# Solved Paper 2006

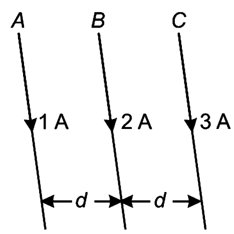
# AMU

## Engineering Entrance Exam

## Physics

1. The difference between the apparent frequency of a source of sound as perceived by the observer during its approach and recession is 2% of the frequency of the source. If the speed of sound in air is 300 $ms^{-1}$, the velocity of the source is
(a) 1.5 $ms^{-1}$ (b) 12 $ms^{-1}$
(c) 6 $ms^{-1}$ (d) 3 $ms^{-1}$

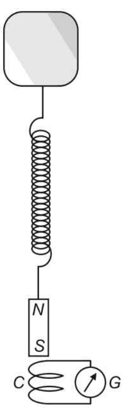
2. Three long straight wires *A*, *B* and *C* are carrying currents as shown in figure. Then, the resultant force on *B* is directed




(a) perpendicular to the plane of paper and outward
(b) perpendicular to the plane of paper and inward
(c) towards *A*
(d) towards *C*

3. Curie-Weiss law is obeyed by iron
(a) at Curie temperature only
(b) at all temperatures
(c) below Curie temperature
(d) above Curie temperature

4. A magnet *N-S* is suspended from a spring and when it oscillates, the magnet moves in and out of the coil *C*. The coil is connected to a galvanometer *G*. Then, as the magnet oscillates

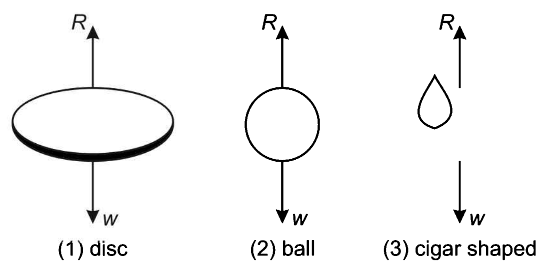



(a) *G* shows no deflection
(b) *G* shows deflection to the left and right but the amplitude steadily decreases
(c) *G* shows deflection to the left and right with constant amplitude
(d) *G* shows deflection on one side

5. The dimensional formula for inductance is
(a) $[ML^2T^{-2}A^{-2}]$ (b) $[ML^2TA^{-2}]$
(c) $[ML^2T^{-1}A^{-2}]$ (d) $[ML^2T^{-2}A^{-1}]$

6. The maximum current that can be measured by a galvanometer of resistance 40 Ω is 10 mA. It is converted into a voltmeter that can read upto 50 V. The resistance to be connected in series with the galvanometer (in ohm) is
(a) 2010 (b) 4050
(c) 5040 (d) 4960

7. When a body falls in air, the resistance of air depends to a great extent on the shape of the body. 3 different shapes are given. Identify the combination of air resistances which truly represents the physical situation?
(The cross-sectional areas are the same).




(a) $1 < 2 < 3$ (b) $2 < 3 < 1$
(c) $3 < 2 < 1$ (d) $3 < 1 < 2$

8. Heavy water is
(a) compound of deuterium and oxygen
(b) water at 4°C
(c) water, in which soap does not lather
(d) compound of heavy oxygen and heavy hydrogen

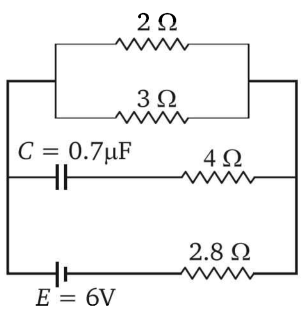
9. A bullet moving with a speed of 100 $ms^{-1}$ can just penetrate two planks of equal thickness. Then, the number of such planks penetrated by the same bullet when the speed is doubled will be
(a) 6 (b) 10
(c) 4 (d) 8

**10.** Absorption coefficient of an open window is
(a) 1 (b) 0.25
(c) zero (d) 0.5

**11.** In Melde's experiment in the transverse mode, the frequency of the tuning fork and the frequency of the waves in the string are in the ratio
(a) 2 : 1 (b) 4 : 1
(c) 1 : 1 (d) 1 : 2

**12.** An unknown resistance $R_1$ is connected in series with a resistance of 10 Ω. This combination is connected to one gap of meter bridge while a resistance $R_2$ is connected in the other gap. The balance point is at 50 cm. Now, when the 10 Ω resistance is removed the balance point shifts to 40 cm. The value of $R_1$ (in ohm) is
(a) 20 (b) 10
(c) 60 (d) 40

**13.** In the circuit shown, the internal resistance of the cell is negligible. The steady state current in the 2 Ω resistor is

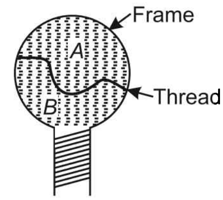



(a) 0.6 A (b) 1.2 A
(c) 0.9 A (d) 1.5 A

**14.** A rectangular coil of 300 turns has an average area of 25 cm × 10 cm. The coil rotates with a speed of 50 cps in uniform magnetic field of strength $4 \times 10^{-2}$ T about an axis perpendicular to the field. The peak value of the induced emf (in volt) is
(a) 300 π (b) 3000 π
(c) 3 π (d) 30 π

**15.** In a *L-C-R* circuit the potential difference between the terminals of their inductance is 60 V, between the terminals of the capacitor is 30 V and that between the terminals of resistance is 40 V. The supply voltage will be equal to
(a) 130 V (b) 10 V
(c) 50 V (d) 70 V

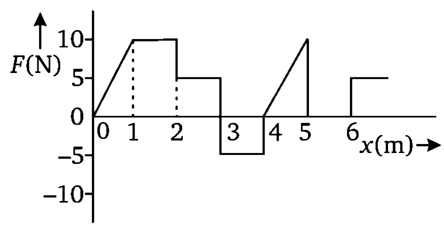
**16.** A thread is tied slightly loose to a wire frame as in figure and the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion *A* is punctured with a pin, the thread




(a) becomes concave towards *A*
(b) becomes convex towards *A*
(c) either (a) or (b) depending on the size of *A* with respect to *B*
(d) remains in the initial position

**17.** Oxygen is 16 times heavier than hydrogen. Equal volumes of hydrogen and oxygen are mixed. The ratio of speed of sound in the mixture to that in hydrogen is
(a) $\sqrt{8}$ (b) $\sqrt{\frac{2}{17}}$
(c) $\sqrt{\frac{1}{8}}$ (d) $\sqrt{\frac{32}{17}}$

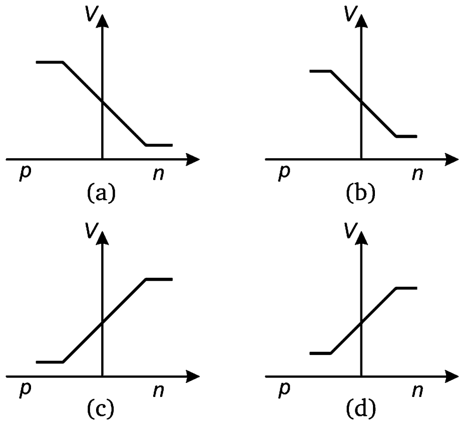
**18.** The relationship between the force *F* and position *x* of a body is as shown in figure. The work done in displacing the body from $x = 1$ m to $x = 5$ m will be



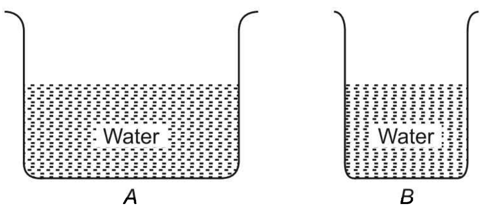

(a) 30 J (b) 15 J
(c) 25 J (d) 20 J

**19.** Specific rotation of sugar solution is 0.01 SI unit. 200 $kgm^{-3}$ of impure sugar solution is taken in a polarimeter tube of length 0.25 m and an optical rotation of 0.4 rad is observed. The percentage of purity of sugar in the sample is
(a) 11% (b) 20%
(c) 80% (d) 89%

**20.** In a forward biased *p-n* junction diode, the potential barrier in the depletion region is of the form




**21.** From the adjacent figure, the correct observation is




(a) the pressure on the bottom of tank *A* is greater than at the bottom of *B*
(b) the pressure on the bottom of tank *A* is smaller than at the bottom of *B*
(c) the pressure depends on the shape of the container
(d) the pressure on the bottom of *A* and *B* is the same

**22.** Two electric bulbs *A* and *B* are rated as 60 W and 100 W. They are connected in parallel to the same source. Then
(a) *B* draws more current than *A*
(b) currents drawn are in the ratio of their resistances
(c) both draw the same current
(d) *A* draws more current than *B*

**23.** A thin plano-convex lens acts like a concave mirror of focal length 0.2 m when silvered from its plane surface. The refractive index of the material of the lens is 1.5. The radius of curvature of the convex surface of the lens will be
(a) 0.1 m (b) 0.75 m
(c) 0.4 m (d) 0.2 m

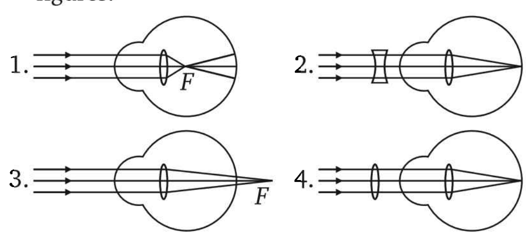
**24.** A balloon is rising vertically up with a velocity of 29 $ms^{-1}$. A stone is dropped from it and it reaches the ground in 10 s. The height of the balloon when the stone was dropped from it is ($g = 9.8\ ms^{-1}$)
(a) 400 m (b) 150 m
(c) 100 m (d) 200 m

**25.** In a Young's double slit experiment, the separation between the two slits is 0.9 mm and the fringes are observed 1 m away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of the monochromatic source of light used is
(a) 450 nm (b) 400 nm
(c) 500 nm (d) 600 nm

**26.** *H*-polaroid is prepared by
(a) orienting herapathite crystal in the same direction in nitrocellulose
(b) using thin tourmaline crystals
(c) stretching polyvinyl alcohol and then heated with dehydrating agent
(d) stretching polyvinyl alcohol and then impregnating with iodine

**27.** A spherical drop of capacitance 1 μF is broken into eight drops of equal radius. Then, the capacitance of each small drop is
(a) $\frac{1}{2}\mu F$ (b) $\frac{1}{4}\mu F$
(c) $\frac{1}{8}\mu F$ (d) $8\mu F$

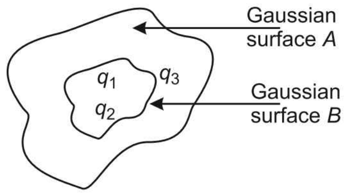
**28.** Identify the wrong description of the below figures.




(a) 1 represents far-sightedness
(b) 2 correction for short-sightedness
(c) 3 represents far-sightedness
(d) 4 correction for far-sightedness

**29.** Threshold wavelength for photoelectric emission from a metal surface is 5200 Å. Photoelectrons will emitted when this surface is illuminated with monochromatic radiation from
(a) 1W IR lamp (b) 50W UV lamp
(c) 50W IR lamp (d) 10W IR lamp

**30.** The electric flux for Gaussian surface $A$ that enclose the charged particles in free space is (Given $q_1 = -14$nC, $q_2 = 78.85$ nC, $q_3 = -56$ nC)




(a) $10^3 \text{Nm}^2 \text{C}^{-1}$

(b) $10^3 \text{CN}^{-1}\text{m}^{-2}$

(c) $6.32 \times 10^3 \text{ Nm}^2 \text{C}^{-1}$

(d) $6.32 \times 10^3 \text{ CN}^{-1}\text{m}^{-1}$

**31.** Which state of triply ionised beryllium ($Be^{3+}$) has the same orbital radius as that of the ground state of hydrogen ?

(a) $n = 3$ (b) $n = 4$

(c) $n = 1$ (d) $n = 2$

**32.** If $M$ is the atomic mass and $A$ is the mass number, packing fraction is given by

(a) $\frac{M}{M-A}$ (b) $\frac{M-A}{A}$

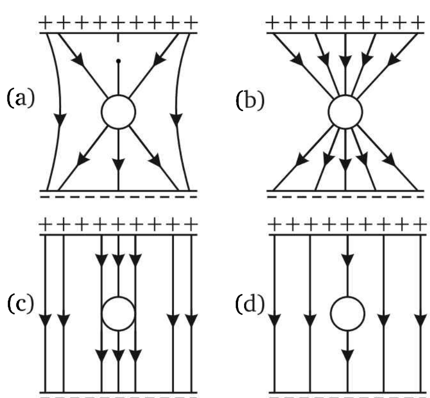
(c) $\frac{A}{M-A}$ (d) $\frac{A-M}{A}$

**33.** A count rate meter shows a count of 240/min from a given radioactive source after in the meter shows a count rate of 30/min. The half-life of the source is

(a) 80 min (b) 120 min

(c) 20 min (d) 30 min

**34.** An uncharged sphere of metal is placed inside a charged parallel plate capacitor. The lines of force will look like




**35.** A current flows in a conductor from east to west. The direction of the magnetic field at a point above the conductor is

(a) towards east

(b) towards west

(c) towards north

(d) towards south

**36.** Excitation energy of a hydrogen like in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is

(a) 40.8 eV (b) 27.2 eV

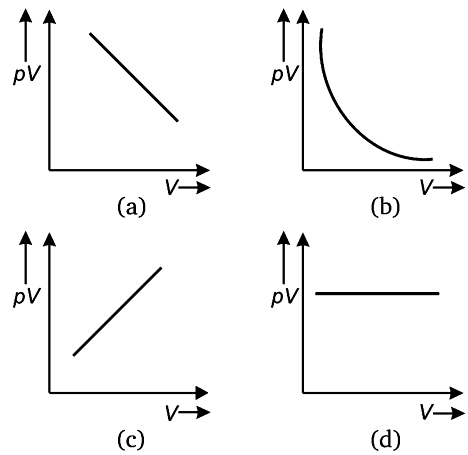
(c) 54.4 eV (d) 13.6 eV

**37.** An ideal gas heat engine operates in a Carnot's cycle between 227°C and 127°C. It absorbs $6 \times 10^4$J at high temperature. The amount of heat converted into work is

(a) $1.6 \times 10^4$J (b) $1.2 \times 10^4$J

(c) $4.8 \times 10^4$J (d) $3.5 \times 10^4$J

**38.** Which one of the following graphs represents the behaviour of an ideal gas ?




**39.** A beam of parallel rays is brought to focus by a plano-convex lens. A thin concave lens of the same focal length is joined to the first lens. The effect of this is

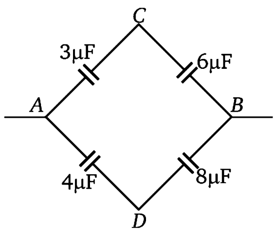
(a) the focus shifts to infinity

(b) the focal point shifts towards the lens by a small distance

(c) the focal point shifts away from the lens by a small distance

(d) the focus remains undisturbed

**40.** Effective capacitance between $A$ and $B$ in the figure, shown is




(a) $\frac{3}{14}\,\mu F$ (b) $\frac{14}{3}\,\mu F$
(c) $21\,\mu F$ (d) $23\,\mu F$

## Chemistry

**41.** Which one of the following is not an amphoteric substance?
(a) $HNO_3$ (b) $HCO_3^-$
(c) $H_2O$ (d) $NH_3$

**42.** When $50\ cm^3$ of 0.2 N $H_2SO_4$ is mixed with $50\ cm^3$ of 1 N KOH, the heat liberated is
(a) 11.46 kJ (b) 157.3 kJ
(c) 573 kJ (d) 573 J

**43.** An artificial radioactive isotope gave $^{14}_{7}N$ after two successive β-particle emissions. The number of neutrons in the parent nucleus must be
(a) 9 (b) 14
(c) 5 (d) 7

**44.** Stainless steel does not rust because
(a) chromium and nickel combine with iron
(b) chromium forms an oxide layer and protects iron from rusting
(c) nickel present in it, does not rust
(d) iron forms a hard chemical compound with chromium present in it

**45.** Which of the following combinations can be used to synthesise ethanol?
(a) $CH_3MgI$ and $CH_3COCH_3$
(b) $CH_3MgI$ and $C_2H_5OH$
(c) $CH_3MgI$ and $CH_3COOC_2H_5$
(d) $CH_3MgI$ and HCHO

**46.** A solution contains $1.2046 \times 10^{24}$ hydrochloric acid molecules in one $dm^3$ of the solution. The strength of the solution is
(a) 6 N (b) 2 N
(c) 4 N (d) 8 N

**47.** Nuclear theory of the atom was put forward by
(a) Rutherford (b) Aston
(c) Neils Bohr (d) J. J. Thomson

**48.** In acetylene molecule, the two carbon atoms are linked by
(a) one sigma-bond and two pi-bonds
(b) two sigma-bonds and one pi-bond
(c) three sigma-bonds
(d) three pi-bonds

**49.** The enthalpy of reaction,

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g)$ is $\Delta H_1$ and that of

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l)$ is $\Delta H_2$.

