Three Dimensional Geometry

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. The co-ordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis is *[CBSE 2020, (65/2/1)]*

   (*a*) (2, 3, 4) (*b*) (– 2, – 3, – 4) (*c*) (0, –3, 0) (*d*) (2, 0, 4)

2. If a line makes angles α, β and γ with the axes respectively, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$ *[CBSE 2020, (65/4/1)]*

   (*a*) –2 (*b*) –1 (*c*) 1 (*d*) 2

3. Distance of the point (α, β, γ) from y-axis is

   (*a*) β (*b*) $|\beta|$ (*c*) $|\beta|+|\gamma|$ (*d*) $\sqrt{\alpha^2+\gamma^2}$

4. Direction cosines of the line $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ are *[CBSE 2023, (65/1/1)]*

   (*a*) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (*b*) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

   (*c*) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (*d*) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

5. The angle between the two diagonals of a cube is

   (*a*) 30° (*b*) 45° (*c*) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (*d*) $\cos^{-1}\left(\frac{1}{3}\right)$

6. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is equal to

   (*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{8}{3}$

7. If a line makes angle $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with x-axis and y-axis respectively, then the angle made by the line with z-axis is

   (*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{4}$ (*d*) $\frac{15\pi}{12}$

8. P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is *[NCERT Exemplar]*

   (*a*) 2 (*b*) 1 (*c*) –1 (*d*) –2

9. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are *[CBSE 2023, (65/5/1)]*

   (*a*) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (*b*) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (*c*) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (*d*) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

10. Equation of a line passing through point (1, 1, 1) and parallel to z-axis is *[CBSE 2023, (65/2/1)]*

    (*a*) $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$ (*b*) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{1}$

    (*c*) $\frac{x}{0}=\frac{y}{0}=\frac{z-1}{1}$ (*d*) $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{1}$

11. **The equations of x-axis in space are** [*NCERT Exemplar*]

(*a*) $x = 0, y = 0$ (*b*) $x = 0, z = 0$ (*c*) $x = 0$ (*d*) $y = 0, z = 0$

12. **A line makes equal angles with co-ordinate axis. Direction cosines of this line are** [*NCERT Exemplar*]

(*a*) $\pm(1, 1, 1)$ (*b*) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (*c*) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (*d*) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

13. **The distance of the point (1, 6, 3) to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is**

(*a*) $\sqrt{13}$ (*b*) 13 (*c*) $2\sqrt{13}$ (*d*) None of these

14. **The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, if and only if**

(*a*) $aa' + cc' + 1 = 0$ (*b*) $aa' + bb' + cc' + 1 = 0$

(*c*) $aa' + bb' + cc' = 0$ (*d*) $(a' + a')(b' + b') + (c' + c') = 0$

15. **The area of the quadrilateral $ABCD$ where A (0, 4, 1), B (2, 3, –1), C (4, 5, 0) and D (2, 6, 2) is equal to**

(*a*) 9 sq units (*b*) 18 sq units (*c*) 27 sq units (*d*) 81 sq units

16. **If the point $P(a, b, 0)$ lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is** [*CBSE 2023 (65/3/2)*]

(*a*) (1, 2) (*b*) $\left(\frac{1}{2}, \frac{2}{3}\right)$ (*c*) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (*d*) (0, 0)

17. **The shortest distance between the lines given by**
$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ **and** $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ **is**

(*a*) 7 units (*b*) 2 units (*c*) 14 units (*d*) 3 units

18. **The image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ is**

(*a*) (2, 0, 5) (*b*) (1, 3, 4) (*c*) (1, 0, 7) (*d*) (– 3, – 2, 0)

19. **The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is** [*CBSE 2023, (65/5/1)*]

(*a*) 0° (*b*) 30° (*c*) 45° (*d*) 90°

20. **The co-ordinates of the foot of perpendicular drawn from point $A(1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$ are**

(*a*) $\left(\frac{-7}{3}, \frac{2}{3}, \frac{11}{3}\right)$ (*b*) $\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ (*c*) $\left(\frac{4}{3}, \frac{2}{3}, \frac{11}{3}\right)$ (*d*) None of these

Answers

1. (*c*)	2. (*b*)	3. (*d*)	4. (*d*)	5. (*d*)	6. (*c*)	7. (*b*)
8. (*a*)	9. (*a*)	10. (*d*)	11. (*d*)	12. (*b*)	13. (*a*)	14. (*a*)
15. (*a*)	16. (*c*)	17. (*c*)	18. (*c*)	19. (*d*)	20. (*b*)	

Solutions of Selected Multiple Choice Questions

1. The x and z co-ordinates on y-axis are 0.

$\therefore$ Required point is (0, –3, 0) on y-axis.

$\therefore$ Option (*c*) is correct.

2. Given α, β, γ are the angles line made with the co-ordinate axes.

$\therefore$ $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$.

$\because\ l^2 + m^2 + n^2 = 1 \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Now $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 = 2 \times 1 - 3 = 2 - 3 = -1$$

$\therefore$ Option (*b*) is correct.

4. Given equation of line be

$$\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$$

$$\Rightarrow \frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-\frac{1}{2}}{6}$$

Its direction cosines are

$$\frac{2}{\sqrt{(2)^2+(-3)^2+(6)^2}}, \frac{-3}{\sqrt{(2)^2+(-3)^2+(6)^2}}, \frac{6}{\sqrt{(2)^2+(-3)^2+(6)^2}}$$

i.e. $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$

$\therefore$ Option (*d*) is correct.

5. Let *ABCDEFG* be a cube with vertices $O(0, 0, 0)$, $A(a, 0, 0)$, $B(a, a, 0)$, $C(0, a, 0)$, $D(0, a, a)$, $E(0, 0, a)$, $F(a, 0, a)$, $G(a, a, a)$

Then the four diagonals are *OG*, *CF*, *BE* and *AD*. Let us consider the two diagonals *OG* and *AD*.

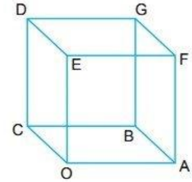
$\therefore\ \overrightarrow{OG} = a\hat{i} + a\hat{j} + a\hat{k}$

$\overrightarrow{AD} = -a\hat{i} + a\hat{j} + a\hat{k}$

$|\overrightarrow{OG}| = \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

$|\overrightarrow{AD}| = \sqrt{(-a)^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

$\overrightarrow{OG} \cdot \overrightarrow{AD} = -a^2 + a^2 + a^2 = a^2$



Let θ be the angle between $\overrightarrow{OG}$ and $\overrightarrow{AD}$

$$\therefore\ \cos\theta = \frac{\overrightarrow{OG} \cdot \overrightarrow{AD}}{|\overrightarrow{OG}||\overrightarrow{AD}|} = \frac{a^2}{\sqrt{3}a \times \sqrt{3}a} = \frac{a^2}{3a^2} = \frac{1}{3}$$

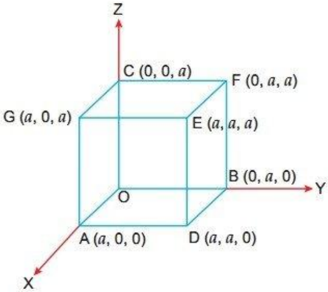
$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

$\therefore$ Option (*d*) is correct.

6. Let *OADBFEGC* be a cube.

Let $OB = OA = OC = a$

Co-ordinate of $O(0, 0, 0)$, $A(a, 0, 0)$, $B(0, a, 0)$, $C(0, 0, a)$, $D(a, a, 0)$, $F(0, a, a)$, $G(a, 0, a)$, $E(a, a, a)$



Dr's of $OE = a - 0, a - 0, a - 0 = a, a, a = 1, 1, 1$

Dr's of $BG = 1, -1, 1$

Dr's of $CD = 1, 1, -1$

Dr's of $AF = -1, 1, 1$

DC's of $OE = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

DC's of $BG = \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

DC's of $CD = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

DC's of $AF = \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Let l, m, n be the dc's of the line and makes $\alpha, \beta, \gamma, \delta$ with the four diagonals of the cube then

$$\cos\alpha = l\left(\frac{1}{\sqrt{3}}\right) + m\left(\frac{1}{\sqrt{3}}\right) + n\left(\frac{1}{\sqrt{3}}\right) = \frac{l+m+n}{\sqrt{3}}$$

Similarly, $\cos\beta = \frac{-l+m+n}{\sqrt{3}}$, $\cos\gamma = \frac{l-m+n}{\sqrt{3}}, \cos\delta = \frac{l+m-n}{\sqrt{3}}$

Now $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1}{3}\left[(l+m+n)^2 + (-l+m+n)^2 + (l-m+n)^2 + (l+m-n)^2\right]$

$$= \frac{1}{3}[l^2 + m^2 + n^2 + 2lm + 2mn + 2nl + l^2 + m^2 + n^2 - 2lm + 2mn - 2nl$$

$$+ l^2 + m^2 + n^2 - 2lm - 2mn + 2nl + l^2 + m^2 + n^2 + 2lm - 2mn - 2nl]$$

$$= \frac{1}{3}\left[4(l^2 + m^2 + n^2) - 0\right] = \frac{4}{3} \times 1 = \frac{4}{3}$$

$\therefore$ Option (*c*) is correct.

7. Given α, β, γ be angles made by the line with the co-ordinate axes.

$\therefore \quad \alpha = \frac{\pi}{3}, \beta = \frac{\pi}{4}$

$\because \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$

$\Rightarrow \quad \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

$\Rightarrow \quad \frac{3}{4} + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \quad \cos\gamma = \pm\frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3}$

$\therefore$ Option (*b*) is correct.

8. Let P divides the line segment in the ratio of $\lambda : 1$, x-coordinate of the point P may be expressed as $x = \frac{6\lambda + 3}{\lambda + 1}$ giving $\frac{6\lambda + 3}{\lambda + 1} = 5$ so that $\lambda = 2$. Thus y-coordinate of P is $\frac{2\lambda + 2}{\lambda + 1} = 2$.

$\therefore$ Option (*a*) is correct.

9. $\alpha = 90°, \beta = 135°, \gamma = 45°$

$l = \cos\alpha = \cos 90° = 0, m = \cos\beta = \cos 135° = -\frac{1}{\sqrt{2}}$

$n = \cos\gamma = \cos 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow$ DC's are $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

$\therefore$ Option (*a*) is correct.

10. Direction cosines of z-axis is 0, 0, 1.

$\therefore$ Equation of a line passing through point (1, 1, 1) and parallel to z-axis, is given by

$$\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{1}$$

$\therefore$ Option (*d*) is correct.

