

# Application of Integrals

## Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

- (a) Both A and R are true and R is the correct explanation for A.
- (b) Both A and R are true but R is not the correct explanation for A.
- (c) A is true but R is false.
- (d) A is false but R is true.

1. **Assertion (A)** : The area of the curve  $y = \sin^2 x$  from 0 to  $\pi$  will be more than that of the curve  $y = \sin x$  from 0 to  $\pi$ .

**Reason (R)** :  $x^2 > x$ , if  $x > 1$

2. **Assertion (A)** : The area of the ellipse  $2x^2 + 3y^2 = 6$  will be more than the area of the circle  $x^2 + y^2 - 2x + 4y + 4 = 0$ .

**Reason (R)** : The length of the semi-major axis of ellipse  $2x^2 + 3y^2 = 6$  is more than the radius of the circle  $x^2 + y^2 - 2x + 4y + 4 = 0$ .

3. **Assertion (A)** : Area enclosed by the circle  $x^2 + y^2 = 36$  is equal to  $36\pi$  sq. units.

**Reason (R)** : Area enclosed by the circle  $x^2 + y^2 = r^2$  is  $\pi r^2$ .

4. **Assertion (A)** : Area enclosed by the curve  $y = x^3$  and the line  $y = x$  in first quadrant is  $\frac{1}{4}$  sq. units.

**Reason (R)** : Area is always the real number.

5. **Assertion (A)** : Area between the curve  $x = y^2$  and the line  $x = 3$  is  $4\sqrt{3}$  sq. units.

**Reason (R)** : Points of intersection of  $y^2 = 4ax$  and the line  $ax + by + c = 0$  is obtained by solving equations of curve and line.

### Answers

1. (d)      2. (b)      3. (a)      4. (b)      5. (b)

## Solutions of Assertion-Reason Questions

- $\therefore \sin^2 x \leq \sin x \quad \forall x \in (0, \pi)$   
 $\therefore$  Area of  $y = \sin^2 x$  will be lesser than the area of  $y = \sin x$  in  $x \in (0, \pi)$ .  
 Clearly, Assertion (A) is false and Reason (R) is true.  
 $\therefore$  Option (d) is correct.
- Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).  
 $\therefore$  Option (b) is correct.
- $\therefore$  Required area =  $4 \times \text{ar}(\text{OABO}) = 4 \times \int_0^6 \sqrt{6^2 - x^2} dx$

$$\begin{aligned}
 &= 4 \times \left[ \frac{x}{2} \sqrt{6^2 - x^2} + \frac{6^2}{2} \sin^{-1} \left( \frac{x}{6} \right) \right]_0^6 \\
 &= 4 \times [(0 + 18 \sin^{-1}(1)) - (0 + 0)] \\
 &= 4 \times 18 \times \sin^{-1}(1) = 72 \times \frac{\pi}{2} = 36 \pi \text{ sq. units}
 \end{aligned}$$

Clearly A is correct statement.

Also R is a correct statement and gives the correct explanation of statement A.

$\therefore$  Option (a) is correct.

- We are given curve and line

$$y = x^3 \quad \dots(i)$$

$$y = x \quad \dots(ii)$$

$$\begin{aligned}
 \text{From (i) and (ii), we have} \quad & x^3 = x \\
 \Rightarrow x^3 - x = 0 & \Rightarrow x(x^2 - 1) = 0 \\
 \Rightarrow x(x-1)(x+1) = 0 & \Rightarrow x = 0, -1, 1
 \end{aligned}$$

From (ii),

$$\text{If } x = 0, y = 0,$$

$$\text{If } x = -1, y = -1$$

$$\text{If } x = 1, y = 1$$

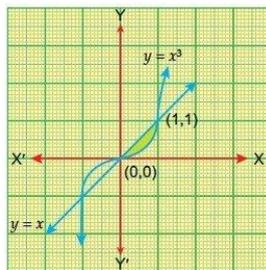
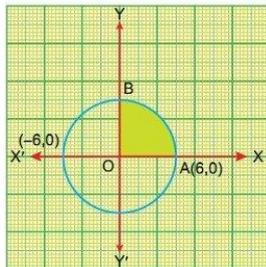
$\therefore$  Points of intersection are  $(-1, -1)$ ,  $(0, 0)$  and  $(1, 1)$ .

$$\begin{aligned}
 \text{Required area} &= \int_0^1 (y_{\text{line}} - y_{\text{parabola}}) dx \\
 &= \int_0^1 x dx - \int_0^1 x^3 dx = \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^4}{4} \right]_0^1 \\
 &= \left( \frac{1}{2} - 0 \right) - \left( \frac{1}{4} - 0 \right) \\
 &= \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ sq. units}
 \end{aligned}$$

Statement A is correct.

Also statement R is correct but does not correctly explain the statement A.

$\therefore$  Option (b) is correct.



5. We are given equations of curve and line as

$$x = y^2 \quad (i)$$

$$\text{and } x = 3 \quad (ii)$$

From (i) & (ii), we have  $y^2 = 3 \Rightarrow y = \pm\sqrt{3}$

$\therefore$  Points of intersection are  $(3, \sqrt{3})$  and  $(3, -\sqrt{3})$ .

$$\text{Required area} = 2 \times \text{ar} (OABO) = 2 \int_0^3 \sqrt{x} \, dx$$

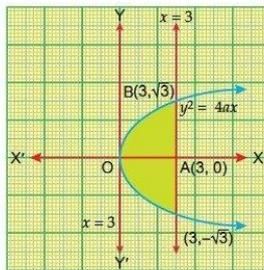
$$= 2 \times \left[ \frac{x^{3/2}}{3/2} \right]_0^3 = 2 \times \frac{2}{3} \left[ x^{3/2} \right]_0^3$$

$$= \frac{4}{3} \left[ 3^{3/2} - 0^{3/2} \right] = \frac{4}{3} \times 3\sqrt{3} = 4\sqrt{3} \text{ sq. units.}$$

Statement A is correct.

Also statement R is correct but does not give correct explanation of statement A.

$\therefore$  Option (b) is correct.



## Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. Read the following passage and answer the following questions.

An architect designs a building whose lift (elevator) is from outside of the building attached to the walls. The floor (base) of the lift (elevator) is in semicircular shape.



The floor of the elevator (lift) whose circular edge is given by the equation  $x^2 + y^2 = 4$  and the straight edge (line) is given by the equation  $y = 0$ .

- (i) Find the point of intersection of the circular edge and straight line edge.  
 (ii) Find the length of each vertical strip of the region bounded by the given curves.  
 (iii) (a) Find the area of a vertical strip between given circular edge and straight edge.  
 (b) Find the area of a horizontal strip between given circular strip and straight edge.

OR

- (iii) Find the area of the region of the floor of the lift of the building (in square units).

Sol. (i) Given curve for circle and straight line are

$$x^2 + y^2 = 4 \quad \dots(1)$$

$$y = 0 \quad \dots(2)$$

∴ From (1) and (2), we have

$$x^2 = 4 \Rightarrow x = \pm 2$$

∴ Points of intersection are (2, 0) and (-2, 0).

(ii) Given curve, is circle whose equation is

$$x^2 + y^2 = 4$$

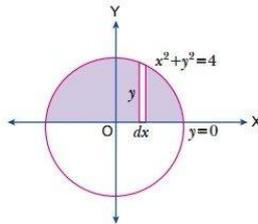
$$\Rightarrow y^2 = 4 - x^2 \Rightarrow y = \sqrt{4 - x^2}$$

$$\text{and } y = 0$$

It represents x-axis.

∴ Length of the vertical strip is  $y = \sqrt{4 - x^2}$ , i.e.,  $\sqrt{4 - x^2}$ .