Then
(a) $\Delta H_1 < \Delta H_2$ (b) $\Delta H_1 + \Delta H_2 = 0$
(c) $\Delta H_1 > \Delta H_2$ (d) $\Delta H_1 = \Delta H_2$

**50.** A radioactive isotope decays at such a rate that after 192 min only 1/16 of the original amount remains left. Its half-life is
(a) 32 min (b) 48 min
(c) 12 min (d) 24 min

**51.** The pressure and temperature of $4\,dm^3$ of carbon dioxide gas are doubled, then the volume of carbon dioxide gas would be
(a) $2\,dm^3$ (b) $3\,dm^3$
(c) $4\,dm^3$ (d) $8\,dm^3$

**52.** 4 g of copper was dissolved in concentrated nitric acid. The copper nitrate solution on strong heating gave 5g of its oxide. The equivalent weight of copper is
(a) 23 (b) 32
(c) 12 (d) 20

**53.** In the manufacture of ammonia by Haber's process,

$N_2(g) + 3H_2 \rightleftharpoons 2NH_3(g) + 92.3\ kJ$

which of the following conditions is unfavourable?

(a) Increasing the temperature
(b) Increasing the pressure
(c) Reducing the temperature
(d) Removing ammonia as it is formed

**54.** The chemical equilibrium of a reversible reaction is not influenced by
(a) pressure
(b) catalyst
(c) concentration of the reactants
(d) temperature

**55.** Cumene process is the most important commercial method for the manufacture of phenol. Cumene is
(a) 1-methylethyl benzene
(b) ethyl benzene
(c) vinyl benzene
(d) propyl benzene

**56.** The reagent which does not give acid chloride on treating with a carboxylic acid, is
(a) $PCl_5$ (b) $Cl_2$
(c) $SOCl_2$ (d) $PCl_3$

**57.** Among the halogens, the one which is oxidised by nitric acid is
(a) fluorine (b) iodine
(c) chlorine (d) bromine

**58.** The metal which forms ammonium nitrate by reaction with dilute nitric acid is
(a) Al (b) Fe
(c) Pb (d) Mg

**59.** The elements with atomic numbers 9, 17, 35, 53, 85 are all
(a) noble gases (b) halogens
(c) heavy metals (d) light metals

**60.** In the electrolytic method of obtaining aluminium from purified bauxite, cryolite is added to the charge in order to
(a) minimise the heat loss due to radiation
(b) protect aluminium produced from oxygen
(c) dissolve bauxite and render it conductor of electricity
(d) lower the melting point of bauxite

**61.** The number of $2p$ electrons having spin quantum number $s = -1/2$ are
(a) 6 (b) 0
(c) 2 (d) 3

**62.** Pick out the alkane which differs from the other members of the group.
(a) 2,2-dimethyl propane
(b) pentane
(c) 2-methyl butane
(d) 2,2-dimethyl butane

**63.** 56 g of nitrogen and 8 g of hydrogen gas are heated in a closed vessel. At equilibrium, 34 g of ammonia are present. At equilibrium number of moles of nitrogen, hydrogen and ammonia are respectively
(a) 1, 2, 2 (b) 2, 2, 1
(c) 1, 1, 2 (d) 2, 1, 2

**64.** A process is taking place at constant temperature and pressure. Then
(a) $\Delta H = \Delta E = 0$ (b) $\Delta H = T\Delta S$
(c) $\Delta H = 0$ (d) $\Delta S = 0$

**65.** In a galvanic cell, the electrons flow from
(a) anode to cathode through the solution
(b) cathode to anode through the solution
(c) anode to cathode through the external circuit
(d) cathode to anode through the external circuit

**66.** The reaction, $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ is carried out in a 1 $dm^3$ vessel and 2 $dm^3$ vessel separately. The ratio of the reaction velocities will be
(a) 1 : 8 (b) 1 : 4
(c) 4 : 1 (d) 8 : 1

**67.** In a mixture of acetic acid and sodium acetate the ratio of concentration of the salt to the acid is increased ten times. Then, the pH of the solution
(a) increases by one
(b) decreases by one
(c) decreases ten fold
(d) increases ten fold

**68.** When a mixture of methane and oxygen is passed through heated molybdenum oxide, the main product formed is
(a) methanoic acid (b) ethanal
(c) methanol (d) methanal

**69.** Benzene can be obtained by heating either benzoic acid with $X$ or phenol with $Y$. $X$ and $Y$ are respectively
(a) zinc dust and soda lime
(b) soda lime and zinc dust
(c) zinc dust and sodium hydroxide
(d) soda lime and copper

**70.** An organic compound is boiled with alcoholic potash. The product is cooled and acidified with HCl. A white solid separates out. The starting compound may be

(a) ethyl benzoate (b) ethyl formate
(c) ethyl acetate (d) methyl acetate

**71.** A nitrogen containing organic compound gave an oily liquid on heating with bromine and potassium hydroxide solution. On shaking the product with acetic anhydride, an antipyretic drug was obtained. The reactions indicate that the starting compound is
(a) aniline
(b) *p*-hydroxy benzamide
(c) acetamide
(d) nitrobenzene

**72.** The silver salt of a fatty acid on refluxing with an alkyl halide gives an
(a) acid (b) ester
(c) ether (d) amine

**73.** Pick out the one which does not belong to the family
(a) pepsin (b) cellulose
(c) ptyalin (d) lipase

**74.** Which one of the following is wrongly matched?
(a) Saponification of $CH_3COOC_2H_5$ – second order reaction
(b) Hydrolysis of $CH_3COOCH_3$ – pseudo unimolecular reaction
(c) Decomposition of $H_2O_2$ – first order reaction
(d) Combination of $H_2$ and $Br_2$ to give HBr – first order reaction

**75.** The diameter of colloidal particles range from
(a) $10^{-6}$ m to $10^{-9}$ m
(b) $10^{-9}$ m to $10^{-12}$ m
(c) $10^{3}$ m to $10^{-3}$ m
(d) $10^{-3}$ m to $10^{-6}$ m

**76.** On treating a mixture of two alkyl halides with sodium metal in dry ether, 2- methyl propane was obtained. The alkyl halides are
(a) 2-chloropropane and chloromethane
(b) 2-chloropropane and chloroethane
(c) chloromethane and chloroethane
(d) chloromethane and 1-chloropropane

**77.** Which of the following statements about benzyl chloride is incorrect?
(a) It is less reactive than alkyl halides
(b) It can be oxidised to benzaldehyde by boiling with copper nitrate solution
(c) It is a lachrymatory liquid and answers Beilstein's test
(d) It gives a white precipitate with alcoholic silver nitrate

**78.** The main product obtained when a solution of sodium carbonate reacts with mercuric chloride is
(a) $Hg(OH)_2$ (b) $HgCO_3 \cdot HgO$
(c) $HgCO_3$ (d) $HgCO_3 \cdot Hg(OH)_2$

**79.** In the electrothermal process, the compound displaced by silica from calcium phosphate is
(a) calcium phosphide
(b) phosphine
(c) phosphorus
(d) phosphorus pentoxide

**80.** The enthalpy of combustion of methane at 25°C is 890 kJ. The heat liberated when 3.2 g of methane is burnt in air, is
(a) 445 kJ (b) 278 kJ
(c) –890 kJ (d) 178 kJ

## Mathematics

**81.** $\sim p \wedge q$ is logically equivalent to
(a) $p \to q$ (b) $q \to p$
(c) $\sim(p \to q)$ (d) $\sim(q \to p)$

**82.** The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45° . If the angle of elevation of the top of the complete pillar at the same point is to be 60°, then the height of the incomplete pillar is to be increased by
(a) $50\sqrt{2}$ m (b) 100 m
(c) $100(\sqrt{3}-1)$ m (d) $100(\sqrt{3}+1)$ m

**83.** What must be the matrix $X$, if
$$2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}?$$
(a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

**84.** The value of $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$ is

(a) 1
(b) 0
(c) $(a-b)(b-c)(c-a)$
(d) $(a+b)(b+c)(c+a)$

**85.** The value of $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$ is

(a) $441 \times 446 \times 4510$
(b) 0
(c) – 1
(d) 1

**86.** $(\vec{\mathbf{a}} \cdot \hat{\mathbf{i}})\hat{\mathbf{i}} + (\vec{\mathbf{a}} \cdot \hat{\mathbf{j}})\hat{\mathbf{j}} + (\vec{\mathbf{a}} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}$ is equal to

(a) $\vec{\mathbf{a}}$ (b) $2\vec{\mathbf{a}}$
(c) $3\vec{\mathbf{a}}$ (d) $\vec{0}$

**87.** Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

**88.** If $|\vec{\mathbf{a}}| = 3, |\vec{\mathbf{b}}| = 4$, then a value of $\lambda$ for which $\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$ is perpendicular to $\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}$, is

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$
(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

**89.** The projection of $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ on $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{2}{\sqrt{14}}$
(c) $\sqrt{14}$ (d) $\frac{-2}{\sqrt{14}}$

**90.** Let $A = \{x : x$ is a multiple of 3$\}$ and $B = \{x : x$ is a multiple of 5$\}$. Then $A \cap B$ is given by
(a) {3, 6, 9, .........}
(b) {5, 10, 15, 20, ..............}
(c) {15, 30, 45, .............}
(d) None of the above

**91.** The maximum of the function $3\cos x - 4\sin x$ is
(a) 2 (b) 3
(c) 4 (d) 5

**92.** If the distance '$s$' metres traversed by a particle in $t$ seconds is given by $s = t^3 - 3t^2$, then the velocity of the particle when the acceleration is zero, (in m/s) is
(a) 3 (b) –2
(c) –3 (d) 2

**93.** For the curve $y^n = a^{n-1}x$ if the subnormal at any point is a constant, then $n$ is equal to
(a) 1 (b) 2
(c) –2 (d) –1

**94.** If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to
(a) $-16x$ (b) $16x$
(c) $x$ (d) $-x$

**95.** If tangent to the curve $x = at^2, y = 2at$ is perpendicular to $x$-axis, then its point of contact is
(a) $(a, a)$ (b) $(0, a)$
(c) $(0, 0)$ (d) $(a, 0)$

**96.** The general solution of the differential equation $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2x} = 0$ is given by
(a) $\tan y + \cot x = c$
(b) $\tan y - \cot x = c$
(c) $\tan x - \cot y = c$
(d) $\tan x + \cot y = c$

**97.** The degree of the differential equation
$$\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$ is
(a) 1/3 (b) 4
(c) 9 (d) 3/4

**98.** The area enclosed between the curves $y = x^3$ and $y = \sqrt{x}$ is,
(a) $\frac{5}{3}$ sq unit (b) $\frac{5}{4}$ sq unit
(c) $\frac{5}{12}$ sq unit (d) $\frac{12}{5}$ sq unit

**99.** $\int_0^{\pi/8} \cos^3 4\theta \, d\theta$ is equal to

(a) $\frac{5}{3}$ (b) $\frac{5}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

100. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

101. If $ax^2 - y^2 + 4x - y = 0$ represents a pair of lines, then $a$ is equal to

(a) – 16 (b) 16

(c) 4 (d) –4

102. What is the equation of the locus of a point which moves such that 4 times its distance from the $x$-axis is the square of its distance from the origin?

(a) $x^2 - y^2 - 4y = 0$

(b) $x^2 + y^2 - 4|y| = 0$

(c) $x^2 + y^2 - 4x = 0$

(d) $x^2 + y^2 - 4|x| = 0$

103. Equation of the straight line making equal intercepts on the axes and passing through the point (2, 4) is

(a) $4x - y - 4 = 0$ (b) $2x + y - 8 = 0$

(c) $x + y - 6 = 0$ (d) $x + 2y - 10 = 0$

104. If the area of the triangle with vertices $(x, 0)$, (1, 1) and (0, 2) is 4 sq unit, then the value of $x$ is

(a) – 2 (b) – 4

(c) – 6 (d) 8

105. $\lim_{\theta \to \frac{\pi}{2}} \frac{\frac{\pi}{2} - \theta}{\cot \theta}$ is equal to

(a) 0 (b) –1

(c) 1 (d) $\infty$

106. The coaxial system of circles given by $x^2 + y^2 + 2gx + c = 0$ for $c < 0$ represents

(a) intersecting circles

(b) non-intersecting circles

(c) touching circles

(d) touching or non-intersecting circles

107. The radius of the circle passing through the point (6, 2) and two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is

(a) 4 (b) 6

(c) 20 (d) $\sqrt{20}$

108. If (0, 6) and (0, 3) are respectively the vertex and focus of a parabola, then its equation is

(a) $x^2 + 12y = 72$ (b) $x^2 - 12y = 72$

(c) $y^2 - 12x = 72$ (d) $y^2 + 12x = 72$

109. For the ellipse $24x^2 + 9y^2 - 120x - 90y + 225 = 0$, the eccentricity is equal to

(a) 2/5 (b) 3/5

(c) $\sqrt{\frac{15}{24}}$ (d) $\frac{1}{5}$

110. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of $b^2$ is

(a) 1 (b) 7

(c) 5 (d) 9

111. The differential coefficient of $f(\sin x)$ with respect to $x$ where $f(x) = \log x$ is

(a) $\tan x$ (b) $\cot x$

(c) $f(\cos x)$ (d) $\frac{1}{x}$

112. If $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of $k$ is

(a) 0 (b) 1/2

(c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

113. If $\omega = \frac{-1 + \sqrt{3}i}{2}$ then $(3 + \omega + 3\omega^2)^4$ is

(a) 16 (b) – 16

(c) 16 $\omega$ (d) 16 $\omega^2$

114. If $y = \tan^{-1}(\sec x - \tan x)$, then $\frac{dy}{dx}$ is equal to

(a) 2 (b) – 2

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

115. If $x + \frac{1}{x} = 2\cos\alpha$ then $x^n + \frac{1}{x^n}$ is equal to

(a) $2^n \cos\alpha$ (b) $2^n \cos n\alpha$

(c) $2i \sin n\alpha$ (d) $2\cos n\alpha$

116. $\int_{-1}^{1} |1 - x|\, dx$ is equal to

(a) –2 (b) 0

(c) 2 (d) 4

117. $\int \frac{dx}{x(x^7 + 1)}$ is equal to

(a) $\log\left(\frac{x^7}{x^7 + 1}\right) + c$ (b) $\frac{1}{7}\log\left(\frac{x^7}{x^7 + 1}\right) + c$

(c) $\log\left(\frac{x^7 + 1}{x^7}\right) + c$ (d) $\frac{1}{7}\log\left(\frac{x^7 + 1}{x^7}\right) + c$