11. On x-axis the y-co-ordinate and z-co-ordinate are zero.

$\therefore$ Option (*d*) is correct.

12. Let the line makes angle α with each of the axis.

Then, its direction cosines are $\cos\alpha$, $\cos\alpha$, $\cos\alpha$.

Since $\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$. Therefore, $\cos\alpha=\pm\frac{1}{\sqrt{3}}$

Dc's are $\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$.

$\therefore$ Option (*b*) is correct.

14. We are given lines

$$x=ay+b,\ z=cy+d$$

$$\Rightarrow\quad \frac{x-b}{a}=y,\ \frac{z-d}{c}=y$$

$$\Rightarrow\quad \frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c} \qquad ...(i)$$

Also $x=a'y+b',\ z=c'y+d'$

$$\Rightarrow\quad \frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'} \qquad ...(ii)$$

Lines (*i*) and (*ii*) are perpendicular to each other.

$$\Rightarrow\quad aa'+1+cc'=0$$

$$\Rightarrow\quad aa'+cc'+1=0$$

$\therefore$ Option (*a*) is correct.

16. Since the point $P(a, b, 0)$ lies on the line

$$\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$$

$$\therefore\quad \frac{a+1}{2}=\frac{b+2}{3}=\frac{0+3}{4}$$

$$\Rightarrow\quad \frac{a+1}{2}=\frac{b+2}{3}=\frac{3}{4} \qquad \Rightarrow \qquad \frac{a+1}{2}=\frac{3}{4} \text{ and } \frac{b+2}{3}=\frac{3}{4}.$$

$$\Rightarrow\quad a=\frac{3}{2}-1 \qquad \text{and} \qquad b+2=\frac{9}{4}$$

$$\Rightarrow\quad a=\frac{1}{2} \qquad \text{and} \qquad b=\frac{9}{4}-2=\frac{1}{4}$$

$$\Rightarrow\quad (a,b)=\left(\frac{1}{2},\frac{1}{4}\right)$$

$\therefore$ Option (*c*) is correct.

19. We are given equation of lines are

$$2x=3y=-z \quad \Rightarrow \quad \frac{x}{\frac{1}{2}}=\frac{y}{\frac{1}{3}}=\frac{z}{-1} \qquad ...(i)$$

and $$6x=-y=-4z \quad \Rightarrow \quad \frac{x}{\frac{1}{6}}=\frac{y}{-1}=\frac{z}{\frac{-1}{4}} \qquad ...(ii)$$

Let a_1, b_1, c_1 and a_2, b_2, c_2 of lines (*i*) and (*ii*) respectively.

$$a_1 = \frac{1}{2}, b_1 = \frac{1}{3}, c_1 = -1$$

$$a_2 = \frac{1}{6}, b_2 = -1, c_2 = \frac{-1}{4}$$

$\therefore$ $$a_1a_2 + b_1b_2 + c_1c_2 = \frac{1}{12} - \frac{1}{3} + \frac{1}{4}$$

$$= \frac{1-4+3}{12} = \frac{4-4}{12} = \frac{0}{12} = 0$$

$\Rightarrow$ Line (*i*) is perpendicular to line (*ii*).

$\therefore$ Option (*d*) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A) :** A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (–1, –2, 1) and (1, 2, 5).

   Reason (R) : Lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$. *[CBSE 2023 (65/3/2)]*

2. **Assertion (A) :** Equation of a line passing through the points (1, 2, 3) and (3, –1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$. *[CBSE 2023 (65/1/1)]*

   Reason (R) : Equation of a line passing through points $(x_1, y_1, z_1), (x_2, y_2, z_2)$, is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

3. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. *[CBSE 2023 (65/2/1)]*

   Reason (R) : The sum of squares of the direction cosines of a line is 1.

4. **Assertion (A) :** The angle between the lines whose direction cosines are $\frac{-\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}; \frac{-\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}$ is 120°.

   Reason (R) : The angle between two lines whose direction ratios are l_1, m_1, n_1 and l_2, m_2, n_2 is given by $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.

5. **Assertion (A) :** Direction cosines of z-axis are 0, 0, 1.

   Reason (R) : If l, m, n be the direction cosines of a line then $l^2 + m^2 + n^2 = 1$.

6. **Assertion (A):** Parametric equation of a line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$ are $x = 1 + 2\lambda$, $y = 2 + 3\lambda$, $z = 4\lambda$, where λ is a parameter.

Reason (R) : The parametric equation of the line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ are $x = x_1 + a\lambda$, $y = y_1 + b\lambda$, $z = z_1 + c\lambda$ where λ is a parameter.

7. **Assertion (A):** Vector equation of the line passes through the points (1, 2, 3) and (5, – 4, – 7) is $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(5\hat{i} - 4\hat{j} - 7\hat{k})$ where λ is a parameter.

Reason (R) : Vector equation of straight line passing through two given points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ where λ is parameter.

Answers

1. (*c*) **2.** (*d*) **3.** (*a*) **4.** (*a*) **5.** (*b*) **6.** (*a*) **7.** (*d*)

Solutions of Assertion-Reason Questions

1. We have equation of a line passing through the points (4, 7, 8) and (2, 3, 4) is

$$\frac{x-4}{2-4}=\frac{y-7}{3-7}=\frac{z-8}{4-8} \quad\Rightarrow\quad \frac{x-4}{-2}=\frac{y-7}{-4}=\frac{z-8}{-4}$$

in vector form $\vec{r} = (4\hat{i} + 7\hat{j} + 8\hat{k}) + \lambda(-2\hat{i} - 4\hat{j} - 4\hat{k})$

Also equation of a line passing through the points (– 1, – 2, 1) and (1, 2, 5) is

$\frac{x+1}{2}=\frac{y+2}{4}=\frac{z-1}{4}$ or, in vector form $\vec{r} = -\hat{i} - 2\hat{j} + \hat{k} + \mu(2\hat{i} + 4\hat{j} + 4\hat{k})$

Clearly, $\frac{-2}{2}=\frac{-4}{4}=\frac{-4}{4}$

$\therefore$ Both lines are parallel.

Assertion (*A*) is true but Reason (*R*) is false.

$\therefore$ Option (*c*) is correct.

2. As we know that equation of line passing through points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \text{ or, } \frac{x-x_2}{x_2-x_1}=\frac{y-y_2}{y_2-y_1}=\frac{z-z_2}{z_2-z_1}$$

Hence, Equation of line passing through the points (1, 2, 3) and (3, –1, 3) is given by

$$\frac{x-1}{3-1}=\frac{y-2}{-1-2}=\frac{z-3}{3-3} \text{ or, } \frac{x-3}{3-1}=\frac{y+1}{-1-2}=\frac{z-3}{3-3}$$

As we know that equation of line passing through points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \text{ or, } \frac{x-x_2}{x_2-x_1}=\frac{y-y_2}{y_2-y_1}=\frac{z-z_2}{z_2-z_1}$$

$\therefore$ Equation of line passing through the points (1, 2, 3) and (3, –1, 3) is given by

$$\frac{x-1}{3-1}=\frac{y-2}{-1-2}=\frac{z-3}{3-3} \text{ or, } \frac{x-3}{3-1}=\frac{y+1}{-1-2}=\frac{z-3}{3-3}$$

$$\Rightarrow \quad \frac{x-1}{2}=\frac{y-2}{-3}=\frac{z-3}{0} \text{ or, } \frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-3}{0}$$

Hence, Assertion (A) is false but Reason (R) is true.

$\therefore$ Option (*d*) is correct.

3. As α, β, γ be the angles made by the line with positive direction of the co-ordinate axes.

$\therefore \quad \cos^2\alpha + \cos^2\beta + \cos^2 r = 1$

$\Rightarrow \quad 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \quad 2 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$\therefore \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Both assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of A.

∴ Option (*a*) is correct.

4. We have, $\cos\theta = \frac{-\sqrt{3}}{4} \times \left(\frac{-\sqrt{3}}{4}\right) + \frac{1}{4} \times \frac{1}{4} + \left(\frac{-\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2}$

$$= \frac{3}{16} + \frac{1}{16} - \frac{3}{4} = \frac{3+1-12}{16}$$

$$= \frac{-8}{16} = -\frac{1}{2} \quad \Rightarrow \quad \theta = 120°.$$

Clearly, both Assertion (*A*) and Reason (*R*) are true Reason (*R*) is the correct explanation of Assertion (*A*).

∴ Option (*a*) is correct.

5. Clearly, both Assertion (*A*) and Reason (*R*) are true but Reason (*R*) is not the correct explanation of Assertion (*A*).

∴ Option (*b*) is correct.

6. For assertion (*A*)

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{4} = \lambda \text{(say)}$$

$\Rightarrow x = 1 + 2\lambda, y = 2 + 3\lambda, z = 4\lambda$ is the parametric from of the given line.

So statement *A* is true.

For reason (*R*)

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \lambda \text{(say)}$$

$\Rightarrow x = x_1 + a\lambda, y = y_1 + b\lambda, z = z_1 + c\lambda$ is the parametric from where λ is a parameter.

So statement *R* is also true and gives the correct explanation of *A*.

∴ Option (*a*) is correct.

7. For assertion (*A*):

We are given points $P(1, 2, 3)$, $Q(5, -4, -7)$

Dr's line joining *P* and *Q* are $5-1, -4-2, -7-3$, *i.e.*, are $4, -6, -10$.

∴ Vector equation of line joining *P* and *Q* is

$$\overline{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(4\hat{i} - 6\hat{j} - 10\hat{k})$$

So statement *A* is not true.

For reason (*R*)

Hence *R* is true but *A* is false.

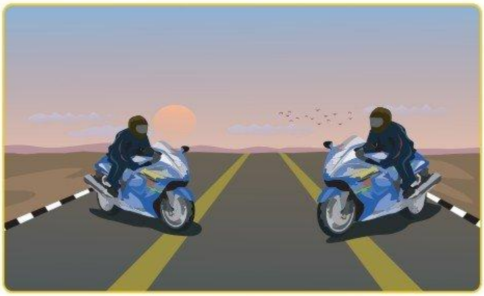
∴ Option (*d*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**

   Two motorcycles *A* and *B* are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.



   (*i*) Write the certesian equation of the line along which motorcycle *A* is running.

   (*ii*) Find the direction cosines of the line along which motorcycle *B* is running.

   (*iii*) (*a*) Find the shortest distance between the given lines.

   Or

   (*iii*) (*b*) Find the point at which the motorcycles may collide.