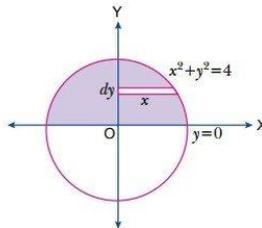
(iii) (a) We have,



Area of one vertical strip

$$= y \cdot dx = \sqrt{4 - x^2} \cdot dx$$

(b) We have,



Area of one horizontal strip

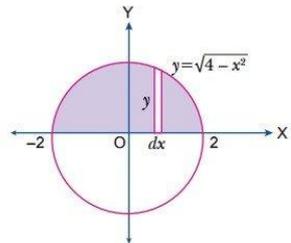
$$= x \cdot dy$$

$$= \sqrt{4 - y^2} \cdot dy$$

OR

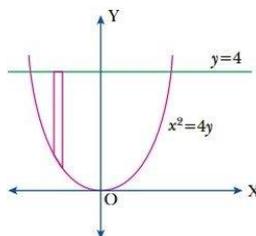
(iii) We have,

$$\begin{aligned} \text{Area of the floor} &= \int_{-2}^2 y \cdot dx = 2 \int_0^2 \sqrt{4 - x^2} \cdot dx \\ &= 2 \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= 4 \sin^{-1} 1 - 0 \\ &= 4 \times \frac{\pi}{2} = 2\pi \text{ sq. units} \end{aligned}$$



2. Read the following passage and answer the following questions.

A student designs an open air Honeybee nest on the branch of a tree, whose plane figure is parabolic and the branch of tree is given by a straight line.



- (i) Find point of intersection of the parabola and straight line.
- (ii) Find the area of each vertical strip.
- (iii) (a) Find the length of each horizontal strip of the bounded region.  
(b) Find the length of each vertical strip.

OR

- (iii) Find the area of region bounded by parabola  $x^2 = 4y$  and line  $y = 4$  (in square units).

Sol. (i) Given equation of parabola is  $x^2 = 4y$  and equation of straight line  $y = 4$ . ... (i)

∴ From (i), we get

$$x^2 = 4 \times 4 = 16 \quad \Rightarrow x = \pm 4$$

∴ Point of intersection are (4, 4) and (-4, 4).

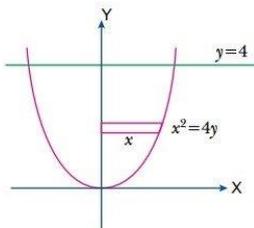
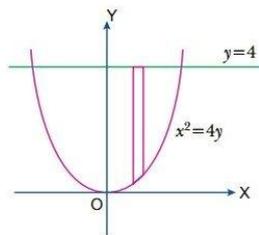
- (ii) Area of each (one) vertical strip

$$= y \cdot dx$$

$$= 4 dx - \frac{x^2}{4} dx$$

$$= \left(4 - \frac{x^2}{4}\right) \cdot dx$$

- (iii) (a) We have,



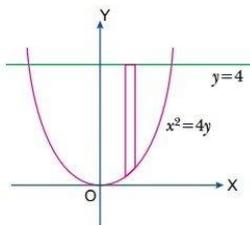
$$x^2 = 4y \quad \dots (i)$$

$$\Rightarrow x = 2\sqrt{y}$$

∴ Length of the horizontal strip be  $2 \times 2\sqrt{y} = 4\sqrt{y}$ .

(b) We have

$$\text{Length of the vertical strip} = 4 - \frac{x^2}{4} = \frac{1}{4}(16 - x^2)$$



OR

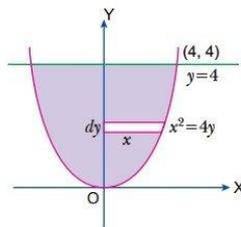
(iii) We have

Area of required bounded region

$$= 2 \int_0^4 x \, dy$$

$$= 2 \int_0^4 2\sqrt{y} \, dy$$

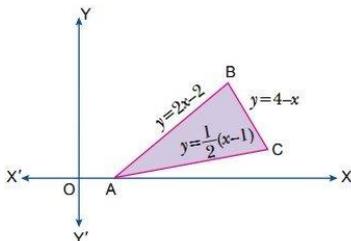
$$= 4 \left[ \frac{2}{3} y^{\frac{3}{2}} \right]_0^4 = \frac{8}{3} \left[ (4)^{\frac{3}{2}} - 0 \right] = \frac{64}{3} \text{ sq units}$$



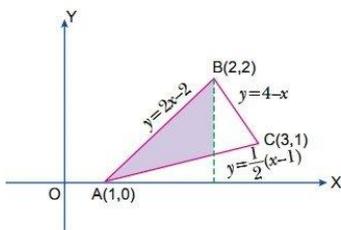
3. Read the following passage and answer the following questions.

A farmer has a triangular shaped field. His son, a science student observes the triangular field has three edges and can be drawn on a plain paper with three lines given by its equations.

(i) Find the area of the shaded region in the figure shown below.



(ii) Find the area of the triangle ABC given below.



Sol. We are given sides

$$y = 2x - 2 \quad \dots(i)$$

$$y = \frac{1}{2}(x - 1) \quad \dots(ii)$$

and  $y = 4 - x \quad \dots(iii)$

From (i) and (ii), we get

$$2x - 2 = \frac{1}{2}(x - 1)$$

$$\Rightarrow 4x - 4 = x - 1$$

$$\Rightarrow 3x = 3 \Rightarrow x = 1$$

Using  $x = 1$  in (i), we get

$$y = 2 \times 1 - 2 = 2 - 2 = 0$$

$$\therefore A \equiv (1, 0)$$

From (i) and (iii), we get

$$2x - 2 = 4 - x$$

$$\Rightarrow 3x = 6 \Rightarrow x = 2$$

Using  $x = 2$  in (iii), we get

$$y = 4 - 2 = 2$$

$$\therefore B \equiv (2, 2)$$

From (ii) and (iii), we get

$$4 - x = \frac{1}{2}(x - 1)$$

$$\Rightarrow 8 - 2x = x - 1$$

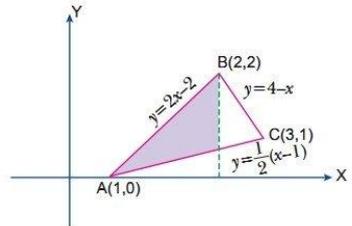
$$\Rightarrow 9 = 3x \Rightarrow x = 3$$

$$\Rightarrow y = 4 - x = 4 - 3 = 1$$

$$\therefore C \equiv (3, 1)$$

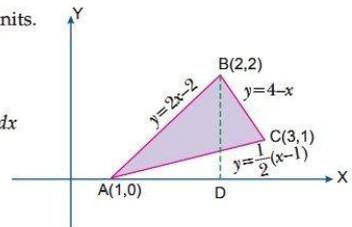
(i) We have area of shaded region

$$\begin{aligned} &= \int_1^2 \left\{ (2x - 2) - \frac{1}{2}(x - 1) \right\} dx \\ &= \int_1^2 \left( \frac{3}{2}x - \frac{3}{2} \right) dx \\ &= \frac{3}{2} \int_1^2 (x - 1) dx = \frac{3}{2} \left[ \frac{x^2}{2} - x \right]_1^2 \\ &= \frac{3}{2} \left[ \frac{4}{2} - 2 - \frac{1}{2} + 1 \right] = \frac{3}{2} \left( \frac{3}{2} - 1 \right) = \frac{3}{4} \text{ sq. units.} \end{aligned}$$



(ii) We have to find area of the triangle ABC.

$$\begin{aligned} &= \int_1^2 (2x - 2) dx + \int_2^3 (4 - x) dx - \int_1^3 \frac{1}{2}(x - 1) dx \\ &= 2 \left[ \frac{x^2}{2} - x \right]_1^2 + \left[ 4x - \frac{x^2}{2} \right]_2^3 - \frac{1}{2} \left[ \frac{x^2}{2} - x \right]_1^3 \\ &= 2 \left[ 2 - 2 - \frac{1}{2} + 1 \right] + \left[ 12 - \frac{9}{2} - 8 + 2 \right] - \frac{1}{2} \left[ \frac{9}{2} - 3 - \frac{1}{2} + 1 \right] = \frac{3}{2} \text{ sq. units.} \end{aligned}$$



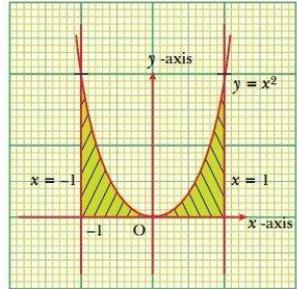
## Very Short Answer Questions

1. Find the area bounded by  $y = x^2$ ,  $x$ -axis and lines  $x = -1$  and  $x = 1$ .

[CBSE Sample Paper 2021]

**Sol.** Area of required region

$$\begin{aligned} &= \int_{-1}^1 x^2 dx \\ &= \left[ \frac{x^3}{3} \right]_{-1}^1 \\ &= \frac{1}{3} + \frac{1}{3} \\ &= \frac{2}{3} \text{ sq. unit} \end{aligned}$$



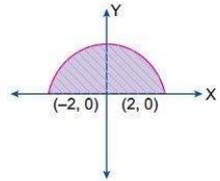
2. Sketch the region  $\{(x, 0) : y = \sqrt{4 - x^2}\}$  and X-axis. Find the area of the region using integration.