**118.** $\int \sqrt{x}\, e^{\sqrt{x}} dx$ is equal to

(a) $2\sqrt{x} - e^{\sqrt{x}} - 4\sqrt{xe^{\sqrt{x}}} + c$

(b) $(2x - 4\sqrt{x} + 4)\, e^{\sqrt{x}} + c$

(c) $(2x + 4\sqrt{x} + 4)e^{\sqrt{x}} + c$

(d) $(1 - 4\sqrt{x})\, e^{\sqrt{x}} + c$

**119.** $\int \frac{dx}{x^2 + 2x + 2}$ is equal to

(a) $\sin^{-1}(x + 1) + c$

(b) $\sin^{-1}(x + 1) + c$

(c) $\tan^{-1}(x - 1) + c$

(d) $\tan^{-1}(x + 1) + c$

**120.** If a tangent to the curve $y = 6x - x^2$ is parallel to the line $4x - 2y - 1 = 0$, then the point of tangency on the curve is

(a) (2, 8) (b) (8, 2)
(c) (6, 1) (d) (4, 2)

**121.** 0.5737373... is equal to

(a) $\frac{284}{497}$ (b) $\frac{284}{495}$
(c) $\frac{568}{999}$ (d) $\frac{567}{990}$

**122.** The number of solutions for the equation $x^2 - 5|x| + 6 = 0$ is

(a) 4 (b) 3
(c) 2 (d) 1

**123.** How many numbers of 6 digits can be formed from the digits of the number 112233?

(a) 30 (b) 60
(c) 90 (d) 120

**124.** The last digit in $7^{300}$ is

(a) 7 (b) 9
(c) 1 (d) 3

**125.** If $\frac{\log x}{a-b} = \frac{\log y}{b-c} = \frac{\log z}{c-a}$, then $xz$ is equal to

(a) 0 (b) 1
(c) –1 (d) 2

**126.** The smallest positive integer $n$ for which $(1 + i)^{2n} = (1 - i)^{2n}$ is

(a) 1 (b) 2
(c) 3 (d) 4

**127.** If $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$ then $p^2 + q^2 + r^2 + 2pqr$ is equal to

(a) 3 (b) 1
(c) 2 (d) – 1

**128.** If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then $x$ is equal to

(a) 1 (b) 4
(c) 3 (d) 5

**129.** If $0 \le x \le \frac{\pi}{2}$ and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$, then $x$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) 0

**130.** The equation of the director circle of the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ is given by

(a) $x^2 + y^2 = 16$ (b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 = 20$ (d) $x^2 + y^2 = 12$

**131.** The normals at the extremities of the latus rectum of the parabola intersects the axis at an angle of

(a) less than 90° (b) greater than 90°
(c) 90° (d) None of these

**132.** The circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches

(a) $x$-axis
(b) $y$-axis
(c) both axes
(d) neither $x$-axis nor $y$-axis

**133.** If $A = \{1, 2, 3\}$ and $B = \{3, 8\}$, then $(A \cup B) \times (A \cap B)$ is

(a) {(3, 1), (3, 3), (3, 8)}
(b) {(1, 3), (2, 3), (3, 3), (8, 3)}
(c) {(1, 2), (2, 2), (3, 3), (8, 8)}
(d) {(8, 3), (8, 2), (8, 1), (8, 8)}

**134.** The condition that one root of the equation $ax^2 + bx + c = 0$ may be double of the other, is

(a) $b^2 = 9ac$ (b) $2b^2 = 9ac$
(c) $2b^2 = ac$ (d) $b^2 = ac$

**135.** The value of $k$ so that $x^2 + y^2 + kx + 4y + 2 = 0$ and $2(x^2 + y^2) - 4x - 3y + k = 0$ cut orthogonally is

(a) $\frac{10}{3}$ (b) $-\frac{8}{3}$
(c) $-\frac{10}{3}$ (d) $\frac{8}{3}$

**136.** $\lim_{x \to \infty} \left(1 - \frac{4}{x-1}\right)^{(3x-1)}$ is equal to

(a) $e^{12}$ (b) $e^{-12}$
(c) $e^4$ (d) $e^3$

**137.** If $A + B + C = 180°$, then $\Sigma \tan \frac{A}{2} \tan \frac{B}{2}$ is equal to
(a) 0 (b) 1
(c) 2 (d) 3

**138.** In a triangle $ABC$, if $b = 2$, $B = 30°$ then the area of the circumcircle of triangle $ABC$ in square unit is
(a) $\pi$ (b) $2\pi$
(c) $4\pi$ (d) $6\pi$

**139.** If $\sin x + \sin^2 x = 1$, then $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$ is equal to
(a) 1 (b) 2
(c) 3 (d) 0

**140.** If $R$ denotes the set of all real number, then the function $f: R \to R$ defined $f(x) = |x|$ is
(a) one-one only
(b) onto only
(c) both one-one and onto
(d) neither one-one nor onto

**141.** If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2,3 are roots of the equation $f(x) = 0$, then the values of $m$ and $n$ are
(a) – 5, – 30 (b) – 5, 30
(c) 5, 30 (d) None of these

**142.** If $p_1, p_2, p_3$ are respectively the perpendiculars from the vertices of a triangle to the opposite sides, then $p_1\, p_2\, p_3$ is equal to
(a) $a^2 b^2 c^2$ (b) $2a^2 b^2 c^2$
(c) $\frac{4a^2 b^2 c^3}{R^2}$ (d) $\frac{a^2 b^2 c^2}{8R^2}$

**143.** If $5\cos 2\theta + 2\cos^2 \frac{\theta}{2} + 1 = 0$, $-\pi < \theta < \pi$, then $\theta$ is equal to
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{3}, \cos^{-1}\left(\frac{3}{5}\right)$
(c) $\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$

**144.** The two forces acting at a point, the maximum effect is obtained when their resultant is 4 N. If they act at right angles, then their resultant is 3N. Then the forces are
(a) $(2 + \frac{1}{2}\sqrt{3})$ N and $(2 - \frac{1}{2}\sqrt{3})$ N
(b) $(2 + \sqrt{3})$ N and $(2 - \sqrt{3})$N
(c) $(2 + \frac{1}{2}\sqrt{2})$ N and $(2 - \frac{1}{2}\sqrt{2})$ N
(d) $(2 + \sqrt{2})$N and $(2 - \sqrt{2})$ N

**145.** The resultant $R$ of two forces $P$ and $Q$ act at right angles to $P$. Then the angle between the forces is
(a) $\cos^{-1}\left(\frac{P}{Q}\right)$ (b) $\cos^{-1}\left(-\frac{P}{Q}\right)$
(c) $\sin^{-1}\left(\frac{P}{Q}\right)$ (d) $\sin^{-1}\left(-\frac{P}{Q}\right)$

**146.** A body starts from rest and moves with a uniform acceleration. The ratio of the distance covered in $n$th sec to the distance covered in $n$ seconds is
(a) $\frac{2}{n} - \frac{1}{n^2}$ (b) $\frac{1}{n^2} - \frac{1}{n}$
(c) $\frac{2}{n^2} - \frac{1}{n}$ (d) $\frac{2}{n} + \frac{1}{n^2}$

**147.** Two points move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity $u$ and the second starts from rest with constant acceleration $f$, the distance between the two points will be maximum at time
(a) $t = \frac{2u}{f}$ (b) $t = \frac{u}{f}$
(c) $t = \frac{u}{2f}$ (d) $t = \frac{u^2}{f}$

**148.** The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0,7,–7) is
(a) $x + y + z = 1$ (b) $x + y + z = 2$
(c) $x + y + z = 0$ (d) None of these

**149.** A plane passes through a fixed point $(p, q)$ and cut the axes in $A, B, C$. Then the locus of the centre of the sphere $OABC$ is
(a) $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$ (b) $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 1$
(c) $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 3$ (d) None of these

**150.** The value of $1^2 \cdot C_1 + 3^2 \cdot C_3 + 5^2 \cdot C_5 + \ldots$ is
(a) $n(n-1)^{n-2} + n \cdot 2^{n-1}$
(b) $n(n-1)2^{n-2}$
(c) $n(n-1) \cdot 2^{n-3}$
(d) None of the above

# Answers

### ➠ PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (d) | **2.** (d) | **3.** (d) | **4.** (b) | **5.** (a) | **6.** (d) | **7.** (c) | **8.** (a) | **9.** (d) | **10.** (c) |
| **11.** (c) | **12.** (a) | **13.** (c) | **14.** (d) | **15.** (c) | **16.** (a) | **17.** (b) | **18.** (d) | **19.** (c) | **20.** (d) |
| **21.** (d) | **22.** (a) | **23.** (d) | **24.** (d) | **25.** (d) | **26.** (d) | **27.** (a) | **28.** (a) | **29.** (b) | **30.** (a) |
| **31.** (d) | **32.** (b) | **33.** (c) | **34.** (a) | **35.** (c) | **36.** (c) | **37.** (b) | **38.** (d) | **39.** (a) | **40.** (b) |

### ➠ CHEMISTRY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **41.** (a) | **42.** (d) | **43.** (a) | **44.** (b) | **45.** (d) | **46.** (b) | **47.** (a) | **48.** (a) | **49.** (c) | **50.** (b) |
| **51.** (c) | **52.** (b) | **53.** (a) | **54.** (b) | **55.** (a) | **56.** (b) | **57.** (b) | **58.** (b) | **59.** (b) | **60.** (d) |
| **61.** (d) | **62.** (d) | **63.** (c) | **64.** (a) | **65.** (c) | **66.** (d) | **67.** (a) | **68.** (d) | **69.** (b) | **70.** (a) |
| **71.** (b) | **72.** (b) | **73.** (b) | **74.** (d) | **75.** (a) | **76.** (a) | **77.** (a) | **78.** (b) | **79.** (d) | **80.** (d) |

### ➠ MATHEMATICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **81.** (d) | **82.** (c) | **83.** (a) | **84.** (c) | **85.** (b) | **86.** (a) | **87.** (d) | **88.** (b) | **89.** (b) | **90.** (c) |
| **91.** (d) | **92.** (c) | **93.** (b) | **94.** (a) | **95.** (c) | **96.** (b) | **97.** (b) | **98.** (c) | **99.** (d) | **100.** (a) |
| **101.** (b) | **102.** (b) | **103.** (c) | **104.** (c) | **105.** (c) | **106.** (a) | **107.** (d) | **108.** (a) | **109.** (c) | **110.** (b) |
| **111.** (b) | **112.** (a) | **113.** (c) | **114.** (d) | **115.** (d) | **116.** (c) | **117.** (b) | **118.** (b) | **119.** (d) | **120.** (a) |
| **121.** (b) | **122.** (a) | **123.** (c) | **124.** (c) | **125.** (b) | **126.** (b) | **127.** (b) | **128.** (c) | **129.** (a) | **130.** (d) |
| **131.** (c) | **132.** (b) | **133.** (b) | **134.** (b) | **135.** (c) | **136.** (b) | **137.** (b) | **138.** (c) | **139.** (a) | **140.** (d) |
| **141.** (b) | **142.** (d) | **143.** (d) | **144.** (c) | **145.** (b) | **146.** (a) | **147.** (b) | **148.** (c) | **149.** (a) | **150.** (d) |

# Hints & Explanations

## Physics

**1. Key Idea** *Apply Doppler effect.*

When source approaches the observer, the apparent frequency heard by observer is

$$\nu' = \nu\left(\frac{v}{v - v_s}\right) \quad \ldots\text{(i)}$$

$v_s$ = speed of source of sound

During its recession, apparent frequency

$$\nu'' = \nu\left(\frac{v}{v + v_s}\right) \quad \ldots\text{(ii)}$$

Accordingly

$$\nu' = \nu'' = \frac{2}{100}\nu \quad \text{(Given)}$$

$$\therefore \quad \nu\left(\frac{v}{v - v_s}\right) - \nu\left(\frac{v}{v + v_s}\right) = \frac{2}{100}\nu$$

or $$\nu\left[\frac{v + v_s - v + v_s}{(v - v_s)(v + v_s)}\right] = \frac{2}{100}$$

or $$\frac{2v\, v_s}{(v - v_s)(v + v_s)} = \frac{2}{100}$$

or $$100 v v_s = v^2 - v_s^2$$

But speed of sound in air $v = 300$ m/s

$$\therefore \quad 30000\, v_s = (300)^2 - v_s^2$$

$$\Rightarrow \quad v_s^2 + 30000\, v_s - 90000$$

$$v_s = \frac{-30000 \pm \sqrt{(30000)^2 + 4 \times 90000}}{2}$$

$$= -\frac{30000 \;\; \pm 30006}{2}$$

$$= \frac{6}{2} = 3\ \text{ms}^{-1} \quad \text{(Taking + ve sign only)}$$

**2. Key Idea** *Force between two parallel conductors carrying current $I_1$ and $I_2$,*

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{d},$$

*force is attractive when the current is flowing in the two wires are in same direction and $F$ is repulsive when the current is flowing in opposite directions.*

Since, the currents in the three wires are flowing in same direction so, the wire $B$ will experience a force of attraction due to both wires $A$ and $C$,

So, $$F_{AB} = \frac{\mu_0}{4\pi} \cdot \frac{2I_A I_B}{d}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2 \times 1 \times 2}{d}$$
$$= \frac{4\mu_0}{4\pi d} \quad \text{...(i)}$$

and $$F_{CB} = \frac{\mu_0}{4\pi} \cdot \frac{2I_B I_c}{d}$$
$$= \frac{\mu_0}{4\pi} \times \frac{2 \times 2 \times 3}{d}$$
$$= \frac{12\mu_0}{4\pi d} \quad \text{...(ii)}$$

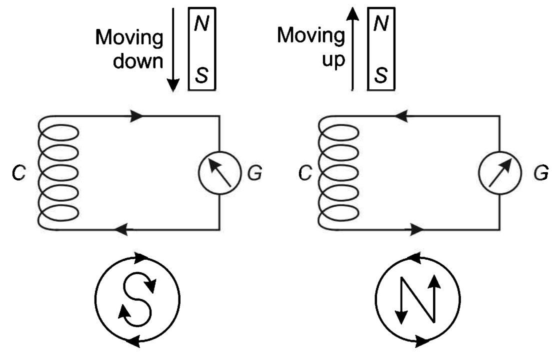
As seen from Eqs. (i) and (ii) $F_{CB} > F_{AB}$ hence, the net force of attraction will be directed towards wire $C$.

**3. Key Idea** *At temperatures above Curie temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to* $(T - T_C)$, $T_C$ *is Curie temperature.*

Iron is a ferromagnetic substance. The Curie point for iron is 1043 K (=770°C). If the temperature of a ferromagnetic material is raised above a certain critical value, called the Curie temperature, the exchange coupling ceases to be effective. Most such materials then become paramagnetic . Also after Curie point, susceptibility of a ferromagntic substance varies inversely with its absolute temperature. Hence, a ferromagnetic substance obeys Curie-Wiess law above its Curie temperature.

**4. Key Idea** *The direction of induced emf or current in a circuit is such as to oppose the cause that produces it.*

When the $S$-pole of a magnet is moved towards the coil, then by Lenz's law the face of coil, towards magnet becomes $S$-pole and the current flows clockwise to cancel change in the magnetic flux. So, to bring the magnet near to the coil, more work has to be done against the force of repulsion produced between them. So, the galvanometer shows deflection to the left. Now, when the $S$-pole is moved away, a current flows in anti-clockwise direction to make the face of the coil towards magnet, a $N$-pole. Thus, will try to attract the magnet. So, the galvanometer shows the deflection to the right. Since, flux varies hence, amplitude will not be constant and will decrease.