Sol. Vector equation of line through which motorcycles *A* and *B* running are

$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ (1)

and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ (2)

(*i*) From (1), we have

$$x\hat{i} + y\hat{j} + z\hat{k} = \vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

$$\Rightarrow x = \lambda, y = 2\lambda, z = -\lambda$$

$$\Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$$

$$\therefore \frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$$, which is required equation.

(*ii*) From (2), we have

$$\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow \vec{r} = \vec{a} + \mu\vec{b} \text{ where } \vec{b} = 2\hat{i} + \hat{j} + \hat{k}$$

$\therefore$ Dr's are 2, 1, 1.

$\therefore$ Dc's are $\frac{2}{\sqrt{2^2+1^2+1^2}}, \frac{1}{\sqrt{2^2+1^2+1^2}}, \frac{1}{\sqrt{2^2+1^2+1^2}}$, *i.e.*, are $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$.

(*iii*) (*a*) Given vector equation of lines

$$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) = 0\hat{i} + 0\hat{j} + 0\hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

We get $\vec{a_1} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ and $\vec{b_1} = \hat{i} + 2\hat{j} - \hat{k}$

and, $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$

We get $\vec{a_2} = 3\hat{i} + 3\hat{j}$ and $\vec{b_2} = 2\hat{i} + \hat{j} + \hat{k}$

Now, $\vec{a_2} - \vec{a_1} = 3\hat{i} + 3\hat{j}$

and $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(2+1) - \hat{j}(1+2) + \hat{k}(1-4)$

$\Rightarrow$ $\vec{b} \times \vec{b_2} = 3\hat{i} - 3\hat{j} - 3\hat{k}$

$\therefore$ Shortest distance between the given lines $= \left| \dfrac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$$= \left| \frac{(3\hat{i} - 3\hat{j} - 3\hat{k}).(3\hat{i} + 3\hat{j})}{\sqrt{9+9+9}} \right|$$

$$= \left| \frac{9-9-0}{3\sqrt{3}} \right| = 0 \text{ units}$$

OR

(*iii*) (*b*) Given lines, $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$

$$= (3 + 2\mu)\hat{i} + (3 + \mu)\hat{j} + \mu\hat{k}$$

For point of intersection, we have

$\lambda = 3 + 2\mu$ ...(*i*)

$2\lambda = 3 + \mu$ ...(*ii*)

and $-\lambda = \mu$ ...(*iii*)

From equation (*i*) and (*iii*), we get

$$\lambda = 3 - 2\lambda \Rightarrow 3\lambda = 3 \Rightarrow \lambda = 1$$

From equation (*iii*), we get

$$\mu = -\lambda = -1$$

$\therefore$ $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$

$$\vec{r} = 1(\hat{i} + 2\hat{j} - \hat{k}) = \hat{i} + 2\hat{j} - \hat{k}$$

$\therefore$ Required point be (1, 2, – 1).

2. Read the following passage and answer the following questions.

The equation of motion of a missile are $x = 3t$, $y = -4t$, $z = t$, where the time 't' is given in seconds, and the distance is measured in kilometres. [*CBSE Question Bank*]

(*i*) **Write the path of the missile.**

(*ii*) **Find the distance of the rocket from the starting point (0, 0, 0) in 5 seconds.**

(*iii*) (*a*) **If the position of the rocket at a certain instant of the time is (5, –8, 10). Find the height of the rocket from the ground. (Ground considered as *xy*-plane)**

OR

(*iii*) (*b*) **Find the value of k for which the lines**

$$\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{k} \text{ and } \frac{x-2}{-2}=\frac{y-3}{-1}=\frac{z-5}{7} \text{ are perpendicular?}$$

Sol. (*i*) Given equation of motion of a missile be

$$x = 3t, y = -4t, z = t$$

$$\Rightarrow \quad \frac{x}{3}=\frac{y}{-4}=\frac{z}{1} \quad \text{which is a straight line.}$$

Hence, the path of the missile is a straight line.

(*ii*) After 5 seconds position of the rocket be

$$x = 3t = 3 \times 5 = 15$$
$$y = -4t = -4 \times 5 = -20$$
$$z = t = 5$$

$\therefore$ Point is (15, –20, 5).

Its distance from origin (0, 0, 0) is $\sqrt{(15-0)^2+(-20-0)^2+(5-0)^2}$

$$= \sqrt{225+400+25} = \sqrt{650} \text{ km}$$

(*iii*) (*a*) Given position of the rocket at a time is (5, –8, 10).

$\therefore$ Height of the rocket from the ground

= Distance between the points (5, –8, 10) and (5, –8, 0).

(Since ground is considered as the *XY*-Plane)

$$= \sqrt{(5-5)^2+(-8+8)^2+(10-0)^2} = 10 \text{ km}$$

OR

(*iii*) (*b*) Given lines are perpendicular if $2 \times (-2) + 3 \times (-1) + 7 \times k = 0$

$$= -7 + 7k = 0 \Rightarrow 7k = 7 \Rightarrow k = 1$$

CONCEPTUAL QUESTIONS

1. If a line has direction ratios 2, –1, –2, then what are its direction cosines? [*CBSE Delhi* 2012]

Sol. Here direction ratios of line are 2, –1, –2.

$\therefore$ Direction cosines of line are $\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$

i.e., $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

[**Note:** If a, b, c are the direction ratios of a line, the direction cosines are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$]

2. Write the direction cosines of a line parallel to *z*-axis. [*CBSE (F) 2012*]

Sol. The angle made by a line parallel to *z*-axis with *x*, *y* and *z*-axis are 90°, 90° and 0° respectively.

$\therefore$ The direction cosines of the line are cos 90°, cos 90°, cos 0° *i.e.,* 0, 0, 1.

3. If a line makes angles 90° and 60° respectively with the positive directions of *x* and *y* axes, find the angle which it makes with the positive direction of *z*-axis. [*CBSE Delhi 2017*]

Sol. Let the angle made by line with positive direction of *z*-axis be θ then,

We know that

$$\cos^2 90° + \cos^2 60° + \cos^2\theta = 1$$

$$\Rightarrow \quad 0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \quad \frac{1}{4} + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \frac{1}{4}$$

$$\Rightarrow \quad \cos^2\theta = \frac{3}{4}$$

$$\Rightarrow \quad \cos\theta = \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 60° \text{ or } \frac{\pi}{3} \text{ if } \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \theta = 150° \text{ or } \frac{5\pi}{6} \text{ if } \cos\theta = -\frac{\sqrt{3}}{2}$$

4. Find the direction ratios of a line passing through the points (1, 0, 0) and (0, 1, 1).

Sol. Given points *A*(1, 0, 0) and *B*(0, 1, 1).

$\therefore$ Dr's of line through *A*, and *B* are 0 – 1, 1 – 0, 1 – 0

i.e. –1, 1 1

5. Find the direction cosines of a line passing through the point (1, 3, 5) and (2, 4, 6).

Sol. We are given points *A*(1, 3, 5) and *B*(2, 4, 6).

$\therefore$ Dr's of the line through *A* and *B* are

2 – 1, 4 – 3, 6 – 5, *i.e.,* 1, 1, 1.

$\therefore$ Dc's are $\frac{1}{\sqrt{1^2+1^2+1^2}}, \frac{1}{\sqrt{1^2+1^2+1^2}}, \frac{1}{\sqrt{1^2+1^2+1^2}}$

i.e., $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Very Short Answer Questions

1. Find the value of p, so that lines $\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{1-z}{7}$ are perpendicular to each other. *[CBSE 2023 (65/3/2)]*

Sol. Given equation of lines

$$\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4} \quad ...(i)$$

and, $$\frac{x-2}{4p}=\frac{y-5}{2}=\frac{1-z}{7}$$

$$\Rightarrow \frac{x-2}{4p}=\frac{y-5}{2}=\frac{z-1}{-7} \quad ...(ii)$$

Since lines (*i*) and (*ii*) are perpendicular to each other

$\therefore -2\times 4p+3p\times 2+4\times -7=0$

$\Rightarrow -8p+6p-28=0 \Rightarrow -2p-28=0 \Rightarrow 2p=-28$

$\Rightarrow p=-14$

2. Find the vector and the cartesian equations of a line that passes through the point $A(1, 2, -1)$ and parallel to the line $5x-25=14-7y=35z$. *[CBSE 2023 (65/5/1)]*

Sol. We are given line

$5x-25=14-7y=35z$

$\Rightarrow 5(x-5)=-7(y-2)=35z$

$$\Rightarrow \frac{x-5}{\frac{1}{5}}=\frac{y-2}{\frac{-1}{7}}=\frac{z}{\frac{1}{35}} \quad ...(i)$$

Dr's of any line parallel to line (*i*) are

$$\frac{1}{5}k, -\frac{1}{7}k, \frac{1}{35}k$$

$\therefore$ Equation of the required line passes through $A(1, 2, -1)$ and parallel to line (*i*) is

$$\frac{x-1}{\frac{1}{5}k}=\frac{y-2}{-\frac{1}{7}k}=\frac{z+1}{\frac{1}{35}k}$$

$$\Rightarrow \frac{x-1}{\frac{1}{5}}=\frac{y-2}{-\frac{1}{7}}=\frac{z+1}{\frac{1}{35}} \Rightarrow 5(x-1)=-7(y-2)=35(z+1) \Rightarrow 5x-5=14-7y=35z+35$$

Vector equation is

$$\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda\left(\frac{1}{5}\hat{i}-\frac{1}{7}\hat{j}+\frac{1}{35}\hat{k}\right)$$

3. Check whether the lines given by equations $x=2\lambda+2$, $y=7\lambda+1$, $z=-3\lambda-3$ and $x=-\mu-2$, $y=2\mu+8$, $z=4\mu+5$ are perpendicular to each other or not. *[CBSE 2023 (65/1/1)]*

Sol. Given equation of lines

$$x=2\lambda+2 \quad \Rightarrow \frac{x-2}{2}=\lambda$$

$$y=7\lambda+1 \quad \Rightarrow \frac{y-1}{7}=\lambda$$

$z = -3\lambda - 3 \quad \Rightarrow \quad \frac{z+3}{-3} = \lambda$

$\therefore$ Equation of one line be

$$\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3} \qquad ...(i)$$

Also, $x = -\mu - 2 \Rightarrow \frac{x+2}{-1} = \mu$

$x = 2\mu + 8 \Rightarrow \frac{y-8}{2} = \mu$

$z = 4\mu + 5 \Rightarrow \frac{z-5}{4} = \mu$

$\therefore$ Equation of other line be

$$\frac{x+2}{-1} = \frac{y-8}{2} = \frac{z-5}{4} \qquad ...(ii)$$

From (*i*) and (*ii*), we have

$a_1a_2 + b_1b_2 + c_1c_2 = 2 \times (-1) + 7 \times 2 + (-3) \times 4$

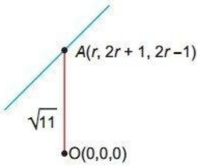
$= -2 + 14 - 12 = 14 - 14 = 0$

$\therefore$ Both lines are perpendicular to each other.

4. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin. [*CBSE 2023 (65/2/1)*]

Sol. Given equation of line be

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$$



$\Rightarrow \quad \frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2} = r$ (let)

$\Rightarrow \quad x = r, y = 2r + 1$ and $z = 2r - 1$

$\therefore \quad$ Any point on the line be $A(r, 2r + 1, 2r - 1)$

Thus, distance between O(0, 0, 0) and point A is given $OA = \sqrt{11}$

$\Rightarrow \quad \sqrt{r^2 + (2r+1)^2 + (2r-1)^2} = \sqrt{11}$

$\Rightarrow \quad r^2 + (2r+1)^2 + (2r-1)^2 = 11$ (on squaring both sides)

$\Rightarrow \quad 9r^2 + 2 = 11 \quad \Rightarrow 9r^2 = 9 \quad \Rightarrow r^2 = 1$

$\Rightarrow \quad r = \pm 1$

When $r = 1$, co-ordinates of point be (1, 3, 1)

When $r = -1$, co-ordinates of point be (–1, –1, –3)

5. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Sol. Let $A(1, -1, 2)$ and $B(3, 4, -2)$ be given points.

Direction ratios of AB are

$(3 - 1), \{(4 - (-1)\}, (-2 - 2)$ *i.e.*, 2, 5, –4.

Let $C(0, 3, 2)$ and $D(3, 5, 6)$ be given points.

Direction ratios of CD are $(3 - 0), (5 - 3), (6 - 2)$ *i.e.*, 3, 2, 4.

We know that two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$\therefore$ $2 \times 3 + 5 \times 2 + (-4) \times 4 = 6 + 10 - 16 = 0$ which is true.

Which shows that lines AB and CD are perpendicular.

6. Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to line through the points (–1, –2, 1), (1, 2, 5).

Sol. Let $A(4, 7, 8)$ and $B(2, 3, 4)$ be given points. Direction ratios of AB are $(2 - 4), (3 - 7), (4 - 8)$

i.e., $-2, -4, -4$.

Let $C(-1, -2, 1)$ and $D(1, 2, 5)$ be given points.

Direction ratios of CD are $1 -(-1), 2 -(-2), (5 - 1)$ *i.e.,* 2, 4, 4.

We know that two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{-2}{2} = \frac{-4}{4} = \frac{-4}{4} = -1, \text{ which is true.}$$

It shows that lines AB and CD are parallel.

7. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

Sol. Let $\vec{a}$ be the position vector of the point (1, 2, 3).

Then $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Now the equation of the line passing through the point having position vector $\vec{a}$ and parallel to vector $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$ is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k}).$$

8. Find the equation of the line in vector and Cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.

Sol. Here $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$.

Now the equation of the line passing through the point having position vector $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda (\hat{i} + 2\hat{j} - \hat{k})$$

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector.

Then $x\hat{i} + y\hat{j} + z\hat{k} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda (\hat{i} + 2\hat{j} - \hat{k})$

$$(2 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} + (4 - \lambda)\hat{k}$$

Comparing coefficients of $\hat{i}, \hat{j}, \hat{k}$ on both sides, we have

$$x = 2 + \lambda, \; y = 2\lambda - 1 \text{ and } z = 4 - \lambda$$

$$\therefore \quad \frac{x-2}{1} = \frac{y+1}{2} = \frac{4-z}{1}$$

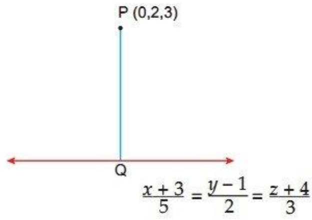
Which is required Cartesian form of the line.

Short Answer Questions

1. Find the coordinates of the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. [CBSE 2023 (65/5/1)]

Sol. We are given point $P(0, 2, 3)$ and the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ ...(i)

Let Q be the foot of perpendicular.



Now $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$ (say)

$\Rightarrow \quad x = -3 + 5\lambda, y = 1 + 2\lambda, z = -4 + 3\lambda$

$\therefore \quad Q \equiv (-3 + 5\lambda, 1 + 2\lambda, -4 + 3\lambda)$

Dr's of PQ are $5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3$

i.e. are $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$

Dr's of line (i) are 5, 2, 3

$\because \quad PQ \perp$ line (i)

$\therefore \quad 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow \quad 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow \quad 38\lambda - 38 = 0 \quad \Rightarrow \quad \lambda = \frac{38}{38} = 1$

$\therefore \quad Q = (-3 + 5, 1 + 2, -4 + 3) = (2, 3, -1)$

Which is the foot of perpendicular.

2. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$. [CBSE Delhi 2017]

Sol. Given line is

$$5x - 25 = 14 - 7y = 35z$$

$$\Rightarrow \quad \frac{x-5}{\frac{1}{5}} = \frac{2-y}{\frac{1}{7}} = \frac{z-0}{\frac{1}{35}} \quad \Rightarrow \quad \frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z-0}{\frac{1}{35}}$$

$$= \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-0}{1} \quad \dots (i)$$

Hence, parallel vector of given line *i.e.*, $\vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$.

Since required line is parallel to given line (i).

$\Rightarrow \quad \vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$ will also be parallel vector of required line which passes through $A(1, 2, -1)$.

Therefore, required vector equation of line is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$$

3. A line passing through the point A with position vector $\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Find the length of the perpendicular drawn on this line from a point P with position vector $\vec{r_1} = \hat{i} + 2\hat{j} + 3\hat{k}$. [CBSE Panchkula 2015]

Sol. The equation of line passing through the point A and parallel to $\vec{b}$ is given in cartesian form as

$$\frac{x-4}{2}=\frac{y-2}{3}=\frac{z-2}{6} \qquad \ldots(i)$$

Let $Q(\alpha, \beta, \gamma)$ be foot of perpendicular drawn from point P to the line (i).

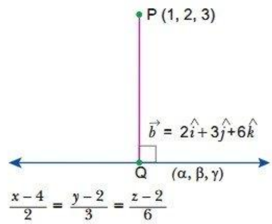
Co-ordinate or point $P \equiv (1, 2, 3)$ $\quad [\because P.V. \text{ of } P \text{ is } \hat{i}+2\hat{j}+3\hat{k}]$

Since, Q lie on line (i)

$$\frac{\alpha-4}{2}=\frac{\beta-2}{3}=\frac{\gamma-2}{6}=\lambda$$

$\Rightarrow \quad \alpha = 2\lambda+4, \beta = 3\lambda+2, \gamma = 6\lambda+2$

Now, $\quad \overrightarrow{PQ} = (\alpha-1)\hat{i}+(\beta-2)\hat{j}+(\gamma-3)\hat{k}$



Obviously, $\overrightarrow{PQ} \perp \vec{b} \quad \therefore \quad \overrightarrow{PQ}\cdot\vec{b}=0$

$\Rightarrow \quad 2(\alpha-1)+3(\beta-2)+6(\gamma-3)=0$

$\Rightarrow \quad 2\alpha-2+3\beta-6+6\gamma-18=0 \quad \Rightarrow \quad 2\alpha+3\beta+6\gamma-26=0$

Putting the value of α, β, γ; we get

$2(2\lambda+4)+3(3\lambda+2)+6(6\lambda+2)-26=0$

$\Rightarrow \quad 4\lambda+8+9\lambda+6+36\lambda+12-26=0$

$\Rightarrow \quad 49\lambda = 0 \quad \Rightarrow \quad \lambda = 0$

Hence, the co-ordinate of $Q \equiv (4, 2, 2)$.

$\therefore$ Length of perpendicular $PQ = \sqrt{(4-1)^2+(2-2)^2+(2-3)^2}$

$= \sqrt{9+0+1} = \sqrt{10}$ units.

4. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1). *[CBSE Sample Paper 2018]*

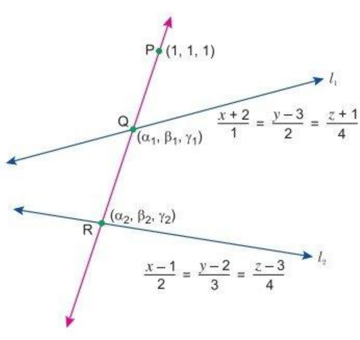
Sol. Let l_1, l_2 be given lines as

$$l_1: \frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4};$$

$$l_2: \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

Let l be the required line, which passes through P (1, 1, 1) and intersect l_1 and l_2 at Q $(\alpha_1, \beta_1, \gamma_1)$ and R $(\alpha_2, \beta_2, \gamma_2)$ respectively.