[NCERT Exemplar]

**Sol.** Given region is  $\{(x, 0) : y = \sqrt{4 - x^2}\}$  and X-axis.

We have,  $y = \sqrt{4 - x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$

$$\begin{aligned} \therefore \text{Area of shaded region, } A &= \int_{-2}^2 \sqrt{4 - x^2} dx = \int_{-2}^2 \sqrt{2^2 - x^2} dx \\ &= \left[ \frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \cdot \sin^{-1} \frac{x}{2} \right]_{-2}^2 \\ &= \frac{2}{2} \cdot 0 + 2 \cdot \frac{\pi}{2} + \frac{2}{2} \cdot 0 - 2 \sin^{-1}(-1) \\ &= 2 \cdot \frac{\pi}{2} + 2 \cdot \frac{\pi}{2} = 2\pi \text{ sq units.} \end{aligned}$$



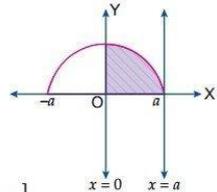
3. Determine the area under the curve  $y = \sqrt{a^2 - x^2}$  included between the lines  $x = 0$  and  $x = a$ .

[NCERT Exemplar]

**Sol.** Given equation of the curve is  $y = \sqrt{a^2 - x^2}$ .

$\Rightarrow y^2 = a^2 - x^2 \Rightarrow y^2 + x^2 = a^2$

$$\begin{aligned} \therefore \text{Required area of shaded region, } A &= \int_0^a \sqrt{a^2 - x^2} dx \\ &= \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\ &= \left[ 0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - \frac{a^2}{2} \sin^{-1} 0 \right] \\ &= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{4} \text{ sq units.} \end{aligned}$$



4. Find the area bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = 2\pi$ . [NCERT Exemplar]

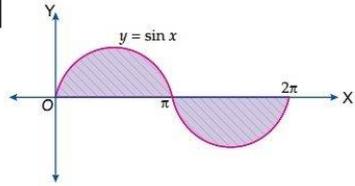
Sol. Required area =  $\int_0^{2\pi} \sin x \, dx = \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right|$

$$= -[\cos x]_0^{\pi} + \left| -[\cos x]_{\pi}^{2\pi} \right|$$

$$= -[\cos \pi - \cos 0] + \left| -[\cos 2\pi - \cos \pi] \right|$$

$$= -[-1 - 1] + \left| -(1 + 1) \right|$$

$$= 2 + 2 = 4 \text{ sq units.}$$



5. Sketch the region bounded by the lines  $2x + y = 8$ ,  $y = 2$ ,  $y = 4$  and the  $y$ -axis. Hence, obtain its area using integration. [CBSE 2023 (65/5/1)]

Sol. We are given lines

$$2x + y = 8 \quad \dots(i)$$

$$y = 2 \quad \dots(ii)$$

and  $y = 4 \quad \dots(iii)$

Form (i) and (ii), we have

$$2x + 2 = 8 \Rightarrow 2x = 6 \Rightarrow x = 3$$

$\therefore$  Point of intersection of lines (i) and (ii) is (3, 2).

From (i) and (iii), we have

$$2x + 4 = 8 \Rightarrow 2x = 4 \Rightarrow x = 2$$

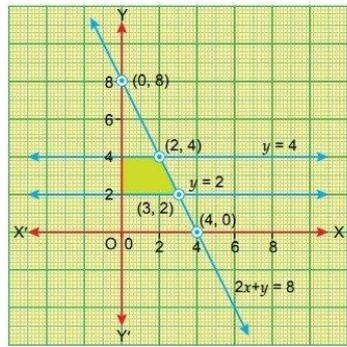
$\therefore$  Point of intersection of lines (i) and (iii) is (2, 4).

$$\therefore \text{Required area} = \int_2^4 x \, dy = \int_2^4 \frac{8-y}{2} \, dy$$

$$= \frac{1}{2} \left[ \int_2^4 8 \, dy - \int_2^4 y \, dy \right] = \frac{1}{2} \left[ 8y - \frac{y^2}{2} \right]_2^4$$

$$= \frac{1}{2} \left[ 8(4-2) - \frac{1}{2}(4^2 - 2^2) \right] = \frac{1}{2} \left[ 16 - \frac{1}{2}(12) \right]$$

$$= \frac{1}{2} [16 - 6] = \frac{1}{2} \times 10 = 5 \text{ sq. units.}$$



6. Find the area bounded by the curves  $y = |x - 1|$  and  $y = 1$ , using integration. [CBSE 2021-22 (Term-2)]

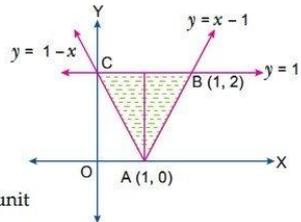
Sol. Given curves,  $y = |x - 1| = \begin{cases} 1 - x, & x \leq 1 \\ x - 1, & x > 1 \end{cases}$

and,  $y = 1$ .

$\therefore$  Area of required (shaded) region is

$$A = \int_0^1 (1-x) \, dx + \int_1^2 (x-1) \, dx$$

$$= \left[ x - \frac{x^2}{2} \right]_0^1 + \left[ \frac{x^2}{2} - x \right]_1^2 = 1 - \frac{1}{2} + \frac{4}{2} - 2 - \frac{1}{2} + 1 = 1 \text{ sq. unit}$$



## Long Answer Questions

1. The area of the region bounded by the line  $y = mx$  ( $m > 0$ ), the curve  $x^2 + y^2 = 4$  and the  $x$ -axis in the first quadrant is  $\frac{\pi}{2}$  units. Using integration, find the value of  $m$ . [CBSE 2023 (65/3/2)]

Sol. Given line  $y = mx$ ,  $m > 0$  ... (i)  
 the curve,  $x^2 + y^2 = 4$  ... (ii)  
 and  $x$ -axis i.e.,  $y = 0$  ... (iii)