**5. Key Idea** *Emf induced in an electrical circuit*

$$e = L\frac{dI}{dt}.$$

$$L = e\frac{dt}{dI}$$
$$= \frac{W}{Q} \cdot \frac{dt}{dI} \qquad \left(\because e = V = \frac{W}{Q}\right)$$
$$= \frac{W \cdot dt}{It \cdot dI}$$
$$= \frac{[ML^2T^{-2}][T]}{[A][T][A]} = [ML^2T^{-2}A^{-2}]$$

**6.** To convert a galvanometer into voltmeter, the necessary value of resistance to be connected in series with the galvanometer is

$$R = \frac{V}{I_g} - G = \frac{50}{10 \times 10^{-3}} - 40$$
$$= 5000 - 40 = 4960\,\Omega$$

**7.** Figure 3 is streamlined, so air resistance of it will be minimum. For figure 1 surface area is maximum, so air resistance for it is maximum. Hence, correct sequence is $3<2<1$.

**8.** The chemical formula of heavy water is $D_2O$. Here, $D$ is the heavy isotope of hydrogen (H) with mass number 2 *ie*, deuterium ${}_1H^2(D)$ and $O$ stands for oxygen. Hence, heavy water is a compound of deuterium and oxygen.

**9. Key Idea** *The problem can be solved using third equation of motion.*

Let the thickness of each plank is $d$.

From equation of motion

$$v^2 = u^2 + 2as$$

**Ist case** ...(i)

$$s = 2d, \quad u = 100 \text{ ms}^{-1}, \quad v = 0$$

$\therefore$ $$0 = (100)^2 + 2a \times 2d$$

$\Rightarrow$ $$4ad = -100 \times 100$$

$\Rightarrow$ $$a = -\frac{100 \times 100}{4d}$$

$\therefore$ $$a = -\frac{2500}{d} \quad \text{...(ii)}$$

(–ve sign stands for retardation)

**IInd case** Let the bullet with double the previous speed will penetrate *n* planks of equal thickness *d*.

Now, $v = 0, \quad u = 200 \text{ ms}^{-1},$

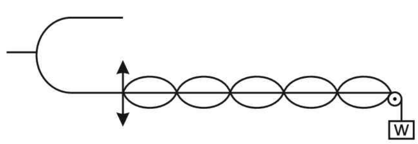
$$a = -\frac{2500}{d}, \quad s = nd$$

Using, Eq. (i) again, we have

$$0 = (200)^2 - 2 \times \frac{2500}{d} \times nd$$

$\Rightarrow$ $$n = \frac{200 \times 200}{2 \times 2500} = 8$$

**11.** In transverse arrangement the tuning fork is placed such that the vibration of the prongs is in direction perpendicular to the length of the string as shown in figure. As the tuning fork completes one vibration, the one vibration of wave on string is completed. Thus, in transverse mode, its frequency is the same as that of the fork. Hence, the required ratio is 1 : 1.




**12.** The balance condition of a meter bridge experiment

$$\frac{R}{S} = \frac{l_1}{(100 - l_1)}$$

**Ist case** $$\frac{R_1 + 10}{R_2} = \frac{50}{50} \quad \text{...(i)}$$

$\Rightarrow$ $$R_1 + 10 = R_2$$

**IInd case** $$\frac{R_1}{R_2} = \frac{40}{60}$$

$\Rightarrow$ $$R_2 = \frac{60}{40} R_1 \quad \text{...(ii)}$$

So, Eqs. (i) and (ii) give

$$R_1 + 10 = \frac{60}{40} R_1$$

$\Rightarrow$ $$\frac{60}{40} R_1 - R_1 = 10$$

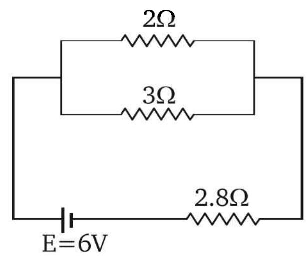
$\Rightarrow$ $$\frac{20}{40} R_1 = 10$$

$\Rightarrow$ $$R_1 = \frac{10 \times 40}{20}$$

$\therefore$ $$R_1 = 20\,\Omega$$

**13. Key Idea** *In the steady state, no current flows through the branch containing the capacitor.*

So, the equivalent circuit will be of the form as shown below

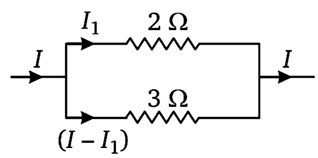



The effective resistance of the circuit is

$$R = \frac{2 \times 3}{2 + 3} + 2.8$$

$$= 1.2 + 2.8 = 4\,\Omega$$

The current through the circuit is




$$I = \frac{E}{R} = \frac{6}{4} = 1.5 \text{ A}$$

Let current $I_1$ flows through $2\,\Omega$ resistance.

$\therefore$ $$2 \times I_1 = (I - I_1) \times 3$$

$\Rightarrow$ $$2I_1 = (1.5 - I_1) \times 3$$

$\Rightarrow$ $$2I_1 = 4.5 - 3I_1$$

$\Rightarrow$ $$5I_1 = 4.5$$

$\Rightarrow$ $$I_1 = 0.9 \text{ A}$$

**14.** Peak value of induced emf in a rectangular coil is $e = n\ BA\,\omega \sin\theta$

$$= 300 \times 4 \times 10^{-2} \times (25 \times 10 \times 10^{-4}) \times (2\pi \times 50) \times \sin 90^\circ$$

$$= 30\pi \text{ V}$$

15. Resultant potential difference (supply voltage of an $L$-$C$-$R$ circuit is)

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(40)^2 + (60-30)^2}$$
$$= \sqrt{(40)^2 + (30)^2}$$
$$= \sqrt{(50)^2} = 50 \text{ V}$$

16. When the portion $A$ is punctured with a pin, the thread becomes concave towards $A$ because film tries to cover minimum surface area.

17. Let one mole of each gas has same volume as $V$. When they are mixed, then density of mixture is

$$\rho_{\text{mixture}} = \frac{\text{mass of } O_2 + \text{mass of } H_2}{\text{volume of } O_2 + \text{volume of } H_2}$$
$$= \frac{32+2}{V+V}$$
$$= \frac{34}{2V} = \frac{17}{V}$$

Also, $\rho_{H_2} = \frac{2}{V}$

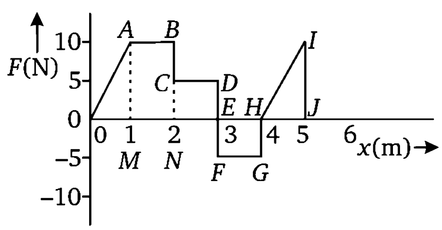
Now, velocity

$$v = \left(\frac{\gamma p}{\rho}\right)^{1/2} \text{ or } v \propto \frac{1}{\sqrt{\rho}}$$

$$\therefore \quad \frac{v_{\text{mixture}}}{v_{H_2}} = \sqrt{\left(\frac{\rho_{H_2}}{\rho_{\text{mixture}}}\right)}$$
$$= \sqrt{\left(\frac{2/V}{17/V}\right)} = \sqrt{\left(\frac{2}{17}\right)}$$

18. **Key Idea** *Work done = Area enclosed by F-x graph*

Work done = area of $ABNO$ + area of $CDEN$ – area of $EFGH$ + area of $HIJ$




$$= \frac{1}{2} \times 1 \times 10 + 1 \times 10 + 1 \times 5 - 1 \times 5 + \frac{1}{2} \times 1 \times 10$$
$$= 5 + 10 + 5 - 5 + 5 = 20 \text{ J}$$

19. **Key Idea** *Specific rotation of sugar solution is*

$$\alpha = \frac{\textit{angle of rotation}}{\textit{length of tube} \times \textit{concentration}}.$$

$$\alpha = \frac{\theta}{l \times C}$$

$$\therefore \quad C = \frac{\theta}{l\alpha}$$
$$= \frac{0.4}{0.25 \times 0.01}$$
$$= 160 \text{ kg/m}^3$$

Thus, purity of sugar solution

$$= \frac{160}{200} \times 100 = 80\%$$

20. In forward biased $p$-$n$ junction, the applied potential is opposite to the junction barrier potential $V_B$. The consequence of this is the effective barrier potential reduces. Hence, the graph (d) is correctly shown.

21. **Key Idea** *Pressure applied by liquid column*

$$p = hdg$$

The pressure depends on the height of liquid column not on its size, so pressure at the bottom of $A$ and $B$ is same.

22. The power drawn by the bulb is

$$P = \frac{V^2}{R}$$
$$\Rightarrow \quad R = \frac{V^2}{P}$$
$$\text{or} \quad R \propto \frac{1}{P}$$

(as $V$ is same in parallel)

It means that greater power will have less resistance and therefore, draws more current. Hence, current flowing in bulb $B$ will be more.

23. After silvering the plane surface, plano-convex lens behaves as a concave mirror of focal length $F = 0.2$

$$\therefore \quad \frac{1}{F} = \frac{2}{f_{\text{lens}}}$$
$$\therefore \quad f_{\text{lens}} = 2F = 2 \times 0.2 = 0.4\text{m}$$

Now, from lens maker's formula

$$\frac{1}{f_{\text{lens}}} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore \quad \frac{1}{0.4} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right)$

$\Rightarrow \quad R_1 = 0.5 \times 0.4 = 0.2\text{ m}$

**24.** **Key Idea** *The problem can be solved using the equation of motion under the effect of gravity.*

For a stone which is thrown downwards from a balloon rising upwards, the equation of motion is

$$h = -ut + \frac{1}{2}gt^2$$

$$= -29 \times 10 + \frac{1}{2} \times 9.8 \times (10)^2$$

$$= -290 + 490 = 200\text{ m}$$

**25.** **Key Idea** *The distance between two consecutive dark or bright fringes is recognised as β (fringe width) and that between central fringe and first dark fringe on either side is* $\frac{\beta}{2}$.

Given, spacing between second dark fringe and central fringe $= \beta + \frac{\beta}{2}$

or $\quad \frac{3\beta}{2} = 1\text{ mm}$

or $\quad \beta = \frac{2}{3} \times 1\text{ mm}$

$$\frac{\lambda D}{d} = \frac{2}{3}\text{ mm}$$

$\therefore \quad \lambda = \frac{2}{3} \times 10^{-3} \times \frac{0.9 \times 10^{-3}}{1}$

$\Rightarrow \quad \lambda = 0.6 \times 10^{-6}\text{ m}$

$\therefore \quad \lambda = 600 \times 10^{-9}\text{ m}$

$$= 600\text{ nm}$$

**26.** When a thin sheet of polyvinyl alcohol is stretched and then impregnated with iodine, *H*-polaroid is obtained.

**27.** Let $R$ and $r$ be the radii of bigger and each smaller drop respectively.

$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad R = 2r \quad \ldots(\text{i})$

The capacitance of a smaller spherical drop is

$$C = 4\pi\varepsilon_0 r \quad \ldots(\text{ii})$$

The capacitance of bigger drop is

$$C' = 4\pi\varepsilon_0 R$$

$$= 2 \times 4\pi\varepsilon_0 r \quad (\because R = 2r)$$

$$= 2C \quad [\text{From Eq. (ii)}]$$

$$C = \frac{C'}{2}$$

$$= \frac{1}{2}\,\mu\text{F} \quad (\because C' = 1\,\mu\text{F})$$

**29.** **Key Idea** *For photoelectronic emission to take place wavelength of incident light must be less that the threshold value.*

The wavelength of infra-red light $= 7800$ Å

The wavelength of ultra-violet light $= 4000$ Å

Thus, it is obvious that wavelength of UV radiation is less than the threshold value. Hence, it can emit photoelectrons from the surface of metal.

**30.** Electric flux $= \frac{q}{\varepsilon_0}$

$$= \frac{(-14 + 78.85 - 56) \times 10^{-9}}{8.85 \times 10^{-12}}$$

$$= \frac{8.85 \times 10^{-9}}{8.85 \times 10^{-12}}$$

$$= 1000\text{ Nm}^2\text{ C}^{-1}$$

**31.** Radius of orbit of electron in $n$th excited state of hydrogen

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

$\therefore \quad r \propto \frac{n^2}{Z} \quad \ldots(\text{i})$

$\therefore \quad \frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \times \frac{Z_2}{Z_1}$

But $\quad r_1 = r_2$

So, $\quad n_2^2 = n_1^2 \times \frac{Z_2}{Z_1}$

Here, $n_1 = 1$ (ground state of hydrogen),

$Z_1 = 1$ (atomic number of hydrogen),

$Z_2 = 4$ (atomic number of beryllium)

$\therefore \quad n_2^2 = (1)^2 \times \frac{4}{1}$

or $\quad n_2^2 = 4$

or $\quad n_2 = 2$

**32.** The mass excess per nucleon of isotopes of atom is known as packing fraction given by

$$P = \frac{M - A}{A}$$

where $M$ is the actual mass of isotope and $A$ is its atomic mass.

**33.** The number of counts left after time $t$

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$\therefore$ $$30 = 240 \left(\frac{1}{2}\right)^{\frac{60}{T_{1/2}}}$$

$[\because t = 1\text{ h} = 60\text{ min}]$

$$\frac{30}{240} = \left(\frac{1}{2}\right)^{\frac{60}{T_{1/2}}}$$

or $$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{60/T_{1/2}}$$

Comparing the powers, we get

$\therefore$ $$\frac{60}{T_{1/2}} = 3$$

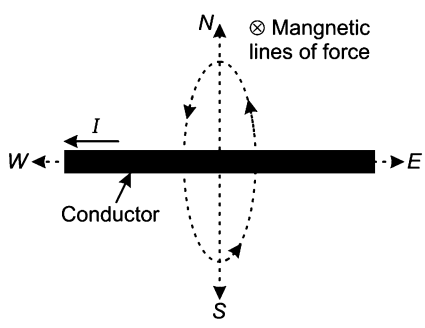
$$T_{1/2} = \frac{60}{3}$$

$$T_{1/2} = 20\text{ min}$$

**34.** When metal sphere is placed inside a charged parallel plate capacitor, the electric lines of force will not enter the metallic conductor as $E=0$ inside a charged conductor. Moreover, the surface of a charged conductor is an equipotential surface and hence, electric lines of force are always perpendicular to equipotential surface.

**35.** **Key Idea** *Apply Maxwell's right hand screw rule.*

The direction of magnetic field at a point above the conductor is towards north and at a point below the conductor is towards south.




**36.** The excitation energy in the first excited state is

$$E = RhcZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= (13.6\text{ eV}) \times Z^2 \times \frac{3}{4}$$

$\therefore$ $$40.8 = 13.6 \times Z^2 \times \frac{3}{4}$$

$\Rightarrow$ $$Z = 2$$

So, the ion in problem is $He^+$. The energy of the ion in the ground state is

$$E = -\frac{RhcZ^2}{1^2}$$

$$= 13.6 \times 4 = 54.4\text{ eV}$$

Hence, 54.4 eV is required to remove the electron from the ion.

**37.** Using the relation

$$\frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

or $$\frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

or $$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \left(\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right)$$

or $$W = Q_1\left(1 - \frac{T_2}{T_1}\right)$$

$\therefore$ $$W = 6 \times 10^4\left(1 - \frac{(127 + 273)}{(227 + 273)}\right)$$

or $$W = 6 \times 10^4\left(1 - \frac{400}{500}\right)$$

$$= 6 \times 10^4 \times \frac{100}{500}$$

$$= 1.2 \times 10^4\text{ J}$$

**38.** For an ideal gas keeping the temperature same throughout,

$$pV = \text{constant}$$

Hence, for a given mass, the graph between $p$-$V$ and $V$ will be a straight line parallel to $V$-axis whatever may be the volume.