Now, Q $(\alpha_1, \beta_1, \gamma_1)$ lie on line l_1



$\therefore \quad \frac{\alpha_1+2}{1}=\frac{\beta_1-3}{2}=\frac{\gamma_1+1}{4}=\lambda$ (say)

$\alpha_1 = \lambda-2, \ \beta_1 = 2\lambda+3, \ \gamma_1 = 4\lambda-1$

Similarly, R $(\alpha_2, \beta_2, \gamma_2)$ lie on line l_2

$$\frac{\alpha_2-1}{2}=\frac{\beta_2-2}{3}=\frac{\gamma_2-3}{4}=\mu \text{ (say)}$$

$\Rightarrow \quad \alpha_2 = 2\mu+1, \ \beta_2 = 3\mu+2, \ \gamma_2 = 4\mu+3$

$\therefore \quad \overrightarrow{PQ} = (\alpha_1-1)\hat{i}+(\beta_1-1)\hat{j}+(\gamma_1-1)\hat{k}$

$= (\lambda-3)\hat{i}+(2\lambda+2)\hat{j}+(4\lambda-2)\hat{k}$

Similarly, $\overrightarrow{PR} = 2\mu\hat{i}+(3\mu+1)\hat{j}+(4\mu+2)\hat{k}$

$\because \quad \overrightarrow{PQ} \parallel \overrightarrow{PR} \quad \Rightarrow \quad \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}=M$ (say)

Now, $\frac{\lambda - 3}{2\mu} = M \Rightarrow \lambda - 3 = 2M\mu \Rightarrow M\mu = \frac{\lambda - 3}{2}$

Also, $\frac{2\lambda + 2}{3\mu + 1} = M \Rightarrow 2\lambda + 2 = 3M\mu + M$

$$\Rightarrow 2\lambda + 2 = \frac{3\lambda - 9}{2} + M \qquad \Rightarrow 2\lambda + 2 - \frac{3\lambda - 9}{2} = M$$

$$\Rightarrow \frac{4\lambda + 4 - 3\lambda + 9}{2} = M \qquad \Rightarrow \frac{\lambda + 13}{2} = M$$

Also, $\frac{4\lambda - 2}{4\mu + 2} = M \Rightarrow 4\lambda - 2 = 4M\mu + 2M$

$$\Rightarrow 4\lambda - 2 = \frac{4\lambda - 12}{2} + 2M \qquad \Rightarrow \frac{8\lambda - 4 - 4\lambda + 12}{2} = 2M$$

$$\Rightarrow \frac{4\lambda + 8}{2} = 2M \qquad \Rightarrow \frac{4(\lambda + 2)}{2} = 2M$$

$$\Rightarrow \lambda + 2 = M \qquad \Rightarrow \lambda + 2 = \frac{\lambda + 13}{2}$$

$$\Rightarrow 2\lambda + 4 = \lambda + 13 \Rightarrow \lambda = 9 \Rightarrow M = 11 \Rightarrow \mu = \frac{3}{11}$$

$\therefore \qquad \overrightarrow{PQ} = 6\hat{i} + 20\hat{j} + 34\hat{k}$

Hence, equation of required line is $\frac{x - 1}{6} = \frac{y - 1}{20} = \frac{z - 1}{34} \Rightarrow \frac{x - 1}{3} = \frac{y - 1}{10} = \frac{z - 1}{17}$

5. Find the vector and cartesian equations of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7}$ and $\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$.

[*CBSE Delhi 2012, 2017*]

OR

Find the equation of a line passing through the point (1, 2, –4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

[*CBSE Allahabad 2015; Delhi 2016*]

Sol. Let the cartesian equation of line passing through (1, 2, – 4) be

$$\frac{x - 1}{a} = \frac{y - 2}{b} = \frac{z + 4}{c} \qquad \ldots(i)$$

Given lines are

$$\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7} \qquad \ldots(ii)$$

$$\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5} \qquad \ldots(iii)$$

Obviously parallel vectors $\vec{b_1}, \vec{b_2}$ and $\vec{b_3}$ of (*i*), (*ii*) and (*iii*) respectively are given as

$$\vec{b_1} = a\hat{i} + b\hat{j} + c\hat{k};\ \vec{b_2} = 3\hat{i} - 16\hat{j} + 7\hat{k};\ \vec{b_3} = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

According to question

$$(i) \perp (ii) \quad \Rightarrow \vec{b_1} \perp \vec{b_2} \quad \Rightarrow \vec{b_1} \cdot \vec{b_2} = 0$$

$$(i) \perp (iii) \quad \Rightarrow \vec{b_1} \perp \vec{b_3} \quad \Rightarrow \vec{b_1} \cdot \vec{b_3} = 0$$

Hence, $3a - 16b + 7c = 0 \qquad \ldots(iv)$

and $\quad 3a + 8b - 5c = 0 \qquad \ldots(v)$

From equation (*iv*) and (*v*), we get

$$\frac{a}{80 - 56} = \frac{b}{21 + 15} = \frac{c}{24 + 48}$$

$\Rightarrow \quad \frac{a}{24} = \frac{b}{36} = \frac{c}{72} \quad \Rightarrow \quad \frac{a}{2} = \frac{b}{3} = \frac{c}{6} = \lambda$ (say)

$\Rightarrow \quad a = 2\lambda, b = 3\lambda, c = 6\lambda$

Putting the value of a, b, c in (i), we get the required cartesian equation of line as

$$\frac{x-1}{2\lambda} = \frac{y-2}{3\lambda} = \frac{z+4}{6\lambda} \quad \Rightarrow \quad \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Hence, vector equation is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

6. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection. **[*CBSE Delhi 2014*]**

Sol. Given lines are

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \quad \ldots(i)$$

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} \quad \ldots(ii)$$

Let two lines (i) and (ii) intersect at a point $P(\alpha, \beta, \gamma)$.

$\Rightarrow \quad (\alpha, \beta, \gamma)$ satisfy line (i)

$\Rightarrow \quad \frac{\alpha+1}{3} = \frac{\beta+3}{5} = \frac{\gamma+5}{7} = \lambda$ (say)

$\Rightarrow \quad \alpha = 3\lambda - 1, \quad \beta = 5\lambda - 3, \quad \gamma = 7\lambda - 5 \quad \ldots(iii)$

Again (α, β, γ) also lie on (ii), we get

$$\frac{\alpha-2}{1} = \frac{\beta-4}{3} = \frac{\gamma-6}{5}$$

$$\Rightarrow \quad \frac{3\lambda-1-2}{1} = \frac{5\lambda-3-4}{3} = \frac{7\lambda-5-6}{5}$$

$$\Rightarrow \quad \underset{\text{I}}{\frac{3\lambda-3}{1}} = \underset{\text{II}}{\frac{5\lambda-7}{3}} = \underset{\text{III}}{\frac{7\lambda-11}{5}}$$

From I and II

$$\frac{3\lambda-3}{1} = \frac{5\lambda-7}{3}$$

$\Rightarrow \quad 9\lambda - 9 = 5\lambda - 7$

$\Rightarrow \quad 4\lambda = 2$

$\Rightarrow \quad \lambda = \frac{1}{2}$

From II and III

$$\frac{5\lambda-7}{3} = \frac{7\lambda-11}{5}$$

$\Rightarrow \quad 25\lambda - 35 = 21\lambda - 33$

$\Rightarrow \quad 4\lambda = 2$

$\Rightarrow \quad \lambda = \frac{1}{2}$

Since, the value of λ in both the cases is same

$\Rightarrow$ Both lines intersect each other at a point.

$\therefore$ Intersecting point $= (\alpha, \beta, \gamma) = \left(\frac{3}{2} - 1, \frac{5}{2} - 3, \frac{7}{2} - 5\right)$ [From (iii)]

$$= \left(\frac{1}{2}, -\frac{1}{2}, \frac{-3}{2}\right)$$

7. A line passes through (2, –1, 3) and is perpendicular to the lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and cartesian form. **[*CBSE (AI) 2014*]**

Sol. Let $\vec{b}$ be parallel vector of required line.

$\Rightarrow \quad \vec{b}$ is perpendicular to both given line.

$\Rightarrow \quad \vec{b} = (2\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = (-4-2)\hat{i} - (4-1)\hat{j} + (4+2)\hat{k} = -6\hat{i} - 3\hat{j} + 6\hat{k}.$$

Hence, the equation of line in vector form is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(-6\hat{i} - 3\hat{j} + 6\hat{k}), \qquad \vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) - 3\lambda(2\hat{i} + \hat{j} - 2\hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \mu(2\hat{i} + \hat{j} - 2\hat{k}) \qquad [\mu = -3\lambda]$$

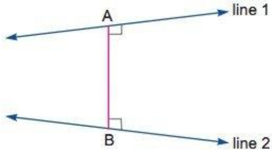
Equation in cartesian form is

$$\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$$

8. Find the shortest distance between the following lines :

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \textbf{and} \quad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$ *[CBSE Delhi 2008, (F) 2013, 2014]*

Sol. Let $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \lambda$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = k$



Now, let's take a point on first line as

$A(\lambda + 3, -2\lambda + 5, \lambda + 7)$ and let

$B(7k - 1, -6k - 1, k - 1)$ be point on the second line

The direction ratio of the line AB

$7k - \lambda - 4, -6k + 2\lambda - 6, k - \lambda - 8$

Now, as AB is the shortest distance between line 1 and line 2 so,

$$(7k - \lambda - 4) \times 1 + (-6k + 2\lambda - 6) \times (-2) + (k - \lambda - 8) \times 1 = 0 \qquad ...(i)$$

and $$(7k - \lambda - 4) \times 7 + (-6k + 2\lambda - 6) \times (-6) + (k - \lambda - 8) \times 1 = 0 \qquad ...(ii)$$

Solving equation (*i*) and (*ii*), we get

$\lambda = 0$ and $k = 0$

$\therefore \quad A \equiv (3, 5, 7)$ and $B \equiv (-1, -1, -1)$

Hence, $AB = \sqrt{(3+1)^2 + (5+1)^2 + (7+1)^2} = \sqrt{16 + 36 + 64} = \sqrt{116}$ units $= 2\sqrt{29}$ units

9. Find the shortest distance between the lines whose vector equations are:

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \textbf{and} \quad \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$ *[CBSE (F) 2014]*

Sol. Comparing the given equations with equations $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$.

We get $\vec{a_1} = \hat{i} + \hat{j}, \vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore, $\vec{a_2} - \vec{a_1} = \hat{i} - \hat{k}$ and

$$\vec{b_1} \times \vec{b_2} = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{9 + 1 + 49} = \sqrt{59}$$

Hence, the shortest distance between the given lines is given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}} \text{ units.}$$

10. Find the distance between the lines l_1 and l_2 given by

$$l_1: \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}); \quad l_2: \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$ *[CBSE (F) 2014]*

Sol. Given lines are

$$l_1: \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$l_2: \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

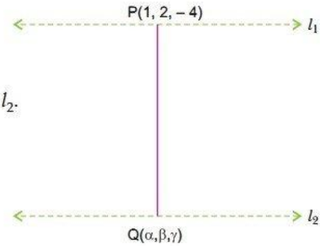
After observation, we get $l_1 \,||\, l_2$

Therefore, it is sufficient to find the perpendicular distance of a point of line l_1 to line l_2.

The coordinate of a point of l_1 is $P(1, 2, -4)$.

Also the cartesian form of line l_2 is

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} \qquad ...(i)$$



Let $Q(\alpha, \beta, \gamma)$ be foot of perpendicular drawn from P to line l_2.

$\because \quad Q(\alpha, \beta, \gamma)$ lie on line l_2.

$\therefore \quad \dfrac{\alpha-3}{4} = \dfrac{\beta-3}{6} = \dfrac{\gamma+5}{12} = \lambda$ (say)

$\Rightarrow \quad \alpha = 4\lambda + 3, \beta = 6\lambda + 3, \gamma = 12\lambda - 5$

Again, $\because \overrightarrow{PQ}$ is perpendicular to line l_2.

