P(0) = \frac{^3C_0 \times {^4C_3}}{^7C_3} = \frac{4 \times 3 \times 2}{7 \times 6 \times 5} = \frac{4}{35} \qquad P(1) = \frac{^3C_1 \times {^4C_2}}{^7C_3} = \frac{3 \times 6 \times 6}{7 \times 6 \times 5} = \frac{18}{35}$$

$$P(2) = \frac{^3C_2 \times {^4C_1}}{^7C_3} = \frac{3 \times 4 \times 6}{7 \times 6 \times 5} = \frac{12}{35} \qquad P(3) = \frac{^3C_3 \times {^4C_0}}{^7C_3} = \frac{1 \times 1 \times 6}{7 \times 6 \times 5} = \frac{1}{35}$$

For calculation of mean

X	$P(X)$
0	4/35
1	18/35
2	12/35
3	1/35
Total	**1**

$$\text{Mean} = \sum XP(X) = 0 \times \frac{4}{35} + 1 \times \frac{18}{35} + 2 \times \frac{12}{35} + 3 \times \frac{1}{35} = \frac{9}{7}$$

12. A box contains 10 tickets 2 of which carry a prize of ₹ 8 each. 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize. *[CBSE 2023 (65/5/1)]*

Sol. Let X denote the random variable of amount of prize. Then X can take values ₹ 2, ₹ 4 and ₹ 8

$$P(X = 2) = \frac{3}{10}$$

$$P(X = 4) = \frac{5}{10}$$

$$P(X = 8) = \frac{2}{10}$$

Probability distribution is given by

X	2	4	8
$P(X)$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{2}{10}$

$$\text{Mean} = \sum X_i P(X_i) = 2 \times \frac{3}{10} + 4 \times \frac{5}{10} + 8 \times \frac{2}{10}$$

$$= \frac{6}{10} + \frac{20}{10} + \frac{16}{10} = \frac{42}{10} = 4.2$$

13. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean of X. *[CBSE 2018]*

Sol.

Let X denote the larger of the two numbers.

$X = 2, 3, 4, 5$

$P(X=2) = \frac{2}{20} = \frac{1}{10}$

$P(X=3) = \frac{4}{20} = \frac{2}{10}$

$P(X=4) = \frac{6}{20} = \frac{3}{10}$

$P(X=5) = \frac{8}{20} = \frac{4}{10}$

Probability Distribution :-

X	2	3	4	5
P(X)	1/10	2/10	3/10	4/10

Mean

$E(X) = \sum p_i x_i$

$\sum p_i x_i = 2 \times \frac{1}{10} + 3 \times \frac{2}{10} + 4 \times \frac{3}{10} + 5 \times \frac{4}{10}$

$\sum p_i x_i = \frac{2}{10} + \frac{6}{10} + \frac{12}{10} + \frac{20}{10} = \frac{40}{10} = \boxed{4}$ ans $\quad \frac{1}{2} + \frac{1}{2}$

[Topper's Answer 2018]

Questions for Practice

■ Objective Type Questions

1. Choose and write the correct option in the following questions.

(*i*) If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals

(*a*) $\frac{4}{15}$ (*b*) $\frac{8}{45}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{2}{9}$

(*ii*) If A and B are two events and $A \neq \phi, B \neq \phi$, then

(*a*) $P(A \mid B) = P(A).P(B)$ (*b*) $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

(*c*) $P(A \mid B).P(B \mid A) = 1$ (*d*) $P(A \mid B) = P(A) \mid P(B)$

(*iii*) If $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A \cup B)' + P(A' \cup B)$ is equal to

(*a*) $\frac{1}{5}$ (*b*) $\frac{9}{5}$ (*c*) $\frac{7}{2}$ (*d*) 1

(*iv*) If the events A and B are independent, then $P(A \cap B)$ equals

(*a*) $P(A) + P(B)$ (*b*) $P(A) - P(B)$ (*c*) $P(A) . P(B)$ (*d*) $P(A)/P(B)$

(*v*) For the following probability distribution:

X	1	2	3	4
P(X)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) 10

(*vi*) Two events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E)$ equals to

(*a*) $\frac{12}{7}$ (*b*) $\frac{31}{35}$ (*c*) $\frac{1}{70}$ (*d*) $\frac{1}{7}$