On plotting lines (i), (ii), and (iii), we have

On solving  $y = mx$  and  $x^2 + y^2 = 4$ , we have

$$\begin{aligned} x^2 + m^2 x^2 &= 4 \\ \Rightarrow x^2 &= \frac{4}{1+m^2} \\ x &= \frac{2}{\sqrt{1+m^2}} \end{aligned}$$

$$\therefore \text{Area of shaded region} = \int_0^{\frac{2}{\sqrt{1+m^2}}} mx \, dx + \int_{\frac{2}{\sqrt{1+m^2}}}^2 \sqrt{4-x^2} \, dx$$

$$\Rightarrow \frac{\pi}{2} = m \left[ \frac{x^2}{2} \right]_0^{\frac{2}{\sqrt{1+m^2}}} + \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\frac{2}{\sqrt{1+m^2}}}^2$$

$$\Rightarrow \frac{\pi}{2} = m \times \frac{4}{2(1+m^2)} + 2 \sin^{-1}(1) - \frac{1}{\sqrt{1+m^2}} \sqrt{4 - \frac{4}{1+m^2}} - 2 \sin^{-1} \frac{1}{\sqrt{1+m^2}}$$

$$\Rightarrow \frac{\pi}{2} = \frac{2m}{1+m^2} - \frac{2m}{1+m^2} + 2 \times \frac{\pi}{2} - 2 \sin^{-1} \left( \frac{1}{\sqrt{1+m^2}} \right)$$

$$\Rightarrow \frac{\pi}{2} = \pi - 2 \sin^{-1} \left( \frac{1}{\sqrt{1+m^2}} \right) \quad \Rightarrow \quad 2 \sin^{-1} \left( \frac{1}{\sqrt{1+m^2}} \right) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} \left( \frac{1}{\sqrt{1+m^2}} \right) = \frac{\pi}{4} \quad \Rightarrow \quad \frac{1}{\sqrt{1+m^2}} = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1+m^2 = 2 \quad \Rightarrow \quad m^2 = 1 \quad \Rightarrow \quad m = 1 \quad (\text{As } m > 0)$$

2. Find the area of the region bounded by the curves  $x^2 = y$ ,  $y = x + 2$  and  $x$ -axis, using integration. [CBSE 2023 (65/1/1)]

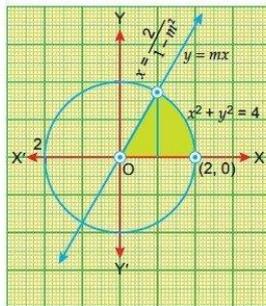
Sol. Given curves,  
 $x^2 = y$  ... (i)  
 $y = x + 2$  ... (ii)

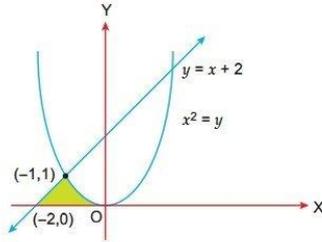
and  $x$ -axis i.e.  $y = 0$

On plotting these two curves, we have the region (shaded).

Points of intersection of the curves (i) and (ii), we get

$$\begin{aligned} x^2 &= x + 2 \quad \Rightarrow \quad x^2 - x - 2 = 0 \quad \Rightarrow \quad x^2 - 2x + x - 2 = 0 \\ &\Rightarrow \quad x(x-2) + 1(x-2) = 0 \\ &\Rightarrow \quad (x+1)(x-2) = 0 \\ &\Rightarrow \quad x = -1, 2 \end{aligned}$$





$$\begin{aligned} \therefore \text{Area of the shaded region} &= \int_{-2}^{-1} (x+2)dx + \int_{-1}^0 x^2 dx = \left[ \frac{x^2}{2} + 2x \right]_{-2}^{-1} + \left[ \frac{x^3}{3} \right]_{-1}^0 \\ &= \frac{(-1)^2}{2} + 2 \times (-1) - \frac{(-2)^2}{2} - 2 \times (-2) + \frac{0}{3} - \frac{(-1)^3}{3} \\ &= \frac{1}{2} - 2 - \frac{4}{2} + 4 + \frac{1}{3} = \frac{1}{2} - 4 + 4 + \frac{1}{3} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \text{ sq. unit} \end{aligned}$$

3. Using integration, find the area of region bounded by line  $y = \sqrt{3}x$ , the curve  $y = \sqrt{4-x^2}$  and  $y$ -axis in first quadrant. [CBSE 2023 (65/2/1)]

Sol. Given equation of curves

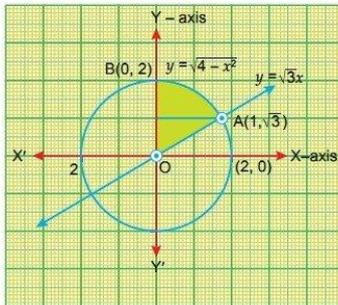
$$y = \sqrt{3}x \quad \dots(i)$$

$$y = \sqrt{4-x^2} \quad \dots(ii)$$

and,  $y$ -axis i.e.  $x = 0$

$\dots(iii)$

on plotting equation (i), (ii) and (iii), we have



Required region is shaded region  $OABO$ .

Point of intersection of  $y = \sqrt{3}x$  and  $y = \sqrt{4-x^2}$  is  $A(1, \sqrt{3})$ .

$$\begin{aligned} \therefore \text{Area of shaded region} &= \int_0^{\sqrt{3}} \frac{y}{\sqrt{3}} dy + \int_{\sqrt{3}}^2 \sqrt{4-y^2} dy \\ &= \frac{1}{\sqrt{3}} \left[ \frac{y^2}{2} \right]_0^{\sqrt{3}} + \left[ \frac{y}{2} \sqrt{4-y^2} + \frac{4}{2} \sin^{-1} \frac{y}{2} \right]_{\sqrt{3}}^2 \\ &= \frac{1}{\sqrt{3}} \left[ \frac{(\sqrt{3})^2}{2} - 0 \right] + \left[ \frac{2}{2} \times 0 + 2 \sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2 \sin^{-1} \frac{\sqrt{3}}{2} \right] \\ &= \frac{1}{\sqrt{3}} \times \frac{3}{2} + 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{3} = \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq. units} \end{aligned}$$

4. Find the area of the following region using integration:

$$\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$$

[CBSE 2023 (65/5/1)]

Sol. We are given region

$$\begin{aligned} \{(x, y) | y^2 \leq 2x \text{ and } y \geq x - 4\} \\ = \{(x, y) | y^2 \leq 2x\} \cap \{(x, y) | y \geq x - 4\} \end{aligned}$$

$$\text{Given curves are } y^2 = 2x \quad \dots(i)$$

$$\text{and } x - y = 4 \quad \dots(ii)$$

Obviously, curve (i) is right handed parabola having vertex at (0, 0) and axis along +ve direction of x-axis while curve (ii) is a straight line.

For intersection point of curve (i) and (ii), we get

$$(x - 4)^2 = 2x$$

$$\Rightarrow x^2 - 8x + 16 = 2x$$

$$\Rightarrow x^2 - 8x - 2x + 16 = 0 \Rightarrow x(x - 8) - 2(x - 8) = 0$$

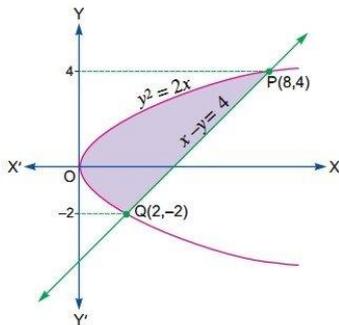
$$\Rightarrow (x - 8)(x - 2) = 0 \Rightarrow x = 2, 8$$

$$\Rightarrow y = -2, 4$$

Intersection points are (2, -2), (8, 4).

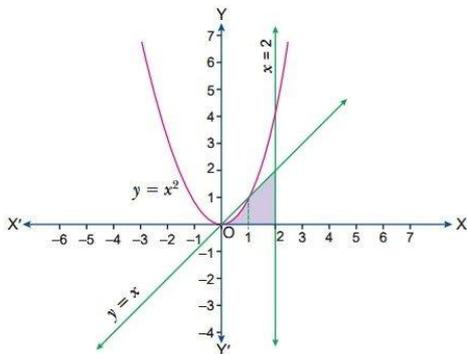
Therefore, required area = area of shaded region

$$\begin{aligned} &= \int_{-2}^4 (y + 4) dy - \int_{-2}^4 \frac{y^2}{2} dy = \left[ \frac{(y + 4)^2}{2} \right]_{-2}^4 - \frac{1}{2} \left[ \frac{y^3}{3} \right]_{-2}^4 \\ &= \frac{1}{2} [64 - 4] - \frac{1}{6} [64 + 8] = 30 - \frac{72}{6} = 18 \text{ sq units.} \end{aligned}$$



5. Make a rough sketch of the region  $\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$  and find the area of the region using integration.