$\Rightarrow \quad \overrightarrow{PQ}.\vec{b} = 0$, where $\vec{b}$ is parallel vector of l_2

$\Rightarrow \quad (\alpha - 1).4 + (\beta - 2).6 + (\gamma + 4).12 = 0 \qquad [\because \overrightarrow{PQ} = (\alpha - 1)\hat{i} + (\beta - 2)\hat{j} + (\gamma + 4)\hat{k}]$

$\Rightarrow \quad 4\alpha - 4 + 6\beta - 12 + 12\gamma + 48 = 0$

$\Rightarrow \quad 4\alpha + 6\beta + 12\gamma + 32 = 0$

$\Rightarrow \quad 4(4\lambda + 3) + 6(6\lambda + 3) + 12(12\lambda - 5) + 32 = 0$

$\Rightarrow \quad 16\lambda + 12 + 36\lambda + 18 + 144\lambda - 60 + 32 = 0$

$\Rightarrow \quad 196\lambda + 2 = 0$

$\Rightarrow \quad \lambda = \dfrac{-2}{196} = \dfrac{-1}{98}$

Coordinate of $Q \equiv \left(4\times\left(-\dfrac{1}{98}\right) + 3, 6\times\left(-\dfrac{1}{98}\right) + 3, 12\times\left(-\dfrac{1}{98}\right) - 5\right)$

$$\equiv \left(-\frac{2}{49} + 3, -\frac{3}{49} + 3, -\frac{6}{49} - 5\right) \equiv \left(\frac{145}{49}, \frac{144}{49}, -\frac{251}{49}\right)$$

Therefore required perpendicular distance is

$$\sqrt{\left(\frac{145}{49} - 1\right)^2 + \left(\frac{144}{49} - 2\right)^2 + \left(\frac{-251}{49} + 4\right)^2} = \sqrt{\left(\frac{96}{49}\right)^2 + \left(\frac{46}{49}\right)^2 + \left(\frac{-55}{49}\right)^2}$$

$$= \sqrt{\frac{96^2 + 46^2 + 55^2}{49^2}} = \sqrt{\frac{9216 + 2116 + 3025}{49^2}}$$

$$= \frac{\sqrt{14357}}{49} = \frac{7\sqrt{293}}{49} = \frac{\sqrt{293}}{7} \text{ units}$$

Long Answer Questions

1. Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines. [*CBSE 2023 (65/3/2)*]

Sol. Given equation of lines be

$$\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$$

and, $$\frac{x-4}{5}=\frac{y-1}{2}=\frac{z-0}{1}$$

Let $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}=k_1$ and, $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z-0}{1}=k_2$

$\therefore$ Any point lies on the line (*i*) be $x = 2k_1 + 1, y = 3k_1 + b, z = 4k_1 + 3$

and any point lies on the line (*ii*) be $x = 5k_2 + 4, y = 2k_2 + 1, z = k_2$

$\therefore$ For point of intersection of lines (*i*) and (*ii*), we have

$2k_1 + 1 = 5k_2 + 4$ and $4k_1 + 3 = k_2$

$\Rightarrow$ $2k_1 - 5k_2 = 3$ ...(*i*) and $4k_1 - k_2 = -3$ ...(*ii*)

on solving equations (*i*) and (*ii*), we have

$k_1 = -1,$ $k_2 = -1$

Also, $y = 3k_1 + b$ and $y = 2k_2 + 1$

$\Rightarrow$ $y = 3 \times (-1) + b$ and $y = 2 \times (-1) + 1 = -1$

$\Rightarrow$ $y = -3 + b$ and $y = -1$

$\therefore$ $-3 + b = -1$ $\Rightarrow$ $b = -1 + 3 = 2$

$\Rightarrow$ $b = 2$

Point of intersection is $(2 \times (-1) + 1, 3 \times (-1) + 2, 4 \times (-1) + 3)$

$= (-1, -1, -1)$

2. Find the equations of all the sides of the parallelogram $ABCD$ whose vertices are $A(4, 7, 8)$, $B(2, 3, 4)$, $C(-1, -2, 1)$ and $D(1, 2, 5)$. Also, find the coordinates of the foot of the perpendicular from A to CD. [*CBSE 2023 (65/3/2)*]

Sol. Given vertices of the parallelogram $ABCD$ are A (4, 7, 8), B (2, 3, 4), C (–1, –2, 1) and D (1, 2, 5)

$\therefore$ Equation of side AB is given by

$$\frac{x-4}{2-4}=\frac{y-7}{3-7}=\frac{z-8}{4-8} \Rightarrow \frac{x-4}{-2}=\frac{y-7}{-4}=\frac{z-8}{-4}$$

$$\Rightarrow \frac{x-4}{1}=\frac{y-7}{2}=\frac{z-8}{2}$$

$\therefore$ Equation of side BC is given by

$$\frac{x-2}{-1-2}=\frac{y-3}{-2-3}=\frac{z-4}{1-3} \Rightarrow \frac{x-2}{-3}=\frac{y-3}{-5}=\frac{z-4}{-2}$$

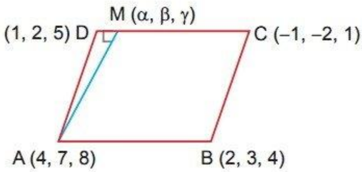
$$\Rightarrow \frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{2}$$

$\therefore$ Equation of side CD is given by

$$\frac{x+1}{2}=\frac{y+2}{4}=\frac{z-1}{4} \Rightarrow \frac{x+1}{1}=\frac{y+2}{2}=\frac{z-1}{2}$$

$\therefore$ Equation of side DA is given by

$$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-5}{3}$$



Let $M(\alpha, \beta, \gamma)$ be the foot of perpendicular.

$$\therefore \quad \frac{\alpha+1}{1}=\frac{\beta+2}{2}=\frac{\gamma-1}{2}=k$$

$$\Rightarrow \quad \alpha=k-1, \beta=2k-2, \gamma=2k+1 \qquad ...(i)$$

Direction ratios of AM are $\alpha-4, \beta-7, \gamma-8$.

Also, direction ratio of CD are 1, 2, 2.

Since, $AM \perp CD$

$$\therefore \quad (\alpha-4)\times 1+(\beta-7)\times 2+(\gamma-8)\times 2=0$$

$$\Rightarrow \quad \alpha+2\beta+2\gamma=34 \qquad ...(ii)$$

$$\Rightarrow \quad (k-1)+2(2k-2)+2(2k+1)=34 \Rightarrow 9k=37 \Rightarrow k=\frac{37}{9}$$

$$\therefore \quad \alpha=k-1=\frac{37}{9}-1=\frac{28}{9}, \beta=2\times\frac{37}{9}-2=\frac{74-18}{9}=\frac{56}{9}$$

and $$\gamma=2\times\frac{37}{9}+1=\frac{74+9}{9}=\frac{83}{9}$$

Required foot of perpendicular is $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$.

3. Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(l, 0, – 11). Hence, find the distance between the two lines. ***[CBSE 2023 (65/1/1)]***

Sol. Given point (1, 2, –4) through which a line is passing, therefore its vector equation be

$$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(1\hat{i}+0\hat{j}-11\hat{k}-3\hat{i}-3\hat{j}+5\hat{k})$$

$$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(-2\hat{i}-3\hat{j}-6\hat{k}) \qquad ...(i)$$

and its cartesian form be

$$\frac{x-1}{-2}=\frac{y-2}{-3}=\frac{z+4}{-6}$$

Also, equation of the line passing through points A(3, 3, –5) and B(1, 0, –11) is given by,

$$\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(1.\hat{i}+0.\hat{j}-11\hat{k}-3\hat{i}-3\hat{j}+5\hat{k})$$

$$\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(-2\hat{i}-3\hat{j}-6\hat{k}) \qquad ...(ii)$$

$\therefore$ Shortest distance between two given lines (*i*) and (*ii*) is given by

$$D=\left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|=\left|\frac{(2\hat{i}+\hat{j}-\hat{k})\times(-2\hat{i}-3\hat{j}-6\hat{k})}{\sqrt{4+9+36}}\right|$$

We have,

$$(\vec{a}_2-\vec{a}_1)\times\vec{b}=\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ -2 & -3 & -6\end{bmatrix}=-9\hat{i}-14\hat{j}-4\hat{k}$$

$$\therefore \quad D=\left|\frac{-9\hat{i}-14\hat{j}-4\hat{k}}{7}\right|=\frac{1}{7}\times\sqrt{81+196+16}=\frac{\sqrt{293}}{7} \text{ units}$$

4. **Find the equations of the line passing through the points $A(1, 2, 3)$ and $B(3, 5, 9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.**

[CBSE 2023 (65/1/1)]

Sol. Equation of the line passing through two points $A(1, 2, 3)$ and $B(3, 5, 9)$ is given by

$$\frac{x-3}{2}=\frac{y-5}{3}=\frac{z-9}{6} \qquad ...(i)$$

Direction ratios of the line are 2, 3, 6.

$\therefore$ Its direction cosines are $\frac{2}{\sqrt{4+9+36}}, \frac{3}{\sqrt{4+9+36}}, \frac{6}{\sqrt{4+9+36}}$ *i.e.* $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$.

Equation of line (i) may be written as

$$\frac{x-3}{\frac{2}{7}}=\frac{y-5}{\frac{3}{7}}=\frac{z-9}{\frac{6}{7}}=r \text{ (Let)} \qquad ...(ii)$$

Co-ordinates of any point on line (ii) may be taken as

$$Q\left(\frac{2}{7}r+3, \frac{3}{7}r+5, \frac{6}{7}r+9\right)$$

Distance of Q from B is 14 units.