(*vii*) A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is [CBSE 2020 (65/5/1)]

(*a*) $\frac{1}{3}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

(*viii*) A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is [CBSE 2020 (65/5/1)]

(*a*) $\frac{2}{5}$ (*b*) $\frac{3}{5}$ (*c*) 0 (*d*) 1

■ Conceptual Questions

2. Two coins are tossed. What is the probability of coming up two heads if it is known that at least one head comes up?
3. Four cards are drawn from 52 cards with replacement. Find the probability of getting at least 3 aces.
4. A bag contains 5 white and 4 red balls. 2 balls are drawn from the bag. Find the probability that both balls are white.
5. The probability that Hari hits a target is $\frac{1}{4}$. He fires 64 times. Find the expected number (μ) of times he will hit the target.
6. If $P(A) = 0.4$, $P(B) = p$, $P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of 'p'. **[*CBSE (F) 2017*]**

■ Very Short Answer Questions

7. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P (not A or not B) $= \frac{1}{4}$. State whether A and B are independent.
8. If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$ then find $P\left(\frac{B}{A}\right) + P\left(\frac{A}{B}\right)$.
9. In a girl's hostel's, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random. If she reads English newspaper, then find the probability that she reads Hindi newspaper also.
10. The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that $P(A) + P(B) = 2 - 2p + q$.

■ Short Answer Questions

11. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins. **[*CBSE Delhi 2016*]**
12. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first. **[*CBSE (North) 2016*]**
13. P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact? **[*CBSE Delhi 2013*]**
14. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y. **[*CBSE (North) 2016*]**
15. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. **[*CBSE (AI) 2013*]**
16. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it is $\frac{2}{5}$. If each one of A and B shoots at the target, what is the probability that

    (*i*) the target is hit? (*ii*) exactly one of them hits the target? **[*CBSE (F) 2009*]**
17. A box has 20 pens of which 2 are defective. Calculate the probability that out of 5 pens drawn one by one with replacement, at most 2 are defective. **[*CBSE (F) 2016*]**

18. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Find the value of C and also calculate mean of the distribution. [*CBSE (East) 2016*]

19. Let, X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0 \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0 & \text{if } x > 4 \end{cases}$$

where k is a positive constant. Find the value of k. Also, find the probability that you will get admission in (*i*) exactly one college (*ii*) at most 2 colleges (*iii*) at least 2 colleges. [*CBSE (F) 2016*]

20. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of the distribution also. [*CBSE (F) 2013*]

21. Of the students in a school; it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain *A* grade and 10% irregular students attain *A* grade in their annual examination. At the end of the year, one student is chosen at random from the school and he has *A* grade. What is the probability that the student has 100% attendance? [*CBSE (AI) 2017*]

22. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean of the distribution.

23. Suppose 10000 tickets are sold in a lottery each for ₹1. First prize is of ₹3000 and the second prize is of ₹2000. There are three third prizes of ₹500 each. If you buy one ticket, then what is your expectation? [*NCERT Exemplar*]

Long Answer Questions

24. An insurance company insured 3,000 scooters, 4,000 cars and 5,000 trucks. The probabilities of the accident involving a scooter, a car and a truck are 0·02, 0·03 and 0·04 respectively. One of the insured vehicles meet with an accident. Find the probability that it is a (*a*) scooter (*b*) car (*c*) truck. [*CBSE (AI) 2008*]

25. Bag *A* contains 3 red and 5 black balls, while bag *B* contains 4 red and 4 black balls. Two balls are transferred at random from bag *A* to bag *B* and then a ball is drawn from bag *B* at random. If the ball drawn from bag *B* is found to be red, find the probability that two red balls were transferred from *A* to *B*. [*CBSE (F) 2016*]

26. In a factory which manufactures bolts, machines *A*, *B* and *C* manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4, 1 per cent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine *B*. [*CBSE Allahabad 2015*]

27. In a bolt factory, three machines *A*, *B* and *C* manufacture 25, 35 and 40 per cent of the total bolts manufactured. Of their output, 5, 4 and 2 per cent are defective respectively. A bolt is drawn at random and is found to be defective. Find the probability that it was manufactured by either machine *A* or *C*. [*CBSE Allahabad 2015*]