**39.** **Key Idea** *The combined focal length is given by*

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

For plano-convex lens $f_1 = \infty$ (for plane surface), $f_2 = f$ (say)

$\therefore$ $$\frac{1}{F} = \frac{1}{\infty} + \frac{1}{f}$$

$\Rightarrow$ $$F = f$$

Now, when concave lens of same focal length is joined to first lens, then combined focal length

$$\frac{1}{F'} = \frac{1}{F_1} + \frac{1}{F_2}$$

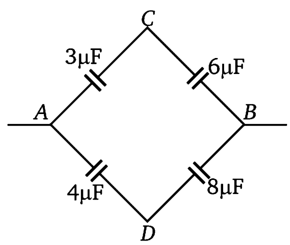
$$= \frac{1}{f} - \frac{1}{f} \quad (\because F_1 = f, F_2 = -f)$$

$$= 0$$

$$F' = \infty$$

Thus, focus shifts to infinity.

**40.** The effective capacitance in upper arm in series, is given by




$$C_1 = \frac{3 \times 6}{3 + 6} = \frac{18}{9}$$

$$= 2\mu F$$

The effective capacitance in lower arm in series is given by

$$C_2 = \frac{4 \times 8}{4 + 8} = \frac{32}{12}$$

$$= \frac{8}{3} \mu F$$

Hence, the resultant capacitance in parallel is given by

$$C = C_1 + C_2$$

$$= 2 + \frac{8}{3}$$

$$= \frac{14}{3} \mu F$$

## Chemistry

**41. Key Idea** *Amphoteric substance can accept as well as donate a proton.*

(a) **$HNO_3$**

$$\underset{\text{acid}}{HNO_3} \longrightarrow H^+ + NO_3^-$$

$\because$ $HNO_3$ can donate a proton (acid) but cannot accept a proton.

$\therefore$ It is a strong acid, not an amphoteric substance.

(b) **$HCO_3^-$**

$$\underset{\text{acid}}{HCO_3^-} \longrightarrow H^+ + CO_3^{2-}$$

$$\underset{\text{base}}{HCO_3^-} + H^+ \longrightarrow H_2CO_3$$

$\because$ $HCO_3^-$ can accept as well as donate a proton.

$\therefore$ It is an amphoteric substance.

(c) **$H_2O$**

$$\underset{\text{acid}}{H_2O} \longrightarrow H^+ + OH^-$$

$$\underset{\text{base}}{H_2O} + H^+ \longrightarrow H_3O^+$$

$\because$ $H_2O$ can accept as well as donate a proton.

$\therefore$ It is an amphoteric substance.

(d) **$NH_3$**

$$\underset{\text{acid}}{NH_3} \longrightarrow NH_2^- + H^+$$

$$NH_3 + H^+ \longrightarrow NH_4^+$$

$\because$ $NH_3$ can accept as well as donate a proton, hence amphoteric.

Hence, $HNO_3$ is not an amphoteric substance.

**42. Key Idea** *57.3 kJ heat is evolved when one gram-equivalent of strong acid (like $H_2SO_4$) and one gram-equivalent of strong base (like KOH) are neutralised.*

The number of gram-equivalents of

$$H_2SO_4 = 0.2 \times \frac{50}{1000} = 1.0 \times 10^{-2}$$

The number of gram-equivalents of

$$KOH = 1 \times \frac{50}{1000} = 5 \times 10^{-2}$$

$\because$ $1.0 \times 10^{-2}$ gram-equivalent of $H_2SO_4$ is neutralised by $1.0 \times 10^{-2}$ gram-equivalent of KOH.

Hence, the heat evolved will be

$$= 57.3 \text{ kJ} \times 1.0 \times 10^{-2}$$

$$= 57.3 \times 10^3 \times 10^{-2} \text{ J} = 573 \text{ J}$$

**43. Key Idea**

*(i) Emission of β-particles increases the atomic number by one but does not affect mass number.*

*(ii) Number of neutron*

*= mass number − atomic number*

Hence, the parent nucleus (which on two successive β-emissions gave ${}_{7}N^{14}$) must be ${}_{5}X^{14}$.

Hence, number of neutrons in parent nucleus

$$= 14 - 5 = 9$$

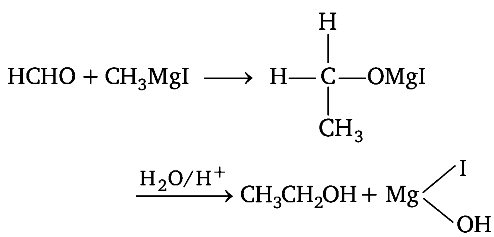
**44. Key Idea** *More electropositive metals protect the less electropositive metals from rusting.*

Chromium is more electropositive metal than iron. In stainless steel, chromium forms an oxide layer and thus it protects steel from rusting.

**45. Key Idea** *Aldehydes on reaction with Grignard reagent give 1° alcohols while ketones and esters give 2° and 3° alcohols. Alcohols with Grignard reagent give alkanes.*

∵ Ethanol is a 1° alcohol.

∴ When HCHO reacts with $CH_3MgI$, ethanol is formed.



**46. Key Idea**

*(i) 1 mol = $6.023 \times 10^{23}$ molecules*

*(ii) Use the following formula to find strength*

$$Strength = \frac{\text{number of moles of solute}}{\text{volume of solution (in } L)}$$

*(iii) 1 L = 1 dm$^3$*

∵ $6.023 \times 10^{23}$ molecules of HCl ≅ 1 mol HCl

∴ $1.2046 \times 10^{24}$ molecules of HCl

$$\cong \frac{1.2046 \times 10^{24} \times 1}{6.023 \times 10^{23}}$$

$$\cong 2 \text{ moles of HCl}$$

Thus, strength $= \frac{2}{1} = 2$ N

Therefore, the solution will be of 2 N.

**47.** Rutherford showed the existence of nucleus in an atom by his α-particles scattering experiment. He postulated that every atom has a small central part which has positive charge and almost all the mass of atom (*ie,* nucleus consists of protons and neutrons).

**48. Key Idea**

*(i) Single bond contains only one σ bond.*

*(ii) Double bond contains one σ and one π-bonds.*

*(iii) Triple bond contains one σ and two π-bonds.*

In acetylene ($HC \equiv CH$) molecule, a triple bond is present, hence it contains one σ and two π bond. This can be explained as follows.

In acetylene molecule, both carbon atoms are *sp*-hybridised. The two unpaired *p*-orbitals form π-bond by overlapping with each other. One *sp*-orbital is σ-bonded to *s*-orbital of H-atom while the rest *sp*-orbital is σ-bonded to *sp*-orbital of other *C*-atom. Hence, the carbon atoms are linked by two π-bonds and one sigma-bond, in acetylene.

**49. Key Idea** *Use the following formula and then compare $\Delta H_1$ and $\Delta H_2$.*

$$\Delta H = \Delta E + \Delta n_g RT$$

*where, $\Delta n_g$ = number of moles of products − number of moles of reactants*

For reaction,

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g)$$

$$\Delta n_g = 1 - 1\frac{1}{2} = -\frac{1}{2}$$

$$\Delta H_1 = \Delta E - \frac{1}{2}RT$$

For reaction,

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l)$$

$$\Delta n_g = 0 - 1\frac{1}{2} = -1\frac{1}{2}$$

$$\Delta H_2 = \Delta E - 1\frac{1}{2}RT$$

∴ $\Delta H_1 > \Delta H_2$ and more energy will be released when $H_2O(l)$ is formed.

**50. Key Idea**

$$t = n \times t_{1/2} \quad \text{...(i)}$$

$$N = N_0 \times \left(\frac{1}{2}\right)^n \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

Given, $t = 192$ min

$$\frac{N}{N_0} = \frac{1}{16}$$

$$t_{1/2} = ?$$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\therefore \quad \frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{192}{t_{1/2}}}$$

$$\text{or} \quad \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{192}{t_{1/2}}}$$

$$\text{or} \quad 4 = \frac{192}{t_{1/2}}$$

$$\therefore \quad t_{1/2} = \frac{192}{4} = 48 \text{ min}$$

**51. Key Idea** *From gas equation,*

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\text{or} \quad \frac{V_1}{V_2} = \frac{p_2}{p_1} \times \frac{T_1}{T_2}$$

Given, $\frac{p_2}{p_1} = 2$, $\frac{T_2}{T_1} = 2$, $V_1 = 4 \text{ dm}^3$, $V_2 = ?$

$$\therefore \quad \frac{4}{V_2} = 2 \times \frac{1}{2} = 1$$

$$\therefore \quad V_2 = 4 \text{ dm}^3$$

**52.** Weight of copper oxide = 5 g

Weight of copper taken = 4 g

∴ Weight of oxygen in copper oxide

$$= 5 - 4 = 1 \text{ g}$$

∵ Weight of copper, reacted with $1 \text{ g } O_2 = 4 \text{ g}$

∴ Weight of copper, which would react with

$$8 \text{ g } O_2 = \frac{4 \times 8}{1} = 32 \text{ g}.$$

Hence, equivalent weight of copper = 32

**53. Key Idea** *According to Le-Chatelier's principle, on changing concentration, temperature and pressure, the equilibrium shifts in a direction that tends to undo their effects.*

For reaction

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + 92.3 \text{ kJ}$$

The favourable conditions are :

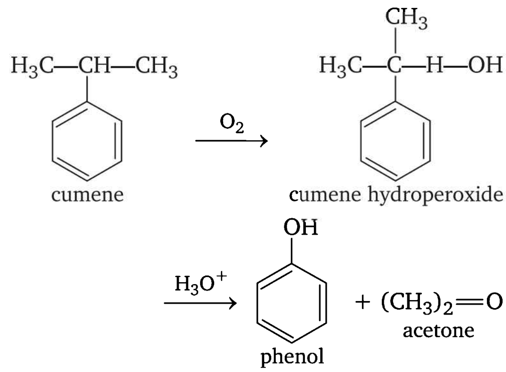
(i) low temperature (as the reaction is exothermic)

(ii) high pressure (volume is decreasing)

(iii) constant removal of ammonia gas as it is formed.

**54.** Catalyst affect both the forward and backward reactions upto same extent, hence it overall does not affect the equilibrium state. However, it brings equilibrium state quickly.

**55.** Cumene is isopropyl benzene (1-methyl ethyl benzene). It on oxidation gives phenol.




**56.** $PCl_3$, $PCl_5$ and $SOCl_2$ are used to replace —OH group of an alcohol or an acid by —Cl group.

$$R—\overset{\overset{\displaystyle O}{\|}}{C}—OH + PCl_5 \longrightarrow R—\overset{\overset{\displaystyle O}{\|}}{C}—Cl + POCl_3 + HCl$$

or $PCl_3$ (giving $H_3PO_3$ in place of $POCl_3 + HCl$)

$$R—\overset{\overset{\displaystyle O}{\|}}{C}—OH + SOCl_2 \longrightarrow R—\overset{\overset{\displaystyle O}{\|}}{C}—Cl + SO_2 + HCl$$

However, when acid reacts with $Cl_2$ in presence of red phosphorus, α-chloro acid is obtained. (Hell-Volhard Zelinsky reaction).

$$CH_3CH_2COOH + Cl_2 \xrightarrow[\Delta]{\text{red P}} CH_3\underset{}{\overset{\overset{\displaystyle Cl}{|}}{C}}HCOOH + HCl$$

α-chloropropionic acid

Hence, $Cl_2$ is not used to prepare acid chloride from carboxylic acid.

**57. Key Idea**

*(i) Less the oxidation potential, less will be the oxidising power.*

*(ii) Oxidation potential decreases down the group.*

∴ The value of oxidation potential is least for iodine among halogens and hence, it has least oxidising power and can be oxidised by nitric acid to give oxy-acid (*ie*, $HIO_3$).

$$10HNO_3 + I_2 \longrightarrow \underset{\text{iodic acid}}{3HIO_3} + 10NO_2 + 4H_2O$$

**58. Key Idea**

(i) *Most reactive metals* (Mg, Mn, Na, *etc.*) *liberates hydrogen with dil.* $HNO_3$ *but* Al *renders passive towards it.*

(ii) *Moderately reactive metals* (Fe, Zn) *forms ammonium nitrate with dil.* $HNO_3$.

(iii) *Least reactive metals* (Cu, Ag, Pb) *forms nitric oxide with dil.* $HNO_3$.

Among the given metals, Fe is moderately reactive, hence forms ammonium nitrate with dil. $HNO_3$.

$$[Fe + 2HNO_3 \longrightarrow Fe(NO_3)_2 + 2H] \times 4$$

$$HNO_3 + 8H \longrightarrow NH_3 + 3H_2O$$

$$NH_3 + HNO_3 \longrightarrow NH_4NO_3$$

$$\underset{\text{dil.}}{4Fe + 10HNO_3} \longrightarrow 4Fe(NO_3)_2 + \underset{\substack{\text{ammonium}\\\text{nitrate}}}{NH_4NO_3} + 3H_2O$$

**59. Key Idea**

(i) *Write the electronic configuration for each atomic number and then count the number of electrons in valence shell.*

(ii) *Group number = number of electrons in valence shell.*

(iii) *Find the general name of this group.*

$$9 = 1s^2, \underbrace{2s^2 2p^5}_{\text{7 valence electrons}}$$

$$17 = 1s^2, 2s^2 2p^6, \underbrace{3s^2 3p^5}_{\text{7 valence electrons}}$$

$$35 = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, \underbrace{4s^2 4p^5}_{\text{7 valence electrons}}$$

$$53 = [Kr]\ 4d^{10}, \underbrace{5s^2 5p^5}_{\text{7 valence electrons}}$$

$$85 = [Xe]\ 4f^{14}, 5d^{10}, \underbrace{6s^2 6p^5}_{\text{7 valence electrons}}$$

All have seven electrons in their valence shell.

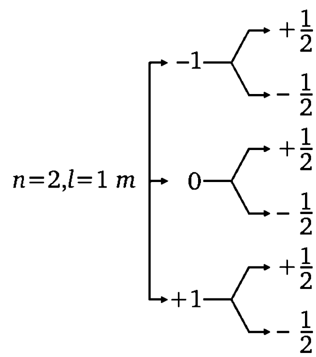
∴ These are VIIA group elements.

VIIA group elements are also known as halogens (which means originating from sea).

**60.** Cryolite ($Na_3AlF_6$) is added to alumina for its electrolysis to decrease its melting point and also increase its conductivity.

**61. Key Idea** *Each orbital contains two electrons with opposite spins* $\left(+\frac{1}{2} \text{ or } -\frac{1}{2}\right)$.

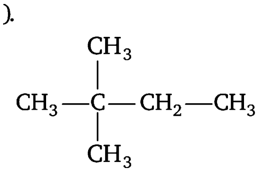
For 2*p*-subshell,



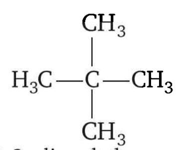

Hence, number of $e^-$ with $s = -\frac{1}{2}$ is 3.

**62. Key Idea** *Draw the structure of each alkane and find the odd one.*

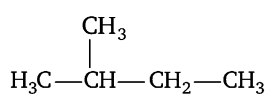
2, 2-dimethyl butane is 6-carbon hydrocarbon ($C_6H_{14}$).



Rest all are the chain isomers of pentane ($C_5H_{12}$).



2, 2–dimethyl propane
(*neo*-pentane)



2–methyl butane
(*iso*-pentane)

$CH_3CH_2CH_2CH_2CH_3$
*n*-pentane

Hence, the odd one is 2, 2-dimethyl butane.

**63.**

| | $N_2$ | + | $3H_2$ | $\rightleftharpoons$ | $2NH_3$ |
|---|---|---|---|---|---|
| Initially | $\frac{56}{28} = 2$ | | $\frac{8}{2} = 4$ | | |
| At equilibrium | $(2-x)$ mol | | $(4-3x)$ mol | | $2x = \frac{34}{17} = 2$ moles |

Hence, $x = 1$ mol

Hence, number of moles of $N_2$ at equilibrium

$$= 2 - 1 = 1 \text{ mol}$$

Number of moles of $H_2$ at equilibrium

$$= 4 - 3 = 1 \text{ mol}$$

Number of moles of $NH_3$ at equilibrium

$$= 2 \text{ moles.}$$

**64.** For an isothermal process, $\Delta E = 0 \quad (\because \Delta T = 0)$

As $p$ is also constant, $\Delta p = 0$

$$\left[\begin{array}{l} pV = \text{constant} \\ \text{On differentiating, we get} \\ p\Delta V + V\Delta p = 0 \\ \qquad p\Delta V = 0 \\ \because p \neq 0, \quad \Delta V = 0 \end{array}\right]$$

$$\Rightarrow \quad \Delta H = \Delta E + p\Delta V$$

$$= p + 0 \times 0 = 0$$

Hence, for the process, $\Delta H = \Delta E = 0$.