$\therefore \qquad |r| = 14$

Putting the value of $|r| = 14 \quad \Rightarrow \quad r = \pm 14$ in Q, we get

$Q(\pm 4+3, \pm 6+5, \pm 12+9) \quad \Rightarrow \quad Q(7, 11, 21)$ or, $Q(-1, -1, -3)$

5. **A line l passes through point $(-1, 3, -2)$ and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.**

[CBSE 2023 (65/2/1)]

Sol. Given lines are

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \qquad ...(i)$$

and, $$\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5} \qquad ...(ii)$$

Let equation of line l passing through $(-1, 3, -2)$ is

$$\frac{x+1}{a}=\frac{y-3}{b}=\frac{z+2}{c} \qquad ...(iii)$$

Since line (iii) is perpendicular to both (i) and (ii)

$\therefore \qquad a\times 1+2\times b+3\times c=0 \qquad \Rightarrow a+2b+3c=0 \qquad ...(iv)$

and, $\qquad a\times(-3)+b\times 2+c\times 5=0 \qquad \Rightarrow -3a+2b+5c=0 \qquad ...(v)$

On solving equation (iv) and (v), we have

$$\frac{a}{2\times5-3\times2}=\frac{-b}{1\times5-3\times(-3)}=\frac{c}{1\times2-2\times-3}$$

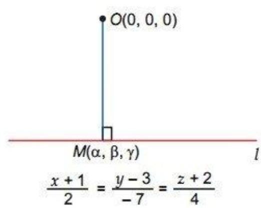
$\Rightarrow \qquad \frac{a}{4}=\frac{-b}{14}=\frac{c}{8} \qquad \Rightarrow \frac{a}{4}=\frac{b}{-14}=\frac{c}{8}=k \text{ (let)}$

$\therefore \qquad a=4k, b=-14k$ and $c=8k$

Putting the value of a, b and c in equation (iii), we get

$$\frac{x+1}{4k}=\frac{y-3}{-14k}=\frac{z+2}{8k}$$

$\Rightarrow \qquad \frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$



$$\Rightarrow \quad \frac{x-(-1)}{2}=\frac{y-3}{-7}=\frac{z-(-2)}{4}$$

Its vector form is

$$\vec{r}=(-\hat{i}+3\hat{j}-2\hat{k})+\lambda(2\hat{i}-7\hat{j}+4\hat{k})$$

Now, let OM perpendicular. l and co-ordinates of point M be (α, β, γ).

Also, direction ratio of lines be 2, – 7, 4.

$$\therefore \quad 2\alpha-7\beta+4\gamma=0 \qquad \text{(since } OM \perp l\text{)}$$

As $M(\alpha, \beta, \gamma)$ lies on the line l

$$\therefore \quad \frac{\alpha+1}{2}=\frac{\beta-3}{-7}=\frac{\gamma+2}{4}=r \text{ (let)}$$

$$\Rightarrow \quad \alpha=2r-1, \beta=-7r+3, \gamma=4r-2$$

$$\therefore \quad 2(2r-1)-7(-7r+3)+4(4r-2)=0$$

$$\Rightarrow \quad 4r-2+49r-21+16r-8=0$$

$$\Rightarrow \quad 69r=31 \quad \Rightarrow r=\frac{31}{69}$$

$$\therefore \quad \alpha=2r-1=2\times\frac{31}{69}-1=\frac{62-69}{69}=\frac{-7}{69}$$

$$\beta=-7r+3=-7\times\frac{31}{69}+3=\frac{-217+207}{69}=\frac{-10}{69}$$

$$\gamma=4r-2=4\times\frac{31}{69}-2=\frac{124-138}{69}=\frac{-14}{69}$$

$\therefore$ Distance from origin to the line is $\sqrt{\alpha^2+\beta^2+\gamma^2}$

$$=\sqrt{\left(\frac{-7}{69}\right)^2+\left(\frac{-10}{69}\right)^2+\left(\frac{-14}{69}\right)^2}$$

$$=\frac{1}{69}\sqrt{49+100+196}$$

$$=\frac{1}{69}\sqrt{345}=\sqrt{\frac{5}{69}} \text{ units}$$

6. Find the shortest distance between the lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

Sol. We are given equation of lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \qquad ...(i)$$

and

$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4} \qquad ...(ii)$$

In vector form

$$\vec{r}=(3\hat{i}+8\hat{j}+3\hat{k})+\lambda(3\hat{i}-\hat{j}+\hat{k})=0 \qquad ...(iii)$$

and

$$\vec{r}=(-3\hat{i}-7\hat{j}+6\hat{k})+\mu(-3\hat{i}+2\hat{j}+4\hat{k})=0 \qquad ...(iv)$$

$$\vec{a_1}=3\hat{i}+8\hat{j}+3\hat{k}, \vec{a_2}=-3\hat{i}-7\hat{j}+6\hat{k}$$

$$\vec{b_1}=3\hat{i}-\hat{j}+\hat{k}, \vec{b_2}=-3\hat{i}+2\hat{j}+4\hat{k}$$

$$\vec{a_2}-\vec{a_1}=-6\hat{i}-15\hat{j}+3\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = \hat{i}(-4-2) - \hat{j}(12+3) + \hat{k}(6-3) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\text{Shortest distance } = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{36+225+9}{\sqrt{36+225+9}} \right| = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30} \text{ units.}$$

7. Find the value of $a + b + c$ where (a, b, c) is the image of $(1, 2, -3)$ in the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$.

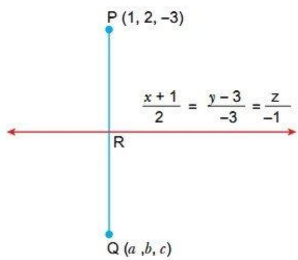
Sol. We are given equation of line

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda \text{ (say)}$$

$\therefore$ Co-ordinate of the point $R = (-1 + 2\lambda, 3 - 2\lambda, -\lambda)$.

Dr's of PR are $2\lambda - 2, 1 - 2\lambda, 3 - \lambda$.

$\because$ PR is perpendicular to the given line.



$\therefore\ 2(2\lambda - 2) - 2(1 - 2\lambda) - 1(3 - \lambda) = 0$

$\Rightarrow\ 4\lambda - 4 - 2 + 4\lambda - 3 + \lambda = 0$

$\Rightarrow\ 9\lambda - 9 = 0 \Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$

$\therefore\ R \equiv (1, 1, -1)$

$\because\ R$ is the mid-point of PQ.

$\therefore\ 1 = \frac{1+a}{2}, 1 = \frac{2+b}{2}, -1 = \frac{-3+c}{2}$

$\Rightarrow\ 2 = 1 + a, 2 = 2 + b, -2 = -3 + c$

$\Rightarrow\ 1 = a, b = 0, c = -2 + 3 = 1$

$\therefore\ Q \equiv (a, b, c) = (1, 0, 1)$

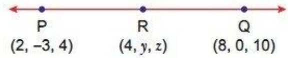
$\therefore\ a + b + c = 1 + 0 + 1 = 2$

8. If a point $R(4, y, z)$ lies on the line segment joining the point $P(2, -3, 4)$ and $Q(8, 0, 10)$. Find the distance of R from origin.

Sol. Equation of the line through the points $P(2, -3, 4)$ and $Q(8, 0, 10)$ is

$$\frac{x-2}{8-2} = \frac{y+3}{0-(-3)} = \frac{z-4}{10-4}$$

i.e., $\frac{x-2}{6} = \frac{y+3}{3} = \frac{z-4}{6} = \lambda \text{ (say)}$



$\therefore$ Co-ordinate of general point on this line be

$(2 + 6\lambda, -3 + 3\lambda, 4 + 6\lambda)$

$\therefore\ R \equiv (2 + 6\lambda, -3 + 3\lambda, 4 + 6\lambda)$

Given $R \equiv (4, y, z)$

$\therefore\ 4 = 2 + 6\lambda, y = -3 + 3\lambda, z = 4 + 6\lambda$

$\Rightarrow\ 2 = 6\lambda, y = -3 + 3\lambda, z = 4 + 6\lambda$

$\Rightarrow\ \lambda = \frac{1}{3}, y = -3 + 1, z = 4 + 2$

$\Rightarrow \quad y = -2, z = 6$

$\therefore \; R \equiv (4, -2, 6)$

$\therefore$ Required distance $= \sqrt{(4-0)^2 + (-2-0)^2 + (6-0)^2}$

$$= \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$

9. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{z-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines. ***[CBSE 2020 (65/5/1)]***

Sol. Let the cartesian equation of the required line be

$$\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c} \qquad ...(i)$$

where, a, b, c are direction ratios and given lines are

$$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4} \qquad ...(ii)$$

and $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \qquad ...(iii)$$

Since the line (*i*) is perpendicular to both the lines (*ii*) and (*iii*)

$\therefore \quad a \times 1 + b \times 2 + c \times 4 = 0 \quad \Rightarrow \quad a + 2b + 4c = 0$

Also, $\quad a \times 2 + b \times 3 + c \times 4 = 0 \quad \Rightarrow \quad 2a + 3b + 4c = 0$

On solving these two equations, we get

$$\frac{a}{8-12} = \frac{-b}{4-8} = \frac{c}{3-4}$$

$$\Rightarrow \quad \frac{a}{-4} = \frac{b}{4} = \frac{c}{-1}$$

$\therefore$ Direction ratios of the required line be $-4, 4, -1$.

$\therefore$ Vector and cartesian equation of required line be

$$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(-4\hat{i} + 4\hat{j} - \hat{k})$$

and, $\frac{x-1}{-4} = \frac{y-1}{4} = \frac{z-1}{-1}$ respectively.

Let θ be the angle between given lines.

$$\therefore \quad \cos\theta = \left|\frac{1\times 2 + 2\times 3 + 4\times 4}{\sqrt{1+4+16}\times\sqrt{4+9+16}}\right| = \frac{24}{\sqrt{609}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$$

10. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P (5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also find the image of P in this line. ***[CBSE (AI) 2012]***

Sol. Given line is

$$\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$$

It can be written in cartesian form as

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} \qquad ...(i)$$

Let $Q(\alpha, \beta, \gamma)$ be the foot of perpendicular drawn from $P(5, 4, 2)$ to the line (*i*) and $P'(x_1, y_1, z_1)$ be the image of P on the line (*i*)

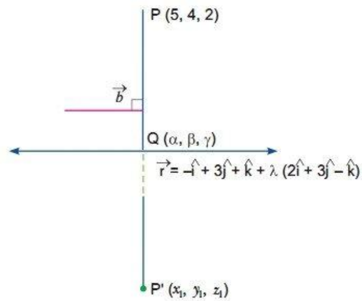
$\because$ $Q(\alpha, \beta, \gamma)$ lie on line (*i*)

$\therefore$ $\dfrac{\alpha+1}{2} = \dfrac{\beta-3}{3} = \dfrac{\gamma-1}{-1} = \lambda$ (say)

$\Rightarrow$ $\alpha = 2\lambda - 1; \beta = 3\lambda + 3$ and $\gamma = -\lambda + 1$ ...(*ii*)

Now, $\overrightarrow{PQ} = (\alpha - 5)\hat{i} + (\beta - 4)\hat{j} + (\gamma - 2)\hat{k}$

Parallel vector of line (*i*) $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.