28. If *A* and *B* are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find $P(A)$ and $P(B)$. [*CBSE Delhi 2015*]

29. A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is 4. Find the probability that it is actually a 4. [*CBSE Bhubaneswar 2015*]

30. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 6 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'. [CBSE 2019 (65/3/1)]

31. A girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she threw 1, 2, 3 or 4 with the die? [CBSE Delhi 2012]

32. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second group will win are 0. 6 and 0 .4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product was introduced by the second group. [CBSE Delhi 2009]

33. Find the probability distribution of the random variable X, which denotes the number of doublets in four throws of a pair of dice.

Hence, find the mean of the number of doublets (X). [CBSE 2020 (65/2/1)]

34. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first. [CBSE (East) 2016]

35. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean of the distribution. [CBSE Delhi 2016]

36. Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean of the distribution. [CBSE (Central) 2016]

37. An urn contains 5 red and 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes, find the mean of X. [CBSE Ajmer 2015]

Answers

1. (*i*) (*d*) (*ii*) (*b*) (*iii*) (*d*) (*iv*) (*c*) (*v*) (*d*) (*vi*) (*c*) (*vii*) (*c*) (*viii*) (*d*)

2. $\frac{1}{3}$ 3. $\frac{49}{28561}$ 4. $\frac{5}{18}$ 5. 16 6. $\frac{1}{3}$ 7. No

8. $\frac{7}{12}$ 9. $\frac{1}{2}$ 11. $\frac{5}{17}$ 12. $\frac{12}{23}, \frac{11}{23}$ 13. $\frac{31}{50}$ or 62% 14. $\frac{9}{17}$

15. $\frac{26}{49}$ 16. (*i*) $\frac{3}{5}$ (*ii*) $\frac{7}{15}$ 17. $\frac{99144}{100000}$ 18. $C = \frac{1}{10}$; 2.66

19. $k = \frac{1}{8}$; (*i*) $\frac{1}{8}$ (*ii*) $\frac{5}{8}$ (*iii*) $\frac{7}{8}$

20.

X	0	1	2
$P(X)$	$\frac{38}{245}$	$\frac{120}{245}$	$\frac{87}{245}$

; mean = $\frac{294}{245}$

21. $\frac{3}{4}$

22.

X	3	4	5	6
$P(X)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

Mean = 5.25

23. 0.65 24. (*a*) $\frac{3}{19}$ (*b*) $\frac{6}{19}$ (*c*) $\frac{10}{19}$ 25. $\frac{18}{33}$ 26. $\frac{11}{31}$ 27. $\frac{41}{69}$

28. $P(A)=\frac{1}{5}, P(B)=\frac{1}{6}$

Hint: Apply $P(A\cap B)=P(A).P(B)$

$P(\overline{A}\cap B)=P(\overline{A}).P(B)$

$P(A\cap \overline{B})=P(A).P(\overline{B})$

29. $\frac{3}{13}$ **Hint:** Let E_1 = 4 occurs; E_2 = 4 does not occur; A = man report that 4 occurs.

$P(E_1)=\frac{1}{6}, P(E_2)=\frac{5}{6}$, $P(A/E_1)=\frac{3}{5}, P(A/E_2)=\frac{2}{5}$

$P(E_1/A)$ = required

30. $n=5$ 31. $\frac{4}{7}$ 32. $\frac{2}{9}$

33.

X	0	1	2	3	4
P(X)	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

; Mean $=\frac{2}{3}$

34. $\frac{81}{217}, \frac{72}{217}, \frac{64}{217}$

35. Mean $=\frac{8}{3}$,

X or x	0	1	2	3	4
P(X) or p(x)	$\frac{1}{81}$	$\frac{8}{81}$	$\frac{24}{81}$	$\frac{32}{81}$	$\frac{16}{81}$

36. Mean $=\frac{4}{5}$,

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

37. X is random variable and have values 0, 1, 2.

Mean $=\frac{4}{7}$

Hint: $P(X=0)=\frac{{}^5C_2}{{}^7C_2}$, $P(X=1)=\frac{{}^2C_1\times{}^5C_1}{{}^7C_2}$, $P(X=2)=\frac{{}^2C_2}{{}^7C_2}$