**65. Key Idea** *Electrons flow from oxidation half cell to reduction half-cell.*

In galvanic cell, oxidation (*ie*, removal of electrons) occurs at anode. These electrons flow through external circuit from anode to cathode. Therefore, the direction of current in external circuit is from cathode (–ve) to anode (+ve).

**66.** $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

For this reaction,

$$\text{Rate } (r_1) = k\,[SO_2]_1^2\,[O_2]_1 \qquad \ldots\text{(i)}$$

On doubling the volume of vessel, concentration would be half. Hence,

$$\text{Rate } (r_2) = k\left(\frac{[SO_2]_1}{2}\right)^2 \left(\frac{[O_2]_1}{2}\right) = \frac{r_1}{8}$$

$$\frac{r_1}{r_2} = 8 : 1$$

**67. Key Idea** *Mixture of acetic acid and sodium acetate is an acidic buffer. Hence, use Henderson equation,*

$$pH = pK_a + \log\frac{[Salt]}{[Acid]}$$

Let initially $\frac{[\text{Salt}]}{[\text{Acid}]} = 1$

$\therefore \quad pH = pK_a$

On increasing [salt] ten times of that of [acid],

$$pH = pK_a + \log\frac{10}{1}$$

$$= pK_a + 1$$

$\therefore$ pH increases by one.

**68. Key Idea** *Alkanes on oxidation give either aldehydes or acids depending upon the nature of catalyst taken.*

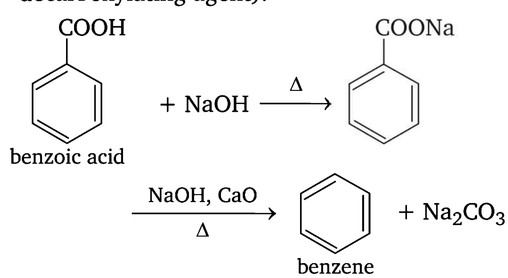
*In presence of molybdenum oxide alkanes give corresponding aldehydes while in presence of manganese acetate they give corresponding acids.*

$\therefore$ When methane is oxidised in presence of molybdenum oxide (MoO), it gives methanal (formaldehyde).

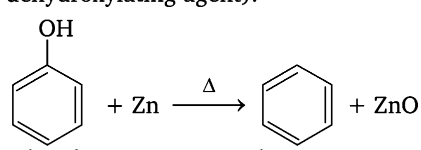
$$CH_4 + 2[O] \xrightarrow{[MoO]} \underset{\text{formaldehyde}}{HCHO} + H_2O$$

**69. Key Idea** *Benzoic acid on decarboxylation gives benzene and phenol on dehydroxylation gives benzene. Hence, use decarboxylating and dehydroxylating reagents respectively.*

Benzene can be obtained by heating benzoic acid with soda lime (soda lime is a decarboxylating agent).




Benzene can also be obtained by heating phenol with zinc dust. (Zn dust is a dehydroxylating agent).




**70. Key Idea** *In options only esters are given and esters are hydrolysed on boiling with alcoholic potash to acid salt and alcohol. Acid salt on acidification gives corresponding acid.*

Among the obtained acids only benzoic acid is a white solid.

∴ The ester must be ethyl benzoate and

The reactions occur as follows :

$$\underset{\text{ethyl benzoate}}{C_6H_5COOC_2H_5} + KOH \xrightarrow{\Delta} C_6H_5COOK + C_2H_5OH$$

$$C_6H_5COOK + HCl \longrightarrow \underset{\substack{\text{white solid}\\ \text{(benzoic acid)}}}{C_6H_5COOH} + KCl$$

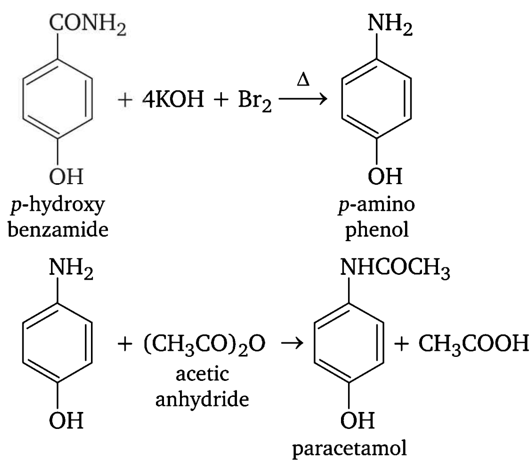
**71. Key Idea**

*(i)* $Br_2$ / KOH *are used to convert* —$CONH_2$ *group into* —$NH_2$ *group.*

*(ii) Aromatic amines are generally liquid while lower members of aliphatic amines are gases.*

∵ An oily liquid is obtained, on heating the compound with $Br_2$/ KOH.

∴ The compound must be benzamide and benzamide undergoes Hofmann-bromamide reaction with $Br_2$/KOH to give aniline. The aniline give paracetamol (antipyretic drug) with acetic anhydride.



**72.** The silver salt of fatty acid on refluxing with an alkyl halide, gives an ester.

$$RCOOAg + R'Cl \xrightarrow{\Delta} \underset{\text{ester}}{RCOOR'} + AgCl$$

**73.** Cellulose is a polysaccharide (carbohydrate) while rest three are enzymes. Enzymes are chemically complex proteins which act as catalyst in biological activities.

**74.** (a) In second order reaction, the rate of reaction depends upon the concentration of two reactants. *eg*, saponification of $CH_3COOC_2H_5$.

(b) When two reactants are involved in a reaction but one reactant is in excess, then rate of reaction depends only upon the concentration of one reactant. Such reactions are pseudo unimolecular reactions. *eg*, hydrolysis of $CH_3COOCH_3$. (In this reaction water is in excess, hence rate does not depend on its concentration).

(c) When the rate of a reaction is directly proportional to concentration of reactant, then the order of reaction is first. *eg*, decomposition of $H_2O_2$.

(d) When the rate of a reaction is not affected by the concentration of reactants, then the reaction is of zero order. *eg*, combination of $H_2$ and $Br_2$ to give HBr.

Combination of $H_2$ and $Br_2$ to give HBr is a zero order reaction.

$$H_2 + Br_2 \xrightarrow{h\nu} 2HBr$$

Hence, it is not correctly matched.

**75. Key Idea** *The size of colloidal particals lies in between that of true solutions and suspension.*

∵ The particle size in true solution < 1 nm

and the particle size of suspension > 100 nm

∴ The particles of size $10^{-6}$m to $10^{-9}$m form colloidal solution.

**76. Key Idea** *When alkyl halide reacts with* Na *in presence of dry ether, alkanes are obtained. In order to obtain a R—R′ type alkane, two different alkyl halides ie, R—X and R′—X are taken.*

This Wurtz reaction. 2-chloropropane and chloromethane are taken in presence of sodium in dry ether to form 2-methyl propane.

$$CH_3\boxed{Cl + 2Na + Cl}—\underset{\substack{|\\ CH_3}}{C}H—CH_3 \xrightarrow{\text{Ether}} CH_3—\underset{\substack{|\\ CH_3}}{C}H—CH_3 + 2NaCl$$

**77.** Benzyl chloride is more reactive than alkyl halides. Benzyl carbocation is stabilised by resonance, hence benzyl chloride easily gives nucleophilic substitution reaction.

**78.** Basic mercuric carbonate is obtained in this reaction.

$$Na_2CO_3 + 2HgCl_2 \longrightarrow HgCO_3 \cdot HgO + 2NaCl + Cl_2 + CO_2$$

**79.** In electrothermal process, silica is heated with calcium phosphate when phosphorus pentoxide is obtained. It is then reduced by coke in electric furnace to get white phosphorus.

$$2Ca_3(PO_4)_2 + 6SiO_2 \xrightarrow{\Delta} 6CaSiO_3 + P_4O_{10}$$

$$P_4O_{10} + 10C \xrightarrow{\Delta} P_4 + 10CO$$

**80.** The combustion of methane can be represented by the following equation

$$\underset{12+4=16}{CH_4} + 2O_2 \longrightarrow CO_2 + 2H_2O + 890 \text{ kJ}$$

$\because$ 16g $CH_4$ burns in air to liberate

$= 890$ kJ of heat

$\therefore$ 3.2 g $CH_4$ will liberate $= \dfrac{890 \times 3.2}{16}$

$= 178$ kJ of heat

## Mathematics

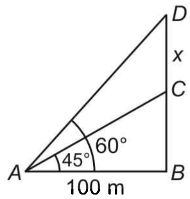
**81.** By using truth table, we know that the proposition $\sim p \wedge q$ is logically equivalent to $\sim (q \rightarrow p)$.

**82.** Let $BC$ be the incomplete pillar and $BD$ be the complete pillar. In $\Delta ABC$,

$$\tan 45° = \frac{BC}{AB}$$

$$\Rightarrow \quad 1 = \frac{BC}{100} \Rightarrow BC = 100 \quad \ldots(\text{i})$$

and in $\Delta ABD$




$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BC + x}{100}$$

$$\Rightarrow \quad x = 100\sqrt{3} - BC$$

$$= 100\sqrt{3} - 100 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad x = 100(\sqrt{3} - 1) \text{ m}$$

**83.** Given that,

$$2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$$

It can be written as,

$$2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

or $$2X = \begin{bmatrix} 3-1 & 8-2 \\ 7-3 & 2-4 \end{bmatrix}$$

or $$2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

**84.** Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ bc & c(a-b) & a(b-c) \\ b+c & (a-b) & (b-c) \end{vmatrix} \begin{matrix} C_2 \rightarrow C_2 - C_1 \\ C_3 \rightarrow C_3 - C_2 \end{matrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 1 & 0 & 0 \\ bc & c & a \\ b+c & 1 & 1 \end{vmatrix}$$

$$= (a-b)(b-c)[1(c-a) - 0 - 0]$$

$$= (a-b)(b-c)(c-a)$$

**85.** Let $\Delta = \begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$

On expanding, we get

$= 441[(201146 - 201150)]$

$- 442[200695 - 200703]$

$+ 443[200250 - 200254]$

$= -1764 + 3536 - 1772$

$= 3536 - 3536 = 0$

**Alternate Solution**

Now,

$$\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix} = \begin{vmatrix} 441 & 1 & 1 \\ 445 & 1 & 1 \\ 449 & 1 & 1 \end{vmatrix} \begin{matrix} C_2 \rightarrow C_2 - C_1 \\ C_3 \rightarrow C_3 - C_2 \end{matrix}$$

$= 0$ $\quad$ ($\because$ two columns are identical)

**NOTE** The value of determinant does nto change on applying any operation.

86. We know that, any vector $\vec{a}$ can be uniquely expressed in terms of three non-coplanar vectors as $\vec{a} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ multiply scalarly in succession by $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$$x = \vec{a}\cdot\hat{i},\quad y = \vec{a}\cdot\hat{j},\quad z = \vec{a}\cdot\hat{k}$$

$$\therefore\ (\vec{a}\cdot\hat{i})\cdot\hat{i} + (\vec{a}\cdot\hat{j})\cdot\hat{j} + (\vec{a}\cdot\hat{k})\cdot\hat{k}$$
$$= x\hat{i} + y\hat{j} + z\hat{k}$$
$$= \vec{a}$$

87. Let $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

Here, cofactors are

$C_{11} = \cos 2\theta, C_{12} = -\sin 2\theta,$
$C_{21} = \sin 2\theta, C_{22} = \cos 2\theta$

$\therefore |A| = |\cos^2 2\theta + \sin^2 2\theta| = 1$

$$\therefore\quad A^{-1} = \frac{1}{|A|}\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}^T$$
$$= \frac{1}{1}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

**Alternate Solution**

For $2\times 2$ matrix inverse of a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ can be determined by $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ c & a \end{bmatrix}$.

$$\therefore A^{-1} = \frac{1}{\cos^2 2\theta + \sin^2 2\theta}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

88. Given that, $|\vec{a}| = 3, |\vec{b}| = 4$ and $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$.

$$\therefore\quad (\vec{a} + \lambda\vec{b})\cdot(\vec{a} - \lambda\vec{b}) = 0$$
$$\Rightarrow\ \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b}\lambda + \lambda\vec{b}\cdot\vec{a} - \lambda^2\vec{b}\cdot\vec{b} = 0$$
$$\Rightarrow\quad (\vec{a})^2 - \lambda^2(\vec{b})^2 = 0$$
$$\Rightarrow\quad \lambda^2 = \frac{\vec{a}^2}{\vec{b}^2} \ \Rightarrow\ \lambda = \frac{|\vec{a}|}{|\vec{b}|} = \frac{3}{4}$$

89. Here, $\vec{a} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ is

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$
$$= \frac{2\times 1 + 3\times 2 - 2\times 3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{14}}$$

90. $\therefore\ A\cap B = \{x : x$ a multiple of 3$\}$ and $\{x : x$ is a multiple of 5$\}$

$= \{x : x$ is a multiple of 15$\}$
$= \{15, 30, 45, \ldots\ldots\ldots\}$

91. **Key Idea** *The maximum and minimum value of the function* $a\cos x \pm b\sin x$ *is* $\sqrt{a^2+b^2}$ *and* $\sqrt{a^2+b^2}$.

We have, $3\cos x - 4\sin x$

$\therefore$ Maximum value is $\sqrt{a^2+b^2}$

where $a = 3$ and $b = 4$. (Here $a = 3, b = 4$)

$\therefore$ Maximum value $= \sqrt{3^2+4^2}$
$= \sqrt{9+16}$
$= \sqrt{25} = 5$

92. We have, $s = t^3 - 3t^2$

On differentiating w.r.t. $t$, we get

$$\frac{ds}{dt} = 3t^2 - 6t \qquad \ldots\text{(i)}$$

Again, differentiating Eq. (i), we get

$$\frac{d^2s}{dt^2} = 6t - 6$$

Since, $\frac{d^2s}{dt^2} = 0$

$\Rightarrow\quad t = 1$

On putting the value of $t = 1$ in Eq. (i), we get

$$\frac{ds}{dt} = 3\times 1 - 6\times 1$$
$$= 3 - 6 = -3\ \text{m/s}$$

93. For the curve $y^n = a^{n-1}x$ putting $n = 2$,

we get $y^2 = ax$

Which is a curve of the form of parabola, where the subnormal at any point is a constant.

$\therefore\quad n = 2$

**94.** Here, $x = A\cos 4t + B\sin 4t$

On differentiating w.r.t. $t$, we get

$$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$$

Again differentiating w.r.t. $t$, we get

$$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$$

$$= -16(A\cos 4t + B\sin 4t)$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -16x$$

**95. Key Idea** *If tangent is perpendicular to the x-axis the* $\frac{dx}{dy} = 0$.

Given, $x = at^2$

$\therefore \quad \frac{dx}{dt} = 2at$

and $y = 2at \Rightarrow \frac{dy}{dt} = 2a$

$\therefore$ Tangent, $\left(\frac{dy}{dx}\right) = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \quad \frac{1}{t} = \infty$ (given)

$\Rightarrow \quad t = 0$

$\therefore$ Point of contact is (0,0).

**Alternate Solution**

Given, $x = at^2$, $y = 2at \Rightarrow y^2 = 4a^2t^2$

$\therefore \quad y^2 = 4a^2 \times \frac{x}{a}$

$\Rightarrow \quad y^2 = 4ax$

We know the tangent to the parabola at the vertex (0, 0).