Sol.



The points of intersection of the parabola  $y = x^2$  and the line  $y = x$  are (0, 0) and (1, 1).

$$\text{Required area} = \int_0^1 y_{\text{parabola}} dx + \int_1^2 y_{\text{line}} dx$$

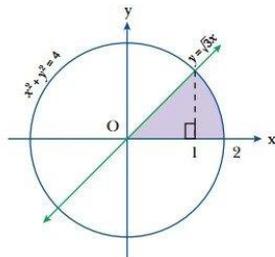
$$= \int_0^1 x^2 dx + \int_1^2 x dx = \left[ \frac{x^3}{3} \right]_0^1 + \left[ \frac{x^2}{2} \right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6} \text{ sq units}$$

6. Find the area of the region bounded by the curves  $x^2 + y^2 = 4$ ,  $y = \sqrt{3}x$  and  $x$ -axis in the first quadrant. [CBSE Sample Paper 2021]

Sol. Solving  $y = \sqrt{3}x$  and  $x^2 + y^2 = 4$ , we get  $x^2 + 3x^2 = 4$

$$\Rightarrow x^2 = 1 \Rightarrow x = 1$$

$$\begin{aligned} \text{Required area} &= \sqrt{3} \int_0^1 x \, dx + \int_1^2 \sqrt{2^2 - x^2} \, dx \\ &= \frac{\sqrt{3}}{2} [x^2]_0^1 + \left[ \frac{x}{2} \sqrt{2^2 - x^2} + 2 \sin^{-1} \left( \frac{x}{2} \right) \right]_1^2 \\ &= \frac{\sqrt{3}}{2} + \left[ 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6} \right] \\ &= \frac{2\pi}{3} \text{ square units.} \end{aligned}$$



7. Using integration, find the area of the region  $\{(x, y) : x^2 + y^2 \leq 9, x + y \geq 3\}$ . [CBSE 2020 (65/1/2)]

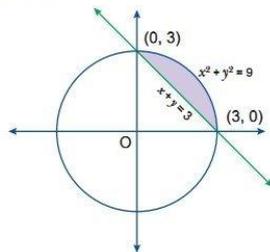
Sol. Given region  $\{(x, y) : x^2 + y^2 \leq 9, x + y \geq 3\}$

We have equation of the curve  $x^2 + y^2 = 9$  which is a circle with centre  $(0, 0)$  and radius 3 units and  $x + y = 3$ , a straight line.

On plotting we have the required region as the shaded region.

$\therefore$  Area of the shaded (required) region

$$\begin{aligned} &= \int_0^3 [\sqrt{9 - x^2} - (3 - x)] \, dx \\ &= \int_0^3 \sqrt{9 - x^2} \, dx - \int_0^3 (3 - x) \, dx \\ &= \int_0^3 \sqrt{3^2 - x^2} \, dx - \left[ 3x - \frac{x^2}{2} \right]_0^3 = \left[ \frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_0^3 - \left[ 3 \times 3 - \frac{9}{2} - 0 \right] \\ &= \left( 0 + \frac{9}{2} \sin^{-1}(1) - 0 \right) - \left( 9 - \frac{9}{2} \right) = \frac{9\pi}{4} - \frac{9}{2} = \frac{9}{4}(\pi - 2) \text{ sq. units.} \end{aligned}$$



8. Using integration, find the area of the region bounded by the triangle whose vertices are  $(2, -2)$ ,  $(4, 5)$  and  $(6, 2)$ . [CBSE 2020 (65/1/1)]

Sol. Let given vertices of the triangle be  $A(2, -2)$ ,  $B(4, 5)$  and  $C(6, 2)$ .

On plotting these points, we get the  $\triangle ABC$ .

Now, equation of line  $AB$  be

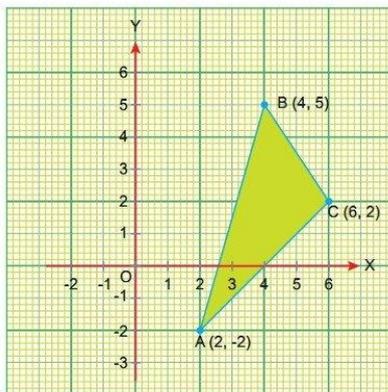
$$\begin{aligned} \frac{y - 5}{x - 4} &= \frac{7}{2} \\ \Rightarrow y - 5 &= \frac{7}{2}(x - 4) = \frac{7}{2}x - 14 \end{aligned}$$

$$\Rightarrow y = \frac{7}{2}x - 9$$

$$\therefore y_{AB} = \frac{7}{2}x - 9$$

Equation of  $BC$  be

$$\begin{aligned} \frac{y - 5}{x - 4} &= \frac{3}{-2} \\ \Rightarrow y - 5 &= -\frac{3}{2}(x - 4) = -\frac{3}{2}x + 6 \end{aligned}$$



$$\Rightarrow y_{BC} = -\frac{3}{2}x + 11$$

Also, equation of AC be

$$\frac{y+2}{x-2} = \frac{4}{4} = 1 \Rightarrow y+2 = x-2 \Rightarrow y = x-4$$

$$\therefore y_{AC} = x-4$$

$\therefore$  Area of required triangle (shaded region  $\triangle ABC$ )

$$\begin{aligned} &= \int_2^4 y_{AB} dx + \int_4^6 y_{BC} dx - \int_2^6 y_{AC} dx \\ &= \int_2^4 \left(\frac{7}{2}x - 9\right) dx + \int_4^6 \left(-\frac{3}{2}x + 11\right) dx - \int_2^6 (x-4) dx \\ &= \left[\frac{7}{4}x^2 - 9x\right]_2^4 + \left[-\frac{3}{4}x^2 + 11x\right]_4^6 - \left[\frac{x^2}{2} - 4x\right]_2^6 \\ &= \frac{7}{4} \times (4^2 - 2^2) - 9(4-2) - \frac{3}{4} \times (6^2 - 4^2) + 11 \times (6-4) - \frac{(6^2 - 2^2)}{2} + 4(6-2) \\ &= \frac{7}{4} \times 12 - 18 - \frac{3}{4} \times 20 + 22 - 16 + 16 \\ &= 21 - 18 - 15 + 22 = 43 - 33 = 10 \text{ sq. units.} \end{aligned}$$

9. Using the method of integration, find the area of the region bounded by the lines  $3x - 2y + 1 = 0$ ,  $2x + 3y - 21 = 0$  and  $x - 5y + 9 = 0$ .

[CBSE Delhi 2012, 2019 (65/4/1)]

Sol. Given lines are

$$3x - 2y + 1 = 0 \quad \dots(i)$$

$$2x + 3y - 21 = 0 \quad \dots(ii)$$

$$x - 5y + 9 = 0 \quad \dots(iii)$$

For intersection of (i) and (ii) applying (i)  $\times 3$  + (ii)  $\times 2$ , we get

$$9x - 6y + 3 + 4x + 6y - 42 = 0$$

$$\Rightarrow 13x - 39 = 0 \Rightarrow x = 3$$

Putting it in (i), we get

$$9 - 2y + 1 = 0$$

$$\Rightarrow 2y = 10 \Rightarrow y = 5$$

Intersection point of (i) and (ii) is (3, 5).

For intersection of (ii) and (iii) applying (ii) - (iii)  $\times 2$ , we get

$$2x + 3y - 21 - 2x + 10y - 18 = 0$$

$$\Rightarrow 13y - 39 = 0 \Rightarrow y = 3$$

Putting  $y = 3$  in (ii), we get

$$2x + 9 - 21 = 0$$

$$\Rightarrow 2x - 12 = 0 \Rightarrow x = 6$$

Intersection point of (ii) and (iii) is (6, 3).