Obviously $\overrightarrow{PQ} \perp \vec{b}$ $\Rightarrow$ $\overrightarrow{PQ}.\vec{b} = 0$

$2(\alpha - 5) + 3(\beta - 4) + (-1)(\gamma - 2) = 0$

$\Rightarrow$ $2\alpha - 10 + 3\beta - 12 - \gamma + 2 = 0$

$\Rightarrow$ $2\alpha + 3\beta - \gamma - 20 = 0$

$\Rightarrow$ $2(2\lambda - 1) + 3(3\lambda + 3) - (-\lambda + 1) - 20 = 0$

[Putting value of α, β, γ from (*ii*)]

$\Rightarrow$ $4\lambda - 2 + 9\lambda + 9 + \lambda - 1 - 20 = 0$

$\Rightarrow$ $14\lambda - 14 = 0 \Rightarrow \lambda = 1$

Hence the coordinates of foot of perpendicular Q are $(2 \times 1 - 1,\ 3 \times 1 + 3, -1 + 1)$, *i.e.*, $(1, 6, 0)$.

$\therefore$ Length of perpendicular $= \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2}$

$= \sqrt{16 + 4 + 4} = \sqrt{24} = 2\sqrt{6}$ units.

Also, since Q is mid-point of PP'

$\therefore$ $1 = \dfrac{x_1 + 5}{2} \Rightarrow x_1 = -3$

$6 = \dfrac{y_1 + 4}{2} \Rightarrow y_1 = 8$

$0 = \dfrac{z_1 + 2}{2} \Rightarrow z_1 = -2$

Therefore required image is $(-3, 8, -2)$.

11. If $l_1, m_1, n_1, l_2, m_2, n_2$ and l_3, m_3, n_3 are the direction cosines of three mutually perpendicular lines, then prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$ and $n_1 + n_2 + n_3$ makes equal angles with them. [*NCERT Exemplar*]

Sol. Let $\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$; $\vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$; $\vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$

$$\vec{d} = (l_1 + l_2 + l_3)\hat{i} + (m_1 + m_2 + m_3)\hat{j} + (n_1 + n_2 + n_3)\hat{k}$$

Also, let α, β and γ are the angles between $\vec{a}$ and $\vec{d}$, $\vec{b}$ and $\vec{d}$, $\vec{c}$ and $\vec{d}$ respectively.

$\therefore$ $\cos\alpha = l_1(l_1 + l_2 + l_3) + m_1(m_1 + m_2 + m_3) + n_1(n_1 + n_2 + n_3)$

$= l_1^2 + l_1l_2 + l_1l_3 + m_1^2 + m_1m_2 + m_1m_3 + n_1^2 + n_1n_2 + n_1n_3$

$= (l_1^2 + m_1^2 + n_1^2) + (l_1l_2 + l_1l_3 + m_1m_2 + m_1m_3 + n_1n_2 + n_1n_3)$

$= 1 + 0 = 1$

$[\because l_1^2 + m_1^2 + n_1^2 = 1$ and $l_1 \perp l_2, l_1 \perp l_3, m_1 \perp m_2, m_1 \perp m_3, n_1 \perp n_2, n_1 \perp n_3]$

Similarly, $\cos\beta = l_2(l_1 + l_2 + l_3) + m_2(m_1 + m_2 + m_3) + n_2(n_1 + n_2 + n_3)$

$= 1 + 0$ and $\cos\gamma = 1 + 0$

$\Rightarrow \quad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad \alpha = \beta = \gamma$

So, the line whose direction cosines are proportional to $l_1 + l_2 + l_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$ makes equal angles with the three mutually perpendicular lines whose direction cosines are $l_1, m_1, n_1, l_2, m_2, n_2$ and l_3, m_3, n_3 respectively.

Questions for Practice

■ Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(i) The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by [*NCERT Exemplar*]

(a) (2, 0, 0) (b) (0, 5, 0)

(c) (0, 0, 7) (d) (0, 5, 7)

(ii) The equation of x-axis in space are

(a) $x = 0, y = 0$ (b) $x = 0, z = 0$

(c) $x = 0$ (d) $y = 0, z = 0$

(iii) If α, β, γ, are the angles that a line makes with a positive direction of x, y, z axes, respectively, then the direction cosines of the line are

(a) $\sin\alpha, \sin\beta, \sin\gamma$ (b) $\cos\alpha, \cos\beta, \cos\gamma$

(c) $\tan\alpha, \tan\beta, \tan\gamma$ (d) $\cos^2\alpha, \cos^2\beta, \cos^2\gamma$

(iv) If a line makes angles α, β, γ with the positive directions of coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(a) 2 (b) 1

(c) -1 (d) none of these

(v) The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is [*CBSE 2020 (65/5/1)*]

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$

(c) -2 (d) 2

(vi) If the line makes an angle of $\frac{\pi}{4}$ with each of y and z axes, then the angle which it makes with x-axis is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

■ Conceptual Questions

2. Cartesian equation of a line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$.

Write the direction ratios of a line parallel to AB. [*CBSE Sample Paper*]

3. If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, find the value of k.

4. If a line makes angles 90°, 135°, 45° with the X, Y and Z axes respectively, find its direction cosines. **[CBSE 2019 (65/1/1)]**

5. Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$. **[CBSE 2019 (65/1/1)]**

6. A line passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction of the vector $\hat{i}+\hat{j}-2\hat{k}$. Find the equation of the line in cartesian form. **[CBSE 2019 (65/2/1)]**

7. If a line has the direction ratios – 18, 12, – 4, then what are its direction cosines? **[CBSE 2019 (65/3/1)]**

8. Find the Cartesian equation of the line which passes through the point (–2, 4, – 5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. **[CBSE Delhi 2013]**

9. If the cartesian equations of a line are $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$, write the vector equation for the line. **[CBSE (AI) 2014]**

10. Find the cartesian equation of line joining the point (–2, 1, 3) and (3, 1, –2).

■ Very Short Answer Questions

11. Find the angle between the lines $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$. **[CBSE (F) 2014]**

12. Find the value of k for which the following lines are perpendicular to each other.
$\frac{x+3}{k-5}=\frac{y-1}{1}=\frac{5-z}{-2k-1}$; $\frac{x+2}{-1}=\frac{2-y}{-k}=\frac{z}{5}$

13. Find the direction cosines of the line passing through two points (–2, 4, –5) and (1, 2, 3).

14. Find the equation of the line passing through the point (2, 1, 3) and having direction ratios 1, 1, – 2.

15. Find the cartesian equation of the line passing through the points having position vector $\hat{i}+2\hat{j}+3\hat{k}$ and $-3\hat{i}+2\hat{j}+4\hat{k}$.

16. Find the vector equation of the line passing through the points $A(3, 4, -7)$ and $B(1, -1, 6)$.

17. Convert the vector equation of line $\vec{r}=(4\hat{i}+3\hat{j}-5\hat{k})+\lambda(-2\hat{i}-5\hat{j}+3\hat{k})$,
where λ is a parameter, into cartesian form.

■ Short Answer Questions

18. Find the shortest distance between the following pair of skew lines:
$\frac{x-1}{2}=\frac{2-y}{3}=\frac{z+1}{4}$, $\frac{x+2}{-1}=\frac{y-3}{2}=\frac{z}{3}$ **[CBSE Sample Paper 2016]**

19. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect. Find their point of intersection. **[CBSE (East) 2016]**

20. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$ Hence find the image of the point A in the line BC. **[CBSE (North) 2016]**

21. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$. **[CBSE (F) 2016]**

22. Find the vector and cartesian equations of a line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 1, 1). **[CBSE Guwahati 2015]**

23. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$. **[CBSE (AI) 2011]**

24. Show that lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection. **[CBSE Delhi 2014]**

25. Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect. Also, find the co-ordinate of the point of intersection.

■ Long Answer Questions

26. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also find the equation of the line joining given points and its image.

27. Find the image of the point (2, 1, 2) in the line $\vec{r} = (4\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$.

28. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and
$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(2\hat{i} + 6\hat{j} + 3\hat{k})$.

29. Find the angle between the following pair of lines
(*i*) $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$
(*ii*) $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

30. Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are $P(4, 2, -6)$, $Q(5, -3, 1)$, $R(12, 4, 5)$ and $S(11, 9, -2)$. Use these equations to find the point of intersection of diagonals. **[CBSE 2013 (65/2/1)]**

Answers

1. (*i*) (*a*) (*ii*) (*d*) (*iii*) (*b*) (*iv*) (*a*) (*v*) (*a*) (*vi*) (*c*)

2. 1, –7, 2 3. $k = \frac{-10}{7}$

4. $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

5. $\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$

6. $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$

7. $l = \frac{-9}{11}, m = \frac{6}{11}, n = \frac{-2}{11}$ 8. $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

9. $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$

10. $\dfrac{x+2}{5}=\dfrac{y-1}{0}=\dfrac{z-3}{-5}$

11. $\theta=\cos^{-1}\left(\dfrac{19}{21}\right)$

12. $k=-1$

13. $\dfrac{3}{\sqrt{77}}, \dfrac{-2}{\sqrt{77}}, \dfrac{8}{\sqrt{77}}$

14. $\dfrac{x-2}{1}=\dfrac{y-1}{1}=\dfrac{z-3}{-2}$

15. $\dfrac{x-1}{-4}=\dfrac{y-2}{0}=\dfrac{z-3}{1}$

16. $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$

17. $\dfrac{x-4}{-2}=\dfrac{y-3}{-5}=\dfrac{z+5}{3}$

18. $\dfrac{42}{\sqrt{390}}$ units

19. (4, 0, –1)

20. (–2, 1, 7); (–3, –6, 10)

22. $\dfrac{x-1}{10}=\dfrac{y+1}{-4}=\dfrac{z-1}{-7}; \vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(10\hat{i}-4\hat{j}-7\hat{k})$

23. $\dfrac{8}{\sqrt{29}}$ units

24. (4, 0, –1)

25. (1, –1, 2)

26. (1, 0, 7); $\dfrac{x-1}{0}=\dfrac{y-6}{-6}=\dfrac{z-3}{4}$

27. (6, 3, 4)

28. $\dfrac{1}{\sqrt{265}}$

29. (*i*) $\cos^{-1}\left(\dfrac{26}{9\sqrt{38}}\right)$ (*ii*) $\cos^{-1}\left(\dfrac{2}{3}\right)$

30. Equation of diagonal *PQ* is $\dfrac{x-4}{8}=\dfrac{y-2}{2}=\dfrac{z+6}{11}$ and

Equation of diagonal *QS* is $\dfrac{x-5}{6}=\dfrac{y+3}{12}=\dfrac{z-1}{-3}$.

Point of intersection is $O\left(8, 3, -\dfrac{1}{2}\right)$.