**96.** Given, $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2x} = 0$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1+\cos 2y}{1-\cos 2x}$$

$$= -\frac{1+2\cos^2 y - 1}{1-(1-2\sin^2 x)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2\cos^2 y}{2\sin^2 x}$$

$$\Rightarrow \quad \frac{dy}{\cos^2 y} = -\frac{dx}{\sin^2 x}$$

$$\Rightarrow \quad \int \sec^2 y\, dy = -\int \operatorname{cosec}^2 x\, dx$$

$$\Rightarrow \quad \tan y = \cot x + c$$

$$\Rightarrow \quad \tan y - \cot x = c$$

**97.** $$\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$

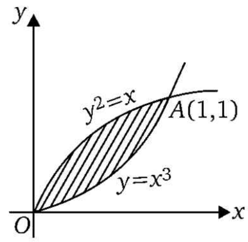
$$\Rightarrow \quad \left(1 + \left(\frac{dy}{dx}\right)^2\right)^9 = \left(\frac{d^2y}{dx}\right)^4$$

Hence, degree is 4.

**NOTE** The degree of transcedental equation cannot be determined.

**98.** We have, $y = \sqrt{x}$ or $y^2 = x\ (y \geq 0)$,

and $y = x^3$




$\therefore$ Points of intersection are (0,0) and (1,1).

$\therefore$ Required area

$$= \int_0^1 (\sqrt{x} - x^3)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{5}{12} \text{ sq unit}$$

**99.** Let $I = \int_0^{\pi/8} \cos^3 4\theta\, d\theta$

$$= \int_0^{\pi/8} \cos^2 4\theta \cos 4\theta\, d\theta$$

$$= \int_0^{\pi/8} \left(\frac{1+\cos 8\theta}{2}\right) \cos 4\theta\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/8} \cos 4\theta\, d\theta + \frac{1}{2}\int_0^{\pi/8} \cos 8\theta \cos 4\theta\, d\theta$$

$$= \frac{1}{2}\left[\frac{\sin 4\theta}{4}\right]_0^{\pi/8} + I_1 \quad \ldots\text{(i)}$$

Now, $I_1 = \frac{1}{2}\int_0^{\pi/8} \cos 8\theta \cos 4\theta\, d\theta$

$$= \frac{1}{2}\left[\cos 8\theta \frac{\sin 4\theta}{4}\right]_0^{\pi/8}$$

$$- \frac{1}{2}\int_0^{\pi/8} (-8\sin 8\theta)\frac{\sin 4\theta}{4}\, d\theta$$

$$= -\frac{1}{8} + \int_0^{\pi/8} \sin 8\theta \sin 4\theta \, d\theta$$

$$= -\frac{1}{8} + \left[\sin 8\theta \left(-\frac{\cos 4\theta}{4}\right)\right]_0^{\pi/8} - \int_0^{\pi/8} 8\cos 8\theta \left(-\frac{\cos 4\theta}{4}\right) d\theta$$

$$= -\frac{1}{8} + 0 + 2\int_0^{\pi/8} \cos 8\theta \ \cos 4\theta \, d\theta$$

$$\Rightarrow \quad I_1 = -\frac{1}{8} + 4I_1$$

$$\Rightarrow \quad -3I_1 = -\frac{1}{8} \Rightarrow I_1 = \frac{1}{24} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$I = \frac{1}{8} + \frac{1}{24}$$

$$= \frac{3+1}{24} = \frac{4}{24} = \frac{1}{6}$$

**Alternate Solution**

$$I = \int_0^{\pi/8} \cos^3 4\theta \, d\theta$$

Put $4\theta = t \Rightarrow d\theta = \frac{1dt}{4}$

$$\therefore \quad I = \int_0^{\pi/2} \frac{\cos^3 t}{4} dt$$

$$= \frac{1}{4}\left[\frac{2}{3\cdot 1}\right] \quad \text{(using Walli's fromula)}$$

$$= \frac{1}{6}$$

**NOTE** If $n$ is positive integer, then

$$\int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx$$

$$= \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\cdot n \text{ is even}$$

$$= \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1, n \text{ is odd}$$

**100.** Let $I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$ ...(i)

Putting $x = \left(\frac{\pi}{2} - x\right)$ in Eq. (i), we get

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \left(\frac{\cos x - \sin x}{1 + \cos x \sin x}\right) dx \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\cos x - \sin x}{1 + \cos x \sin x} - \frac{\cos x - \sin x}{1 + \cos x \sin x}\right) dx$$

$$= \int_0^{\pi/2} 0 \, dx = 0$$

$$\Rightarrow \quad I = 0$$

**101. Key Idea** *If*

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

*represents a pair of straight lines, then*

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

We have, $ax^2 - y^2 + 4x - y = 0$

Here, $a = a$, $h = 0$, $b = -1$,

$f = -\frac{1}{2}$, $g = 2$, $c = 0$

$$\therefore \quad \begin{vmatrix} a & 0 & 2 \\ 0 & -1 & -1/2 \\ 2 & -1/2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad a\left[0 + \left(-\frac{1}{4}\right)\right] - 0 + 2[2] = 0$$

$$\Rightarrow \quad -\frac{a}{4} + 4 = 0$$

$$\Rightarrow \quad a = 16$$

**102.** Let $(h, k)$ be any point.

According to the question

$$4\sqrt{(h-h)^2 + k^2} = h^2 + k^2$$

$$\Rightarrow \quad 4|k| = h^2 + k^2$$

Locus of the point is

$$4|y| = x^2 + y^2$$

$$\Rightarrow \quad x^2 + y^2 - 4|y| = 0$$

**103.** Line making equal intercepts therefore, its equation is

$$x \pm y = a \quad \text{...(i)}$$

Since, it passes through (2, 4).

$$\therefore \quad 2 \pm 4 = a$$

$$\Rightarrow \quad a = -2, 6$$

$\therefore$ Equation of the required lines are

$$x \pm y = a$$

*ie,* $x + y = -2$ or $x + y = 6$

$$\Rightarrow \quad x + y - 6 = 0$$

**104. Key Idea** *If $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ are the vertices of a triangle, then area of triangle is* $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

Given that, $x_1 = x, x_2 = 1, x_3 = 0,$

$$y_1 = 0, \quad y_2 = 1, \quad y_3 = 2$$

$\therefore$ Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[x(1-2) + 1(2-0) + 0(0-1)]$$

$$= \frac{1}{2}[-x + 2 + 0]$$

$$= \frac{1}{2}(2 - x)$$

According to the question

$$\frac{1}{2}(2 - x) = 4 \text{ sq unit} \qquad \text{(given)}$$

$$\Rightarrow \quad 2 - x = 8$$

$$\Rightarrow \quad x = -6$$

**105.** We have, $\lim_{\theta \to \frac{\pi}{2}} \frac{\frac{\pi}{2} - \theta}{\cot \theta}$

Using L'Hospital's rule, we get

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{-1}{-\text{cosec}^2 \theta}$$

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{-1}{-1/\sin^2 \theta}$$

$$= \lim_{\theta \to \frac{\pi}{2}} \sin^2 \theta = 1$$

**NOTE** L' Hospital's rule can be applied only when it is of the form, $\frac{0}{0}, \frac{\infty}{\infty}$.

**106.** $x^2 + y^2 + 2g\,x + c = 0$

$\because$ Family of circle passing through the point of intersection of two circles is given by

$$S + \lambda S' = 0$$

$\therefore$ Option (a) is correct.

**107. Key Idea** *The intersection of two diameters lines is the centre of the circle.*

Centre is a point of intersection of two diameters is C(8, –2).

Since, the circle passing through the point $P(6, 2)$.

$$\therefore \quad r = CP = \sqrt{(6-8)^2 + (-2-2)^2}$$

$$= \sqrt{4 + 16} = \sqrt{20}$$

**108. Key Idea** *If x-coordinates of foci and vertex are same, then axis of parabola is parallel to x-axis and if y-coordinates are same, then it is parallel to y-axis.*

Since, the focus and vertex of the parabola are on $y$-axis ,therefore its directrix is parallel to $x$-axis and axis of the parabola is $y$-axis.

Let the equation of the directrix be $y = k$ the directrix meets the axis of the parabola at $(0, k)$. But vertex is the mid point of the line segment joining the focus to the point where directrix meets axis of the parabola

$$\therefore \quad \frac{k+3}{2} = 6 \quad \Rightarrow \quad k = 9$$

Thus the equation of directrix is $y = 9$.

$\therefore$ Equation of parabola is

$$(x-0)^2 + (y-3)^2 = (y-9)^2$$

$$\Rightarrow \quad x^2 + y^2 - 6y + 9 = y^2 - 18y + 81$$

$$\Rightarrow \quad x^2 + 12y - 72 = 0$$

**109.** Given equation can be written as

$$24\left(x^2 - 5x + \frac{25}{4}\right) + 9(y^2 - 10y + 25) + 225 - 150 - 225 = 0$$

$$\Rightarrow \quad 24\left(x - \frac{5}{2}\right)^2 + 9(y-5)^2 = 150$$

$$\Rightarrow \quad \frac{\left(x - \frac{5}{2}\right)^2}{\frac{150}{24}} + \frac{(y-5)^2}{\frac{150}{9}} = 1$$

Here, $a^2 = \frac{150}{24}$, $b^2 = \frac{150}{9}$ and $b > a$

$$\therefore \quad e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{9}{24}}$$

$$= \sqrt{\frac{15}{24}}$$

**110.** Given, $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ ...(i)

and $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ ...(ii)

For hyperbola,

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$= 1 + \frac{81}{144} = \frac{225}{144}$$

$$\Rightarrow \quad e = \frac{15}{12} = \frac{5}{4} \text{ ie, } \quad e > 1$$

Also, $a^2 = \frac{144}{25}$

Hence, the foci are ($\pm$ *ae*, 0).

*ie,* $\left(\pm \frac{12}{5} \cdot \frac{5}{4}, 0\right) = (\pm 3, 0)$

Since, the foci coincide.

$\therefore$ For ellipse $ae' = 3$ or $a^2e^2 = 9$

$$\Rightarrow \quad a^2\left(1 - \frac{b^2}{a^2}\right) = 9$$

$$\Rightarrow \quad a^2 - b^2 = 9$$

$$\Rightarrow \quad 16 - 9 = b^2$$

$$\Rightarrow \quad b^2 = 7$$

**111.** Given, $f(x) = \log x$

Let $y = f(\sin x)$

$\therefore$ $y = \log(\sin x)$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = \frac{1}{\sin x} \times \cos x$$

$$= \cot x$$

**112.** Given that, $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1 - \cos x}{x}$$

$$= \lim_{x \to 0} \frac{2\sin^2\left(\frac{x}{2}\right)}{4\left(\frac{x}{2}\right)^2} \cdot x = 0$$

and $f(0) = k$

Since, function is continuous at $x = 0$.

$$\therefore \quad \lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \quad k = 0$$

**113.** Given that, $\omega = \frac{-1 + \sqrt{3}i}{2}$

$$\therefore \quad (3 + \omega + 3\omega^2)^4 = [3(1 + \omega^2) + \omega]^4$$

$$= 16\,\omega$$

**114.** Given, $y = \tan^{-1}(\sec x - \tan x))$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1 - \sin x}{\cos x}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$$

**115.** Given that, $x + \frac{1}{x} = 2\cos\alpha$

$$\Rightarrow \quad x^2 - 2x\cos\alpha + 1 = 0$$

$$\Rightarrow \quad x = \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - 4}}{2}$$

$\Rightarrow \quad x = \cos\alpha + i\sin\alpha$ (take +ve sign)

Now, $x^n = (\cos\alpha + i\sin\alpha)^n$

$$= \cos n\alpha + i\sin n\alpha$$

and $\frac{1}{x^n} = (\cos\alpha - i\sin\alpha)^n$

$$= \cos n\alpha - i\sin n\alpha$$

$$\therefore x^n + \frac{1}{x^n} = \cos n\alpha + i\sin n\alpha + \cos n\alpha - i\sin n\alpha$$

$$= 2\cos n\alpha$$

**116.** Let $I = \int_{-1}^{1} |1 - x|\,dx$

Here, $-1 \leq x \leq 1$

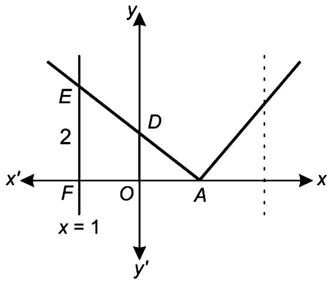
$$\Rightarrow \quad 1 - x \geq 0$$

$$\therefore \quad I = \int_{-1}^{1} (1 - x)\,dx$$

$$= \left[x - \frac{x^2}{2}\right]_{-1}^{1}$$

$$= 1 - \frac{1}{2} + 1 + \frac{1}{2} = 2$$

**Alternate Solution**




$\int_{-1}^{1} |1 - x|\, dx$ = Area of $\Delta AEF$

$$= \frac{1}{2} \times 2 \times 2 = 2$$

**117.** Let $I = \int \frac{dx}{x(x^7 + 1)}$

On putting $x^7 = t \Rightarrow 7x^6\, dx = dt$

$\Rightarrow \quad x^6\, dx = \frac{dt}{7}$

$\therefore \quad I = \frac{1}{7} \int \frac{1}{t} \frac{dt}{(t + 1)}$

$$= \frac{1}{7} \int \left( \frac{1}{t} - \frac{1}{t + 1} \right) dt$$

$$= \frac{1}{7} \log \left( \frac{t}{t + 1} \right) + c$$

$$= \frac{1}{7} \log \left( \frac{x^7}{x^7 + 1} \right) + c$$

**118.** Let $I = \int \sqrt{x}\, e^{\sqrt{x}}\, dx$

Putting $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\therefore \quad I = 2 \int t^2 e^t\, dt$

$$= 2[t^2 e^t - (2t)e^t + 2e^t] + c$$

$$= (2x - 4\sqrt{x} + 4)\, e^{\sqrt{x}} + c$$

**119.** Let $I = \int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{(x + 1)^2 + 1}$

$\therefore \quad I = \int \frac{dx}{1 + (x + 1)^2} = \tan^{-1}(x + 1) + c$

**120.** Since, tangent is parallel to $y = 2x - \frac{1}{2}$.

$\therefore$ Equation of tangent is $y = 2x + \lambda$.

The point of tangency will be the point of intersections of tangent and curve but in the given options only option (a) satisfied the equation of curve, then point (2, 8) will be the point of tangency.

**121.** Let $y = 0.5\overline{73}$ ...(i)

$\therefore \quad 10y = 5.\overline{73}$ ...(ii)

Again multiply Eqs. (ii) by 100 Then.

$1000y = 573.\overline{73}$ ...(iii)

Now, subtracting Eq. (ii) from (iii), we get

$$990y = 568$$

$\Rightarrow \quad y = \frac{568}{990} = \frac{284}{495}$

**122.** **Case I** When $|x| = -x$,

$$x^2 + 5x + 6 = 0$$

$\Rightarrow \quad x^2 + 3x + 2x + 6 = 0$

$\Rightarrow \quad x(x + 3) + 2(x + 3) = 0$

$\therefore \quad (x + 2)(x + 3) = 0$

$\Rightarrow \quad x = -2, -3$

**Case II** When $|x| = x$,

$\therefore \quad x^2 - 5x + 6 = 0$

$$x^2 - 3x - 2x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

$\Rightarrow \quad x = 2, 3$

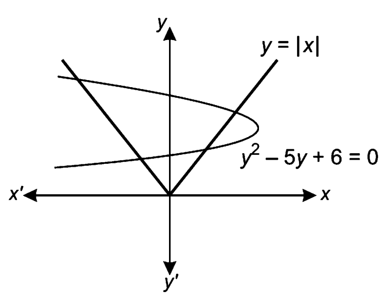
Hence, there are four roots *ie,* $\pm 2, \pm 3$.