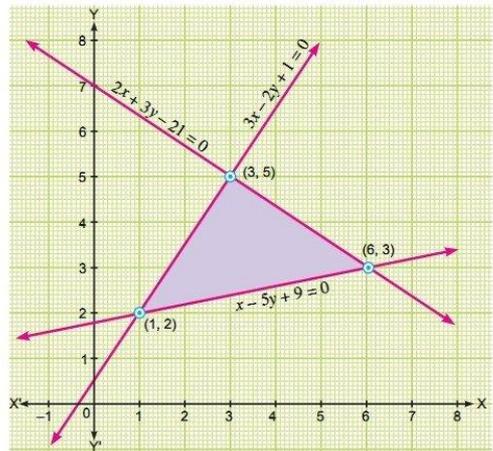
For intersection of (i) and (iii) applying (i) - (iii)  $\times 3$ , we get

$$3x - 2y + 1 - 3x + 15y - 27 = 0$$

$$\Rightarrow 13y - 26 = 0 \Rightarrow y = 2$$

Putting  $y = 2$  in (i), we get

$$3x - 4 + 1 = 0 \Rightarrow x = 1$$



Intersection point of (i) and (iii) is (1, 2).

With the help of point of intersection we draw the graph of lines (i), (ii) and (iii).

Shaded region is required region.

$$\begin{aligned} \therefore \text{Area of required region} &= \int_1^3 \frac{3x+1}{2} dx + \int_3^6 \frac{-2x+21}{3} dx - \int_1^6 \frac{x+9}{5} dx \\ &= \frac{3}{2} \int_1^3 x dx + \frac{1}{2} \int_1^3 dx - \frac{2}{3} \int_3^6 x dx + 7 \int_3^6 dx - \frac{1}{5} \int_1^6 x dx - \frac{9}{5} \int_1^6 dx \\ &= \frac{3}{2} \left[ \frac{x^2}{2} \right]_1^3 + \frac{1}{2} [x]_1^3 - \frac{2}{3} \left[ \frac{x^2}{2} \right]_3^6 + 7[x]_3^6 - \frac{1}{5} \left[ \frac{x^2}{2} \right]_1^6 - \frac{9}{5} [x]_1^6 \\ &= \frac{3}{4}(9-1) + \frac{1}{2}(3-1) - \frac{2}{6}(36-9) + 7(6-3) - \frac{1}{10}(36-1) - \frac{9}{5}(6-1) \\ &= 6 + 1 - 9 + 21 - \frac{7}{2} - 9 = 10 - \frac{7}{2} = \frac{20-7}{2} = \frac{13}{2} \text{ sq units.} \end{aligned}$$

10. Find the area of the region  $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$ . [CBSE (AI) 2012]

Sol. Let  $R = \{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$

$$\Rightarrow R = \{(x, y) : x^2 + y^2 \leq 4\} \cap \{(x, y) : x + y \geq 2\}$$

i.e.,  $R = R_1 \cap R_2$ , where  $R_1 = \{(x, y) : x^2 + y^2 \leq 4\}$  and

$R_2 = \{(x, y) : x + y \geq 2\}$

For region  $R_1$

Obviously  $x^2 + y^2 = 4$  is a circle having centre at (0,0) and radius 2.

Since (0,0) satisfy  $x^2 + y^2 \leq 4$ . Therefore region  $R_1$  is the region lying interior of circle  $x^2 + y^2 = 4$ .

For region  $R_2$

$x$	0	2
$y$	2	0

$x + y = 2$  is a straight line passing through (0, 2) and (2, 0). Since (0, 0) does not satisfy  $x + y \geq 2$  therefore  $R_2$  is that region which does not contain origin (0, 0) i.e., above the line  $x + y = 2$ .

Hence, shaded region is required area.

Now, area of required region

$$\begin{aligned} &= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx = \left[ \frac{1}{2} x \sqrt{4-x^2} + \frac{1}{2} 4 \sin^{-1} \left( \frac{x}{2} \right) \right]_0^2 - 2[x]_0^2 + \left[ \frac{x^2}{2} \right]_0^2 \\ &= [2 \sin^{-1} 1 - 0] - 2[2-0] + \left[ \frac{4}{2} - 0 \right] \\ &= 2 \times \frac{\pi}{2} - 4 + 2 = (\pi - 2) \text{ sq units.} \end{aligned}$$

11. Using integration, find the area of the triangle ABC, co-ordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4). [CBSE (AI) 2010, 2017]

Sol. Given triangle ABC, coordinates of whose vertices are

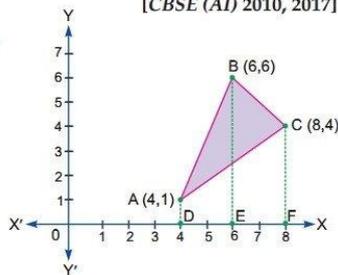
A(4, 1), B(6, 6) and C(8, 4).

Equation of AB is given by

$$y - 6 = \frac{6-1}{6-4}(x-6) \text{ or } y = \frac{5}{2}x - 9$$

Equation of BC is given by

$$y - 4 = \frac{4-6}{8-6}(x-8) \text{ or } y = -x + 12$$



Equation of AC is given by

$$y - 4 = \frac{4-1}{8-4}(x-8) \text{ or } y = \frac{3}{4}x - 2$$

$\therefore$  Area of  $\triangle ABC$  = area of trap. DABE + area of trap. EBCF - area of trap. DACF

$$\begin{aligned} &= \int_4^6 \left( \frac{5}{2}x - 9 \right) dx + \int_6^8 (-x + 12) dx - \int_4^8 \left( \frac{3}{4}x - 2 \right) dx \\ &= \frac{5}{2} \left[ \frac{x^2}{2} \right]_4^6 - 9[x]_4^6 - \left[ \frac{x^2}{2} \right]_6^8 + 12[x]_6^8 - \frac{3}{4} \left[ \frac{x^2}{2} \right]_4^8 + 2[x]_4^8 \\ &= \frac{5}{4}(36 - 16) - 9(6 - 4) - \frac{1}{2}(64 - 36) + 12(8 - 6) - \frac{3}{8}(64 - 16) + 8 \\ &= \frac{5}{4} \times 20 - 18 - \frac{28}{2} + 24 - \frac{3}{8} \times 48 + 8 \\ &= 25 - 18 - 14 + 24 - 18 + 8 = 7 \text{ sq units.} \end{aligned}$$

12. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

[CBSE 2019 (65/1/1)]

Sol.

Q22

Eq<sup>n</sup> of general line:  
 $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Eq<sup>n</sup> of line AB:  $(y - 5) = \frac{2}{2}(x - 2)$   
 $y = x + 3$

Eq<sup>n</sup> of line BC:  $(y - 7) = \frac{-5}{2}(x - 4)$   
 $y = -\frac{5}{2}x + 17$

Eq<sup>n</sup> of line AC:  $(y - 5) = \frac{-3}{4}(x - 2)$   
 $y = -\frac{3x}{4} + \frac{13}{2}$

Area of shaded region is required area  $A = \text{ar}(DABE) + \text{ar}(EBCF) - \text{ar}(ACF)$

$$A = \int_2^4 y_1 dx + \int_4^6 y_2 dx - \int_2^6 y_3 dx$$

$$= \int_2^4 (x+3) dx + \int_4^6 \left( -\frac{5x}{2} + 17 \right) dx - \int_2^6 \left( -\frac{3x}{4} + \frac{13}{2} \right) dx$$

$$A = \left[ \frac{x^2}{2} + 3x \right]_2^4 + \left[ -\frac{5x^2}{4} + 17x \right]_4^6 - \left[ \frac{3x^2}{8} - \frac{13x}{2} \right]_2^6$$

$$= \left[ 8 + 12 - 2 - 6 \right] + \left[ 102 - 45 - 68 + 20 \right] - \left[ 27 - \frac{27}{2} - 13 + \frac{13}{2} \right]$$

$$= 12 + 9 - 14$$

$A = 7 \text{ sq. units}$

[Topper's Answer 2019]

13. Find the area of the region in the first quadrant enclosed by the x-axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .  
[CBSE Delhi 2014, 2018, CBSE 2020 (65/4/1)]

Sol.

Circle,  
 $x^2 + y^2 = 32$  — (i)  
 centre  $(0,0)$   
 radius  $= 4\sqrt{2}$  cm.

line,  $y = x$  — (ii)

On solving (i) and (ii) eq:  
 $x^2 + x^2 = 32$   
 $2x^2 = 32$   
 $x^2 = 16$   
 $x = \pm 4$   
 $y = \pm 4$

Required Area :-  
 $\int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2} \, dx$

$= \left(\frac{x^2}{2}\right)_0^4 + \left(\frac{x}{2}\sqrt{32-x^2} + \frac{32}{2} \sin^{-1} \frac{x}{4\sqrt{2}}\right)_4^{4\sqrt{2}}$

$= \left(\frac{x^2}{2}\right)_0^4 + \left(\frac{x}{2}\sqrt{32-x^2} + \frac{16}{4\sqrt{2}} \sin^{-1} \frac{x}{4\sqrt{2}}\right)_4^{4\sqrt{2}}$

$= \left(\frac{16-0}{2}\right) + \left(\frac{4\sqrt{2}}{2}\sqrt{32-32} + 16 \frac{\sin^{-1} 4\sqrt{2}}{4\sqrt{2}} - \left[\frac{4}{2}\sqrt{32-16} + 16 \frac{\sin^{-1} 4}{4\sqrt{2}}\right]\right)$

$= (8-0) + (0 + 16 \sin^{-1}(1) - 2\sqrt{16} - 16 \frac{\sin^{-1} 1}{\sqrt{2}})$

$= 8 + (16 \times \frac{\pi}{2} - 2 \times 4 - 16 \times \frac{\pi}{4})$

$= 8 + 8\pi - 8 - 4\pi$

$= 4\pi$  units  $\text{cm}^2$

[Toppers Answer 2018]

14. Find the area of the region included between the parabola  $4y = 3x^2$  and the line  $3x - 2y + 12 = 0$ .  
[CBSE (AI) 2009, (F) 2013]

Sol. Given equation of parabola  $4y = 3x^2 \Rightarrow y = \frac{3x^2}{4}$  ... (i)

and the line  $3x - 2y + 12 = 0 \Rightarrow \frac{3x+12}{2} = y$  ... (ii)

The line intersect the parabola at  $(-2, 3)$  and  $(4, 12)$ .

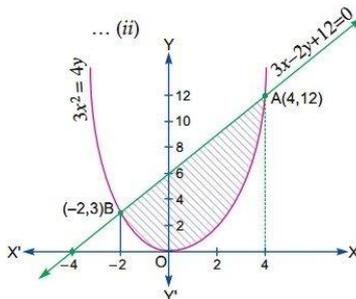
Hence, the required area will be the shaded region.

Required area =  $\int_{-2}^4 \frac{3x+12}{2} \, dx - \int_{-2}^4 \frac{3x^2}{4} \, dx$

$= \left[\frac{3}{4}x^2 + 6x - \frac{x^3}{4}\right]_{-2}^4$

$= (12 + 24 - 16) - (3 - 12 + 2)$

$= 20 + 7 = 27$  square units.



15. Using integration, find the area bounded by the tangent to the curve  $4y = x^2$  at the point (2, 1) and the lines whose equations are  $x = 2y$  and  $x = 3y - 3$ . [CBSE Sample Paper 2016]

Sol. Obviously  $4y = x^2$  is upward parabola having vertex at origin.

$$\text{Now } 4y = x^2$$

$$\Rightarrow 4 \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{1}{2}x \Rightarrow \left. \frac{dy}{dx} \right|_{(2,1)} = \frac{1}{2} \times 2 = 1$$

$\Rightarrow$  Slope of tangent at (2, 1) to given curve  $4y = x^2$  is 1.

$$\text{Equation of tangent} = \frac{y-1}{x-2} = 1$$

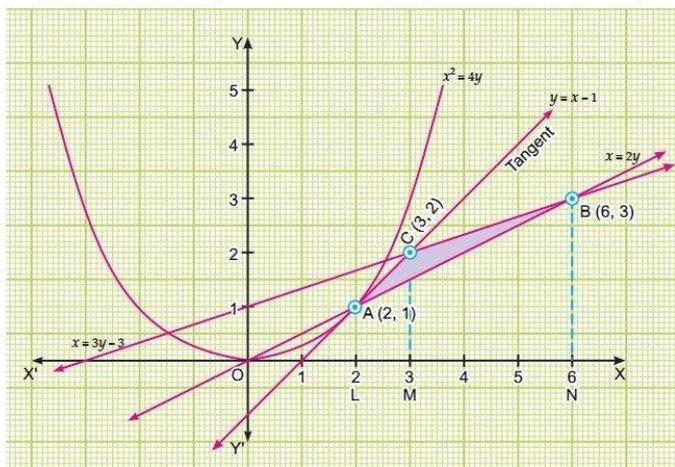
$$\Rightarrow y - 1 = x - 2 \quad \Rightarrow y = x - 1$$

Now, for graph of  $x = 2y$

x	0	2
y	0	1

Also for graph of  $x = 3y - 3$

x	0	3
y	1	2



After plotting the graph, we get shaded region ABC as required region, area of which is to be calculated.

After solving the respective equation, we get

Coordinate of  $A \equiv (2, 1)$ ;  $B \equiv (6, 3)$ ;  $C \equiv (3, 2)$

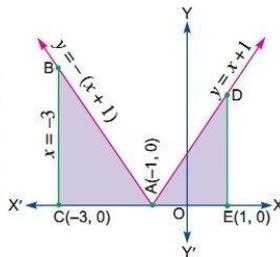
Now, the required area = area of shaded region ABC

$$\begin{aligned} &= ar(\text{region ALMC}) + ar(\text{region CMNB}) - ar(\text{region ALNB}) \\ &= \int_2^3 (x-1) dx + \int_3^6 \frac{x+3}{3} dx - \int_2^6 \frac{x}{2} dx = \left[ \frac{x^2}{2} - x \right]_2^3 + \frac{1}{3} \left[ \frac{x^2}{2} + 3x \right]_3^6 - \frac{1}{2} \left[ \frac{x^2}{2} \right]_2^6 \\ &= \left[ \left( \frac{9}{2} - 3 \right) - \left( \frac{4}{2} - 2 \right) \right] + \frac{1}{3} \left[ \left( \frac{36}{2} + 18 \right) - \left( \frac{9}{2} + 9 \right) \right] - \frac{1}{4} (36 - 4) \\ &= \frac{3}{2} + \frac{1}{3} \left( 36 - \frac{27}{2} \right) - 8 = \frac{3}{2} + \frac{45}{6} - 8 = 1 \text{ square unit.} \end{aligned}$$

16. Sketch the graph  $y = |x + 1|$ . Evaluate  $\int_{-3}^1 |x + 1| dx$ . What does this value represent on the graph?

Sol. We have,  $y = |x + 1| = \begin{cases} x + 1, & \text{if } x + 1 \geq 0 \text{ i.e., } x \geq -1 \\ -(x + 1), & \text{if } x + 1 < 0 \text{ i.e., } x < -1 \end{cases}$

So, we have  $y = x + 1$  for  $x \geq -1$  and  $y = -x - 1$  for  $x < -1$ . Clearly,  $y = x + 1$  is a straight line cutting  $x$  and  $y$ -axes at  $(-1, 0)$  and  $(0, 1)$  respectively. So,  $y = x + 1, x \geq -1$  represents that portion of the line which lies on the right side of  $x = -1$ . Similarly,  $y = -x - 1, x < -1$  represents that part of the line  $y = -x - 1$  which is on the left side of  $x = -1$ . A rough sketch of  $y = |x + 1|$  is shown in fig.