**Alternate Solution**

Let $y = |x|$, then

$$y^2 - 5y + 6 = 0$$

$\Rightarrow \quad y = 2, 3$




It is clear from the graph that two curves intersect at four points.

**123.** From 112233, the number of 6 digits that can be formed the digits

$$= \frac{6!}{2!\,2!\,2!} = \frac{720}{8} = 90$$

**124.** Now, $7^1 = 7, 7^2 = 7 \times 7 = 49$

$$7^3 = 7 \times 7 \times 7 = 343$$
$$7^4 = 7 \times 7 \times 7 \times 7 = 2401$$
$$7^5 = 7 \times 7 \times 7 \times 7 \times 7 = 16807$$

and $7^{300} = 7^{4 \times 75}$

Hence, last digit of $7^{300}$ will be 1.

**125.** Given that, $\frac{\log x}{a-b} = \frac{\log y}{b-c} = \frac{\log z}{c-a}$

Let each ratio be $k$

then, $\log x = k(a-b), \log y = k(b-c)$

and $\log z = k(c-a)$

Let $A = xyz$

$\therefore \log A = \log x + \log y + \log z$

$$= k(a-b) + k(b-c) + k(c-a)$$
$$= k[a-b+b-c+c-a]$$
$$= k[0]$$

$\therefore \log A = \log(xyz) = 0 \quad [\because A = xyz]$

$\Rightarrow xyz = e^0$

$\Rightarrow xyz = 1$

**126.** Given that, $(1+i)^{2n} = (1-i)^{2n}$

$\Rightarrow 2^n i^n = 2^n (-1)^n i^n$

$\Rightarrow 1 = (-1)^n$

$\therefore$ The smallest value of $n$ is 2.

**127.** Given, $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$

Hence, the given equation will hold only when each is $\frac{\pi}{3}$.

$\therefore p = q = r = \cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore p^2 + q^2 + r^2 + 2pqr$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{2}{2 \cdot 2 \cdot 2}$$
$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$$

**128.** Given, $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left[\frac{x}{5} \times \sqrt{1 - \left(\frac{4}{5}\right)^2} + \frac{4}{5} \times \sqrt{1 - \left(\frac{x}{5}\right)^2}\right] = \frac{\pi}{2}$$

$\Rightarrow \frac{x}{5} \times \frac{3}{5} + \frac{4}{5} \times \sqrt{\frac{25 - x^2}{25}} = \sin\frac{\pi}{2}$

$\Rightarrow 3x + 4\sqrt{25 - x^2} = 25$

$\Rightarrow 4\sqrt{25 - x^2} = 25 - 3x$

On squaring both sides, we get

$$16(25 - x^2) = 625 + 9x^2 - 150x$$

$\Rightarrow 16x^2 + 9x^2 - 150x + 625 - 400 = 0$

$\Rightarrow 25x^2 - 150x + 225 = 0$

$\Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow (x-3)^2 = 0$

$\therefore x = 3$

**129.** Given, $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

$\Rightarrow 81^{\sin^2 x} + 81^{1 - \sin^2 x} = 30$

$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$

Let $81^{\sin^2 x} = y$

$\therefore y + \frac{81}{y} = 30$

$\Rightarrow y^2 - 30y + 81 = 0$

$\Rightarrow (y - 27)(y - 3) = 0$

$\Rightarrow y = 27$ or $y = 3$

$\Rightarrow 81^{\sin^2 x} = 27$ or $81^{\sin^2 x} = 3$

$\Rightarrow 3^{4\sin^2 x} = 3^3$ or $3^{4\sin^2 x} = 3^1$

$\Rightarrow \sin^2 x = \frac{3}{4}$ or $4\sin^2 x = 1$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$ or $\sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3}$ or $x = \frac{\pi}{6}$

**130.** **Key Idea** *The equation of director circle to the hyperbola* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ *is* $x^2 + y^2 = b^2 - a^2$.

Equation of director circle of the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ (where $a^2 = 16, b^2 = 4$) is

$$x^2 + y^2 = a^2 - b^2$$

$\Rightarrow x^2 + y^2 = 16 - 4$

$\Rightarrow x^2 + y^2 = 12$

**131.** It intersect at right angle.

**132.** **Key Idea** *Let $(h, k)$ be the centre of the circle. If circle touches x-axis then $r = k$ and if circle touches y-axis, then $r = h$.*

Given, $x^2 + y^2 - 8x + 4y + 4 = 0$

On comparing with standard equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$$g = -4, \quad f = 2 \quad \text{and} \quad c = 4$$
$$-g = 4, -f = -2$$

$\therefore$ Coordinate of the centre $= (-g, -f)$
$= (4, -2)$

$\therefore$ Radius of the circle $= \sqrt{g^2 + f^2 - c}$
$= \sqrt{(-4)^2 + (2)^2 - 4}$
$= \sqrt{16 + 4 - 4} = 4$

Here, radius of circle is equal to $x$ coordinate of the centre.

$\therefore$ Circle touches $y$-axis.

**133.** Given, $A = \{1, 2, 3\}$ and $B = \{3, 8\}$

Now, $(A \cup B) = \{1, 2, 3\} \cup \{3, 8\}$
$= \{1, 2, 3, 8\}$

Now, $(A \cap B) = \{1, 2, 3\} \cap \{3, 8\}$
$= \{3\}$

$\therefore (A \cup B) \times (A \cap B) = \{1, 2, 3, 8\} \times \{3\}$
$= \{(1, 3), (2, 3), (3, 3), (8, 3)\}$

**134.** Let the roots be $\alpha, 2\alpha$ of the equation $ax^2 + bx + c = 0$.

$\therefore \quad \alpha + 2\alpha = -\dfrac{b}{a}$

$\Rightarrow \quad \alpha = -\dfrac{b}{3a}$ ...(i)

and $\quad \alpha \cdot 2\alpha = \dfrac{c}{a}$

$\Rightarrow \quad \alpha^2 = \dfrac{c}{2a}$

$\Rightarrow \quad \left(-\dfrac{b}{3a}\right)^2 = \dfrac{c}{2a}$ [from Eq. (i)]

$\Rightarrow \quad 2b^2 = 9ac$

**135.** **Kye Idea** *Two circles*

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

*and* $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

*cut orthogonally, then*

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

Given, $x^2 + y^2 + kx + 4y + 2 = 0$ ...(i)

and $x^2 + y^2 - 2x - \dfrac{3}{2}y + \dfrac{k}{2} = 0$ ...(ii)

From Eq. (i)

$$g_1 = \frac{k}{2}, f_1 = 2 \text{ and } c_1 = 2$$

and from Eq. (ii)

$$g_2 = -1, f_2 = -\frac{3}{4} \text{ and } c_2 = \frac{k}{2}$$

Condition for two circles cut orthogonally is

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

$\therefore 2 \times \dfrac{k}{2} \times (-1) + 2 \times 2 \times \left(-\dfrac{3}{4}\right) = 2 + \dfrac{k}{2}$

$\Rightarrow \quad -k + 4 \times \left(-\dfrac{3}{4}\right) = 2 + \dfrac{k}{2}$

$\Rightarrow \quad \dfrac{3}{2}k = -5$

$\Rightarrow \quad k = -\dfrac{10}{3}$

**136.** **Key Idea** $\lim\limits_{x \to a} \left(1 + \dfrac{1}{f(x)}\right)^{f(x)} = e$

*Provided* $\lim\limits_{x \to a} f(x) = \infty$

Given that,

$$\lim_{x \to \infty} \left(1 - \frac{4}{x-1}\right)^{3x-1}$$

$$= \lim_{x \to \infty} \left[\left(1 - \frac{4}{x-1}\right)^{\frac{-(x-1)}{4}}\right]^{-4\left(\frac{3x-1}{x-1}\right)}$$

$$= e^{-4 \lim\limits_{x \to \infty} (3 - 1/x)/(1 - 1/x)}$$

$$= e^{-4 \times 3} = e^{-12}$$

**137.** Given that, $A + B + C = 180°$

$\Rightarrow \quad A + B = 180° - C$

$\Rightarrow \quad \dfrac{A}{2} + \dfrac{B}{2} = 90° - \dfrac{C}{2}$

$\Rightarrow \quad \tan\left(\dfrac{A}{2} + \dfrac{B}{2}\right) = \tan\left(90° - \dfrac{C}{2}\right)$

$\Rightarrow \quad \dfrac{\tan\dfrac{A}{2} + \tan\dfrac{B}{2}}{1 - \tan\dfrac{A}{2}\tan\dfrac{B}{2}} = \cot\dfrac{C}{2}$

$$\Rightarrow \left(\tan\frac{A}{2} + \tan\frac{B}{2}\right)\tan\frac{C}{2} = 1 - \tan\frac{A}{2}\tan\frac{B}{2}$$

$$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$$

**138.** Given, $b = 2$, $B = 30°$

We known,

$$R = \frac{b}{2\sin B} = \frac{2}{2\sin 30°} = \frac{2}{1}$$

$$\Rightarrow \quad R = 2$$

Area of circumcircle $= \pi R^2$

$$= \pi \times (2)^2$$

$$= 4\pi \text{ sq unit}$$

**139.** Given, $\sin x + \sin^2 x = 1$ ...(i)

$$\Rightarrow \quad \sin x = 1 - \sin^2 x$$

$$\Rightarrow \quad \sin x = \cos^2 x$$

Now, $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$

$$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x$$

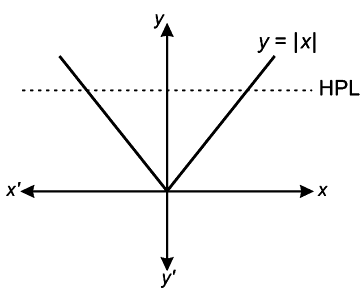
$$= (\sin^2 x + \sin x)^3$$

$$= (1)^3 = 1 \quad \text{[from Eq. (i)]}$$

**140.** The given function is

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

and $f: R \to R$,




It is clear from the graph that, by HPL test graph will intersect more than one point so it is not one-one and also graph is above $x$-axis, so it is not onto.

**141.** Since, 2 and 3 are the roots of the equation $f(x) = 2x^3 + mx^2 - 13x + n$, therefore

$$2(2)^3 + m(2)^2 - 13(2) + n = 0$$

and $2(3)^3 + m(3)^2 - 13(3) + n = 0$

$$\Rightarrow \quad 16 + 4m - 26 + n = 0$$

and $54 + 9m - 39 + n = 0$

$$\Rightarrow \quad 4m + n = 10 \quad \text{and} \quad 9m + n = -15$$

$$\Rightarrow \quad m = -5, n = 30$$

**142.** **Key Idea** *If $p_1$, $p_2$ and $p_3$ are perpendicular, then*

$$\Delta = \frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3$$

Now, $p_1 = \frac{2\Delta}{a}$, $p_2 = \frac{2\Delta}{b}$, $p_3 = \frac{2\Delta}{c}$

$$\therefore \quad p_1 p_2 p_3 = \frac{8\Delta^3}{abc} = \frac{8}{abc}\left(\frac{abc}{4R}\right)^3$$

$$= \frac{a^2 b^2 c^2}{8R^2}$$

**143.** Given, $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$

$$\Rightarrow \quad 5(2\cos^2\theta - 1) + (1 + \cos\theta) + 1 = 0$$

$$\Rightarrow \quad 10\cos^2\theta + \cos\theta - 3 = 0$$

$$\Rightarrow \quad (5\cos\theta + 3)(2\cos\theta - 1) = 0$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}, \cos\theta = -\frac{3}{5}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$$

**144.** When maximum effect, resultant force

$$F_1 + F_2 = 4 \quad \text{...(i)}$$

If forces act at right angles, then
Resultant force

$$\sqrt{F_1^2 + F_2^2} = 3 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$F_1 = \left(2 + \frac{1}{2}\sqrt{2}\right)\text{N}$$

and $F_2 = \left(2 - \frac{1}{2}\sqrt{2}\right)\text{N}$

**145.** **Key Idea** *If $\alpha$ be the angle between the forces P and Q. Then*

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Here, $\tan\frac{\pi}{2} = \frac{Q\sin\alpha}{P + Q\cos\alpha}$

$$\Rightarrow \quad P + Q\cos\alpha = 0$$

$$\Rightarrow \quad \cos\alpha = -\frac{P}{Q}$$

$$\therefore \quad \alpha = \cos^{-1}\left(-\frac{P}{Q}\right)$$

**146. Key Idea** *The distance covered in nth second is*

$$S_n = u + \frac{1}{2} f \cdot (2n-1)$$

Body started from rest, so $u = 0$ and

$$S_n\text{th} = \frac{1}{2} f \cdot (2n-1)$$

and $$S_n = \frac{1}{2} fn^2$$

$\therefore$ $$\frac{S_n\text{th}}{S_n} = \frac{\frac{1}{2} f(2n-1)}{\frac{1}{2} fn^2}$$

$$= \frac{2n-1}{n^2} = \frac{2}{n} - \frac{1}{n^2}$$

**147.** Let $s_1$ and $s_2$ be the distances moved by the first and second particle in time $t$. Then $s_1 = ut$ and $s_2 = \frac{1}{2} ft^2$.

Let $s$ be the distance between the particle at time $t$, then $s = s_1 - s_2$

$\Rightarrow$ $$s = ut - \frac{1}{2} ft^2$$

$\Rightarrow$ $$\frac{ds}{dt} = u - ft \quad \text{and} \quad \frac{d^2s}{dt^2} = -f$$

For maximum or minimum $\frac{ds}{dt} = 0$

$\Rightarrow$ $$t = \frac{u}{f}$$

Clearly, $$\frac{d^2s}{dt^2} = -f < 0$$

$\therefore$ $s$ is maximum at $t = \frac{u}{f}$.

**148. Key Idea** *If any line lie in a plane, then normal to the plane is perpendicular to the line.*

The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is

$$a(x+1) + b(y-3) + c(z+2) = 0 \quad \ldots\text{(i)}$$

and $$-3a + 2b + c = 0 \quad \ldots\text{(ii)}$$

This passes through (0, 7, –7)

$\therefore$ $$a + 4b - 5c = 0 \quad \ldots\text{(iii)}$$

On solving Eqs. (ii) and (iii), we get

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{-14}$$

or $$\frac{a}{1} = \frac{b}{1} = \frac{c}{1}$$

$\therefore$ The required equation of plane is

$$x + y + z = 0$$

**149.** Let the coordinates of $A$, $B$ and $C$ be $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ respectively.

Then equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Also, it passes through $(p, q, r)$.

So, $$\frac{p}{a} + \frac{q}{b} + \frac{r}{c} = 1 \quad \ldots\text{(i)}$$

Also, equation of sphere passes through 0, $A$, $B$, $C$ will be $x^2 + y^2 + z^2 - ax - by - cz = 0$.

If its centre $(x_1, y_1, z_1)$, then

$$x_1 = \frac{a}{2}, y_1 = \frac{b}{2}, \ z_1 = \frac{c}{2}$$

$\therefore$ $$a = 2x_1, \ b = 2y_1, \ c = 2z_1$$

From Eq. (i),

$$\frac{p}{2x_1} + \frac{q}{2y_1} + \frac{r}{2z_1} = 1$$

$\therefore$ Locus of centre of sphere is

$$\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$$

**150.** We have

$$\sum_{r=1}^{n} r^2 \cdot {}^nC_r = n(n-1)2^{n-2} + n \cdot 2^{n-1}$$

and $$\sum_{r=1}^{n} (-1)^{r-1} r^2 \, {}^nC_r = 0$$

On adding above equation, we get

$$2\,[1^2 C_1 + 3^2 C_3 + 5^2 C_5 + \ldots] = n(n-1)\, 2^{n-2} + n \cdot 2^{n-1}$$

$$\Rightarrow 1^2 C_1 + 3^2 C_3 + 5^2 C_5 + \ldots . = n(n-1)\, 2^{n-3} + n \cdot 2^{n-2}$$