Now,  $\int_{-3}^1 |x + 1| dx = \int_{-3}^{-1} -(x + 1) dx + \int_{-1}^1 (x + 1) dx$

$$= -\left[\frac{(x + 1)^2}{2}\right]_{-3}^{-1} + \left[\frac{(x + 1)^2}{2}\right]_{-1}^1 = -\left[0 - \frac{4}{2}\right] + \left[\frac{4}{2} - 0\right] = 4 \text{ sq units}$$

This value represents the area of the shaded portion shown in figure.

17. Using integration find the area of the triangular region whose sides have equations  $y = 2x + 1$ ,  $y = 3x + 1$  and  $x = 4$ . [CBSE Delhi 2008; 2011]

Sol. The given lines are

$$y = 2x + 1 \quad \dots(i)$$

$$y = 3x + 1 \quad \dots(ii)$$

$$x = 4 \quad \dots(iii)$$

For intersection point of (i) and (iii)

$$y = 2 \times 4 + 1 = 9$$

Coordinates of intersecting point of (i) and (iii) is  $(4, 9)$ .

For intersection point of (ii) and (iii)

$$y = 3 \times 4 + 1 = 13$$

i.e., coordinates of intersection point of (ii) and (iii) is  $(4, 13)$ .

For intersection point of (i) and (ii)

$$2x + 1 = 3x + 1 \quad \Rightarrow \quad x = 0$$

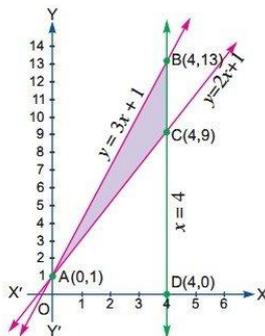
$\therefore y = 1$

i.e., coordinates of intersection point of (i) and (ii) is  $(0, 1)$ .

Shaded region is required triangular region.

$\therefore$  Required area = area of trapezium  $OABDO$  - area of trapezium  $OACDO$

$$\begin{aligned} &= \int_0^4 (3x + 1) dx - \int_0^4 (2x + 1) dx = \left[3\frac{x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4 \\ &= [(24 + 4) - 0] - [(16 + 4) - 0] = 28 - 20 = 8 \text{ sq units.} \end{aligned}$$



## Questions for Practice

### ■ Objective Type Questions

1. Choose and write the correct option in each of the following questions.

(i) The area bounded by the curve  $y = x|x|$ ,  $x$ -axis and the ordinates  $x = -1$  and  $x = 1$  is given by

- (a) 0 sq. units      (b)  $\frac{1}{3}$  sq. unit      (c)  $\frac{2}{3}$  sq. unit      (d)  $\frac{4}{3}$  sq. units

(ii) The area bounded by the curve  $y = |\sin x|$ ,  $x$ -axis and ordinates  $x = \pi$  and  $x = 10\pi$  is equal to

- (a) 8 sq. units      (b) 10 sq. units      (c) 18 sq. units      (d) 20 sq. units

(iii) The area of the region bounded by the parabola  $y^2 = x$  and the straight line  $2y = x$  is

- (a)  $\frac{4}{3}$  sq. units      (b) 1 sq. unit      (c)  $\frac{2}{3}$  sq. unit      (d)  $\frac{1}{3}$  sq. unit

(iv) The area of the region enclosed by the parabola  $x^2 = y$  and the line  $y = x + 2$  is

- (a)  $\frac{9}{2}$  sq. units      (b) 4 sq. units      (c) 2 sq. units      (d) None of these

(v) Area lying in the first quadrant and bounded by the circle  $x^2 + y^2 = 4$  and the lines  $x = 0$  and  $x = 2$  is

- (a)  $\pi$  sq. units      (b)  $\frac{\pi}{2}$  sq. units      (c)  $\frac{\pi}{3}$  sq. units      (d)  $\frac{\pi}{4}$  sq. units

### ■ Very Short Answer Questions

- Find the area bounded by the curve  $y = x^2$ ,  $x = 2$ ,  $x = 3$  and  $x$ -axis.
- Calculate the area under the curve  $y = 2\sqrt{x}$  included between the lines  $x = 0$  and  $x = 1$ .
- Find the area under the curve  $y = \sqrt{x-1}$  between the lines  $x = 1$  and  $x = 5$ .

### ■ Long Answer Questions

- Find the area bounded by the lines  $y = 4x + 5$ ,  $y = 5 - x$  and  $4y = x + 5$ . [NCERT Exemplar]
- Find the area bounded by the curve  $x^2 = 4y$  and the straight line  $x = 4y - 2$ . [CBSE Delhi 2010, 2013]
- Using integration, find the area of the region  $\{(x, y) : 9x^2 + y^2 \leq 36 \text{ and } 3x + y \geq 6\}$ . [CBSE (F) 2009]
- Find the area of the region  $\{(x, y) : x^2 \leq y \leq x\}$ .
- Find the area of the region bounded by the curve  $y = \frac{3}{4}x^2$  and the line  $3x - 2y + 12 = 0$ .
- Using integration, find the area of the triangle  $ABC$ , where  $A$  is  $(2, 3)$ ,  $B$  is  $(4, 7)$  and  $C$  is  $(6, 2)$ .
- Make a rough sketch of the region given below and find its area, using integration:  
 $\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3, 0 < x \leq 3\}$
- Using integration, find the area of the triangle  $ABC$ , whose vertices have coordinates  
 $A(2, 0)$ ,  $B(4, 5)$  and  $C(6, 3)$ . [CBSE (F) 2012]

13. Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $\frac{x}{3} + \frac{y}{2} = 1$ .  
[CBSE Delhi 2010; (F) 2014]
14. Using integration, find the area of the triangle formed by negative  $x$ -axis and tangent and normal to the circle  $x^2 + y^2 = 9$  at  $(-1, 2\sqrt{2})$ .  
[CBSE East 2016]
15. If the area bounded by the parabola  $y^2 = 16ax$  and the line  $y = 4mx$  is  $\frac{a^2}{12}$  sq units, then using integration, find the value of  $m$ .  
[CBSE Ajmer 2015]
16. Using the method of integration, find the area of the triangular region whose vertices are  $(2, -2)$ ,  $(4, 3)$  and  $(1, 2)$ .  
[CBSE (North) 2016]
17. Find the area of the triangle whose vertices are  $(-1, 1)$ ,  $(0, 5)$  and  $(3, 2)$ , using integration.  
[CBSE 2019 (65/3/1)]
18. Using integration, find the area of the triangle whose vertices are  $(2, 3)$ ,  $(3, 5)$  and  $(4, 4)$ .  
[CBSE 2019 (65/1/3)]
19. Using integration, find the area of the triangle  $ABC$  with vertices as  $A(-1, 0)$ ,  $B(1, 3)$  and  $C(3, 2)$ .  
[CBSE (F) 2009]
20. Using integration, find the area of the triangle  $ABC$ , where  $A$  is  $(2, 3)$ ,  $B$  is  $(4, 7)$  and  $C$  is  $(6, 2)$ .

## Answers

- |                             |                              |                              |   |         |
|-----------------------------|------------------------------|------------------------------|---|---------|
| 1. (i) (c)                  | (ii) (c)                     | (iii) (a)                    | (iv) (a)  | (v) (a) |
| 2. $\frac{19}{3}$ sq. units | 3. $\frac{4}{3}$ sq. units   | 4. $\frac{16}{3}$ sq. units  | 5. $\frac{15}{2}$ sq. units                     |         |
| 6. $\frac{9}{8}$ sq. units  | 7. $3(\pi - 2)$ sq. units    | 8. $\frac{1}{6}$ sq. unit    | 9. 27 sq. units                                 |         |
| 10. 9 sq. units             | 11. $\frac{50}{3}$ sq. units | 12. 7 sq. units              | 13. $\left(\frac{3\pi}{2} - 3\right)$ sq. units |         |
| 14. $9\sqrt{2}$             | 15. $m = 2$                  | 16. $\frac{13}{2}$ sq. units | 17. $\frac{15}{2}$ sq. units                    |         |
| 18. $\frac{3}{2}$ sq. units | 19. 4 sq. units              | 20. 9 sq. units              |   |         |

